SOME CURRENT RESEARCH PROBLEMS

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1. Low-temperature properties of glasses (with Di Zhou)
2. Energy saving in the cuprate superconductors (with D. Pouliot)
3. Toppological quantum computing in $(p + ip)$ Fermi superfluids (with Yiruo Lin)

1. LOW TEMPERATURE ($\lesssim$ 1K) PROPERTIES OF GLASSES

	Crystals (the anomaly)	Glasses (the norm)
Specific heat c_v	$\propto T^3$	$\propto T$ (roughly)
Thermal conductivity κ	$\propto c_V e^{T_o/T}$	$\propto T^2$ (roughly)
Saturation of sound absorption?	No	Yes
Echoes in sound absorption?	No	Yes

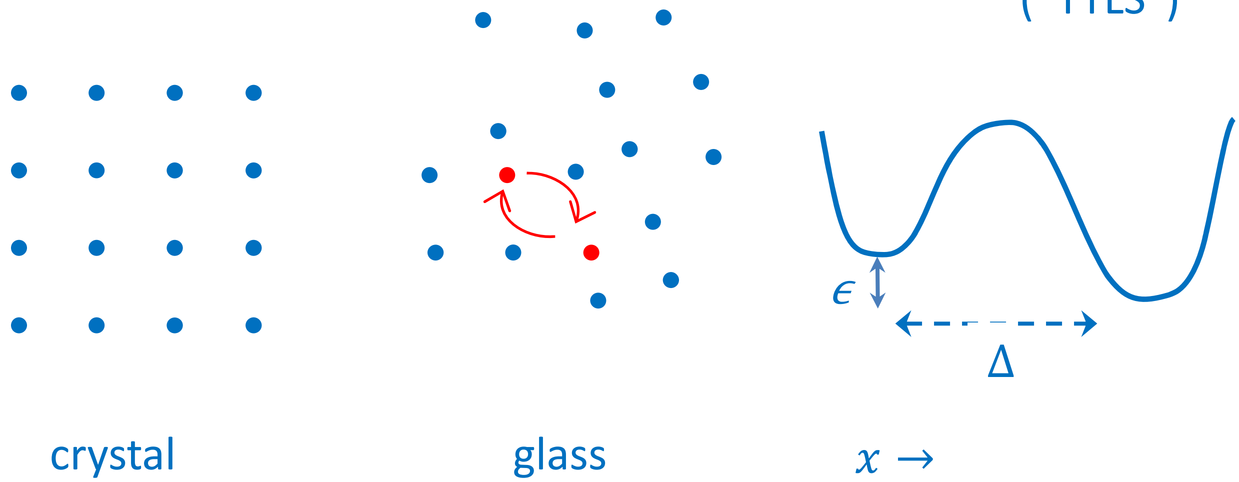
etc…

A bet:	Measure	Predict
 ↑ not crystalline, not metallic	Dimensionless transverse ultrasonic absorption Q^{-1}(MHz-GHz)	$Q^{-1} = 3 \times 10^{-4}$ $\pm 50\%$

verified: ~30 different materials
falsified ~1-2

WHY?

Standard model of LT properties of glasses: tunnelling two-level systems ("TTLS")



with suitable choice of parameters, can "explain" C_V, κ, echoes ---

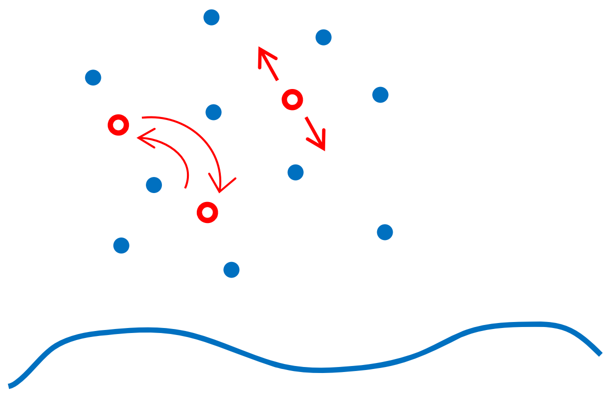
: in TTLS model, ultrasound absorption Q^{-1} is product of 3 independent factors:

$$Q^{-1} = \frac{\pi}{2} \frac{N(O)\gamma^2}{(\rho c^2)}$$

- $N(O)$: density of TTLS states
- γ^2: coupling of sound to TTLS states
- (ρc^2): compressibility

These 3 factors are mutually independent! So how come they always conspire to give $Q^{-1} \approx 3 \times 10^{-4}$?

