"BASIC QUANTUM MECHANICS, AND SOME SURPRISING CONSEQUENCES"

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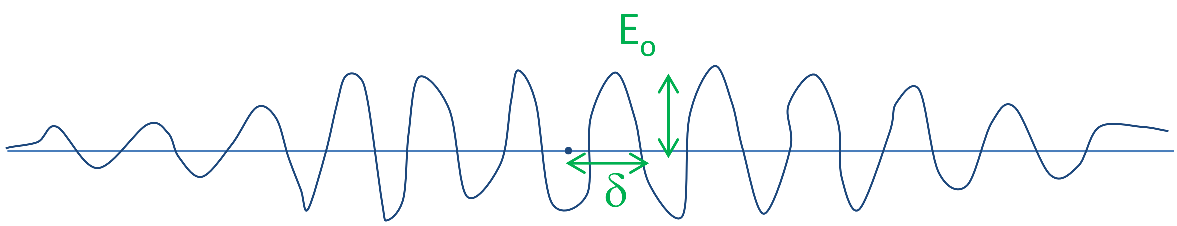
CLASSICAL LIGHT WAVES

A (classical) light beam is a wave in which the electric field oscillates:

if I could "photograph" the electric field at a given time, it might look something like



and if I sit at a given point in space, it would have the same behavior in time: formally,

$$E(t) \cong E_o \cos(\omega t + \delta)$$

frequency (ω), amplitude (E_o), phase (δ)

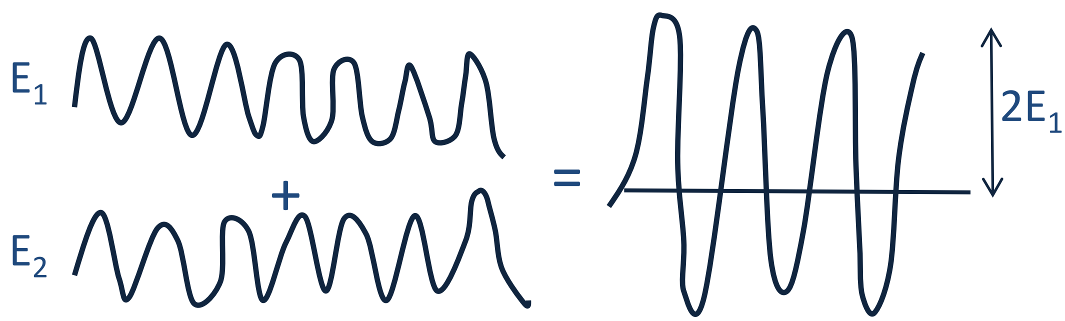
The energy of the wave is proportional to the square of the amplitude:

energy $\propto E_o^2$

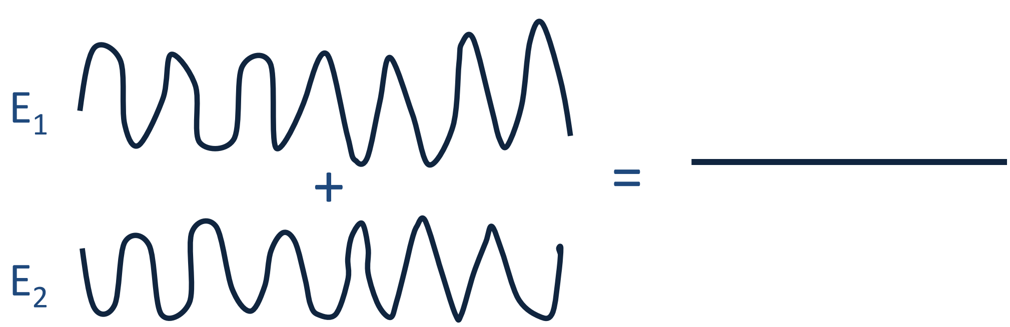
INTERFERENCE

Suppose we try to combine ("superpose") two light waves. If we sit at a particular point, then at any given time the electric field will be the sum of the fields on the two waves. So it depends on their relative phase: e.g.

