SOME HISTORY

	BOSE-EINSTEIN CONDENSATION ("BEC")	COOPER PAIRING ("BCS")
Originators	Einstein 1925 London 1938	Bardeen et al. 1957
what?	(spinless) bosons	degenerate fermions
applied to	Liquid ^4He Dilute alkali gases	Superconductors Liquid ^3He Neutron stars
interactions must be…	nonexistent or repulsive	attractive
"fraction" of condensed particles	~1	$1 \sim T_c/T_F \ll 1$
main excitations	phonons, $E(k) = \hbar ck$ (bosons)	quasiparticles, $E(k) = \sqrt{(\varepsilon_k - \mu)^2 + \lvert\Delta\rvert^2}$ (fermions)
transition temperature T_c	$\sim T_{deg}$ $\sim$"T"$_F$	$\sim T_{deg} \exp - 1/N_0V_0$ $\sim T_F$
consequences	superfluidity	superfluidity (or superconductivity)

A UNIFYING CONCEPT: ODLRO

(Penrose-Onsayer, Yang)

Consider a general system of N indistinguishable particles (bosons or fermions) occupying N-particle states $\Psi_n(\boldsymbol{r}_1\sigma_1, \boldsymbol{r}_2\sigma_2 \ldots \boldsymbol{r}_N\sigma_N)$ with probability ρ_n.

spin may be absent (0)

Define:

(a) Single-particle reduced density matrix (RDM)

$$\rho_1(\boldsymbol{r}\,\sigma, \boldsymbol{r}'\sigma') \equiv \sum_{\sigma_2 \ldots \sigma_N} \int d\boldsymbol{r}_2 \ldots d\boldsymbol{r}_N \bullet$$

$$\sum_n \rho_n \Psi_n(\boldsymbol{r}_1\sigma_1, \boldsymbol{r}_2\sigma_2 \ldots \boldsymbol{r}_N\sigma_N)\, \Psi_n^*(\boldsymbol{r}_1'\sigma_1', \boldsymbol{r}_2\sigma_2 \ldots \boldsymbol{r}_N\sigma_N)$$

$$\sim \sigma \equiv \boldsymbol{r}_1\sigma_1,\ \boldsymbol{r}'\sigma' \equiv \boldsymbol{r}_1'\sigma_1'$$

$$\equiv \overline{\psi(r\sigma)\psi^*(r'\sigma')}$$

Can diagonalize:

$$\rho_1(\boldsymbol{r}\sigma, \boldsymbol{r}'\sigma') = \sum_i n_i \chi_i(\boldsymbol{r}\sigma)\chi_i^*(\boldsymbol{r}'\sigma')$$

For bosons, can have $n_0 \sim N \equiv N_0$ (condensate)

(b) 2-particle RDM:

$$\rho_2(r_1\sigma_1, r_2\sigma_2 : r_1'\sigma_1', r_2'\sigma_2') \equiv \sum_{\sigma_3 \ldots \sigma_N} \int dr_3 \ldots dr_N \cdot$$

$$\sum_n \rho_n \Psi_n(r_1\sigma_1, r_2\sigma_2, r_3\sigma_3 \ldots r_N\sigma_N)\Psi_n^*(r_1'\sigma_1', r_2'\sigma_2', r_3\sigma_3 \ldots r_N\sigma_N)$$

$$= \sum_i n_i \chi_i(\boldsymbol{r}_1\sigma_1, \boldsymbol{r}_2\sigma_2)\chi_i^*(\boldsymbol{r}_1'\sigma_1', \boldsymbol{r}_2'\sigma_2')$$

For bosons or fermions, can have $n_0 \sim N \equiv N_0$

Eagles (1969):

Consider fermions of spin ± ½ ($N_\uparrow = N_\downarrow \equiv$ ½ N), with attractive interaction.

If interaction **strong**, form diatomic molecules (spin 0 ⇒ bosons!) ⇒ undergo BEC

If interaction **weak**, form Cooper pairs.

