How to detect an anomalous dimension of the current in the Strange Metal of the Cuprates?

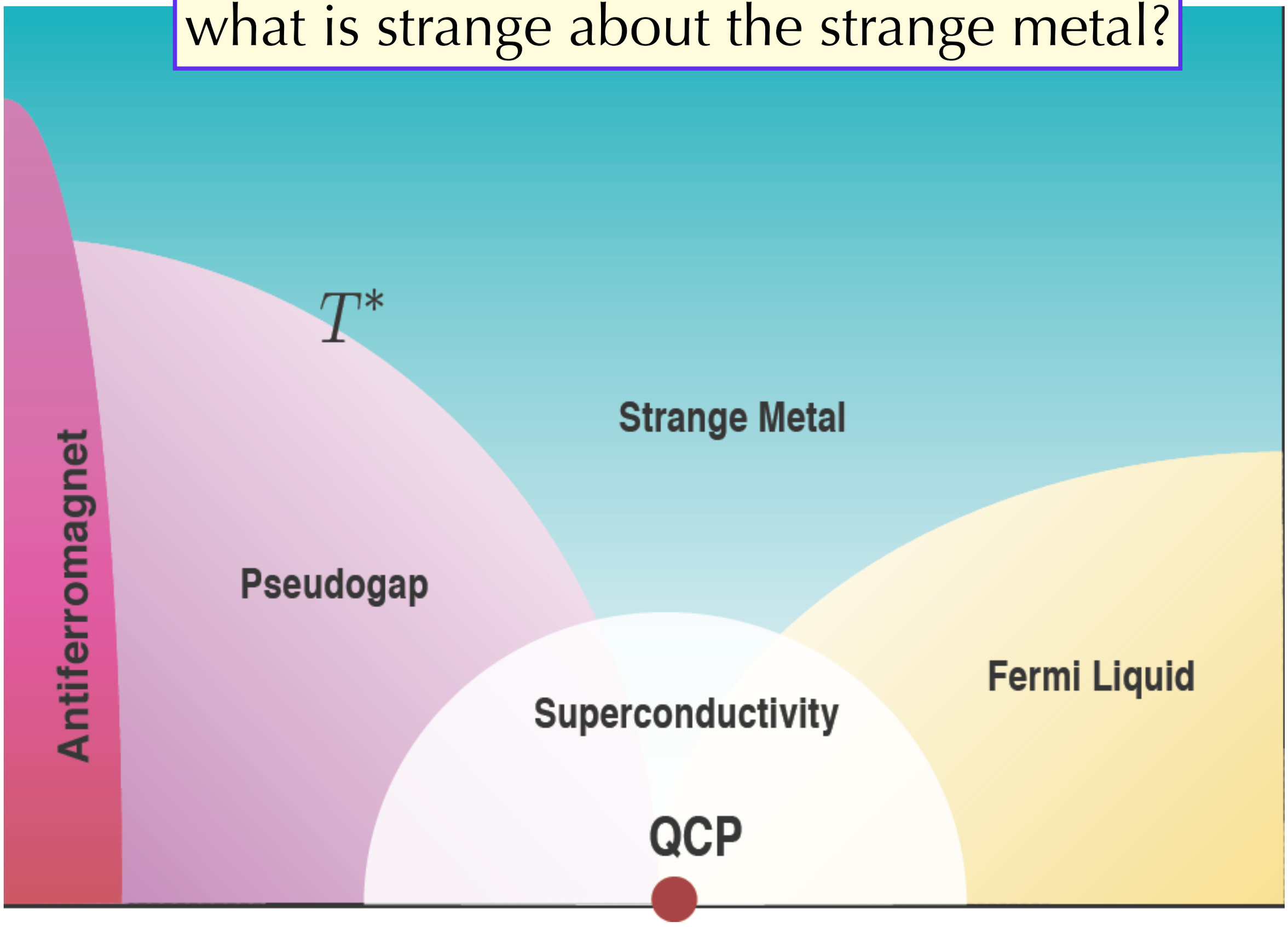
thanks to

Gabriele La Nave

Kridsangaphong Limtragool

NSF+EFRC

what is strange about the strange metal?



