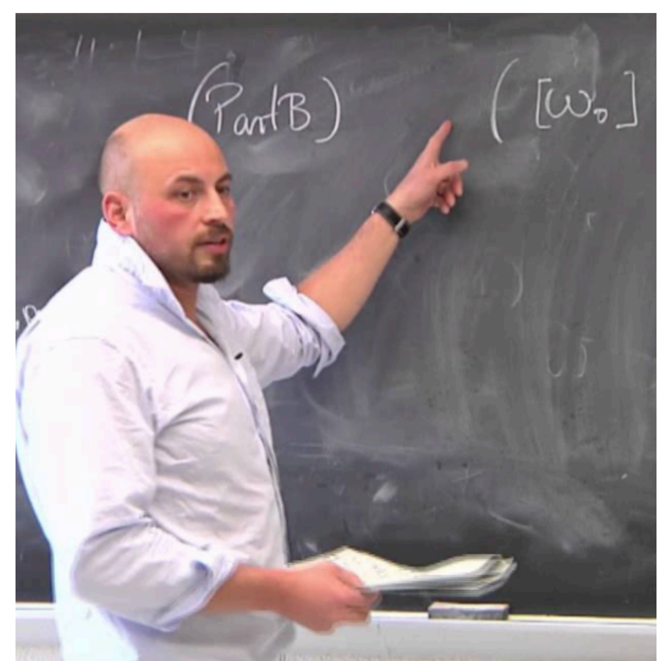
Superconductivity and Mottness: Exact Results

Nature Physics, vol.16, 1175-1180 (2020); vol. 18, 511-516 (2022); PRB, 105, 184509.

Luke Yeo | Edwin Huang | G. La Nave | Jinchao Z.



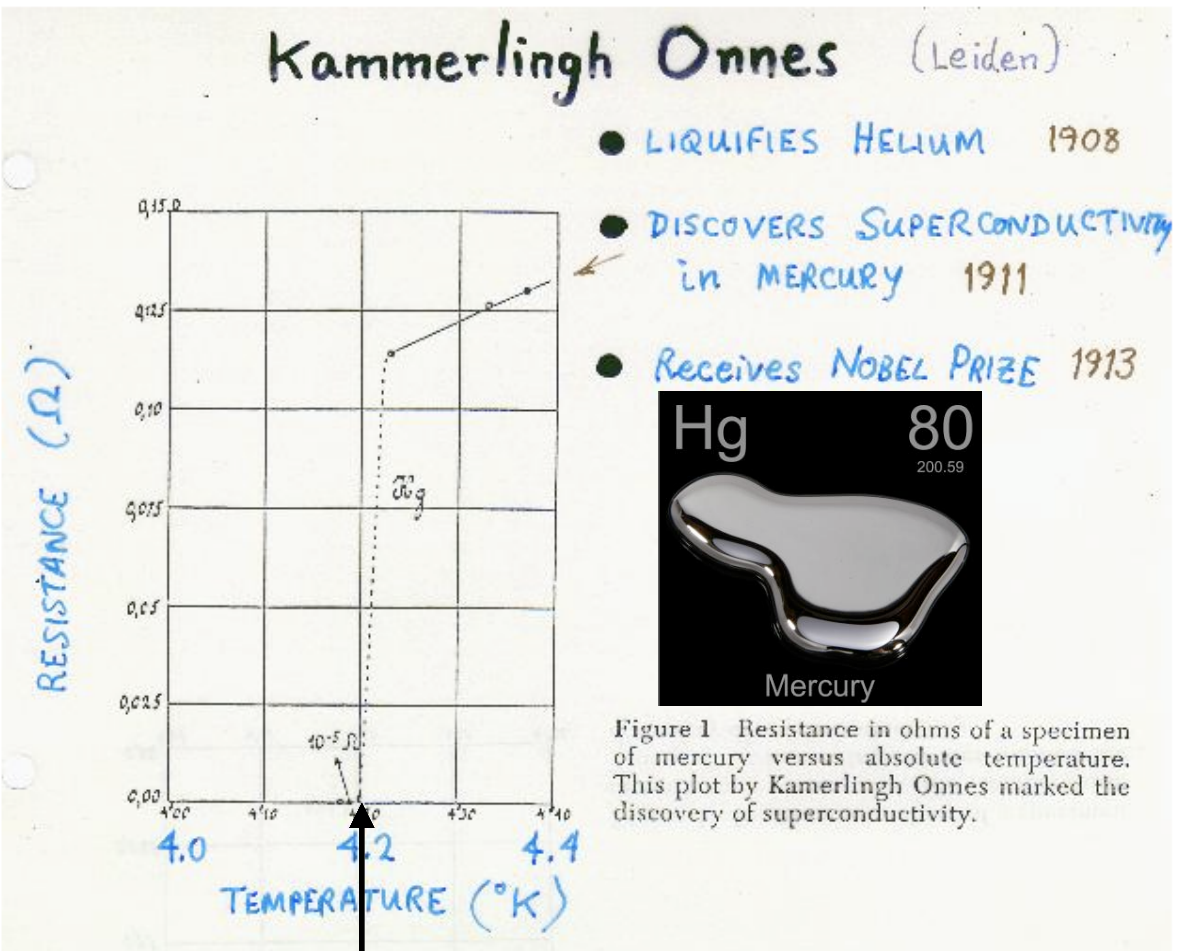
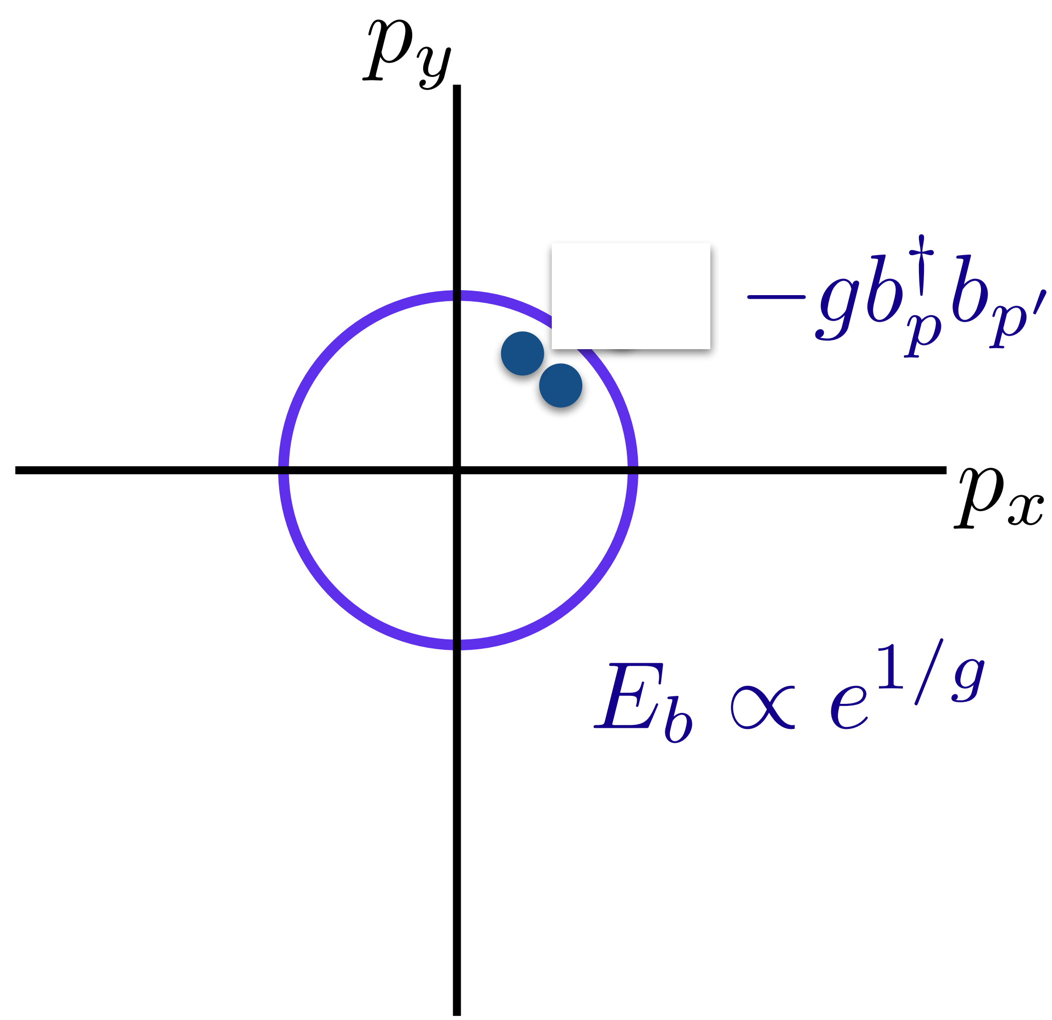


Figure 1 Resistance in ohms of a specimen of mercury versus absolute temperature. This plot by Kamerlingh Onnes marked the discovery of superconductivity.

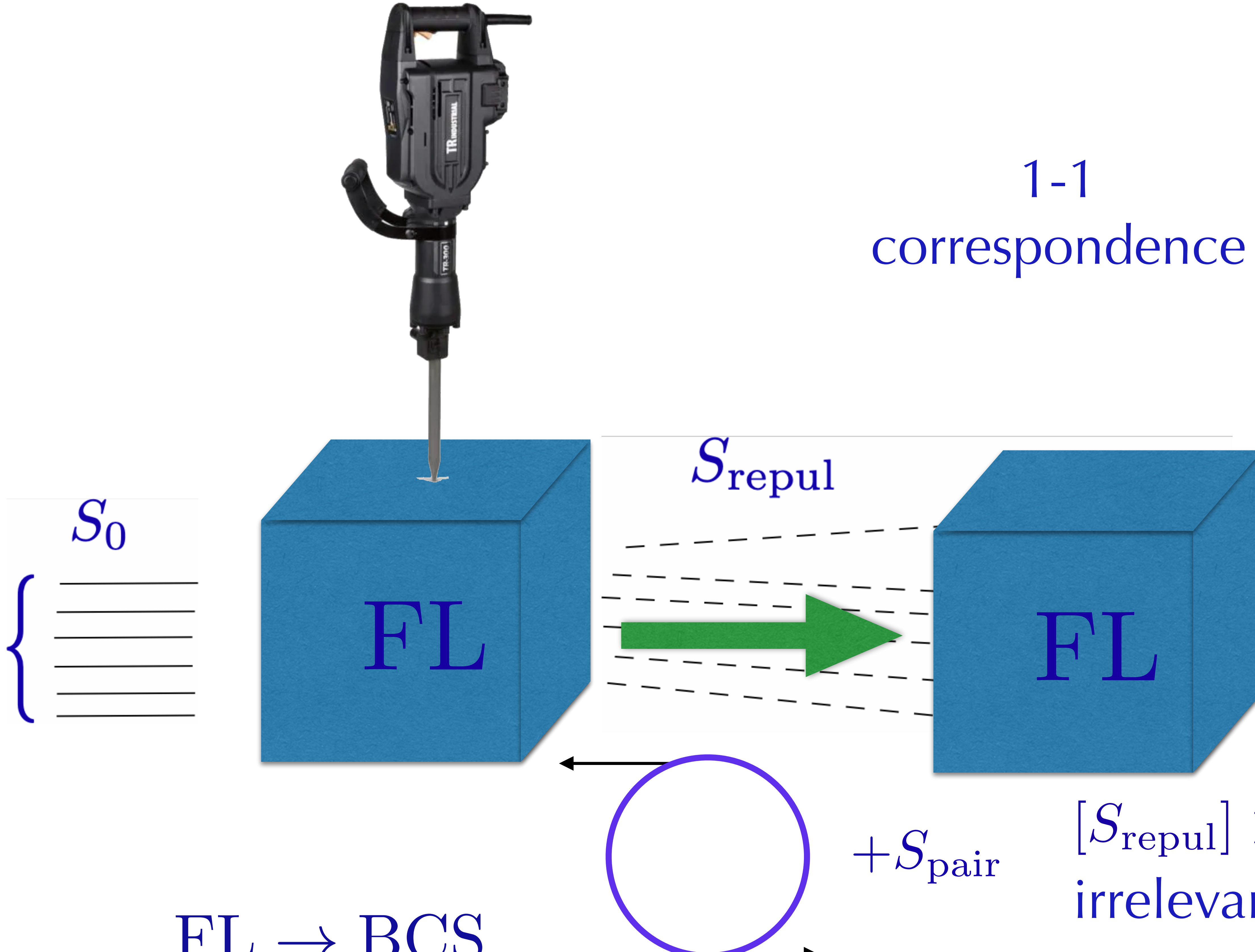
T_c

Cooper instability



$$E_b \propto e^{1/g}$$

$$\langle \psi^\dagger \psi^\dagger \rangle = \Delta e^{i\phi}$$



1-1
correspondence

$+S_{\mathrm{pair}}$

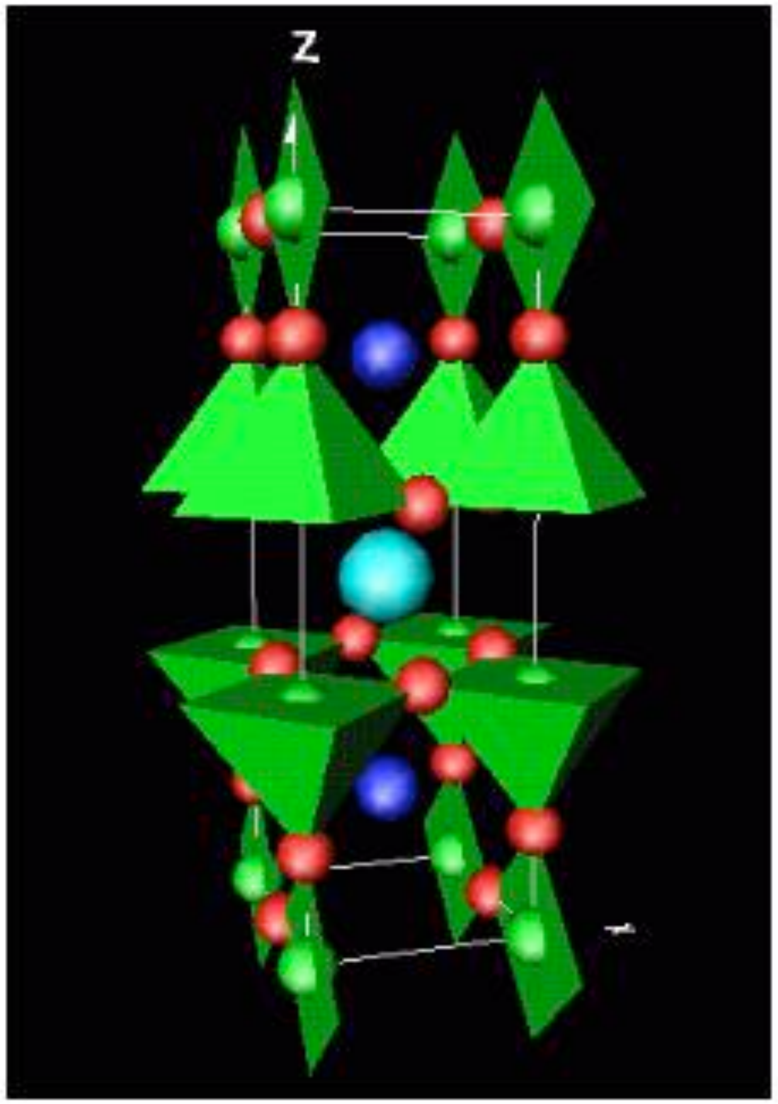
$[S_{\mathrm{repul}}] > 0$
irrelevant

$\mathrm{FL} \rightarrow \mathrm{BCS}$

superconductivity $\frac{2\Delta}{T_c} = 3.5$ $s^{\alpha} \propto \left(\frac{1}{N}\right)^{\alpha>0}$

fixed
point beyond
FL?

quartic
interacting
theory?



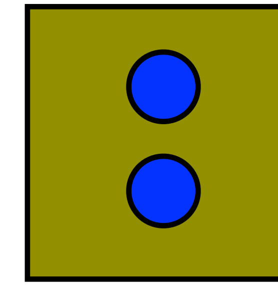
$YBa_2Cu_3O_7$

Cuprate Superconductors

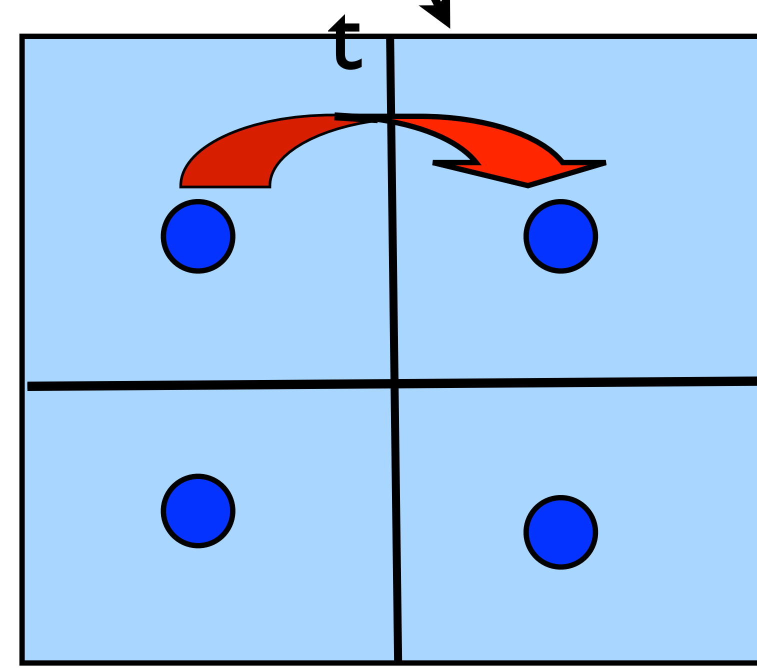
NiO insulates

d^8 ?

