SQUIDS AND RELATED SYSTEMS: THE INTERACTION OF THEORY AND EXPERIMENT OVER FIVE DECADES*

Tony Leggett
Department of Physics
University of Illinois at Urbana-Champaign

DALEFEST

Urbana, IL Sept. 30, 2022

Concentrate on 2 main topics:

1. Macroscopic quantum tunnelling (MQT) and Macroscopic quantum coherence (MQC)
2. Symmetry of the superconducting order parameter

*based in part on

AJL, Josephson Devices as Tests of Quantum Mechanics towards the Everyday Level, in F. Tafuri, ed., Fundamentals and Frontiers of the Josephson Effect, Springer 2019

AJL, Josephson Experiments on the High-Temperature Superconductors, Phil. Mag. **74**, 509 (1996).

Part 1. Squids (and current-biassed junctions) as test-beds for macroscopic quantum behavior (NOT just “many particles/pairs doing the same thing” – that is already exemplified by liquid helium/superconductors/lasers…)

We would like to see

QUANTUM-MECHANICAL BEHAVIOR OF A MACROSCOPIC VARIABLE

e.g. flux Φ trapped in a SQUID ring (“flux qubit”) or phase drop $\Delta\varphi$ across current-biassed Josephson junction (“CBJ”).
Classical Lagrangian:

junction capacitance (C)

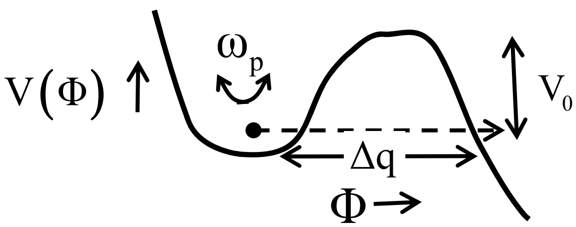
$$L\left(\Phi,\dot{\Phi}\right)=\frac{1}{2}C\dot{\Phi}^2-V\left(\Phi\right),$$

ring self-inductance (L); junction critical current (I_C)

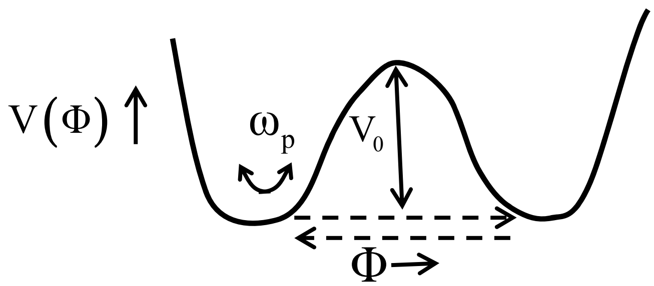
$$V\left(\Phi\right)=\left(\Phi-\Phi_{ext}\right)^2/\left(2L\right)-\left(I_C\Phi_0/2\pi\right)\cos\left(2\pi\Phi/\Phi_0\right)$$

“Naïve” quantization $\rightarrow \exists\Psi\left(\Phi:t\right)$ which satisfies

$$-\hbar\frac{d\Psi\left(\Phi:t\right)}{dt}=-\frac{\hbar^2}{2C}\left(\frac{d^2\Psi\left(\Phi:t\right)}{d\Phi^2}\right)+V\left(\Phi\right)\Psi\left(\Phi:t\right)$$



MACROSCOPIC QUANTUM TUNNELLING (flux qubit or CBJ)



MACROSCOPIC QUANTUM COHERENCE (flux qubit only)

(also quantized energy levels, etc.)

Theoretical predictions for isolated systems (near lability):

Escape rate by tunnelling:

$$\Gamma_{QM} = \underset{\uparrow \atop \sim\,\omega_p}{\omega_0} \exp - 7.2 V_0 / \hbar\omega_p$$

cf:

$$\Gamma_{TH} = \omega_{TH} \exp - V_0 / k_B T$$

$$\Rightarrow T \to T^* \equiv \hbar\omega_p / (7 \cdot 2 k_B)$$

NH_3-type oscillation rate:

$$\Delta = \omega_0 \exp - \frac{16}{3}\left(\frac{V_0}{\hbar\omega_\rho}\right)$$

First theoretical predictions of MQT (in CBJ): Ivanchenko and Zil'berman 1968 (6 years from Josephson!)

