SUPERFLUID ^{3}HE: UNDERSTANDING THE EXPERIMENTS

Anthony J. Leggett

Department of Physics
University of Illinois at Urbana-Champaign
Urbana, IL

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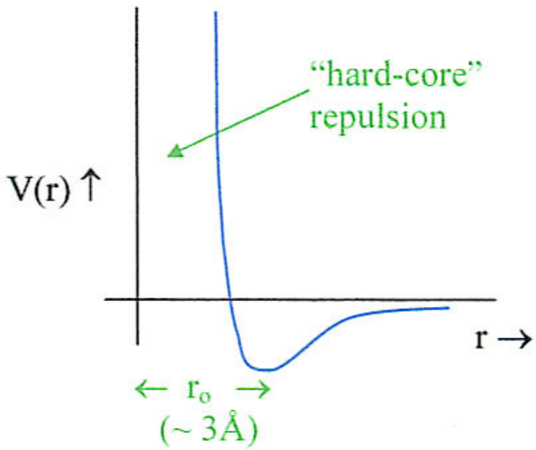
EARLY THEORETICAL WORK ON POSSIBLE COOPER PAIRING IN LIQUID ^{3}HE



$r \sim r_0$, $p \sim p_F$ $(\equiv \sqrt{2mk_BT_F})$
$\Rightarrow$ relative angular momentum
$\ell \equiv (p_F r_0/\hbar) \neq 0$

(prob. 1 or 2)

Pauli principle: $\begin{cases} \ell = 0,2,4\ldots & S = 0 \quad \text{(singlet)} \\ \ell = 1,3,5\ldots & S = 1 \quad \text{(triplet)} \end{cases}$

in general, $\ell \neq 0 \Rightarrow$ relative (internal) wave function of pair
has orientational degree(s) of freedom!

"equal spin pairing"

Anderson & Morel (1961): explore in detail case $\ell = 2$, and a special case of $\ell = 1$: only ↑↑ and ↓↓ pairs form, and have the same orbital ang. momentum in direction $\hat{\underset{\sim}{\ell}}$ ("ABM" state). Physical properties anisotropic.

Vdovin
Balian & Werthamer } (1963): in $\ell = 1$ case all spin components

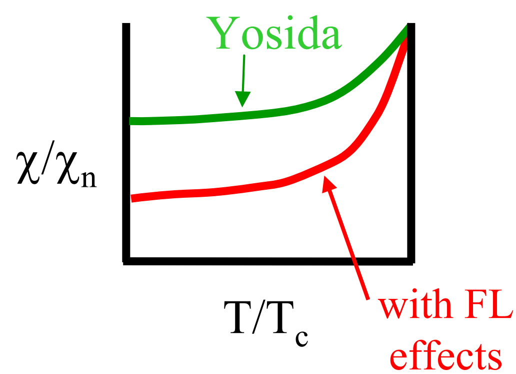
"^{3}P

$\left(\uparrow\uparrow, \downarrow\downarrow, \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)\right)$ can form: in fact for any given pair, $\underset{\sim}{L} = -\underset{\sim}{S} \Rightarrow J = 0$.

("BW" state). All physical properties isotropic. More stable than any ESP state.

FURTHER PRE-1972 THEORETICAL DEVELOPMENTS

1. Combination of Landau Fermi-liquid theory of (strongly interacting) normal phase with BCS theory of (weakly interacting) superfluid phase ("superfluid Fermi liquid") (Larkin-Migdal, Fulde-Ferrell, AJL)
   $\Rightarrow$ FL effects change T-dependence of ρn(T), χ(T) from simple Yosida function.
   e.g. χ(T) of BW phase:
   ↑
   spin susceptibility