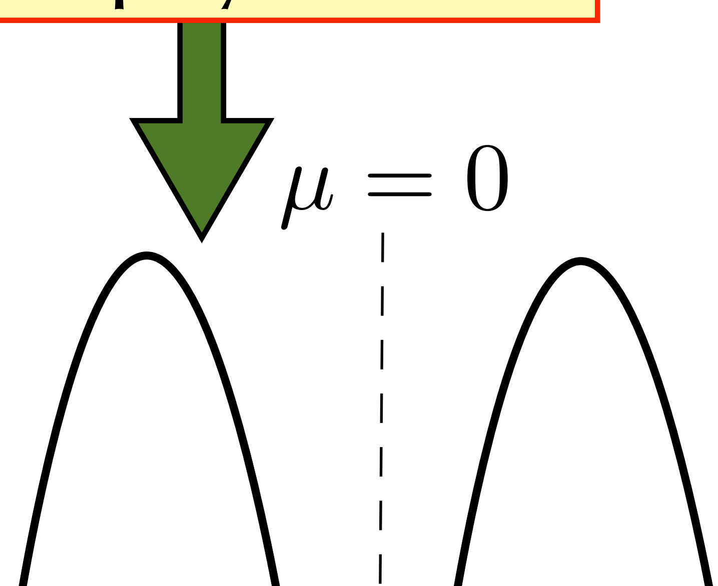
perhaps this costs energy



$U \gg t$



local real-space physics



no change in size of Brillouin zone

solve the Hubbard Model!!

Cooper instability??

Progress thus far?

DMFT

QMC

disputes

Sept. 1997

A Critique of Two Metals

R. B. Laughlin
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Stanford, California 94305

idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.

Nov. 1997

A Critique of "A Critique of Two Metals"

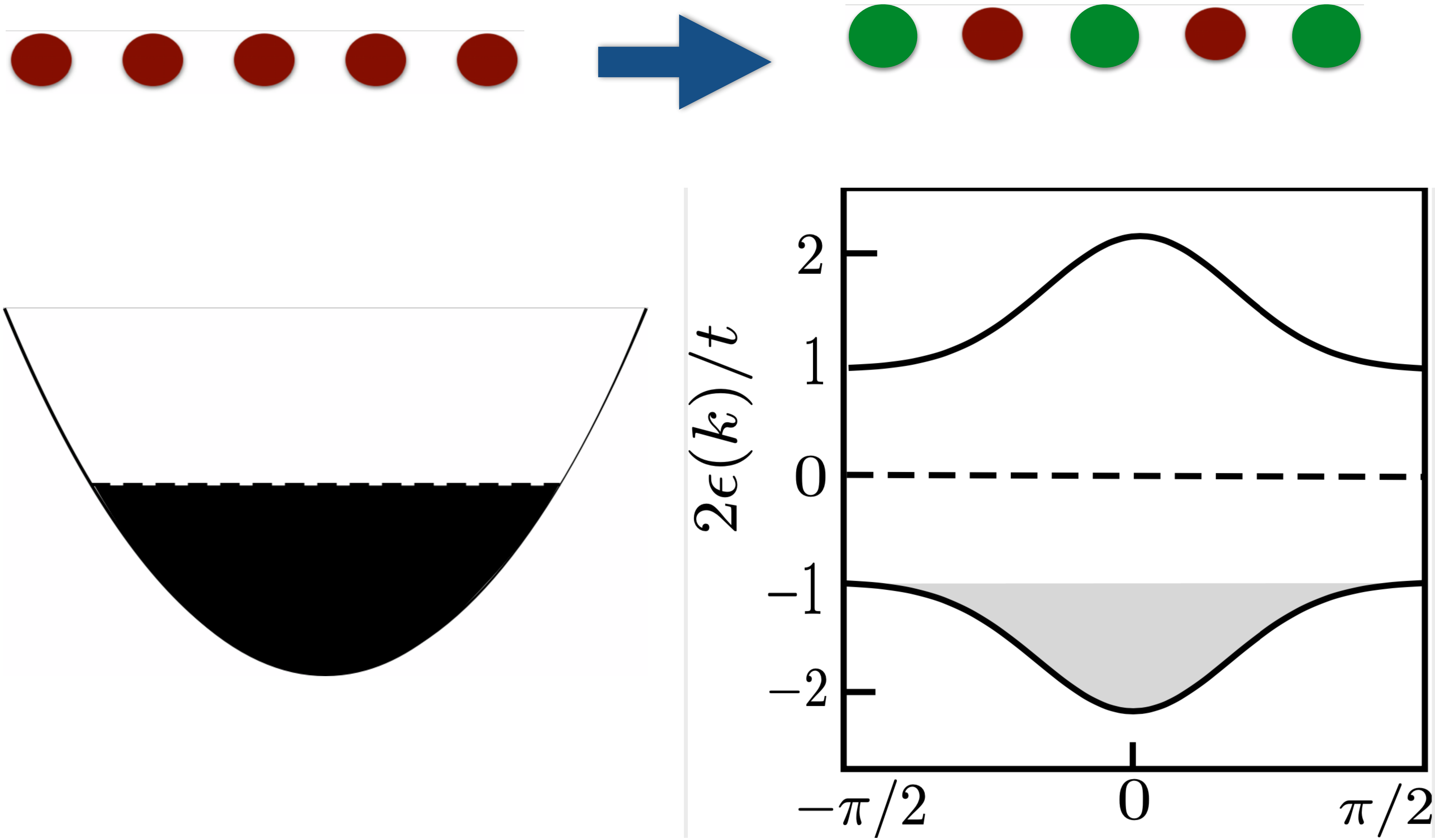
Philip W. Anderson and G. Baskaran

Joseph Henry Laboratories of Physics

Princeton University, Princeton, NJ 08544

The fundamental argument is presented in the second paragraph: "Ten years of work by some of the best minds in theoretical physics have failed to produce any formal demonstration"...of the Mott insulating state. The statement would be ludicrous if it were not so influential. The proviso "at zero temperature" is added, because of course most Mott

concern. It is the tragedy of Mott that although he almost certainly won his Nobel prize for the Mott insulator, Slater, who couldn't think clearly about finite temperature, won the publicity battle.



No Mott Problem

$-$

$=$ Mottness

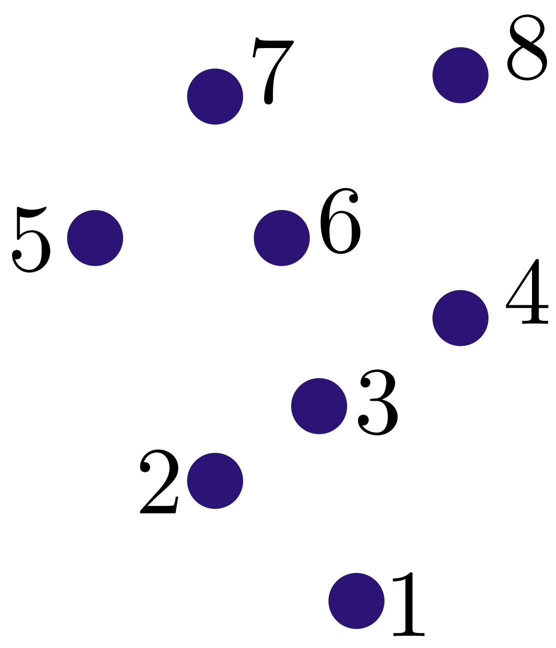
zeros

$= 0$

$\neq 0$

$\mathrm{DetReG}(\omega = 0, \mathrm{p})$

counting particles



is there a more efficient way?

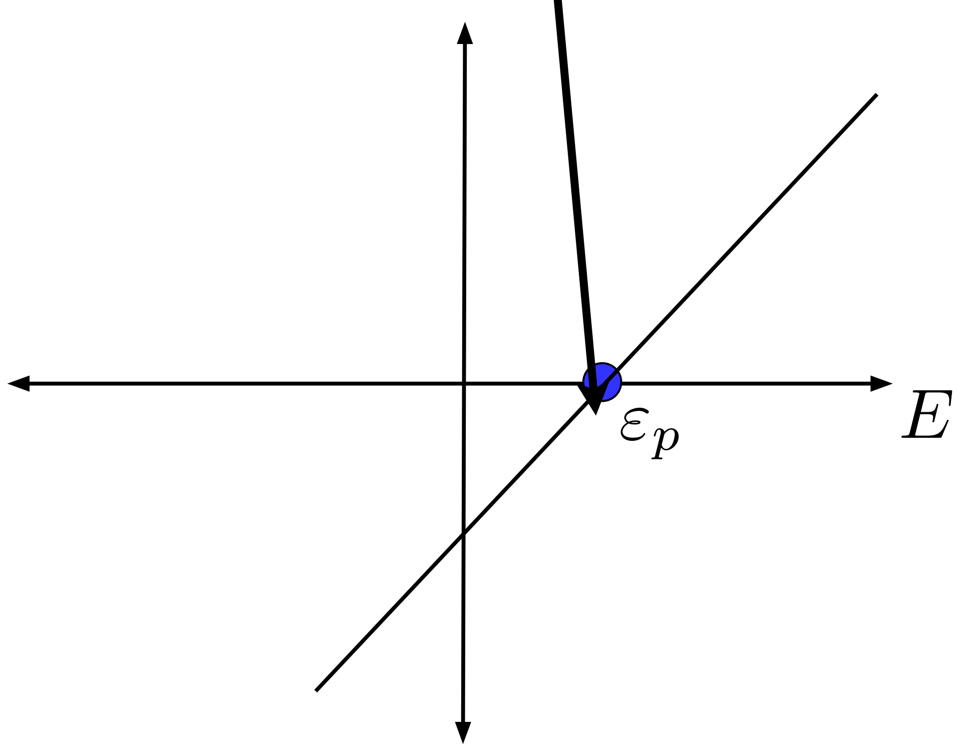
Luttinger counting theorem

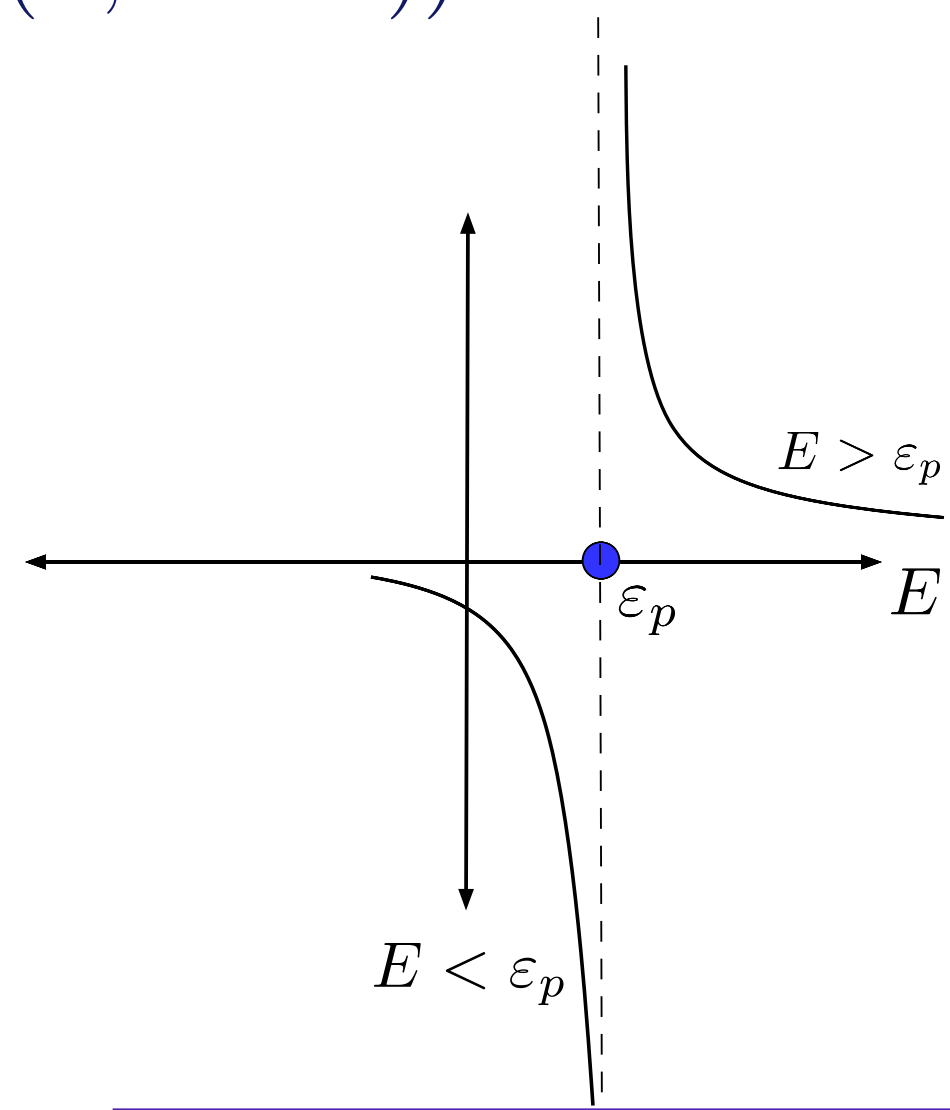
$$G(E) = \frac{1}{E - \varepsilon_p}$$

$$n = 2\sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

zero-crossing

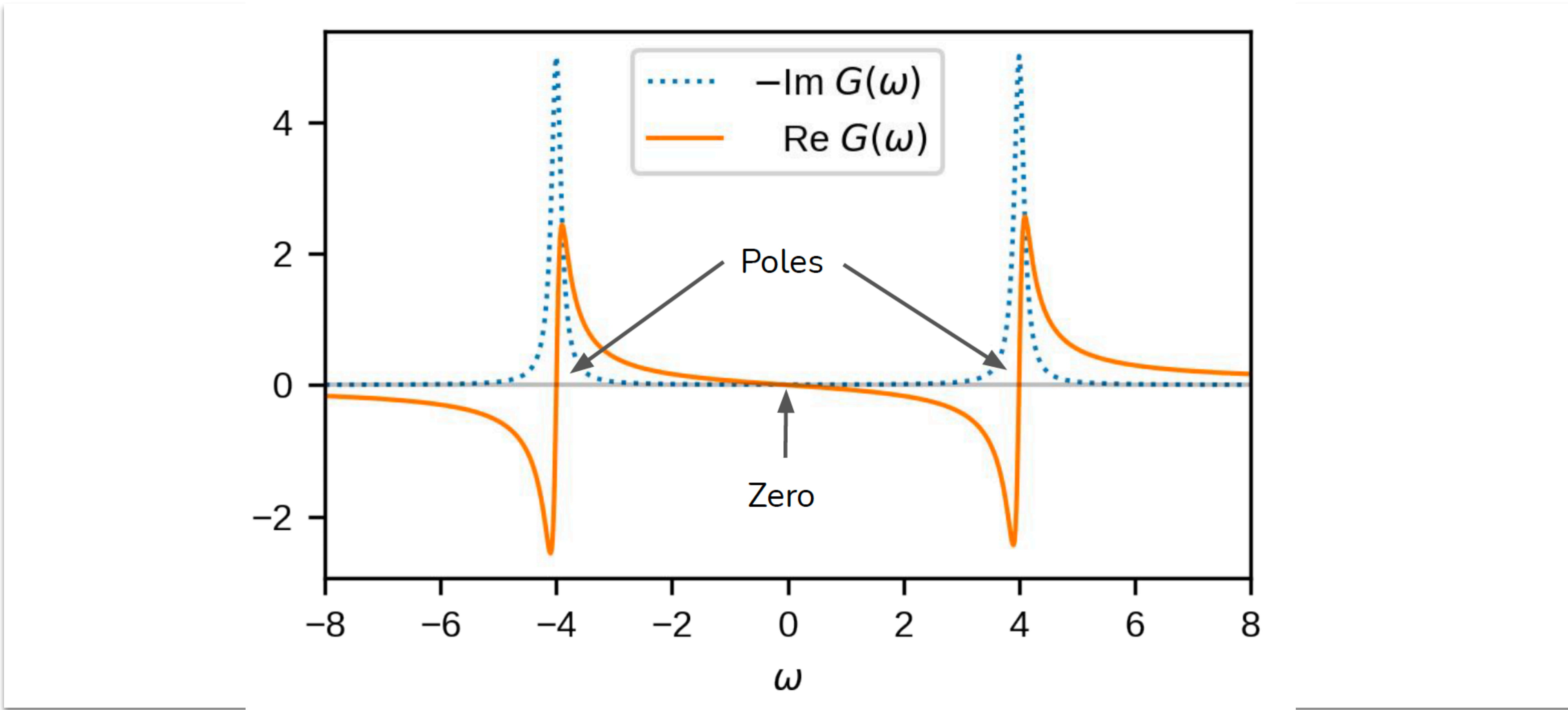
$$\mathrm{Det}G(\omega = 0, \vec{p}) = 0$$





counting poles (qp)

How do zeros obtain?