Experimental non-observation of MQT: Fulton and Dunkelberger 1974

First explicit claim of experimental observation of MQT: Den Boer and de Bruyn Ouboter 1980

Some doubts re MQT in early 80's:

Experimental:

(1) crucial role played by junction capacitance C, which in some experiments is unknown

(2) main evidence for MQT flattening of $\Gamma(T)$, but this could be due to decoupling of macroscopic degree of freedom from thermometer

Theoretical:

(1) is "naïve" quantization of classical equations of motion legitimate? (N.D. Mermin: "can you quantize the equations of mathematical economics?")

(2) effects of (external) decoherence and (internal) dissipation.

A.O. Caldeira and AJL 1981: what is difference between tunnelling escape rate of system which classically satisfies conservative eqn. of motion

$$M\ddot{q}(t) + \partial V(q)/\partial q = F_{est}(t) \qquad \left(\Gamma_{QM} = \text{const.}\exp. - B_0\right)$$

and one which classically satisfies dissipative eqn. of motion of form

$$M\ddot{q}(t) + \eta\dot{q}(t) + \partial V(q)/\partial q = F_{ext}(t)$$

Answer (near lability):

distance under barrier
↓

$$B_0 \to B(\eta) \equiv B_0 + A\eta(\Delta q)^2/\hbar$$

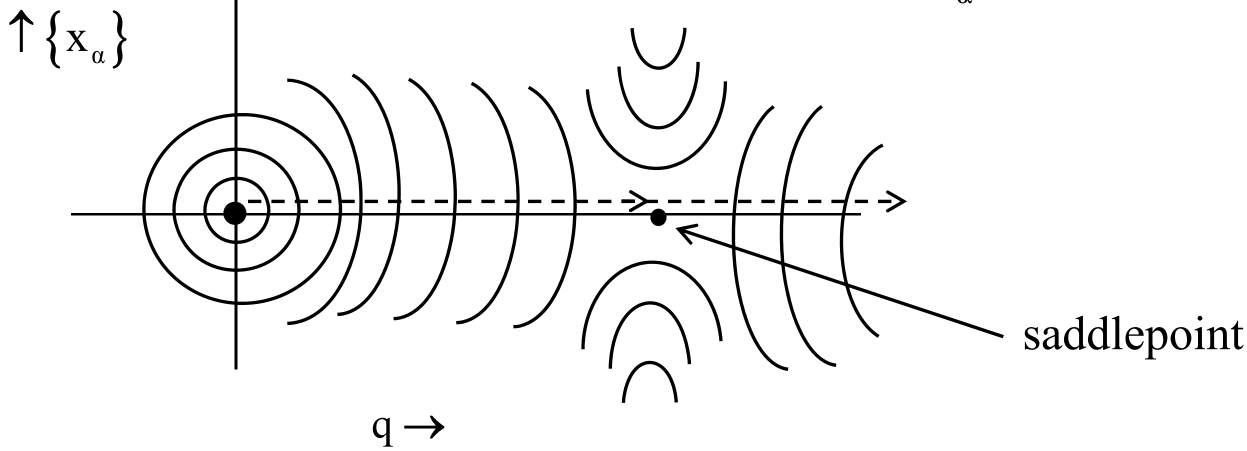
Microscopic confirmation: Ambegaokar et al. 1982.

How to understand intuitively?

Describe environment which gives rise to dissipation by Feynman-Vernon (oscillator-bath) technique, but MUST supplement the linear coupling $\left(q\sum_\alpha c_\alpha x_\alpha, \text{ with } \eta = \frac{\pi}{2}\sum_\alpha \frac{c_\alpha^2}{m_\alpha \omega_\alpha^2}\delta(\omega - \omega_\alpha)\right)$ by a "counterterm" $\left(q^2\sum_\alpha \left(c_\alpha^2/2m_\alpha\omega_\alpha^2\right)\right)$, then energy contours look like

