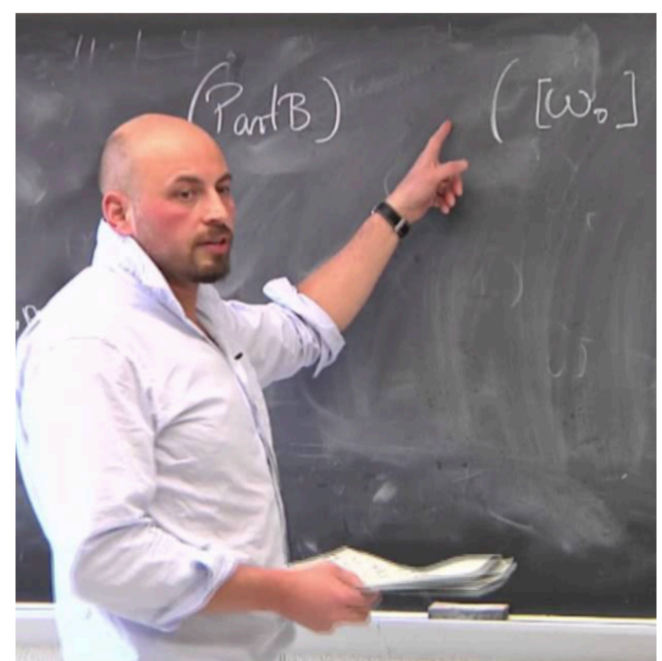
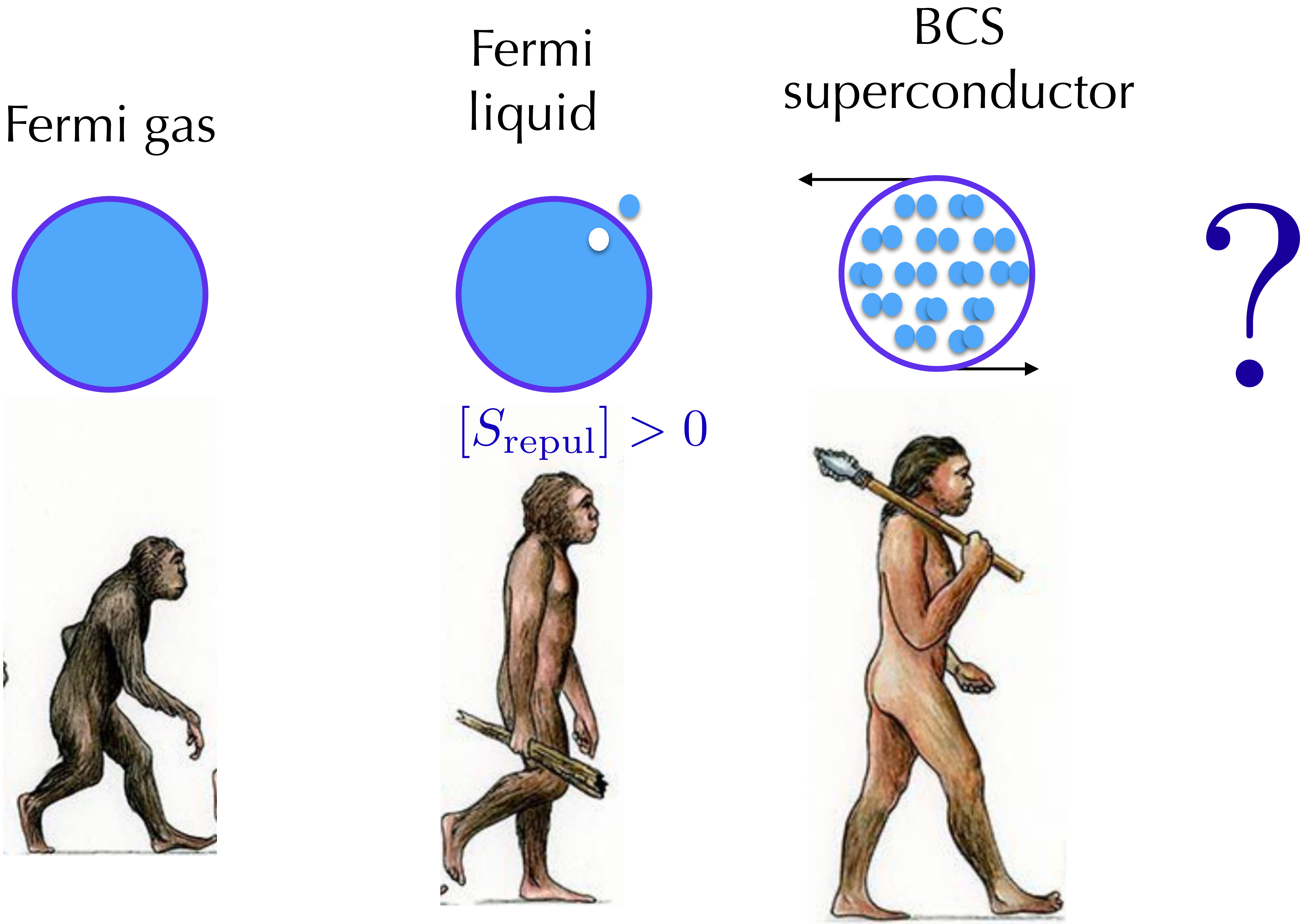
Superconductivity and Mottness: Exact Results

Nature Physics, vol.16, 1175-1180 (2020)
with N&V by J. Zaanen

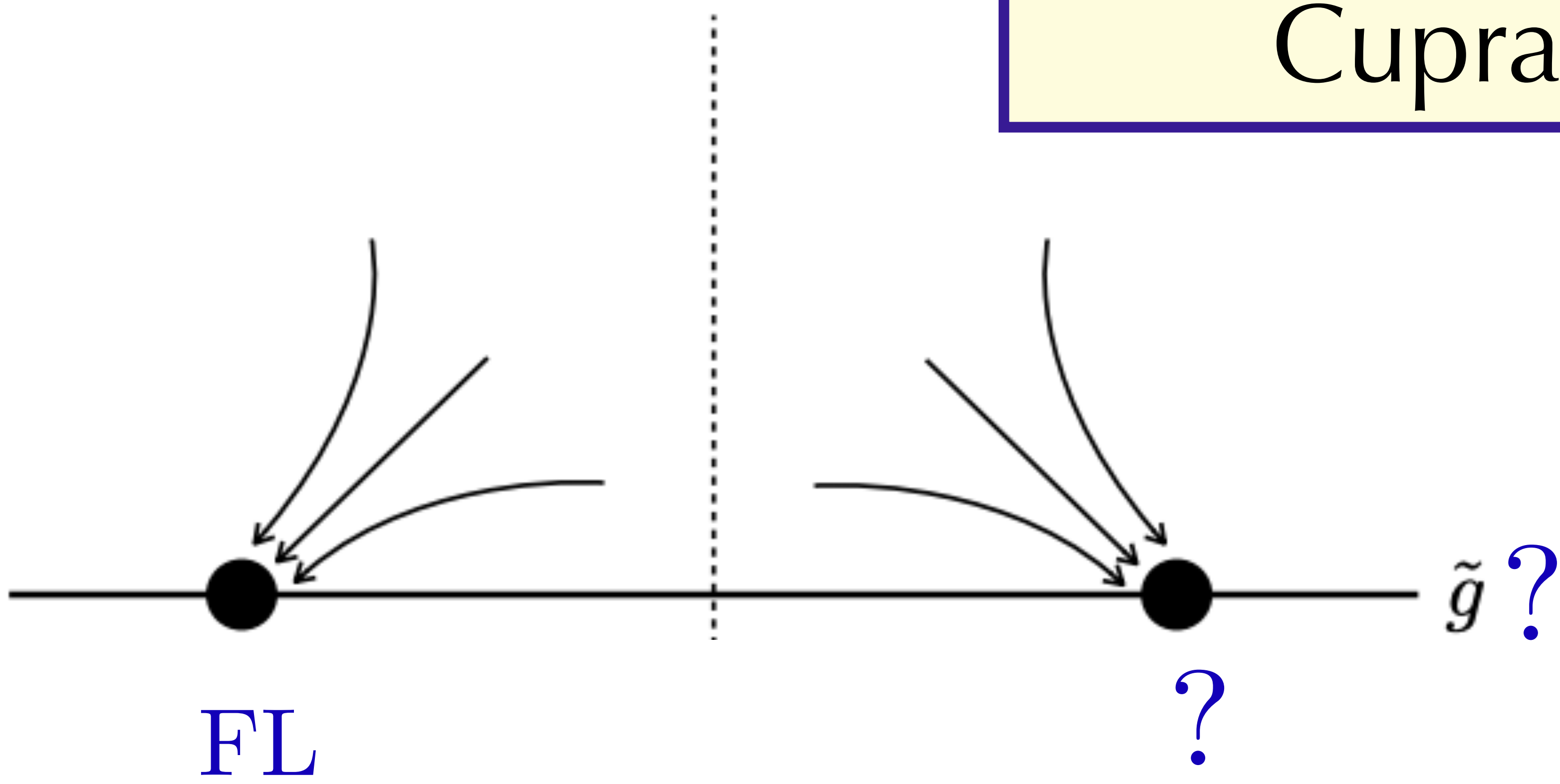
arxiv.org/abs/2103.03256 (to appear NP 2022)

Luke Yeo	Edwin Huang	G. La Nave	Jinchao Z.




Fermi gas
Fermi liquid
$[S_{\rm repul}] > 0$
BCS superconductor
?

fixed point beyond FL? Mottness? Cuprates?

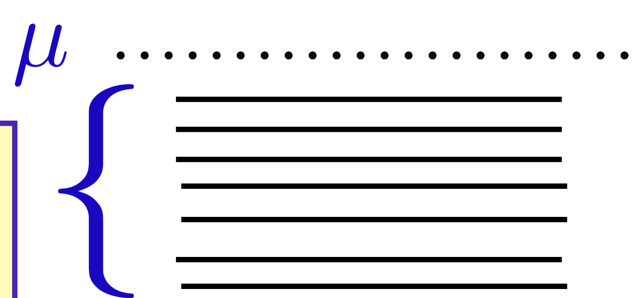


quartic interacting theory?

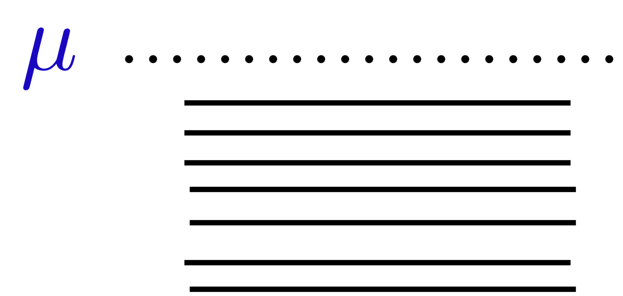
NFL

Fermi liquids

Is single occupancy below chemical potential possible?



doubly occupied



with time-reversal symmetry in tact?

Anderson
Haldane
2000

3 citations

Fermi liquids

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + \cdots \quad (\cdots \to 0)$$

$(n_{p\uparrow}, n_{p\downarrow})$ conserved currents

$(c_{p\uparrow}, c_{p\downarrow}, \text{h.c.})$ 4 objects

$O(4)$

$\text{DetM} = 1$ → SO(4) proper rotations

$\text{DetM} = -1$ → improper rotations

$$\text{Det}\,M = \pm 1 \implies Z_2 = O(4) \div SO(4)$$

$$\epsilon(p) = \epsilon_F$$

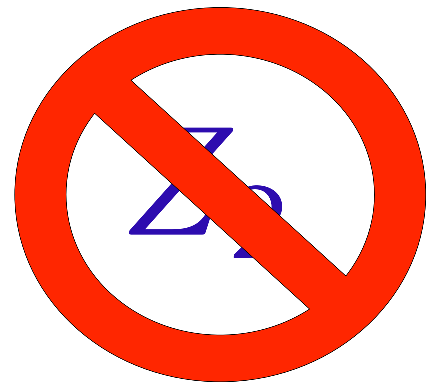
Fermi Surface

$$H = 0$$

$$\left.\begin{aligned} n_{p\uparrow} &\to 1 - n_{p\uparrow} \\ n_{p\downarrow} &\to n_{p\downarrow} \end{aligned}\right\} Z_2$$

at Fermi surface only

How to destroy Fermi liquids?



$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + U n_{p\uparrow} n_{p\downarrow}$$

odd under Z_2

scaling dimension

$[n_{p\uparrow} n_{p\downarrow}] = -2$

relevant interaction

New fixed point!

Hatsugai-Kohmoto model

Hubbard not necessary!

