Mottness and Holography

Thanks to:
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What is Mottness?
What is Mottness?

physics of a non-rigid band:
UV-IR mixing
What is Mottness?

2-particle probe

1-particle

YBa$_2$Cu$_3$O$_y$

E//CuO$_2$

$\sigma(\omega)$

$\Omega^{-1}$cm$^{-1}$

$\nu=6.6$

$\nu=6.1$

Physics of a non-rigid band: UV-IR mixing

SC state

non-BCS
What is Mottness?

2-particle probe

1-particle

YBa$_2$Cu$_3$O$_y$

E//CuO$_2$

E$_c$

$\sigma(\omega)$

$\Omega^{-1}$cm$^{-1}$

$\nu=6.6$

$\nu=6.1$

$0$

$10000$ $30000$

$0$

$1200$

$800$

$400$

$10^4$cm$^{-1}$

$\nu$

physics of a non-rigid band:

UV-IR mixing

SC state

non-BCS

$\nu$

$8\pi^2 A_{h,d}(eV^2)$

$T^2 (10^4 K^2)$

$T_c = 88 K$

$T_c = 66 K$

current =

what particle
What is Mottness?

2-particle probe

1-particle

physics of a non-rigid band: UV-IR mixing

Wilsonian program

current = what particle

SC state

does

non-BCS holography
\[ H = -t \sum_{ij\sigma} g_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \]

Wilsonian program
\[ H = -t \sum_{ij\sigma} g_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \]

Wilsonian program

density of states

\[ \int d(UHB) \]
$H = -t \sum_{ij\sigma} g_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$

Wilsonian program

What is transferred down?

density of states

$\int \cdot \cdot \cdot d(UHB)$
classical limit
classical limit
atomic limit: $x$ holes


\[ G(\omega, k) = \frac{1 + x}{\omega - \mu + U/2} + \frac{1 - x}{\omega - \mu - U/2} \]

density of states

spectral weight: $x$-dependent
atomic limit: $x$ holes

density of states

spectral weight: $x$-dependent

$$G(\omega, k) = \frac{1 + x}{\omega - \mu + U/2} + \frac{1 - x}{\omega - \mu - U/2}$$
atomic limit: \( x \) holes

\[ G(\omega, k) = \frac{1 + x}{\omega - \mu + U/2} + \frac{1 - x}{\omega - \mu - U/2} \]
quantum Mottness: $U$ finite

$U \gg t$
quantum Mottness: $U$ finite  \[ U \gg t \]

double occupancy in ground state!!
quantum Mottness: $U$ finite

$U \gg t$

double occupancy in ground state!!

$W_{PES} > 1 + x$
Beyond the atomic limit: any real system

\[ 1 + x + \alpha(t/U, x) \]

\[ \alpha = \frac{t}{U} \sum_{ij} \langle c_i^\dagger c_j \rangle > 0 \]

Density of states

\[ 1 - x - \alpha \]
$\alpha = \frac{t}{U} \sum_{ij} \langle c^\dagger_{i\sigma} c_{j\sigma} \rangle > 0$

Harris & Lange, 1967

beyond the atomic limit: any real system

density of states

1 + $x + \alpha(t/U, x)$

1 - $x - \alpha$

dynamical spectral weight transfer
Beyond the atomic limit: any real system

\[ 1 + x + \alpha(t/U, x) \]

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Density of states

Harris & Lange, 1967

Intensity > 1 + x

dynamical spectral weight transfer
density of states

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Harris & Lange, 1967

dynamical spectral weight transfer

Intensity > 1 + x

# of charge e states

beyond the atomic limit: any real system
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dynamical spectral weight transfer

Intensity > 1 + x

# of charge e states

# of electron states in lower band
Beyond the atomic limit: any real system

\[ 1 + x + \alpha(t/U, x) \]

\[ \alpha = \frac{t}{U} \sum_{ij} \langle c_i^{\dagger} c_j \rangle > 0 \]

Intensity $> 1 + x$

Number of charge $e$ states

Number of electron states in lower band

Not exhausted by counting electrons alone?

Harris & Lange, 1967
What are the extra states (degrees of freedom)?
Key Equation

1 = 2 - 1
Key Equation

\[ 1 = 2 - 1 \]
\[ e = 2e - e \]
Key Equation

\[ 1 = 2 - 1 \]

\[ e = 2e - e \]

Hubbard bands are not rigid: Mottness
We now know what $\cdot \cdot \cdot$ is
We now know what $d$ is.

$\int d(UHB)$

1 + x + \alpha(t/U, x)$

charge 2e stuff
We now know what $\rho$ is.