strange metal: experimental facts

Quantum critical behaviour in a high-T_c superconductor

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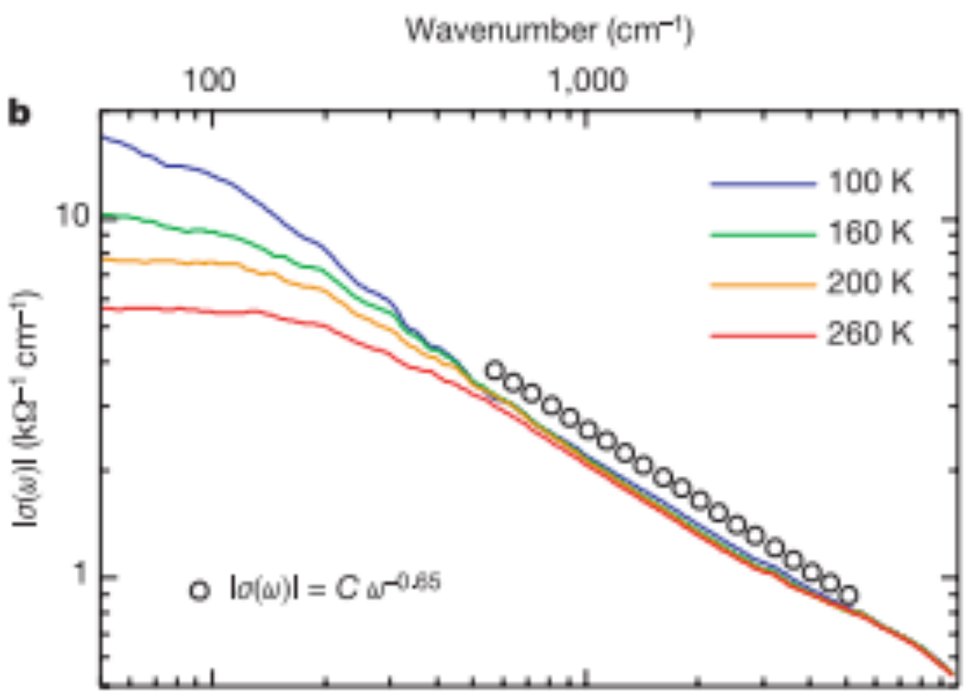
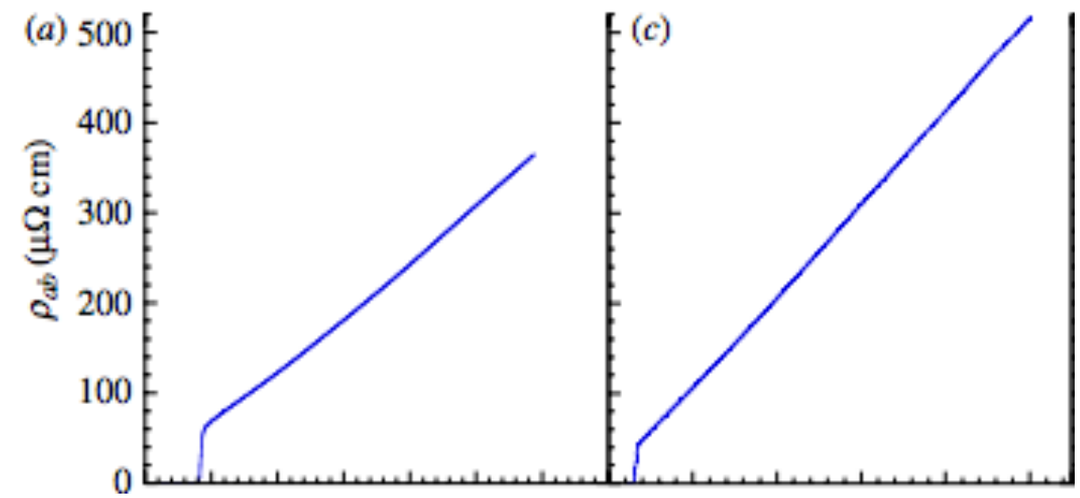


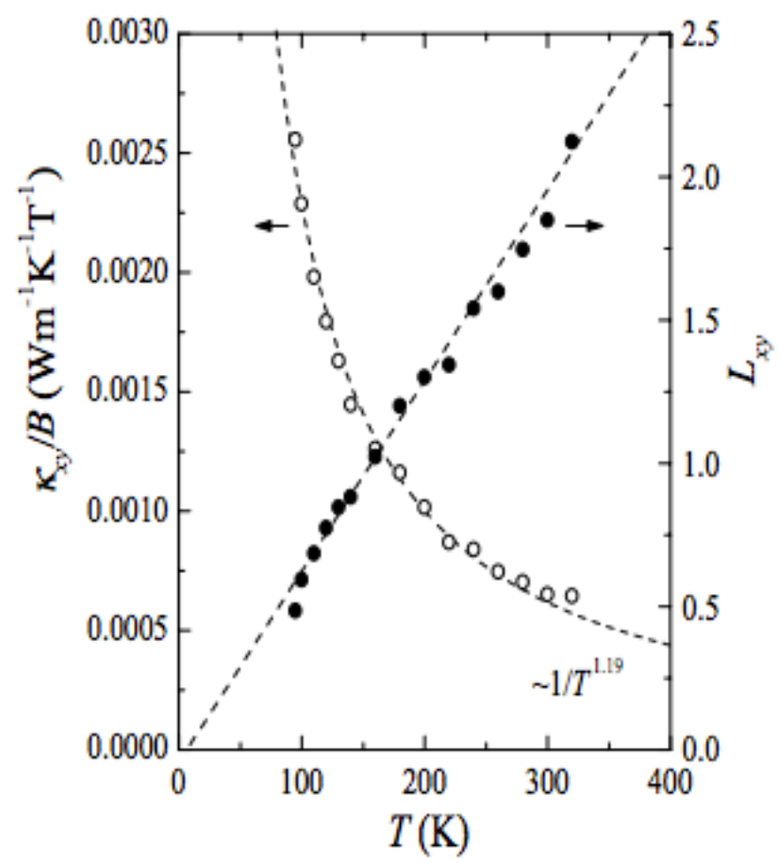
$$\sigma(\omega) = C\omega^{-\frac{2}{3}}$$