General HK Model

$$\sum_k (\xi_k (n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow})$$

Solvable Mott transition:U>W

$$G_{k\sigma}(i\omega_n \rightarrow z) = \frac{1 - \langle n_{k\bar{\sigma}} \rangle}{z - \xi_k} + \frac{\langle n_{k\bar{\sigma}} \rangle}{z - (\xi_k + U)} \neq \frac{1}{z - \omega_k}$$

lower Hubbard band

upper Hubbard band

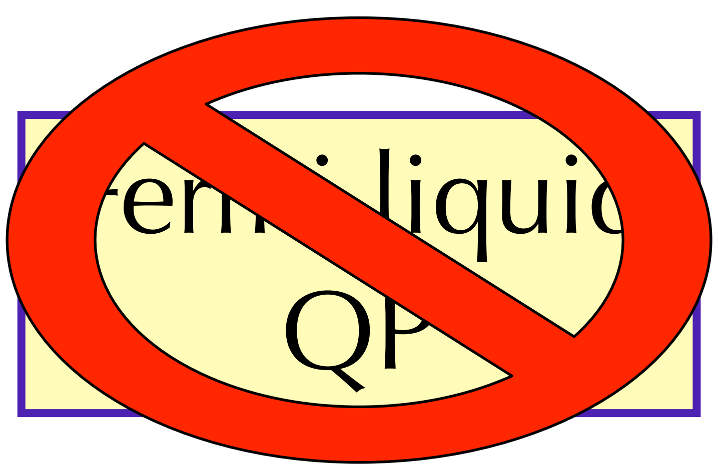
zeros

$$\langle n_{k\sigma} \rangle = \frac{1}{2}$$

$$G^R_\sigma(k,\omega) = \frac{1}{\omega + i0^+ - (\xi_k + U/2) - \frac{(U/2)^2}{\omega + i0^+ - (\xi_k + U/2)}}$$

$\Sigma(k,\omega)$

$\omega = \xi_k + U/2$



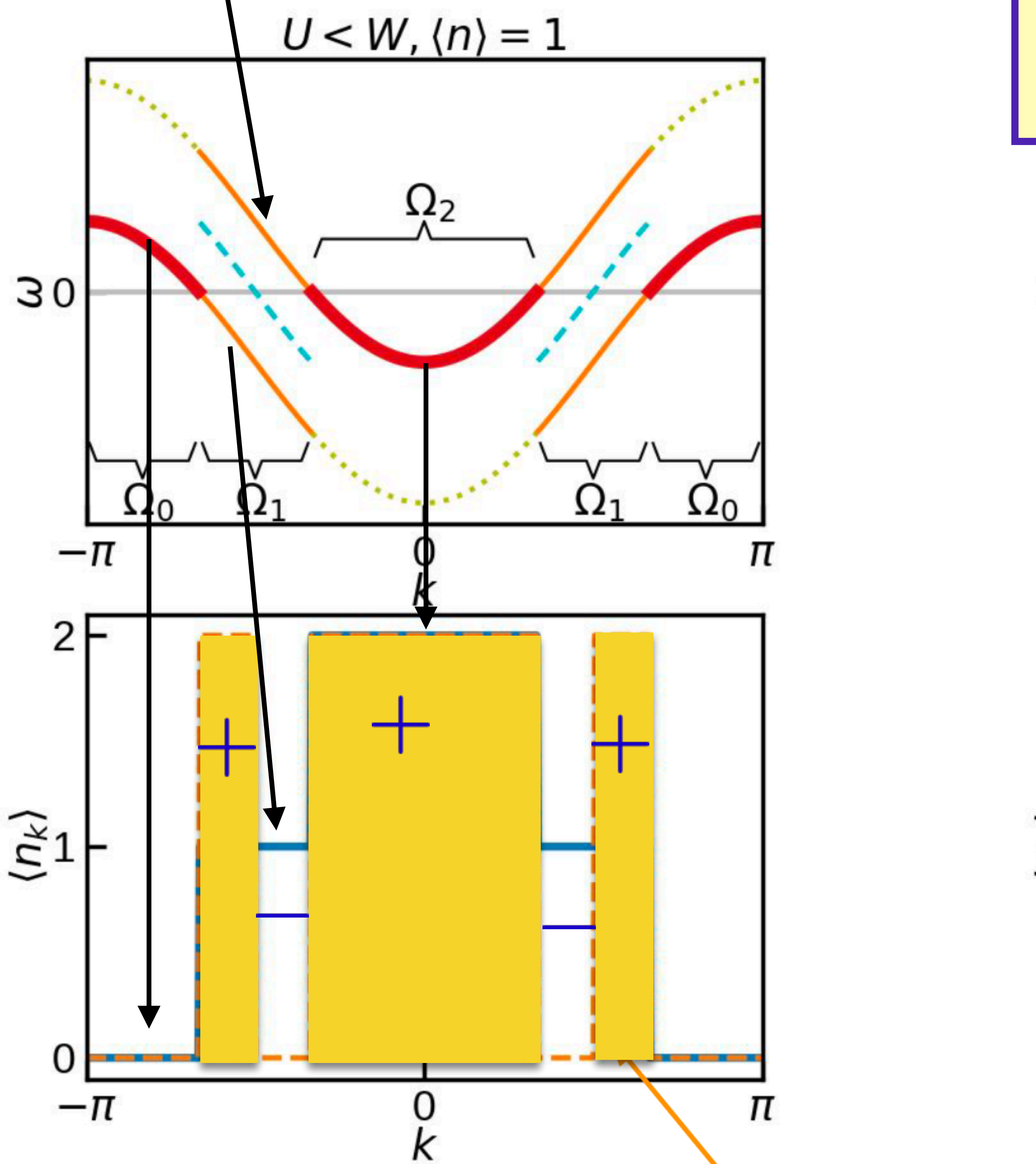
zeros

$\Re\Sigma = \Im\Sigma = \infty$

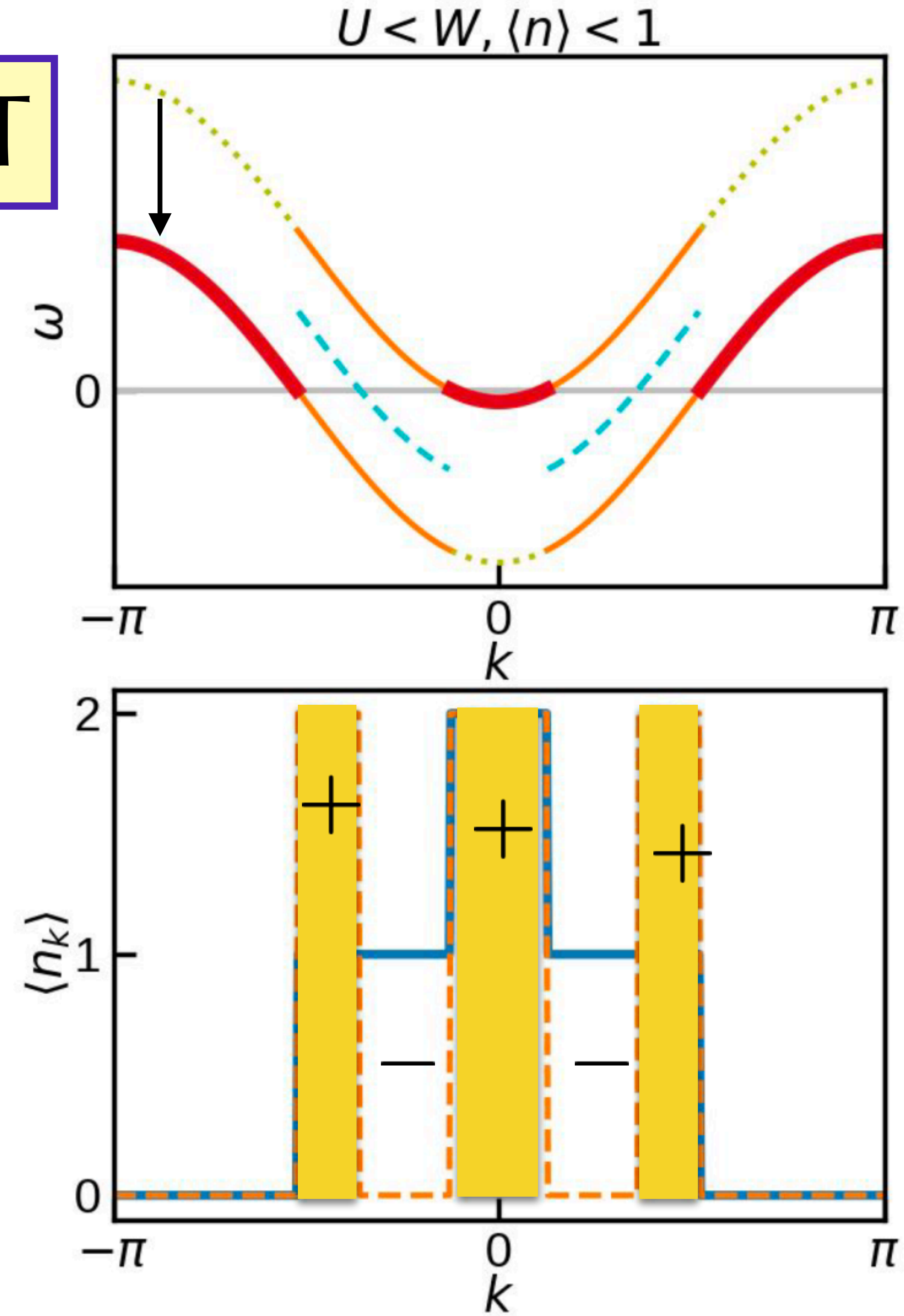
single occupancy

counting charges

SWT



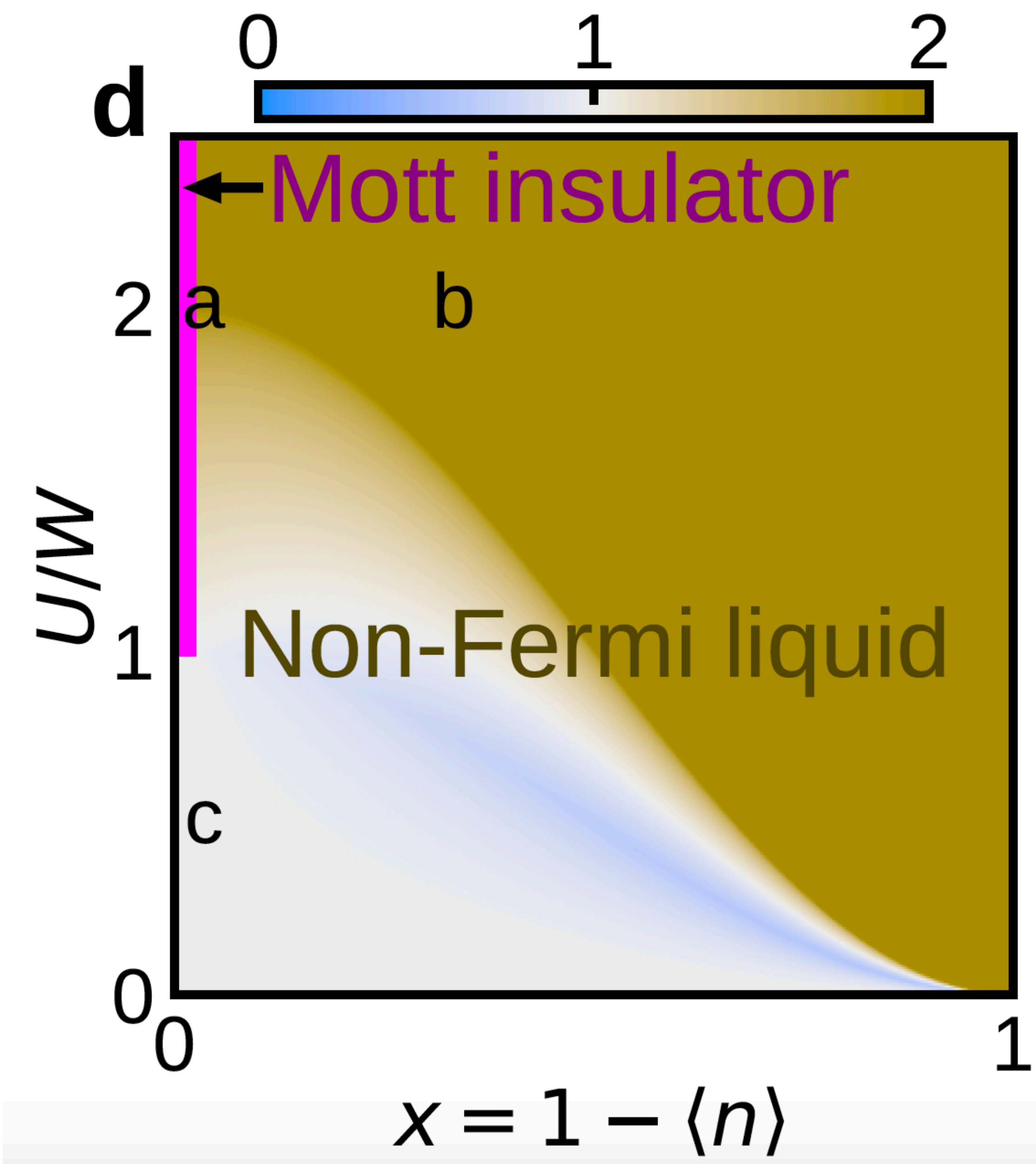
$n_{\mathrm{Lutt}} = \langle n \rangle$ $2\theta(\mathrm{Re}\, G(\mathbf{k}, \omega = 0))$



$n_{\mathrm{Lutt}} \neq \langle n \rangle$

zeros ≠ particles

Why NFL?



