Superconductivity and Mottness: Exact Results

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with N&V by J. Zaanen

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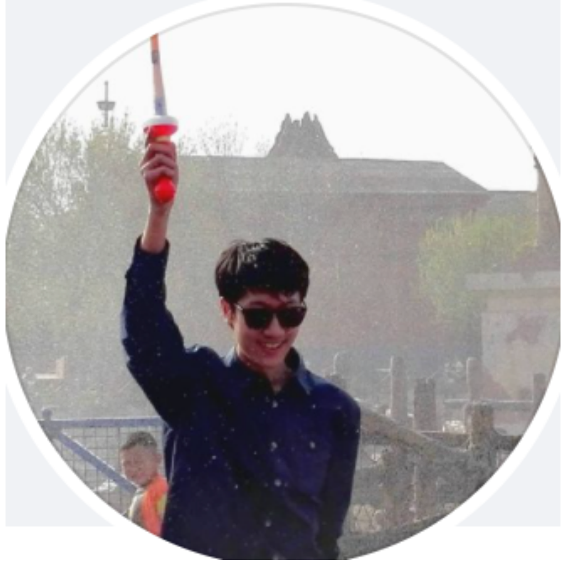
Luke Yeo

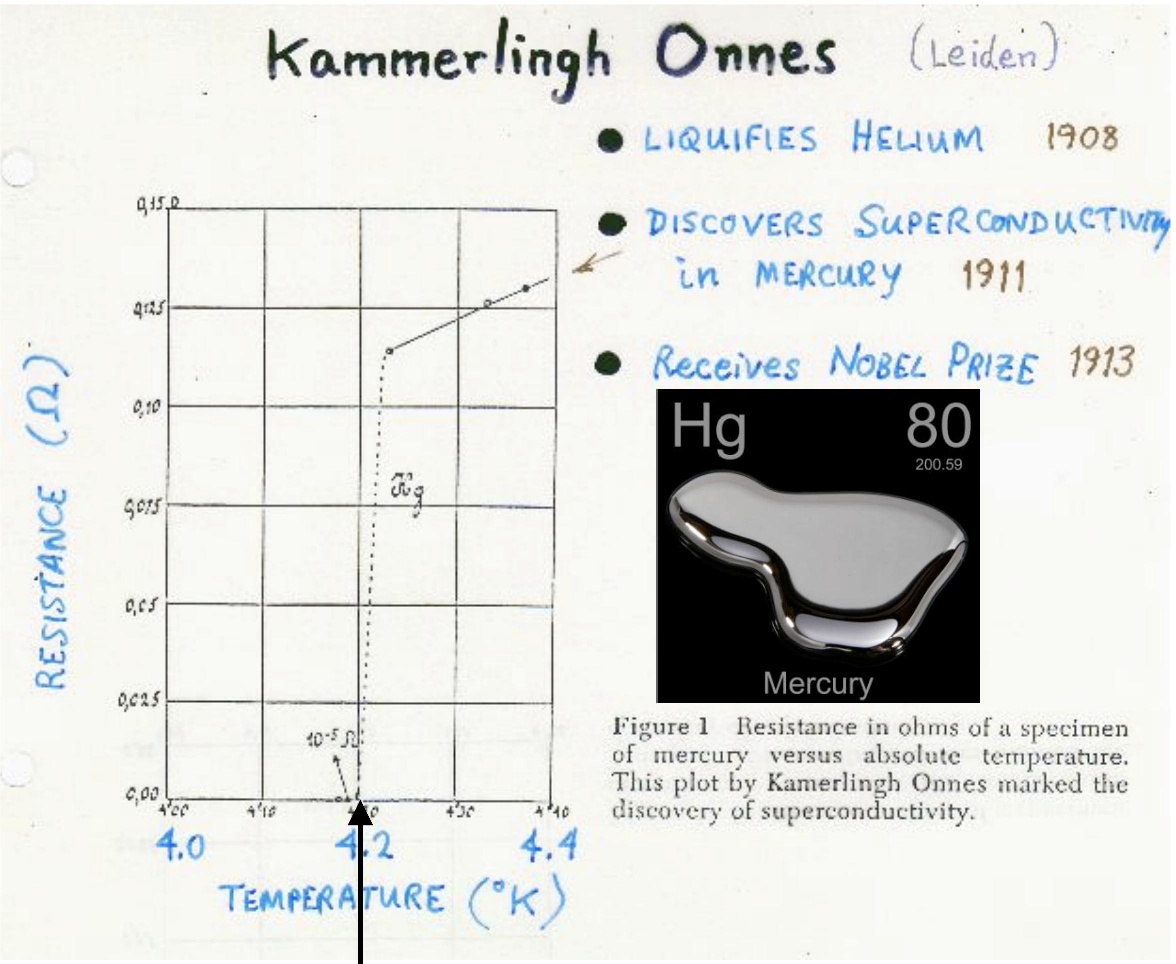
Edwin Huang

G. La Nave

Jinchao Z.



Cooper instability

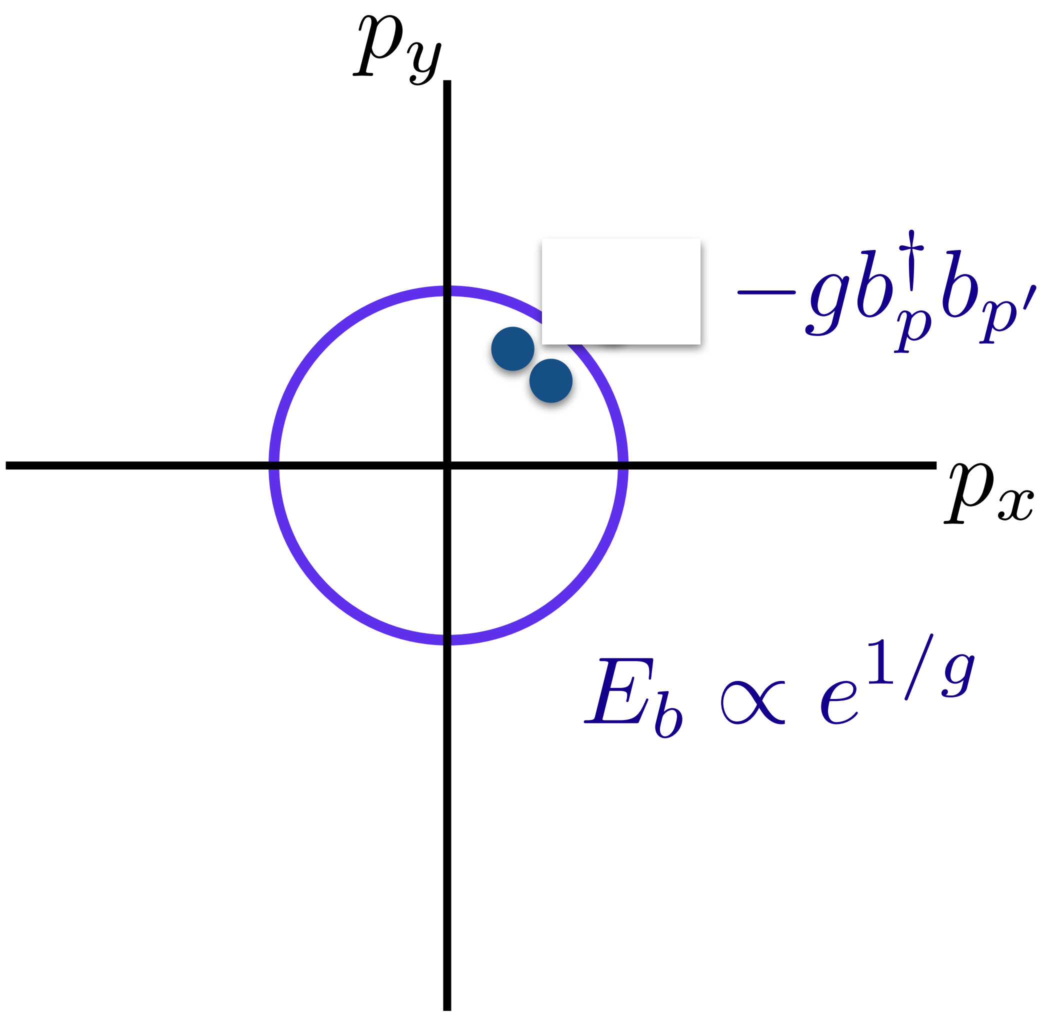


Kammerlingh Onnes (Leiden)

- LIQUIFIES HELIUM 1908
- DISCOVERS SUPERCONDUCTIVITY in MERCURY 1911
- Receives NOBEL PRIZE 1913

Figure 1 Resistance in ohms of a specimen of mercury versus absolute temperature. This plot by Kamerlingh Onnes marked the discovery of superconductivity.

T_c

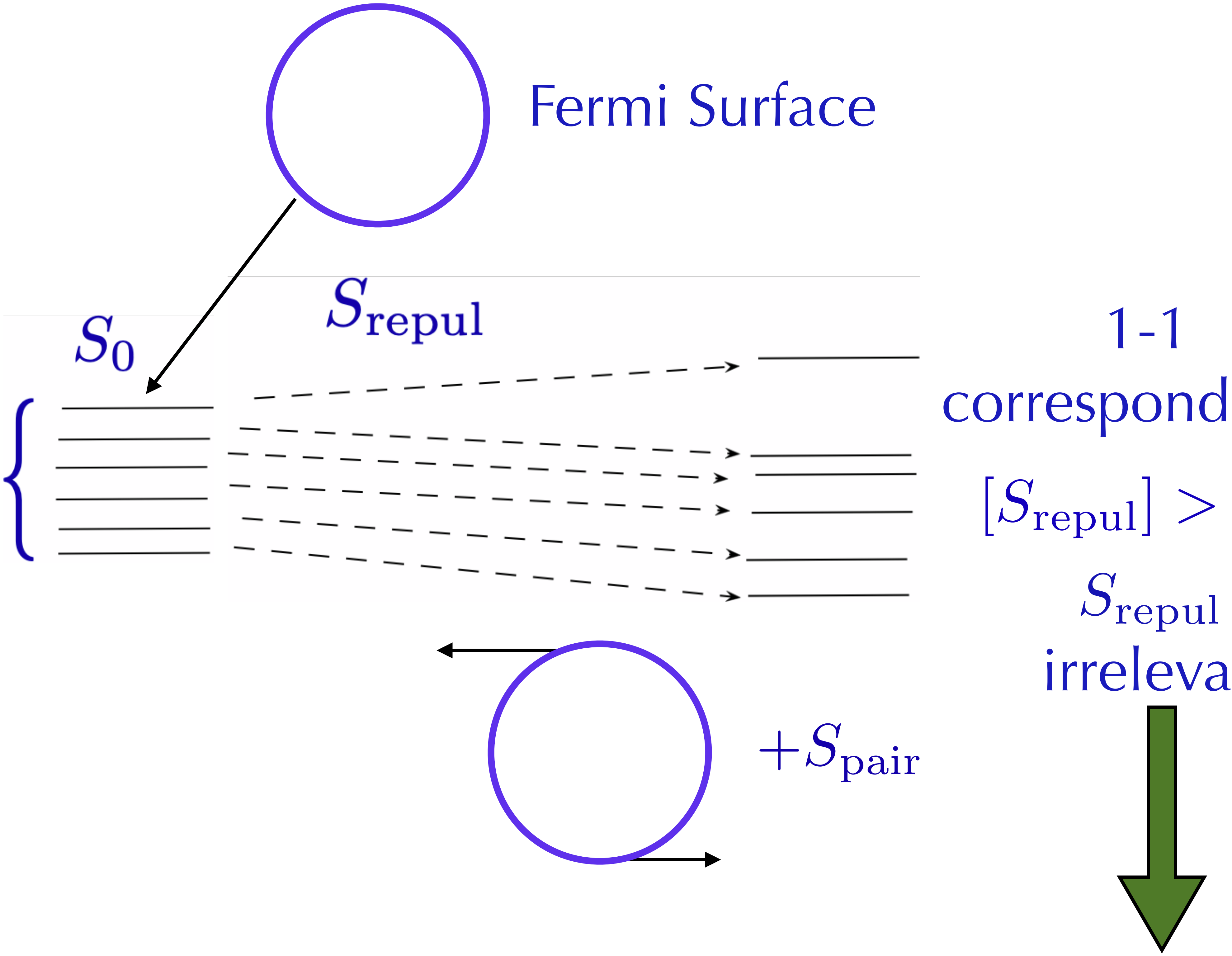


$-gb_p^\dagger b_{p'}$

$E_b \propto e^{1/g}$

$\langle \psi^\dagger \psi^\dagger \rangle = \Delta e^{i\phi}$

FL $\rightarrow$ BCS



Fermi Surface

S_0

S_{repul}

1-1 correspondence

$[S_{\text{repul}}] > 0$

S_{repul} irrelevant

$+S_{\text{pair}}$

FL theory
(free/quadratic)

superconductivity $\frac{2\Delta}{T_c} = 3.5$

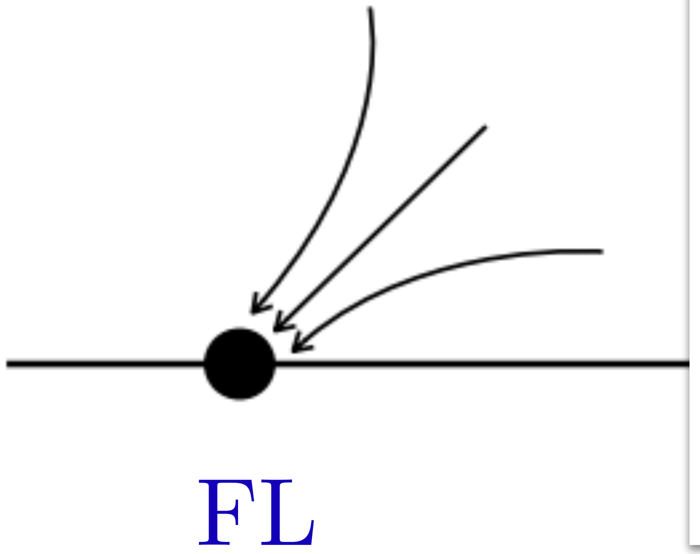
Fermi gas

Fermi
liquid

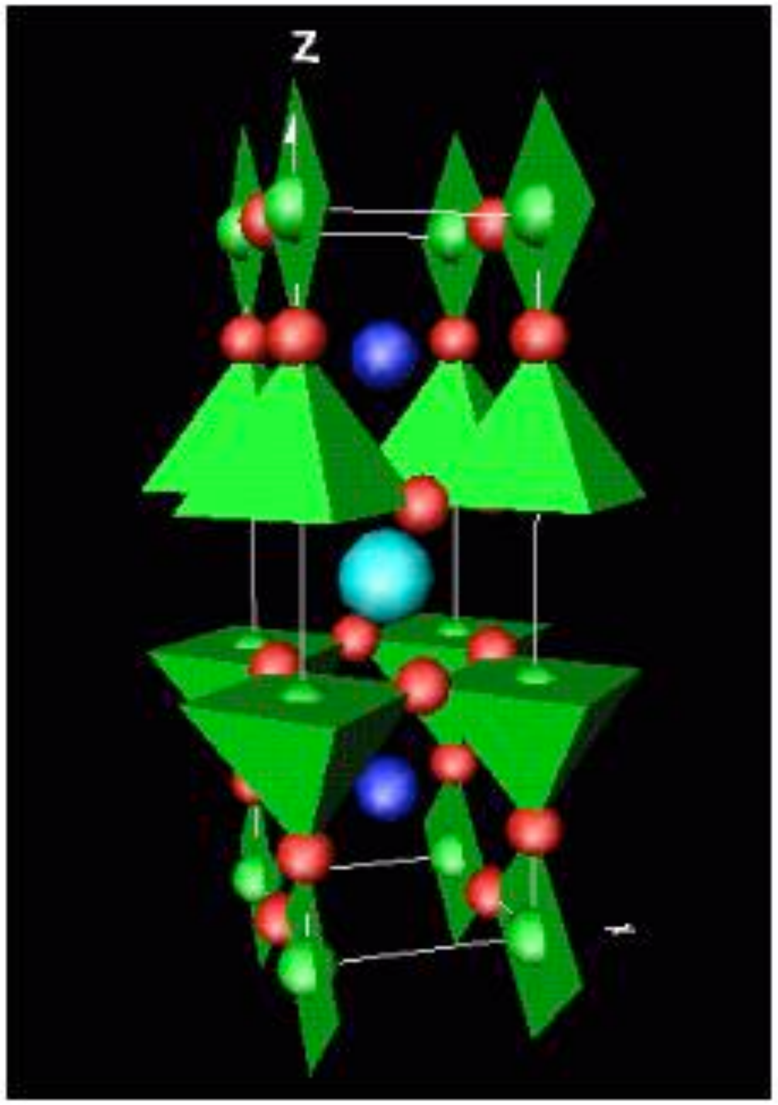
BCS
superconductor

?

fixed
point beyond
FL?



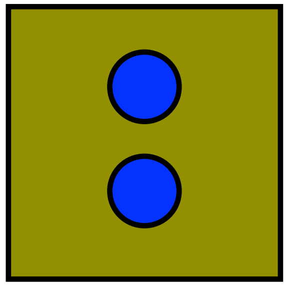
quartic
interacting
theory?



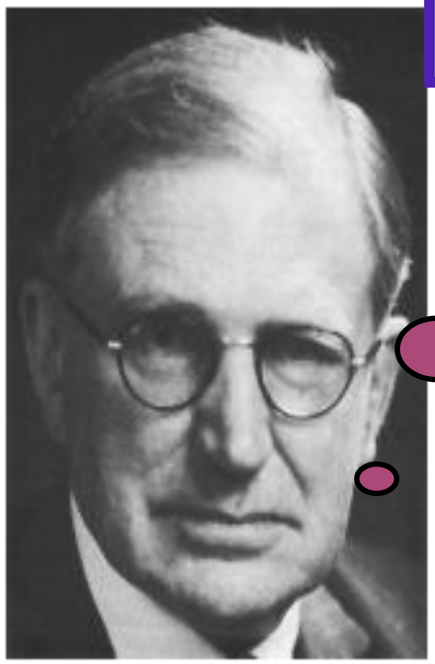
$Y Ba_2Cu_3O_7$
Cuprate Superconductors

NiO insulates
d^8 ?

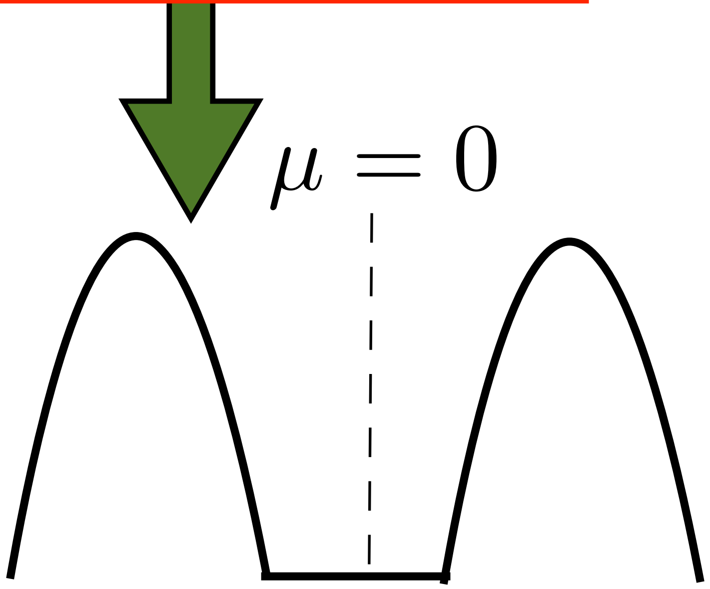
perhaps this
costs energy

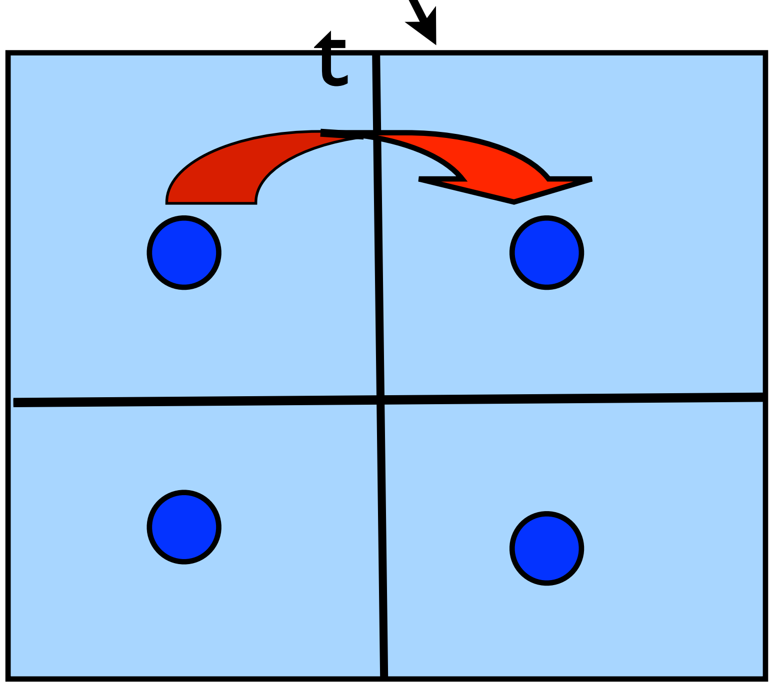


$U \gg t$



local real-space physics





no change in
size of
Brillouin zone

solve the Hubbard Model!!