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

T-linear resistivity



$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$



$$\frac{\text{Theories of cuprates}}{\text{Theories of strange metal}} = \infty$$

starting point?

basic action

$$S = \int d^d x J_\mu A^\mu + \cdots$$

gauge invariance

$$A \to A + \partial\Lambda$$

$$[J_\mu] = d - 1$$

has no units

$$[A_\mu] = \frac{1}{L} = 1$$

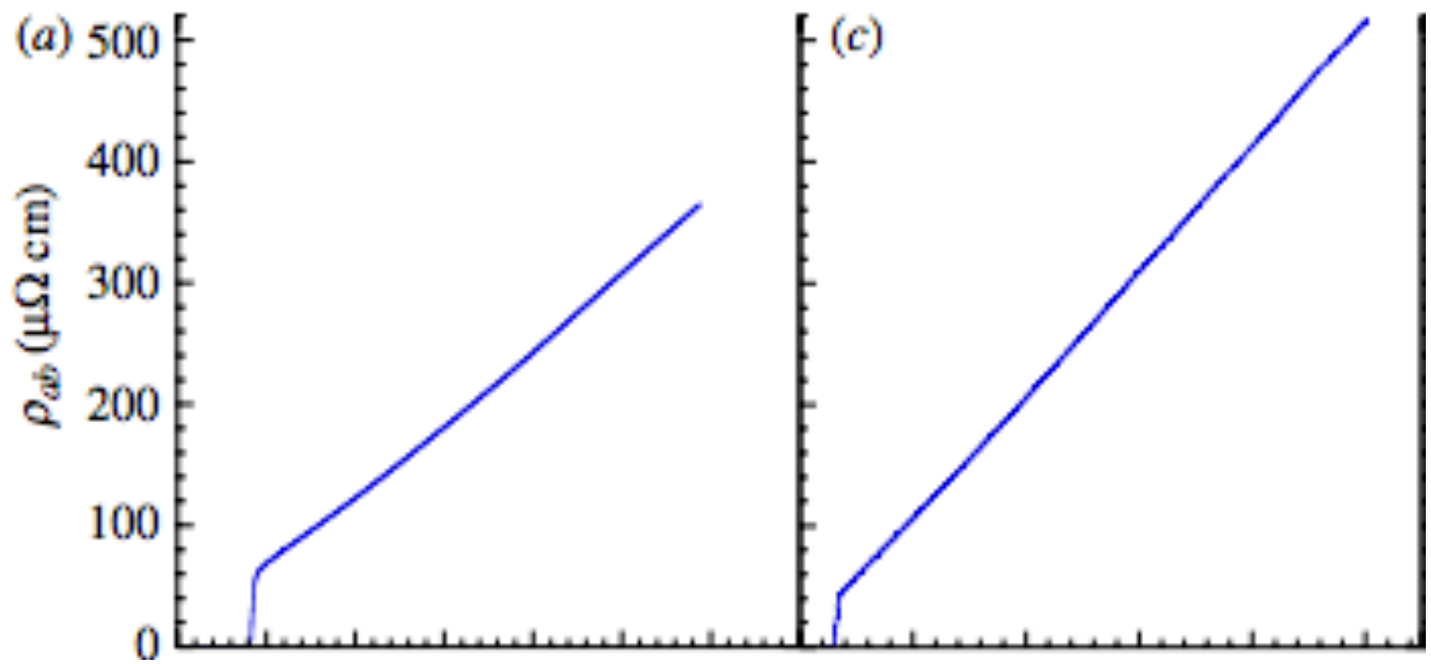
conserved currents lack anomalous dimensions

