THE BERRY PHASE OF A BOGOLIUBOV QUASIPARTICLE IN AN ABRIKOSOV VORTEX*

(how far can we trust arguments based on the Bogoliubov-de Gennes equations?)

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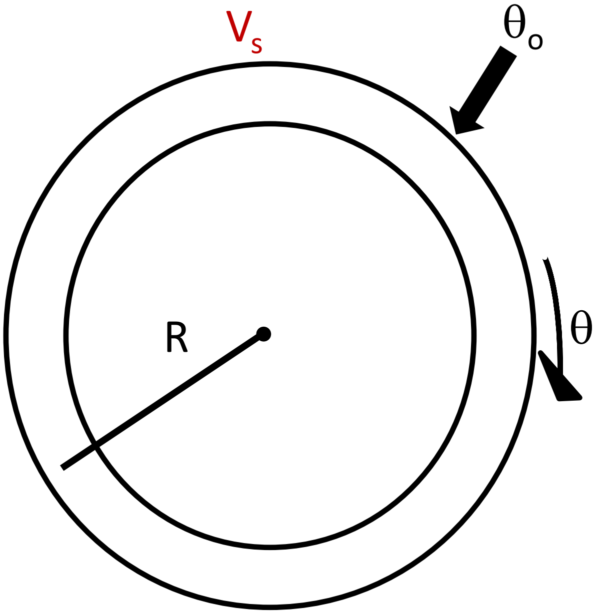
A TRIVIAL-LOOKING PROBLEM



Consider a neutral s-wave Fermi superfluid in an annular geometry: single quantum of circulation

$$\mathbf{v}_s = \hbar/2mR$$

$$(\oint \boldsymbol{v}_s \cdot \boldsymbol{dl} = h/2m).$$

Create Zeeman magnetic field trap:

$$\hat{H}_z = -\mu_B \sigma_z B(r)$$

$$B(r)=B_o f(\theta-\theta_o)$$

pair radius

$\mu_B B_o \ll \Delta$, range of $f(\theta) \gg \xi/R$ but $\ll \mu\ 1$.

So effect on condensate $o(\mu_B B_o/\Delta)^2$

GS of 2N+1-particle system presumably has single Bogoliubov quasiparticle trapped in Zeeman trap.

Now, let's move θ_o adiabatically once around annulus.

Question:

What Berry phase is picked up?

Possible conjectures:

(a) π
(b) 0
(c) something else

Why is this an interesting question?

Motivation:

Possibility of topological quantum computing (TQC) in (p+*i*p) Fermi superfluid (Sr_2RuO_4).

Basic ingredient (Ivanov 2001): Majorana fermions (MF's) trapped on half-quantum vortices (HQV's)

The crucial claim: consider 2 vortices, then 2 states of interest:

Dirac-Bogoliubov

a. 2N-particle GS (no MF's)

b. 2N+1-particle GS (2 MF's = 1 E = O DB fermion)

("braided")

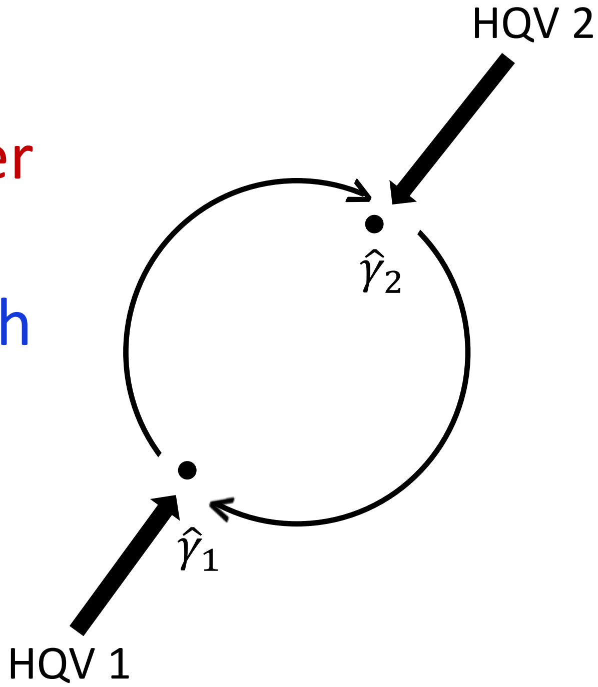
Then if vortices are adiabatically interchanged ^ and we define the Berry phase accumulated by b relative to that accumulated by a as $\Delta\varphi_B$, then (Ivanov 2001)