$\delta_1 - \delta_2 = 0$
("in phase"):



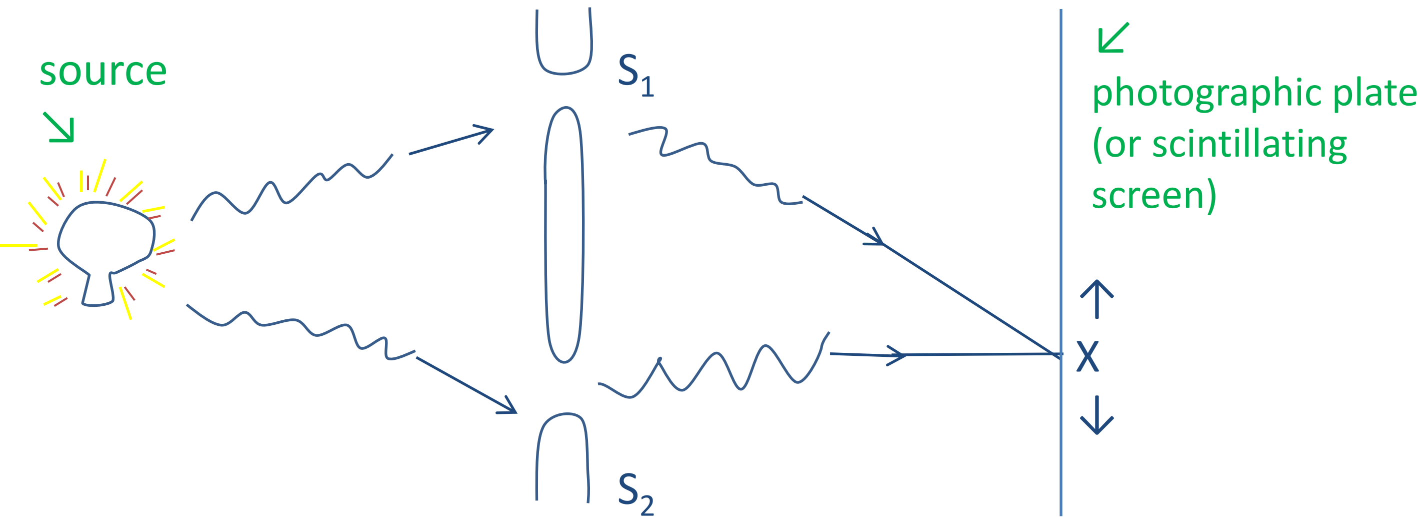
$\delta_1 - \delta_2 = \pi$
("out of phase"):



If $|E_1| = |E_2|$, then energy of 1st wave = energy of second wave $\propto E_1^2$. But in first case, energy of combined wave $\propto (2E_1)^2 \propto 4E_1^2$, and in second case it is 0!

INTERFERENCE, cont.

Famous example of interference: Young's slits:

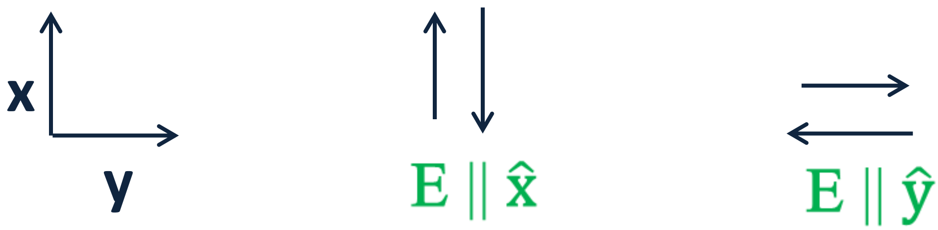


Amplitude E of total field at X depends on difference of phase $(\delta_1 - \delta_2)$, which in turn depends on difference in path length traversed $\Rightarrow$ E, hence energy deposited, depends on position of X on plate $\Rightarrow$ pattern of light (E = max, high-energy) and dark (E = 0, no energy) bands on plate.

