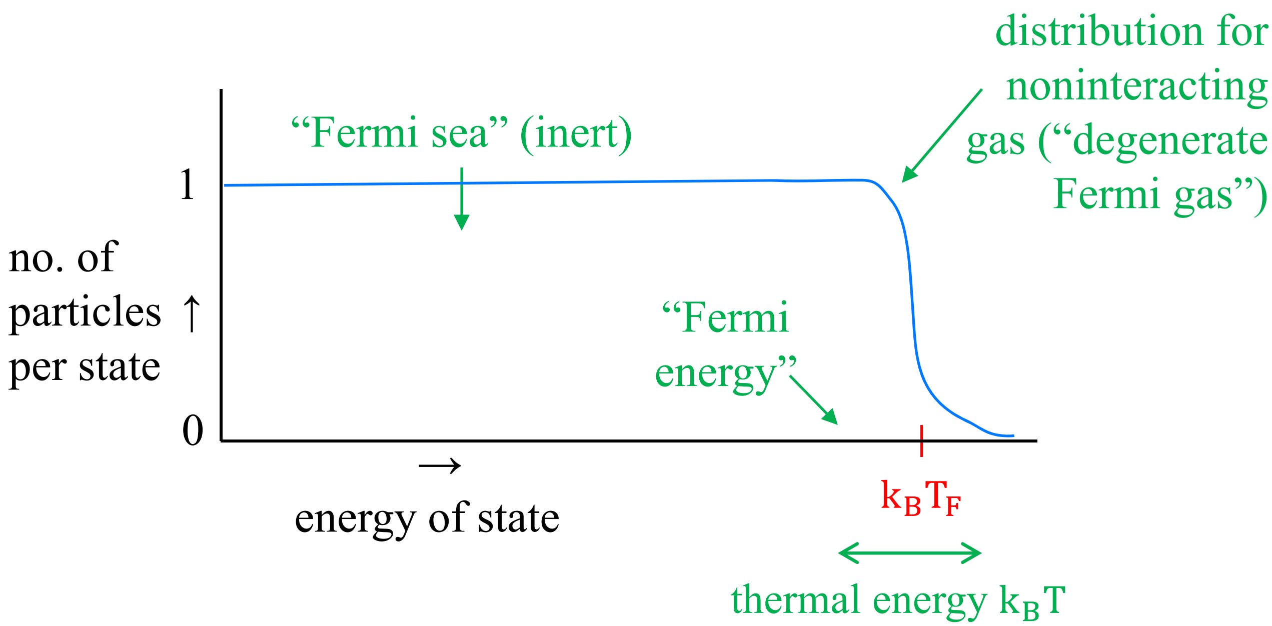
SUPERFLUID ^{3}HE: SOME PRE-HISTORY

^{4}He: below 2K, superfluid

ELECTRONS IN METALS	LIQUID ^{3}HE
since ancient times	since ~ 1950
charged	neutral

Particles of spin ½ ⇒ Fermi statistics



Landau (1957): interactions don't change picture qualitatively (in "normal" phase) ("degenerate Fermi liquid")

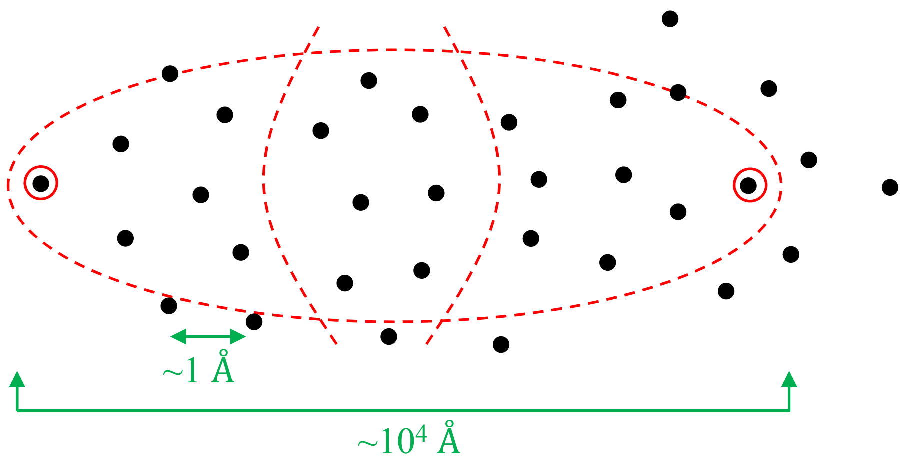
Electrons in metals	Liquid ^{3}He
$T_F \sim 10^4 - 10^5$ K	$T_F \sim 5$ K
at $T \lesssim 20$K.	at $T \lesssim 10^{-3}$K.
superconductivity	superfluidity??
(in some metals)	

