Journal of Low Temperature Physics, Vol. 98, Nos. 3/4, 1995

Nucleation of Superconductivity in a Thin Film with a Lattice of Circular Holes

A. Bezryadin*,† and B. Pannetier*

**Centre de Recherches sur les Très Basses Températures CNRS, 38042, B.P. 166, Grenoble, France*
†Institute of Solid State Physics, Chernogolovka, 142432, Moscow distr., Russia

(Received June 21, 1994; revised September 19, 1994)

We investigate experimentally the nucleation of superconductivity in a thin Aluminium film with a square lattice of microholes in a uniform perpendicular magnetic field H. It is shown that in a non-zero magnetic field, this system has an elevated critical temperature $T_c^*(H)$ in comparison to the reference film without holes. This effect can be considered as a generalisation of the well known surface superconductivity effect for the case of a finite radius of the surface and a multiply connected geometry of the sample. Quantization of the fluxoid around each hole leads to oscillations in the $T_c^*(H)$ dependence with a period approximately corresponding to one flux quantum through a hole. Also another type of oscillation with a smaller period which is equal to one flux quantum per unit cell was observed. We discuss the second type of oscillation in terms of an interaction between the holes. We believe that our results can be useful in the analysis of high-T_c superconductors with columnar defects because, as it is shown here, a comparison of the $T_c(H)$ dependence before and after irradiation can give some special information on the properties of the amorphous tracks produced by high-energy ions. While many experimental parameters are different in our system and in HTSC we present arguments why this analogy should be correct.

1. INTRODUCTION

In this work we investigate experimentally a superconducting film with a lattice of circular holes. Such a system was discussed already in some theoretical and experimental publications.[1,2,3] Among other things it is interesting because a hole in a superconducting film or a cylindrical cavity in a bulk superconductor is a simple model of a pinning centre. Moreover such well known type of pinning centres as the columnar defects can be considered as dielectric cylinders inside the superconductor.

0022-2291/95/0200-0251$07.50/0 © 1995 Plenum Publishing Corporation

In a recent paper of Buzdin[1] an infinite superconductor with a cylindrical cavity is considered in a magnetic field which is applied parallel to the cylinder axis. This system is equivalent to a thin enough film with a circular hole in perpendicular magnetic field at a temperature near T_c^*. A non-monotonous behaviour of the critical temperature T_c^* versus magnetic field H is derived from the linearized Ginsburg-Landau equation for such a system. According to the calculations $T_c^*(H)$ is higher than the critical temperature of the film without holes $T_c(H)$. This result can be considered as a generalisation of the surface superconductivity effect[4] for the case of a finite radius of the surface curvature. For the plane surface in a parallel magnetic field it is well known that the critical field is $H_{c3} = 1.7 \cdot H_{c2}$. In our case the boundary between the superconductor and the vacuum (empty cylinder) is not plane so the critical field, $H_{c3}^*(T)$, will be bound by $H_{c2} < H_{c3}^* < 1.7 \cdot H_{c2}$.[1] Another important feature is the multiply connected geometry of the sample. Due to this, the critical temperature T_c^*, must be an oscillating function of the magnetic field in accordance with the fluxoid quantization condition observed for example, in the Little and Parks experiment.[5]

Experimentally we use a film with a square lattice of holes (the total number is about $2 \cdot 10^5$). If the magnetic field is constant and the temperature is going down then the superconducting order parameter will appear first of all around the holes in the form of thin rings of a thickness $\sim\xi(T_c^*)$ at the temperature $T_c^*(H)$. At the bulk critical temperature $T_{c2}(H)$ superconductivity will appear everywhere in the film. For observation of these two transitions we measure the resistance $R(T)$ of the sample. If the distance between the holes a is large in comparison with $\xi(T_c^*)$ then one can observe two steps at $T = T_c^*(H)$ and $T = T_{c2}(H)$ in the $R(T)$ curve while if the holes are close to each other then due to the proximity effect the resistance of the whole sample will drop to zero at $T = T_c^*$.

Another interesting property of the system under consideration is the interaction between the holes. If a is small enough the supercurrent between the holes (caused by the magnetic field for example) can differ from zero and the whole system will be similar to a superconducting network. In accordance with this we observe another type of oscillation in $T_c^*(H)$ with the period corresponding to one flux through a unit cell.

2. EXPERIMENT

All the measurements are carried out with an aluminium film of a thickness 800 Å prepared by thermal evaporation of pure aluminium in vaccum $3 \cdot 10^{-7}$ *mbar*. Four spirals and the square lattice of holes shown schematically in Fig. 1a are produced using lift-off electron-beam lithography with Negative-Tone Shipley Microposit SAL-601-ER7 E-Beam Resist. The radius of each hole is $r_0 = 2.13\mu m$ and the lattice parameter is $a = 9.05\mu m$. One-half of the same film (right part in Fig. 1a) electrically disconnected from the main sample is used for reference measurements. The total array of holes consists of square fields: each field contains 33×33 holes and there are no holes between the fields. In the schematic drawing (Fig. 1a) one can see these fields of holes (in the real sample there are five fields between the spirals). The distance between the field edges is $d_f = 20\mu m$.