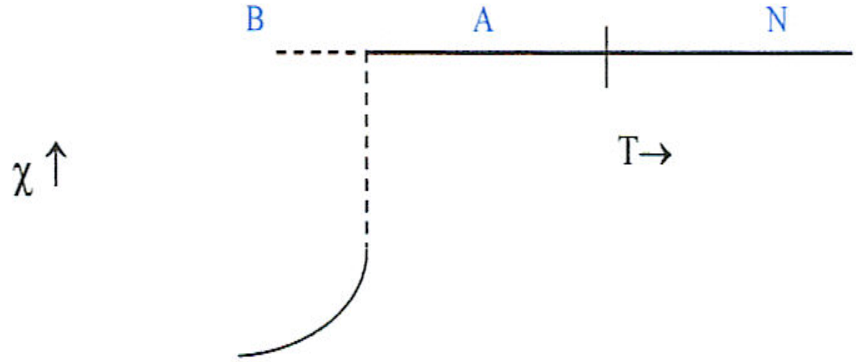
2. Spin-fluctuation-mediated interaction (Layzer-Fay): if $V_{\sigma\sigma}(r) \sim gf(r)\underset{\sim}{\sigma}\cdot\underset{\sim}{\sigma}'$ ↑↑ ~~~~ ↑↓ generates (indirect) repulsion between anti || spins (singlet) but attraction between || over (triplet).

   (No consideration of NMR in paired phase)

Theoretical expectation in spring of 1972:

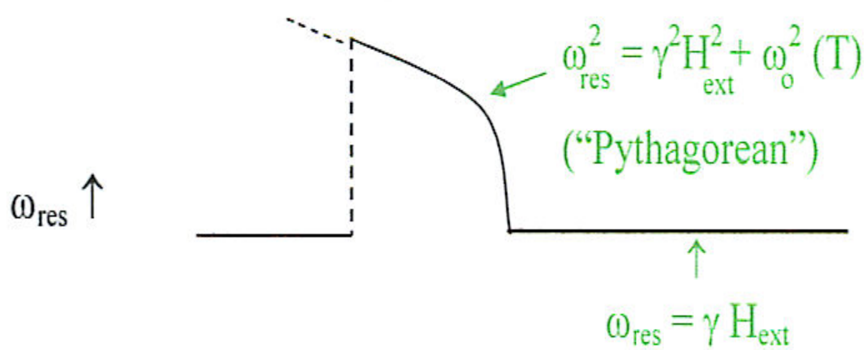
Liquid ^{3}He may form Cooper pairs, either with ℓ = even (spin singlet) or with ℓ = odd (BW state). In either case, χ reduced and all magnetic properties isotropic. T_c difficult to predict.

NMR in the new phases:

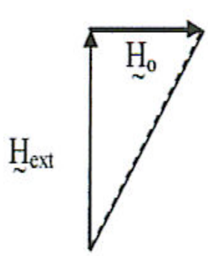


Not necessarily mysterious: e.g. A phase could be an ESP state (only $\uparrow\uparrow,\downarrow\downarrow$ pairs $\Rightarrow$ no reduction in χ), B could be singlet or BW (some $\uparrow\downarrow$ pairs, so χ reduced) [but: why is ESP ever stable?] -

But: what about the resonance frequency?



$$\omega_0^2(T) \approx A(1 - T/T_A), \qquad \frac{A}{(2\pi)^2} \cong 5\times 10^{10}\ \mathrm{Hz}^2$$



$(\equiv \omega_o(T)/\gamma)$

Need $H_o \sim 30$G. But, only spin-nonconserving force in problem is nuclear dipole-dipole interaction, and max. associated field is < 1G!

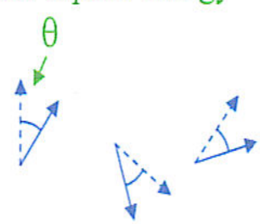
WHAT CAN BE INFERRED FROM SUM RULES?

