Quasiparticles in Normal and Superfluid Fermi Liquids (more questions than answers …)

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QUASIPARTICLES IN NORMAL PHASE

Landau (1956), Nozières, Theory of Interacting Fermi Systems (1964):

Start from noninteracting Fermi gas,

then energy eigenstates specified by $\{n(\boldsymbol{p},\sigma)\}, n(p,\sigma) = 0 \text{ or } 1$

groundstate has $n(\boldsymbol{p},\sigma) = \Theta(p_F - |\boldsymbol{p}|)$, $p_F = \hbar(3\pi^2 n)^{1/3}$

switch on inter-particle interaction $\hat{V}$ adiabatically:

$$\hat{V}(t) = \hat{V}\exp(\alpha t) \qquad t < 0, \alpha \to 0\big(\hat{V}(t=0) = \hat{V}\big)$$

provided perturbation theory, converges, states of fully interacting system can be labelled by the noninteracting states $\{n(p,\sigma)\}$ from which they evolved.

then define "no. of quasiparticles in state p,σ" as $\mathrm{n}(\boldsymbol{p},\sigma)$

(⇒ Luttinger theorem trivial)

Suppose $\hat{Q} = \sum_{p\sigma} q\,(p\sigma)\alpha^{+}_{p\sigma}\alpha_{p,\sigma}$

then in original noninteracting system $\hat{Q} = \sum_{p\sigma} q\,(p\sigma)n(p,\sigma)$.

Is it true that in fully interacting system also $\hat{Q} = \sum_{p\sigma} q\,(p\sigma)n(p,\sigma)$?

Answer: yes, if and only if $\big[\hat{Q},\hat{H}(t)\big] = 0$

What quantities are conserved $\left(\left[\hat{Q}, \hat{H}(t)\right] = 0\right)$?

(a) liquid ^3He:

$$\hat{N} = \sum_{p\sigma} \alpha^+_{p\sigma}\, \alpha_{p\sigma}$$ yes — total number

$$\hat{S} \equiv \sum_{p\sigma} \sigma \alpha^+_{p\sigma}\, \alpha_{p\sigma}$$ yes — total spin

$$\hat{\boldsymbol{J}} = m^{-1} \sum_{p\sigma} p \alpha^+_{p\sigma}\, \alpha_{p\sigma}$$ yes — total current

$$\boldsymbol{J}_\sigma \equiv m^{-1} \sum_{p\sigma} (p\sigma) \alpha^+_{p\sigma}\, \alpha_{p\sigma}$$ no — total spin current

$\Rightarrow$ in real liquid ^3He,

$$\hat{S} = \sum_{p\sigma} \sigma n\,(p\sigma)$$ (etc.)

$$J_\sigma \neq \sum_{p\sigma} \frac{p}{m} \sigma n(p\sigma)$$

(b) metallic system (e.g. cuprates)

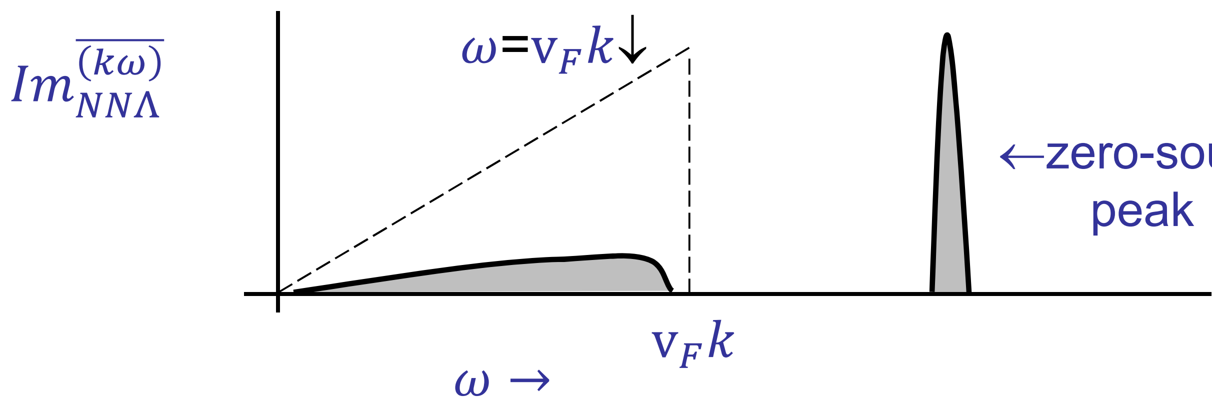
N, S conserved but $\boldsymbol{J}$ not conserved (even after transformation to Bloch states, because of U-processes).