The film resistance is measured by an AC four-terminal resistance bridge at a frequency 33 Hz with a low enough current $I = 4\mu A$ (current density $5 A/cm^2$). For accurate measurements of the critical temperature as a function of magnetic field a feedback system is used which maintains the constant resistance R_0 of the sample by changing the temperature during

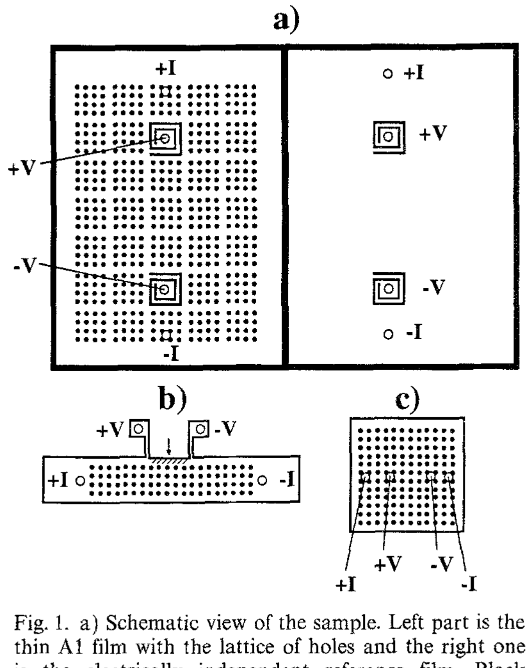

Fig. 1. a) Schematic view of the sample. Left part is the thin Al film with the lattice of holes and the right one is the electrically independent reference film. Black colour indicates the regions where the film was removed using lift-off process. Four open circles in each part of the sample show the voltage (inside the spirals) and the current contacts; b) and c) two usual configurations of the sample for the four-probe resistive measurements. Their disadvantages are discussed in the text.

the sweep of the magnetic field (see the details in [6]). If the $R(T)$ curve exhibits two transitions then one can measure corresponding critical temperatures independently by choosing the parameter of the feed-back circuit R_0 at the middle of the corresponding step.

Fig. 1b illustrates the standard sample configuration for four-probe measurements. It is not acceptable for the measurements of the surface critical field because the superconducting transition of the edge of the film (dashed region marked by an arrow in Fig. 1b) at $H = H_{c3}$ will short out the voltage probes. Another possible geometry which is free from the previous disadvantage is shown in Fig. 1c. Here the boundary of the film coincides with the edges of the substrate and it can be as far as we want from the voltage and current probes: Al wires with the diameter $30 \mu m$ ultrasonically soldered to the film in the points shown by open circles in the figure. These regions which are ultrasonically soldered usually have critical temperature T_{al} different from that of the film. The current between the current leads partially flows through the regions of the voltage contacts (of course this is true independently on the voltmeter input resistance) so their superconducting transition can cause a jump in the voltage. In this case the experimentally measured $R(T)$ dependence has an additional step at $T = T_{al}$. Interpretation of the results is difficult in such a case.

Taking into account all these difficulties a special geometry is used for the measurements (Fig. 1a). On the one hand the edges of the film coincide again with the edges of the substrate so they are far from the voltage and current probes. On the other hand we can neglect the anomalous properties of the voltage contacts (open circles in the figure) only if the current through them is equal to zero. This condition is satisfied by using spirals which are parts of the film (Fig. 1a). It is clear that the centre of such a spiral (where the voltage contact is located) while being electrically connected to the other parts of the film still has zero current through itself. It should be mentioned also that the spirals naturally have the same critical temperature as the film except perhaps their edges. In non-zero magnetic field the edges start to be superconducting at the higher temperature $T_{c3}(H) = T_0 - (T_0 - T_{c2}) * (H_{c2}/H_{c3}) = T_0 - (T_0 - T_{c2})/1.7$ due to the ordinary effect of surface superconductivity. This transition can cause some change in the voltage on the potential probes at $T = T_{c3}$ and it was observed experimentally (the peak in the $R(T)$ curves of the reference sample in non-zero field (Fig. 2, dashed lines)). One can speculate that the decrease of the edge's resistance to zero causes some increase in the density of the current between the spirals (the spirals attract the current lines) and therefore to some increase in the voltage on the potential probes. This parasitic effect is proportional to the size of the spirals and can be predicted quantitatively. It is hidden in the resistive curves of the sample with

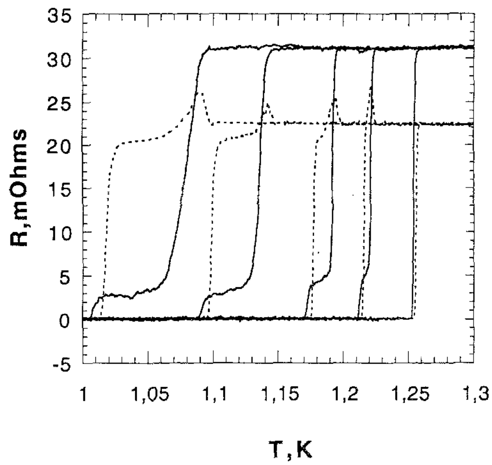

Fig. 2. Dependence of the samplme resistance and the resistance of the reference film versus the temperature at different values of magnetic field (from the right to the left): 0, 0.19, 0.37, 0.56, and 0.75 mT (each value corresponds to one solid and one dashed curve). The increase of the field leads to monotonous decrease of the transition temperatures. The solid curves correspond to the sample with holes and the dashed curves correspond to the reference film without holes.

holes probably because it occurs approximately at the same temperature as the strong decrease of the resistance of the perforated film caused by the nucleation of the surface superconducting states ($T_{c3}(H) \to T_c^*(H)$ when $H \to 0$). In other words the difference between two critical temperatures which correspond to the nucleation of the surface superconductivity near the plane surface (spirals) and near the surface with finite curvature (holes) is smaller than the broadness of the superconducting transition (at least at the weak field which is used in our measurements). Eventually we believe that the configuration with the spirals is the best for our application and it is used for all the measurements.

