

THE LONDON MOMENT OF A ROTATING
SUPERCONDUCTOR:
SOME UNSUSPECTED SUBTLETIES

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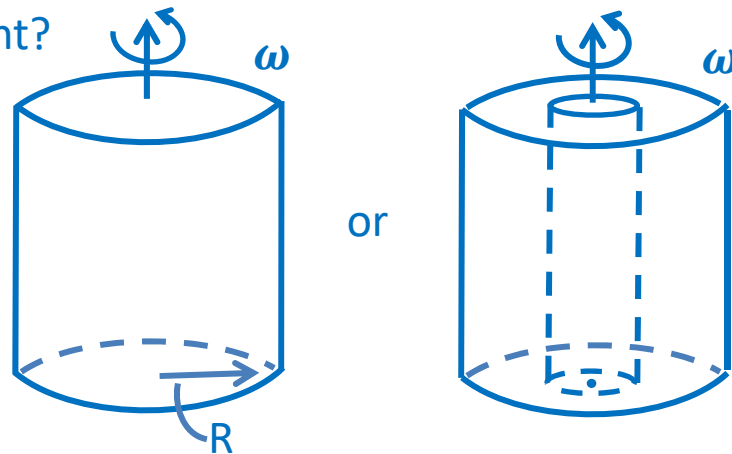
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(very much “work in progress”!)



What is the London moment?



If cylinder is superconducting and is rotated, then in the bulk of the sample a **spontaneous magnetic field** \mathbf{B}_0 will be generated:

$$\mathbf{B}_0 = -\frac{2m}{e} \boldsymbol{\omega} \quad (\text{so since } e < 0, \mathbf{B}_0 \parallel \boldsymbol{\omega})$$

Why?

Electrons (both “normal” and superfluid, i.e. Cooper pairs) “would like” to rotate with lattice. For normal component, $\mathbf{v}_n = \boldsymbol{\omega} \times \mathbf{r}$ without problem.

For superfluid,

$$\mathbf{v}_s = -\frac{e}{m} \mathbf{A}(r) \quad (+nh/2mr)$$

So need $\mathbf{A}(\mathbf{r}) = \frac{-m}{e} \boldsymbol{\omega} \times \mathbf{r} \Rightarrow \mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) = \frac{-2m}{e} \boldsymbol{\omega} \equiv B_0$

However, $\mathbf{B}(\mathbf{r})$ must $\rightarrow 0$ as $r \rightarrow R \Rightarrow$ falls off as $(r \rightarrow R)$,

$$B(r) = (1 - e^{-(R-r/\lambda)})B_0,$$

$\lambda \equiv$ London penetration depth $= (\mu_0 n_s e^2 / m)^{-1/2}$

London moment \approx Meissner effect as viewed from rotating frame.

Apparently most recent experiment: Hendricks et al., JLTIP **4**, 209 (1971) on Sn (type-I)

I Field is very small (3000 Hz \approx 10 μ G)

When is London moment (not) observed?

ZF 3

	Solid	hollow
spin-up	yes	yes
“Meissner”	yes	no*

*that is, magnetometer **in hole** observes $\mathbf{B} \cong 0$, consistent with $\mathbf{B} = \mathbf{B}_0$ in bulk.

kinetic energy



Statics of London moment well understood: need to minimize KE as observed from rotating frame, leading to generalized London equation

$$\mathbf{j}(\mathbf{r}) = -e\mathbf{A}(\mathbf{r}) - m\boldsymbol{\omega} \times \mathbf{r}$$

cylindrical polars



consistently with Maxwell's equation

$$\nabla^2 \mathbf{A}(\mathbf{r}) = \frac{1}{r} \frac{d}{dr} \left(r \frac{dA}{dr} \right) = -\mu_0^{-1} \mathbf{j}(\mathbf{r})$$

and subject to $\mathbf{B}(\mathbf{R}) = (\nabla \times \mathbf{A}(\mathbf{r}))_{r=R} = \mathbf{0}$. This gives

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dA}{dr} \right) = -\lambda^{-2} A(r) = \lambda^{-2} \frac{m}{e} \omega r$$

with the approximate solution ($\lambda \ll R$)

