What Can We Do With a Quantum Liquid?

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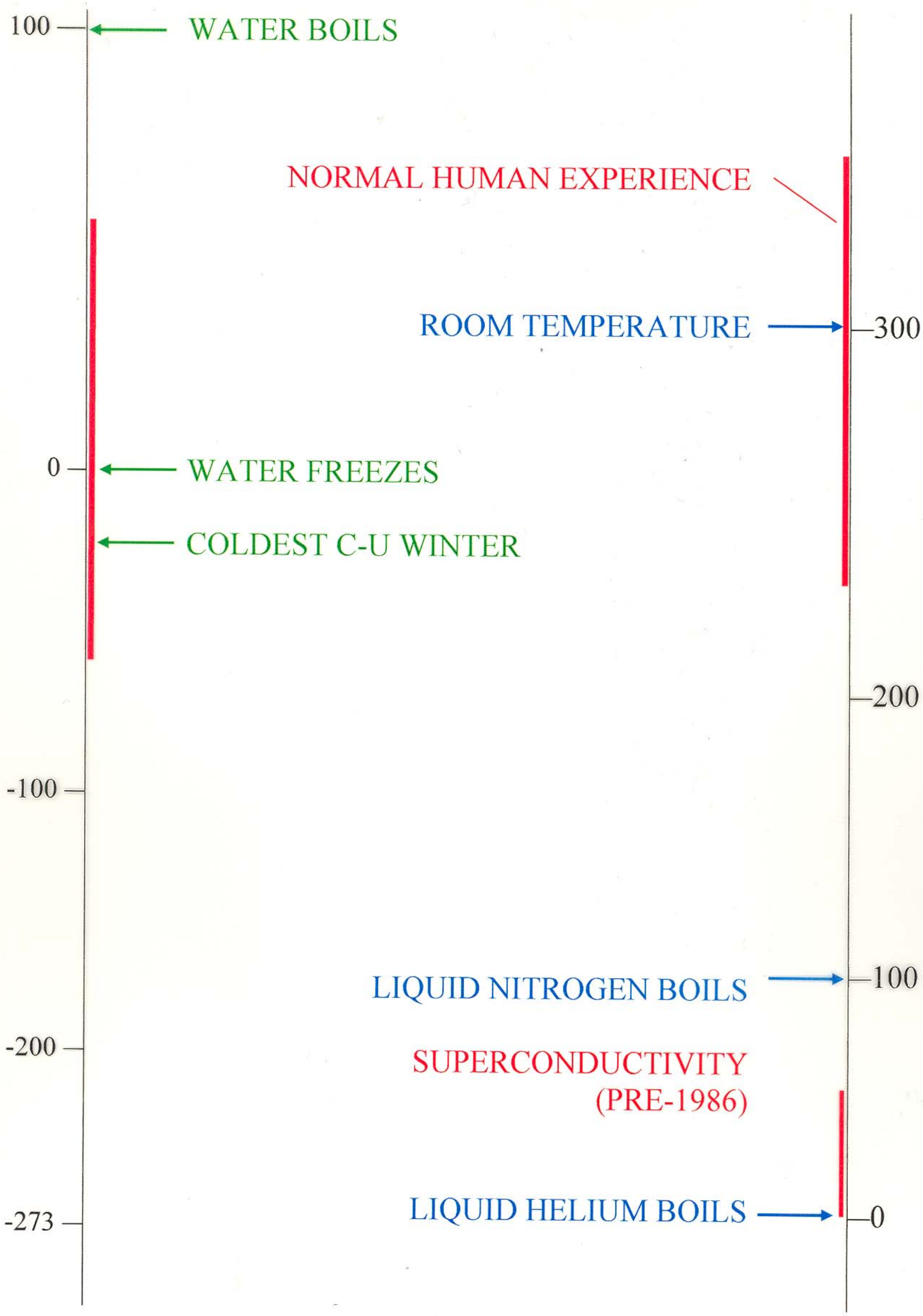
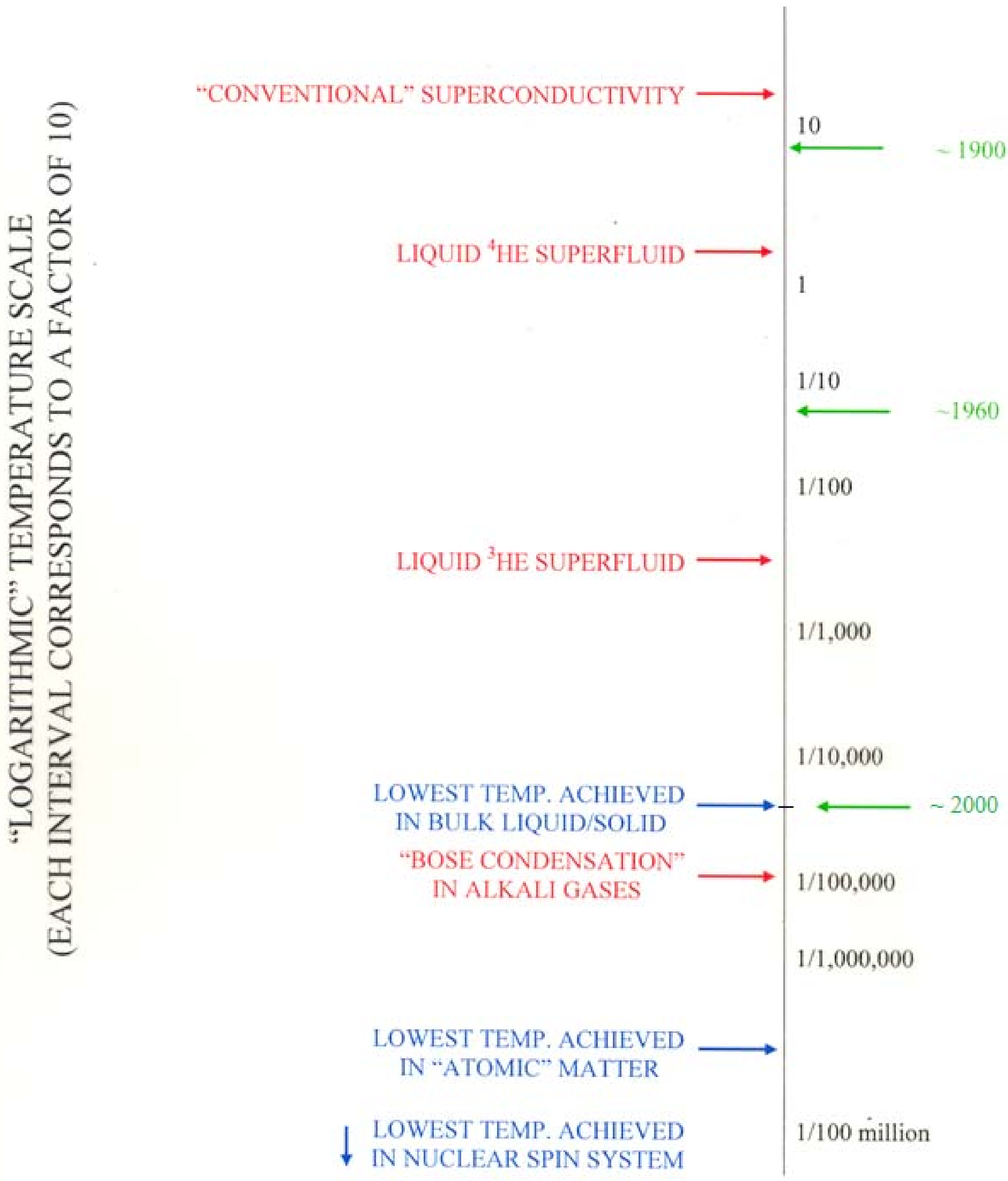
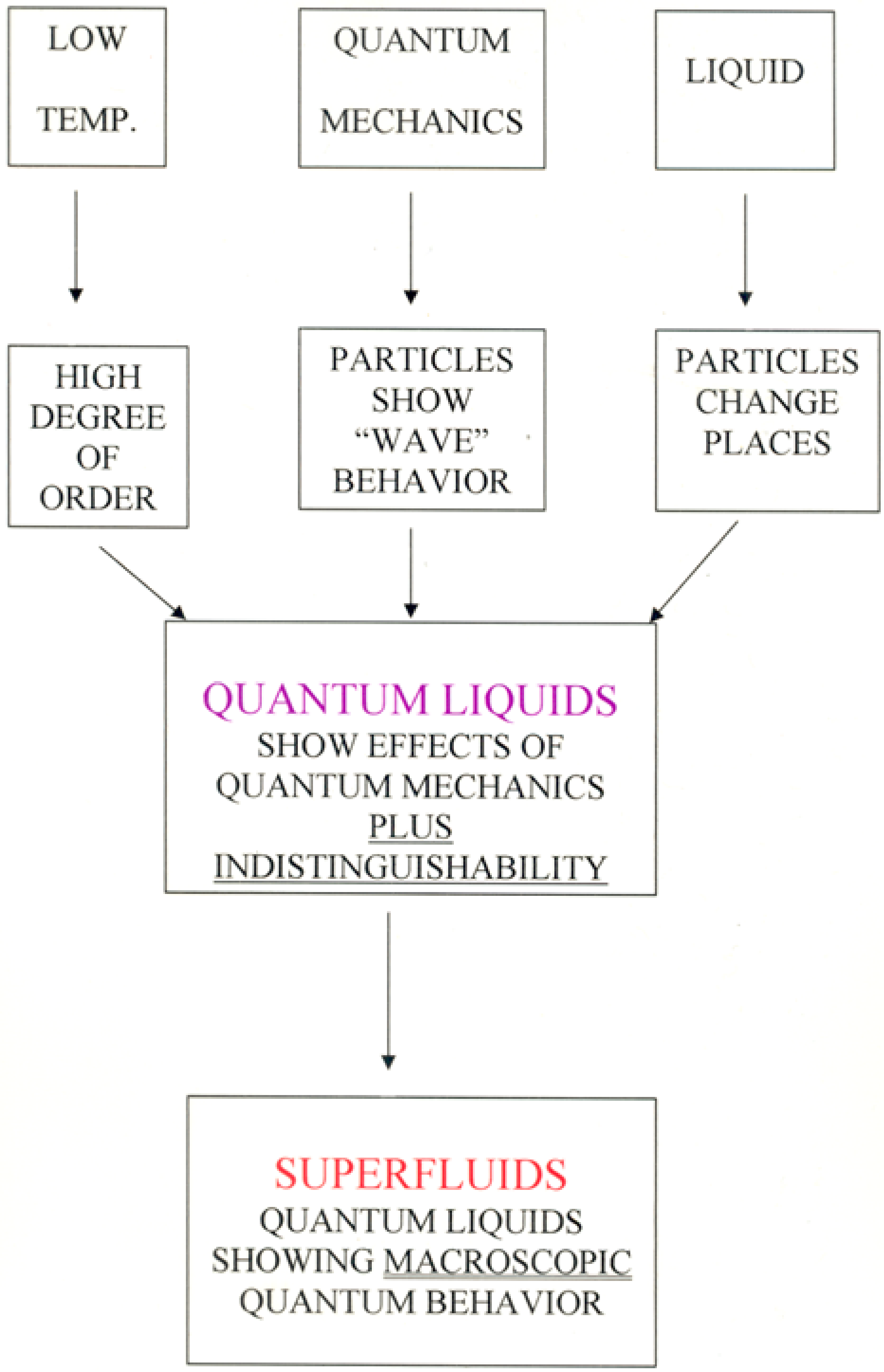
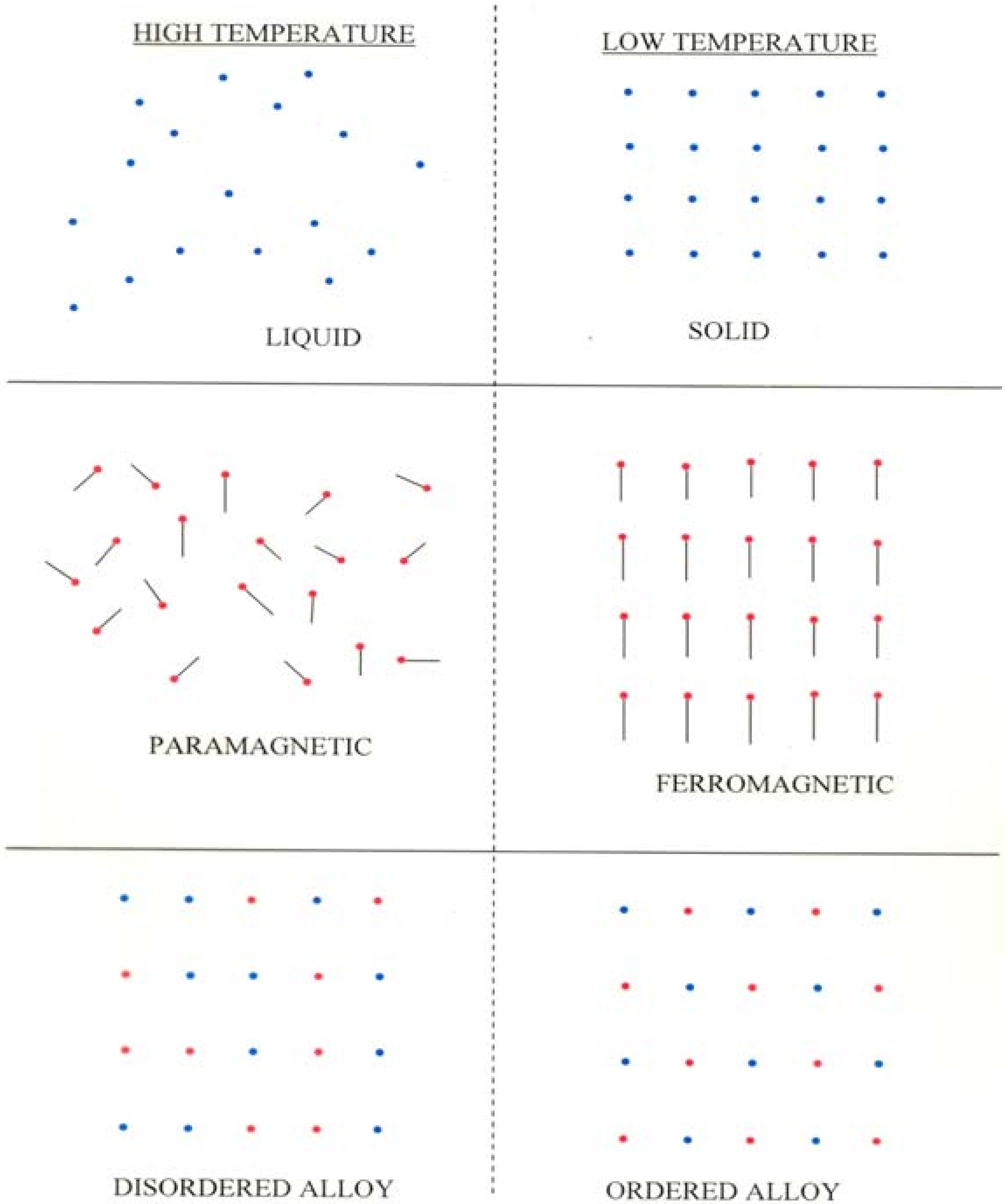

