ULTRACOLD FERMI ALKALI GASES: BOSE CONDENSATION MEETS COOPER PAIRING

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SOME HISTORY

	BOSE-EINSTEIN CONDENSATION ("BEC")	COOPER PAIRING ("BCS")
Originators	Einstein 1925 London 1938	Bardeen et al. 1957
what?	(spinless) bosons	degenerate fermions
applied to	Liquid ^{4}He Dilute alkali gases	Superconductors Liquid ^{3}He Neutron stars
interactions must be…	nonexistent or repulsive	attractive
"fraction" of condensed particles	~1	$\sim T_c/T_F \ll 1$
main excitations	phonons, $E(k) = \hbar ck$ (bosons)	quasiparticles, $E(k) = \sqrt{(\varepsilon_k - \mu)^2 + \lvert\Delta\rvert^2}$ (fermions)
transition temperature T_c	$\sim T_{deg}$ ~"T_F"	$\sim T_{deg} \exp - 1/N_0V_0$ $\sim T_F$
consequences	superfluidity	superfluidity (or superconductivity)

A UNIFYING CONCEPT: ODLRO

(Penrose-Onsager, Yang)

Consider a general system of N indistinguishable particles (bosons or fermions) occupying N-particle states $\Psi_n(\mathbf{r}_1\sigma_1, \mathbf{r}_2\sigma_2 \ldots \mathbf{r}_N\sigma_N)$ with probability p_n.

spin may be absent (0)

Define:

(a) Single-particle reduced density matrix (RDM)

$$\rho_1(\mathbf{r}_1\sigma_1, \mathbf{r}_1'\sigma_1') \equiv \sum_{\sigma_2 \ldots \sigma_N} \int d\mathbf{r}_2 \ldots d\mathbf{r}_N \bullet$$

$$\sum_n p_n \Psi_n(\mathbf{r}_1\sigma_1, \mathbf{r}_2\sigma_2 \ldots \mathbf{r}_N\sigma_N)\, \Psi_n^*(\mathbf{r}_1'\sigma_1', \mathbf{r}_2\sigma_2 \ldots \mathbf{r}_N\sigma_N)$$

Can diagonalize:

$$\rho_1(\mathbf{r}_1\sigma, \mathbf{r}_1\sigma') = \sum_i n_i \chi_i(\mathbf{r}_1\sigma_1) \chi_i^*(\mathbf{r}_1'\sigma_1')$$

For bosons, can have $n_0 \sim N \equiv N_0$ (condensate)

(b) 2-particle RDM:

$$\rho_2(r_1\sigma_1, r_2\sigma_2 : r_1'\sigma_1', r_2'\sigma_2') \equiv \sum_{\sigma_3 \ldots \sigma_N} \int dr_3 \ldots dr_N \cdot$$

$$\sum_n p_n \Psi_n(r_1\sigma_1, r_2\sigma_2, r_3\sigma_3 \ldots r_N\sigma_N) \Psi_n^*(r_1'\sigma_1', r_2'\sigma_2', r_3\sigma_3 \ldots r_N\sigma_N)$$

$$= \sum_i n_i \chi_i(\mathbf{r}_1\sigma_1, \mathbf{r}_2\sigma_2) \chi_i^*(\mathbf{r}_1'\sigma_1', \mathbf{r}_2'\sigma_2')$$

For bosons or fermions, can have $n_0 \sim N \equiv N_0$

Eagles (1969):

Consider fermions of spin ± ½ ($N_\uparrow = N_\downarrow \equiv \frac{1}{2} N$), with attractive interaction.

If interaction strong, form diatomic molecules (spin 0 ⇒ bosons!) ⇒ undergo BEC.

If interaction weak, form Cooper pairs.

⇒ Cooper pairing, and BEC of diatomic molecules, are opposite ends of single spectrum!

Formally:

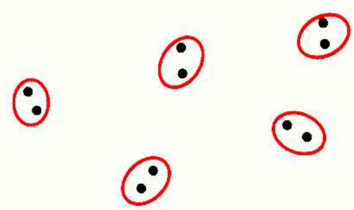
same function for each pair

"naïve" ansatz → $\Psi_N = \mathcal{N}\mathcal{A}\, \varphi(r_1 - r_2 : \sigma_1 \sigma_2)\, \varphi(r_3 - r_4 : \sigma_3 \sigma_4) \ldots\ldots \varphi(r_{N-1} - r_N : \sigma_{N-1} \sigma_N)$

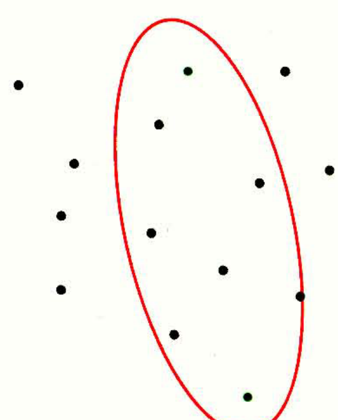
norm^n

antisymmetrizer

Strong attraction: range of φ « interparticle spacing ⇒ n_k «1, $\forall\, \underset{\sim}{k}$ ⇒ Pauli principle unimportant ⇒ BEC of diat. mols.



