Superconductivity and Mottness: Exact Results

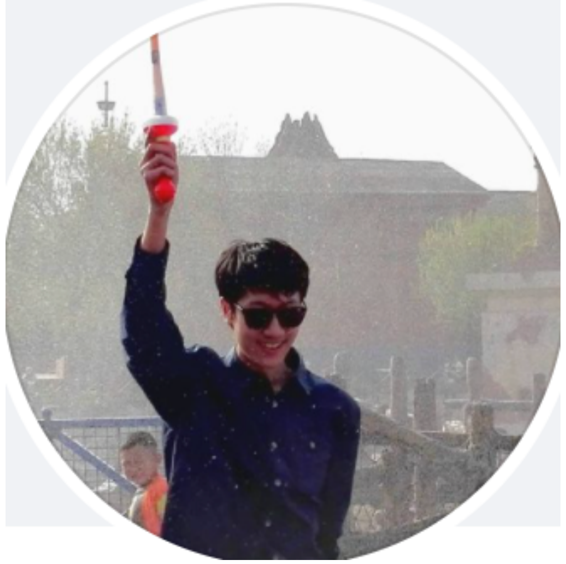
Nature Physics, vol.16, 1175-1180 (2020);
vol. 18, 511-516 (2022); PRB, 105, 184509.

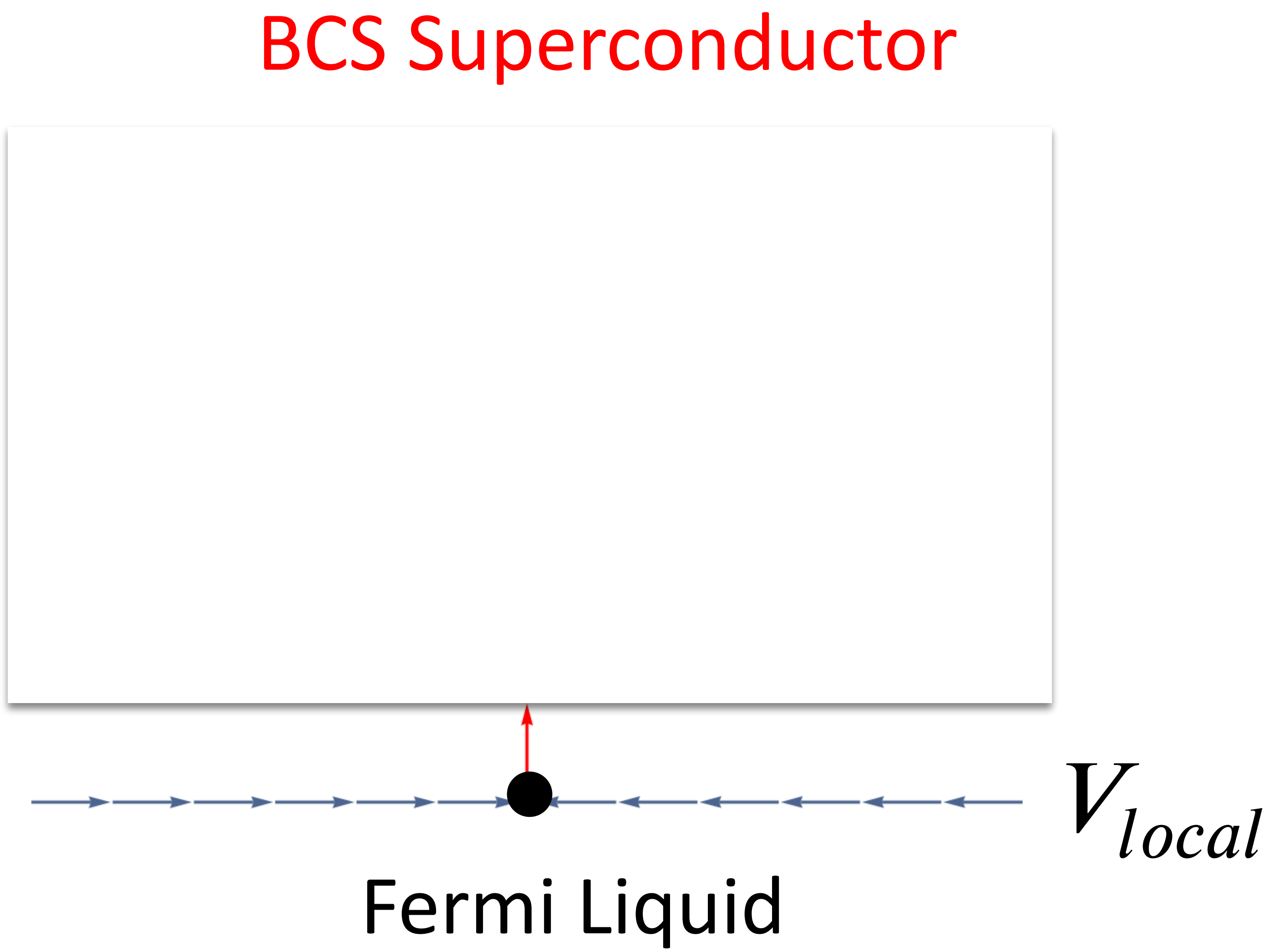
Luke Yeo

Edwin Huang

G. La Nave

Jinchao Z.





BCS Superconductor

V_{local}

Fermi Liquid

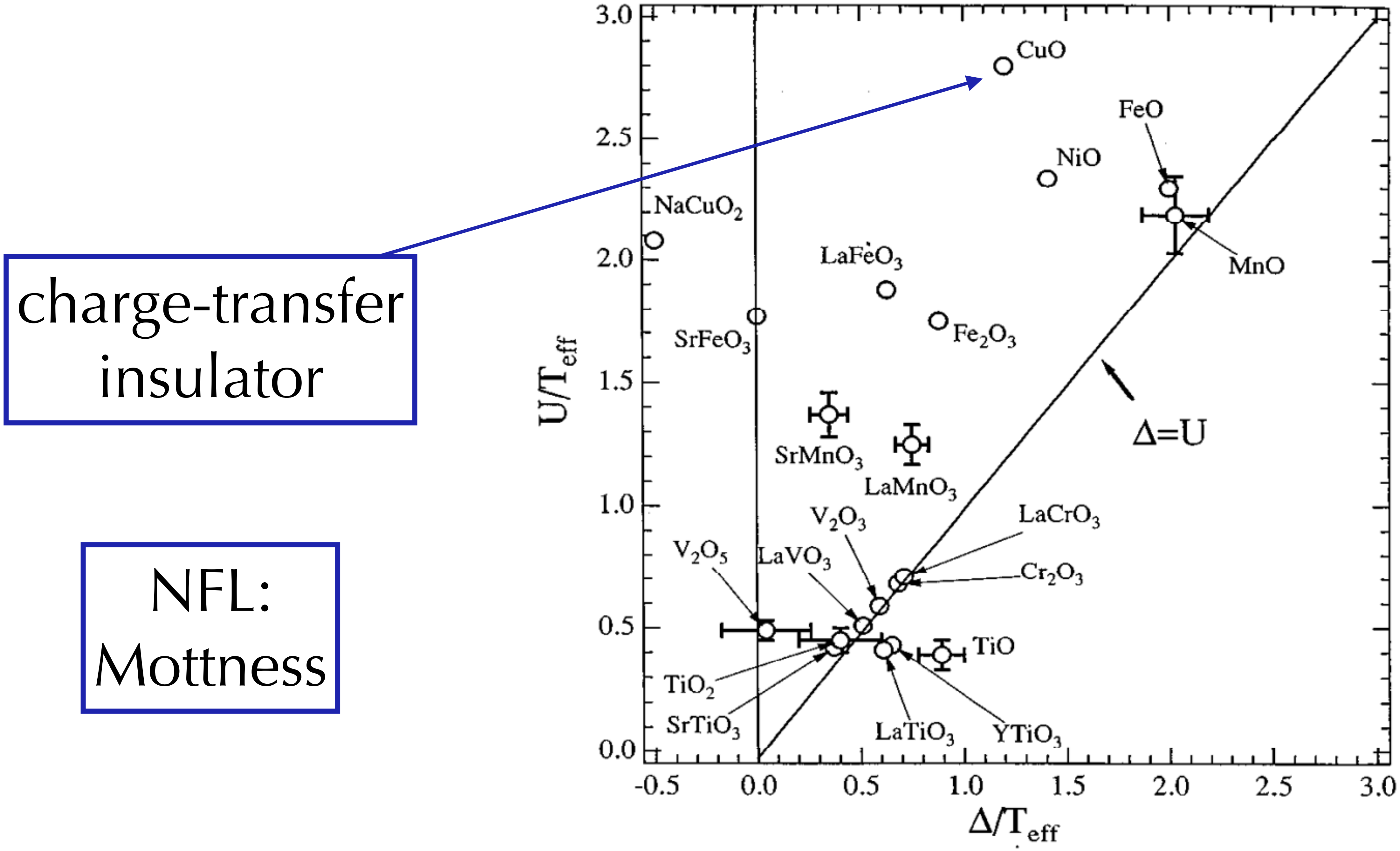
fixed
point beyond
FL?

quartic
interacting
theory?

Discovery of Orbital Ordering in $Bi_2Sr_2CaCu_2O_{8+x}$

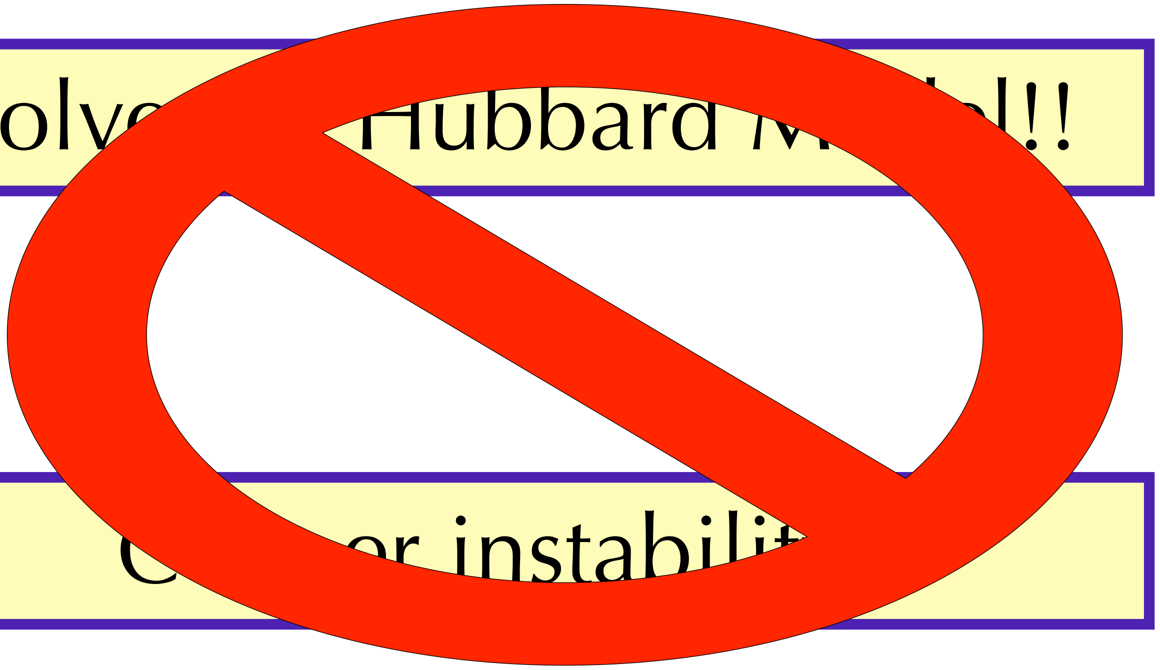
Shuqiu Wang[1§], Niall Kennedy[1,2§], K. Fujita[3], S. Uchida[4], H. Eisaki[5],
P.D. Johnson[1,3], J.C. Séamus Davis[1,2,6,7] and S.M. O'Mahony[2]

Overall, these data reveal a Q=0 orbitally ordered state that lifts the energy degeneracy of p_x/p_y oxygen orbitals at separate CuO_2 oxygen sites, in striking analogy to the ordering of d_{zx}/d_{zy} iron orbitals of the iron-based superconductors.



charge-transfer insulator

NFL: Mottness



solv Hubbard M l!!