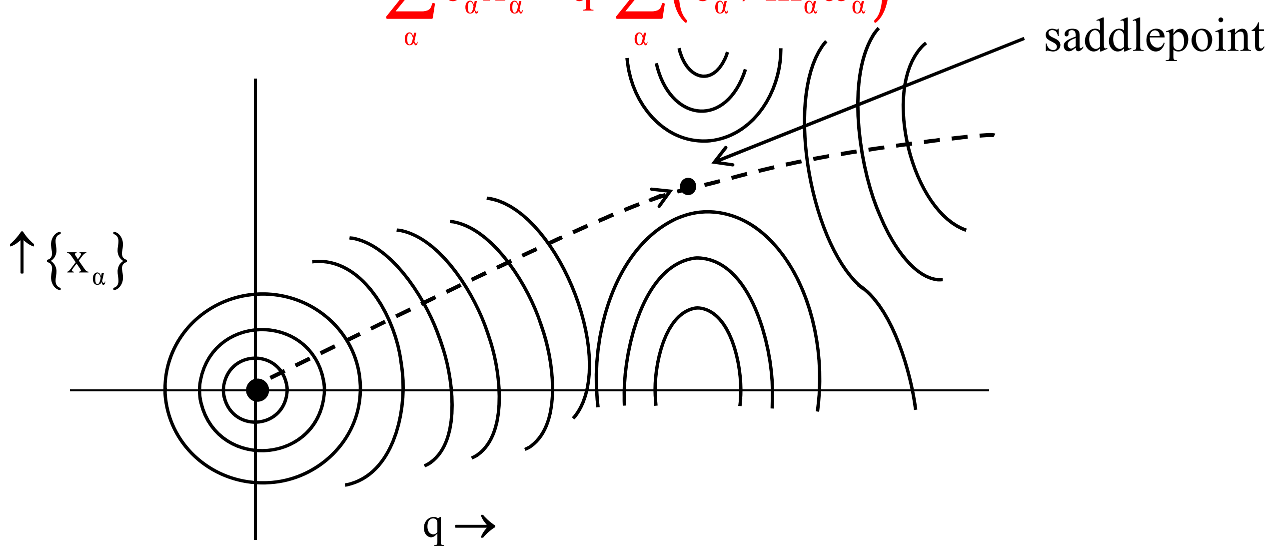
A. Zero dissipation $\left(\eta \to 0 \Rightarrow \text{ all } c_\alpha = 0\right)$

$$V\left(q, x_\alpha\right) = \frac{1}{2} M\omega_0^2 q^2 - kq^3 + \sum_\alpha m_\alpha \omega_\alpha^2 x_\alpha^2$$



B. Nonzero dissipation $\left(\eta \neq 0 \Rightarrow c_\alpha \neq \upsilon\right)$

$$V\left(q, x_\alpha\right) = \frac{1}{2} m\omega_0^2 q^2 - kq^3 + \sum_\alpha m_\alpha \omega_\alpha^2 x_\alpha^2$$
$$+^q \sum_\alpha c_\alpha x_\alpha + q^2 \sum_\alpha \left(c_\alpha^2 / m_\alpha \omega_\alpha^2\right)$$



Height of saddlepoint unchanged, **distance to it increased**!
In thermal activation, exponent of rate Γ_{th} sensitive only to barrier height $\to$ unaffected by dissipation (Kramers)
In quantum tunnelling, exponent of Γ_{QM} is affected by both height of barrier **and distance to it** $\left(B \sim \int \sqrt{V}\, dx / \hbar\right) \to$ reduced by dissipation.

Meanwhile, in the Clarke group at Berkeley, including DVH:
detailed consideration of the voltage noise in SQUIDs due to a parallel resistor $\left(R^{-1} \rightleftarrows \text{CL's}\ \eta\right)$:

Koch, Van Harlingen, Clarke 1980

VIth International Conference on Noise in Physical Systems,
Gaithersburg, Md. April 6-10, 1981
(AJL paper, p. 355: Koch et al. paper, p. 359)

Early '80s: several experiments (Voss & Webb, Jackel…) on MQT in CBJ's

better control over junction capacitance, but "noise temperature" problem persists

Two milestone papers in Oct. 1985:
Martinis, Devoret, Clarke: energy-level quantization in zero-voltage state of a CBJ
Devoret, Martinis, Clarke: MQT out of zero-voltage state: all relevant parameters of junction measured in situ
(3rd International Conference on SQUIDs, West Berlin, June '85)