FL

HK Mottness

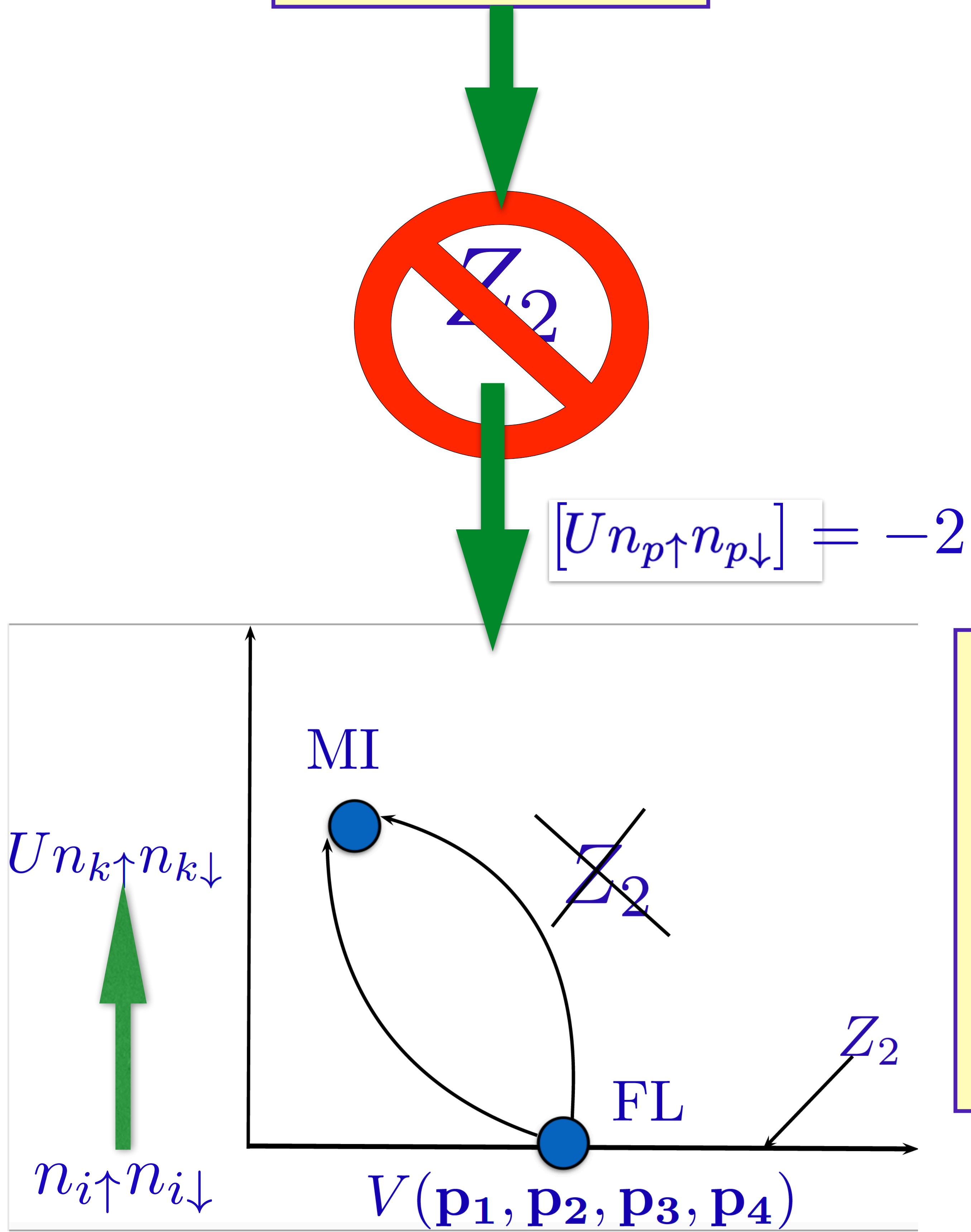
$$c^\dagger_{k\uparrow} c^\dagger_{k\downarrow}|G\rangle \quad \not\leftrightarrow \quad ?$$

$$c^\dagger_{k\uparrow}|G\rangle, c^\dagger_{k\downarrow}|G\rangle \quad \leftrightarrow \quad \zeta^\dagger_{k\uparrow}|G\rangle, \zeta^\dagger_{k\downarrow}|G\rangle$$

$$|G\rangle \qquad\qquad |G\rangle$$

$$\zeta^\dagger_{k\uparrow}\zeta^\dagger_{k\downarrow}|G\rangle = 0$$

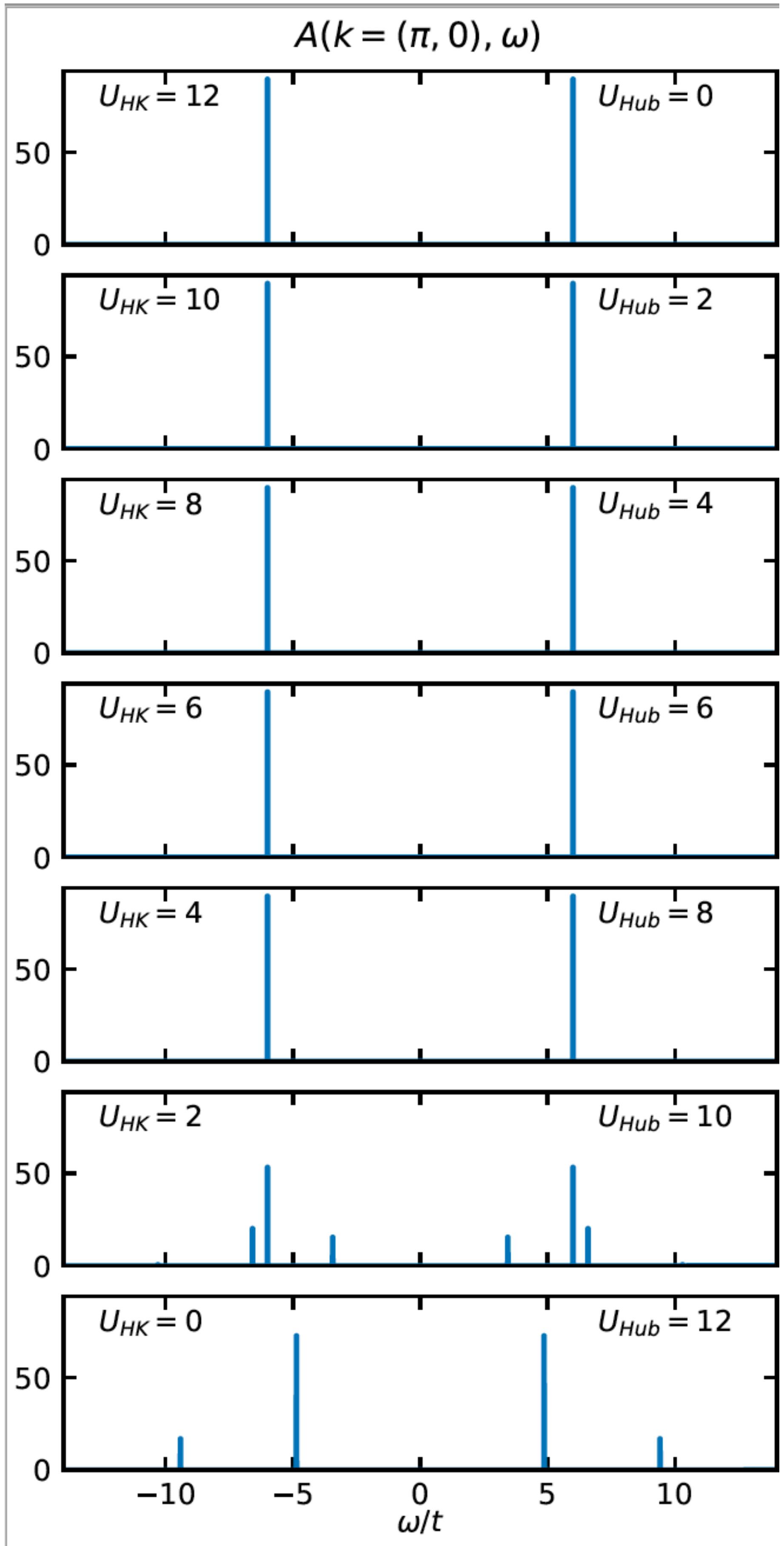
Fermi liquids



Hubbard not necessary (universality class)

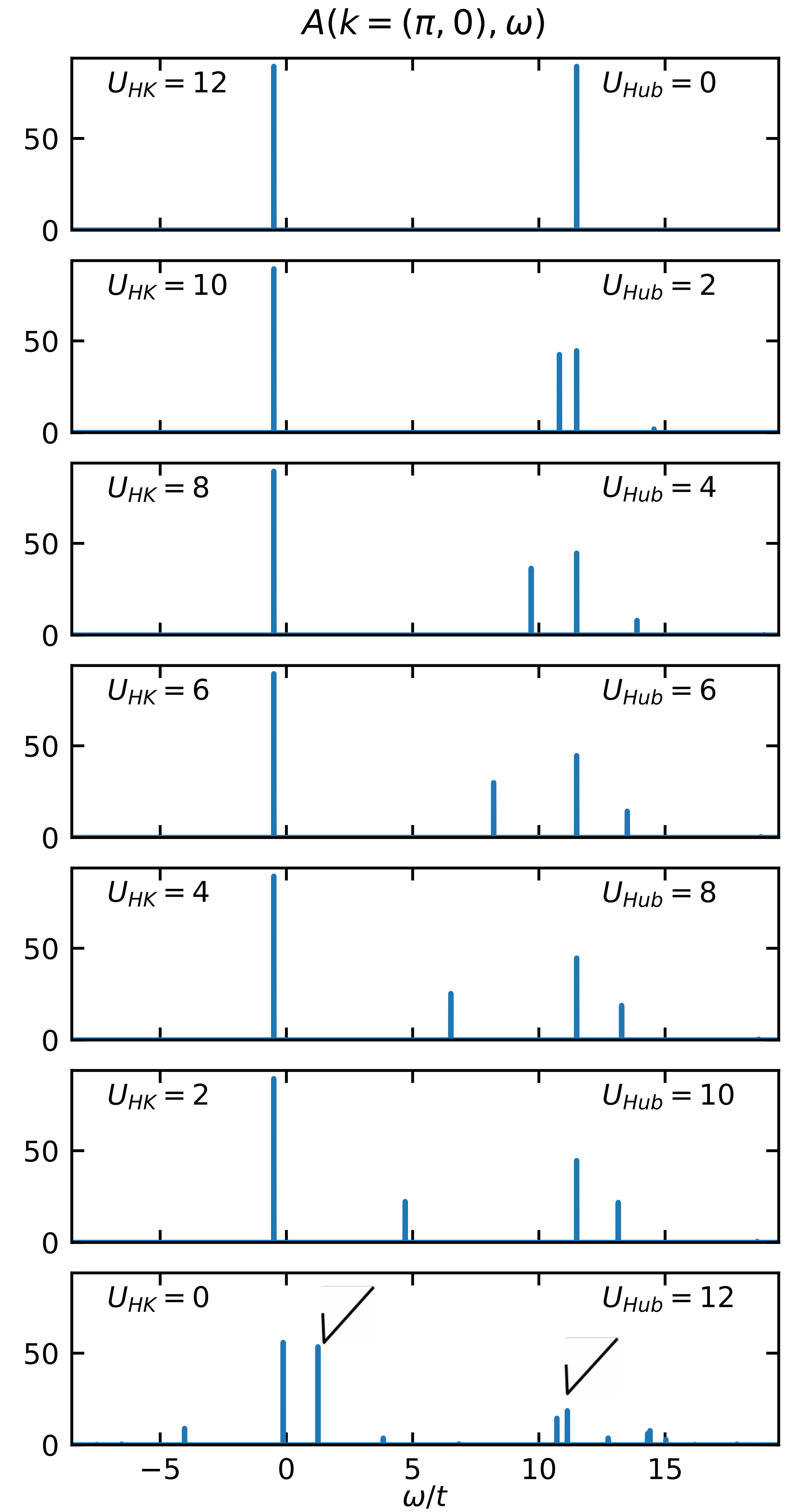
$n = 1.0$

$H_{\mathrm{HK}} \approx H_{\mathrm{Hubb}}$



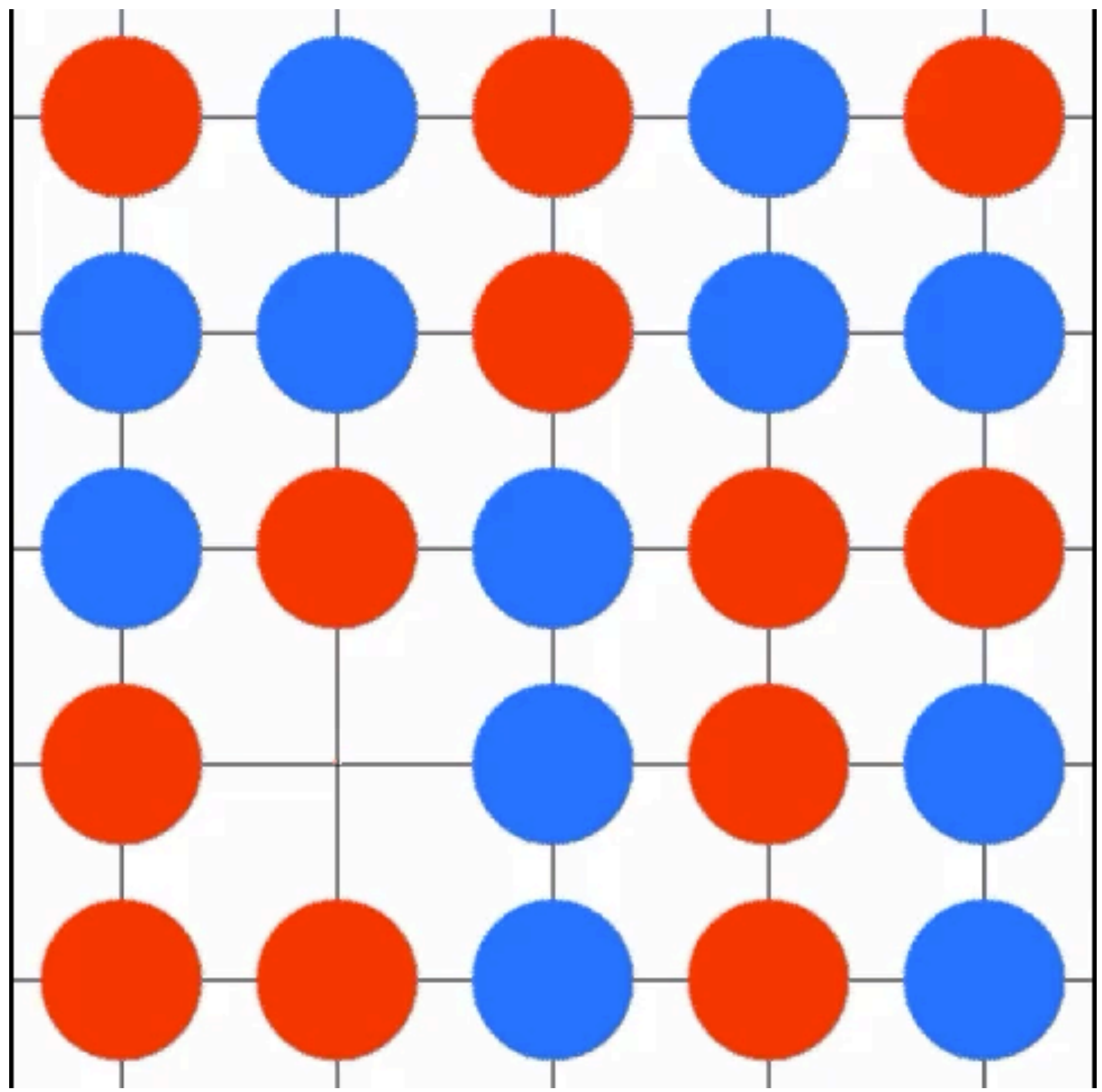
$n = 0.875$

DSWT



what does the HK model leave out??

