SUPERFLUIDITY, PHASE COHERENCE AND THE NEW BOSE-CONDENSED ALKALI GASES

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1. Superfluidity in liquid ^{4}He

2. Explanation thereof in terms of
   BOSE-EINSTEIN CONDENSATION (BEC)

3. BEC in the alkali-gas systems

4. Differences and similarities to liquid ^{4}He

* 5. What NEW things can we do with the BEC alkali gases?

SUPERFLUIDITY IN LIQUID ^{4}HE

^{4}He liquefied: 1908

$T < T_\lambda$ (2.17 K): 1920

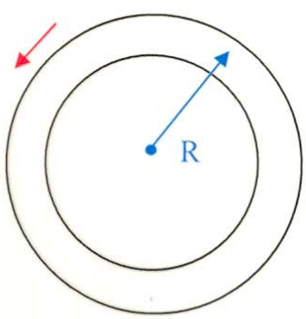
Frictionless flow below T_λ: 1938

~ 20 YEARS!

Modern point of view:

Define

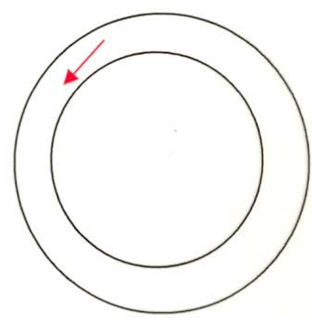
$\omega_c \equiv \hbar/mR^2 \equiv$ quantum unit of rotation ($\sim 10^{-4}$ Hz for $R \sim 1$cm)



EXPT. A
("Hess-Fairbank" effect)

walls rotate with
ang. velocity $\lesssim \omega_c$,
liquid stationary

EQUILIBRIUM
EFFECT



EXPT B
(Persistent currents)

walls at rest,
liquid rotating with
ang. velocity $\gg \omega_c$.

METASTABLE
EFFECT

BEC IN A NONINTERACTING BOSE GAS:

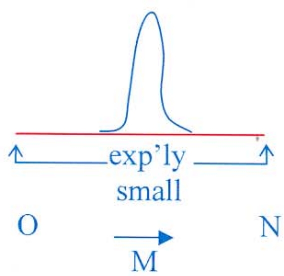
THE EFFECTS OF STATISTICS

I. Qualitative argument:

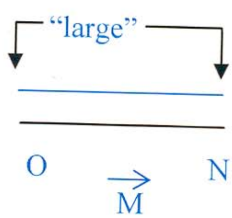
Distribute N objects between 2 boxes: what is probability P(M) of finding M in one box?

A. Objects distinguishable (≡ coin toss):

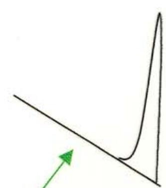
$$P(M) = \frac{N!}{M!\ N\text{-}M!}$$



B. Objects indistinguishable (bosons):



$P(M) = 1/N$



all objects in one box

II. Quantitative argt. (Einstein, 1925):

chemical potential, ≤ 0

$$n_i(T) = [\exp(\varepsilon_i - \mu(T)/k_B T - 1)]^{-1}$$

$\mu(T)$ fixed by: $\sum_i n_i(T{:}\ \mu(T)) = N$ ← total no. of particles

$T \to \infty \Rightarrow \mu \to -\infty$: $T\downarrow \Rightarrow \mu\uparrow$. But what if

$$\sum_i [\exp(\varepsilon_i/k_B T) - 1]^{-1} < N?$$

Einstein: Macroscopic no. of particles occupy lowest ($\varepsilon = 0$) state!

BEC IN A GENERAL (INTERACTING, NONEQUILIBRIUM) SYSTEM:

Can always find set of "single-particle" states $\chi_i(\mathbf{r},t)$ st. average no. of atoms in state i is $n_i(t)$ (and $\langle a_i^+ a_j \rangle \equiv 0, i \neq j$)

Df of ("simple") BEC:

one and only one single-particle state
i (say i = 0) has $n_i(t) = 0(N)$, rest $o(1)$

Then,

$N_0(t) \equiv$ "condensate number"
$\chi_0(\mathbf{r},t) \equiv$ "condensate wave function"

WHY BEC IN GENERAL CASE?

