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14 May 2021

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THE

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 115, NO. 3 AUGUST 1, 1959

Significance of Electromagnetic Potentials in the Quantum Theory

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(Received May 28, 1959; revised manuscript received June 16, 1959)

In this paper, we discuss some interesting properties of the electromagnetic potentials in the quantum domain. We shall show that, contrary to the conclusions of classical mechanics, there exist effects of potentials on charged particles, even in the region where all the fields (and therefore the forces on the particles) vanish. We shall then discuss possible experiments to test these conclusions; and, finally, we shall suggest further possible developments in the interpretation of the potentials.

1. INTRODUCTION

IN classical electrodynamics, the vector and scalar potentials were first introduced as a convenient mathematical aid for calculating the fields. It is true that in order to obtain a classical canonical formalism, the potentials are needed. Nevertheless, the fundamental equations of motion can always be expressed directly in terms of the fields alone.

In the quantum mechanics, however, the canonical formalism is necessary, and as a result, the potentials cannot be eliminated from the basic equations. Nevertheless, these equations, as well as the physical quantities, are all gauge invariant; so that it may seem that even in quantum mechanics, the potentials themselves have no independent significance.

In this paper, we shall show that the above conclusions are not correct and that a further interpretation of the potentials is needed in the quantum mechanics.

2. POSSIBLE EXPERIMENTS DEMONSTRATING THE ROLE OF POTENTIALS IN THE QUANTUM THEORY

In this section, we shall discuss several possible experiments which demonstrate the significance of potentials in the quantum theory. We shall begin with a simple example.

Suppose we have a charged particle inside a "Faraday cage" connected to an external generator which causes the potential on the cage to alternate in time. This will add to the Hamiltonian of the particle a term $V(x,t)$ which is, for the region inside the cage, a function of time only. In the nonrelativistic limit (and we shall assume this almost everywhere in the following discussions) we have, for the region inside the cage, $H=H_0+V(t)$ where H_0 is the Hamiltonian when the generator is not functioning, and $V(t)=e\phi(t)$. If $\psi_0(x,t)$ is a solution of the Hamiltonian H_0, then the solution for H will be

$$\psi=\psi_0 e^{-iS/\hbar},\quad S=\int V(t)dt,$$

which follows from

$$i\hbar\frac{\partial\psi}{\partial t}=\left(i\hbar\frac{\partial\psi_0}{\partial t}+\psi_0\frac{\partial S}{\partial t}\right)e^{-iS/\hbar}=[H_0+V(t)]\psi=H\psi.$$

The new solution differs from the old one just by a phase factor and this corresponds, of course, to no change in any physical result.

Now consider a more complex experiment in which a single coherent electron beam is split into two parts and each part is then allowed to enter a long cylindrical metal tube, as shown in Fig. 1.

After the beams pass through the tubes, they are combined to interfere coherently at F. By means of time-determining electrical "shutters" the beam is chopped into wave packets that are long compared with the wavelength λ, but short compared with the length of the tubes. The potential in each tube is determined by a time delay mechanism in such a way that the potential is zero in region I (until each packet is well inside its tube). The potential then grows as a function of time, but differently in each tube. Finally, it falls back to zero, before the electron comes near the

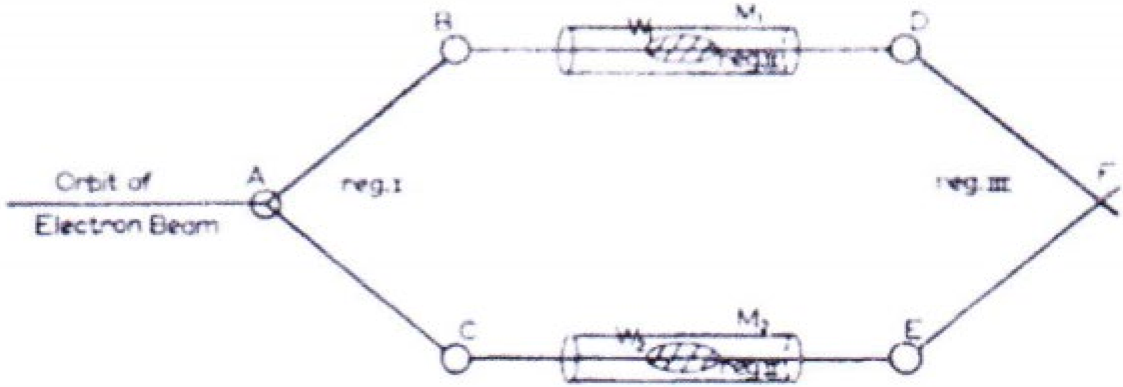

FIG. 1. Schematic experiment to demonstrate interference with time-dependent scalar potential. A, B, C, D, E: suitable devices to separate and divert beams. W_1, W_2: wave packets. M_1, M_2: cylindrical metal tubes. F: interference region.