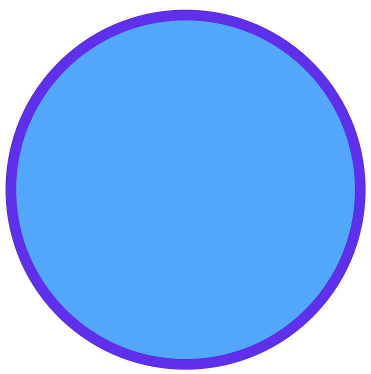
$$[H_t, H_U] \neq 0$$



dynamical spectral weight transfer

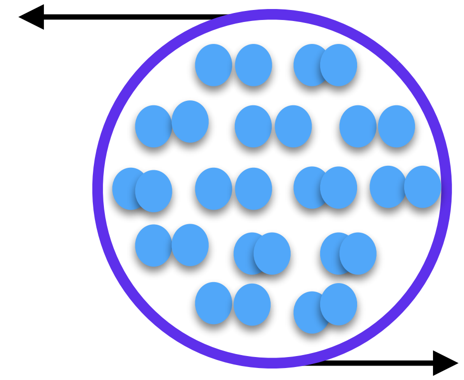
Mottness

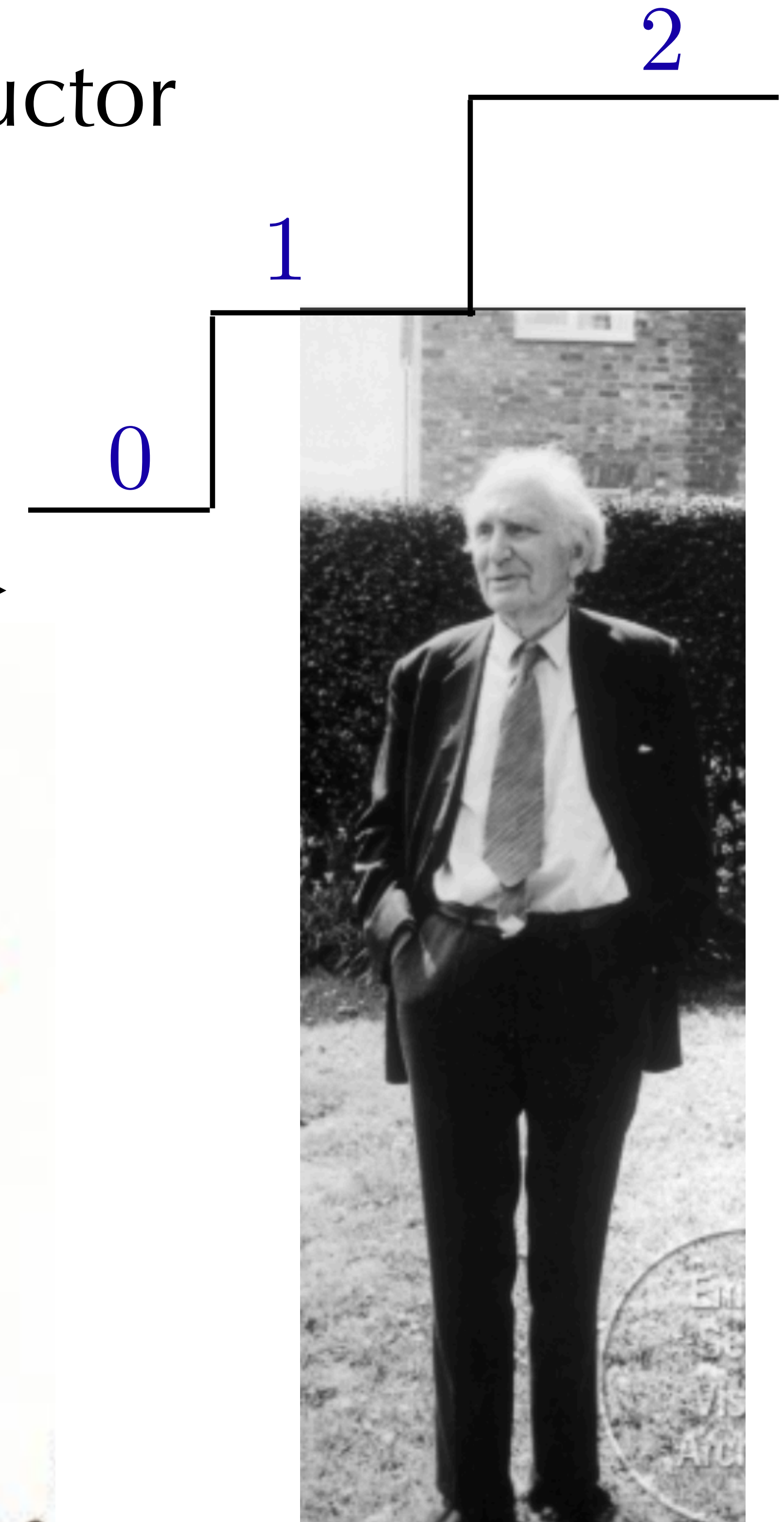
Fermi gas



Fermi liquid

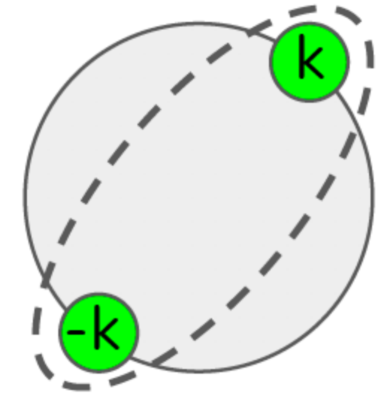
BCS superconductor





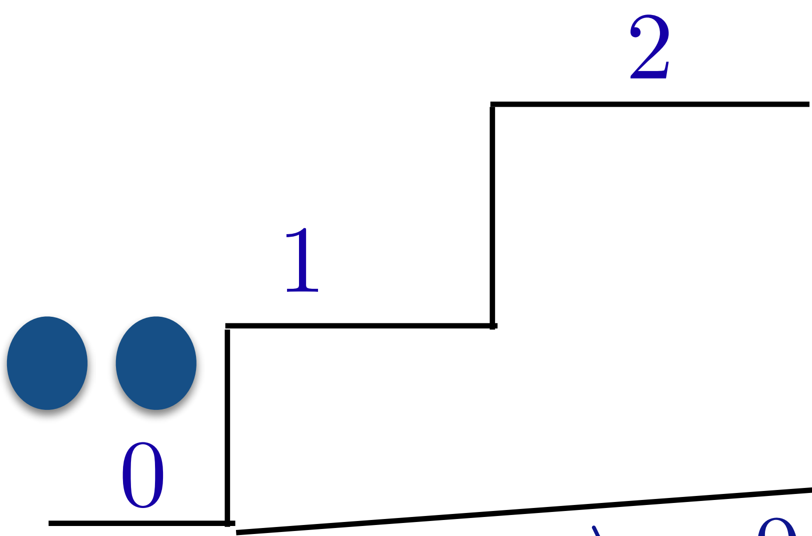
Superconductivity?

Cooper Instability



$$H = H_{\mathrm{HK}} - g H_p$$

$$= \sum_{k,k'} b_k^\dagger b_{k'}$$



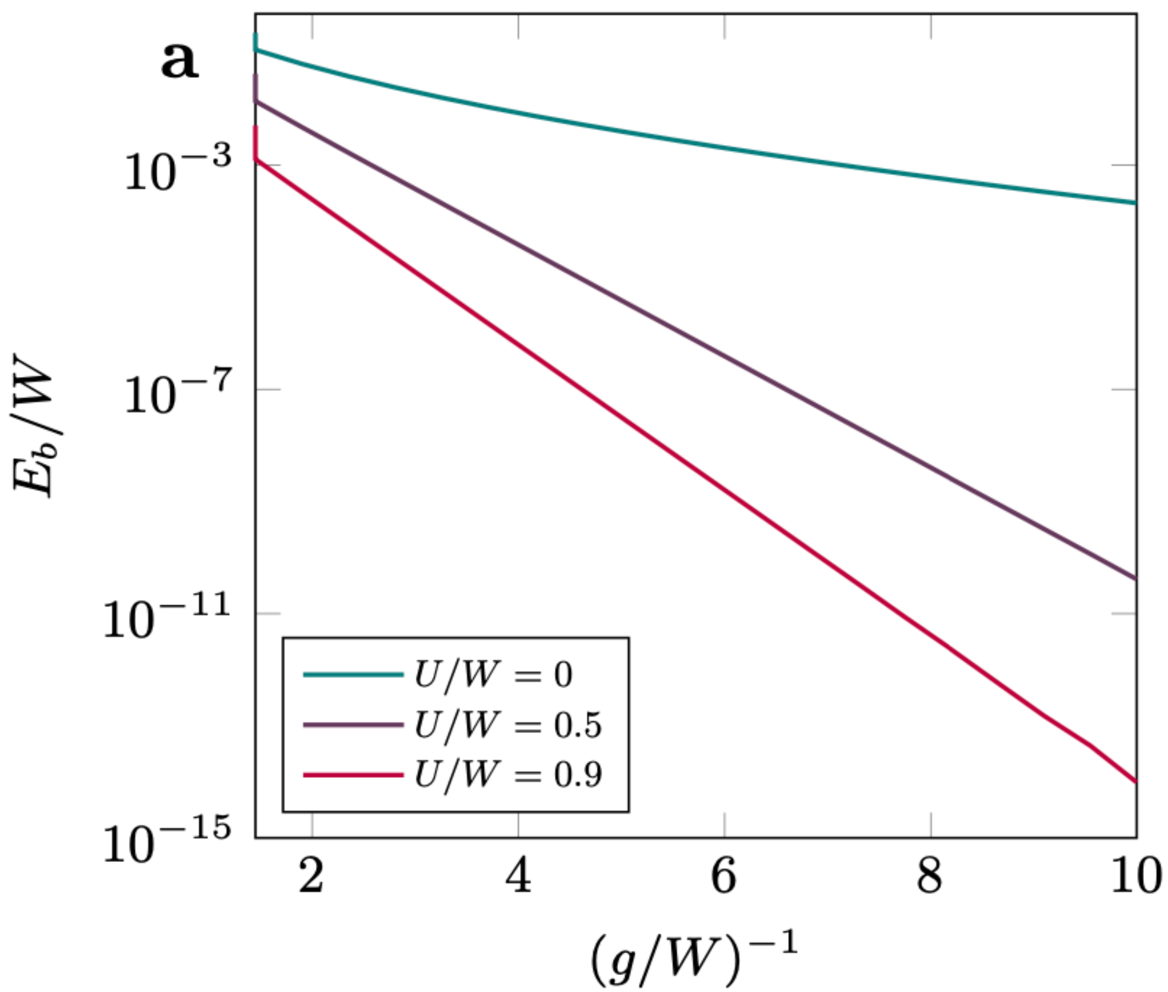
$$|\psi\rangle = \sum_{k \in \Omega_0} \alpha_k b_k^\dagger |\mathrm{GS}\rangle$$

$$\langle n_{k\sigma} \rangle = 0$$

$$E_b = \langle \mathrm{GS}|\mathrm{H}|\mathrm{GS}\rangle - \langle \psi|\mathrm{H}|\psi\rangle \lessgtr 0$$