= below gap+above gap = 0

$\mathrm{DetReG}(k, \omega = 0) = 0$ (single band)

strongly correlated gapped systems

zeros

no propagation

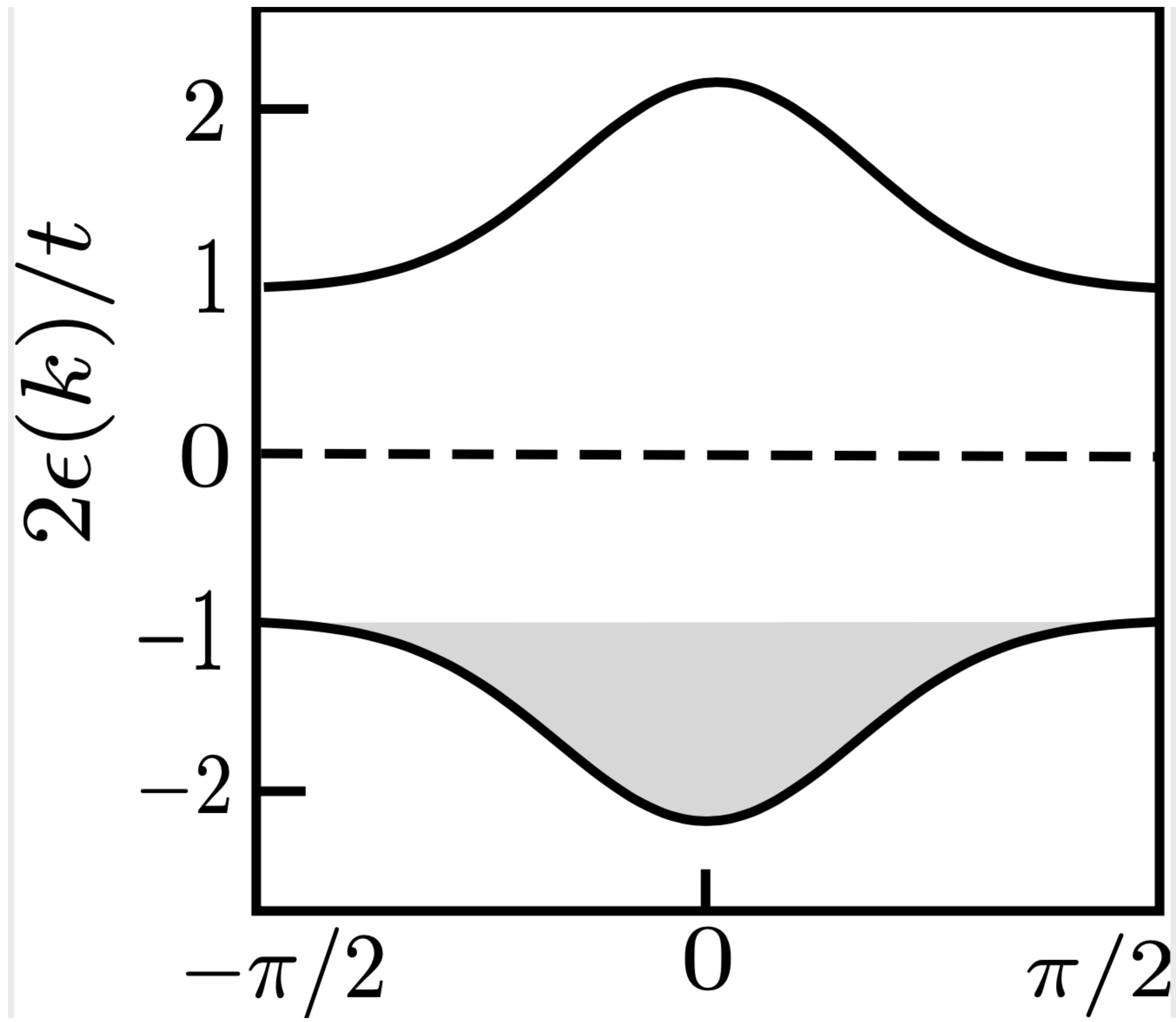
breakdown of particle concept

Mottness

n=zeros+poles?

Symmetry Breaking

$$G_k(\omega) = \begin{pmatrix} \frac{1}{\omega - E_k^+} & 0 \\ 0 & \frac{1}{\omega - E_k^-} \end{pmatrix}$$

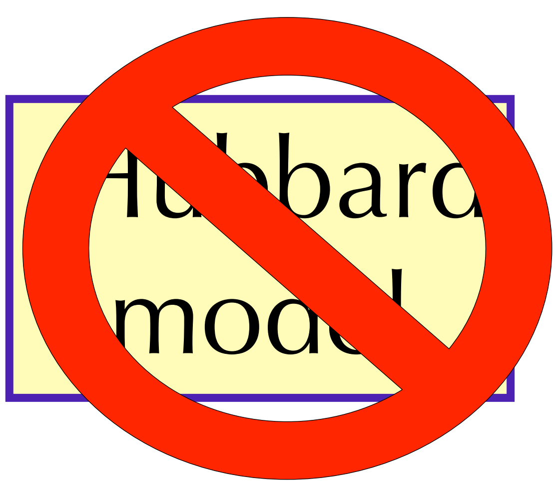


$$\mathrm{Det}G \neq 0$$

no Mottness

Laughlin

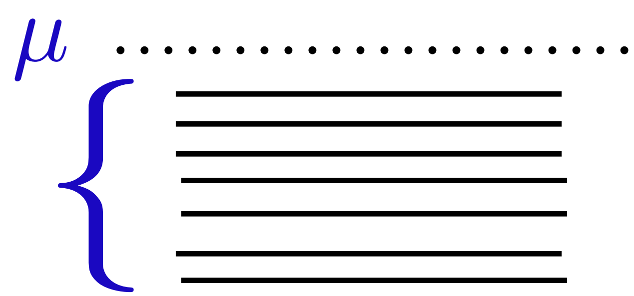
Minimal model for Mottness?



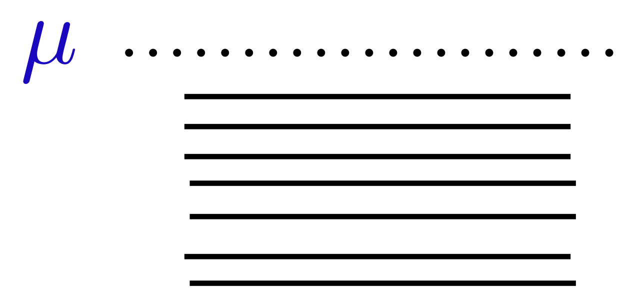
Fermi liquids

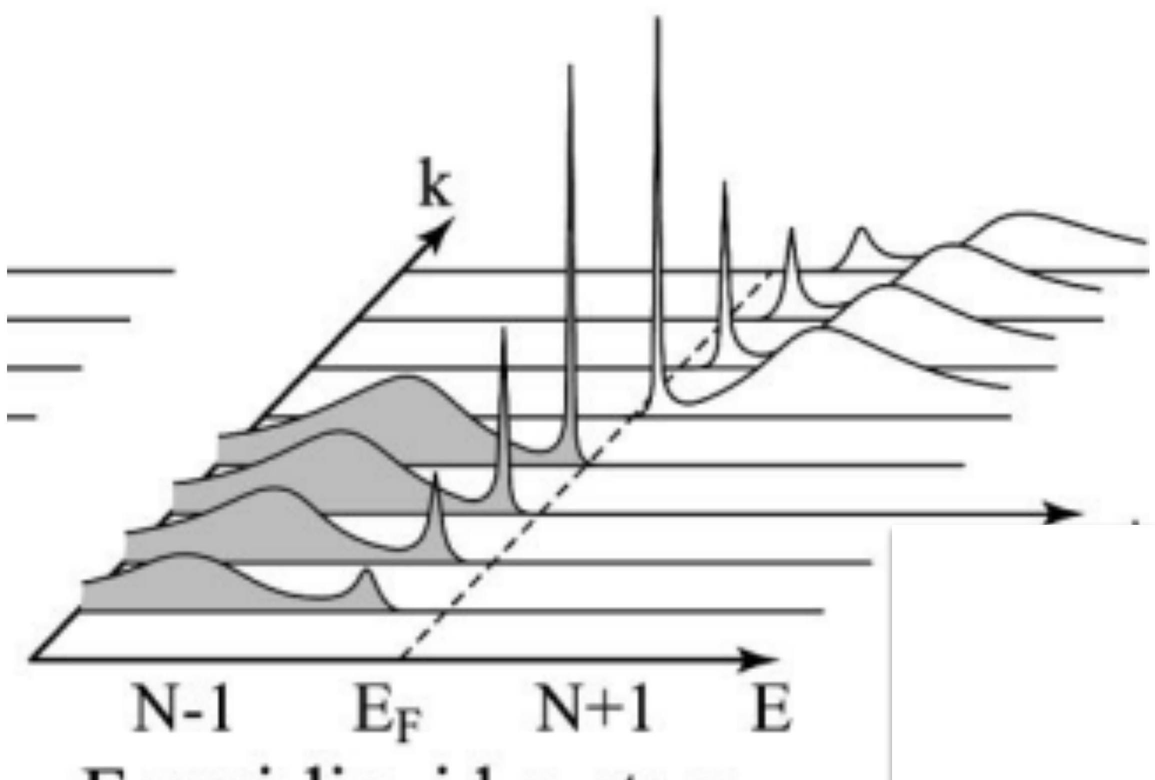
NFL

doubly occupied



Is single occupancy below chemical potential possible?





Fermi-liquid system

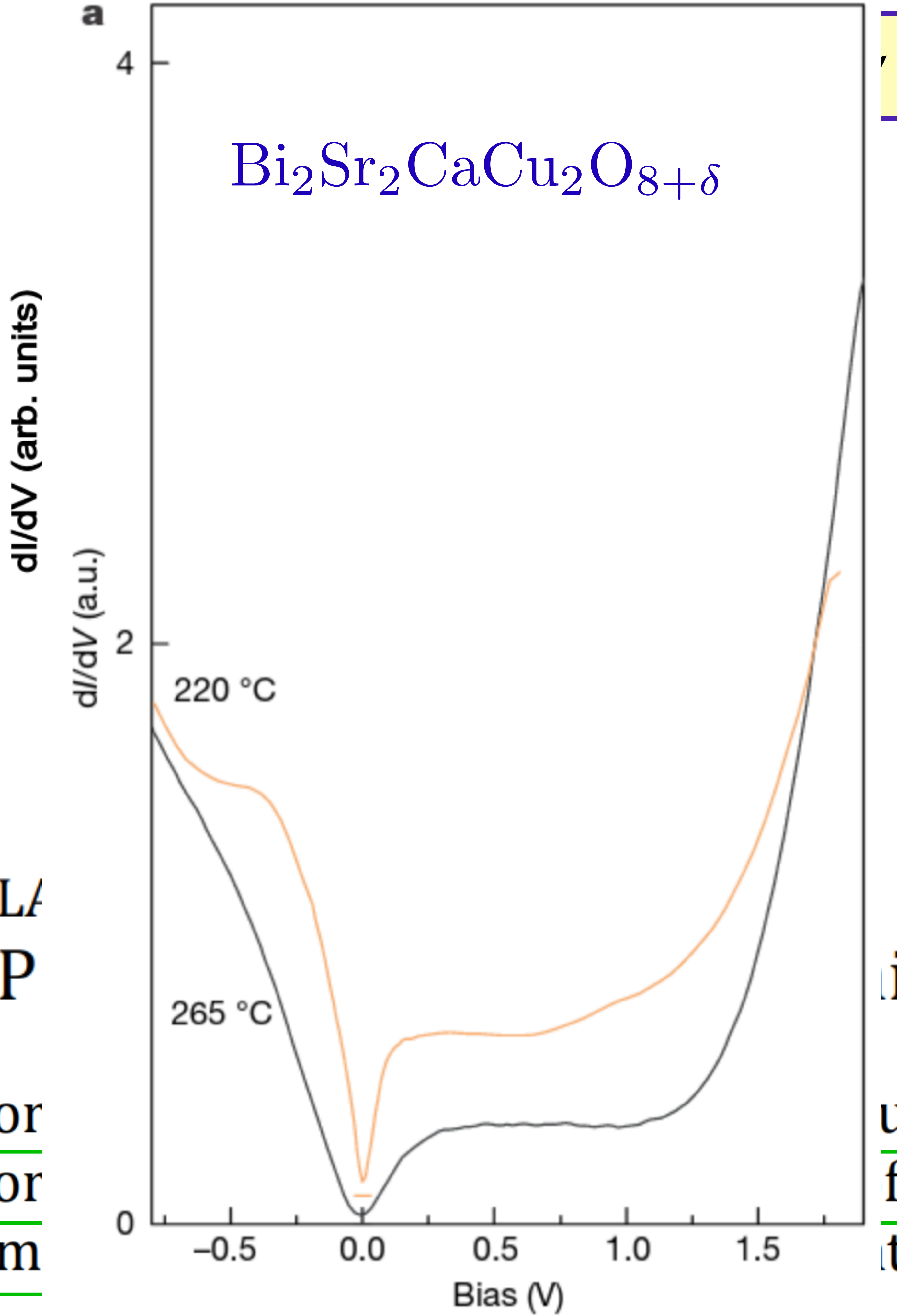
with time-reversal symmetry in tact?

p-h symmetry

LA

P iversity

theor universal refusal of
theor f hole-particle
asym t the first step of any



single occupancy

?

particle-hole asymmetry

Anderson
Haldane
2000

3 citations

Fermi liquids

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + \cdots \quad (\cdots \to 0)$$

$(n_{p\uparrow}, n_{p\downarrow})$ conserved currents

$(c_{p\uparrow}, c_{p\downarrow}, \mathrm{h.c.})$ 4 objects

$O(4)$

$\mathrm{Det M} = 1$ → SO(4) proper rotations

$\mathrm{Det M} = -1$ → improper rotations

$$\mathrm{Det}\, M = \pm 1 \implies Z_2 = O(4) \div SO(4)$$

Improper Rotations

Majorana basis

$$\begin{pmatrix} c_{p\uparrow} \\ c^\dagger_{p\uparrow} \\ c_{p\downarrow} \\ c^\dagger_{p\downarrow} \end{pmatrix} \longrightarrow \begin{pmatrix} c_{p\uparrow} + c^\dagger_{p\uparrow} \\ i(c_{p\uparrow} - c^\dagger_{p\uparrow}) \\ c_{p\downarrow} + c^\dagger_{p\downarrow} \\ i(c_{p\downarrow} - c^\dagger_{p\downarrow}) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} c_{p\uparrow} + c^\dagger_{p\uparrow} \\ i(c_{p\uparrow} - c^\dagger_{p\uparrow}) \\ c_{p\downarrow} + c^\dagger_{p\downarrow} \\ i(c_{p\downarrow} - c^\dagger_{p\downarrow}) \end{pmatrix} \Rightarrow c_{p\downarrow} \to c^\dagger_{p\downarrow}$$

p-h transformation

$$\epsilon(p) = \epsilon_F$$

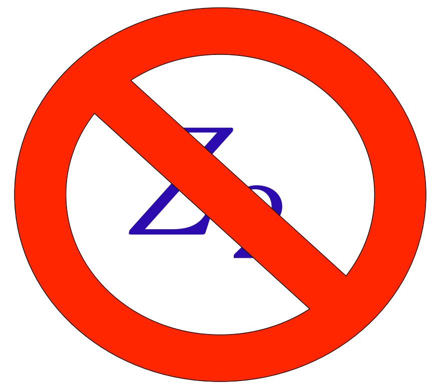
Fermi Surface

$$H = 0$$

$$\left.\begin{aligned} n_{p\uparrow} &\to 1 - n_{p\uparrow} \\ n_{p\downarrow} &\to n_{p\downarrow} \end{aligned}\right\} Z_2$$

at Fermi surface only

How to destroy Fermi liquids?



$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + U n_{p\uparrow} n_{p\downarrow}$$

odd under Z_2

scaling dimension

$[n_{p\uparrow} n_{p\downarrow}] = -2$

relevant interaction

New fixed point!

Hatsugai-Kohmoto or Baskaran model

Hubbard not necessary!