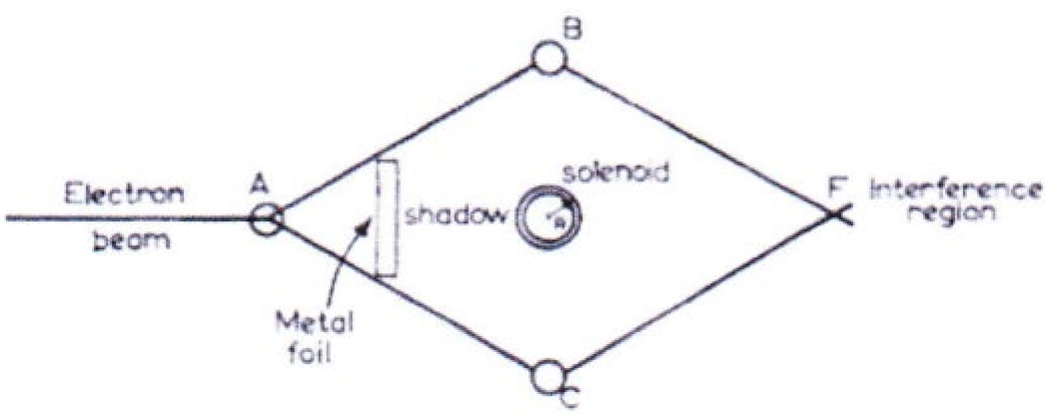

FIG. 2. Schematic experiment to demonstrate interference with time-independent vector potential.

L. Marton *et al.* (private communication).

Sec also, \\'. Ehr 11lrnrg and R. E. Siday, Proc. l'hys. . Soc (London) U6:?, 8 (19-19), who, on the basis ui :t ,cmiclnssi(al treatment, obtained some oi our results ; viz., the predict ion of . fringe 'hiit due to magnetic ,· ,-tor i><•lcnlinls in a field ir c multipl connected region

The Refractive Index in Electron Optics and the Principles of Dynamics

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MS. received 23rd February 1948, *and in amended form 28th July* 1948; *read 3rd December* 1948

ABSTRACT. In view of mis-statements made in the literature, the origin of the refractive index in electron optics is discussed in some detail, and the uniqueness of an expression previously given is demonstrated. On this basis, some general properties of electron optics are investigated.

A relation between ray direction and wave normal is obtained. Whereas the refractive index is unique in terms of the magnetic vector potential **A**, this itself is arbitrary to some extent. It is shown that **A** must, for purposes of electron optics, be chosen so as to satisfy Stokes' theorem and that, if it does, no observable effects result from the arbitrariness of **A**. An expression for the optical path difference is given in terms of the magnetic flux enclosed. The results are applied to a number of questions, viz. the differential equations for trajectories, the focusing properties of an axially symmetric field and the interference pattern produced by two converging bundles of rays which enclose a magnetic flux.

§ 1. INTRODUCTION

Whereas in light optics the refractive index of a medium is in the first instance an experimental datum, and its accurate value is the basis of any detailed discussion of the performance of optical instruments, all geometrical electron optics is entirely contained in Lorentz's equation for the forces acting on a moving charge. As a result, all equations for trajectories can be derived directly from that equation by specifying the electric and magnetic fields. Thus the rôle of the refractive index in electron optics is far less obvious than that of its counterpart for light, and in fact different authors have proposed essentially different values of the refractive index for the same field without arousing much perturbation (Glaser 1933, 1937, Opatowski 1943). Only through the persistent

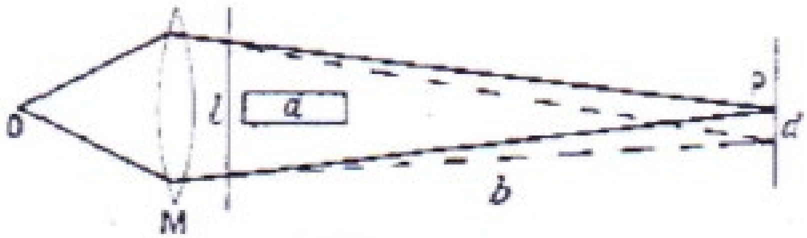

Figure 3.

Consider now an arrangement as in Figure 3. O denotes a point source of electrons which is focused by the lens M at the point P. Through the pair of slits separated by l a set of interference fringes will arise so that the distance of the nth maximum from P is given by $d = b\lambda_0 n/l$. If, now, a magnetic flux is established normal to the plane of the paper through the area a, then, according to (43), the order of interference at any point of the focal plane is changed by

$$N = \frac{1}{\lambda_0(H\rho)}\iint H_n d\sigma. \quad \text{or with (35) by} \quad N = (\mathbf{e}/\mathbf{ch})\iint H_n d\sigma. \quad \ldots\ldots(50)$$

Thus, a flux of $3{\cdot}9 \times 10^{-7}$ gauss cm^2 is required to change the order of interference by 1, and half of this flux will change the maximum at P to a minimum.

It is very curious that equation (50) associates a phenomenon observable at least in principle with a flux; one expects a change of flux, but not steady flux, to have observable effects. The effect has, however, a certain analogy in the existence of a permanent current in a superconducting ring due to a magnetic flux through it.

PHYSICAL REVIEW VOLUME 140, NUMBER 6A 13 DECEMBER 1965

Theory of a Superfluid Fermi Liquid. I. General Formalism and Static Properties*

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(Received 28 June 1965)