Weak attraction: range of φ » interparticle spacing ⇒ Pauli principle dominant ⇒ BCS theory (collective bound state)



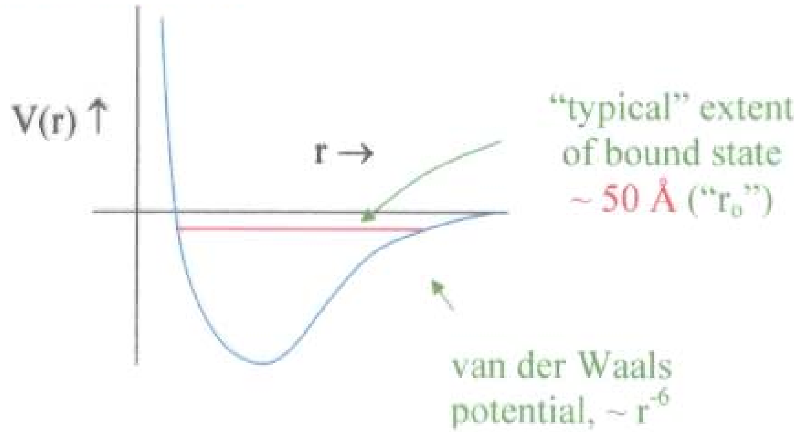
Can we study "crossover"?
(HTS ?)

DILUTE ALKALI FERMI GASES (^{6}Li, ^{40}K ...)

(very cold!)

Z = odd

2 atoms in different internal (hyperfine) states ⇒ possibility of relative s-wave



Typical densities ~ 10^{12} cm^{-3} ⇒ $n^{-1/3} \sim 10^4$ Å ("k_F^{-1}")

⇒ $k_F r_o \ll 1$ (always)

However:

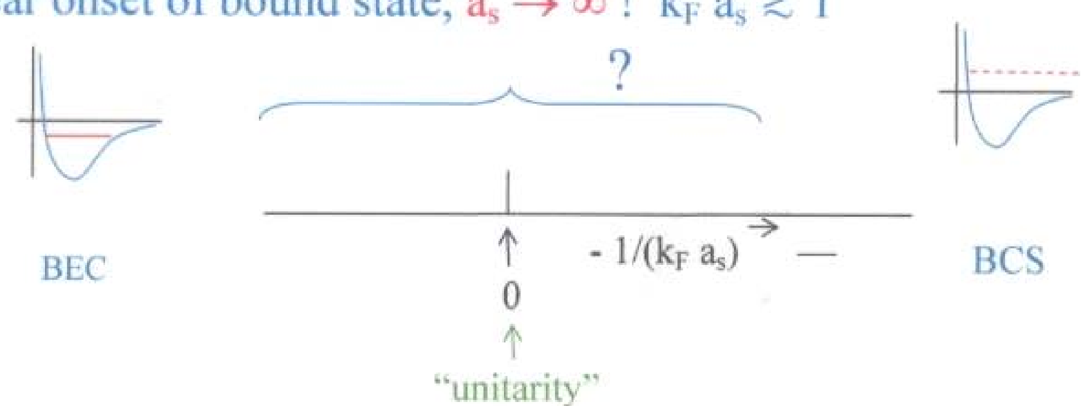
s-wave scattering:

s-wave sc. length

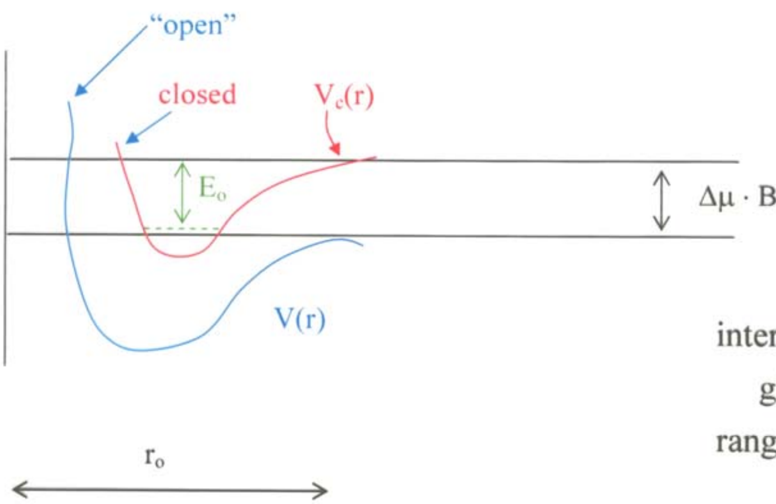
$$\psi(r) = 1 - \frac{a_s}{r}$$

relative w.f.

