

# THE BERRY PHASE OF A BOGOLIUBOV QUASIPARTICLE IN AN ABRIKOSOV VORTEX\*

(how far can we trust arguments based on the  
Bogoliubov-de Gennes equations?)

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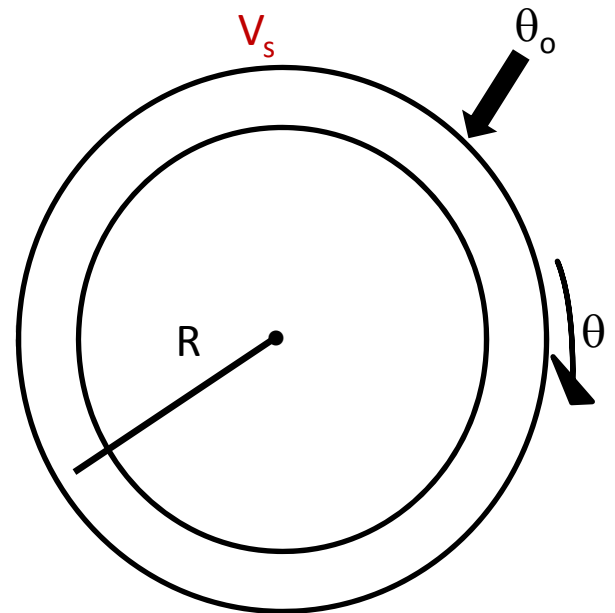
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# A TRIVIAL-LOOKING PROBLEM

Consider a neutral s-wave  
Fermi superfluid in an  
annular geometry: single  
quantum of circulation

$$\mathbf{v}_s = \hbar / 2mR$$

$$(\oint \mathbf{v}_s \cdot d\mathbf{l} = h/2m).$$



Create Zeeman magnetic field trap:

$$\hat{H}_z = -\mu_B \sigma_z B(r)$$

$$B(r) = B_0 f(\theta - \theta_0)$$

$$\mu_B B_0 \ll \Delta, \text{ range of } f(\theta) \gg \xi/R \text{ but } \ll 1.$$

So effect on condensate  $\propto (\mu_B B_0 / \Delta)^2$

GS of  $2N+1$ -particle system presumably has **single Bogoliubov quasiparticle** trapped in Zeeman trap.

Now, let's move  $\theta_0$  **adiabatically** once around annulus:

Question:

What Berry phase is picked up?

Possible conjectures:

- (a)  $\pi$
- (b) 0
- (c) something else

Why is this an interesting question?

## Motivation:

Possibility of topological quantum computing (TQC) in (p+ip) Fermi superfluid ( $\text{Sr}_2\text{RuO}_4$ ).

Basic ingredient (Ivanov 2001): Majorana fermions (MF's) trapped on half-quantum vortices (HQV's)

The crucial claim: consider 2 vortices, then 2 states of interest:

- a.  $2N$ -particle GS (no MF's)
  - b.  $2N+1$ -particle GS (2 MF's = 1  $E = 0$  DB fermion)
- Dirac-Bogoliubov  
↓

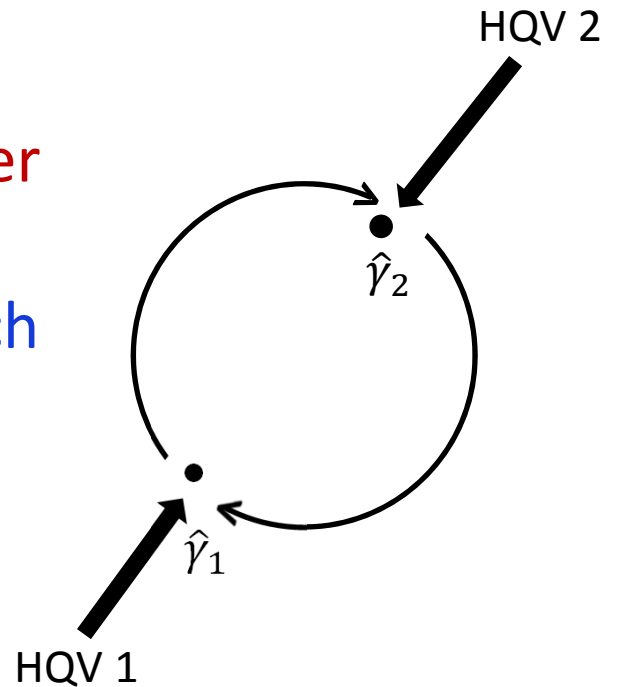
Then if vortices are adiabatically interchanged (“braided”) and we define the Berry phase accumulated by b relative to that accumulated by a as  $\Delta\varphi_B$ , then (Ivanov 2001)