General HK Model

$$\sum_k (\xi_k (n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow})$$

Solvable Mott transition:U>W

$$G_{k\sigma}(i\omega_n \to z) = \frac{1 - \langle n_{k\bar{\sigma}} \rangle}{z - \xi_k} + \frac{\langle n_{k\bar{\sigma}} \rangle}{z - (\xi_k + U)} \neq \frac{1}{z - \omega_k}$$

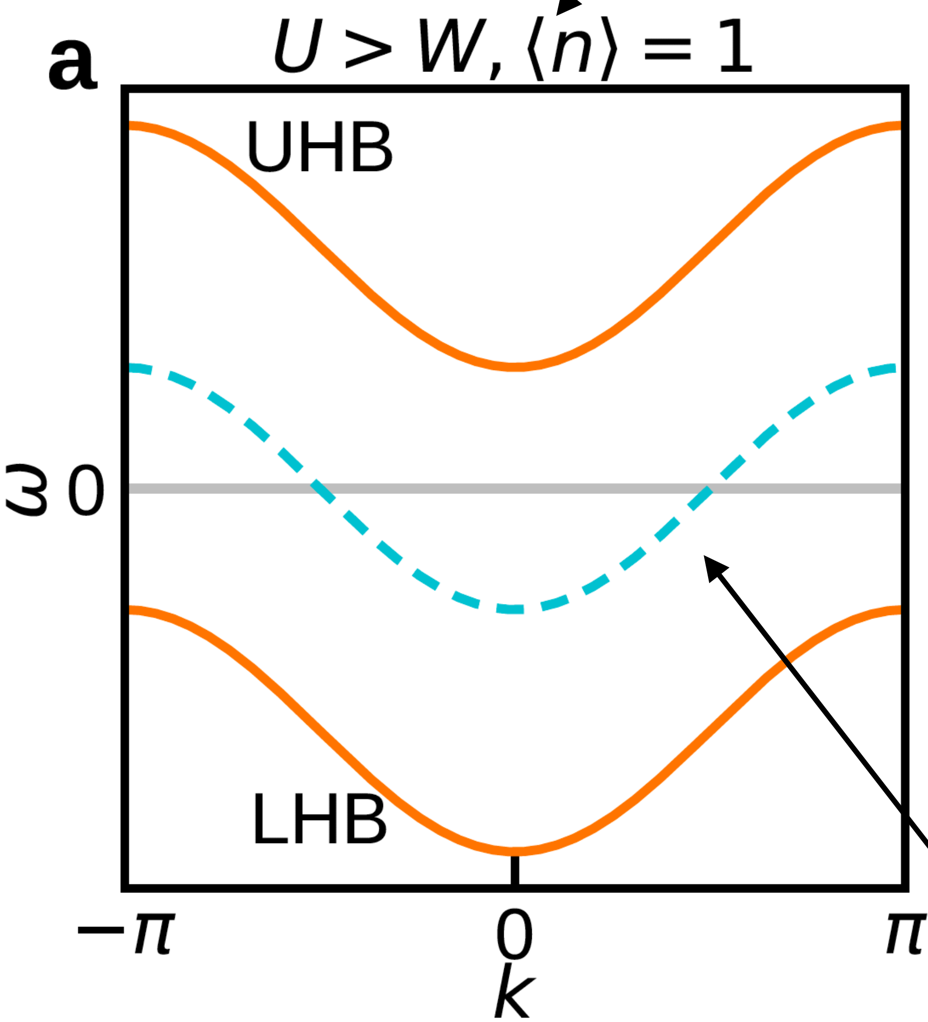
lower Hubbard band

upper Hubbard band

zeros

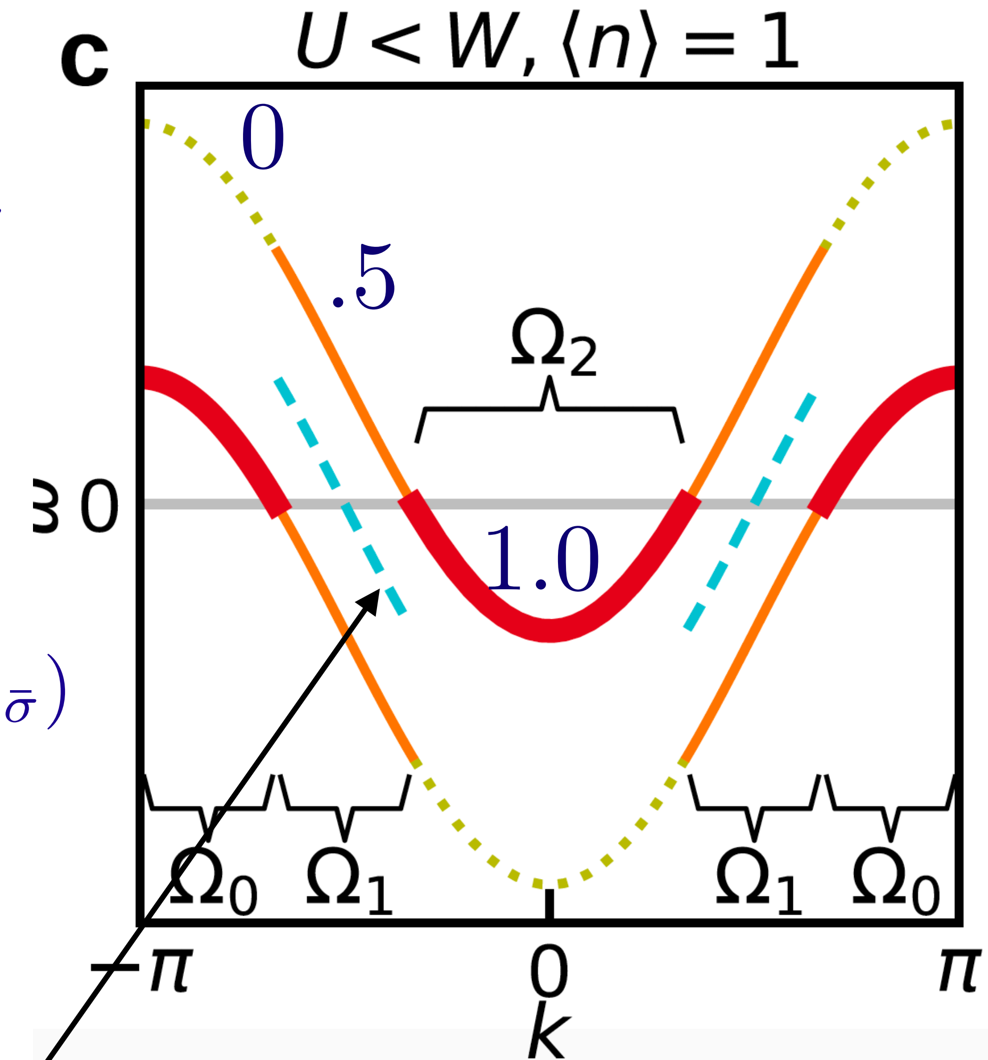
Mott transition: composite excitations

$$\Delta E = U - 4dt = U - W$$



$$\eta^\dagger_{k\sigma} = c^\dagger_{k\sigma} n_{k\bar{\sigma}}$$

$$\zeta^\dagger_{k\sigma} = c^\dagger_{k\sigma}(1 - n_{k\bar{\sigma}})$$



insulator

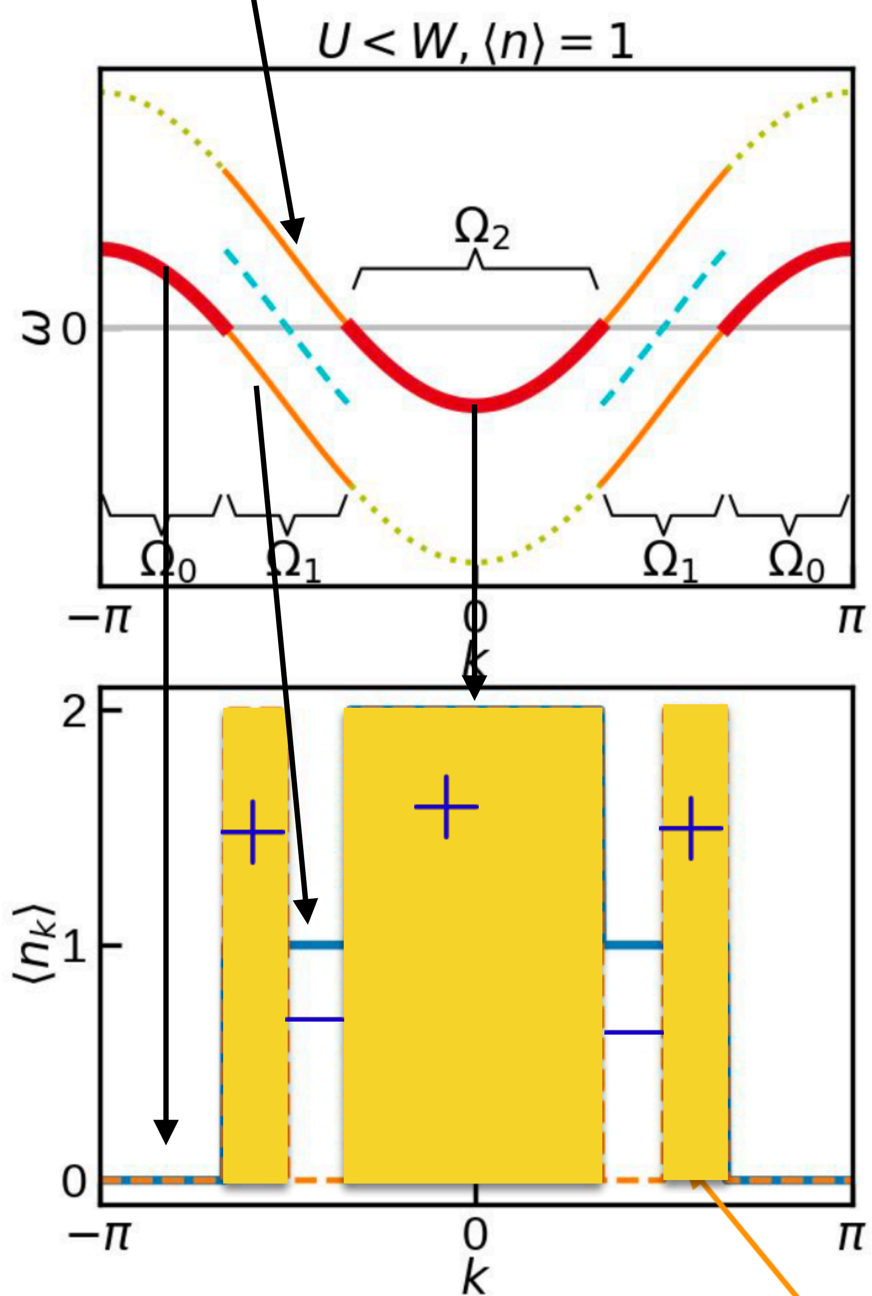
Luttinger Surface

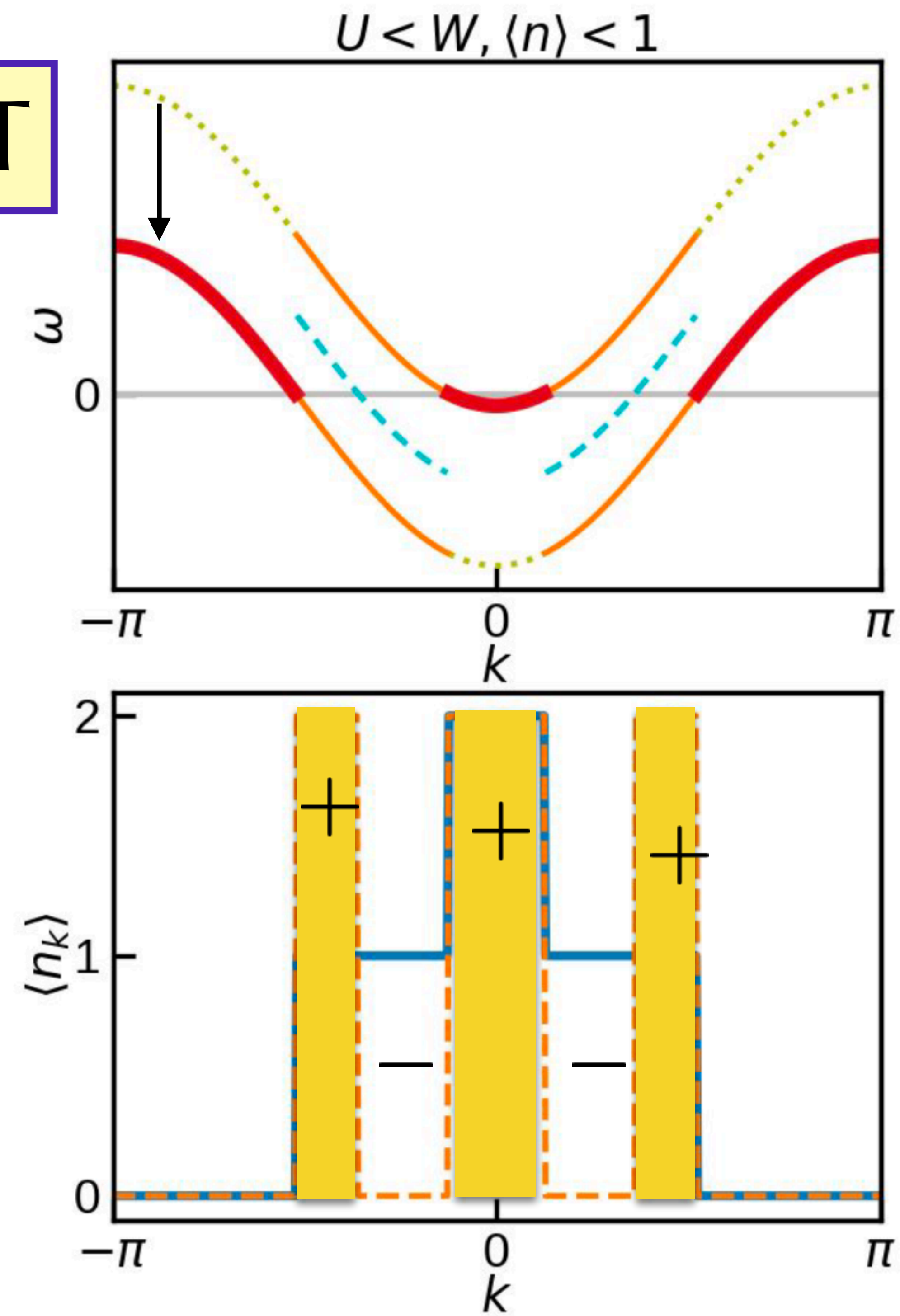
metal

single occupancy

counting charges

SWT



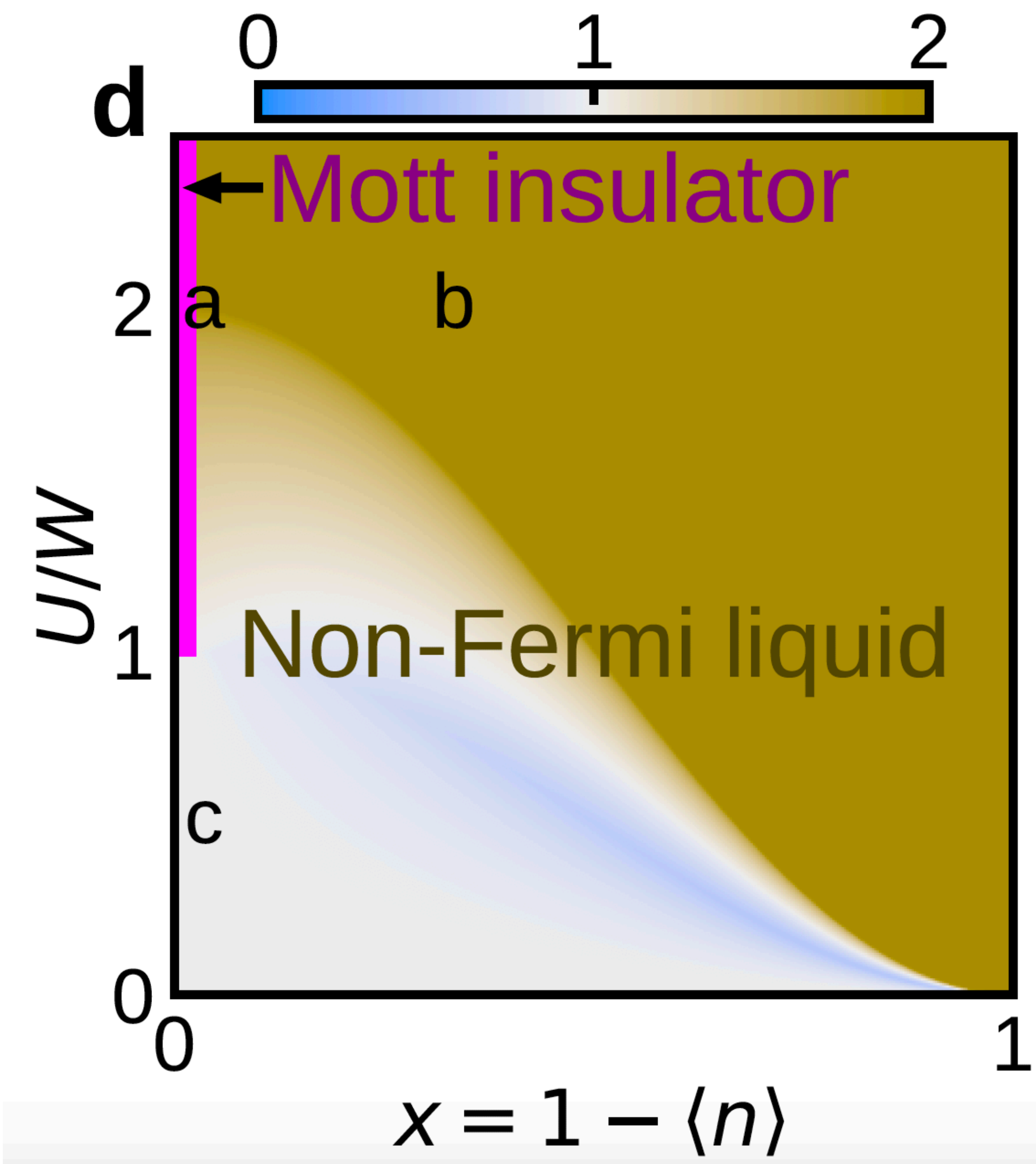


$n_{\mathrm{Lutt}} = \langle n \rangle$

$n_{\mathrm{Lutt}} \neq \langle n \rangle$

zeros$\neq$particles

Why NFL?