The microscopic theory of a superfluid Fermi liquid at finite temperature is developed for the case of a pure system with S-wave pairing, and applied to the calculation of the static properties. As a function of $\theta \equiv T/T_c$ these properties are determined entirely by the Landau parameters F_0, F_1, Z_0, etc., characterizing quasiparticle interactions in the normal phase. In particular the spin susceptibility χ and the density of the normal component ρ_n are given by

$$\chi(\theta)/\chi(1) = (1+\tfrac{1}{4}Z_0)\; f(\theta)/[1+\tfrac{1}{4}Z_0 f(\theta)],$$
$$\rho_n/\rho = (1+\tfrac{1}{3}F_1)\; f(\theta)/[1+\tfrac{1}{3}F_1 f(\theta)],$$

where the universal function $f(\theta) \equiv -[\nu(0)]^{-1}\Sigma_{\mathbf{p}}(dn/dE_{\mathbf{p}})$ is the "effective density of states near the Fermi surface" relative to its value $\nu(0)$ in the normal phase. Thus the often-quoted expression $\rho_n = \frac{1}{3}\Sigma_{\mathbf{p}}\mathbf{p}^2(dn/dE_{\mathbf{p}})$ is valid for an interacting system only in the limit $T \to 0$. In the latter part of the paper a simple phenomenological theory of "Fermi-liquid" effects on χ and ρ_n is developed for arbitrary conditions (including the presence of impurities and pairing with $l \neq 0$); it is found that under most circumstances explicit expressions for χ and ρ_n may be obtained which involve only the Landau parameters and a suitably generalized effective density of states. The theory should apply to the possible superfluid phase of He^3 and to most superconductors. It is suggested that the Knight shift in nontransition-metal superconductors should display some "Fermi-liquid" effects. The weak-field dc penetration depth $\lambda(T)$ is shown to be insensitive to such effects both in the Pippard limit and near T_c; however, in a London superconductor at lower temperatures the correction to $\lambda(T)$ should be observable and yield a direct estimate of F_1.

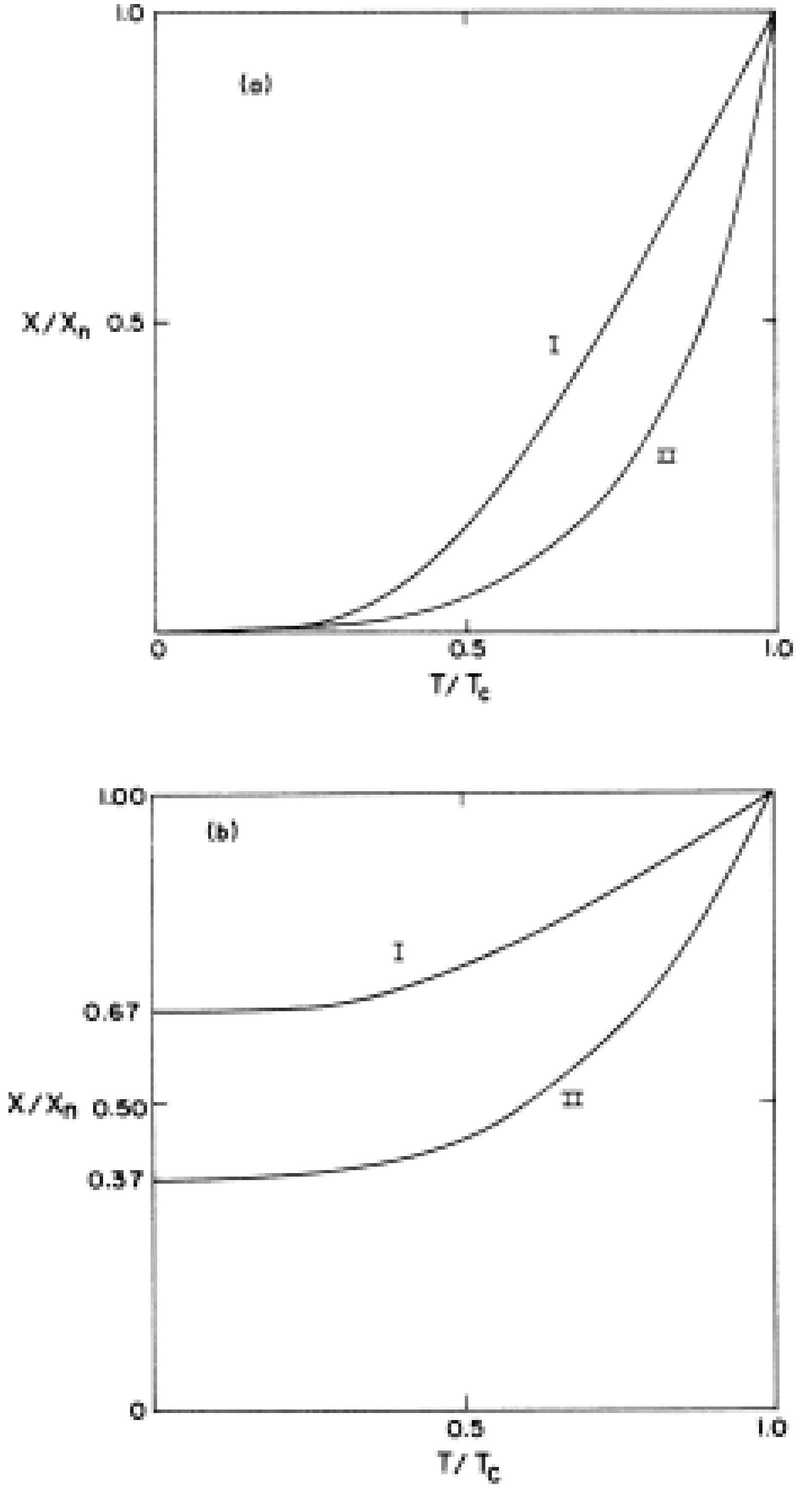

FIG. 1. $\chi(T)/\chi_n$ vs T/T_c for (a) S-state and (b) P-state condensation. (I): weak-coupling predictions (references 2 and 4, respectively); (II): present theory with $Z_0 = -2.8$.