100
WATER BOILS
NORMAL HUMAN EXPERIENCE
ROOM TEMPERATURE
300
0
WATER FREEZES
COLDEST C-U WINTER
200
-100
LIQUID NITROGEN BOILS
100
-200
SUPERCONDUCTIVITY
(PRE-1986)
LIQUID HELIUM BOILS
-273
0

SUPERCONDUCTIVITY
(1997)

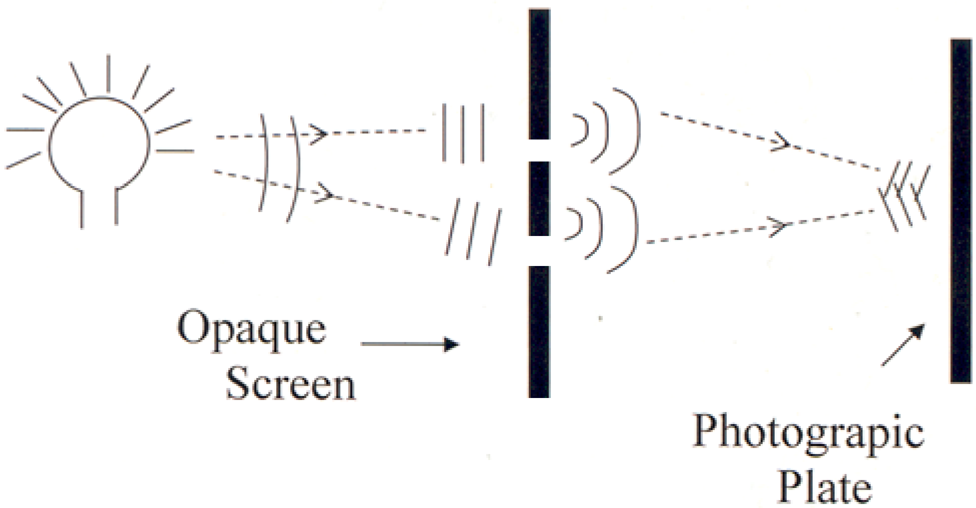

"LOGARITHMIC" TEMPERATURE SCALE
(EACH INTERVAL CORRESPONDS TO A FACTOR OF 10)
"CONVENTIONAL" SUPERCONDUCTIVITY
10
~ 1900
LIQUID ^{4}HE SUPERFLUID
1
1/10
~1960
1/100
LIQUID ^{3}HE SUPERFLUID
1/1,000
1/10,000
LOWEST TEMP. ACHIEVED IN BULK LIQUID/SOLID
~ 2000
"BOSE CONDENSATION" IN ALKALI GASES
1/100,000
1/1,000,000
LOWEST TEMP. ACHIEVED IN "ATOMIC" MATTER
LOWEST TEMP. ACHIEVED IN NUCLEAR SPIN SYSTEM
1/100 million


LOW TEMP.
QUANTUM MECHANICS
LIQUID
HIGH DEGREE OF ORDER
PARTICLES SHOW "WAVE" BEHAVIOR
PARTICLES CHANGE PLACES
QUANTUM LIQUIDS
SHOW EFFECTS OF QUANTUM MECHANICS PLUS INDISTINGUISHABILITY
SUPERFLUIDS
QUANTUM LIQUIDS SHOWING MACROSCOPIC QUANTUM BEHAVIOR