⇒ Cooper pairing, and BEC of diatomic molecules, are **opposite ends of single spectrum!**

Formally:

same function for each pair

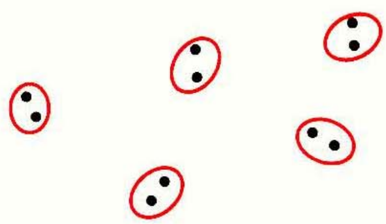
"naïve" ansatz ⟶

$$\Psi_N = \mathcal{N}\mathcal{A}\, \varphi(r_1 - r_2 : \sigma_1 \sigma_2)\, \varphi(r_3 - r_4 : \sigma_3 \sigma_4) \ldots \varphi(r_{N-1} - r_N : \sigma_{N-1} \sigma_N)$$

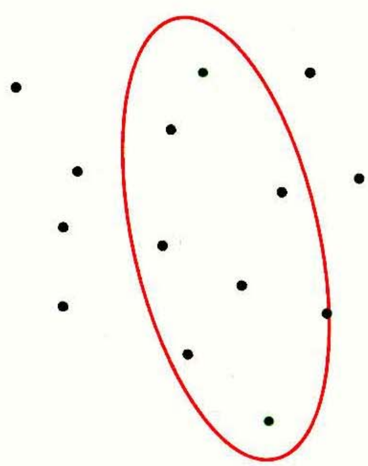
$\mathcal{N}$: norm^n

$\mathcal{A}$: antisymmetrizer

Strong attraction: range of φ « interparticle spacing ⇒ n_k «1, ∀ $\underset{\sim}{k}$ ⇒ Pauli principle unimportant ⇒ BEC of diat. mols.



Weak attraction: range of φ » interparticle spacing ⇒ Pauli principle dominant ⇒ BCS theory (<u>collective</u> bound state)



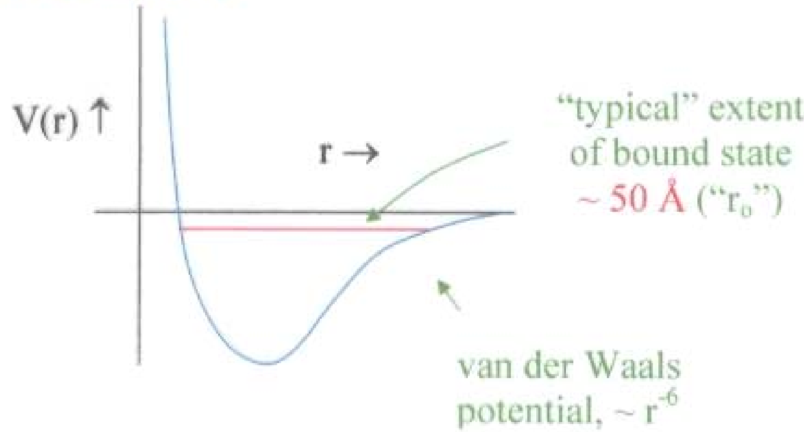
Can we study "crossover"?
(HTS ?)

DILUTE ALKALI FERMI GASES (^{6}Li, ^{40}K …)

(very cold!)

Z = odd

2 atoms in different internal (hyperfine) states ⇒ possibility of relative s-wave



Typical densities ~ 10^{12} cm^{-3} ⇒ $n^{-1/3} \sim 10^4$ Å ("k_F^{-1}")

⇒ $k_F r_o \ll 1$ (always)

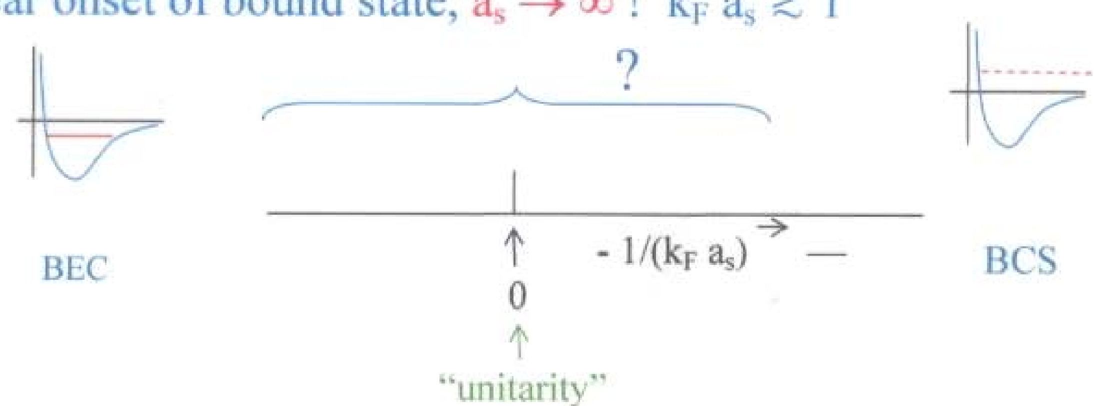
However:

s-wave scattering: s-wave sc. length

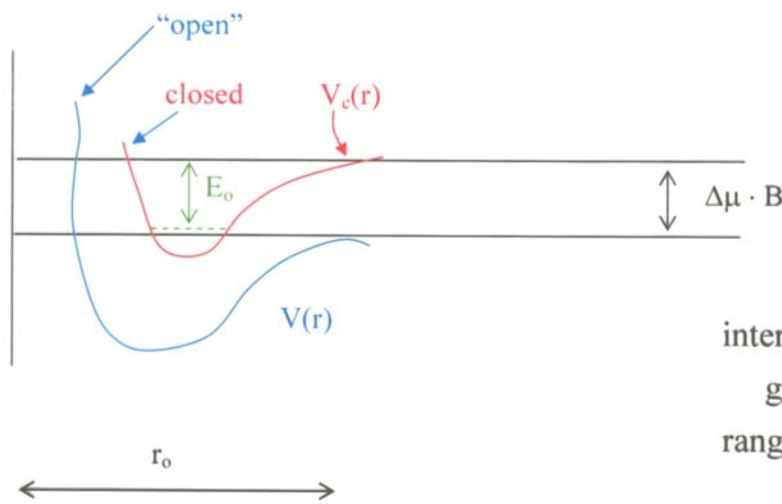
$$\psi(r) = 1 - \frac{a_s}{r}$$

relative w.f.