In the above work the interaction between a pair of electrons has been included, but the interaction of unpaired electrons neglected. The same potential which produces scattering between pairs of electrons and leads to the establishment of the energy gap will also give exchange scattering between any two normal electrons and thereby modify the energy of the depaired state. This effect decreases the stability of the depaired state relative to the BCS state, but its effect on the normal state is even greater, because of the larger magnetization. The effect of the interaction on the spin susceptibility is already well known in nonsuperconducting metals. For the repulsive Coulomb potential this is the usual exchange scattering which favors ferromagnetism, and for paramagnetic metals tends to increase the paramagnetic spin susceptibility. It is an effect which is included by Landau[13] in his discussion of the quasi-particle treatment of the spin susceptibility of a Fermi liquid, and has also been discussed from a somewhat different point of view by one of the present authors.[14] A useful picture for this effect is that of the molecular field, the basis of the Weiss theory of ferromagnetism. In the present case of interest, because of the attractive short-range potential of the BCS theory, the molecular field is opposite in sign relative to its usual direction and is unfavorable to the polarization of electrons.[15] It subtracts from the actual external exchange field applied to the sample, H_{ex}. Thus, the net magnetic field which effectively serves to act on any given electron spin is

$$H_{eff}=H_{ex}-\tfrac{1}{2}NV(M/N\Delta_0). \qquad (24)$$

[13] L. D. Landau, Zh. Eksperim. i Teor. Fiz. **30**, 1058 (1956) [English transl.: Soviet Phys.—JETP **3**, 920 (1957)].
[14] J. J. Quinn and R. A. Ferrell, Plasma Phys. **2**, 18 (1961).

P. Felde & R.A. Ferrell. Superconductivity in Strong Spin-Exchange Field. Phys. Rev. A 135:3, 550-563 (1964).

EFFECT OF COLLECTIVE EXCITATIONS ON THE ELECTRODYNAMICS OF SUPERCONDUCTORS

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Submitted to JETP editor December 30, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) 46, 2188-2201 (June, 1964)

1. INTRODUCTION

IN the study of the electrodynamics of superconductors,[1-3] only that part of the interaction between particles is usually taken into account which leads to pairing and to a gap in the spectrum of one-particle excitations. The quasiparticles are assumed to be noninteracting. The effect of the remaining interaction on the electrodynamics of superconductors in a constant field leads to a renormalization of the constants which determine the penetration depth. It will be shown below that the number of free electrons, which enters into the London constant depends on the remaining interaction and on the form of the periodic potential, and is identical with the corresponding constant in the dielectric permittivity of metals in the infrared region. In the Pippard limiting case, the penetration depth is determined by the momentum on the Fermi boundary and does not depend on the interaction.

3. LONDON AND PIPPARD LIMITS

Equation (21) becomes simplified in the limiting cases of small and large **k**. In the London limit we have $\mathbf{k} = 0$, $L = 1$, $O = 0$; consequently, $\tilde{T} = 0$ and we get

$$K_{\alpha\beta}(0, \omega) = 4\pi e^2 c^{-2} \rho \langle v_\alpha T_\beta \rangle, \qquad \mathbf{T} = \mathbf{v} + f^k \mathbf{T}. \quad (22)$$

For a spheroidal Fermi surface

$$K_{\alpha\beta} = \delta_{\alpha\beta} K, \qquad K = \lambda_L^{-2} = 4\pi/c^2\Lambda = 4\pi N_0 e^2/mc^2, \quad (23)$$

where Λ is the London parameter, λ_L is the London penetration depth, and N_0 is the so-called number of free electrons

$$N_0 = \frac{\rho v^2 m}{3(1 - f_1^k)} = \frac{p_0^2 v m}{3\pi^2 (1 - f_1^k)}. \quad (24)$$

This number is identical with the corresponding quantity entering into the dielectric constant $\epsilon = -4\pi N_0 e^2/m\omega^2$ in the infrared region,[10] and depends on the periodic field and on the interelectron interaction. Only in the absence of a periodic field, when the Galilean invariance leads to the following relation between the velocity and the momentum

$$mv = p_0(1 - f_1^k) = p_0/(1 + f_1^\omega),$$

does the number N_0 coincide with the electron density $p_0^3/3\pi^2$.

In the Pippard limiting case $kv \gg \Delta$ we have $L \sim M \sim N \sim O \sim \Delta/kv \ll 1$, $\mathbf{T} = \mathbf{v}$ in Eqs. (21), and we get

$$K_{\alpha\beta} = 4\pi e^2 c^{-2} \rho \langle v_\alpha L v_\beta \rangle. \quad (25)$$

For an isotropic Fermi surface and a transverse vector potential, we get

$$K_{\alpha\beta} = K\delta_{\alpha\beta}, \qquad K = \frac{e^2 p_0^2}{\pi c^2 k} \int_{-\infty}^{\infty} L\, d\mathbf{k}\mathbf{v}. \quad (26)$$

This expression is identical with that obtained in the weak coupling model.[2,3] It does not depend either on the velocity on the Fermi surface or on the interaction, and is determined only by the momentum on the Fermi surface, which is expressed in terms of the number of electrons in the conduction band. The integral in Eq. (26) can be reduced to an elliptic integral. We set down, for various frequencies ω, the limiting values needed for what follows:

$$K(0) = \frac{e^2 p_0^2}{c^2} \frac{\pi\Delta}{k}, \quad K(2\Delta) = \frac{e^2 p_0^2}{c^2} \frac{2\Delta}{k},$$

$$K(\omega \gg \Delta) = \frac{e^2 p_0^2}{c^2} \frac{i\omega}{k}. \quad (27)$$

The latter case corresponds to the normal state.

