

SUPERFLUID ^3He : SOME PRE-HISTORY
 ^4He : below 2K, **superfluid**

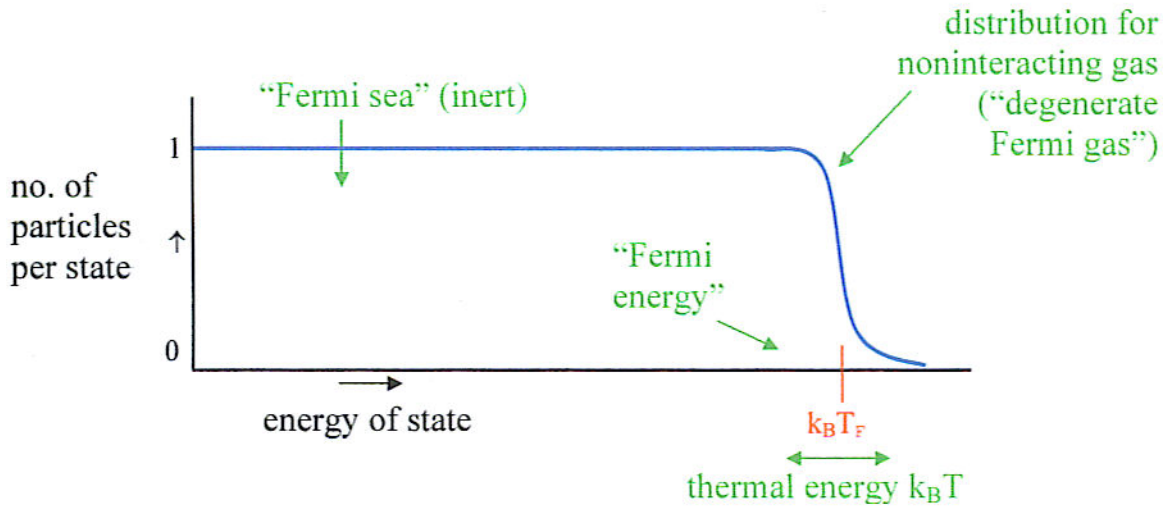
ELECTRONS IN METALS

since ancient times
charged

LIQUID ^3He

since ~ 1950
neutral

Particles of spin $\frac{1}{2} \Rightarrow$ **Fermi statistics**



Landau (1957): interactions don't change picture qualitatively (in "normal" phase) ("degenerate Fermi liquid")

$T_F \sim 10^4 - 10^5 \text{ K}$
 at $T \lesssim 20 \text{ K}$.
superconductivity
 (in some metals)

$T_F \sim 5 \text{ K}$
 at $T \lesssim 10^{-3} \text{ K}$.
superfluidity??

MORE PREHISTORY

Theory of superconductivity:

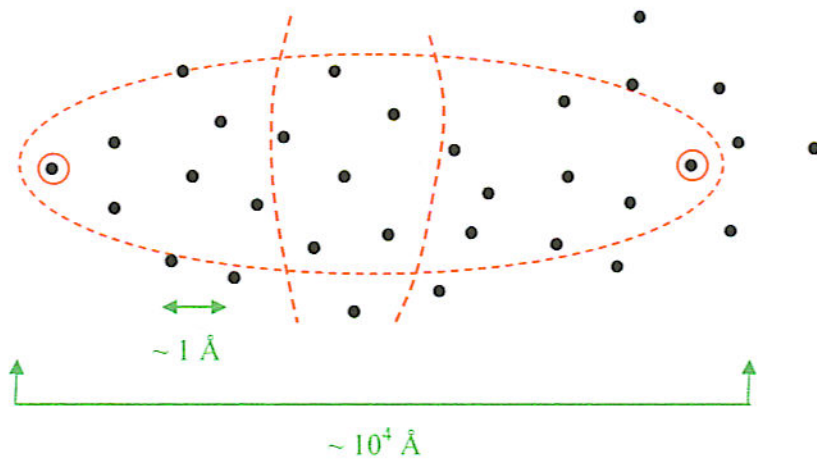
- (a) phenomenological (V. L. Ginzburg, A. A. Abrikosov, et al., 1950-1955):