A. Statistics
B. Interactions ("Fock" term):
e.g. if $V(\mathbf{r}) = V_0\, \delta(\mathbf{r})$:

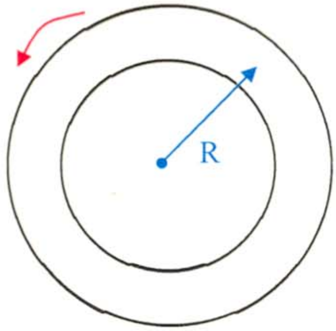
N atoms in $\chi_0(rt)$: $\langle V \rangle(t) = \frac{1}{2} V_0 N^2 \cdot \int |\chi_0(\mathbf{rt})|^4\, dr$

N_1 in χ_1, N_2 in χ_2: $\langle V \rangle(t) = 2V_0 N_1 N_2 \int |\chi_1(rt)|^2 |\chi_2(rt)|^2\, dr$

$\Rightarrow$ if $V_0 > 0$, advantageous to have all in one state

what if $V_0 < 0$?

EXPLANATION OF HESS-FAIRBANK EFFECT IN TERMS OF BEC:



Walls rotating with ang. velocity

$\omega \lesssim \omega_c \Leftarrow \equiv \hbar/m R^2$

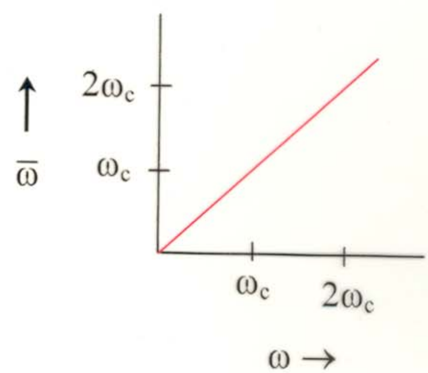
What does liquid do?

General principle: Average ang. velocity of atoms ($\overline{\omega}$) as close as possible to ω

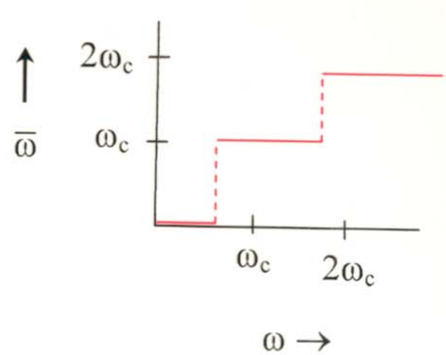
↑ : Single-atom states must obey quantization condition: $\omega = n\omega_c$ ($\ell = n\hbar$)

A. "Normal" (non-BEC) system: many different single-particle states occupied (typical value of $n \sim (kT/\hbar\omega_c)^{1/2} \sim 10^7$)

$\Rightarrow$ to get $\overline{\omega} = \omega$, just shift atoms slightly between states.



B. BEC system ($T \ll T_c$) (almost) all atoms in condensate $\rightarrow$ must have same value of n (n_o) $\Rightarrow$ $\overline{\omega} \cong n_o \omega_c$



INTERACTIONS "OPTIONAL"

^{4}He: PERSISTENT CURRENTS

Initially, after walls stopped,

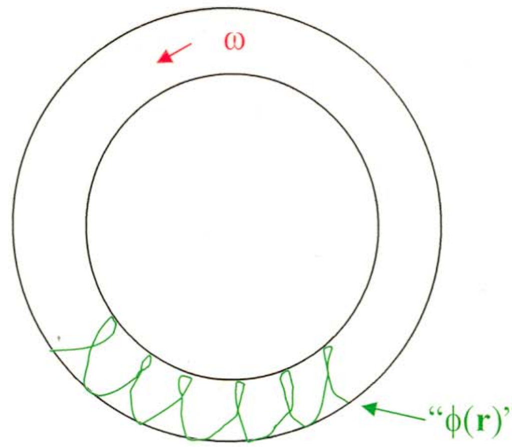
$\langle L \rangle = N_o \ell_o \hbar, \quad \ell_o \gg 1 \quad (\bar{\omega} \gg \omega_c)$

But groundstate has $\langle L \rangle = 0$. $(\omega = 0)$

Why no relaxation?

