What Makes Superfluid ^{3}He Special?

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SOME EARLY THEORETICAL WORK ON POSSIBLE COOPER PAIRING IN LIQUID ^{3}HE

"equal spin pairing"

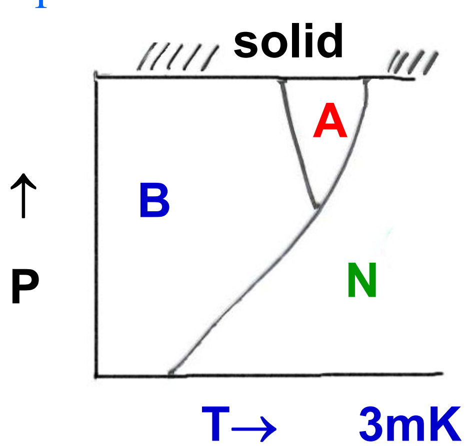
Anderson & Morel (1961): explore in detail case $\ell = 2$, and a special case of $\ell = 1$: only ↑↑ and ↓↓ pairs form, and have the same orbital angular momentum in direction $\hat{\underset{\sim}{\ell}}$ ("ABM" state) Physical properties anisotropic.

Vdovin
Balian & Werthamer

(1963): in $\ell = 1$ case all spin components "^3P_o"

$(\uparrow\uparrow, \downarrow\downarrow, \frac{1}{\sqrt{2}}\uparrow\downarrow + \downarrow\uparrow)$ can form: in fact for any given pair, $\tilde{L} = -\tilde{S} \Rightarrow J = 0$. ("BW" state). All physical properties isotropic. More stable than any ESP state.

Theoretical expectation c. 1964:
Liquid ^{3}He may form Cooper pairs, either ℓ = even (spin singlet) or with ℓ = odd (BW state). In either case, χ reduced and all magnetic properties isotropic. T_c difficult to predict.



In the event, see <u>both</u> ABM and BW phases!   How come?

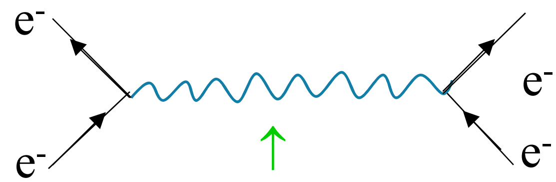
RESOLUTION OF THE PARADOX OF TWO NEW PHASES.

(Anderson & Brinkman, Phys. Rev. Letters **30**, 1108 (1973))

In BCS (weak-coupling) theory for ℓ=1, BW phase is always stable, independently of pressure and temperature.

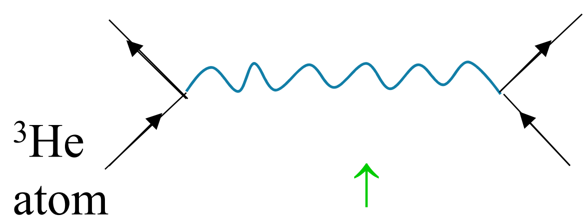
Crucial difference between Cooper pairing in superconductors and ^{3}He:

Superconductor:



↑
lattice vibration,
insensitive to onset of
pairing electrons

liquid ^{3}He:



↑
spin fluctuation of
^{3}He system ⇒ sensitive to
onset of pairing

⇒ "feedback" effects: Over most of the phase diagram, BW state stable as in BCS theory. But at high temperature and pressure, feedback effects uniquely favor ABM phase.

major qualitative leap beyond BCS!

CONCLUSION (by summer of 1973):

Both a priori stability considerations and NMR experimental data are consistent with hypothesis that both new phases are Cooper-paired (“superfluid”) phases. Specifically,

A phase = ABM

B phase = BW

What has superfluid ^{3}He been good for (1972-2022)?

What may it be (2022- …)?