TEMPERATURE, ORDER and DISORDER



PARTICLES AS WAVES



Quantitative particle-wave relation
("de Broglie relation"):

mass

velocity

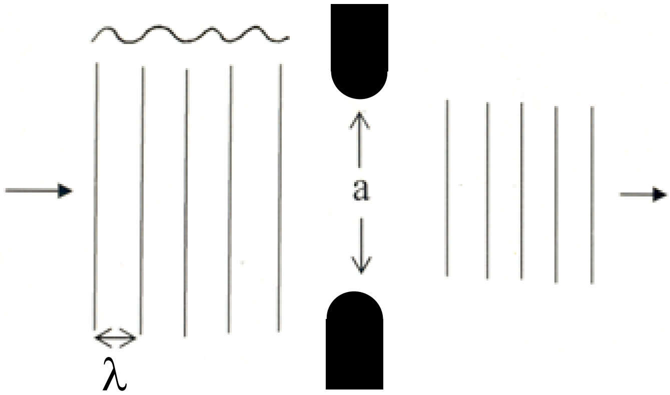
(particle
properties)

$$\boxed{\lambda = h/mv}$$

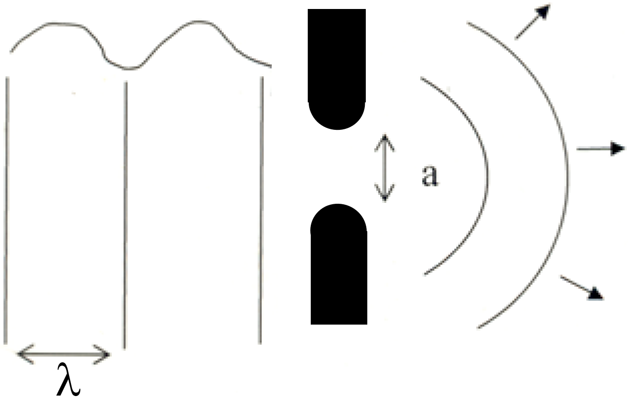
wavelength
(wave property)

Planck's constant,
~ 6 x 10^{-34} joule secs

When does a "wave" behave like a "particle"?



$\lambda \ll a$:
No diffraction:
"Particle-like"



$\lambda \gtrsim a$
Diffraction and interference

since $\lambda = h/mv$ (De Broglie) to get $\lambda \gtrsim a$ need

$$v \lesssim h/ma: \text{ but } \frac{1}{2}mv^2 \sim k_B T$$

so to see "wave" effects need

$$\boxed{T \lesssim h^2/2mk_B a^2}$$

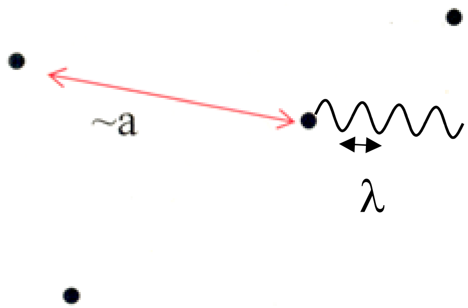
Boltzmann's constant, $\sim 10^{-23}$ joules/degree

In a gas/liquid/solid, take "slit width" a ~ interparticle spacing

⇒ to get "wavelike" behavior, need (for atoms)
$T \lesssim 20°K$/(atomic number)

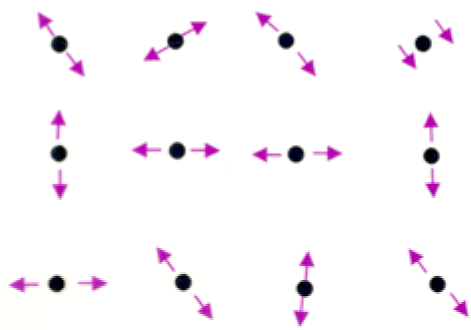
(electrons show "wavelike" behavior for all T in liquid/solid phase)

Why "Quantum Liquids"?



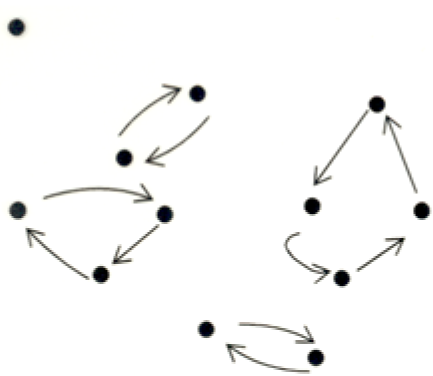
Gas: (usually)

$\lambda \ll a$
so no "wave"
(quantum) effects



Solid at low T:

$\lambda \gtrsim a$ but atoms
don't change places



Liquid at low T:

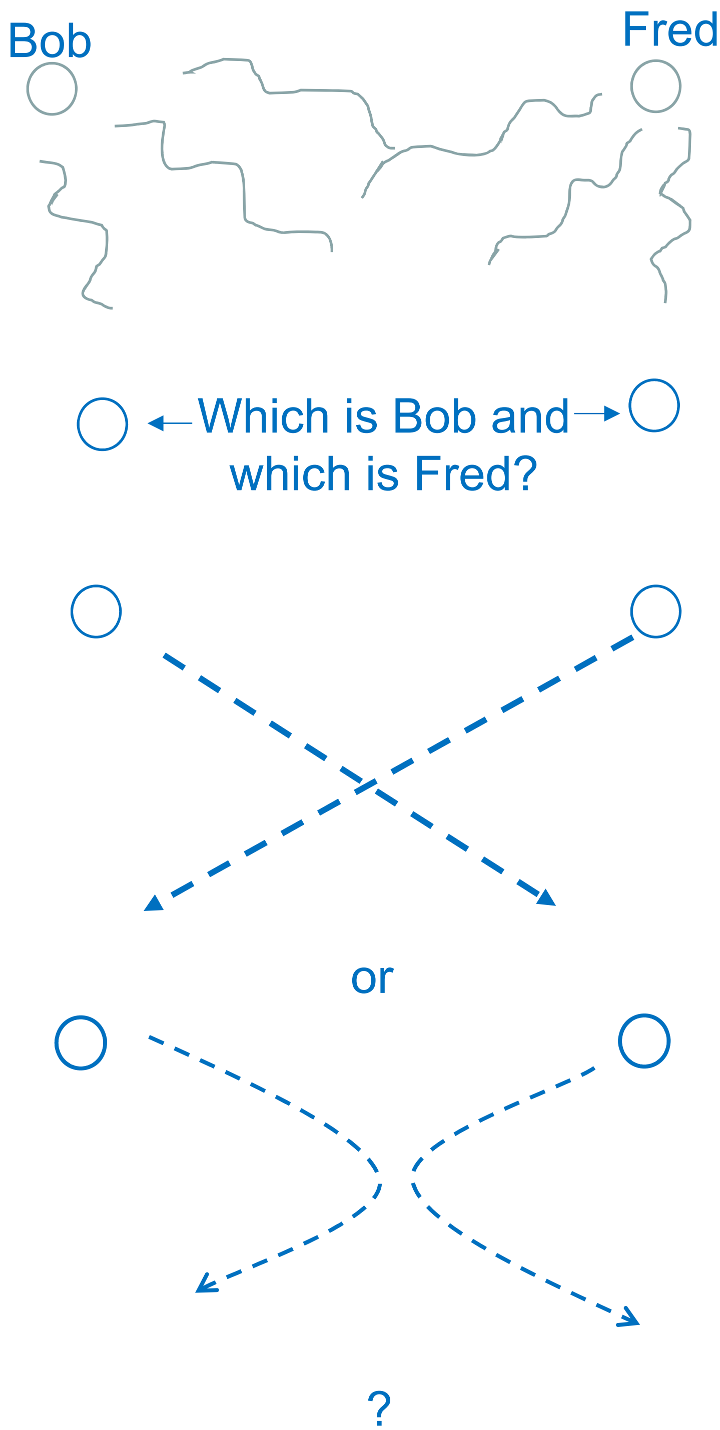
$\lambda \gtrsim a$ <u>and</u>
atoms change places

need: $T \lesssim 20°$ K/(atomic no.) and liquid!

Atoms: helium (and ultracold atomic gases)
Electrons: all liquid/solid metals

Indistinguishablity of elementary particles

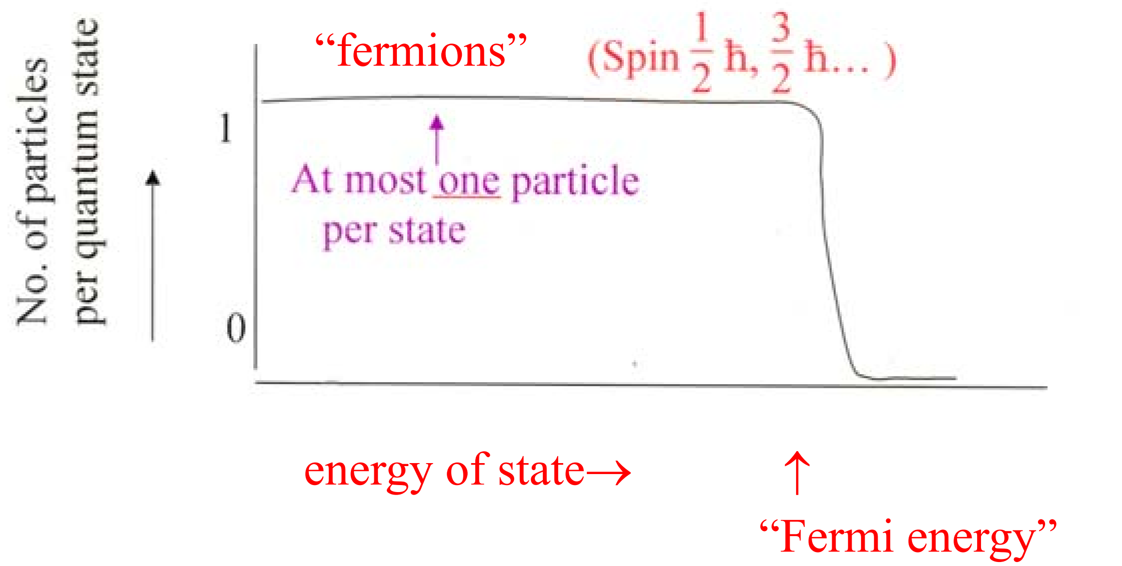
Because particles behave like waves, impossible to “tag” them.