Near onset of bound state, $a_s \to \infty$! $k_F a_s \gtrsim 1$



FESHBACH RESONANCE

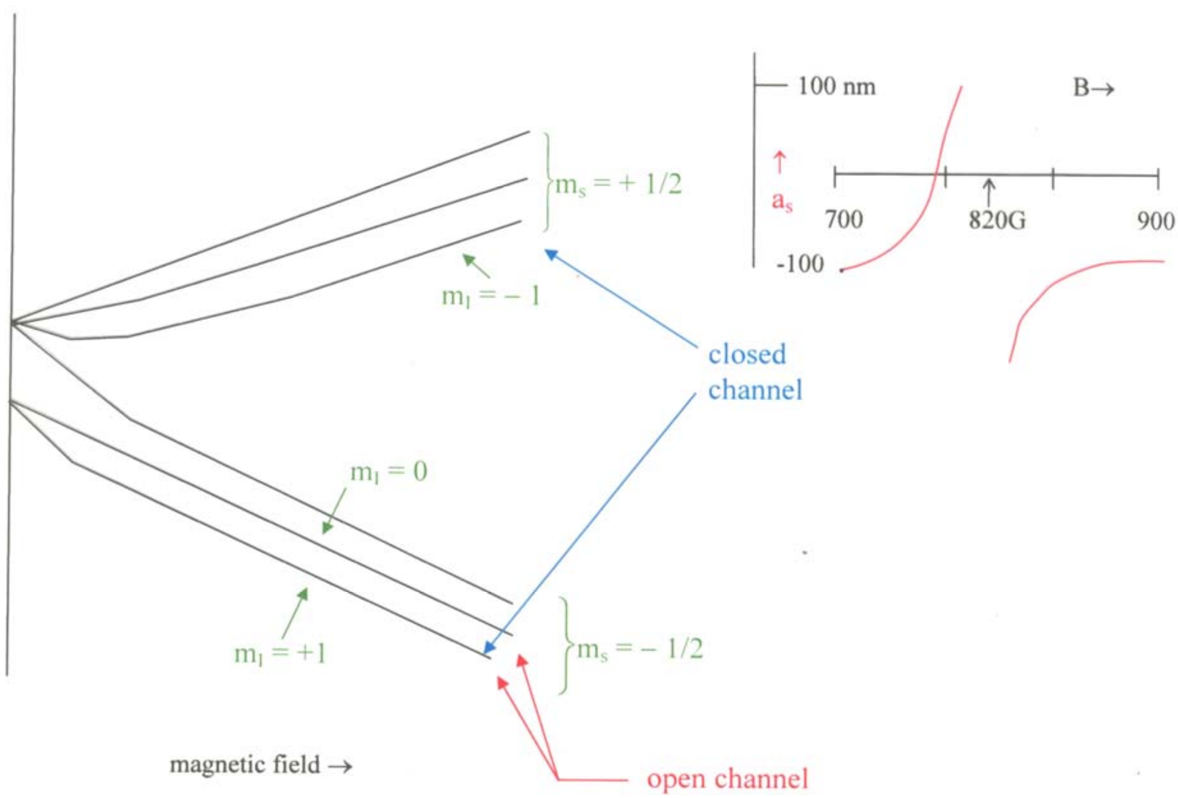


interchannel coupling

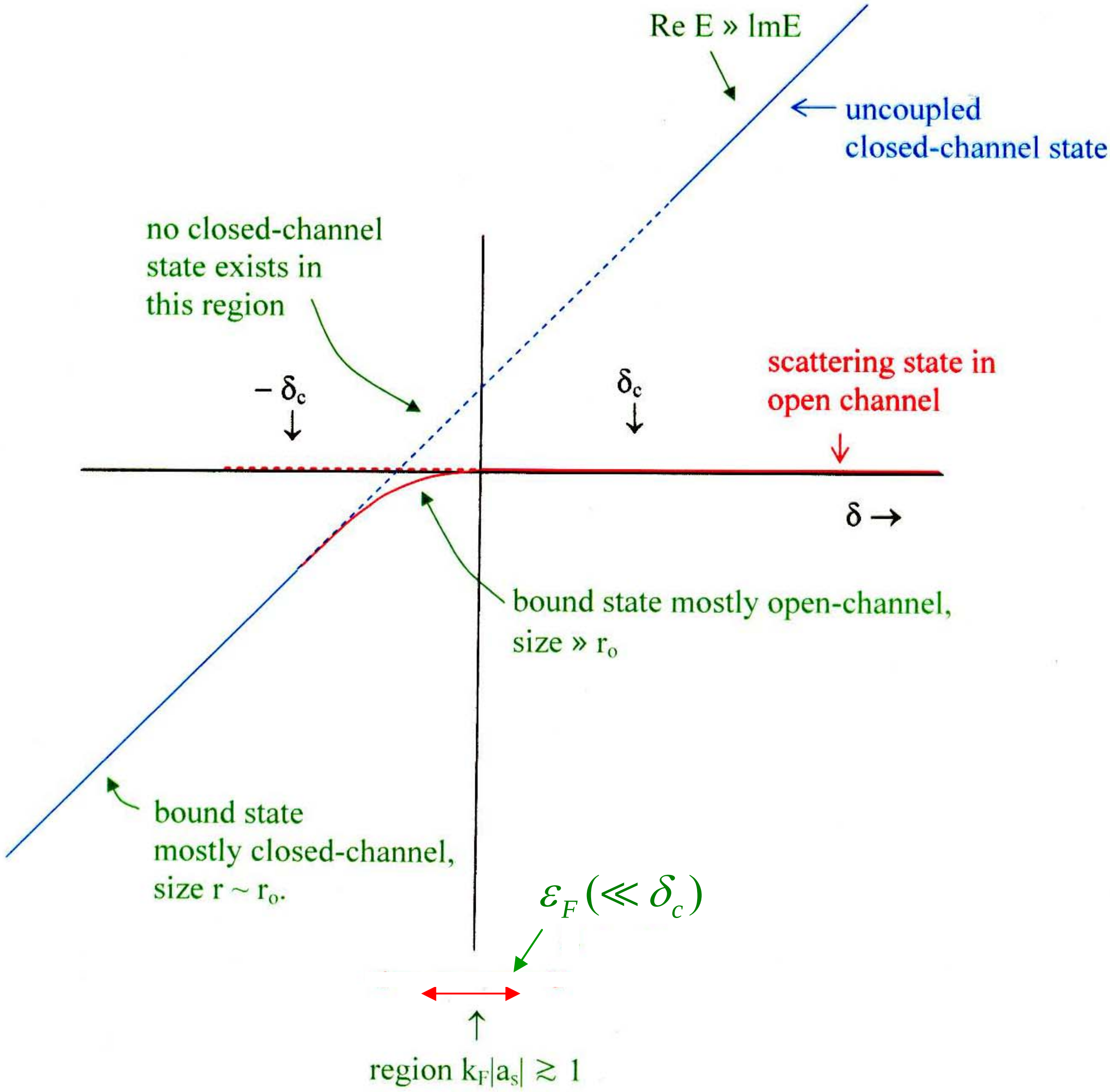
$gf(r)$, $f(r) \lesssim 1$.