Consequences of conservation for response functions

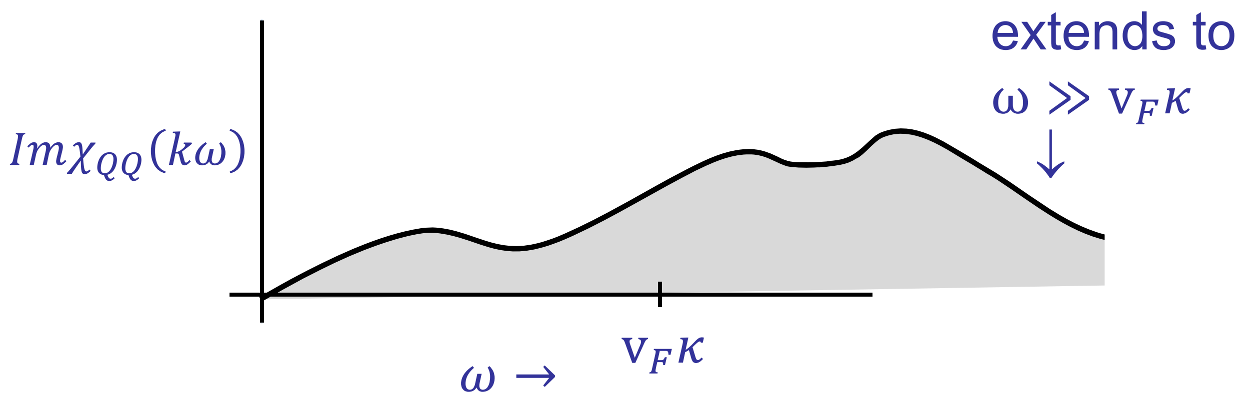
Consider $\chi_{QQ}(k,\omega) \equiv FT$ of $\ll \hat{Q}(0{,}0)\hat{Q}(rt) \gg$

If $\hat{Q}$ is conserved, then in limit $k \to 0$, support of $Im\chi_{QQ}(k\omega)$ comes entirely from quasiparticle states and is limited to $\omega \lesssim \mathrm{v}_F k$

Ex: $\hat{Q} = \hat{N}$ (density response function) of liquid ^3He



If $\hat{Q}$ is not conserved, e.g. $\hat{Q} = \hat{J}_\sigma$ in ^3He, quasiparticle states do not exhaust sum rule for $\chi_{QQ}(k\omega)$ and even in limit $\mathrm{k} \to 0$ get incoherent background.



In liquid ^3He, can infer quasiparticle contribution from Landau parameter F_a^1 (measurable in spin-echo experiments). Conclusion:

In ^3He, incoherent background contributes >80% of sum rule!

Moral: Even if system is a "decent" Fermi liquid, correlation function of non-conserved quantity can have large contribution from incoherent background.

Application to cuprates (and maybe other SCES):

N, S conserved but $\boldsymbol{J}$ not conserved (because of U-processes)

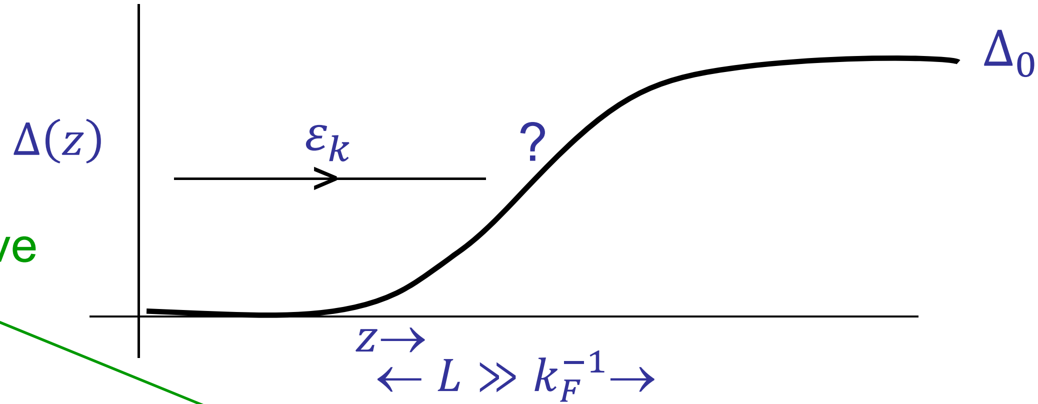
$\Rightarrow$ sum rule for $\omega Im\chi_{JJ}(\omega)$ $\left(= Re\ \sigma(\omega)\right)$ can have large contribtion from incoherent background (MIR peak).