$\rho = 1 + x + \alpha(t/U, x)$

Charge $2e$ stuff:

$\int d(UHB)$

Collective excitation from double occupancy: incoherence at low energies.
Key idea: similar to Bohm/Pines

Extend the Hilbert space:
Associate with U-scale new Fermionic oscillators

\[ N(\omega) \]

\[ \frac{U}{2} \tilde{D}_i \tilde{D}_i + \frac{U}{2} D_i^\dagger D_i \]
transforms as a boson.
$D_i^\dagger$ transforms as a boson

one per site (fermionic)

transforms as a boson

Fermionic
\( D_i^\dagger \) transforms as a boson

\[ \delta(D_i - \theta c_i^{\uparrow} c_i^{\downarrow}) \]

one per site (fermionic)

Fermionic

Grassmann
$D_i^\dagger$ transforms as a boson

Grassmann

$\theta \varphi_i^\dagger$ charge $2e$ boson

$\delta(D_i - \theta c_{i\uparrow} c_{i\downarrow})$
Exact low-energy Lagrangian

\[ L = \#L_{\text{bare}}(\text{electrons}) + \#L_{\text{bare}}(\text{bosons}) \]

\[ + f(\omega)L_{\text{int}}(c, \varphi) + \tilde{f}(\omega)L_{\text{int}}(c, \tilde{\varphi}) \]

\[ \Psi^\dagger \Psi \quad \tilde{\Psi}^\dagger \tilde{\Psi} \]

quadratic form: composite or bound excitations of

\[ \varphi^\dagger c_i \sigma \]
Exact low-energy Lagrangian

\[ L = \#L_{\text{bare}}(\text{electrons}) + \#L_{\text{bare}}(\text{bosons}) \]

\[ + f(\omega) L_{\text{int}}(c, \varphi) + \tilde{f}(\omega) L_{\text{int}}(c, \tilde{\varphi}) \]

\[ f(\omega) = 0 \]

dispersion of propagating light modes

quadratic form: composite or bound excitations of

\[ \varphi^\dagger c_{i\sigma} \]
composite excitations determine spectral density: Mott gap

\[ \gamma_{\vec{p}}(\omega) = \frac{U - t\varepsilon_{\vec{p}} - 2\omega}{U} \sqrt{1 + 2\omega/U} \]

\[ \tilde{\gamma}_{\vec{p}}(\omega) = \frac{U + t\varepsilon_{\vec{p}} + 2\omega}{U} \sqrt{1 - 2\omega/U}. \]

\[ \Delta = U - 4dt \]

each momentum has SD at two distinct energies
hole-doping?
Extend the Hilbert space:
Associate with U-scale a new Fermionic oscillator

$$N(\omega)$$

$$UD_i^\dagger D_i$$
\[ \mathcal{L}_{\text{IR}} = L_{\text{spinstuff}}(c_i, c_i^\dagger) + L_{\text{charge}}(c_i, c_i^\dagger, \varphi_i) \]
Electron Dispersion

Fermi liquid
Electron Dispersion

\[ \phi_i = \phi \quad \forall \quad i \]

Fermi liquid

states cross chemical potential

no electron-like quasi-particles anywhere!!

homogeneous
What is seen experimentally?
Are there 0-energy excitations?
What is seen experimentally? Are there 0-energy excitations?
What is seen experimentally?
Are there 0-energy excitations?

seen
not seen

Fermi arcs: no double crossings (PDJ, JCC, ZXS)
What is seen experimentally? Are there 0-energy excitations?

seen

not seen

Fermi arcs: no double crossings (PDJ, JCC, ZXS)
What is seen experimentally? Are there 0-energy excitations?

<table>
<thead>
<tr>
<th>seen</th>
<th>not seen</th>
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Fermi arcs: no double crossings (PDJ, JCC, ZXS)

meaning?

can we explain this?
yes
\[ \varphi_j = e^{ij \cdot \pi} \varphi \]

\[ \varphi_j = e^{ij \cdot 9\pi} \varphi \]
why choose these funky solutions?
Why choose these funky solutions?
why choose these funky solutions?

they minimize the free energy (no order)
what causes the ghost states?
what causes the ghost states?

- Ghost states
- Incoherent feature
- No ghost states
- Band crossing
- No fermi arcs
what causes the ghost states?

- **Ghost states**
- **Incoherent feature**
- **Bound**

No ghost states
No Fermi arcs
Band crossing

$\varphi_j^\dagger c_i \bar{\sigma}$
what causes the ghost states?