$\chi_o(\mathbf{r}) = |\chi_o(\mathbf{r})| \exp i\, \phi(\mathbf{r})$

condensate w.f.



Df: "winding no." $n \equiv \oint \dfrac{\nabla\phi \cdot d\mathbf{l}}{2\pi}$

Initially, $n = \ell_o$: eq^m state has $n = 0$.

To change n, must depress $|\chi_o(\mathbf{r})|$ to zero somewhere!

(a) Electron in atom:

Schrödinger eqn. linear $\Rightarrow$ nodes cost no extra energy, e.g.

$$\psi(t) = a(t)\, \psi_p + b(t)\, \psi_s \qquad \begin{cases} t \to -\infty: & a=1,\ b=0 \\ t \to +\infty: & a=0,\ b=1 \end{cases}$$

$\langle E \rangle(t) = |a(t)|^2 E_p + |b(t)|^2 E_s$ = monotonically decreasing

(b) BEC (^{4}He):

Extra term in energy: $\langle V \rangle = \overset{>0}{V_o} \int |\chi_o(\mathbf{r}t)|^4 \, d\mathbf{r}$

$\Rightarrow$ energy NOT monotonically decreasing!

(REPULSIVE) INTERACTIONS ESSENTIAL!

BEC IN THE ALKALI GASES (1995-…)

What? Atomic gases of ^{87}Rb, ^{23}Na, ^{7}Li, ^{1}H, ^{85}Rb, ^{4}He*

No recombination

Even no. of fermions!

How? magnetic or laser confinement

Only certain hyperfine species

Any hyperfine species

Typical parameter values:

Density*	$\lesssim 10^{15}$ cm^{-3}	[air: ~ 10^{18}cm^{-3}]
Temperature	100-500 nK	[H: ~ 50μK]
No. of atoms in condensate	$10^3 - 10^9$	
Atomic velocity*	$\lesssim$ 1 mm/sec	("subcochlear")

*In BEC phase

Interactions parameterized uniquely by s-wave

scattering length a_s:

$$V(\mathbf{r}_1 - \mathbf{r}_2) = U_0\delta(\mathbf{r}_1 - \mathbf{r}_2) \ , \ U_0 \equiv \frac{4\pi \hbar^2 a_s}{M}$$