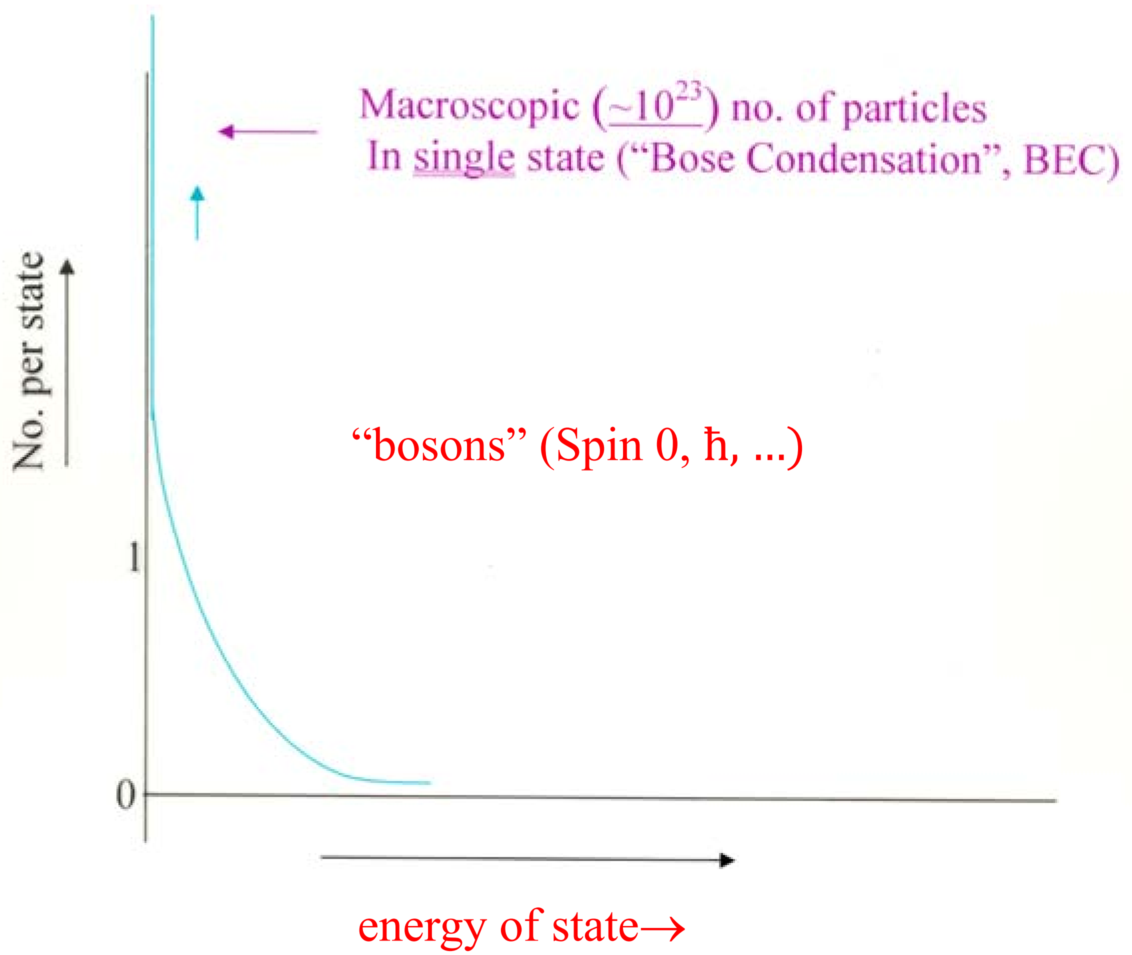


Evidently, for this property to be important, must be able to change places

Result of indistinguishability:

<u>"QUANTUM STATISTICS"</u>





WHY BEC?

I. Qualitative argument:

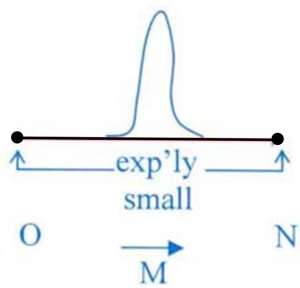
Distribute N objects between 2 boxes: what is probability P(M) of finding M in one box?

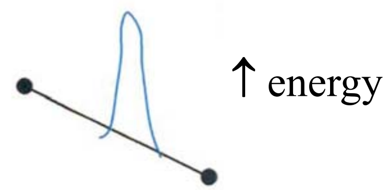
A. Objects distinguishable ($\equiv$ coin toss):

P(M)

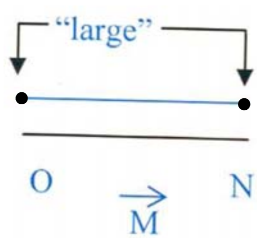
=N!

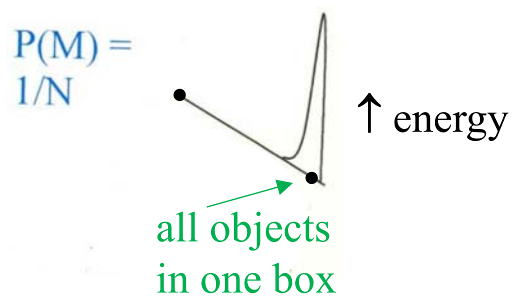
M! N=M!





B. Objects indistinguishable (bosons):





II. Quantitative argt. (Einstein, 1925):

chemical potential, ≤ 0

$$n_i(T) = [\exp(\varepsilon_i - \mu(T)/k_B T - 1)]^{-1}$$

$\mu(T)$ fixed by: $\sum_i n_i\,(T{:}\,\mu(T)) = N \longleftarrow$ total no. of particles

$T \to \infty \Rightarrow \mu \to -\infty$: $T\downarrow \Rightarrow \mu\uparrow$. But what if

$$\sum_i [\exp(\varepsilon_i/k_B T) - 1]^{-1} < N?$$

Einstein: Macroscopic no. of particles occupy lowest (ε=0) state!