macroscopic wave function

- (b) microscopic (Bardeen et al., 1957):

electrons in energy shell of width $\sim k_B T_C$ around Fermi energy form Cooper pairs

critical temp., $\lesssim 20K$



Crucial feature of BCS theory: ALL COOPER PAIRS MUST BEHAVE IN EXACTLY THE SAME WAY!

(GL “macroscopic wave function” is just the common center-of-mass wave function of all the pairs)

In BCS theory, “internal” wave function of pairs trivial: (“ 1S_0 ”)

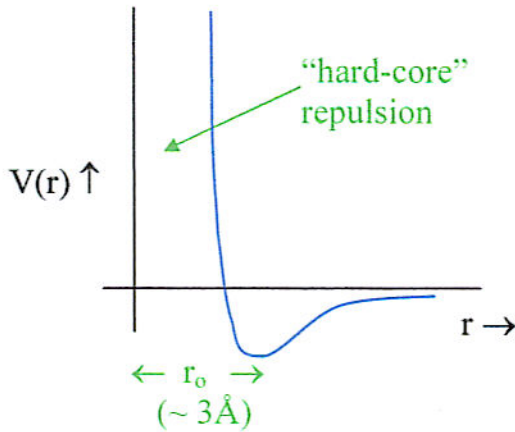
$$\psi(\underline{r}_1 \underline{r}_2; \sigma_1 \sigma_2) \sim \frac{1}{\sqrt{2}} (\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2) f(|\underline{r}_1 - \underline{r}_2|)$$

spin singlet

spherically symmetric ($\ell = 0$)

NO INTERNAL (“ORIENTATIONAL”) DEGREES OF FREEDOM

EARLY THEORETICAL WORK ON POSSIBLE COOPER PAIRING IN LIQUID ³HE



$$r \sim r_0, \quad p \sim p_F \quad (\equiv \sqrt{2mk_B T_F})$$

$$\Rightarrow \text{relative angular momentum}$$

$$\ell \equiv (p_F r_0 / \hbar) \neq 0$$

(prob. 1 or 2)

Pauli principle: $\begin{cases} \ell = 0, 2, 4, \dots & S = 0 \quad (\text{singlet}) \\ \ell = 1, 3, 5, \dots & S = 1 \quad (\text{triplet}) \end{cases}$

in general, $\ell \neq 0 \Rightarrow$ relative (internal) wave function of pair
has orientational degree(s) of freedom!

“equal spin pairing”

Anderson & Morel (1961): explore in detail case $\ell = 2$, and a special case of $\ell = 1$: only $\uparrow\uparrow$ and $\downarrow\downarrow$ pairs form, and have the same orbital angular momentum in direction $\hat{\ell}$ (“ABM” state) Physical properties **anisotropic**.

Vdovin
 Balian & Werthamer } (1963): in $\ell = 1$ case all spin components “³P₀”
✓
 $(\uparrow\uparrow, \downarrow\downarrow, \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow))$ can form: in fact for any given pair, $\underline{L} = -\underline{S} \Rightarrow J = 0$.
 (“BW” state). All physical properties **isotropic**. More stable than any ESP state.

