Mottness and Holography

Thanks to:
NSF, EFRC (DOE)

M. Edalati

Seungmin Hong
and
S. Chakraborty

T.-P. Choy

R. G. Leigh

What is Mottness?

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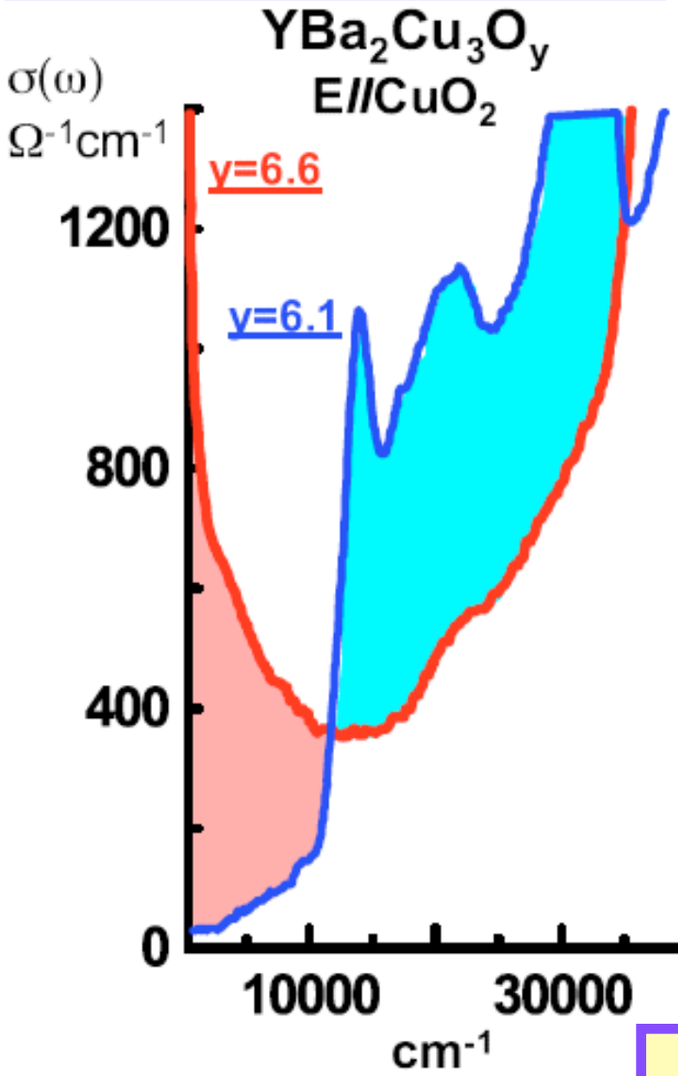
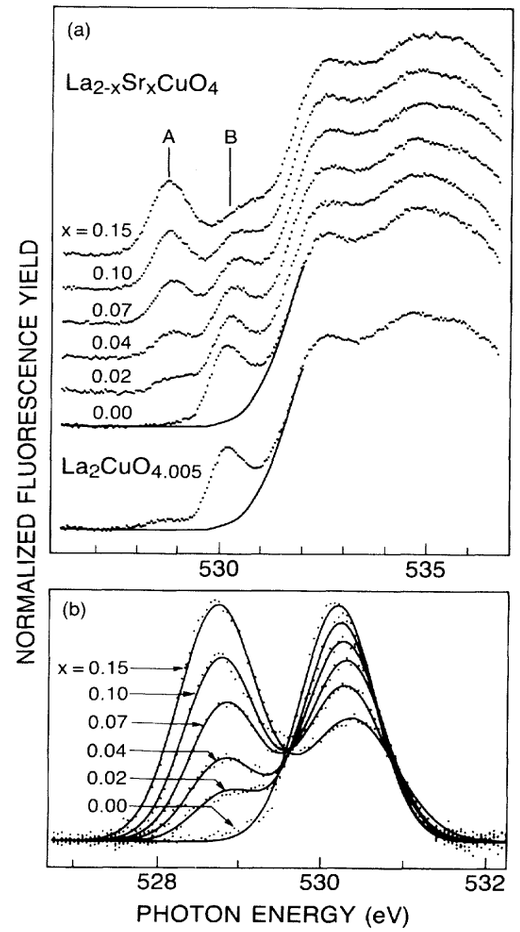
physics of a non-rigid band:
UV-IR mixing

What is Mottness?

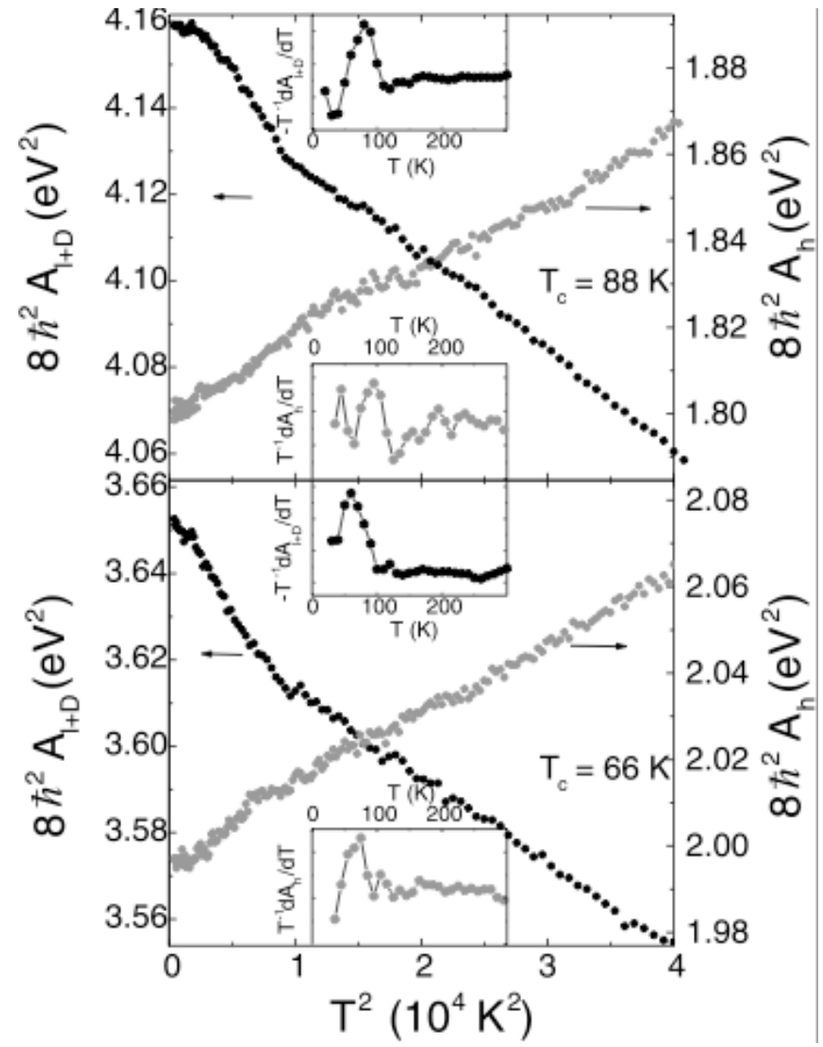
2-particle probe



1-particle



SC state



non-BCS

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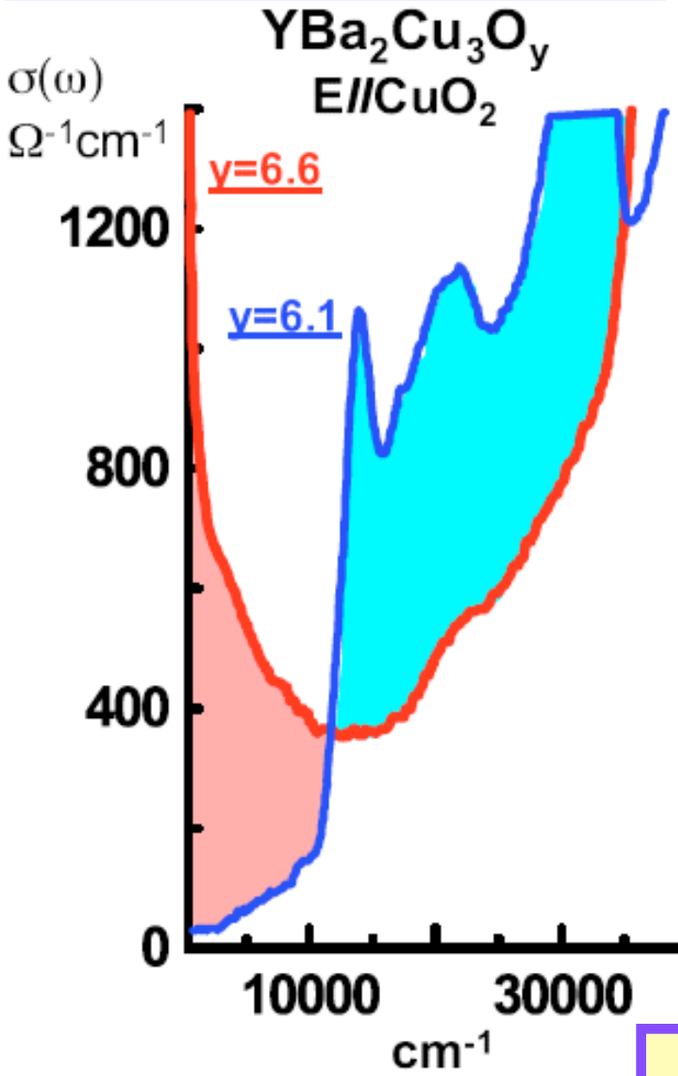

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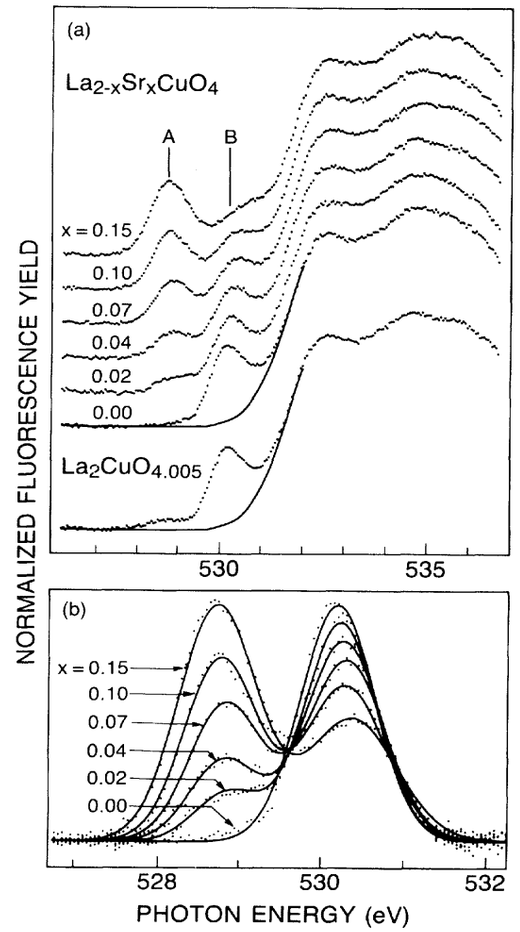

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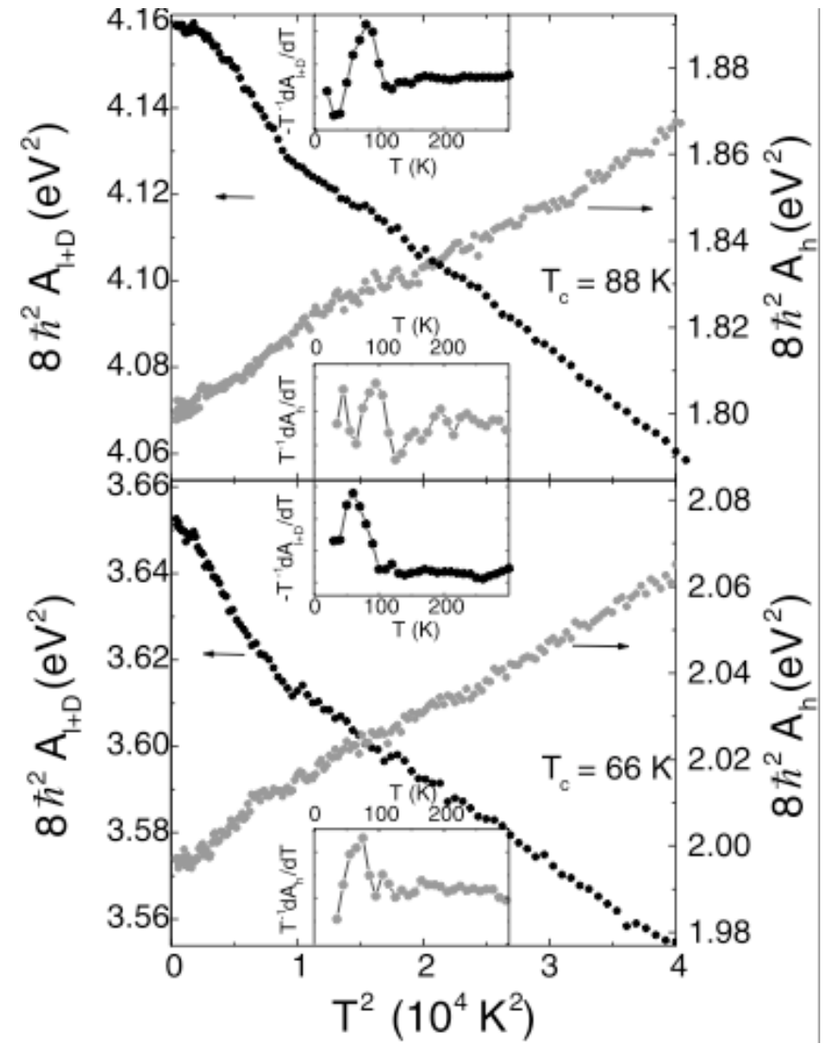

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physics of a non-rigid band:
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current=
what particle

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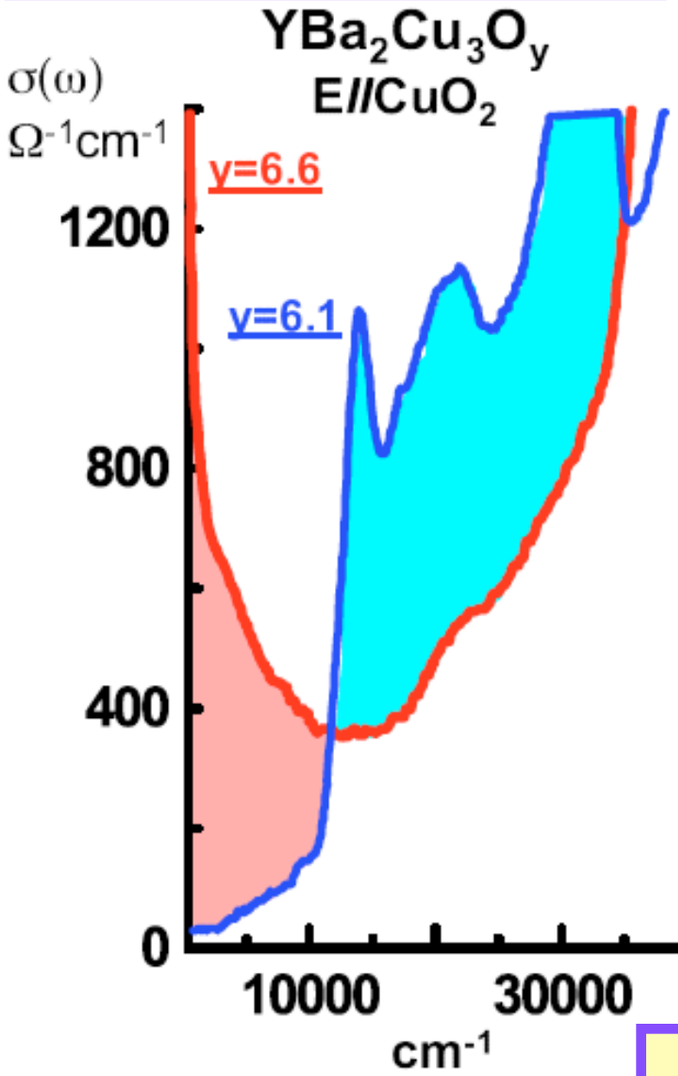

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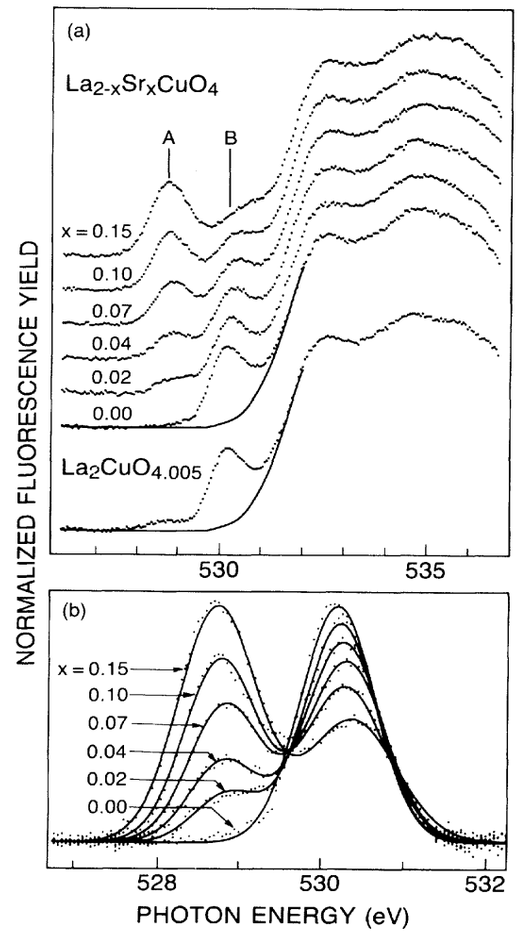

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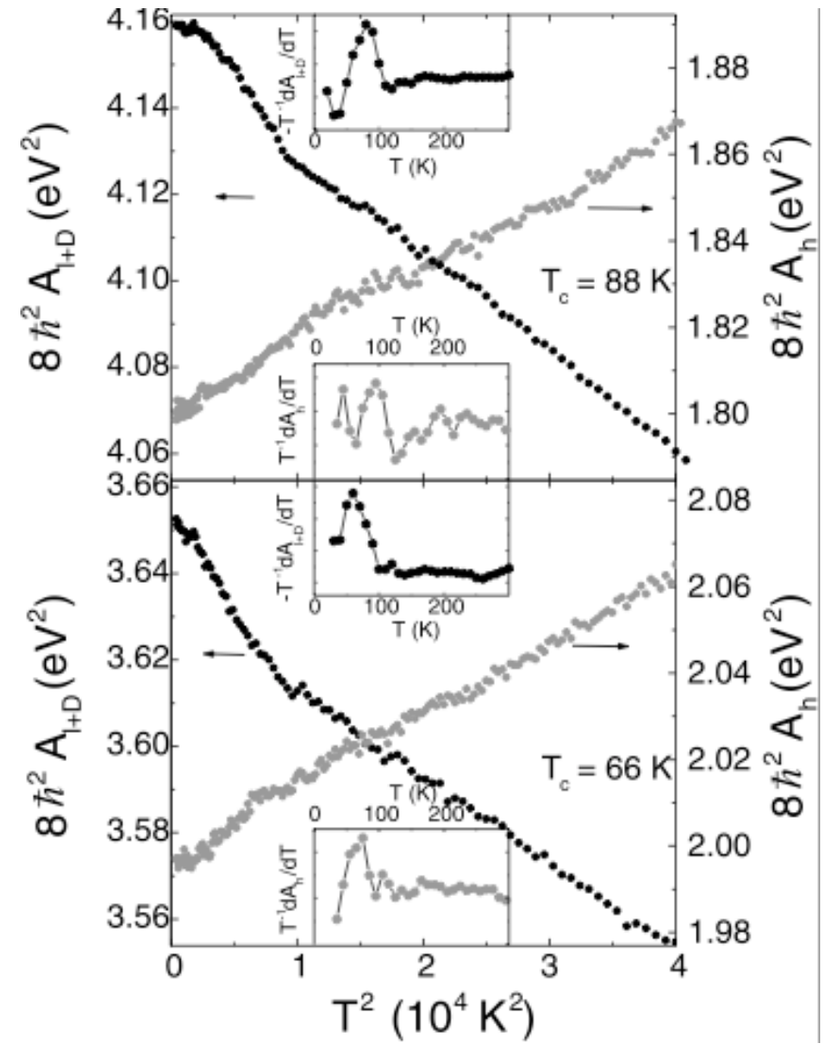

non-BCS

physics of a non-rigid band:
UV-IR mixing

Wilsonian program

current=
what particle

holography

$$H = -t \sum_{ij\sigma} g_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Wilsonian program

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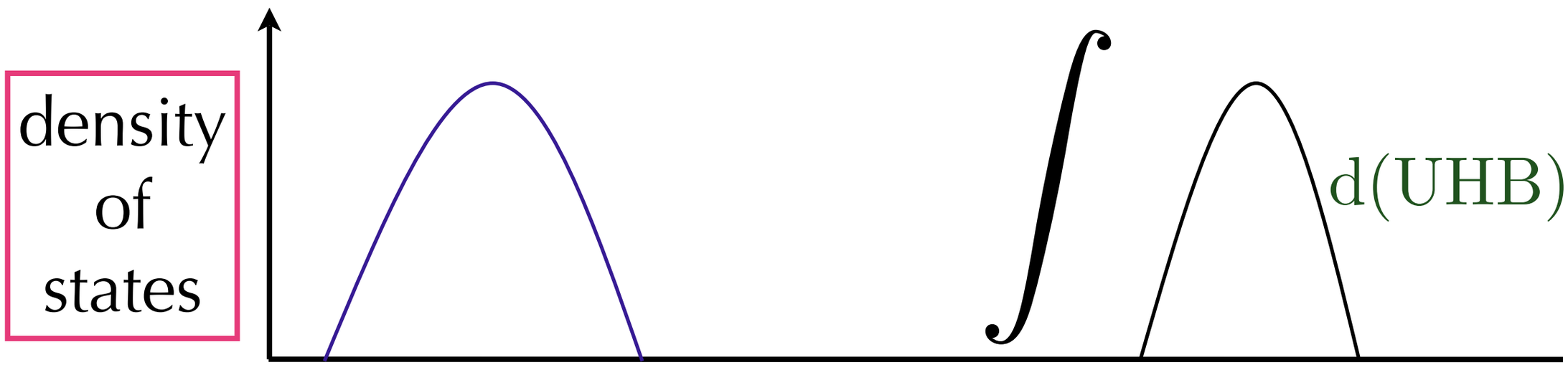

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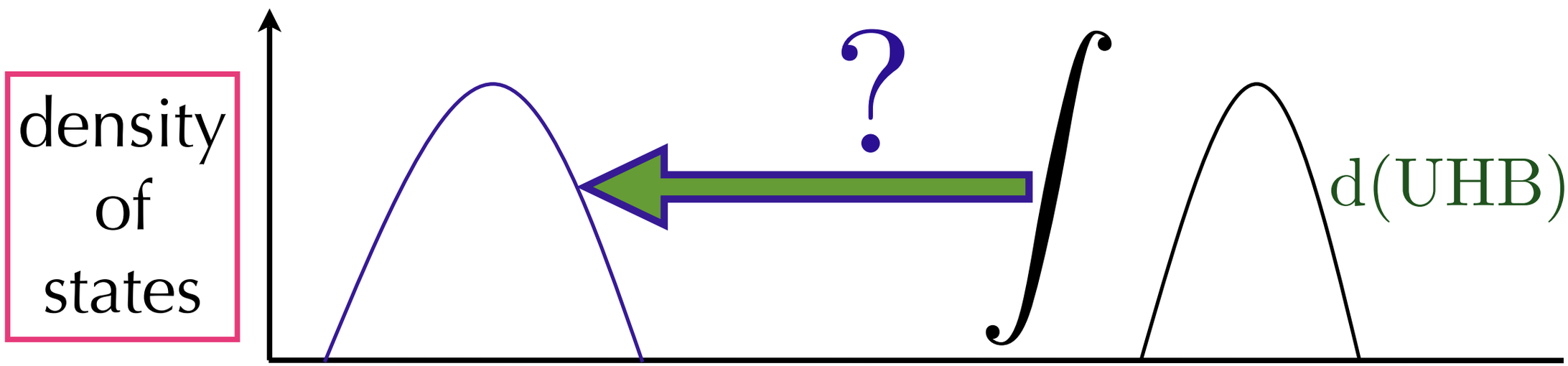
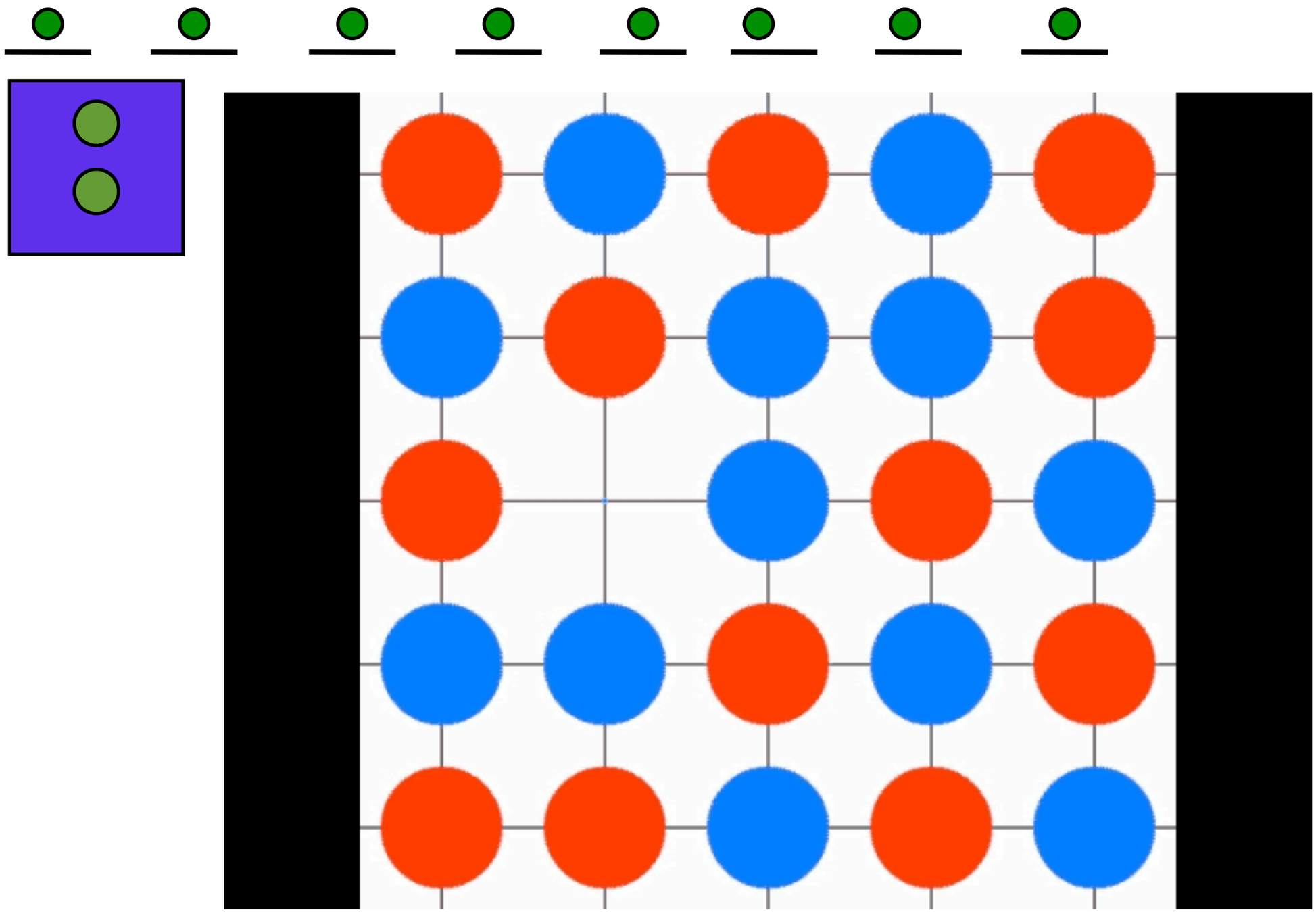


What is transferred down?

classical limit



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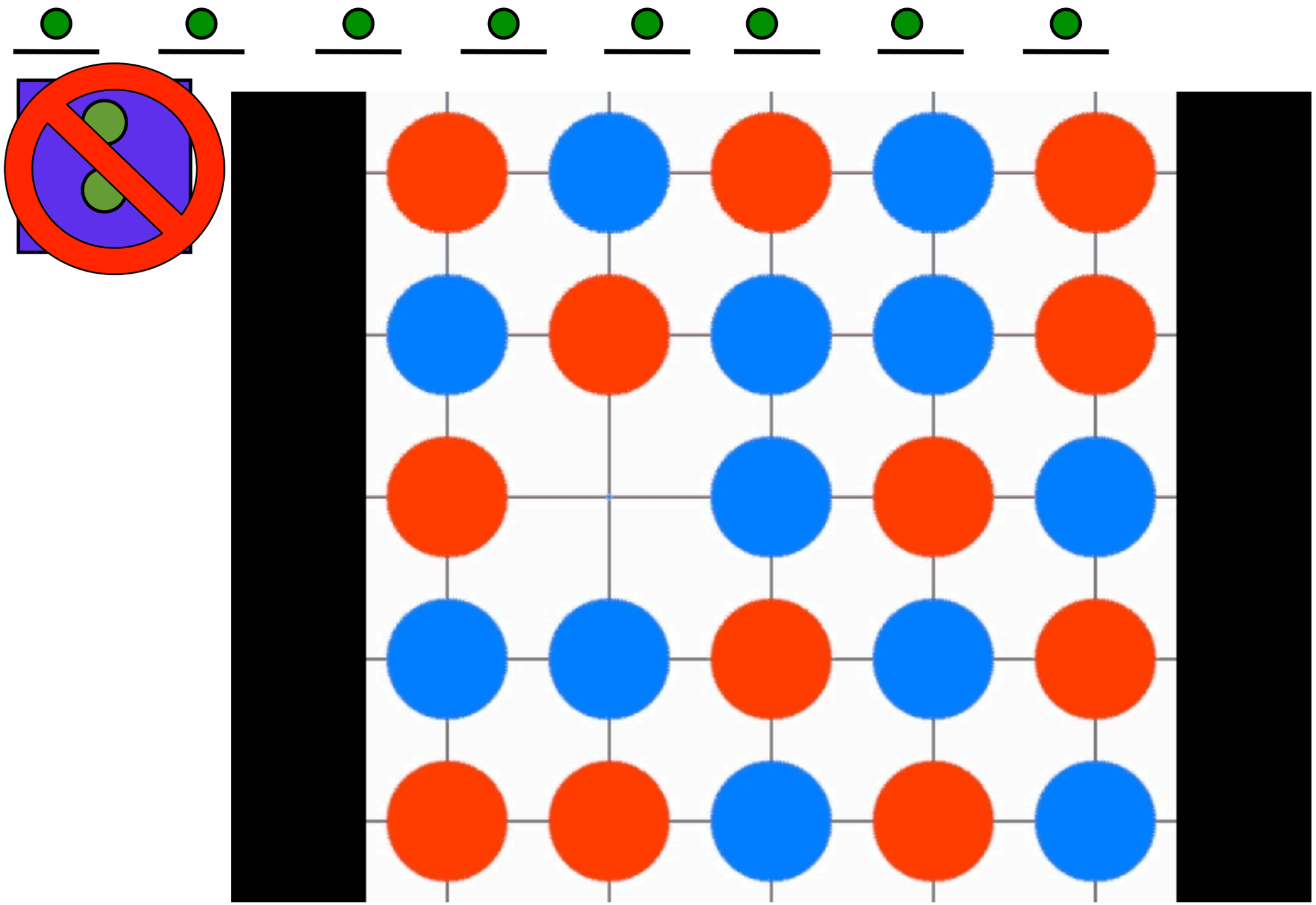
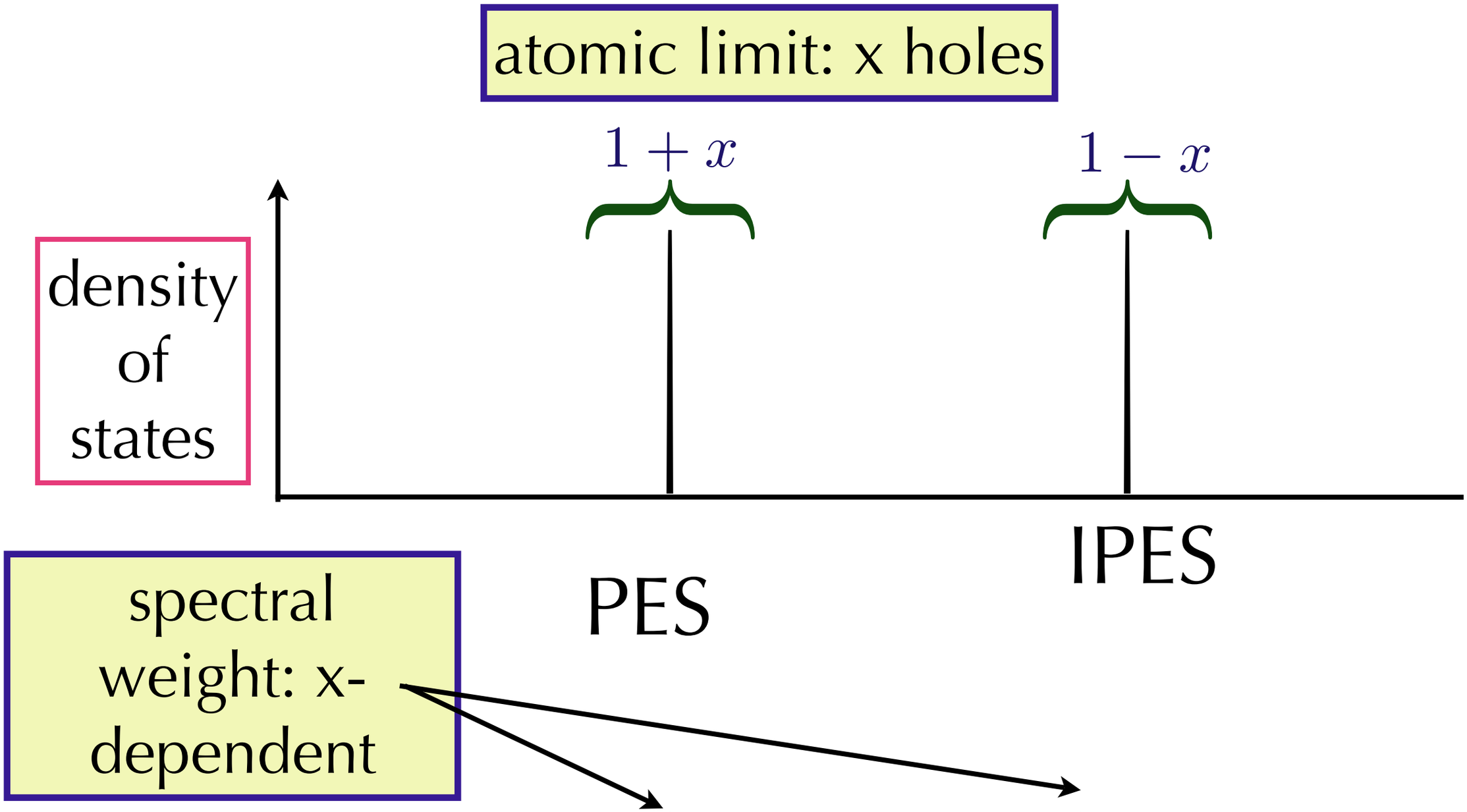
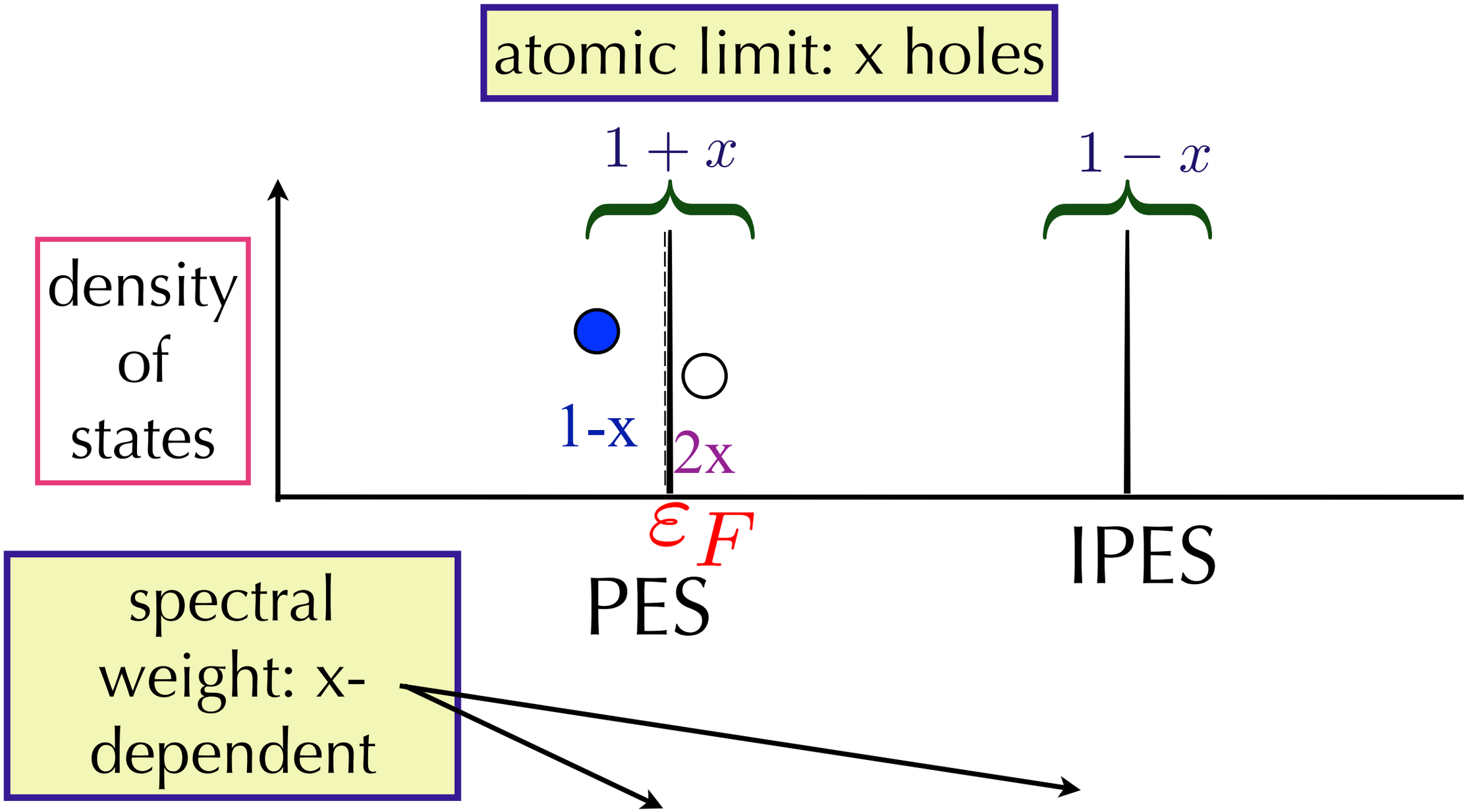




$$G(\omega, k) = \frac{1+x}{\omega - \mu + U/2} + \frac{1-x}{\omega - \mu - U/2}$$



