

Superconductivity and Mottness: Exact Results

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Luke Yeo



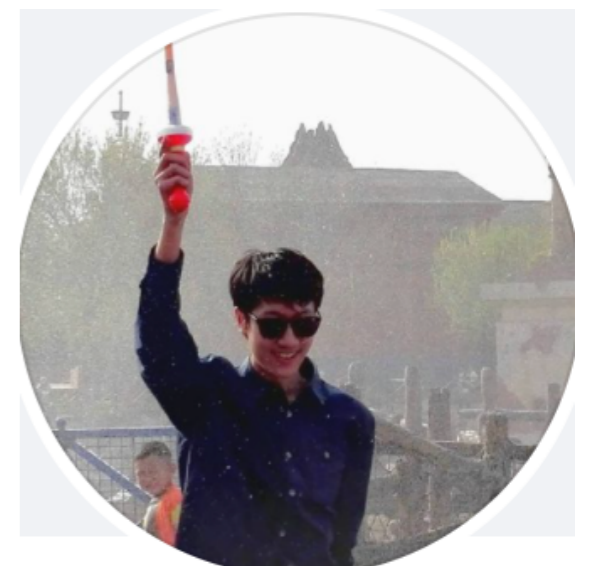
Edwin Huang



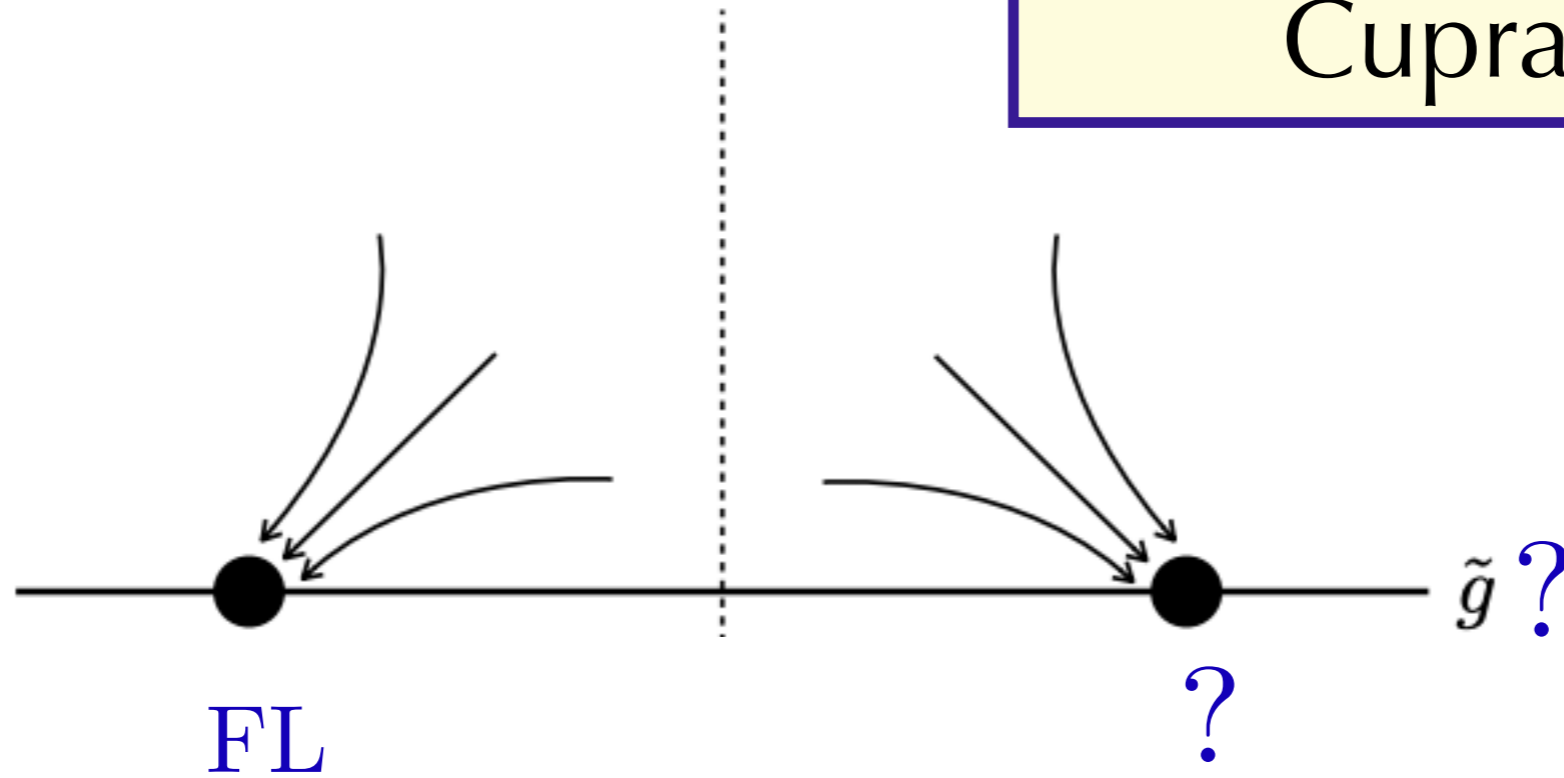
G. La Nave



Jinchao Z.



fixed point beyond
FL? Mottness?
Cuprates?

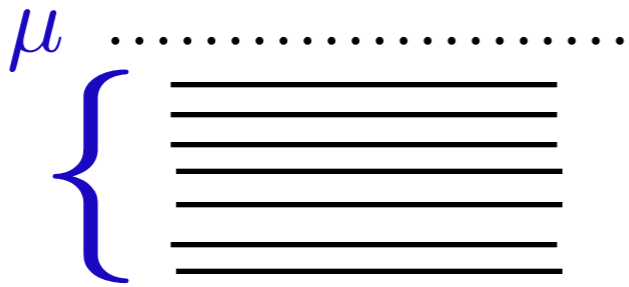


quartic
interacting
theory?

