TOPOLOGICAL QUANTUM COMPUTING IN (p+ip) FERMI SUPERFLUIDS AND STRONTIUM RUTHENATE: PROSPECTS AND PROBLEMS

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support: NSF-DMR-0906921

thanks: G. Baskaran, R. Budakian, M. Stone, V. Vakaryuk. . . .

Brief Review of "Established Wisdom"

1. <u>p + ip Fermi superfluids</u>

order parameter ↓

For a general spin-1/2 Fermi superfluid, OP df. by

$$F_{\alpha\beta}(\underset{\sim}{r},\underset{\sim}{r}') \equiv \left\langle \psi_\alpha^\dagger(r)\psi_\beta^\dagger(r')\right\rangle \leftarrow \text{"anomalous average"}$$

p + ip:

$$F_{\alpha\beta}(\underset{\sim}{r},\underset{\sim}{r}') = \delta_{\alpha\beta}F_\alpha(\underset{\sim}{r},\underset{\sim}{r}')$$

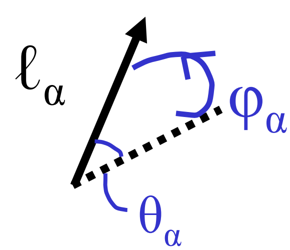
(ESP) "equal spin pairing"

$$F_{\alpha\beta}(\underset{\sim}{r},\underset{\sim}{r}') \equiv F_\alpha(\underset{\sim}{R},\underset{\sim}{\rho}) \cong F_\alpha(\underset{\sim}{R})f_\alpha(\underset{\sim}{\rho})$$

COM → $\underset{\sim}{R}$, rel. coord. → $\underset{\sim}{\rho}$

$$f_\alpha(\underset{\sim}{\rho}) \equiv \sin\theta_\alpha \exp i\varphi_\alpha$$

↑ breaks TRI



thus if ℓ_α taken as z-axis, F. T. is

$$F_\alpha(\underset{\sim}{p}) = p_x + ip_y \quad \leftarrow \text{"}p+ip\text{"}$$

Standard ansatz for MBWF (COM's of ↑, ↓ at rest):

$$\Psi = \Psi_{\uparrow}\Psi_{\downarrow}$$

$$\Psi_{\alpha} \equiv \left(\sum_{\boldsymbol{k}} c^{\alpha}_{\underset{\sim}{k}} a^{+}_{\underset{\sim}{k}\alpha} a^{+}_{-\underset{\sim}{k}\alpha}\right)^{N/2} |\,\mathrm{vac}\rangle$$

where, if z-axis chosen along ℓ_{α}.

$$c^{\alpha}_{k} = f(|\underset{\sim}{k}|)\cdot(k_x + ik_y) \sim \exp i\varphi_{\underset{\sim}{k}}$$

Note dependence on φ_k extends to whole Fermi sea

Some properties of "standard" ansatz:

1. Ang. Momentum along $\ell_{\alpha} \cong N_{\alpha}\hbar/2\left(-0(\Delta/E_F)\right)$ even in limit $\Delta \to 0$ or $T \to T_c$.
2. For 2D ("planar") case ($\underset{\sim}{\ell}_{\alpha} \perp$ plane), put $z = x + iy$, then for all $|z_i - z_j| \gg \xi$, coord-space MBWF is of form

   ↗ pair radius

$$\Psi_N(z_1 z_2 \ldots z_N) = \text{const.}\ \mathcal{A}\left\{\frac{1}{z_1 - z_2}\cdot\frac{1}{z_3 - z_4}\cdot\frac{1}{z_5 - z_6}\cdots\right\}$$

"Pfaffian"

Cf MR ansatz for $\nu = 5/2\ QH$ state

↗ Moore-Read

ESTABLISHED WISDOM (cont.)

Examples of (p + ip) Fermi superfluids:

1. ^{3}He-A

   Evidence:

   (a) ESP: χ unchanged, NMR

   (b) p-wave: sp. ht., ultrasound attenuation

   (c) broken TRI: (literal) ferromagnetism (?)