Cooper Instability

$$E_b = -E \sim W(1-(U/W)^2)e^{-\pi W\sqrt{1-(U/W)^2}/g}$$



Pair Susceptibility

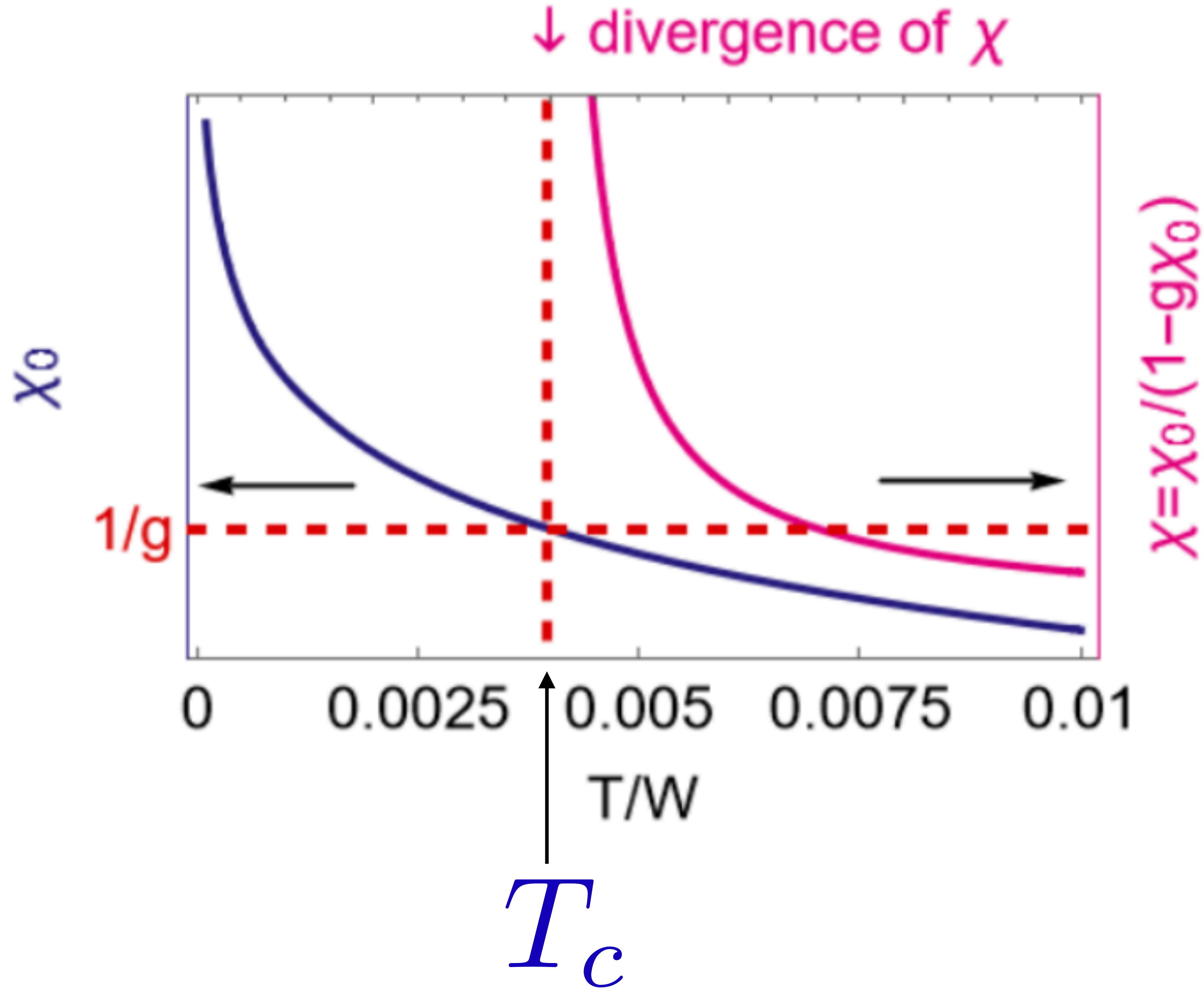
$$\chi(i\nu_n) \equiv \frac{1}{L^d} \int_0^\beta d\tau e^{i\nu_n \tau} \langle T \Delta(\tau) \Delta^\dagger \rangle_g$$

↓

$$= \frac{\chi_0}{1 - g\chi_0}$$

↓ $g\chi_0 = 1$

solve for T_c



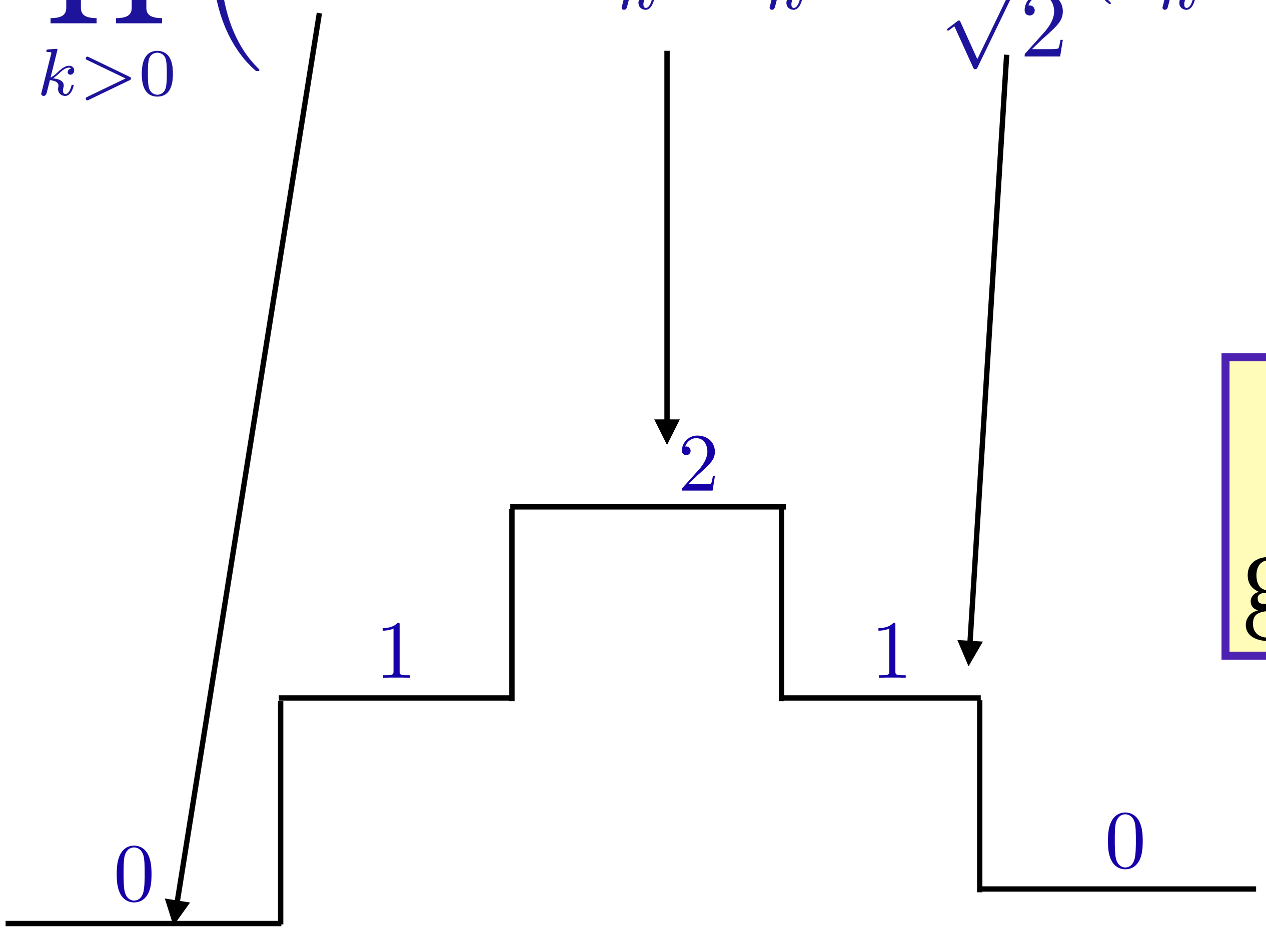
↓ divergence of χ
χ0
1/g
χ=χ0/(1−gχ0)
0
0.0025
0.005
0.0075
0.01
T/W
T_c

variational wave function

$$|\psi_{\mathrm{BCS}}\rangle = \prod_{k}(u_k + v_k b_k^{\dagger}|0\rangle$$

$$|\psi_{\mathrm{BCS}}\rangle = \prod_{k>0}(u_k^2 + v_k^2 b_k^{\dagger} b_{-k}^{\dagger} + u_k v_k (b_k^{\dagger} + b_{-k}^{\dagger}))|0\rangle$$

$$|\psi\rangle = \prod_{k>0} \left(x_k + y_k b^\dagger_k b^\dagger_{-k} + \frac{z_k}{\sqrt{2}} (b^\dagger_k + b^\dagger_{-k}) \right) |0\rangle$$