Near onset of bound state, $a_s \to \infty$! $k_F a_s \gtrsim 1$



FESHBACH RESONANCE

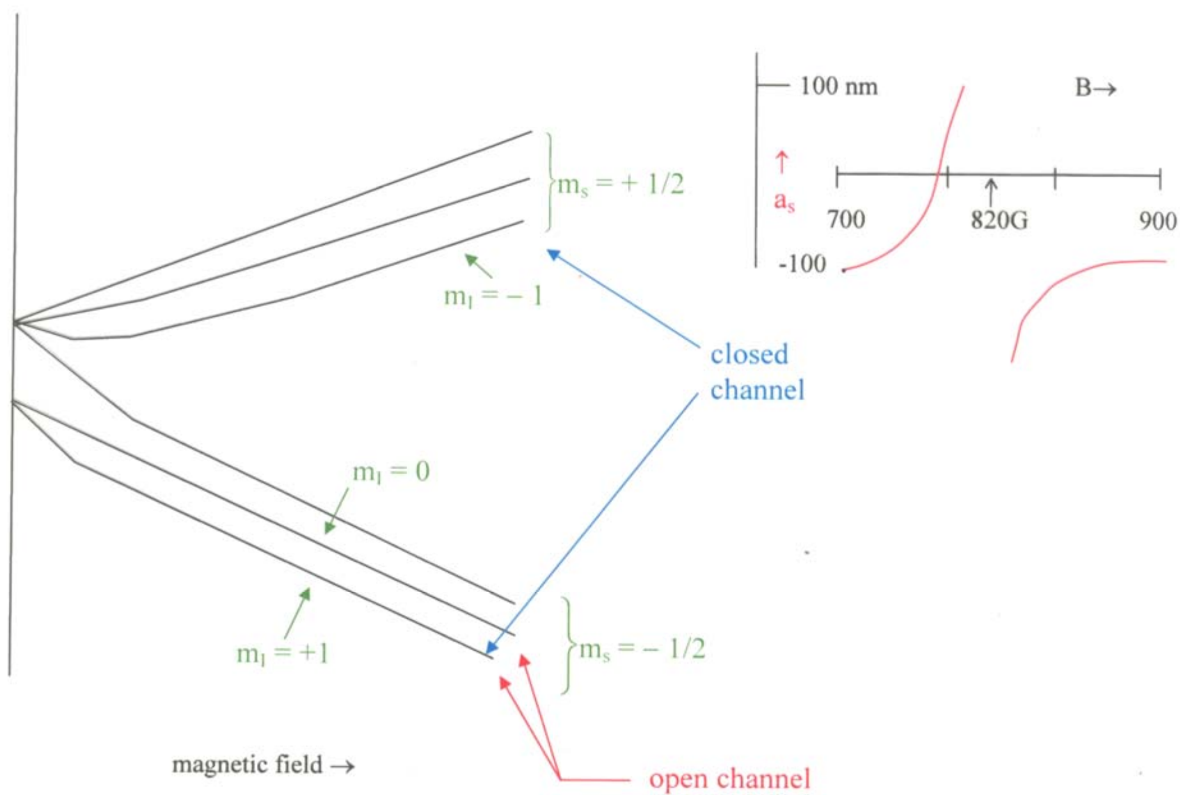


interchannel coupling

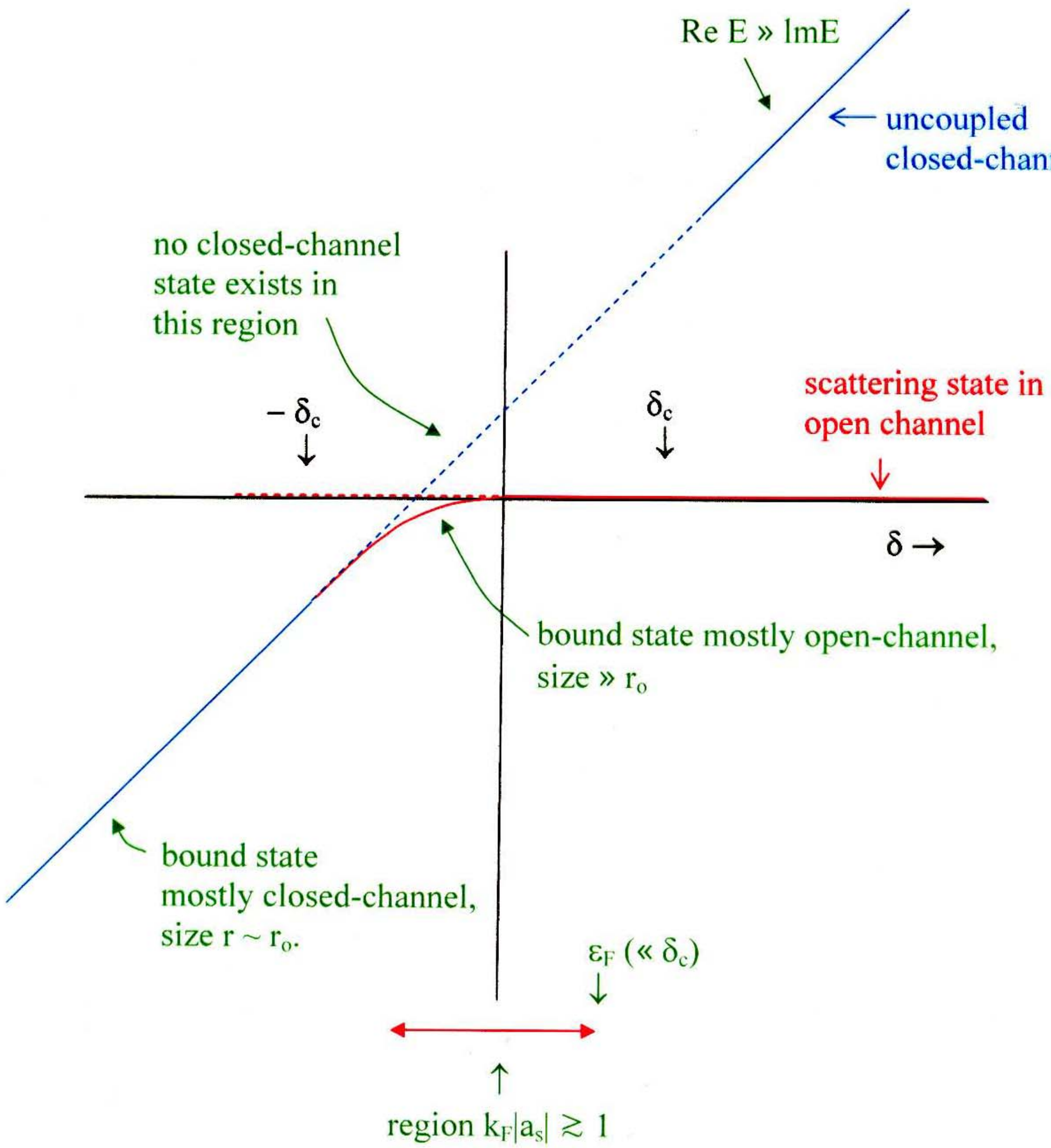
$gf(r)$, $f(r) \lesssim 1$.