$$\Delta\varphi_B = \pi/2 \quad (*)$$

From this, in case of 4 vortices, braiding induces non-Abelian statistics  $\Rightarrow$  possibility of (Ising) TQC.

How does (\*) result in Ivanov's argument?

Considers way in which MF creation operator  $\gamma_1, \gamma_2$  depends on **phase of Cooper pair order parameter  $\Delta(\mathbf{r})$** , then works out way in which  $\Delta(r_1)$  and  $\Delta(r_2)$  behave under interchange.



Concludes,  $\gamma_1 \rightarrow \gamma_2$  but  $\gamma_2 \rightarrow -\gamma_1 \Rightarrow (*)$ .

So, \$64K question:

**is (\*) correct?**

## Some possible problems with Ivanov argument:

1. Based on BdG equation.  $\Rightarrow$  total



Bogoliubov de Gennes

particle no. not conserved. However, for real TQC applications, must compare states of (same) **definite particle no.**

2. Thus condensate wave function is fixed, independently of behavior of excitations.

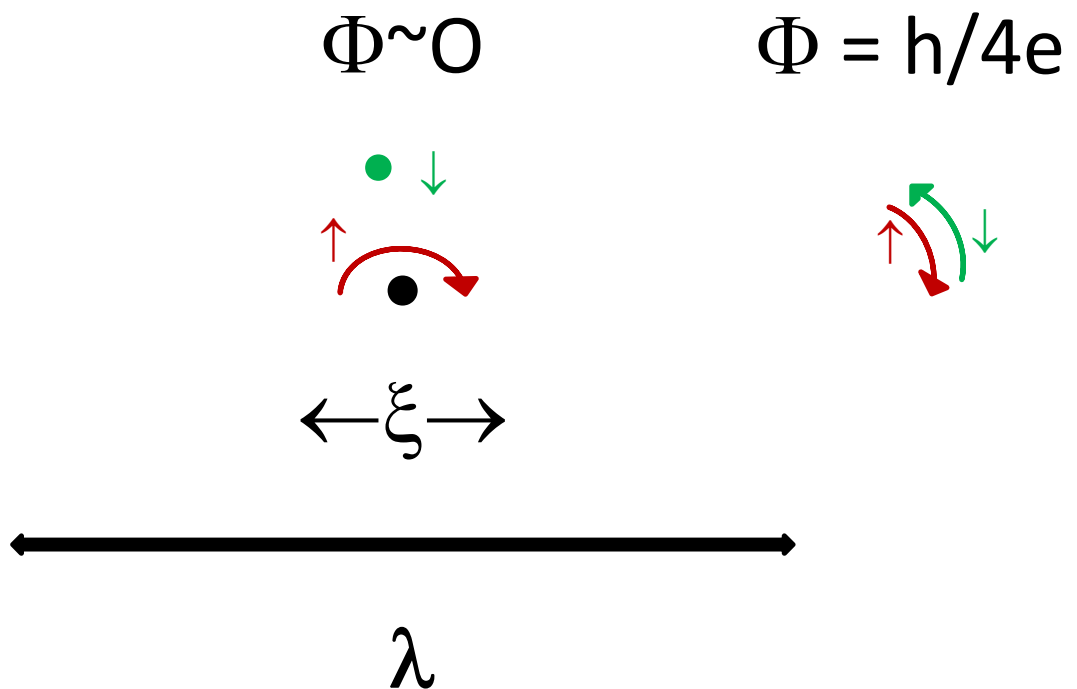
3. No specification of **how** HQV's are "adiabatically" braided.