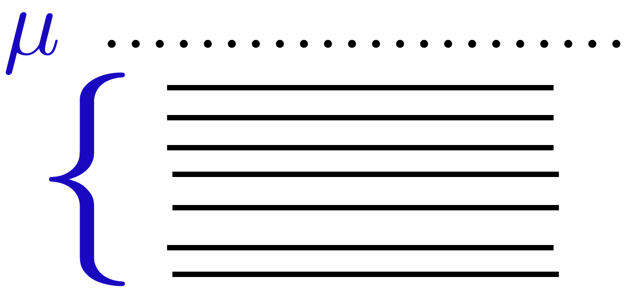
C or instabilit

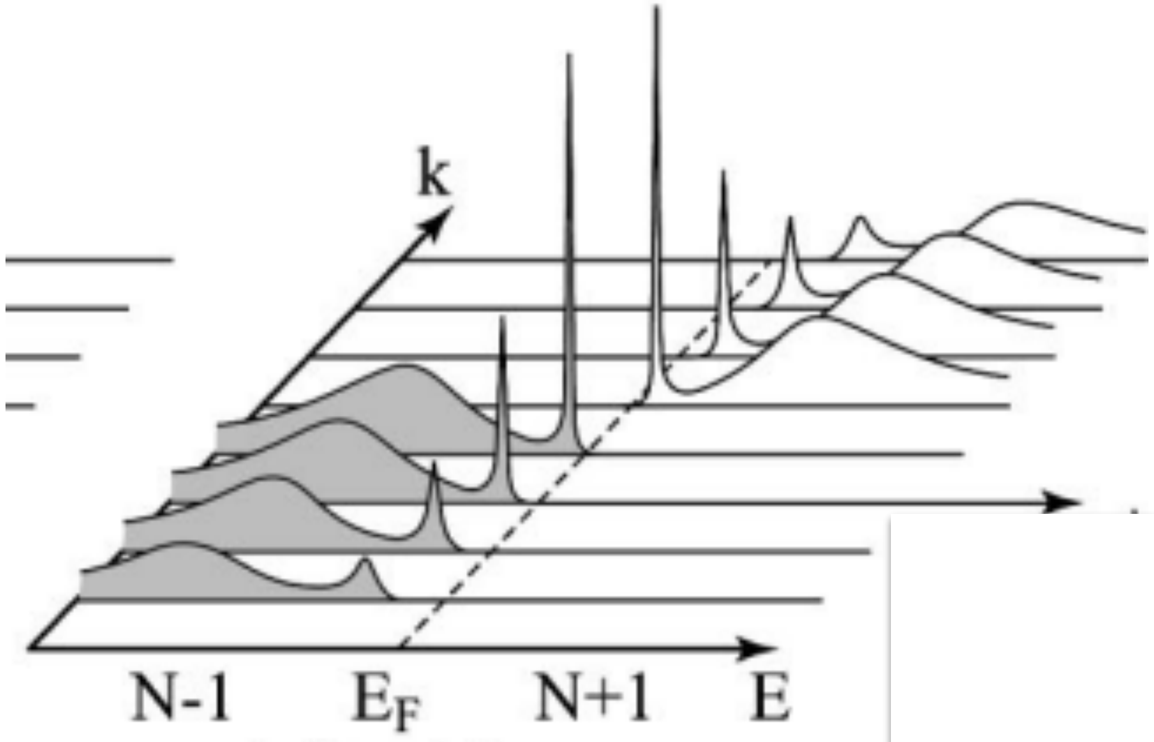
Is there a simpler
approach that has Mottness?

Yes!

Fermi liquids

doubly occupied



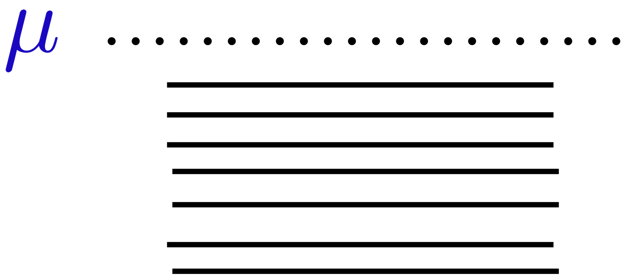


Fermi-liquid system

p-h symmetry

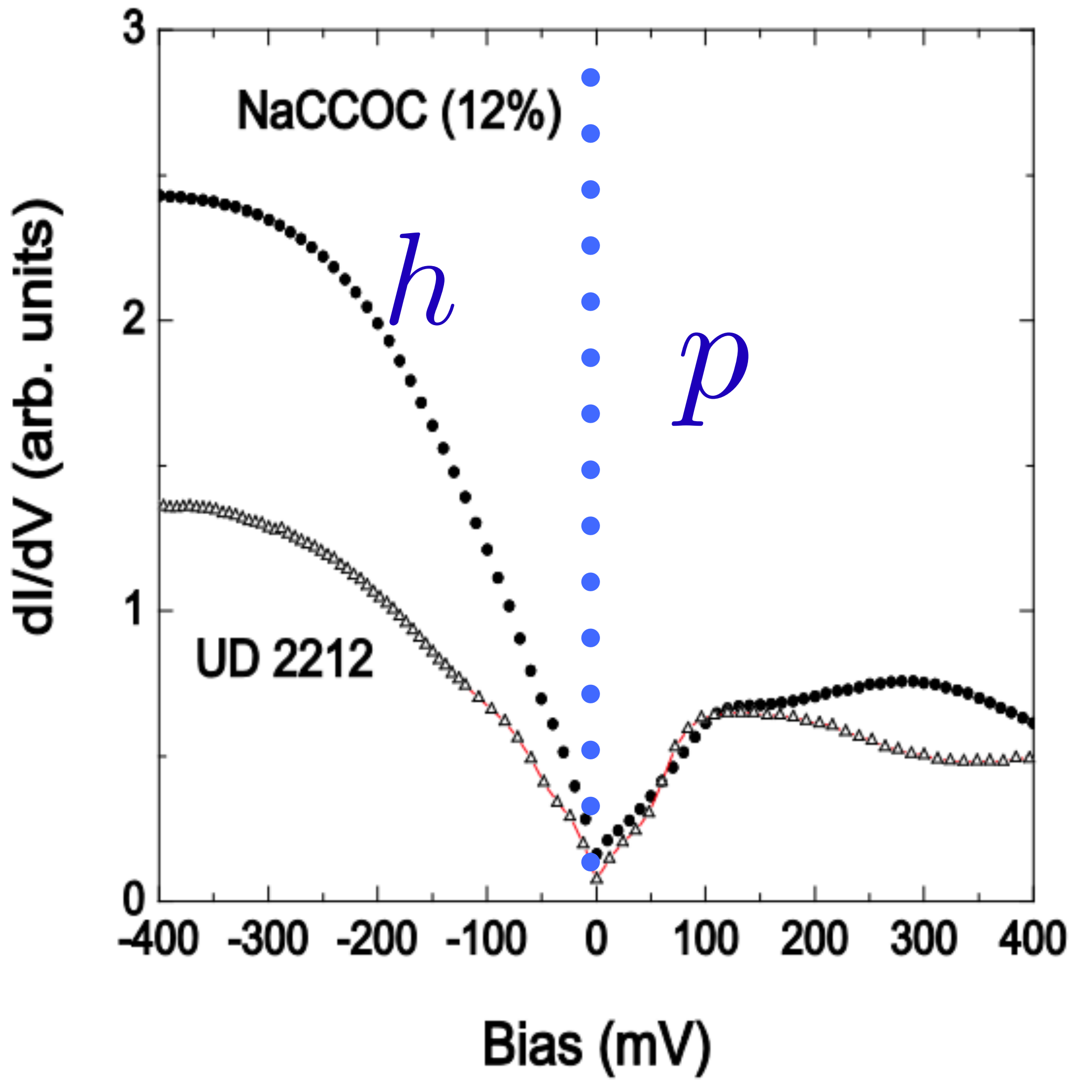
NFL

Is single occupancy below chemical potential possible?



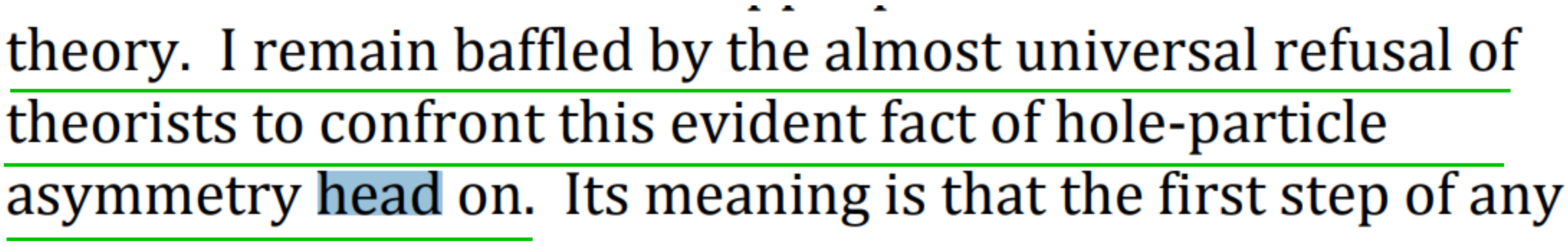
with time-reversal symmetry in tact?

p-h asymmetry



LAST WORDS ON THE CUPRATES

P W Anderson, Princeton University



theory. I remain baffled by the almost universal refusal of theorists to confront this evident fact of hole-particle asymmetry head on. Its meaning is that the first step of any single occupancy

?

particle-hole asymmetry

Anderson
Haldane
2000

3 citations

Fermi liquids

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + \underset{\swarrow\;\rightarrow 0}{\cdots}$$

$(n_{p\uparrow}, n_{p\downarrow})$ conserved currents

$(c_{p\uparrow}, c_{p\downarrow}, \text{h.c.})$ 4 objects

$O(4)$

$\text{DetM} = 1$ $\qquad$ $\text{DetM} = -1$

SO(4) proper rotations $\qquad$ improper rotations

$$\text{Det}\,M = \pm 1 \implies Z_2 = O(4) \div SO(4)$$

Improper Rotations

Majorana basis

$$\begin{pmatrix} c_{p\uparrow} \\ c^{\dagger}_{p\uparrow} \\ c_{p\downarrow} \\ c^{\dagger}_{p\downarrow} \end{pmatrix} \longrightarrow \begin{pmatrix} c_{p\uparrow} + c^{\dagger}_{p\uparrow} \\ i(c_{p\uparrow} - c^{\dagger}_{p\uparrow}) \\ c_{p\downarrow} + c^{\dagger}_{p\downarrow} \\ i(c_{p\downarrow} - c^{\dagger}_{p\downarrow}) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} c_{p\uparrow} + c^{\dagger}_{p\uparrow} \\ i(c_{p\uparrow} - c^{\dagger}_{p\uparrow}) \\ c_{p\downarrow} + c^{\dagger}_{p\downarrow} \\ i(c_{p\downarrow} - c^{\dagger}_{p\downarrow}) \end{pmatrix} \Longrightarrow \quad c_{p\downarrow} \to c^{\dagger}_{p\downarrow}$$

p-h transformation

$$\epsilon(p) = \epsilon_F$$

Fermi
Surface

$$H = 0$$

$$\left.\begin{array}{r} n_{p\uparrow} \to 1 - n_{p\uparrow} \\ n_{p\downarrow} \to n_{p\downarrow} \end{array}\right\} Z_2$$

at Fermi
surface only

How to destroy Fermi liquids?