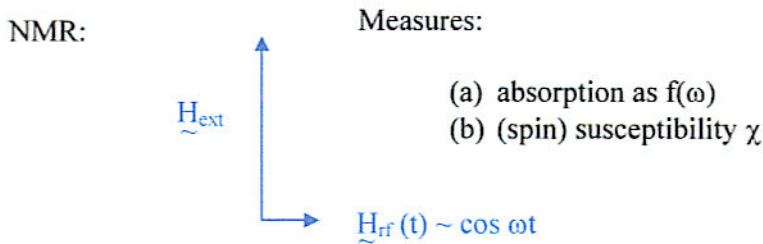
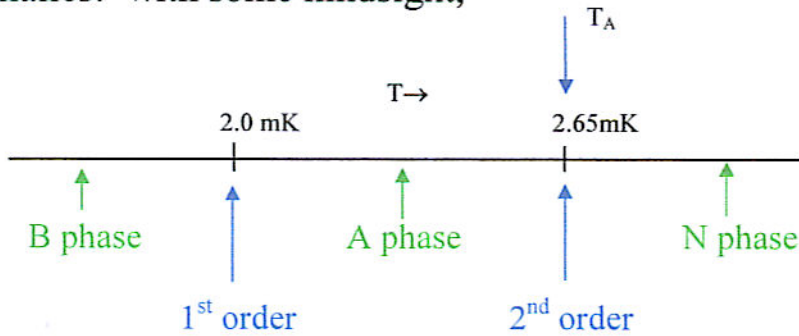
Theoretical expectation c. 1964:

Liquid ³He may form Cooper pairs, either with $\ell = \text{even}$ (**spin singlet**) or with $\ell = \text{odd}$ (**BW state**). In either case, χ reduced and all magnetic properties isotropic. T_c difficult to predict.

THE EXPERIMENTS OF 1971-72 (D. D. Osheroff, R. C. Richardson,
D. M. Lee...(Nobel prize 1996)):

Mixture of liquid and solid ^3He , $T < 3 \text{ mK}$.
(so only temperature varied).

First expts: pressurization (P as $f(t)$)
2 anomalies: with some hindsight,

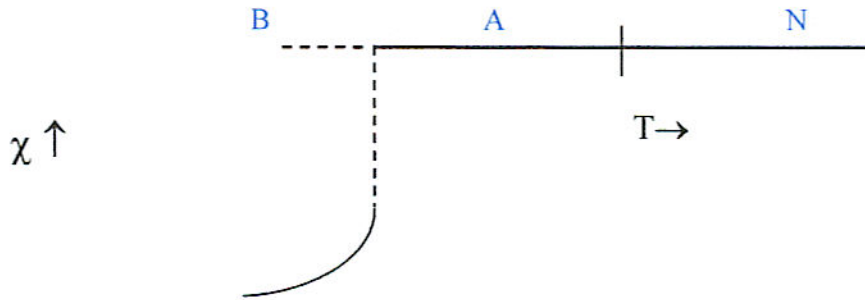


In N state:

χ independent of temperature, value as expected
for degenerate Fermi liquid

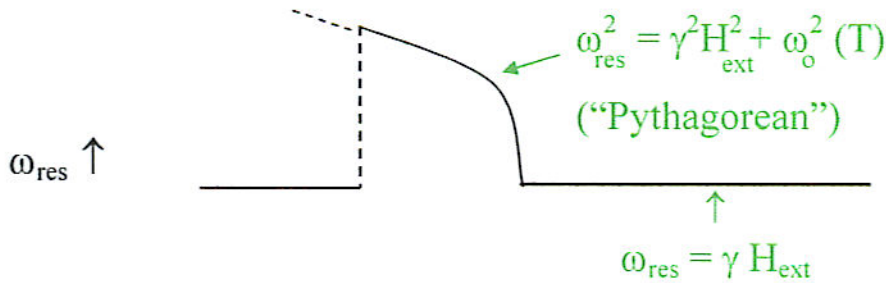
Absⁿ shows v. sharp peak at free-atom Larmor frequency: $\omega_{\text{res}} = \gamma H_{\text{ext}}$
gyromagnetic ratio of free ^3He atom, $\sim 3000 \text{ Hz/G}$

NMR in the new phases:

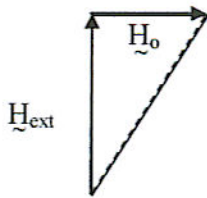


Not necessarily mysterious: e.g. A phase could be an ESP state (only $\uparrow\uparrow, \downarrow\downarrow$ pairs \Rightarrow no reduction in χ), B could be singlet or BW (some $\uparrow\downarrow$ pairs, so χ reduced) [but: why is ESP ever stable?] -

But: what about the resonance frequency?



$$\omega_0^2 (T) \approx A(1 - T/T_A), \quad \frac{A}{(2\pi)^2} \approx 5 \times 10^{10} \text{ Hz}^2$$



$(\equiv \omega_0 (T)/\gamma)$
 Need $H_0 \sim 30\text{G}$. But, only spin-nonconserving force in problem is **nuclear dipole-dipole interaction**, and max. associated field is $< 1\text{G}$!

IS THIS THE FIRST INDICATION OF A RADICAL BREAKDOWN OF QUANTUM MECHANICS?

WHAT CAN BE INFERRED FROM SUM RULES?

IF a single sharp resonance is observed (as in expt.) then:

$$\omega_{\text{res}}^2 = \gamma^2 H_{\text{ext}}^2 + \omega_0^2$$

$$\omega_0^2 = \gamma^2 \chi^{-1} \partial^2 \langle H_D \rangle / \partial \theta^2$$

nuclear dipole energy

angle of simultaneous rot.ⁿ of all spins

But $\partial^2 \langle H_D \rangle / \partial \theta^2 \sim \langle H_D \rangle$:

So, exptl. value of $\omega_0^2(T) \Rightarrow$

$$\langle H_D \rangle (T) \sim K(1 - T/T_A), \quad K \sim 10^{-3} \text{ ergs/cm}^3$$

HOW CAN THIS BE?

$$\left\{ \begin{array}{l} \uparrow \text{ ("bad")} \quad \uparrow \\ \rightarrow \text{ ("good")} \quad \rightarrow \end{array} \right.$$

$$\Delta E \lesssim \frac{\mu_0 \mu_N^2}{3 r_0} \sim 10^{-7} \text{ K} \ll k_B T$$

So, prima facie, preference for "good" orientation over "bad" is at most

$$\sim \Delta E / k_B T \sim 10^{-4} \quad [\text{actually, } \sim \Delta E / k_B T_F \sim 10^{-7}]$$

\Rightarrow expectation value of dipole energy much too small!

SPONTANEOUSLY BROKEN SPIN-ORBIT SYMMETRY

Ferromagnetic analogy:

FERROMAGNET

$$\hat{H} = \hat{H}_0 + \hat{H}_z$$

↑
invariant under simult.
rotation of all spins

$$\hat{H}_z = -\mu_B \mathcal{H} \sum_i S_{zi}$$

↑
breaks spin-rot.ⁿ symmetry

↑
extl. field

Paramagnetic phase ($T > T_c$):
spins behave independently,
 kT competes with $\mu_B \mathcal{H} \Rightarrow$
polarization $\sim \mu_B \mathcal{H} / kT \ll 1 \Rightarrow$
 $\langle H_z \rangle \sim N(\mu_B \mathcal{H})^2 / kT$

Ferromagnetic phase ($T < T_c$):
 \hat{H}_0 forces all spins to **lie parallel**
 $\Rightarrow k_B T$ competes with $N\mu_B \mathcal{H}$
 $\Rightarrow \langle S_z \rangle \sim 1 \Rightarrow \langle H_z \rangle \sim N\mu_B \mathcal{H}$

LIQUID ^3He

$$\hat{H} = \hat{H}_0 + \hat{H}_D$$

↑
invariant under relative
rotation of spin + orbital
coordinate systems

$$\hat{H}_D \equiv \mu_0 \mu_n^2 / r_0^3$$

$$\hat{H}_D = g_D \sum_{ij} \left(\frac{\sigma_i \cdot \sigma_j - 3\sigma_i \cdot \hat{L}_{ij} \sigma_j \cdot \hat{L}_{ij}}{(r_{ij}^3 / r_0^3)} \right)$$