2. RESULTS

In Fig. 2 the dependence of the resistance $R = V/I$ versus temperature is shown for the sample with holes (solid lines) and for the reference uniform sample (dashed lines). The parameter is the uniform magnetic field H which is directed perpendicular to the film (each pair of curves is taken at the same field). At $H = 0$ we have a single narrow transition ($\approx 3mK$) for both films at approximately the same critical temperature $T_0 = 1.25K$. In increasing fields the transitions move to lower temperatures and the form of the $R(T)$ curve corresponding to the sample with holes starts to be more complex. In particular, one can see two transitions: the first one takes place at the upper critical temperature T_c^* and the second one at $T_{c2} < T_c^*$ when the resistance drops to zero. We neglect in this discussion the small peak in the resistance of the reference sample at the temperature close to T_{c3}. As it was mentioned earlier we believe that this feature could be explained if one takes into account the nucleation of superconductivity at the edges of the spirals. Let us consider the sample with holes. During the first transition at $T = T_c^*$ the resistance drops to a finite value which is much smaller (approximately by a factor of 10) than the initial resistance but still it is not zero. It is important to note that the lower critical temperature $T_{c2}(H)$ when the resistance of the sample goes to zero is approximately equal to the transition temperature of the reference film $T_c(H)$. We conclude that at T_{c2} we observe the ordinary bulk superconducting transition. The difference of about one percent between T_{c2} and T_c could be explained by the difference in the sample quality. We should note in this context that RRR (the ratio of the resistance at $T = 300$ K and $T = 4.2$ K) is equal to 4.9 and 5.1 for the part with holes and for the reference part of the film correspondingly.

The results of measurements of the critical temperatures T_c^* and T_{c2} versus H by the feed-back control of the sample resistance are shown in Fig. 3a. The values of the parameter R_0 of the feed-back circuit (in other words it is the resistance of the sample which is considered as a definition of the corresponding critical temperature) are: $R_0 = 20$ mOhms for T_c^* and $R_0 = 1$ mOhms for T_{c2}. This figure can be considered as a phase diagram and therefore each curve maps out the dependence of the critical field versus the temperature. There are three regions in the diagram: normal state (N) where the order parameter is equal to zero everywhere; ordinary mixed state (S) where the whole sample is superconducting except vortex cores; and a state of localised superconductivity (LS) in which the order parameter differs from zero only in the thin rings around the holes. Generally speaking the LS state can have zero or non-zero resistance depending on the distance between the holes in comparison to the coherence length. In the dependence $T_c^*(H)$ one can see an oscillating component which was predicted for the critical temperature of the film with the hole[1] (see also the derivative in Fig. 3b). On the contrary no oscillations were observed in $T_{c2}(H)$ except one or two cusps near zero with the small period corresponding to one flux quantum through a unit cell.

4. DISCUSSION

To clearly see the oscillations and for comparison with the theory it is convenient to reconstruct the dependence $T_c^*(H)$ (Fig. 3a), which can be considered as the $H_{c3}^*(T)$ dependence at the same time. First of all we should mention that for each point of the $H-T$ plane one can put uniquely into correspondence the values of the normalised flux $\phi/\phi_0 = H\pi r_0^2/\phi_0$ (where $\phi_0 = 2.07 \cdot 10^{-7} Oe \cdot cm^2$ is the flux quantum) and the second critical field $H_{c2} = \phi_0/(2\pi\xi^2(T)) = (1 - T/T_0) \cdot \phi_0/(2\pi\xi^2(0))$. So if we follow the curve $H_{c3}^*(T)$ we can plot the value of the reduced critical field $h_{c3}^* \equiv H_{c3}^*/H_{c2}$ as a function of ϕ/ϕ_0 (Fig. 4, solid curve). In fact the reduced critical field is a function of the critical temperature $T_c^*(H)$: $h_{c3}^* = 2\pi T_0 \xi^2(0)/\phi_0 \cdot H/(T_0 - T_c^*)$. The coherence length $\xi(0) = 2500$ Å was determined from the initial linear part of the $T_c^*(H)$ dependence using the relation $T_c^*(H) = T_0(1 - H/H_{c2}(0)) = T_0(1 - 2\pi\xi^2(0)H/\phi_0)$ and taking into account that when $H \to 0$ one expects the same dependence of the critical

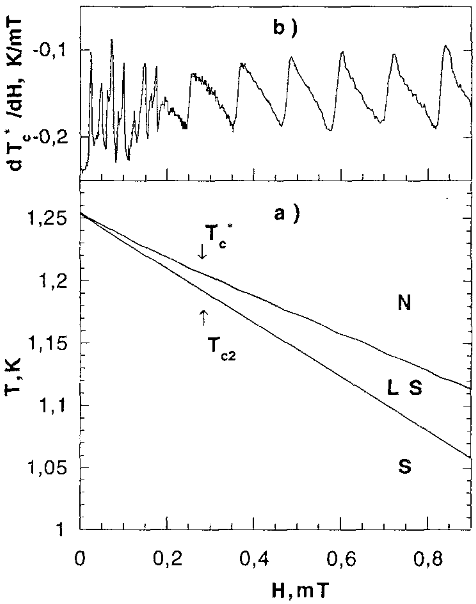

Fig. 3. a) The experimental critical temperatures as functions of the magnetic field. Two curves correspond to the two steps on the $R(T)$ curve of the sample with holes. They are measured near the middle points of the two transitions, namely at 20 mOhms (upper critical temperature T_c^*) and 1 mOhms (lower critical temperature T_{c2}); b) derivative of the upper critical temperature which shows two types of oscillations.

temperature versus field for the films with and without holes (Fig. 3a). The result of calculations made for the case of a single hole in the infinite film (see below) is shown by the dashed line in Fig. 4. In the inset one can see the same two curves at a different scale and also an additional experimental dependence (the lowest curve) $h^*_{c3}(\phi/\phi_0)$ taken at the lower part of the first transition, namely at $R_0 = 6$ mOhms. The monotonously decreasing dependence of the coherence length taken at T^*_c versus the flux through the hole is also shown. One can see that the coherence length is of the same order of magnitude as the distance between the holes ($a = 9\mu m$).