strange metal: experimental facts

Hall Angle

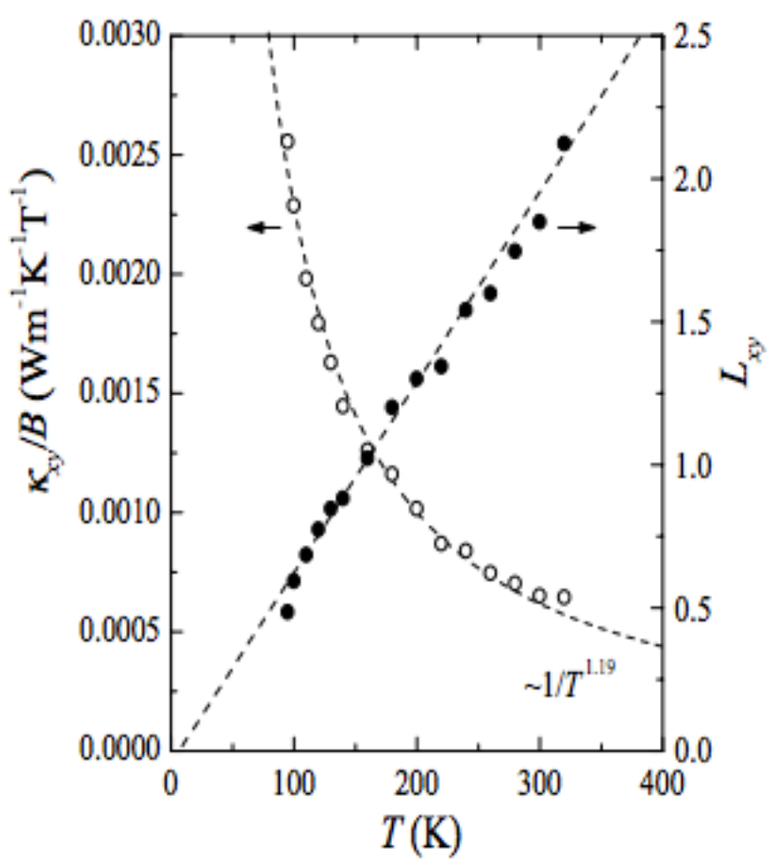
$$\cot\theta_H \equiv \frac{\sigma_{xx}}{\sigma_{xy}} \approx T^2$$

T-linear resistivity



Hall Lorenz ratio

$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$



all explained if

$$[J_\mu] = d - \theta + \Phi + z - 1$$

Hartnoll/Karch

$$[A_\mu] = 1 - \Phi$$

$$\Phi = -2/3$$

is there a direct
experimental probe of this anomaly?

how can A have an
anomalous dimension?

How can $\mathcal{A}_\mu$ have an anomalous dimension?