   (d) specific (p + ip) state (3D): NMR

2. Sr_2RuO_4 ("SRO")

   (structure similar to cuprates, $T_c \sim 1{\cdot}5$ K)

   Evidence:

   (a) ESP: χ unchanged in sup^g phase

   (b) p-wave-like (i.e. odd-parity): Josephson (PSU)

   (c) broken TRI: muon spin resonance, Josephson (UIUC), Kerr rotation (but ↑: also in N state of cuprates)

ESTABLISHED WISDOM (cont.)

3. Half-quantum vortices ("HQV")

Should occur in any ESP Fermi superfluid, provided coupling between ↑↑ and ↓↓ sufficiently weak.

e.g. (neutral case):

vortex in ↑↑ components, nothing in ↓↓ component, i.e.

$\Delta_{\uparrow\uparrow} \propto \exp i\Phi$, $\Delta_{\downarrow\downarrow} \propto$ const. ("half-quantum" vortex)

Note, however, that quantization condition for ↑↑ pair velocity is still

$$K \equiv \int_0^{2\pi} \underset{\sim}{\upsilon}_{s\uparrow} \cdot \underset{\sim}{dl} = h/2m$$

Can tolerate Majorana fermions.

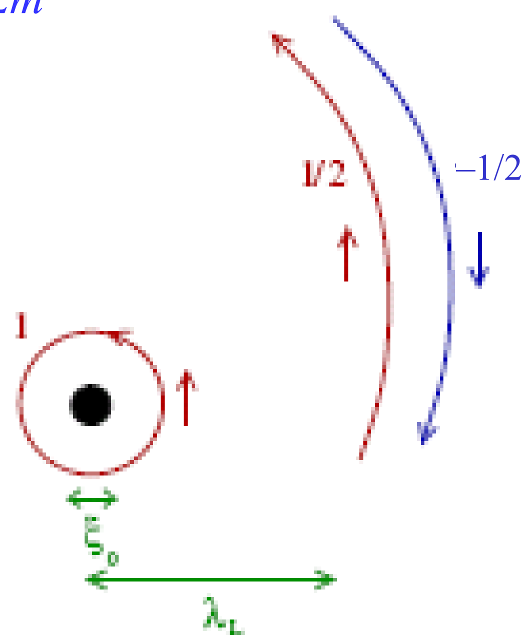
What about charged case?

At $r \ll \lambda_L$, current only in ↑↑ component → total $j \neq 0$.

However, for $r \gtrsim \lambda_L$, ↓↓'s are involved :

$$0 \neq \underset{\sim}{j}_{\uparrow} = \underset{\sim}{j}_{\downarrow}$$

$$\underset{\sim}{j}_{tot} = 0$$



Total trapped flux = $\Phi_0/2 = nh/4e$.

Hence, vortex with Dirac (non-Majorana) fermion circling another picks up phase π/2, not π as for BCS)

Note: HQV in charged system carries circulating spin current as r→∞ ⇒ energetically disadvantaged relative to simple (h/2e) vortex.

ESTABLISHED WISDOM (cont.)

4. Majorana Fermions and TQC ← topological quantum computation

Consider a HQV in a 2D (p + ip) Fermi superfluid. Model "gap function" $\Delta_\uparrow(\underset{\sim}{r},\underset{\sim}{r}')\left(\equiv V(\underset{\sim}{r}-\underset{\sim}{r}')F_\uparrow(rr')\right)$ by

$$\Delta_\uparrow(\underset{\sim}{r},\underset{\sim}{r}') \equiv \Delta_\uparrow(\underset{\sim}{R},\underset{\sim}{\rho}) = \Delta(\underset{\sim}{R})(\nabla_x + i\nabla_y)\delta(\underset{\sim}{\rho})$$

COM → $\underset{\sim}{R}$, $\underset{\sim}{\rho}$ ← rel. coord

Then (Kopnin-Salomaa, Volovik, Moore-Read) ∃ a single solution of the BdG (Bogoliubov-deGennes) equations for the particle/hole amplitudes $u(\underset{\sim}{r})\,/\,\mathrm{v}(r)$ s.t.

$$\underbrace{E = 0,\; u(\underset{\sim}{r}) = \mathrm{v}^*(\underset{\sim}{r})}_{\text{"Majorana fermion" (M.F.)}}$$

These M.F.'s satisfy the braiding and recombination rules of Ising anyons, and that can be used (Ivanov) for (partially) topologically protected quantum computation, by braiding the HQV's appropriately.