Condition for this to happen: roughly, $T \lesssim h^2/2mk_B a^2$ (as above)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Science
22 December 1995
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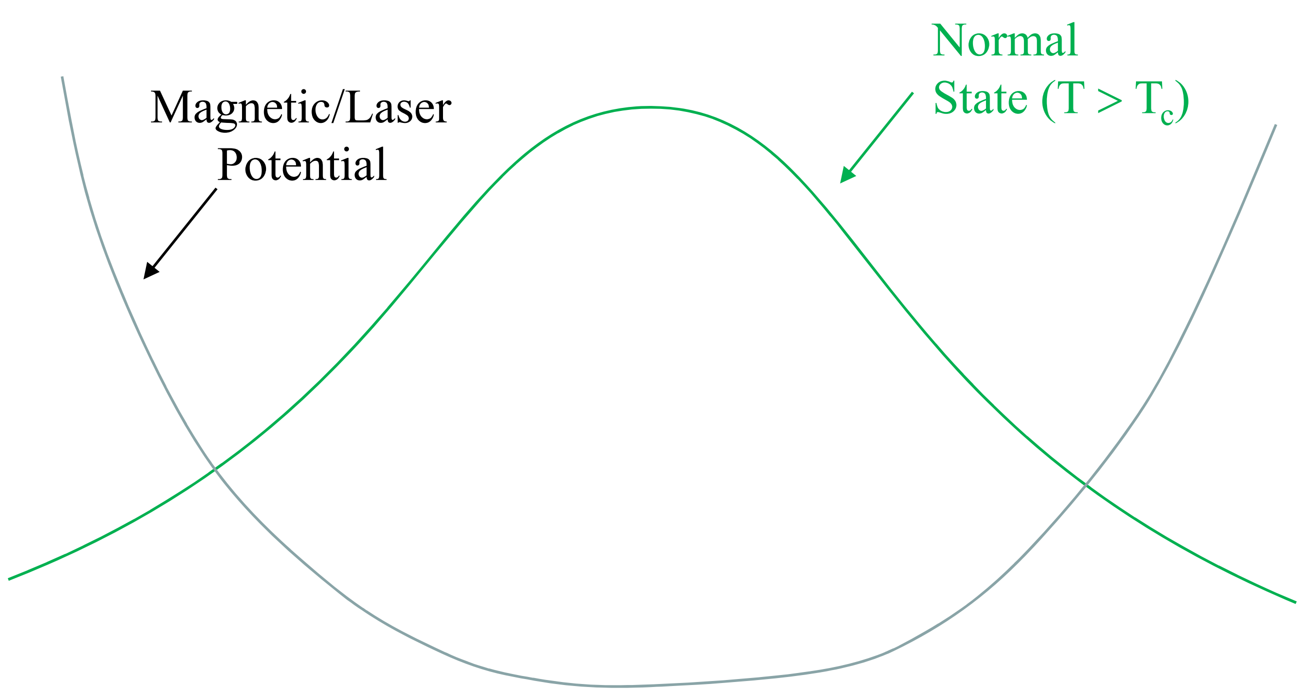
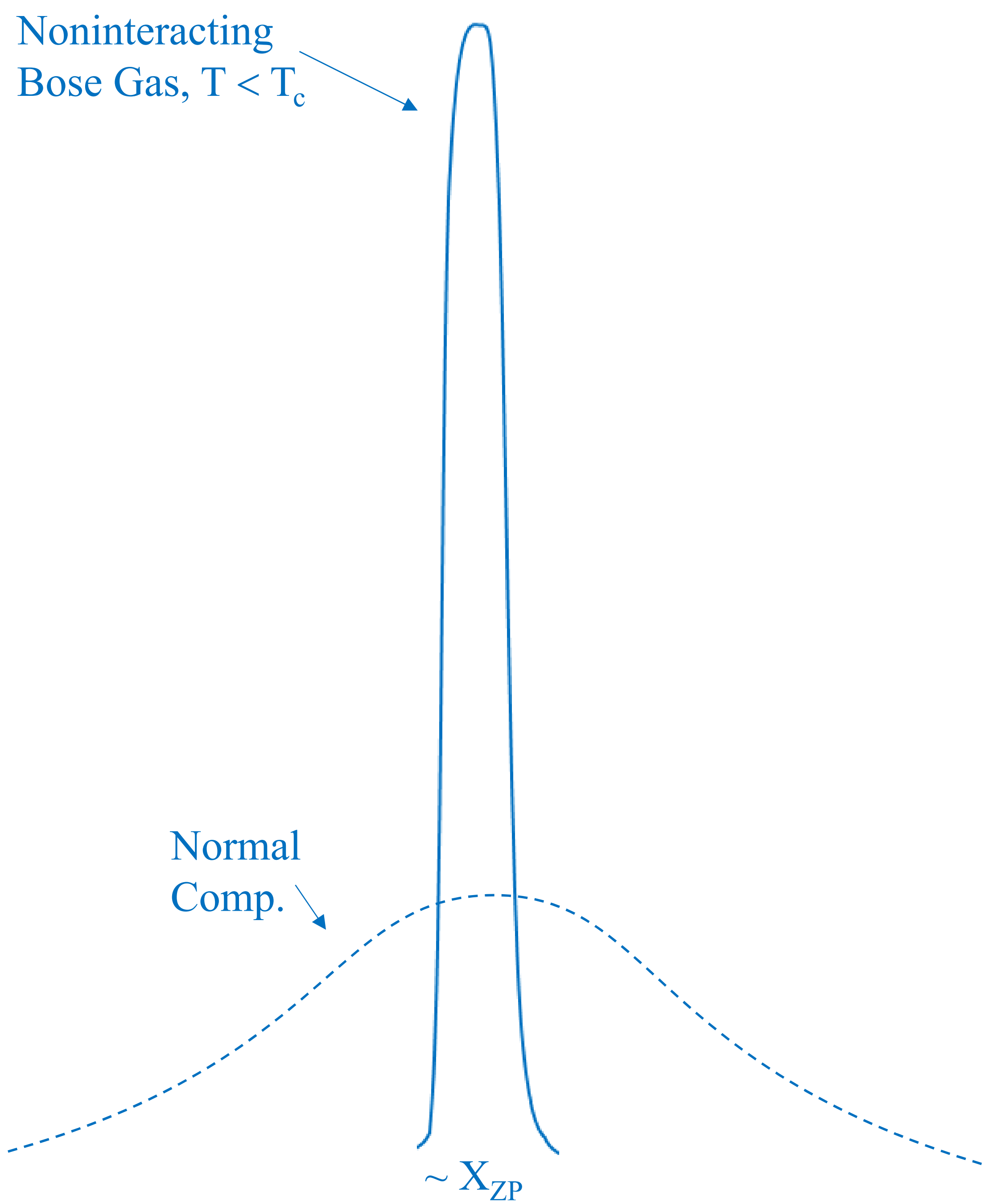
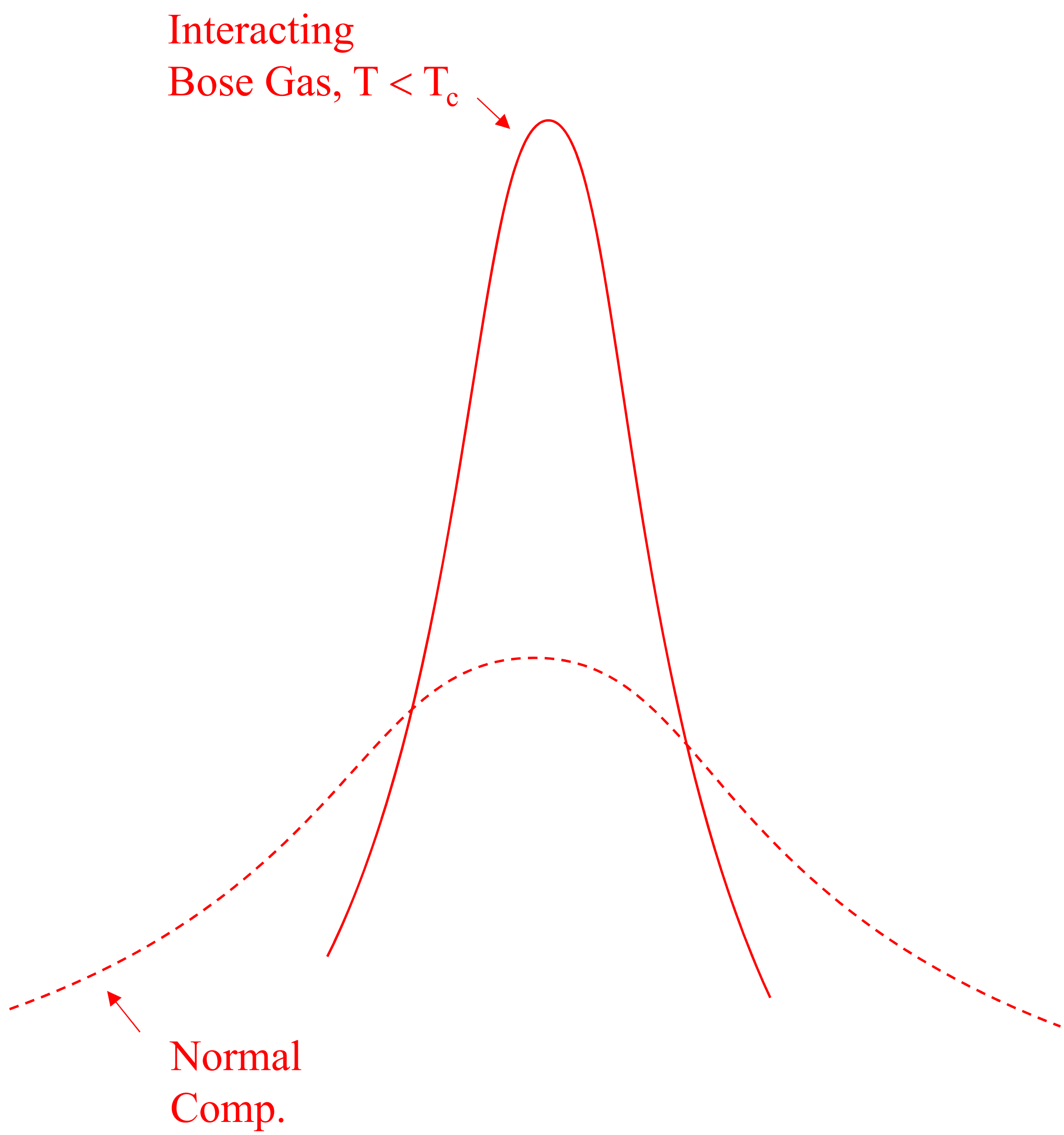
$7.00
Molecule of the Year
the Bose-Einstein Condensate

HOW TO SEE BEC OCCURRING?

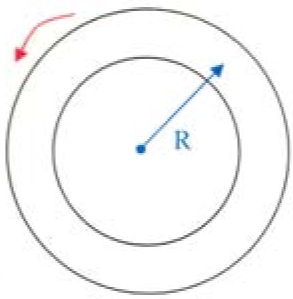
LITERALLY




Noninteracting
Bose Gas, T < T$_c$
Normal
Comp.
~ X$_{ZP}$


Interacting
Bose Gas, T < T_c
Normal
Comp.

"NO-ROTATION" EFFECT IN LIQUID ^{4}HE



Walls rotating with ang. velocity

$\omega \lesssim \omega_c \Leftarrow \equiv \hbar/m R^2$ (typically ~ 1 revolution/hour!)

What does liquid do?