range of $f(r) \sim \ell_0 \lesssim r_o$

^{6}Li :



QUALITATIVE PICTURE OF FESHBACH RESONANCE (2-body problem):



In "interesting" region for many-body effects, "molecules" almost entirely in open channel ⇒ expect behavior identical to single-channel case

The problem: N fermions, equal nos. ↑ and ↓,

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 \qquad N_{tot} = (k_F^3/3\pi^2)$$

subject to b.c.

Ψ_N~ const. ($1-a_s/r_{ij}$) for antiparallel-spin particles *i, j*

(in dilute limit, parallel-spin particles noninteracting)

All (equilibrium) props. must be functions only of $\zeta = -1/k_F a_S$

"Naïve" Ansatz (Eagles 1969, AJL 1980, Randeria et al. 1985, Stajic et al. 2005 . . .):

$$\Psi_N = \mathcal{N} \cdot \mathcal{A} \cdot \{\varphi(r_1 - r_{2:} : \sigma_1\sigma_2)\varphi(r_3 - r_{4:} : \sigma_3\sigma_4)\ldots\varphi(r_{N-1} - r_{N:} : \sigma_{N-1}\sigma_N)\}$$

$\langle\Psi_N | \hat{H} | \Psi_N\rangle =:$

1. Pairing terms ← fully taken into account
2. Fock terms ← vanish in dilute limit
3. Hartree terms ← ??

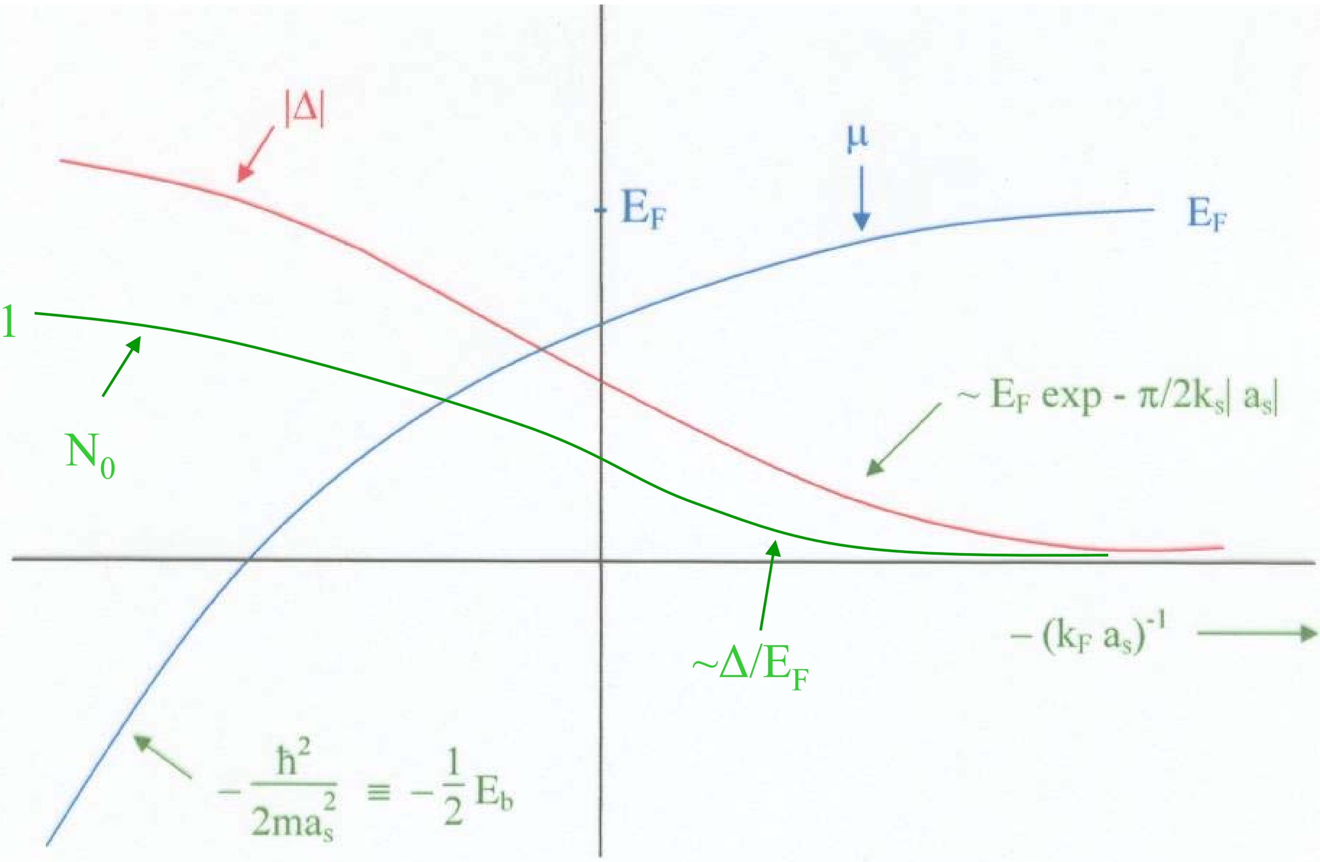
equivalently: each term of $\Psi_N^{(\text{naïve})}$ satisfies b.c. for paired particles only, e.g. 1st term satisfies it for 1, 2 but not (e.g.) for 1, 3.