$\cancel{Z_2}$

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + \boxed{U n_{p\uparrow} n_{p\downarrow}}$$

odd under Z_2

New fixed point!

Hatsugai-Kohmoto model

scaling dimension

$[n_{p\uparrow} n_{p\downarrow}] = -2$

relevant interaction

Hubbard not necessary!

General HK Model

$$\sum_k (\xi_k(n_{k\uparrow}+n_{k\downarrow})+Un_{k\uparrow}n_{k\downarrow})$$

Solvable Mott transition:U>W

$$G_{k\sigma}(i\omega_n \to z) = \frac{1-\langle n_{k\bar{\sigma}}\rangle}{z-\xi_k} + \frac{\langle n_{k\bar{\sigma}}\rangle}{z-(\xi_k+U)} \neq \frac{1}{z-\omega_k}$$

lower Hubbard band

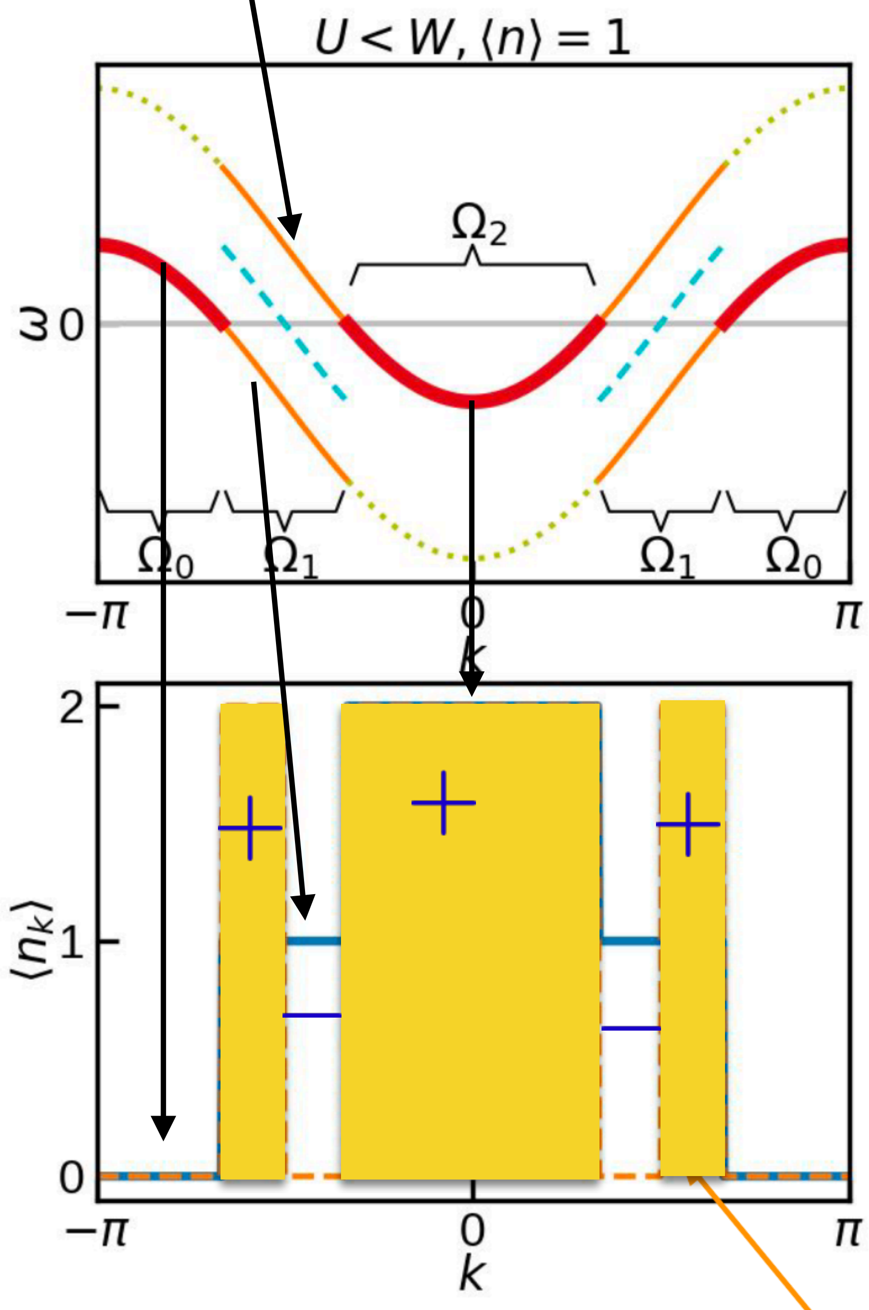
upper Hubbard band

zeros

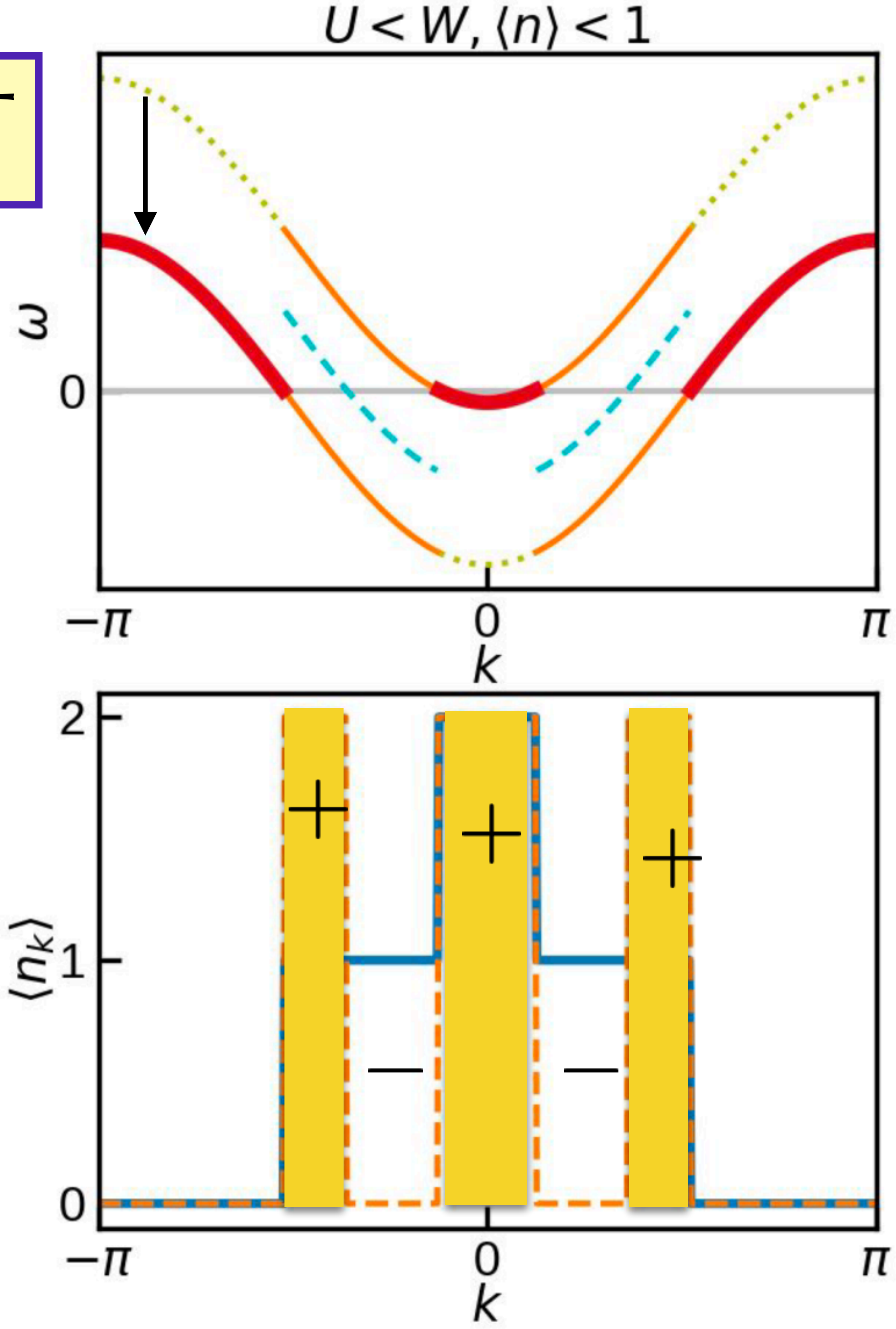
single occupancy

counting charges

SWT



$$n_{\mathrm{Lutt}} = \langle n \rangle$$



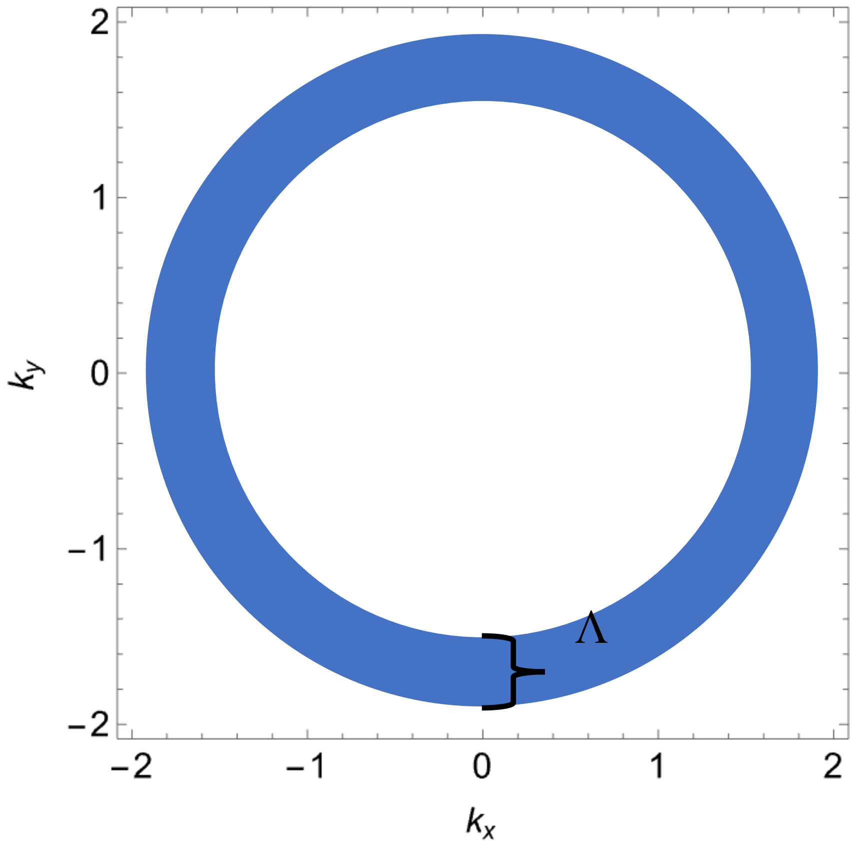
$$n_{\mathrm{Lutt}} \neq \langle n \rangle$$