HK generalization

three variational parameters

$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation

$\Delta \ll U, W$

$$\Delta = (W - U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

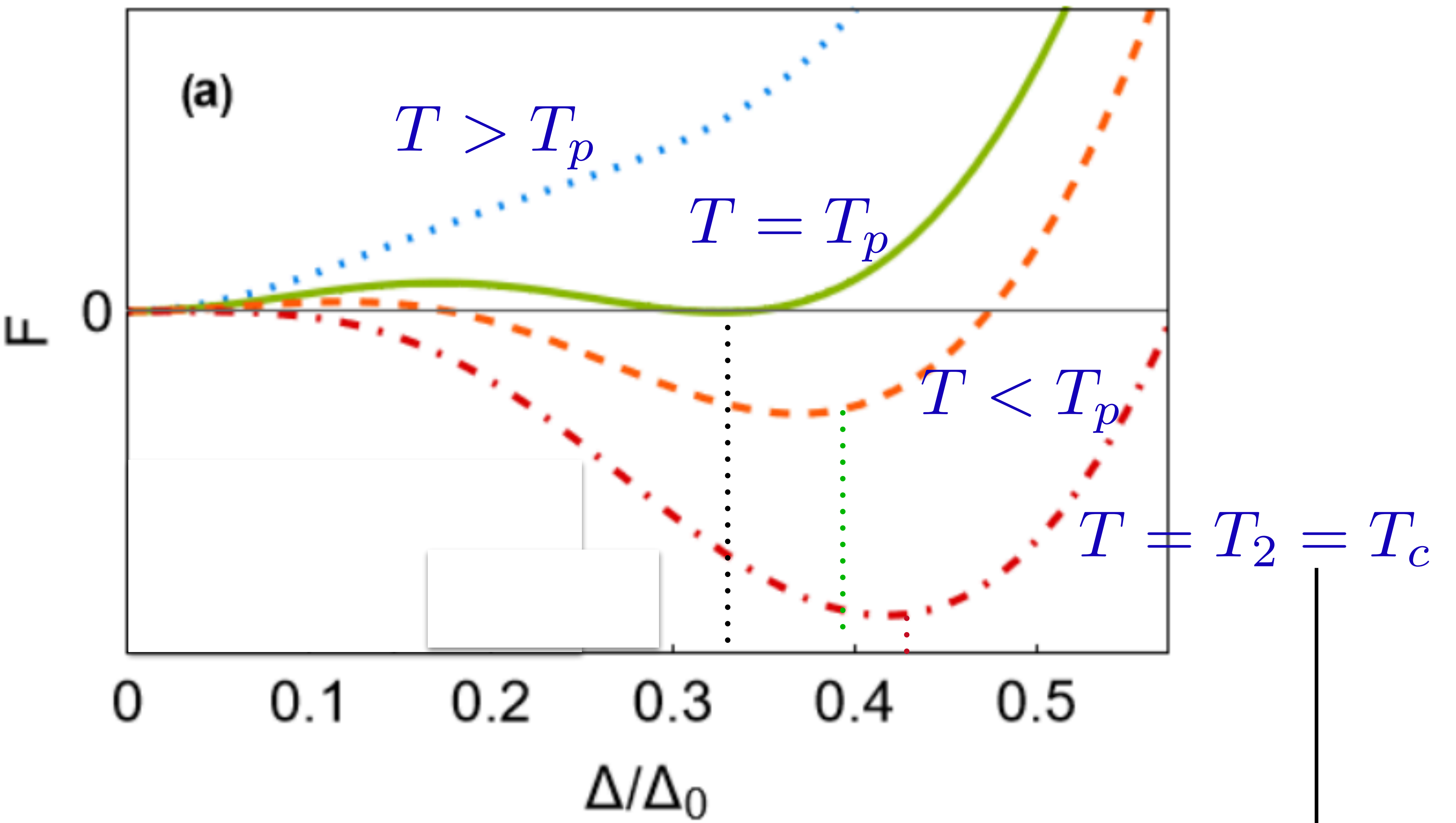
gap/T_c ratio

$$\Delta = (W-U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

$$T_c = (W-U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}}.$$

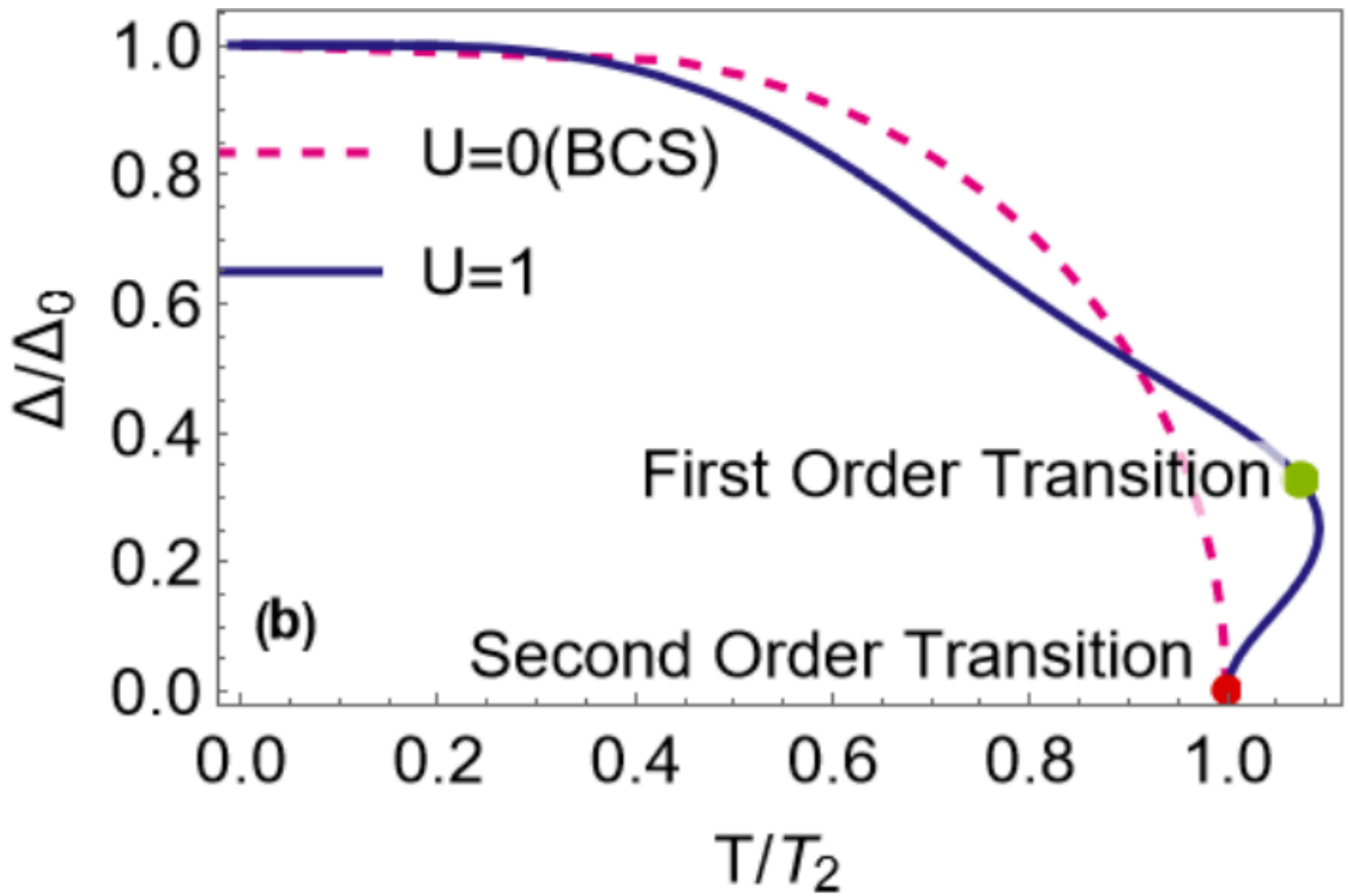
$$\lim_{g\to 0} \frac{\Delta}{T_c} \to \infty$$

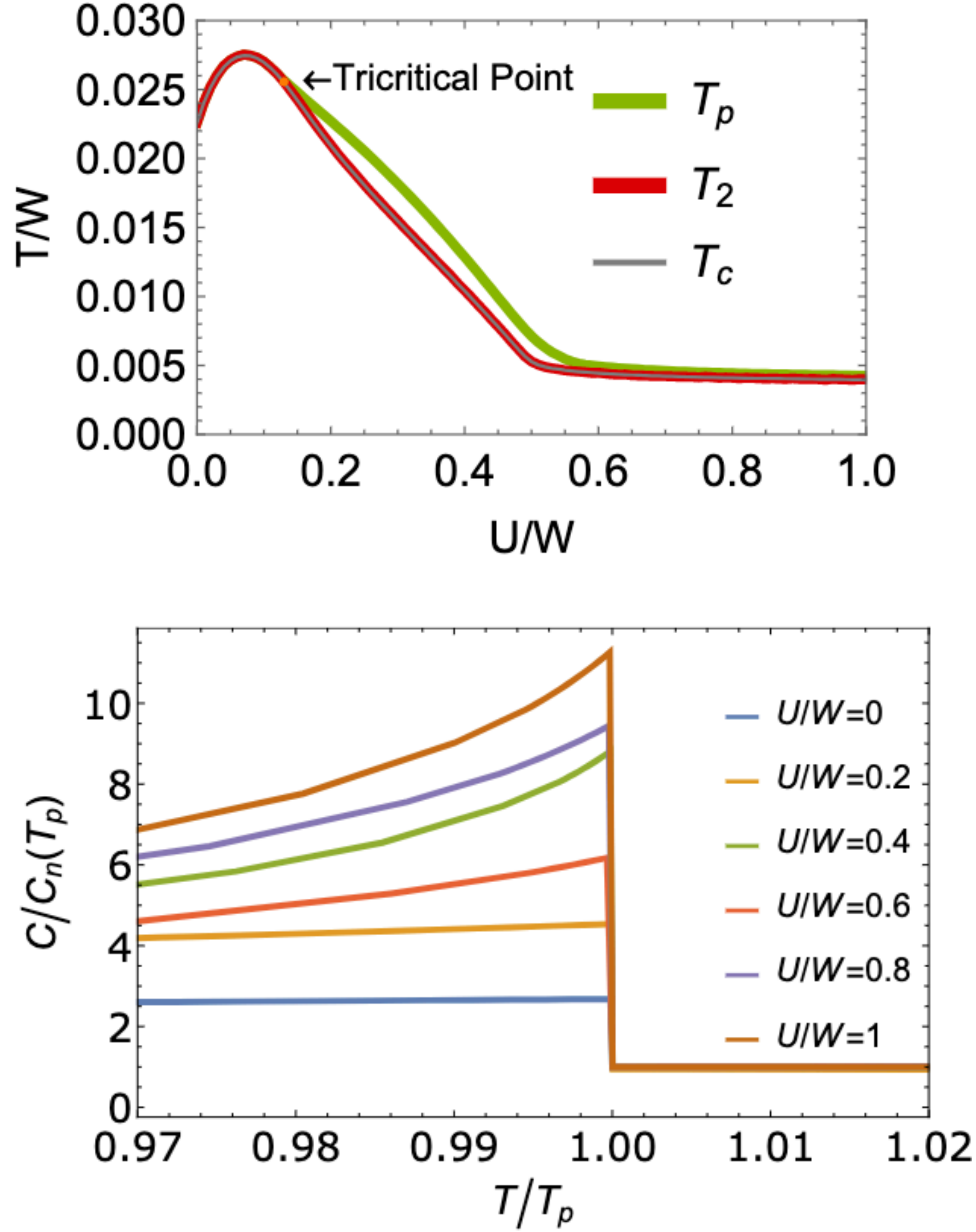
non-BCS superconductivity



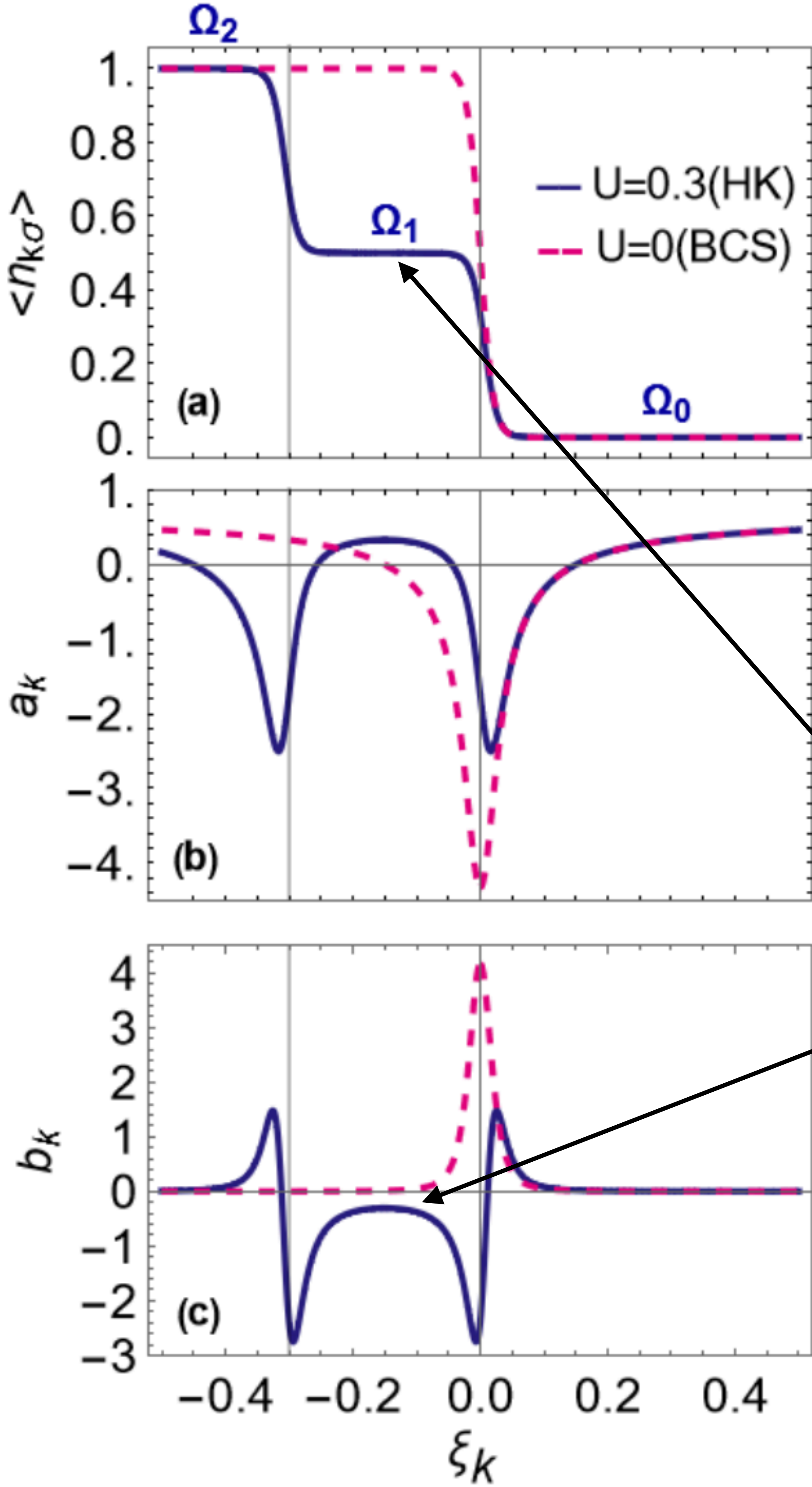
$t_G \approx 10^{-11}$

MF theory
is accurate!


1.0
0.8
0.6
0.4
0.2
0.0
Δ/Δ0
U=0(BCS)
U=1
First Order Transition
(b)
Second Order Transition
0.0
0.2
0.4
0.6
0.8
1.0
T/T2



T/W
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0.0
0.2
0.4
0.6
0.8
1.0
U/W
←Tricritical Point
T_p
T_2
T_c
$C/C_n(T_p)$
0
2
4
6
8
10
0.97
0.98
0.99
1.00
1.01
1.02
T/T_p
U/W=0
U/W=0.2
U/W=0.4
U/W=0.6
U/W=0.8
U/W=1