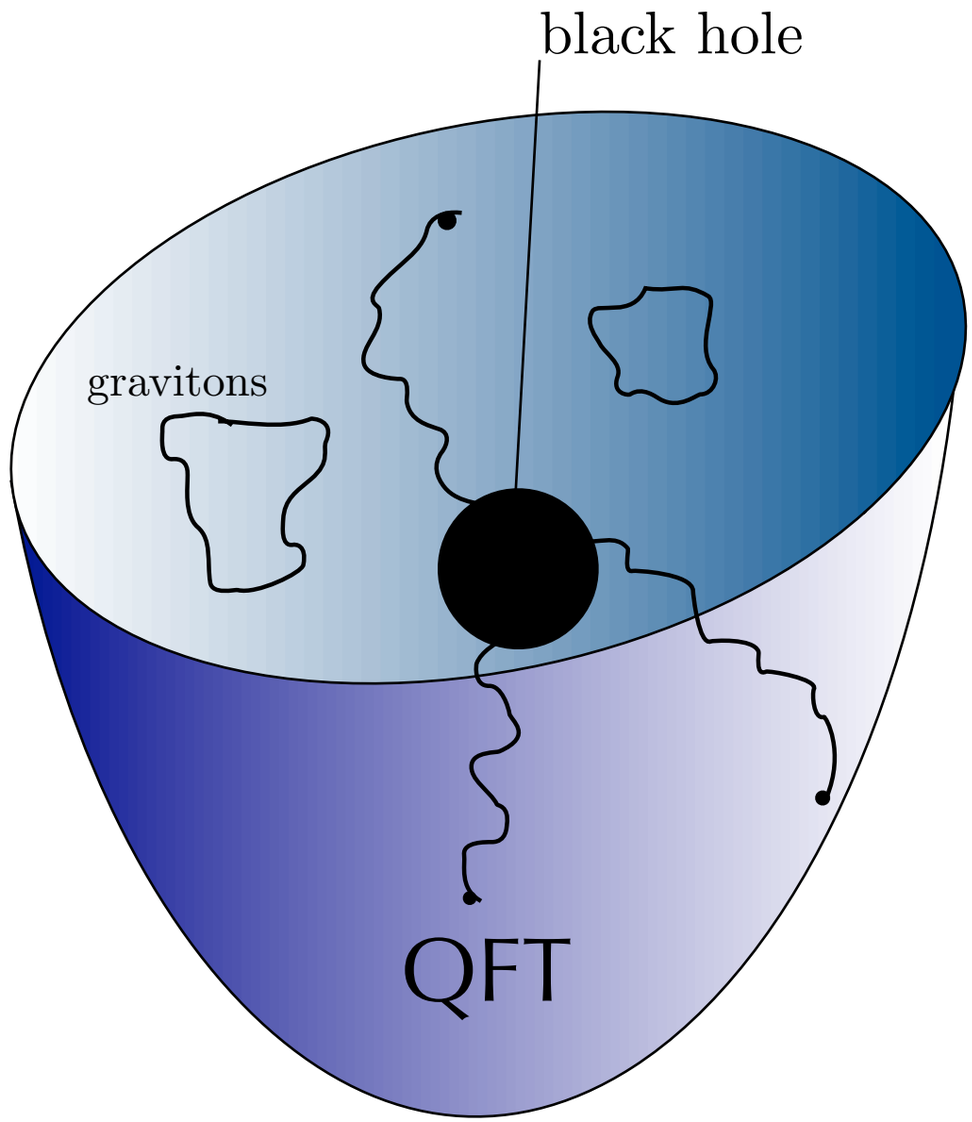
solution

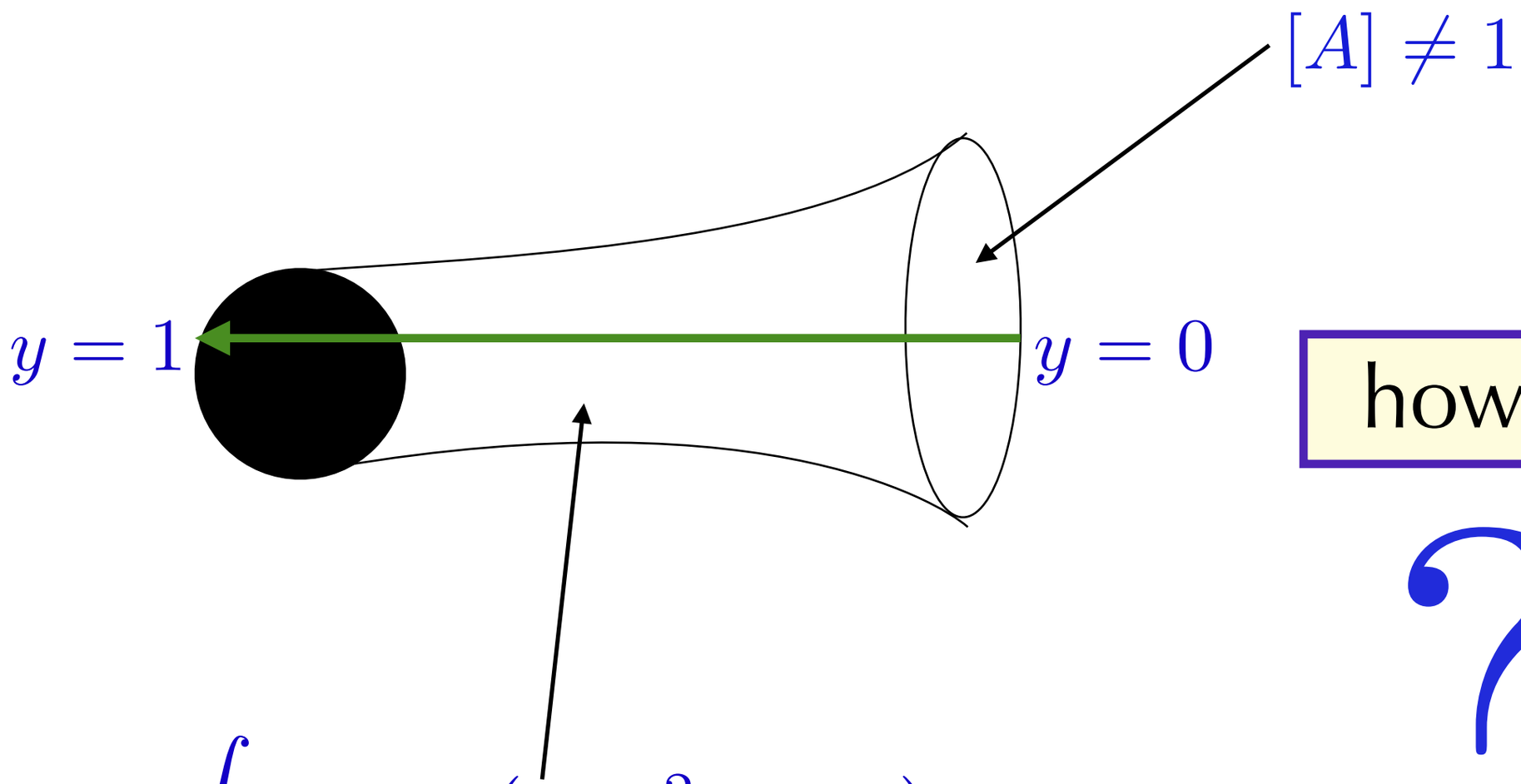
$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial_\mu^{\alpha} \mathcal{G}$$

not rotationally invariant

how can anomalous dimensions be engineered?

certain strongly coupled theories have gravity duals (holography)





how?