4. (In context of implementation in  $\text{Sr}_2\text{RuO}_4$ ): **at what distance** (relative to  $\lambda$ ) are HQV's braided?



London penetration depth

# Reminder re currents, etc., in HQVs in charged system ( $\text{Sr}_2\text{RuO}_4$ ):



How to resolve these problems?

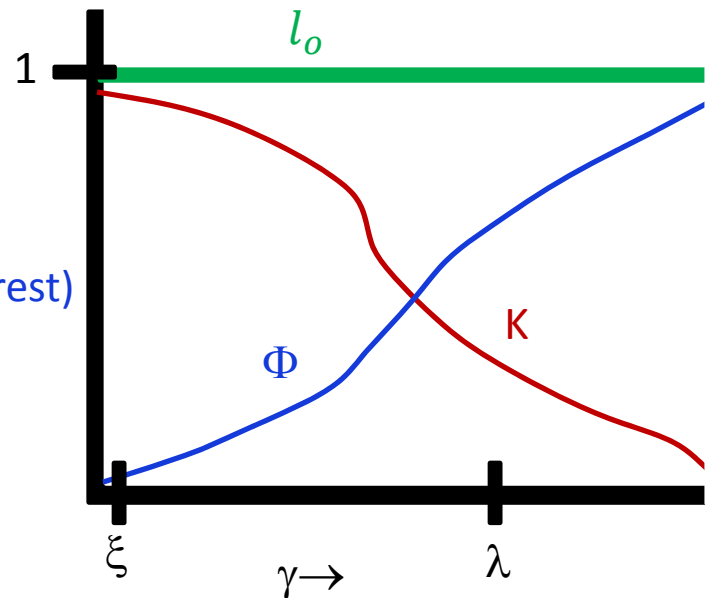
- (a) approach from Kitaev quantum-wire model (Alicea et al.,...)
- (b) try to resolve simpler problem first

# A "TOY" PROBLEM: DB QUASIPARTICLE IN SIMPLE ABRIKOSOV VORTEX

$$\mathbf{v}_s = \frac{\hbar}{2m} \left( \nabla\phi - \frac{2e}{\hbar} \mathbf{A}(\mathbf{r}) \right)$$

$$\oint \nabla\phi \cdot d\boldsymbol{\ell} = \ell_0 \quad (\equiv 1 \text{ for case of interest})$$

+ Maxwell



$$\text{Df. } \left. \begin{array}{l} K \equiv \oint \mathbf{v}_s \cdot d\boldsymbol{\ell} / \left( \frac{\hbar}{2m} \right) \\ \Phi \equiv \oint \mathbf{A} \cdot d\boldsymbol{\ell} / \left( \frac{\hbar}{2e} \right) \end{array} \right\} , \text{ then } K(r) = \ell_0 - \Phi(r)$$

Transport single DB qp adiabatically around core at distance  $r$ .

Is Berry phase picked up

- (a)  $\propto \ell_0$  (i.e. constant)
  - (b)  $\propto \Phi(r)$
  - (c)  $\propto K(r)$
- } r-dependent ?



A. “Ultra-naïve” argument : Berry phase is topological property, thus must depend on only topological invariant, namely  $l_o$ .

Now it seems certain that for  $r \rightarrow \infty$  correct answer is  $\pi$ , so if this argument is correct then also for  $r \rightarrow 0$  (but  $\gg \xi$ ) result should be  $\pi$ .

Is this right? ( $\Rightarrow$  “annular” problem)

B. Somewhat-less-naïve argument:

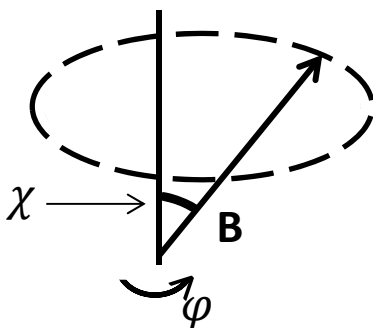
Treat  $B(\theta - \theta_o)$  as simply fixing qp position near  $\theta_o$ , then problem formally analogous to particle of spin  $\frac{1}{2}$  in magnetic field whose direction varies in space.

