Shanghai Jiao Tong University
Lecture 11
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Lecture 11. "Exotic" superconductivity

property	Class							
	Classic	BKBO MgB2	Heavy-fermions	organics	Ruth-enates	Fuller-enes	Ferroni-ctids	Cupr-ates
Tc < 25K	(√)	×	√	√	√	×	×	×
FL normal state	√	√	×	×	×	√	(√)	×
No neighboring phase trans.	√	√	×	√	√	√	×	×
OP s-wave	√	√	?	?	×	√	(×)	×
Phonon mechanism	√	√	×	?	?	√	×	×
Crystal structure simple	√	×	√	×	×	×	×	×
Stoichiometry-insensitive	√	√	√	√	√	×	×	×

How do we know?
1,3,6,7 by direct inspection
2 mostly from T-dependence of (eg) R
that leaves most important differences, 4 and 5

Phonon versus non-phonon mechanisms

Original BCS prediction: $T_c = 2\omega_D \exp - 1/\underbrace{N(0)|V|}$

Debye freq, $\propto M^{-1/2}$ independent of isotopic mass

thus predict: if superconductors which are chemical identical but have different isotopic masses are compared

$$T_c \propto M^{-\alpha} \qquad \alpha = 1/2$$

Prediction satisfied by most "classic" superconductors (Al, Sn, Pb, ...): a few exceptions, but understood by more sophisticated phonon-plus-Coulomb theory (McMillan) giving

$$\alpha = \frac{1}{2}(1 - A) \qquad A > 0 \text{ , may be } > 1$$

No examples of classic superconductor with $\alpha > 1/2$ known.

So $\alpha \cong 0$ (eg cuprates) suggests non-phonon mechanism

However, all evidence (flux quantization, Josephson effect...) suggests even exotic superconductivity still based on Cooper pairing.

If phonons don't play a role, must be "all-electronic", ie Coulomb mechanism!

But Coulomb interaction, even when screened, is intrinsically repulsive!

$$\langle\hat{H}\rangle = \langle\hat{H}_0\rangle + \langle\hat{V}\rangle \qquad \hat{H}_0 \equiv \hat{T} + \hat{U} \qquad \hat{V} \equiv \hat{V}_{\text{coul}}$$

kinetic energy   potential of static lattice

so prima facie only 2 possibilities:

1. Cooper pairing reduces $\langle\hat{H}_0\rangle$
2. """" $\langle\hat{V}\rangle$

Option 1 ⇒ $\langle H_0\rangle$ in N state (simple Fermi sea) already considerably > noninteracting-electron volume

Option 2 ⇒ $\langle V_{\text{coul}}\rangle$ already large in N state

So in either case

exotic superconductivity ⇒ "strongly correlated" normal state

Symmetry of order parameter (OP)

Recall: in BCS theory, general form of gap equation (at T=0) is

$$\Delta_k = -\sum_{k'} V_{kk'}\,\Delta_{k'}/2\,\varepsilon_{k'}$$

If $V_{kk'} = \text{const.} \equiv V_0$, solution is simply $\Delta_k \equiv \text{const.} \equiv \Delta$.
Then expression for pair wave function (order parameter) in k-space, F_k , is

$$F_k = \Delta/2(\varepsilon_k^2 + |\Delta|^2)^{1/2}) \equiv F(|\vec{k}|)$$

(i.e. independent of direction $\hat{\vec{k}}$ of $\vec{k}$)

and so real-space form is

$$F\left(\vec{r}\right) = \sum_k F_k e^{i\vec{k}\cdot\vec{r}} = \int k^2 F(k) \exp(ikr\cos\theta)\, dk\, d\Omega$$

$$= \int k^2 F(k) \frac{\sin kr}{kr} dk \equiv F(|r|) \quad \theta \equiv \angle \hat{\vec{k}} \cdot \hat{\vec{r}}$$

(i.e. independent of direction $\hat{\vec{r}}$ of $\vec{r}$)

Equivalently, pairs form in spin singlet with $l = 0$ ("s-state")

relative angular momentum

If $V_{kk'} \equiv V_{\vec{k}-\vec{k}'}$ is slowly varying for $|\vec{k} - \vec{k}' \lesssim \mathrm{k_F}$ (true in almost all "classic" superconductors) this result still holds.

But if $V_{\vec{k}-\vec{k}'}$ varies substantially on scale $\lesssim$ (as e.g. in liquid $^3\mathrm{He}$), Δ_k and hence F_k (and $F(r)$) can be anisotropic,

i.e. $\Delta_k = \Delta(\vec{k}) =$ function of direction $\hat{\vec{k}}$ as well as of $|k|$, and $F(\vec{r}) =$ function of direction of $\hat{\vec{r}}$. Also, spin state not necessarily singlet. This is a generic feature, which remains true even when ?? of BCS theory inapplicable. Terminology: superconductor with pairing different from simple s-wave "exotic" - usually more interesting than s-wave!

