Application of BCS-like Ideas to Superfluid 3-He

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Electrons in Metals (BCS):

Fermions of spin ½, $T_F \sim 10^4 K,\ T_c \sim 10K$

$\Rightarrow$ strongly degenerate at onset of superconductivity

Normal state: in principle described by Landau Fermi-liquid theory, but "Fermi-liquid" effects often small and generally very difficult to see.

BCS: model normal state as

Weakly interacting gas with weak "fixed"

attractive interaction

Liquid ^{3}He:

also fermions of spin ½ $T_F \sim 1K,\ T_c \sim 10^{-3} K$

$\Rightarrow$ again, strongly degenerate at onset of superfluidity

Normal state: must be described by Landau Fermi-liquid theory, Fermi-Liquids effects very strong. (e.g. Wilson ratio ~4)

$\Rightarrow$ low-lying states (inc. effects of pairing) must be described in terms of Landau quasiparticles.

What is Common:

2-particle density matrix has single macroscopic (~N) eigenvalue, with associated eigenfunction

$$F(\boldsymbol{r}_1\boldsymbol{r}_2\sigma_1\sigma_2) \equiv F(\boldsymbol{R} : \boldsymbol{r}\sigma_1\sigma_2)$$

"wave function of Cooper pairs"

(for $\boldsymbol{r}, \sigma_1, \sigma_2$ fixed: GL "macroscopic wave function Ψ(**R**))

STRUCTURE OF COOPER-PAIR WAVE FUNCTION

(in original BCS theory of superconductivity, for fixed **R**, σ_1, σ_2)

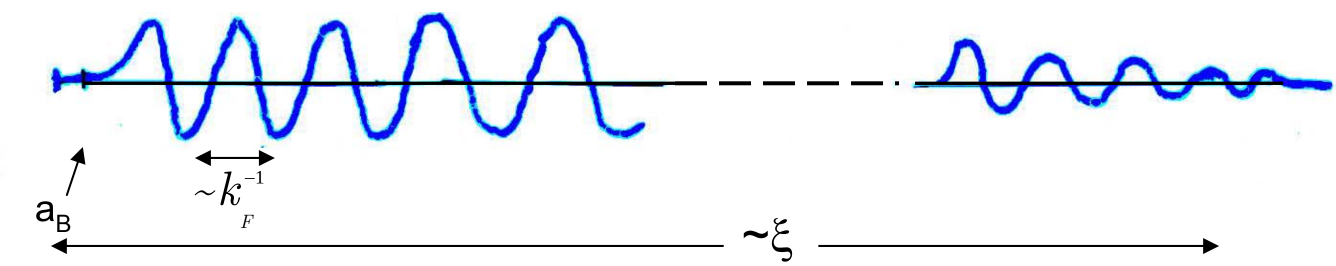
$$F(\boldsymbol{r}) = F(|\boldsymbol{r}|) = \Delta \sum_{\boldsymbol{k}} (2E_k)^{-1} \exp i\boldsymbol{k}\cdot\boldsymbol{r}$$

Energy gap

$$E_k = \left(\varepsilon_k^2 + |\Delta|^2\right)^{1/2}$$

$$\cong \text{const.}\left(N\Delta / E_F \Omega^{1/2}\right) \frac{\sin k_F r}{k_F r} \exp -r/\xi$$

$$\xi = \text{"pair radius"} \sim \hbar v_F / \Delta \left(\sim 10^4 \text{Å}\right)$$



"Number of Cooper pairs" (N_o) = norm^n of $F(\boldsymbol{r})$

$$\equiv \int |F(\boldsymbol{r})|^2 \, d\boldsymbol{r} \sim \frac{N^2}{\Omega}\frac{\Delta^2}{E_F^2}\frac{1}{k_F^2}\xi \sim N(\Delta / E_F) \sim 10^{-4} N$$

(cf: $N_0 / N \sim 10\%$ in ^{4}He)

In original BCS theory of superconductivity,

$$F(\boldsymbol{r} : \sigma_1\sigma_2) = \frac{1}{\sqrt{2}}\left(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\right) F(|\boldsymbol{r}|)$$

