THE "ARROW OF TIME"
IN
QUANTUM MECHANICS

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Standard “textbook” formulation of QM:

Time-dependent Schrödinger eqn. (TISE):

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H}(t)|\Psi(t)\rangle$$

state vector Hamiltonian

$$\Rightarrow \quad |\Psi(t)\rangle = \hat{U}(t)|\Psi(0)\rangle,$$

$$\hat{U}(t) \equiv \exp - i\int_0^t \hat{H}(t')dt'/\hbar$$

$\Rightarrow$ state of system at initial time ($|\Psi(0)\rangle$)

determines subsequent behavior at time t > 0 ($|\Psi(t)\rangle$)

But:

since TISE is first order in time, value $|\Psi(t)\rangle$ at any time t

determines state of system $|\Psi(t')\rangle$ for all times

$t' \neq t$!

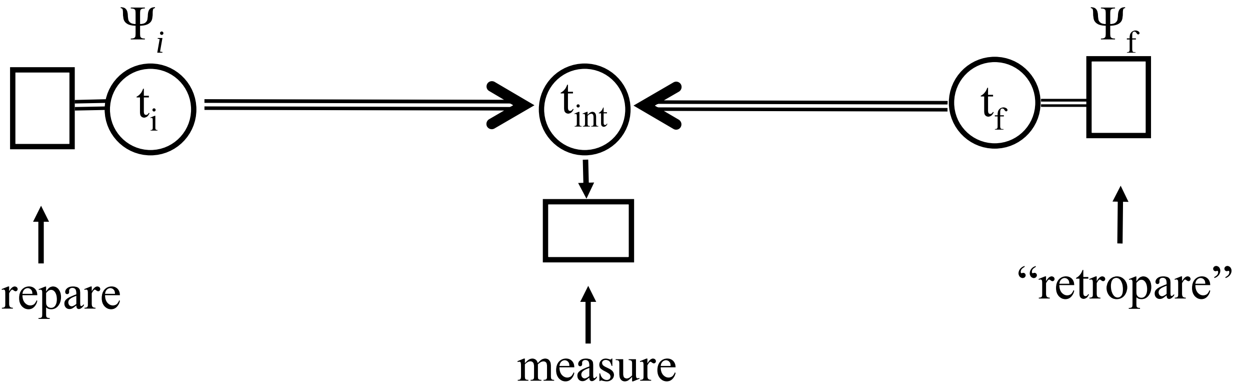
in particular, value $|\Psi(t_f)\rangle$ at final time t_f determines behavior

for all previous times

“Classic” Treatment of Time (A) Symmetry in QM:

Y. Aharonov, P. G. Bergmann, J. Lebowitz, Phys. Rev. **134** B1410 (1964).

see also: F. J. Belinfante, *Measurements and Time Reversal in Objective Quantum Theory*, Pergamon, Oxford 1975.



Given Ψ_i *and* Ψ_f, what is probability of obtaining Ψ_{int}?
If we throw away all knowledge of Ψ_f (“postgarble”) formula reduces to standard (“predictive”) QM.

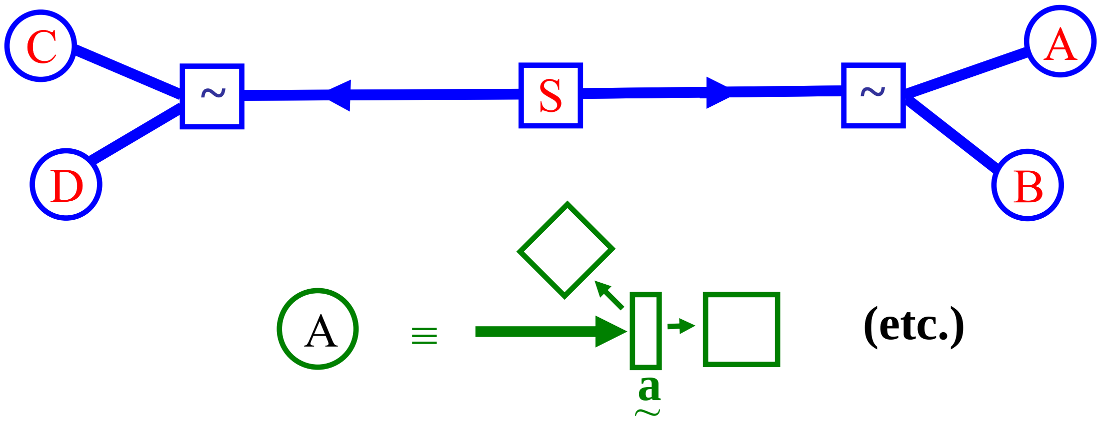
If we throw away all knowledge of Ψ_i (“pregarble”) get “retrodictive” QM ($\Psi(t)$ det. by Ψ_f)

WHY SHOULD WE POSTGARBLE?