range of $f(r) \sim \ell_0 \lesssim r_o$

^{6}Li :



QUALITATIVE PICTURE OF FESHBACH RESONANCE (2-body problem):



In "interesting" region for many-body effects, "molecules" almost entirely in open channel ⇒ expect behavior identical to single-channel case

The problem: N fermions, equal nos. ↑ and ↓,

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 \qquad N_{tot} = (k_F^3/3\pi^2)$$

subject to b.c.

$\Psi_N \sim$ const. $(1 - a_s/r_{ij})$ for antiparallel-spin particles *i, j*

(in dilute limit, parallel-spin particles noninteracting)

All (equilibrium) props. must be functions only of $S = -1/k_F a_S$

"Naïve" Ansatz (Eagles 1969, AJL 1980, Randeria et al. 1985, Stajic et al. 2005 . . .):

$$\Psi_N = \mathcal{N} \cdot \mathcal{A} \cdot \{\varphi(r_1 - r_2 : \sigma_1\sigma_2)\varphi(r_3 - r_4 : \sigma_3\sigma_4)\ldots\varphi(r_{N-1} - r_N : \sigma_{N-1}\sigma_N)\}$$

$\langle \Psi_N | \hat{H} | \Psi_N \rangle =:$

1. Pairing terms ← fully taken into account
2. Fock terms ← vanish in dilute limit
3. Hartree terms ← ??

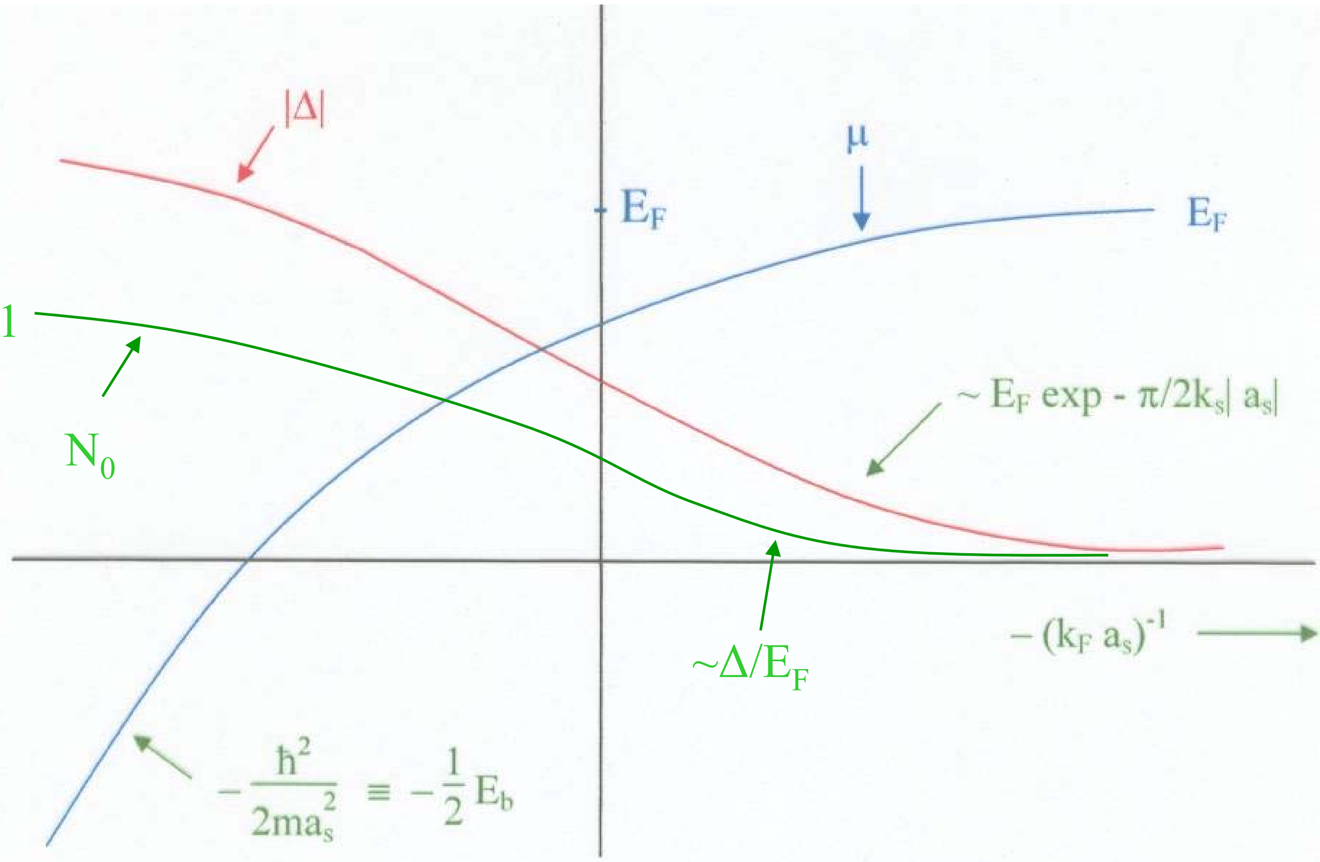
equivalently: each term of $\Psi_N^{(\text{naïve})}$ satisfies b.c. for paired particles only, e.g. 1st term satisfies it for 1, 2 but not (e.g.) for 1, 3.

Output of naïve ansatz:

$\mu(\zeta), \Delta(\zeta)$

Hence also $(E/N)(\zeta)$.

(calc^n analytic except for 2 |D numerical integrals)



Excitation energy of quasiparticle with momentum <u>k</u>
(normal-state energy $\xi_k \equiv \hbar^2 k^2/2m$):

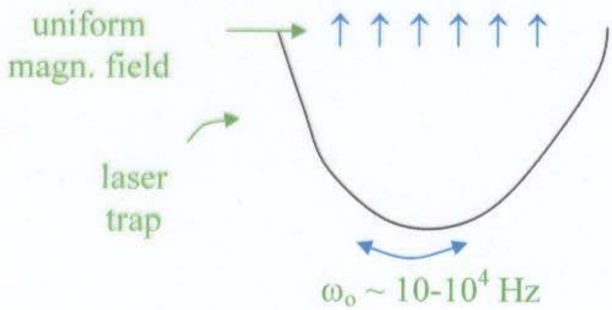
$$E_k = \sqrt{(\xi_k - \mu)^2 + |\Delta|^2}$$

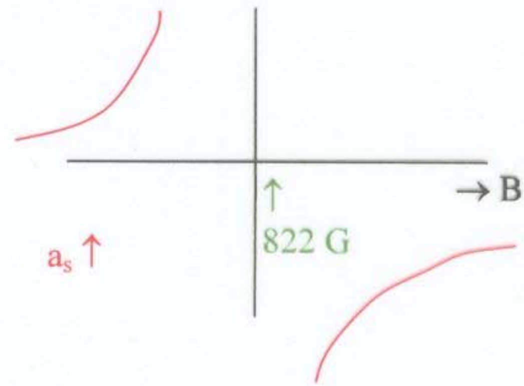
$\mu > 0$: $\min E_k = |\Delta|$

$\mu < 0$: $\min E_k = \sqrt{|\mu|^2 + |\Delta|^2}$

EXPERIMENTS ON BEC-BCS CROSSOVER

JILA:	^{40}K, $m_F = -\ 9/2$ and $-\ 7/2$	F.R. at 202G
others:	^{6}Li, $m_F = ½$ and $-\ ½$	F.R. at 822G