If we neglect crystal lattice & spin-orbit (SO) coupling, then $\hat{H}$ (or F) is invariant separately under spatial & spin rotations ->

free energy

both total spin S and total (internal) angular momentum L of pairs are good quantum numbers:

$$S = 0, 1$$

$$L = 0, 1, 2, 3 \ldots \; (\equiv l)$$

However, Pauli principle

$$\Longrightarrow \begin{cases} S = 0 \text{ associated with even } l\ (0, 2, 4 \ldots) \\ S = 1 \text{ associated with odd } l\ (1, 3, 5 \ldots) \end{cases}$$

but even within these assignments, different possibilities: example, liquid $^3\mathrm{He}$:
both A and B phases have $S = L = 1$, but

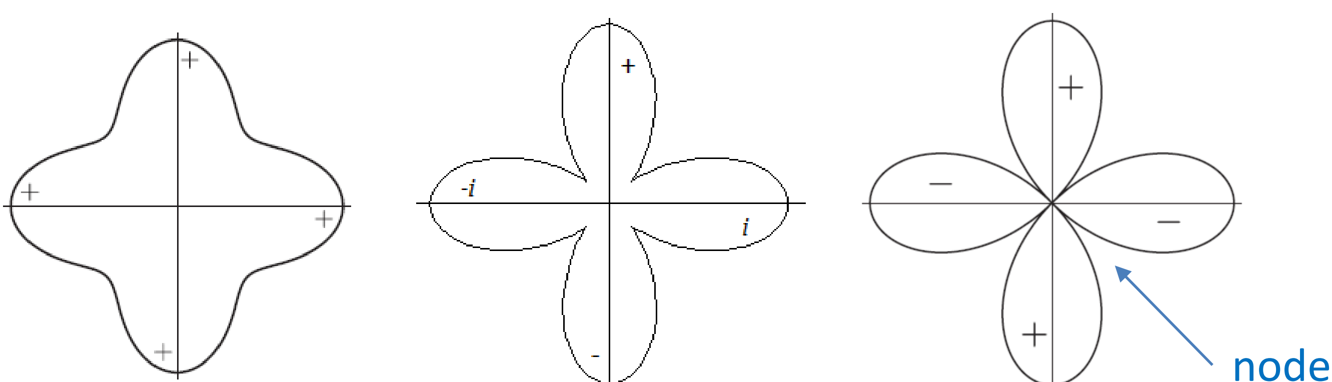
A phase has $S_z = 0, L_z = 1$
(with some convenient choice of axes)

B phase has $S = 1, L = 1$ coupled to that $\vec{J} \equiv \vec{L} + \vec{S} = 0$
(in absence of SO coupling)

In the real-life "exotic" superconductors symmetry considerations modified:
(1) many are quasi-2D -> symmetry of OP defined only within plane
(2) crystal lattice breaks symmetry, e.g. in square lattice $\mathrm{O}(3) \rightarrow D_2$.

(only symmetry operations are reflections $+\pi/2$ rotations)

Square lattice ($\mathrm{Sr_2RuO_4}$, cuprates, ferropnictides,...): some possibilities:



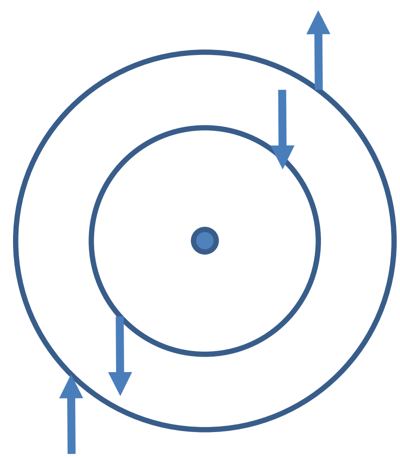
s-wave $\cong l = 0$ | p+*i*p $\cong l = 1$ | d-wave $\cong l = 2$

How do we tell?

(1) spin susceptibility χ : if $S = 0$ ($l = \text{even}$), χ is reduced in S state because to polarize system must break up Cooper pairs (c.f. lecture 7). For $S = 1$ pairing effect is either absent or reduced, e.g. if $S_z = \pm 1$ (or in $^3\text{He-A}$)

$$\chi_S = \chi_N$$

(2)s-wave state usually has non-zero minimum value of excitation energy ("gap") ⇒ as $T \to 0$ number of excitations $\propto \exp(-\Delta/T)$ ⇒ specific heat, etc., exponentially small.
By contrast most (but not all) exotic pairing states have "nodes" in gap ($\Delta \to 0$ for some $\vec{k}$) -> substantial number of excitations as $T \to 0$ $\to$ specific heat, etc. proportional to some power of T.



(3) Effect of nonmagnetic impurities: for a simple s-wave state in free space (BCS case) T_c is virtually unaffected. For the case of an s-wave state in a lattice, expect some depression but not to 0. However, for an "exotic" state (p-wave, d-wave, ...) nonmagnetic impurities have a qualitatively similar effect to magnetic impurities in BCS, i.e.