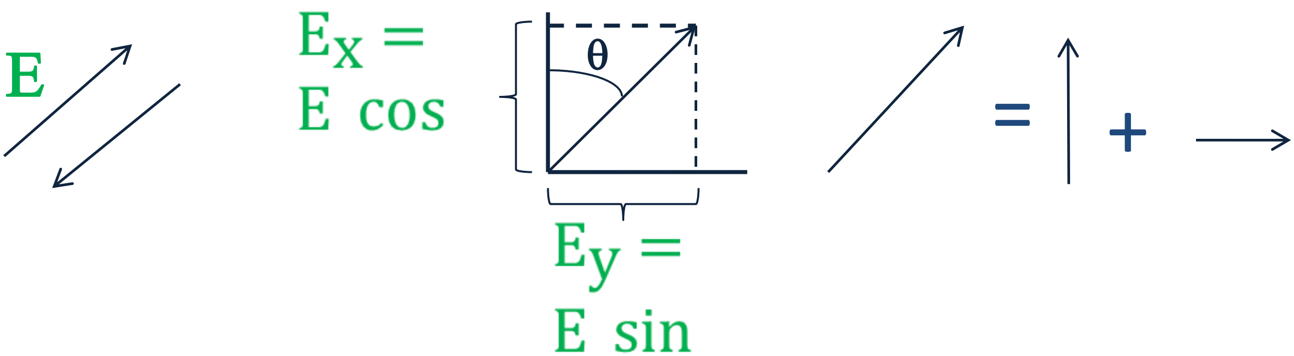
Note: If either slit S_1 or S_2 is closed, electric field E, and hence distribution of energy on plate is (nearly) uniform (independent of position X)

POLARIZATION

The electric field also has a direction associated with it: e.g. for the wave propagating into the screen, we can have



But we can also have e.g.



We can express **E** as a "superposition" of its two components ("polarizations")

$$\mathbf{E} = \hat{x}\, E_x + \hat{y}\, E_y \equiv E\,(\hat{x}\,\cos\theta + \hat{y}\,\sin\theta)$$

Actually, E_x and E_y can have different phases: e.g.

$$E_x = E_0\,\cos\omega t\,,\; E_y = E_0\cos(\omega t + \pi/2) = E_0\sin\omega t$$

Then, if we sit at a particular point, the electric field rotates as a function of time ("circular polarization"). This kind of wave carries angular momentum.

POLARIZERS

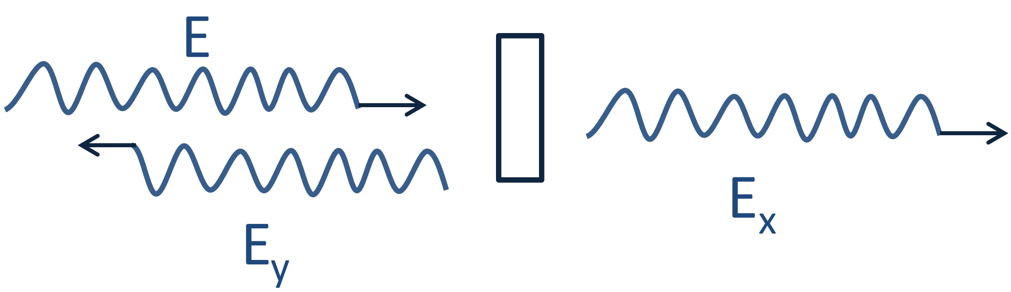
A polarizer is a crystal which (e.g.) transmits a light wave completely when its polarization is vertical ($\|\hat{\mathbf{x}}$) and reflects it completely when the polarization is horizontal ($\|\hat{\mathbf{y}}$)

But what if the polarization is at an angle θ in the xy-plane?.

Answer: resolve ↗ into its components!

$$\mathbf{E} = \hat{\mathbf{x}}\, E_x + \hat{\mathbf{y}}\, E_y \equiv E_o\,(\hat{\mathbf{x}}\, \cos\theta + \hat{\mathbf{y}}\, \sin\theta)$$

then the x-component is completely transmitted and the y-component completely reflected:



Since energy $\propto E^2$,

fraction of energy transmitted = $E_x^2 / E^2 = \cos^2\theta$

("Malus's law")

SUPERPOSITIONS ARE NOT "MIXTURES"!

In the Young's slits experiment, the effect of light coming through both slits is not simply the sum of the effects of the two beams coming through the two slits individually.