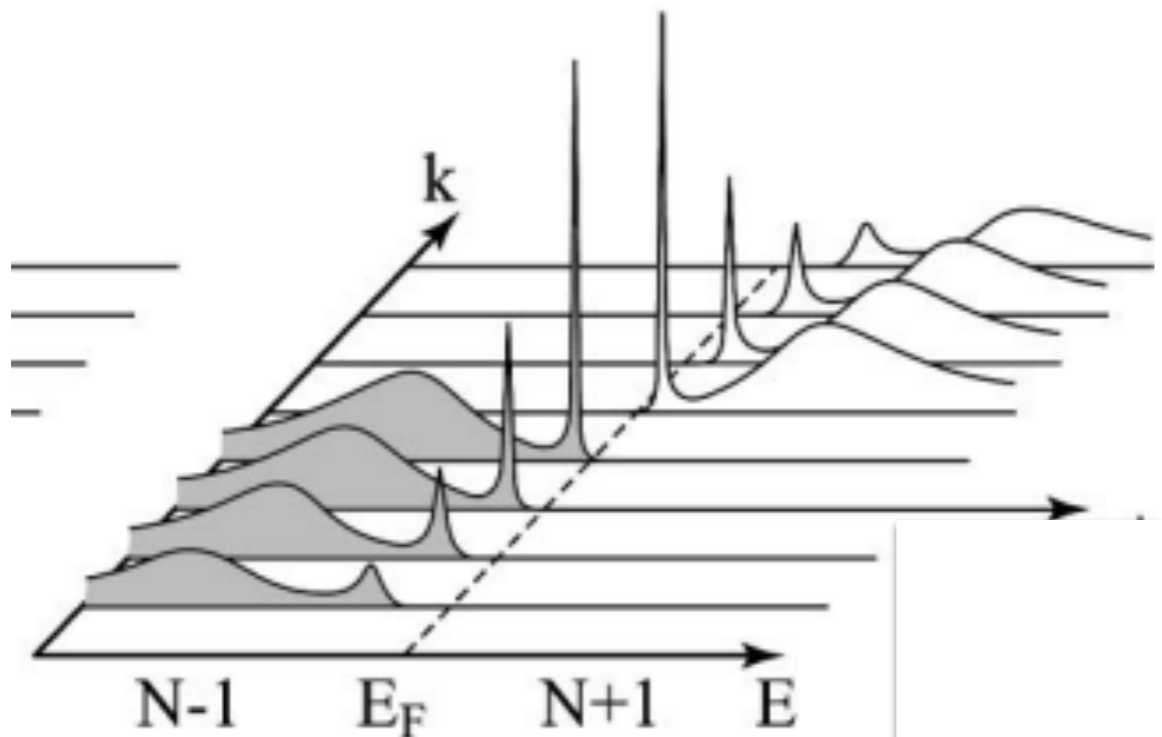
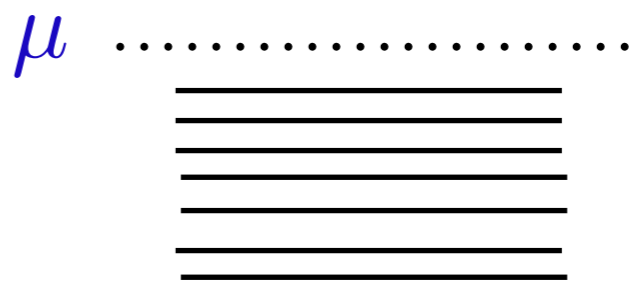
Fermi liquids