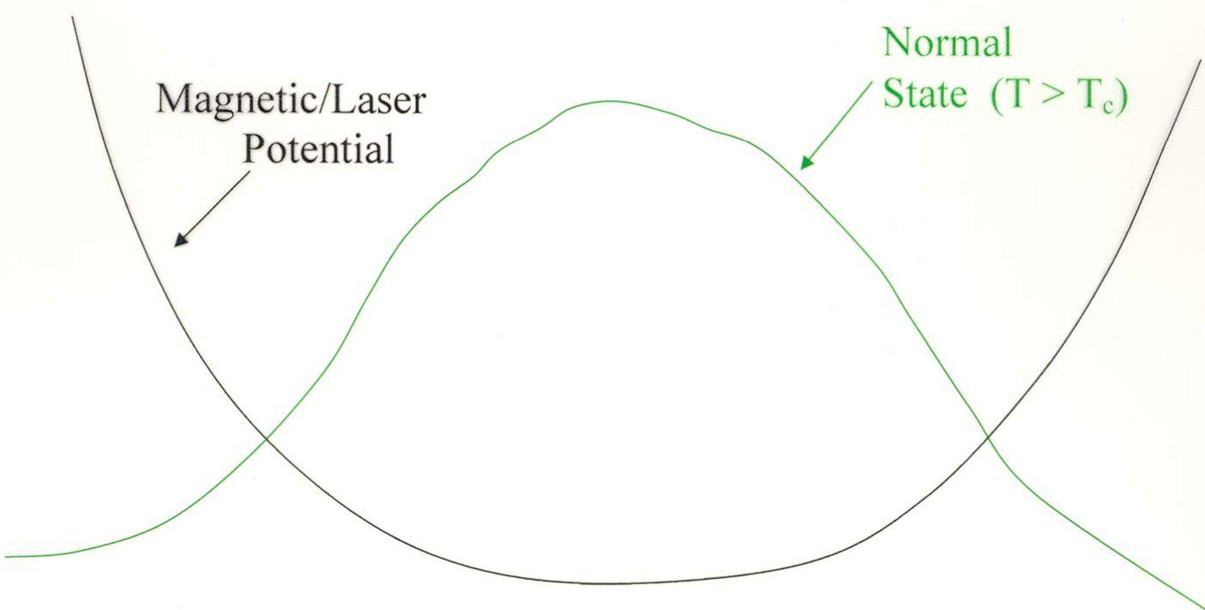
^{87}Rb, ^{23}Na, ^{1}H: $a_s > 0$

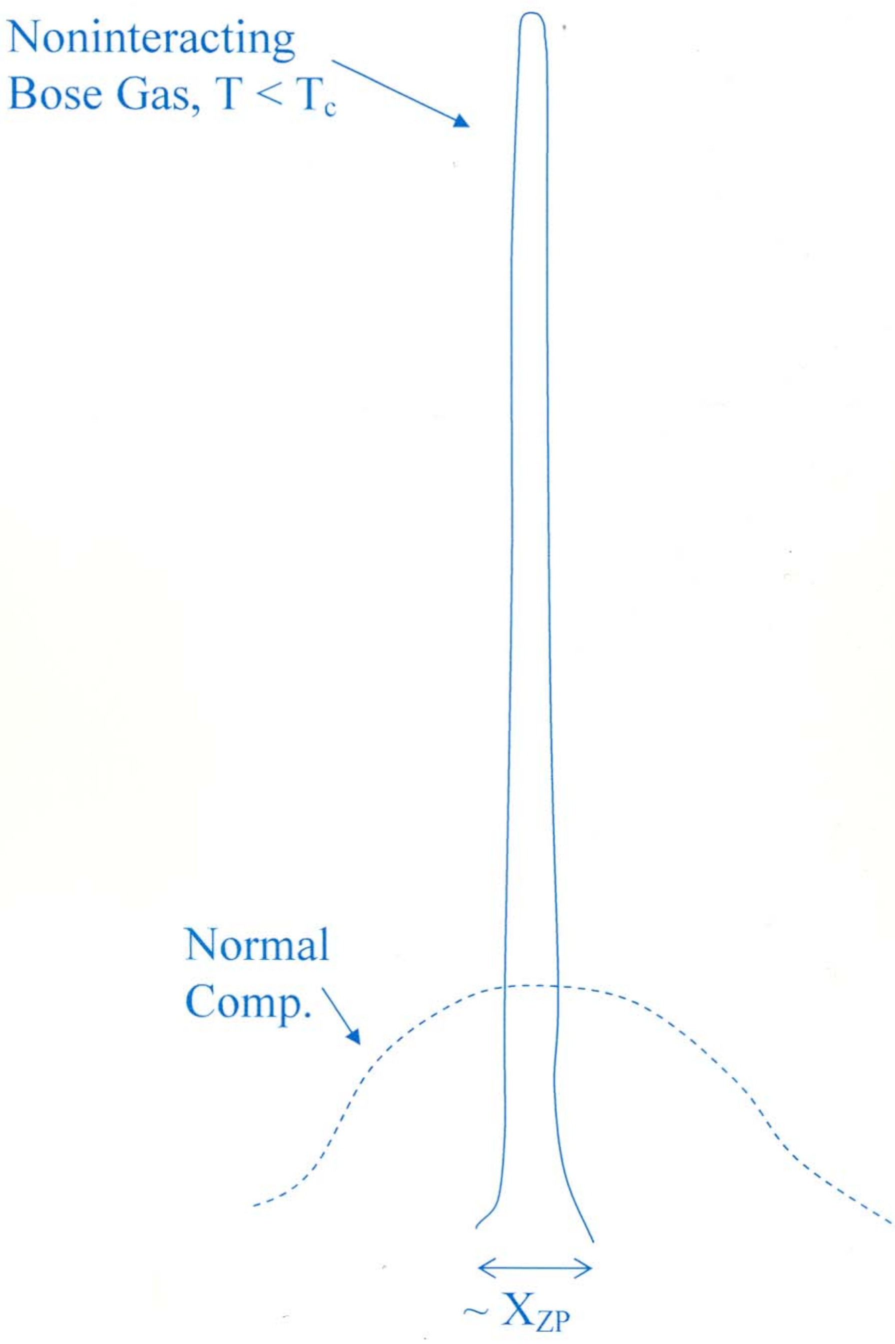
^{7}Li: $a_s < 0$ ⇐ Interaction attractive