zeros≠particles

$$H_{\mathrm{HK}} + V_{\mathrm{local}}$$

$$\delta S_4 = \frac{1}{4}\int u(4,3,2,1)\bar{\psi}(4)\bar{\psi}(3)\psi(2)\psi(1)\times$$

$$\prod_{i=1}^{4}\frac{d^d k_i d\omega_i}{(2\pi)^{d+1}}\delta(\mathbf{k_1}+\mathbf{k_2}-\mathbf{k_3}-\mathbf{k_4})\delta(\omega_\mathbf{1}+\omega_\mathbf{2}-\omega_\mathbf{3}-\omega_\mathbf{4})$$



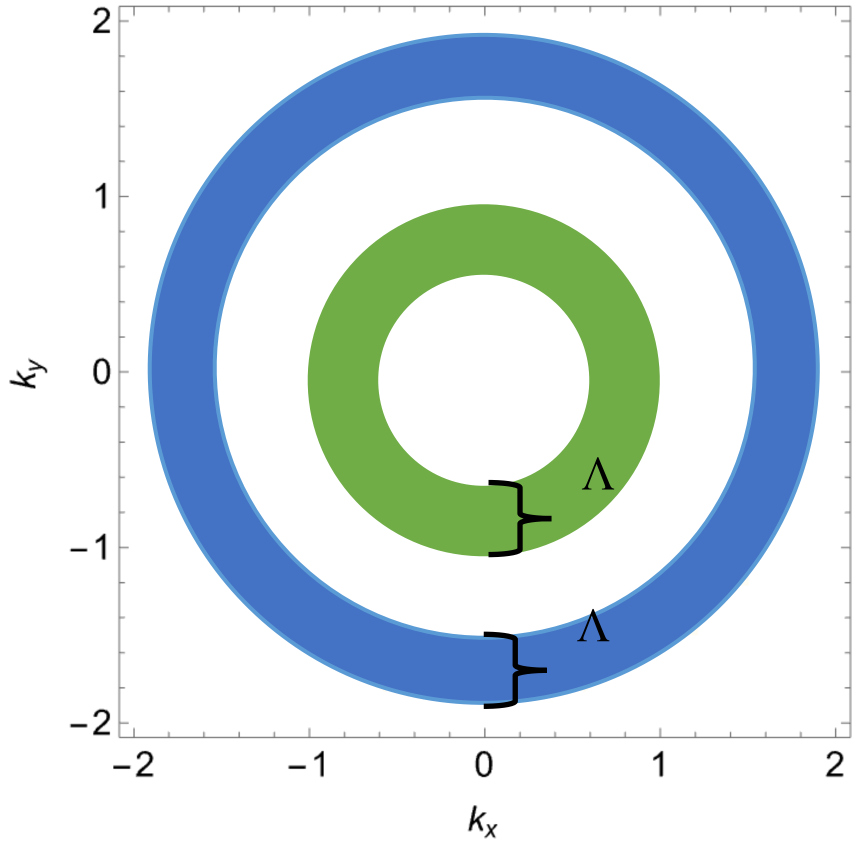
$\Lambda \to \Lambda/s$

$q' = qs$

$\omega' = \omega/s$

$\psi'(k',\omega') = s^{-3/2}\psi(k,\omega)$

$U' = s^2 U$

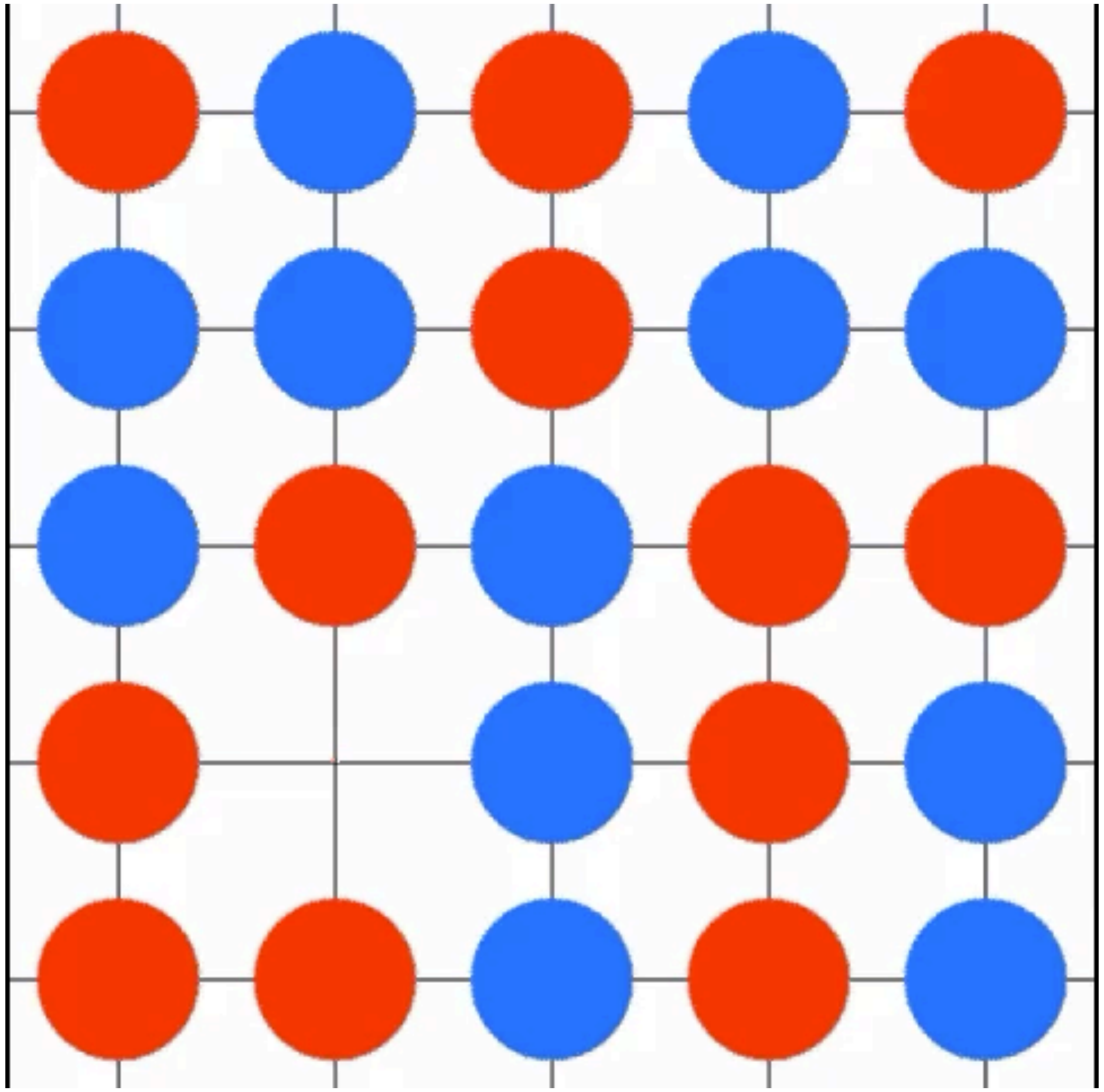


Local (Hubbard) part of 4-Fermion interactions are still irrelevant

			$\cancel{Z_2}$
		FL	**HK**
quantum numbers		k	k
stability	V_{local}	$\beta(V_{\text{local}}) = 0$	$\beta(V_{\text{local}}) = 0$
	V_{SC}	$\beta(V_{\text{SC}}) \to \infty$	?

what does the HK model leave out??