?

$$S = \int dV_d dy \left(y^a F^2 + \cdots \right)$$

$$F = dA$$

$$S = \int dV_d dy \left(y^a F^2 + \cdots \right) \quad F = dA$$

equations of motion

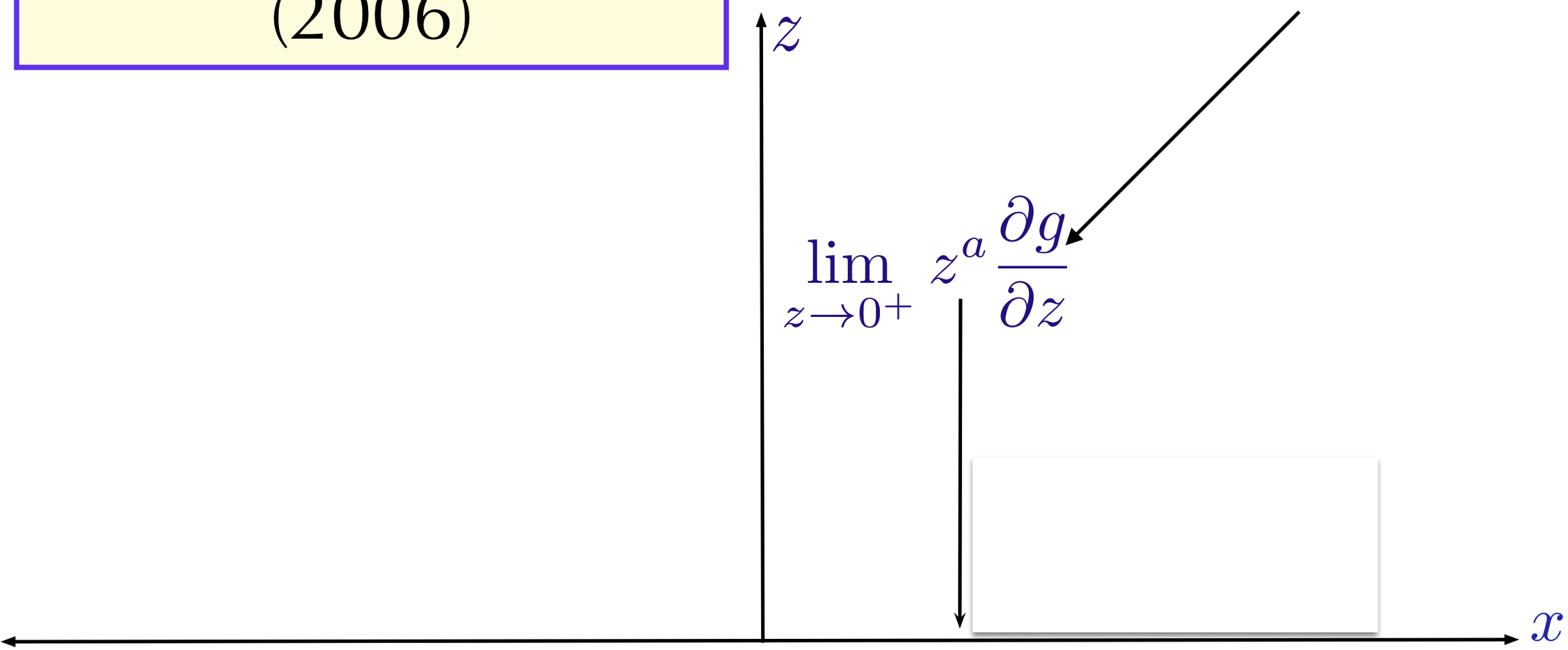
$$d(y^a \star dA) = 0$$

$$y \neq 0 \longrightarrow A \to A + \partial\Lambda$$

boundary (y=0)?

Caffarelli-Silvestre extension theorem (2006)

$$\nabla \cdot (z^a \nabla g(x,z))$$



$$g(z=0,x) = f(x)$$

$$\gamma = \frac{1-a}{2}$$

closer look

$$\nabla \cdot (y^a \nabla u) = 0$$

scalar field
(use CS theorem)

$$d(y^a \star dA) = 0$$

holography

similar equations

generalize CS
theorem to p-forms
GL,PP:1708.00863

boundary action: fractional Maxwell equations

$$\Delta^{\gamma} A_{\perp} = 0$$

boundary action has anomalous dimension (non-locality)

$$F_{ij} = \partial_i^{\gamma} A_j - \partial_j^{\gamma} A_i \equiv d_{\gamma} A = d\Delta^{\frac{\gamma-1}{2}} A,$$

new gauge transformation

$$A \to A + d_\gamma \Lambda,$$

$$d_\gamma \equiv (\Delta)^{\frac{\gamma-1}{2}} d$$

$$[A] = \gamma$$

non-locality
circumvents the Ward
identities

physical consequences of anomalous dimension for $\mathcal{A}_\mu$

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial_\mu^{\alpha} \mathcal{G}$$