Cooper instability??

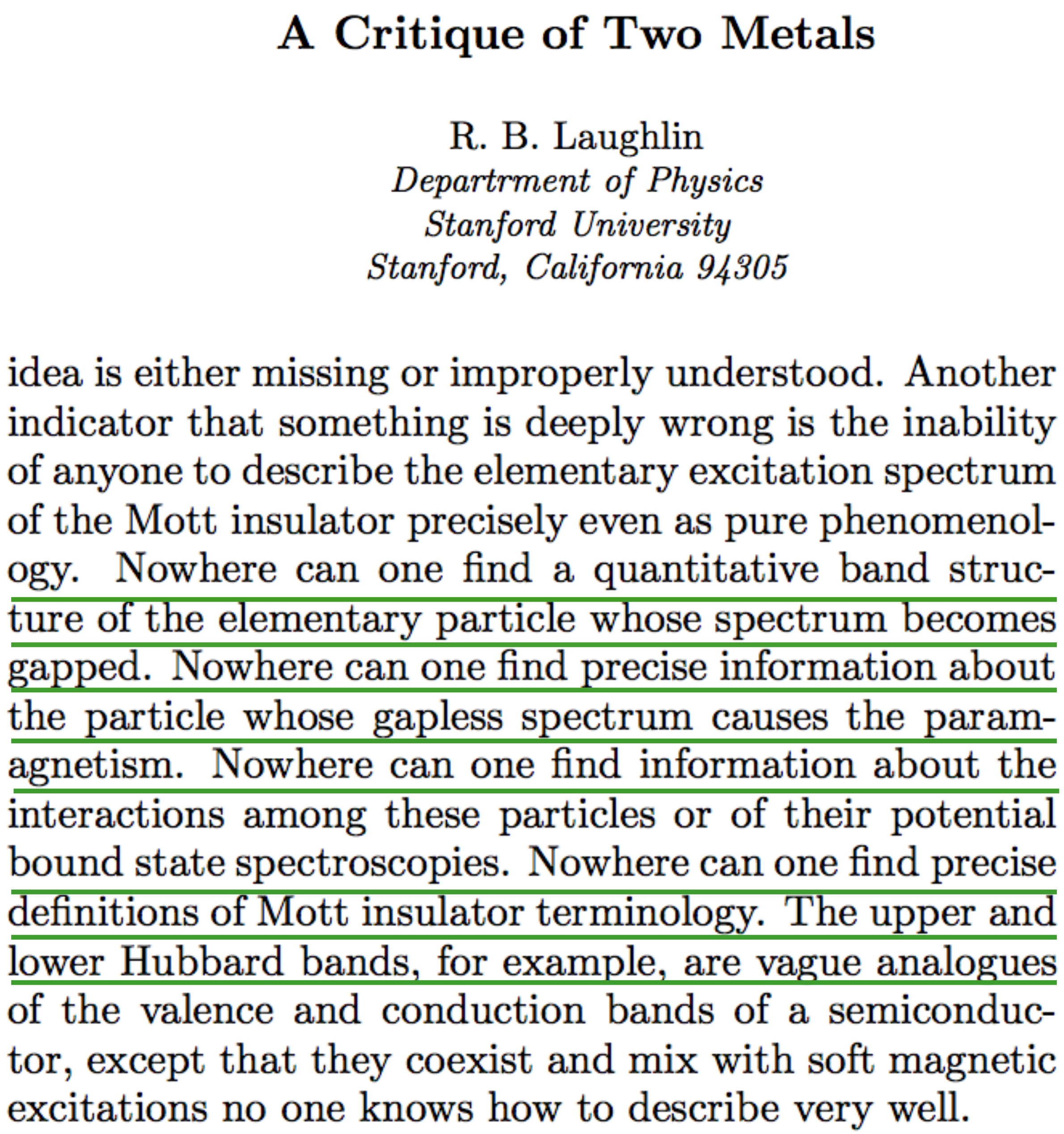
Progress thus far?

DMFT

QMC

disputes

Sept. 1997

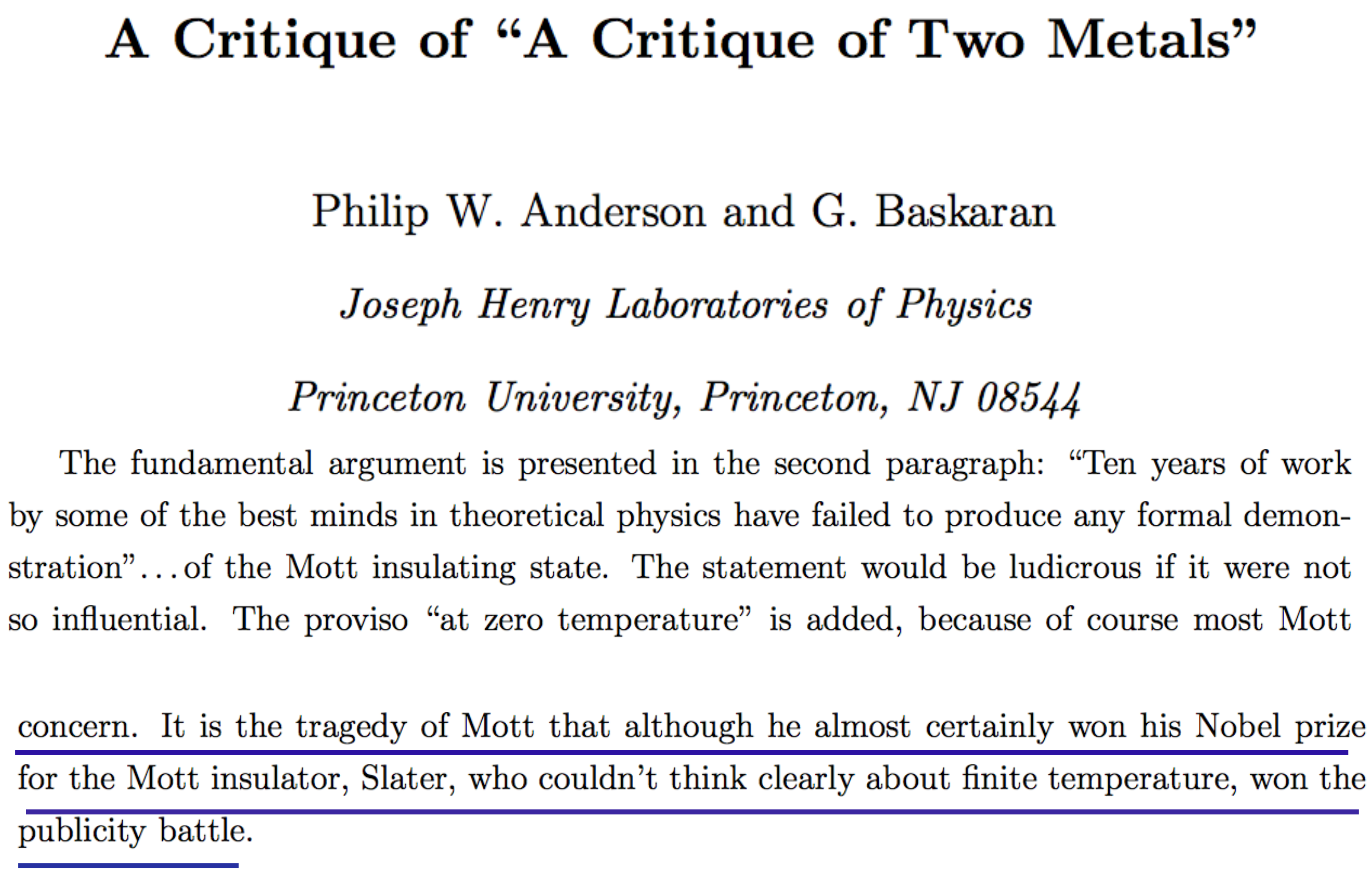


A Critique of Two Metals

R. B. Laughlin
Departrment of Physics
Stanford University
Stanford, California 94305

idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.

Nov. 1997



A Critique of "A Critique of Two Metals"

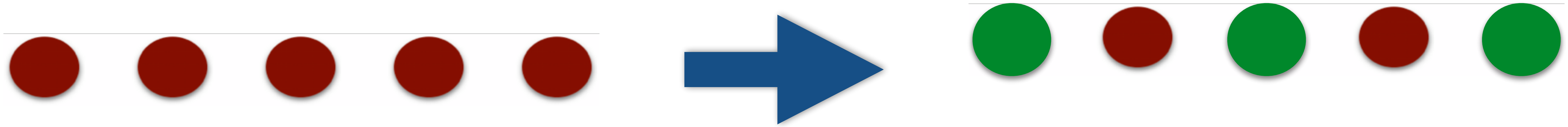
Philip W. Anderson and G. Baskaran

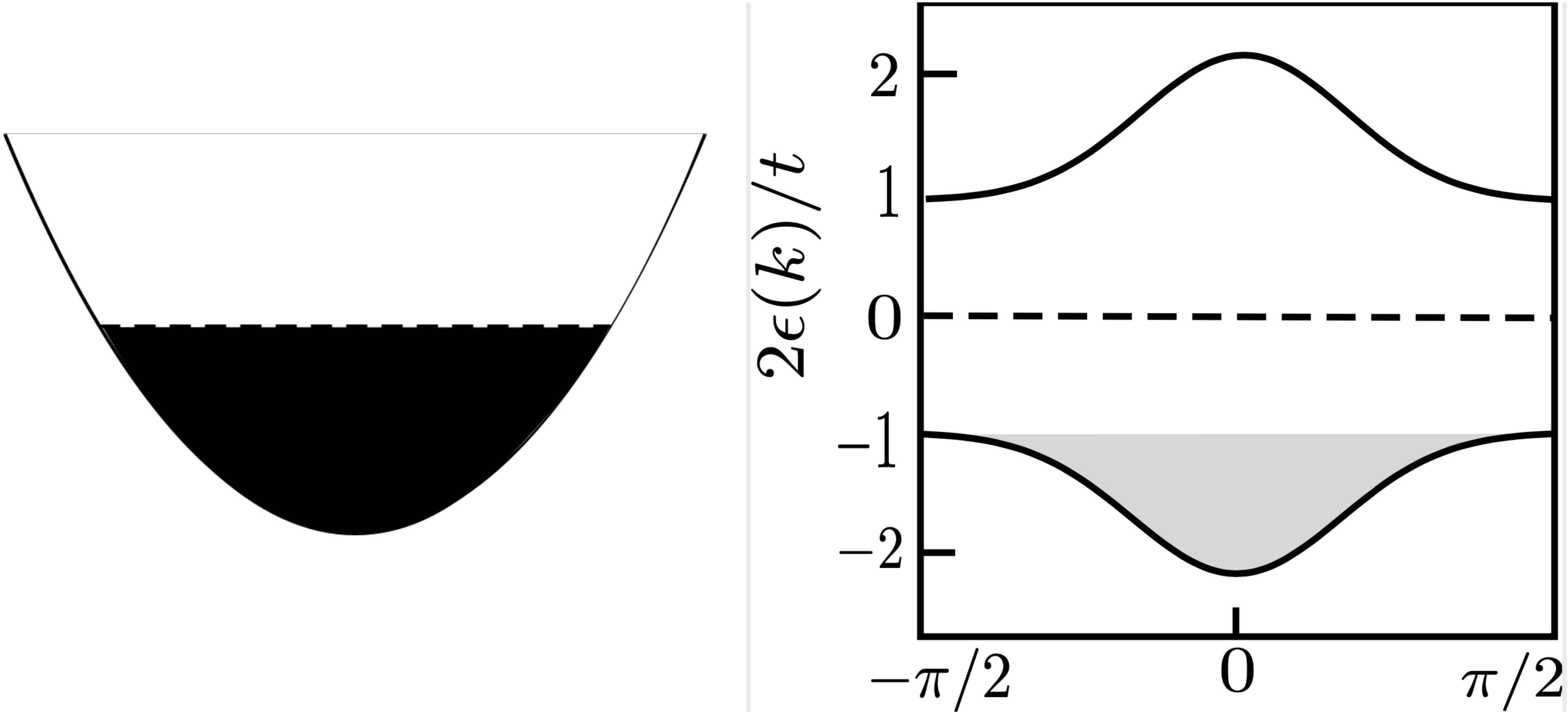
Joseph Henry Laboratories of Physics

Princeton University, Princeton, NJ 08544

The fundamental argument is presented in the second paragraph: "Ten years of work by some of the best minds in theoretical physics have failed to produce any formal demonstration"...of the Mott insulating state. The statement would be ludicrous if it were not so influential. The proviso "at zero temperature" is added, because of course most Mott

concern. It is the tragedy of Mott that although he almost certainly won his Nobel prize for the Mott insulator, Slater, who couldn't think clearly about finite temperature, won the publicity battle.





No Mott Problem

$-$

$= \boxed{\text{Mottness}}$

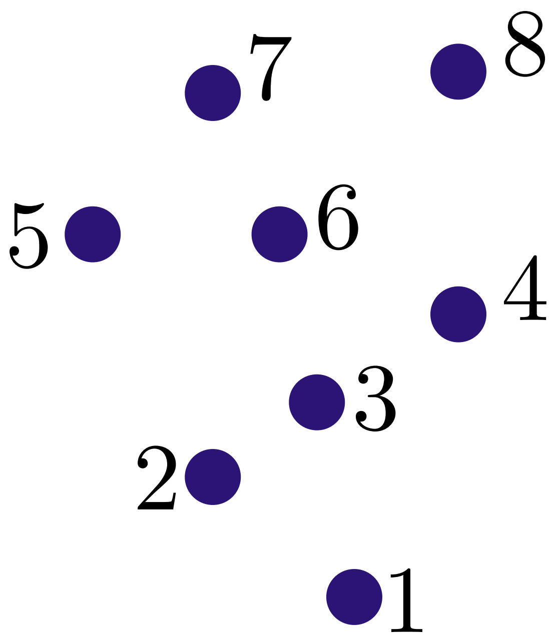
$\boxed{\text{zeros}}$

$= 0$

$\neq 0$

$$\mathrm{DetReG}(\omega = 0, \mathrm{p})$$

counting particles



is there a more efficient way?