(a) most sophisticated physical system of which we can claim detailed quantitative understanding. E.g. textures, orientational dynamics, topological singularities…

(b) analogies with systems in particle physics, cosmology… (G. E. Volovik)

(c) studies of (some aspects of) turbulence

(d) A phase is “topological superfluid” ⇒ if can form in sufficiently thin slab and create “half-quantum” vortices (HQV’s), expect to see (in)famous Majorana fermions.
(Unfortunately, HQV’s so far not seen in bulk ^{3}He-A)

(e) The combination of
1) “Superfluid amplification”
2) exotic pairing
3) no lattice pinning

main subject of this talk

(1) Superfluid amplification

Superconducting state of metal: Cooper pairs form, i.e. :

2-particle density matrix ρ_2 has single macroscopic (~N) eigenvalue, with associated eigenfunction

COM

$$\underbrace{F(\boldsymbol{r}_1\boldsymbol{r}_2\sigma_1\sigma_2)}_{\text{“wave function of Cooper pairs”}} \equiv F(\boldsymbol{R} : \boldsymbol{r}\sigma_1\sigma_2)$$

relative

$$\left(\equiv \left\langle \psi^{\dagger}\left(\boldsymbol{R}+\boldsymbol{r}/2:\sigma\right)\psi^{\dagger}\left(\boldsymbol{R}-\boldsymbol{r}/2:\sigma'\right)\right\rangle\right)$$

in words: a sort of “Bose condensation of diatomic (quasi-) molecules” = a macroscopic number of pairs of atoms are all doing the same thing at the same time (“superfluid amplification”)


American Association for the Advancement of Science
SCIENCE
22 December 1995
$7.00
Vol. 270 • Pages 1893–2064
Molecule of the Year
the
Bose-Einstei
Condensate

but in metals, internal state of pairs usually boring ($\ell = S = 0$)
(and anyway, any anisotropy pinned by crystal lattice)

The First Anisotropic Cooper-Paired System: Superfluid ^{3}He

as in metals, fermions of spin ½ $T_F \sim 1K,\ T_c \sim 10^{-3}K \Rightarrow T_C / T_F \sim 10^{-3}$

$\Rightarrow$ and, strongly degenerate at onset of superfluidity, but also strongly interacting.

$\Rightarrow$ low-lying states (inc. effects of pairing) must be described in terms of Landau quasiparticles.
(and Fermi-liquid effects v. impt.)

2-PARTICLE DENSITY MATRIX $\hat{\rho}_2$
still has one and only one macroscopic
(~N) eigenvalue
$\Rightarrow$ can still define "pair wave function" $F(\boldsymbol{R},\boldsymbol{r}:\sigma_1\sigma_2)$
However, even when $F \neq F(\boldsymbol{R})$,

(2) $F(\boldsymbol{r}\sigma_1\sigma_2)$ HAS ORIENTATIONAL DEGREES OF FREEDOM!

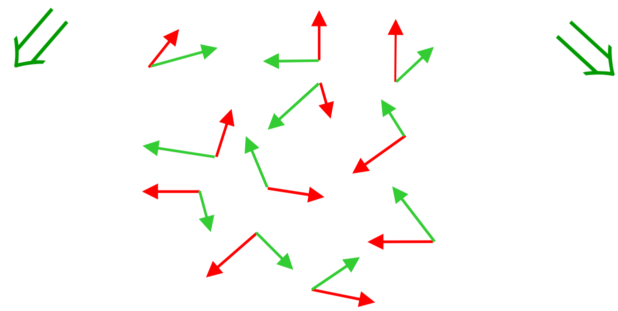
(i.e. depends nontrivially on $\hat{\boldsymbol{r}}, \sigma_1\sigma_2$.)

Standard identifications (from spin susceptibility, ultrasound absorption, NMR… plus theory):

In both A and B phases, Cooper pairs have $\ell = S = 1$

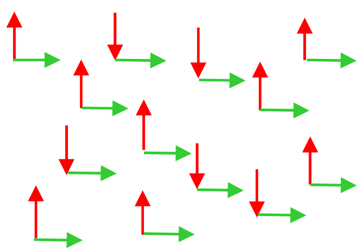
SPIN-ORBIT : ORDERING MAY BE SUBTLE

NORMAL PHASE



$\Leftarrow$ ORDERED PHASE $\Rightarrow$

A

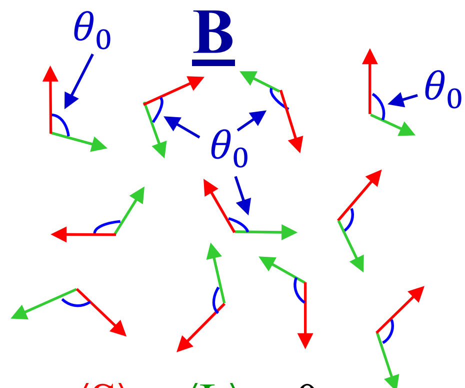


$\langle \mathbf{S}^2 \rangle \neq 0$ (but $\langle \mathbf{S} \rangle = 0$),
$\langle \mathbf{L} \rangle \neq 0$ ($\hat{d} \cdot \hat{\ell}$)