<u>MORE PREHISTORY</u>

Theory of superconductivity:

a) phenomenological (V. L. Ginzburg, A. A. Abrikosov, et al., (1950-1955):
macroscopic wave function

b) microscopic (Bardeen et al., 1957):
critical temp., $\lesssim$ 20K
electrons in energy shell of width ~ $k_B T_C$ around Fermi energy form Cooper pairs



Crucial feature of BCS theory: ALL COOPER PAIRS MUST BEHAVE IN EXACTLY THE SAME WAY!

(GL "macroscopic wave function" is just the common center-of-mass wave function of all the pairs)

In BCS theory, "internal" wave function of pairs trivial: ("^1S_o")

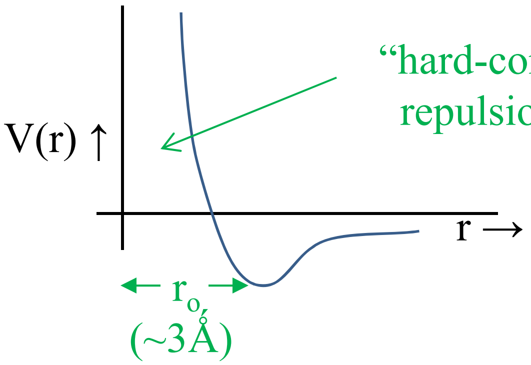
$$\psi(\underset{\sim}{r}_1\underset{\sim}{r}_2 : \sigma_1\sigma_2) \sim \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2)f(|\underline{r}_1 - \underline{r}_2|)$$

spin singlet

spherically symmetric ($\ell = 0$)

NO INTERNAL ("ORIENTATIONAL") DEGREES OF FREEDOM

EARLY THEORETICAL WORK ON POSSIBLE COOPER PAIRING IN LIQUID ^{3}HE



$r \sim r_0$, $p \sim p_F$ ($\equiv \sqrt{2mk_BT_F}$)
$\Rightarrow$ relative angular momentum
$\ell \equiv (p_F r_0/\hbar) \neq 0$
(prob. 1 or 2)

Pauli principle: $\begin{cases} \ell = 0,2,4\ \ldots & S = 0 \text{ (singlet)} \\ \ell = 1,3,5\ \ldots & S = 1 \text{ (triplet)} \end{cases}$

in general, $\ell \neq 0 \Rightarrow$ relative (internal) wave function of pair has orientational degree(s) of freedom!

"equal spin pairing"

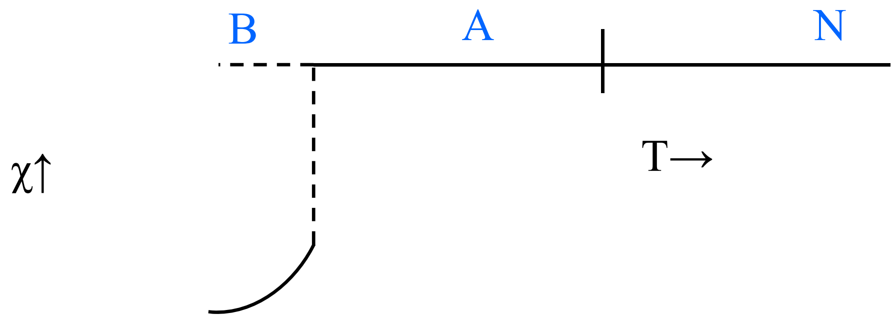
Anderson & Morel (1961): explore in detail case $\ell = 2$, and a special case of $\ell = 1$: only ↑↑ and ↓↓ pairs form, and have the same orbital angular momentum in direction $\hat{\underset{\sim}{\ell}}$ ("ABM" state) Physical properties anisotropic.

Vdovin
Balian & Werthamer (1963): in $\ell = 1$ case all spin components ($\uparrow\uparrow, \downarrow\downarrow, \frac{1}{\sqrt{2}}\uparrow\downarrow + \downarrow\uparrow$) can form: in fact for any given pair, $\underset{\sim}{L} = -\underset{\sim}{S} \Rightarrow J = 0$. ("BW" state). All physical properties isotropic. More stable than any ESP state.