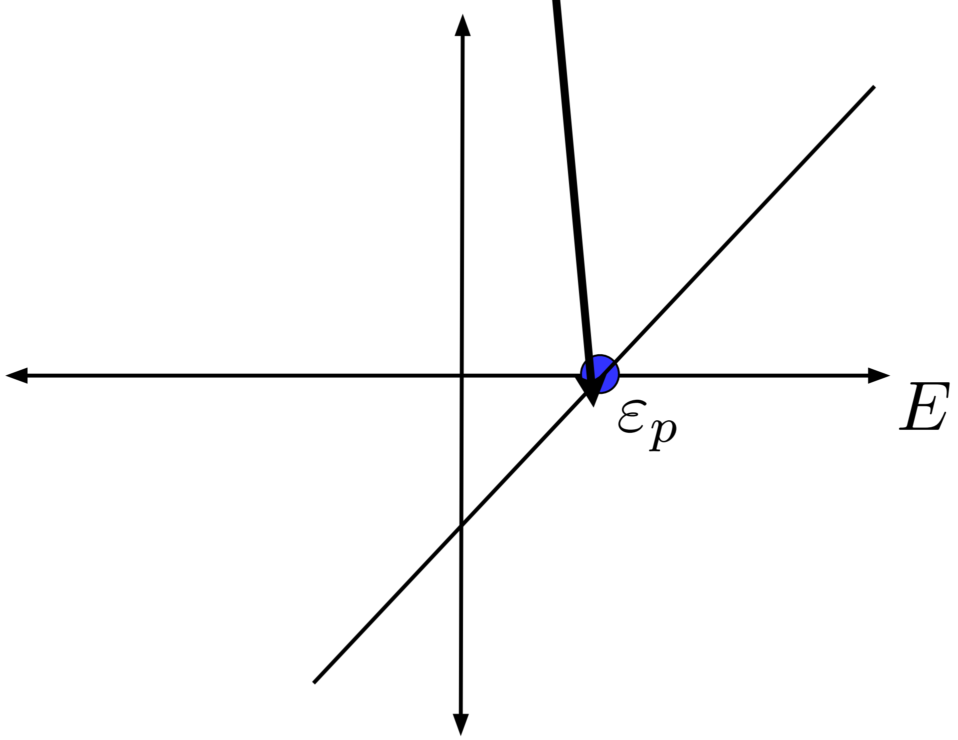
Luttinger counting theorem

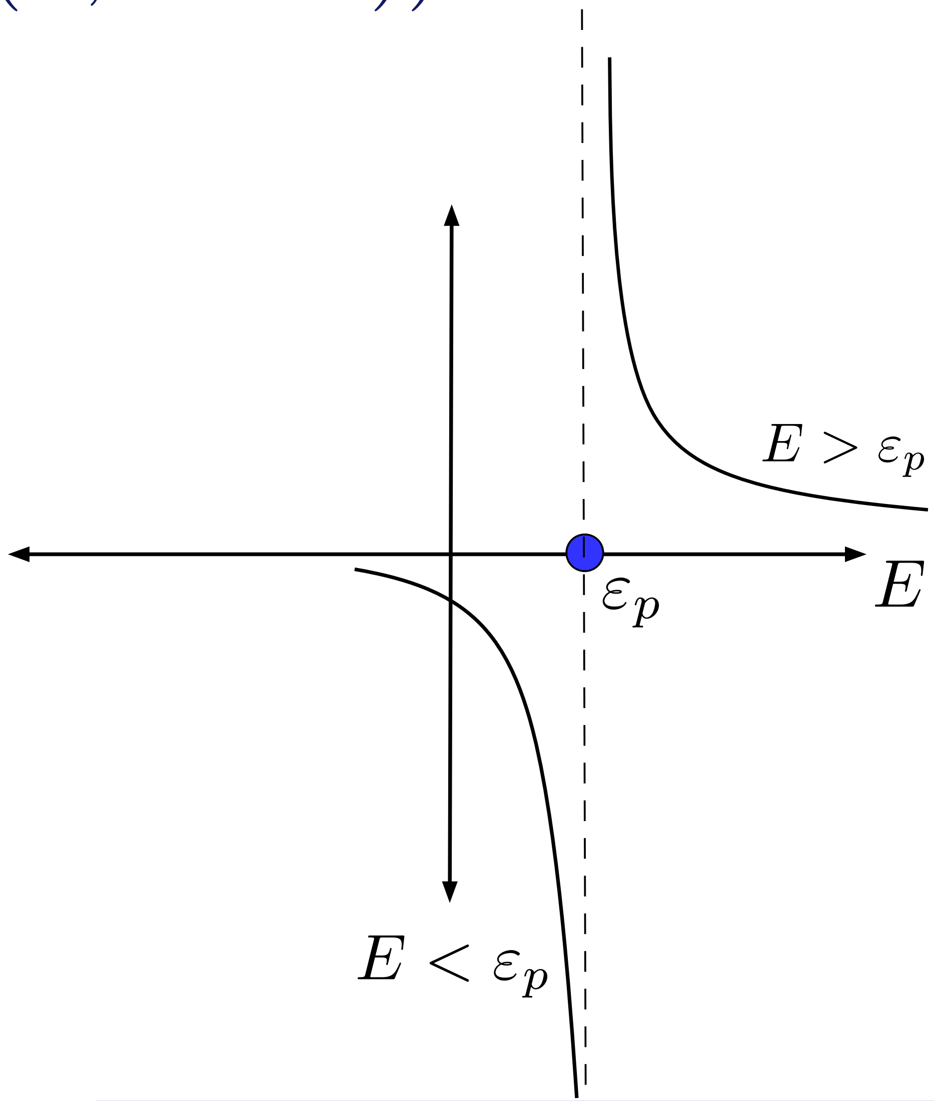
$$G(E) = \frac{1}{E - \varepsilon_p}$$

$$n = 2\sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

zero-crossing

$$\mathrm{Det}G(\omega = 0, \vec{p}) = 0$$



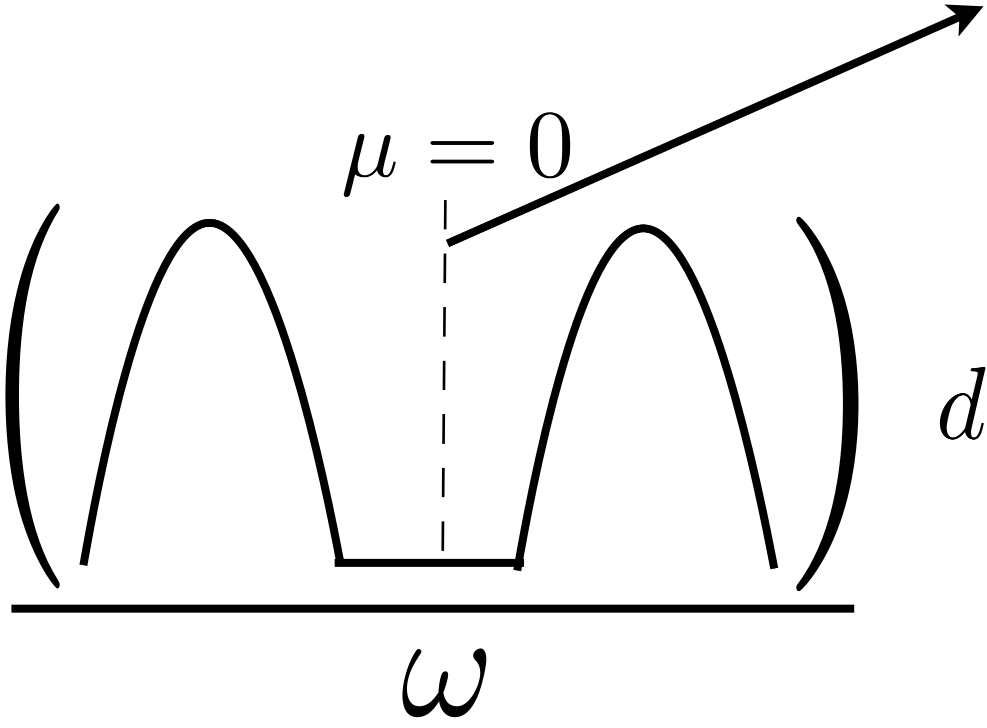


counting poles (qp)

How do zeros obtain?

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty}$$



$d\omega$

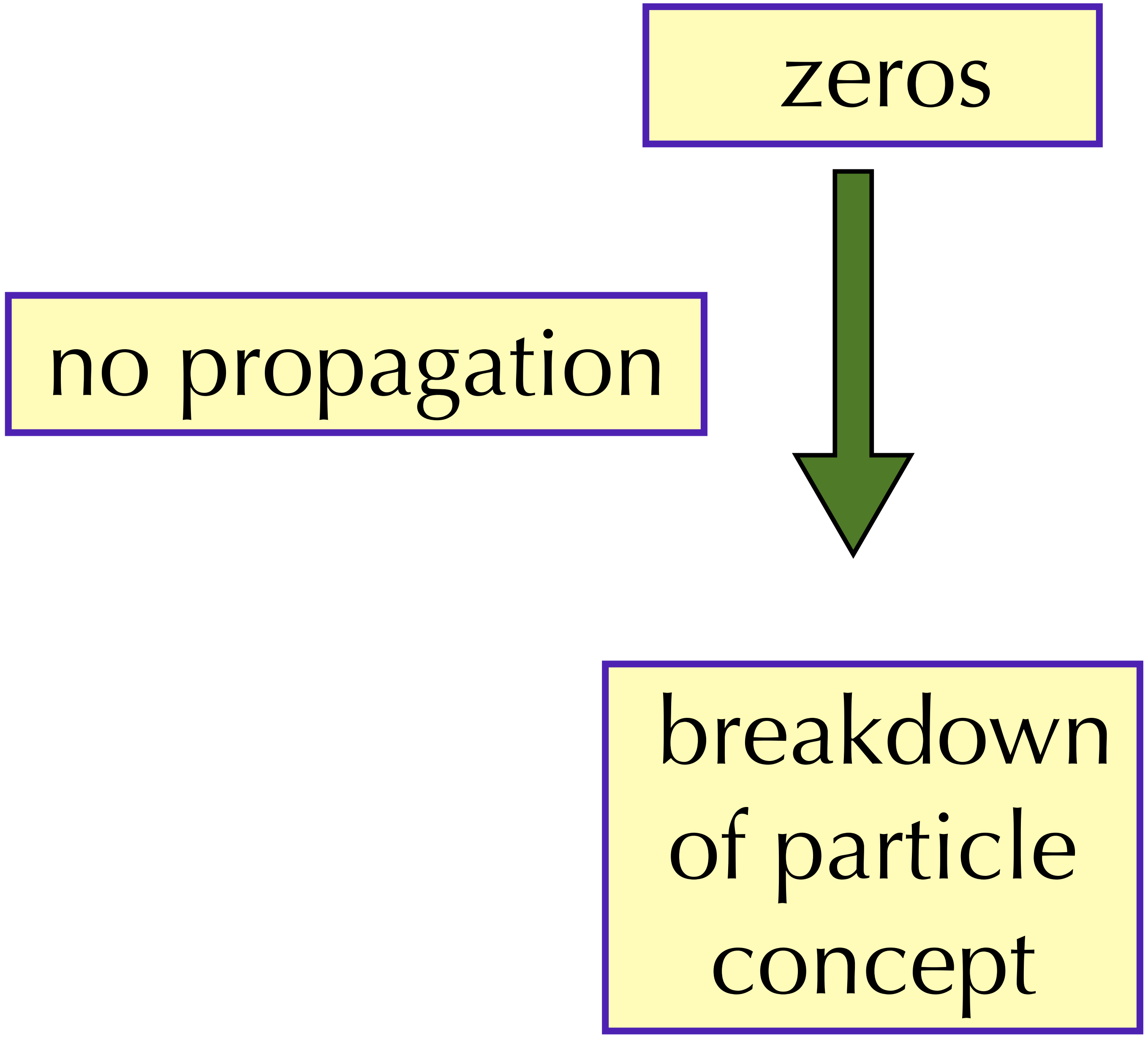
Kramers
-Kronig

$=$ below gap+above gap

$= 0$

$\mathrm{DetReG}(k, \omega = 0) = 0$ (single band)

strongly correlated gapped systems



zeros

no propagation

breakdown of particle concept

Mottness

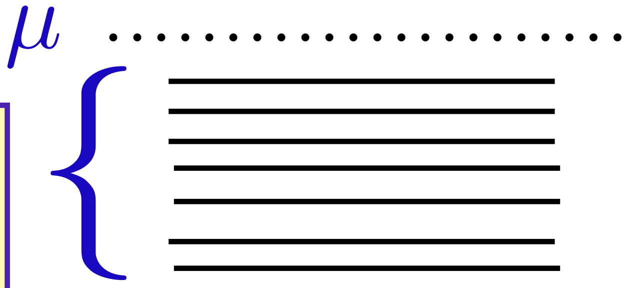
n=zeros+poles?

Minimal model
for Mottness?

~~Hubbard
model~~

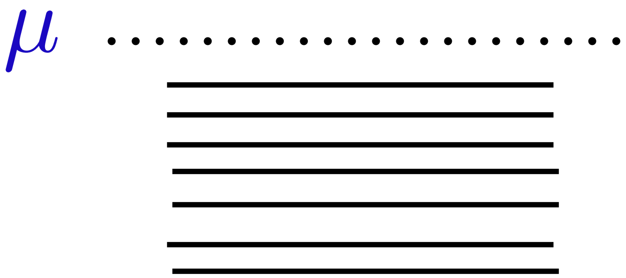
Fermi liquids

doubly occupied



NFL

Is single occupancy below chemical potential possible?



with time-reversal symmetry in tact?

Fermi liquids

Anderson
Haldane
2000

3 citations

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + \overset{\to 0}{\cdots}$$

$(n_{p\uparrow}, n_{p\downarrow})$ conserved currents

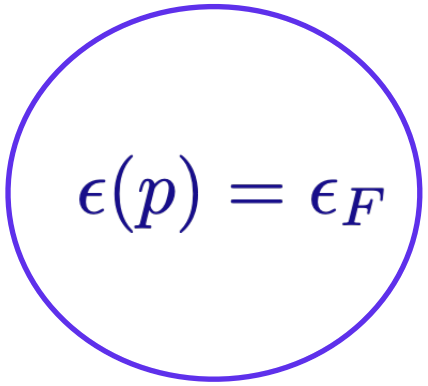
$(c_{p\uparrow}, c_{p\downarrow}, \text{h.c.})$ 4 objects

$$\begin{tikzcd} & O(4) \arrow[dl, "\text{DetM} = 1"'] \arrow[dr, "\text{DetM} = -1"] & \\ \text{proper rotations} & & \text{improper rotations} \end{tikzcd}$$

SO(4) proper rotations

improper rotations

$$\text{Det}\, M = \pm 1 \implies Z_2 = O(4) \div SO(4)$$



$\epsilon(p) = \epsilon_F$

Fermi
Surface

$$H = 0$$

$$\left.\begin{array}{l} n_{p\uparrow} \to -n_{p\uparrow} \\ n_{p\downarrow} \to n_{p\downarrow} \end{array}\right\} Z_2$$

at Fermi
surface only

How to destroy Fermi liquids?

$\cancel{Z_2}$

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + \boxed{U n_{p\uparrow} n_{p\downarrow}}$$

odd under Z_2

New fixed point!

Hatsugai-Kohmoto model

scaling dimension

$[n_{p\uparrow} n_{p\downarrow}] = -2$

relevant interaction

Hubbard not necessary!

General HK Model

$$\sum_k (\xi_k (n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow})$$

relevant perturbation

$$[H_t, H_U] = 0$$

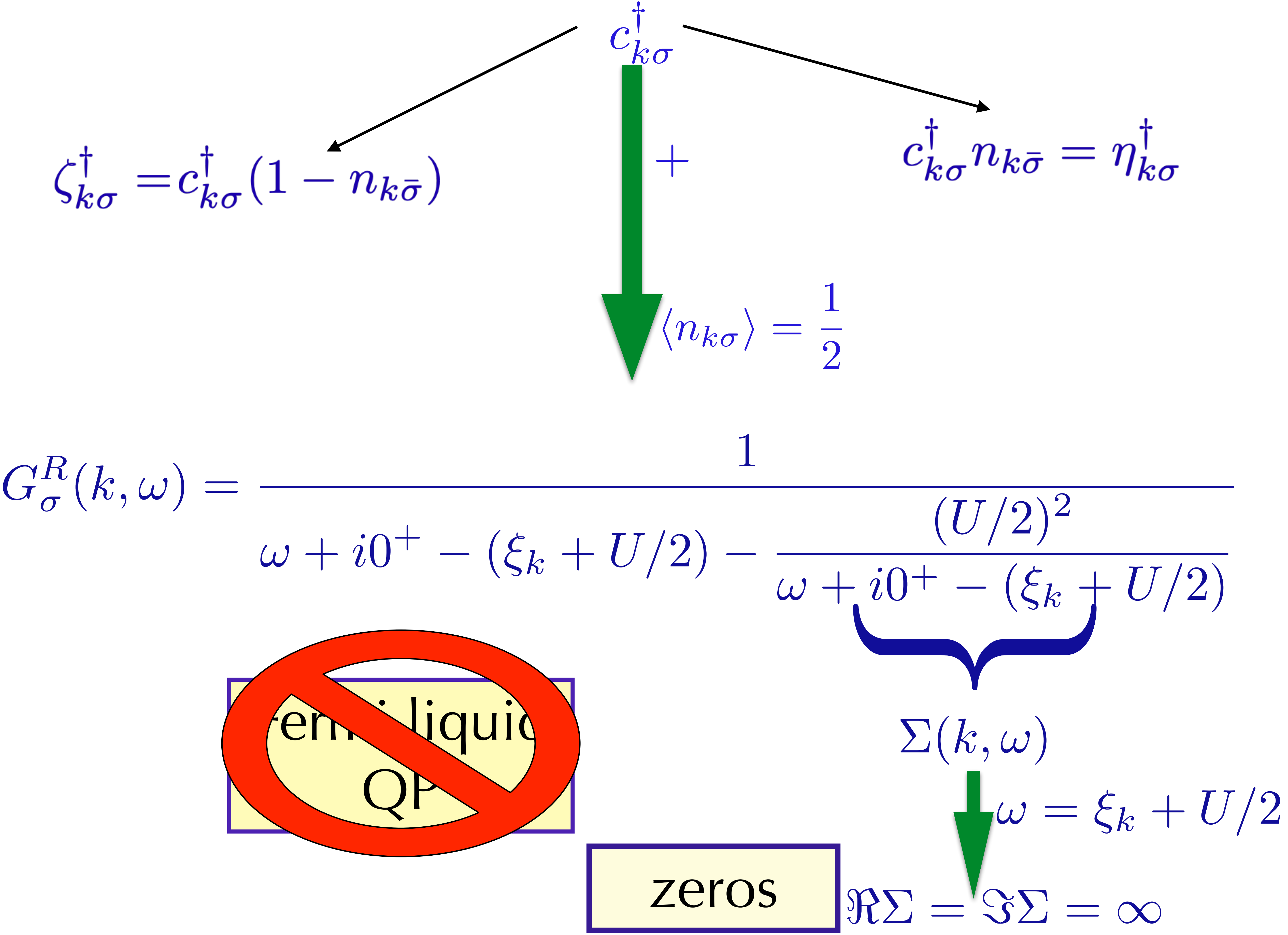
Solvable Mott transition

$$G_{k\sigma}(i\omega_n \to z) = \frac{1 - \langle n_{k\bar{\sigma}} \rangle}{z - \xi_k} + \frac{\langle n_{k\bar{\sigma}} \rangle}{z - (\xi_k + U)} \neq \frac{1}{z - \xi_k}$$

lower Hubbard band

upper Hubbard band

Hubbard band operators



$c^{\dagger}_{k\sigma}$

$\zeta^{\dagger}_{k\sigma} = c^{\dagger}_{k\sigma}(1 - n_{k\bar{\sigma}})$

$+$

$c^{\dagger}_{k\sigma} n_{k\bar{\sigma}} = \eta^{\dagger}_{k\sigma}$

$\langle n_{k\sigma} \rangle = \frac{1}{2}$

$$G^R_\sigma(k,\omega) = \frac{1}{\omega + i0^+ - (\xi_k + U/2) - \frac{(U/2)^2}{\underbrace{\omega + i0^+ - (\xi_k + U/2)}_{\Sigma(k,\omega)}}}$$