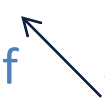
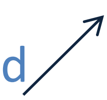
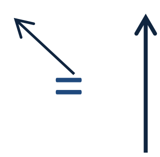
Another example: transmission through a polarizer (↑ transmitted, →reflected):

for a beam with polarization 

50% of energy is reflected ($\cos^2 45^\circ = {}^1/_2$)

for a beam with polarization 

50% is also reflected ($\cos^2 45^\circ = {}^1/_2$)

But, a superposition of  and  is just  for which

no energy is reflected at all!

So, in dealing with waves, must first add amplitudes, and only then calculate energies etc.

Notation (for future reference): $\mathrm{E_x} \equiv$ amplitude of electric field for light wave with vertical ($\hat{x}$ —) polarization, $\mathrm{E_y} \equiv$ amplitude for wave with horizontal ($\hat{y}$ —) polarization. Then it turns out that amplitude for RH circularly polarized light wave is $\mathrm{E_x} + i|\mathrm{E_y}\rangle$ and state of LH circularly polarized one is $\mathrm{E_x} - i|\mathrm{E_y}\rangle$

(i = $\sqrt{-1}$)

QUANTUM MECHANICS: PHOTONS

In quantum mechanics, the energy of a light wave of frequency ω comes in irreducible packets (p "photons") of energy $\hbar\omega$.

$\hbar$: $h/2\pi$, $h \equiv$ Planck's constant

The "amplitude" of a photon is proportional to the amplitude of the electric field of the corresponding classical light wave, so e.g. if

$|x\rangle$ represents amplitude for a photon polarized in x-direction
$|y\rangle$ represents amplitude for a photon polarized in y-direction

Then

$\cos\theta|x\rangle + \sin\theta|y\rangle \equiv |\mathbf{a}\rangle$ represents amplitude for photon polarized in direction **a** in xy-plane, making <θ with x-axis, and

$|x+iy\rangle$ represents amplitude for RH circularly polarized photon

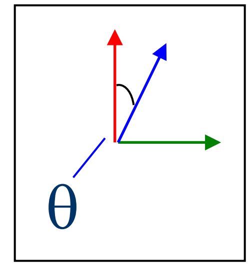
$|x-iy\rangle$ represents amplitude for LH circularly polarized photon
polarized photon

Note: RH (LH) circularly polarized photons carry angular momentum $\hbar(-\hbar)$

Two-photon processes: Suppose an atom emits two photons with definite polarizations, say $|\mathbf{a}\rangle$for 1 and $|\mathbf{b}\rangle$ for 2. Then the combined state is described by an amplitude which is the product of the amplitude for the two photons separately:

$$\boxed{\text{amplitude} = |\mathbf{a}\rangle_1\,|\mathbf{b}\rangle_2}$$

Single photon incident on birefringent crystal ("polarizer"):

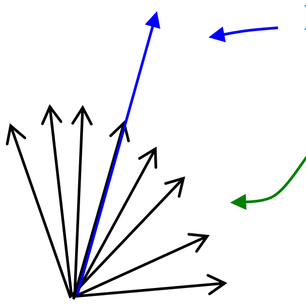


Probability of transmission = $\cos^2\theta$

(quantum version of Malus' law)

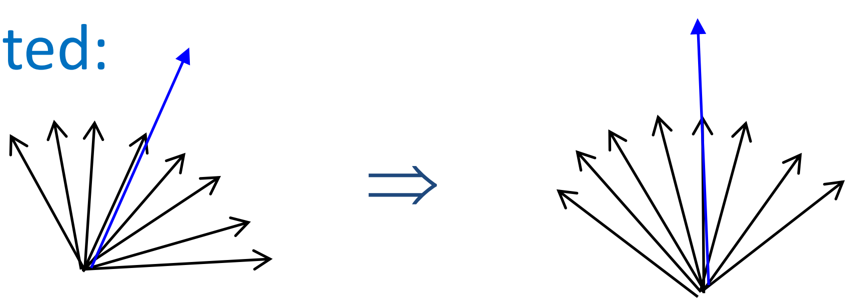
Digression: Can a classical probabilistic theory explain this?

YES!