1985 – 2000: much theoretical work on effects of dissipation on MQC (e.g. AJL et al. 1987). Also, blueprints for MQC experiment (Tesche, Rome group) culminating in:
2000: first generally accepted observation of MQC in SQUIDs (Stony Brook, Delft)

but in the meantime…

Cuprates (high-T_c) superconductors 1986

early 1990's: order parameter

What is structure of OP

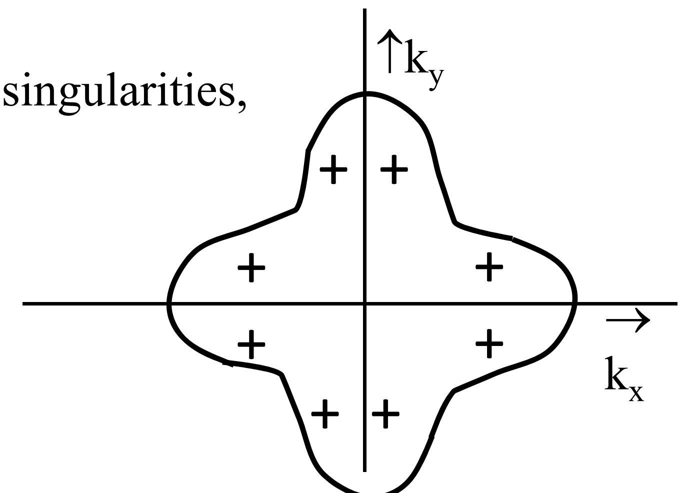
$$F(\mathbf{r},\mathbf{r}':\sigma\sigma') \equiv \langle \hat{\psi}(\mathbf{r}\sigma)\hat{\psi}(\mathbf{r}'\sigma')\rangle$$

as function of relative coord. $\boldsymbol{\rho} \equiv \mathbf{r}-\mathbf{r}'$ (or F.T. $\mathbf{k}$)?

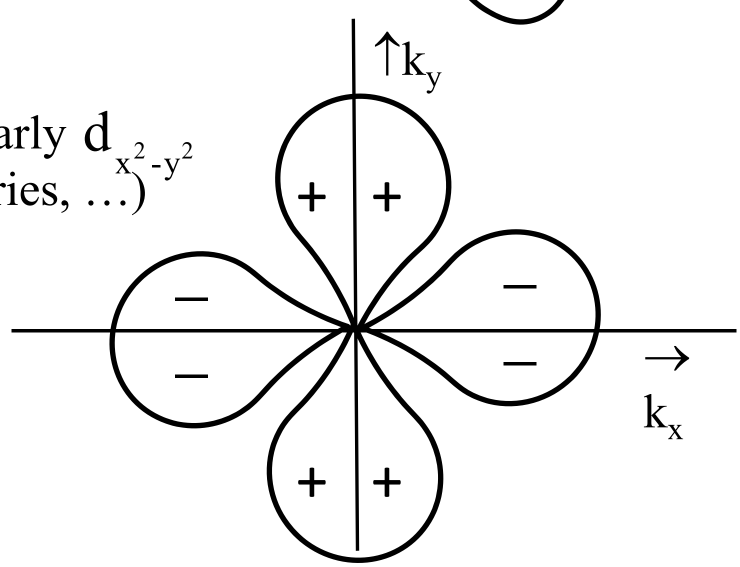
early experiments: $\chi \to 0$ in superconducting phase $\to$ spin singlet $\to$ even parity in $\boldsymbol{\rho}$ ($\ell = 0,2,\ldots$)

2 main contenders:

A. s-wave (phonons, van Hove singularities, Anderson ILT model ….)



B. d-wave, and particularly $d_{x^2-y^2}$ (spin fluctuation theories, …)



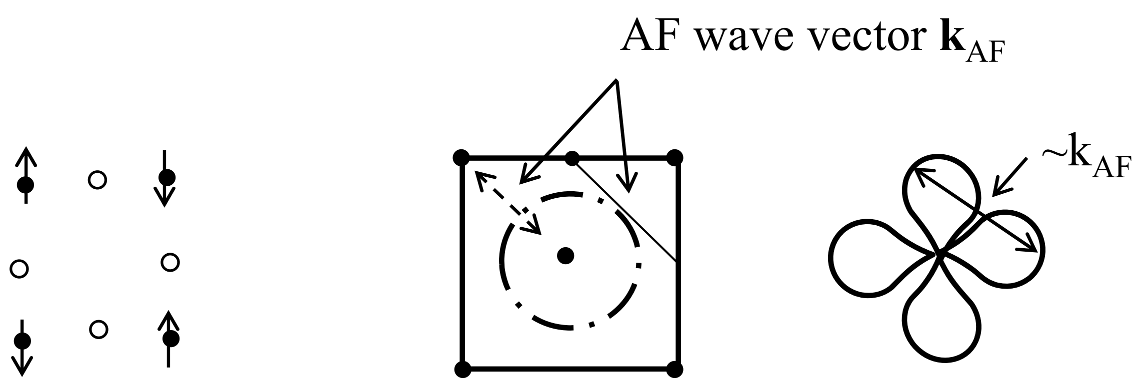
Why do spin-fluctuation theories of cuprate superconductivity favor $d_{x^2-y^2}$ symmetry of OP (Scalapino, Moriya, Pines…)?

Generally, pairing energy given by

$$\langle V_{eff} \rangle = \sum_{\sigma\sigma'} \int_{FS} d\mathbf{k} \int_{FS^*} d\mathbf{k}' \, V_{eff}(\mathbf{k}-\mathbf{k}', \text{spins}) \, F(\mathbf{k}, \text{spins}) \, F^*(\mathbf{k}', \text{spins})$$

In phonon case, V_{eff} ~ ind. of ($\mathbf{k}-\mathbf{k}'$, spins) and (mostly) attractive, so $F(\mathbf{k},\sigma\sigma')$ ~ const.($\mathbf{k}$) × spin singlet (BCS). What about cuprates?

In cuprate phase diagram, superconductivity occurs next to AF state:



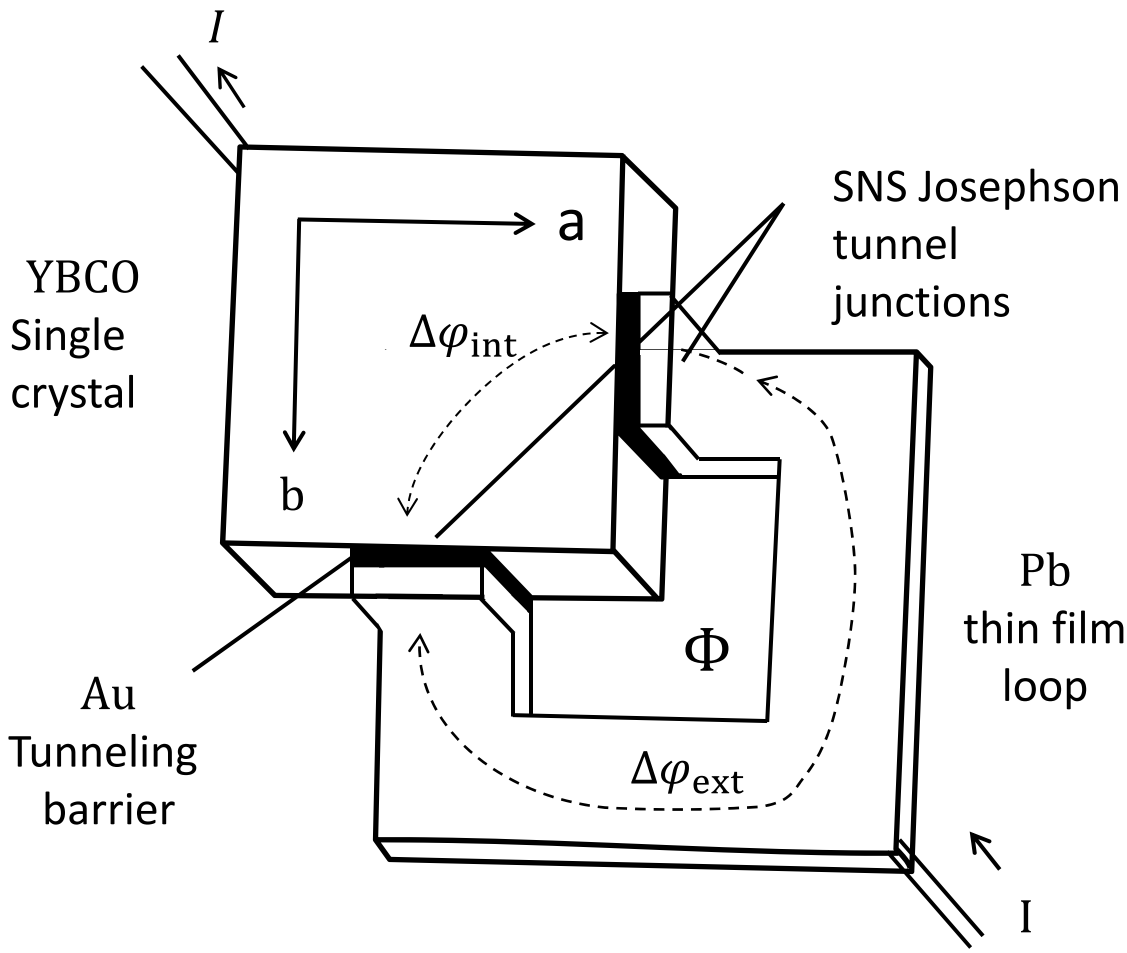
Low-energy spin waves AF, and attraction due to their exchange mostly around $\mathbf{k}_{AF}$ ~ connects antinodes of F. So what should be relative sign of F on antinodes so connected?