magnetic case :  $\hat{H} = -\boldsymbol{\sigma} \cdot \mathbf{B}$

$$B_x + iB_y = |B_{\perp}| \exp i\varphi$$

superfluid case :  $\hat{H}_{MF} = -\text{“}\boldsymbol{\sigma}\text{”} \cdot \mathcal{H}$

$$\mathcal{H}_x + i\mathcal{H}_y \propto |\Delta| \exp i\theta$$



$$|\uparrow\rangle \Leftrightarrow |p\rangle, \quad |\downarrow\rangle \Leftrightarrow |h\rangle$$

↑ particle                      ↑ hole

In spin  $-\frac{1}{2}$  case, standard result is (mod  $2\pi$ ) :

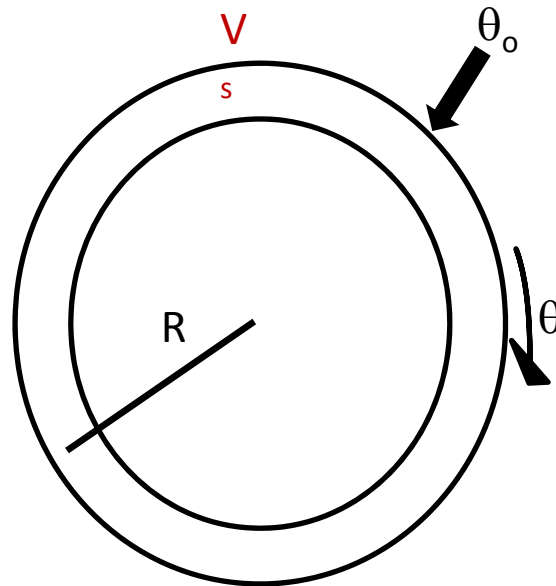
$$\varphi_B = 2\pi \cos^2 \chi/2 \leftarrow \text{“weight” of } |\uparrow\rangle \text{ component}$$

in particular, for equal weights of  $|\uparrow\rangle$  and  $|\downarrow\rangle$  ( $\chi = \pi/2$ ),  $\varphi_B = \pi$

In our case, weight of  $|p\rangle$  and  $|h\rangle$  should be equal for bound state (Andreev reflection)  $\Rightarrow$  if analogy with spin  $-\frac{1}{2}$  is valid, then

$$\varphi_B = \pi$$

### C. Microscopic (N-conserving) argument (executive summary)



Ansatz:

2N-particle “ground” state for  $\ell_0 \neq 0$  is

$$\Psi_{2n} = (\sum_{\ell} c_{\ell} a_{\ell\uparrow}^{\dagger} a_{-\ell+\ell_0}^{\dagger})^{N/2} |\text{vac}\rangle \quad (c_{\ell} \equiv v_{\ell} / u_{\ell})$$

$$\equiv (\hat{\Omega}_{\ell_0})^{N/2} |\text{vac}\rangle$$

In presence of Zeeman trap, 2N+1-particle “ground” state with  $l_0 \neq 0$  is of general form

$$\Psi_{2N+1} = \sum_{\ell} f_{\ell} \alpha_{\ell}^{\dagger} \Psi_{2N} \quad \alpha_{\ell}^{\dagger} \equiv u_{\ell} a_{\ell\uparrow}^{\dagger} - v_{\ell}^* a_{-\ell+\ell_0\downarrow} \hat{\Omega}_{\ell_0}$$

↑  
Conserves N!

Then easy to show that

$$\varphi_B = 2\pi \sum_{\ell} (\ell |f_{\ell}|^2.)$$

Evaluation of RHS tricky, but

(a) for  $v_s \rightarrow 0$  ( $r \rightarrow \infty$ ), certainly  $\varphi_B = \pi$  (AB result)

(b) for  $\Phi \rightarrow 0$  ( $r \rightarrow 0$ ), most plausible arguments give

$$\varphi_B = 0$$

If this is correct, “naïve” BdG-based arguments (e.g. B above) may be qualitatively misleading.

The \$64K question: does any of this affect the “established” conclusions re TQC in a  $(p + ip)$  Fermi superfluid?