"true" polarization

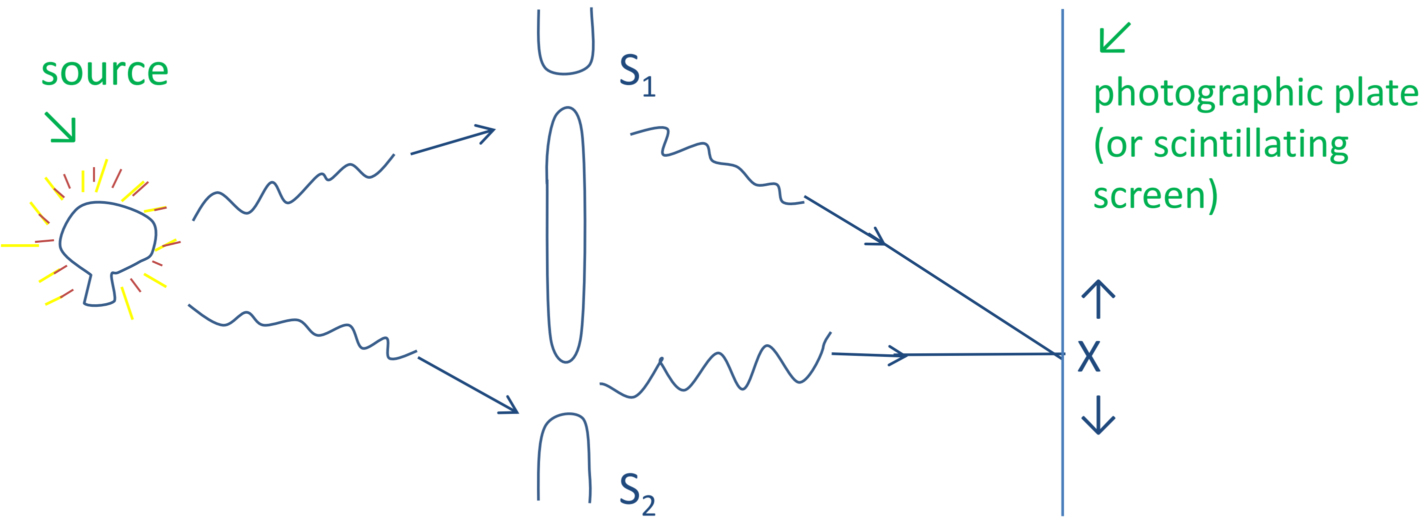
If ↗ closer to ↑, transmitted: if closer to →, reflected. If transmitted, distribution of "hidden" variable is adjusted:



With suitable choice of random distribution,

can reproduce $P_T(\theta) = \cos^2\theta$ $\quad f(\chi) = \cos 2(\theta - \chi)$

PHOTONS IN THE YOUNG'S SLITS EXPERIMENT



If we turn down the strength of the source until photons come through one at a time, we can see them arriving individually on the screen. Initially, arrival seems random, but eventually a pattern builds up, with

probability of arrival at X ∝ brightness of classical interference pattern

Since classically the brightness ∝ energy $\propto E^2$ this suggests that in quantum mechanics

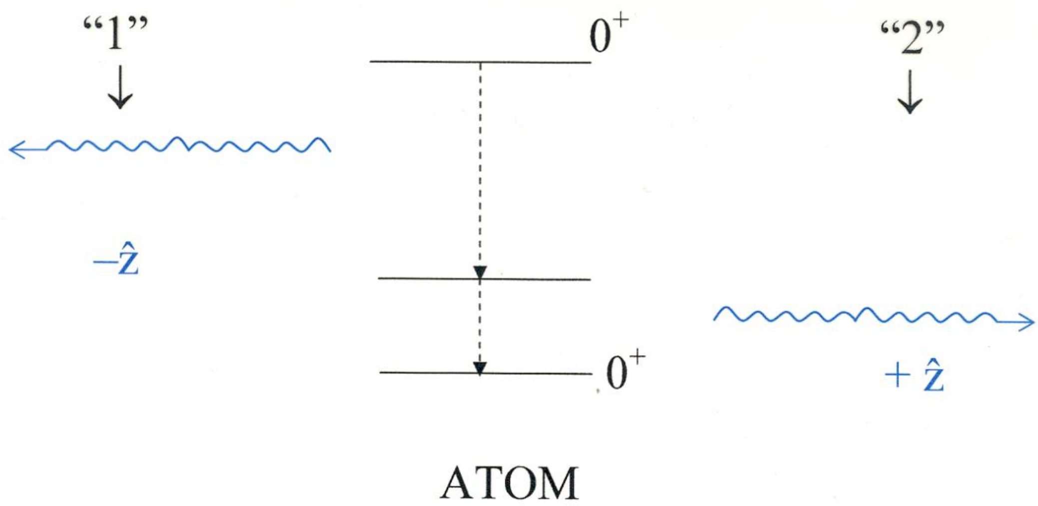
$$\text{PROBABILITY} \propto (\text{AMPLITUDE})^2$$