REVIEWS OF MODERN PHYSICS VOLUME 41, NUMBER 1 JANUARY 1969

ANL 10

Kapitza Resistance*

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Kapitza resistance (R_K) is the thermal boundary resistance which occurs at the interface when heat flows from a solid into liquid helium. This review attempts a comprehensive presentation of experimental and theoretical knowledge since the discovery of Kapitza resistance in 1941. The experiments discussed and data presented include measurements of R_K at interfaces between liquid ^{4}He and: copper, lead, mercury, tin, indium, nickel, constantan, gold, silver, platinum, tungsten, silicon, quartz, lithium fluoride, and sapphire. The experiments between solids and liquid ^{3}He are also discussed. The treatments include discussion of the dependence of R_K on these variables: temperature, pressure, surface structure and preparation, and elastic properties of the solid. The principal experimental problems are associated with the surface properties, so these are discussed in detail. The principal theoretical discussion is of the acoustic impedance theory, following Khalatnikov and Mazo and Onsager, and the results are compared with the experiments. Modifications of the theory connected with improved matching at the interface, due, for example, to condensed He are also considered. When the theory is applied to interfaces between metals and liquid He, then it must be modified to take into account phonon electron interactions. The theory gives a temperature dependence $R_K \propto T^{-3}$, which is approximately what is experimentally, observed. However, the observed R_K's are usually an order of magnitude or more smaller than theoretical values. The source of the disagreement lies either in a lack of knowledge of the surface physics or in another, dominant, mechanism for thermal energy exchange across the interface. Evidence for each possibility and some suggestions aimed at resolving the discrepancies are offered.

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I. INTRODUCTION

When heat is conducted from a solid into a liquid, the temperature is not continuous at the boundary. Instead there is a small temperature difference (ΔT) across the interface. If the heat flow ($\dot{Q}$) is small, the temperature difference is proportional to it. The ratio, $\Delta T/\dot{Q}$, is effectively a thermal resistance for the boundary, and it is inversely proportional to the interfacial area A. Kapitza resistance (R_K) is the thermal boundary resistance between a solid and liquid helium. It is defined as

$$R_K = A\Delta T/\dot{Q}(\text{cm}^2\,{}^\circ\text{K/W}). \tag{1}$$

The units shown are those most commonly used.

Thermal boundary resistance is probably associated with heat flow across all solid–solid as well as all solid–liquid interfaces, but Kapitza resistance is an extremely interesting special case. Generally, thermal boundary resistances are smaller, less well defined, and more difficult to measure than for interfaces between solids and liquid helium.

The study of Kapitza resistance is about 30 years old. Interest in the subject has been quickened recently for two reasons: First is for its fundamental interest in the study of the physics of solids, surfaces, and liquid helium. Second is for its application to recent experiments to attain temperatures of a few millidegrees Kelvin and below.

The phenomenon was discovered by Kapitza in 1941[1] during his classic experimental investigation of superfluidity of He II, the superfluid phase of liquid ^{4}He. In an effort to understand why the apparent thermal conductivity of He II in capillaries could be much larger than the thermal conductivity of bulk He II, Kapitza measured the temperature distribution in the neighborhood of heated metal surfaces freely suspended in He II. He observed, in the range between 1.6°K and T_λ, the lambda point temperature 2.1720°K, a temperature jump between the solid and the He II of the order of 2 m°K for each milliwatt/cm² of thermal flux crossing the interface. This thermal boundary resistance decreased with increasing temperature approximately as T^{-3}. From these measurements and from study of the behavior of the discontinuity when the surfaces were surrounded with emery powder, Kapitza deduced that the discontinuity took place within a few hundredths of a millimeter of the interface and not in the bulk helium.

In practice, the Kapitza resistance depends not only on temperature but also on the pressure and on the solid itself. It is also expected to be especially sensitive to the nature of the carriers of thermal energy in the

* Work supported in part by the U.S. Atomic Energy Commission.