[End of Established Wisdom]

Some Questions about the Established Wisdom

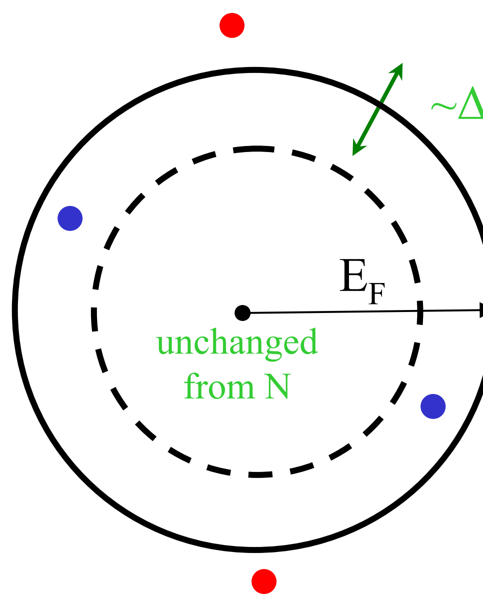
1. Nature of MBWF of (p + ip) Fermi superfluid

Recap: standard ansatz is (for say ↑↑)

$$\Psi \sim \left(\sum_k c_k a_k^+ a_{-k}^+ \right)^{N/2} |\,\mathrm{vac}\rangle, c_k \sim \exp i\varphi_k$$

i.e. all pairs of states in Fermi sea have anyon momentum $\hbar$.

Alternative ansatz:



first shot:

$$\Psi(N_p, N_h)$$

$$\sim \left(\sum_{k>k_F} c_k a_k^+ a_{-k}^+ \right)^{N_p/2},$$

$$\left(\sum_{k<k_F} d_k a_{-k} a_k \right)^{N_h/2} |\,\mathrm{vac}\rangle$$

⚠: keeps pp→pp and hh→hh, but not (e.g.) pp→hh.

Remedy:

$$\Psi \sim \sum_{N_p, N_K} Q_{N_p N_k} \Psi(N_p, N_k),$$

$$Q \text{ slowly varying as } f(N_p, N_k)$$

degenerate with standard ansatz to $0(N^{-1/2})$, but

$$L \sim (N\hbar / 2) \cdot (\Delta / E_F)^2$$

IS GS OF (p + ip) UNIQUE?

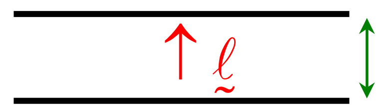
QUESTIONS (cont.)

2. Are the physical systems suitable?

^{3}He-A

Almost certainly (p + ip), but can it be made "2D"?

e.g. "slab" geometry:



Δ: en. gap has modes along $\underset{\sim}{\ell}$

⇒ low-lying excitations (v. bad for TQC!)

To eliminate these, need $k_F d \ll \varepsilon_F / k_B T$. But also need $d \gg \xi$. Compatible for $T \ll T_c$.

SRO

Ev. for ESP and violation of TRI fairly strong, but not entirely obvious that latter is due to (p + ip) (e.g. μSR signal could be due to nonunitary spin state?)

In any case, is it sufficiently "2D"?

(a) single layer?

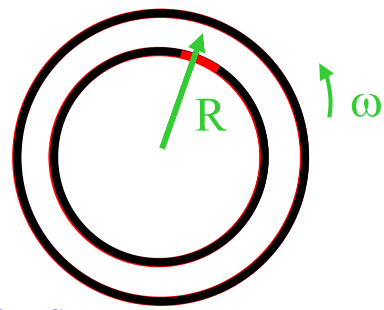
(b) thin macroscopic slab?

QUESTIONS (cont.)

3. Half-quantum vortices

Problems:

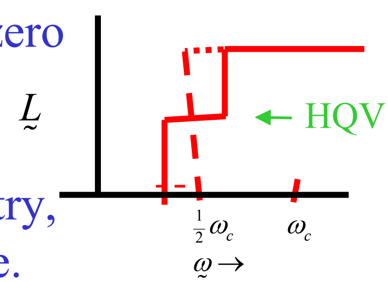
(A) Non-observation of HQV's in ^{3}He-A:
Consider thin annulus rotating at ang. velocity ω, and df. $\omega_c \equiv \hbar / 2mR^2$



At $\omega = \frac{1}{2}\omega_c$ exactly, the nonrotating state and the ordinary "vortex" (p-state) with both spins rotating are degenerate.