Are the optimally doped and underdoped cuprates "bad" Fermi liquids? (cf. e.g. Berthod et al., PR B **87**, 115109 (2013)).

QUASIPARTICLES IN THE SUPERFLUID STATE

1. Andreev reflection

e.g.:



energy relative to Fermi energy

Incident particle, $0 < \varepsilon_k < \Delta_0$

for normal reflection on needs $\Delta k \sim 2k_F$ so amplitude $\sim \exp - 2k_F L \ll 1$, and Fermi sea blocked

$\Rightarrow$ reflected as hole: by conservation of energy $E_k = |\epsilon_k| \Rightarrow$ hole energy is $-\epsilon_k$.

What is momentum transfer? $\epsilon_k \sim \hbar v_F (k - k_F) \Rightarrow$

$$\Delta p = \frac{2\epsilon_k}{v_F} \ll p_F$$ (normal incidence)

Is there direct experimental evidence for this? Yes!

Buchanan et al., (PRL 57, 341 (1986) measure terminal velocity of $^3He \;\; A - B$ interface.

$v_{term} = \Delta G_{AB}/\Gamma \leftarrow$ frictional force due to reflection of qps

If we assume reflection is “normal”, $v_{term} \lesssim 1 \text{ mm/sec}$

Experimentally, $v_{term} \sim 0.1 - 1 \text{ m/sec} \Rightarrow$ Andreev reflection

(S-K Yip and AJL, PRL **57**, 345 (1986))

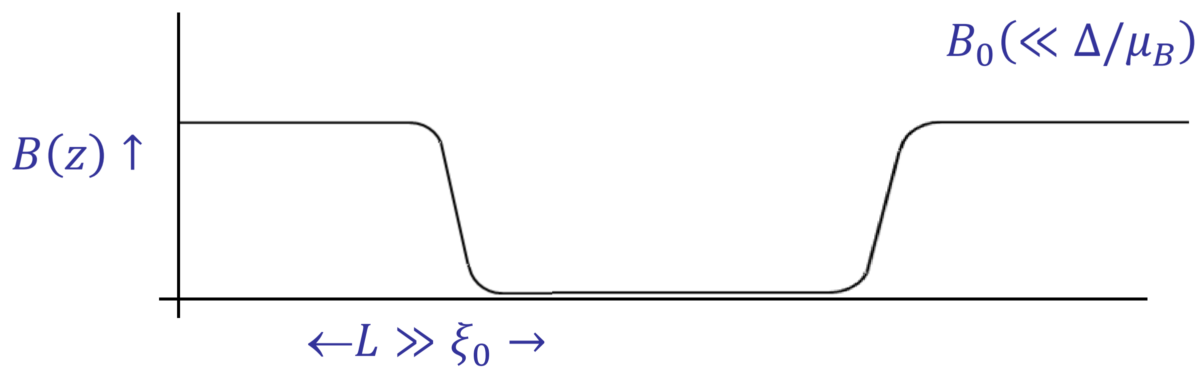
2. The "Zeeman-dimple" problem

Note: spatial variation of gap $\Delta(z)$ not a necessary condition for AR!

Andreev reflection

Can alternatively result from spatial variation of "diagonal" potential $V(r)$, provided this is the same for particle and hole (e.g. Zeeman potential $-\mu_B \sigma B(z)$)

Ex*: neutral Fermi superfluid with Zeeman coupling to external field $B(z)$ with "dimple"



Even (number) – parity ground state Ψ_0 has $\Delta(z) = const \equiv \Delta$ to linear order in B.

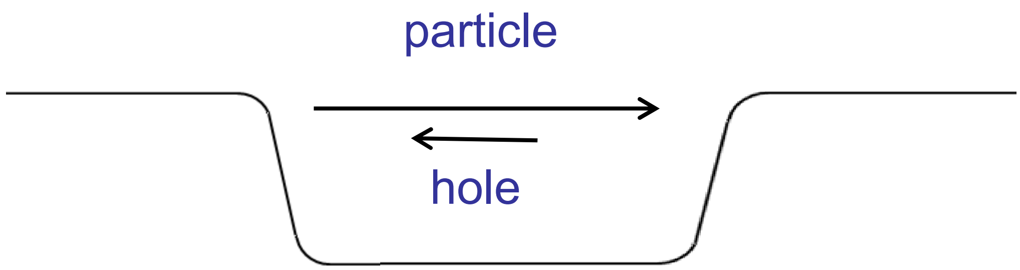
What is nature of lowest-energy odd-parity state?