NOTE: If either slit closed, probability is uniform (ind. of X). Thus in QM, cannot add probabilities of two mutually exclusive events (paths), but

must add amplitudes not probabilities!

Couldn't we arrange to detect which slit a particular photon came through? Yes (in principle), but then we destroy the interference pattern! (i.e. prob. of arrival is ind. of X)

2-PHOTON STATES FROM CASCADE DECAY OF ATOM



What is polarization state of photons?

(note: $|x> \pm i|\hat{y}>$ corr. photon angular momentum $\pm \hbar$)

General principle of QM; if process can happen either of two ways, and we don't (can't) know which, must add amplitudes!

Here, we know that <u>total</u> angular momentum of 2 photons is zero, but we don't know whether photon 1 carried off $+\hbar$ and 2 $-\hbar$ (intermediate atomic state has m = 1) or vice versa (int. state m = -1). Hence must write

crucial!

$$|\psi> = |x+iy>_1 |x-iy>_2 + \text{(phase factor)}. \; |x-iy>_1 |x+iy>_2$$

Actually (parity $\Rightarrow$) phase factor = + 1, so

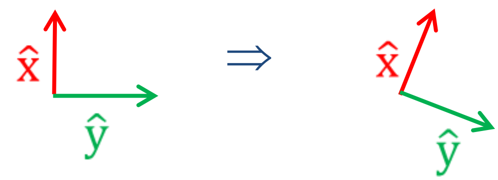
$$|\psi> = |x>_1 |x>_2 + |y>_1 |y>_2 \quad \left(\text{x} \frac{1}{\sqrt{2}}\right)$$

POLARIZATION STATE OF 2 PHOTONS EMITTED BACK TO BACK IN ATOMIC $0^+ \to 1^- \to 0^+$ TRANSITION (recap):

$$\Psi_{2\gamma} = \frac{1}{\sqrt{2}}\left(|x>_1 |x>_2 + |y>_1 |y>_2\right)$$

So, if photon 1 is measured to have polarization x(y) so inevitably will photon 2!

But, state is rotationally invariant:



$$\Psi_{2\gamma} = \frac{1}{\sqrt{2}}\left(|x'>_1 |x'>_2 + |y'>_1 |y'>_2\right)$$

so if 1 measured to have polarization x' (y') so does 2!

(**NOT** true for "mixture" of $|x>_1 |x>_2$ and $|y>_1 |y>_2$)

Now:

What if photon 1 is incident on polarizer with "transmission" axis $\underset{\sim}{\hat{a}}$. and photon 2 on one with a <u>differently</u> oriented **transmission axis** $\underset{\sim}{\hat{b}}$?

Since choice of axes for $\Psi_{2\gamma}$ arbitrary, choose $\underset{\sim}{\hat{x}} = \underset{\sim}{\hat{a}}$.