ferm i liquic
QP

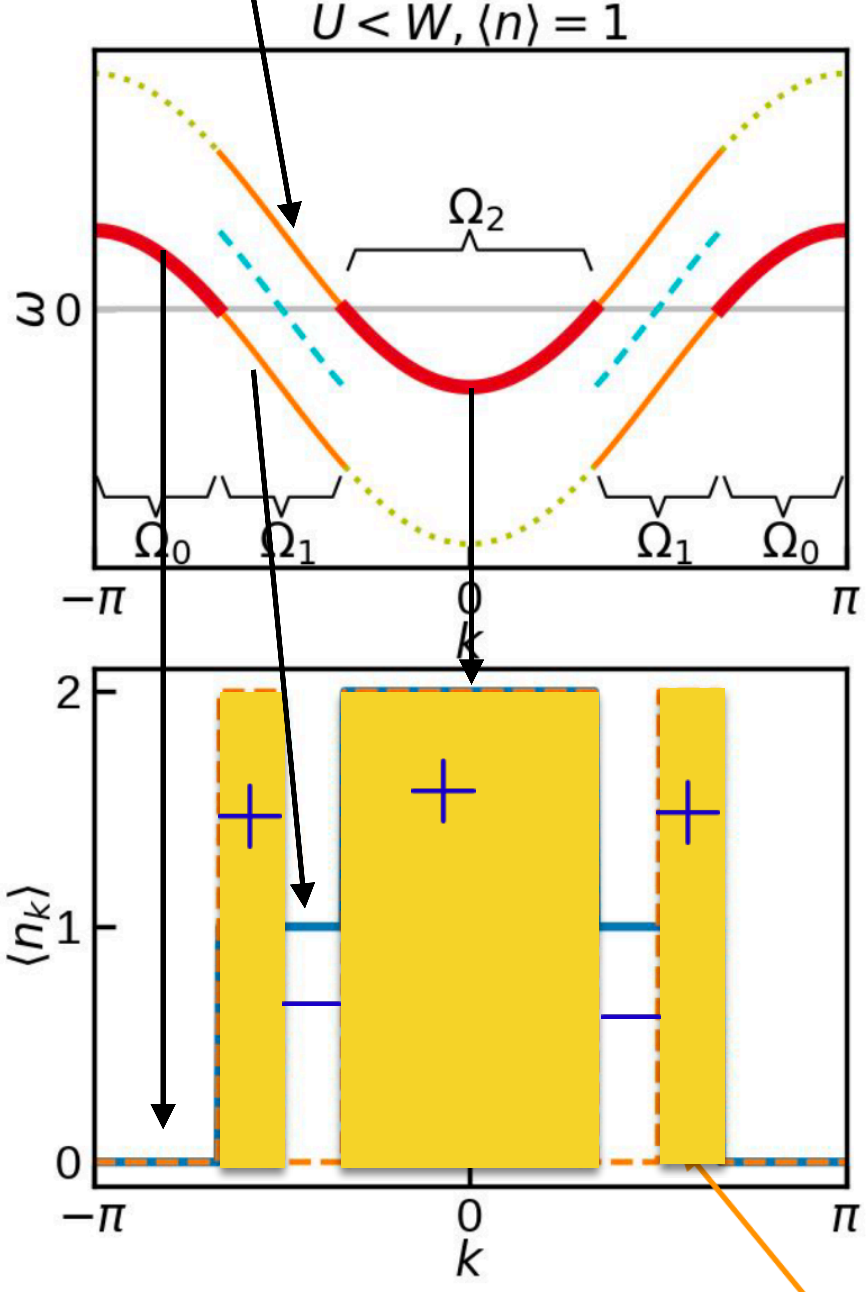
$$\xrightarrow{\omega = \xi_k + U/2}$$

zeros

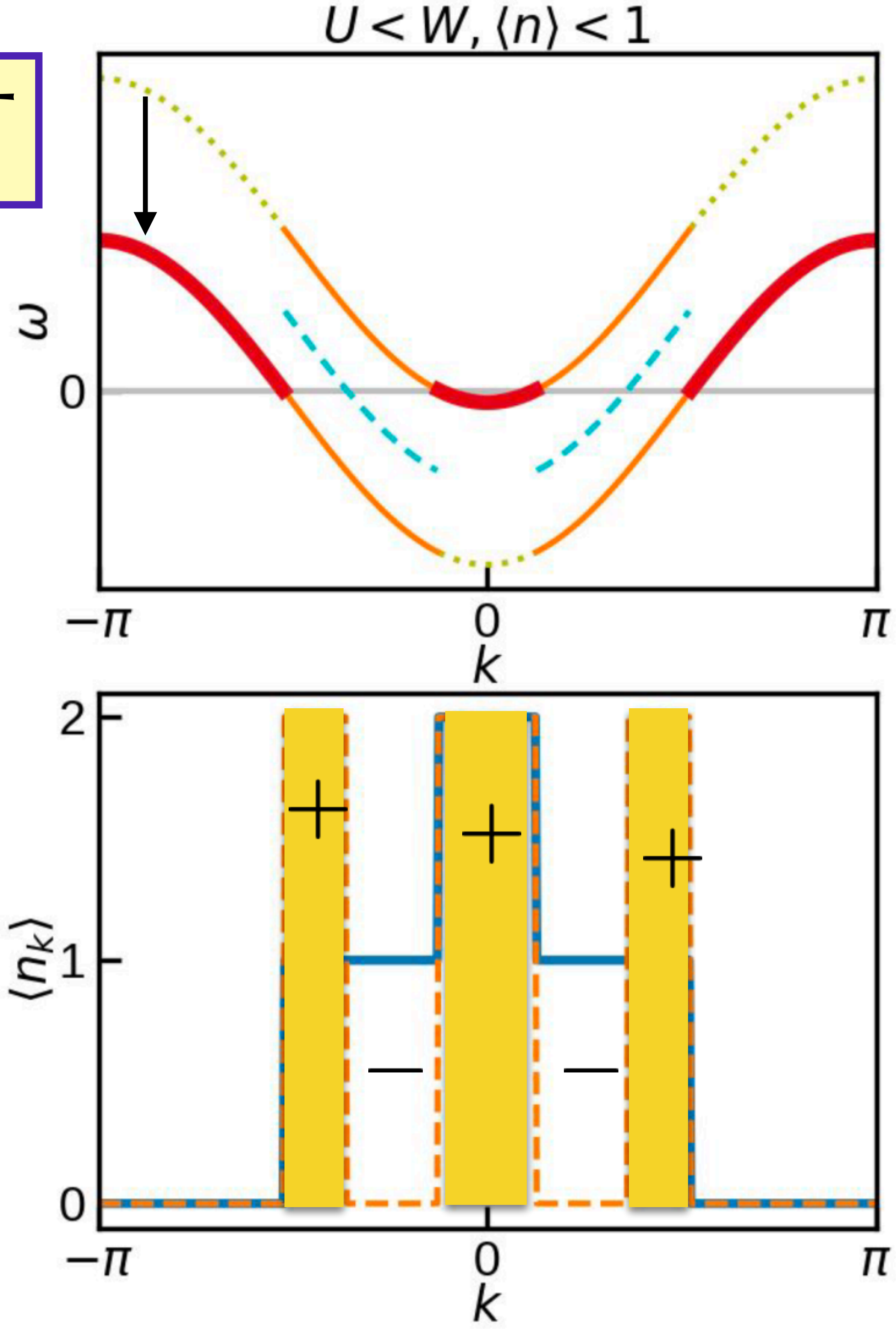
$\Re\Sigma = \Im\Sigma = \infty$

single occupancy

counting charges



SWT



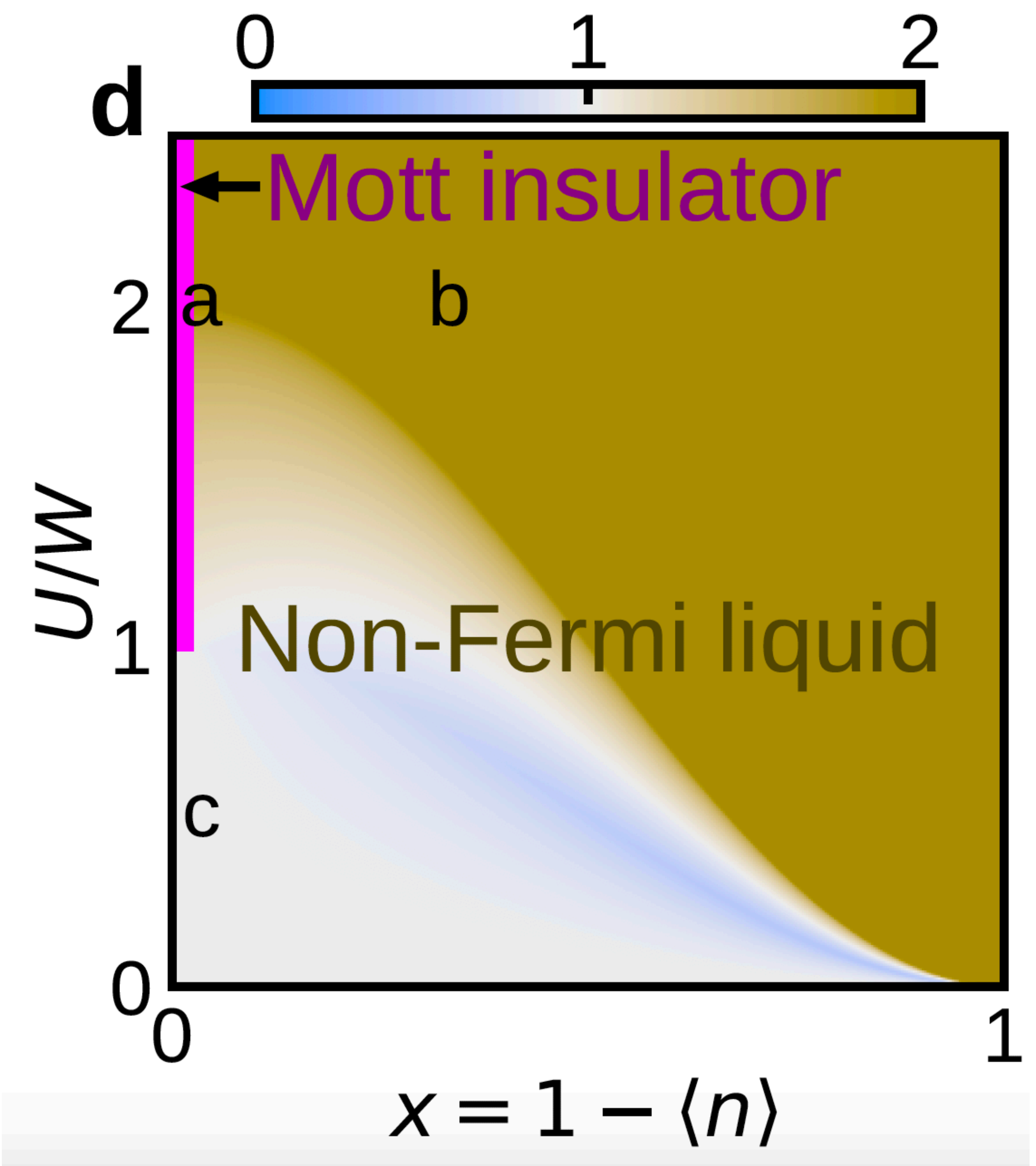
$n_{\mathrm{Lutt}} = \langle n \rangle$

$2\theta(\mathrm{Re}\, G(\mathbf{k}, \omega = 0))$

$n_{\mathrm{Lutt}} \neq \langle n \rangle$

zeros≠particles

Why NFL?



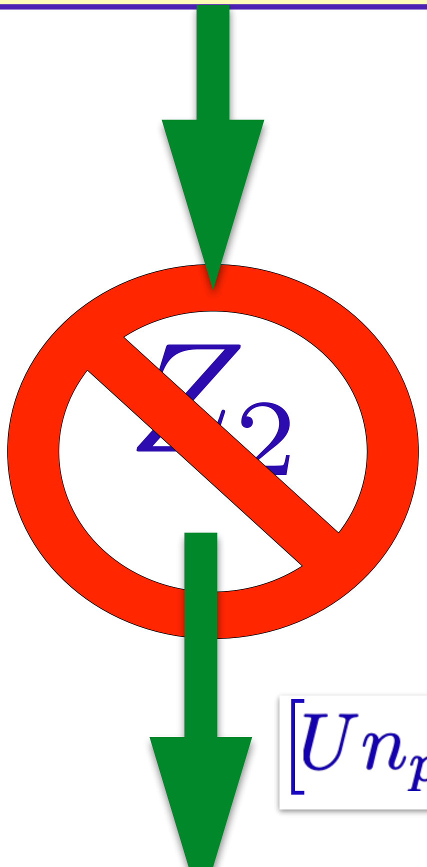
FL

HK
Mottness

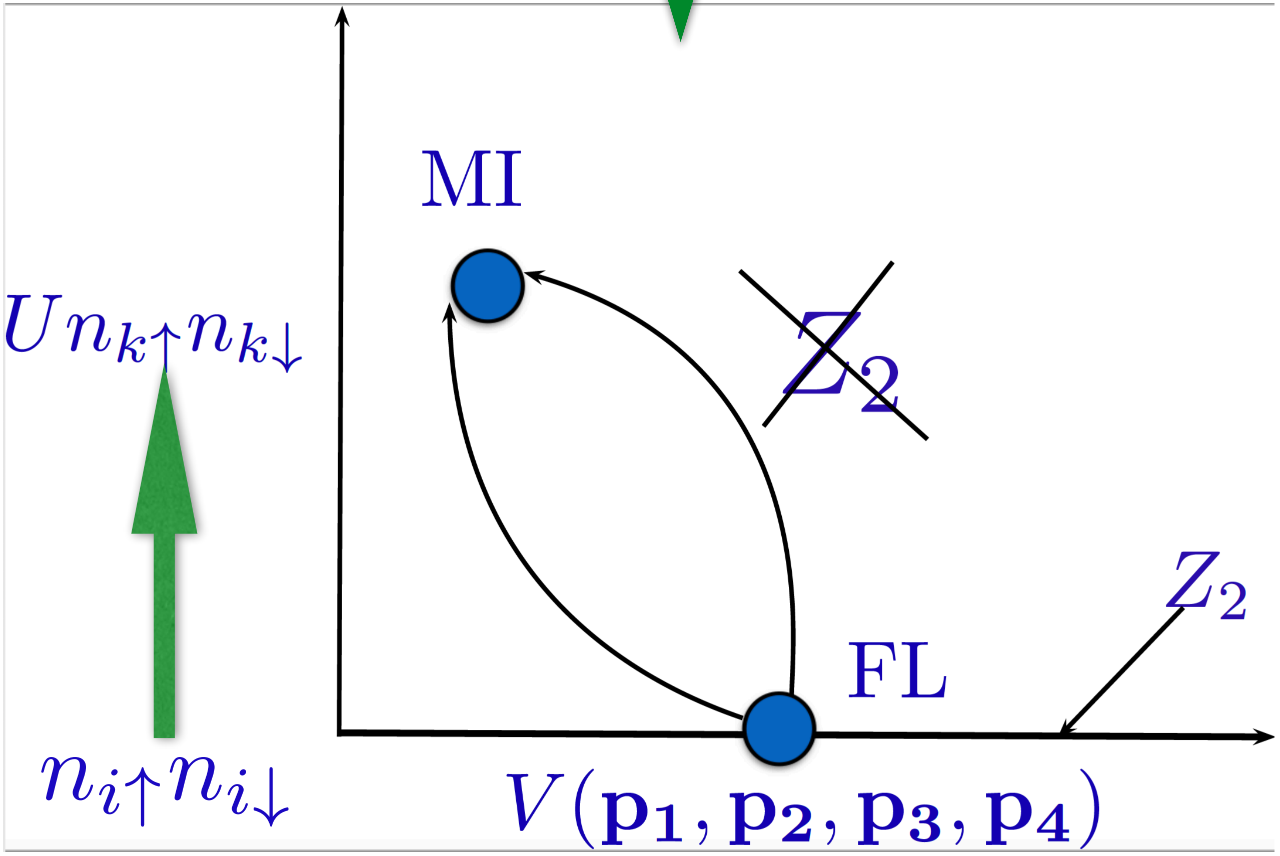
$$\begin{tikzcd} c^\dagger_{k\uparrow}c^\dagger_{k\downarrow}|G\rangle \arrow[rr, leftrightarrow, "\times" description] & & ? \\ c^\dagger_{k\uparrow}|G\rangle, c^\dagger_{k\downarrow}|G\rangle \arrow[rr, leftrightarrow] & & \zeta^\dagger_{k\uparrow}|G\rangle, \zeta^\dagger_{k\downarrow}|G\rangle \\ |G\rangle \arrow[u] & & |G\rangle \arrow[u] \end{tikzcd}$$

$$\zeta^\dagger_{k\uparrow}\zeta^\dagger_{k\downarrow}|G\rangle = 0$$

Fermi
liquids

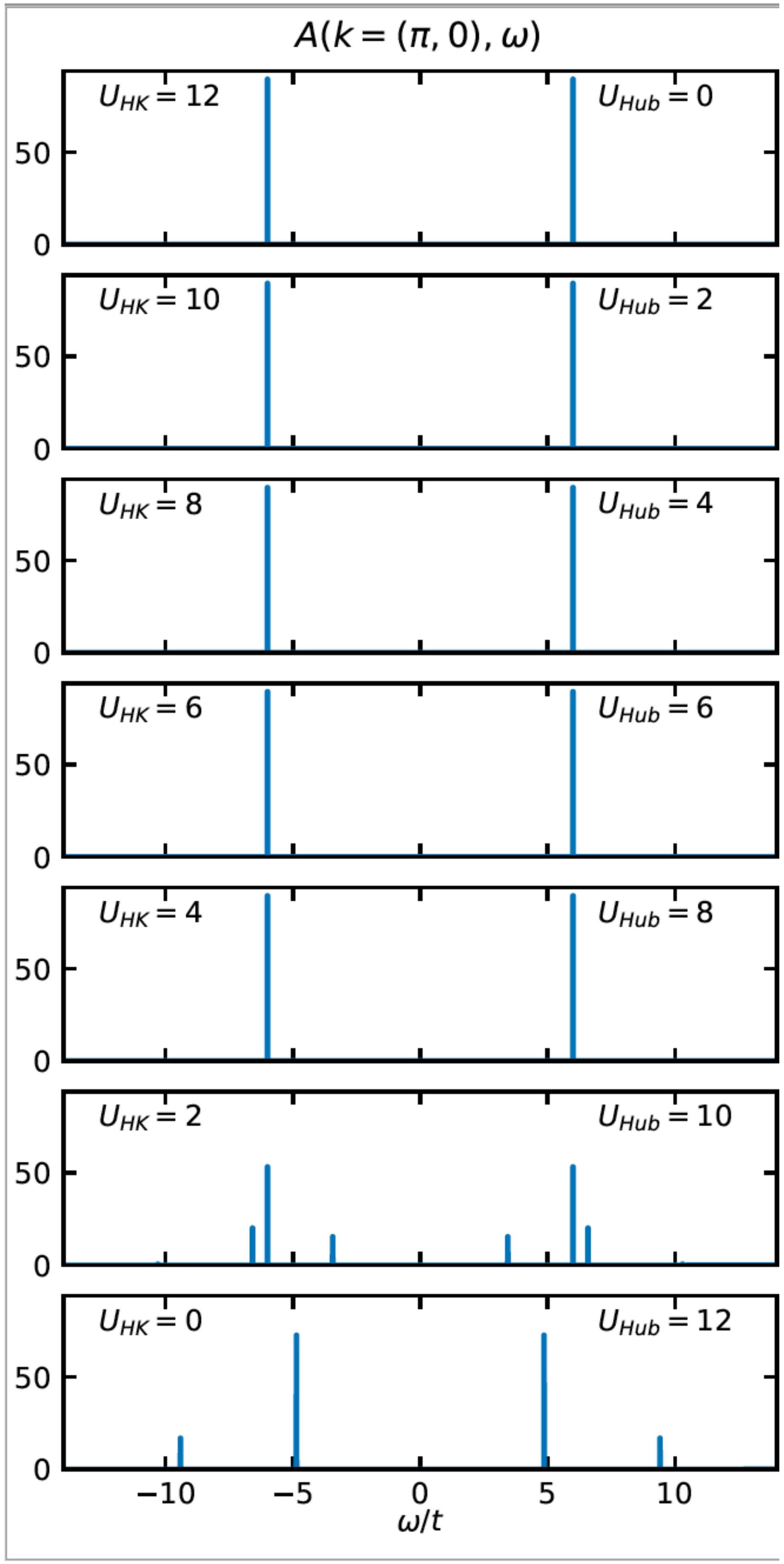


$$[U n_{p\uparrow} n_{p\downarrow}] = -2$$

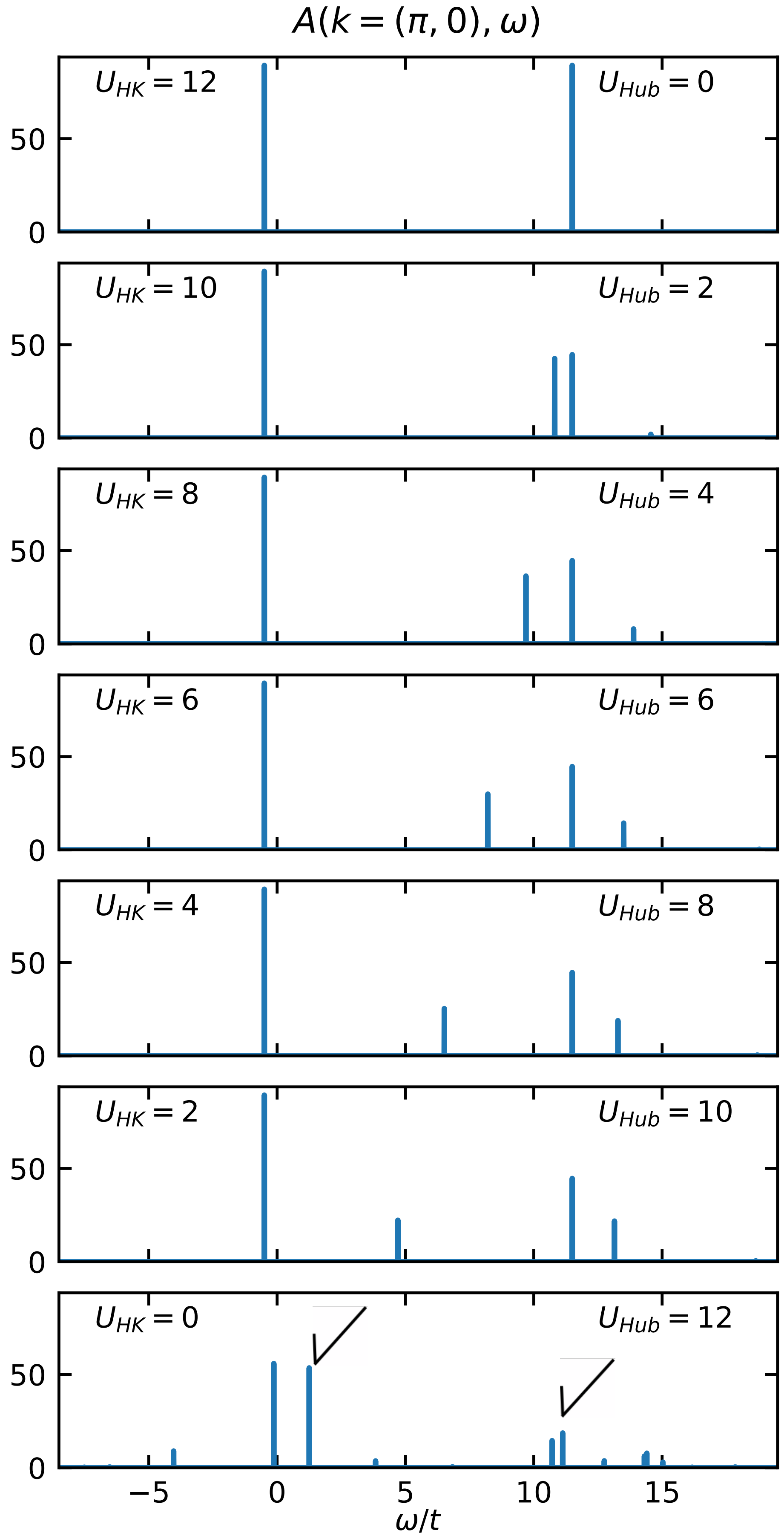


Hubbard
not
necessary
(universality
class)