But a simple variational argument shows that barring pathology, there exists a nonzero range of ω close to $\frac{1}{2}\omega_c$ where the HQV is more stable than either!

In a simply connected flat-disk geometry, argument is not rigorous but still plausible.



Δ: Yamashita et al. (2008) do experiment in flat-disk geometry, find NO EVIDENCE for HQV!

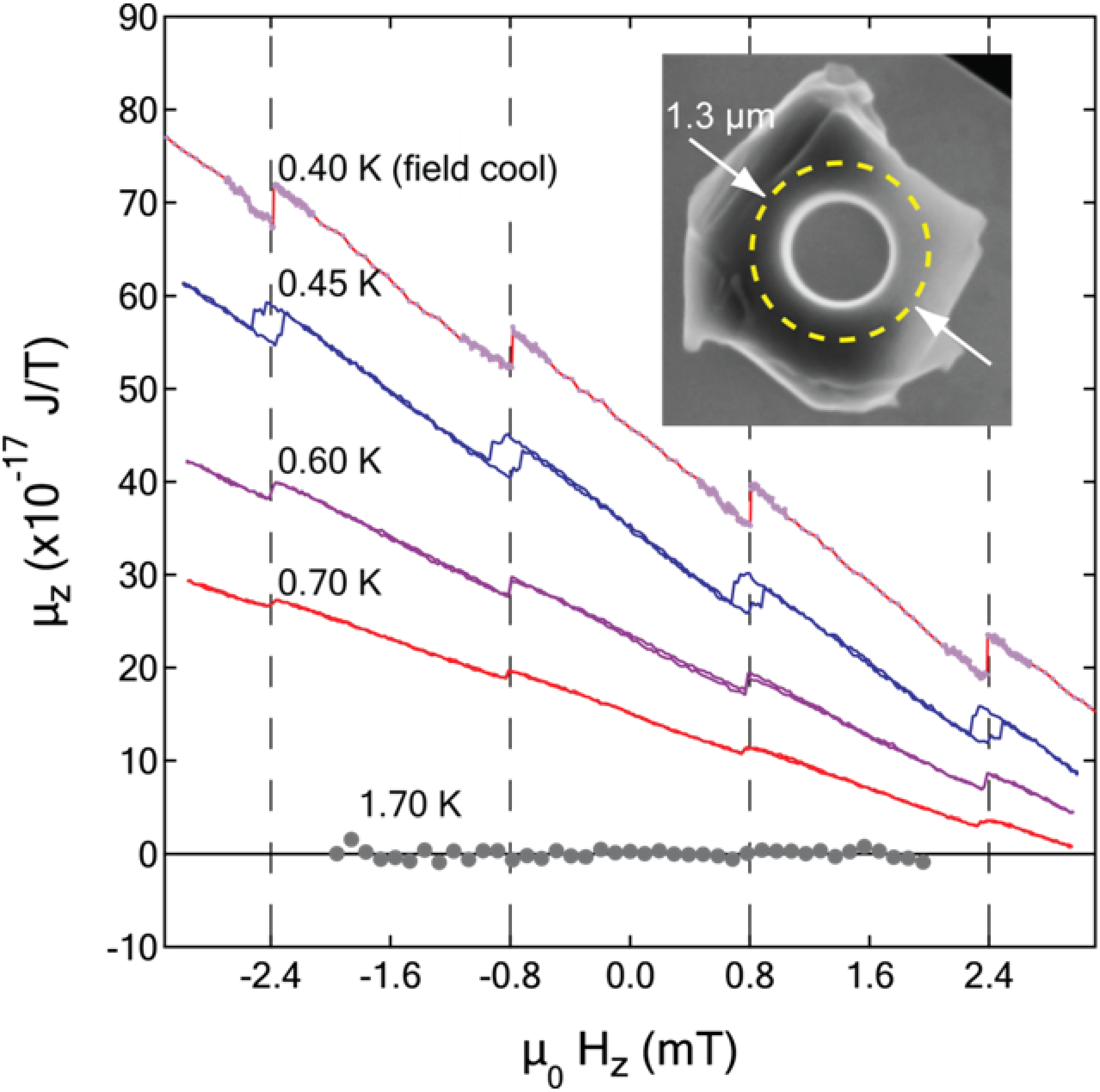
Possible explanations:

(1) HQV is not stable under experimental conditions (Kawakami et al., 2009)

(2) HQV did occur, but NMR detection technique insensitive to it.