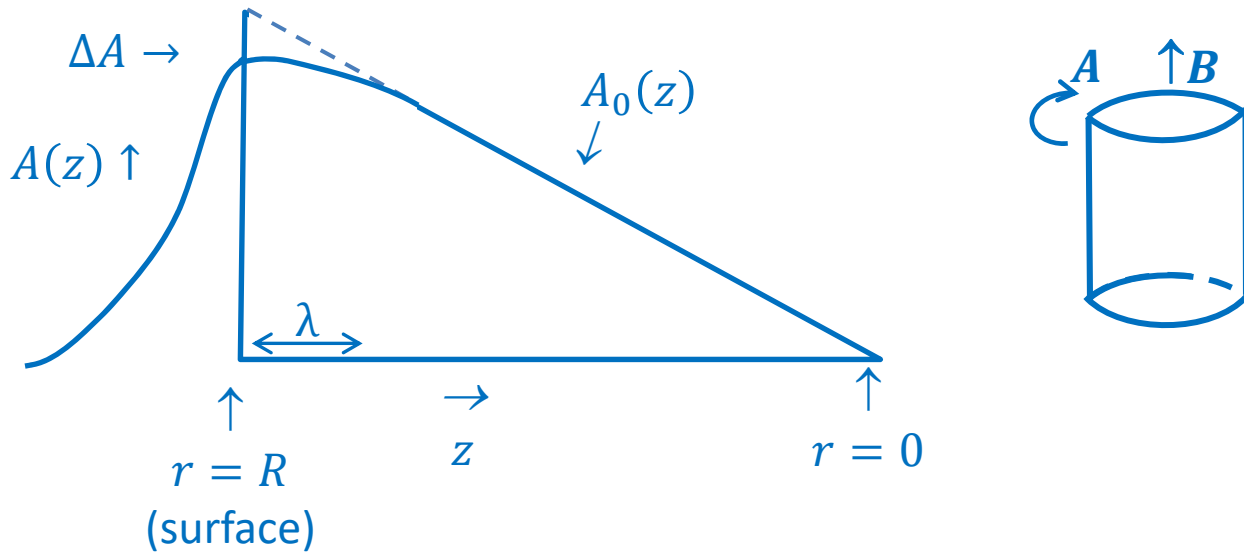
$$\begin{aligned} A(r) &= \frac{1}{2} B_0 r + \Delta A(r), r < R \\ &= \frac{1}{2} B_0 R^2 / r \left(+0 \left(\lambda / R \right) \right) r > R \end{aligned}$$

where $\Delta A(r) = 2B_0 \lambda \exp -(R - r)/\lambda \equiv 2B_0 \lambda \exp -z/\lambda$



R-r





Note: total energy in rotating frame = magnetic energy $\frac{1}{2\mu_0} \int \mathbf{B}^2(\mathbf{r}) d\mathbf{r}$
 + (r.f.) KE = $\frac{1}{2} m \int (\mathbf{v}_s - \boldsymbol{\omega} \times \mathbf{r})^2 d\mathbf{r}$, which relative to its $\lambda = 0$ value
 $\frac{1}{2\mu_0} B_0^2 V$ is $E(\omega, \lambda) = -\frac{1}{4} \mu_0^{-1} B_0^2 \lambda$

note this is an incredibly tiny fraction ($\sim 10^{-12}$) of the characteristic thermal energy

Now, question (Hirsch):

what is the **kinetics** of the formation of the London moment (in particular under “Meissner” conditions)?

can discuss (a) initial nucleation

(b) development with change of temperature

I
$$\left(\lambda \equiv \left(\frac{\rho_s(T) e^2 \mu_0}{m} \right)^{-1/2} = \lambda(T) \right)$$

Take $A(zt)$ to be correction to $\lambda \rightarrow 0$ value $\frac{1}{2}rB_0$, measure v_s and v_n in rotating frame, then in equilibrium

$$v_n(z) = 0, \quad v_s(z) = -eA(z)/m \Rightarrow j_s(z) = \rho_s e^2 A(z)/m$$

Quite generally*, at London level,

$$\text{Maxwell:} \quad \frac{d^2 A(zt)}{dz^2} = -\mu_0 j(zt)$$

$$\text{Generalized London: } j(zt) = -\rho_s e^2 A(zt)/m + j_n(zt)$$

\Rightarrow dynamics of superfluid controlled entirely by that of normal component.