$$\Delta\varphi_B = \pi/2 \qquad (*)$$

From this, in case of 4 vortices, braiding induces non-Abelian statistics $\Rightarrow$ possibility of (Ising) TQC.

How does (*) result in Ivanov's argument?

Considers way in which MF creation operator γ_1, γ_2 depends on phase of Cooper pair order parameter Δ ($\mathbf{r}$), then works out way in which $\Delta(r_1)$ and $\Delta(r_2)$ behave under interchange.



Concludes, $\gamma_1 \rightarrow \gamma_2$ but $\gamma_2 \rightarrow - \gamma_1 \Rightarrow$ (*).
So, $64K question:
is (*) correct?

Some possible problems with Ivanov argument:

1. Based on BdG equation. ⇒ total

Bogoliubov de Gennes

particle no. not conserved. However, for real TQC applications, must compare states of (same) definite particle no.

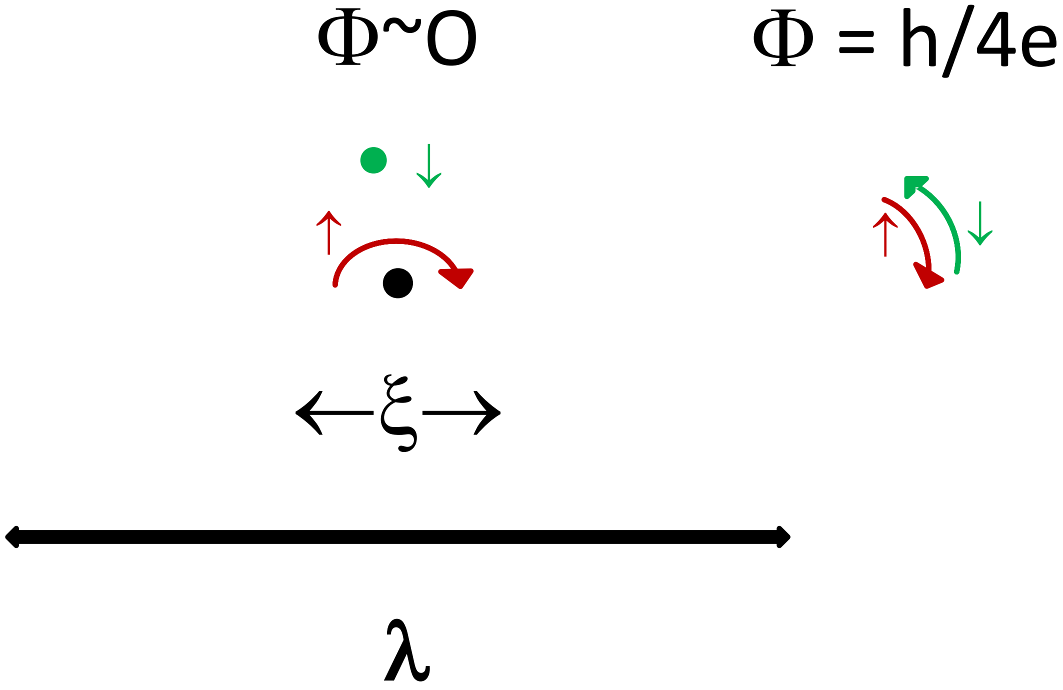
2. Thus condensate wave function is fixed, independently of behavior of excitations.