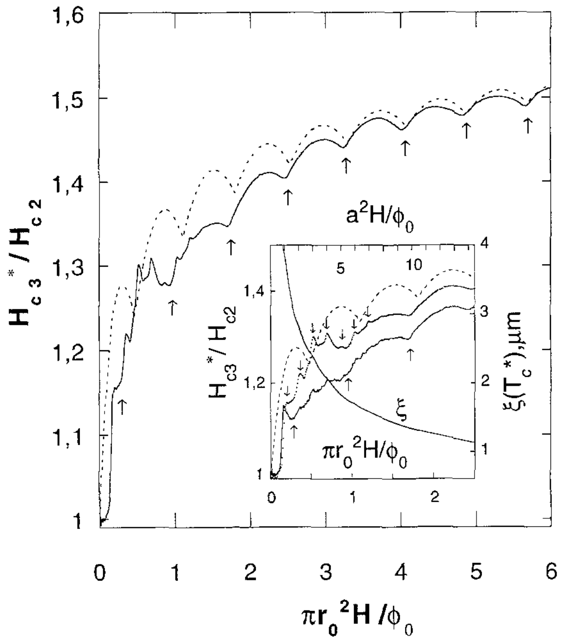

Fig. 4. The reduced critical field $h^*_{c3} \equiv H^*_{c3}/H_{c2}$ versus the normalised flux through a hole. The dashed curve represents the calculation for the case of the boundary between a superconductor and a perfect dielectric ($b \to \infty$), solid curve is reconstructed from the experimental $T^*_c(H)$ dependence (see Fig. 3a). Inset: magnification of the low field part. Upper scale is the normalised flux through a unit cell of the square lattice. The additional curve (lowest solid line) is measured at the lower part of the first superconducting transition, namely at 6 mOhms. The cusps of the "collective" and "single-object" types are shown by down-directed and up-directed arrows correspondingly. The monotonously decreasing curve is the value of the coherence length at the upper critical temperature (see the right y-axis).

Let us consider now the main experimental $h^*_{c3}(\phi/\phi_0)$ dependence (Fig. 4, solid curve). The envelope of this curve is an increasing function which is equal to unity when $\phi/\phi_0 = 0$ and has as the theoretical limit the value $h^*_{c3}(\infty) = 1.7$. This is natural because we consider the critical point $(T \to T^*_c)$, so when $\phi/\phi_0 \to 0$ the radius of the hole is much smaller then $\xi(T)$ and the effect of surface superconductivity is negligible.[1] The opposite situation is realised when ϕ/ϕ_0 is big: $h^*_{c3} \equiv H^*_{c3}/H_{c2} \to H_{c3}/H_{c2} = 1.7$.

The most interesting fact is the oscillations in h^*_{c3}. In Fig. 4 (see also the inset) one can see two types of oscillations with different frequencies. Firstly these are well pronounced cusps (shown by up-arrows) with a big period in the field. Their amplitude decreases slowly with the field so it was possible to observe them in the whole range of fields under investigation (0 – 1.8 mT). We will name them "single-object" oscillations because they reflect individual properties of the holes and in principle can be observed in films with a single hole. The first peak (shown by the left up-arrow in the inset) of this type can be resolved clearly on the curve measured at $R_0 = 6$ mOhms where the peaks of the other type with a higher frequency are suppressed. This second type of oscillation (shown by down-arrows) reflects collective properties of the array of holes or in other words the interaction between the holes (we will name them "collective" oscillations). They are exactly periodic with the field with the period $\Delta H = 0.0247$ mT which is in good agreement with the expected value $\phi_0/a^2 = 0.0253$ mT where a^2 is the unit cell area (see the upper x-axis). Their amplitude decreases with the field much quicker in comparison to the "single-object" oscillations.

It is obvious that we deal here with two general types of the critical temperature oscillations simultaneously. "Single-object" oscillations were discovered by Little and Parks in their experiment with a hollow thin-walled cylinder. It was shown in [7] that a superconducting disk also exhibits the same type of oscillations. Their general properties are the following (Fig. 5d, solid line): the critical temperature $T_c(\tilde{\phi}/\phi_0)$ is maximum when $\tilde{\phi}/\phi_0 = m$, where $m = 0, \pm 1, \pm 2, \ldots$ and its derivative is equal to zero in those points; when $\tilde{\phi}/\phi_0 = m + \frac{1}{2}$ the critical temperature $T_c(\tilde{\phi}/\phi_0)$ is minimum and it has a jump in the derivative (cusp). Under $\tilde{\phi}$ we understand here the flux through some characteristic area of the object. In many cases this area is determined only by the geometry as for example in the case of a thin-walled cylinder where $\tilde{\phi} = \pi r_0^2 H$ (here r_0 is the radius of the cylinder). But in other cases (as an example one can consider the film with a hole or the disk) this characteristic area depends on $\xi(T_c)$ because the order parameter has its maximum not exactly at the geometrical edge of the object but it is shifted on the distance $\xi(T)$. In this situation the "single-object" oscillations are not periodic with the field. "Collective" oscillations

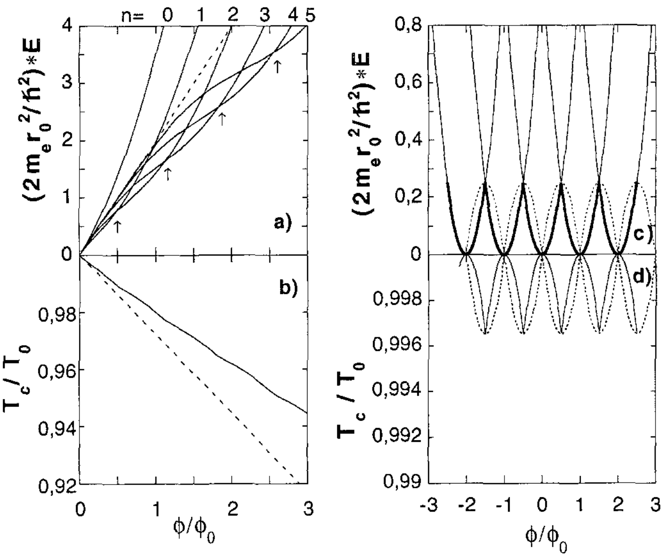

Fig. 5. a) Energy of a single particle (with the charge $2e$ and the mass m_e) in the film with a single hole at different values of the orbital momenta n (solid lines), the dashed line is the lowest Landau level in a uniform film without holes; b) corresponding normalised critical temperatures for the uniform (dashed line) and holed (solid line) films for the case when $\xi(0)/r_0 = 0.117$. c) single-particle energy for a circular wire loop (solid lines) at different values of the orbital momenta, thick solid line is the minimum energy, and the dashed line is the average energy per a unit cell of the superconducting network (see text); d) corresponding normalised critical temperatures of the loop (solid line) and of the network (dashed line) for the case when $\xi(0)/r_0 = \xi(0)/\tilde{r}_0 = 0.117$.