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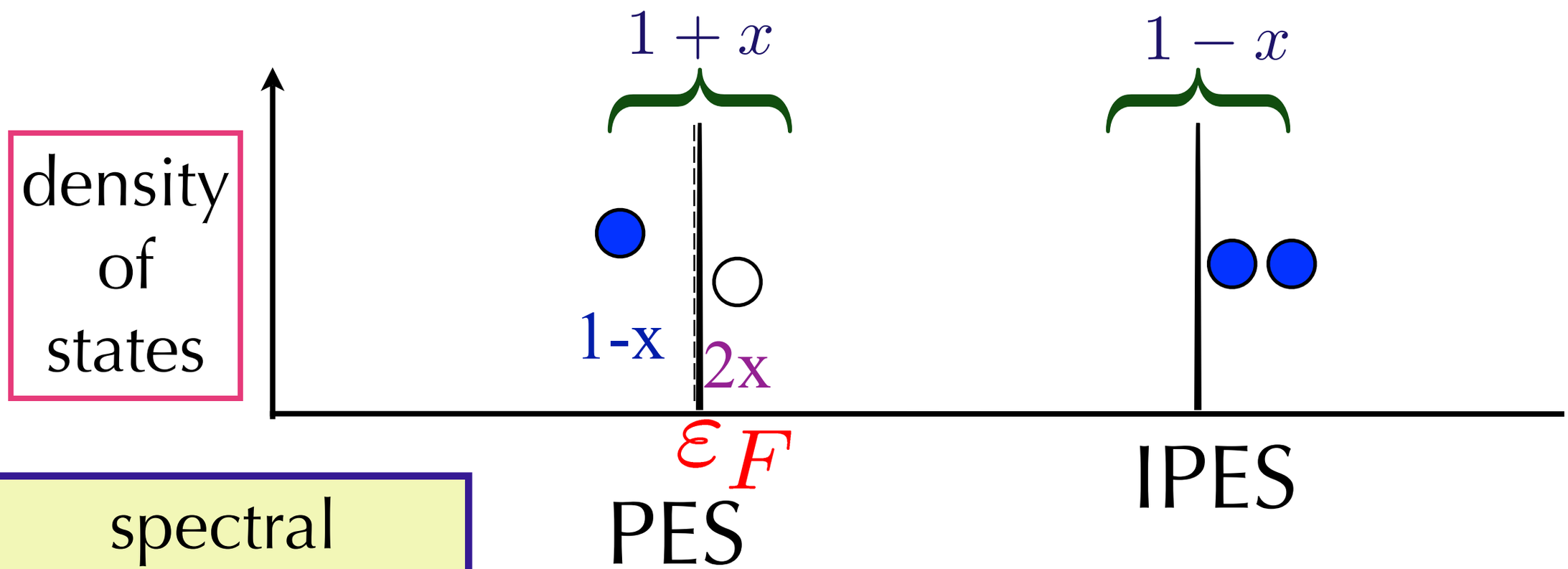
atomic limit: x holes

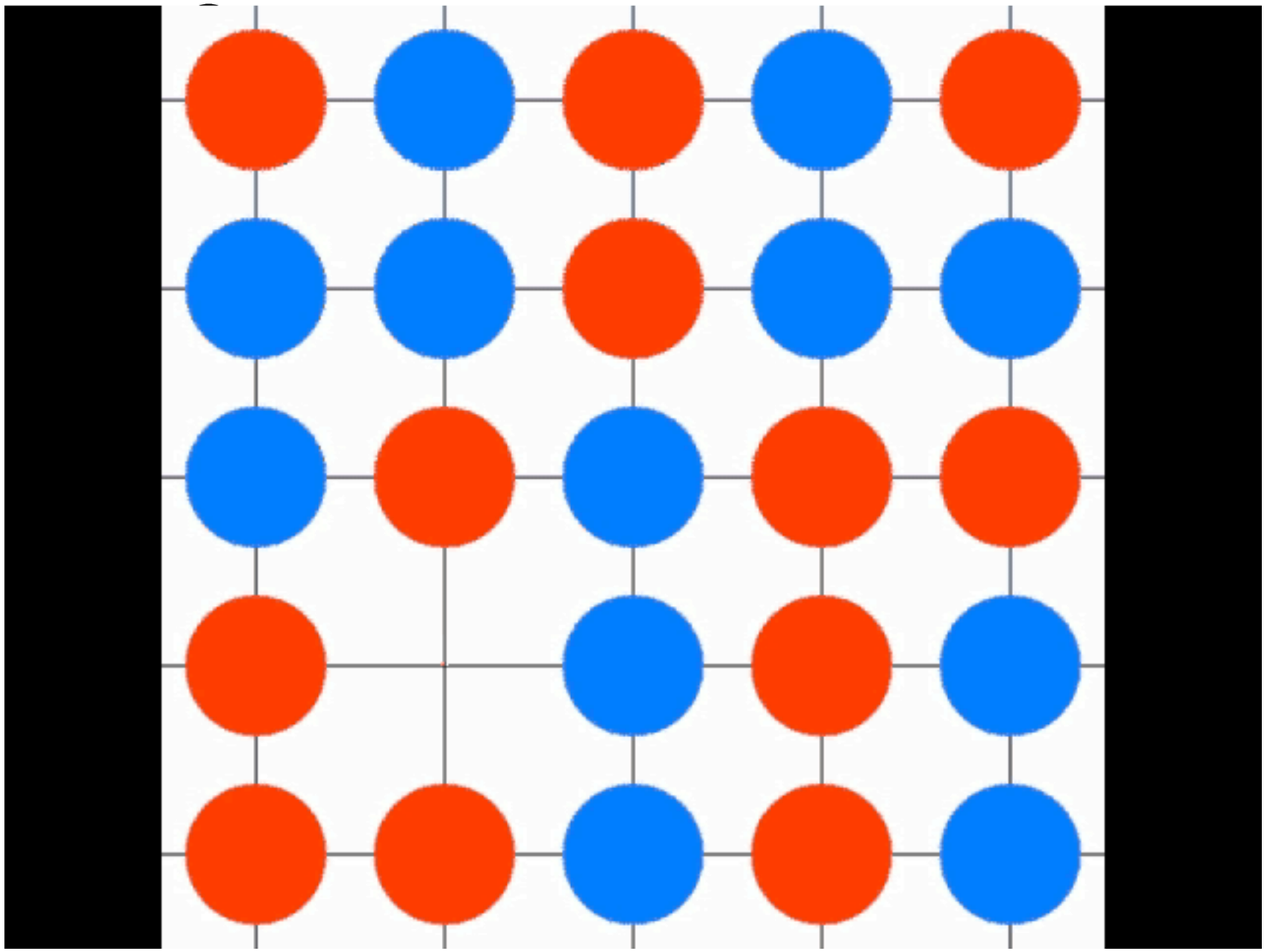


spectral weight: x-dependent

$$G(\omega,k) = \frac{1+x}{\omega-\mu+U/2} + \frac{1-x}{\omega-\mu-U/2}$$

quantum Mottness: U finite

$U \gg t$



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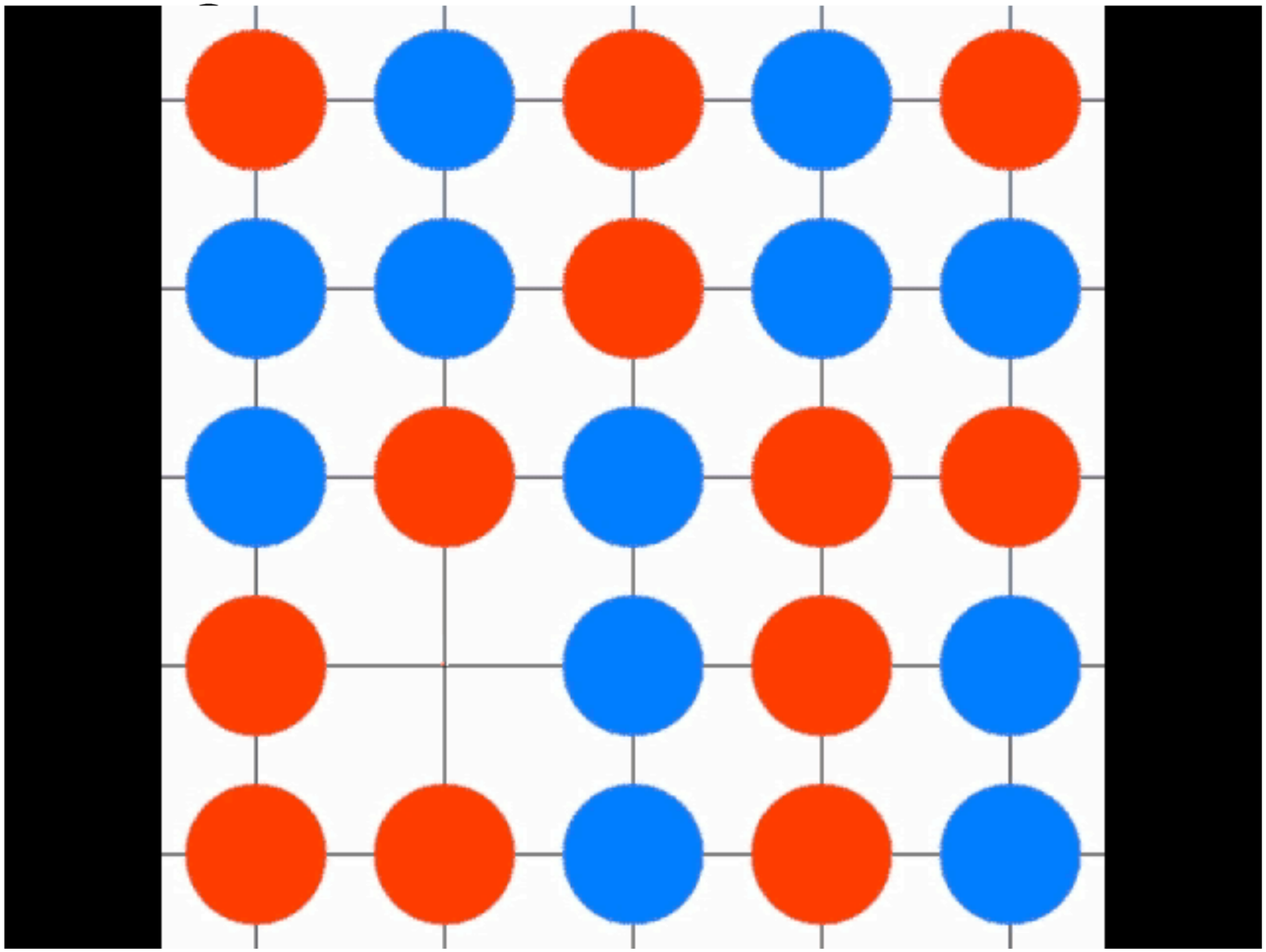

double occupancy in ground state!!

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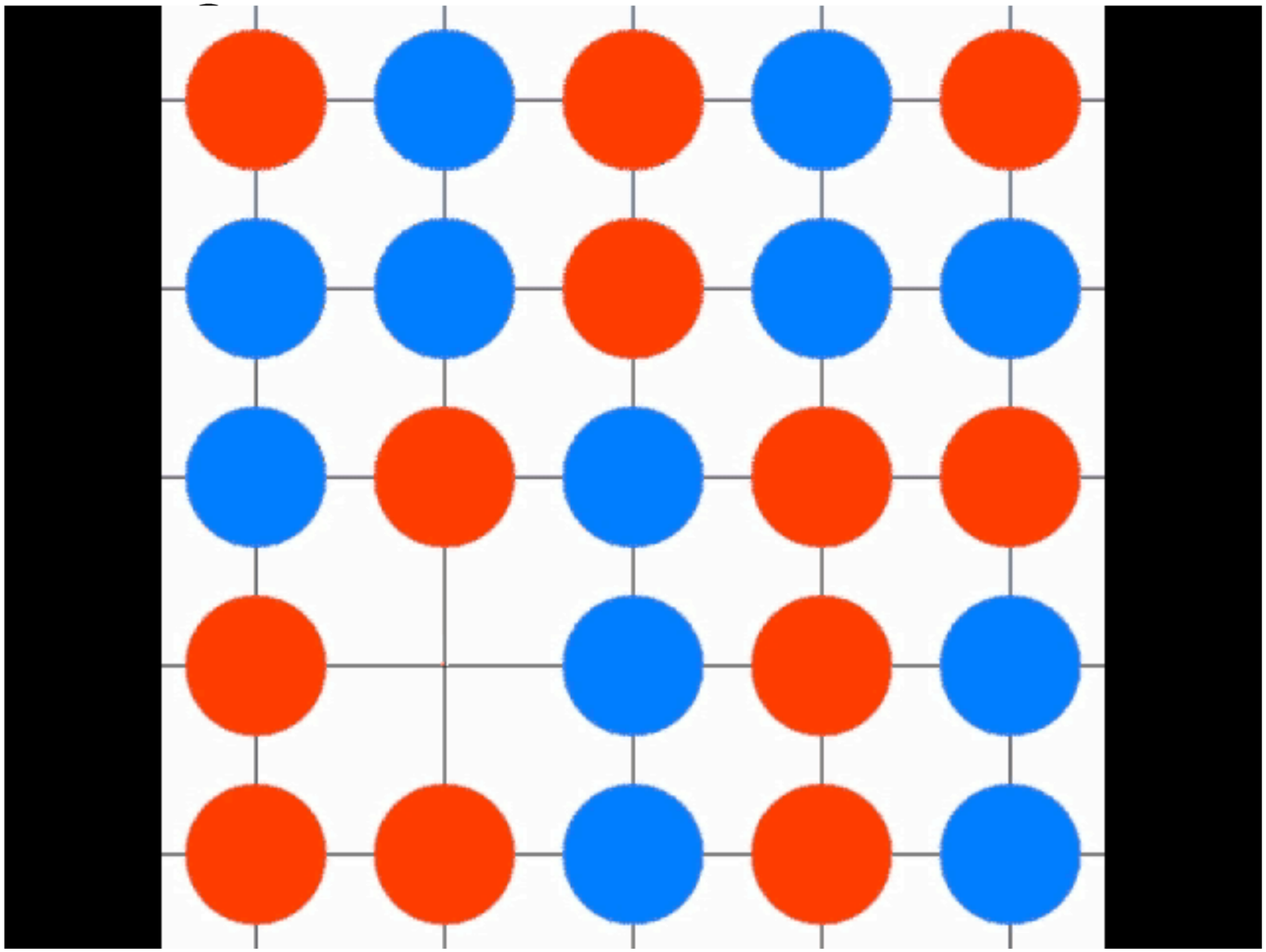
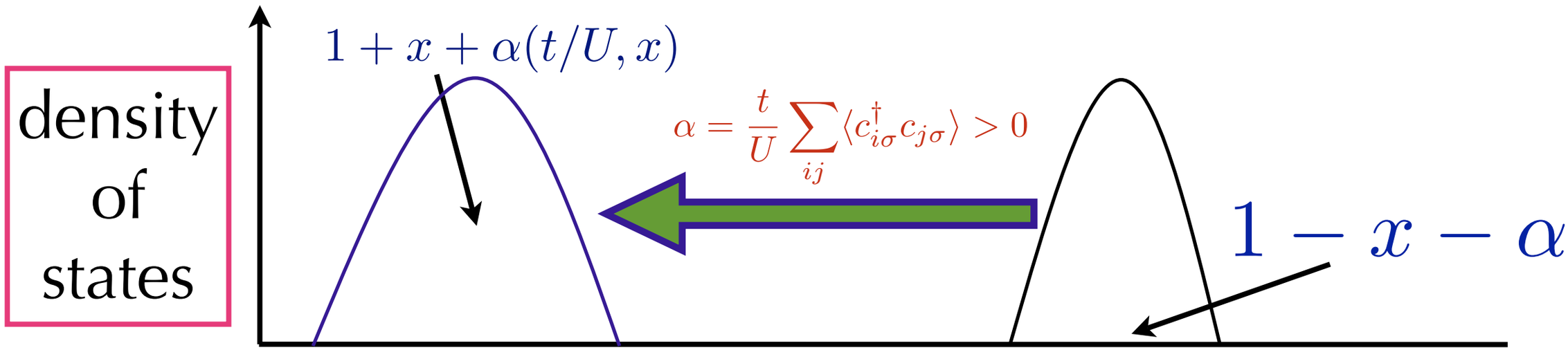


double occupancy in ground state!!

$W_{\mathrm{PES}} > 1 + x$

beyond the atomic limit: any real system



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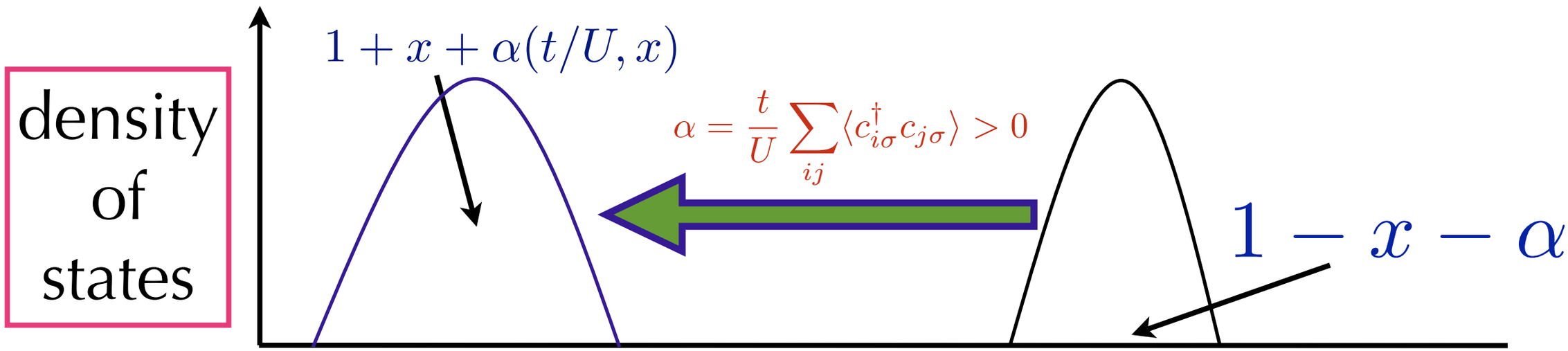

Harris & Lange, 1967

dynamical spectral weight transfer

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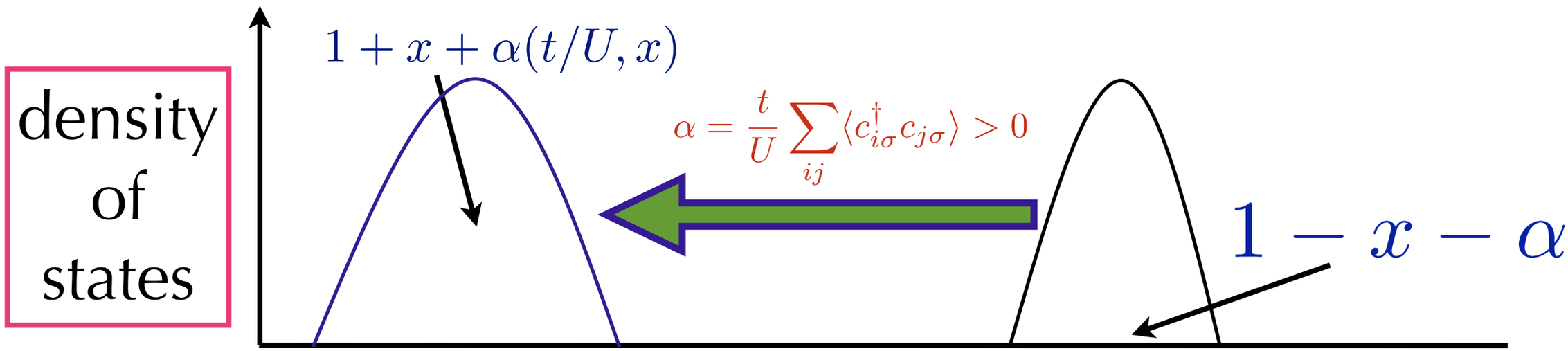

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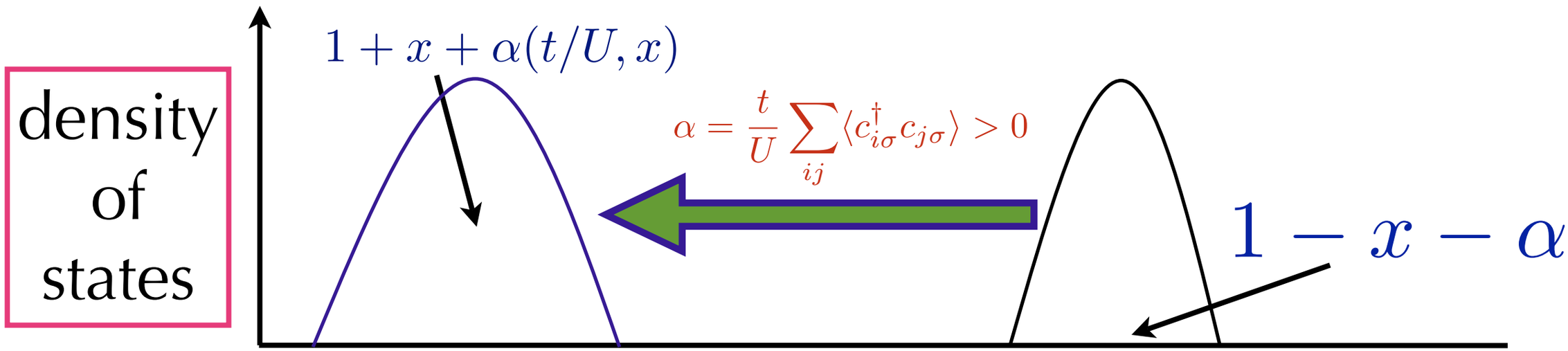

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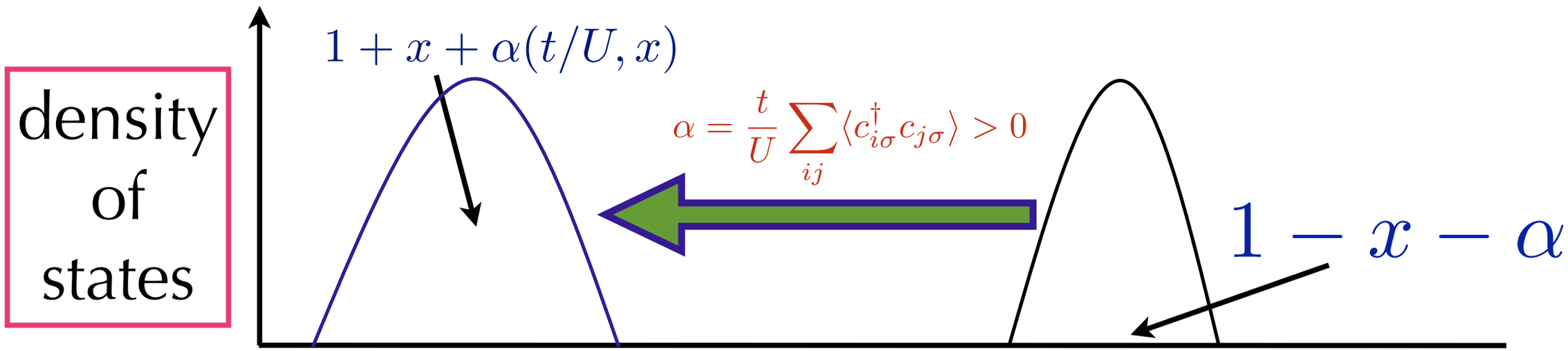

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# of electron states in lower band

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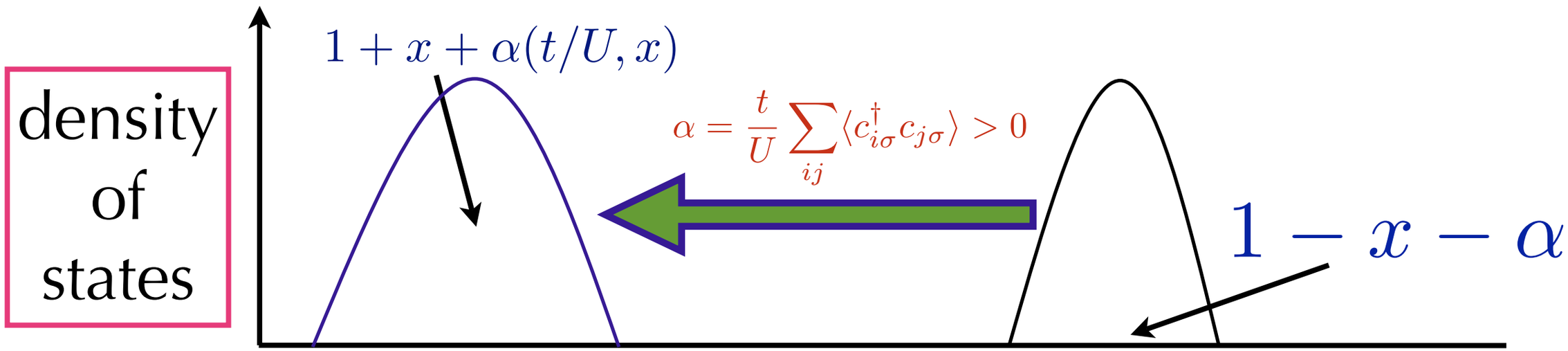

Harris & Lange, 1967

dynamical spectral weight transfer

Intensity>1+x

# of charge e states

# of electron states in lower band

not exhausted by counting electrons alone?

What are the extra states (degrees of freedom)?

Key Equation

$$1 = 2 - 1$$

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$$e = 2e - e$$

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$$1 = 2 - 1$$

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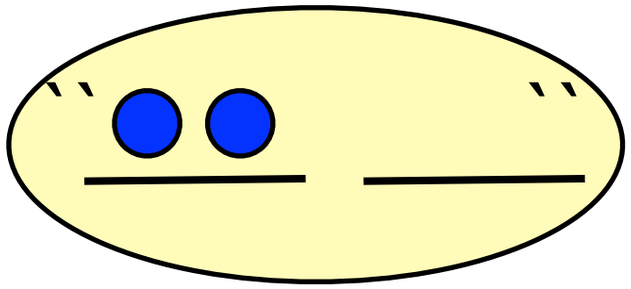
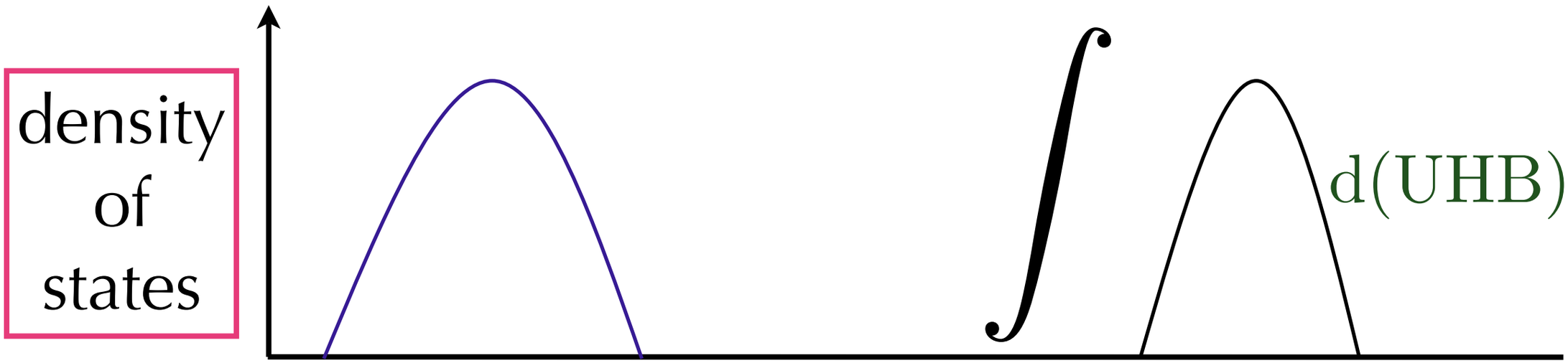
composite (bound) excitation

2e(boson)
+
hole



Hubbard bands are not rigid: Mottness

We now know what ? is



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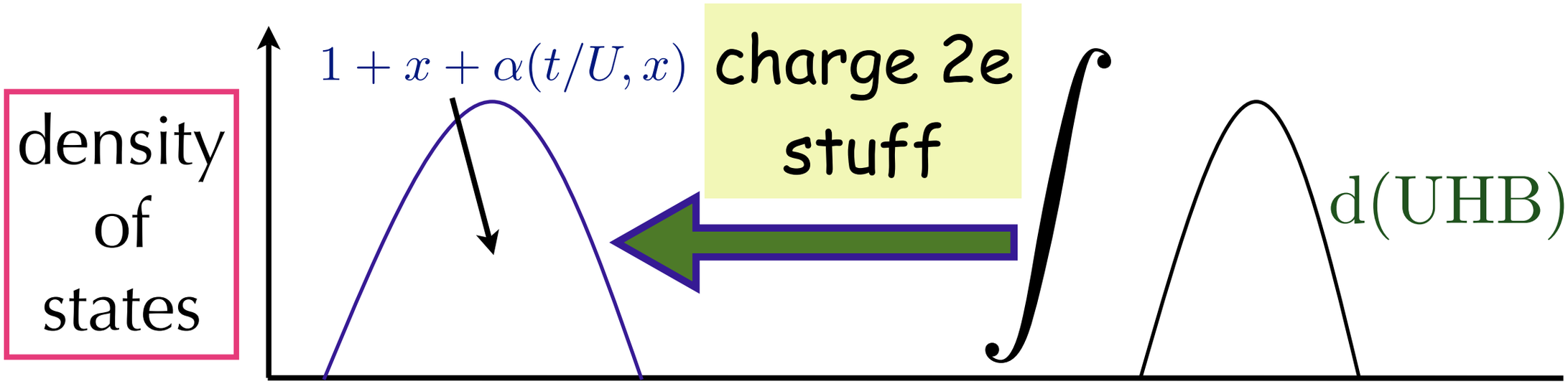