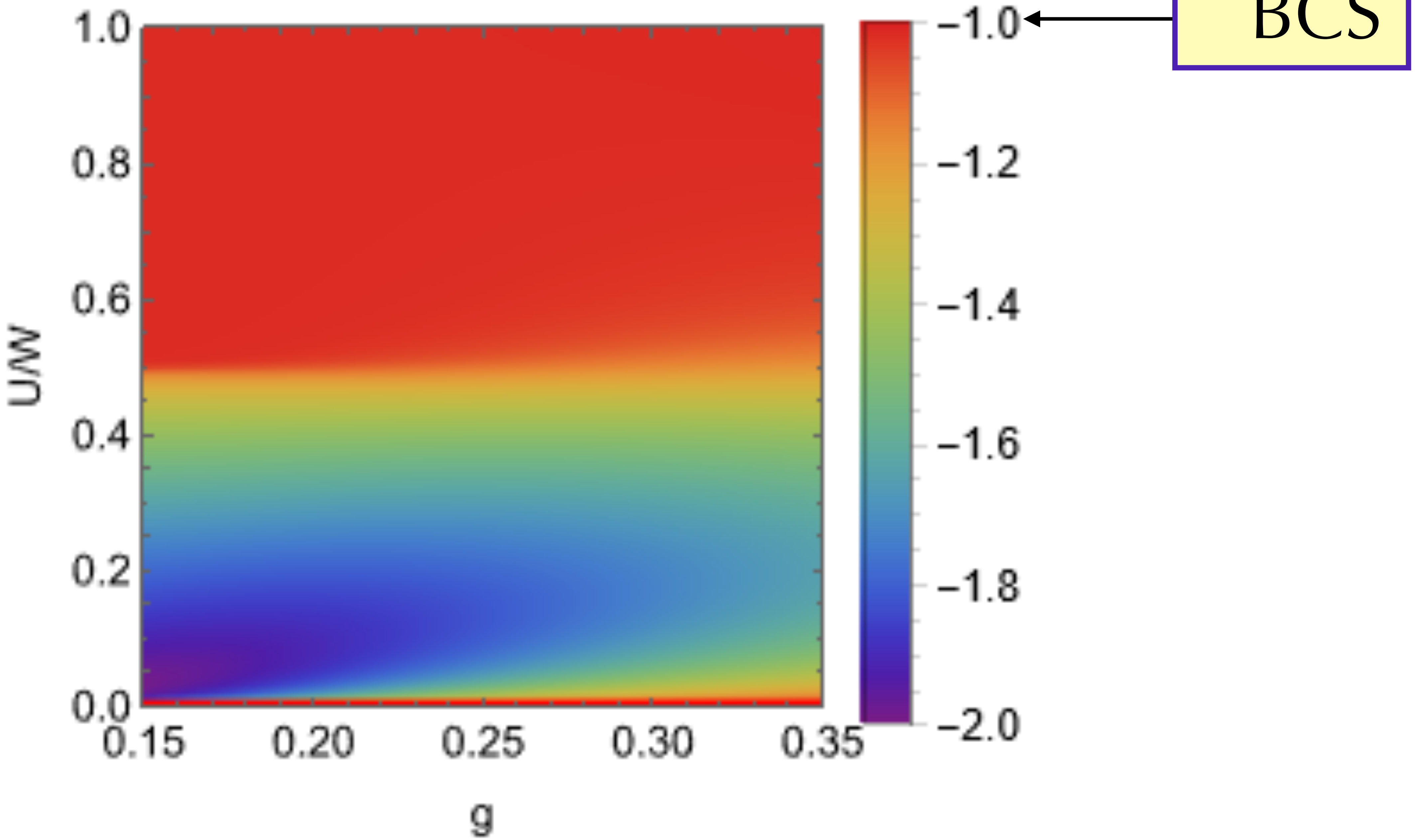


Landau parameters

Mottness

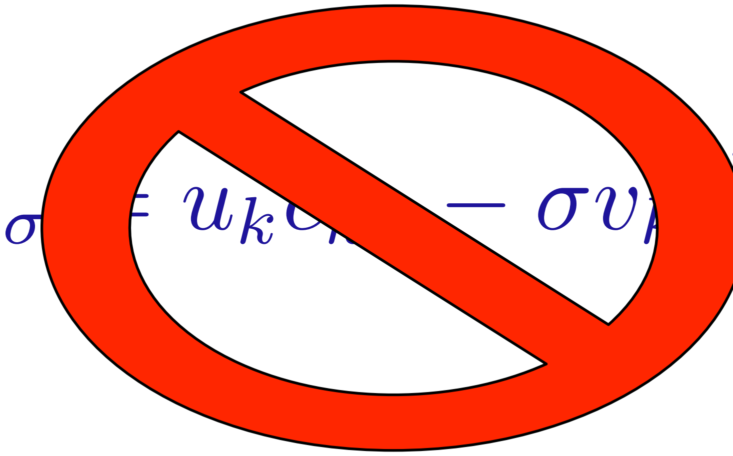
Condensation Energy

$$E_{\rm cond}/N(0)\Delta^2 < 0$$



Bogoliubov excitations

$$\gamma_{k\sigma}|\psi_{\mathrm{BCS}}\rangle = 0$$

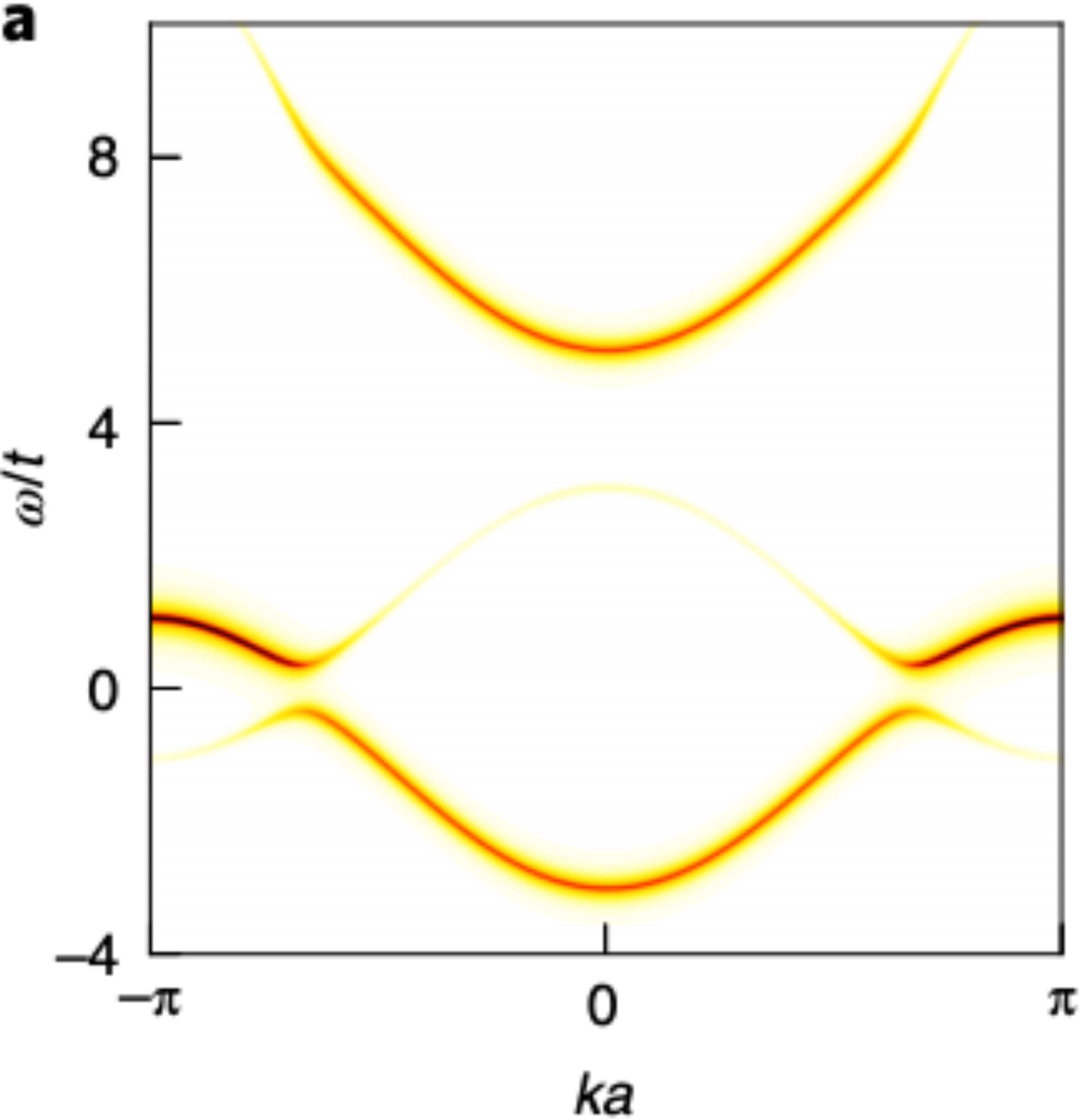


$$\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c^\dagger_{-k\bar{\sigma}}$$

PYHons excitations

$$\gamma^l_{k\sigma} \propto \sqrt{2} x_k \zeta^\dagger_{k\sigma} - \sigma z_k \zeta_{-k\bar{\sigma}}$$

$$\gamma^u_{k\sigma} \propto z_k \eta^\dagger_{k\sigma} - \sigma \sqrt{2} y_k \eta_{-k\bar{\sigma}}$$



PYHon band

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$

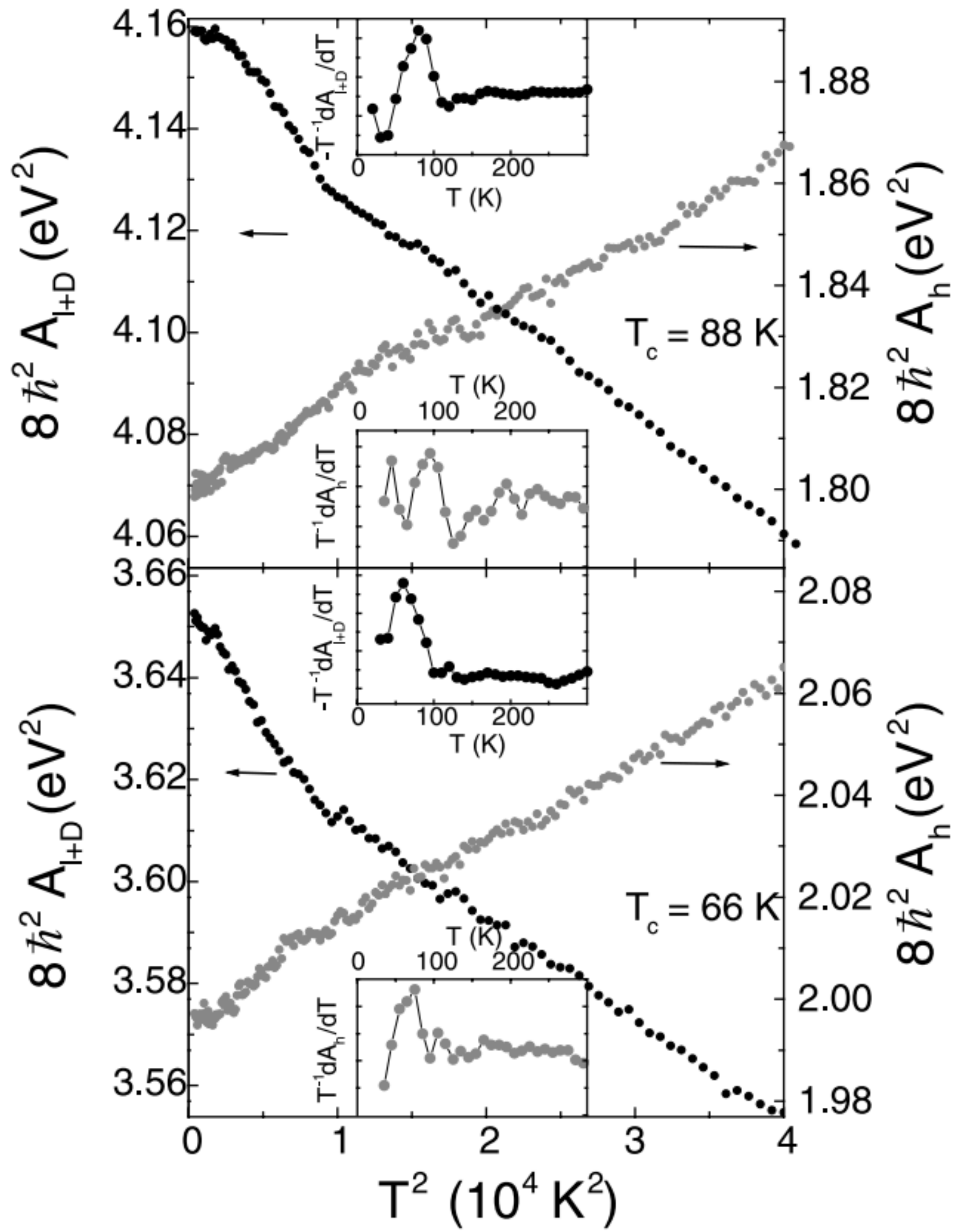
H. J. A. Molegraaf[1], C. Presura[1], D. van der Marel[1,*], P. H. Kes[2], M. Li[2]
+ See all authors and affiliations

Science 22 Mar 2002:
Vol. 295, Issue 5563, pp. 2239-2241
DOI: 10.1126/science.1069947

$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

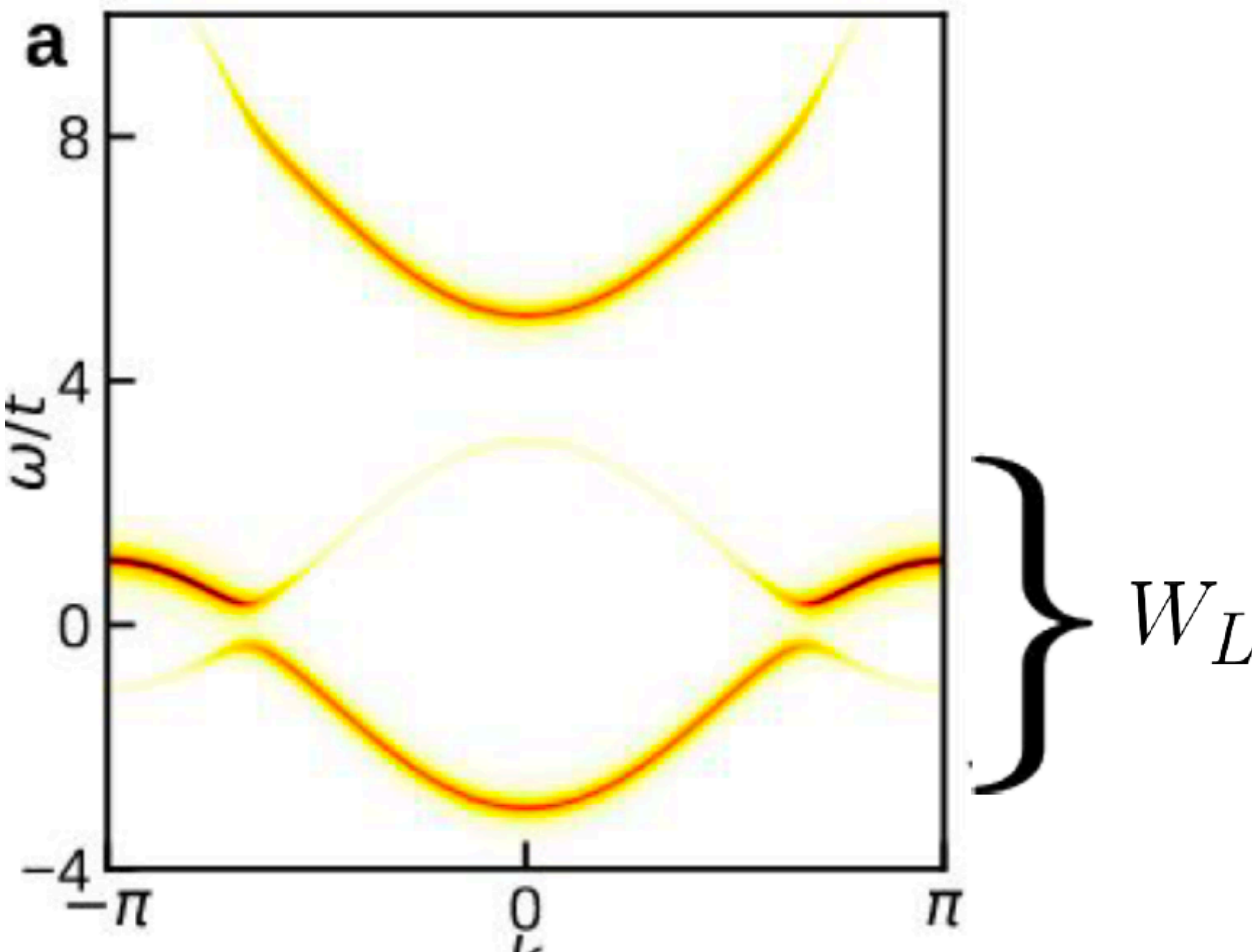
$$\frac{\Delta A_l}{A_l} \propto 3\%$$



condensation energy

Optical data are reported on a spectral weight transfer over a broad frequency range of $Bi_2Sr_2CaCu_2O_{8+\delta}$, when this material became superconducting. Using spectroscopic ellipsometry, we observed the removal of a small amount of spectral weight in a broad frequency band from 10^4 cm^{-1} to at least 2×10^4 cm^{-1}, due to the onset of superconductivity. We observed a blue shift of the *ab*-plane plasma frequency when the material became superconducting, indicating that the spectral weight was transferred to the infrared range. Our observations are in agreement with models in which superconductivity is accompanied by an increased charge carrier spectral weight. The measured spectral weight transfer is large enough to account for the condensation energy in these compounds.