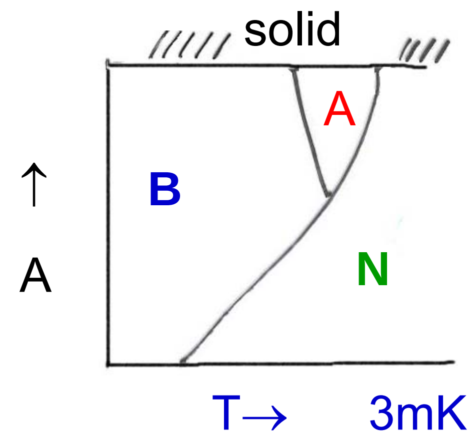
spin singlet — orbital s-wave

⇒PAIRS HAVE NO "ORIENTATIONAL" DEGREES OF FREEDOM

(⇒stability of supercurrents, etc.)

THE FIRST ANISOTROPIC COOPER-PAIRED SYSTEM: SUPERFLUID ^{3}HE

2-PARTICLE DENSITY MATRIX ρ_2 still has one and only one macroscopic eigenvalue
$\Rightarrow$ can still define "pair wave function" $F(\boldsymbol{R},\boldsymbol{r}:\sigma_1\sigma_2)$
However, even when $F \neq F(\boldsymbol{R})$,



$F(\boldsymbol{r}\sigma_2\sigma_2)$ HAS ORIENTATIONAL DEGREES OF FREEDOM!

(i.e. depends nontrivially on $\hat{\boldsymbol{r}},\sigma_1\sigma_2$)

All three superfluid phases have $\ell = S = 1$

A phase ("ABM")

Spin triplet

$$F(\boldsymbol{r}:\sigma_1\sigma_2) = \frac{1}{\sqrt{2}}\left(\uparrow_1\downarrow_2 + \downarrow_1\uparrow_2\right)_{\hat{\boldsymbol{d}}} \times f(\boldsymbol{r})$$

char. "spin axis"

$$f(\boldsymbol{r}) = f_o(|\boldsymbol{r}|)\times\left(\sin\,\theta\cdot\exp\,i\varphi\right)_{\hat{\ell}}$$

char. "orbital axis"

Properties anisotropic in orbital and spin space separately,

e.g. $|\Delta_K| = \left|\Delta\left(\hat{k}\right)\right| = \Delta_o\left|\hat{\boldsymbol{k}}\times\hat{\ell}\right| \Leftarrow$ nodes at $\pm\hat{\ell}$!

WHAT IS TOTAL ANG. MOMENTUM? $\left(N / N\left(\Delta / E_F\right) / N\left(\Delta / E_F\right)^2\right)$?

B phase ("BW")

For any particular direction $\hat{\boldsymbol{n}}$ (in real or **k**-space) can always choose spin axis s.t.

$$F(\hat{\boldsymbol{n}}:\sigma_1\sigma_2) \sim \frac{1}{\sqrt{2}}\left(\uparrow_1\downarrow_2 + \downarrow_1\uparrow_2\right)_{\hat{\boldsymbol{d}}}$$

i.e. $\hat{\boldsymbol{d}} = \hat{\boldsymbol{d}}(\hat{\boldsymbol{n}})$. Alternative description:

BW phase is ^3P_o state "spin-orbit rotated" by 104^o.

L=S=J=O   because of dipole force $\cos^{-1}(-1/4)=\theta_o$

Note: rotation (around axis $\hat{\omega}$) breaks P but not T

$\|$ extℓ field $\boldsymbol{H}_o$ inversion   time reversal

Orbital and spin behavior individually isotropic, but: properties involving spin-orbit correlations anisotropic!