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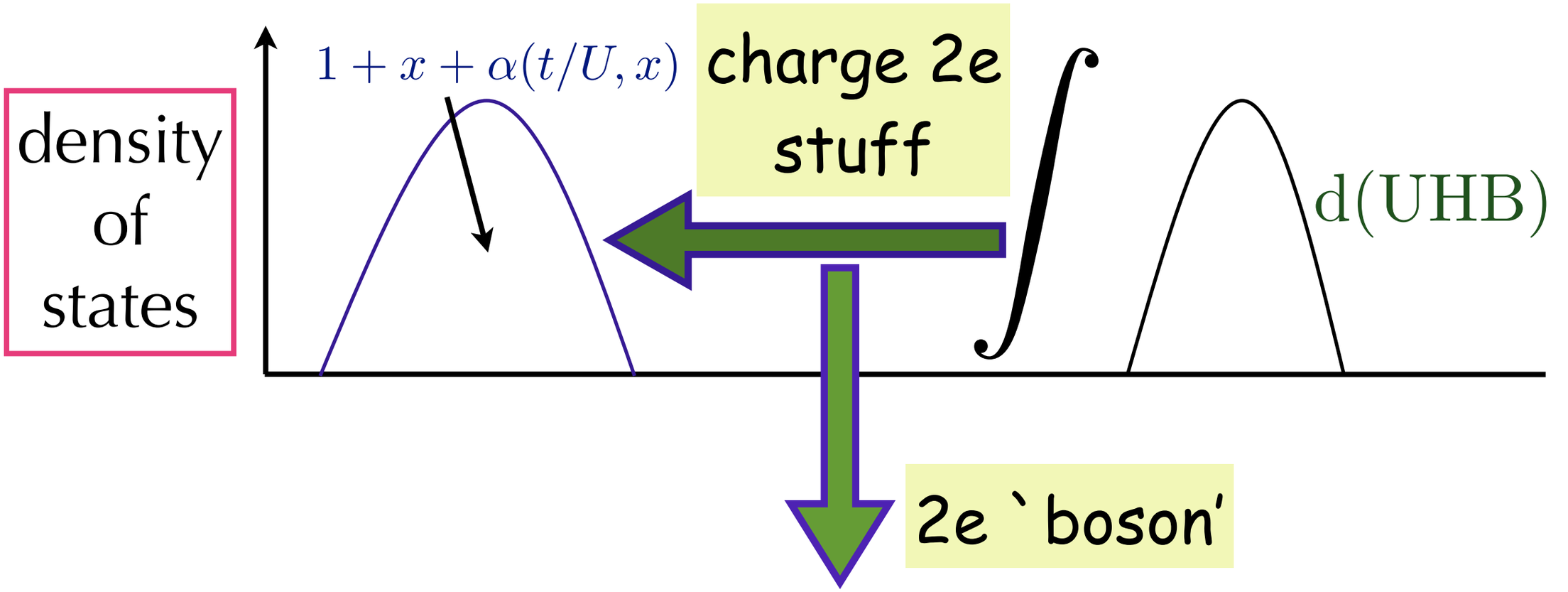
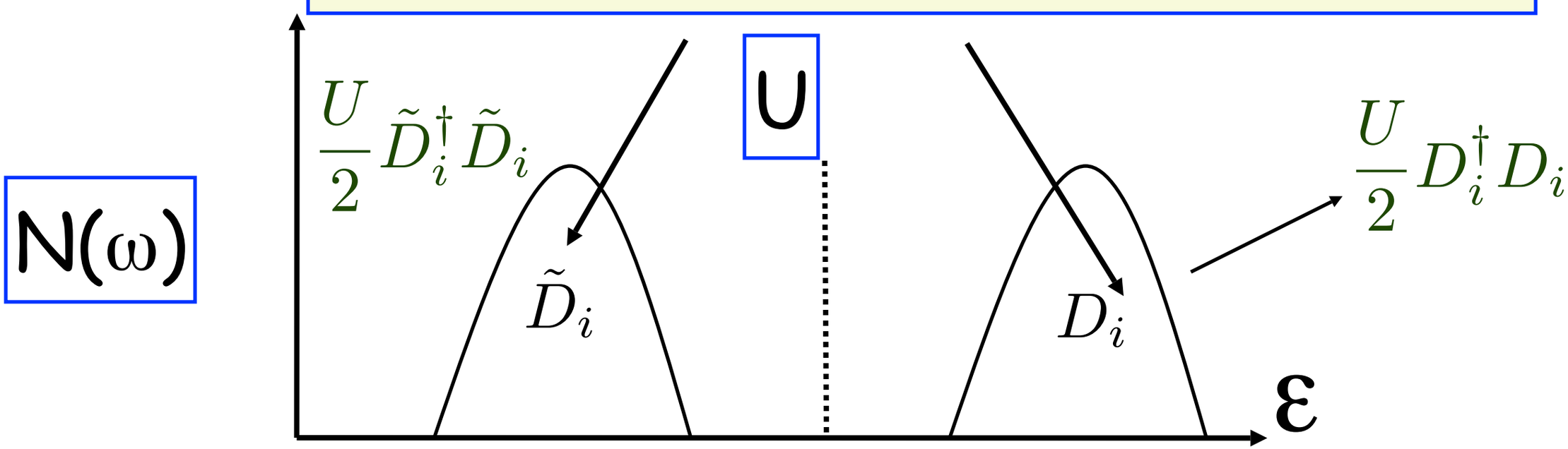
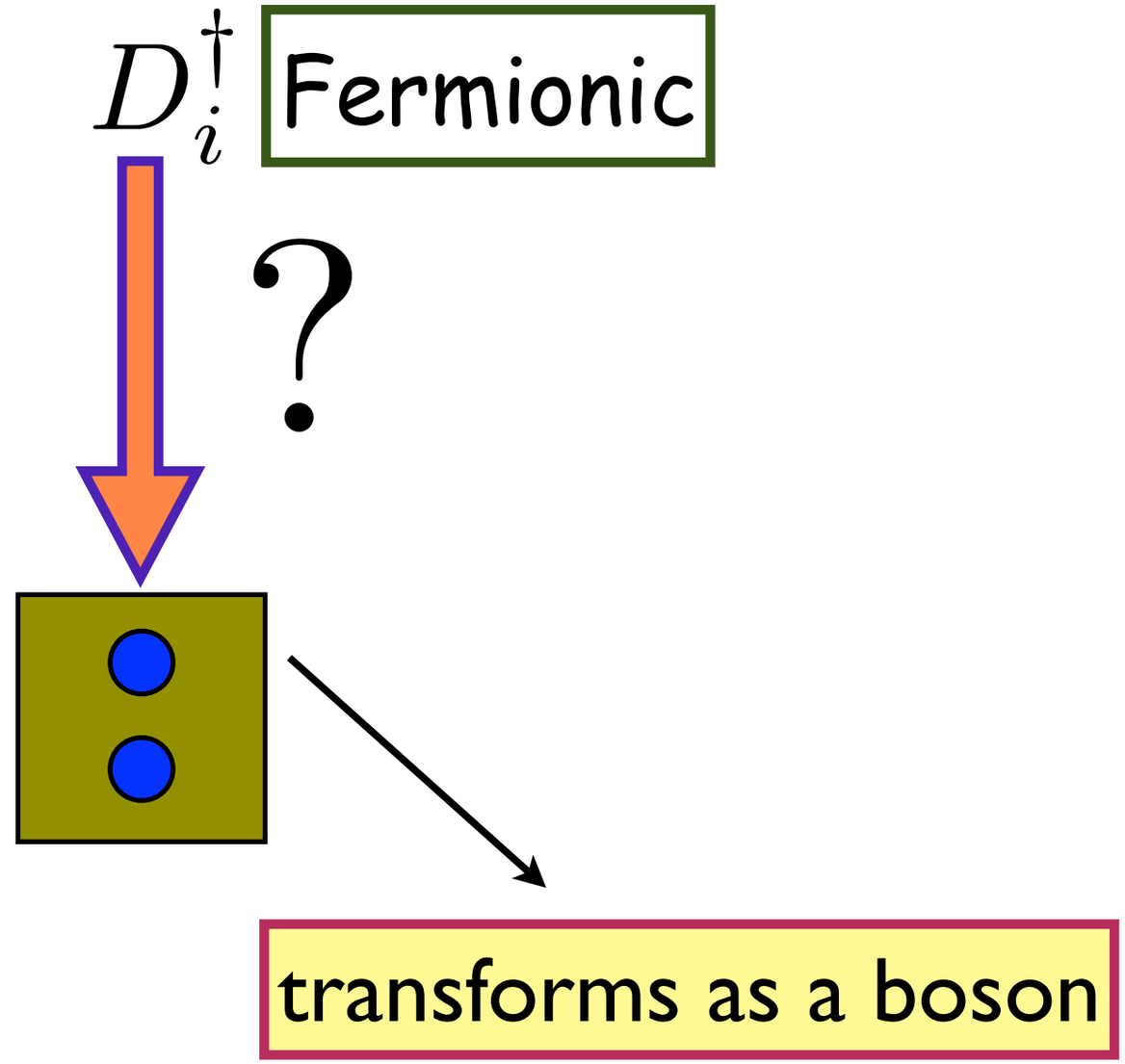


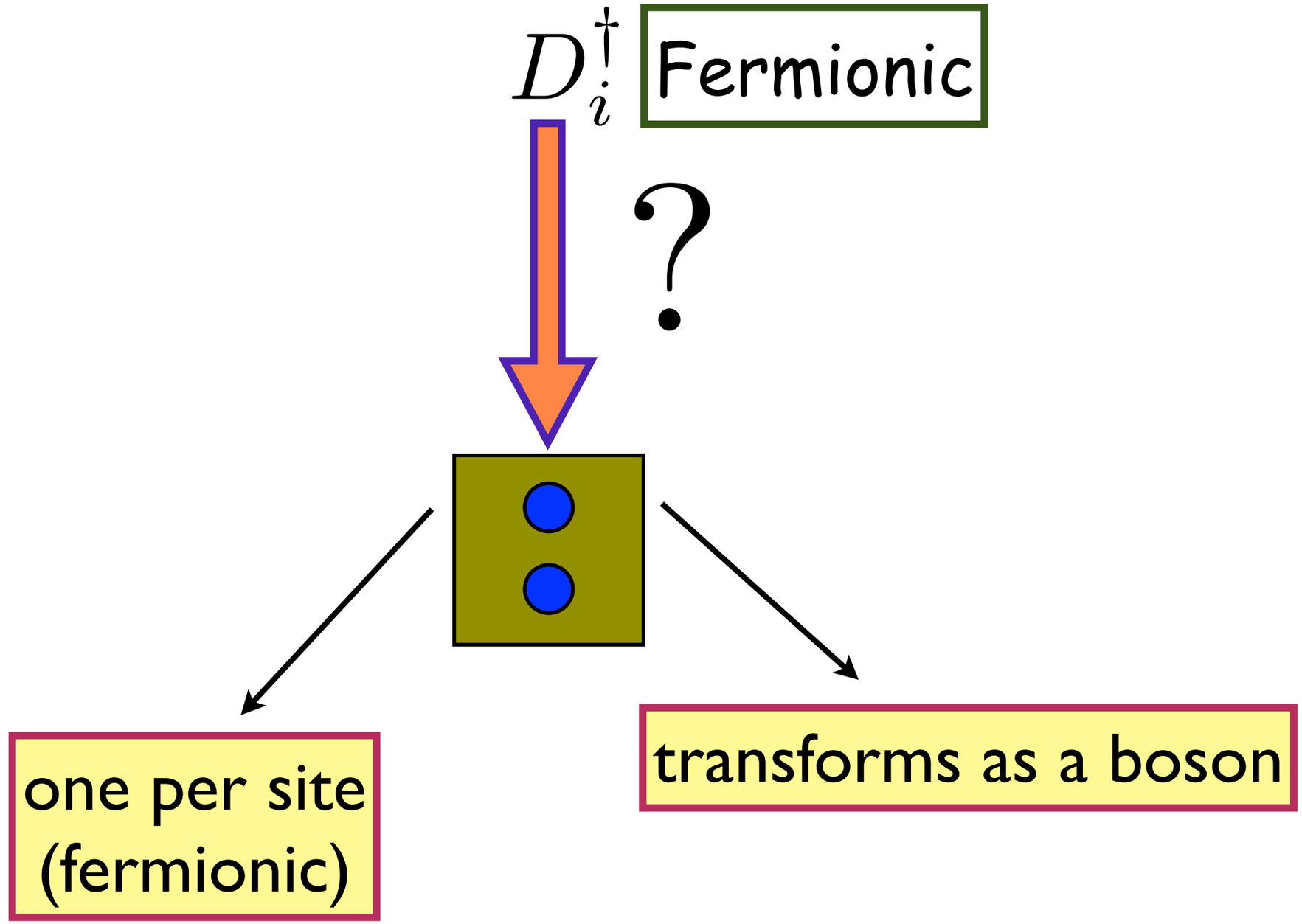
collective excitation from double occupancy:
incoherence at low energies

Key idea: similar to Bohm/Pines

Extend the Hilbert space:
Associate with U-scale new
Fermionic oscillators




$D_i^\dagger$
Fermionic
?
transforms as a boson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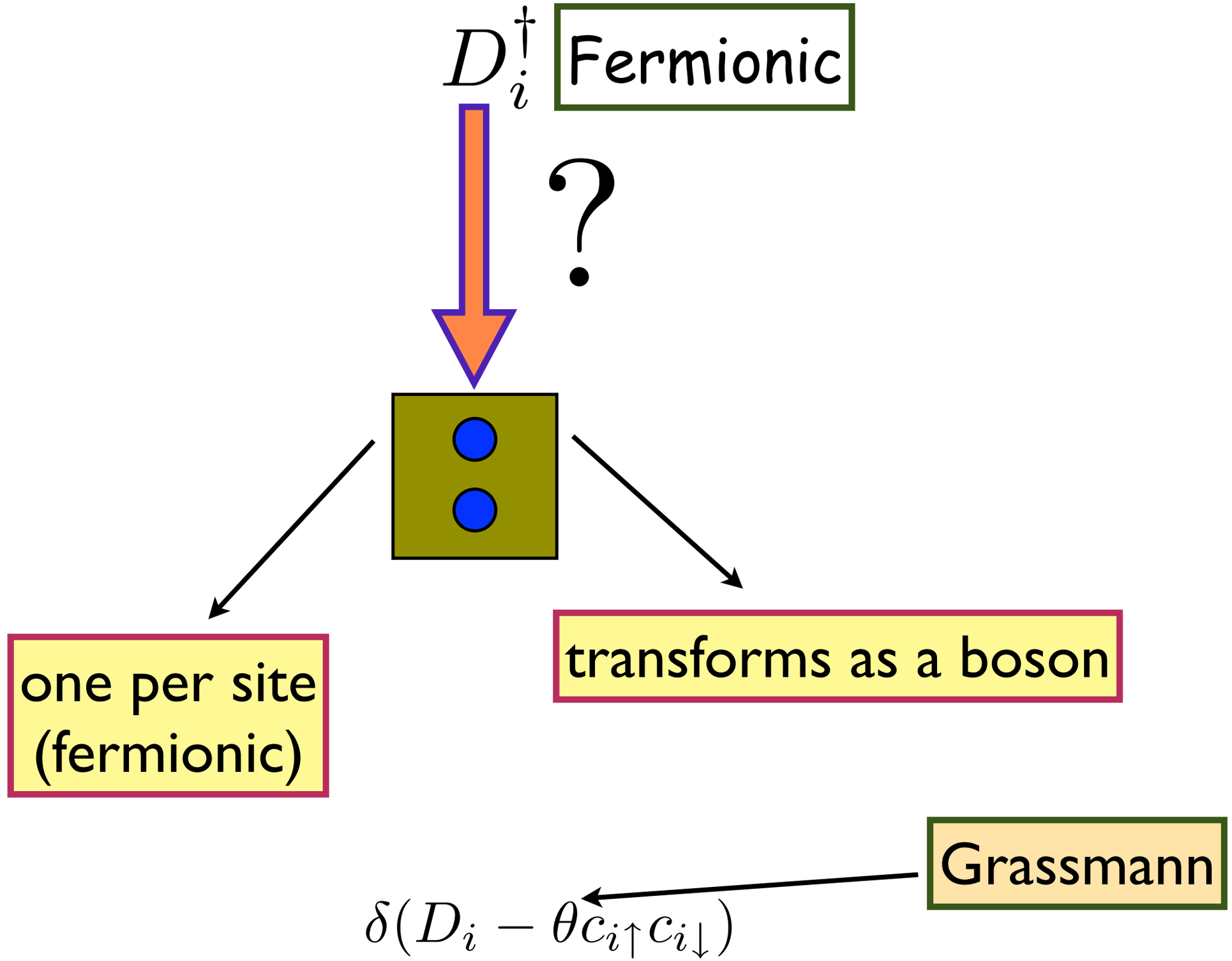
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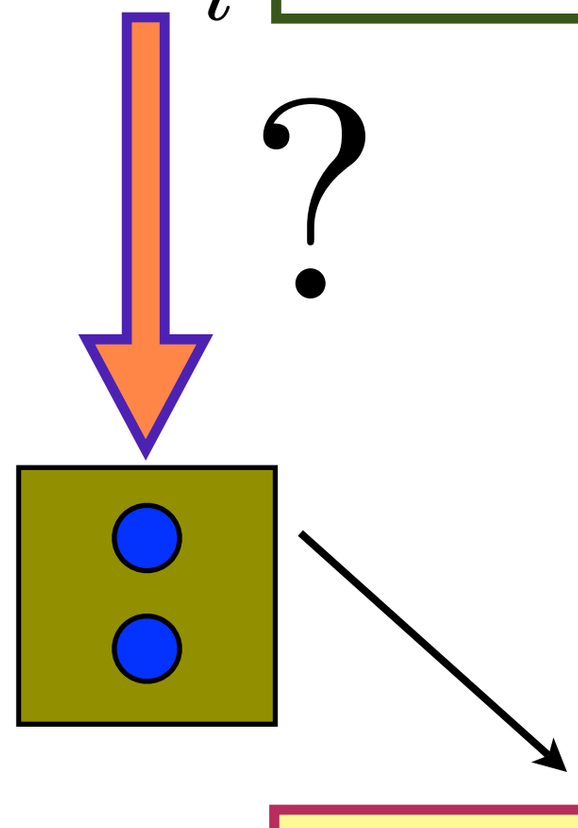
Grassmann
$\delta(D_i - \theta c_{i\uparrow} c_{i\downarrow})$

$D_i^\dagger$ Fermionic

?



one per site (fermionic)

transforms as a boson

Grassmann

$$\delta(D_i - \theta c_{i\uparrow} c_{i\downarrow})$$

$$\theta \varphi_i^\dagger \longrightarrow$$ charge 2e boson

Exact low-energy Lagrangian

$$L = \#L_{\text{bare}}(\text{electrons}) + \#\text{L}_{\text{bare}}(\text{bosons})$$

$$+f(\omega)L_{\text{int}}(c,\varphi) \quad +\tilde{f}(\omega)L_{\text{int}}(c,\tilde{\varphi})$$

$$\downarrow \qquad\qquad \downarrow$$

$$\Psi^\dagger\Psi \qquad\qquad \tilde{\Psi}^\dagger\tilde{\Psi}$$

quadratic form:
composite or bound
excitations of
$\varphi^\dagger c_{i\sigma}$

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$f(\omega) = 0$

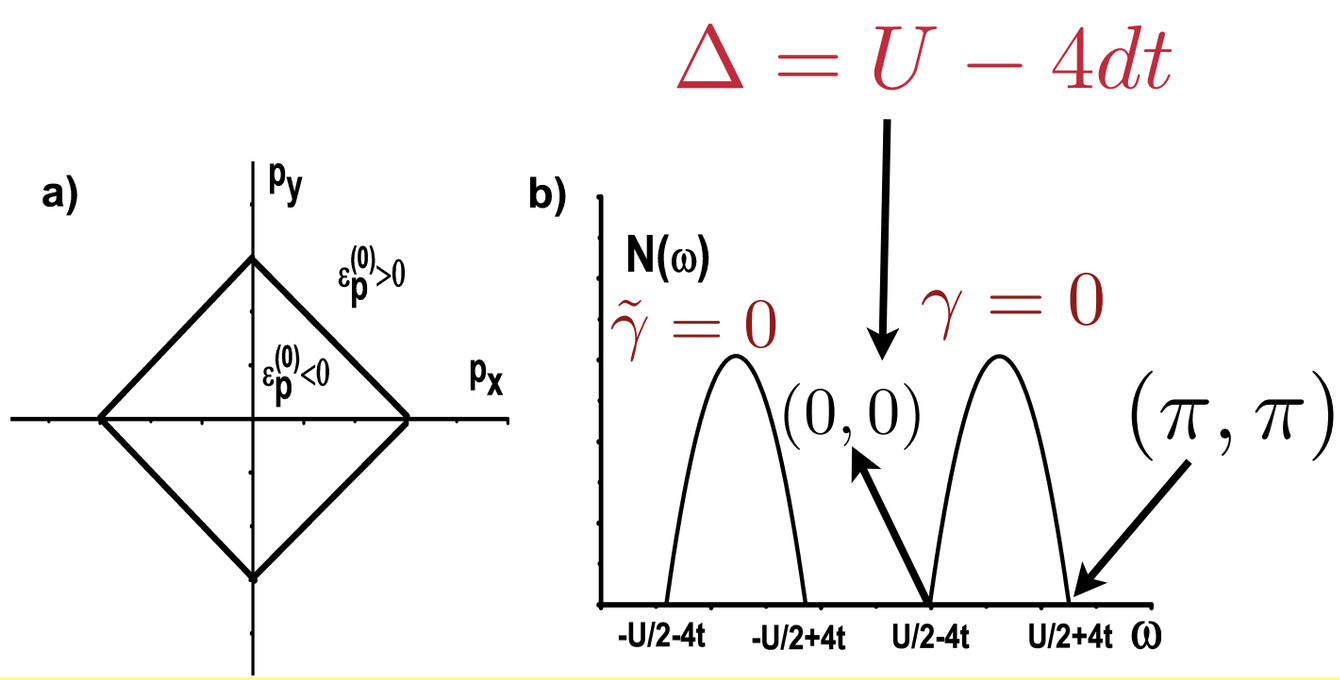
dispersion of propagating light modes

quadratic form: composite or bound excitations of $\varphi^\dagger c_{i\sigma}$

composite excitations determine spectral density: Mott gap

$$\gamma_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U - t\varepsilon_{\vec{p}}^{(\vec{k})} - 2\omega}{U}\sqrt{1 + 2\omega/U}$$

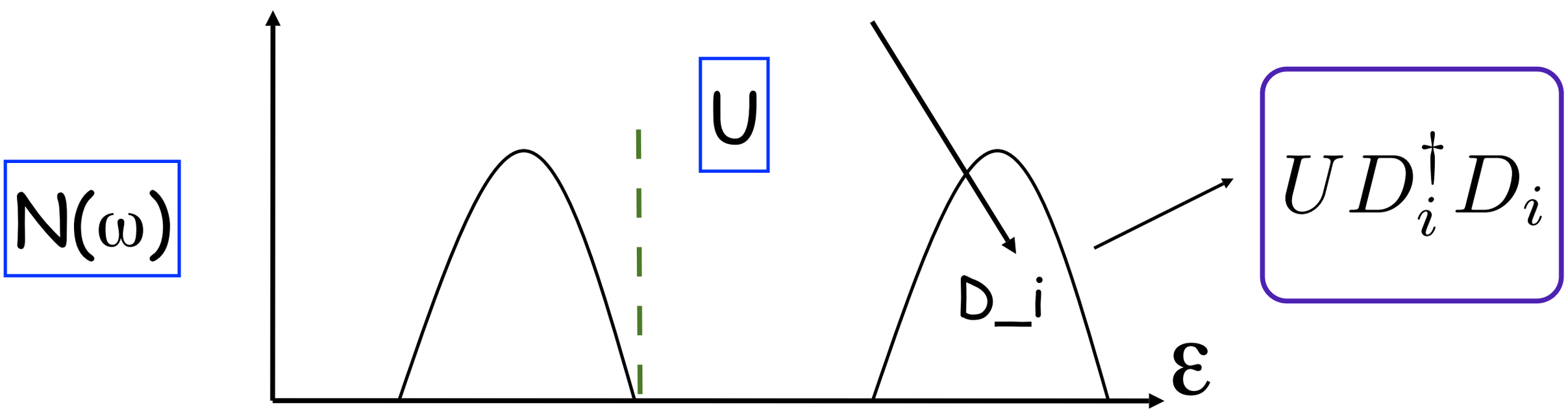
$$\tilde{\gamma}_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U + t\varepsilon_{\vec{p}}^{(\vec{k})} + 2\omega}{U}\sqrt{1 - 2\omega/U}.$$



each momentum has SD at two distinct energies

hole-doping?

Extend the Hilbert space:
Associate with U-scale a new
Fermionic oscillator

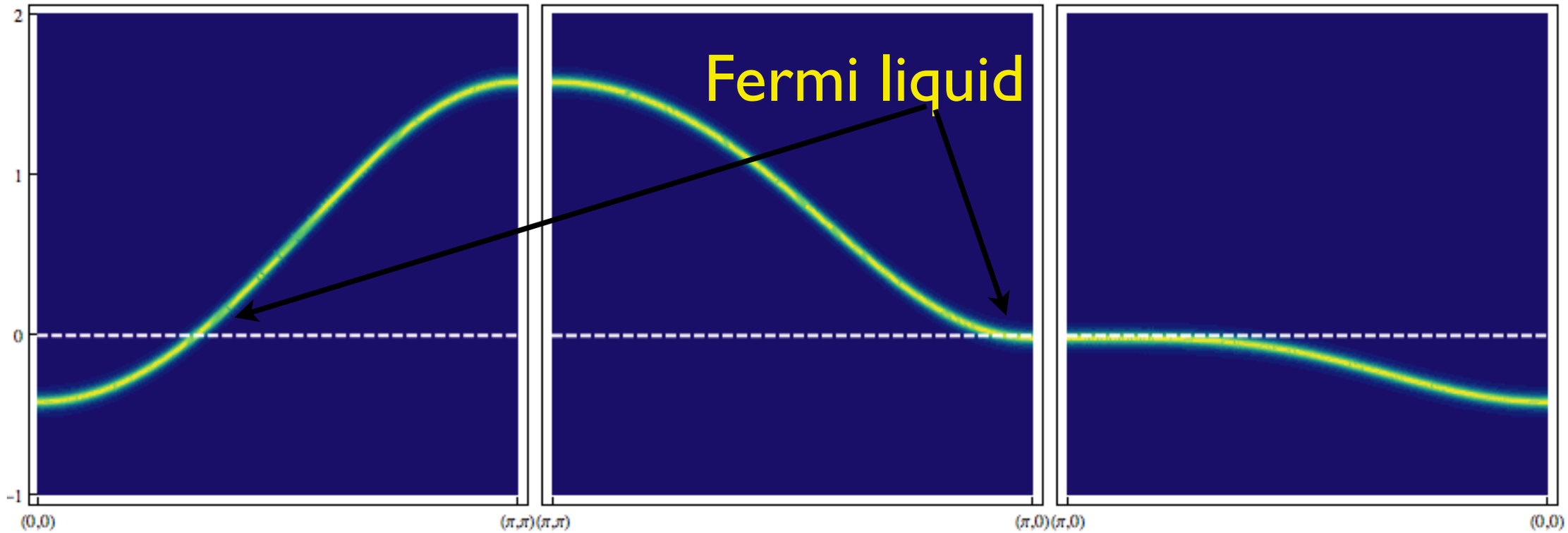


$$\mathcal{L}_{\mathrm{IR}} = L_{\mathrm{spinstuff}}(c_i, c_i^\dagger) + \boxed{L_{\mathrm{charge}}(c_i, c_i^\dagger, \varphi_i)}$$

quadratic

spectral function

Electron Dispersion



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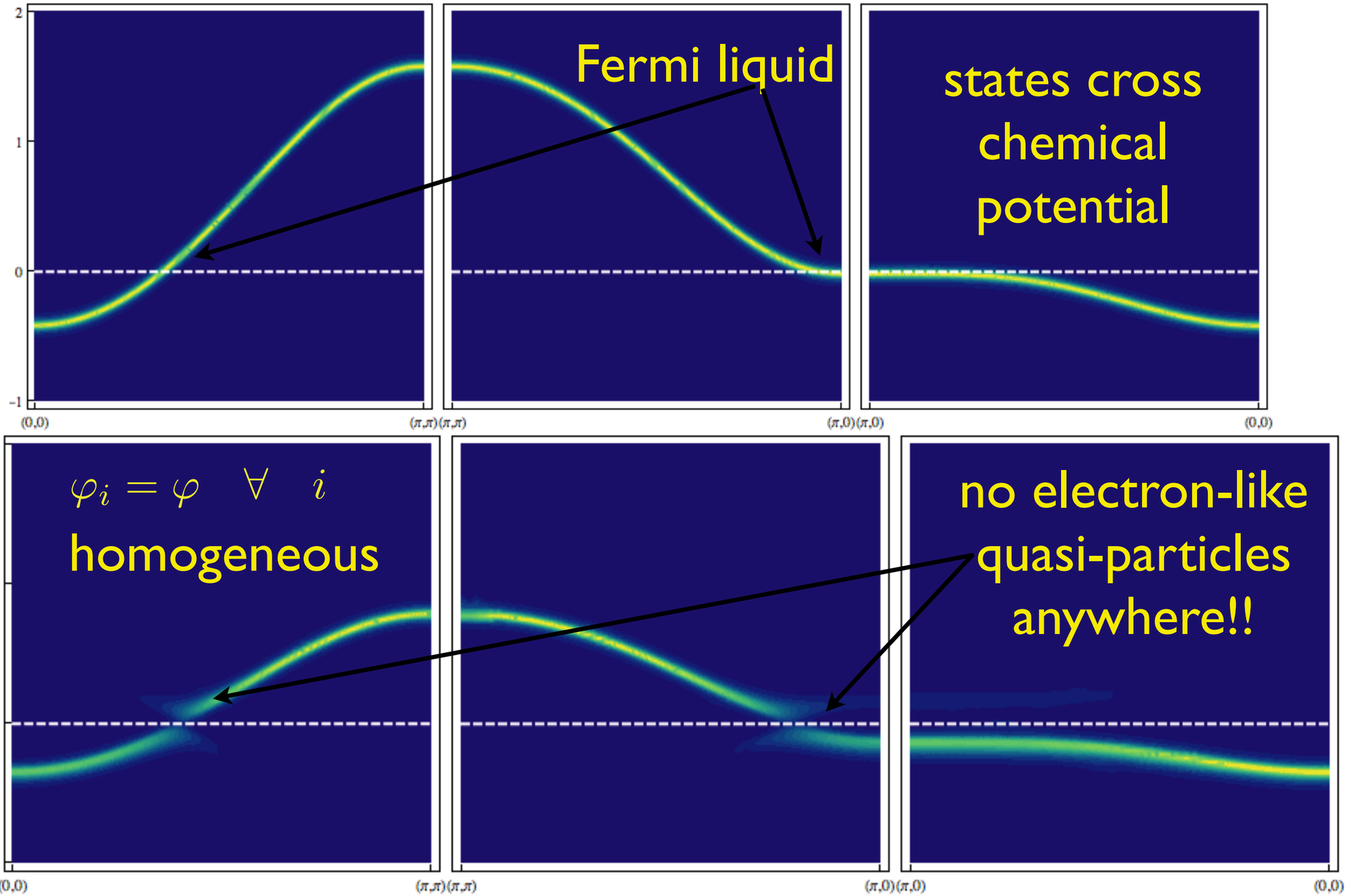

What is seen experimentally?
Are there 0-energy excitations?

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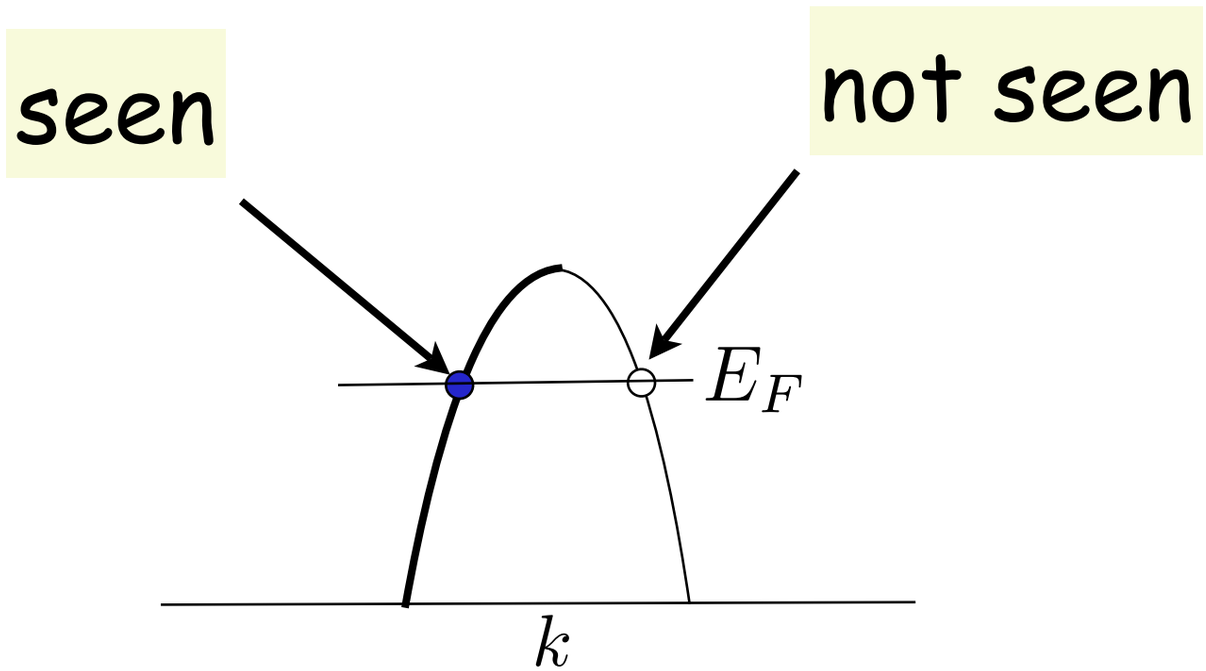

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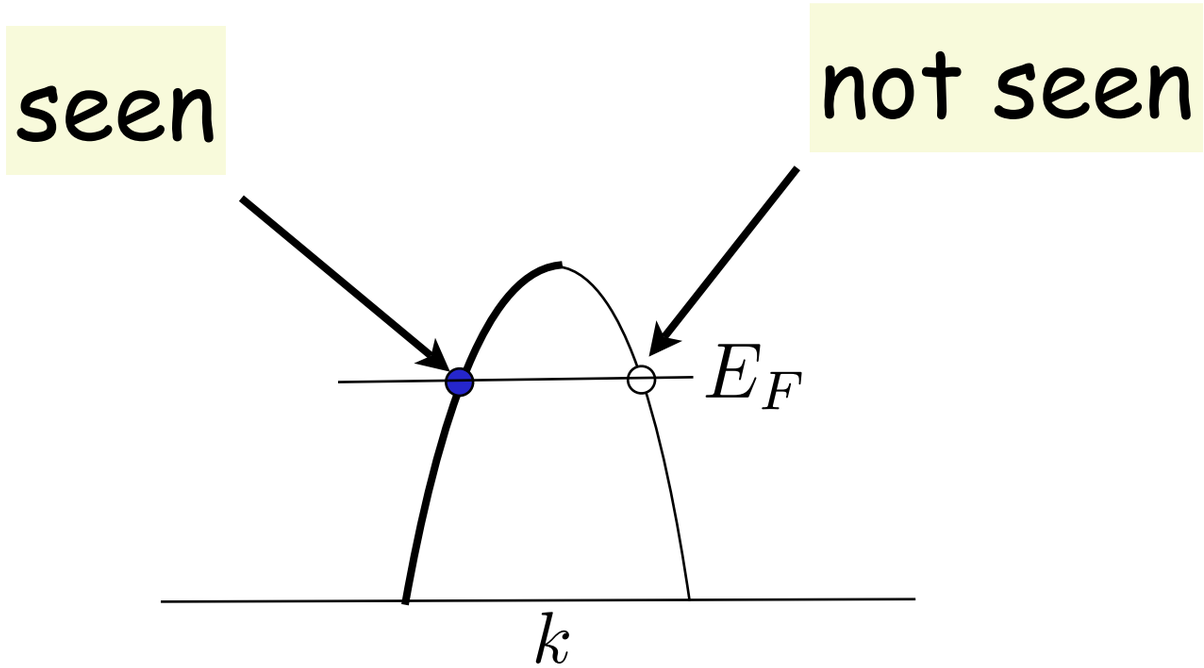
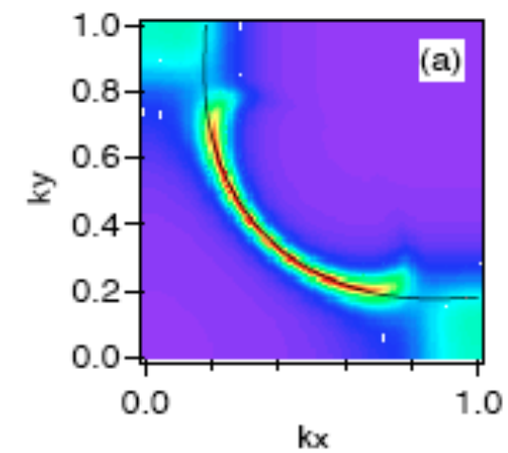


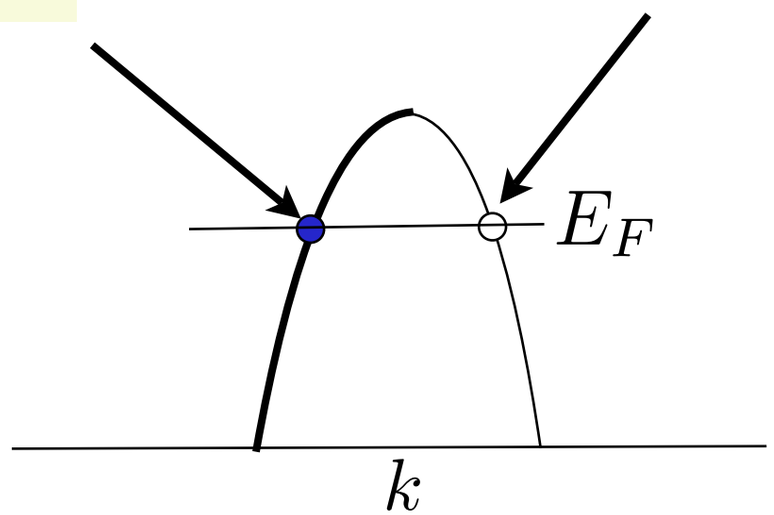


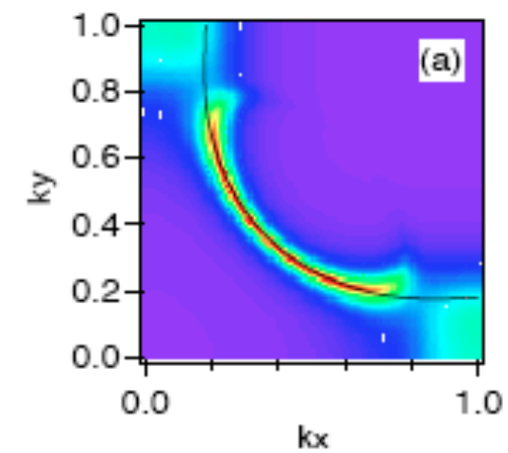
Fermi arcs: no double crossings (PDJ,JCC,ZXS)

What is seen experimentally?
Are there 0-energy excitations?

seen

not seen





meaning?