FL

HK Mottness

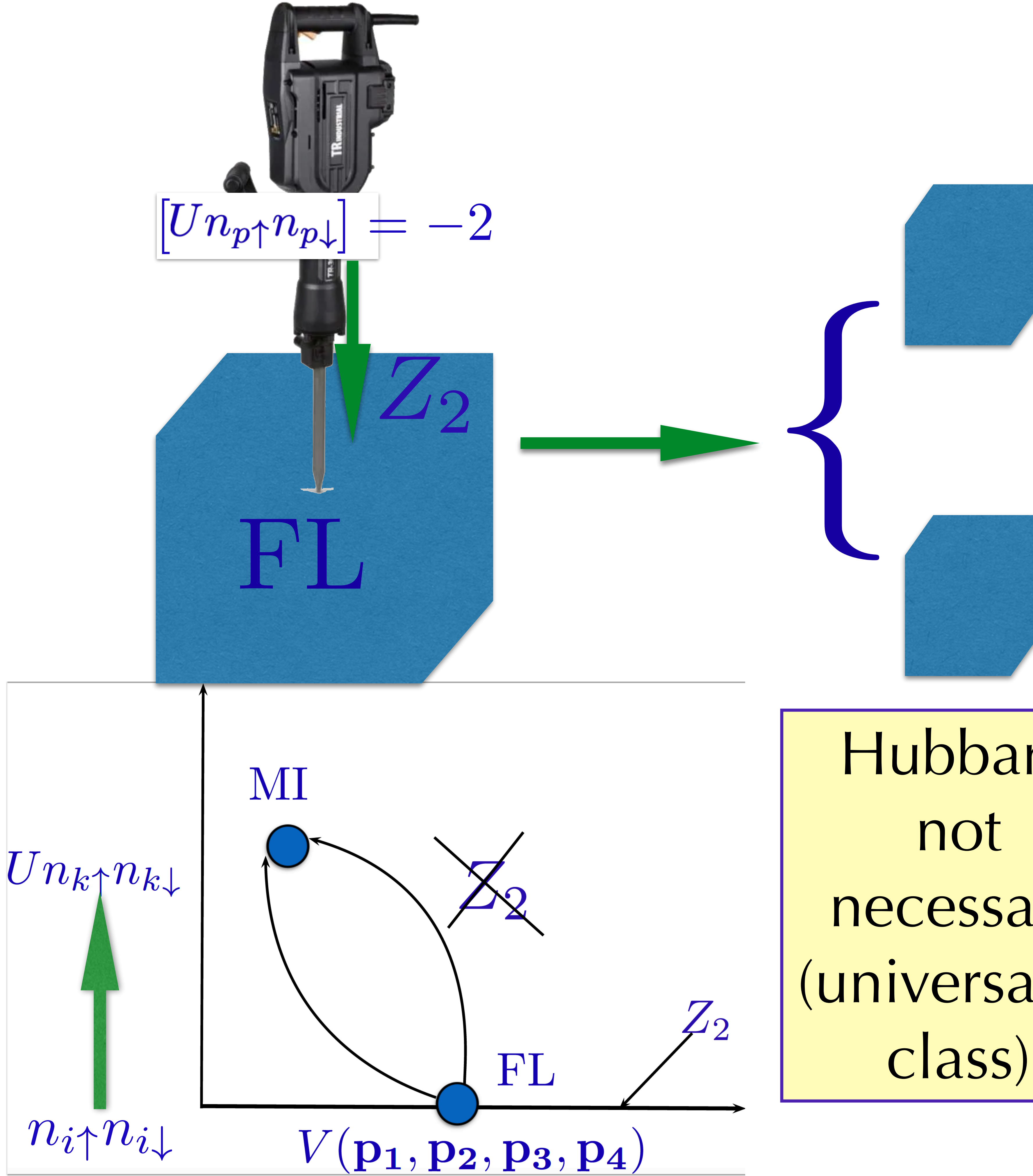
$$c^\dagger_{k\uparrow}c^\dagger_{k\downarrow}|G\rangle \quad \not\leftrightarrow \quad ?$$

$$c^\dagger_{k\uparrow}|G\rangle, c^\dagger_{k\downarrow}|G\rangle \leftrightarrow \zeta^\dagger_{k\uparrow}|G\rangle, \zeta^\dagger_{k\downarrow}|G\rangle$$

$$|G\rangle \qquad\qquad |G\rangle$$

$$\zeta^\dagger_{k\uparrow}\zeta^\dagger_{k\downarrow}|G\rangle = 0$$

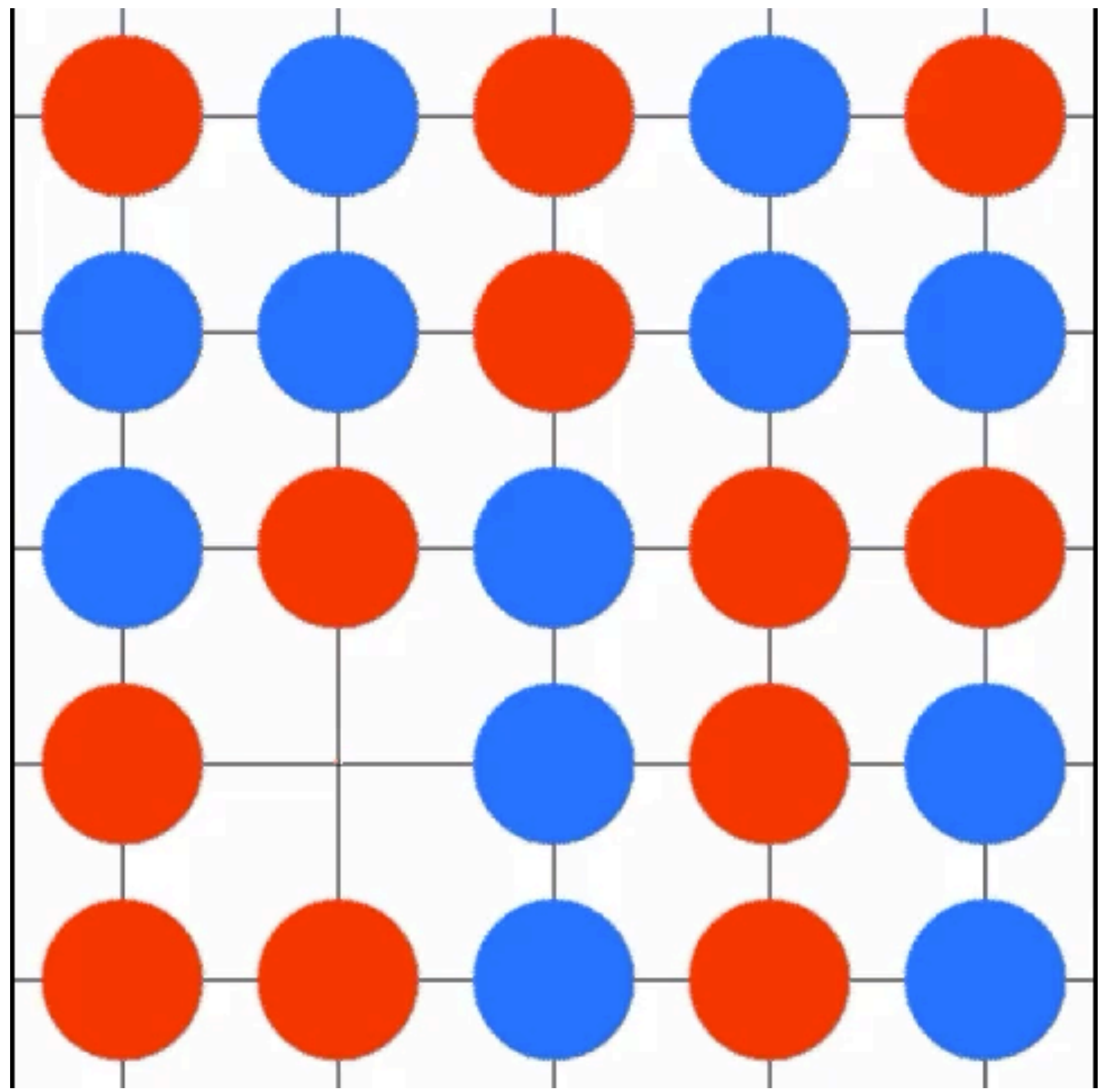
$$\frac{1}{N}\left(\frac{1}{s}\right)^2 \propto \frac{1}{s}$$



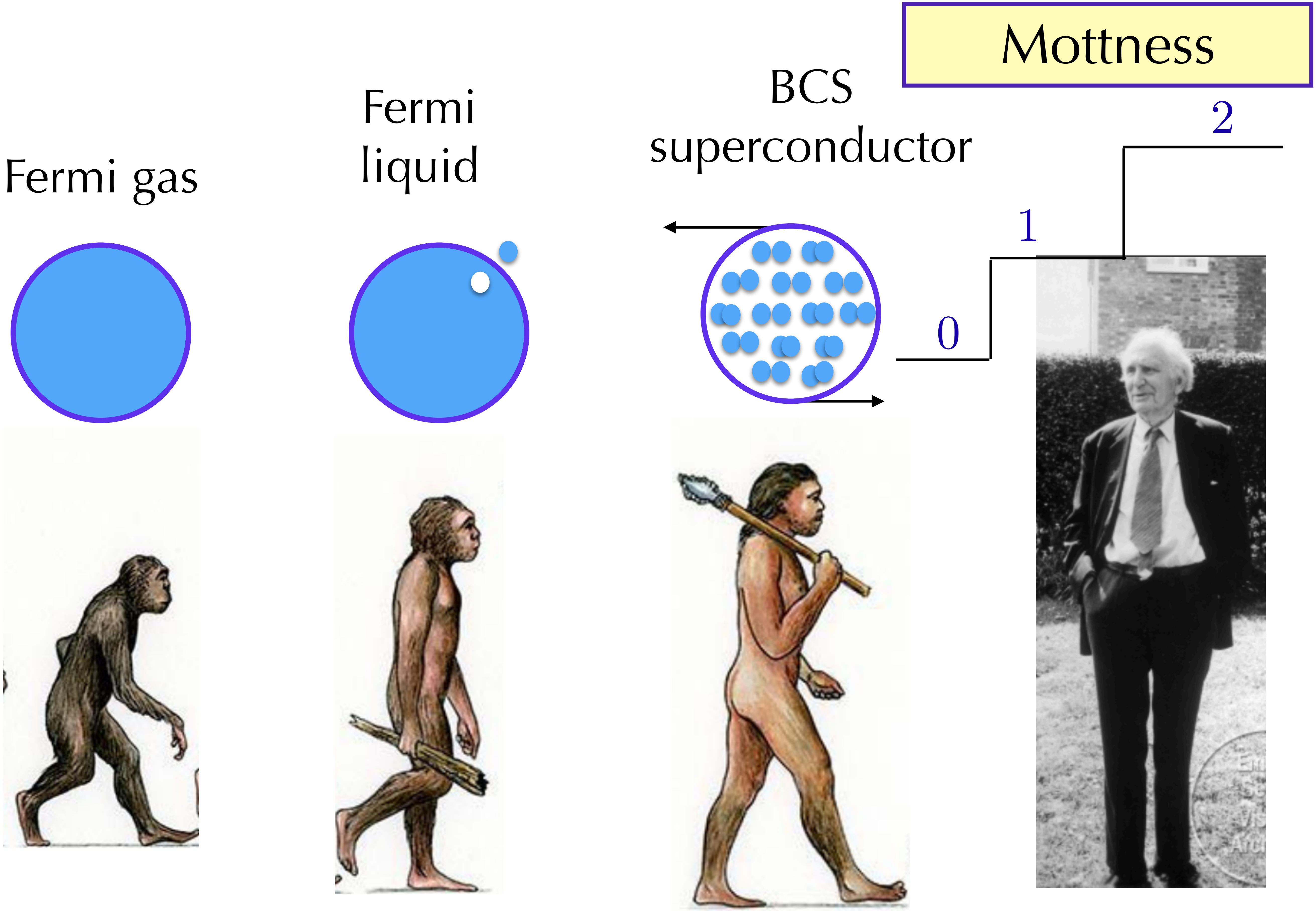
Hubbard not necessary (universality class)

what does the HK model leave out??

$$[H_t, H_U] \neq 0$$

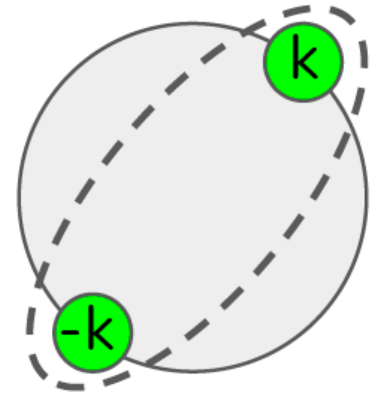


dynamical spectral weight transfer

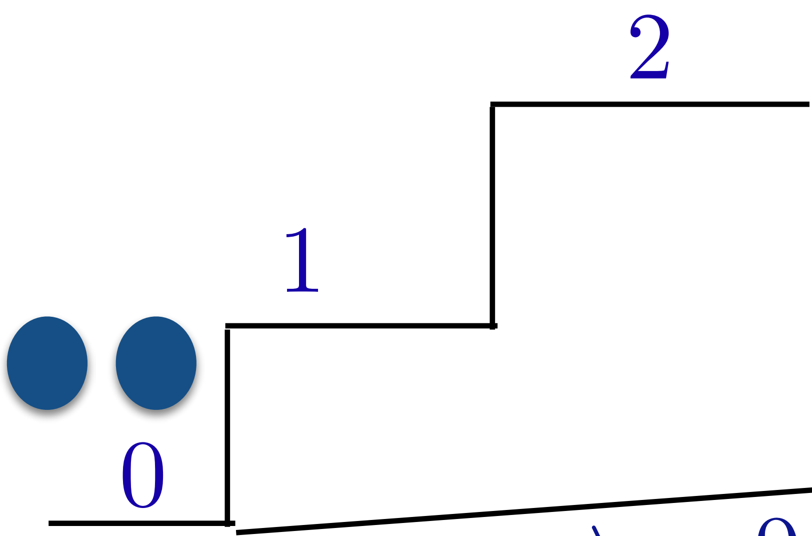

Fermi gas
Fermi
liquid
BCS
superconductor
Mottness
0
1
2

Superconductivity?

Cooper Instability



$$H = H_{\text{HK}} - gH_p$$



$$|\psi\rangle = \sum_{k\in\Omega_0} \alpha_k b_k^\dagger |\text{GS}\rangle$$

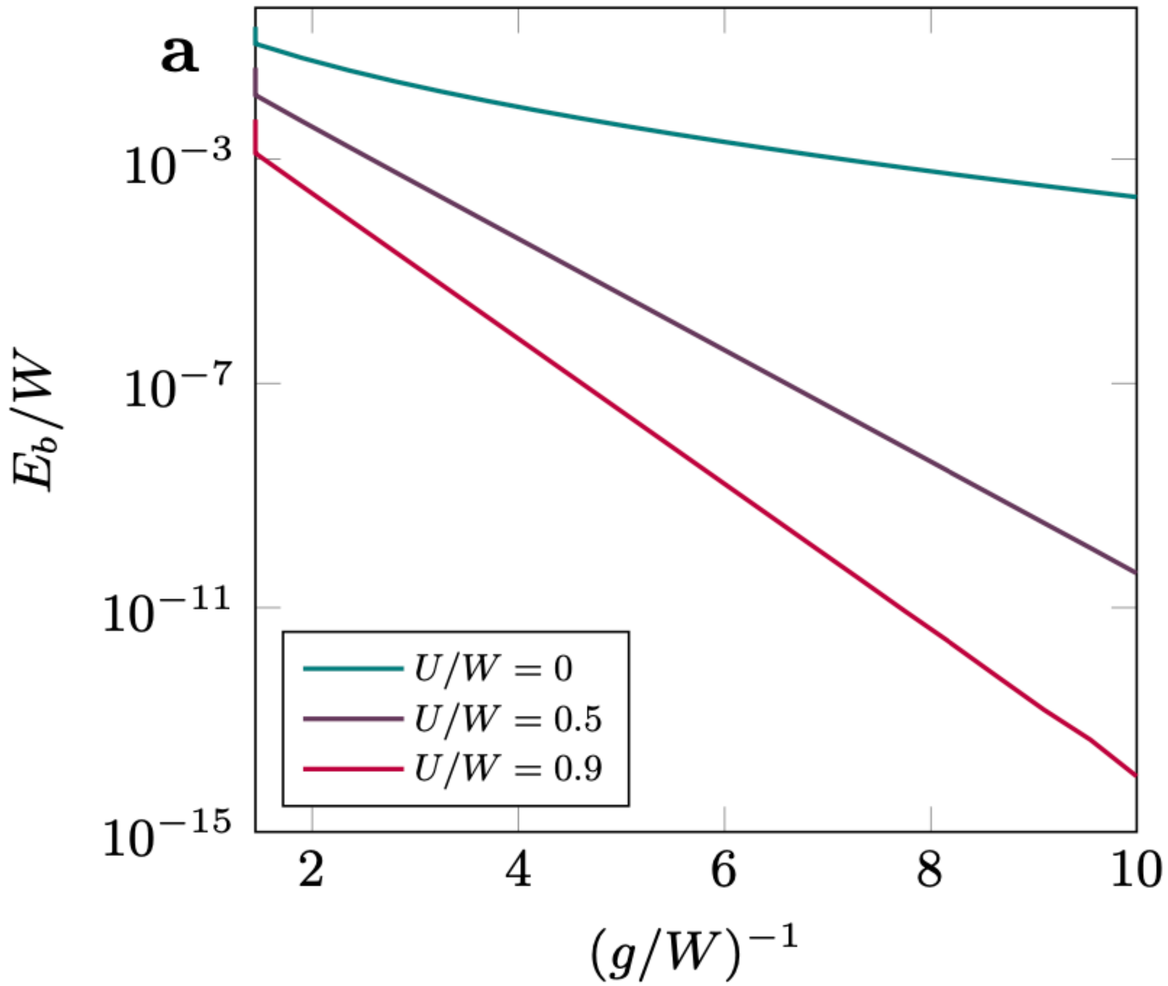
$$= \sum_{k,k'} b_k^\dagger b_{k'}$$

$$\langle n_{k\sigma}\rangle = 0$$

$$E_b = \langle \text{GS}|\text{H}|\text{GS}\rangle - \langle\psi|\text{H}|\psi\rangle \lessgtr 0$$