= total spin of pair (**S**)

= relative orbital
ang. momentum (**L**)

B



$\langle \mathbf{S} \rangle = \langle \mathrm{L} \rangle = 0$
but $\langle \mathrm{L} \times \mathrm{S} \rangle \neq 0$!
out of screen,
along
characteristic
axis $\boldsymbol{\omega}$.

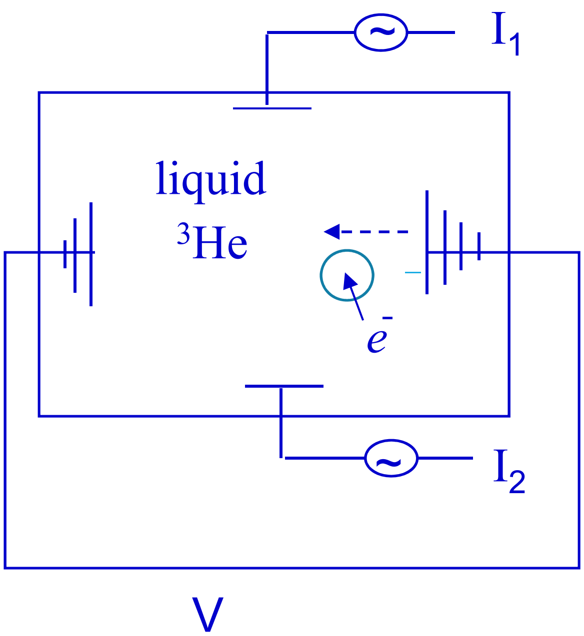
Dipole energy depends on relative angle of ↑ and ↑ $\Rightarrow$ determines
$\hat{d} \cdot \hat{\ell}$ (A phase) or θ_o (B phase)

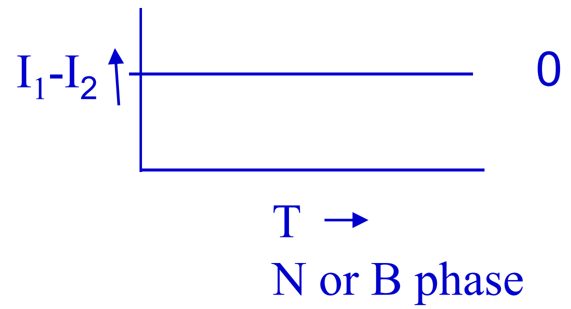
(3) No (strong) pinning of ℓ, $\boldsymbol{d}$, or $\boldsymbol{\omega}$ in bulk

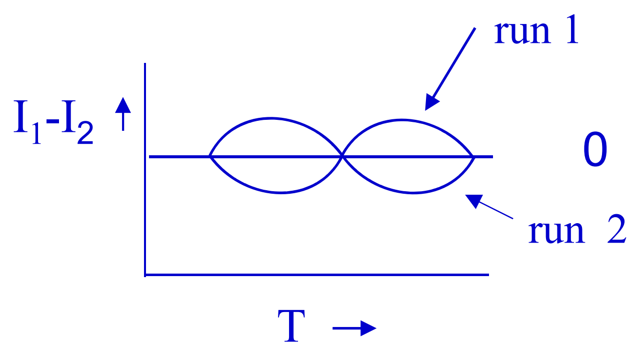
How to "see" the exotic nature of the pairing?
Use superfluid amplification!