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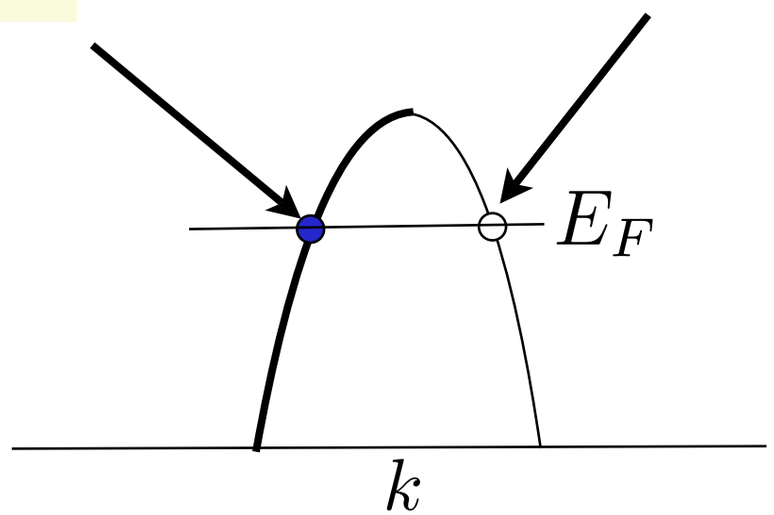
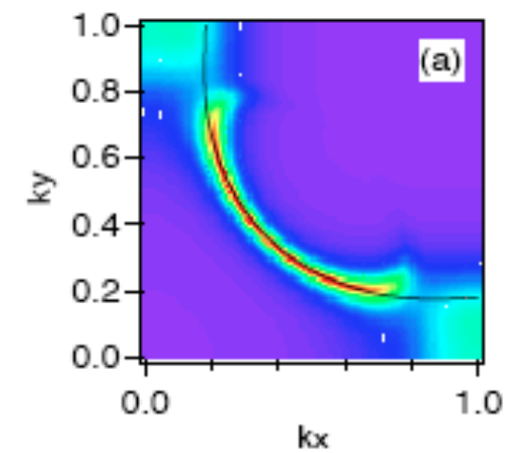


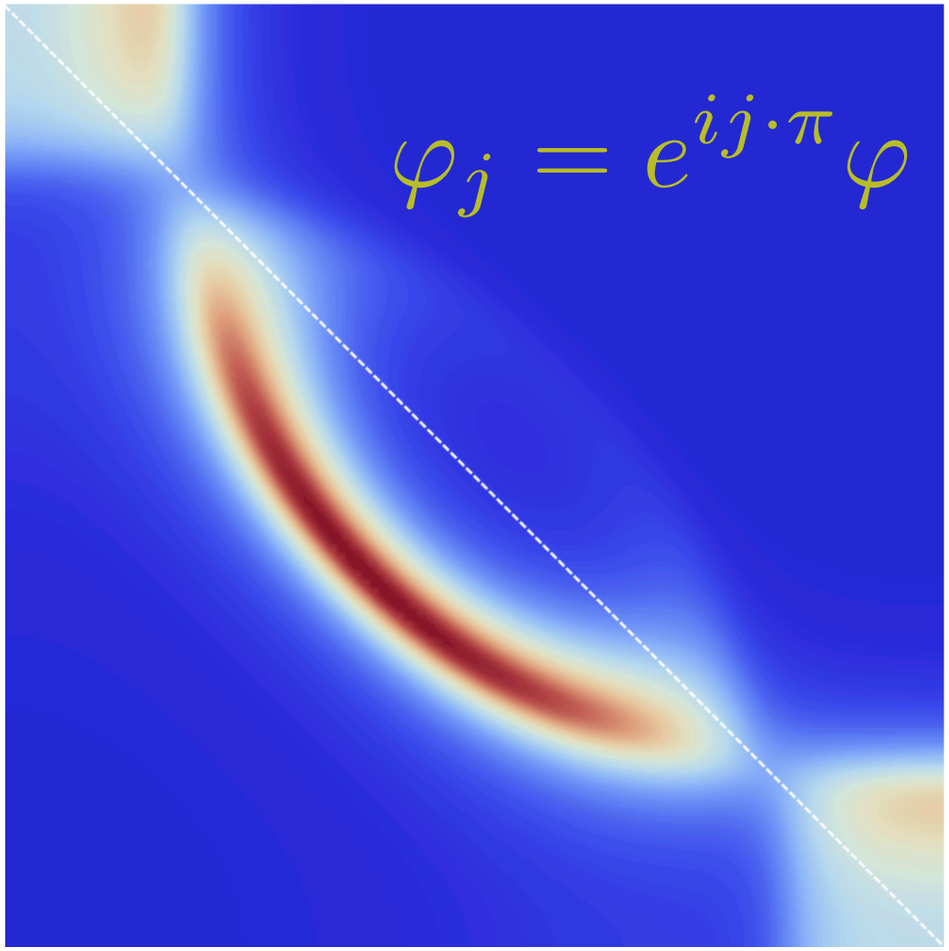


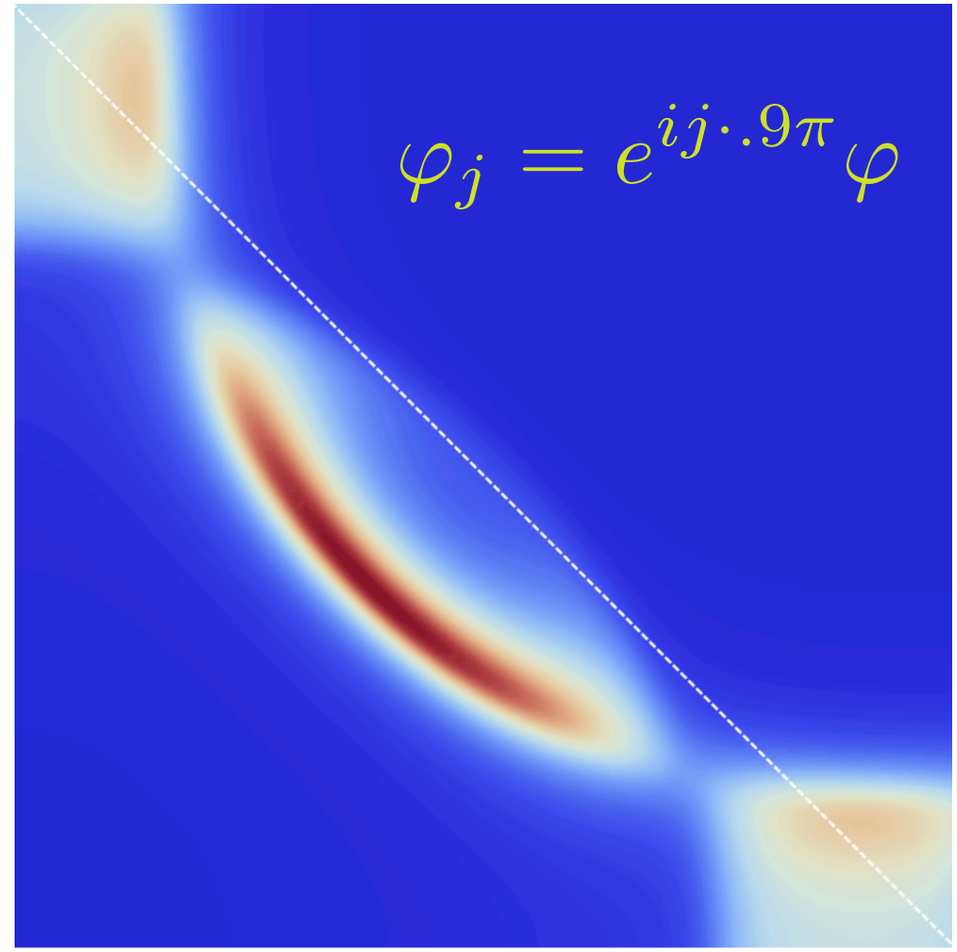
Fermi arcs: no double crossings (PDJ,JCC,ZXS)

meaning?

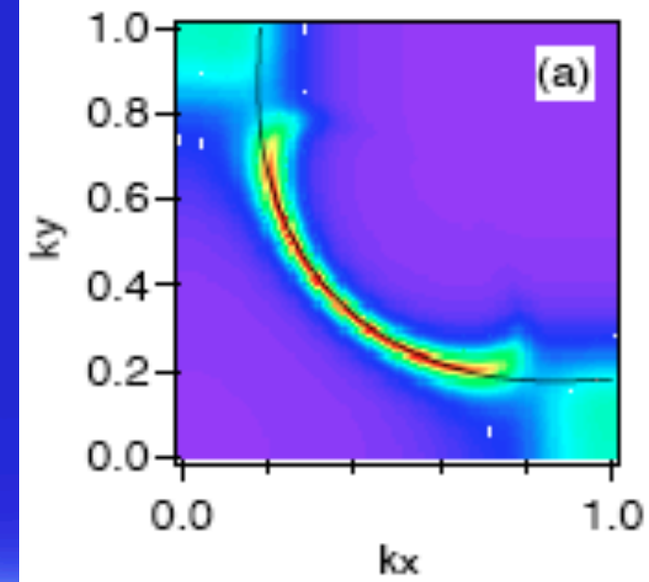
can we explain this?

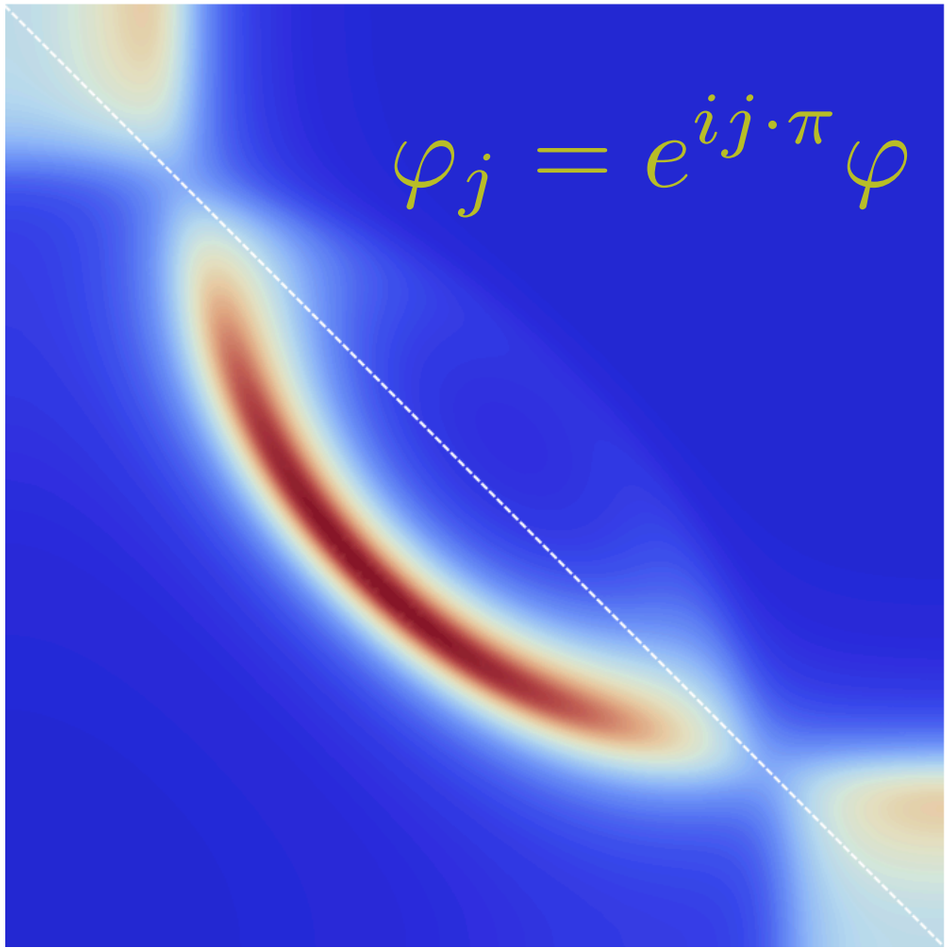
yes

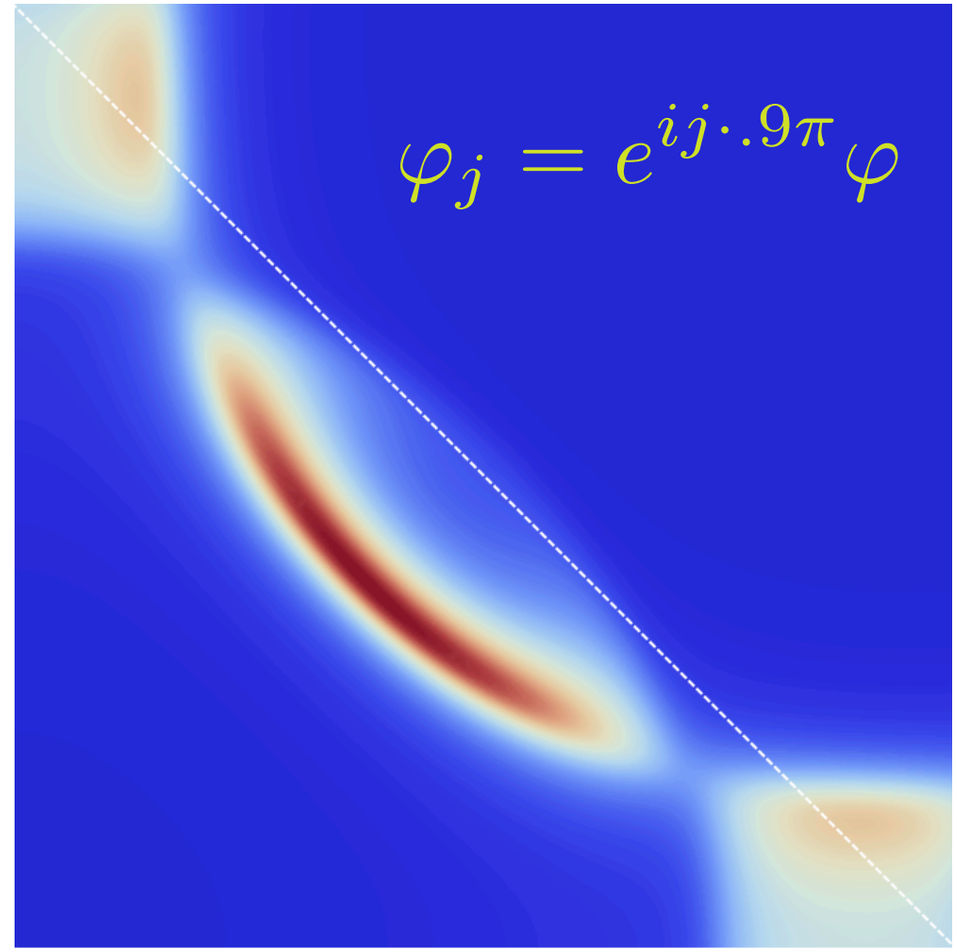

$\varphi_j = e^{ij\cdot\pi}\varphi$


$\varphi_j = e^{ij\cdot.9\pi}\varphi$

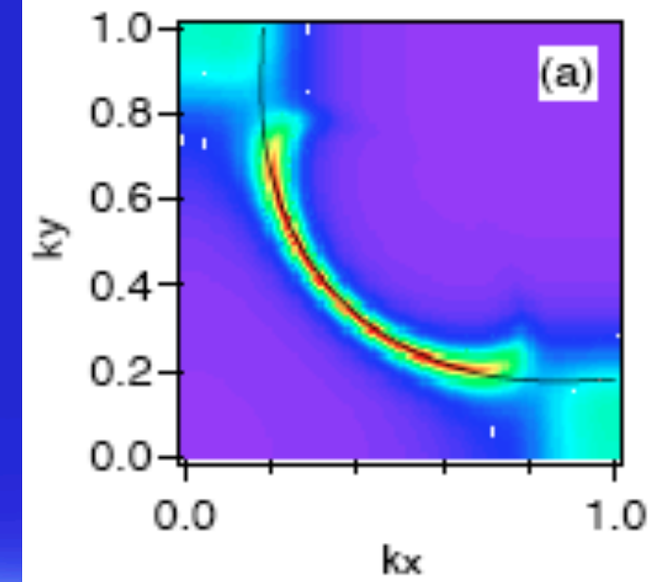
Expt.


(a)
ky
kx
0.0
0.2
0.4
0.6
0.8
1.0
0.0
1.0

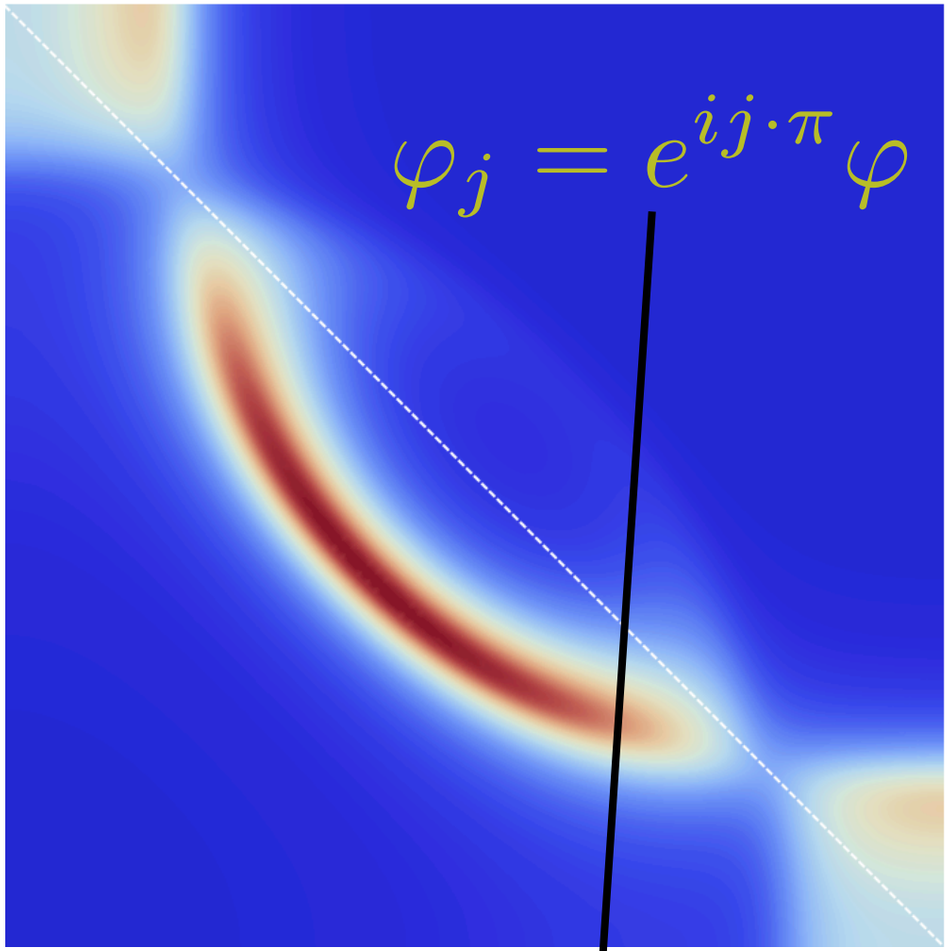


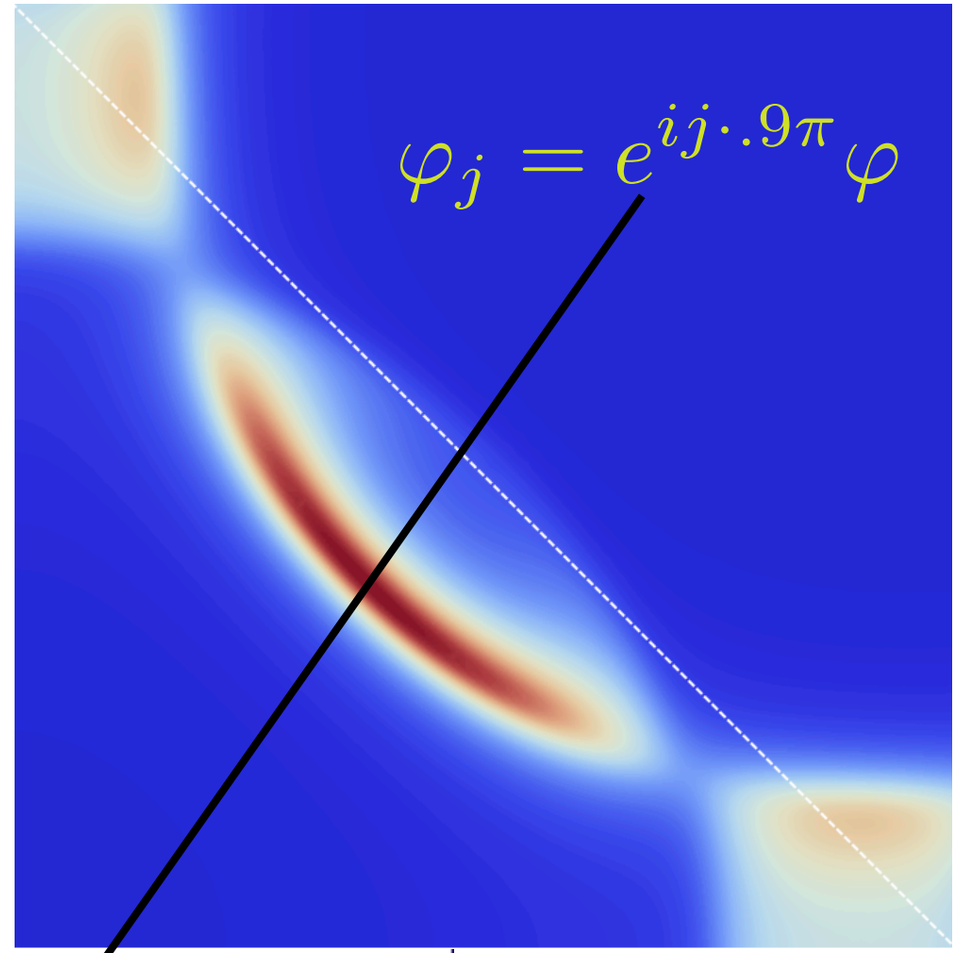


Expt.

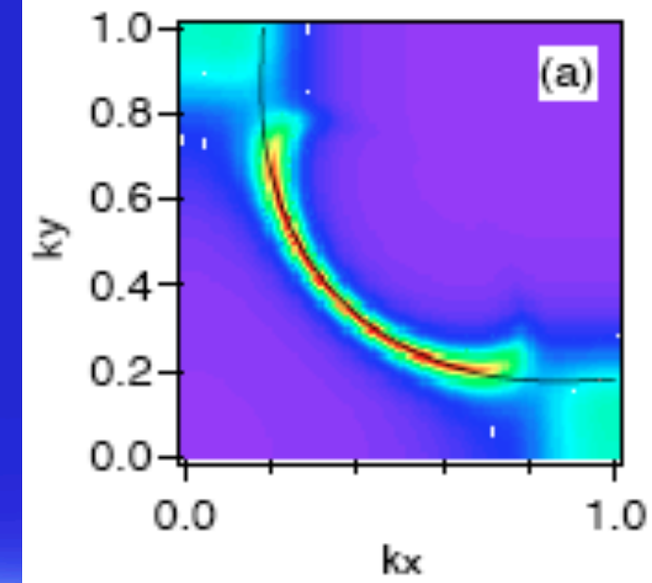


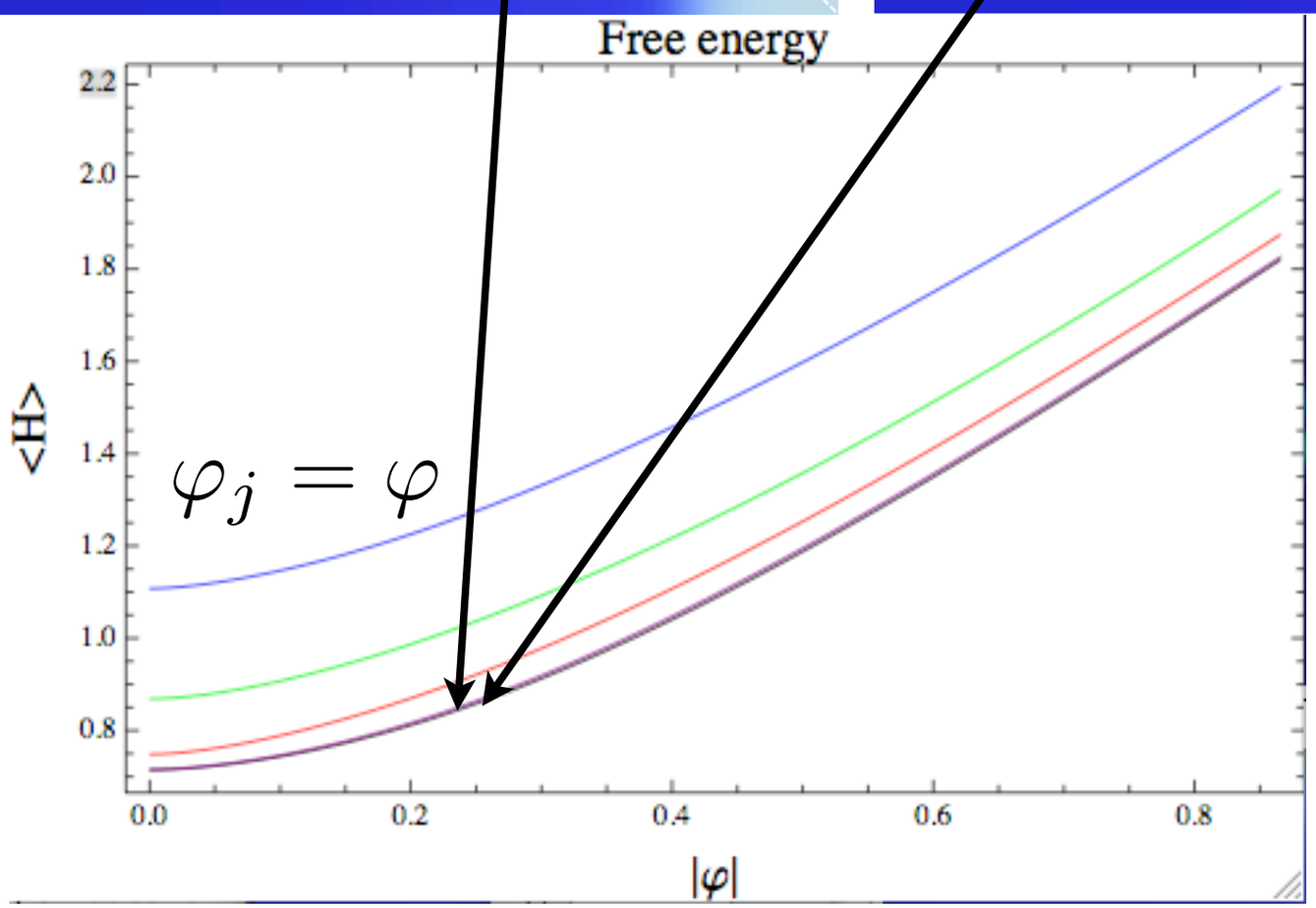
why choose
these funky solutions?



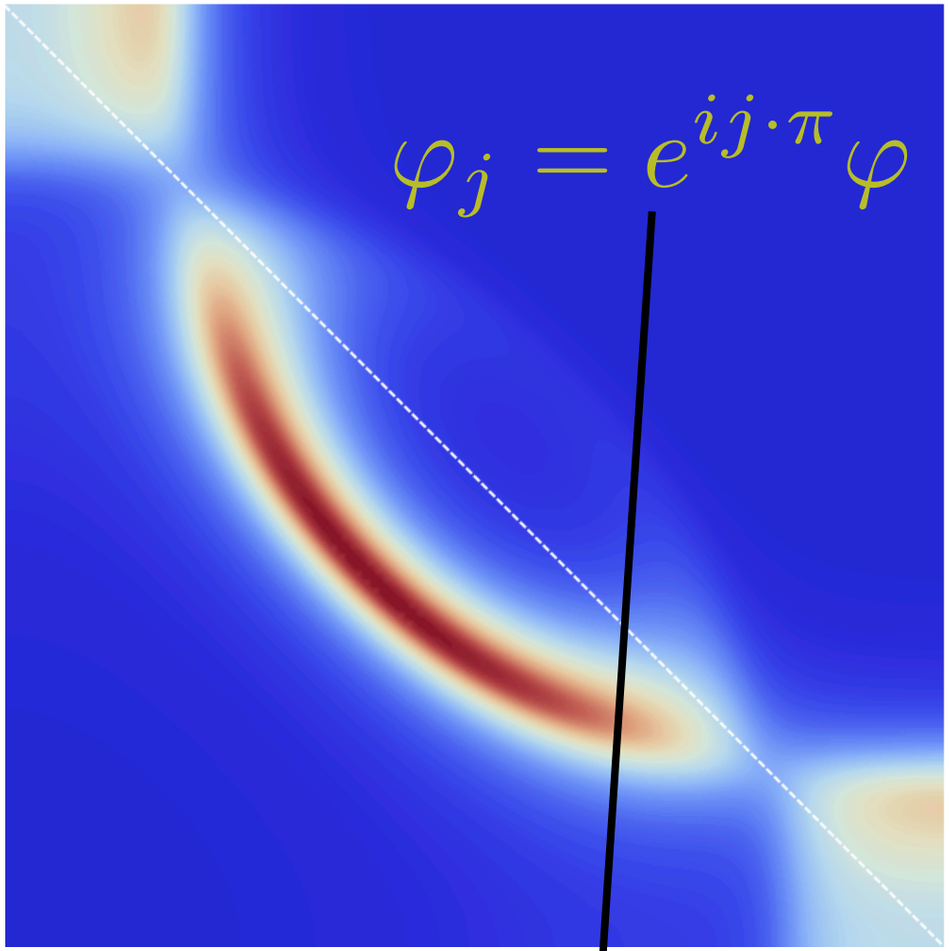


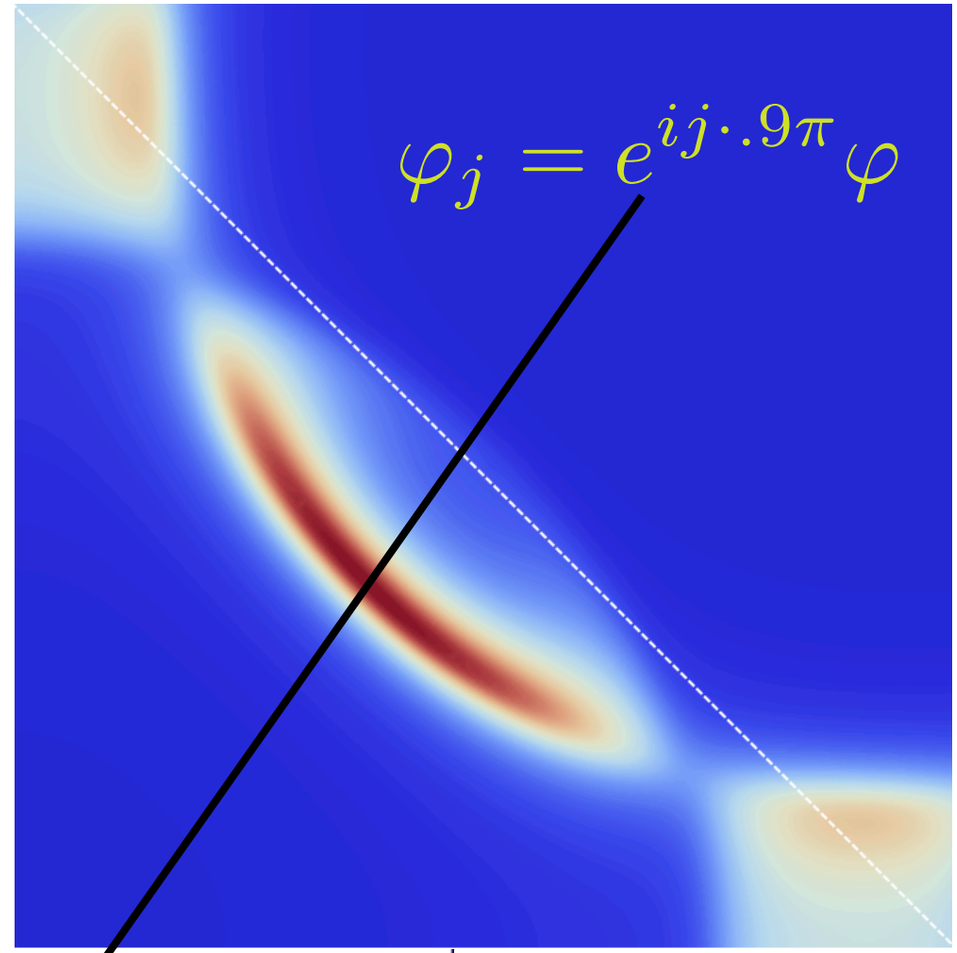
Expt.



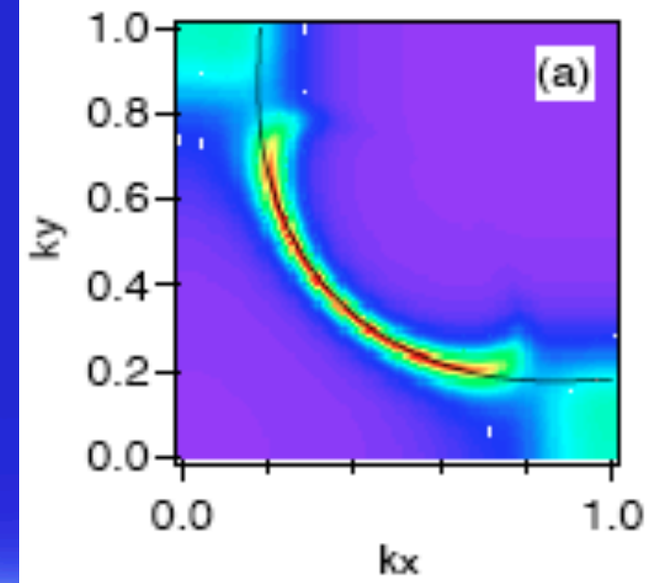


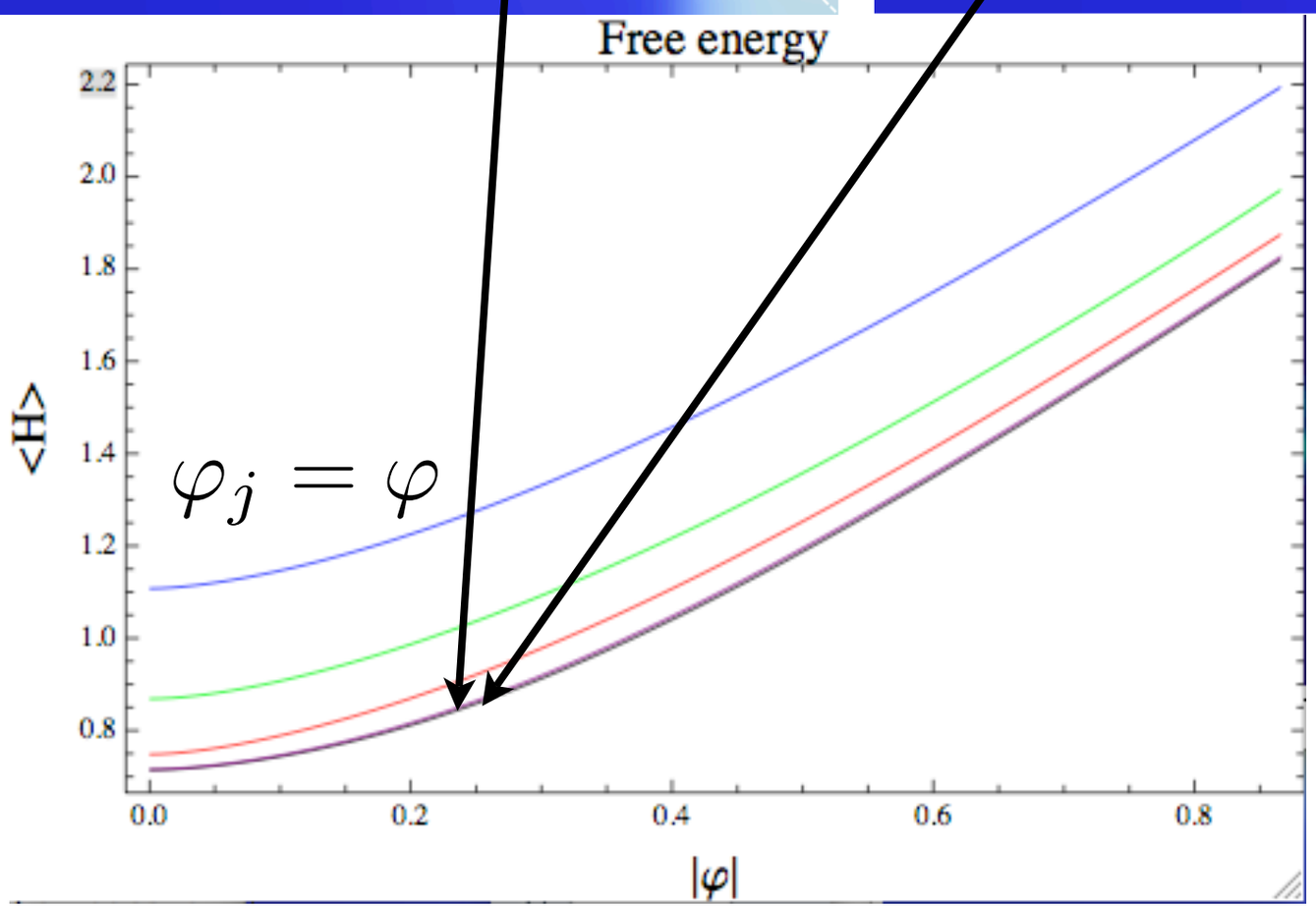
why choose
these funky solutions?