Consider "step" in temperature ($T_{\text{old}} \rightarrow T_{\text{new}}$) which conserves total local current:

$$\begin{aligned} \rho_s(t=0+) &= \rho_s(t=0-) + \Delta\rho_s, \\ \uparrow & \quad \uparrow \\ \rho_s^{\text{new}} & \quad \rho_s^{\text{old}} \end{aligned} \quad \begin{aligned} j_n(z:t=0+) &= -\Delta\rho_s v_s(z, t=0-) \\ &= -2\Delta\rho_s \cdot \omega \cdot \lambda \cdot \exp -z/\lambda_{\text{old}} \end{aligned}$$

thereafter $\rho_s \equiv \rho_s^{\text{new}} = \text{ind. of } t. : df \lambda_{\text{new}} \equiv (\mu_0 \rho_s^{\text{new}} e^2 / m)^{-1/2}$.

Total work W_{tot} done on normal component by Faraday electric field $\mathcal{E}(zt)$ arising from $\partial B / \partial t$:

$$W_{\text{tot}} = \int_{D+}^{\infty} dt \int_0^{\infty} dz \mathcal{E}(zt) j_n(zt)$$

where from Maxwell and gen^d. London

$$\frac{d^2 \mathcal{E}(zt)}{dz^2} - \lambda_{\text{new}}^{-2} \mathcal{E}(zt) = \mu_0 \frac{dj_n}{dt}(zt) \leftarrow \text{total rate of change of } j_n(zt)$$



*neglecting displacement-current terms and terms $0(\lambda/R)$

After several integrations by parts:

$$(\pm)W_{\text{tot}} = E(t = \infty) - E(t = 0+) \quad (+ \text{ other } f(t = \infty) - f(t = 0+))$$

independently of details of normal-component dissipation mechanisms. (\Rightarrow everything consistent).

Prima facie simplest ansatz: local relaxation,

$$\left. \frac{dj_n(zt)}{dt} \right|_{\text{relax}} = -j_n(zt)/\tau_0$$

then

$$v_s(zt) = 2\omega\lambda_{\text{old}} \exp -z/\lambda_{\text{old}} \cdot \exp -t/\tau_{\text{old}} + 2\omega\lambda_{\text{new}} \exp -z/\lambda_{\text{new}} (1 - \exp -t/\tau_{\text{new}})$$

What are $\tau_{\text{old}}, \tau_{\text{new}}$?

$$\frac{d^2\mathcal{E}(zt)}{dz^2} - \lambda_{\text{new}}^{-2}\mathcal{E}(zt) = \left. \frac{dj_n(zt)}{dt} \right|_{\text{total}}$$

however

$$\left. \frac{dj_n(zt)}{dt} \right|_{\text{total}} = \left. \frac{dj_n(zt)}{dt} \right|_{\text{relax}} + \frac{n_n e^2}{m} \mathcal{E}(zt)$$

so

$$\frac{d^2}{dz^2} \mathcal{E}(zt) - \lambda_0^{-2} \mathcal{E}(zt) = \left. \frac{dj_n(zt)}{dt} \right|_{\text{relax}} \quad \lambda_0^2 \equiv (n_{\text{tot}} e^2 \mu_0 / m)^{-1/2}$$

hence for any $\mathcal{E}(zt) \sim e^{-z/\lambda}$

$$\tau(\lambda) = \tau_0 \left(\frac{\lambda^{-2} - \lambda_{\text{new}}^{-2}}{\lambda^{-2} - \lambda_0^{-2}} \right)$$



OK for “old” term: what about “new” one?