"^3P_o"

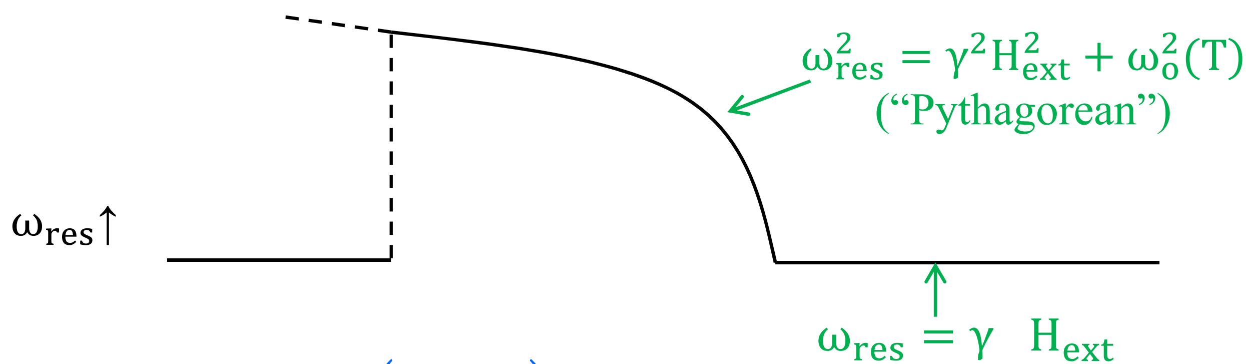
Theoretical expectation c. 1964:
Liquid ^{3}He may form Cooper pairs, either ℓ = even (spin singlet) or with ℓ = odd (BW state). In either case, χ reduced and all magnetic properties isotropic. T_c difficult to predict.

NMR in the new phases:

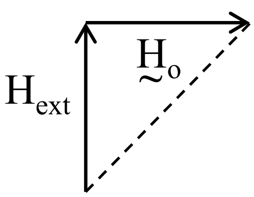


Not necessarily mysterious: e.g. A phase could be an ESP state (only ↑↑,↓↓ pairs ⇒ no reduction in χ), B could be singlet or BW (some ↑↓ pairs , so χ reduced) [but: why is ESP ever stable?]-

But: what about the resonance frequency?



$$\omega_0^2(T) \approx A\left(1 - \frac{T}{T_A}\right), \qquad \frac{A}{(2\pi)^2} \cong 5 \times 10^{10}\,\mathrm{Hz}^2$$



$(\equiv \omega_o(T)/\gamma)$

Need $H_o \sim 30$G. But, only spin-nonconserving force in problem is nuclear dipole-dipole interaction, and max. associated field is < 1G!

IS THIS THE FIRST INDICATION OF A RADICAL BREAKDOWN OF QUANTUM MECHANICS?

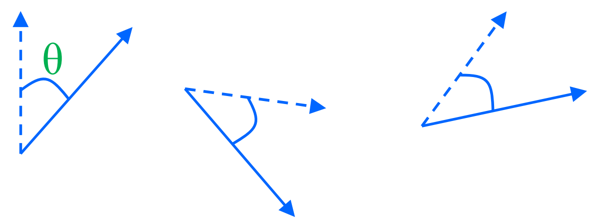
WHAT CAN BE INFERRED FROM SUM RULES?

IF a single sharp resonance is observed (as in expt.) then:

$$\omega_{res}^2 = \gamma^2 H_{ext}^2 + \omega_0^2$$

nuclear dipole energy

$$\omega_0^2 = \frac{\gamma^2 \chi^{-1} \partial^2 \langle H_D \rangle}{\partial \theta^2}$$



angle of simultaneous rot.ⁿ of <u>all</u> spins

But $\dfrac{\partial^2 \langle H_D \rangle}{\partial \theta^2} \sim \langle H_D \rangle$:

So, exptl. value of $\omega_0^2(T) \Rightarrow$

$$\langle H_D \rangle (T) \sim K \left(1 - \frac{T}{T_A}\right), K \sim 10^{-3} \frac{\text{ergs}}{\text{cm}^3}$$

HOW CAN THIS BE?