UV-IR mixing


a
8
4
0
−4
ω/t
−π
0
π
k
W_L

why?

$$H = H_{\mathrm{HK}} + H_p$$

$$[H_{\mathrm{HK}}, H_p] \neq 0$$

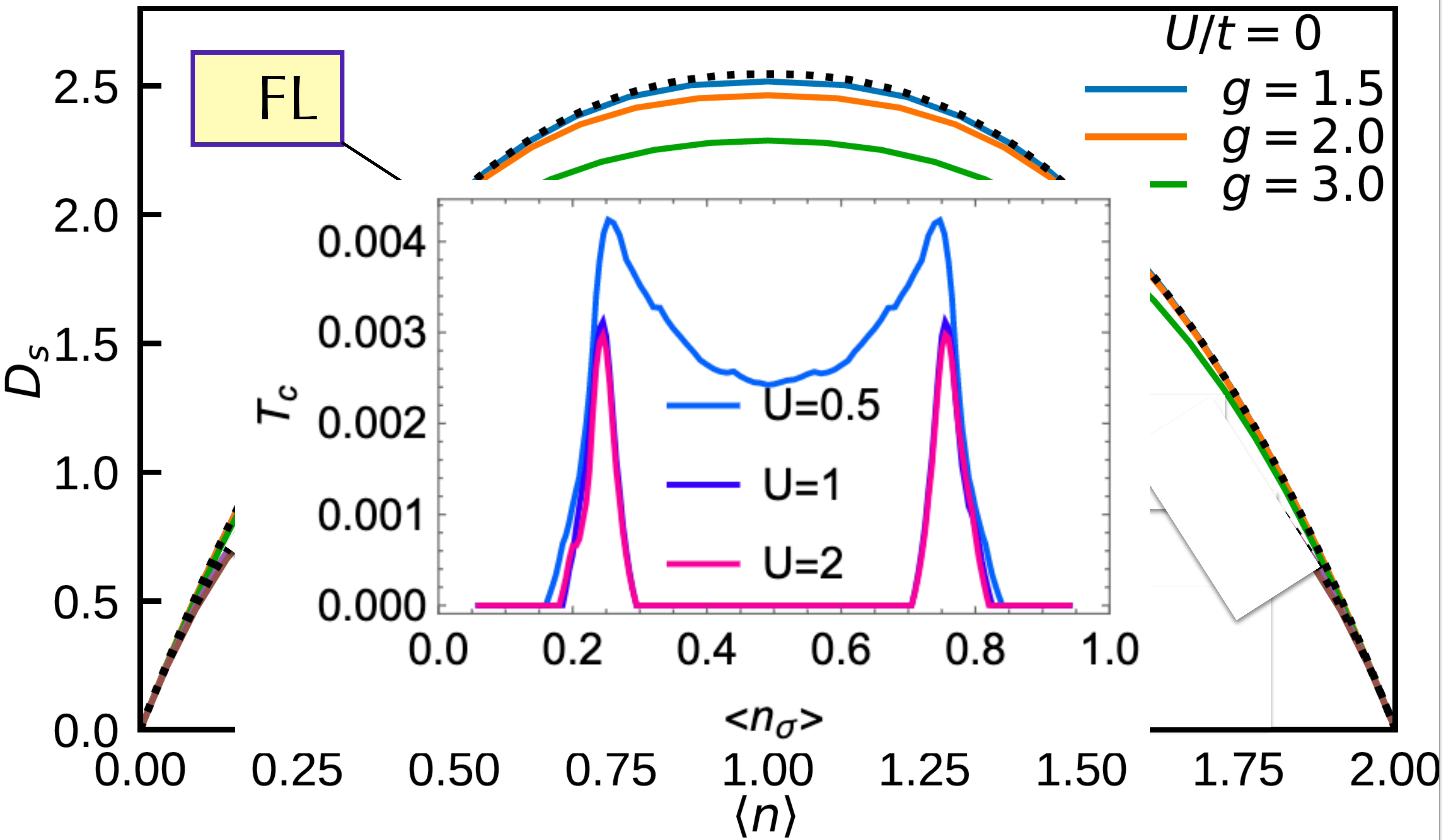
↓

dynamical spectral weight transfer

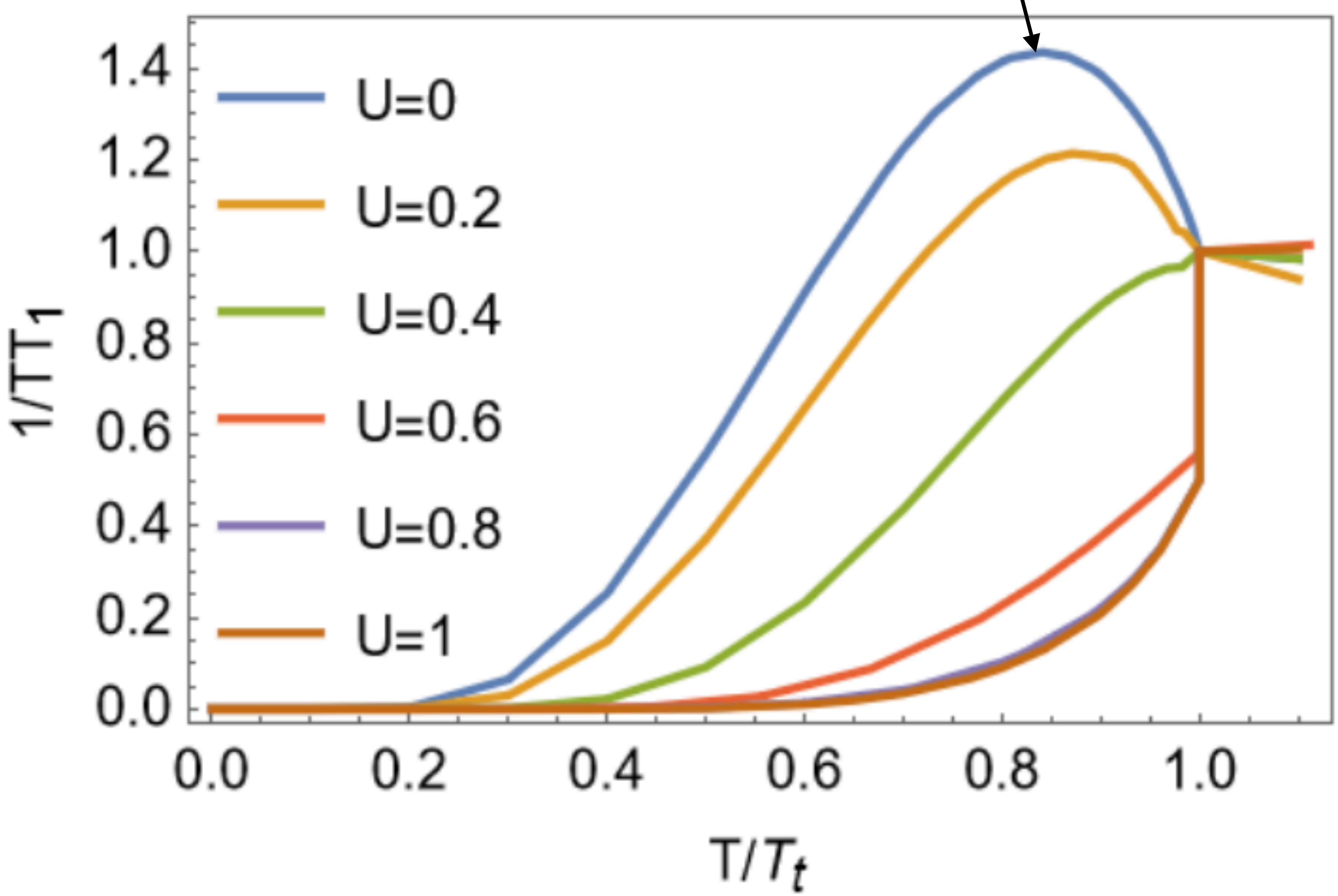
is this the general mechanism of the color change?

Superfluid Density

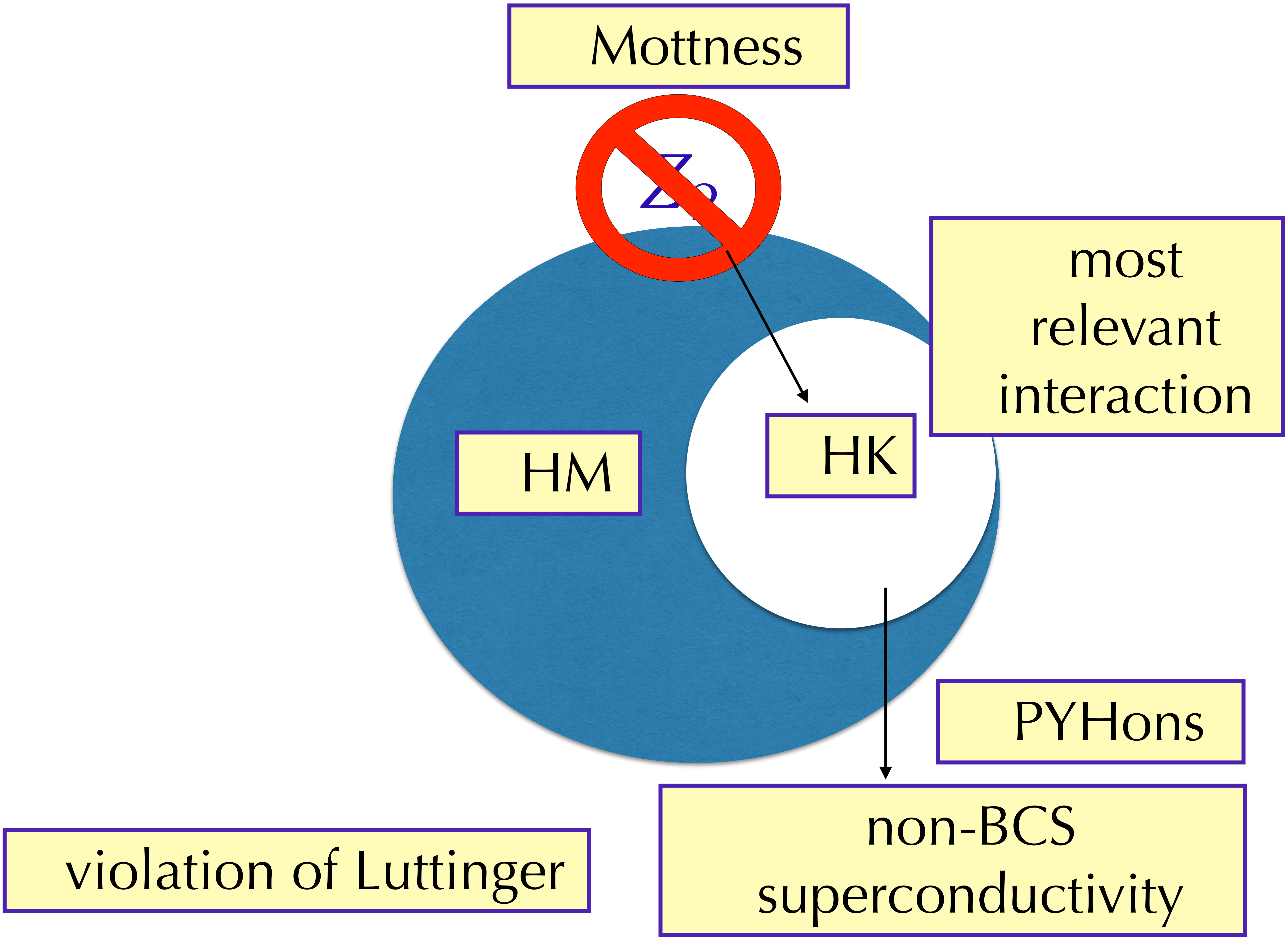
Mottness-induced suppression



Hebel-Slichter peak killed by Mottness



observable	Superconductivity BCS/FL	Mottness PYHZ/HK
$\chi \to \infty$	T_c	$T_c (= T_2)$
$\Delta \neq 0$	T_c	$T_p (> T_2)$
$\lim_{g \to 0} 2\Delta_0 / k_B T_c$	3.52	∞
$\mathrm{quasi-particles}$	Bogoliubons	PYHons
t_G(Ginzburg)	$\approx 10^{-12}$	$\approx 10^{-11}$
$1/\mathrm{TT}_1$	HS peak	no HS peak
Landau Expansion	$a = \alpha t,\ b > 0$	$a = \alpha t,\ b < 0$, $c > 0$
$E_{\mathrm{cond}} / N(0)\Delta^2$	-1	$[-2, -1]$


Mottness
most relevant interaction
HM
HK
PYHons
non-BCS superconductivity
violation of Luttinger