NFL

doubly occupied

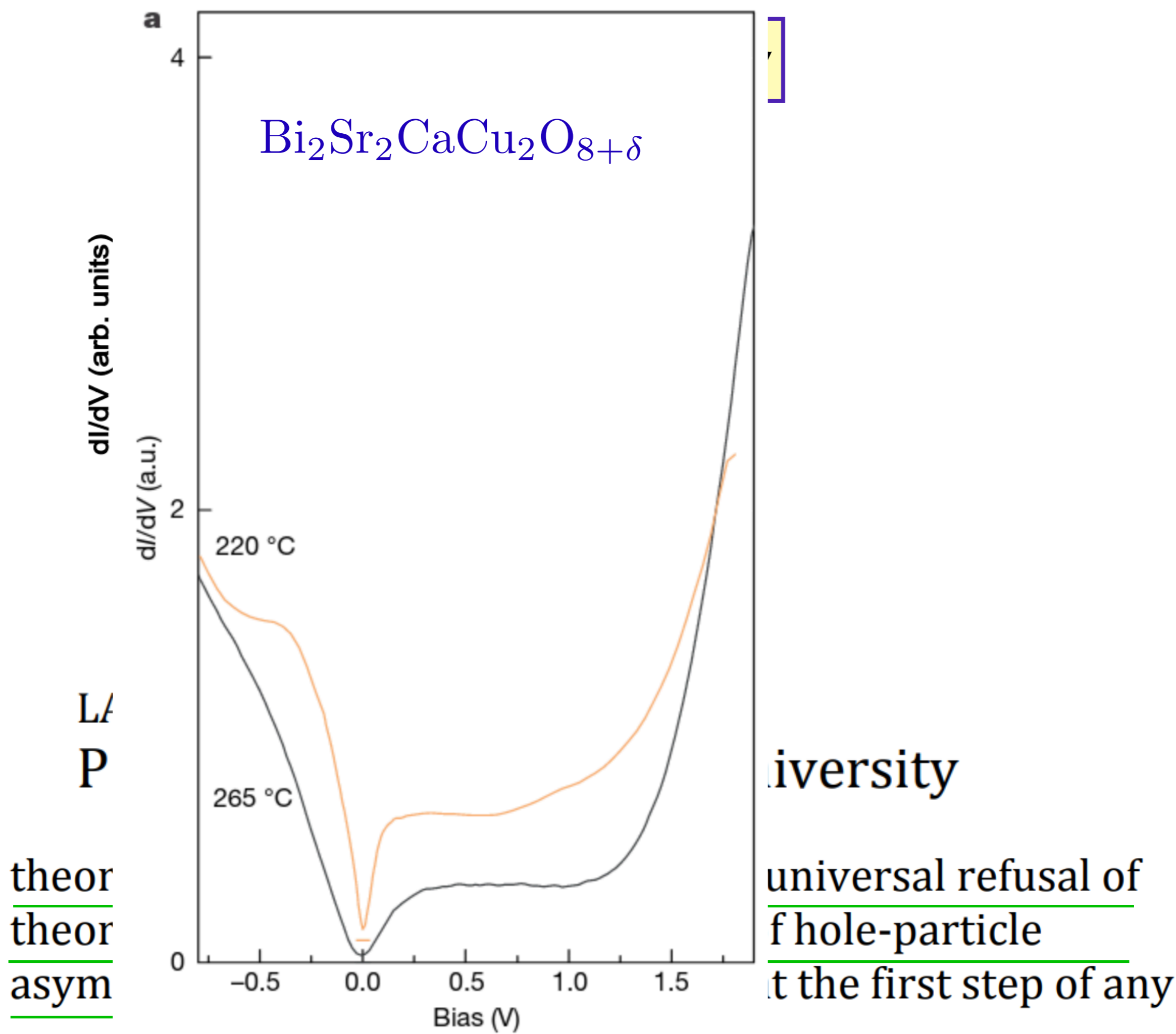


Is single occupancy below chemical potential possible?



Fermi-liquid system
p-h symmetry

with time-reversal symmetry intact?



single
occupancy

?

particle-hole
asymmetry



Anderson
Haldane
2000

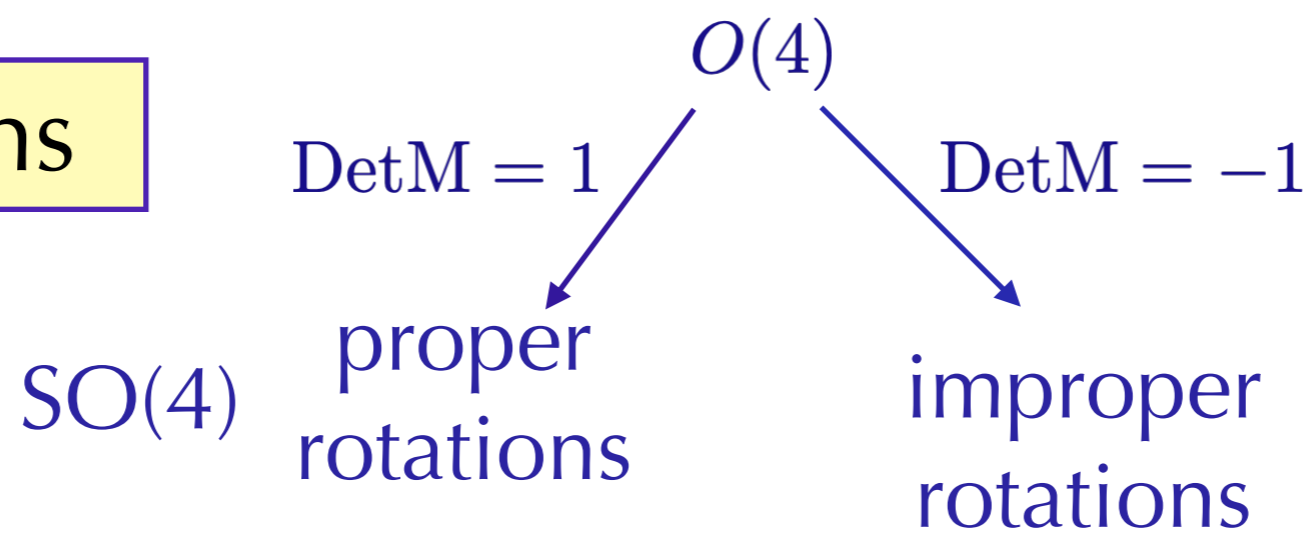
3 citations

Fermi liquids

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F) n_{p\sigma} + \dots \rightarrow 0$$

$(n_{p\uparrow}, n_{p\downarrow})$ conserved currents

$(c_{p\uparrow}, c_{p\downarrow}, \text{h.c.})$ 4 objects



$$\text{Det}M = \pm 1 \implies Z_2 = O(4) \div SO(4)$$

Improper Rotations

Majorana basis

$$\begin{pmatrix} c_{p\uparrow} \\ c_{p\uparrow}^\dagger \\ c_{p\downarrow} \\ c_{p\downarrow}^\dagger \end{pmatrix} \longrightarrow \begin{pmatrix} c_{p\uparrow} + c_{p\uparrow}^\dagger \\ i(c_{p\uparrow} - c_{p\uparrow}^\dagger) \\ c_{p\downarrow} + c_{p\downarrow}^\dagger \\ i(c_{p\downarrow} - c_{p\downarrow}^\dagger) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} c_{p\uparrow} + c_{p\uparrow}^\dagger \\ i(c_{p\uparrow} - c_{p\uparrow}^\dagger) \\ c_{p\downarrow} + c_{p\downarrow}^\dagger \\ i(c_{p\downarrow} - c_{p\downarrow}^\dagger) \end{pmatrix} \longrightarrow c_{p\downarrow} \rightarrow c_{p\downarrow}^\dagger$$

p-h transformation

$$\epsilon(p) = \epsilon_F$$

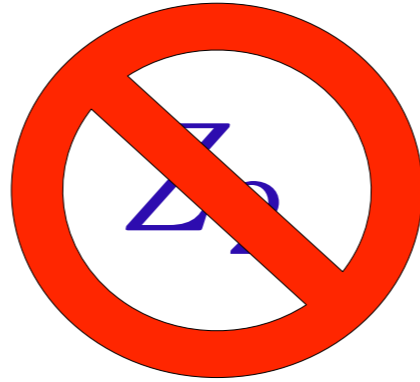
Fermi
Surface

$$H = 0$$



$$\left. \begin{array}{l} n_{p\uparrow} \rightarrow 1 - n_{p\uparrow} \\ n_{p\downarrow} \rightarrow n_{p\downarrow} \end{array} \right\} \mathbb{Z}_2 \text{ at Fermi surface only}$$

How to destroy Fermi liquids?