$n = 1.0$
$H_{\mathrm{HK}} \approx H_{\mathrm{Hubb}}$

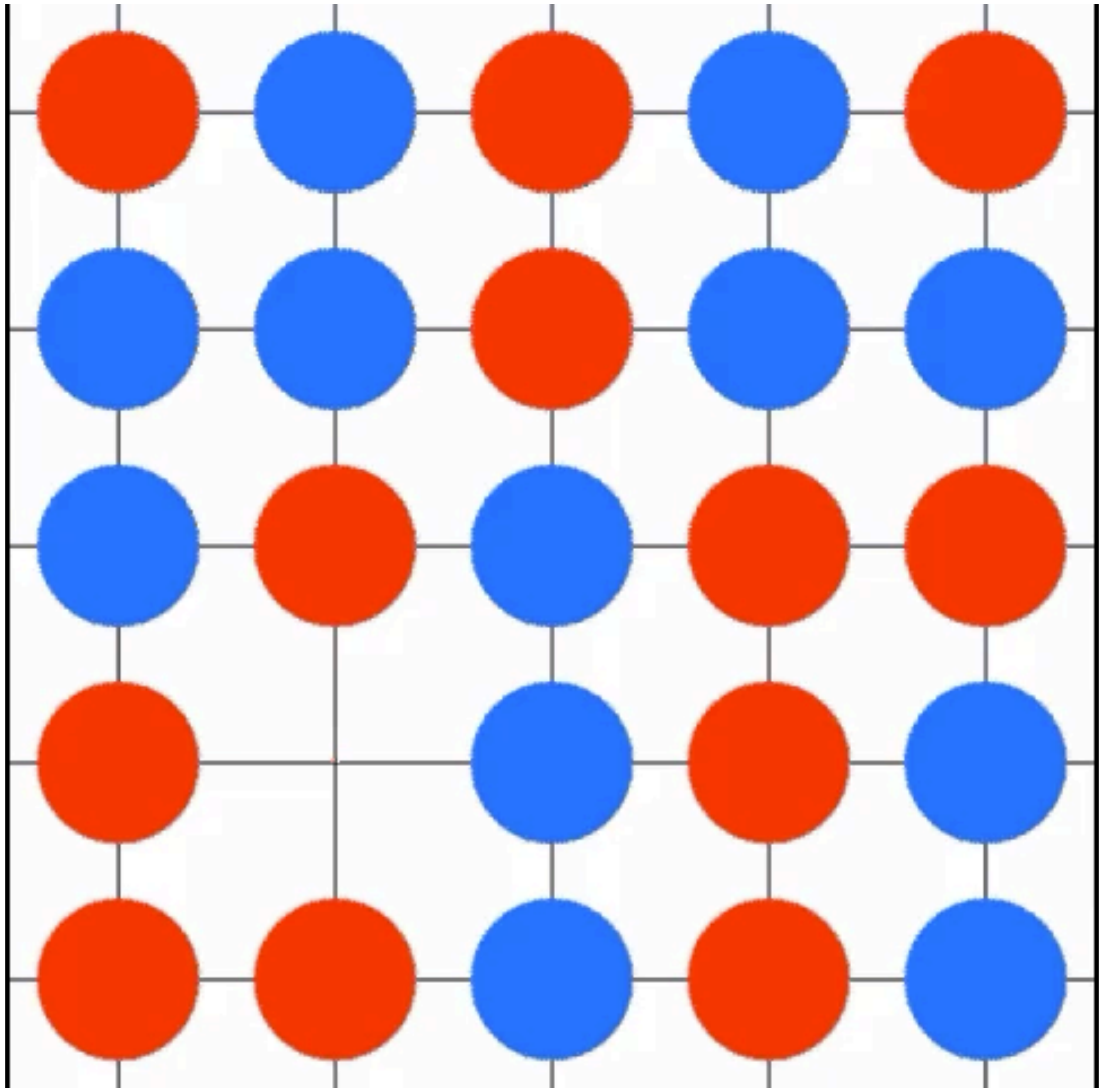


$n = 0.875$
DSWT



what does the HK model leave out??

$$[H_t, H_U] \neq 0$$



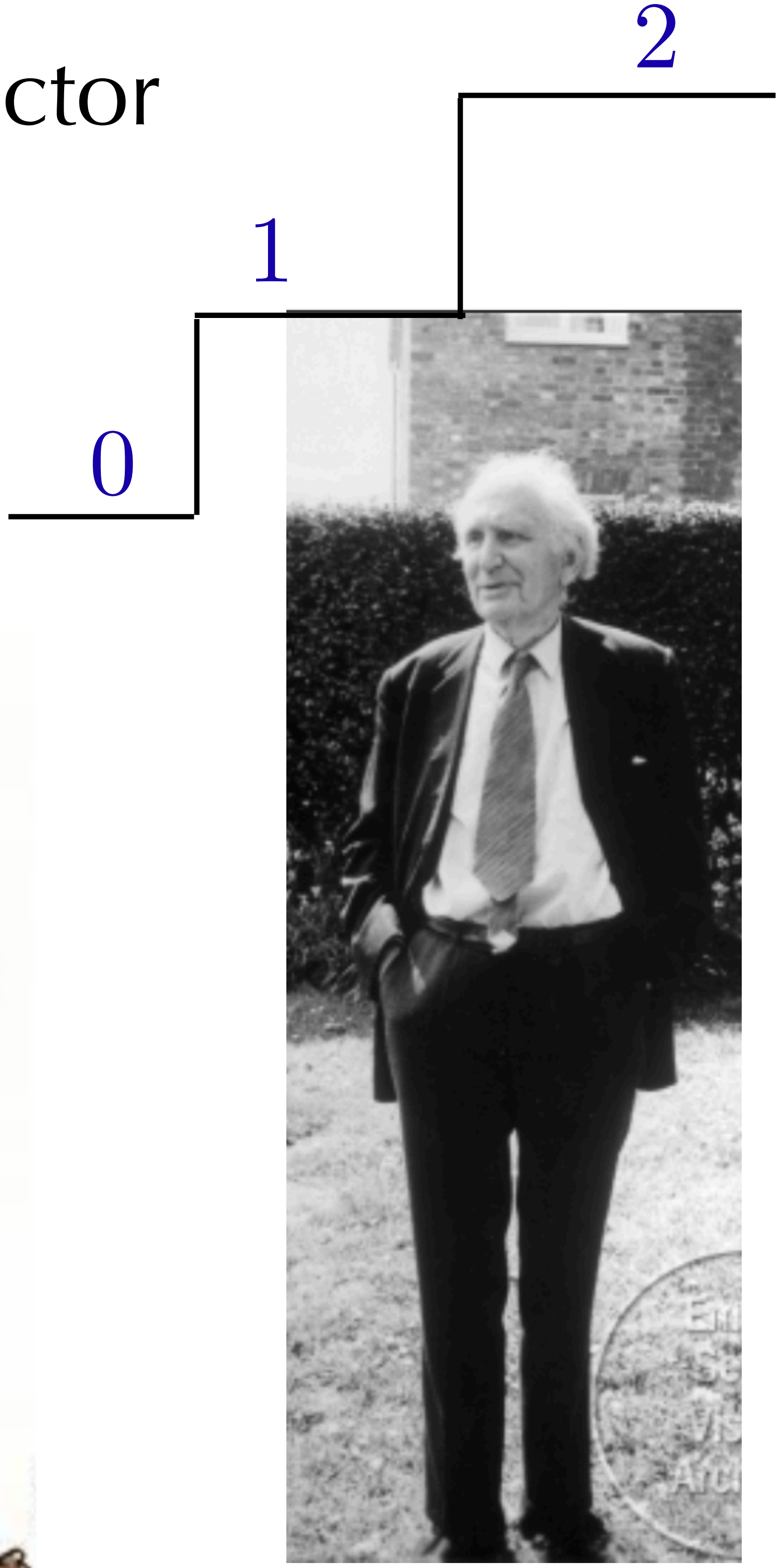
dynamical spectral weight transfer

Fermi gas

Fermi liquid

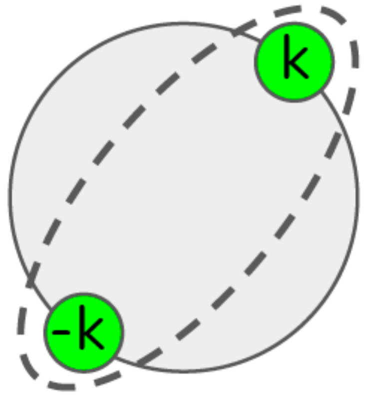
BCS superconductor

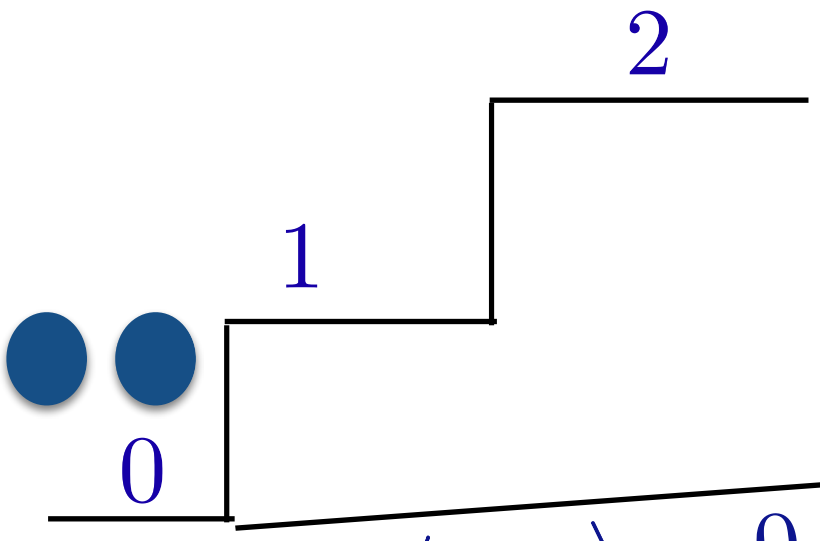
Mottness



Superconductivity?

Cooper Instability





$$H = H_{\mathrm{HK}} - gH_p$$

$$|\psi\rangle = \sum_{k\in\Omega_0} \alpha_k b_k^\dagger |\mathrm{GS}\rangle$$

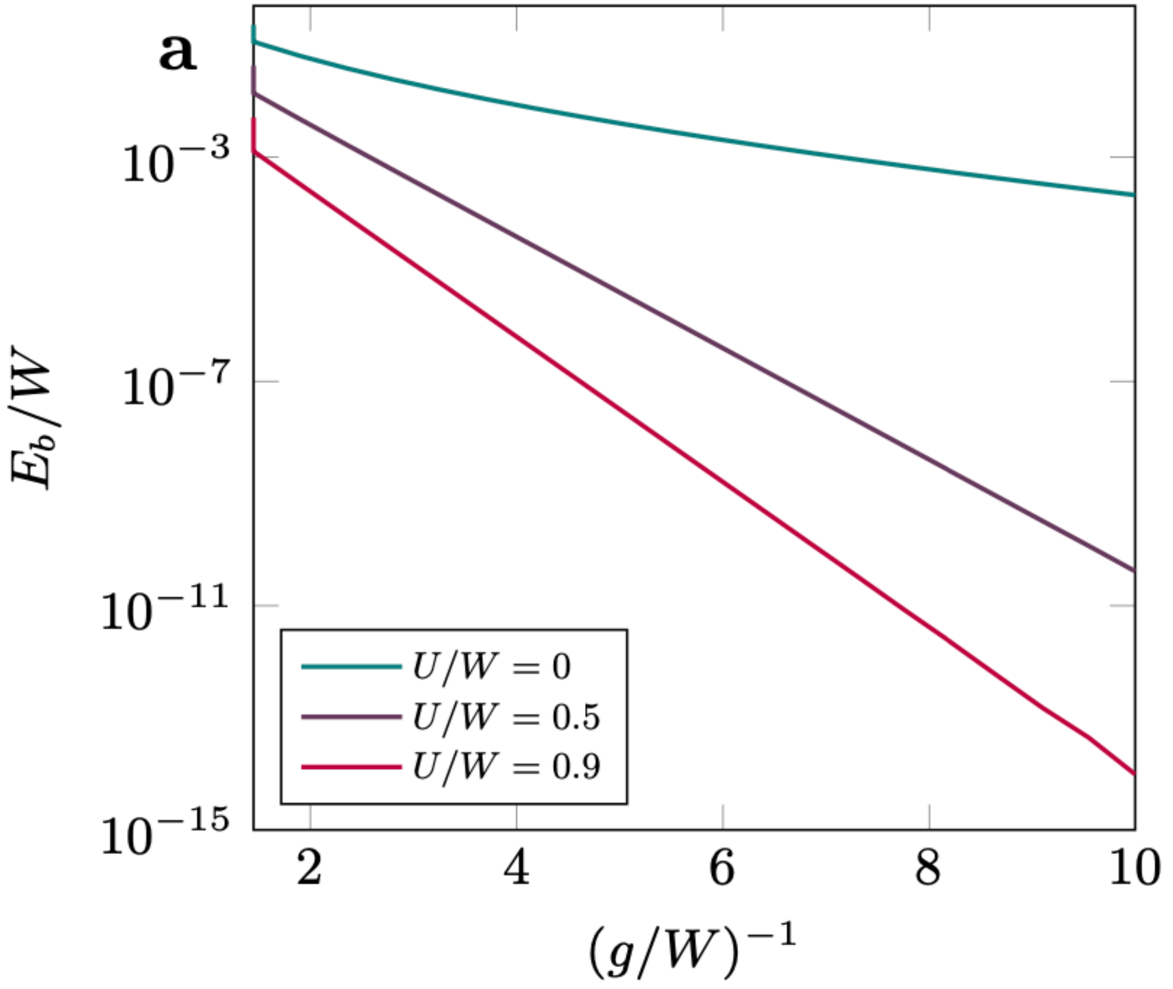
$$= \sum_{k,k'} b_k^\dagger b_{k'}$$

$$\langle n_{k\sigma}\rangle = 0$$

$$\boxed{E_b = \langle \mathrm{GS}|\mathrm{H}|\mathrm{GS}\rangle - \langle\psi|\mathrm{H}|\psi\rangle \lessgtr 0}$$

Cooper Instability

$$E_b = -E \sim W(1-(U/W)^2)e^{-\pi W\sqrt{1-(U/W)^2}/g}$$



Pair Susceptibility

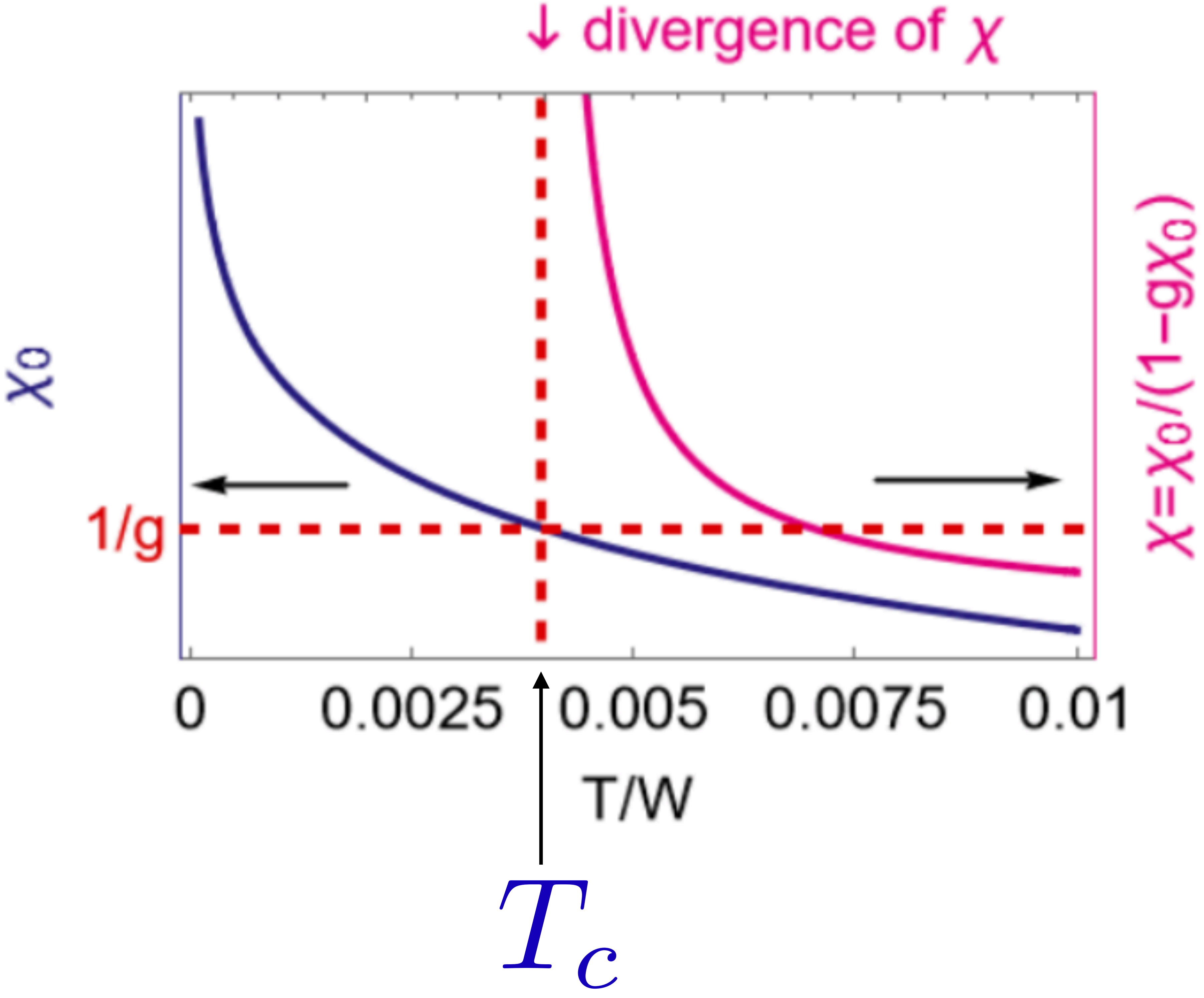
$$\chi(i\nu_n) \equiv \frac{1}{L^d}\int_0^\beta d\tau e^{i\nu_n\tau}\langle T\Delta(\tau)\Delta^\dagger\rangle_g$$

$$\Big\downarrow$$

$$= \frac{\chi_0}{1 - g\chi_0}$$

$$\Big\downarrow \quad g\chi_0 = 1$$

solve for T_c


↓ divergence of χ
χ_0
$\chi=\chi_0/(1-g\chi_0)$
1/g
0
0.0025
0.005
0.0075
0.01
T/W
T_c