LT PROPERTIES OF GLASSES: THE "COLLECTIVE" SCENARIO

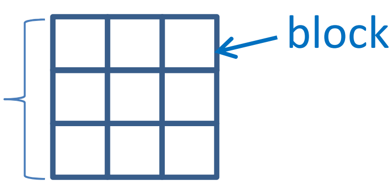


1 At microscale ($\lesssim 20\text{Å}$, say) excitations are
(1) Phonons (harmonic oscillators)
(2) Something else ("junk")

The junk can be anything except harmonic oscillators.

2 Now consider "small" block (size $\sim(100\text{Å})^3$, say). The junk contributes to the (non-phonon) stress tensor of the block, which will interact with the phonon degrees of freedom s.t. $\lambda \gtrsim 100\text{Å}$.

$$\text{interaction} = \sum_i T_{\alpha\beta}^{(i)} u_{\alpha\beta}^{(i)}$$



3 As a result, one gets an interaction between stress tensors of neighboring blocks:

$$\hat{H}_{\text{superblock}} \sum_i \hat{H}_i + \sum_{ij} V_{\alpha\beta\gamma\delta}^{(ij)} T_{\alpha\beta}^{(i)} T_{\gamma\delta}^{(j)}$$

where $V_{\alpha\beta\gamma\delta}^{(ij)} = \dfrac{1}{|r_i - r_j|^3} \Lambda_{\alpha\beta\gamma\delta}^{ij}$ ←nasty 4th rank tensor

$\Rightarrow$ $\hat{H}$ and $\hat{T}$ generated for superblock
$\Rightarrow$ repeat process (real-space RG).

Crucial point: because of r^{-3} behavior of interaction, problem looks identical at each scale $\Rightarrow$ one might anticipate properties iterate to a fixed point.

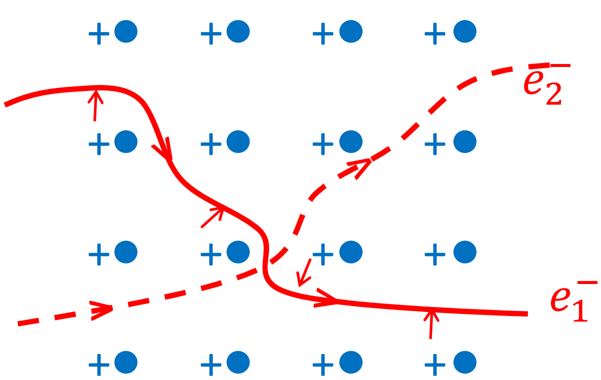
Quantization implementation: very hard! 50 years… partial success, still trying…

2. Energy Saving in Cuprate Superconductivity

Quite generally, if a system makes a transition from a disordered state to a more "ordered" one as T decreases towards 0, can conclude that at $T = 0 \quad \epsilon_{\mathrm{ord}} < \epsilon_{\mathrm{dis}}$. So in particular, if a metal makes a transition from normal (N) state to superconducting (S) state as $T \to 0$, then at $T = 0$ $\epsilon_S < \varepsilon_N$. So:

Where does the energy saving come from?

(a) "Classic" superconductors (Al, Sn, Nb…):
standard textbook picture: (from isotope effect)



$+ve$ ions drawn towards path of e_1^-, relax slowly $\Rightarrow$ e_2^- attracted towards past position of e_1^- $\Rightarrow$ "retarded" $e^- - e^-$ attraction, may outweigh direct Coulomb repulsion. (Some of) this attraction then saved when e^-'s form Cooper pairs.

Chester (1956): saved total Coulomb energy $(e-e, n-n, e-n)$ exactly balanced by expenditure of electronic kinetic energy. Balance is tipped by nuclear kinetic energy!

(b) Cuprate superconductors (since 1986): lack of isotope effect, etc.

$\Rightarrow$ ionic motion (phonons) unimportant. Then only remaining energies which might be saved are
(a) electron kinetic energy and
(b) inter-electron Coulomb repulsion.
Default assumption is latter, then:

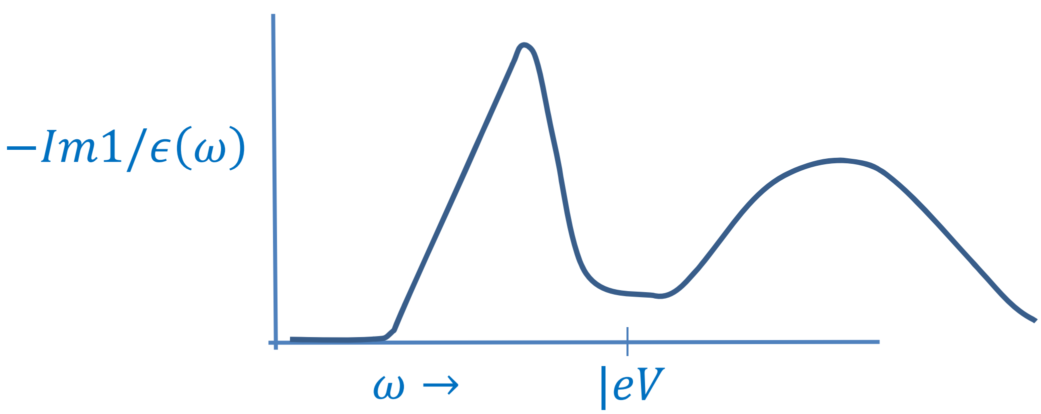
$$\Delta V_{\text{total}} = \int d^3 q \int d\omega \, \Delta V(q\omega),$$