Notions of “preparation,” “measurement” (and absence of “retroparation”) implicitly involve 2nd law of thermodynamics?

When Might “Retroparation” Play a Role?

Ex: “EPR-Bell” experiments



Measure: correlations ⟨AC⟩, ⟨AD⟩, ⟨BC⟩, ⟨BD⟩

Experimental result: correlations consistent with predictions of QM, but inconsistent with any theory (“objective local theory”) embodying conjunction of

(1) $\underbrace{\text{Macroscopic counterfactual definiteness}}_{\text{“MCD”}}$ (or microscopic realism)

(2) Local causality

(3) Induction (i.e. state of photon ensemble det. by conditions at S only, not by subsequent switching or measurement events) (no “retroparation”)

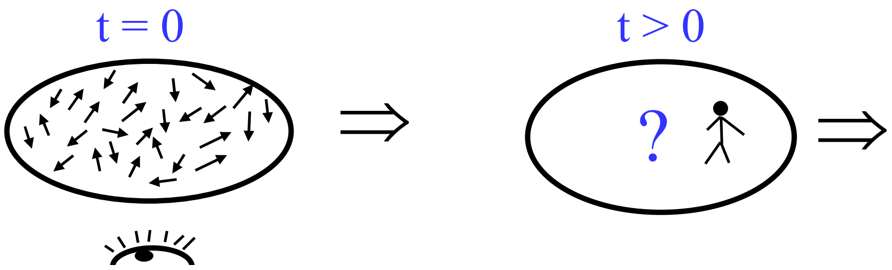
SHOULD WE CHALLENGE INDUCTION?

(Costa de Beauregard, Cramer, ’t Hooft, Price . . .)

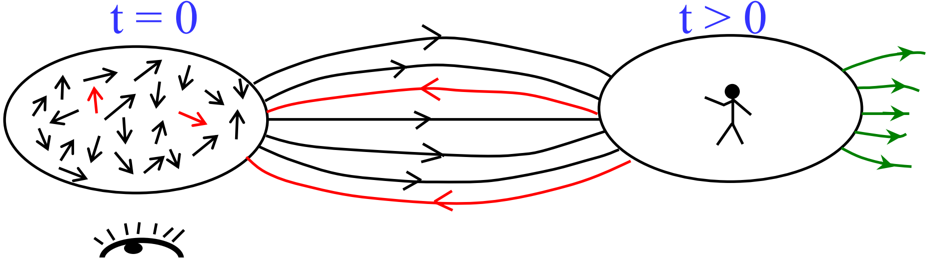
IS (A LIMITED AMOUNT OF) "RETROPARATION" INCONSISTENT WITH THE 2ND LAW OF THERMODYNAMICS?

(can the present partially "determine" the past?)

Classic thought-experiment (classical physics): Laplace's demon



but suppose:



OK provided demon can only obtain thermodynamic, not microscopic, information! Also in QM.

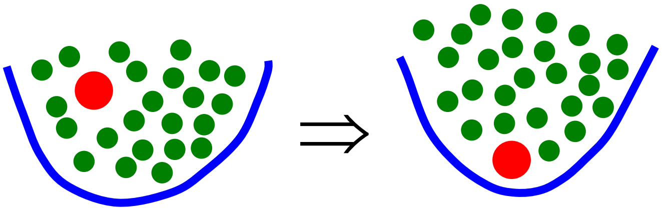
⚠: in e.g. Bell-EPR situation, normally assume we control (prepare) exact initial quantum state of photon ensemble:

For an isolated system (perfect) preparation and (partial) "retroparation" seemingly irreconcilable. But ….

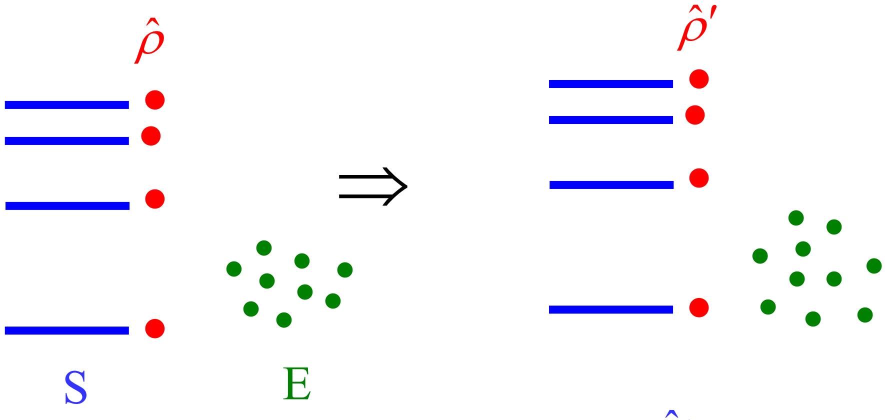
Nothing forbids “violation” of 2nd law for a subsystem (system in context with “environment”)!