Expt.



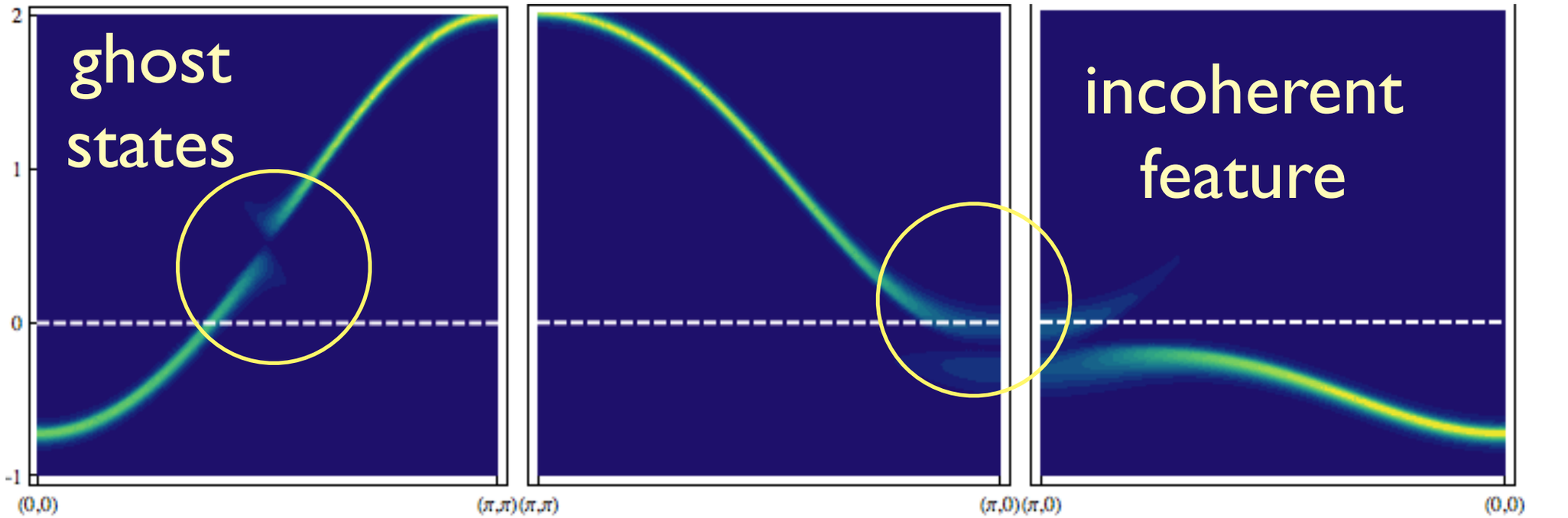


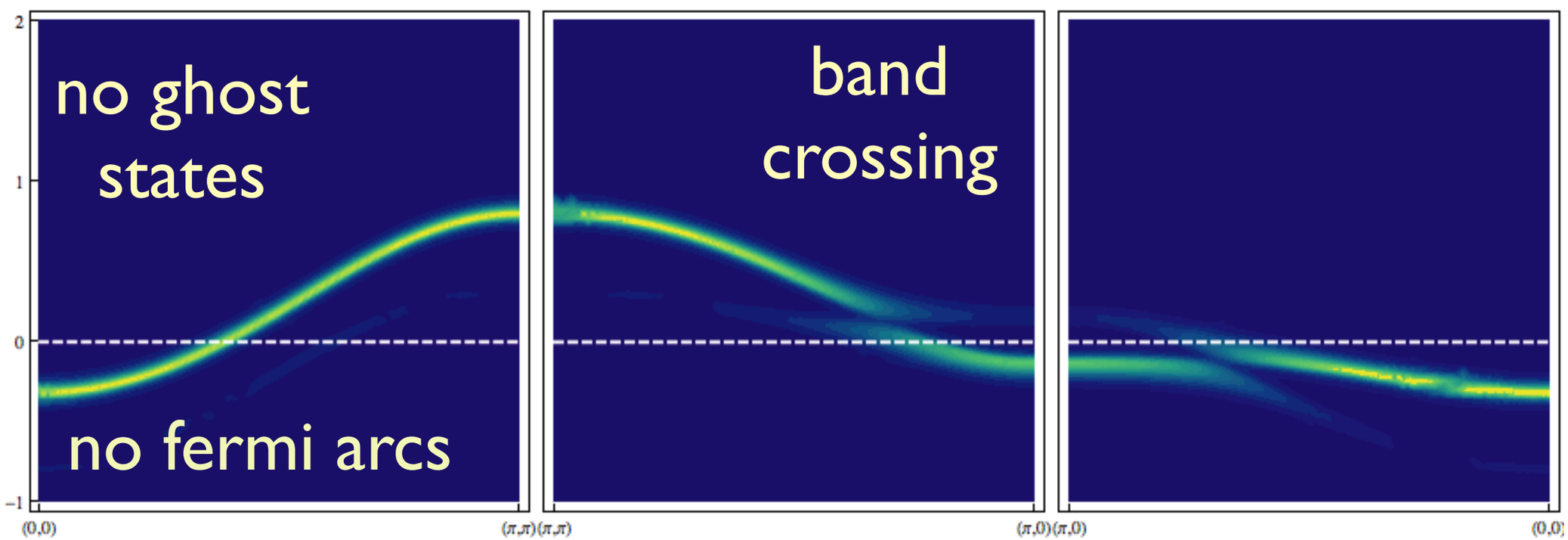
why choose
these funky solutions?

they minimize the free energy (no order)

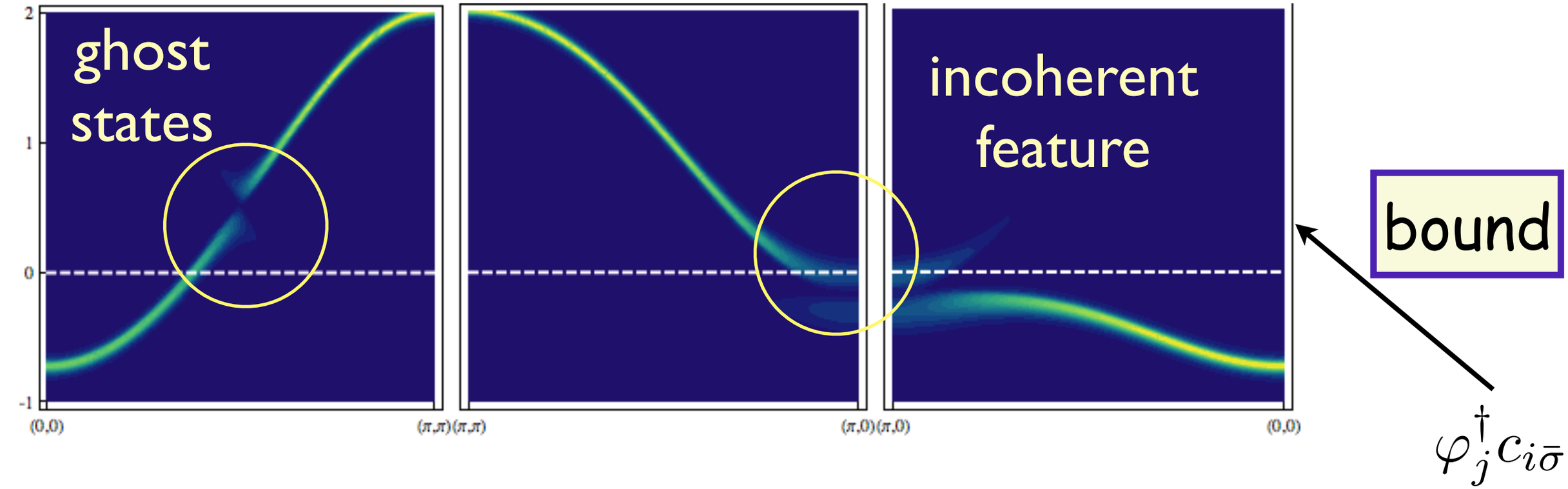
what causes the ghost states?

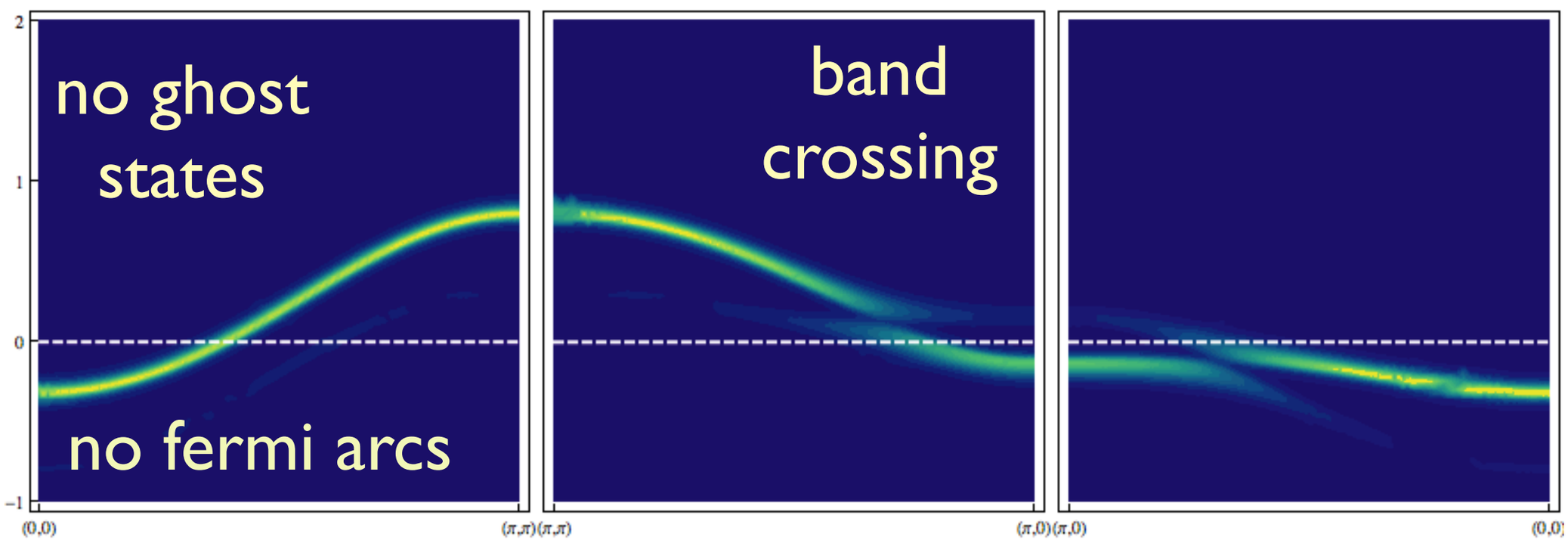
what causes the ghost states?



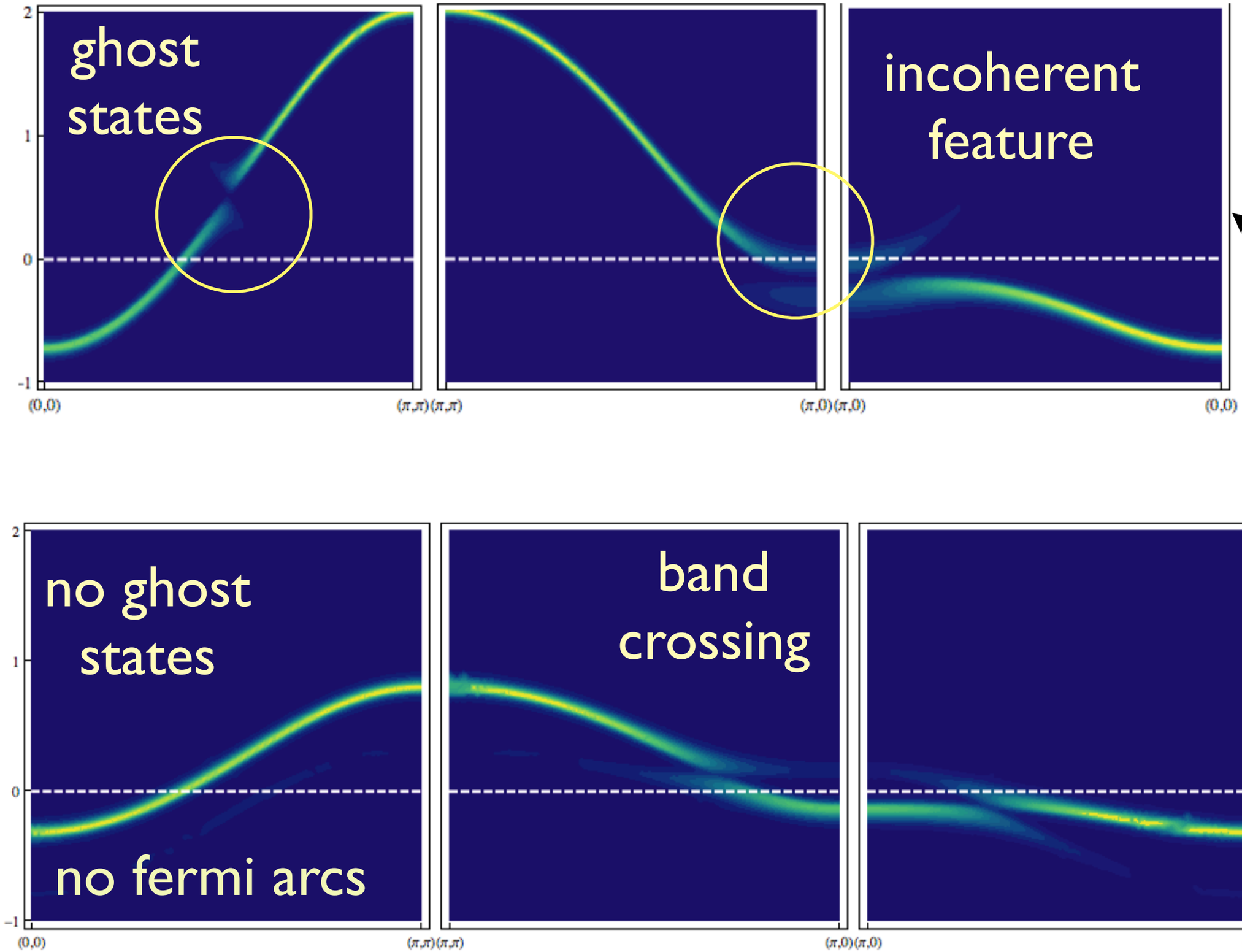


what causes the ghost states?





what causes the ghost states?

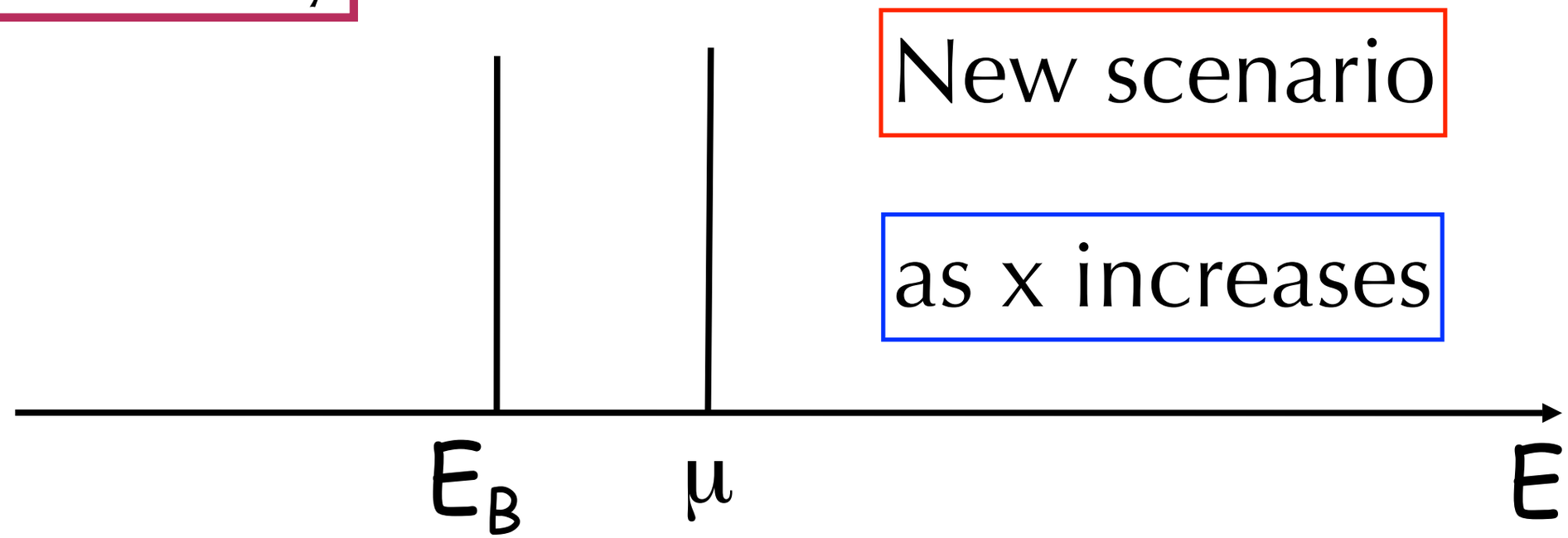


bound

$\varphi_j^\dagger c_{i\bar{\sigma}}$

unbound

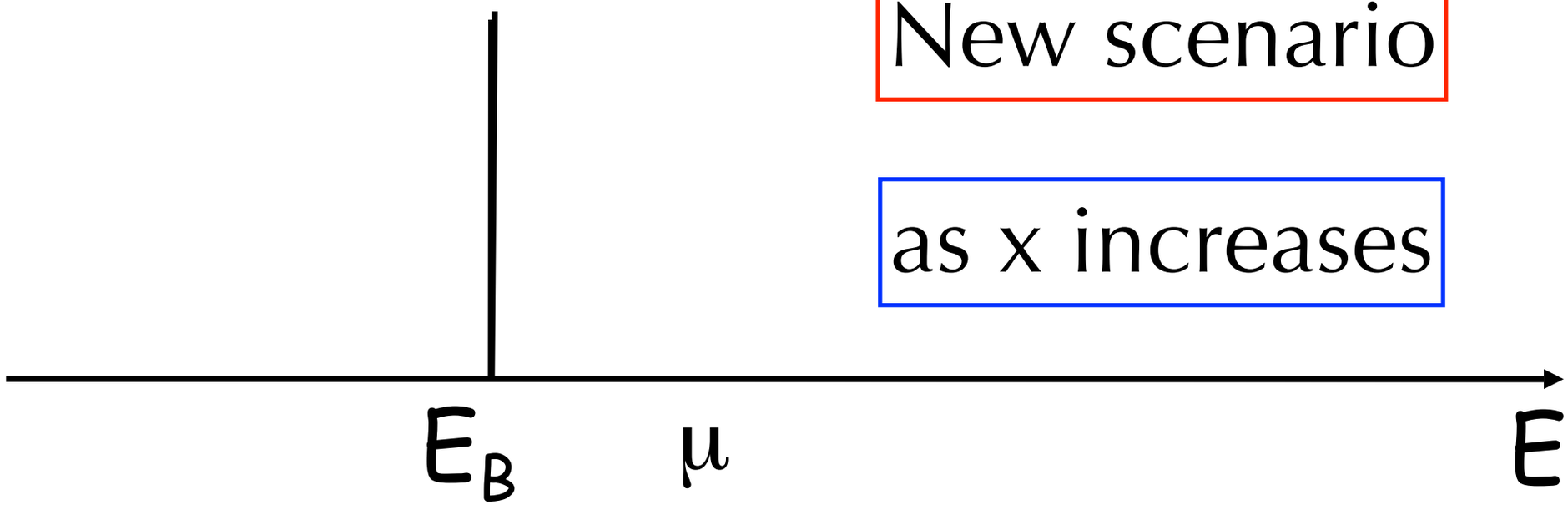
T-linear resistivity



T-linear resistivity

New scenario

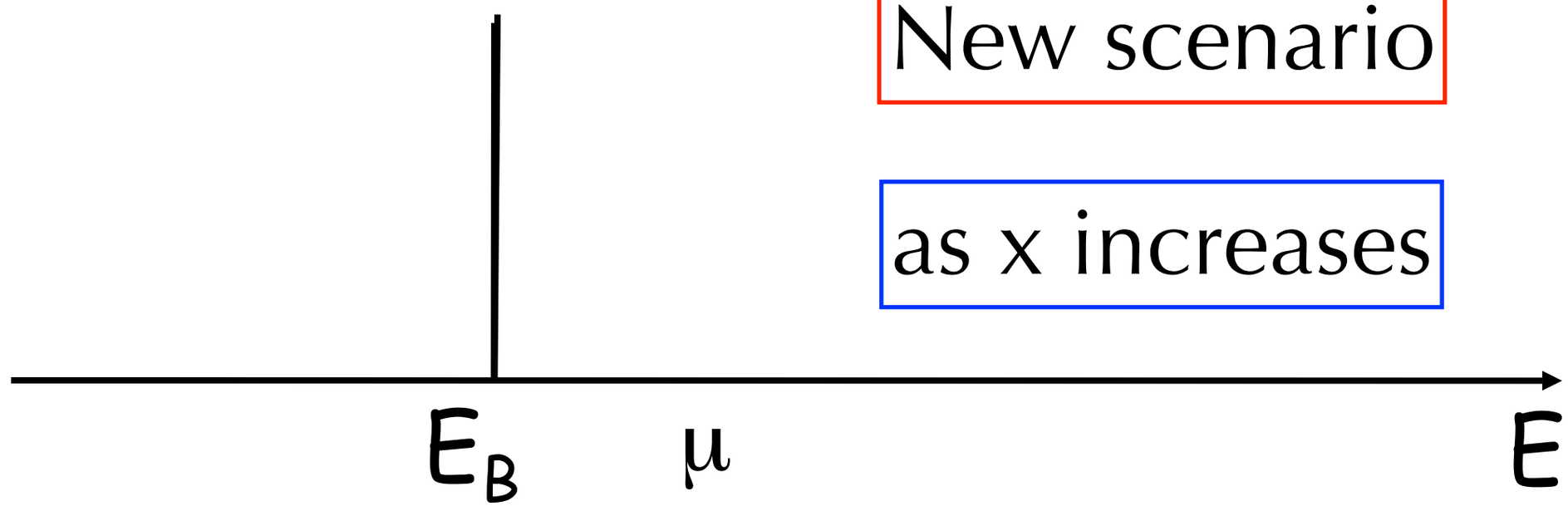
as x increases

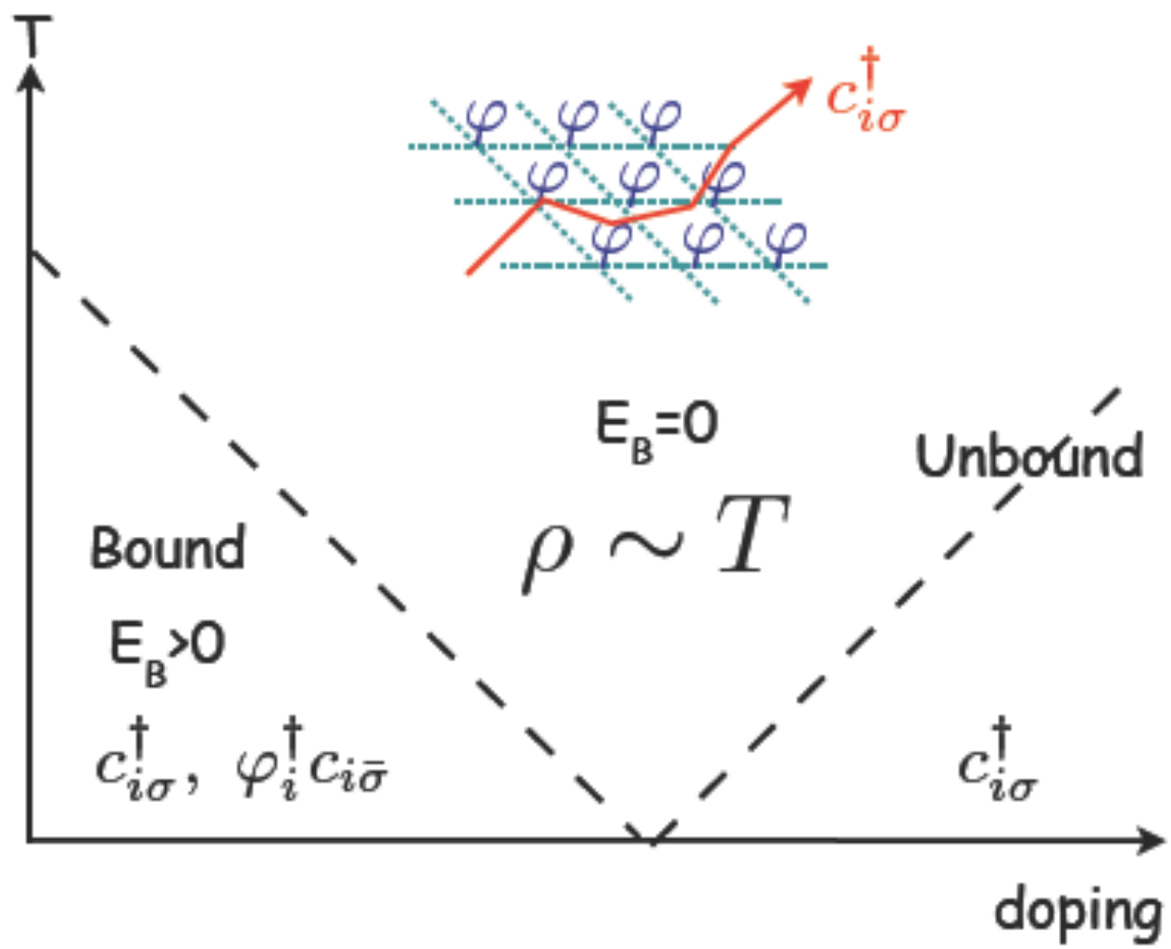


T-linear resistivity

New scenario

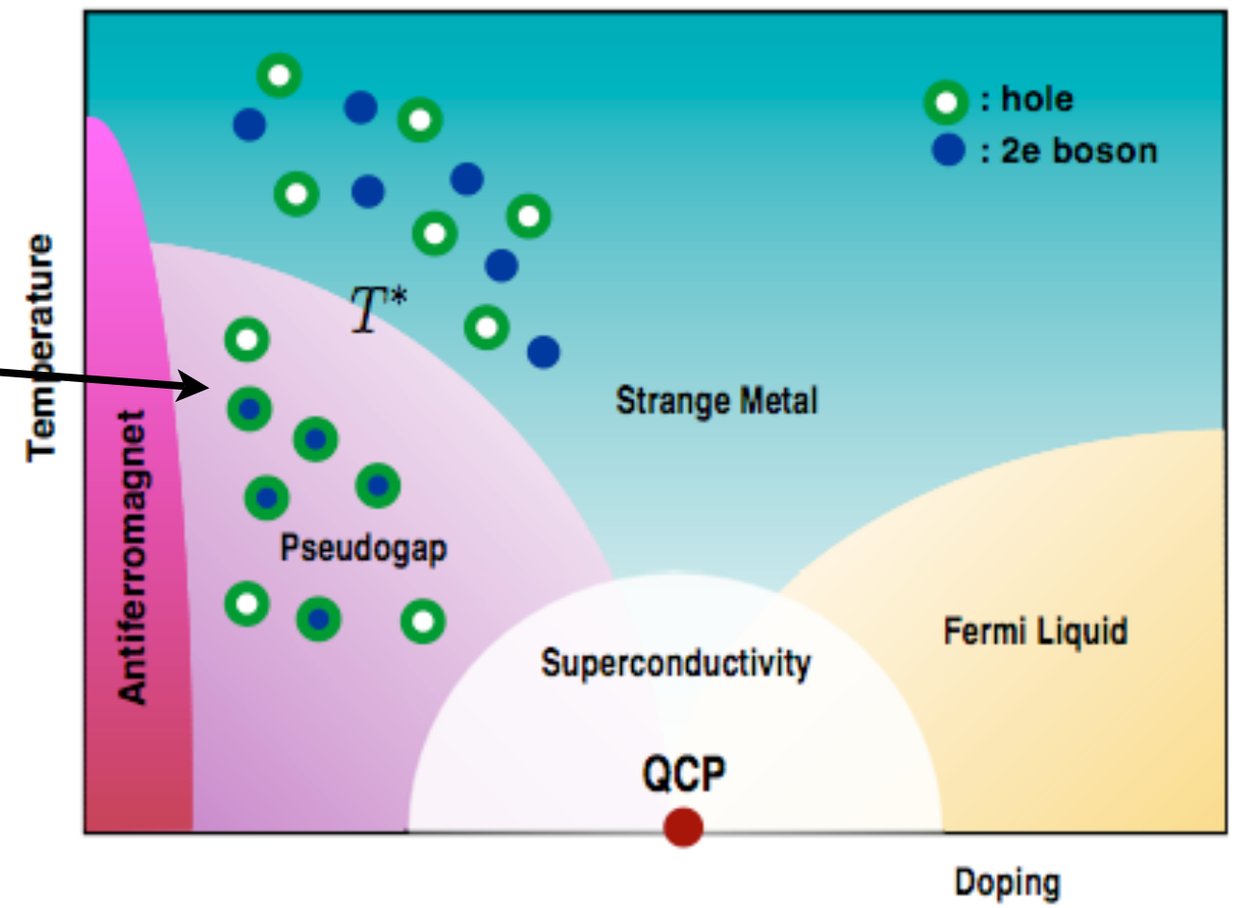
as x increases





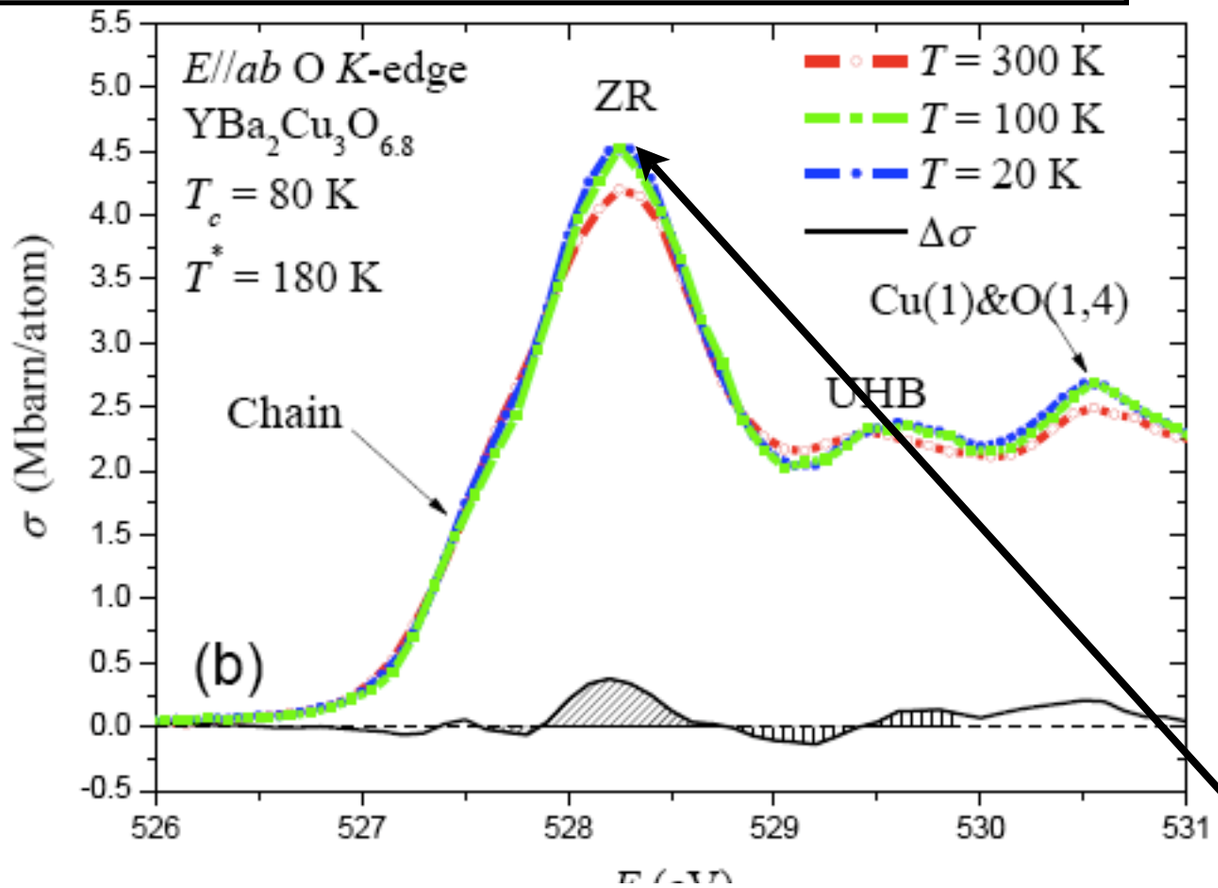
Pseudogap=`confinement'

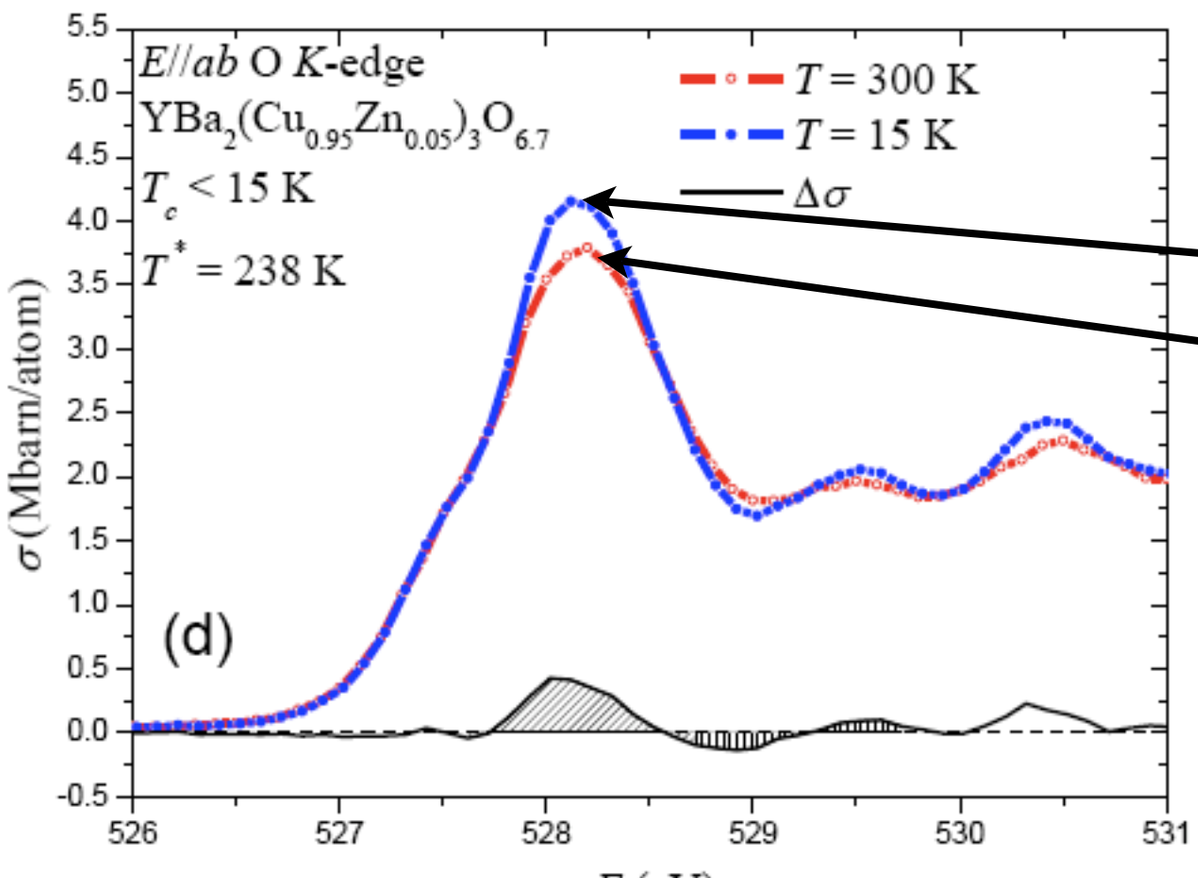
More `e' states at lower temperature



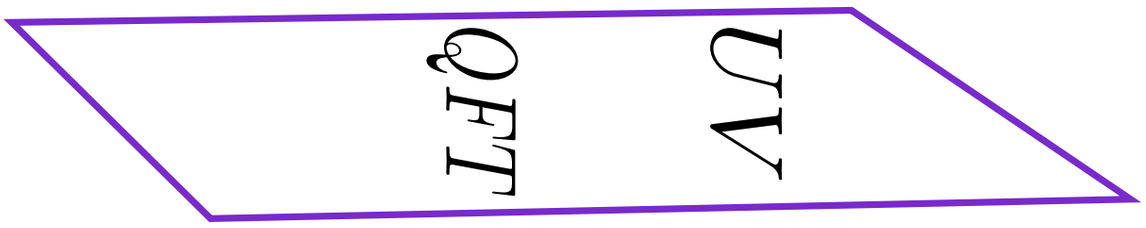
composite or bound states not in UV theory