$$\Delta V(q\omega) = \Delta\left(Im\left(-1/\epsilon(q\omega)\right)\right)$$

Dielectric constant, measurable in experiments

loss function

Where in space of wave vector q and frequency ω is Coulomb energy saved? Use "Willie Sutten principle":



Conjecture – small $(\ll q_F)q$, midinfrared ω!

In principle, verifiable by EELS experiments (e.g. P. Abbamonte, UIUC) or (less unambiguously) optics (e.g. D. van der Marel (Geneva))

3. Topological Quantum Computing

Currently, there is a huge interest in the possibility of building a quantum computer, which (theoretically) might be able to perform in a few minutes calculations which would take a classical computer the age of the universe. The underlying principle is that while the basic element of a classical computer, a "bit", can only take one of the 2 values 0 or 1, the corresponding element in a quantum computer (sometimes called a "qubit") can exist in an arbitrary quantum superposition of the two quantum states $|0\rangle, |1\rangle$ corresponding to the classical 0(1):

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad \alpha, \beta \text{ complex numbers, } |\alpha|^2 + |\beta|^2 = 1$$

This might not look like a dramatic advantage for a single qubit, but for N qubits it has the effect of replacing 2N variables by 2^N variables, permitting massive parallel processing which the quantum computer is designed to carry out. If it is to do so successfully, we must be able (as a minimum) to

(a) change the values of α and β at will when we want to
(b) preserve the values of α and β when we do not want to affect them

Both of these operations are subject to random interference by the environment of the system, which in particular is likely to scramble the relative phase and magnitude of α and β in a random way, ("decoherence") so to build a working quantum computer we have to fight this tendency (point (b)) while at the same time being able to manipulate the relative phase and amplitude when we want to (point (a)). In other words, we should be able to do something Nature can't.

Example: trapped ions (we have laser, Nature doesn't!)

The principle involved in topological quantum computing (TQC) is to choose the states $|0\rangle$ and $|1\rangle$ to be very complicated states of a many-particle system, so that the phase and amplitude information is "topologically protected" and can be recovered only by some complicated operation which is available to us but not to nature.

One kind of system which is believed to be suitable for TQC is a topological superconductor such as Sr_2RuO_4. Superconductivity occurs when the system forms Cooper pairs, but these may have different symmetries (relative wave function):

"classic" superconductors (e.g. Al, Sn, Pb…)	s-wave	(boring)
high-temperature superconductors (e.g. YBCO)	d-wave	(somewhat more interesting)
topological superconductors (Sr_2RuO_4)	$p + ip$	very interesting

In a $p + ip$ superconductor, it is possible to produce a peculiar kind of vortex, which comes in pairs. Given such a pair, it is believed that we can associate with them 2 states $|0\rangle$ and $|1\rangle$, as follows:

In state $|0\rangle$, nothing "sits" on either vortex: total number of particles in system is even.

In state $|1\rangle$, a single extra "Bogoliubov quasiparticle" is shared between the two vortices (even if they are a mile apart!): total number of particles is odd. Often this Bogoliubov quasiparticle is said to split itself into 2 independent Majorana fermions. (much sought after in experiment)

The standard picture relaxes condition of conservation of particle number, and postulates that the extra particle is a quantum superposition of an extra electron and an extra hole. Since these have equal weight, on average there is no extra mass or charge density, so this extra particle (or the individual Majorana fermions) is completely undetectable by any local probe. Moreover, (it is claimed) if one constructs the quantum superposition $\alpha|0\rangle + \beta|1\rangle$ and then "braids" (interchanges) the vortices, the relative phase of α and β changes by $\pi/2$, implying nonabelian ("Ising") statistics and hence the possibility of topologically protected quantum computing.

↑: standard lore relies on "mean-field" approach, which violates conservation of particle number. In reality, when one creates the "hole" component of $|1\rangle$, one has to add an extra Cooper pair is conserve particle number.

The $64K question: Is this extra Cooper pair "harmless", or does it spoil the whole show?