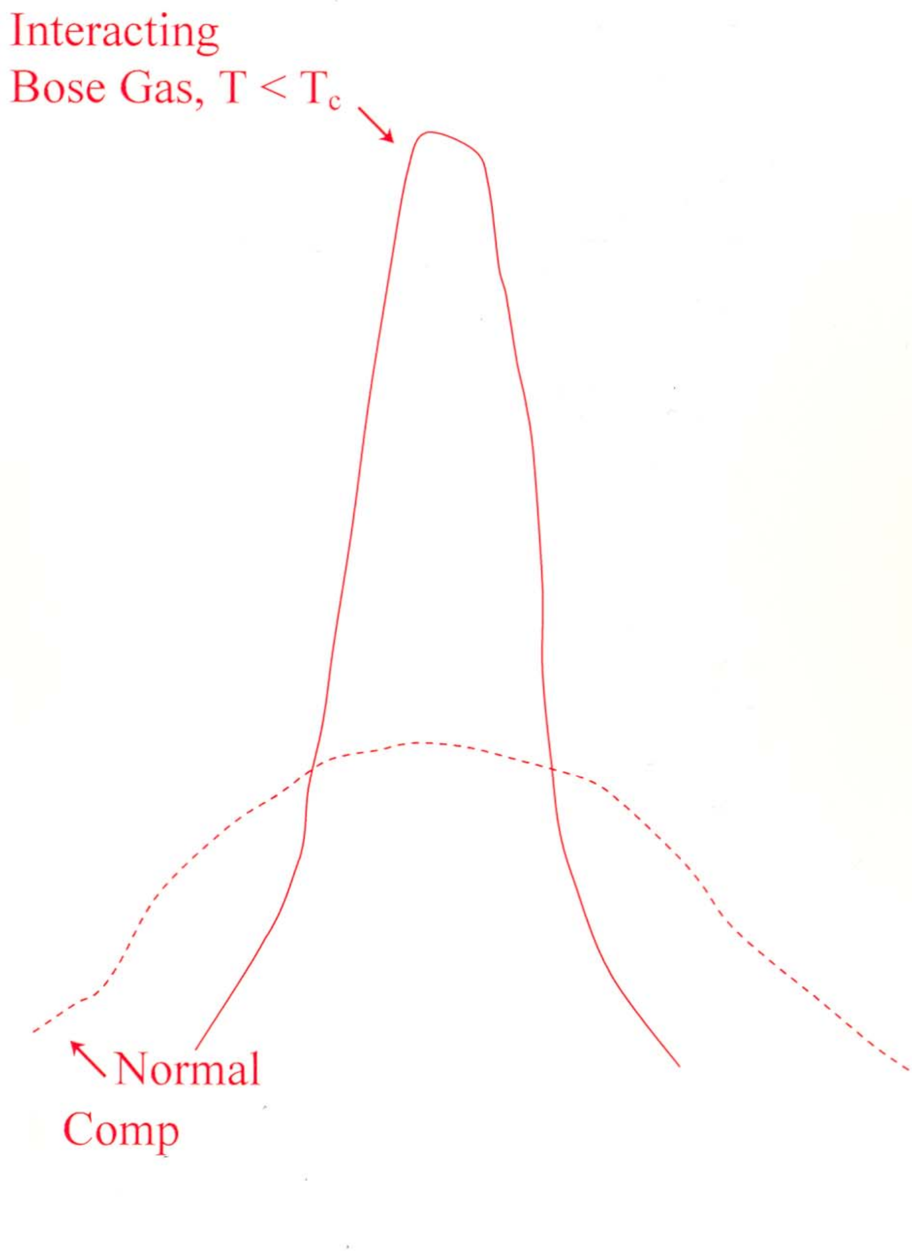
^{85}Rb: a_s changes sign as function of magnetic field

HOW TO SEE BEC OCCURRING?