were observed first in superconducting wire networks.[6,8] Generally speaking it is a property of any correlated in phase periodic structure. Their period exactly equal to one flux through a unit cell (in particular this is true in our case of the lattice of holes). As it is schematically shown in Fig. 5d (dashed line), each maximum of $T_c(\phi/\phi_0)$ again corresponds to the integer values of the flux $\phi/\phi_0 = m$ (here $\phi = Ha^2$ is the flux through a unit cell) but it coincides now with the jump in derivative. On the contrary in the points of minimum $(\phi/\phi_0 = m + \frac{1}{2})$ the derivative is equal to zero. The amplitude of the both types of oscillations is of the order of $(\xi^2(0)/L^2) \cdot T_0$ (here L is the lattice parameter for the array or the characteristic size of the single object). The two types of the critical temperature oscillations shown in Fig. 5d can be illustrated by following simplest examples.

The single-particle energy (6) of a superconducting thin wire loop in perpendicular magnetic field (see [9]) is equal:

$$E_n = \frac{\hbar^2}{2m_e r_0^2}(n - \phi/\phi_0)^2 \tag{1}$$

(the same is true for a thin-walled cylinder as in the Little and Parks experiment). Here r_0-radius of the loop, $\phi = \pi r_0^2 H$, n-integer number, m_e- the mass of the electron, $\hbar$-Plank constant. This energy for some values of the orbital momenta n is shown in Fig. 5c by thin solid lines. The thick solid line is the minimum energy E^0, and the corresponding critical temperature calculated using the relation (7) is shown in Fig. 5d (solid line). The second example is a superconducting network. It can be roughly considered as an array of loops with the same relation for the energy (1) (r_0 should be replaced by the value $\tilde{r}_0 \sim a$). But now due to the phase correlation between the loops it is not possible for all of them to be in the energetically favourable state, for example in the state E_0 when $0 < \phi/\phi_0 < 0.5$. On the contrary some of them (namely the part ϕ/ϕ_0) must be in the state E_1 so the average energy per unit cell $\langle E \rangle$ can be written approximately as

$$\langle E \rangle = \frac{\phi}{\phi_0} E_1 + \left(1 - \frac{\phi}{\phi_0}\right) E_0 = \frac{\hbar^2}{2m_e \tilde{r}_0^2}\left[\frac{1}{4} - \left(\frac{\phi}{\phi_0} - \frac{1}{2}\right)^2\right] \tag{2}$$

It is true when $0 < \phi/\phi_0 < 1$. In general when $n < \phi/\phi_0 < n+1$ then there are two possible states for a unit cell: E_n and E_{n+1} so it is necessary to replace in (2) ϕ/ϕ_0 by $\phi/\phi_0 - n$. The average energy is shown in Fig. 5c by the dashed line and corresponding critical temperature (7) in Fig. 5d (dashed line).

Now we will consider in more details our particular problem: the infinite film with a hole of a radius r_0 at the origin with a uniform magnetic field perpendicular to the film.[1] The order parameter ψ satisfy the linearized Ginsburg-Landau (GL) equation (we neglect the non-linear term and assume the field to be uniform because we are interested only in the critical temperature now):

$$\frac{1}{2m_e}\left(-i\hbar\vec{\nabla} - \frac{2e}{c}\vec{A}\right)^2 \psi = \frac{\hbar^2}{2m_e \xi^2(T)}\psi \tag{3}$$

where

$$\vec{A} = \tfrac{1}{2}[\vec{H} \times \vec{r}], \tag{4}$$

and the boundary condition:

$$\frac{1}{\hbar}\vec{n}\left(-i\hbar\vec{\nabla}-\frac{2e}{c}\vec{A}\right)\psi\bigg|_{\rho=r_0}=-\frac{i}{b}\psi \tag{5}$$

where b is the characteristic boundary length, $\vec{n}=\vec{r}/|\vec{r}|$ is the unit vector perpendicular to the boundary of the superconductor.[10] One can solve the Schrodinger equation for a single particle with the mass m_e and the charge $2e$ in the same magnetic field and with the same boundary conditions as for the order parameter:

$$\frac{1}{2m_e}\left(-i\hbar\vec{\nabla}-\frac{2e}{c}\vec{A}\right)^2\psi^i=E^i\psi^i \tag{6}$$

The energy of the ground state $E^0=\min(E^i)$ (we refer E^i as a single-particle energy) can be used to find the critical temperature (see for example[7]):

$$T_c=T_0\left(1-\frac{2m_e\xi^2(0)}{\hbar^2}E^0\right) \tag{7}$$

Moreover the order parameter just near T_c is proportional to the ψ-function of the particle in the ground state.

The two-dimensional GL equation can be reduced to a one-dimensional equation if we choose the order parameter in the form $\psi=f(\rho)\cdot\exp(in\theta)$ (here (ρ,θ) are the polar coordinates and n is the orbital momentum, $\xi\equiv\xi(T)$):

$$\frac{d^2f}{dx^2}+\frac{1}{x}\frac{df}{dx}-\left(\frac{n}{x}-\frac{\phi}{\phi_0}x\right)^2f+\frac{r_0^2}{\xi^2}f=0$$

$$\frac{df}{dx}\bigg|_{x=1}=\frac{r_0}{b}f$$

where $x=\rho/r_0$; $\phi=\pi r_0^2H$-flux through the hole.