IF a single sharp resonance is observed (as in expt.) then:

nuclear dipole energy

$$\omega_{res}^2 = \gamma^2 H_{ext}^2 + \omega_0^2$$

$$\omega_0^2 = \gamma^2 \chi^{-1} \partial^2 \langle H_D \rangle / \partial \theta^2$$



angle of simultaneous rot^n of <u>all</u> spins

But $\partial^2 \langle H_D \rangle / \partial\theta^2 \sim \langle H_D \rangle$:

So, exptl. value of $\omega_0^2\,(T) \Rightarrow$

$$\langle H_D \rangle\,(T) \sim K(1 - T/T_A), \quad K \sim 10^{-3}\ \text{ergs/cm}^3$$

HOW CAN THIS BE?

$$\begin{cases} \uparrow \quad (\text{"bad"}) \quad \uparrow \\ \rightarrow \quad (\text{"good"}) \quad \rightarrow \end{cases}$$

$$\Delta E \lesssim \frac{\mu_0 \mu_n^2}{r_0^3} \sim 10^{-7}\,\text{K} \ll k_B T$$

So, prima facie, preference for "good" orientation over "bad" is <u>at most</u>

$$\sim \Delta E / k_B T \sim 10^{-4} \quad [\text{actually, } \sim \Delta E / k_B T_F \sim 10^{-7}]$$

$\Rightarrow$ expectation value of dipole energy $\sim n(\Delta E)^2/k_B T$: much too small!

But: what if "spin-orbit" symmetry is spontaneously broken? Then $\langle H_D \rangle \sim n\Delta E$: about right!

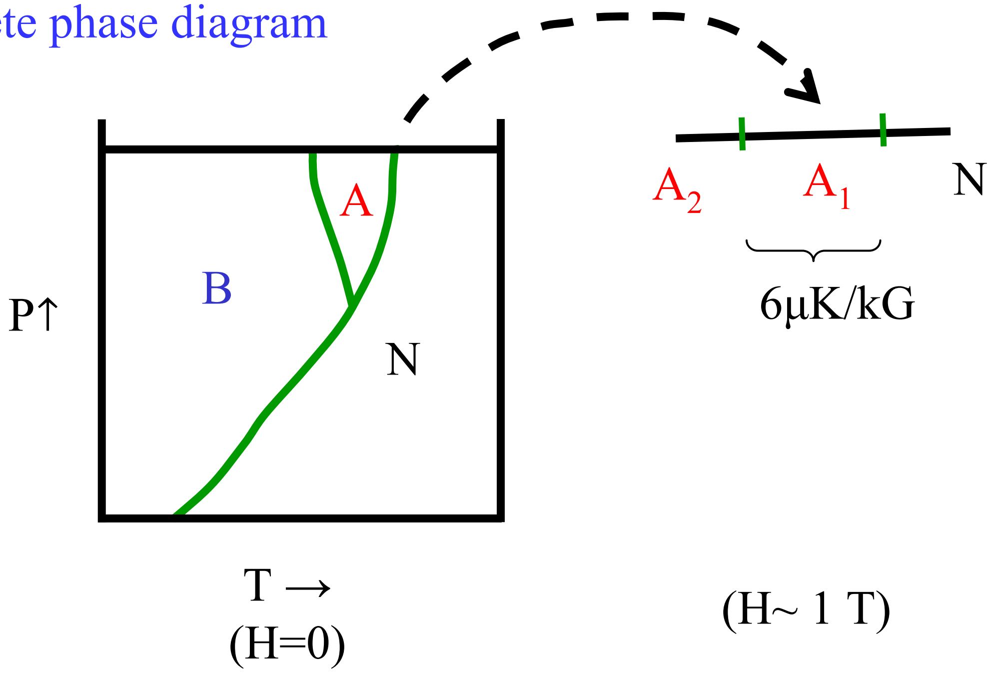
What could break SO symmetry? One possibility:

anisotropic BCS pairing!

⚠: what about B phase?

SOME EXPERIMENTAL DEVELOPMENTS June 72–June 73

Complete phase diagram



Ev. for superfluidity (4th sound, viscosity, velocity of collisionless sound …)

SOME THEORETICAL INSIGHTS

A_1 phase: most likely ↑↑ only paired (A_2: ↑↑, ↓↓) (Ambegaokar-Mermin). (confirms ESP assignment of A phase)
GL analysis (Mermin-Stare, Anderson-Brinkman): allows stability of ABM or BW, but not of "planar" phase $(\underset{\sim}{\ell}_{\uparrow} = -\ell_{\downarrow})$.

MAJOR OUTSTANDING PUZZLES:

(a) WHY A and B?

(b) B-phase NMR.