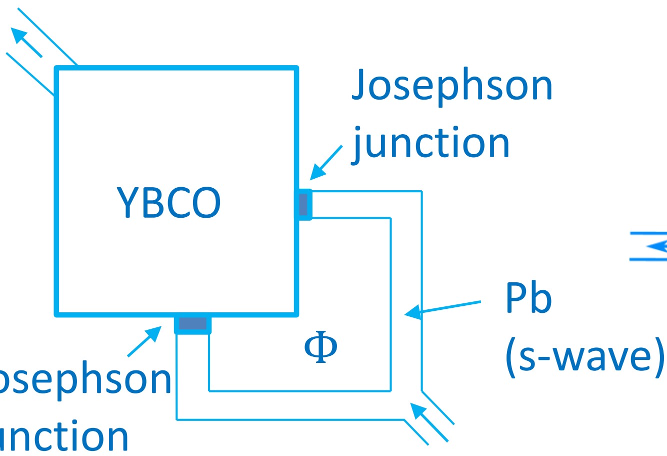
$$T_c \to 0 \text{ for } \Gamma \gtrsim \Delta_0$$

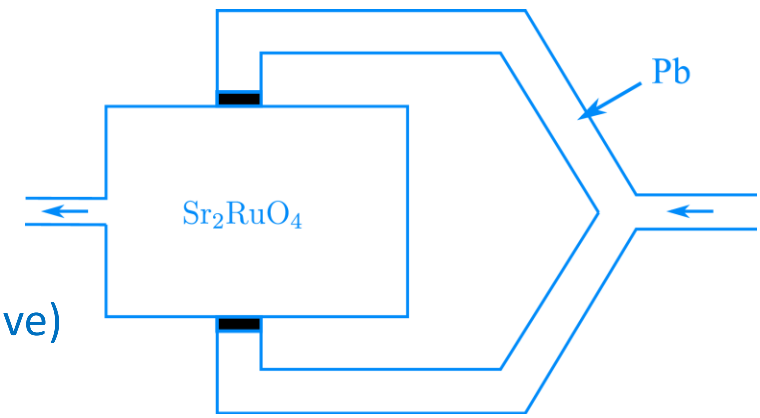
relevant scattering rate (rms) gap for pure case

Thus, e.g., the fact that very small concentrations of impurities in $\mathrm{Sr_2RuO_4}$ drive $\mathrm{T_c}$ to 0 usually taken as evidence for exotic pairing.

4. Phase-sensitive (Josephson) experiments

refinement of SQUID geometry

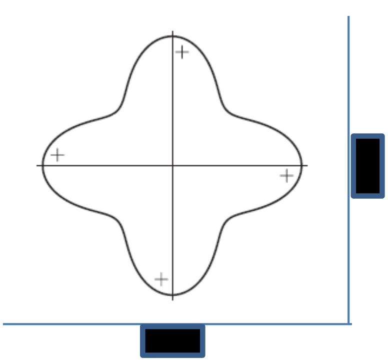




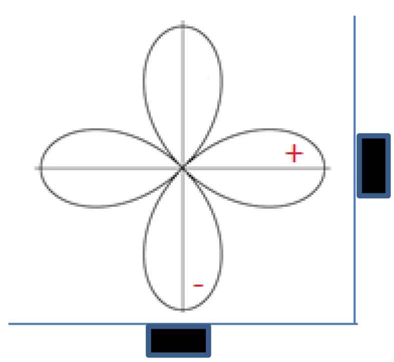
General principle:

$\Phi = \sum_i \Delta\varphi_i$ where $\Delta\varphi_i$ includes "internal" phase differences due to "roation" of pair wave function

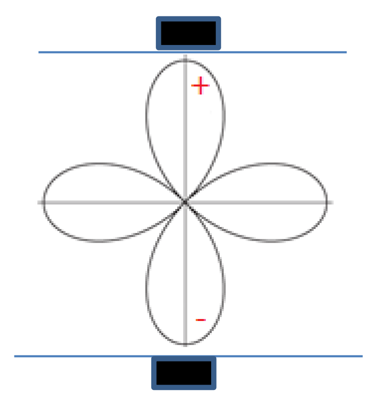
e.g. in YBCO



s-wave



d-wave



similarly for Sr_2RuO_4 if p-wave

s-wave: $\Delta\varphi_1 + \Delta\varphi_2 = \Phi \Rightarrow$ max. of I_c at $\Phi = (n + 1/2)\Phi_0$
d-wave: $\Delta\varphi_1 + \Delta\varphi_2 = \Phi + \pi \Rightarrow$ max of I_c at $\Phi = (n + 1/2)\Phi_0$

"internal" phase drop

Generally accepted conclusions:

cuprates d-wave

$\mathrm{Sr_2RuO_4}$ p-wave (may be $p + ip$)

ferropnictides (complicated) s-wave