How to detect an anomalous dimension of the current in the Strange Metal of the Cuprates?

thanks to

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NSF+EFRC
what is strange about the strange metal?
Quantum critical behaviour in a high-$T_c$ superconductor

$\sigma(\omega) = C \omega^{-\frac{2}{3}}$

$\frac{n\tau e^2}{m} \frac{1}{1 - i\omega\tau}$

$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$

Strange metal: experimental facts
Theories of cuprates
Theories of strange metal = ∞

starting point?
The basic action is given by:

\[ S = \int d^d x J_\mu A^\mu + \cdots \]

Gauge invariance is described by:

\[ A \rightarrow A + \partial \Lambda \]

The conserved currents have the following dimensions:

\[ [J_\mu] = d - 1 \]

\[ [A_\mu] = \frac{1}{L} = 1 \]

Conserved currents lack anomalous dimensions.

The action has no units.
strange metal: experimental facts

Hall Angle
\[ \cot \theta_H \equiv \frac{\sigma_{xx}}{\sigma_{xy}} \approx T^2 \]

Hall Lorenz ratio
\[ L_{xy} = \frac{\kappa_{xy}}{T\sigma_{xy}} \neq \# \propto T \]

all explained if
\[ [J_\mu] = d - \theta + \Phi + z - 1 \]
\[ [A_\mu] = 1 - \Phi \]
\[ \Phi = -2/3 \]

Hartnoll/Karch
is there a direct experimental probe of this anomaly?

how can A have an anomalous dimension?
How can $A_\mu$ have an anomalous dimension?

Solution

$A_\mu \rightarrow A_\mu + \partial_\mu G$

Not rotationally invariant

How can anomalous dimensions be engineered?
certain strongly coupled theories have gravity duals (holography)
$S = \int dV_d dy \left( y^a F^2 + \cdots \right)$

$F = dA$

$[A] \neq 1$
\[ S = \int dV dy \left( y^a F^2 + \cdots \right) \quad F = dA \]

**equations of motion**

\[ d(y^a \times dA) = 0 \]

\[ y \neq 0 \quad \rightarrow \quad A \rightarrow A + \partial \Lambda \]

**boundary (y=0)?**
Caffarelli-Silvestre extension theorem (2006)

\[ g(z = 0, x) = f(x) \]

\[ \gamma = \frac{1 - a}{2} \]

\[ \nabla \cdot (z^a \nabla g(x, z)) \]

\[ \lim_{z \to 0^+} z^a \frac{\partial g}{\partial z} \]
closer look

\[ \nabla \cdot (y^a \nabla u) = 0 \quad \text{scalar field (use CS theorem)} \]

\[ d(y^a \star dA) = 0 \quad \text{holography} \]

similar equations

generalize CS theorem to p-forms
GL,PP:1708.00863
boundary action: fractional Maxwell equations

\[ \Delta^\gamma A_\perp = 0 \]

boundary action has anomalous dimension (non-locality)

\[ F_{ij} = \partial_i^\gamma A_j - \partial_j^\gamma A_i \equiv d_\gamma A = d\Delta^{\frac{\gamma-1}{2}} A, \]
new gauge transformation

\[ A \rightarrow A + d_\gamma \Lambda, \]

\[ d_\gamma \equiv (\Delta)^{\frac{\gamma-1}{2}} d \]

\[ [A] = \gamma \]

non-locality circumvents the Ward identities
physical consequences of anomalous dimension for $A_\mu$

\[
A_\mu \rightarrow A_\mu + \partial_\mu G
\]

\[
F_{\mu\nu} = \partial_{[\mu} A_{\nu]}
\]

\[
\tilde{\nabla}^\alpha \times \tilde{A} = \tilde{B}
\]

no Stokes' theorem

\[
\oint A \cdot d\ell \neq \int_S B \cdot d\tilde{S}
\]

Aharonov-Bohm Effect must change
\[ \Delta \phi_D = \frac{e}{\hbar} \pi r^2 B R^{2\alpha-2} \left( \frac{\sqrt{\pi} 2^{1-\alpha} \Gamma(2-\alpha) \Gamma\left(1-\frac{\alpha}{2}\right)}{\Gamma(\alpha) \Gamma\left(\frac{3}{2} - \frac{\alpha}{2}\right)} \sin^2 \frac{\pi \alpha}{2} \ _2F_1(1-\alpha, 2-\alpha; 2; \frac{r^2}{R^2}) \right) \]
is the correction large?

\[ \alpha = 1 + \frac{2}{3} = \frac{5}{3} \]

\[ \Delta \Phi_R = \frac{eB\ell^2}{\hbar} L^{-5/3}/(0.43)^2 \]

yes!
Planckian dissipation

\[ \tau = \frac{\hbar}{k_B T} \]

\[ \tau \approx 10^{-14} \text{s} \]

Table 11.1

<table>
<thead>
<tr>
<th>Element</th>
<th>77 K</th>
<th>273 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li</td>
<td>$7.3 \times 10^{-14}$ s</td>
<td>$8.8 \times 10^{-15}$ s</td>
</tr>
<tr>
<td>Na</td>
<td>$1.7 \times 10^{-13}$ s</td>
<td>$3.2 \times 10^{-14}$ s</td>
</tr>
<tr>
<td>K</td>
<td>$1.8 \times 10^{-13}$ s</td>
<td>$4.1 \times 10^{-14}$ s</td>
</tr>
<tr>
<td>Rb</td>
<td>$1.4 \times 10^{-13}$ s</td>
<td>$2.8 \times 10^{-14}$ s</td>
</tr>
<tr>
<td>Cs</td>
<td>$8.6 \times 10^{-14}$ s</td>
<td>$2.1 \times 10^{-14}$ s</td>
</tr>
</tbody>
</table>
Strange Metal?

\[ [A] = \gamma \]

Fractional Aharonov-Bohm in strange metal in cuprates?