LITERALLY




Noninteracting
Bose Gas, $T < T_c$
Normal
Comp.
$\sim X_{ZP}$


Interacting
Bose Gas, T < T_c
Normal
Comp

SOME IMPORTANT DIFFERENCES BETWEEN ALKALI GASES AND ^{4}HE

1. Very low density $\Rightarrow$ kinetics may dominate energetics
2. Multiple species
3. "Raw data" consists almost entirely of $\rho_\alpha(\mathbf{r}t)$ $\leftarrow$ density of hf species α at $(\mathbf{r}, t)$
4. N_o very easy to measure, ρ_s difficult.

   $\underset{\text{condensate fraction}}{\underset{\uparrow}{N_o}}$ $\qquad$ $\underset{\text{superfluid density}}{\underset{\uparrow}{\rho_s}}$

5. (Laser trapping): potentials very flexible
6. (Laser trapping): potentials "instantaneously" tunable *

thermal d.B. wavelength

$\downarrow$

At low T, $\lambda_T \gg r_o$ $\leftarrow$ range of atomic potential

$\Rightarrow$ potential uniquely parameterized by s-wave scattering length a_s

Effective interaction:

$\uparrow$ $\quad V_{eff}(r) = U_o\,\delta(\mathbf{r}) \qquad U_o \equiv 4\pi\hbar^2 a_s/m$

$\leftarrow$ $\operatorname{sgn} U_o = \operatorname{sgn} a_s$

valid for bosons and (2 different species of) fermions

mostly, $na_s^3 \ll 1$ $\leftarrow$ "dilute" limit

THE SIMPLEST POSSIBLE THEORETICAL DESCRIPTION: TIME-DEPENDENT GROSS PITAEVSKII EQUATION

(≡ HARTREE EQN. FOR CONDENSED BOSONS)

Ansatz: $\Psi(r_1\, r_2 \ldots r_N : t) = \prod_{i=1}^{N} \chi_o(r_i : t)$ ("GP" state)

In absence of interactions, $\chi_o\,(r : t)$ obeys TDSE:

$$i\hbar \frac{\partial \chi_0(rt)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \chi_o + V_{ext}\,(rt)\,\chi_o\,(rt)$$

In presence of interactions, try

$$V_{ext}(rt) => V_{ext}(rt) + V_{int}(rt)$$

$$V_{int}(rt) = U_o\,\rho(rt) = U_o\,N_o|\chi_o\,(rt)|^2$$

so,

$$i\hbar \frac{\partial \chi_0(rt)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \chi_o + V_{ext}\,(rt)\,\chi_o\,(rt) + U_o\,N_o|\chi_o\,(rt)|^2\,\chi_o\,(rt)$$

alternate notation: define

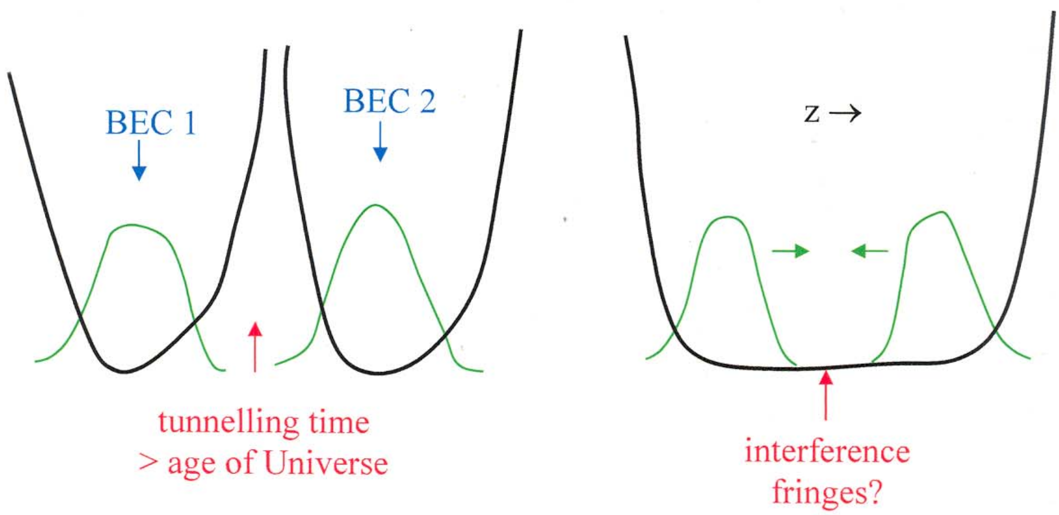
$$\sqrt{N_o}\,\chi_o\,(rt) \equiv \Psi\,(rt) \leftarrow \text{"order parameter"}$$