- ghost states
- incoherent feature
- bound

- no ghost states
- band crossing
- unbound

\[ \varphi_j^\dagger C_i \sigma \]
T-linear resistivity

New scenario as $x$ increases
T-linear resistivity

New scenario

as $x$ increases
T-linear resistivity

New scenario as $x$ increases
More `e` states at lower temperature

Pseudogap=``confinement’’

composite or bound states not in UV theory
More addition states in PG: new charge e states
$IR \quad ?? \quad UV$
$g = 1/\text{ego}$ coupling constant
gauge-gravity duality (Maldacena, 1997)

coupling constant

\[ g = \frac{1}{\text{ego}} \]
`Holography'

$\text{AdS}_{d+1}$

$\beta(g)$ is local geometrize RG flow

$\Lambda_1 \rightarrow \Lambda_2 \rightarrow \Lambda_3 \rightarrow \Lambda_4 \rightarrow \text{IR}$

$\Lambda_1 \rightarrow \Lambda_2 \rightarrow \Lambda_3 \rightarrow \Lambda_4 \rightarrow \text{IR}$

$\beta(g)$ is local geometrize RG flow

$\{t, \bar{x}, r\} \rightarrow \{\lambda t, \lambda \bar{x}, \lambda r\}$

RN black hole

$ds^2 = L^2 \left( -\frac{dt^2}{r^2} + \frac{d\bar{x}^2}{r^2} + \frac{dr^2}{r^2} \right)$
Charged system

$J_\mu$

$S_0$

probe

$J_\psi O_\psi$

RN-AdS

$ds^2, A_t$

geometry
Charged system

\[ J^\mu \]

\[ S_0 \]

probe

\[ J_\psi O_\psi \]

\[ \text{RN-AdS} \]

\[ ds^2, A_t \]

\[ \text{geometry} \]

\[ \psi \text{ Dirac Eq.} \]
Retarded Green function: \( G = \frac{b}{a} = f(\text{UV,IR}) \)

\[ \psi(r \to \infty) \approx ar^m + br^{-m} \]

in-falling boundary conditions

Dirac Eq.

\( J^{\mu} \)

UV

charged system

probe

\( S_0 \)

\( J_\psi O_\psi \)

RN-AdS

d\( s^2, A_t \)

geometry
What gravitational theory gives rise to a gap in $\text{Im}G$ without spontaneous symmetry breaking?

dynamically generated gap: Mott gap (for probe fermions)
bottom-up schemes

$\sqrt{-g} \bar{\psi} (D - m) \psi$ `non-Fermi liquids'

AdS-RN
MIT, Leiden group
\[
\sqrt{-g i \bar{\psi} (D - m) \psi}
\]

bottom-up schemes

\text{`non-Fermi liquids'}

\text{AdS-RN}
\text{MIT, Leiden group}

\text{Mott Insulator}
fermions in RN Ads_{d+1} coupled to a gauge field through a dipole interaction

consider \( \sqrt{-g}i\bar{\psi}(\mathcal{D} - m - ip\mathcal{F})\psi \)

bottom-up schemes

`non-Fermi liquids'

AdS-RN
MIT, Leiden group
How is the spectrum modified?

P=0
How is the spectrum modified?

Fermi surface peak
How is the spectrum modified?

$P=0$  

$P > 4.2$

Fermi surface peak
How is the spectrum modified?

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dynamically generated gap:

$P = 0$

$P > 4.2$
How is the spectrum modified?

P=0

spectral weight transfer

Fermi surface peak

dynamically generated gap:

P > 4.2
How is the spectrum modified?

Fermi surface peak

dynamically generated gap: confirmed by Gubser, Gauntlett, 2011

spectral weight transfer

$P = 0$

$P > 4.2$
Finite Temperature Mott transition

\[
\frac{T}{\mu} = 5.15 \times 10^{-3}
\]

\[
\frac{T}{\mu} = 3.92 \times 10^{-2}
\]
Finite Temperature Mott transition

\[ \frac{\Delta}{T_{\text{crit}}} \approx 20 \]

vanadium oxide

\[ T/\mu = 5.15 \times 10^{-3} \]

\[ T/\mu = 3.92 \times 10^{-2} \]
Finite Temperature Mott transition

\[ \frac{\Delta}{T_{\text{crit}}} \approx 10 \]

\[ \frac{\Delta}{T_{\text{crit}}} \approx 20 \text{ vanadium oxide} \]
spectral weight
transfer
UV-IR mixing
black hole

gravitons

electrons

boundary: Mott insulator
Mottness

Hubbard

Holography

dynamical
spectral
weight
transfer
Mottness
Hubbard
Holography
dynamical
spectral
weight
transfer