Cooper Instability

$$E_b = -E \sim W(1-(U/W)^2)e^{-\pi W\sqrt{1-(U/W)^2}/g}$$



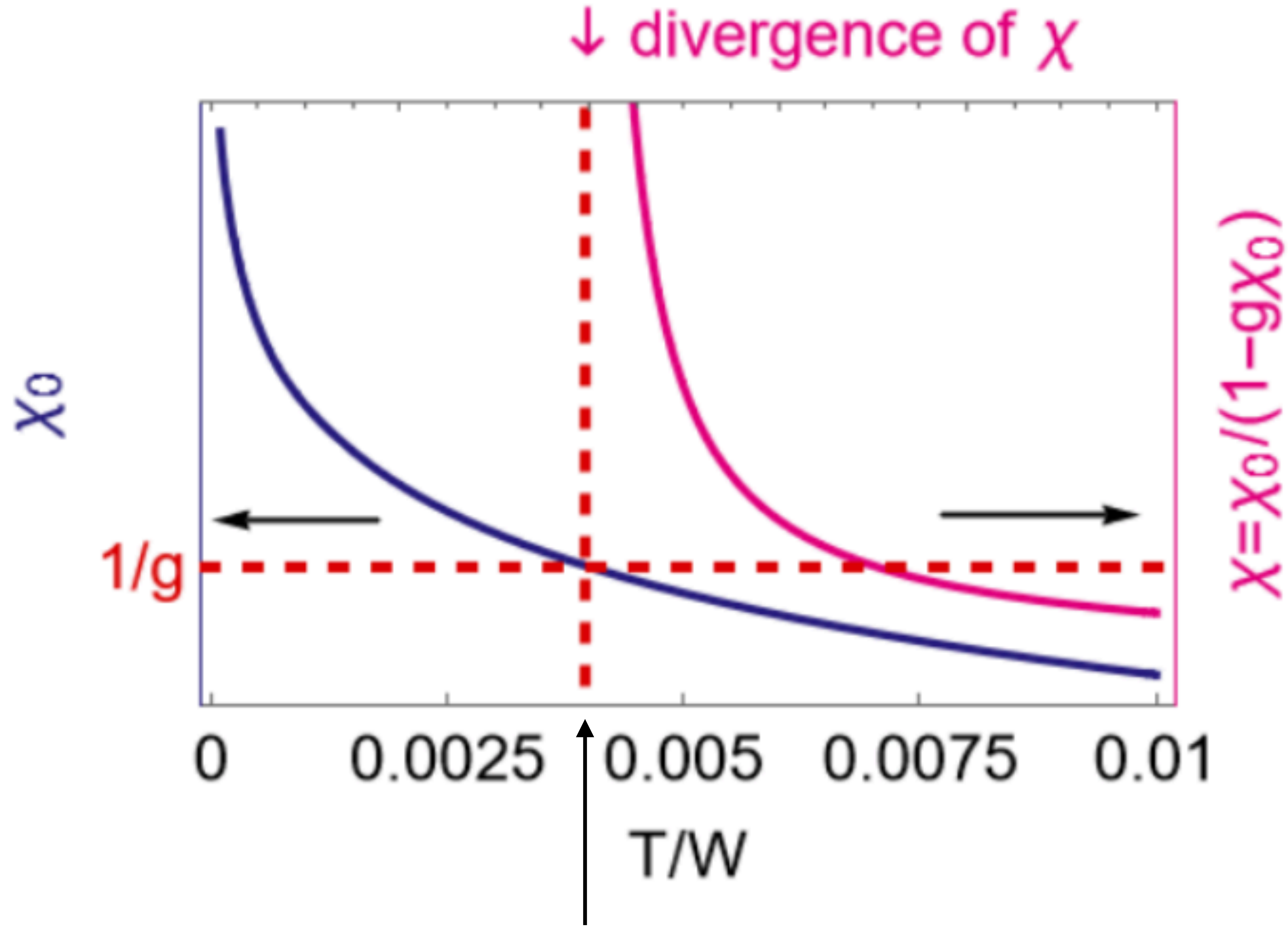
Pair Susceptibility

$$\chi(i\nu_n) \equiv \frac{1}{L^d}\int_0^\beta d\tau e^{i\nu_n\tau}\langle T\Delta(\tau)\Delta^\dagger\rangle_g$$

$$= \frac{\chi_0}{1-g\chi_0}$$

$$g\chi_0 = 1$$

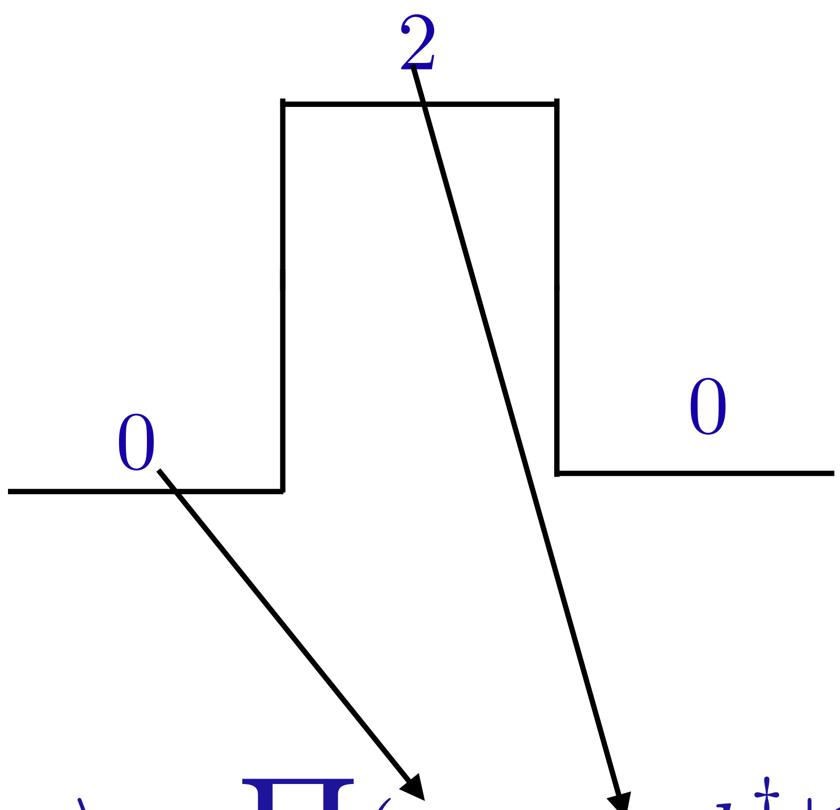
solve for T_c



$$T_c$$

$$T_c = (W-U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}}.$$

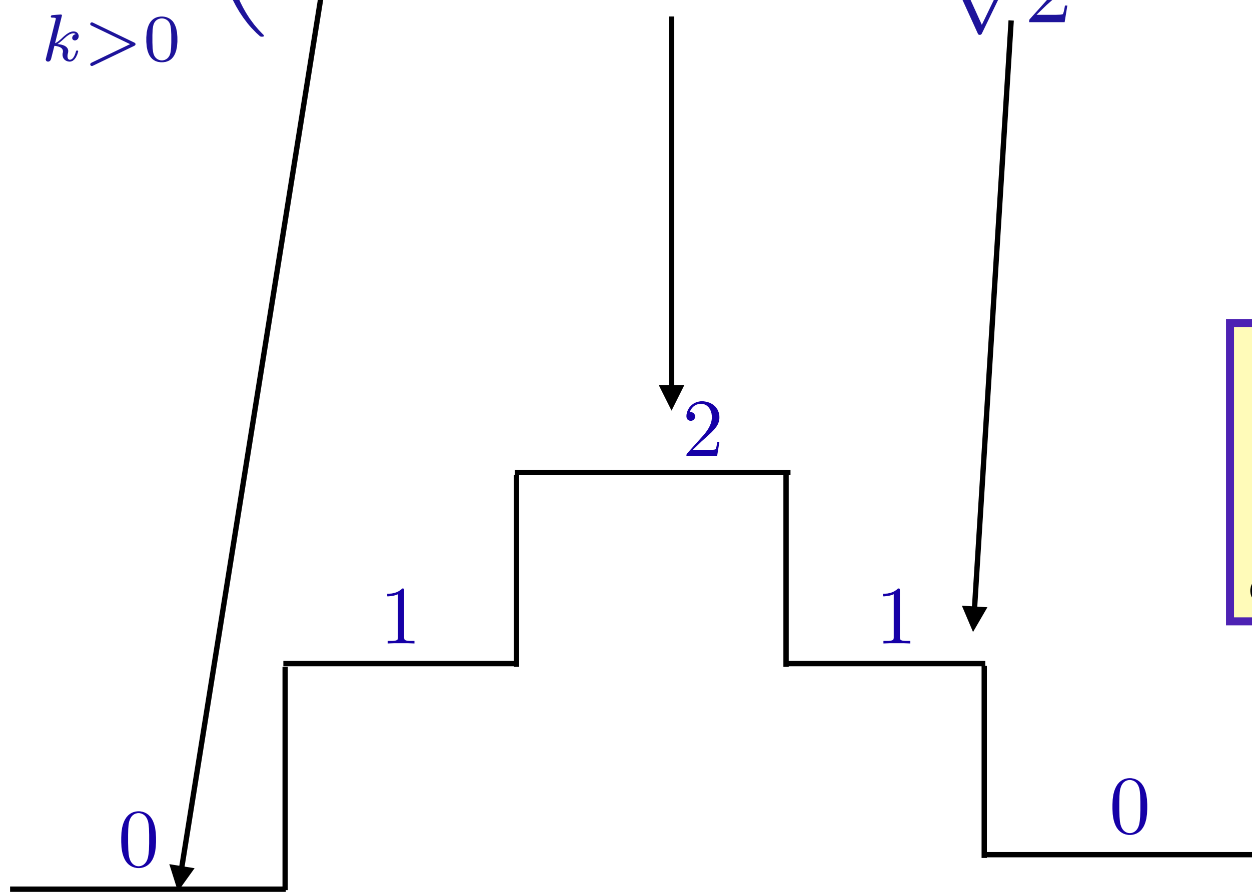
BCS variational wave function



$$|\psi_{\text{BCS}}\rangle = \prod_k (u_k + v_k b_k^\dagger |0\rangle$$

variational MF wave function

$$|\psi\rangle = \prod_{k>0}\left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}}(b_k^\dagger + b_{-k}^\dagger)\right)|0\rangle$$



HK generalization

three variational parameters

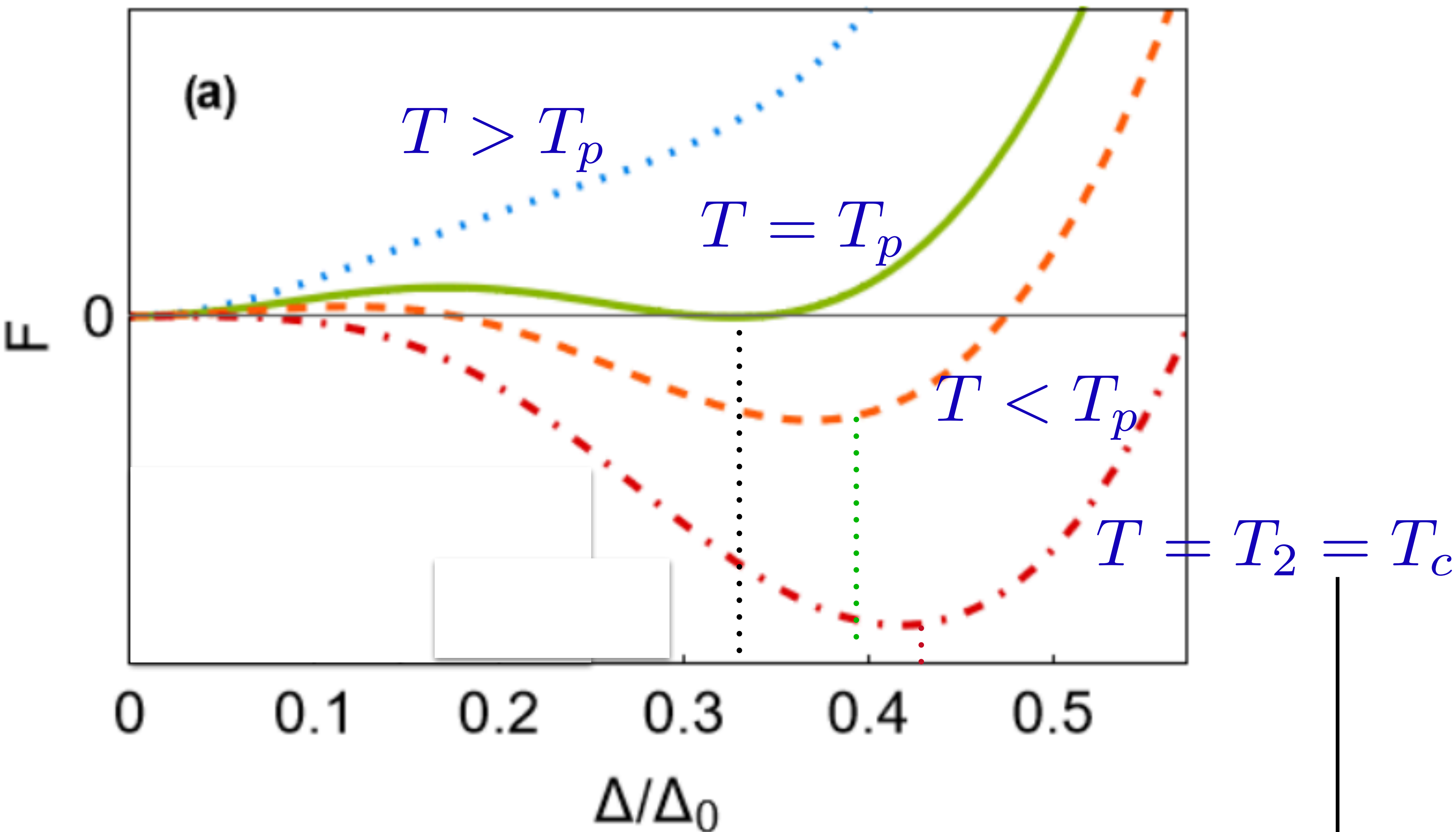
$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation

$$\Delta \ll U, W$$

$$\Delta = (W - U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

compute free energy



$t_G \approx 10^{-11}$

$$\lim_{g \to 0} \frac{\Delta}{T_c} \to \infty$$

MF theory
is accurate!