A better solution: from

$$\left. \begin{aligned} \frac{d^2 E(zt)}{dz^2} - \lambda_{\text{new}}^{-2} \varepsilon(zt) &= \frac{dj_n(zt)}{dt} \Big|_{\text{total}} \\ \frac{dj_n(zt)}{dt} \Big|_{\text{total}} &= \frac{n_n e^2}{m} E(zt) - j_n(zt)/\tau_0 \end{aligned} \right\}$$

with “naïve” version of Fourier transform, derive dispersion relation

$$\omega(q) = \frac{i}{\tau_0} \left(\frac{q^2 + \lambda_{\text{new}}^{-2}}{q^2 + \lambda_0^{-2}} \right) \equiv \lambda(T=0)$$

Moreover, from initial condition $j_n(z, 0) = -2\Delta\rho_S \omega \lambda_{\text{old}} \exp - z/\lambda_{\text{old}}$

derive

$$E(q, t=0) = \left(\frac{2\Delta\rho_S \omega}{q^2 + \lambda_{\text{old}}^{-2}} \frac{(1 + iq\lambda_{\text{old}})}{q^2 + \lambda_0^{-2}} \right) \frac{1}{\tau_0}$$



Thus, complete solution for $\varepsilon(qt)$ is

$$E(qt) = \frac{2\Delta\rho_S\omega(1+iq\lambda_{\text{old}})}{(q^2 + \lambda_{\text{old}}^{-2})(q^2 + \lambda_0^{-2})} \frac{1}{\tau_0} \exp - \frac{t}{\tau_0} \left(\frac{q^2 + \lambda_{\text{new}}^{-2}}{q^2 + \lambda_0^{-2}} \right)$$

and

$$\delta A(q) \Big|_{t=0}^{t=\infty} = - \int_0^{\infty} E(qt) dt = \frac{2\Delta\rho_S\omega(1+iq\lambda_{\text{old}})}{(q^2 + \lambda_{\text{old}}^{-2})(q^2 + \lambda_{\text{new}}^{-2})} = \begin{matrix} \text{ind. of } \tau \\ \text{(and of } \lambda_0) \end{matrix} \checkmark$$

but is it equal to difference of static solutions $A(q)_{\text{new}} - A(q)_{\text{old}}$?

For real part, \checkmark

but for imaginary part, X !

Possible hint: $j_n(zt)$ is defined only for $z \geq 0$, but $A(z)$ (and $\Delta A(z)$) is defined also for $z < 0$!

??



Back to question (a) (nucleation): winding no.



Recall: $\mathbf{v}_s(rt) = -eA(rt)/m + n\hbar/2mr$

So far, have set $n = 0$. But is this right? What determines n ?

Plausible conjecture: under “Meissner” conditions,

n is determined by “fitting” to rotation of cylinder **at point where superconductivity first nucleates.**

i.e. if nucleation occurs at radius r , then $n =$ nearest integer to $2m\omega r^2/\hbar$.

So, choice $n = 0$ corresponds to nucleation **at origin.**

But... coldest point is $r = R$ (surface)!

What if nucleation were to occur on surface + propagate inwards?

Then since winding no. conserved

$$\mathbf{v}_s(r) = -eA(r)/m + n\hbar/2mr \quad \text{for all } r \Rightarrow \text{singularity at origin.}$$

For hollow cylinder, can accommodate at cost of extra KE \Rightarrow metastable state

But what about solid case? Conjecture: normal “core” forms near $r = 0$. (“pseudo-London” state)



Is there an analogy for Meissner effect?

A further question (Hirsch):

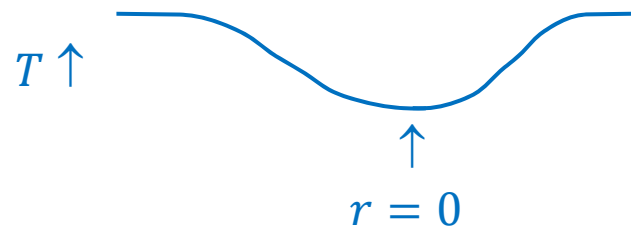
is $N \rightarrow S$ transition in rotating cylinder 1st or 2nd order?

Within London approxⁿ, if nucleation occurs at origin, should be 2nd order.

However, superconductor used in experiments (S_n) is type-I!

\Rightarrow need to go beyond London (at least to GL level)

Question: independently of rotation, in presence of thermal gradient, is $S \rightarrow N$ transition 1st or 2nd order?



Definitely work in progress...