$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F) n_{p\sigma} + U n_{p\uparrow} n_{p\downarrow}$$

odd
under Z_2

scaling dimension

$$[n_{p\uparrow} n_{p\downarrow}] = -2$$

relevant
interaction

New fixed point!

Hatsugai-Kohmoto
model

Hubbard
not
necessary!

General HK Model

$$\sum_k (\xi_k (n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow})$$

Solvable Mott transition: $U > W$

$$G_{k\sigma}(i\omega_n \rightarrow z) = \frac{1 - \langle n_{k\bar{\sigma}} \rangle}{z - \xi_k} + \frac{\langle n_{k\bar{\sigma}} \rangle}{z - (\xi_k + U)} \neq \frac{1}{z - \omega_k}$$

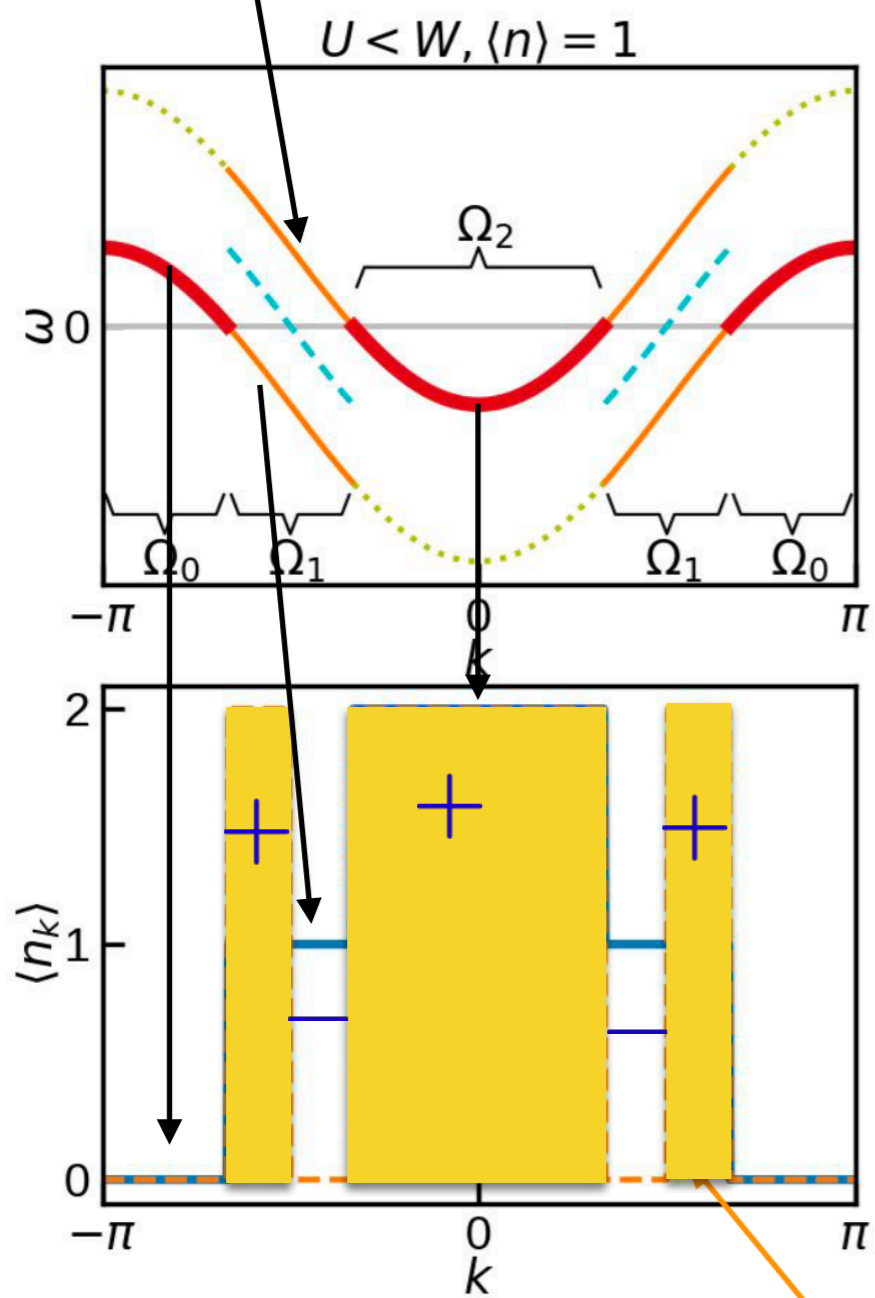
lower Hubbard band

upper Hubbard band

zeros

single occupancy

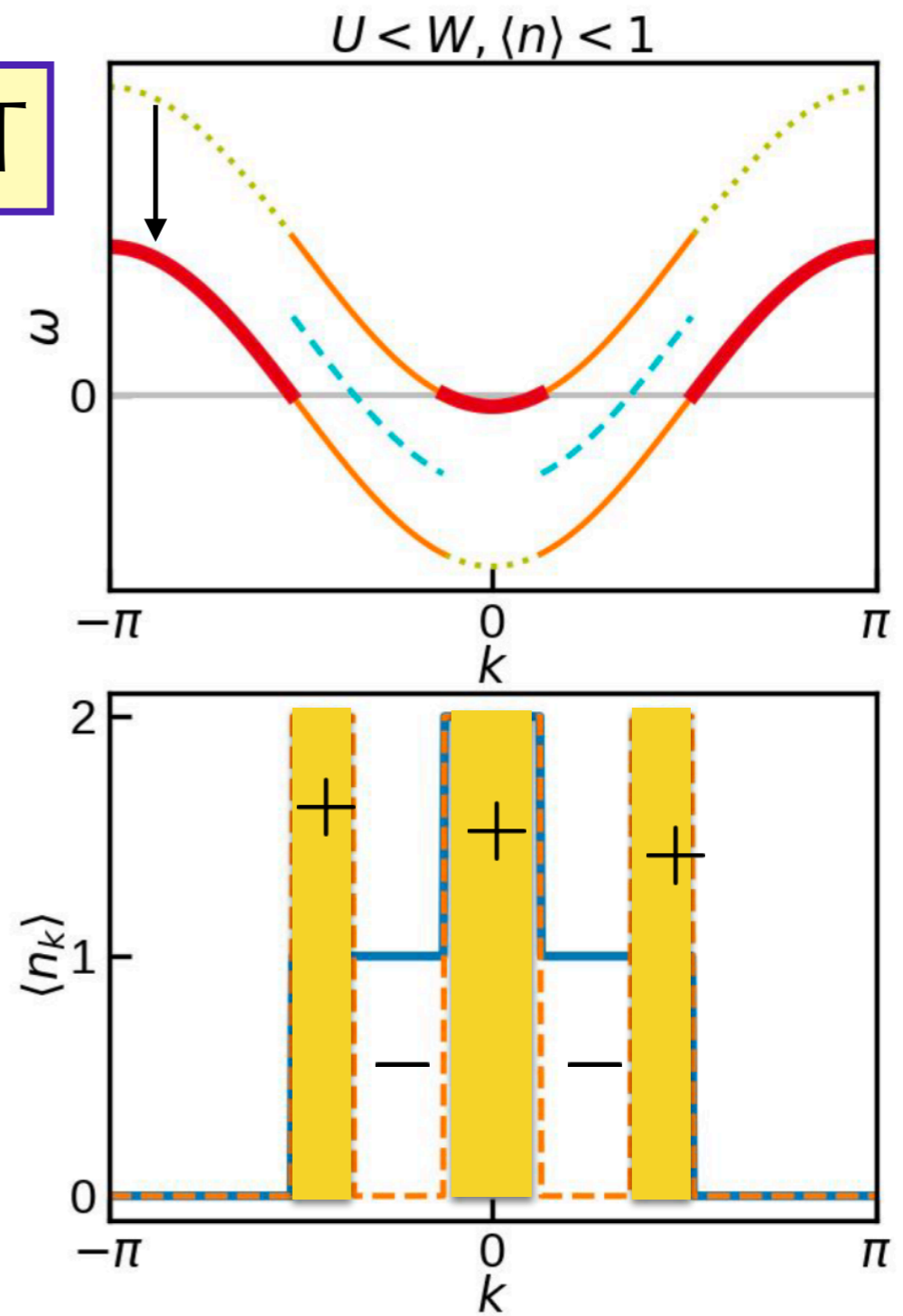
counting charges



$$n_{\text{Lutt}} = \langle n \rangle$$

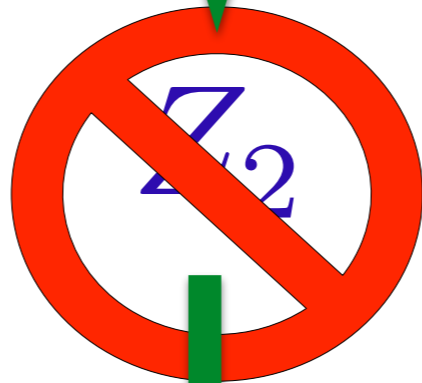
zeros \neq particles

SWT



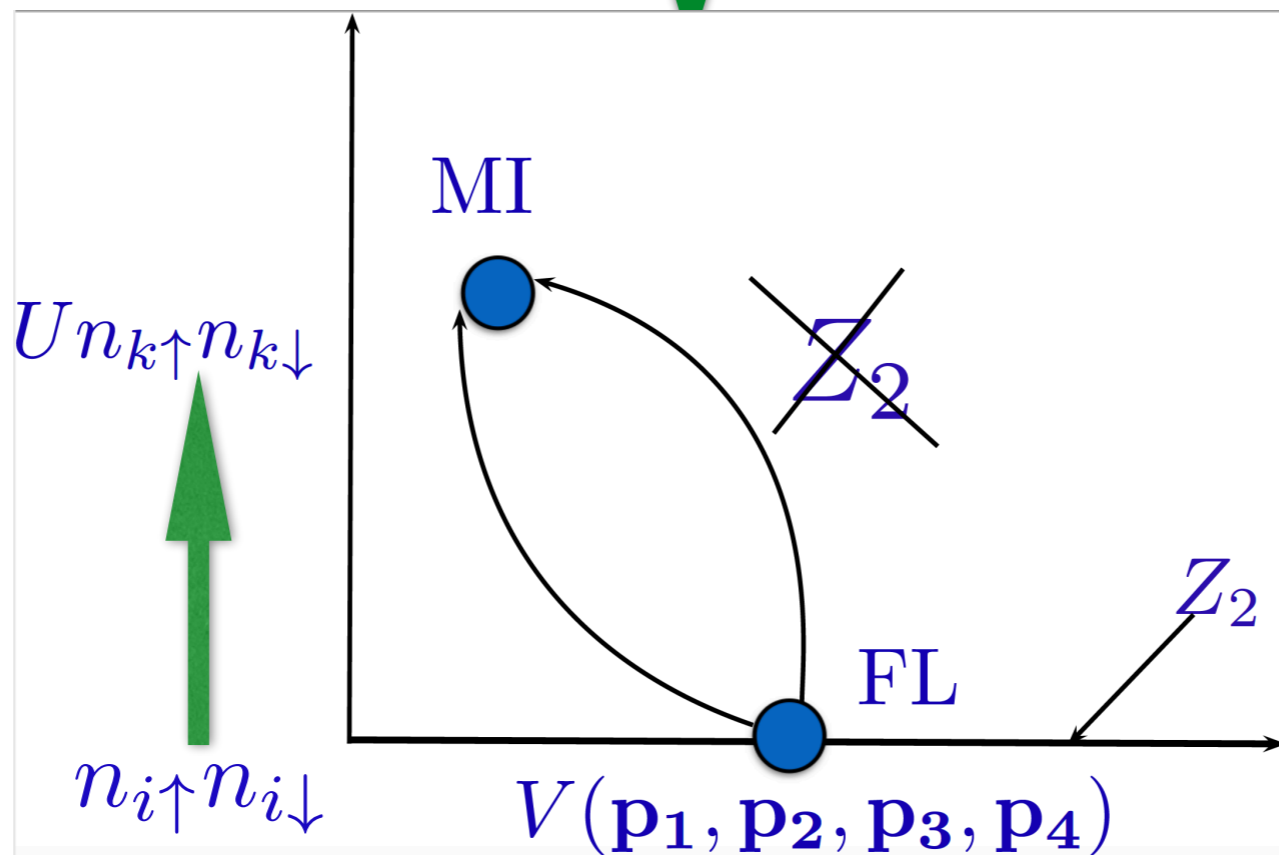
$$n_{\text{Lutt}} \neq \langle n \rangle$$