Output of naïve ansatz:

$\mu(\zeta), \Delta(\zeta)$

Hence also $(E/N)(\zeta)$.

(calc^n analytic except for 2 |D numerical integrals)



Excitation energy of quasiparticle with momentum $\underline{k}$ (normal-state energy $\xi_k \equiv \hbar^2 k^2/2m$):

$$E_k = \sqrt{(\xi_k - \mu)^2 + |\Delta|^2}$$

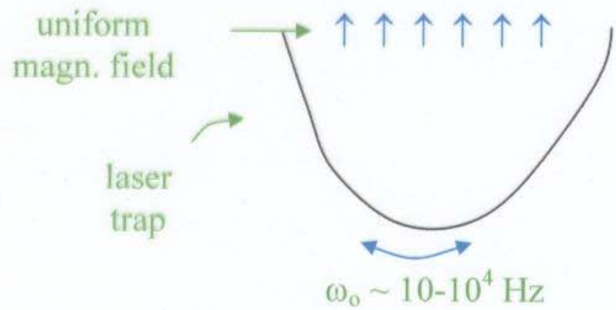
$\mu > 0$: $\min E_k = |\Delta|$

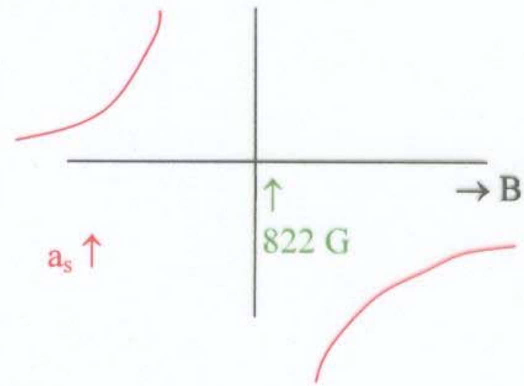
$\mu < 0$: $\min E_k = \sqrt{|\mu|^2 + |\Delta|^2}$

EXPERIMENTS ON BEC-BCS CROSSOVER

JILA: ^{40}K, $m_F = -\ 9/2$ and $-\ 7/2$ F.R. at 202G

others: ^{6}Li, $m_F = ½$ and $-\ ½$ F.R. at 822G





↑ : need "balanced" populations! $(N_\uparrow \cong N_\downarrow)$

Preparation technique: ($|m_I = 1\rangle$ = "$|\downarrow\rangle$", $|m_I = 0\rangle$ = "$|\uparrow\rangle$":

1. start with all $|\downarrow\rangle$
2. $\pi/2$ rf pulse at 76 MHz $\Rightarrow$ all in $|\rightarrow\rangle \equiv 2^{-1/2}(|\uparrow\rangle + |\downarrow\rangle)$
3. inhomogeneous precession $\Rightarrow$ incoherent mixture of $|\uparrow\rangle$ and $|\downarrow\rangle$ with equal weight (+ heating!)
4. evaporation (to cancel heating)

Some Experiments on the BEC-BCS Crossover

(mostly ^{6}Li : some ^{40}K) (s-wave, unpolarized)

EXPERIMENT	SHOWS/MEASURES
in-situ imaging imaging after expansion	(fermionic statistics)
Lifetimes of atoms + molecules	(fermionic statistics)
Collective excitations in trap Sound velocity	crossover thermodynamics
Specific heat	"3-fluid" model
NMR (ESR)	energy gap (on both sides of unitarity)
Field sweep	pairing on BCS side
Persistence of vorticity under BEC $\rightarrow$ BCS $\rightarrow$ BEC sweep	pairing on BCS side
Optical absorption	Nonzero closed-channel compt. (on both sides of unitarity)

All these experiments appear qualitatively

consistent with "naïve" ansatz.