then

$$\underbrace{i\hbar \frac{\partial \Psi(rt)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \Psi + V_{ext}\,\Psi + U_o|\Psi(rt)|^2\,\Psi(rt)}_{\text{TDGP EQN. (NONLINEAR SCHR. EQN.)}}$$

Amazingly successful description of BEC alkali gases!
(to extent that creation of real quasiparticles negliglble)

INTERFERENCE EXPT. (Andrews et al., Science Jan. '97)



In initial state, 2 condensates have never "seen" one another $\Rightarrow \Psi_o \sim [\psi_1(r)]^{N_1} \bullet [\psi_2(r)]^{N_2} \Rightarrow$ after expansion

$$\Psi_o\,(rt) \sim [\psi_1(rt)]^{N_1}\,[\psi_2(rt)]^{N_2}$$

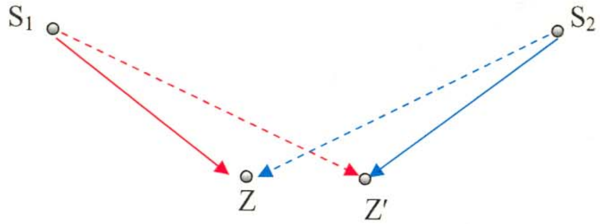
2 independent condensates, no phase relation; different from simple GP state $\left[\left(\psi_1(rt) + \psi_2(rt)\right)^N\right]$

Calculate expectation value of single-particle density $\rho(zt)$:

$$\langle\rho(zt)\rangle \cong \text{const. (i.e. no oscillations)}$$

$\Rightarrow$ no interference fringes?

"MACROSCOPIC HANBURY BROWN-TWISS" EXPT!



Single-particle density (intensity):

$$(\langle\rho(z)\rangle = \overline{|(\psi_1(z) + \psi_2(z))|^2} = |\psi_1(z)|^2 + |\psi_2(z)|^2$$

$$+ \overline{2\mathrm{Re}\,\psi_1^*(z)\,\psi_2(z)} \qquad \Leftarrow = 0$$

$\Rightarrow$ no interference

Two-particle density (intensity) correlation:

$$\langle\rho(z)\rho(z')\rangle = \overline{|\psi_1(z)\,\psi_2(z') + \psi_2(z)\psi_1(z')|^2}$$

$$= |\psi_1(z)|^2 \cdot |\psi_2(z')|^2 + |\psi_2(z)|^2 \cdot |\psi_1(z')|^2 + 2\mathrm{Re}\,\overline{\psi_1^*(z)\,\psi_2^*(z')\,\psi_1(z')\psi_2(z)} \quad \Leftarrow \neq 0$$

$$= \text{const.} + A.\ \cos 2k(z-z')$$

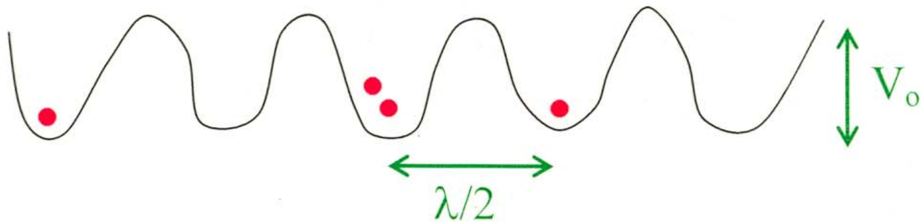
RELATIVE POSITION OF FRINGES FIXED: ABSOLUTE POSITION "AS IF" GP STATE WITH PHASE RANDOM FROM SHOT TO SHOT!