$$\begin{cases} \uparrow \ (\text{“bad”}) & \uparrow \\ \rightarrow \ (\text{“good”}) & \rightarrow \end{cases}$$

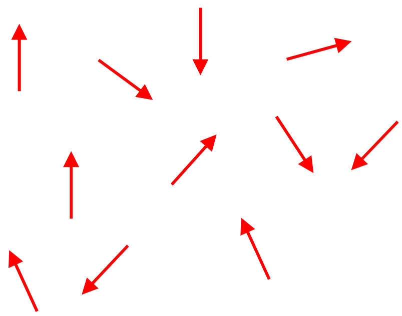
$$\Delta E \lesssim \frac{\mu_o \mu_n^2}{r_o^3} \sim 10^{-7}\,\text{K} \ll k_B T$$

So, prima facie, preference for “good” orientation over “bad” is <u>at most</u> $\sim \dfrac{\Delta E}{k_B T} \sim 10^{-4}$ [actually, $\sim \dfrac{\Delta E}{k_B T_F} \sim 10^{-7}$]

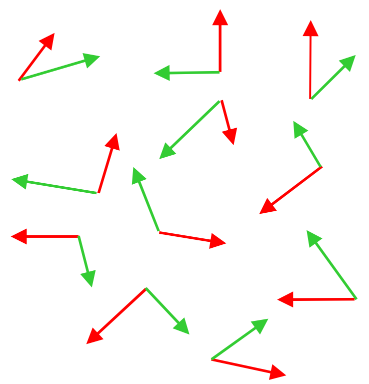
$\Rightarrow$ expectation value of dipole energy much too small!

SBSOS: ORDERING MAY BE SUBTLE

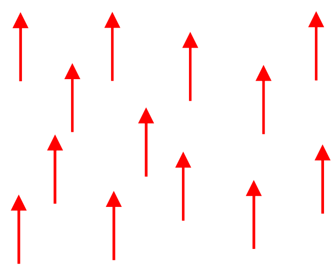
FERROMAGNET | **LIQUID ^{3}HE**



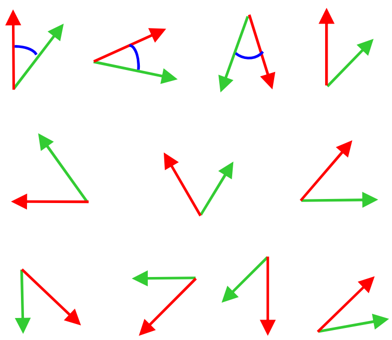
⇐ NORMAL PHASE ⇒

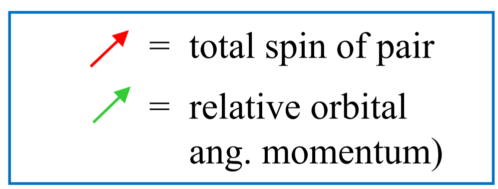


⇓ ⇓



⇐ ORDERED PHASE ⇒





$\langle \underset{\sim}{S} \rangle \neq 0$

$\langle \underset{\sim}{S} \rangle = \langle \underset{\sim}{L} \rangle = 0$

but $\langle \underset{\sim}{L} \times \underset{\sim}{S} \rangle \neq 0$!

"Absolute" direction chosen by (ultraweak) magnetic field

Relative direction chosen by (ultraweak) nuclear dipole force

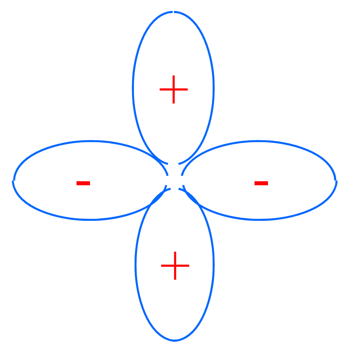
MORAL OF ^{3}HE NMR THEORY

"Behavioral conformism" of Cooper pairs applies not just to COM behavior but to relative motion of the components of a pair

⇒ in systems with "exotic" (non-s-wave) OP's, spectacular amplification of the effects of ultra-weak forces (dipole, ? magnetic, ? PNC)

Other systems with "exotic" Cooper pairing:

U Pt_3 (etc.)	odd-parity
Sr_2RuO_4	p + ip (?)
ferropnictides	$s_\pm$ (?)
cuprates	$d_{x^2-y^2}$



most "spectacular" amplification effects in cuprates:

Josephson effect