Fermi liquids



$$[U n_{p\uparrow} n_{p\downarrow}] = -2$$

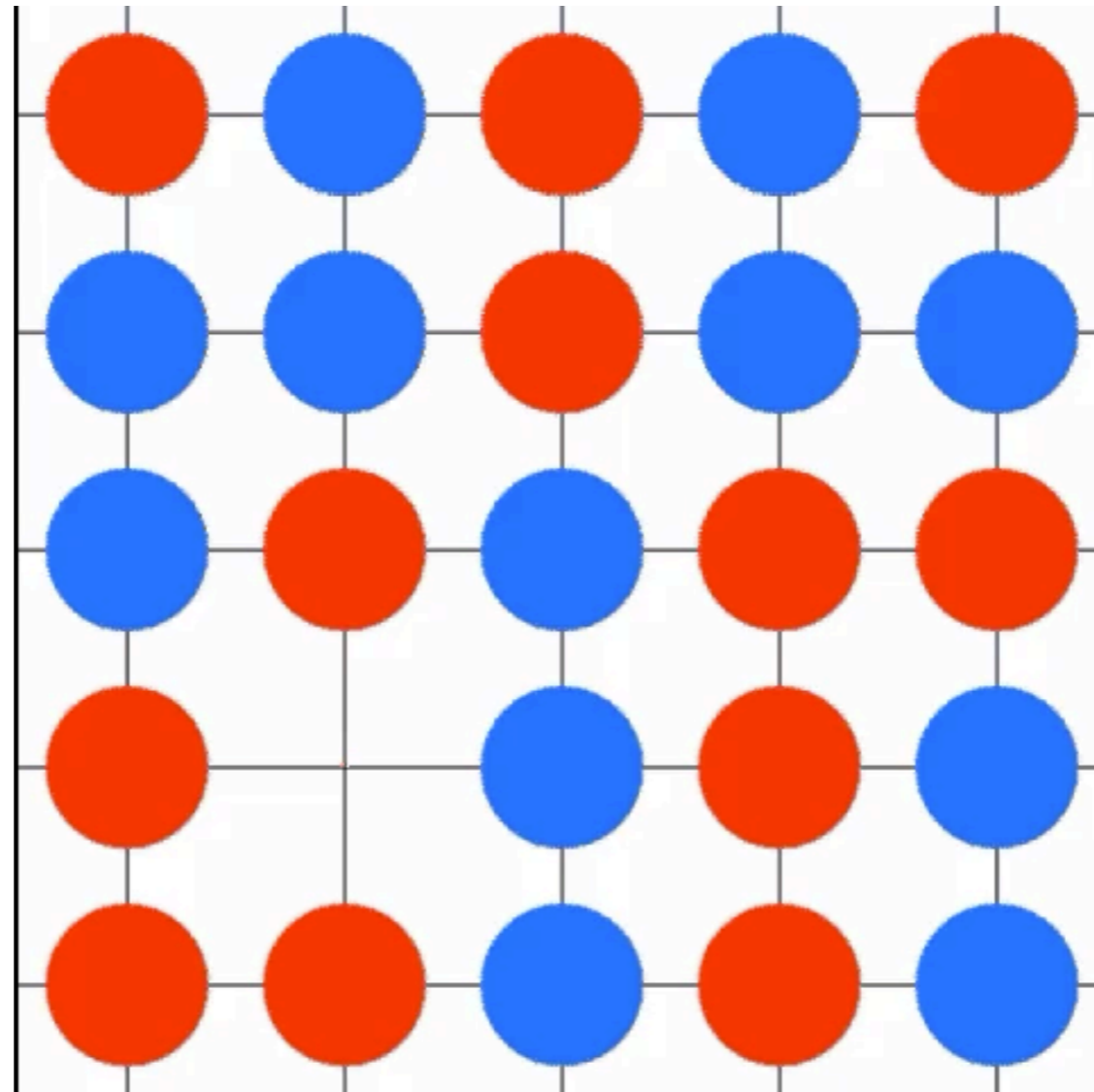
$$\left(\frac{1}{s}\right)^2 \propto \frac{1}{s}$$



Hubbard
not
necessary
(universality
class)

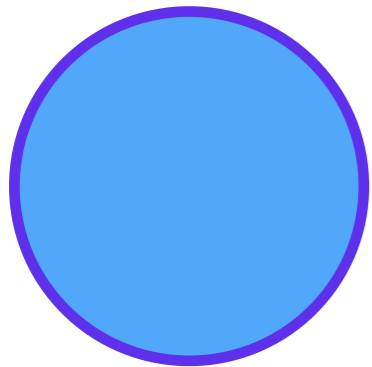
what does the HK model leave out??