Answer: Single Bogoliubov quasiparticle trapped in "dimple".

Extra spin localized in/close to dimple = 1.

*Y.-R. Lin and AJL, JETP **119**, 1034 (2014)

What is extra charge?



In quasiclassical approximation with only Andreev reflection:

$$E_{hole} = E_{particle} \Rightarrow \epsilon_{hole} = -\epsilon_{particle}$$

but in formula

$$\psi_{qp} = (u_k a_{k\uparrow}^{+} - v_k a_{-k\downarrow}^{+})|\Psi_0\rangle$$

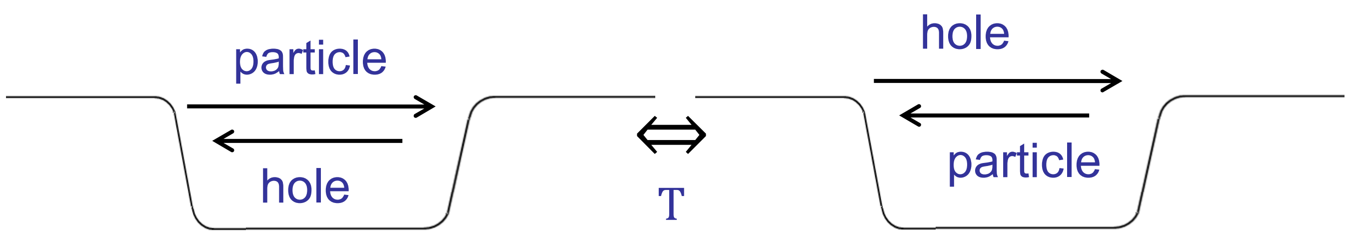
$$u_k = \frac{1}{\sqrt{2}}(1 + \varepsilon_k/E_k), v_k = \frac{1}{\sqrt{2}}(1 - \varepsilon_k/E_k)$$

so $\varepsilon_k \rightarrow -\varepsilon_k \Rightarrow u_k \rightleftarrows u_k$. Also, $v_k = \hbar^{-1}\dfrac{\partial E_k}{\partial k} k$

$= \hbar^{-1}\,(E_k/E_k)(\partial E_k/\partial k)$, so to the extent that N-state spectrum is particle-hole symmetric, $(\partial\epsilon_k/\partial k = \hbar v_F = const.)$,

extra charge = 0

Further complication: in this approximation, ground state of odd-number-parity sector is doublet related by time reversal!



“Normal” (non-Andreev) reflection splits doublet into even and odd combinations with exponentially small splitting. However, this does not change situation with regard to C-symmetry.

$\Rightarrow$ zero extra charge is not robust. (even in quasiclassical approximation)

3. Effect of taking particle number conservation seriously

With assumption of SBU(1)S ← spontaneously broken U(1) symmetry

standard formula for creation of Bogoliubov quasiparticle from even-parity groundstate $|\Psi_0\rangle \sim \hat{C}^{N/2}|vac\rangle$ is

$$\psi_u = (u_k a_{k\uparrow}^{+} - v_k a_{-k\downarrow})|\Psi_0\rangle$$

or more generally (BDG) ← Bogoliubov-de Gennes

$$\psi_u = \gamma^\dagger|\Psi_0\rangle$$

$$\gamma^\dagger = \int \{u(r)\hat{\psi}^\dagger(r) + v(r)\hat{\psi}(r)\}\,dr$$

This does not conserve particle number. Remedy:

$$\gamma^\dagger = \int \{u(r)\hat{\psi}^\dagger(r) + v(r)\hat{\psi}(r)\hat{C}\}\,dr$$

↑ creates extra Cooper Pair

Question 1: Is the "extra" pair the same as those in the even-parity GS?

Question 2: Irrespective of answer to 1,

does it matter?

Conjecture: for “usual” case (e.g. Zeeman-dimple problem), effect is nonzero but probably small.

but for case where Cooper pairs have “interesting” properties (e.g. intrinsic angular momentum) effect may be qualitative.

The crunch case: Majorana fermions in (*p*+*ip*) Fermi superfluid (Sr_2RuO_4?): does extra Cooper pair change results of “standard theory (e.g. Ivanov 2001) qualitatively?

-the $64K (actually $6.4M!) question…