$$\partial_\mu^\alpha = \Delta^{\frac{\gamma-1}{2}} \partial_\mu$$

$$F_{\mu\nu} = \partial_{[\mu}^\alpha A_{\nu]}$$

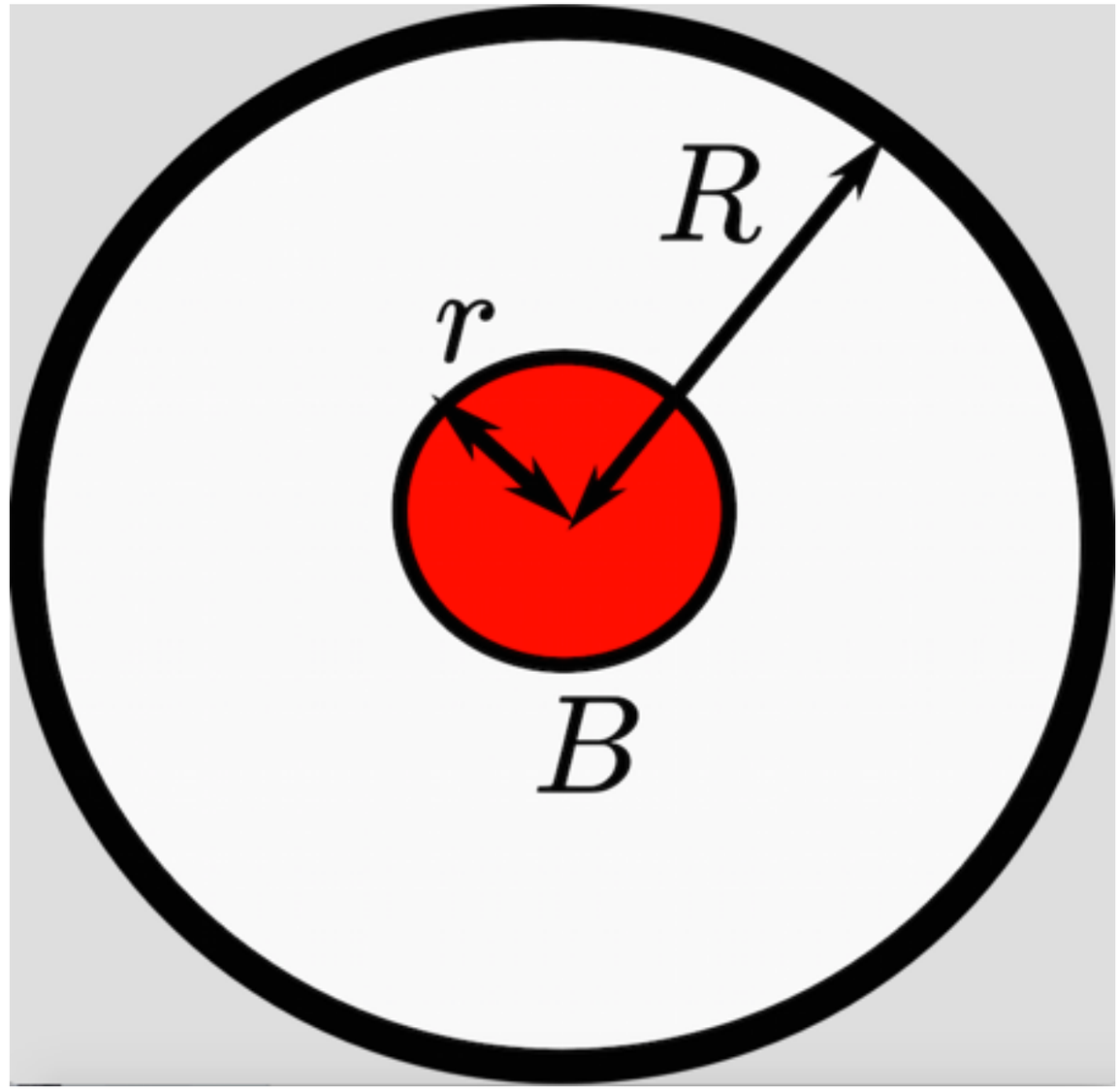
rotationally invariant

$$\vec{\nabla}^\alpha \times \vec{A} = \vec{B}$$

no Stokes' theorem

$$\oint \vec{A} \cdot d\ell \neq \int_S B \cdot d\vec{S}$$

Aharonov-Bohm Effect must change



$$\Delta\phi_{\mathrm{D}} = \frac{e}{\hbar}\pi r^2 B \boxed{R^{2\alpha-2}\left(\frac{\sqrt{\pi}2^{1-\alpha}\Gamma(2-\alpha)\Gamma(1-\frac{\alpha}{2})}{\Gamma(\alpha)\Gamma(\frac{3}{2}-\frac{\alpha}{2})}\sin^2\frac{\pi\alpha}{2}\,{}_2F_1(1-\alpha,2-\alpha;2;\frac{r^2}{R^2})\right)}$$