General principle: Average ang. velocity of atoms ($\bar{\omega}$) as close as possible to ω

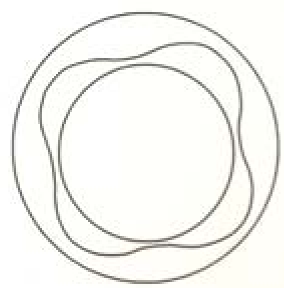
↑ : Single-atom states must obey quantization condition:

$$\underbrace{n\lambda = 2\pi R \qquad + \text{d.B. } \lambda =}\ h/mv$$

$$\Rightarrow \quad L = mvR = n\hbar \qquad (\hbar \equiv h/2\pi)$$

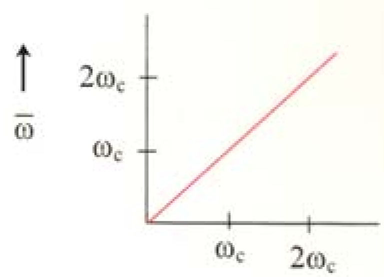
ang. momentum

$$\Rightarrow \quad L/mR^2 \equiv \omega = n\omega_c$$

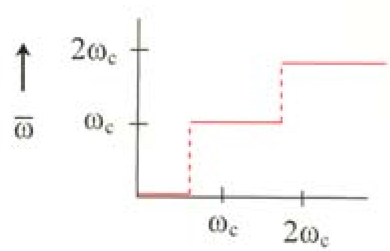


A. "Normal" (non-BEC) system: many different single-particle states occupied (typical value of $n \sim (kT/\hbar\omega_c)^{1/2} \sim 10^7$)

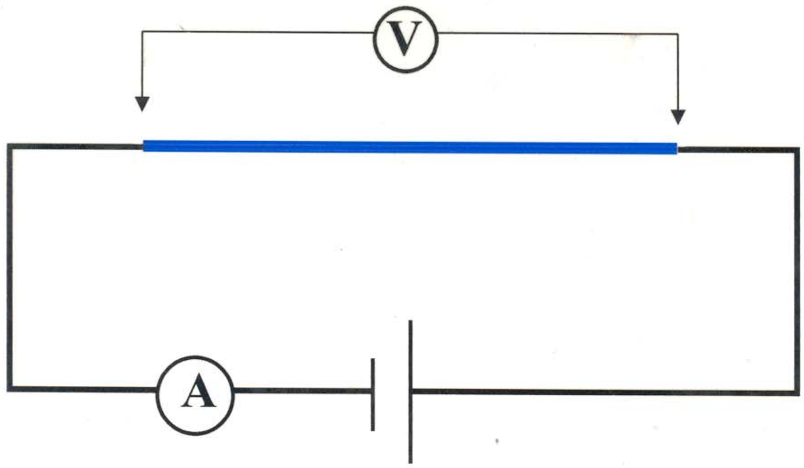
$\Rightarrow$ to get $\bar{\omega} = \omega$, just shift atoms slightly between states.



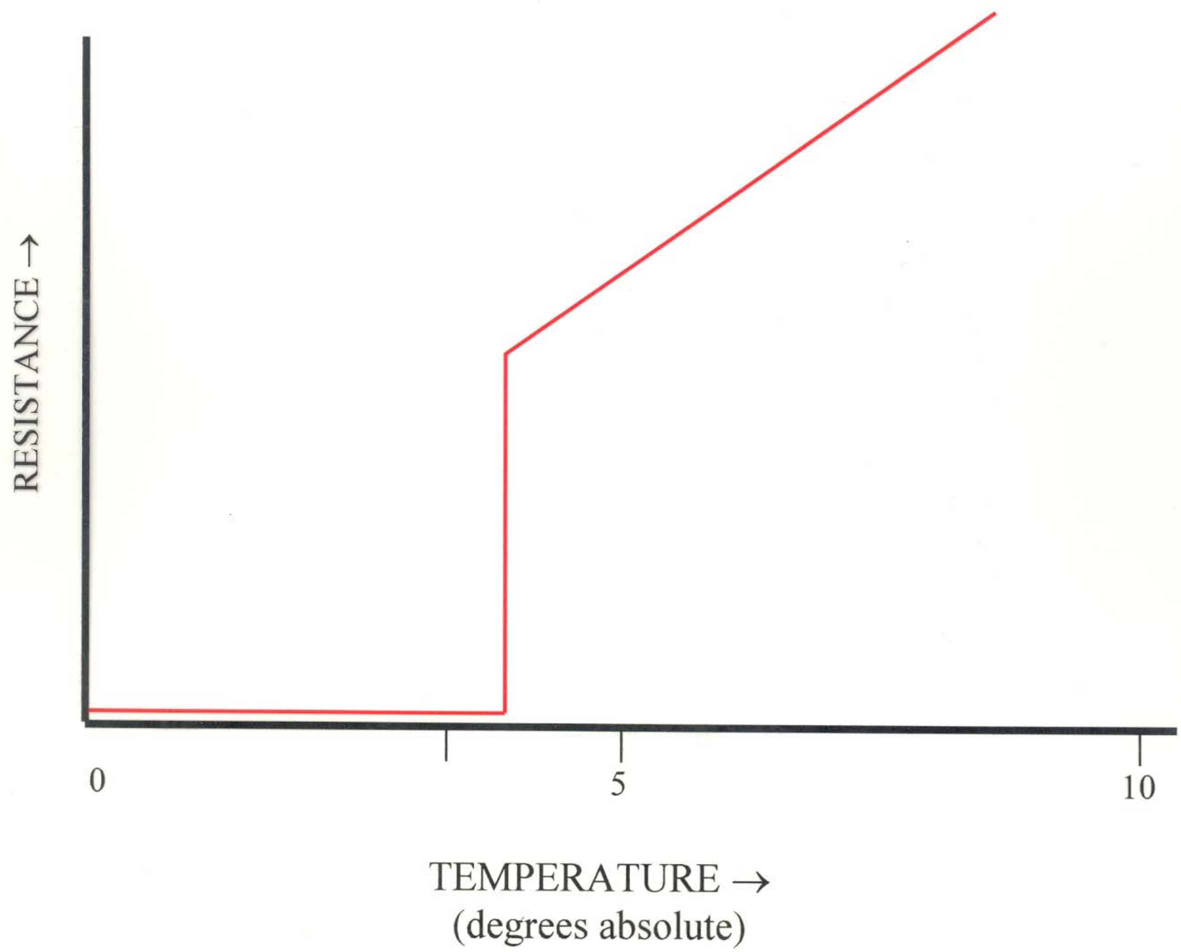
B. BEC system ($T \ll T_c$): (almost) all atoms in condensate must have same value of n. $(n_o) \Rightarrow \bar{\omega} \cong n_o\,\omega_c$