Example*: Spontaneous violation of P- and T-symmetry in A phase

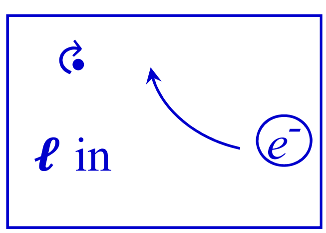
$$\left(f(r) = (\sin\theta e^{i\varphi})_{\hat{\ell}} \right)$$



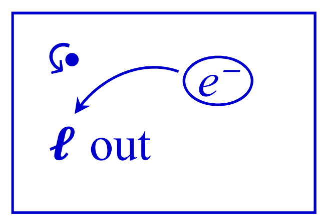




Intrinsic Magnus force:



(run 1)



(run 2)

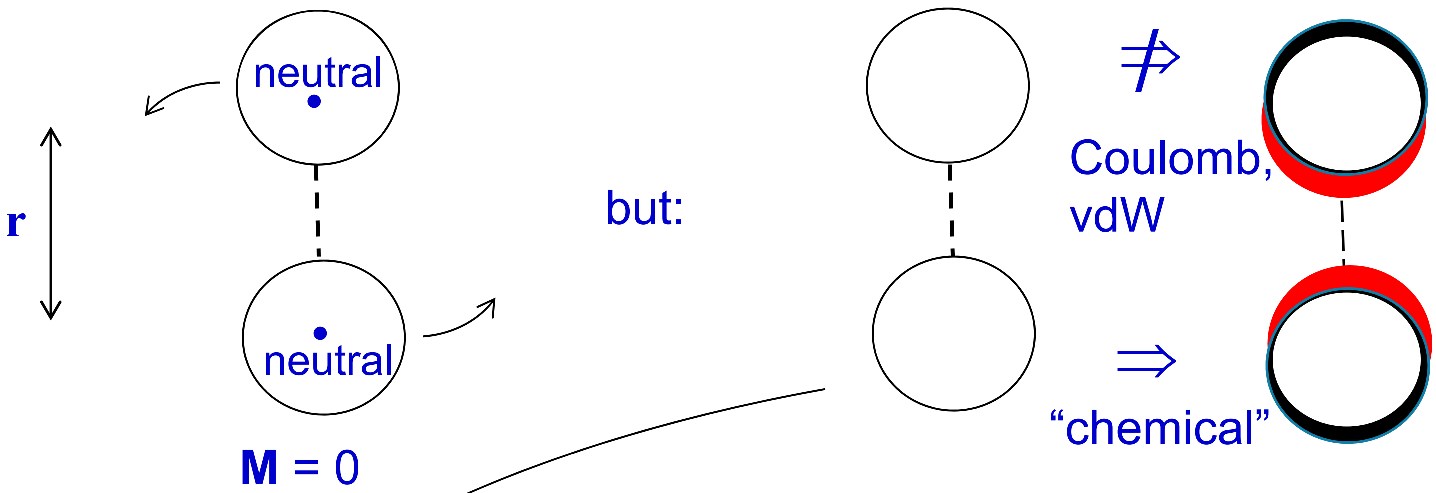
(Somewhat) unexpected effect: magnetic field can orient $\boldsymbol{\ell}$ – vector "in" or "out"!
indicates coupling of $\boldsymbol{\ell}$ to field, i.e. ^{3}He is a weak orbital ferromagnet, with magnetic moment along $(\pm)\ \boldsymbol{\ell}$.

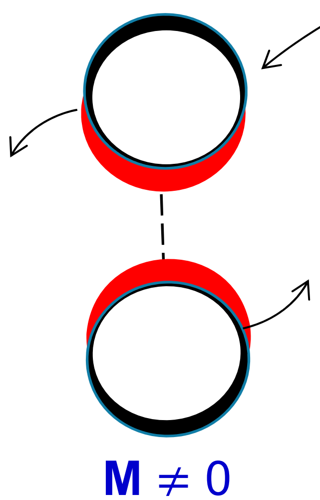
But…. ^{3}He atoms are neutral! How can this be?

*H. Ikegami et al., Science **341**, 59 (2013)

Weak ferromagnetism in ^{3}He – A*

Known effect in chemical physics†: rotation even of homonuclear diatomic molecule gives rise to magnetic moment!





Even in covalent homonuclear diatomic molecules, (e.g. $^2C_{12}$) very tiny effect, moreover falls off exponentially with r:

$$\mu = \mu(r), \mu_{mol} = \int r^2 P(r) \mu(r) dr$$

distribution of radial prob.