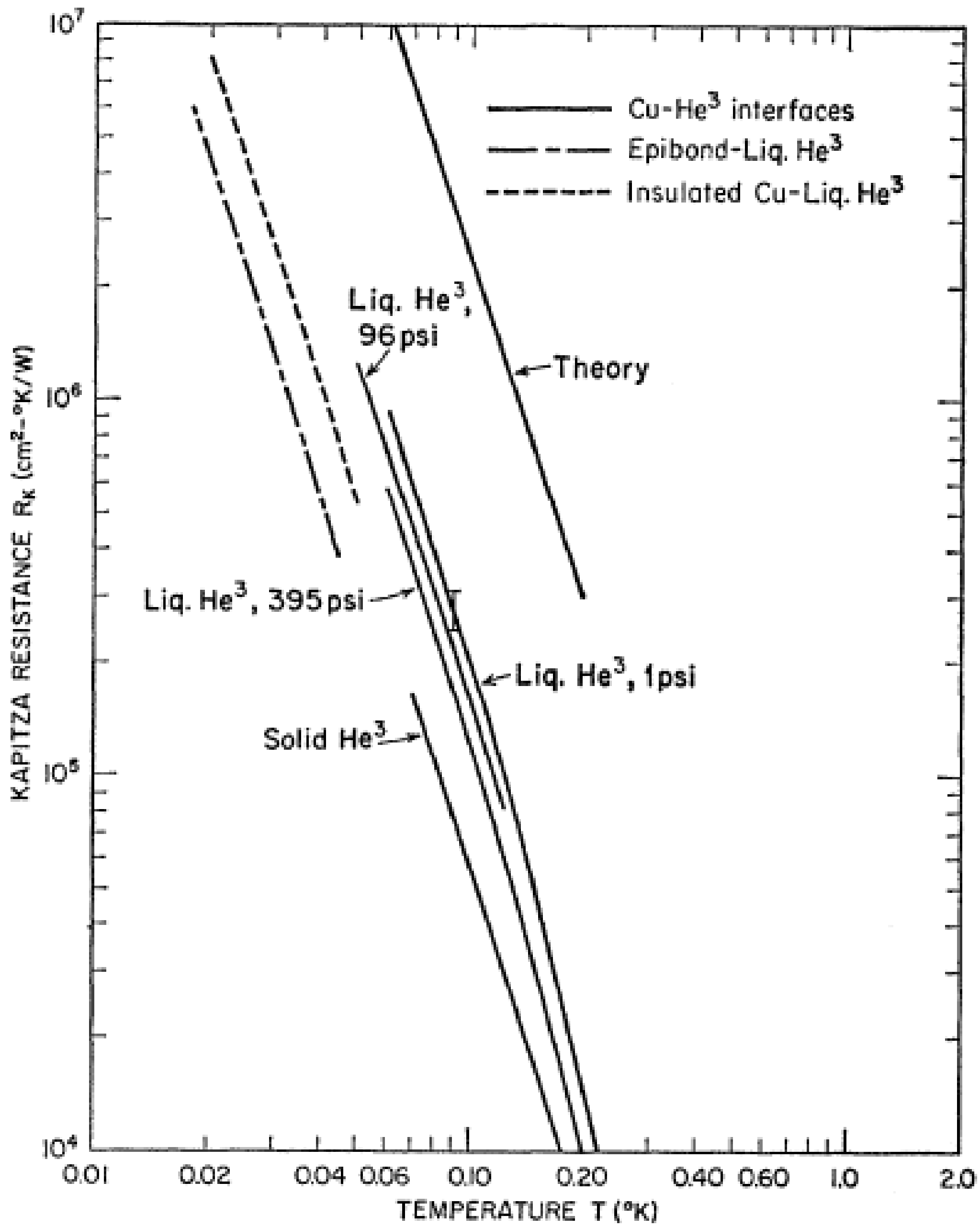


FIG. 8. (a) Experimental data on Kapitza resistance for: Cu-liquid-^{3}He, Cu-solid-^{3}He, Epibond 100-A-liquid-^{3}He, and Formex-insulated-Cu-wire-liquid-^{3}He, interfaces.

The net flux $\dot{Q}/A$ from the solid into the liquid for small temperature differences is calculated by the methods used earlier for Eqs. (29) and (30), so that the result for the Kapitza boundary resistance between a solid and liquid ^{3}He at low temperatures becomes[52]

$$R_K = \frac{5h^3 \rho_S c_t^3 m}{8\pi^5 k^4 \rho p_s [aF(c_l/c_t) + b\Phi(c_l/c_t)] T^3}. \qquad (3)$$

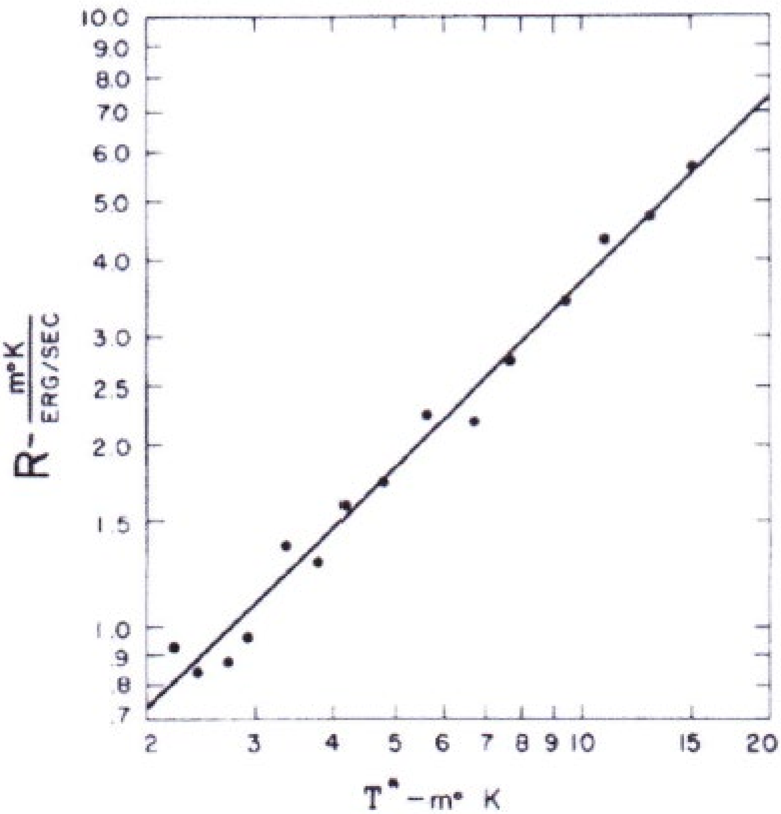
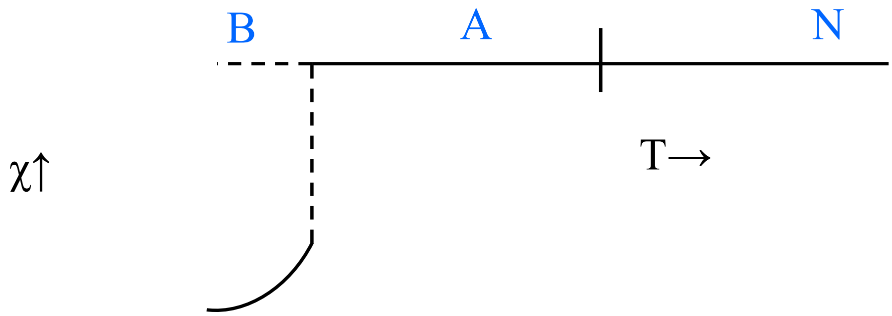