Example: NMR

dipole energy

$$\frac{d\boldsymbol{S}}{dt} = \boldsymbol{S} \times \boldsymbol{H}_o + \frac{\delta E_D}{\delta\theta}$$

$\angle$ of rotation about *rf* field

direction $\hat{\mathscr{H}}_{rf}$

$\boldsymbol{H}_o, \hat{\omega}$

$\uparrow \mathscr{H}_{rf}$ (long)

$\rightarrow \mathscr{H}_{rf}$ (transverse)

In transverse resonance, rotation around $\hat{\mathscr{H}}_{rf}$ equiv. rotation of $\hat{\boldsymbol{\omega}}$ with θ_o unchanged

$\Rightarrow$ No dipole torque.

In longitudinal resonance, rotation changes θ_o

$\Rightarrow$ finite-frequency resonance!

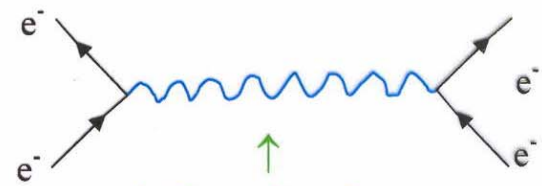
RESOLUTION OF THE PARADOX OF TWO NEW PHASES.

(Anderson & Brinkman, Phys. Rev. Letters **30**, 1108 (1973))

In BCS (weak-coupling) theory for $\ell = 1$, BW phase is always stable, independently of pressure and temperature.

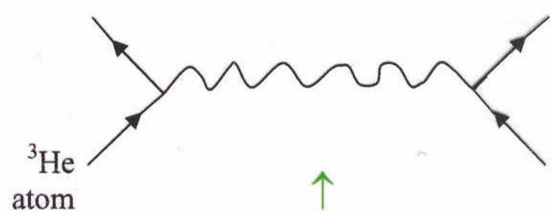
Crucial difference between Cooper pairing in superconductors and ^{3}He:

Superconductor:



↑
lattice vibration, insensitive to onset of pairing of electrons

liquid ^{3}He:



↑
spin fluctuations of ^{3}He system ⇒ sensitive to onset of pairing

⇒ "feedback" effects: Over most of the phase diagram, BW state stable as in BCS theory. But at high temperature and pressure, feedback effects uniquely favor ABM phase.

“Exotic” Properties of Superfluid ^{3}He

A. Orientation const. in space, varying in time:
— spin dynamics (NMR)
— orbital dynamics (“normal locking”) (A phase)
— effect of macroscopic ang. momentum? (A phase)

B. Orientation const. in time, varying in space
— spin textures (^{3}He-A) ($\hat{d} = \pm \hat{l}$) in equation

$\hat{\ell}$ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

$\hat{d}$ ↑ ↑ ↗ ↗ → ↘ ↓ ↓ ↓

(carries spin current)

— orbital textures
— topological singularities (boojums, “half-quantum” vortices. . . .)
— instability of supercurrents in ^{3}He-A.

C. Orientation varying in both space and time
— spin waves
— orbital waves
—”flapping” and “clapping” modes

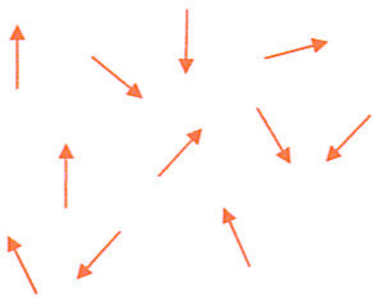
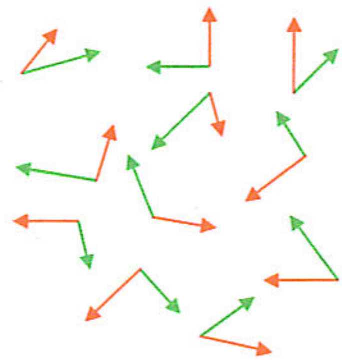
D. Amplification of ultra-weak effects

SPONTANEOUSLY BROKEN SPIN-ORBIT SYMMETRY

Ferromagnetic analogy:

FERROMAGNET	LIQUID ^{3}HE
$\hat{H} = \hat{H}_o + \hat{H}_z$	$\hat{H} = \hat{H}_o + \hat{H}_D$
$\uparrow$ invariant under simult. rotation of all spins	$\uparrow$ invariant under relative rotation of spin + orbital coordinate systems
$\hat{H}_z = -\mu_B \mathcal{H} \sum_i S_{zi}$ (extl. field: $\mathcal{H}$); breaks spin-rot^n symmetry	$\hat{H}_D = g_D \sum_{ij} \left(\frac{\underline{\sigma}_i \cdot \underline{\sigma}_j - 3\underline{\sigma}_i \cdot \hat{\underline{r}}_{ij}\, \underline{\sigma}_j \cdot \hat{\underline{r}}_{ij}}{(r_{ij}^3 / r_o^3)} \right)$, $g_D \equiv \mu_o \mu_n^2 / r_o^3$; breaks relative spin-orbit rot^n symmetry
Paramagnetic phase ($T > T_c$): spins behave independently, kT competes with $\mu_B \mathcal{H} \Rightarrow$ polarization $\sim \mu_B \mathcal{H}/kT \ll 1 \Rightarrow$ $\langle H_z \rangle \sim N(\mu_B \mathcal{H})^2/kT$	Normal phase ($T > T_A$): pairs of spins behave independently $\Rightarrow$ polarization $\sim g_D/kT \ll 1 \Rightarrow$ $\langle H_D \rangle \sim N g_D^2/kT$
Ferromagnetic phase ($T < Tc$): $\hat{H}_o$ forces all spins to lie parallel $\Rightarrow k_BT$ competes with $N\mu_B\mathcal{H}$ $\Rightarrow \langle S_z \rangle \sim 1 \Rightarrow \langle H_z \rangle \sim N\mu_B\mathcal{H}$	Ordered phase ($T < T_A$): $\hat{H}_o$ forces all pairs to behave similarly $\Rightarrow$ kT competes with Ng_D $\Rightarrow \langle H_D \rangle \sim Ng_D$ $\sim 10^{-3}$ ergs/cm^3 !

SBSOS: ORDERING MAY BE SUBTLE

FERROMAGNET		LIQUID ^{3}HE
	⇐ NORMAL PHASE ⇒	
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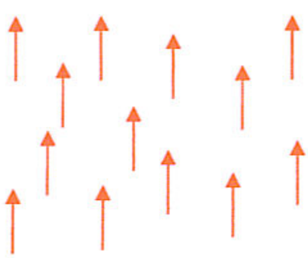
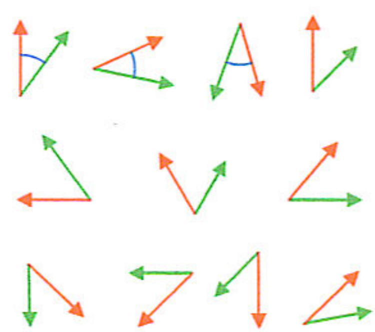
	⇐ ORDERED PHASE ⇒	

(↗ = total spin of pair

↗ = relative orbital ang. momentum)

$\langle \underset{\sim}{S} \rangle \neq 0$

$\langle \underset{\sim}{S} \rangle = \langle \underset{\sim}{L} \rangle = 0$

but $\langle \underset{\sim}{L} \times \underset{\sim}{S} \rangle \neq 0$!

Amplification of ultra-weak effects (cf NMR):
Example: P- (but not T-) violating effects of neutral current part of weak interaction:
For single elementary particle, any EDM $\underset{\sim}{d}$ must be of form

$\underset{\sim}{d} = \text{const.}\ \underset{\sim}{J}$ ← violates T as well as P.

But for $^3He - B$, can form

$$d \sim \text{const.}\ \underset{\sim}{L} \times \underset{\sim}{S} \sim \text{const.}\ \hat{\underset{\sim}{\omega}}$$

↑

violates P but not T.

Effect is tiny for single pair, but since all pairs have same value of $\underset{\sim}{L} \times \underset{\sim}{S}$, is multiplied by factor of $\sim 10^{23} \Rightarrow$

macroscopic P-violating effect?