Superconductivity

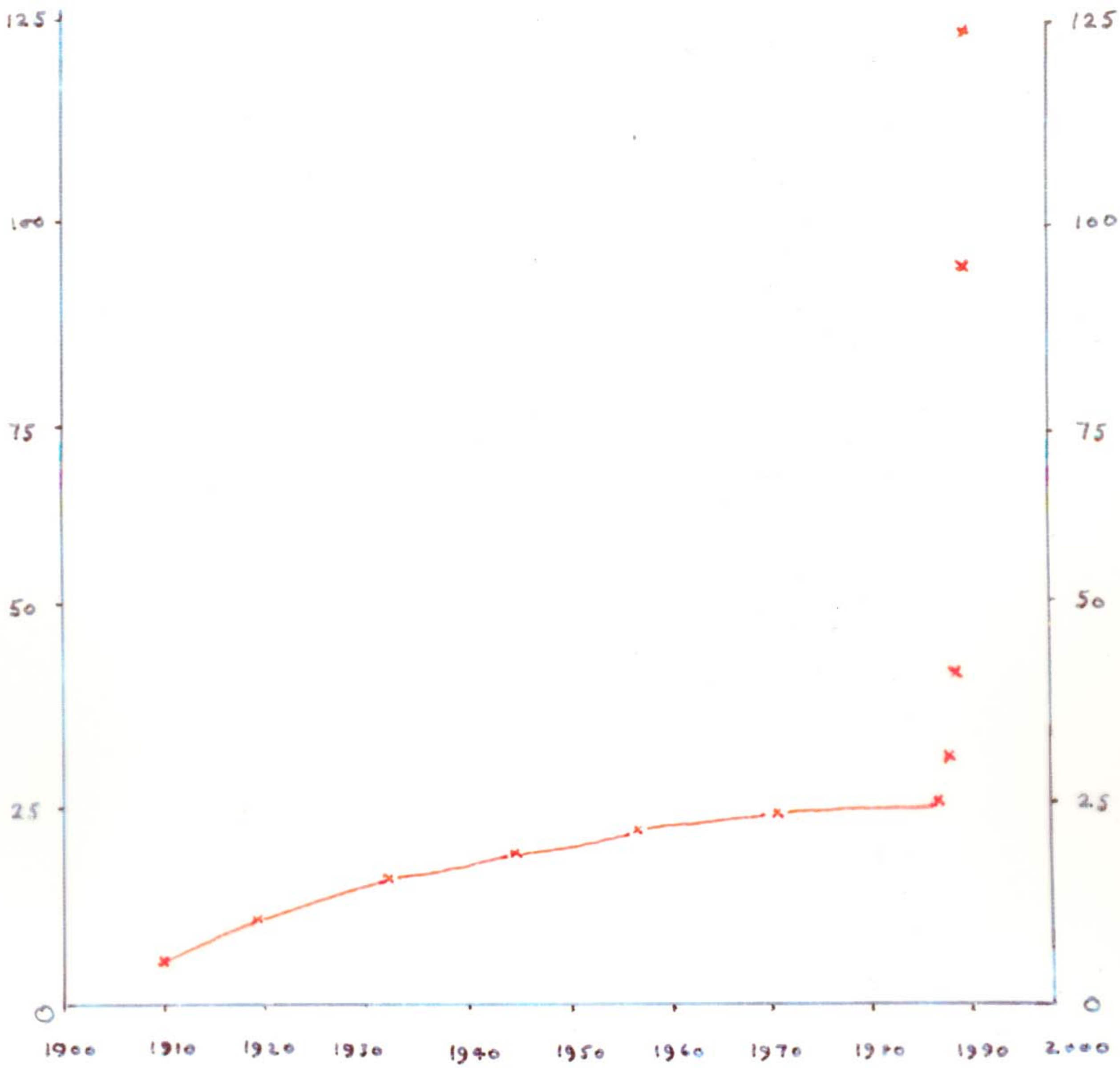


resistance of ▬ = V/A = voltage/current



2014

HISTORY OF THE HIGHEST TEMPERATURE ("T_c") AT WHICH SUPERCONDUCTIVITY KNOWN



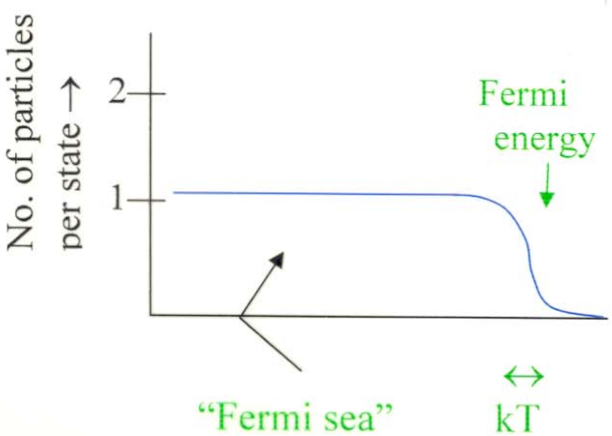
PHYSICS OF SUPERCONDUCTIVITY

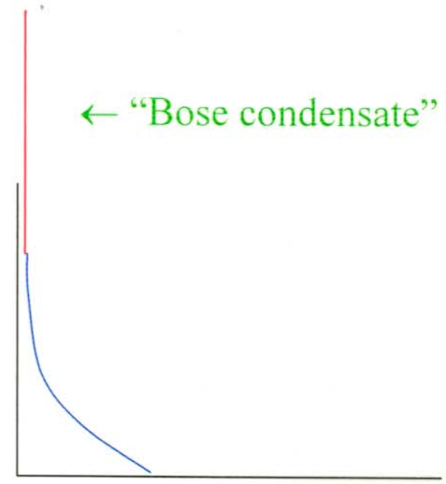
"Spin" of elementary particles $= \frac{n}{2}\hbar$

$0, 1, 2....$ bosons

$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}....$ fermions

At low temperatures:





Electrons in metals: spin ½ ⇒ fermions

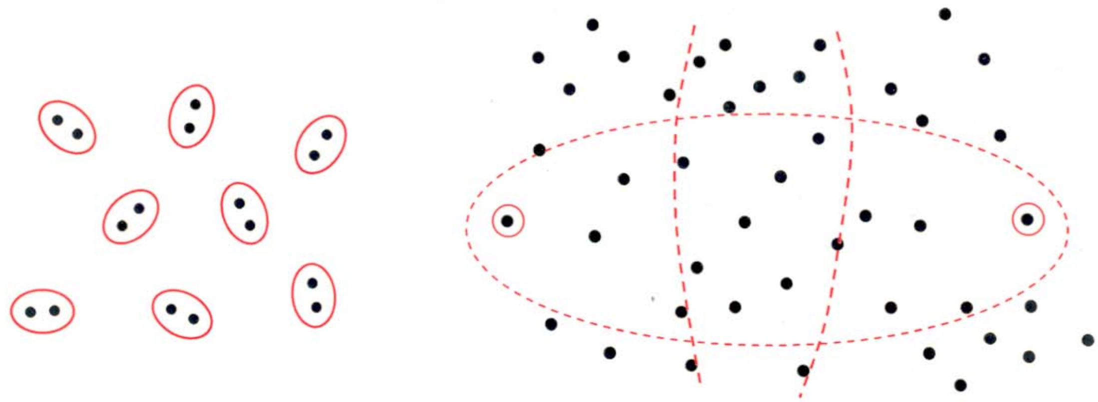
But a compound object consisting of an even no.

of fermions has spin 0, 1, 2 … ⇒ boson.

(Ex: $2p + 2n + 2c = {}^4He$ atom)

⇒ can undergo Bose condensation

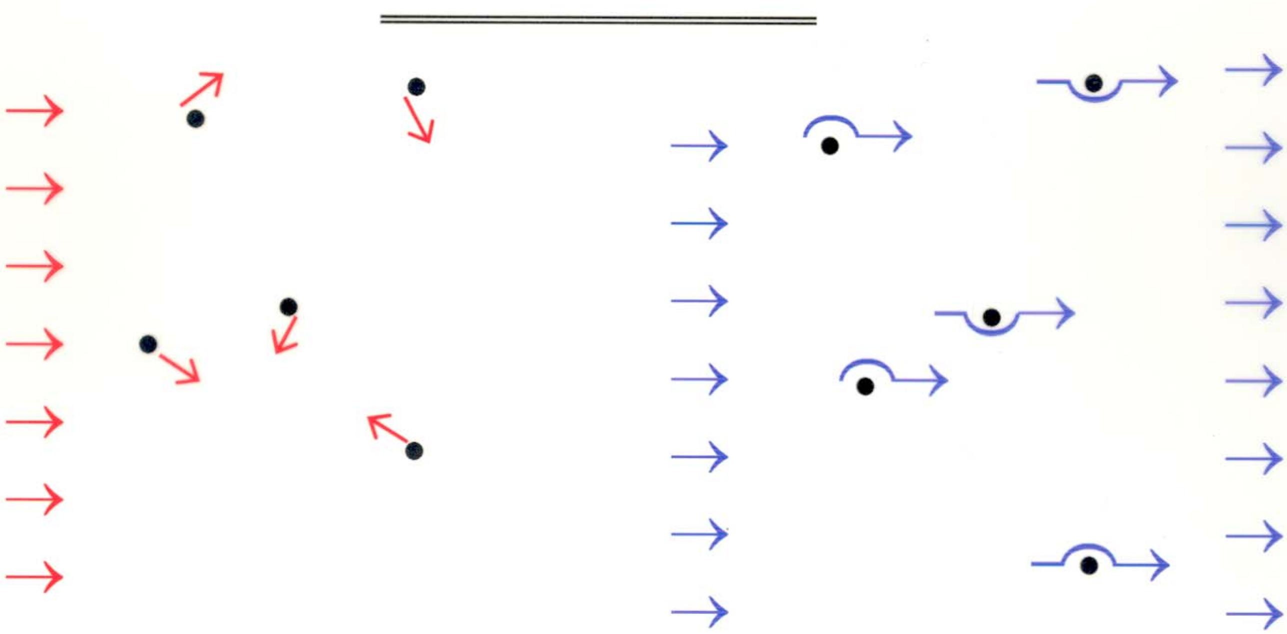
Pairing of electrons:



"di-electronic molecules" Cooper Pairs

In simplest ("BCS") theory, Cooper pairs, once formed, must automatically undergo Bose condensation!

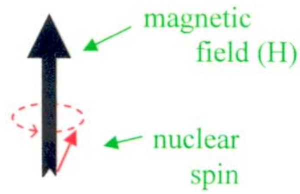
⇒ must all do exactly the same thing at the same time (also in nonequilibrium situation)



Single electrons in normal metal

Cooper pairs in superconductor

NUCLEAR MAGNETIC RESONANCE

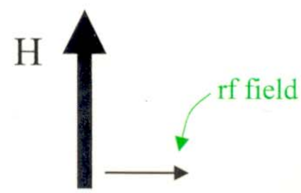


Rate of "precession"

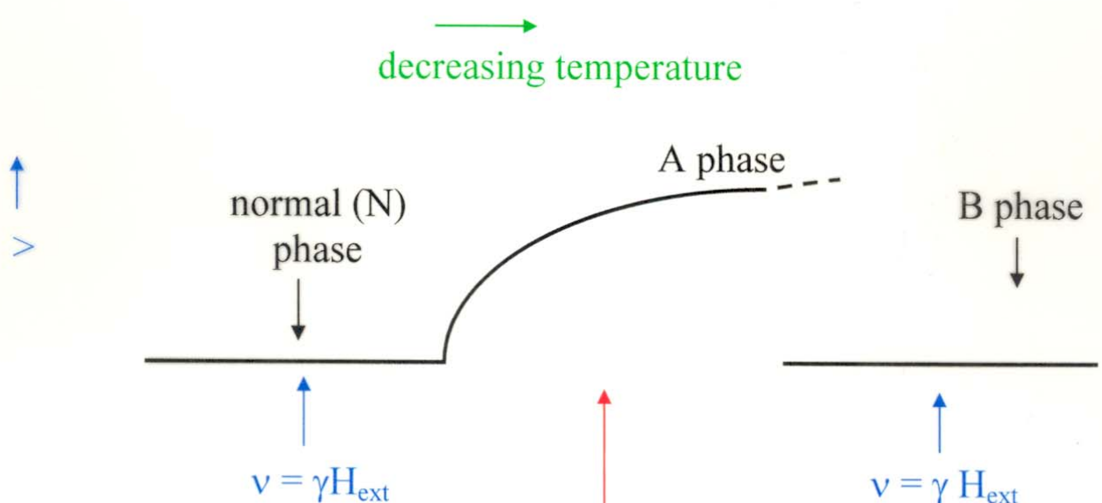
$\nu = \gamma H$

"gyromagnetic ratio"

γ is known, (in ^{3}He, ~ 3 kHz/gauss)
so, rate of precession (ν) measures magn. field (H)
To measure ν, apply
oscillating (r.f.) field $\perp$ H:
field is strongly absorbed when its frequency is ν.