$$[H_t, H_U] \neq 0$$



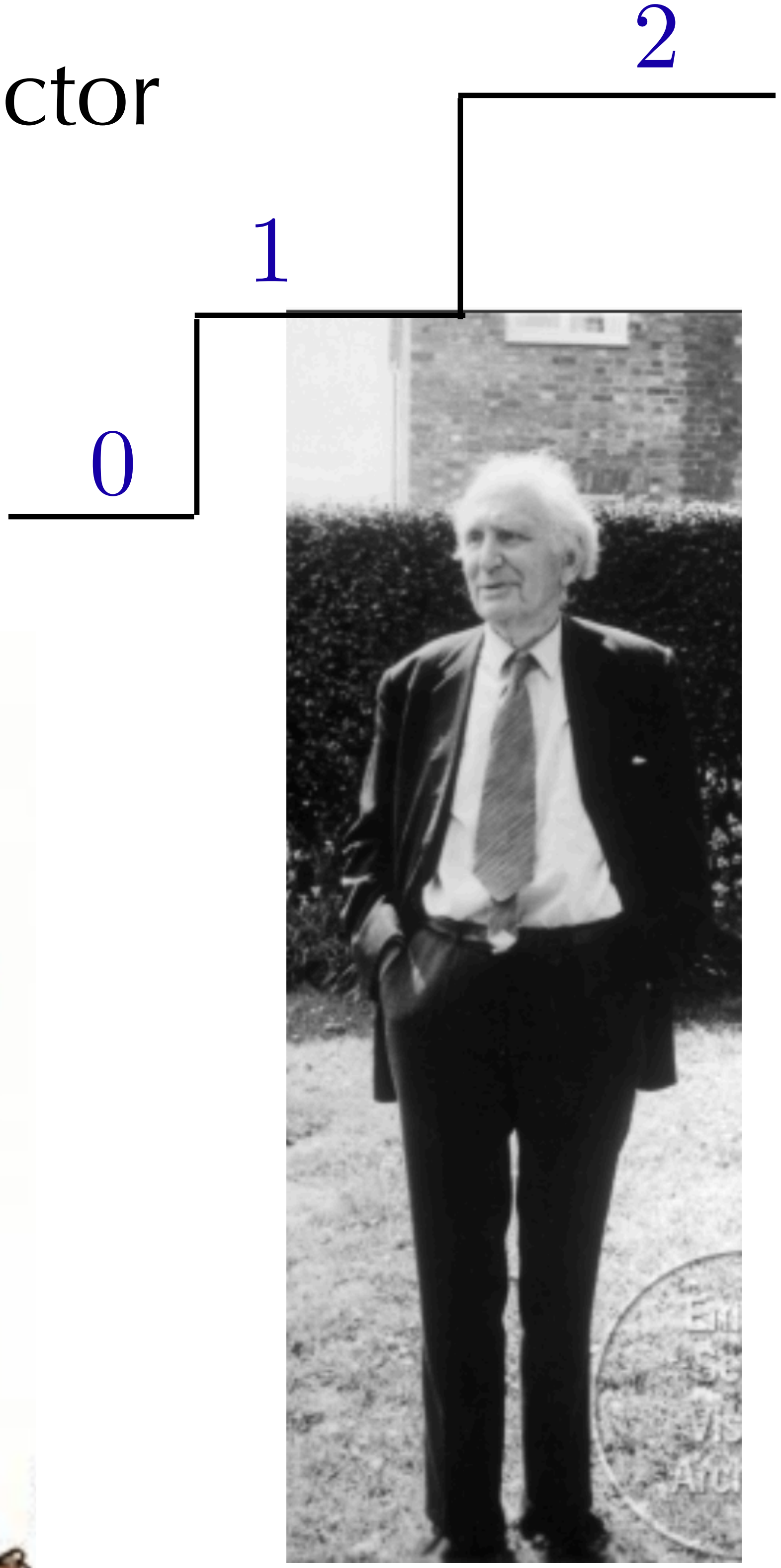
dynamical spectral weight transfer

Mottness

Fermi gas

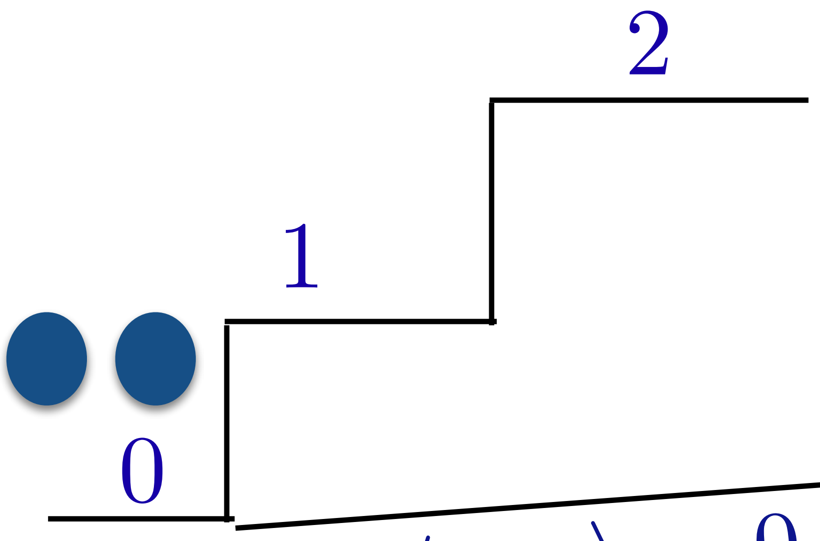
Fermi liquid

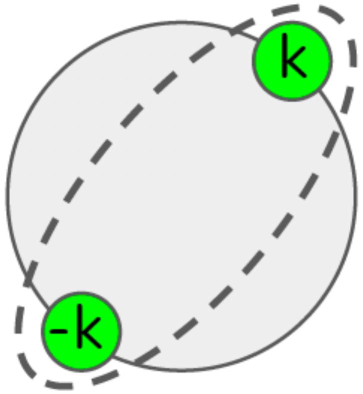
BCS superconductor



Superconductivity?

Cooper Instability





$$H = H_{\rm HK} - gH_p$$

$$|\psi\rangle = \sum_{k\in\Omega_0} \alpha_k b_k^\dagger |{\rm GS}\rangle$$

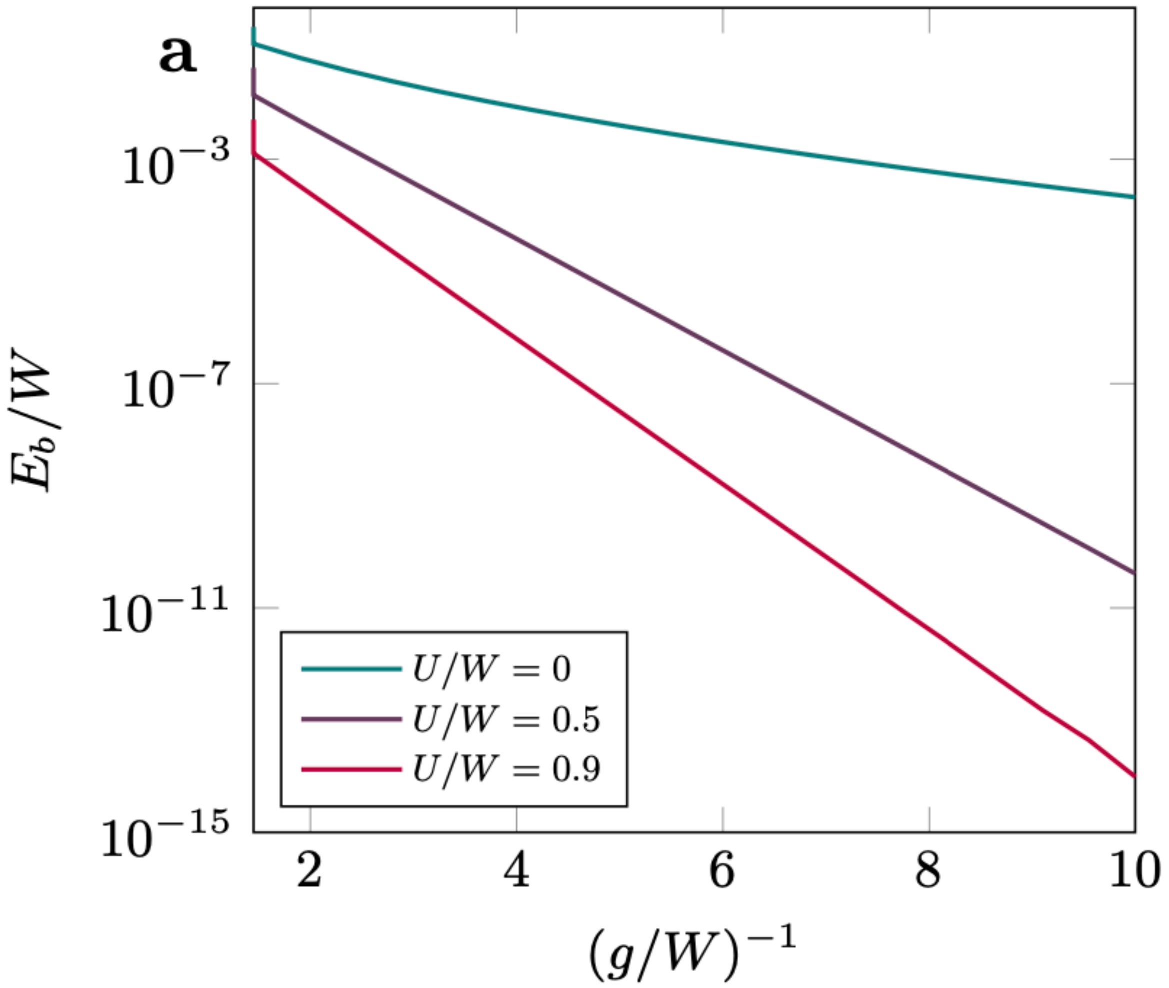
$$= \sum_{k,k'} b_k^\dagger b_{k'}$$

$$\langle n_{k\sigma}\rangle = 0$$

$$\boxed{E_b = \langle {\rm GS}|{\rm H}|{\rm GS}\rangle - \langle\psi|{\rm H}|\psi\rangle \lessgtr 0}$$

Cooper Instability

$$E_b = -E \sim W(1-(U/W)^2)e^{-\pi W\sqrt{1-(U/W)^2}/g}$$



Pair Susceptibility

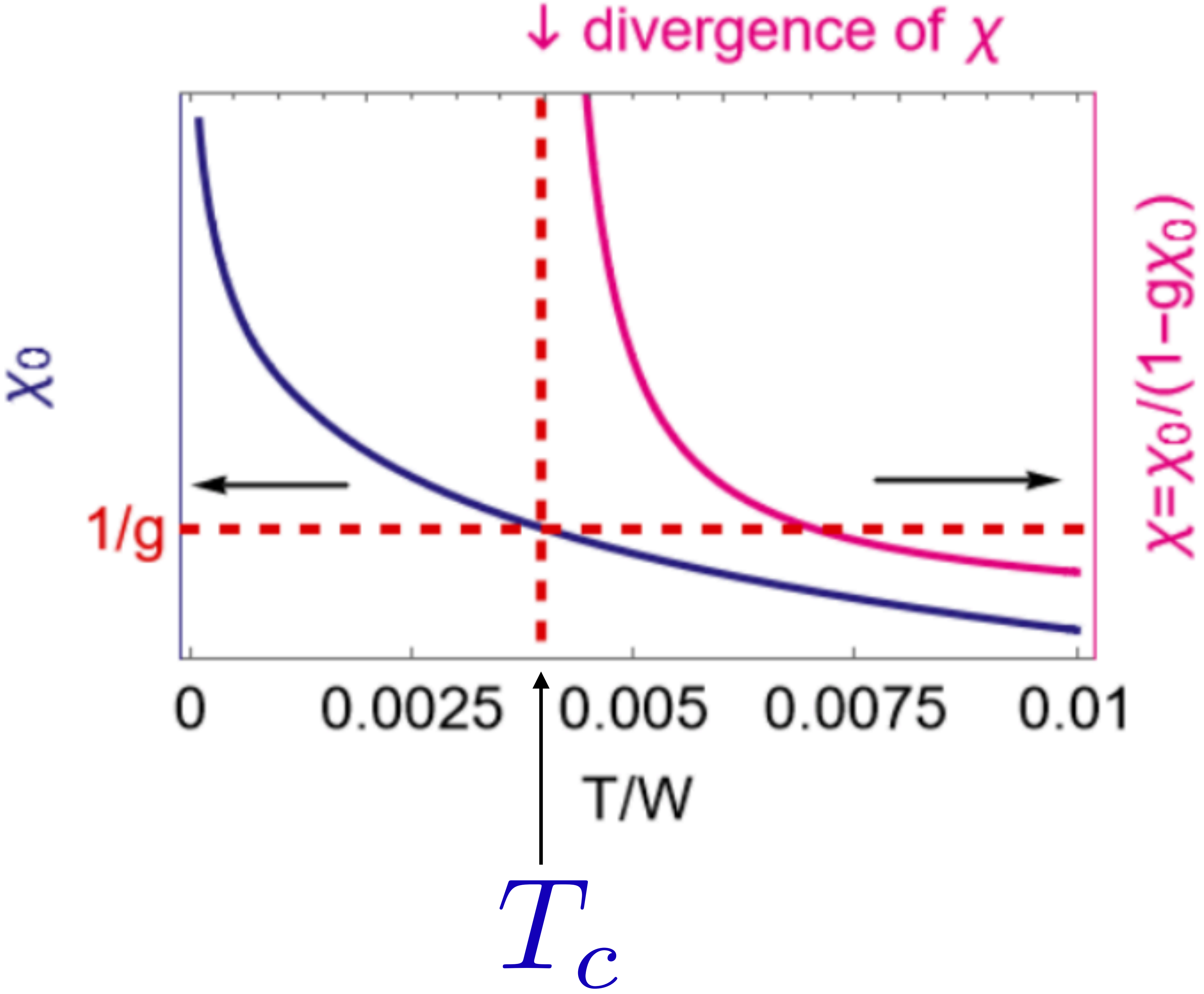
$$\chi(i\nu_n) \equiv \frac{1}{L^d}\int_0^\beta d\tau e^{i\nu_n\tau}\langle T\Delta(\tau)\Delta^\dagger\rangle_g$$