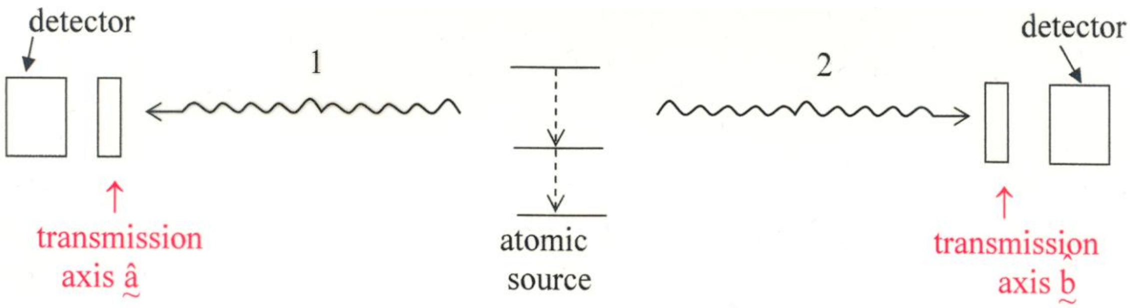
Then:

Prob. of transmission of 1 = ½.

But if 1 transmitted, then polarization of 2 is $\underset{\sim}{\hat{a}}$, so probability of transmission of polarizer set in direction $\underset{\sim}{\hat{b}} = \cos^2 \theta_{ab}$

(Malus's law)

$$P(\text{both transmitted}) = \tfrac{1}{2}\cos^2 \theta_{ab}$$



$$\text{Prob. (both detected)} = \tfrac{1}{2}\cos^2\theta_{ab} = \tfrac{1}{4}(1 + \cos 2\theta_{ab})$$

"Isotropic mixture" $\Rightarrow \frac{1}{4}(1 + \frac{1}{2}\cos 2\theta_{ab})$

$$\text{Prob. (neither detected)} = \tfrac{1}{2}\cos^2\theta_{ab}$$

$$\left.\begin{array}{l}\text{Prob. (1 detected, 2 not)}\\ \text{Prob. (2 detected, 1 not)}\end{array}\right\} = \tfrac{1}{2}\sin^2\theta_{ab}$$

THESE ARE THE PREDICTIONS OF STANDARD QUANTUM MECHANICS. CAN THEY BE EXPLAINED BY A CLASSICAL PROBABILISTIC THEORY?

Df: If for a given pair, with polarizer 1 set at $\underset{\sim}{\hat{a}}$, photon 1 is detected,

df. $A \equiv +1$: if rejected, $A \equiv -1$. Similarly with polarizer 2 set at $\underset{\sim}{\hat{b}}$, if

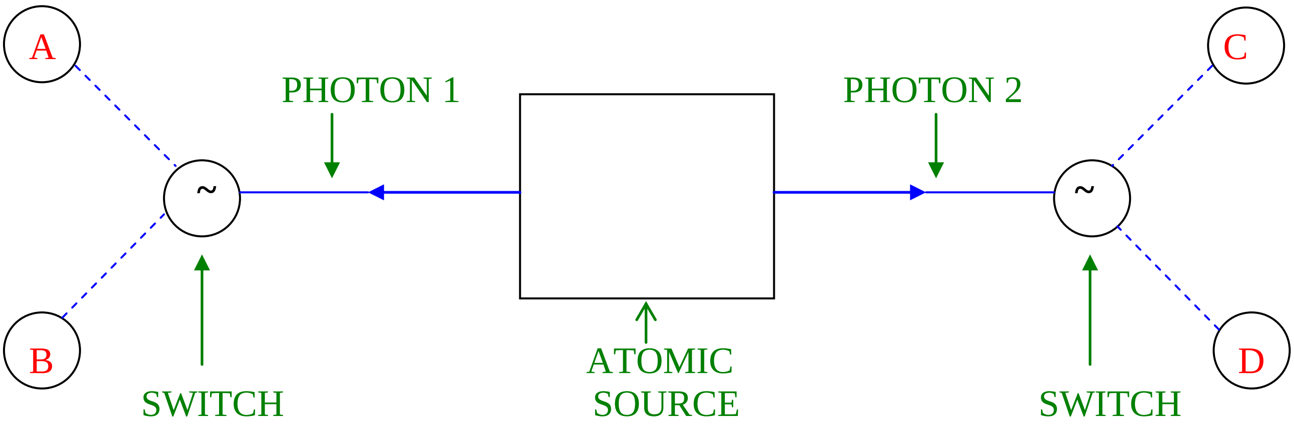
photon 2 detected, df. $B \equiv +1$: if rejected, then $B \equiv -1$. Then above is equivalent to the statement that for the average over the ensemble of pairs,