A SIMPLIFYING CONSIDERATION IN UNDERSTANDING (SOME OF) THE EXPERIMENTS: DECOUPLING OF "2-PARTICLE" AND "MANY-BODY" EFFECTS*

Consider a general quantity of the form

$$\Omega \equiv \frac{1}{2}\sum_{ij} S(\boldsymbol{r}_i - \boldsymbol{r}_j : \sigma_i \sigma_j)$$

with the range of $S(r) \leq r_0$. (Exx: potential energy, closed-channel fraction, 1st moment of ESR spectrum). Intuitively, $\langle\Omega\rangle$ should depend only on the prob. of finding two atoms within $\lesssim r_0$ of one another. Formally:

$$\rho_2(\boldsymbol{r}_1\sigma_1\boldsymbol{r}_2\sigma_2 : \boldsymbol{r}_1'\sigma_1'\boldsymbol{r}_2'\sigma_2') = \sum_i n_i \chi_i(\boldsymbol{r}_1\sigma_1\boldsymbol{r}_2\sigma_2)\chi_i^*(\boldsymbol{r}_1'\sigma_1'\boldsymbol{r}_2'\sigma_2')$$

then

$$\langle\Omega\rangle = \sum_i n_i \sum_{\sigma_1\sigma_2} \iint d\boldsymbol{r}_1 d\boldsymbol{r}_2\, S(\boldsymbol{r}_1 - \boldsymbol{r}_2 : \sigma_1\sigma_2) \,|\, \chi_i(\boldsymbol{r}_1\boldsymbol{r}_2, \sigma_1\sigma_2)\,|^2$$

However, in the limit $k_F r_0 \ll 1$ the functional form of $\chi_i(\boldsymbol{r}_1\boldsymbol{r}_2\sigma_1\sigma_2)$ at distances $|\boldsymbol{r}_1 - \boldsymbol{r}_2| \lesssim r_0$ is simply that of the 2-particle (free-space) wave function, and the only dependence on *i* is through the normalization. So, writing

$$\chi_i(r_1 r_2 \sigma_1 \sigma_{2|\boldsymbol{r}_1 - \boldsymbol{r}_2| \lesssim r_0}) \equiv C_i \cdot \chi_{fs}(\boldsymbol{r}_1 - \boldsymbol{r}_2 : \sigma_1\sigma_2)$$

←appropriately normalized 2-p w.f.

we can write

$$\langle\Omega\rangle = h(\xi, \tau)\cdot\varphi_\Omega$$

$$\varphi_\Omega \equiv \sum_{\sigma_1\sigma_2} \int dr\, S(r : \sigma_1\sigma_2)\,|\,\chi_{fs}(r : \sigma_1\sigma_2)\,|^2$$ ← 2-body quantity

$$h(\zeta, \tau) \equiv \sum_i n_i(\zeta, \tau)\,|\,C_i(\zeta, \tau)\,|^2$$ ← incorporates ALL "many-body" effects

*S. Zhang and A. J. Leggett, Phys. Rev. A **79**, 023601 (2009): cf. S. Tan, Ann. Phys. (NY) *323*, 2952 (2008)

Some obvious questions:

1a. Statics (T=0): how good is "naïve'' ansatz?

In particular, at unitarity have "simple" problem: (Bertsch)

Min. e. v. of

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2$$

subject to b.c.

$$\Psi\left(\underset{\sim}{r}_1\sigma_1\underset{\sim}{r}_2\sigma_2\ldots\underset{\sim}{r}_N\sigma_N\right) \sim r_{ij}^{-1}$$

whenever $r_{ij} \to 0$ for $\sigma_i \neq \sigma_j$.