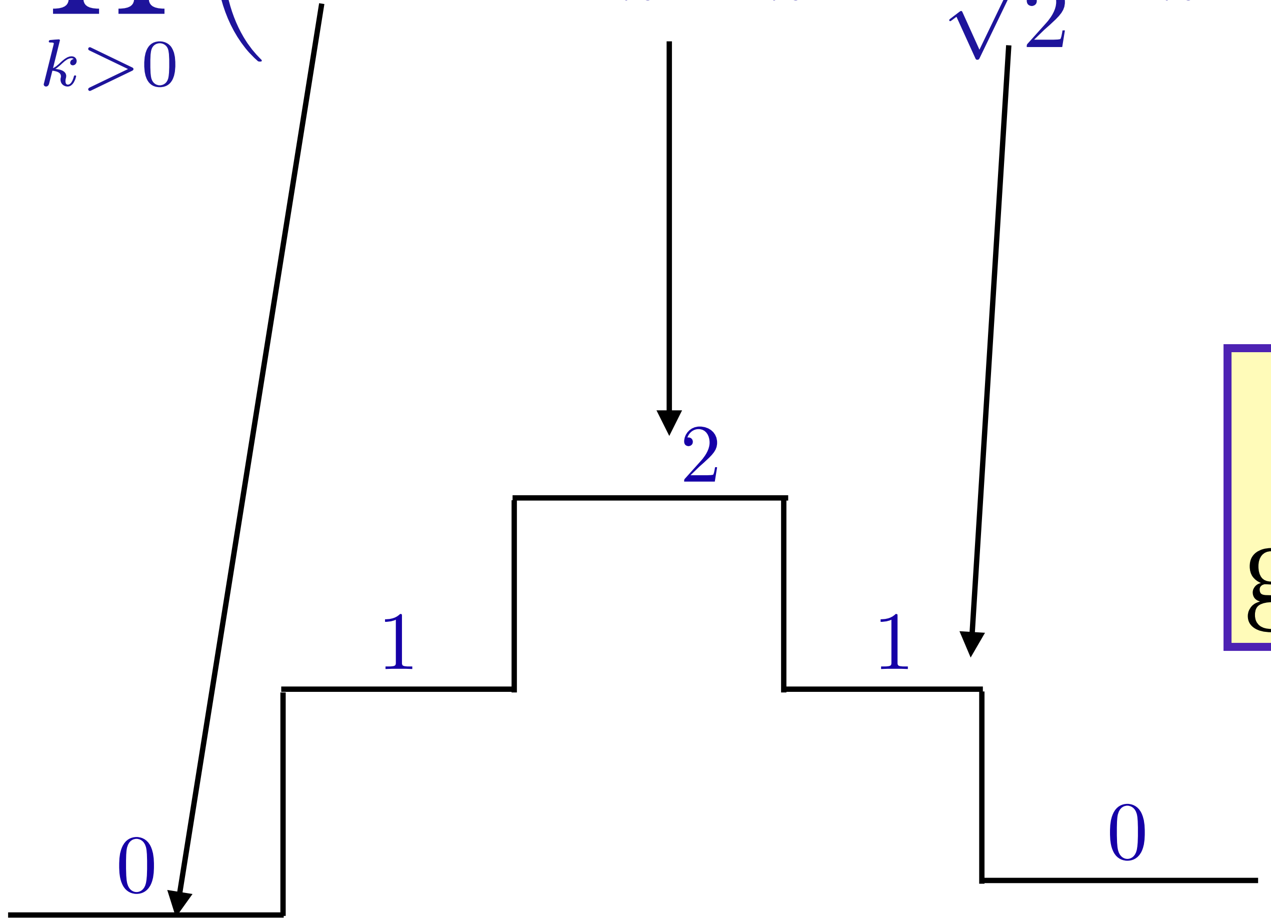
variational wave function

$$|\psi_{\mathrm{BCS}}\rangle = \prod_k (u_k + v_k b_k^\dagger |0\rangle$$

↓

$$|\psi_{\mathrm{BCS}}\rangle = \prod_{k>0} (u_k^2 + v_k^2 b_k^\dagger b_{-k}^\dagger + u_k v_k (b_k^\dagger + b_{-k}^\dagger))|0\rangle$$

$$|\psi\rangle = \prod_{k>0}\left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}}(b_k^\dagger + b_{-k}^\dagger)\right)|0\rangle$$



HK
generalization

three variational parameters

$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation

$$\Big\downarrow \; \Delta \ll U, W$$

$$\Delta = (W-U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

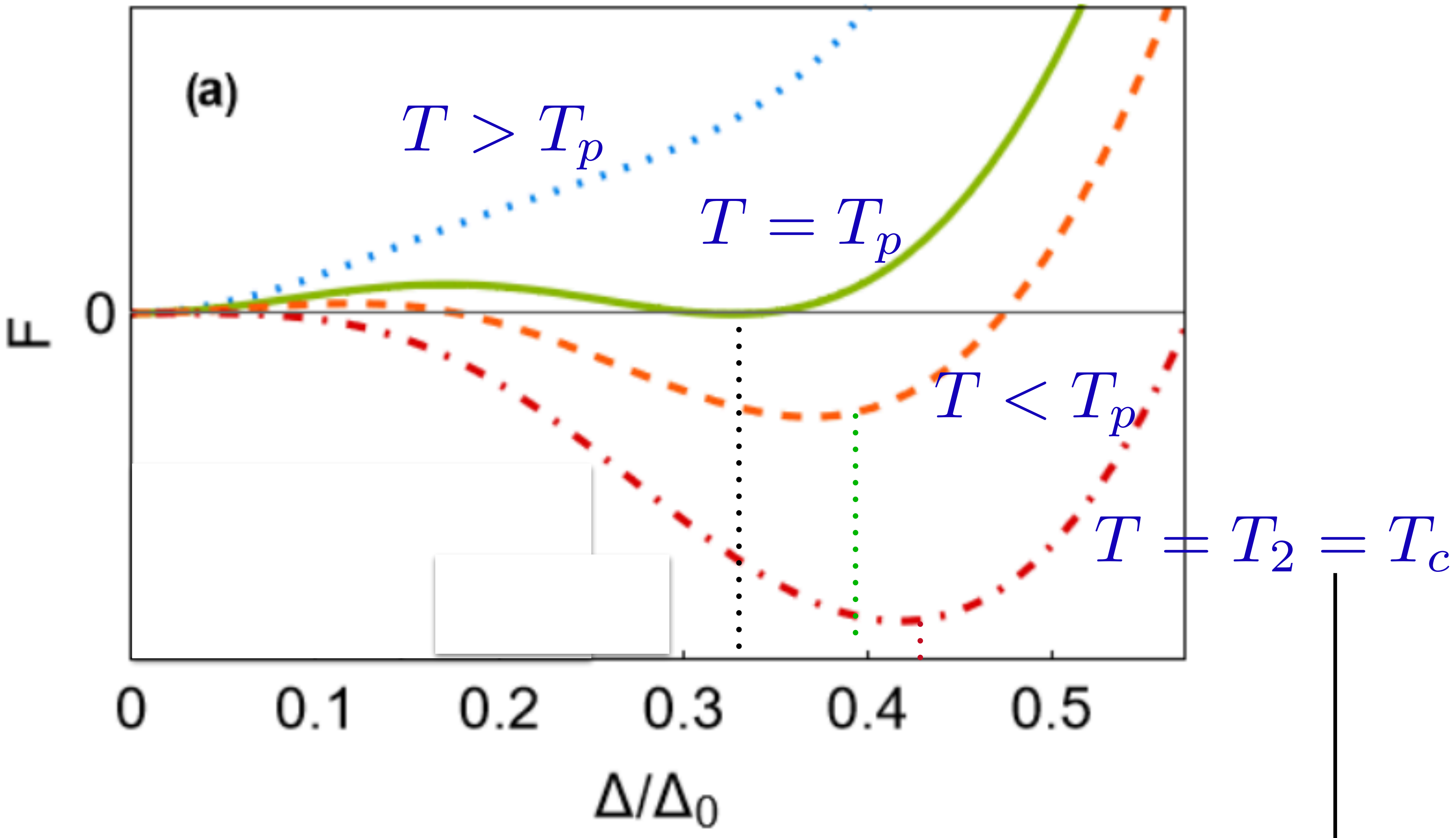
gap/T_c ratio

$$\Delta = (W-U)^{1/2}U^{1/2}e^{-\frac{W}{2g}}$$

$$T_c = (W-U)^{4/5}U^{1/5}\frac{e^{\gamma}}{\pi}e^{-\frac{4}{5}\frac{W}{g}}.$$

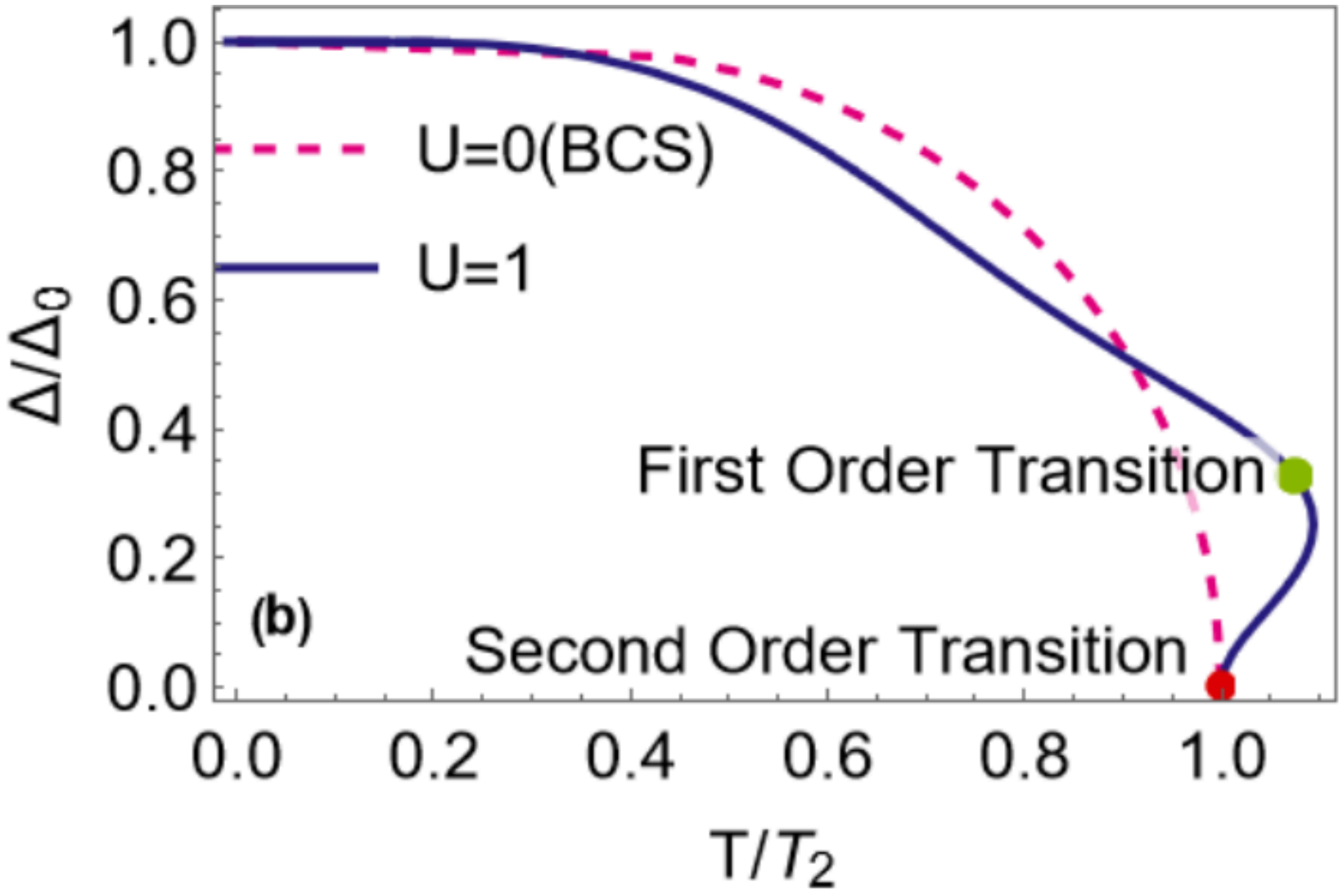
$$\lim_{g\to 0}\frac{\Delta}{T_c}\to\infty$$

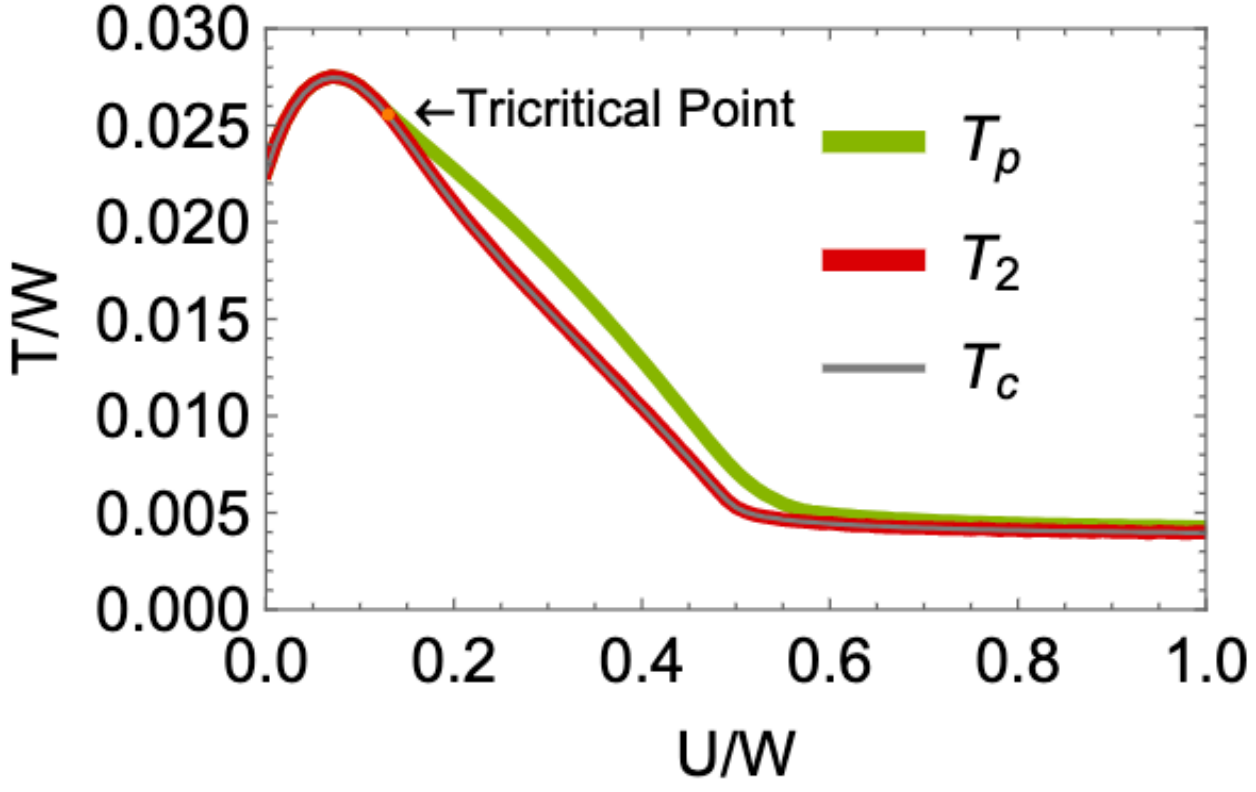
non-BCS superconductivity

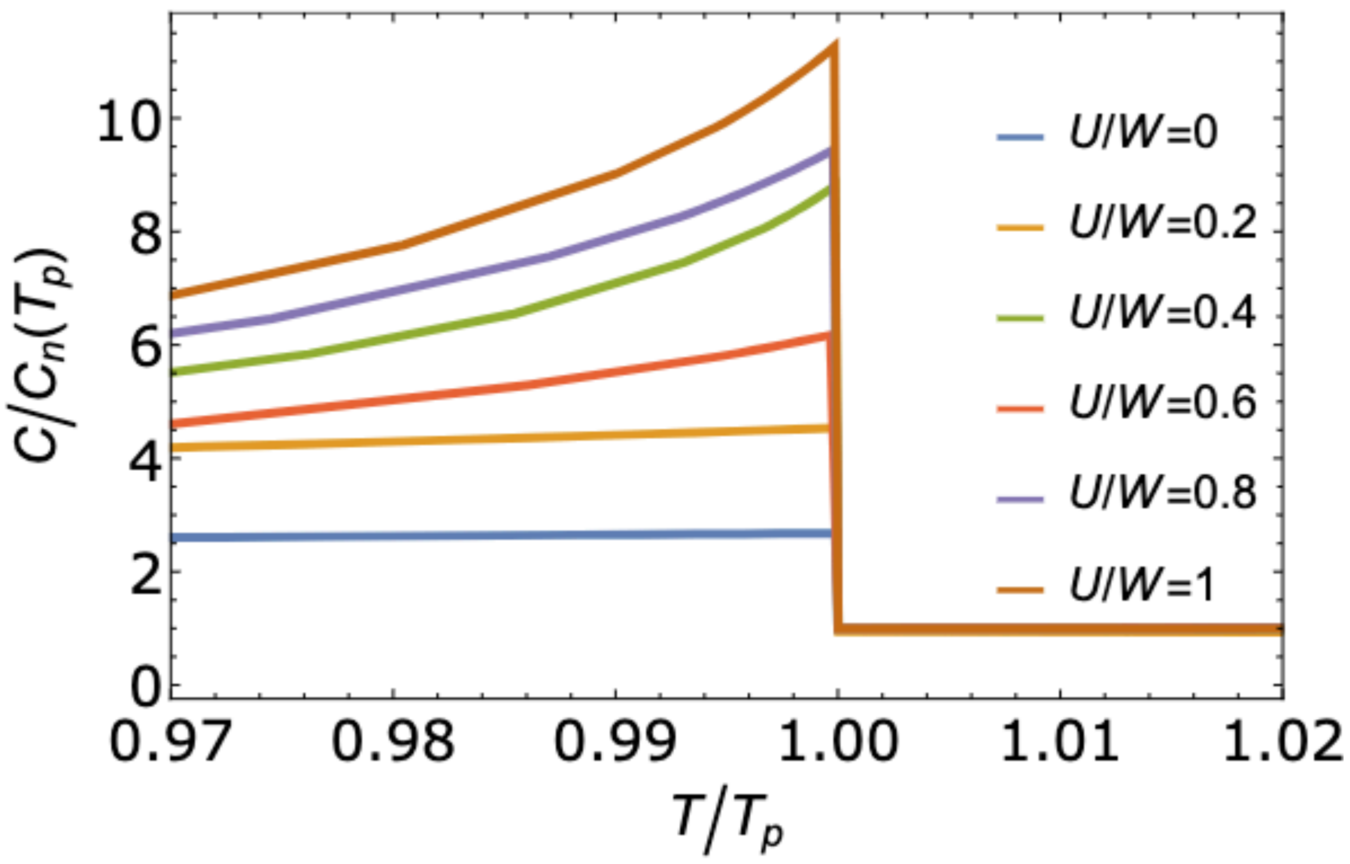


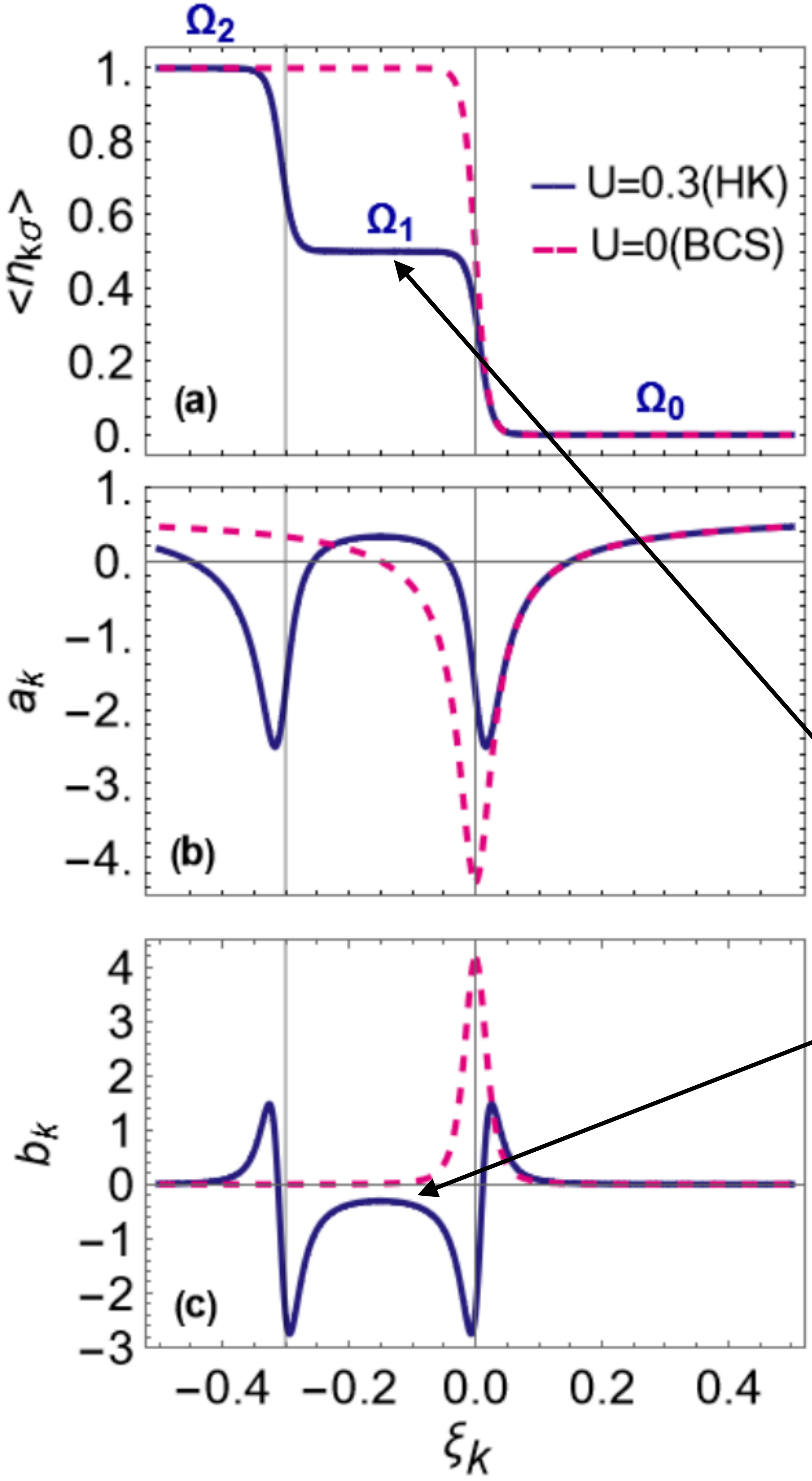
$t_G \approx 10^{-11}$

MF theory
is accurate!