In free space, 2 ^{3}He atoms do not even form a bound state! For Cooper pair, vast bulk of $P(r) \equiv |F(r)|^2$ lies at $r \gg a_o, k_F^{-1}$

Hence, for single Cooper pair calculate (lots of exotic chemical physics!) $\mu_{CP} \sim 10^{-11} \mu_B$. (almost certainly immeasurably small). Certainly, in N phase completely unobservable.

What saves us is the principle of superfluid amplification – all Cooper pairs do same thing at same time! As a result, estimate effective equivalent field $H_{eq} = n_{cp}\mu_{CB} / \chi \sim 10 - 20 mG$.. Paulson et al. find circumstantial evidence for spontaneous field of just this o. of m.

*AJL, Nature **270**, 585 (1977): Paulson & Wheatley, PRL 40, 557 (1978)
†GC Wick, Phys. Rev **73**, 51 (1948)

More spectacular (but less direct) example of superfluid amplification: NMR

Recall: dipole energy depends on angle between $\uparrow$ and $\uparrow$

dipole energy

$$\frac{d\boldsymbol{S}}{dt} = \boldsymbol{S} \times \boldsymbol{H}_o + \frac{\delta E_D}{\delta \theta}$$

$\angle$of rotation about *rf* field direction $\hat{\mathscr{H}}_{rf}$

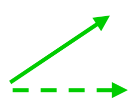
In normal phase, $\delta E_D / \delta\theta$ negligible.

$\uparrow \boldsymbol{H}_o, \hat{\omega}$

$\uparrow \mathscr{H}_{rf}$ (long) (L)

$\rightarrow \mathscr{H}_{rf}$ (transverse) (T)

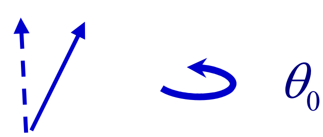
For A phase, dipole energy locks $\boldsymbol{d} \,||\, \boldsymbol{\ell}$ in equilibrium, and usually $\boldsymbol{d} \perp \boldsymbol{H}_o \Rightarrow$ both T and L fields move $\boldsymbol{d}$ away from $\ell \Rightarrow$ T frequency shift + L resonance (√)



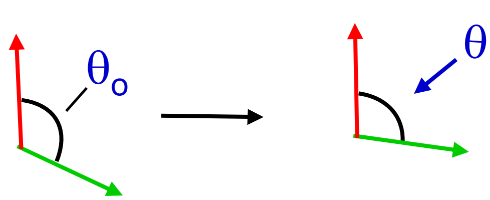
($\boldsymbol{H}_{rf}$ into screen)

For B phase:

in transverse resonance, rotation around $\hat{\mathscr{H}}_{rf}$ equiv. rotation of $\hat{\boldsymbol{\omega}}$ with θ_o unchanged $\Rightarrow$ no dipole torque, $\Rightarrow$ no resonance shift. (√)



In longitudinal resonance, rotation changes θ away from θ_o



$\Rightarrow$ finite-frequency resonance! (√)

One more proposed* (but so far unrealized!) example of superfluid amplification:

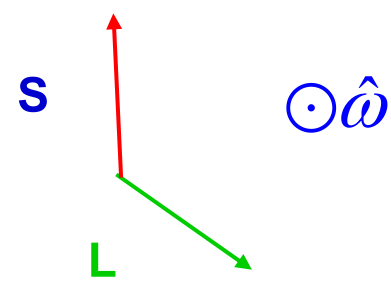
P-(but not T-) violating effects of neutral current part of weak interaction:
For single elementary particle, by Wigner-Eckart theorem, any EDM $\boldsymbol{d}$ must be of form

$$\boldsymbol{d} = \text{const. } \boldsymbol{J} \qquad \leftarrow \text{violates T as well as P.}$$

But for ^{3}He – B, can form

$$\boldsymbol{d} \sim \text{const. } \mathrm{L}\times\mathrm{S} \sim \text{const. } \hat{\omega}$$

$\uparrow$
violates P but not T.



Calculation involves factors similar to that of A-phase ferromagnetism (lots of even more exotic chemical physics!):

Effect is tiny for single pair, but since all pairs have same value of $\mathrm{L}\times\mathrm{S}$, is multiplied by factor of $\sim 10^{23} \Rightarrow$

macroscopic P-violating effect?

(maybe in 10-20 years…)

*AJL, PRL **39**, 587 (1977)