3. No specification of how HQV's are "adiabatically" braided.

4. (In context of implementation in Sr_2RuO_4): at what distance (relative to λ) are HQV's braided?

London penetration depth

Reminder re currents, etc., in HQVs in charged system (Sr_2RuO_4):



Φ~O

$\Phi = h/4e$

$\uparrow$ $\downarrow$

$\uparrow$ $\downarrow$

$\leftarrow\xi\rightarrow$

λ

How to resolve these problems?

(a) approach from Kitaev quantum-wire model (Alicea et al.,…)

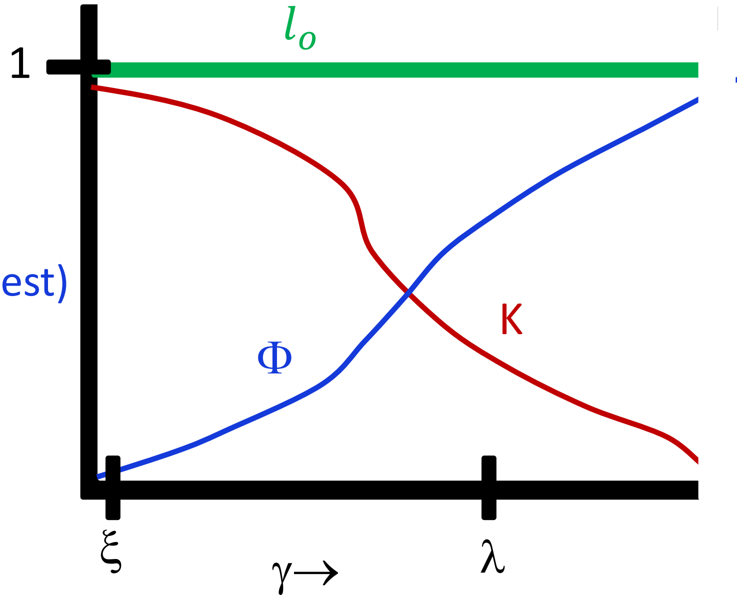
(b) try to resolve simpler problem first

A "TOY" PROBLEM: DB QUASIPARTICLE IN SIMPLE ABRIKOSOV VORTEX

$$\mathbf{v}_s = \frac{\hbar}{2m}\left(\nabla\varphi - \frac{2e}{\hbar}A(\boldsymbol{r})\right)$$

$\oint \nabla\varphi \cdot \boldsymbol{d\ell} = \ell_o$ (≡ 1 for case of interest)

+ Maxwell



Df.
$$\left.\begin{array}{l} K \equiv \oint \boldsymbol{v_s}\cdot\boldsymbol{d\ell}/(\frac{h}{2m}) \\ \Phi \equiv \oint \boldsymbol{A}\cdot\boldsymbol{d\ell}/(\frac{h}{2e}) \end{array}\right\}, \text{ then } K(r) = l_o - \Phi\,(r)$$

Transport single DB qp adiabatically around core at distance r.

Is Berry phase picked up

(a) $\propto \ell_o$ (i.e. constant)

$$\left.\begin{array}{l}\text{(b)} \propto \Phi\,(r) \\ \text{(c)} \propto K\,(r)\end{array}\right\} \text{ r-dependent } \quad ?$$

A. “Ultra-naïve” argument : Berry phase is topological property, thus must depend on only topological invariant, namely l_o.

Now it seems certain that for r→∞ correct answer is π, so if this argument is correct then also for r→0 (but ≫ ξ) result should be π.