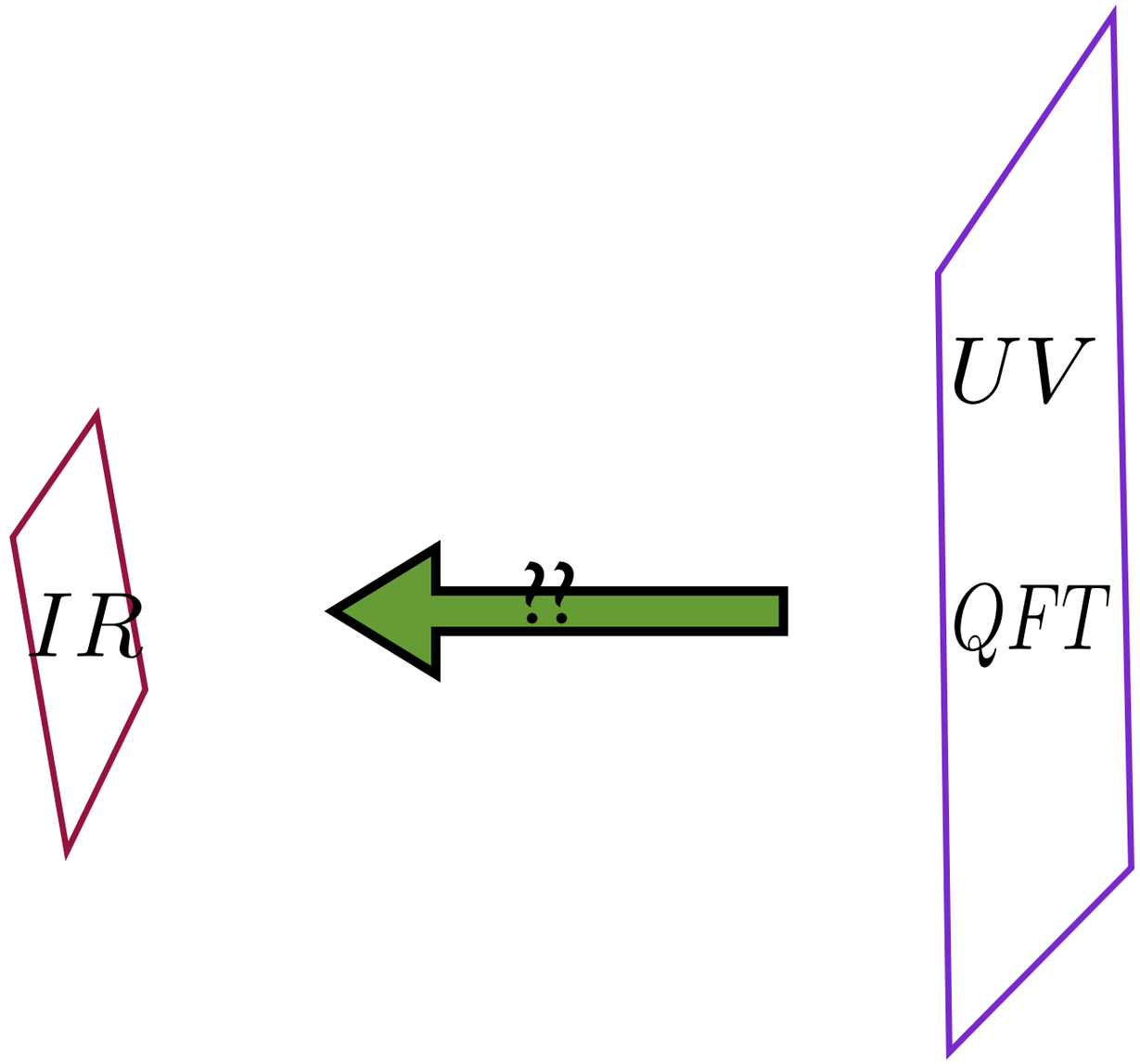
Jiunn-Yuan Lin, et al. (Taiwan)

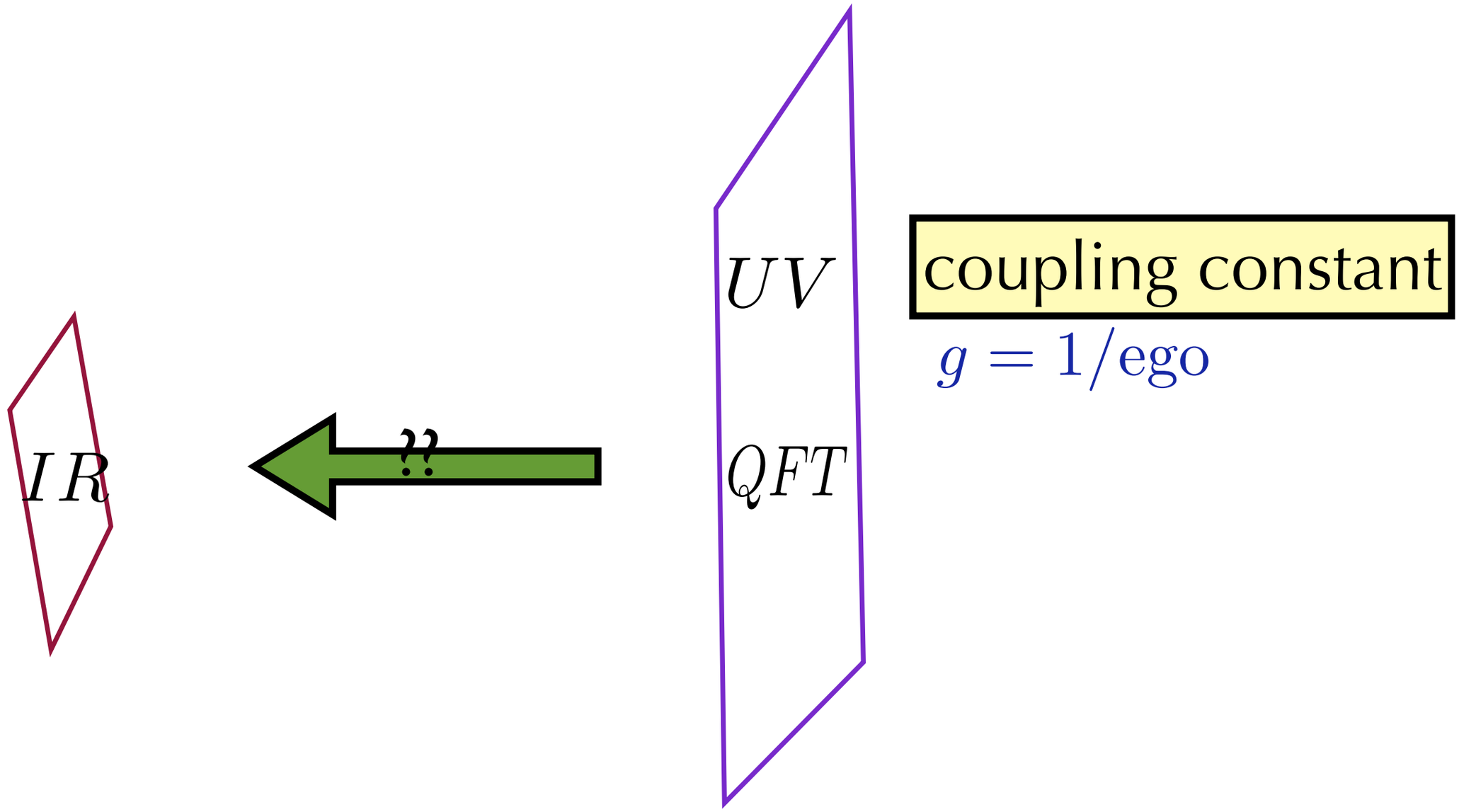




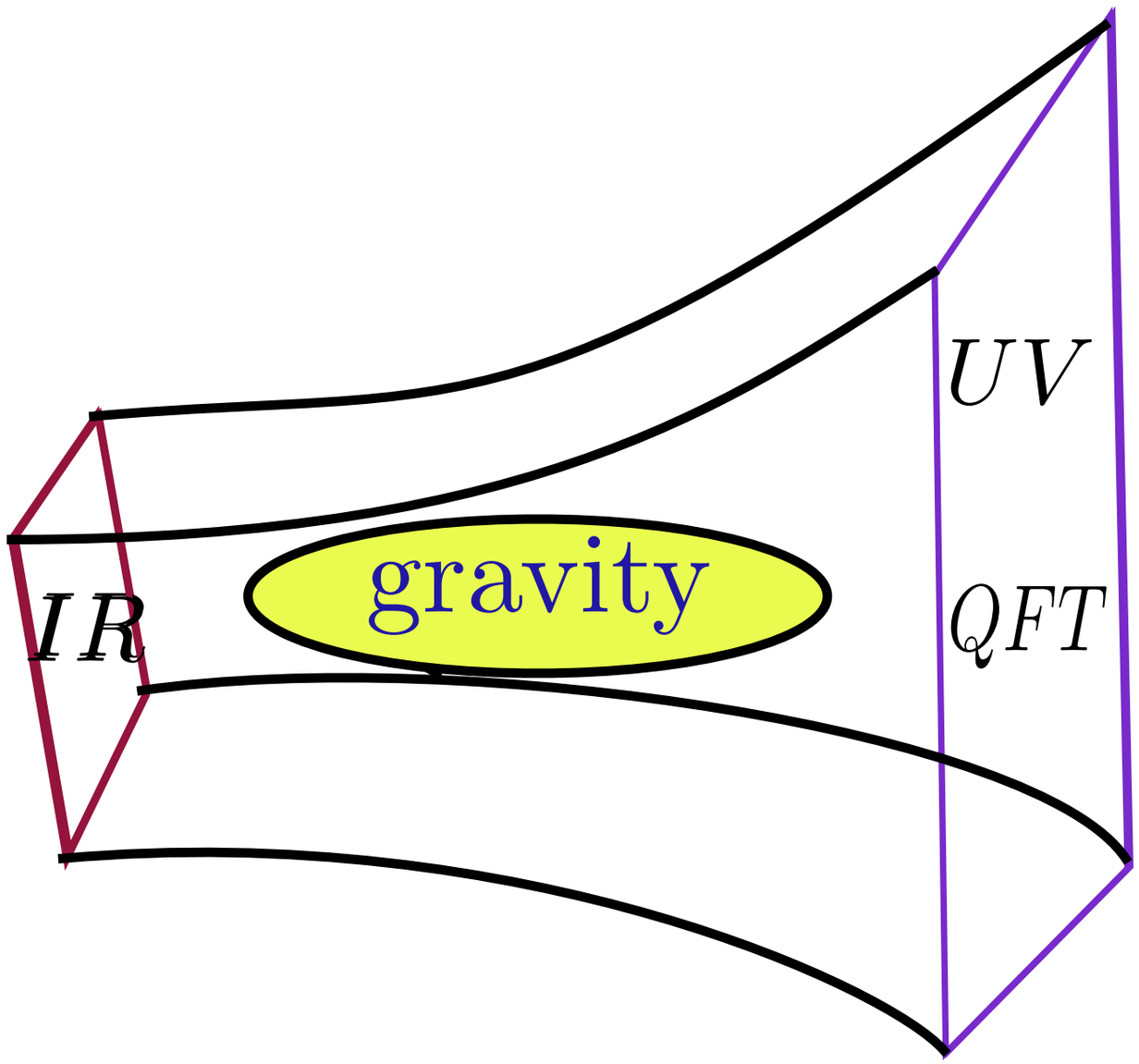
More addition states in PG: new charge e states


UV
QFT


UV
QFT
??
IR


UV
coupling constant
$g = 1/\mathrm{ego}$
IR
??
QFT

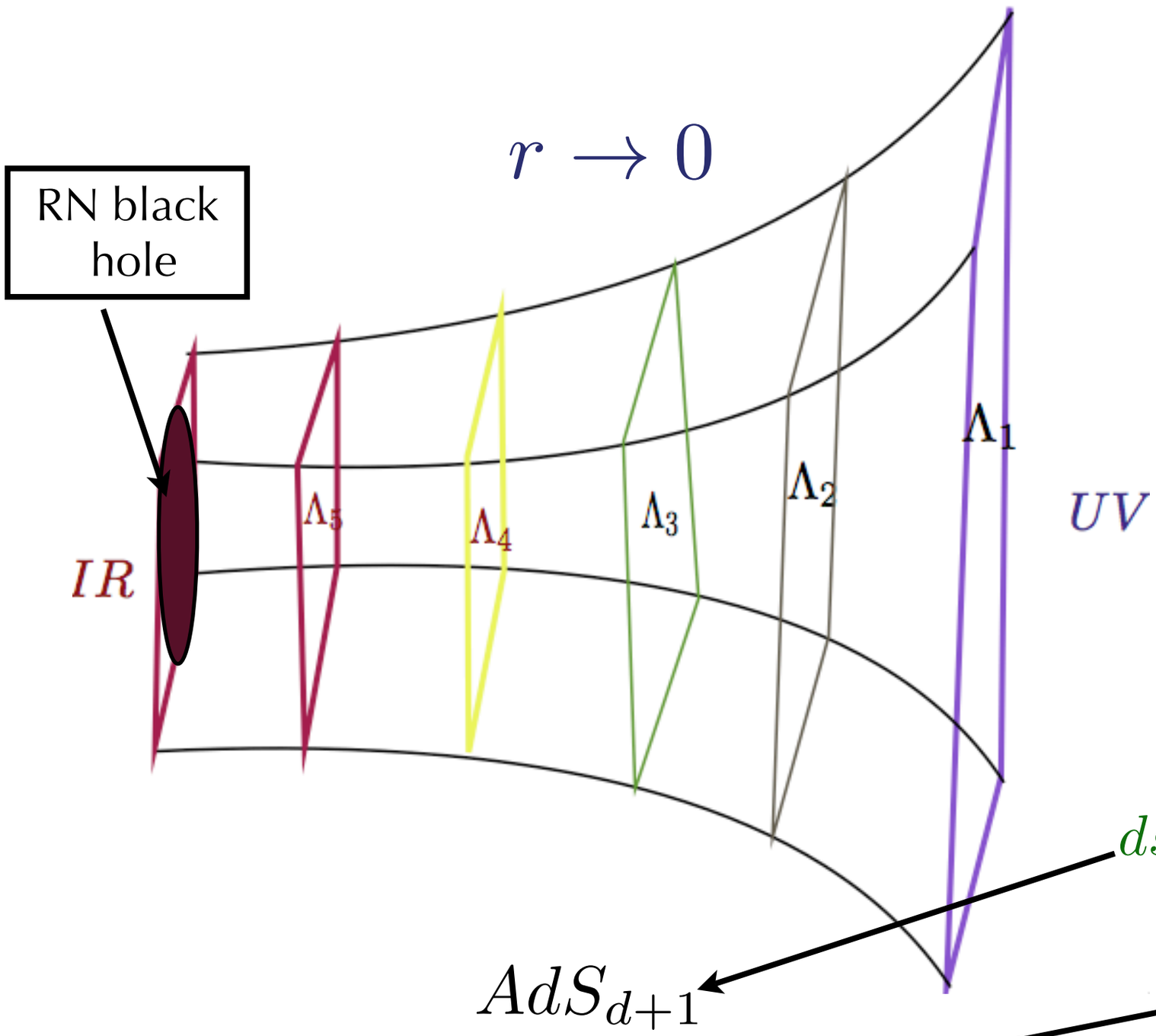
gauge-gravity duality
(Maldacena, 1997)



coupling constant

$g = 1/\text{ego}$

`Holography'



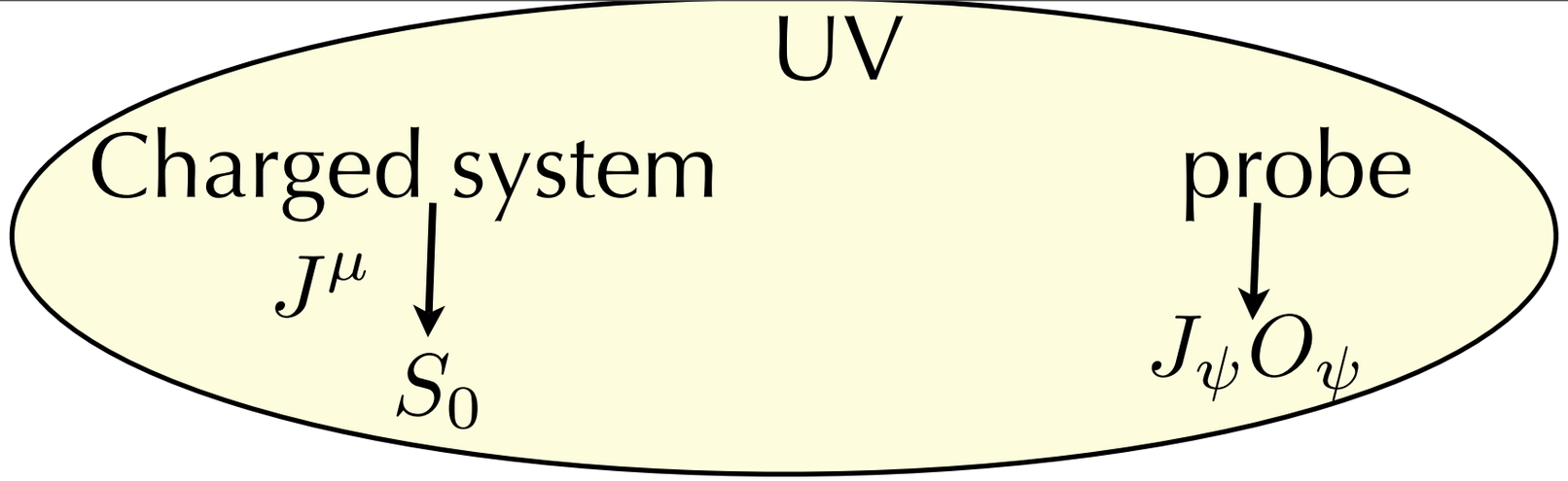
$$ds^2 = L^2 \left(\frac{-dt^2}{r^2} + \frac{d\vec{x}^2}{r^2} + \frac{dr^2}{r^2} \right)$$

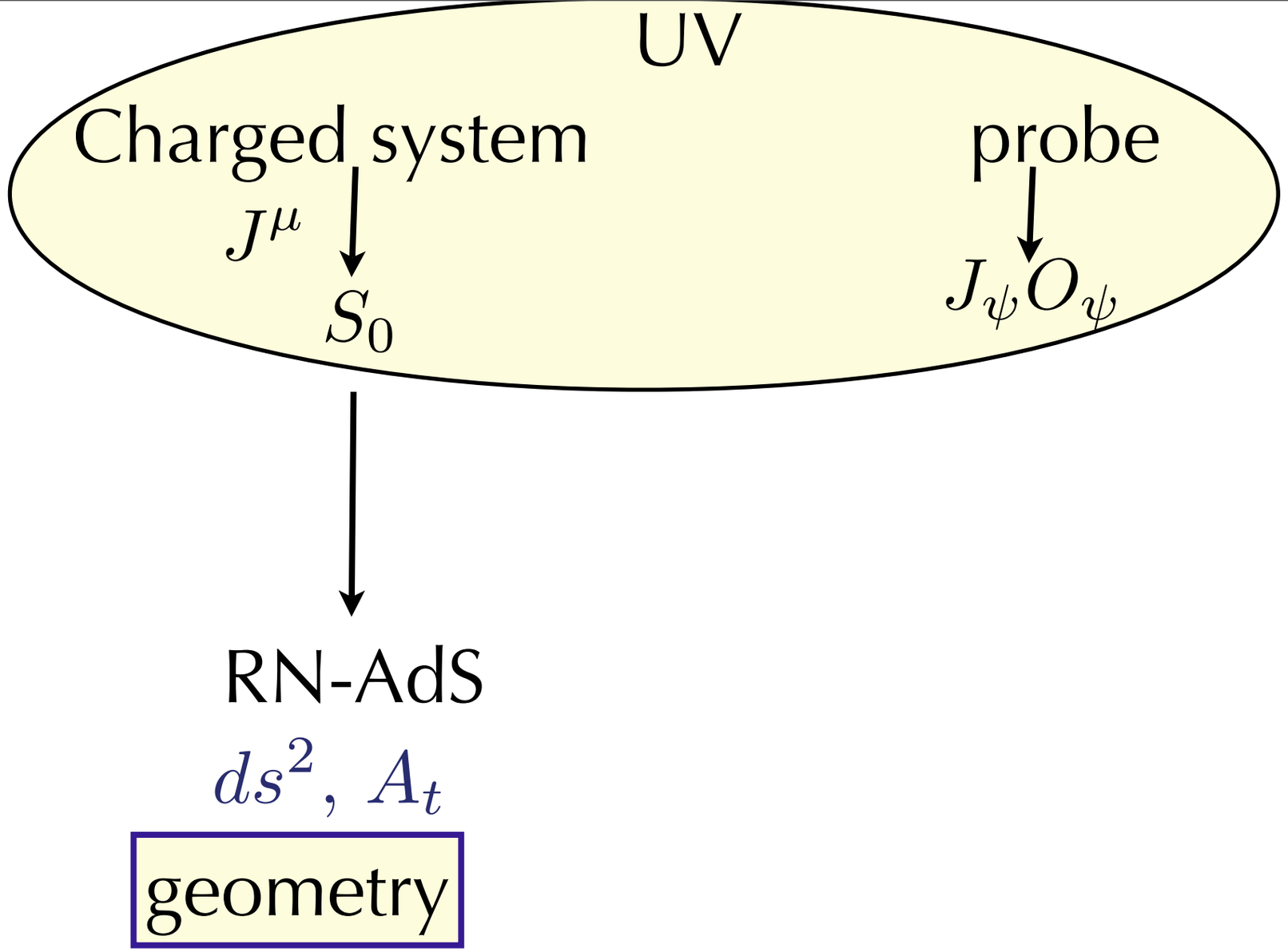
$\beta(g)$ is local

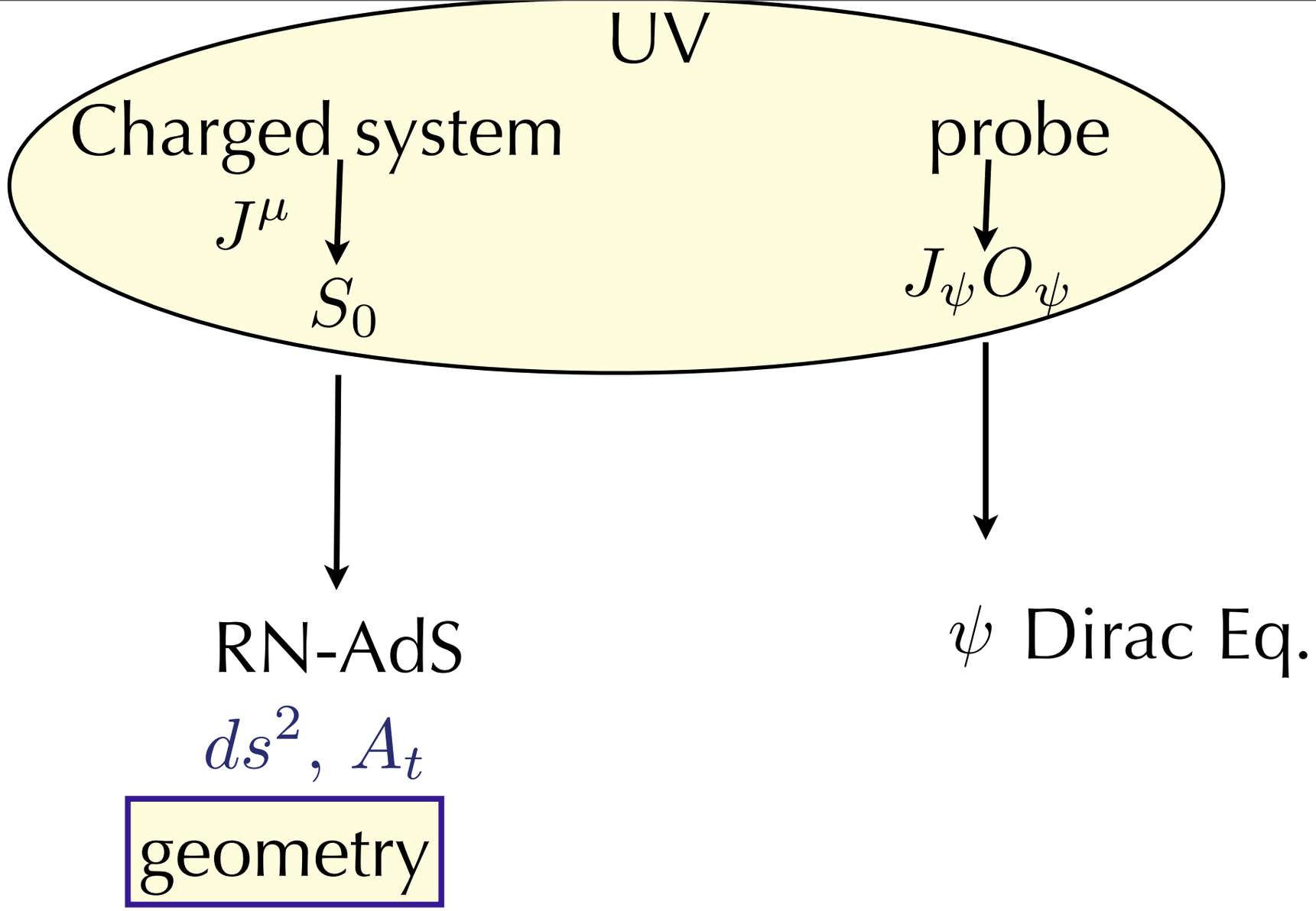
geometrize RG flow

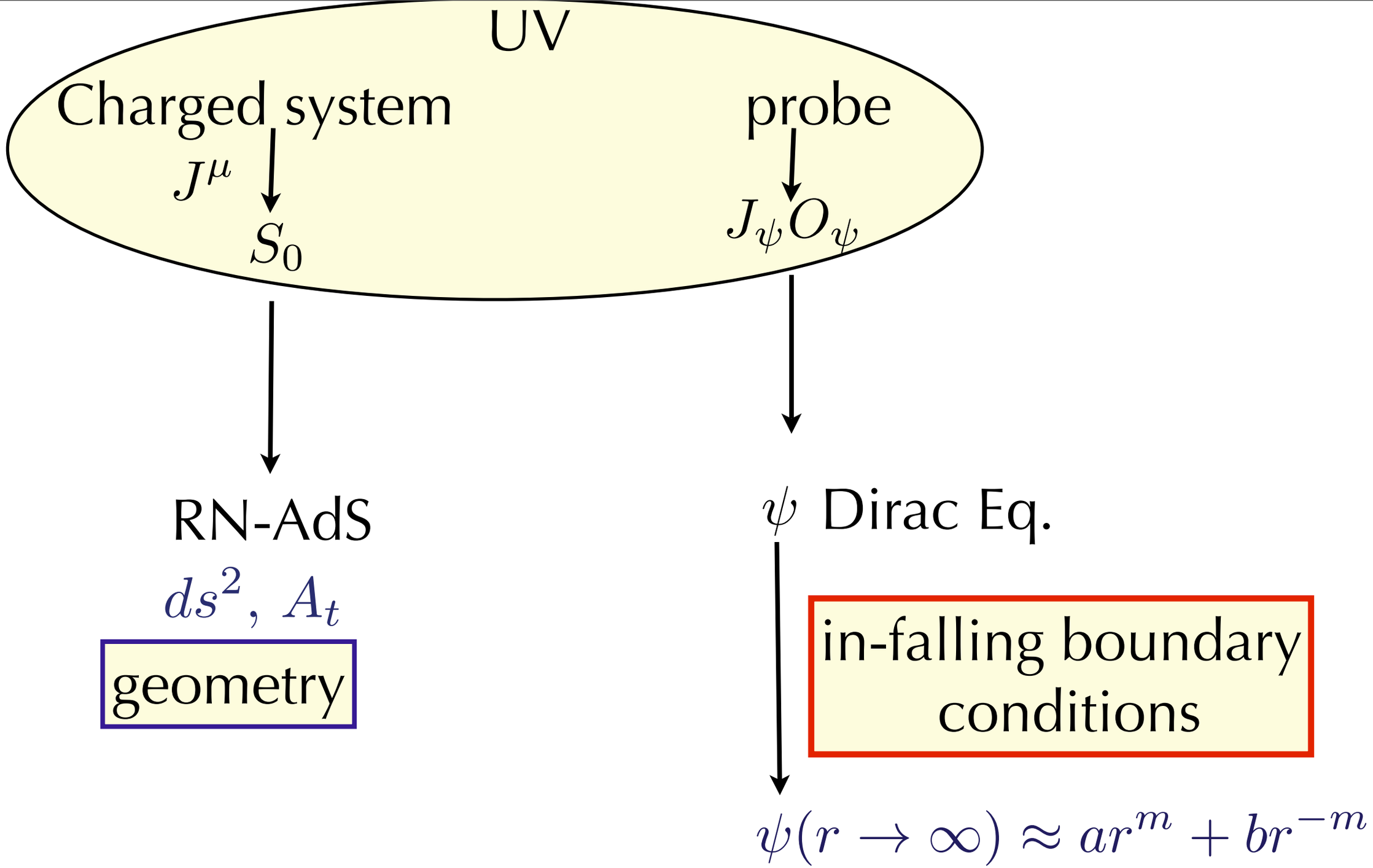
symmetry:

$$\{t, \vec{x}, r\} \to \{\lambda t, \lambda \vec{x}, \lambda r\}$$


UV
Charged system
probe
J^μ
S_0
$J_\psi O_\psi$

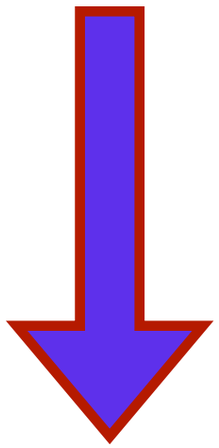

UV
Charged system
probe
J^μ
S_0
$J_\psi O_\psi$
RN-AdS
$ds^2,\ A_t$
geometry


UV
Charged system
probe
J^μ
S_0
$J_\psi O_\psi$
RN-AdS
ψ Dirac Eq.
$ds^2,\ A_t$
geometry



Retarded Green function: $G = \dfrac{b}{a}$ =f(UV,IR)

What gravitational theory gives rise to a gap in ImG without spontaneous symmetry breaking?



dynamically generated gap: Mott gap
(for probe fermions)

bottom-up schemes

$$\sqrt{-g}i\bar{\psi}(D-m)\psi$$

AdS-RN
MIT, Leiden group

`non-Fermi liquids'

bottom-up schemes

$$\sqrt{-g}i\bar{\psi}(D-m)\psi$$

AdS-RN
MIT, Leiden group

`non-Fermi liquids'

??

Mott Insulator

bottom-up schemes

$$\sqrt{-g}i\bar{\psi}(D-m)\psi$$

AdS-RN
MIT, Leiden group

→ `non-Fermi liquids'

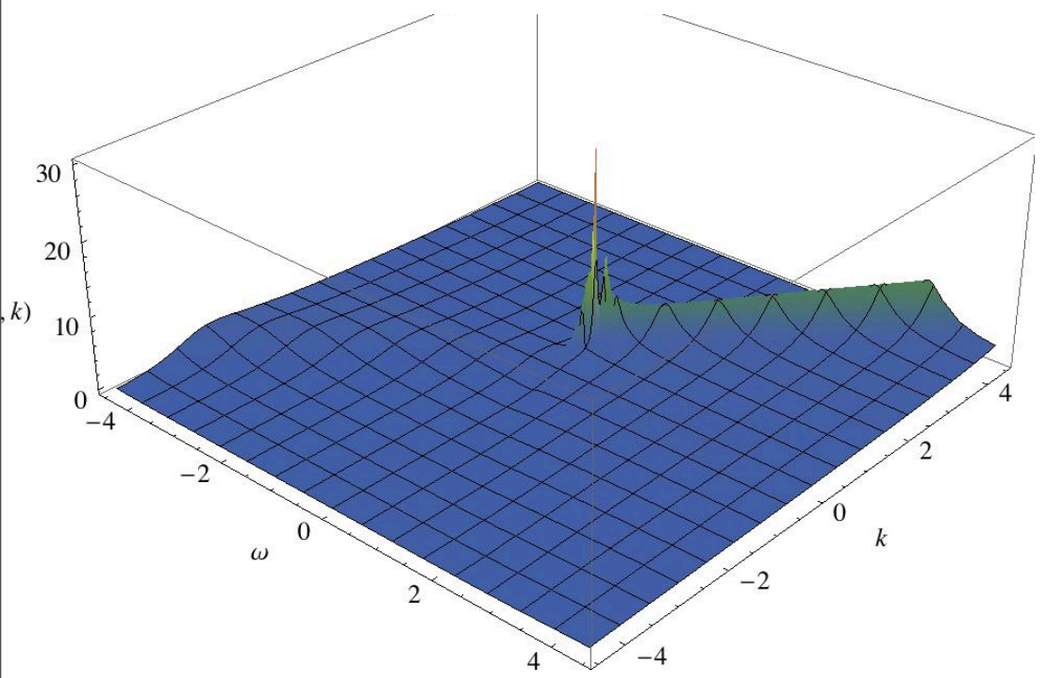
?? → Mott Insulator

consider $\sqrt{-g}i\bar{\psi}(\not{D}-m-ip\not{F})\psi$

fermions in RN Ads_{d+1} coupled to a gauge field through a dipole interaction

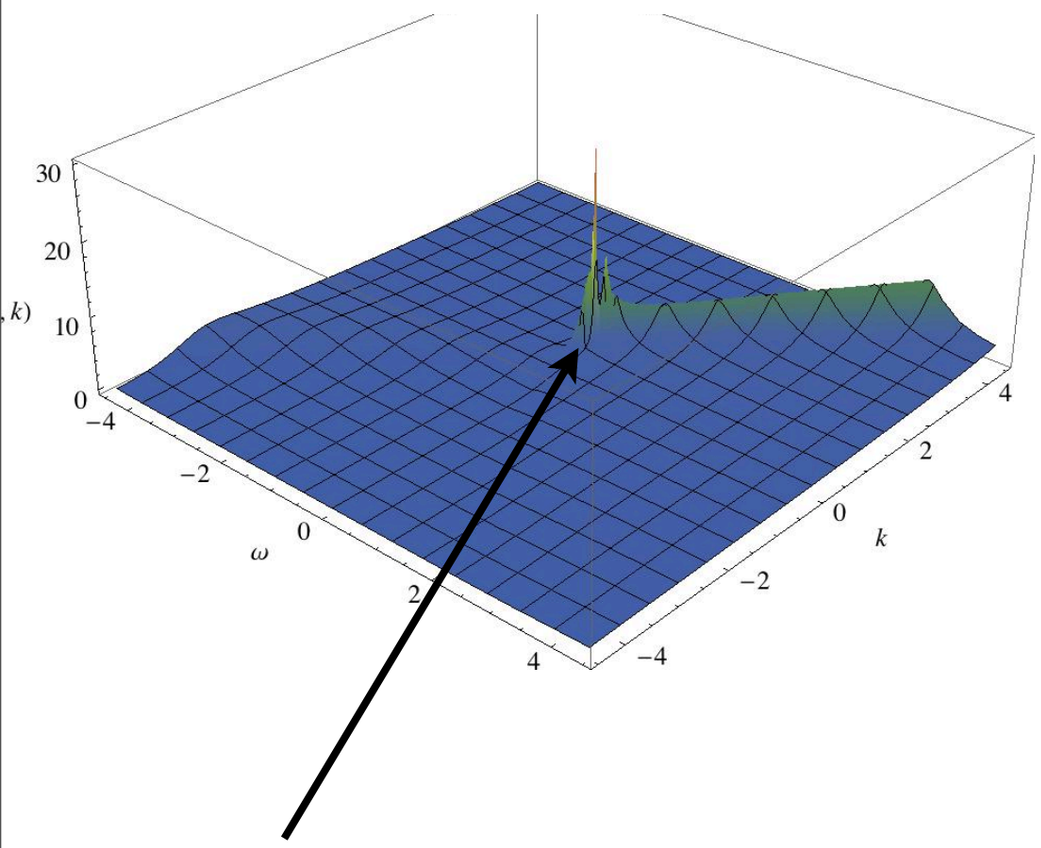
How is the spectrum modified?