$$[H_t, H_U] \neq 0$$

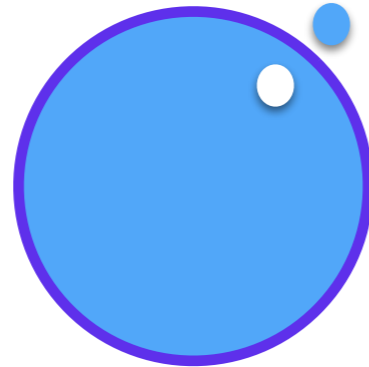


dynamical spectral weight transfer

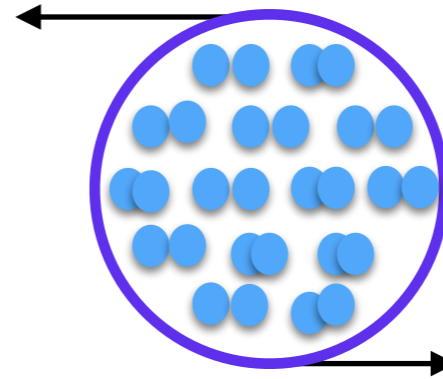
Fermi gas



Fermi liquid



BCS
superconductor



Mottness

2

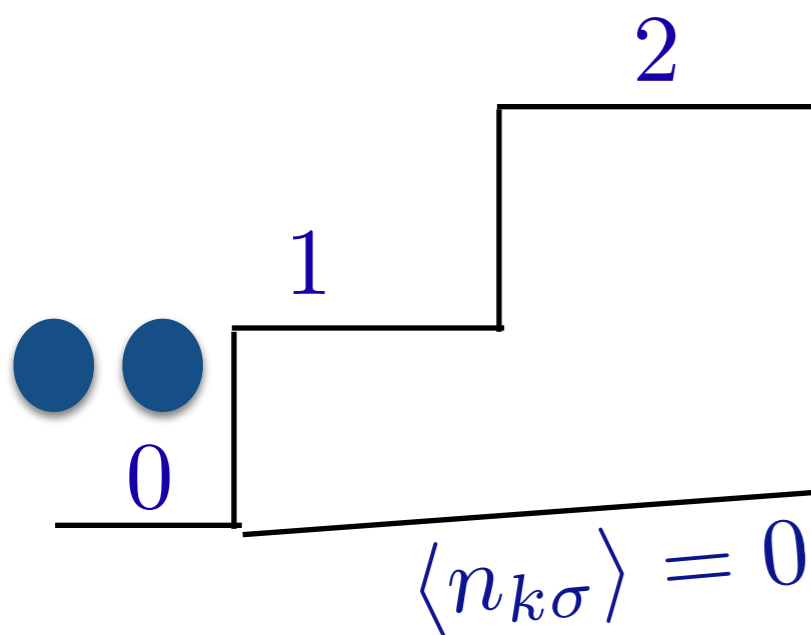
1

0



Superconductivity?