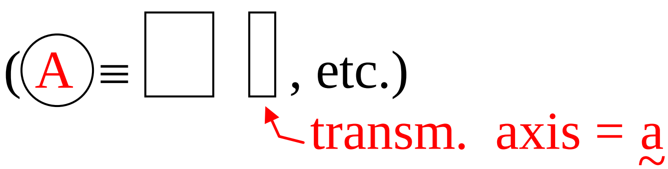
$$\langle AB\rangle = \cos^2\theta_{ab} - \sin^2\theta_{ab} = \cos 2\theta_{ab}$$

["Mixture": $\frac{1}{2}\cos 2\theta_{ab}$]

as special cases, for $\theta_{ab} = 0$ $\langle AB\rangle = +1$, and for $\theta_{ab} = \pi/2$. $\langle AB\rangle = -1$. (EPR). These two special cases can be accounted for by a classical probabilistic model. But…

EXPERIMENTS ON CORRELATED PHOTONS





DEFINITION: If photon 1 is switched into counter "A", then:

If counter "A" clicks, A = + 1 (DF.)

If counter "A" does not click, A = – 1 (DF.)

NOTE:

If photon 1 switched into counter "B", then A is NOT DEFINED.

Experiment can measure

$\langle AC \rangle_{exp}$ on one set of pairs (1→ "A", 2 → "C")

$\langle AD \rangle_{exp}$ on another set of pairs (1→"A", 2→"D")

etc.

Of special interest is

$$K_{exp} \equiv \langle AC \rangle_{exp} + \langle AD \rangle_{exp} + \langle BC \rangle_{exp} - \langle BD \rangle_{exp}$$

for which Q.M. makes clear predictions.

POSTULATES OF "OBJECTIVE LOCAL" THEORY:

(1) Local causality
(2) Induction
(3) Microscopic realism OR macroscopic "counter-factual definiteness"

BELL'S THEOREM

1. (3) $\rightarrow$ For each photon 1, EITHER $A = +1$ OR $A = -1$, independently of whether or not A is actually measured.

2. (1) $\rightarrow$ Value of A for any particular photon 1 unaffected by whether C or D measured on corresponding photon 2. : etc.

3. $\therefore$ For each pair, quantities AC, AD, BC, BD exist, with A, B, C, D, $= \pm 1$ and A the same in (AC, AD) (etc.)

4. Simple algebra then $\rightarrow$ for each pair, $AC + AD + BC + - BD \leq 2$

5. Hence for a single ensemble,
$$\langle AC\rangle_{ens} + \langle AD\rangle_{ens} + \langle BC\rangle_{ens} - \langle BD\rangle_{ens} \leq 2$$

6. (2) $\rightarrow \langle AC\rangle_{exp} = \langle AC\rangle_{ens}$, hence the measurable quantity
$$K_{exp} \quad \langle AC\rangle_{exp} + \langle BC\rangle_{exp} + \langle BC\rangle_{exp} - \langle BD\rangle_{exp}$$
satisfies

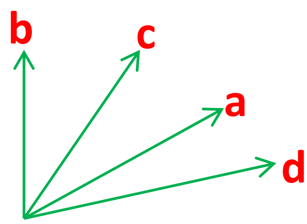
$K_{exp} \leq 2$, Obj. Local Theory

OBJECTIVE LOCAL THEORY: $K_{exp} \leq 2$.

QM: If polarizer settings are **a**, **b**, **c**, **d**
Then e.g. for a 0^+ transition predict

$$\langle AC \rangle = \cos\left(2\theta_{\underline{a}\cdot\underline{c}}\right), \text{ etc.}$$

$\Rightarrow$ for



QM predicts (ideal case)

$$K_{exp} = 2\sqrt{2}$$

$\Rightarrow$ Exptl. Predictions of QM incompatible with those of an theory embodying

- Local causality
- Induction
- Macroscopic counter-factual definiteness

1. "It is a fact that either A would have clicked or A would not have clicked"

2. "Either it is a fact that A would have clicked, or it is a fact that A would not have clicked"