$$\Big\downarrow$$

$$= \frac{\chi_0}{1 - g\chi_0}$$

$$\Big\downarrow \quad g\chi_0 = 1$$

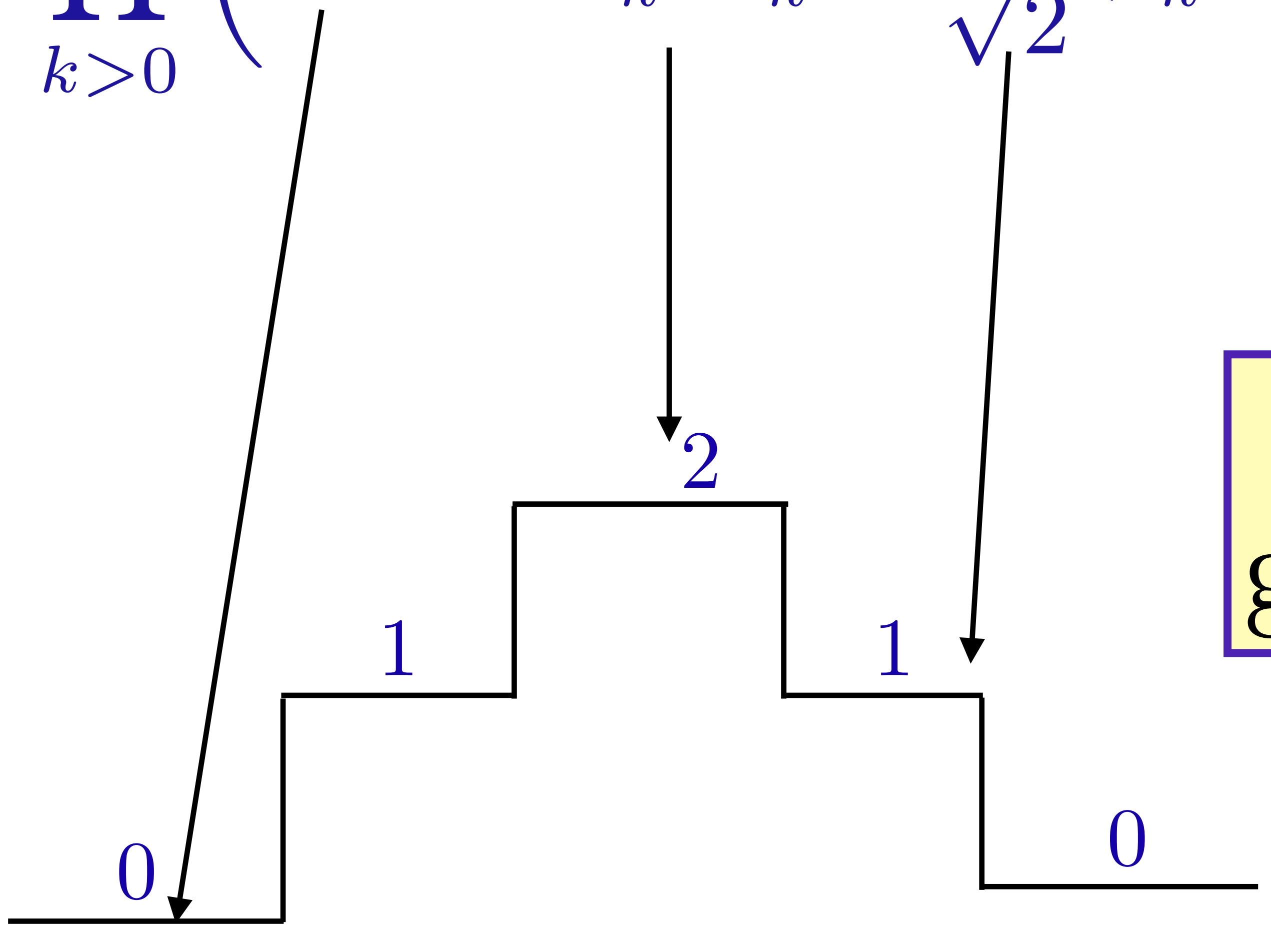
solve for T_c



$$T_c = (W-U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}}.$$

variational MF wave function

$$|\psi\rangle = \prod_{k>0}\left(x_k + y_k b^\dagger_k b^\dagger_{-k} + \frac{z_k}{\sqrt{2}}(b^\dagger_k + b^\dagger_{-k})\right)|0\rangle$$



HK
generalization

three variational parameters

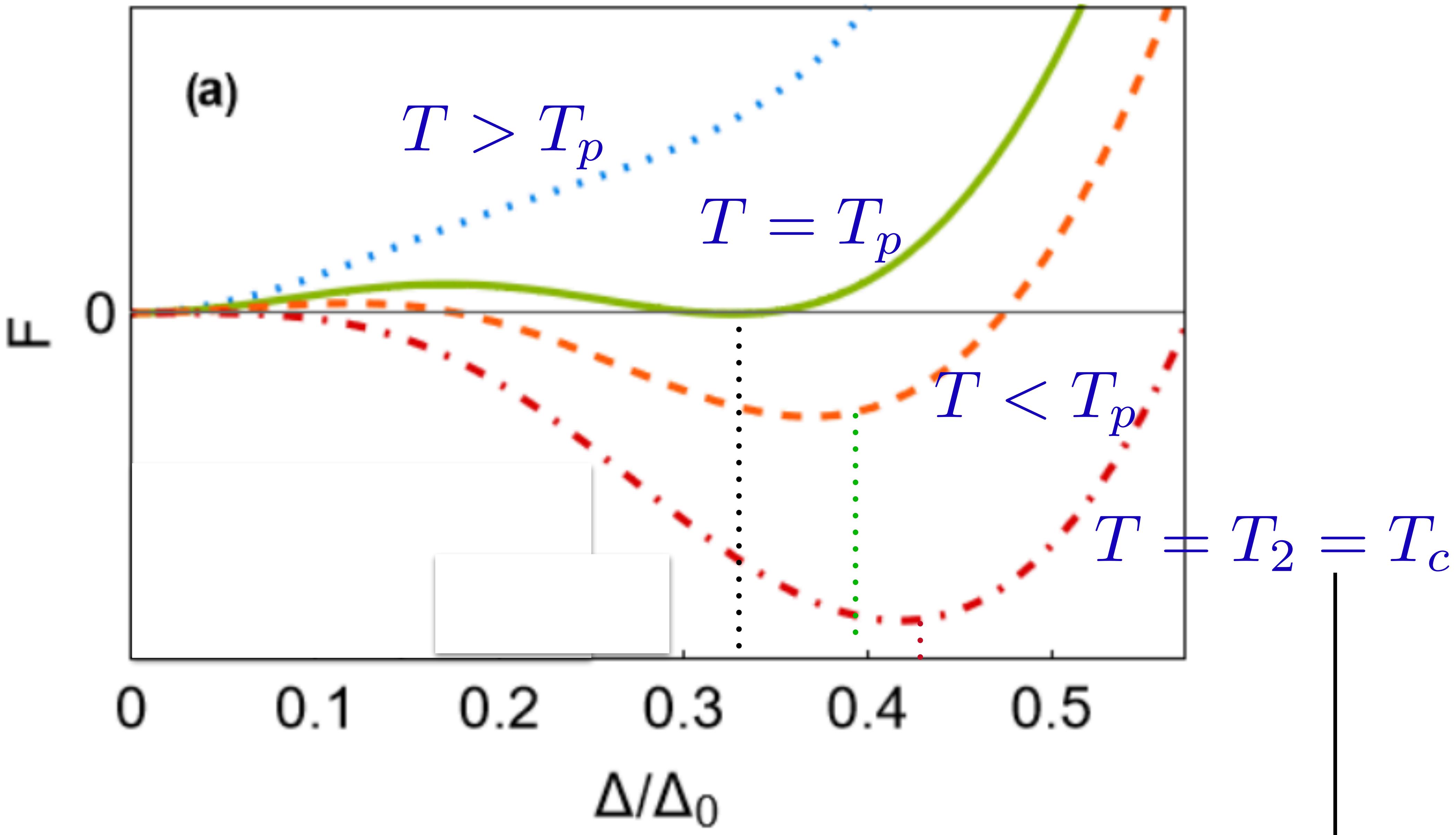
$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation

$$\Big\downarrow \quad \Delta \ll U, W$$

$$\Delta = (W - U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

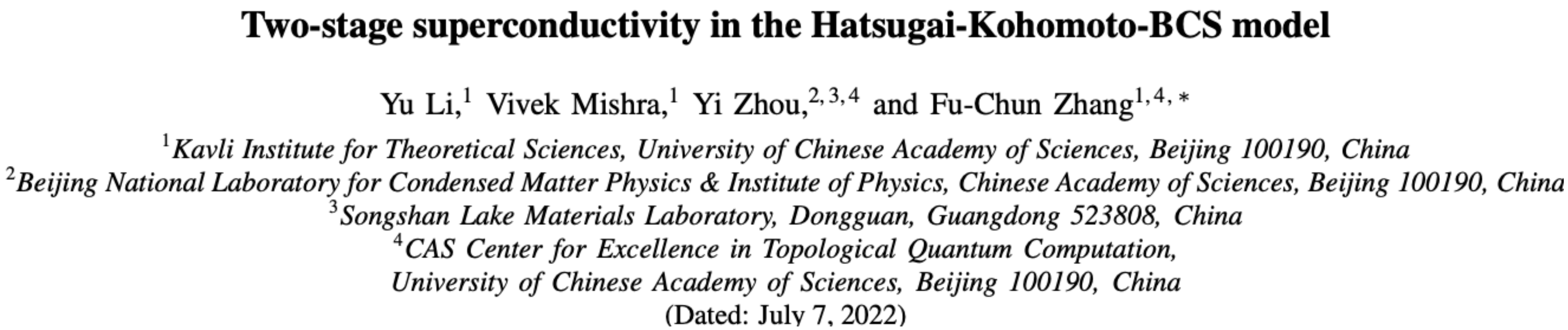
compute free energy



$t_G \approx 10^{-11}$

$$\lim_{g \to 0} \frac{\Delta}{T_c} \to \infty$$

MF theory
is accurate!