FIG. 2. Effective thermal resistance deduced from the data of Fig. 1 using a two-bath model and the measured heat capacities of He^3 and CMN.

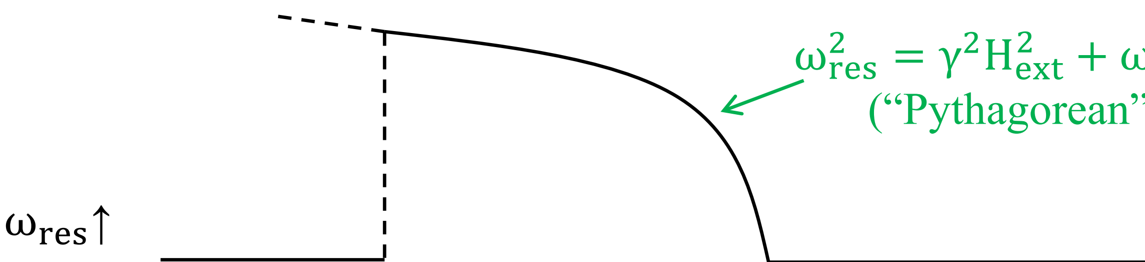
From W.R.Abel et al.,Phys.Rev.Letters 16, 273 (1966)

NMR in the new phases:



Not necessarily mysterious: e.g. A phase could be an ESP state (only ↑↑,↓↓ pairs ⇒ no reduction in χ), B could be singlet or BW (some ↑↓ pairs , so χ reduced) [but: why is ESP ever stable?]-

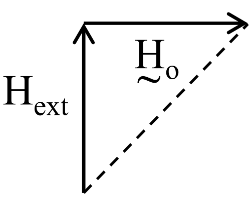
But: what about the resonance frequency?



$\omega_{res}^2 = \gamma^2 H_{ext}^2 + \omega_0^2(T)$ ("Pythagorean")

$\omega_{res} = \gamma \; H_{ext}$

$$\omega_0^2(T) \approx A\left(1 - \frac{T}{T_A}\right), \qquad \frac{A}{(2\pi)^2} \cong 5 \times 10^{10}\,\mathrm{Hz}^2$$



$(\equiv \omega_o\,(T)/\gamma)$

Need $H_o \sim 30$G. But, only spin-nonconserving force in problem is nuclear dipole-dipole interaction, and max. associated field is < 1G!

IS THIS THE FIRST INDICATION OF A RADICAL BREAKDOWN OF QUANTUM MECHANICS?

PHYSICAL REVIEW B VOLUME 36, NUMBER 1 1 JULY 1987

Vortices with half magnetic flux quanta in "heavy-fermion" superconductors

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(Received 23 June 1986)

It is shown that in "heavy-fermion" superconductors a new vortex state can occur characterized by the existence of half magnetic flux quanta. Vortices in polycrystals should exist even in the absence of an externally applied magnetic field. The internal structure of the vortices is also investigated.

INTRODUCTION

The so-called "heavy-fermion" behavior observed in various substances is one of the most stimulating topics of condensed matter physics which have attracted the attention of both theoretical and experimental investigators during the last years. Experimental data on superconducting cerium- and uranium-based intermetallic compounds such as $CeCu_2Si_2$, UBe_{13}, and UPt_3 (Ref. 1) suggest that an "unusual" pairing of electrons can take place.

In this context tunneling between spin "singlet" and hypothetical "triplet" superconductors has been investigated for many years[2] together with proximity effect between such systems.[3] More recently, the limits of using pair tunneling as a probe to differentiate between even- and odd-parity states (e.g., Ref. 4) and results of rather sophisticated experiments[5] have been discussed by several authors.

In the case of unusual pairing, the order parameter should belong to the nontrivial representation of the crystal symmetry group.[4–8] This circumstance leads to quite interesting consequences. In this article we show that in these superconductors there exist vortices with a half magnetic flux quantum.[9] We shall demonstrate that in a single crystal these vortices may exist only on the domain walls between different degenerate superconducting states whereas in a polycrystalline sample, vortices can occur at the intersection of the three boundaries between three crystal grains (the line L at which three crystalline grains are in contact). In this case these vortices are energetically favored and they exist even in the absence of an externally applied magnetic field. We shall refer only to nonmagnetic superconducting phases.

Let us show first that a system of an *S-P-S* sandwich (*S* and *P* stand for usual and unusual superconductor, respectively) closed by a superconducting loop (Fig. 1) will contain half magnetic flux quanta, namely $(n+\frac{1}{2})\Phi_0$.