1.0
0.8
0.6
0.4
0.2
0.0
Δ/Δ_0
U=0(BCS)
U=1
First Order Transition
Second Order Transition
(b)
0.0
0.2
0.4
0.6
0.8
1.0
T/T_2


←Tricritical Point
T_p
T_2
T_c
T/W
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0.0
0.2
0.4
0.6
0.8
1.0
U/W


U/W=0
U/W=0.2
U/W=0.4
U/W=0.6
U/W=0.8
U/W=1
$C/C_n(T_p)$
10
8
6
4
2
0
0.97
0.98
0.99
1.00
1.01
1.02
T/T_p

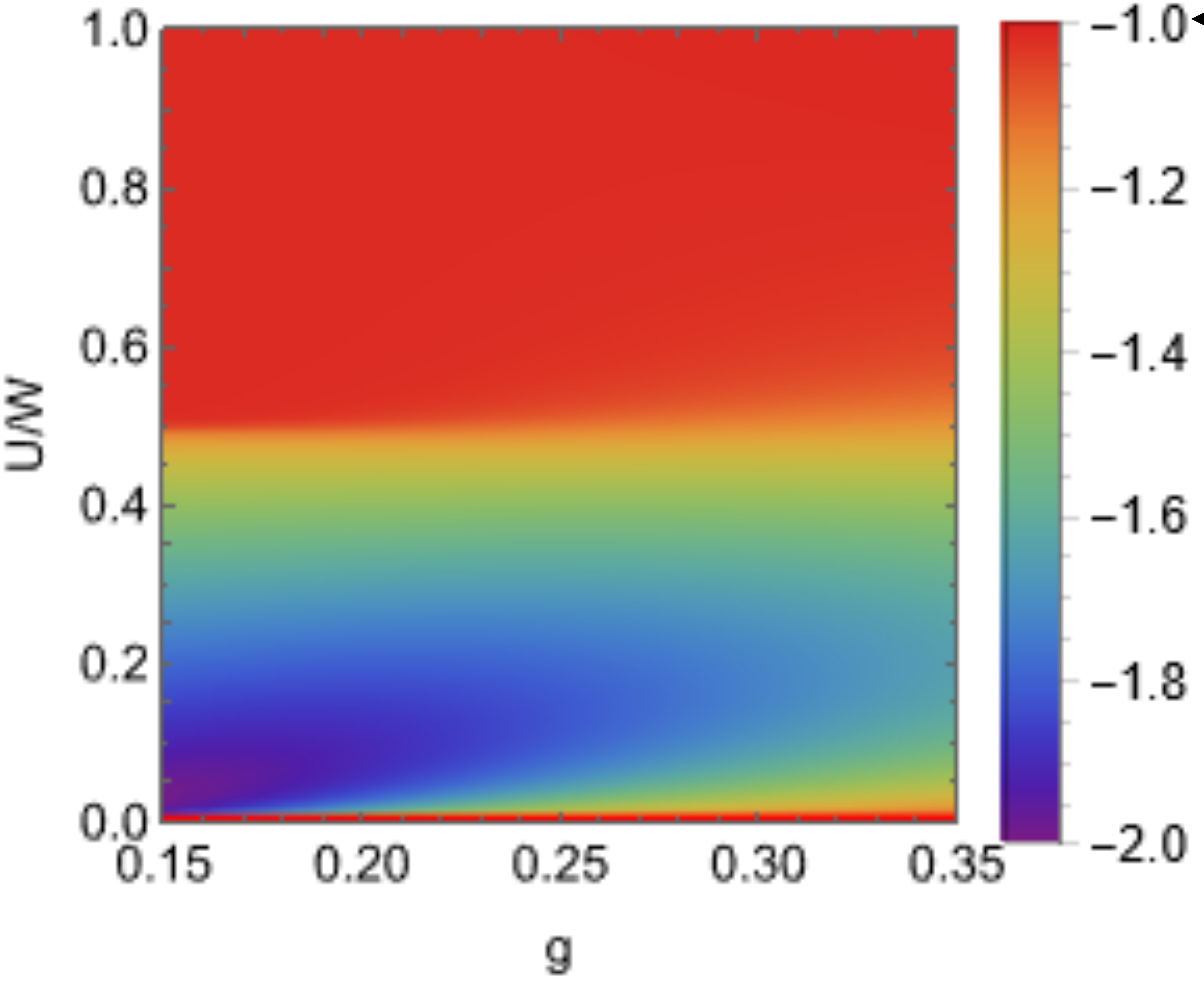


Landau parameters

Mottness

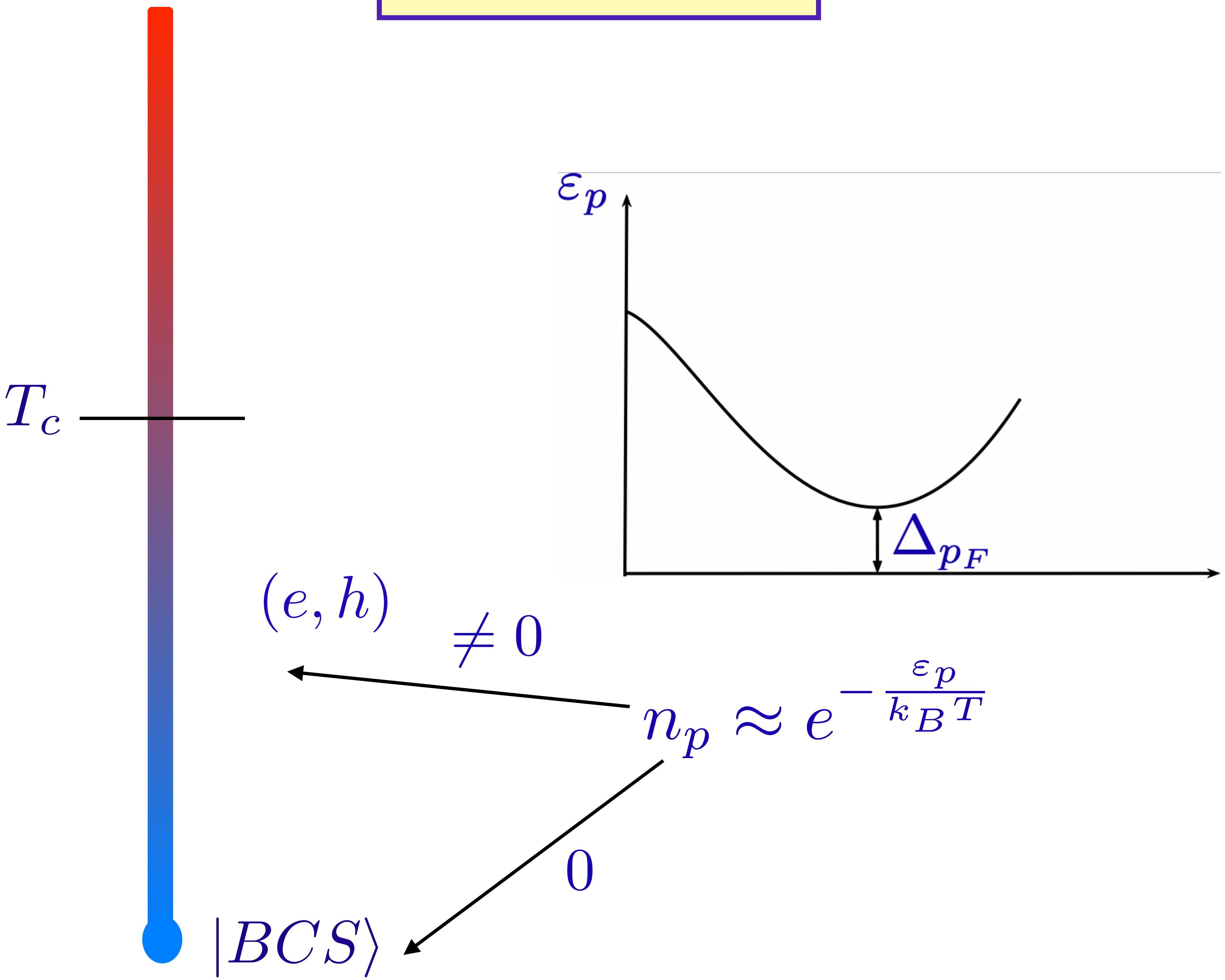
Condensation Energy

$$E_{\text{cond}}/N(0)\Delta^2 < 0$$



BCS

excited states



$$n_p \approx e^{-\frac{\varepsilon_p}{k_B T}}$$

Bogoliubov excitations

$$\gamma_{k\sigma}|\psi_{\mathrm{BCS}}\rangle = 0$$

$$\cancel{\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c^\dagger_{-k\overline{\sigma}}}$$

PYHons excitations

$$\gamma^l_{k\sigma} \propto \sqrt{2} x_k \zeta^\dagger_{k\sigma} - \sigma z_k \zeta_{-k\overline{\sigma}}$$

$$\gamma^u_{k\sigma} \propto z_k \eta^\dagger_{k\sigma} - \sigma\sqrt{2} y_k \eta_{-k\overline{\sigma}}$$

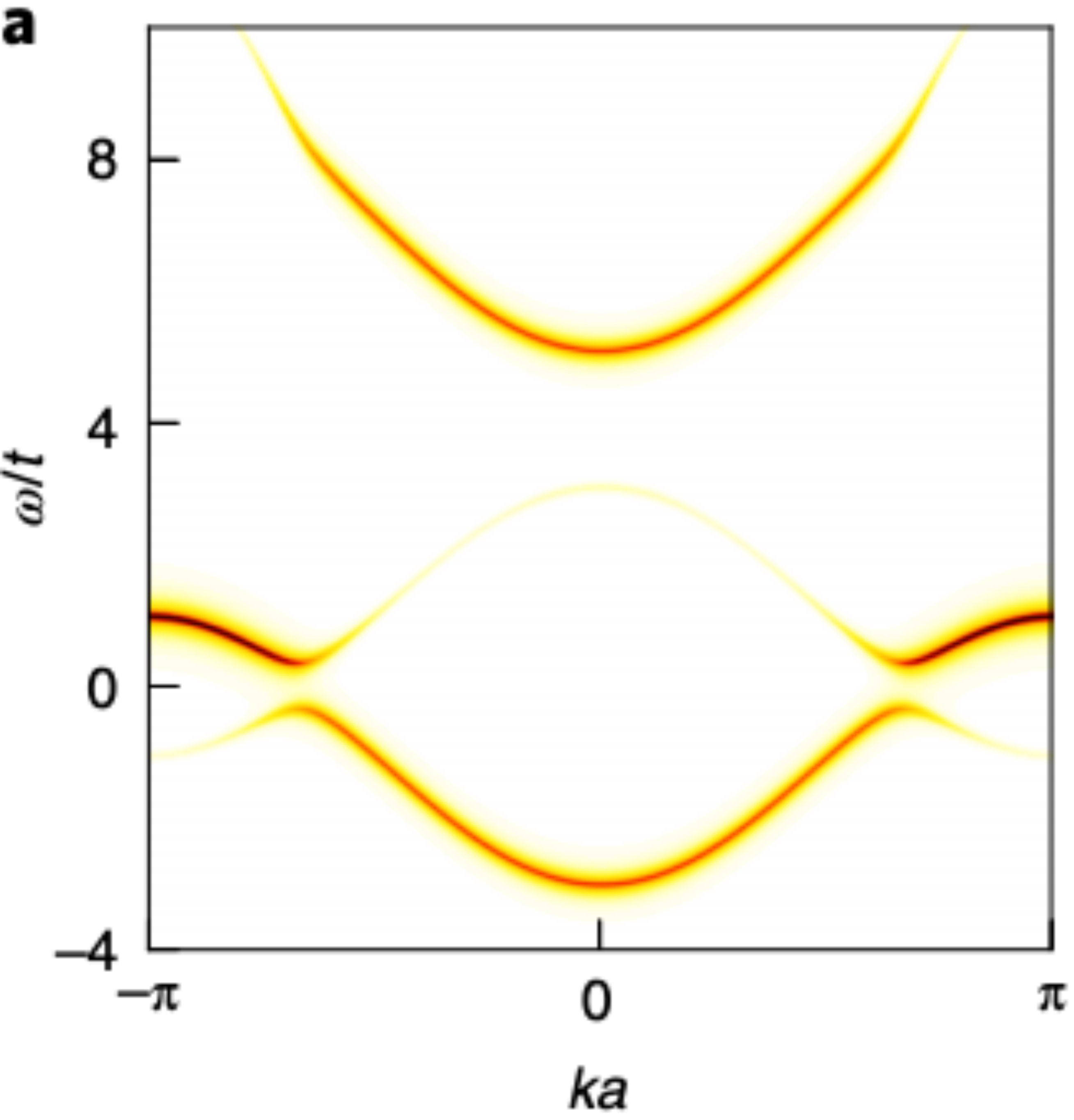
Excitation spectrum

$$\gamma_{k\sigma}^{u/l}|\psi\rangle = 0$$

$$\langle\psi|\gamma_{k\sigma}^{u/l} H \gamma_{k\sigma}^{u/l})^{\dagger}|\psi\rangle = \langle\psi|H|\psi\rangle + E_k^{u/l}$$

$$E_k^{u/l} = \sqrt{{\xi_k^{u/l}}^2 + \Delta^2}$$

superconductivity affects both bands!



PYHon band

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$

H. J. A. Molegraaf[1], C. Presura[1], D. van der Marel[1,*], P. H. Kes[2], M. Li[2]
+ See all authors and affiliations

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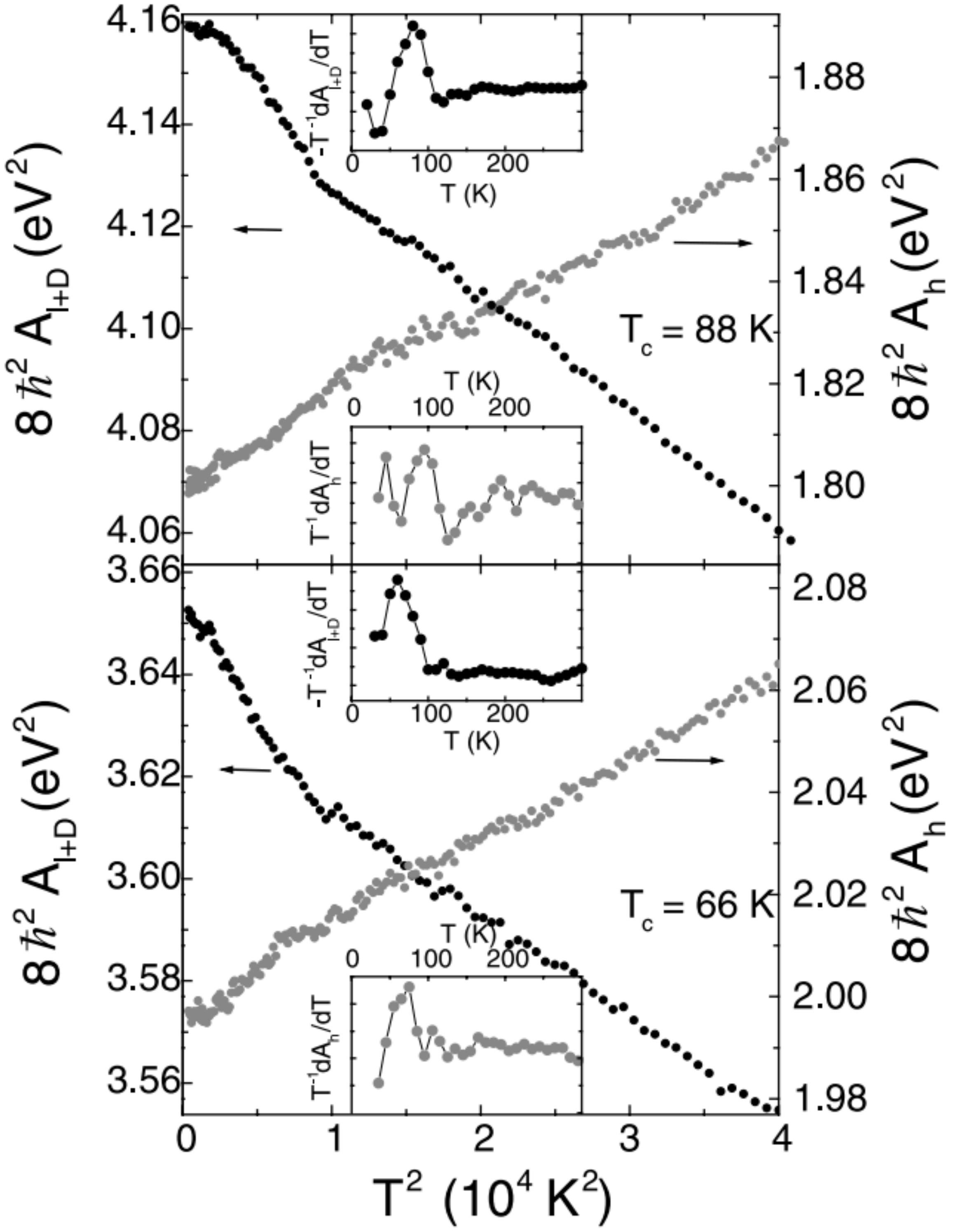
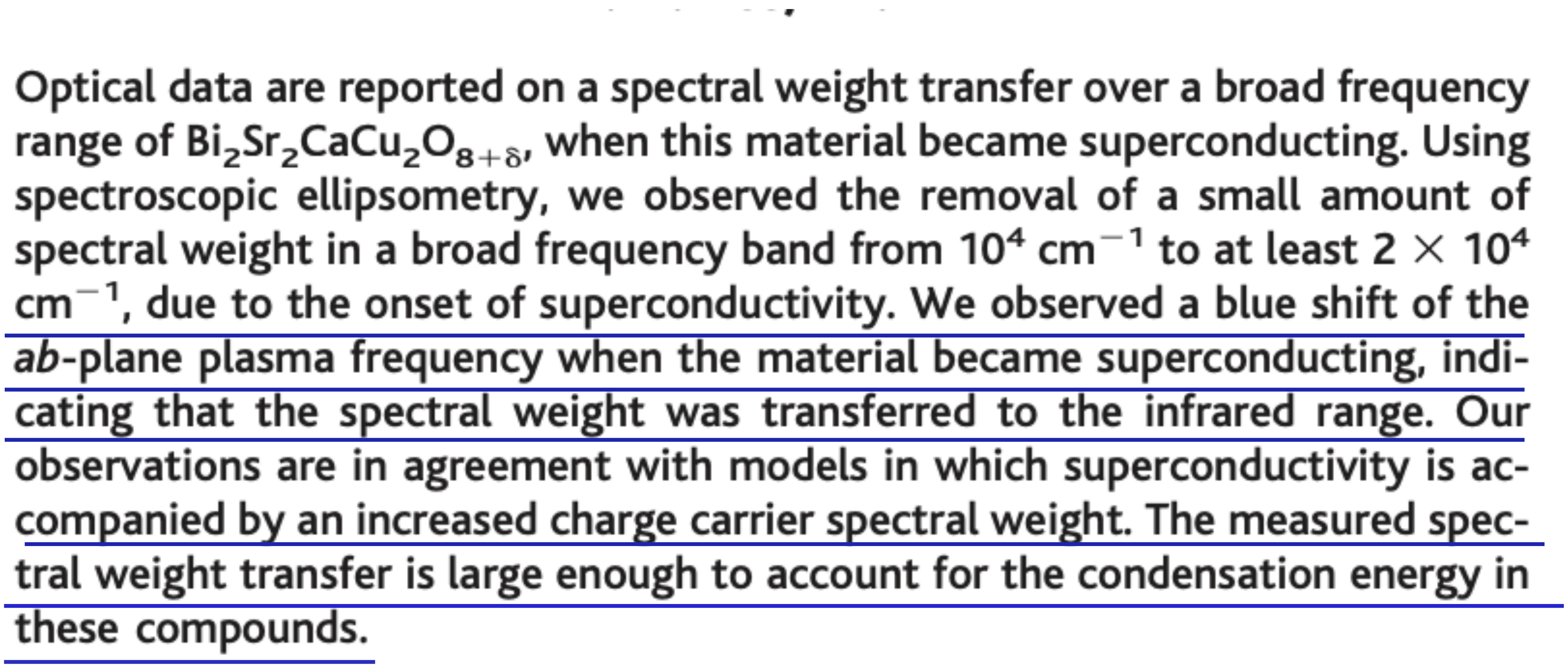
$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000cm^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000cm^{-1}$$