P=0



How is the spectrum modified?

P=0

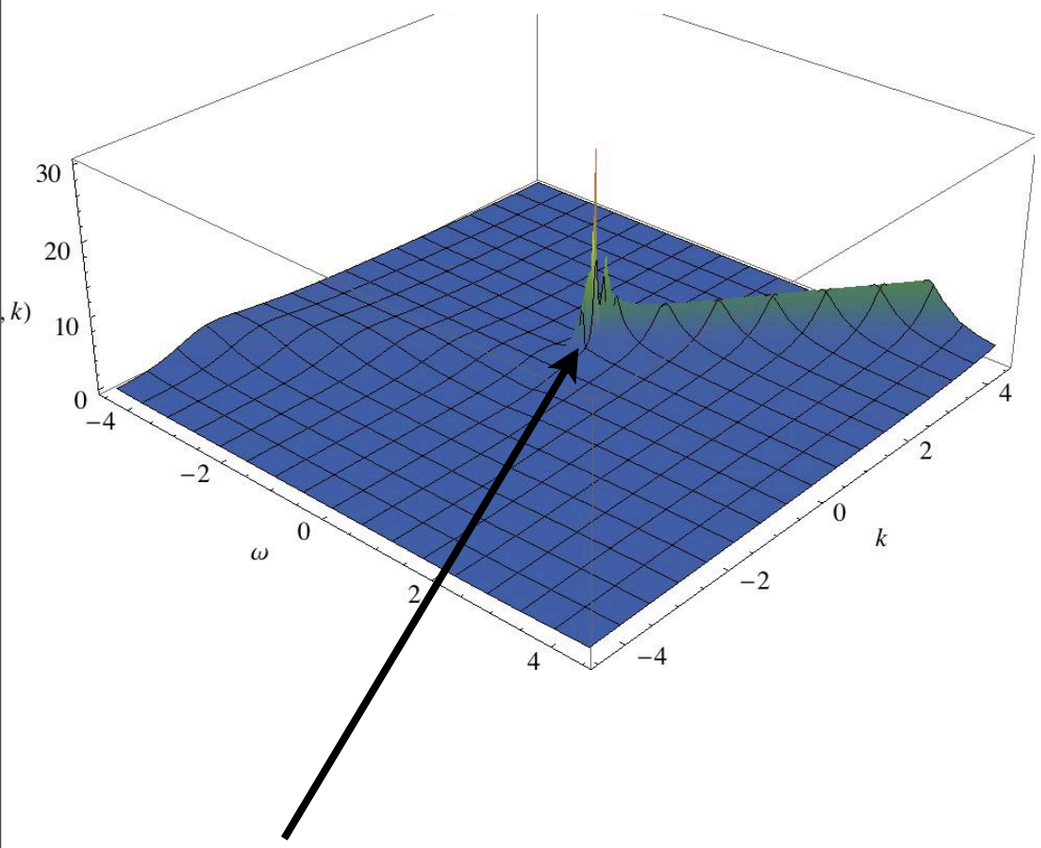


Fermi surface peak

How is the spectrum modified?

P=0

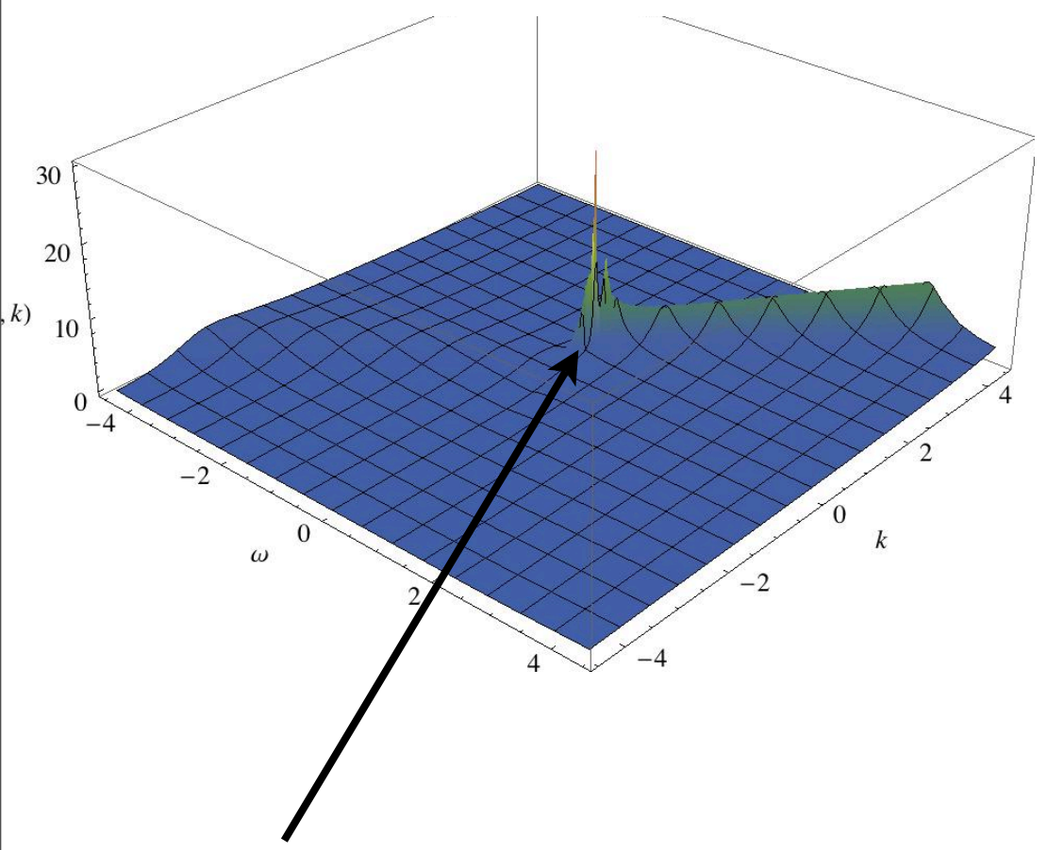
$P > 4.2$



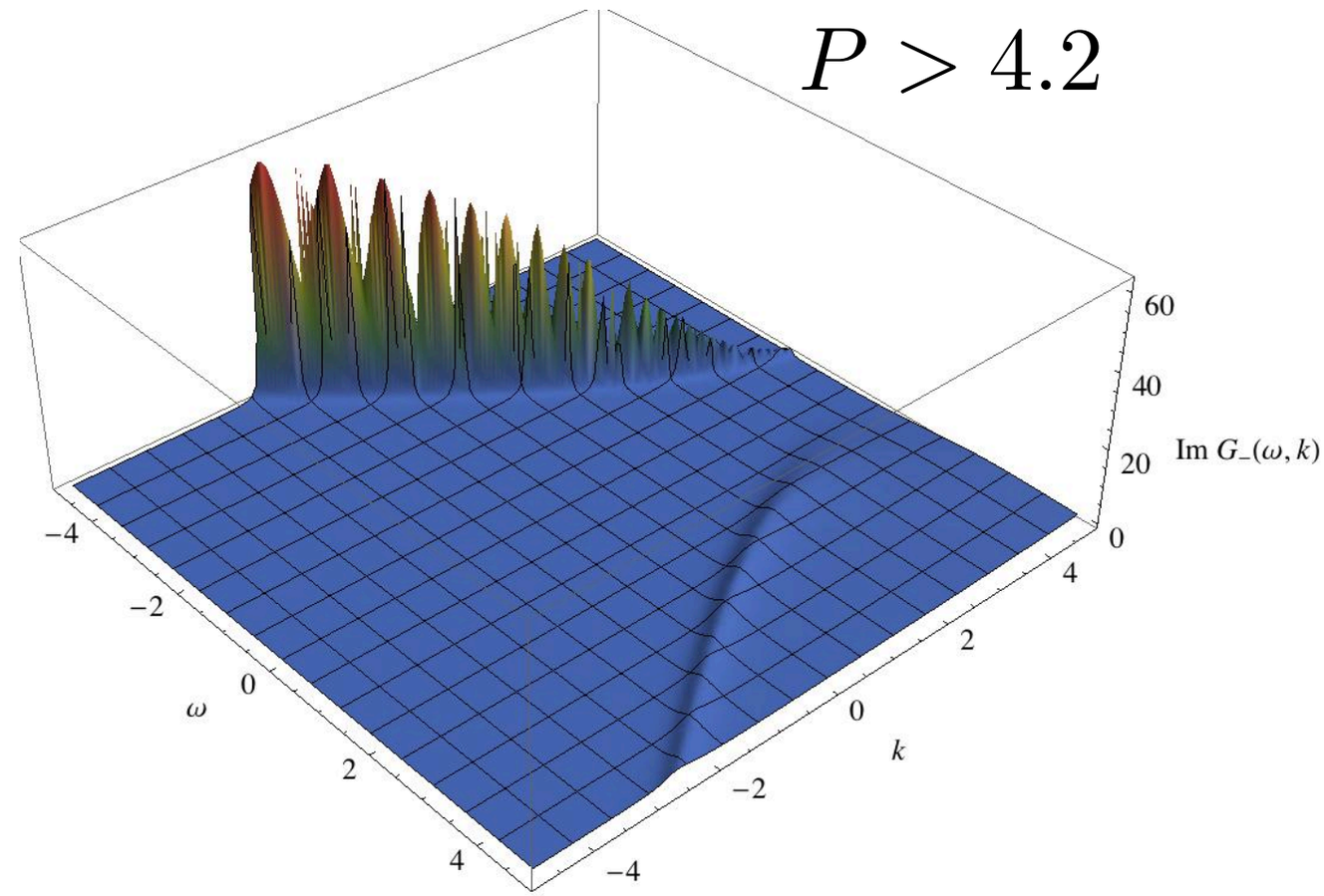
Fermi surface peak

How is the spectrum modified?

P=0



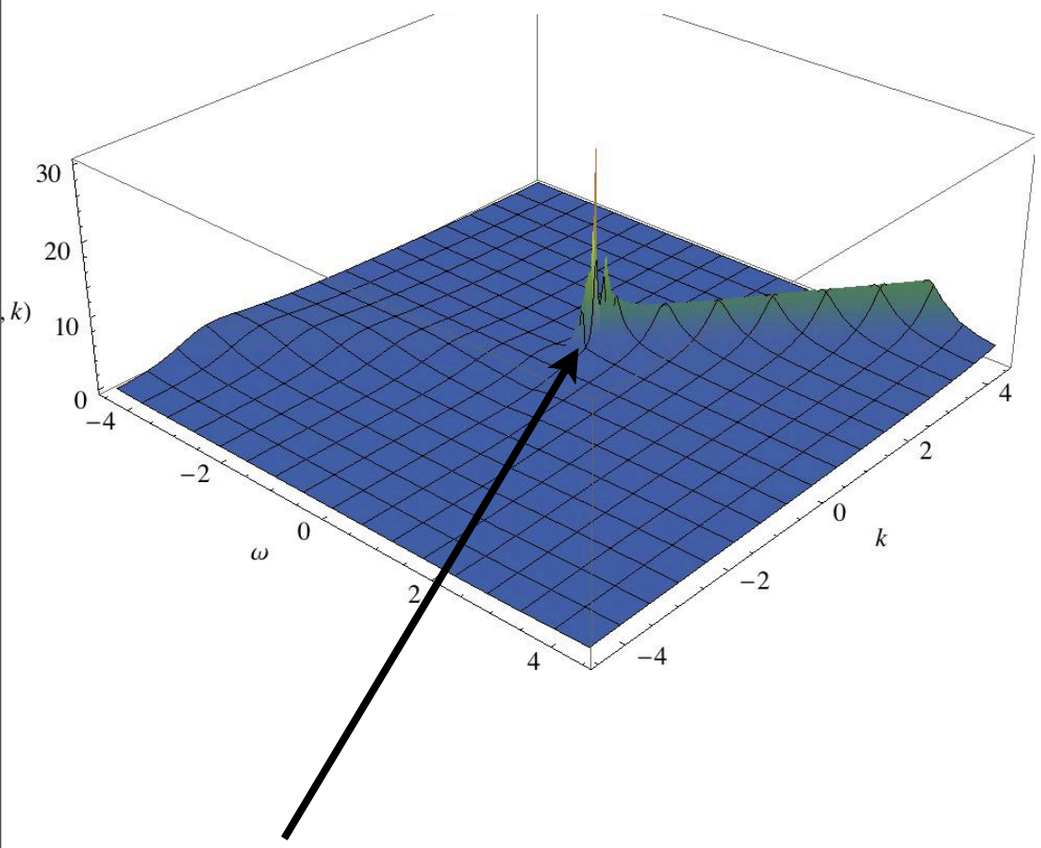
$P > 4.2$



Fermi surface peak

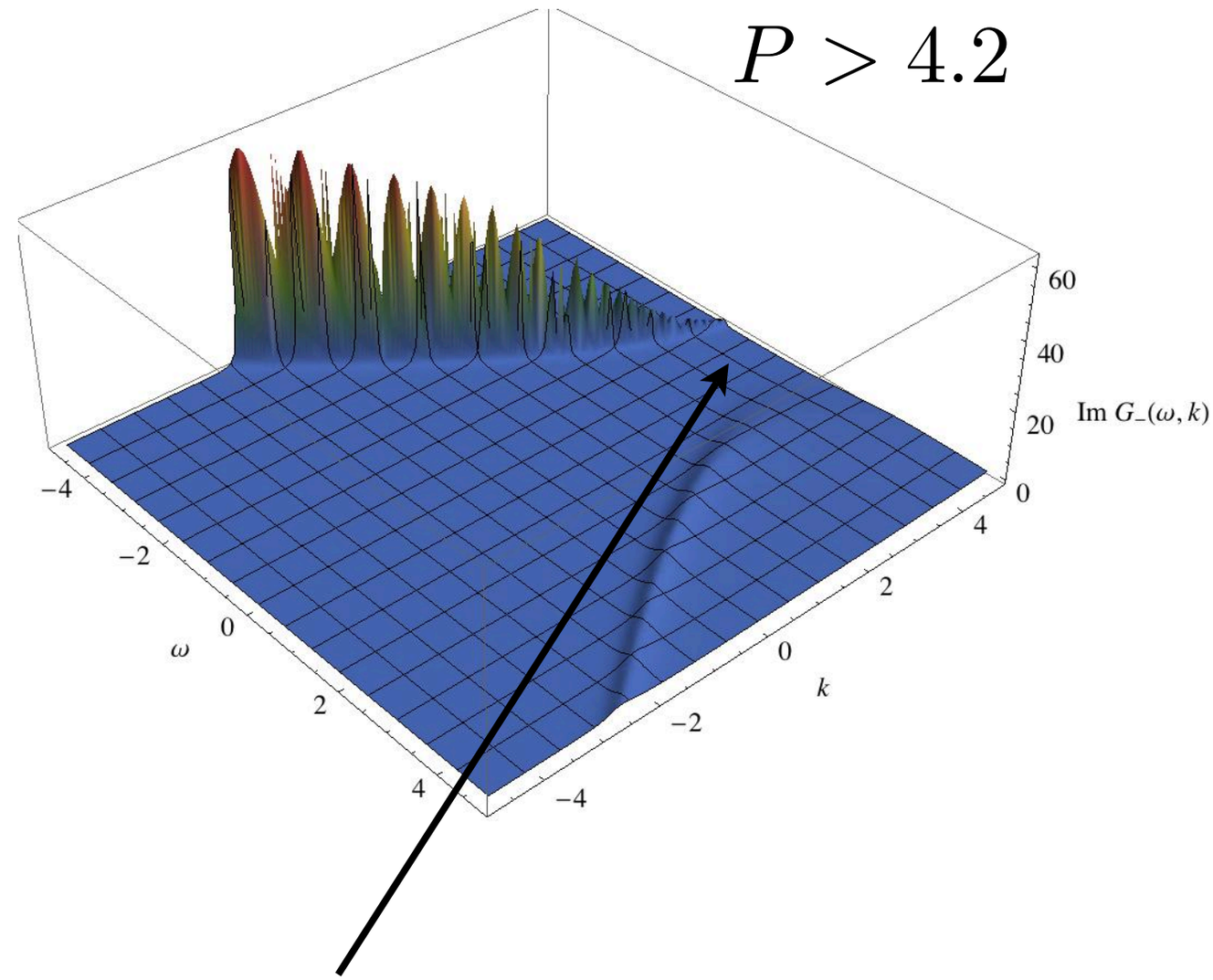
How is the spectrum modified?

P=0



Fermi surface peak

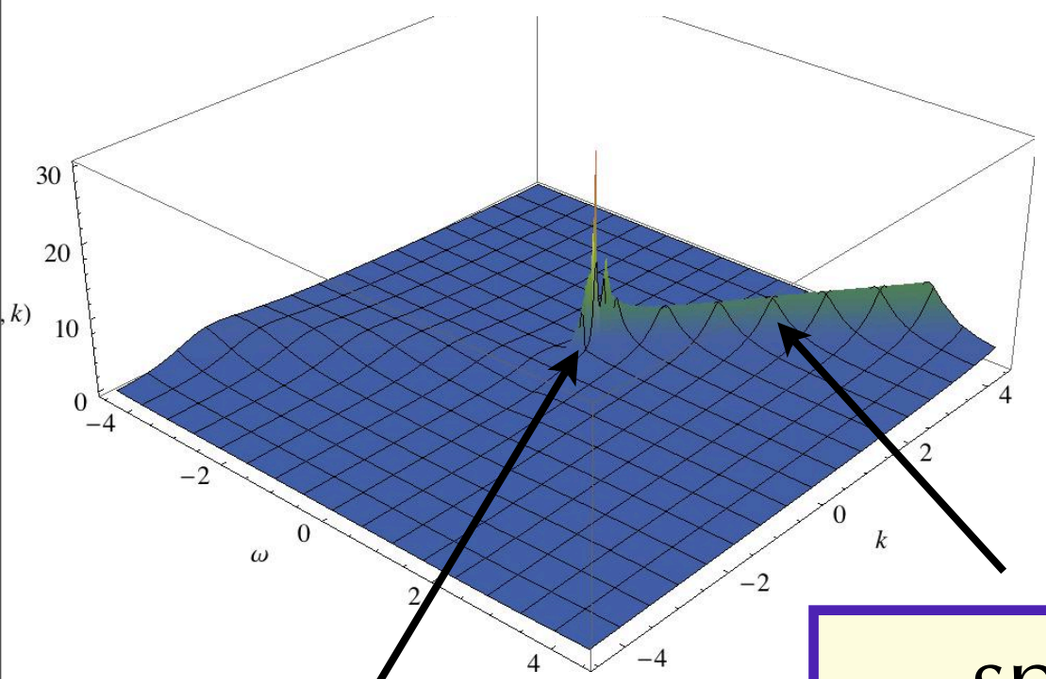
$P > 4.2$



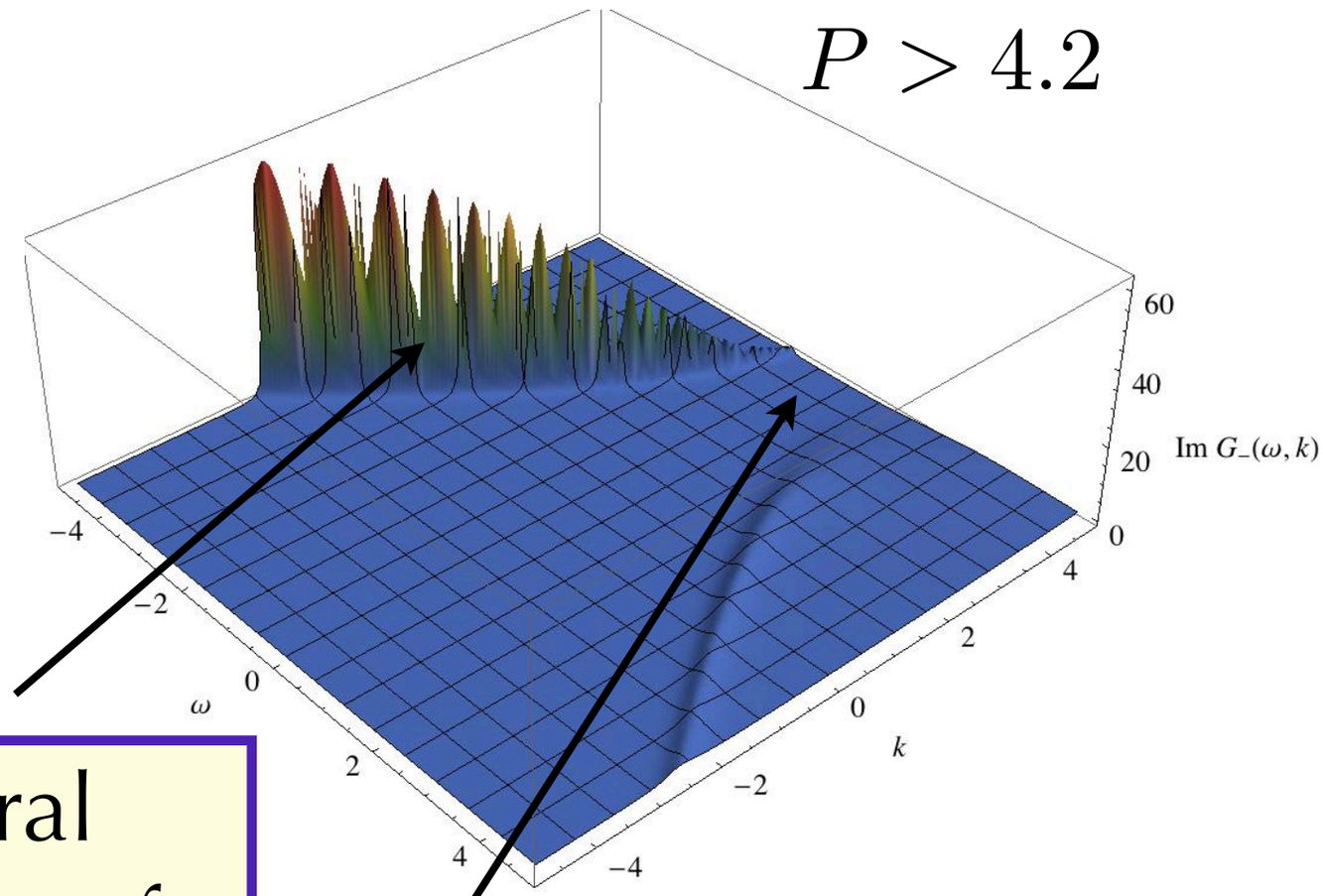
dynamically generated gap:

How is the spectrum modified?

P=0



$P > 4.2$



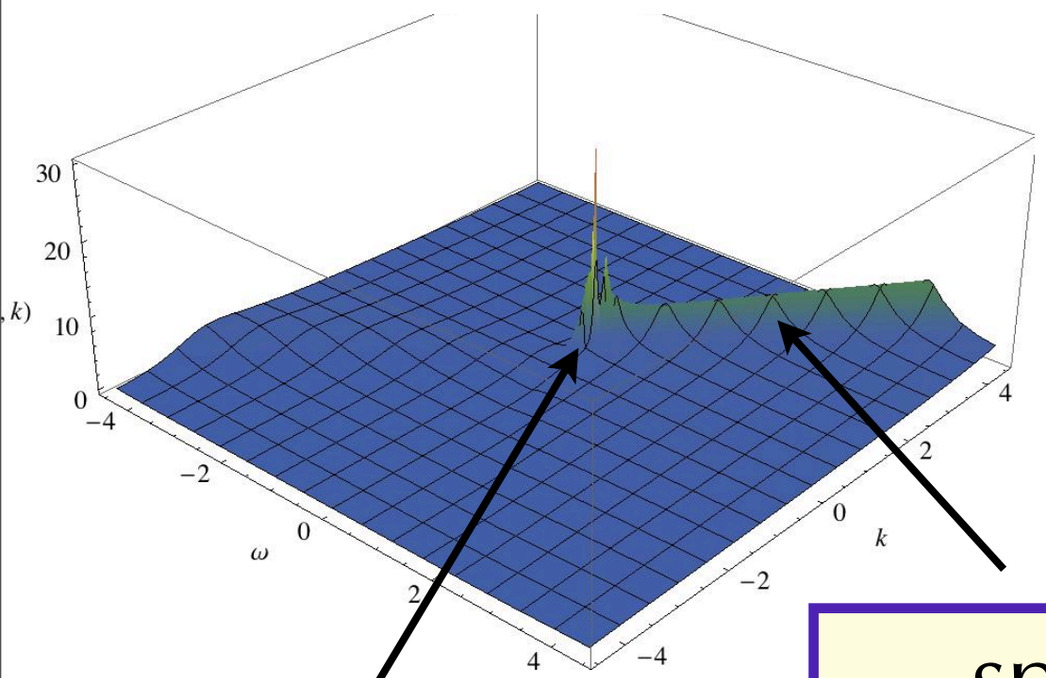
spectral weight transfer

Fermi surface peak

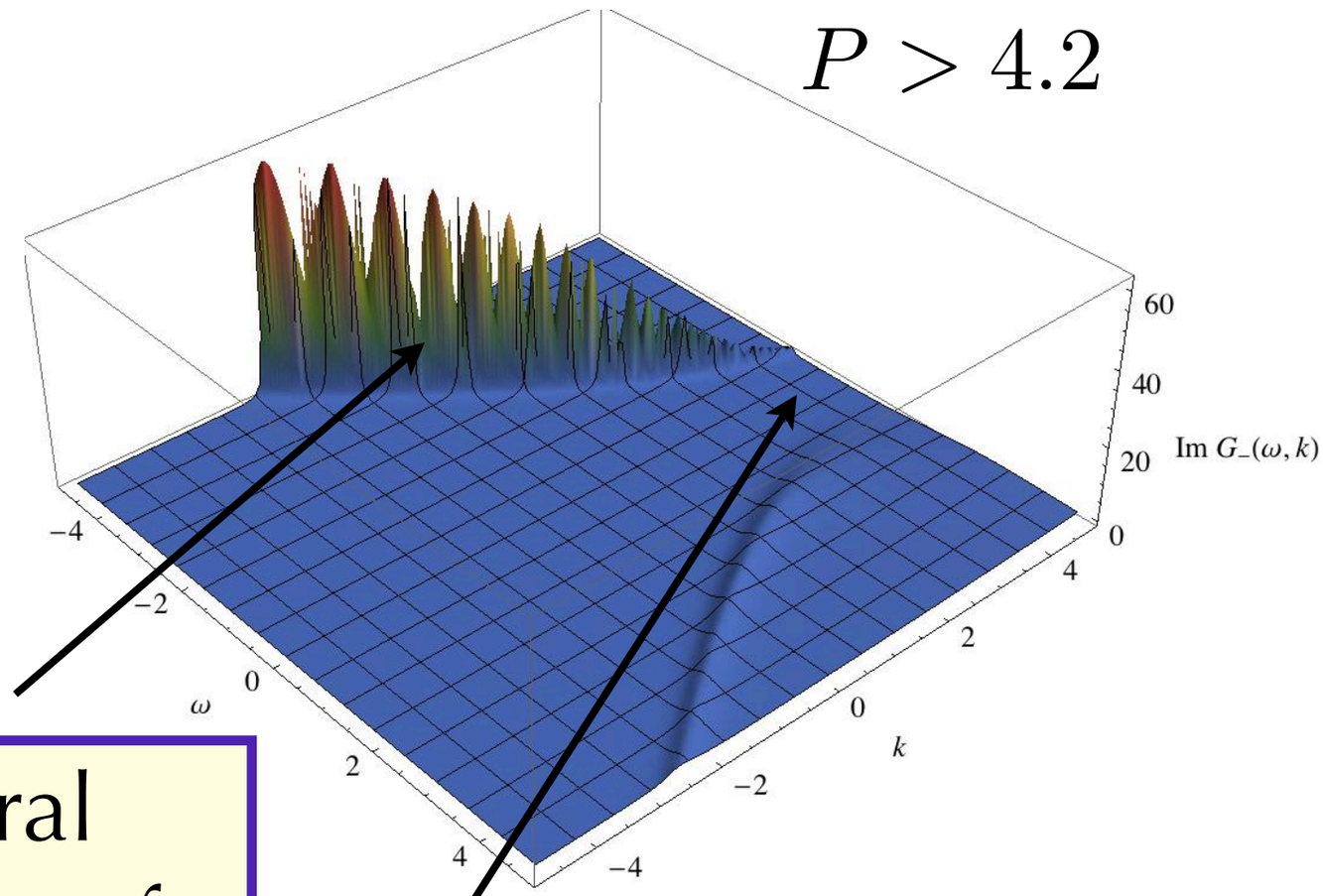
dynamically generated gap:

How is the spectrum modified?

P=0



$P > 4.2$



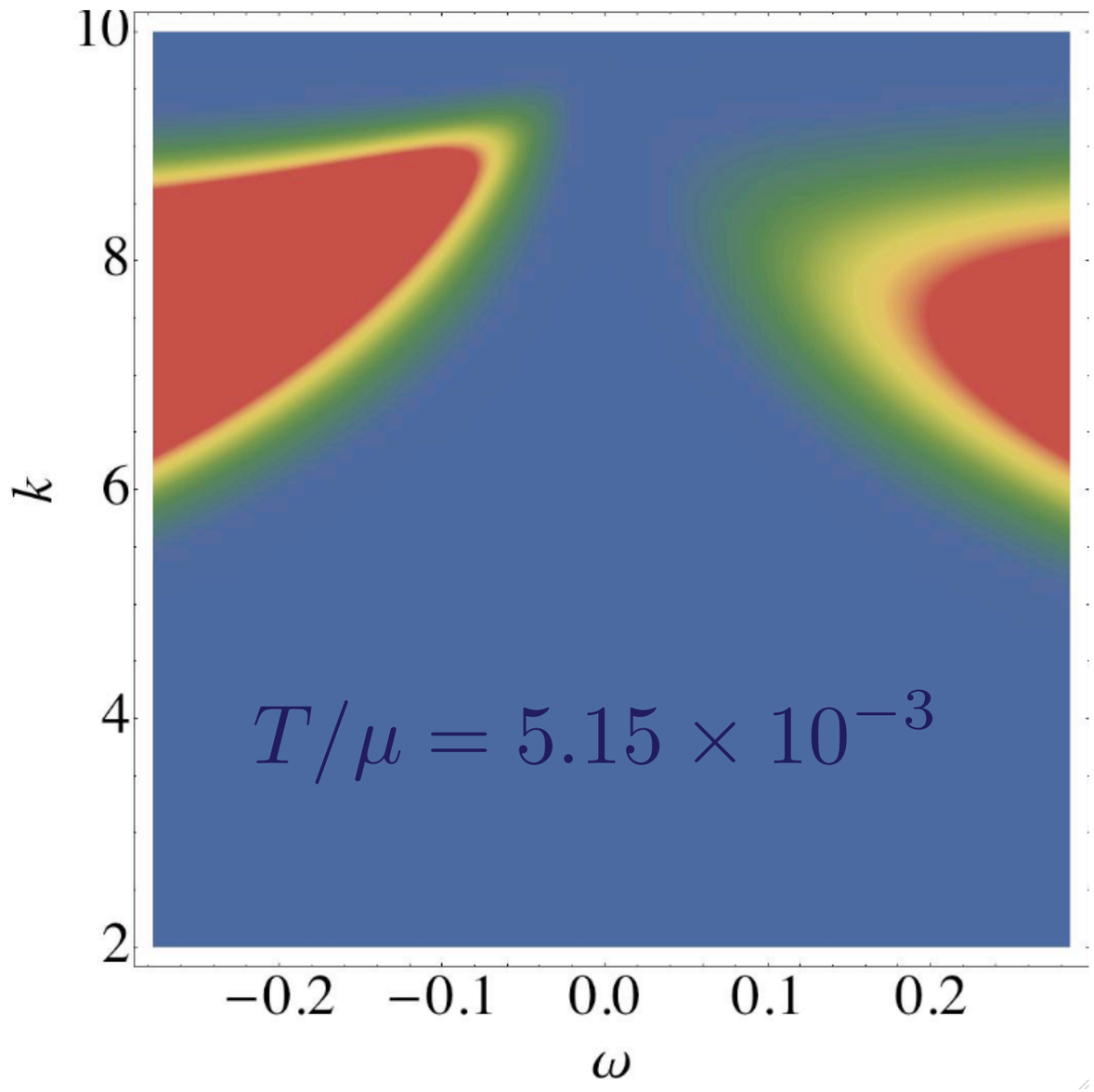
spectral weight transfer

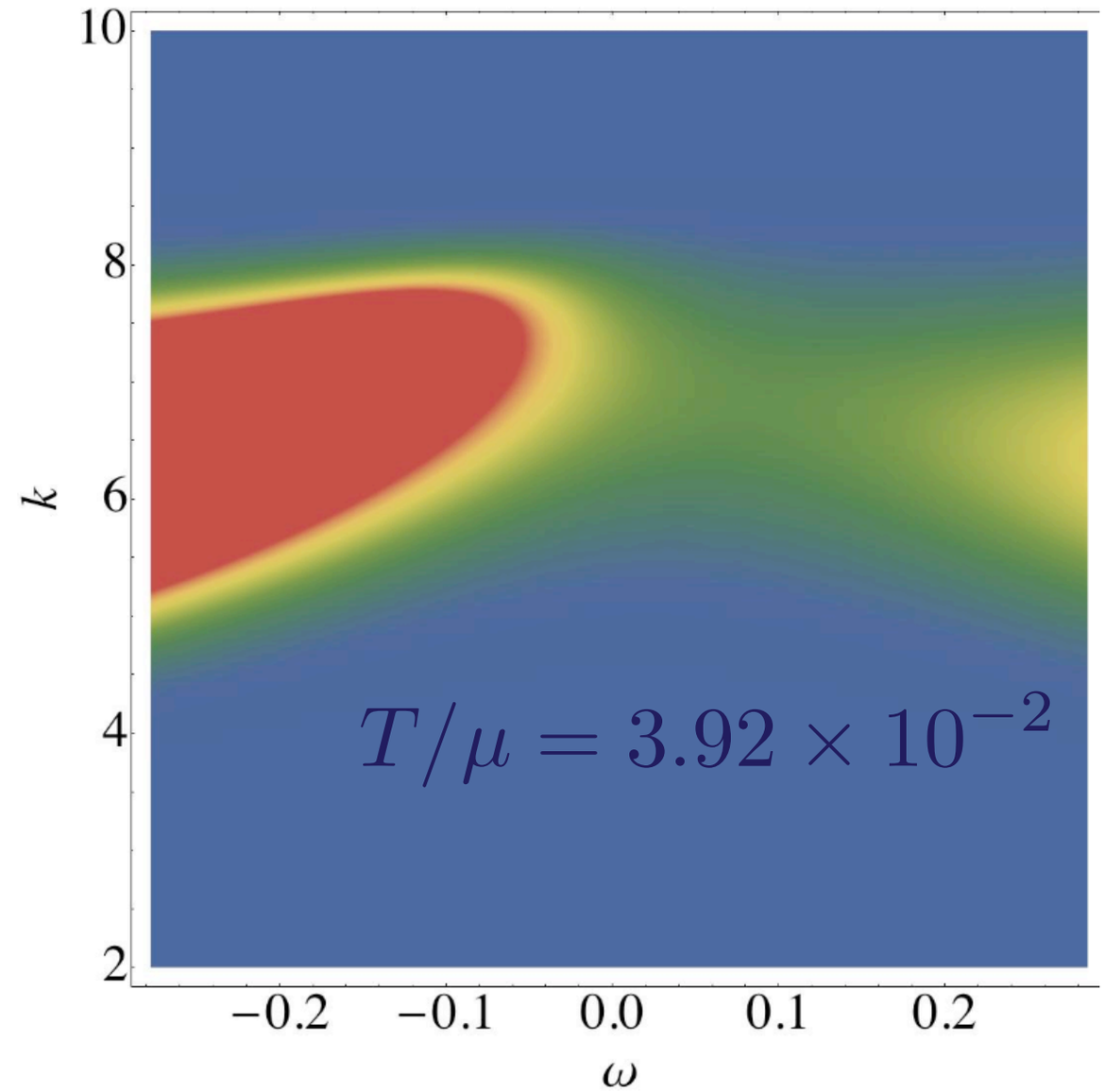
Fermi surface peak

dynamically generated gap:

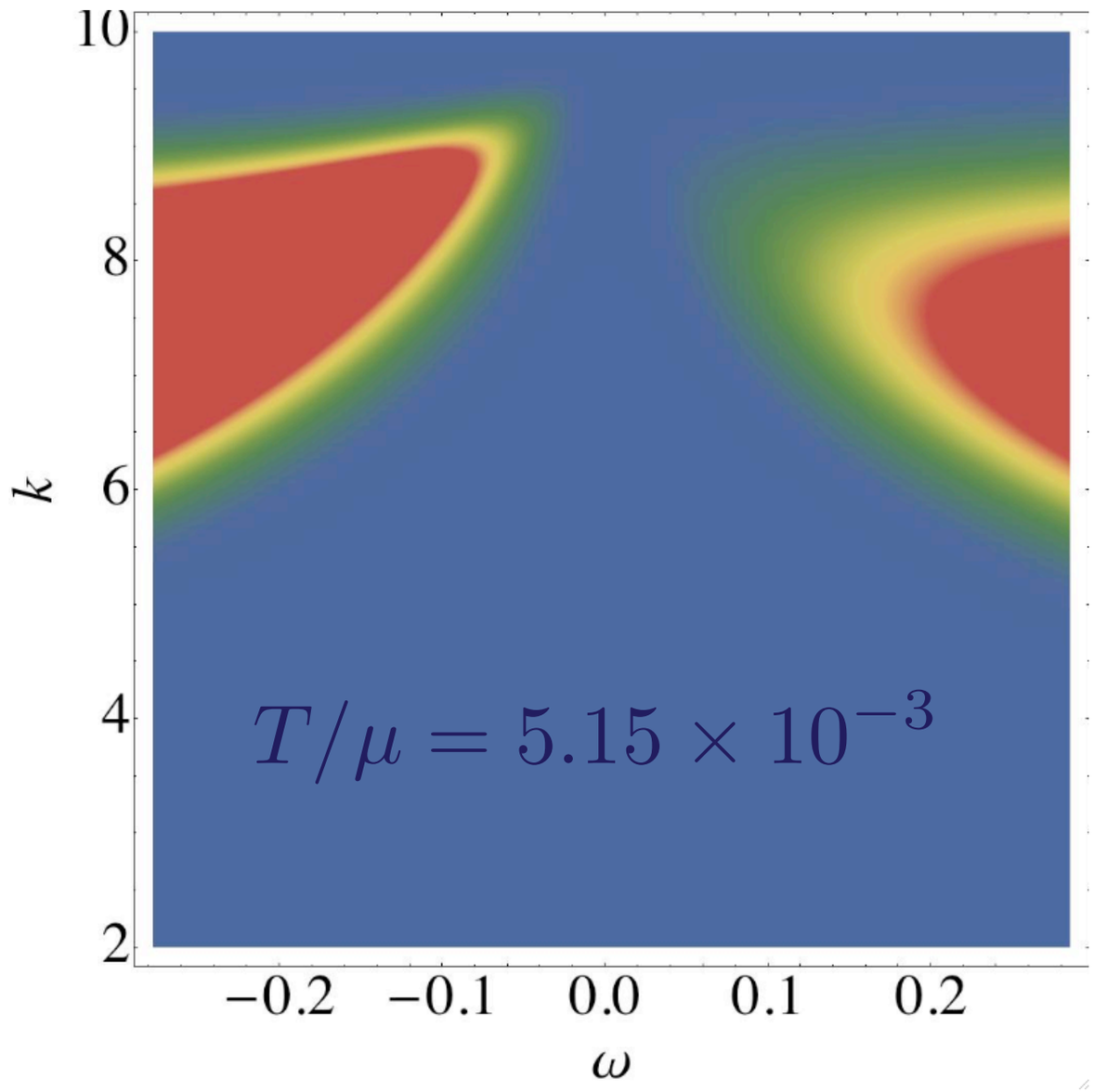
confirmed by Gubser, Gauntlett, 2011

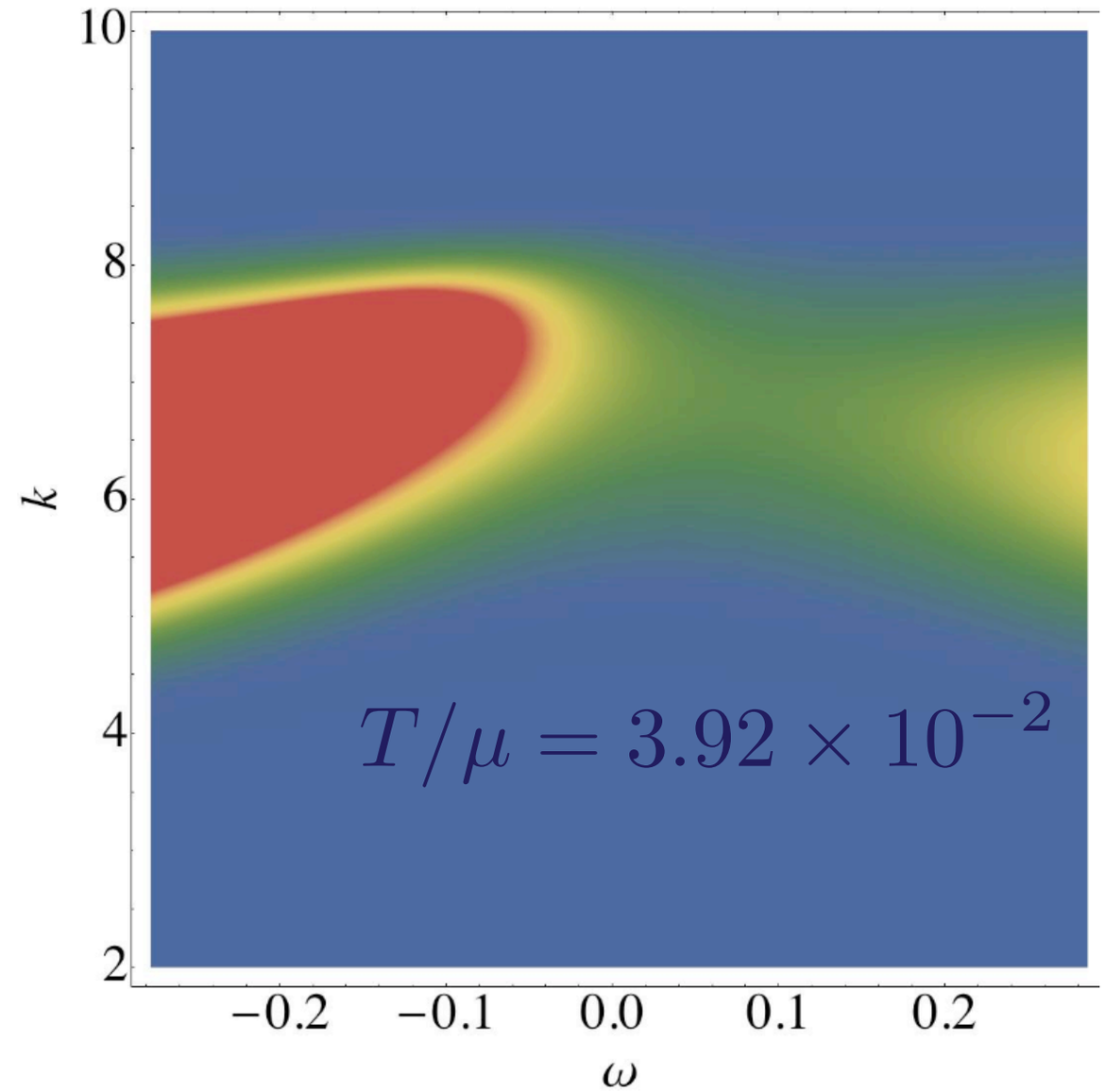
Finite Temperature Mott transition





Finite Temperature Mott transition

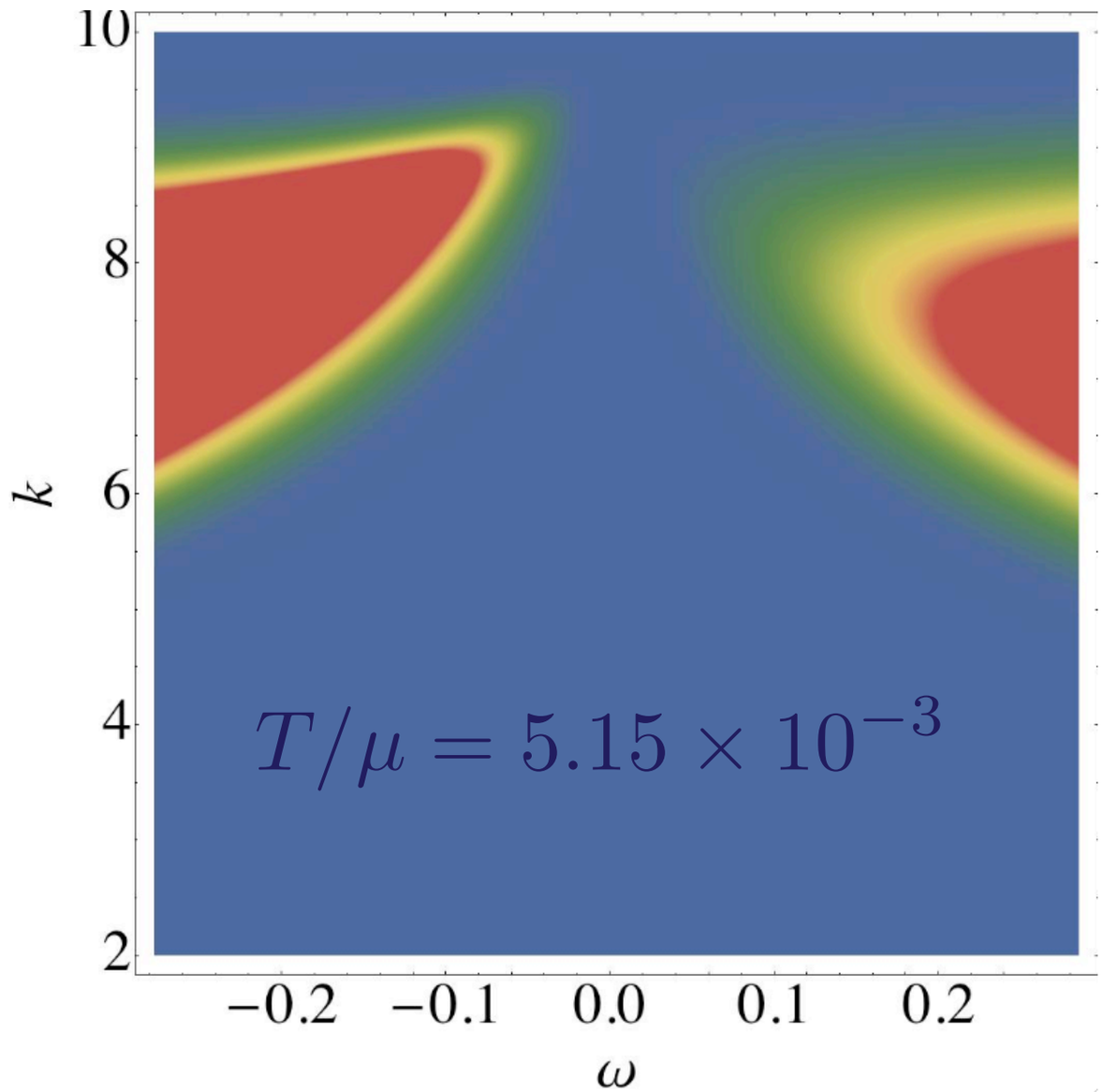


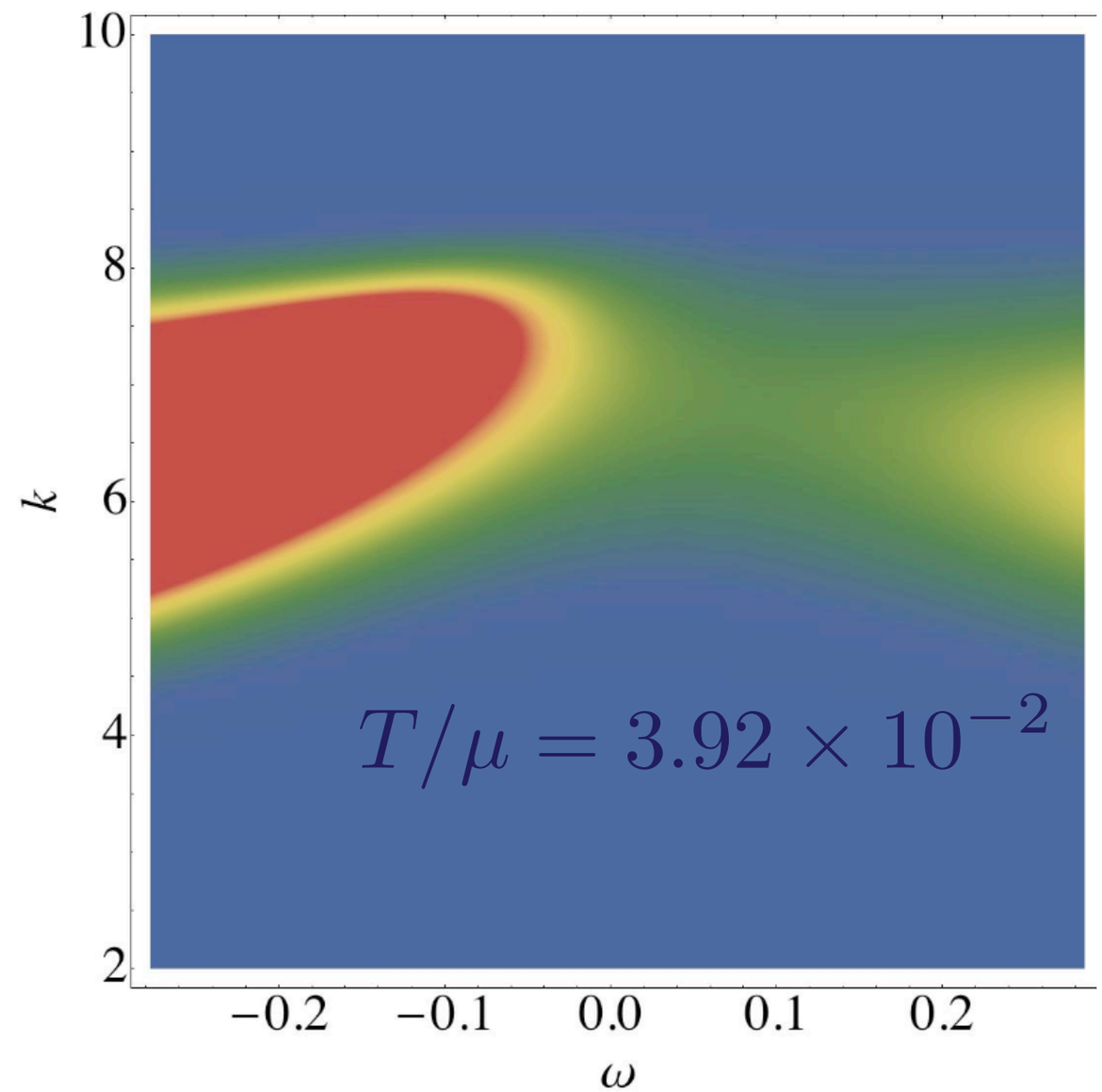


$$\frac{\Delta}{T_{\rm crit}} \approx 20$$

vanadium oxide

Finite Temperature Mott transition

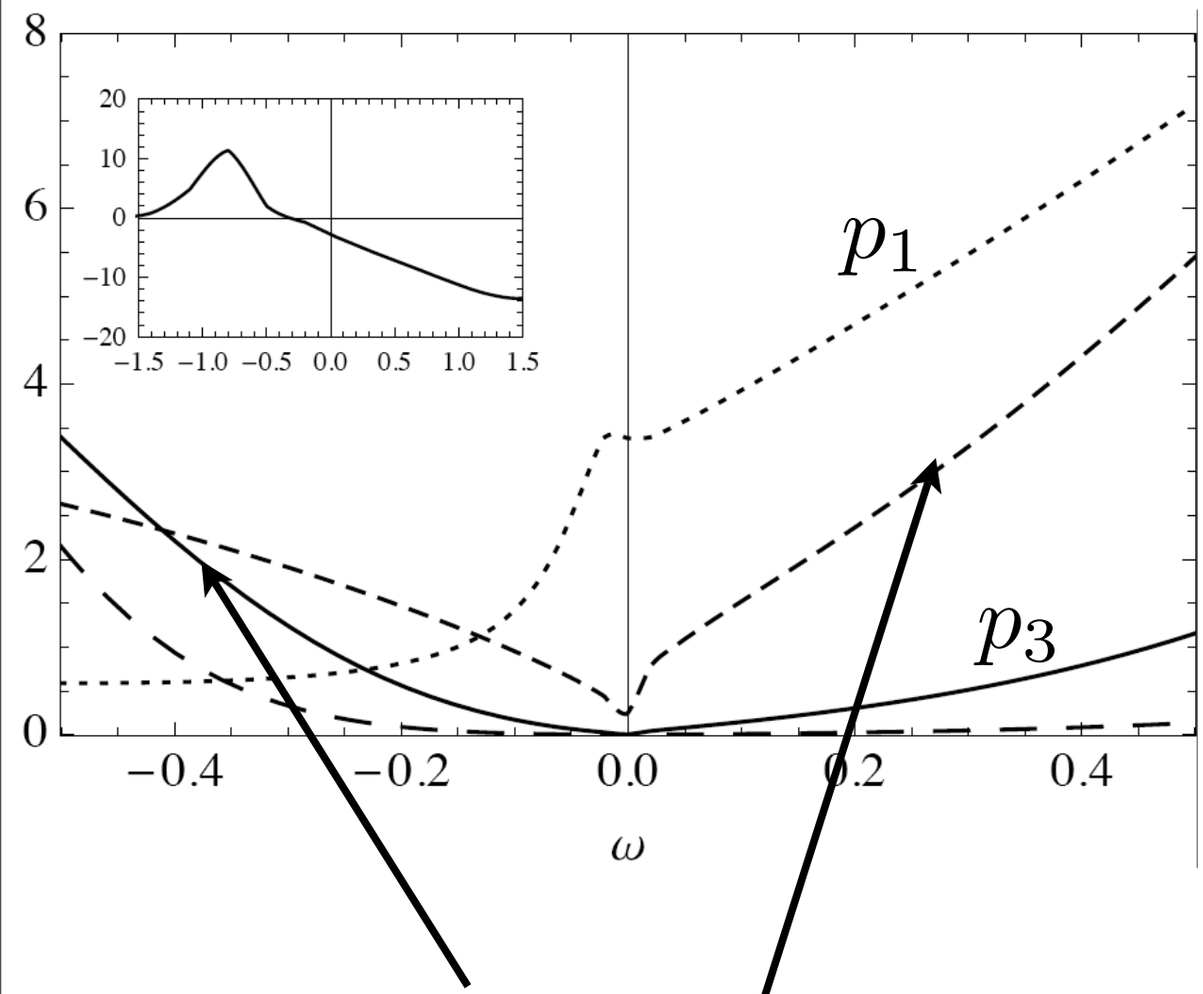




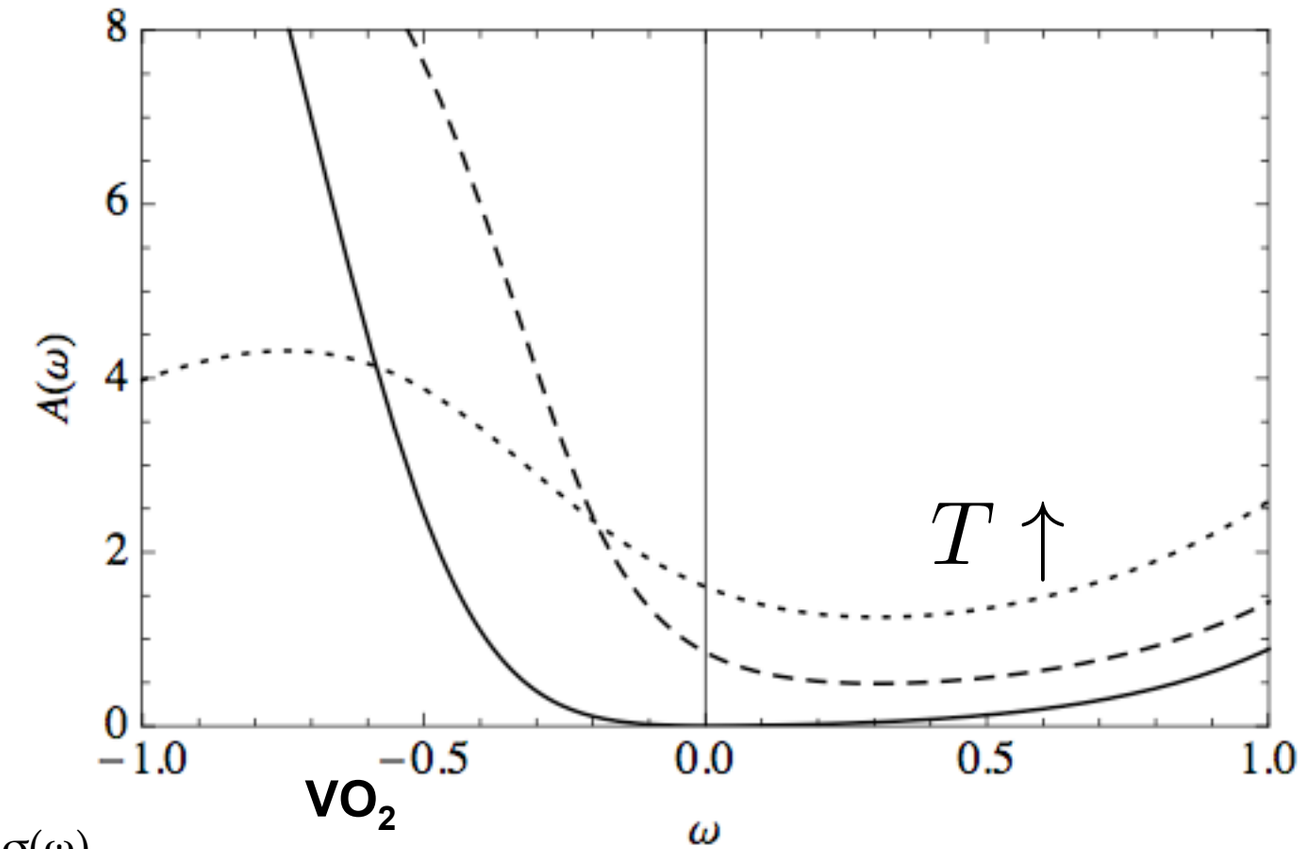
$$\frac{\Delta}{T_{\rm crit}} \approx 10$$

$$\frac{\Delta}{T_{\rm crit}} \approx 20$$

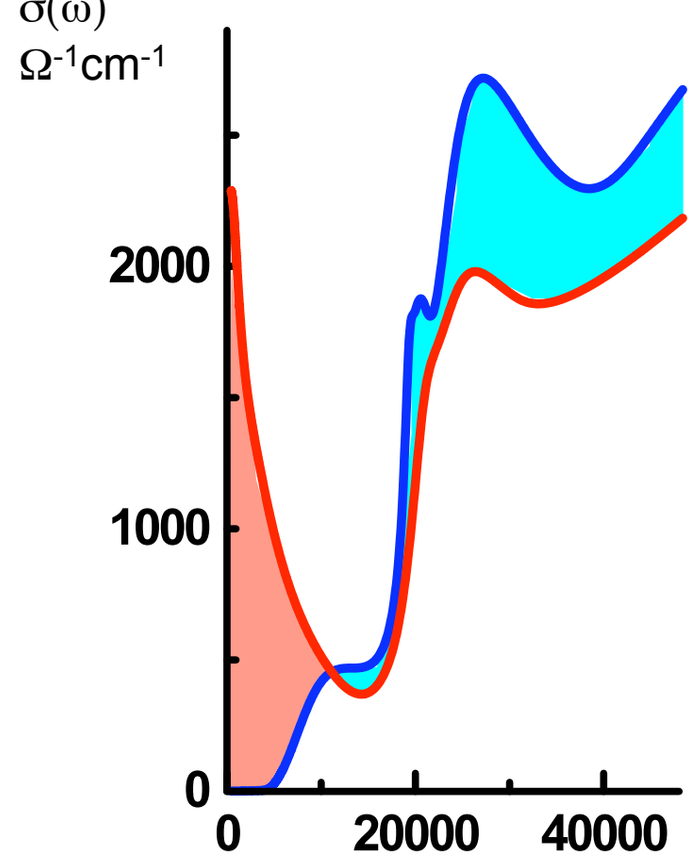
vanadium oxide

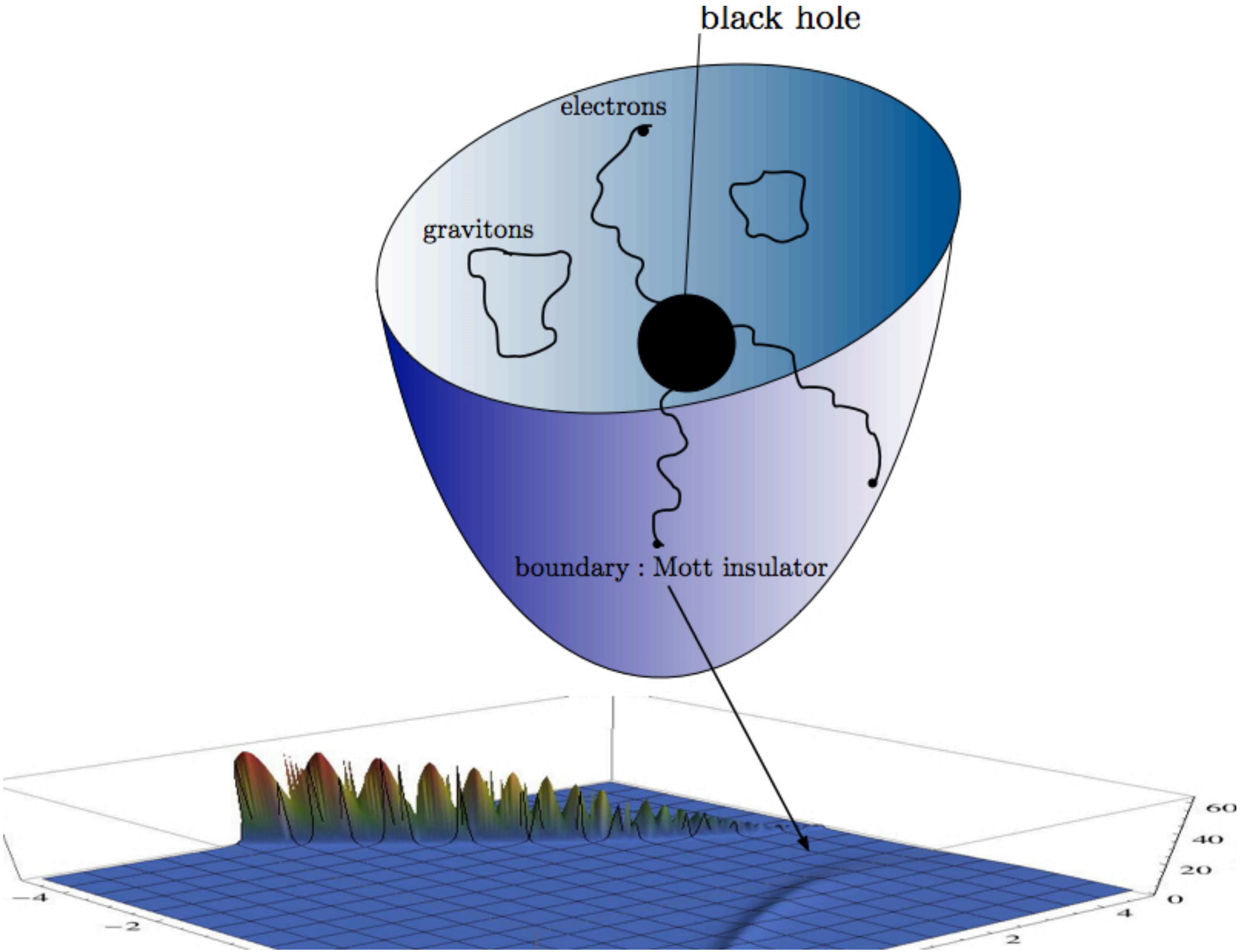


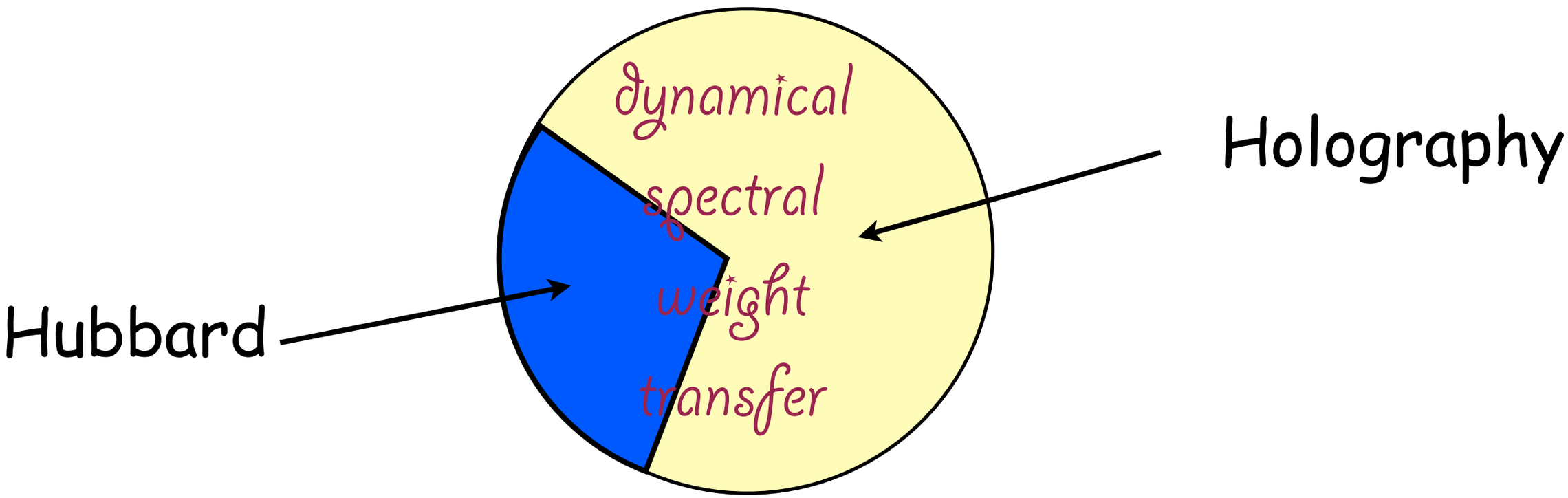
spectral weight transfer
UV-IR mixing



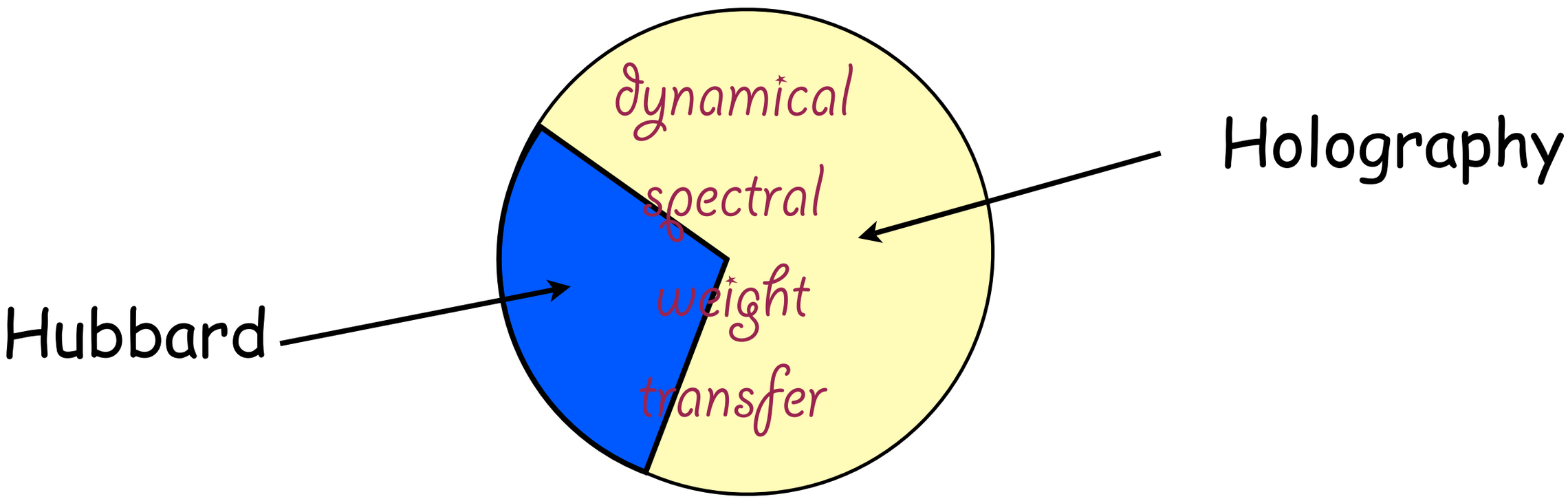
VO_2

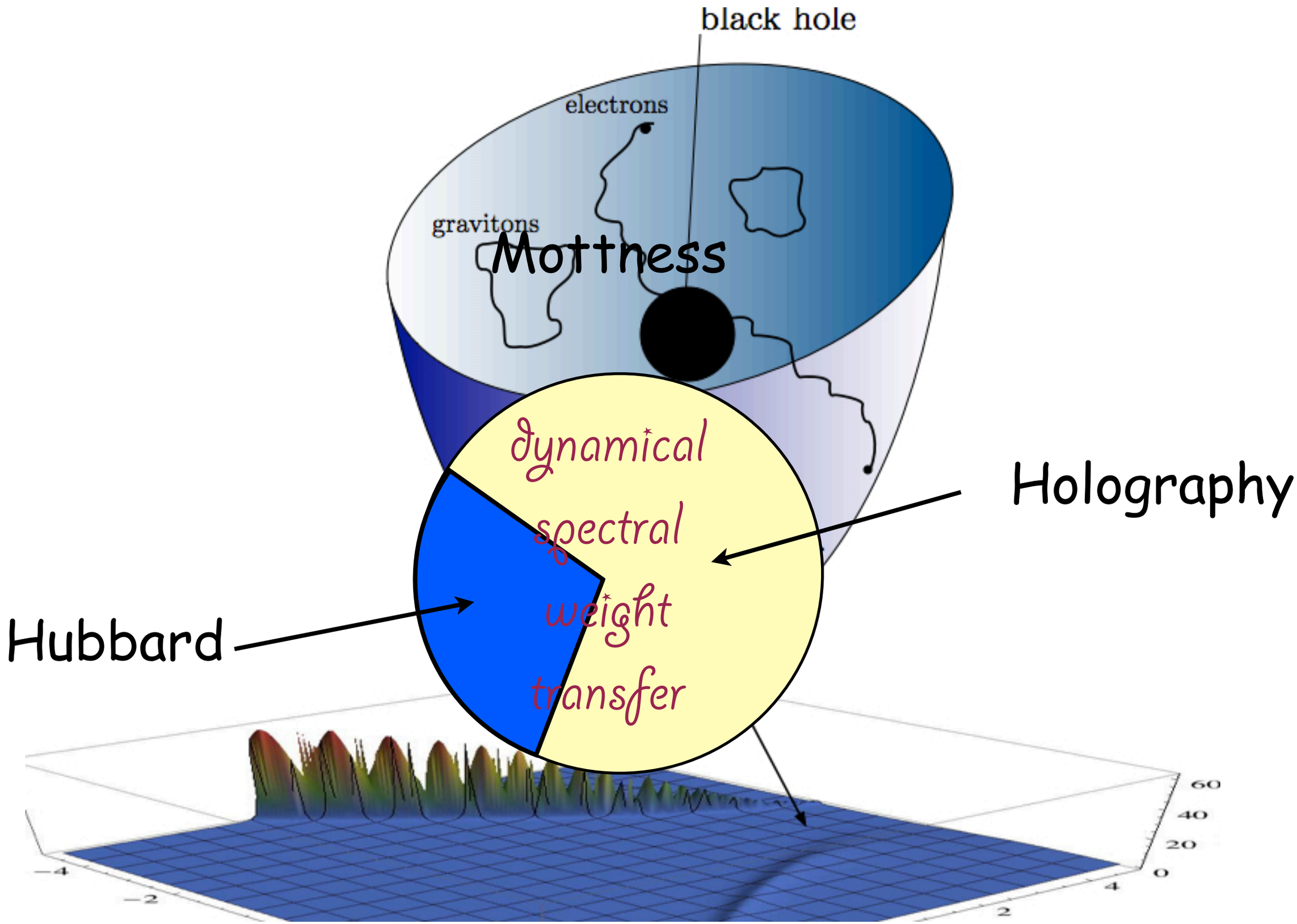



black hole
electrons
gravitons
boundary : Mott insulator
60
40
20
0
−4
−2
2
4


dynamical
spectral
weight
transfer
Holography
Hubbard

Mottness




black hole
electrons
gravitons
Mottness
dynamical
spectral
weight
transfer
Holography
Hubbard
60
40
20
0
−4
−2
2
4