Two-stage superconductivity in the Hatsugai-Kohomoto-BCS model

Yu Li,[1] Vivek Mishra,[1] Yi Zhou,[2,3,4] and Fu-Chun Zhang[1,4,*]

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[3]Songshan Lake Materials Laboratory, Dongguan, Guangdong 523808, China
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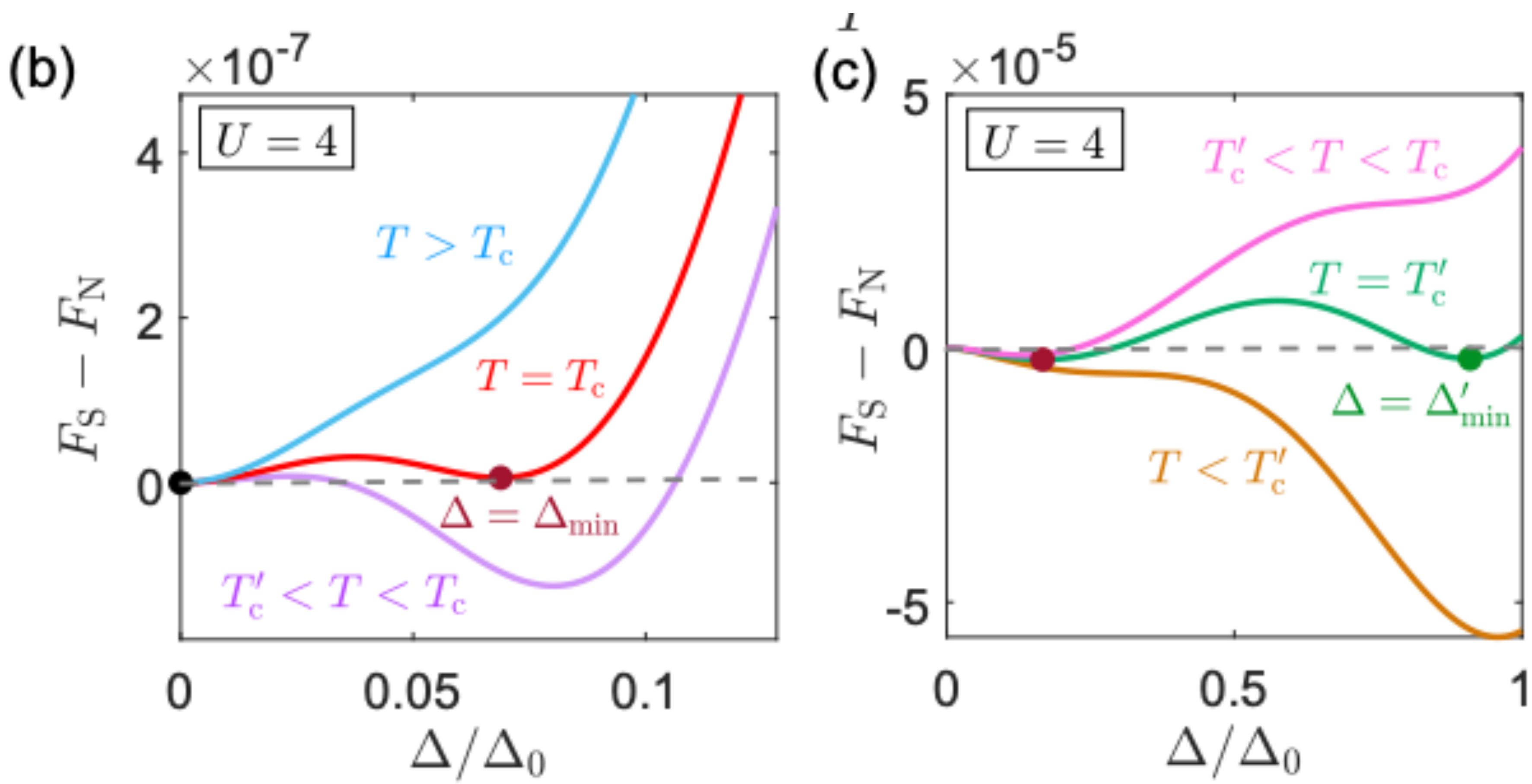
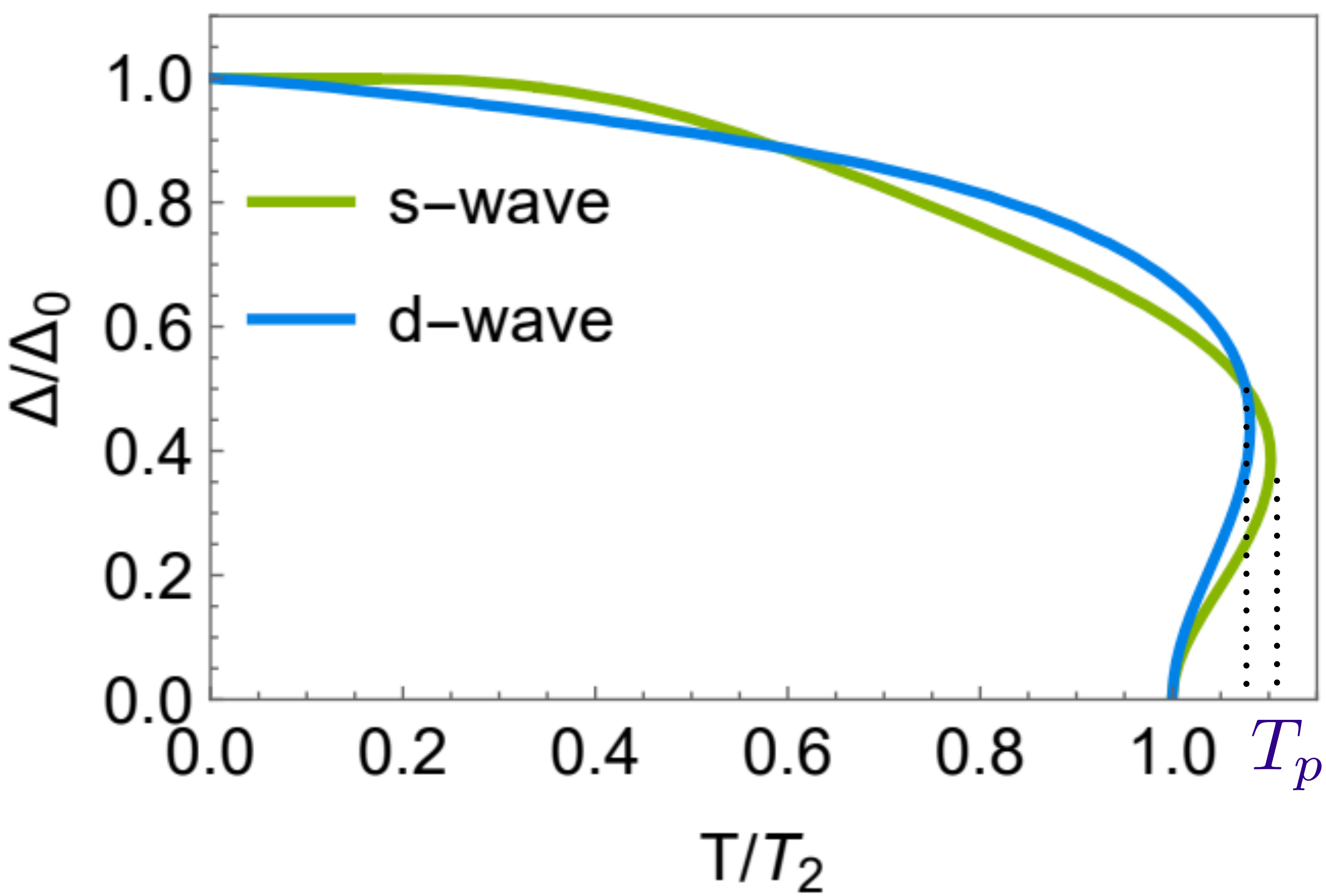
(Dated: July 7, 2022)

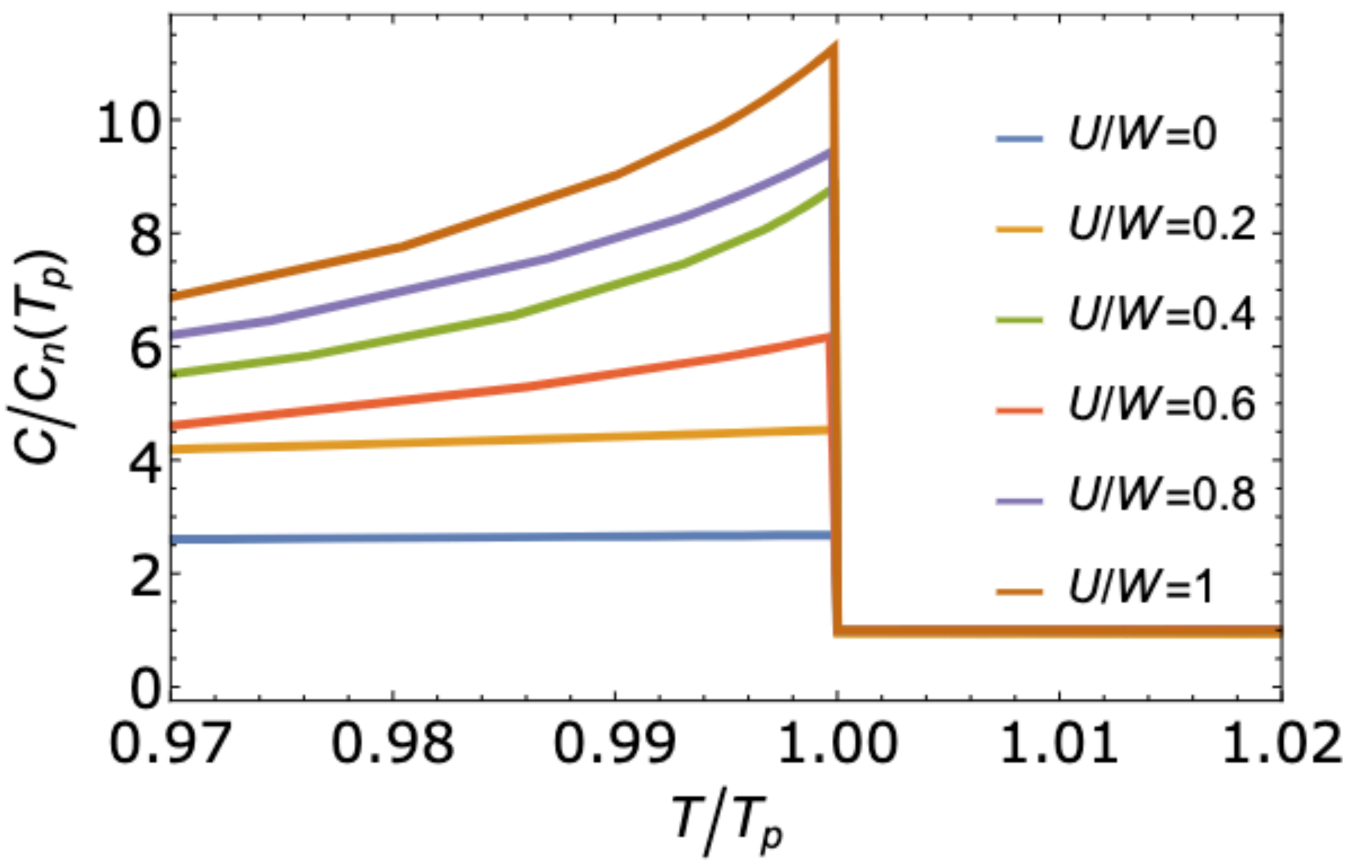
https://arxiv.org/pdf/2207.01904.pdf





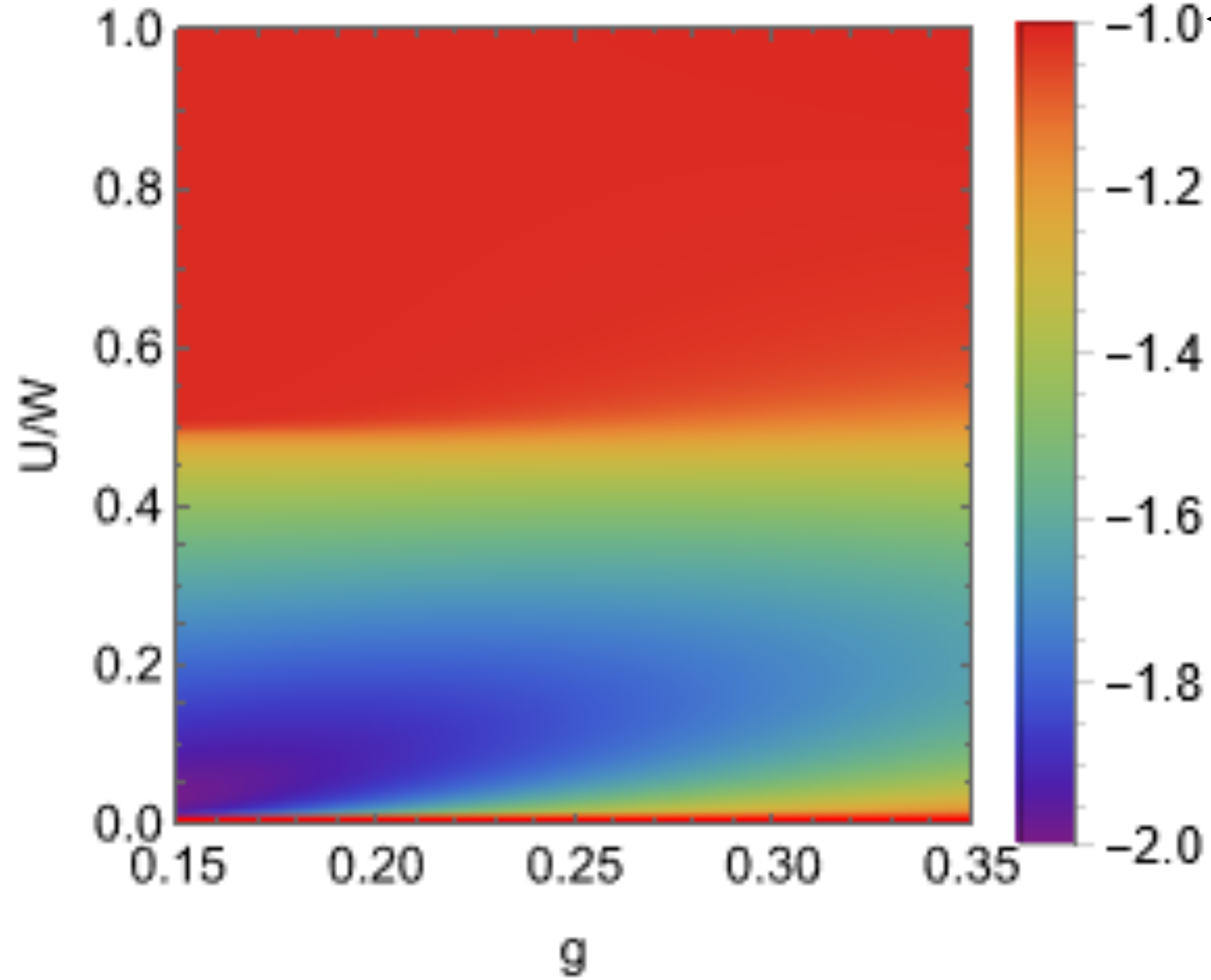
1.0
0.8
0.6
0.4
0.2
0.0
Δ/Δ_0
s–wave
d–wave
0.0
0.2
0.4
0.6
0.8
1.0
T_p
T/T_2