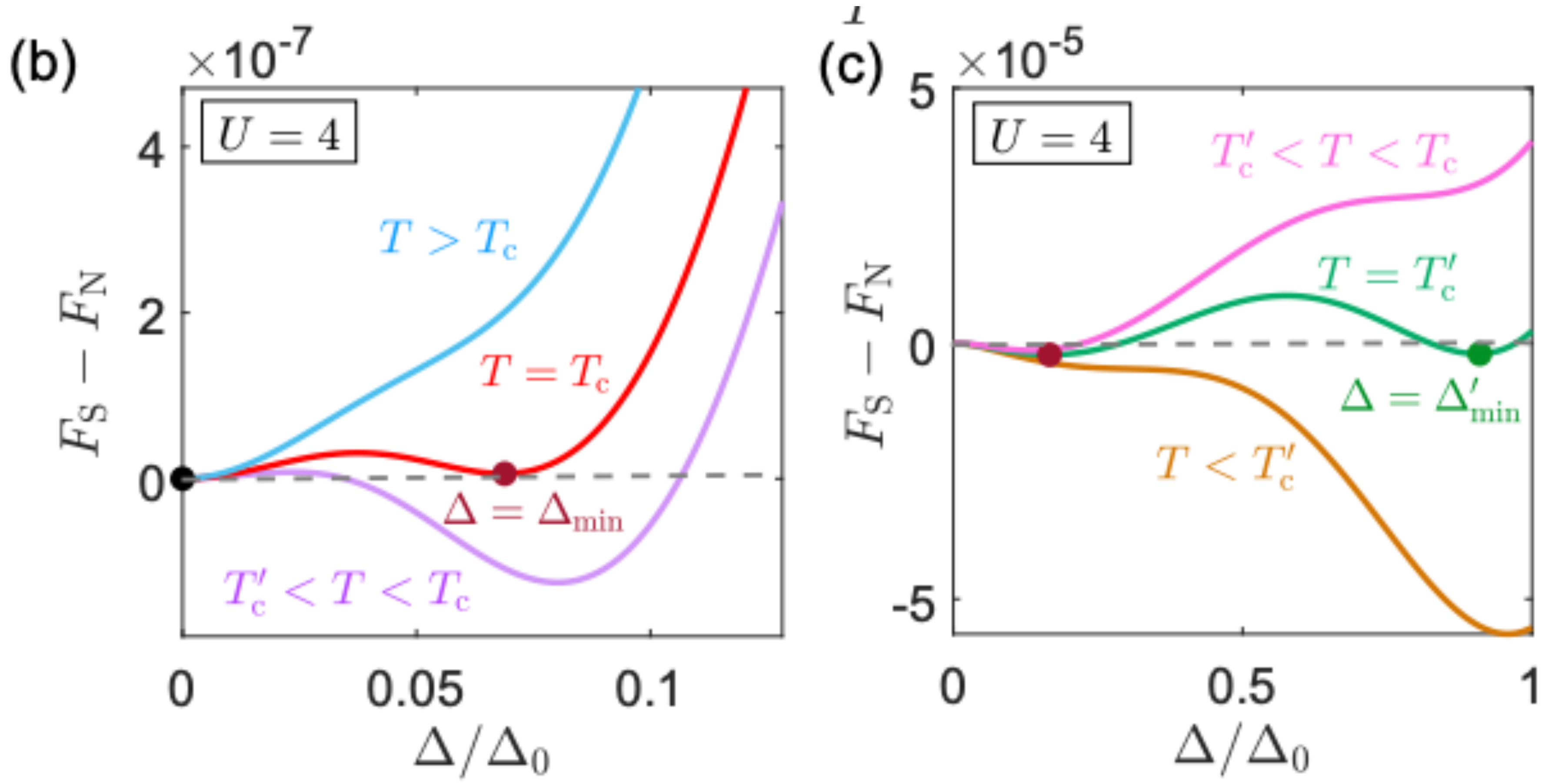
Two-stage superconductivity in the Hatsugai-Kohomoto-BCS model

Yu Li,[1] Vivek Mishra,[1] Yi Zhou,[2,3,4] and Fu-Chun Zhang[1,4,*]

[1]*Kavli Institute for Theoretical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China*
[2]*Beijing National Laboratory for Condensed Matter Physics & Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*
[3]*Songshan Lake Materials Laboratory, Dongguan, Guangdong 523808, China*
[4]*CAS Center for Excellence in Topological Quantum Computation,*
University of Chinese Academy of Sciences, Beijing 100190, China

(Dated: July 7, 2022)

https://arxiv.org/pdf/2207.01904.pdf



Bogoliubov excitations

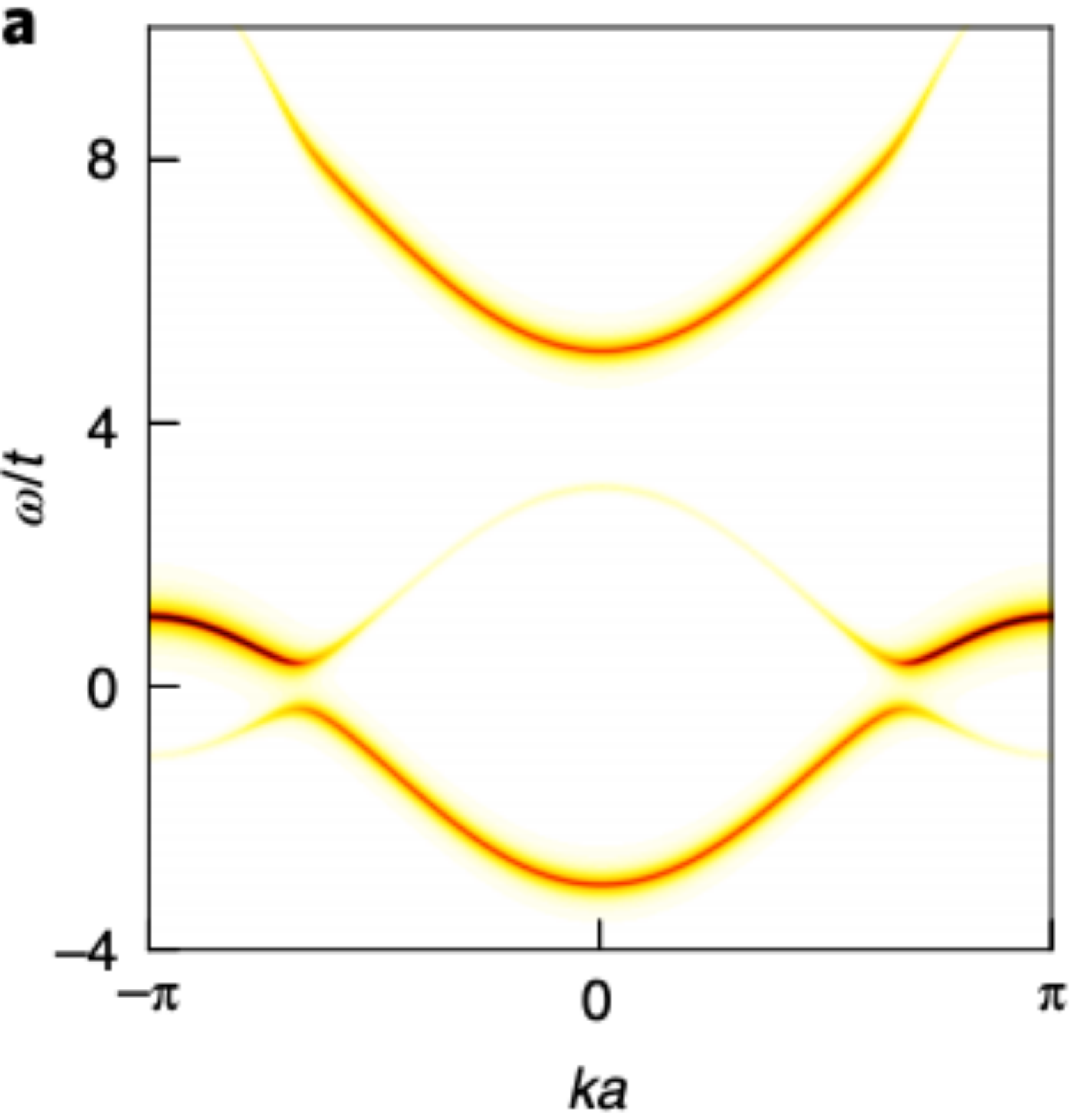
$$\gamma_{k\sigma}|\psi_{\mathrm{BCS}}\rangle = 0$$

$$\cancel{\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c^\dagger_{-k\overline{\sigma}}}$$

PYHons excitations

$$\gamma^l_{k\sigma} \propto \sqrt{2} x_k \zeta^\dagger_{k\sigma} - \sigma z_k \zeta_{-k\overline{\sigma}}$$

$$\gamma^u_{k\sigma} \propto z_k \eta^\dagger_{k\sigma} - \sigma \sqrt{2} y_k \eta_{-k\overline{\sigma}}$$



PYHon band

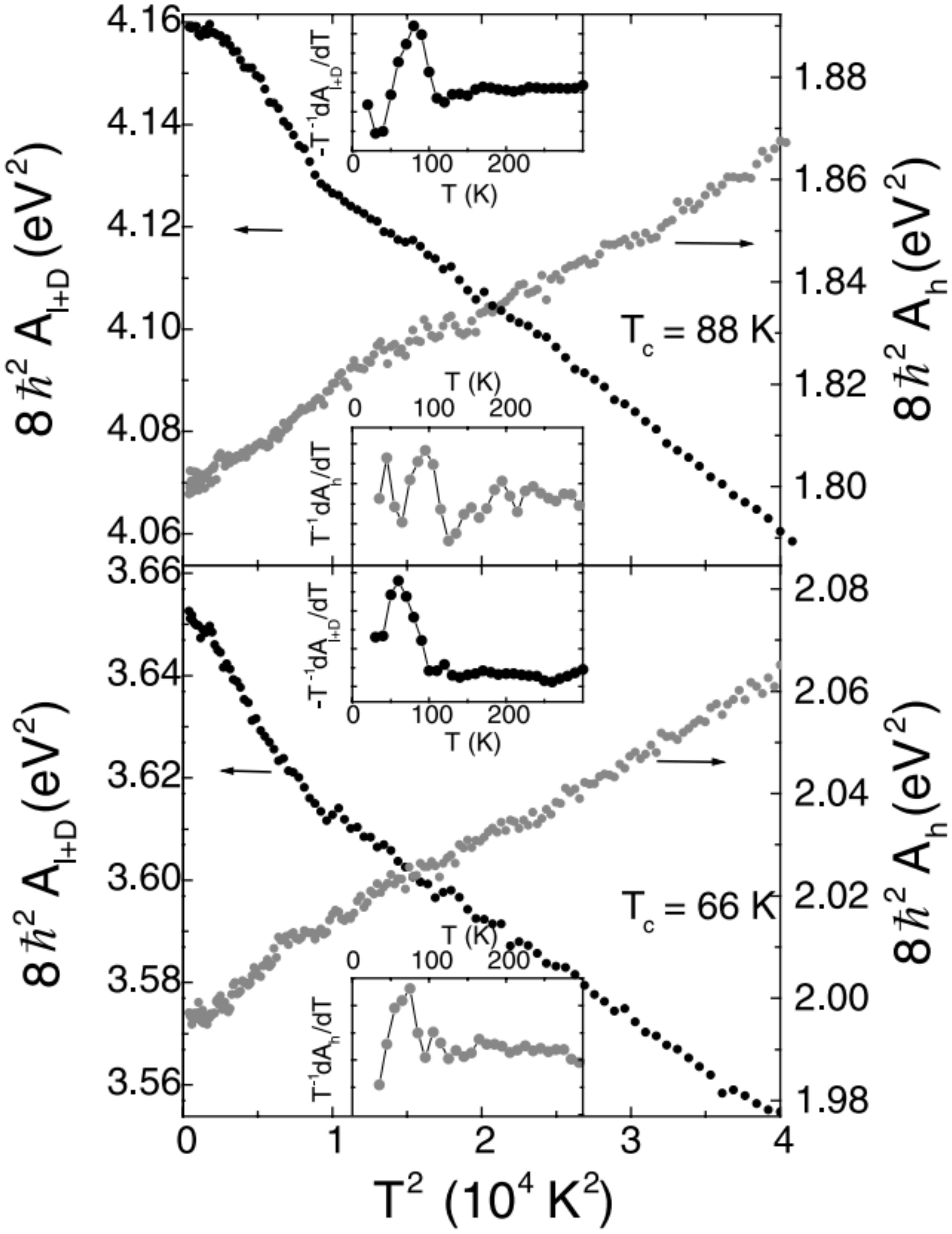
can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$