NMR IN LIQUID ^{3}He BELOW 3mK:



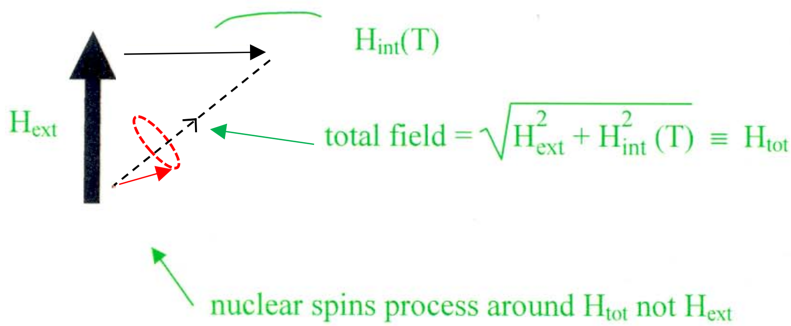
$$\nu = \gamma\ \sqrt{H_{ext}^2 + H_{int}^2\,(T)}$$

THE ^{3}HE NMR PUZZLE (cont.)

In A phase, precession freq. ν is larger than value (γH_{ext}) in N phase, and given be expression of form

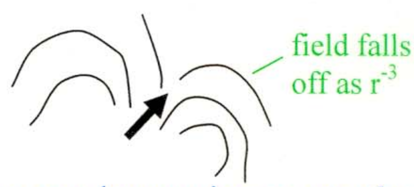
$$\nu = \gamma \sqrt{H_{ext}^2 + H_{int}^2(T)}$$

Simplest interpretation:



Problem:

Only possible origin of H_{int} (T) is other nuclear spins



Max. value of field of one nuclear spin on another (at distance of closest approach of atoms) < 1 gauss.

But, experimental value of $H_{int}(T)$ is ~ 30 gauss!

FIRST EVIDENCE FOR BREAKDOWN OF QUANTUM MECHANICS?

RESULT OF MORE SOPHISTICATED APPROACH:

A. Simple classical argument too naive.

(no "transverse" internal field)

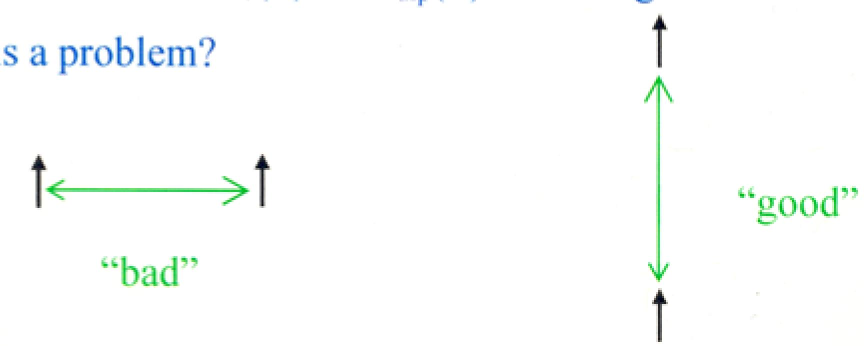
B. Nevertheless, indeed predict formula

$$\nu = \gamma \sqrt{H_{ext}^2 + H_o^2(T)}$$

where $H_o^2(T)$ is proportional to average value of nuclear dipole interaction energy $E_{dip}(T)$.

Experimental value of $H_o(T) \rightarrow E_{dip}(T) \sim 10^{-3}$ ergs/cm^3

Why is this a problem?



— energy difference (ΔE) between "good" and "bad" orientations $< 10^{-7}$ K per pair.

— thermal energy (E_{th}) $(= k_B T) \sim 10^{-3}$ K.

$\Rightarrow$ preference for "good" orientation over "bad" only $\sim \Delta E/E_{th} < 10^{-4}$

$\Rightarrow$ resulting value of $E_{dip}(T)$ much too small to fit experiment.

Need preference for "good" over "bad" ~1 not $\sim \Delta E/E_{th}$!

SPONTANEOUSLY BROKEN SPIN-ORBIT SYMMETRY:

the analogy with ferromagnetism

FERROMAGNET

difference in energy per spin = ΔE (small)

Above Curie temp. ("paramagnetic" phase), spins behave independently $\Rightarrow$ thermal energy E_{th} competes with ΔE $\Rightarrow$ polarization only $\sim \Delta E/E_{eth} \ll 1$

Below T_c ("ferromagnetic" phase): strong (exchange) forces constrain all spins to lie parallel:

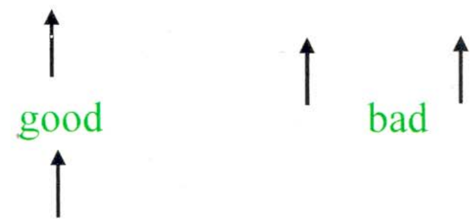
↑↑↑↑↑... or ↓↓↓↓↓...

"good" "bad"

$E_{good} - E_{bad} \sim N\Delta E \gg E_{th}$

$\Rightarrow$ polarization ~ 1

LIQUID ^{3}HE



difference in energy per pair $\equiv \Delta E < 10^{-7}$ K

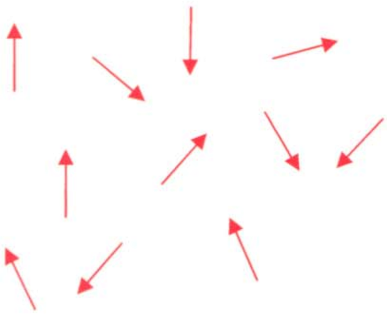
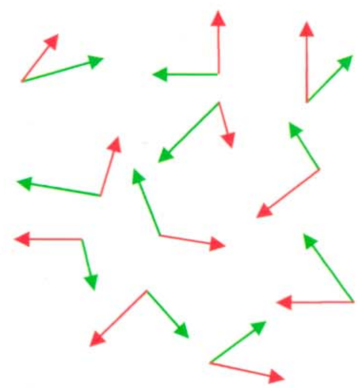
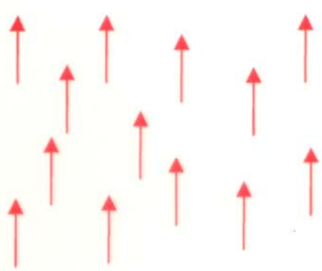
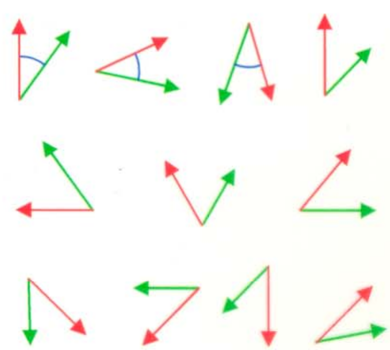
In normal phase, pairs behave independently $\Rightarrow E_{th}$ competes with $\Delta E \Rightarrow$ "polarization" (pref. for good orientation over bad) only $\sim \Delta E/E_{th} \ll 1$.

In A phase, assume: strong (kinetic-energy, VDW) forces constrain all pairs to behave in same way $\Rightarrow$ either all "good" or all "bad"

$E_{good} - E_{bad} \sim \underbrace{N}_{\sim 10^{23}!}\,\Delta E \gg E_{th}$

$\Rightarrow$ polarization can be ~ 1

SBSOS: ORDERING MAY BE SUBTLE

FERROMAGNET		LIQUID ^{3}HE
	$\Leftarrow$ NORMAL PHASE $\Rightarrow$	
$\Downarrow$		$\Downarrow$
	$\Leftarrow$ ORDERED PHASE $\Rightarrow$	

(↗ = total spin of pair

↗ = relative orbital ang. momentum)

$\langle \underset{\sim}{S} \rangle = \langle \underset{\sim}{L} \rangle = 0$

$\langle \underset{\sim}{S} \rangle \neq 0$

but $\langle \underset{\sim}{L} \times \underset{\sim}{S} \rangle \neq 0!$

Amplification of ultra-weak effects by BEC (cf NMR):

Example: P- (but not T-) violating effects of neutral current part of weak interaction:

For single elementary particle, any EDM $\underset{\sim}{d}$ must be of form

$\underset{\sim}{d} = \text{const.}\ \underset{\sim}{J}$ ← violates T as well as P.

But for ^{3}He – B, can form

$$d \sim \text{const.}\ \underset{\sim}{L} \times \underset{\sim}{S} \sim \text{const.}\ \hat{\underset{\sim}{\omega}}$$

↑

violates P but not T.

Effect is tiny for single pair, but since all pairs have same value of $\underset{\sim}{L} \times \underset{\sim}{S}$, is multiplied by factor of $\sim 10^{23} \Rightarrow$

macroscopic P-violating effect?