THEORETICAL WORK ON SUPERCONDUCTIVITY UP TO 1956*

A.J. Leggett
Dept. of Physics,
University of Illinois at
Urbana-Champaign

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Superconductivity: theory up to 1957

A. Pre-Meissner

Discovery of "superconductivity" (= zero resistance):
Kamerlingh Onnes & Holst, 1911 (Hg at 4.3K)

1911-1933: Superconductivity ≡ zero resistance

Theories: (Bloch, Bohr…): Epstein, Dorfman, Schachenmeier, Kronig, Frenkel, Landau…

Landau, Frenkel: groundstate of superconductor characterized by spontaneous current elements, randomly oriented but aligned by externally imposed current (cf. domains in ferromagnetism)

Bohr, Kronig, Frenkel: superconductivity must reflect correlated motion of electrons (but correlation crystalline…)

Meissner review, 1932:

— are the "superconducting" electrons the same ones that carry current in the normal phase?

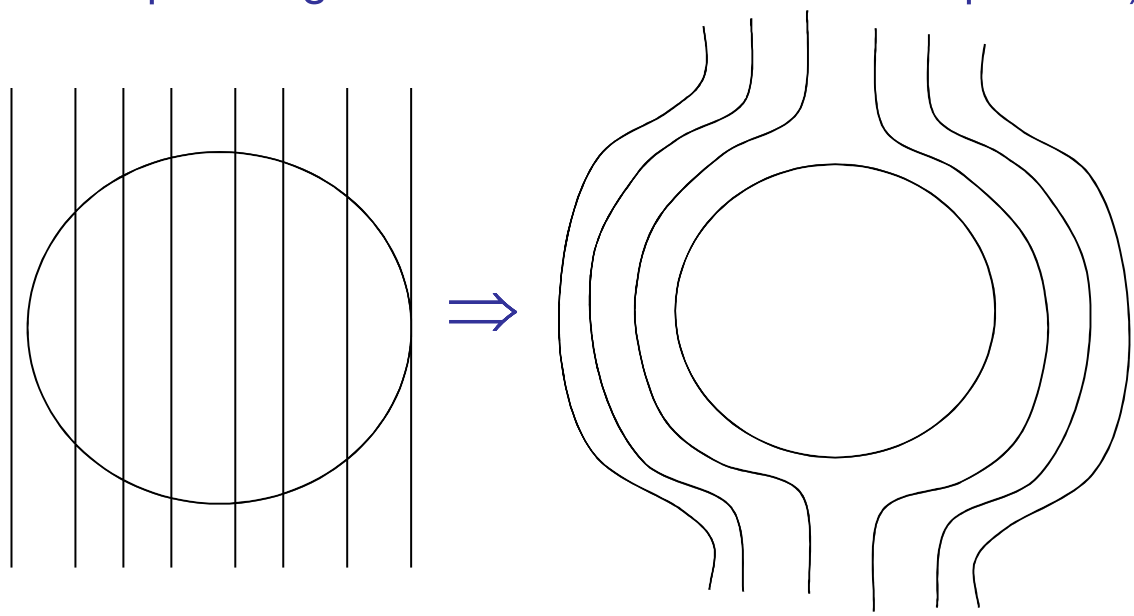
— is superconductivity a bulk or a surface effect?

— why do properties other than R show no discontinuity at T_c? In particular, why no jump in K? ← thermal cond^y.

— maybe "only a small fraction of ordinary electrons superconducting at T_c?"

Meissner and Ochsenfeld, 1933:

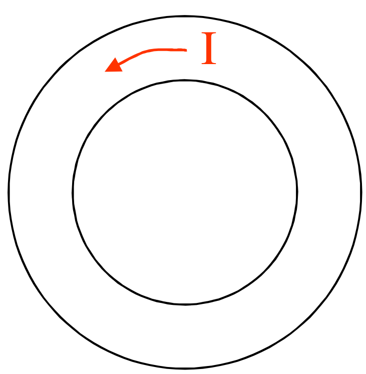
(note: expt. designed to answer "bulk/surface "question)



$T > T_c$ (H fixed) $T < T_c$

$\Rightarrow$ superconductivity is (also) equilibrium effect!
("perfect diamagnetism")

contrast:

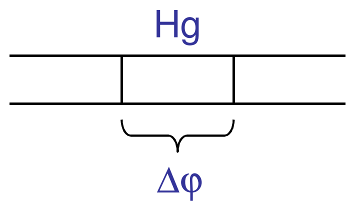


$\leftarrow$ cannot be stable groundstate

$$\text{if} \quad I > \frac{e}{mR}\left(\frac{N\hbar}{2}\right)$$

$\Rightarrow$ metastable effect (Bloch)

Digression:
which did K.O. see?



Answer (with hindsight!)
depends on whether or not $\Delta\varphi > \pi$! cannot tell from data given in original paper.