H. J. A. Molegraaf[1], **C. Presura**[1], **D. van der Marel**[1,*], **P. H. Kes**[2], **M. Li**[2]
+ See all authors and affiliations

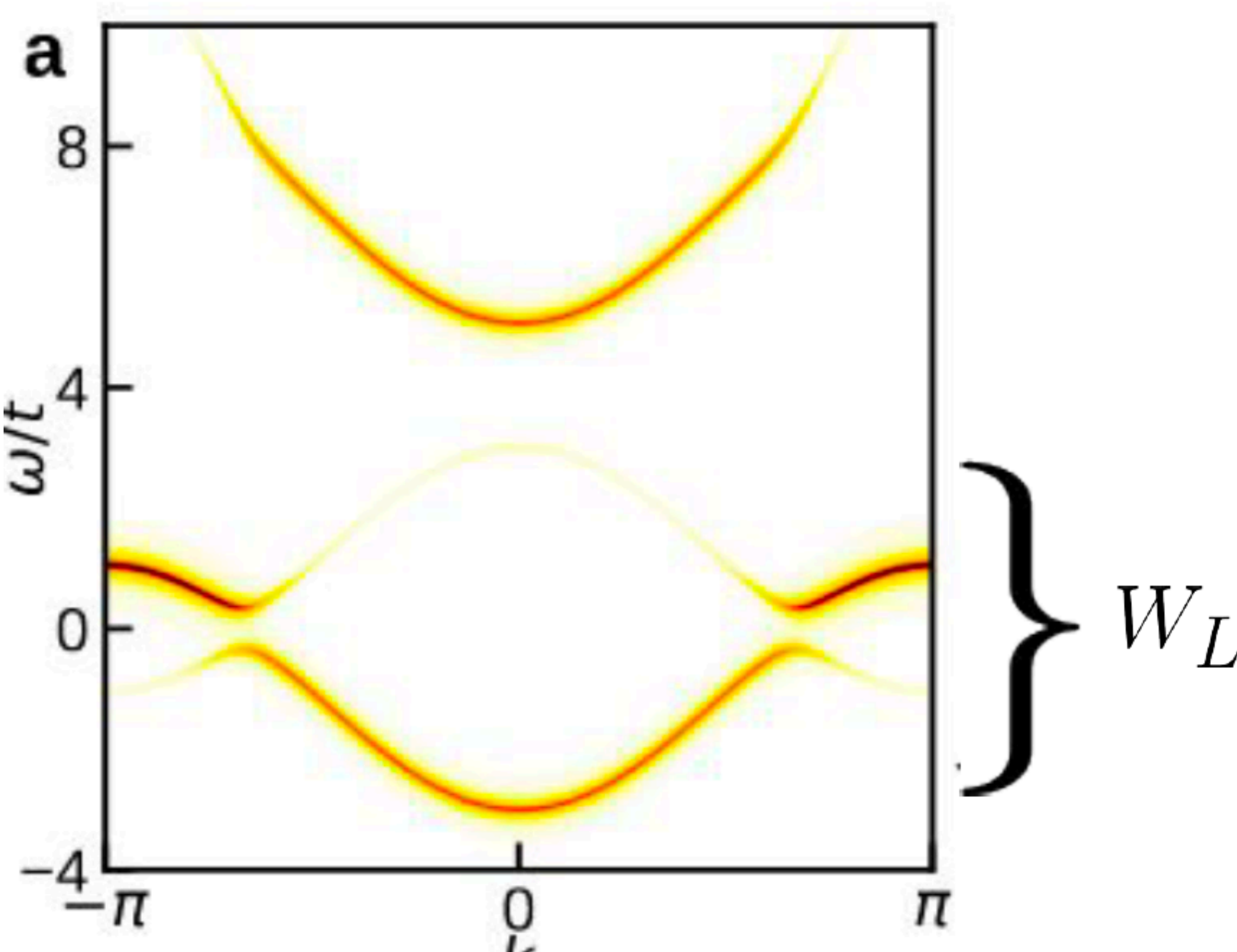
Science 22 Mar 2002:
Vol. 295, Issue 5563, pp. 2239-2241
DOI: 10.1126/science.1069947



$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000cm^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000cm^{-1}$$

$$\frac{\Delta A_l}{A_l} \propto 3\%$$


a
8
4
0
−4
ω/t
−π
0
π
k
W_L

why?

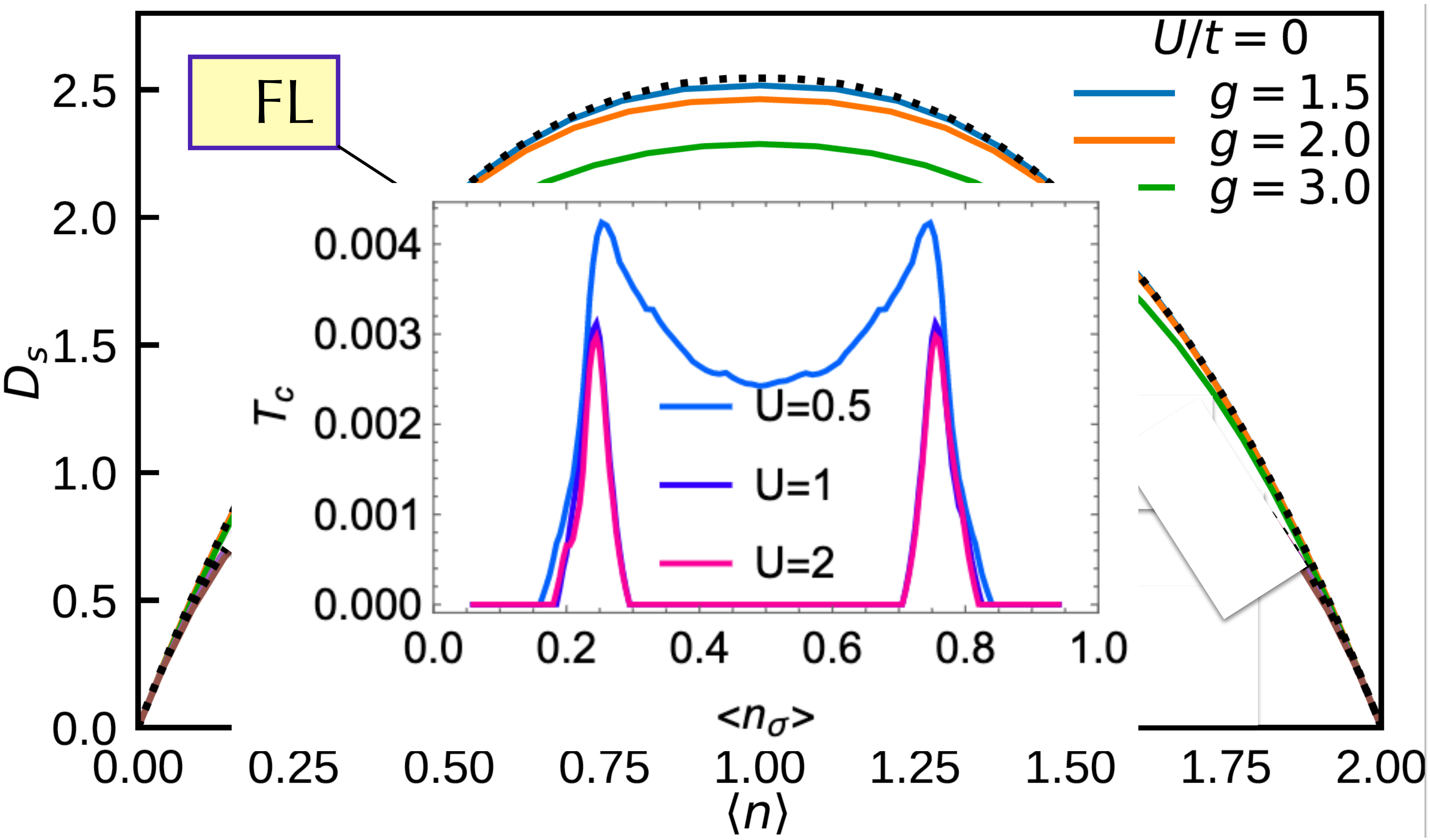
$$H = H_{\mathrm{HK}} + H_p$$

$$[H_{\mathrm{HK}}, H_p] \neq 0$$

dynamical
spectral weight
transfer

Superfluid Density

Mottness-induced suppression



	Superconductivity	Mottness
observable	BCS/FL	PYHZ/HK
$\chi \to \infty$	T_c	$T_c (= T_2)$
$\Delta \neq 0$	T_c	$T_p (> T_2)$
$\lim_{g \to 0} 2\Delta_0 / k_B T_c$	3.52	∞
$\mathrm{quasi-particles}$	Bogoliubons	PYHons
$t_G(\mathrm{Ginzburg})$	$\approx 10^{-12}$	$\approx 10^{-11}$
$1/\mathrm{TT}_1$	HS peak	no HS peak
Landau Expansion	$a = \alpha t,\ b > 0$	$a = \alpha t,\ b < 0$ $c > 0$
$E_{\mathrm{cond}} / N(0) \Delta^2$	-1	$[-2, -1]$

			$\cancel{Z_2}$
		FL	HK
quantum numbers		k	k
stability	V_{local}	$\beta(V_{\text{local}}) = 0$	$\beta(V_{\text{local}}) = 0$
	V_{SC}	$\beta(V_{\text{SC}}) \to \infty$	$\beta(V_{\text{SC}}) \to \infty$?

Topology + Strong Correlations?

Are Exact Statements Possible?

Haldane +HK model

TI

1/4-filled MI