After the substitution into the first equation: $f(x)=x^n\exp(-\frac{1}{2}(\phi/\phi_0)x^2)\cdot w(x)$ and $z=(\phi/\phi_0)x^2$ one can get:

$$z\frac{d^2w}{dz^2}+(1+n-z)\frac{dw}{dz}-\left(\frac{1}{2}-\frac{r_0^2}{4\xi^2}\frac{\phi_0}{\phi}\right)w=0$$

It is the Kummer's equation which has as its solutions the confluent hypergeometric functions (see their properties in ref. 11 for example):

$$M(A,B,z)={}_1F_1(A;B;z)\text{ and }U(A,B,z)=z^{-A}\cdot{}_2F_0\left(A,1+A-B;;-\frac{1}{z}\right)$$

where $A = \frac{1}{2} - (r_0^2/4\xi^2)(\phi_0/\phi)$ or $A = \frac{1}{2} - (m_e r_0^2/2\hbar^2)(\phi_0/\phi)E^0$ (A should not be confused with the vector potential $\vec{A}$), E^0 is the ground state energy (6), and $B = n+1$. The general solution is the linear combination of the two functions: $w = C_1 M(A, B, z) + C_2 U(A, B, z)$, but for a hole in the infinite film one should keep only the second term to have a finite solution for the order parameter in the infinity. Finally we get:

$$\psi_n(\rho, \theta) = \exp(in\theta) \cdot x^n \cdot \exp\left(-\frac{1}{2}\frac{\phi}{\phi_0}x^2\right) \cdot U\left(A, n+1, \frac{\phi}{\phi_0}x^2\right) \qquad (8)$$

Using (8) and a differential property of the U-function[11] one can transform the boundary condition (5) to the form:

$$\left(n - \frac{\phi}{\phi_0} - \frac{r_0}{b}\right) \cdot U\left(A, n+1, \frac{\phi}{\phi_0}\right) - 2\frac{\phi}{\phi_0} \cdot A \cdot U\left(A+1, n+2, \frac{\phi}{\phi_0}\right) = 0 \qquad (9)$$

This equation can be solved numerically. As a result for each value of the orbital momenta one can get $A(\phi/\phi_0)$ and therefore the dependence of the single-particle energy versus the reduced flux: $E_n(\phi/\phi_0)$ (see Fig. 5a, solid lines; $b = \infty$). Substitution of the minimum energy into (7) give us the critical temperature $T_c^*(\phi/\phi_0)$ (Fig. 5b, solid line; $b = \infty$, $\xi(0)/r_0 = 0.117$) which can be used to get $h_{c3}^*(\phi/\phi_0)$ dependence shown in Fig. 4 (dashed curve). In fact h_{c3}^* does not depend on the ratio $\xi(0)/r_0$ and can be obtained directly from the energy:

$$h_{c3}^*\left(\frac{\phi}{\phi_0}\right) = \frac{\phi}{\phi_0} \cdot \frac{\hbar^2}{m_e r_0^2} \cdot \frac{1}{E^0(\phi/\phi_0)} = \frac{1}{1-2A} \qquad (10)$$

It is important that A is the solution of (9) which depends only on ϕ/ϕ_0 and b/r_0 but not on the ratio $\xi(0)/r_0$. Here we can mention that our approach to the problem gives in general the same results as the variational method used in 1. The only exception is that we found that the state with $n = 0$ is not favourable even at a small field ($E_0 > E_1$ at any field) as it is clear from Fig. 5a so the first "single-object" cusp corresponds to the transition ($n = 1 \to n = 2$) while in 1 there is an additional cusp ($n = 0 \to n = 1$).[12] This discrepancy can be explained by the choice of the trial function $\psi = \exp[-\text{const} \cdot (\rho - r_0)^2]$ in ref. 1 which has its maximum exactly at the hole edge while in fact the maxima of all the states with $n > 0$ are shifted on the distance $\sim\xi(T)$ from the edge of the hole at least when $\phi/\phi_0 < 0.2$ (it follows from (8)). This shift decreases the energy of the superconducting state. It may be useful to mention also that from (9) one can derive (using some recurrence relations of the confluent hypergeometric functions[11]) that the position of the cusp corresponding to the transition $(n-1 \to n)$ can be found from the system of equations:

$$\left(n-\frac{\phi}{\phi_0}-\frac{r_0}{b}\right)\cdot U\left(A, n+1, \frac{\phi}{\phi_0}\right)=2\frac{\phi}{\phi_0}\cdot A\cdot U\left(A+1, n+2, \frac{\phi}{\phi_0}\right)$$

$$\frac{\phi}{\phi_0}=n+\frac{1}{2}-2A-\sqrt{2n(1-2A)+\left(2A-\frac{1}{2}\right)^2+\frac{r_0}{b}+\left(\frac{r_0}{b}\right)^2}$$

While at large values of ϕ/ϕ_0 there is a quite good correlation between the experiment and the theory, it is not the case near zero (Fig. 4, inset). First of all one can see that in zero field the theoretical curve has a big derivative while the experimental one has zero derivative and moreover $h^*_{c3}=1$ in a finite range of the flux $(\phi/\phi_0<0.1)$. We think that it is a collective effect which can be observed when the field is small enough. As concerns to one independent hole it was discussed already that in any small field the localised state with $n=1$ (in other words when there is one vortex in the hole) has smaller energy (and therefore higher critical temperature) than the delocalised state (Fig. 5a, dashed line) which is realised in a uniform film without holes at $H=H_{c2}$ (in other words E_1 is lower than the lowest Landau level in a uniform film). On the contrary the localised state which contains no vortices (ψ_0) has its energy higher with respect to the delocalised one (Fig. 5a). Let us return to our lattice of holes and consider the region of a weak field when $a \leqslant 2\xi(T)$. The holes are not independent in this case and there phases are correlated. Therefore in the case of a small enough field when we have less than one vortex per a unit cell (and therefore less than one vortex per hole in the case of the square lattice) it is not possible to have the state $n=1$ in all the holes. It means that only those that have a vortex inside will have an elevated critical temperature. Therefore in small field (considerably smaller than the field corresponding to one flux per two cells) when vortices are far from each other most of the holes (in fact the regions around these holes) will pass into superconducting state only at $T=T_{c2}$ or $H=H_{c2}$. This is a qualitative explanation of the plateau in the experimental curve when $H \to 0$.