↑
breaks relative spin-orbit
rot.ⁿ symmetry

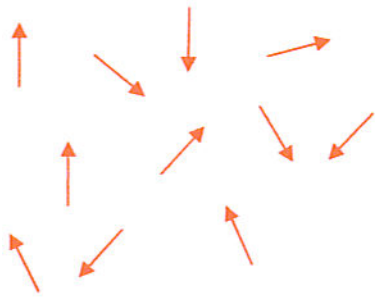
Normal phase ($T > T_A$):
pairs of spins behave
independently \Rightarrow
polarization $\sim g_D / kT \ll 1 \Rightarrow$
 $\langle H_D \rangle \sim N g_D^2 / kT$

Ordered phase ($T < T_A$):
 \hat{H}_0 forces all pairs to
behave similarly \Rightarrow
 kT competes with $N g_D$
 $\Rightarrow \langle H_D \rangle \sim N g_D$
 $\sim 10^{-3} \text{ ergs/cm}^3!$

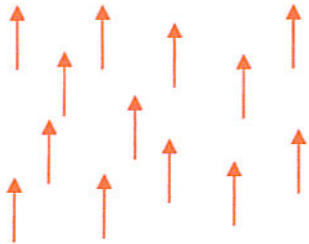
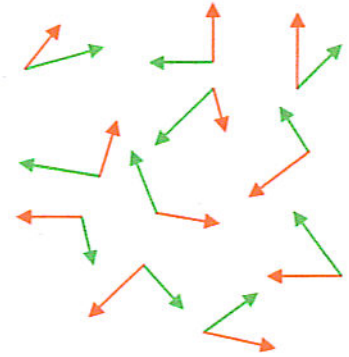
SBSOS: ORDERING MAY BE SUBTLE

FERROMAGNET

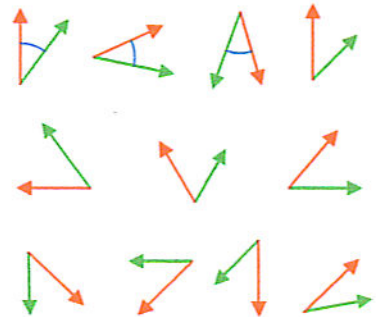
LIQUID ^3He



⇐ NORMAL
PHASE ⇒



⇐ ORDERED
PHASE ⇒



(= total spin of pair

= relative orbital ang. momentum)

$$\langle \tilde{S} \rangle = \langle \tilde{L} \rangle = 0$$

$$\text{but } \langle \tilde{L} \times \tilde{S} \rangle \neq 0!$$

$$\langle \tilde{S} \rangle \neq 0$$

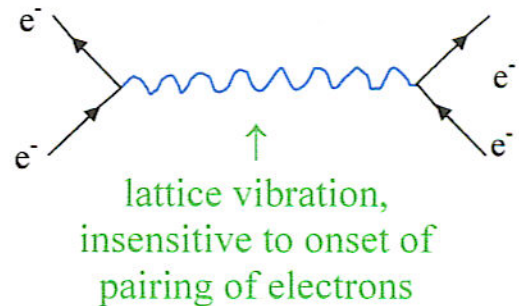
RESOLUTION OF THE PARADOX OF TWO NEW PHASES.

(Anderson & Brinkman, Phys. Rev. Letters **30**, 1108 (1973))

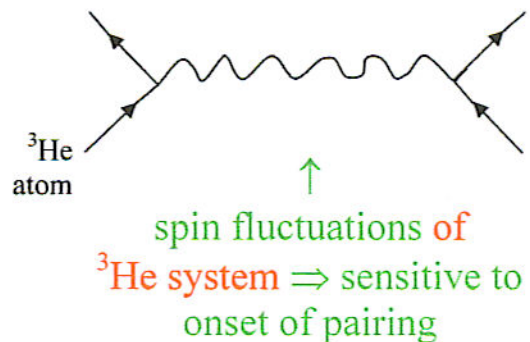
In BCS (weak-coupling) theory for $\ell=1$, BW phase is always stable, independently of pressure and temperature.