$$\frac{\Delta A_l}{A_l} \propto 3\%$$

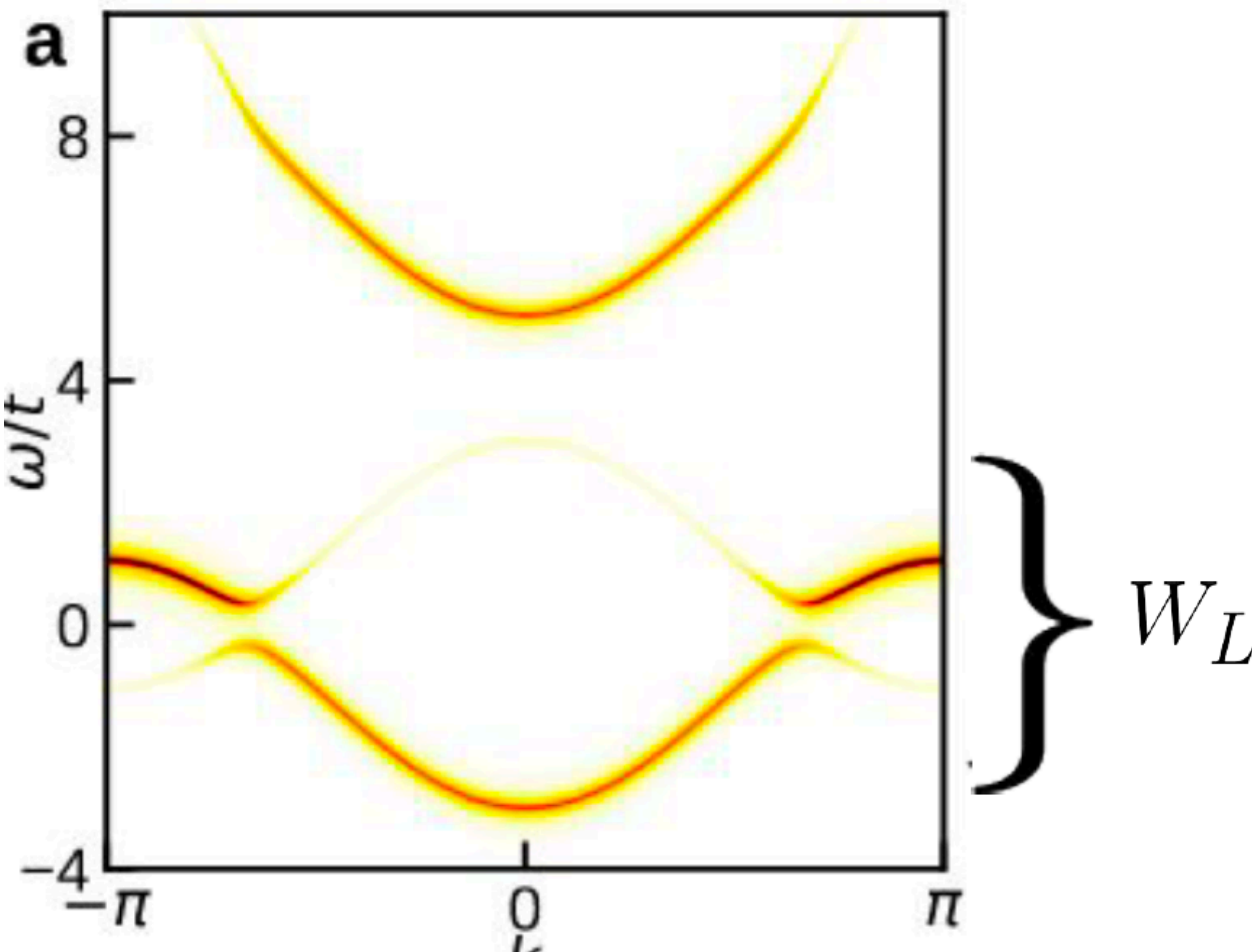


condensation energy



Optical data are reported on a spectral weight transfer over a broad frequency range of $Bi_2Sr_2CaCu_2O_{8+\delta}$, when this material became superconducting. Using spectroscopic ellipsometry, we observed the removal of a small amount of spectral weight in a broad frequency band from 10^4 cm^{-1} to at least 2×10^4 cm^{-1}, due to the onset of superconductivity. We observed a blue shift of the *ab*-plane plasma frequency when the material became superconducting, indicating that the spectral weight was transferred to the infrared range. Our observations are in agreement with models in which superconductivity is accompanied by an increased charge carrier spectral weight. The measured spectral weight transfer is large enough to account for the condensation energy in these compounds.

UV-IR mixing


a
8
4
0
−4
ω/t
−π
0
π
k
W_L

why?

$$H = H_{\rm HK} + H_p$$

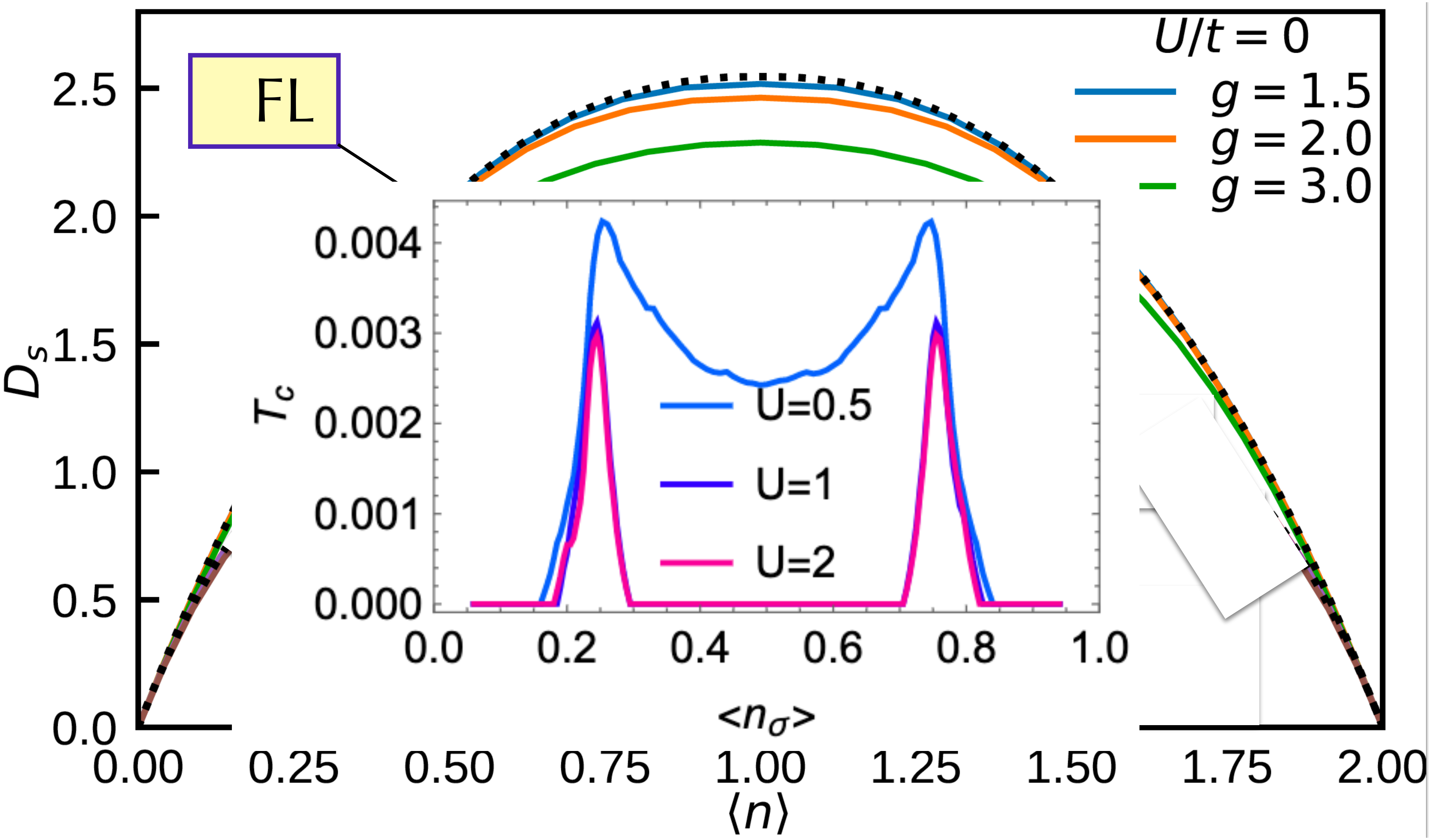
$$[H_{\rm HK}, H_p] \neq 0$$

dynamical
spectral weight
transfer

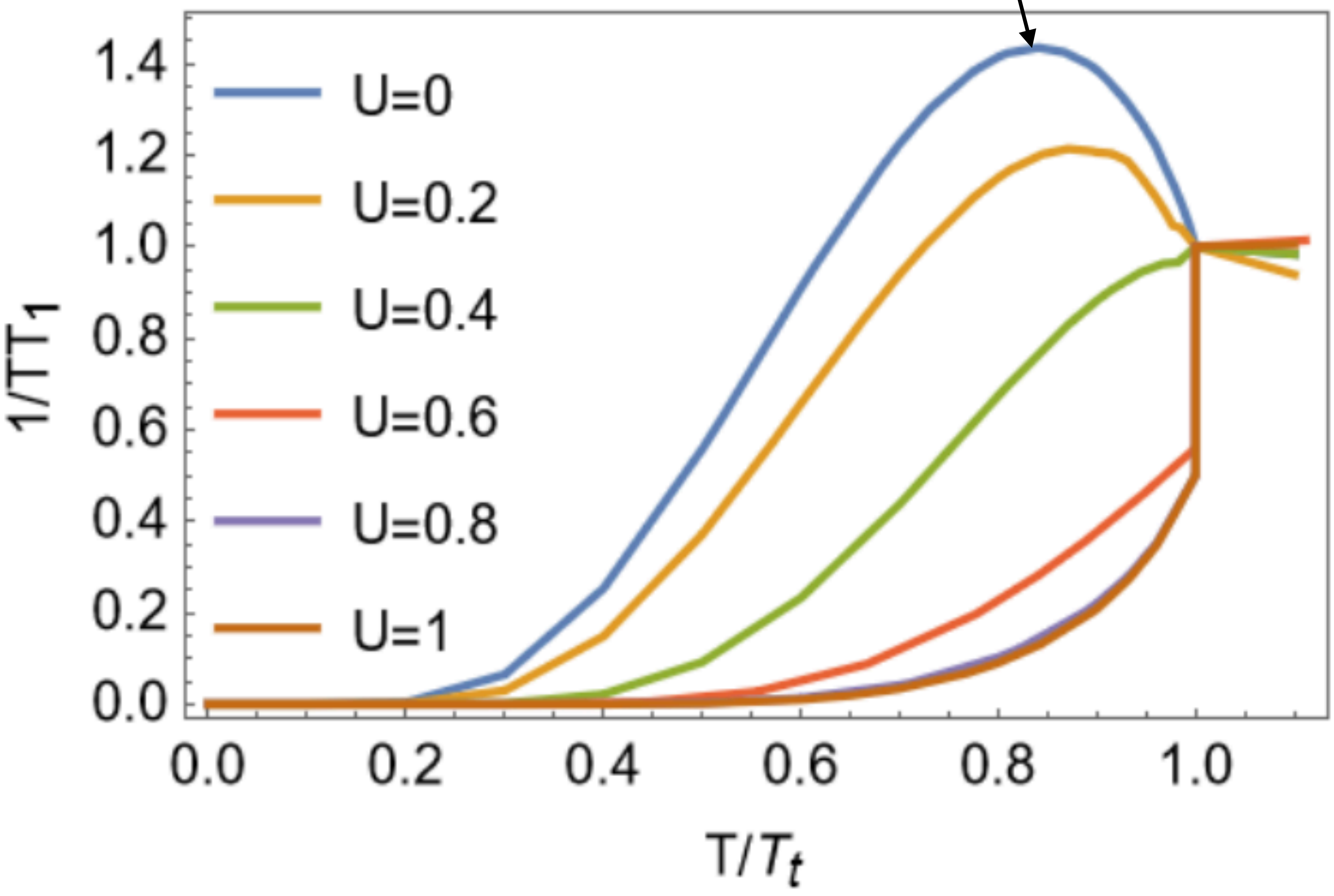
is this the general mechanism of the color change?

Superfluid Density

Mottness-induced suppression

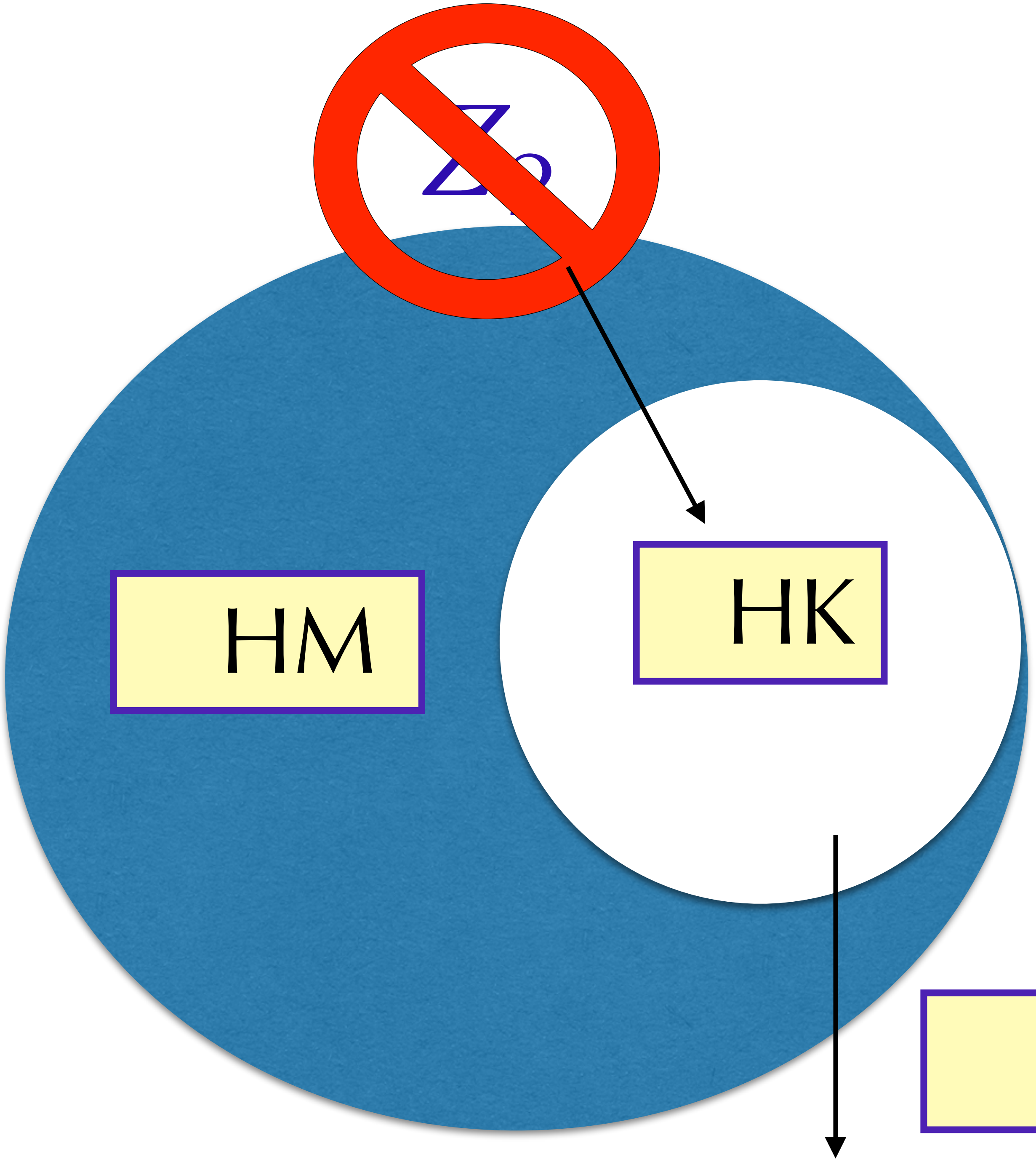


Hebel-Slichter peak killed by Mottness



	Superconductivity	Mottness
observable	BCS/FL	PYHZ/HK
$\chi \to \infty$	T_c	$T_c(= T_2)$
$\Delta \neq 0$	T_c	$T_p(> T_2)$
$\lim\limits_{g\to 0} 2\Delta_0/k_B T_c$	3.52	∞
$\mathrm{quasi-particles}$	Bogoliubons	PYHons
t_G(Ginzburg)	$\approx 10^{-12}$	$\approx 10^{-11}$
$1/\mathrm{TT}_1$	HS peak	no HS peak
Landau Expansion	$a = \alpha t,\ b > 0$	$a = \alpha t,\ b < 0$ $c > 0$
$E_{\mathrm{cond}}/N(0)\Delta^2$	-1	$[-2, -1]$

Mottness



HM

HK

PYHons

non-BCS superconductivity

violation of Luttinger