The positions of the first few cusps of the "single-object" type are also far from those theoretically predicted (Fig. 4). Namely experimental peaks occur at smaller values of ϕ/ϕ_0 and at lower values of h^*_{c3}. To explain this fact we try (following[7]) to take into account a non-perfect boundary of the superconductor inside the holes, namely we assume that b is not infinite. In Fig. 6 one can see the experimental positions of the peaks (crosses) with respect to the theoretical positions calculated for different values of b. Generally speaking each calculated peak (black squares) moves to the origin with decreasing b so it is possible to improve correlation between the theoretical and experimental cusps (taking $b/r_0 \approx 15$ for example). Even better correlation can be achieved assuming that b decreases with increasing temperature (or decreasing flux through a hole). For each peak there is a critical value of b when it disappears. For example at $b/r_0 = 1$ we observe disappearance of the first cusp. After this the minimum possible value of the orbital momenta start to be $n = 2$ (at any small field) and the first cusp corresponds now to the transition $(n = 2 \to n = 3)$. In the limiting case $b = 0$ the effect of the surface superconductivity is absent and $h_c^* = 1$ at any value of the flux. Such a situation can be realised if the hole in the film is replaced by a normal metal disk.

It may be interesting to consider now a high-Tc superconductor with columnar defects. These defects which are in fact amorphous tracks produced by high-energy ions can be considered as dielectric cylinders inside the superconductor. While many parameters of HTSC with columnar defects and of the thin Al film with holes are strongly different, we believe that our results can be used for analysis of HTSC also. The only assumption which we should make is that GL-equation is valid in HTSC

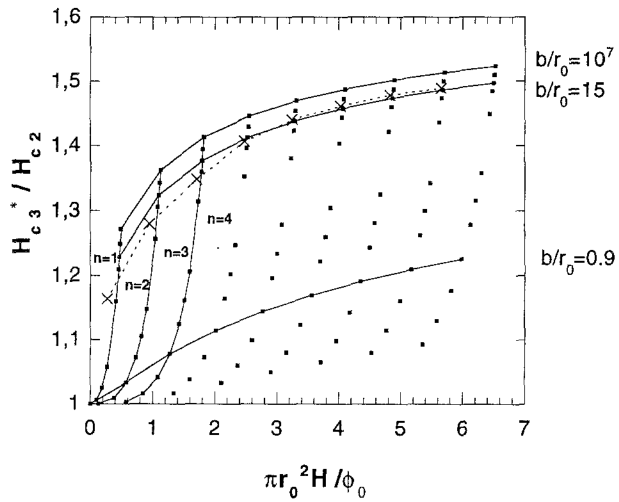

Fig. 6. Positions of the "single-object" type cusps taken from the $h_{c3}^*(\phi/\phi_0)$ dependence (they are indicated by up-arrows in Fig. 4). Crosses show the experimental data while the blak dotes are the results of the calculation at different values of the characteristic boundary length $b/r_0 = 10^7$, 30, 15, 10, 5, 2, 1.5, 1.2, 0.9, 0.7, 0.55, and 0.45. Vertical lines show the change in the cusp positions with changing b-parameter. Three horizontal lines connect the cusps corresponding to different orbital momenta transitions with the same b-parameter.

near the critical temperature. As was discussed already the dependence of the reduced critical field versus the flux through the hole (or through the area of the track in the case of HTSC) shown in Fig. 4 (dashed line) is independent on the ratio $\xi(0)/r_0$ (see 10) so in principle the same dependence can be observed in superconductor with columnar defects. A possible difficulty is the small radius of the defects: $r_{cd} \leqslant 50$ Å. Therefore to observe at least one cusp in the critical field (which takes place at $H \cdot \pi r_{cd}^2/\phi_0 \approx 0.5$) it is necessary to apply field $H \geqslant 13$ T. At the same time when $H \to 0$ the theoretical curve $h_{c3}^*(\phi/\phi_0)$ has a big positive slope (Fig. 4) which can be observed experimentally if the distance between the tracks is so large that there is no phase coherence between the amorphous region and superconductor can be characterised in this case by some finite value of the boundary length b (see the boundary condition (5)). As one can see in the Fig. 6 (see also previous discussion) the decrease of b cause the motion of the cusps to the origin and when $b = 0$ the effect of the surface superconductivity is absent and one can observe no oscillations and no increase of the critical temperature due to the irradiation. In practice the ratio b/r_{cd} should be big. This conclusion is based on the following relation of the theory of de Gennes and Werthammer[13]:

$$\frac{b}{d_N} = \frac{\sigma_S}{\sigma_N} \cdot \left[\frac{\xi_N}{d_N} \coth\left(\frac{d_N}{\xi_N}\right) \right]$$

To make a rough estimation one can replace the thickness of the normal layer d_N by the radius of the columnar defect. Taking into account that the function in the square brackets always bigger than unity (independently on the value ξ_N/d_N) one can see that $b/r_{cd} > \sigma_S/\sigma_N \gg 1$. This is true because the mean free path in the crystal l_S always larger than the mean free path inside the amorphous columnar defect and the same is true for the conductivity: $\sigma_S/\sigma_N \sim l_S/l_N \gg 1$. Note that the roughness of the track boundary can also suppress the surface effect.