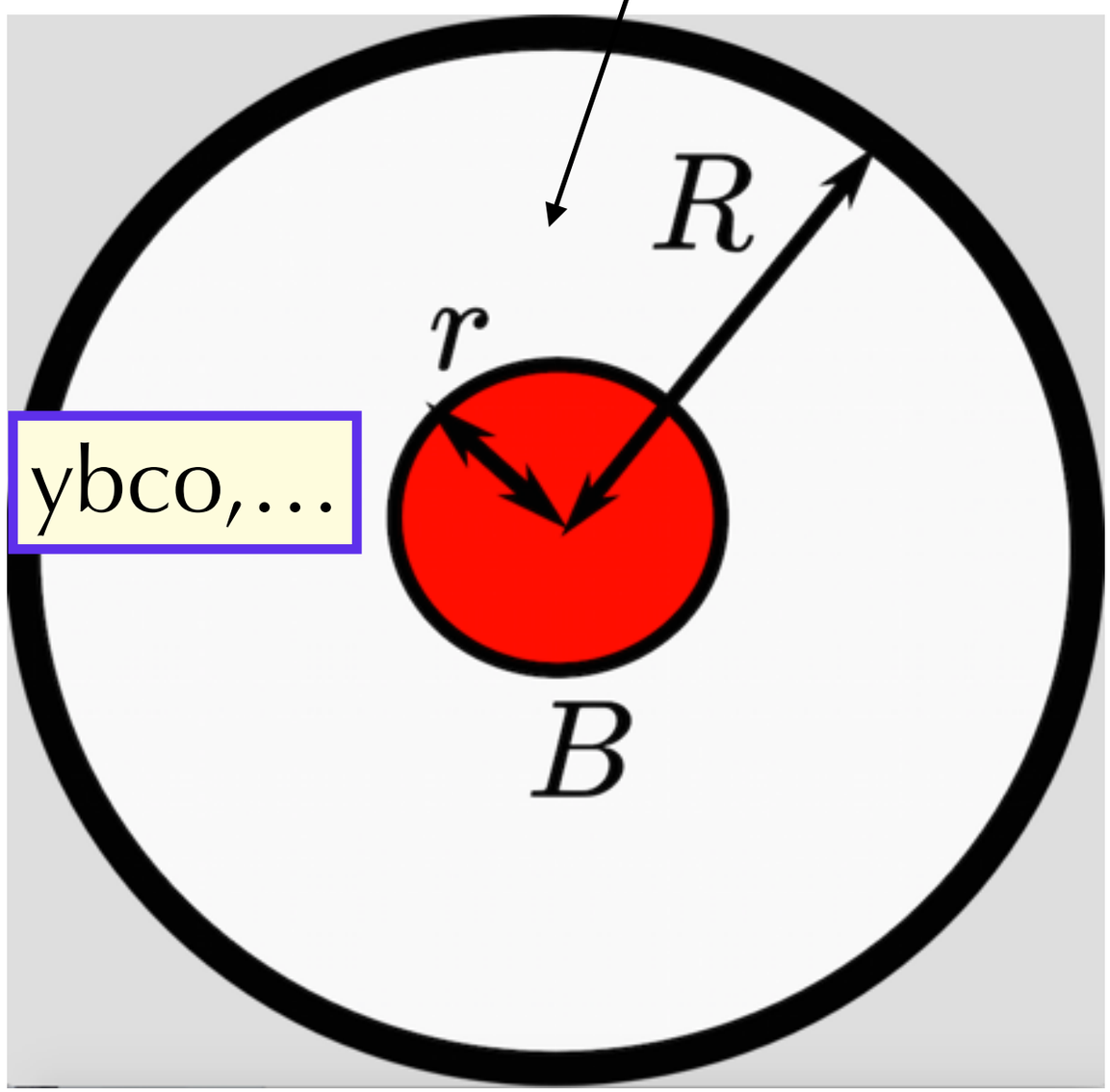
is the correction large?

$$\alpha = 1 + 2/3 = 5/3$$

$$\Delta\Phi_R = \frac{eB\ell^2}{\hbar} L^{-5/3}/(0.43)^2$$

yes!

experiment



Planckian dissipation

$$\tau = \frac{\hbar}{k_B T}$$

$$\tau \approx 10^{-14} s$$

Table 11.1

Element	77 K	273 K
Li	$7.3 \times 10^{-14} s$	$8.8 \times 10^{-15} s$
Na	$1.7 \times 10^{-13} s$	$3.2 \times 10^{-14} s$
K	$1.8 \times 10^{-13} s$	$4.1 \times 10^{-14} s$
Rb	$1.4 \times 10^{-13} s$	$2.8 \times 10^{-14} s$
Cs	$8.6 \times 10^{-14} s$	$2.1 \times 10^{-14} s$

Strange Metal?

$$[A] = \gamma$$

Fractional Aharonov-Bohm
in strange metal
in cuprates?