As is known,[5] the order parameter of a superconductor can be expressed as

$$\Delta_{\alpha\beta}(\mathbf{k},X)=\sum_i \eta^i \psi^i_{\alpha,\beta}(\mathbf{k}) \; , \qquad (1)$$

where η^i is an order parameter which can be in general function of temperature T and coordinate X_i, and $\psi^i_{\alpha\beta}(\mathbf{k})$ is the basis function of the representation which contains the angular dependence of the momentum. The usual normalization condition for $\psi(\mathbf{k})$ is $\int |\psi(k)|^2 d\Omega=1$. If in the superconductive phase there exists only one function $\psi^i(i=1)$, then η belongs to the one-dimensional representation. Let us consider a system of two superconductors, one usual (S) and one unusual (P), coupled by tunneling. The free energy of such a system of two weakly-coupled superconductors is given by

$$F=\mathrm{Re}\int \Delta_1\Delta_2^* G_1G_1 |T|^2 G_2G_2\, d\mathbf{k}_1\, d\mathbf{k}_2 = A\, \mathrm{Re}\eta_1\eta_2^* \; , \qquad (2)$$

where Δ_1 and Δ_2 are the order parameters in the *S*- and *P*-type superconductors, respectively. The coefficient A is a function of the orientation of the barrier with respect to the crystal axes (it is also temperature dependent). A is an odd function of n (unit vector normal to the surface of contact) since Δ_1 and Δ_2 are even and odd functions of $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively. Here our reference system is provided by the crystal axes. For example if we assume the order parameter Δ_2 to be a pseudoscalar $[\Delta_2 \sim (\mathbf{k},\boldsymbol{\sigma})]$ we can write for the case of a crystal of cubic symmetry[8]

$$A(n)=Af(\mathbf{n}) \; ,$$

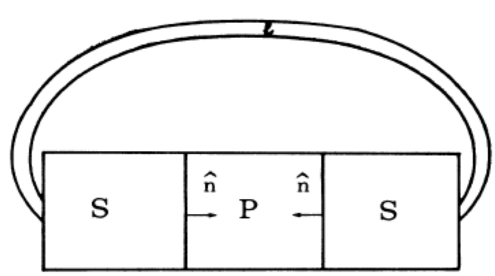

FIG. 1. Sketch of a sandwhich structure *S-P-S* (see text) closed by a superconducting loop.

Fenton[5] pointed out that the spin-orbit interaction accounts for the nonvanishing Josephson current. He ignored, however, the crystal structure, and the effect he obtained was small compared with the usual effect, at least because of the small ratio of the interatomic distances to the size of the pair, ξ. The ratio is small because the order parameter varies slowly near the surface of the sample. A considerably stronger effect occurs as a result of the nondiagonal nature of the tunnel Hamiltonian $T_{\alpha\nu}$. For the tunnel Hamiltonian to be nondiagonal, the spin-orbit interaction need not necessarily occur in the insulating layer: the insulating layer need only be present in the superconductor. This effect, which is similar to the spin-orbit scattering by light impurities in metals with heavy atoms, is associated with the fact that the index ν describes the projection of the pseudospin, rather than the spin. In a microscopic calculation of the tunnel Hamiltonian $T_{\alpha\nu}$, the wave functions in the insulator should be matched with the wave functions of the type in (3), which describe the behavior of an electron in a metal with heavy atoms. Since the heavy atoms are found only on one side of the boundary, the mean vector **r** in expression (4) is directed along the vector **n**. As a result, we have

$$T_{\alpha\nu}(\mathbf{k}) = T(\delta_{\alpha\nu} + \lambda/k_F([\mathbf{kn}]\vec{\sigma}_{\alpha\nu})), \qquad (6)$$

where the dimensionless parameter λ in a metal with heavy atoms is on the order of unity. Substituting expression (5) and (6) into expression (2), we find the following expression for the odd pairing:

$$I(\varphi) = 2e\,\mathrm{Im}\int \lambda T^2 F^* K(\mathbf{x})([\mathbf{d}(\mathbf{k},\mathbf{x})\,\mathbf{k}]\mathbf{n})\,d\mathbf{x}\,d\mathbf{k}. \qquad (7)$$

The **k** dependence of the order parameter **d** for different representations of several groups was found in Refs. 2 and 3. In the integration of expression (7) over the angles of the vector **k**, one should take into account that T^2 and $K(\mathbf{x})$, and also $\mathbf{d}(\mathbf{k},\mathbf{x})$ for representations other than the one-dimensional, are complex functions of $(\mathbf{kn})$. As a result, we find expression (1) for the pseudoscalar representation A_1 of the cubic group. At low temperatures the constant A differs by only a factor on the order of unity from the standard expression for the current that passes through the contact. This factor is associated with the spin-orbit interaction and the anisotropy. An additional small parameter on the order of $\xi_0/\xi(T)$ appears near T_c because the order parameter of the uncommon pairing falls off as the surface is approached within $x \sim \xi(T)$, while the kernel $K(\mathbf{x})$ decreases at a distance of ξ_0. The parameter A is of the same order of magnitude for the other phases.

The dependence of the Josephson current on the angles of the vector **n** for the other phases can be found from Eqs. (2) and (7) and from the symmetry considerations. The spin-orbit interaction is unimportant for the even phases. The dependence $I(\mathbf{n})$ is the same as the dependence of the scalar order parameter $\psi(\mathbf{k})$ on the angles of the momentum **k**. The current vanishes when the normal to the surface runs in the direction along which the gap in the excitation spectrum vanishes. These directions have been determined in Ref. 2.

For odd phases the current vanishes in two cases: 1) if the symmetry axis in the plane of the contact is such that a 180° rotation around this axis does not change the order parameter and 2) if the normal to the surface is directed along the axis the