So, how to interpret (correct) result

$\langle\rho\ (z,t)\ \rangle \cong$ const. (i.e. no fringes)?

Ans: Relevant QM ensemble is not ensemble of atoms, but ensemble of different runs!

Optical lattices



2 characteristic energies:

1. Single-particle tunneling matrix element t

   (exponentially sensitive to V_o)

2. Intra-well 2-particle interaction U

   (algebraically sensitive to V_o)

$$\hat{H} = -t \sum_{\substack{ij \\ =nn}} (a_i^+ a_j + H.c.) + U \sum_i n_i (n_i - 1)$$

= (BOSE)-HUBBARD MODEL!

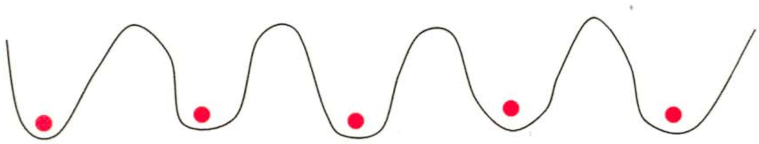
Crucial difference from condensed matter systems: can tune ratio U/t virtually instantaneously!

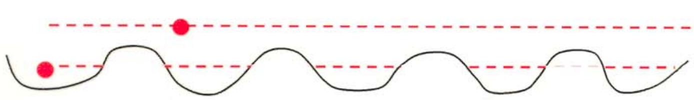
WHAT QUALITATIVELY NEW QUESTIONS ARISE?

Kinetics of BEC (and related things):

ex: Mott-insulator

kinetics of "MI↔superfluid" transition (LMU group)

"MI"  ← U »t

"sup"  ← U « t

Experiment sees not superfluidity as such, but BEC. (i.e. macroscopic occupation of $\mathbf{k} = 0$ Bloch state) [superposition of states at all sites]

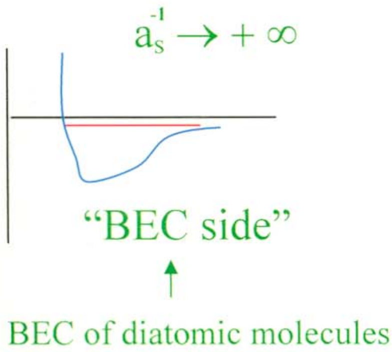
Prima facie, to "establish" BEC a single atom has to be able to explore the whole lattice. But, experimentally, time scale is $\sim \hbar/t$! ⇒ indistinguishability of atoms essential.

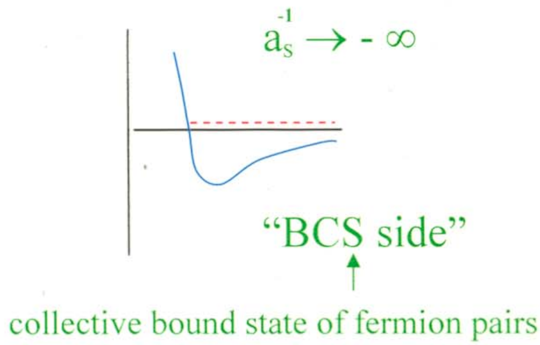
? Connection with PSU "supersolid" expts?

Can one redo with "impurities"?

THE "BEC-BCS CROSSOVER" PROBLEM

2 fermion species interacting by potential which either just does or just doesn't tolerate a bound state.





Theory: since 1969, almost entirely (until ~ 2000) on "single-channel" (potential-resonance) problems, and on static properties

Experiments (2003-…): use Feshbach resonance ("2-channel problem") and are often on kinetic properties

Some new questions arising:

1) What is "proper" definition of "degree of Cooper pairing" at BCS end? ($N_o/\rho_s/\ldots$?)
2) How does condensate behave as we sweep from BCS to BEC (or vice versa)? are N_0, ρ_s, … conserved?
3) How does normal component behave in the sweep? What happens if normal component is badly out of equilibrium with condensate?

(situation unique to alkali gases)