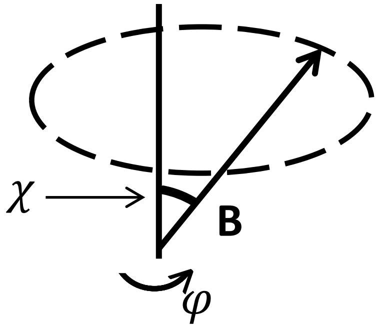
Is this right? (⇒ “annular” problem)

B. Somewhat-less-naïve argument:

Treat $B(\theta-\theta_o)$ as simply fixing qp position near θ_o, then problem formally analogous to particle of spin ½ in magnetic field whose direction varies in space.

magnetic case : $\hat{\mathrm{H}} = -\,\boldsymbol{\sigma}\cdot\,\mathbf{B}$ $\qquad$ $B_x + B_y = |B_\perp|\ \exp i\varphi$

superfluid case : $\hat{\mathrm{H}}_{MF} = -\,\text{“}\boldsymbol{\sigma}\text{”}\cdot\mathcal{H}$ $\qquad$ $\mathcal{H}_x + i\mathcal{H}_y = \propto |\Delta|\ \exp i\theta$



|↑> ⇄ |p> , |↓> ⇄ |h>

⬆ particle

⬆ hole

In spin – ½ case, standard result is (mod 2 π) :

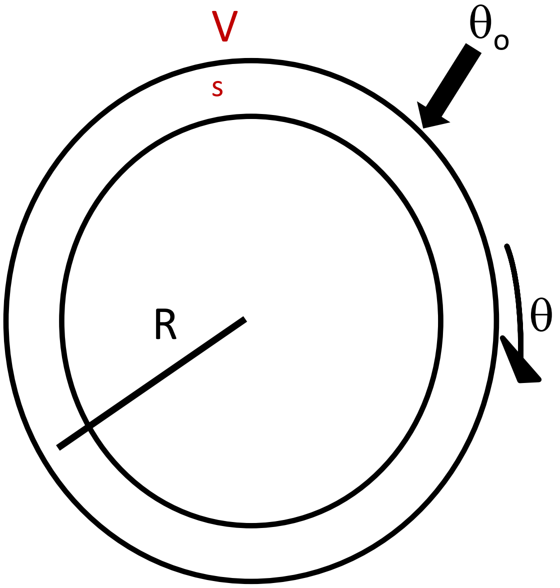
$\varphi_B = 2\pi \cos^2 \chi/2 \leftarrow$ “weight” of |↑> component

in particular, for equal weights of |↑> and |↓> ($\chi = \pi/2$), $\varphi_B = \pi$

In our case, weight of |p> and |h> should be equal for bound state (Andreev reflection) ⇒ if analogy with spin – ½ is valid, then

$$\varphi_B = \pi$$

C. Microscopic (N-conserving) argument (executive summary)



Ansatz:

2N-particle "ground" state for $\ell_o \neq 0$ is

$$\Psi_{2n} = \left(\sum_\ell c_\ell \; a^+_{\ell\uparrow} a^+_{-\ell+\ell_o}\right)^{N/2} |\text{vac}> \qquad (C_\ell \equiv v_\ell / u_\ell)$$

$$\equiv (\hat{\Omega}_{\ell o})^{N/2} |\text{vac}>$$

In presence of Zeeman trap, 2N+1-particle "ground" state with $l_o \neq 0$ is of general form

$$\Psi_{2N+1} = \sum_\ell f_\ell \alpha^+_\ell \Psi_{2N} \qquad \alpha^+_\ell \equiv u_\ell a^+_{\ell\uparrow} - \mathrm{v}^*_\ell a_{-\ell+\ell_o\downarrow} \underset{\uparrow}{\hat{\Omega}_{\ell_o}}$$

Conserves N!

Then easy to show that

$$\varphi_B = 2\pi \sum_\ell (\ell |f_\ell|^2.)$$

Evaluation of RHS tricky, but

(a) for $v_s \rightarrow o$ ($r \rightarrow \infty$), certainly $\varphi_B = \pi$

(b) for $\Phi \rightarrow o$ ($r \rightarrow o$), with high confidence,

$\varphi_B = O$

If this is correct, “naïve” BdG-based arguments (e.g. B above) may be qualitatively misleading.

The $64K question: does any of this affect the “established” conclusion re TQC in a (p + *i*p) Fermi superfluid?