Example:

(a) classical:



(b) quantum:



$(\hat{T}_E \ll \Delta E / k_B)$

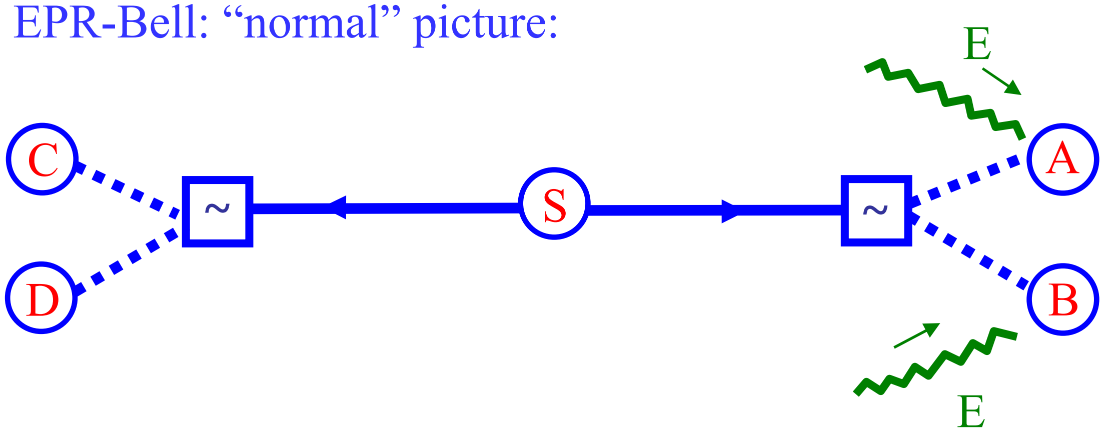
$(\hat{T}'_E > T_E$

but still $\ll \Delta E / k_B)$

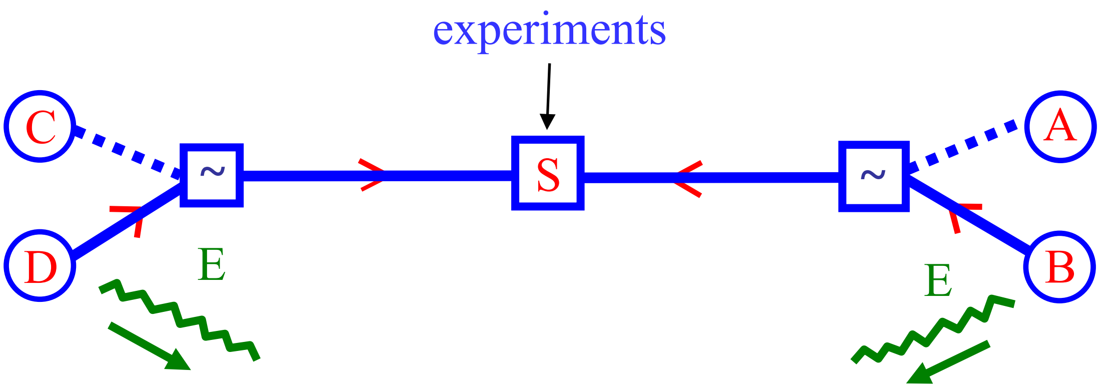
$$S' \equiv Tr\ \hat{\rho}' \ln \hat{\rho}' < S = Tr\ \hat{\rho} \ln \hat{\rho}$$

(but $S'_E > S_E$)

Could this sort of process work “in reverse”?
e.g. EPR-Bell: “normal” picture:



EPR-Bell experiments: a possible reinterpretation?



reversal **partial** "causality" ("retroparation") may be consistent with preparation by experiments provided only **thermodynamic** state of environment is open to inspection.

⚠: needs working out at microscopic level

⚠: not "worth it" to avoid MCD?

Irrespective of that:

Conclusion: problem of "arrow of time" in QM not obviously conceptually different from that in classical physics.