Crucial difference between Cooper pairing in superconductors and ^3He

Superconductor:



liquid ^3He :



\Rightarrow “feedback” effects: Over most of the phase diagram, BW state stable as in BCS theory. But at high temperature and pressure, feedback effects **uniquely favor ABM phase**.

major qualitative leap beyond BCS!

MICROSCOPIC SPIN DYNAMICS (SCHEMATIC)

Basic variables:

- | | | |
|--|---|----------------------------------|
| (a) Total spin \underline{S} | } | $[S_i, \theta_j] = i\delta_{ij}$ |
| (b) Orientation $\underline{\theta}$ of spin of Cooper pairs | | |

$$\hat{H} = \hat{H}_o(\underline{S}) + \hat{H}_D(\underline{\theta})$$



hydrodynamic (Born-Oppenheimer) approximation

Semiclassical equations of motion:

$$\frac{d\underline{\theta}}{dt} = \frac{\partial \langle \hat{H}_o \rangle}{\partial \underline{S}} = \mathcal{H}_{\text{ext}} - \chi^{-1} \underline{S}, \quad \frac{d\underline{S}}{dt} = \underline{S} \times \mathcal{H}_{\text{ext}} - \frac{\partial \langle \hat{H}_D \rangle}{\partial \underline{\theta}}$$

dipole torque



zero in eq^m!

⇒ linear NMR behavior completely determined by eigenvalues of quantity

$$\Omega_{ij}^2 \equiv \partial^2 \langle H_D \rangle / \partial \theta_i \partial \theta_j$$

so, can "fingerprint"
A and B phases by
NMR!

ABM: **single resonance line**

axial: split resonance

BW: original BW state is $\underline{L} = -\underline{S}$, i.e. $J = 0$. But dipole torque rotates \underline{S} relative to \underline{L} by $\angle \cos^{-1}(-1/4) = 104^\circ$ around axis $\hat{\underline{\omega}}$ whose "best" choice is \mathcal{H}_{ext} .

Result: **no shift in transverse resonance**, but finite-frequency **longitudinal resonance!**
(also in ABM phase)

$$\mathcal{H}_{\text{ext}} \uparrow \quad \uparrow \mathcal{H}_{\text{rf}} \sim \cos \omega t$$

CONCLUSION (by summer of 1973):

Both a priori stability considerations and NMR experimental data are consistent with hypothesis that both new phases are Cooper-paired (“superfluid”) phases. Specifically,

A phase = ABM

B phase = BW

What is superfluid ^3He good for?

- (a) most sophisticated physical system of which we can claim detailed quantitative understanding. E.g. textures, orientational dynamics, topological singularities...
- (b) analogies with systems in particle physics, cosmology... (G. E. Volovik)
- (c) studies of (some aspects of) turbulence
- (d) Amplification of ultra-weak effects (cf NMR):
Example: P- (but not T-) violating effects of neutral current part of weak interaction:

For single elementary particle, any EDM \underline{d} must be of form

$$\underline{d} = \text{const. } \underline{J} \leftarrow \text{violates T as well as P.}$$

But for $^3\text{He} - \text{B}$, can form

$$d \sim \text{const. } \underline{L} \times \underline{S} \sim \text{const. } \hat{\omega}$$

↑

violates P but not T.

Effect is tiny for single pair, but since all pairs have same value of $\underline{L} \times \underline{S}$, is multiplied by factor of $\sim 10^{23} \Rightarrow$

macroscopic P-violating effect?