Recently there were published some results on the anomalous proximity effect in aluminium.[14] It was observed that the resistive curve $R(T)$ of a modulated S-N-S structure (S and N perpendicular to the current strips are the parts of the Al film with a little bit different T_c produced by reactive-ion etching) shows a single homogeneous transition while the calculated coherence length (in the normal regions) is smaller by a factor of 60 than the period of the structure and therefore the order parameter in the centre of the N-region is smaller by a factor of 10^{13} than at its edge. We think that our measurements can also be considered as a test of the proximity effect in aluminium. To see this we should remind that the total array of holes (see Fig. 1a) consists of the square fields with the distance between their edges $d_f = 20\mu m$ (which is much smaller than the size of each field). It was observed that the $R(T)$ curve starts to have two steps when $H = 0.09mT$ and $T_c = 0.087T_0$ (this transition is not shown in Fig. 2). At the same time the order parameter at the centre of the normal region between the fields is smaller only by a factor of 100 than at the edges: $d_f/2\xi(T_c) = d_f/2\xi(0)\sqrt{1 - T_c/T_0} = 4.6$ therefore $\psi_{\text{center}} \approx \psi_{\text{edge}} \cdot \exp(-d_f/2\xi(T_c)) = 10^{-2}$. We conclude that our result is in contradiction with ref. 14 and probably can be described by the ordinary theory of the proximity effect which takes into account fluctuation phenomena. Note that the plateau in the $R(T)$ curve is higher when the field is lower. This feature seems to be in a contradiction with the fact that the coherence length is larger at higher temperature (or what is the same at lower field) so the width of the normal region should be smaller. We see a qualitative explanation of this fact in the fast decrease of the order parameter amplitude near the hole edge when $T \to T_c^*$. Due to this the width of the regions with non-zero resistance (which can be determined by taking into account thermal fluctuations) can be an increasing function of the temperature.

Up to now we discussed only the nucleation of superconductivity in a film with a hole. In particular each cusp in the curve $T_c^*(H)$ gives origin to two lines corresponding to the second order phase transitions from the normal state into a superconducting state with the orbital momenta equals n or $n+1$ where n is the cusp number. Another principally different question is the possibility of transitions below T_c^* between the states with different values of the orbital momenta. For analysis of such a question it is necessary to take into account the non-linear term in the Ginsburg-Landau equation. As it is shown in ref. 15 these are the first order transitions and corresponding phase boundary is not parallel to the temperature axis in contrast to the case of a thin-walled hollow cylinder. Experimentally the mentioned above transition can be observed below T_c^* by measuring the magnetisation (or the heat capacitance) versus temperature at fixed magnetic field.

In conclusion we present the experimental results on the surface superconductivity in the case of the finite radius of the superconductor boundary and the multiply connected geometry of the sample. Namely we observe some increase of the critical temperature and its oscillations in perpendicular magnetic field on the aluminium film with the lattice of circular holes. Another type of oscillations caused by the interaction between the holes are also observed. Comparison of our experimental data with the results of the calculation based on the linearized Ginsburg-Landau equation shows some discrepancy which is discussed in terms of the interaction between the holes. We consider also a possible application of our results for the analysis of high-Tc materials with columnar defects.

The authors are grateful to A. Buzdin for useful discussions and for kind provision of his results before publication. We acknowledge also several stimulating conversations with V. Kopylov. The work was supported by the CEE "SUPNET" contract ERBCGRCT920068.

We are grateful also for Yu. N. Ovchinnikov who has called our attention (after the paper was ready for publication) to the fact that the nucleation of superconductivity in a cylinder and near a cylindrical cavity was considered theoretically in his work (see ref. 16) and a non-monotonous dependence for the critical field versus the temperature was predicted.

REFERENCES

1. A. I. Buzdin, *Phys. Rev. B* **47**, 11416 (1993).
2. G. S. Mkrtchyan and V. V. Shmidt, *Soviet Phys. JETP* **44**, 195 (1972).
3. A. T. Fiory, A. F. Hebard, and S. Somekh, *Appl. Phys. Lett.* **32**, 73 (1978).
4. D. Saint-James and P. G. de Gennes, *Phys. Lett.* **7**, 306 (1963).
5. W. A. Little and R. Parks, *Phys. Rev. A* **133**, 97 (1964).
6. B. Pannetier, J. Chaussy, R. Rammal, J. C. Villegier, *Phys. Rev. Lett.* **53**, 1845 (1984).
7. O. Buisson, P. Gandit, R. Rammal, Y. Y. Wang, and B. Pannetier, *Physics Letters A* **150**, 36 (1990); R. Rammal (1989), unpublished.
8. B. Pannetier, J. Chaussy and R. Rammal, *J. Physique-LETTERS* **44**, L-853 (1983).
9. M. Tinkham, Introduction to Superconductivity (McGraw-Hill, New York, 1975).
10. P. G. de Gennes, Superconductivity of Metals and Alloys (Benjamin, New York, 1966).
11. Handbook of Mathematical Functions, Edited by M. Abramowitz and I. A. Stegun (Dover Publications, Inc., New York, 1970), p. 504.
12. This additional peak corresponding to the transition ($n=0 \rightarrow n=1$) will appear if one consider a film of a finite size, for example in the form of a disk with a single hole in the centre.
13. G. Deutscher and P. G. de Gennes, in *Superconductivity*, Vol. 2, ed. Parks (Dekker, New York, 1969), p. 1005.
14. Y. K. Kwong, K. Lin, M. S. Isaacson, and J. M. Parpia, *Phys. Rev. Lett.* **65**, 2905 (1990).
15. A. Bezryadin, A. Buzdin, and B. Pannetier, *Phys. Rev. B* **51** (1995), to be published.
16. Yu. N. Ovchinnikov, *Sov. Phys. JETP* **52**, 755 (1980).