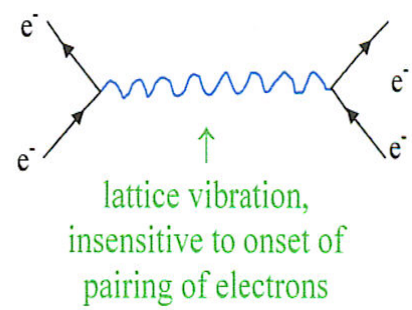
Resolution of the Paradox of Two New Phases

(Anderson & Brinkman, Phys. Rev. Letters **30**, 1108 (1973): cf. Brinkman, Serene and Anderson, Phys. Rev. A **10**, 2386 (1974))

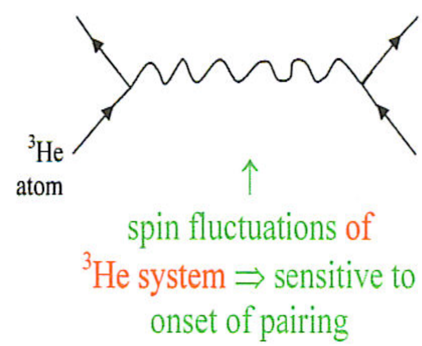
In BCS (weak-coupling) theory for $\ell = 1$, BW phase is always stable, independently of pressure and temperature.

Crucial difference between Cooper pairing in superconductors and ^{3}He:

Superconductor:



liquid ^{3}He:



⇒"feedback" effects: ⇒ $$\Delta F_{\text{feedback}} \sim \Delta^4 \Big(1 - 2\sum_{ij} \left\langle \mathrm{Re}\, d_i^* d_j \right\rangle^2\Big) \leftarrow \alpha$$

$$\alpha = +1/3 \text{ for BW}, \ -1 \text{ for ABM}$$

Over most of the phase diagram, BW state stable as in BCS theory. But at high temperature and pressure, feedback effects uniquely favor ABM phase

major qualitative leap beyond BCS!

MICROSCOPIC SPIN DYNAMICS (SCHEMATIC)

Basic variables:

(a) Total spin $\underset{\sim}{S}$

(b) Orientation $\underset{\sim}{\theta}$ of spin of Cooper pairs

$$\left.\begin{array}{l}\text{(a)}\\ \text{(b)}\end{array}\right\}\quad [S_i, \theta_j] = i\delta_{ij}$$

$$\hat{H} = \hat{H}_o(\underset{\sim}{S}) + \hat{H}_D(\underset{\sim}{\theta})$$

↑

hydrodynamic (Born-Oppenheimer) approximation

Semiclassical equations of motion:

zero in eq^{m}!

dipole torque

$$\frac{d\underset{\sim}{\theta}}{dt} = \frac{\partial\langle\hat{H}_o\rangle}{\partial\underset{\sim}{S}} = \mathcal{H}_{ext} - \chi^{-1}\underset{\sim}{S}, \qquad \frac{d\underset{\sim}{S}}{dt} = \underset{\sim}{S}\times\mathcal{H}_{ext} - \frac{\partial\langle\hat{H}_D\rangle}{\partial\underset{\sim}{\theta}}$$

⇒ linear NMR behavior completely determined by eigenvalues of quantity

$$\Omega^2_{ij} \equiv \partial^2\langle H_D\rangle/\partial\theta_i\,\partial\theta_j$$

so, can "fingerprint" A and B phases by NMR!

ABM: single resonance line

axial: split resonance

BW: original BW state is $\underset{\sim}{L} = -\underset{\sim}{S}$, i.e. $J = 0$. But dipole torque rotates $\underset{\sim}{S}$ relative to $\underset{\sim}{L}$ by $\angle\cos^{-1}(-1/4) = 104°$ around axis $\hat{\underset{\sim}{\omega}}$ whose "best" choice is $\mathcal{H}_{ext}$.

Result: no shift in transverse resonance, but finite-frequency longitudinal resonance!
(also in ABM phase)

$\mathcal{H}_{ext}$ ↑

↑ $\mathcal{H}_{rf} \sim \cos\omega t$

(expt: Osheroff, 1974)