On dimensional grounds,

$$E/N = AE_{FG} \leftarrow = \frac{3}{5}\varepsilon_F$$

$$\Delta = BE_{FG}$$

Chang + Pandharipande: Jastrow-BCS ansantz,

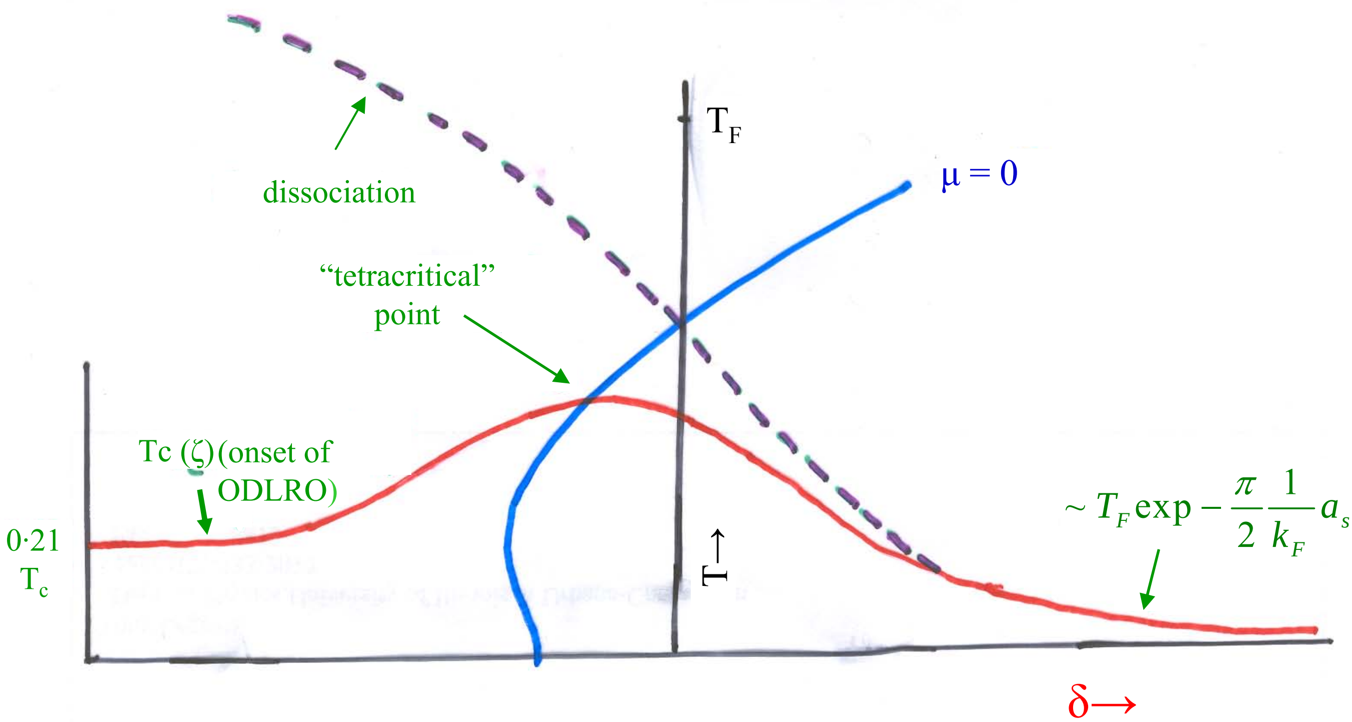
$$\Psi = \prod_{ij} f(r_{ij})\Psi_{BCS}\{r_i,\sigma_i\}$$

accomodates "Hartree" affect

	CP	Naive	Expt
A	0•44	0•59	$0\cdot 32 \pm 0\cdot 12$ $0\cdot 36 \pm 0\cdot 15$ $0\cdot 51 \pm 0\cdot 04$ $0\cdot 46 \pm 0\cdot 05$
B	0•99	1•13	—

1b. More questions on statics:

Behavior of Crossover in (ζ, T) Plane



$64K question: how to go beyond the naïve ansatz?

(and why does it seem to be qualitatively correct?)

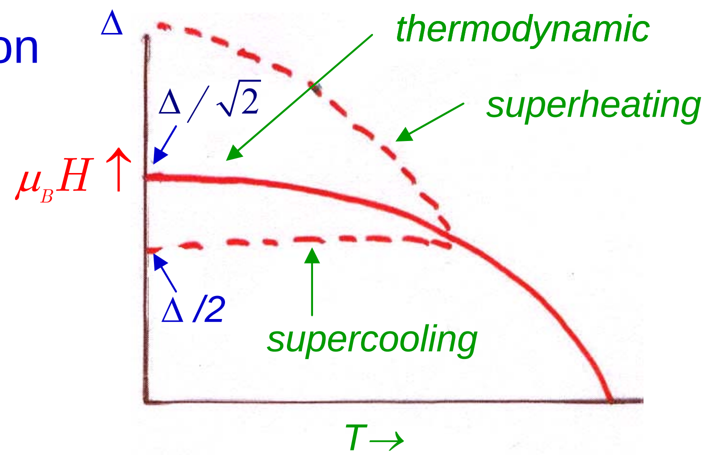
rigorous upper limit on T_c? On $N_o(T)/\rho_s(T)$?

Other questions: Dynamics, kinetics . . .

SOME GENERALIZATIONS

A. S-wave pairing, unequal spin populations

Effect of magnetic field on pairing in "neutral" superconductor (Clogston, Chandrasekhar, Maki and Tsuzuki. . .)



Effect observed, in real superconductor, by Meissner effect (and small polarizability)

Experiments on ^{6}Li with unequal spin populations (separate detection of 2 species)

— phase separation into "pure" paired regions and normal (nonzero-spin) regions

— profiles sometimes nonmonotonic

— critical polarization for pairing at unitarity

≈70%

Fully polarized system described by noninteracting Fermi sea (for $k_F r_0 \ll 1$). What is MBWF for a single reversed spin?

GENERALIZATIONS (cont.)

B. The $\ell \neq 0$ case

1. Qualitative difference from s-wave case: (2-body prob). In s-wave case, general E=0 solution outside potential is

$$\Psi(\boldsymbol{r}) = 1 - a_s / r$$

and in particular, at unitarity, $\Psi(\boldsymbol{r}) \sim r^{-1}$ $\Rightarrow$ in many-body cases expect strong 3, 4 . . . -body interaction effects.