: need "balanced" populations! ($N_\uparrow \cong N_\downarrow$)

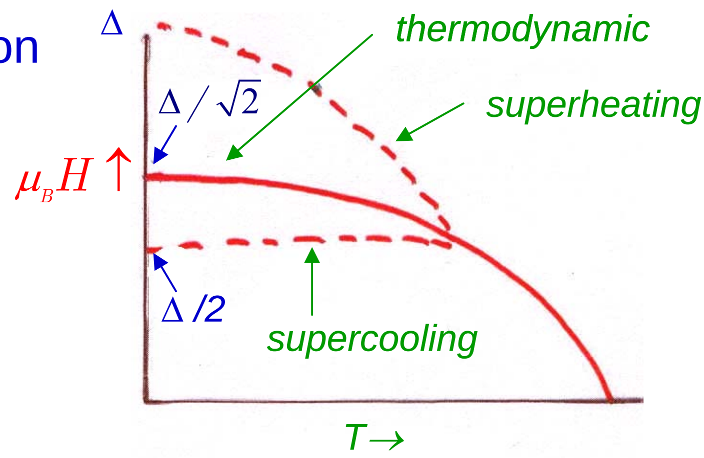
Preparation technique: ($|m_I = 1\rangle$ = "$|\downarrow\rangle$", $|m_I = 0\rangle$ = "$|\uparrow\rangle$":

1. start with all $|\downarrow\rangle$
2. $\pi/2$ rf pulse at 76 MHz $\Rightarrow$ all in $|\rightarrow\rangle \equiv 2^{-1/2}(|\uparrow\rangle + |\downarrow\rangle)$
3. inhomogeneous precession $\Rightarrow$ incoherent mixture of $|\uparrow\rangle$ and $|\downarrow\rangle$ with equal weight (+ heating!)
4. evaporation (to cancel heating)

SOME GENERALIZATIONS

A. S-wave pairing, unequal spin populations

Effect of magnetic field on pairing in “neutral” superconductor (Clogston, Chabrasckher, Makin and Tsuzuhi. . .)



Effect observed, in real superconductor, by Meissner effect (and small polarizability)

Experiments on ^{6}Li with unequal spin populations (separate detection of 2 species)

— phase separation into “pure” paired regions and normal (nonzero-spin) regions

— profiles sometimes nonmontaric

— critical polarization for pairing at unitarity

$\approx$70%

Fully polarized system described by noninteracting Fermi sea (for $k_F r_0 \ll 1$). What is MBWF for a single normal spin?

GENERALIZATIONS (cont.)

B. The $\ell \neq 0$ case

1. Qualitative difference from s-wave case: (2-body prob). In s-wave case, general E=0 solution outside potential is

$$\Psi(\boldsymbol{r}) = 1 - a_s / r$$

   and in particular, at unitarity, $\Psi(\boldsymbol{r}) \sim r^{-1} \Rightarrow$ in many-body cases expect strong 3, 4 . . . -body interaction effects.

   In $\ell \neq 0$ case,

$$\Psi(\boldsymbol{r}) \sim + \frac{c_2}{r^{\ell+1}}$$

   suggests unitary limit may be (almost) trivial in lim $r_o \ll a \; n^{-1/3}$!

2. The angular momentum problem:

   In BEC of tightly bound $\ell \neq 0$ diatonic modules, overwhelmingly plausible that

$$\boldsymbol{L} = \frac{N}{2} \hbar \hat{\underset{\sim}{\ell}}$$

   What is situation in BCS limit?
   Most “obvious” number-conserving ansatz:

$$\Psi \sim \left(\sum_k c_k a_k^+ a_{-k}^+ \right)^{N/2}, \qquad c_k \equiv v_k / u_k$$

   with (e.g.) $c_k \sim \exp i\varphi_k$. This has $\boldsymbol{L} = \frac{N}{2} \hbar \hat{\ell}$ just as in BEC limit, inoperative of magn. of |Δ|.

   Problem: macroscopic discontinuity at transition to normal state $(\boldsymbol{L} = 0)$!