Theory, post-Meissner:

Ehrenfest, 1933: classification of phase transitions,
superconductivity (and λ–transition of ^{4}He) is second-order
(no discontinuity in S, but discontinuity in C_v)

↑ entropy ↑ specific heat

Landau, 1937 (?): idea of order parameter: OP for superconductivity is critical current. (Gorter and Casimir: fraction of superconducting electrons, → 1 as T→ 0, → 0 as T →T_c)

F. and H. London, 1935:

for n electrons per u.v. not subject to collisions,

$$\left.\begin{array}{rr} & \dot{\mathbf{J}} = ne^2\ \mathbf{E}/m \\ \text{(Faraday)} & \nabla \times \mathbf{E} = -\mu_o \dot{\mathbf{H}} \end{array}\right\} \Rightarrow \lambda_L^2\ \nabla \times \dot{\mathbf{J}} + \dot{\mathbf{H}} = 0$$

$$+\ \text{(Maxwell)}\ \nabla \times \mathbf{H} = \mathbf{J} \Rightarrow\ \nabla^2 \dot{\mathbf{J}} = \lambda_L^2\ \dot{\mathbf{J}}$$

⇒when accelerated, current (+ field) falls off in metal as exp $-z/\lambda_L$
(de Haas-Lorentz, 1925)

Londons: drop time derivative! i.e.

$$(\lambda_L^2\ \nabla \times\ \mathbf{J} + \mathbf{H} = 0\ \overset{\text{(+ Maxwell)}}{\Rightarrow}\ \nabla^2 \mathbf{J} = \lambda_L^2\ \mathbf{J} \Rightarrow \text{Meissner effect}$$

In simply connected geometry, equiv to

$$\mathbf{J} = -\ \frac{ne^2}{m}\mathbf{A}$$

Londons, cont.:

why $\mathbf{J} = -\dfrac{ne^2}{m}\mathbf{A}$? (*)

for a single particle,

$$\mathbf{J} = -\frac{i\hbar e}{2m}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - \frac{e^2}{m}|\psi|^2\mathbf{A}$$

In normal metal, application of **A** deforms s.p.w.f's by mixing in states of arb. low energy $\Rightarrow$ second term cancelled. But if no such states exist, $\psi = \psi_o \Rightarrow \nabla\psi = 0 \Rightarrow$ only 2nd term survives. Then $\sum_i = n$, so, obtain(*).

Why no low-energy states? Idea of energy gap (supported by expts. of late 40's and early 50's)

(London, 1948: flux quantization in superconducting ring, with unit h/e)

Ginzburg & Landau, 1950:

order parameter of Landau theory is quantum-mechanical wave function $\Psi(\mathbf{r})$ ("macroscopic wave function"): interaction with magnetic field just as for Schrodinger w.f., i.e.

$$\text{K.E.} = \frac{\hbar^2}{2m}\left|\left(\nabla - ie^*\mathbf{A}/\hbar\right)\Psi(\mathbf{r})\right|^2 \qquad e^*=e\ ?$$

vector potential

$$\text{P.E.} = \alpha|\Psi(r)|^2 + \frac{1}{2}\beta|\Psi(r)|^4 \qquad \text{(as in general Landau theory)}$$

+Maxwell $\Rightarrow$ 2 char. lengths:

$\xi(T)$ = length over which OP can be "bent" before en. exceeds condens^n en.

$\lambda(T)$ = London penetration depth

If $\kappa \equiv \lambda(T)/\xi(T) > 1/\sqrt{2}$, eff. surface en. between S and N regions –ve.

Abrikosov (1957): for $\kappa > 1/\sqrt{2}$, vortex lattice forms!

Post-Meissner microscopics 1945-50: various attempts at microscopic theory (Frenkel, Landau, Born & Cheng…)

Heisenberg, Koppe (1947): Coulomb force ⇒ electron wave-packets localized, move in correlated way; predicted energy gap with (assumed) exponential dependence on materials properties.

Pippard, 1950: exptl. penetration depth much less dependent on magnetic field than in London theory and increases dramatically with alloying ⇒ nonlocal relation between $\mathbf{J}(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$, with characteristic length ("coherence length") ~ 10^{-4} Å.

Frohlich, 1950: indirect attraction between electrons due to exchange of virtual phonons.

Bardeen & Pines, 1955: combined treatment of screened Coulomb repulsion and phonon-induced attraction ⇒ net interaction at low ($\omega \lesssim \omega_D$) frequencies may (or may not) be attractive.

Schafroth, 1954: superconductivity results from BEC of electron pairs (cf. Ogg 1946)

1957: BCS theory

ARE THERE ANY LESSONS FOR CUPRATES (etc.)?

Perhaps: importance of asking specific questions which can by answered by experiment without reliance on a specific microscopic model.

Pre-1957 examples:

Bulk or surface effect? (von Laue) ⇒ Meissner effect

Are phonons relevant? (Fröhlich) ⇒ isotope effect

Why is effect of magnetic field so weak?
(Pippard)) ⇒ nonlocal **J** - **A** relation

Examples in cuprates:

Can we understand macroscopic EM properties?

Symmetry of order parameter

Nature of Fermi surface

Where is the Coulomb energy saved (or expended)?

$$\langle V \rangle \sim - \int d\mathrm{q} \int d\omega \, \mathrm{Im} \left\{ \frac{1}{1 + V_q \, \chi_o \, (\mathrm{q}\omega)} \right\}$$

FT of Coulomb potential (V_q)

"bare" density response ($\chi_o(\mathrm{q}\omega)$)