In $\ell \neq 0$ case,

$$\Psi(\boldsymbol{r}) \sim + \frac{c_2}{r^{\ell+1}}$$

suggests unitary limit may be (almost) trivial in lim $r_o \ll a\, n^{-1/3}$!

2. The angular momentum problem:

In BEC of tightly bound $\ell \neq 0$ diatonic modules, overwhelmingly plausible that

$$\boldsymbol{L} = \frac{N}{2} \hbar \underset{\sim}{\hat{\ell}}$$

What is situation in BCS limit?
Most "obvious" number-conserving ansatz:

$$\Psi \sim \left(\sum_k c_k a_k^+ a_{-k}^+ \right)^{N/2}, \qquad c_k \equiv \upsilon_k / u_k$$

with (e.g.) $c_k \sim \exp i\varphi_k$. This has $\boldsymbol{L} = \frac{N}{2} \hbar \hat{\ell}$ just as in BEC limit, irrespective of magn. of |Δ|.

Problem: macroscopic discontinuity at transition to normal state $(\boldsymbol{L} = 0)$!

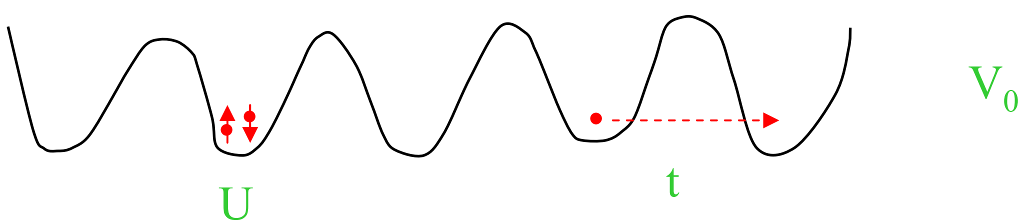
ULTRACOLD FERMI ALKALI GASES: SOME APPLICATIONS

1. Simulation of other systems (nuclear matter, quark-gluon plasma, excitons…) when parameters not adjustable
   : none of these is in "dilute" limit $k_F r_0 \ll 1$.

2. Simulation of specific models:
   case of most interest is 2D Hubbard model (believed by many to describe cuprate superconductors)
   This is a lattice model:

$$\hat{H} = -t \sum_{ij=nn} a_i^+ a_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

   To simulate, need optical lattice:



   U/t tunable via V_0 or via Feshbach resonance.

   : may not be model of real cuprate.

3. Topological quantum computing:
   requires p-wave pairing (Feshbach resonance?)
   According to "standard" picture, a vortex in a (single-spin-component) p-wave Fermi superfluid can accommodate Majorana fermions, which behave as nonabelian anyons and can thus be used for TQC.

   : is "standard" picture correct?

Some Questions about the Established Wisdom

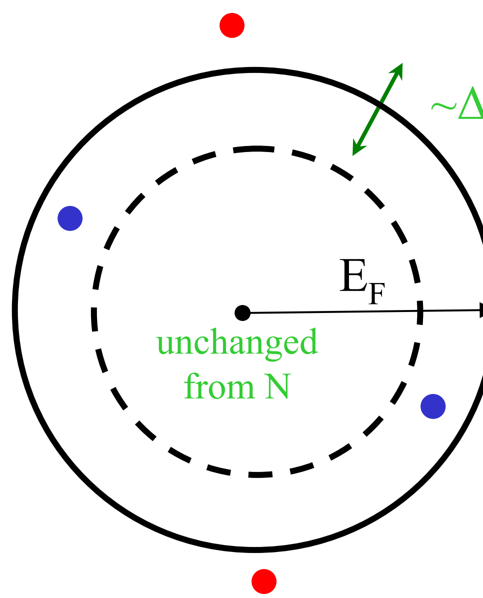
1. Nature of MBWF of (p + ip) Fermi superfluid

Recap: standard ansatz is (for say ↑↑)

$$\Psi \sim \left(\sum_k c_k a_k^+ a_{-k}^+ \right)^{N/2} |\,\text{vac}\rangle, c_k \sim \exp i\varphi_k$$

i.e. all pairs of states in Fermi sea have anyon momentum $\hbar$.

Alternative ansatz:



first shot:

$$\Psi(N_p, N_h)$$

$$\sim \left(\sum_{k>k_F} c_k a_k^+ a_{-k}^+ \right)^{N_p/2},$$

$$\left(\sum_{k<k_F} d_k a_{-k} a_k \right)^{N_h/2} |\,\text{vac}\rangle$$

⚠: keeps pp→pp and hh→hh, but not (e.g.) pp→hh.

Remedy:

$$\Psi \sim \sum_{N_p, N_K} Q_{N_p N_k} \Psi(N_p, N_k),$$

$$Q \text{ slowly varying as } f(N_p, N_k)$$

degenerate with standard ansatz to $0(N^{-1/2})$, but

$$L \sim (N\hbar / 2) \cdot (\Delta / E_F)^2$$

IS GS OF (p + ip) UNIQUE?