(3) HQV is thermodynamically stable, but inaccessible in experiment.

(4) Nature does not like HQV's.



90
80
70
60
50
40
30
20
10
0
-10
μ_z (x10^{-17} J/T)
-2.4
-1.6
-0.8
0.0
0.8
1.6
2.4
μ_0 H_z (mT)
0.40 K (field cool)
0.45 K
0.60 K
0.70 K
1.70 K
1.3 μm

QUESTIONS (cont.)

4. Do MF's "exist"?

A. Does existence of a single "Majorana" mode survive replacement of model form $\Delta(\underset{\sim}{r},\underset{\sim}{r}')=\Delta(\underset{\sim}{R})(\nabla_x+i\nabla_y)\delta(\rho)$ by physical (nonzero-range) form?

B. What is an MF?

(Established approach: 2 MF/s = 1 "real" (Dirac-Bogoliubov) fermion)

Any "completely paired" even-N MBGS can be written

$$\Psi_N \sim \left(\sum_n c_n a_n^+ a_{\bar{n}}^+\right)^{N/2} |\,\mathrm{vac}\rangle \quad n+\bar{n} = \text{ orthonormal set}$$

Write

$$u_n \equiv (1+|c_n|^2)^{-1/2}, \mathrm{v}_n \equiv c_n(1+|c_n|^2)^{-1/2}$$

then:

Any operator of form (or linear combination thereof)

$$\mathrm{v}_n a_n^+ + u_n a_{\bar{n}} C^\dagger \;\leftarrow\cong \sum_n c_n a_n^+ a_{\bar{n}}^+$$

identically annihilates GS ("pure annihilator") while any operator of form (or linear combination thereof)

$$u_n^* a_n^+ - \mathrm{v}_n a_{\bar{n}} C^\dagger$$

creates (N+1)-particle state ("DB fermion") (but in general not energy eigenstate).

The (N+1)-particle energy eigenstates are the particular combinations of DB states that satisfy the BdG equations.

Suppose now we have found a solution of the BdG equations corresponding with energy eigenvalue E = 0. What does it represent?

(a) It can create an energy eigenstate of the N + 1-particle system, (DB fermion) with excitation energy 0 relative to the N-particle GS.

(b) It can simply correspond to a pure annihilator! But, in neither case can $u(r) = v^*(r)$ as required for an M. F.

Conclusion: An M. F. is nothing but a quantum superposition of an E= 0 DB fermion and a pure annihilator!

If GS is strongly entangled, DB fermion may be "split" between 2 spatially separated regions (e.g. 2 mutually remote vortices): then MF's may be localized around single vortices.

So:

E = 0 DB fermion + pure annihilator $\Rightarrow$ 2 MF's.

The $64K question: DOES THE CONVERSE HOLD?

i.e. does existence of 2 MF's establish existence of single "split" DB fermion?

Can we mimic the real-life 2-vortex problem with an exactly soluble toy model?

Plausible shot: "directed-link" model (Kitaev):

• → • → • → • → • → •

$$\hat{H} = -\sum_j \left(t a^+_{j+1} a_j + \Delta a^+_{j+1} a^+_j \right) + H.c.$$

$$(\text{"}1D\ p + ip\text{"})$$

$$\Delta = (\pm) t$$

Exactly soluble, sustains MF's on ends:

x • • • • • • x

But: obvious way to form "vortex" is to reverse sign of Δ relative to t

"vortex"

↓

• → • → • → • → • → • ← • ← • ← • ← • ← •

Still exactly soluble, but now retains 2 MF's at "vortex"!

x • • • • x x • • • • x

If true for real case, disaster for TQC!

? ?

Challenge: find exactly soluble model of (p + ip) Fermi superfluid with 2 vortices, and establish existence/nonexistence of single DB fermion "split" between them.