Prima facie, should be + → s-wave. However, need to consider spin structure of interaction induced by exchange of AF spin waves! (e.g. transverse case: ↑+↓→↓, ↑+↓→↑) introduces extra – sign, hence: sign of F(k) should be opposite to that of F(k′) , i.e. $d_{x^2-y^2}$.

Early '90's: various experiments, mostly to investigate presence of absence of gap nodes. No unique conclusion (cf Annett et al. 1990)

Question: could one determine relative sign of nodes?

(Geshkenbein et al. 1987, for p-wave case)
Wollman et al. 1993 (inc. DVH, AJL)



total phase drop around circuit ↓

$$I_c(\Phi) = 2I_c \cos(\Delta\varphi_{tot})$$

$$\Delta\varphi_{tot} = \Delta\varphi_{ext} + \Delta\varphi_{int} \qquad \Delta\varphi_{ext} \equiv 2\pi\Phi/\Phi_0$$

$$\Delta\varphi_{int} = 0 \text{ for s} \Rightarrow I_c \text{ max. at } \Phi = n\,\Phi_0$$

$$= \pi \text{ for } d_{x^2-y^2} \quad " \quad \Phi = \left(n + \tfrac{1}{2}\right)\Phi_0$$

Conclusion: OP is $d_{x^2-y^2}$
(Tsuei et al., Mathai et al., …)

The Strontium ruthenate (SRO) saga

1994 Superconductivity in SRO at ~ 1K
Rice-Sigrist, Baskaran: (in analogy with ^{3}He–A):
OP F(**k**) is $\propto(k_x+ik_y)$ × triplet spin state
↑
"chiral"

1998 Knight-shift experiments appear to show χ = const. in sup. state
triplet spin state ⇒ odd parity

2000-2019 : many experiments, including some at Illinois, consistent with chiral state

in particular,

Kidwingira et al. (DVH group) 2006 – $(k_x+ik_y) \rightleftarrows (k_x-ik_y)$ fluctuations

Jang et al. (Budakian group) 2011 – "half-quantum" vortices.

In parallel, phase interference experiments similar to Wollman et al. (as originally suggested by Geshkenbein et al. 1987): Nelson et al. (Liu group, PSU) 2004.

One important difference:

for single-junction tunnelling between singlet and singlet, simple "scalar" (Bardeen-Josephson) tunnelling gives nonzero result.

for tunnelling between singlet and triplet ($s \rightleftarrows p$) need to invoke SOI (Geshkenbein & Larkin 1986) ⇒ experiments even more informative

⇧: does SOI need to be in junction itself ?

Cat thrown among pigeons (UCLA 2019): Knight-shift seems to drop towards 0 in sup phase!

Current unknowns:

do experiments measure the true χ?

can spin singlet be reconciled with odd-parity orbital state?

HAPPY RETIREMENT DALE!