heat capacity jump



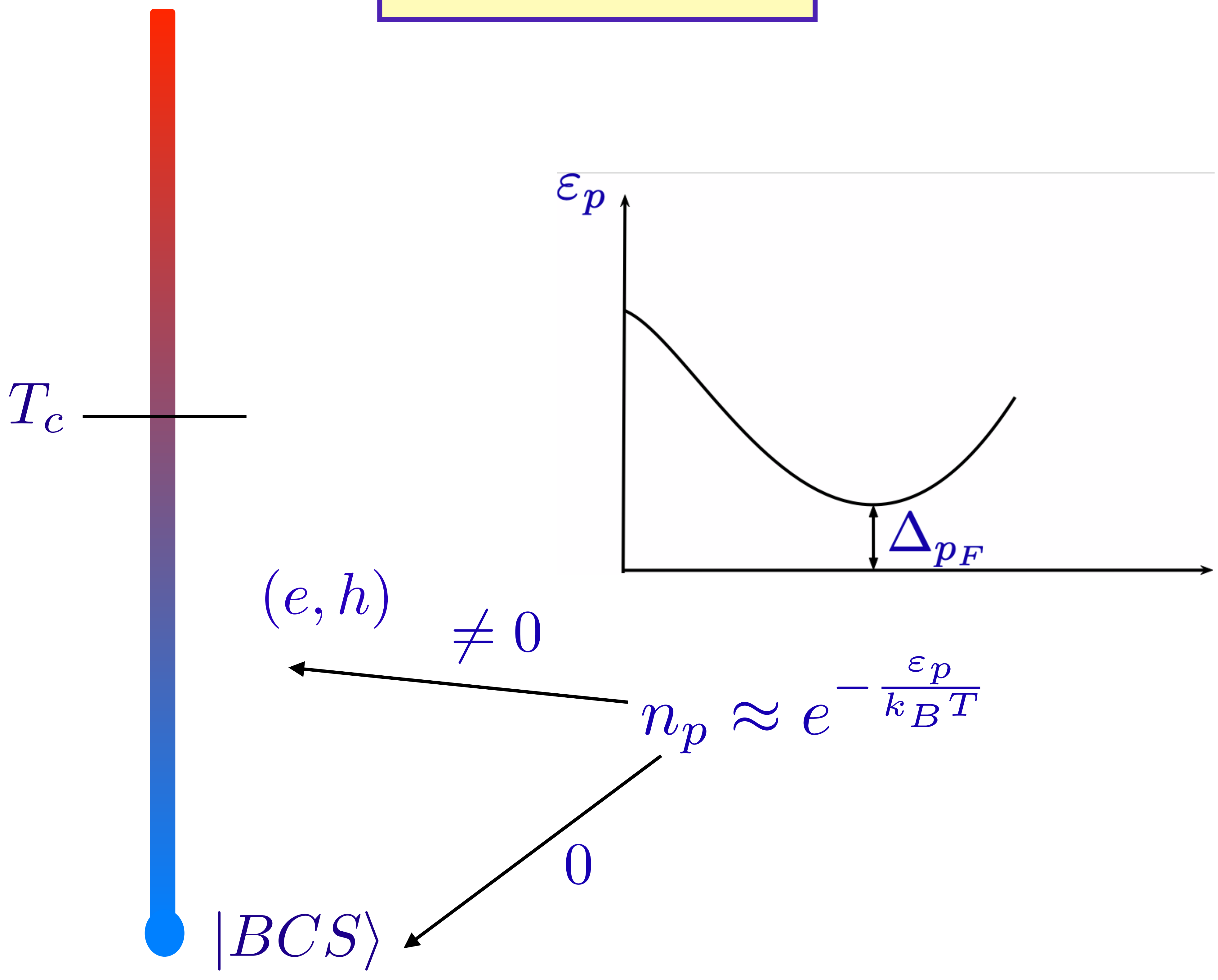
Condensation Energy

$$E_{\text{cond}}/N(0)\Delta^2 < 0$$



BCS

excited states



Bogoliubov excitations

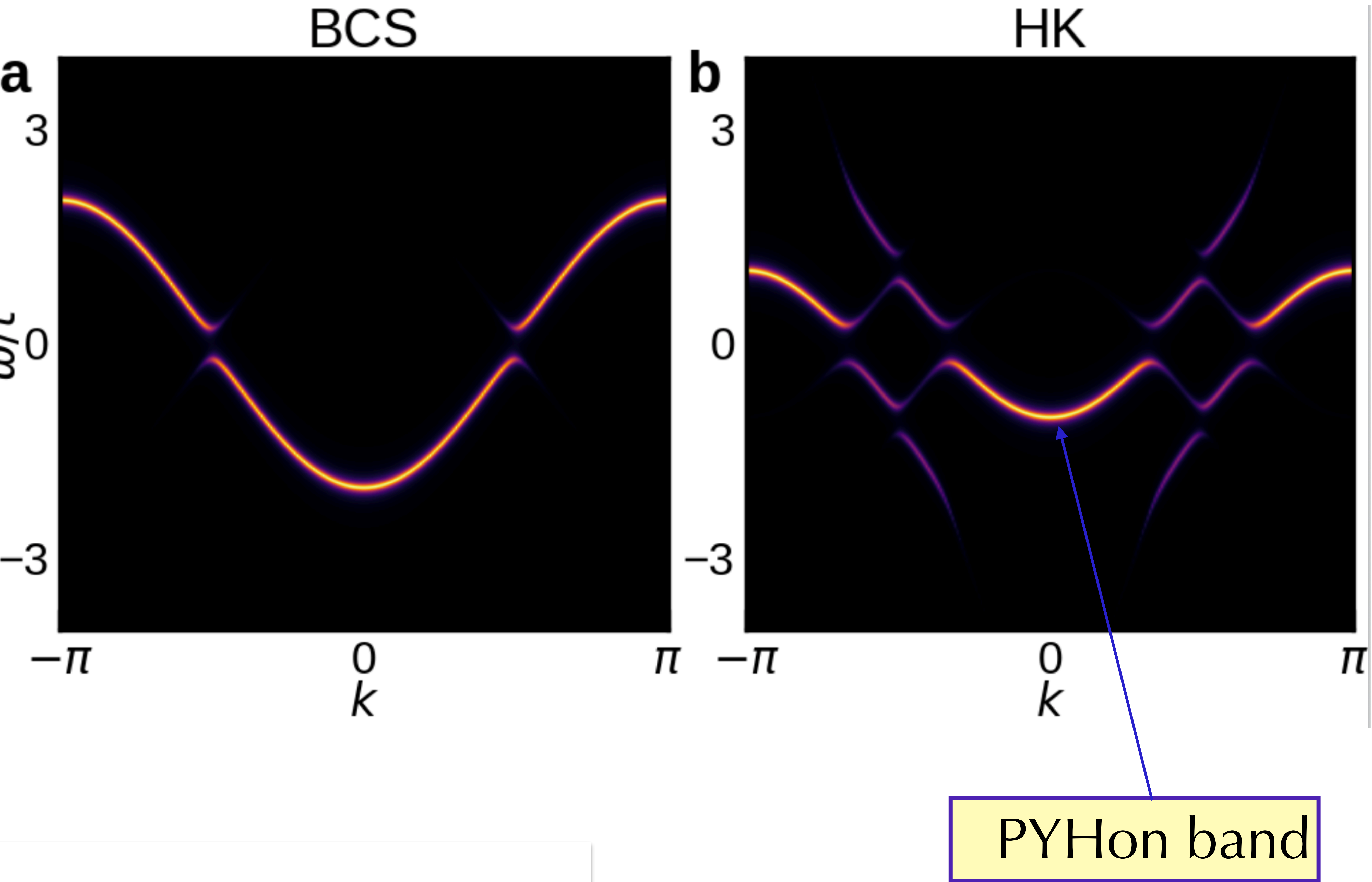
$$\gamma_{k\sigma}|\psi_{\mathrm{BCS}}\rangle = 0$$

$$\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c^\dagger_{-k\overline{\sigma}}$$

PYHons excitations

$$\gamma^l_{k\sigma} \propto \sqrt{2} x_k \zeta^\dagger_{k\sigma} - \sigma z_k \zeta_{-k\overline{\sigma}}$$

$$\gamma^u_{k\sigma} \propto z_k \eta^\dagger_{k\sigma} - \sigma \sqrt{2} y_k \eta_{-k\overline{\sigma}}$$


BCS
HK
a
b
3
0
−3
ω/t
−π
0
π
k
PYHon band

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$

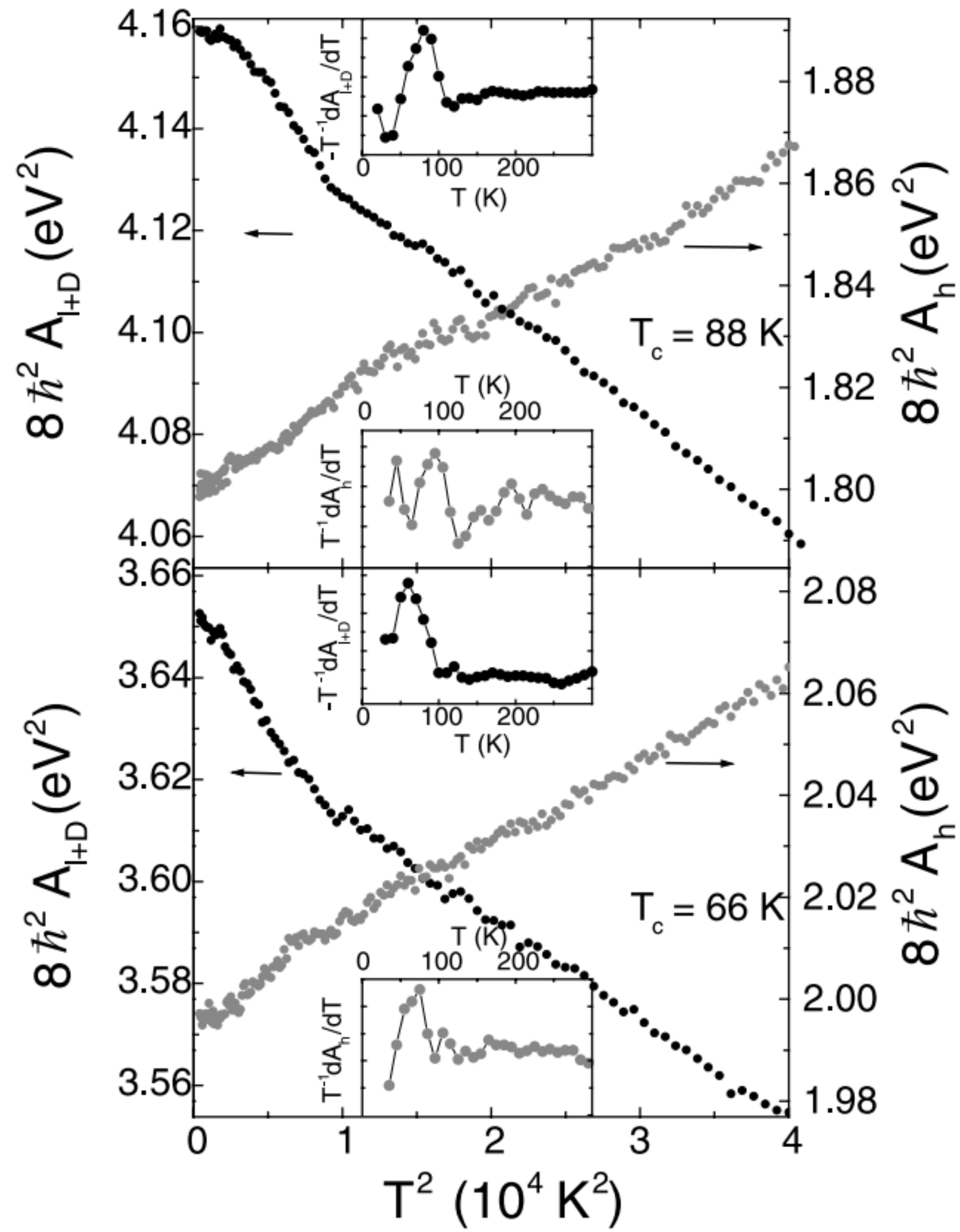
H. J. A. Molegraaf[1], C. Presura[1], D. van der Marel[1,*], P. H. Kes[2], M. Li[2]
+ See all authors and affiliations

Science 22 Mar 2002:
Vol. 295, Issue 5563, pp. 2239-2241
DOI: 10.1126/science.1069947

$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

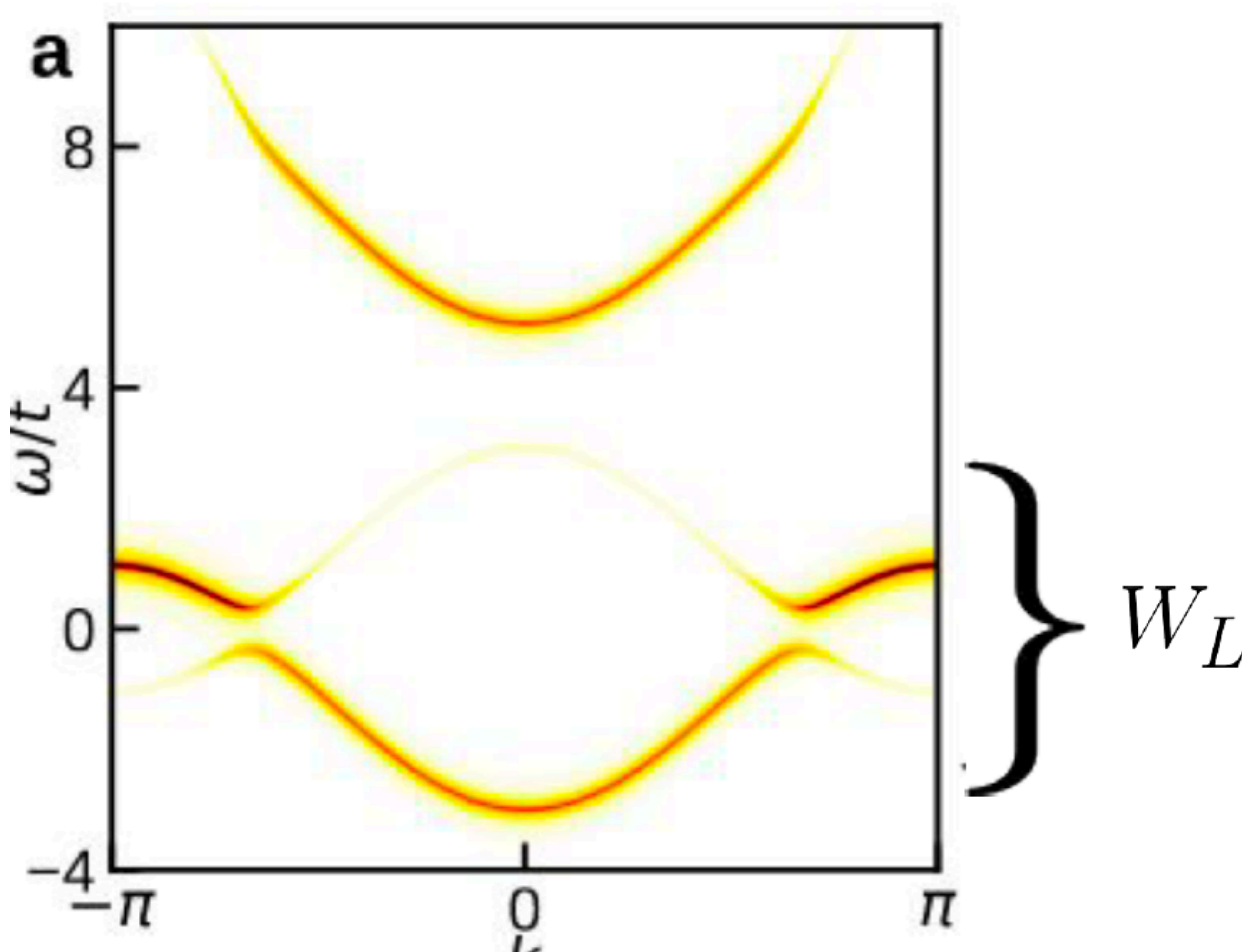
$$\frac{\Delta A_l}{A_l} \propto 3\%$$



condensation energy

Optical data are reported on a spectral weight transfer over a broad frequency range of $Bi_2Sr_2CaCu_2O_{8+\delta}$, when this material became superconducting. Using spectroscopic ellipsometry, we observed the removal of a small amount of spectral weight in a broad frequency band from 10^4 cm^{-1} to at least 2×10^4 cm^{-1}, due to the onset of superconductivity. We observed a blue shift of the *ab*-plane plasma frequency when the material became superconducting, indicating that the spectral weight was transferred to the infrared range. Our observations are in agreement with models in which superconductivity is accompanied by an increased charge carrier spectral weight. The measured spectral weight transfer is large enough to account for the condensation energy in these compounds.

UV-IR mixing



a
8
4
0
−4
ω/t
−π
0
π
k
W_L

why?

$$H = H_{\mathrm{HK}} + H_p$$

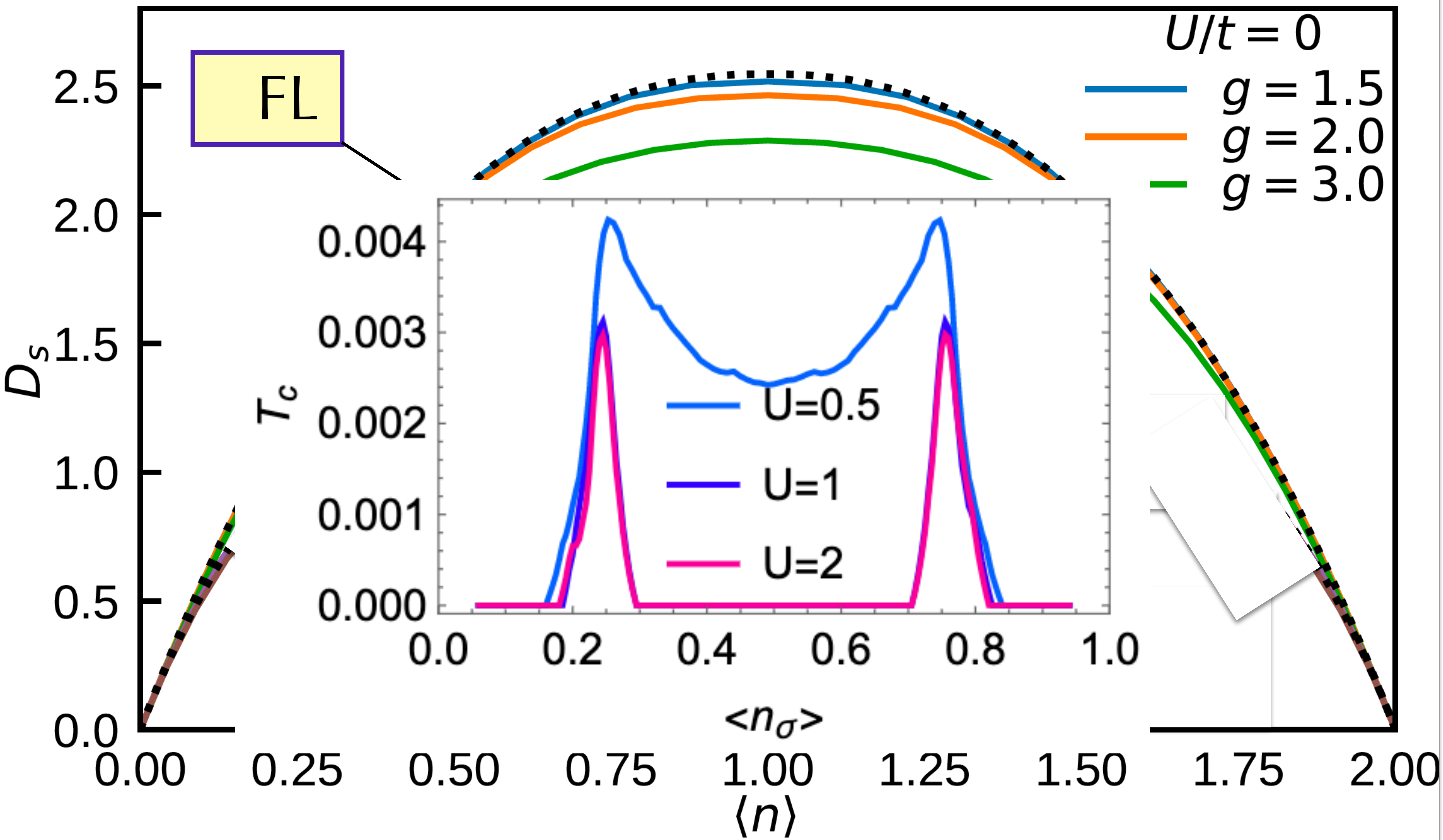
$$[H_{\mathrm{HK}}, H_p] \neq 0$$

↓

dynamical spectral weight transfer

Superfluid Density

Mottness-induced suppression



Superconductivity

observable	BCS/FL
$\chi \to \infty$	T_c
$\Delta \neq 0$	T_c
$\lim_{g\to 0} 2\Delta_0/k_B T_c$	3.52
$\mathrm{quasi-particles}$	$\mathrm{Bogoliubons}$
$t_G(\mathrm{Ginzburg})$	$\approx 10^{-12}$
$1/\mathrm{TT}_1$	$\mathrm{HS\,peak}$
$\mathrm{Landau\,Expansion}$	$a = \alpha t,\ b > 0$
$E_{\mathrm{cond}}/N(0)\Delta^2$	-1

Topology + Strong Correlations?

Are Exact Statements Possible?

Haldane +HK model

TI

1/4-filled MI