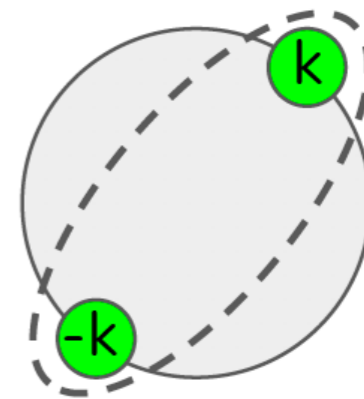
Cooper Instability



$$\langle n_{k\sigma} \rangle = 0$$

$$H = H_{\text{HK}} - gH_p$$

$$|\psi\rangle = \sum_{k \in \Omega_0} \alpha_k b_k^\dagger |\text{GS}\rangle$$

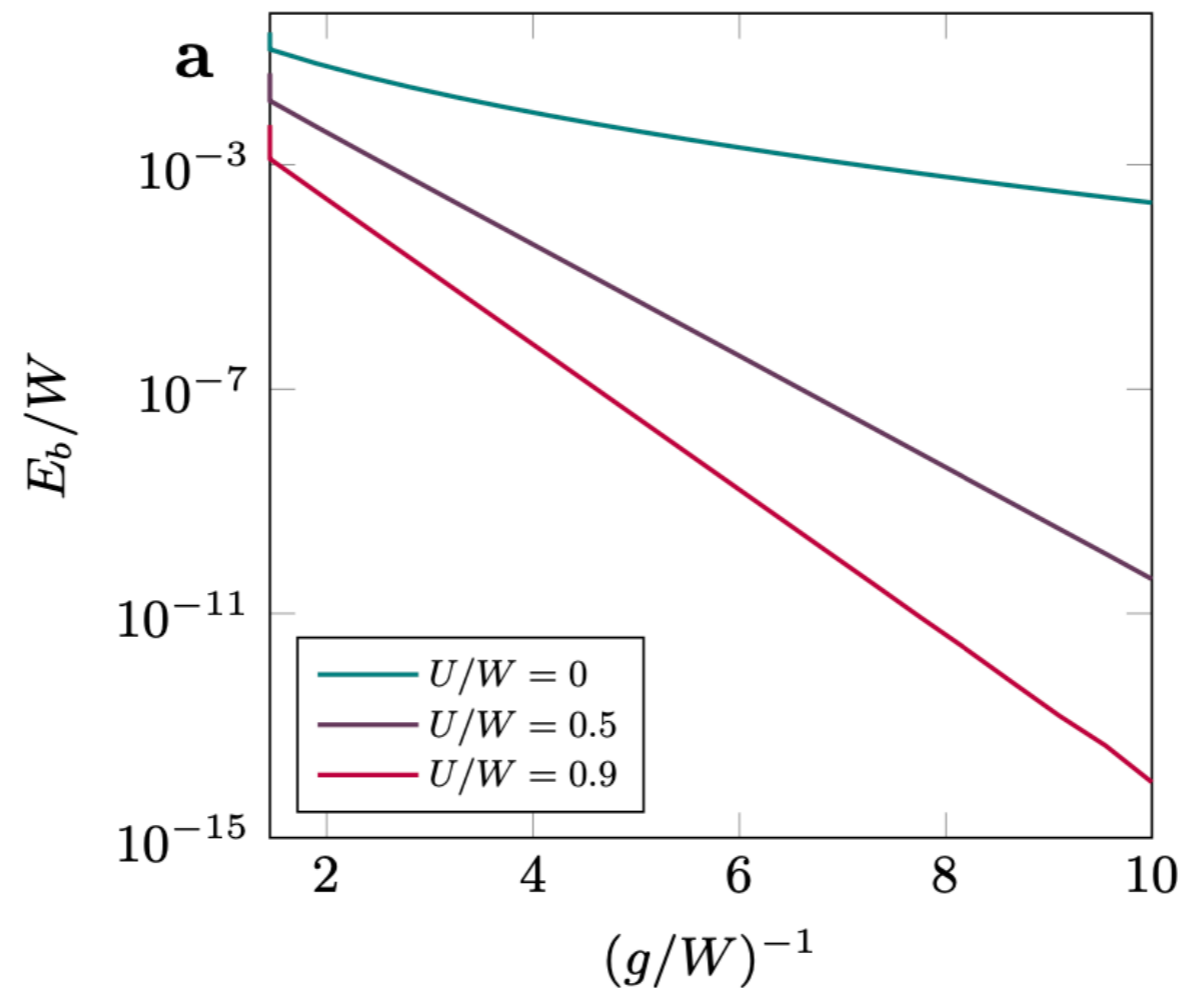


$$= \sum_{k, k'} b_k^\dagger b_{k'}$$

$$E_b = \langle \text{GS} | H | \text{GS} \rangle - \langle \psi | H | \psi \rangle \leq 0$$

Cooper Instability

$$E_b = -E \sim W(1 - (U/W)^2)e^{-\pi W \sqrt{1 - (U/W)^2} / g}$$



Pair Susceptibility

$$\chi(i\nu_n) \equiv \frac{1}{L^d} \int_0^\beta d\tau e^{i\nu_n \tau} \langle T \Delta(\tau) \Delta^\dagger \rangle_g$$

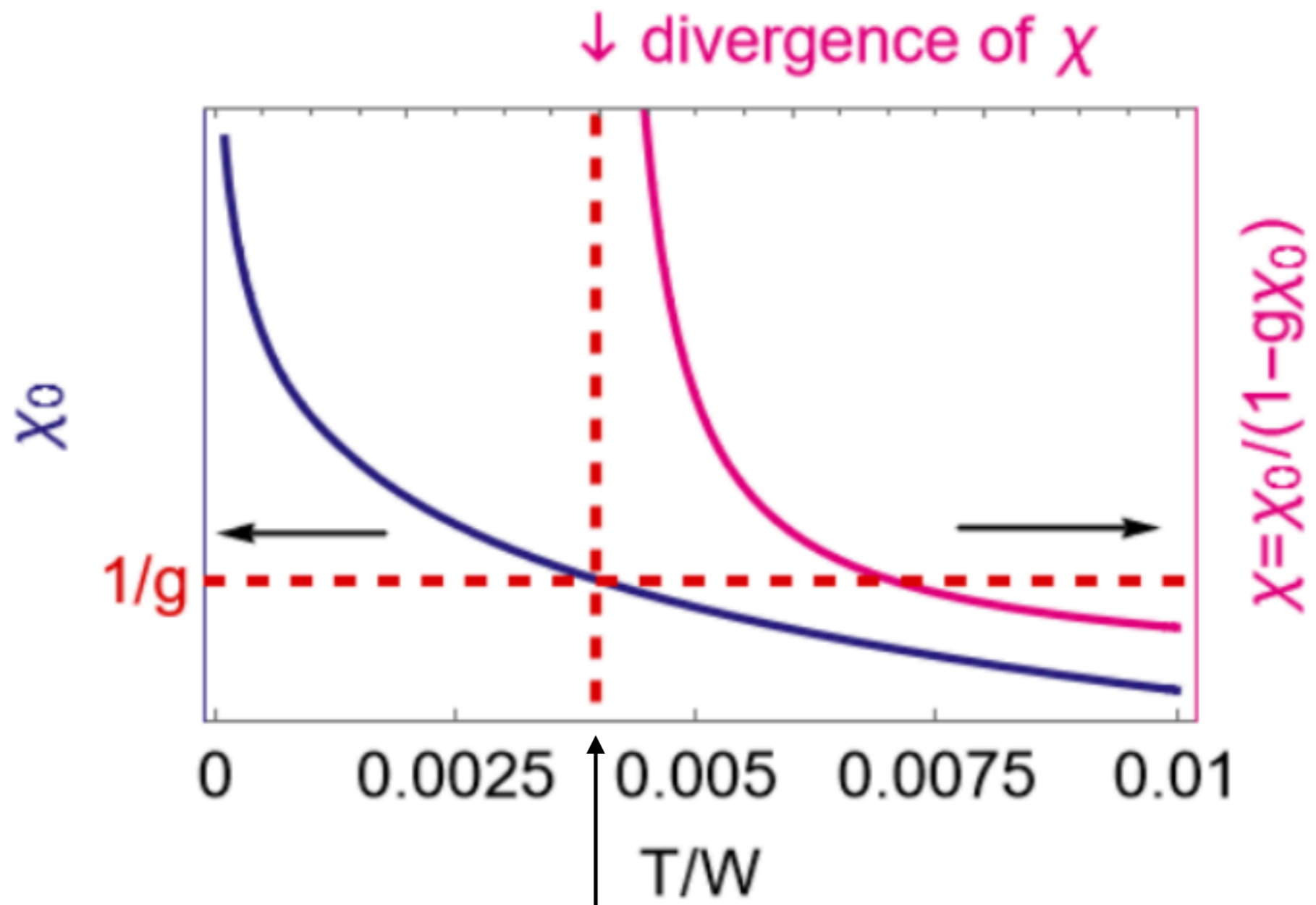


$$= \frac{\chi_0}{1 - g\chi_0}$$



$$g\chi_0 = 1$$

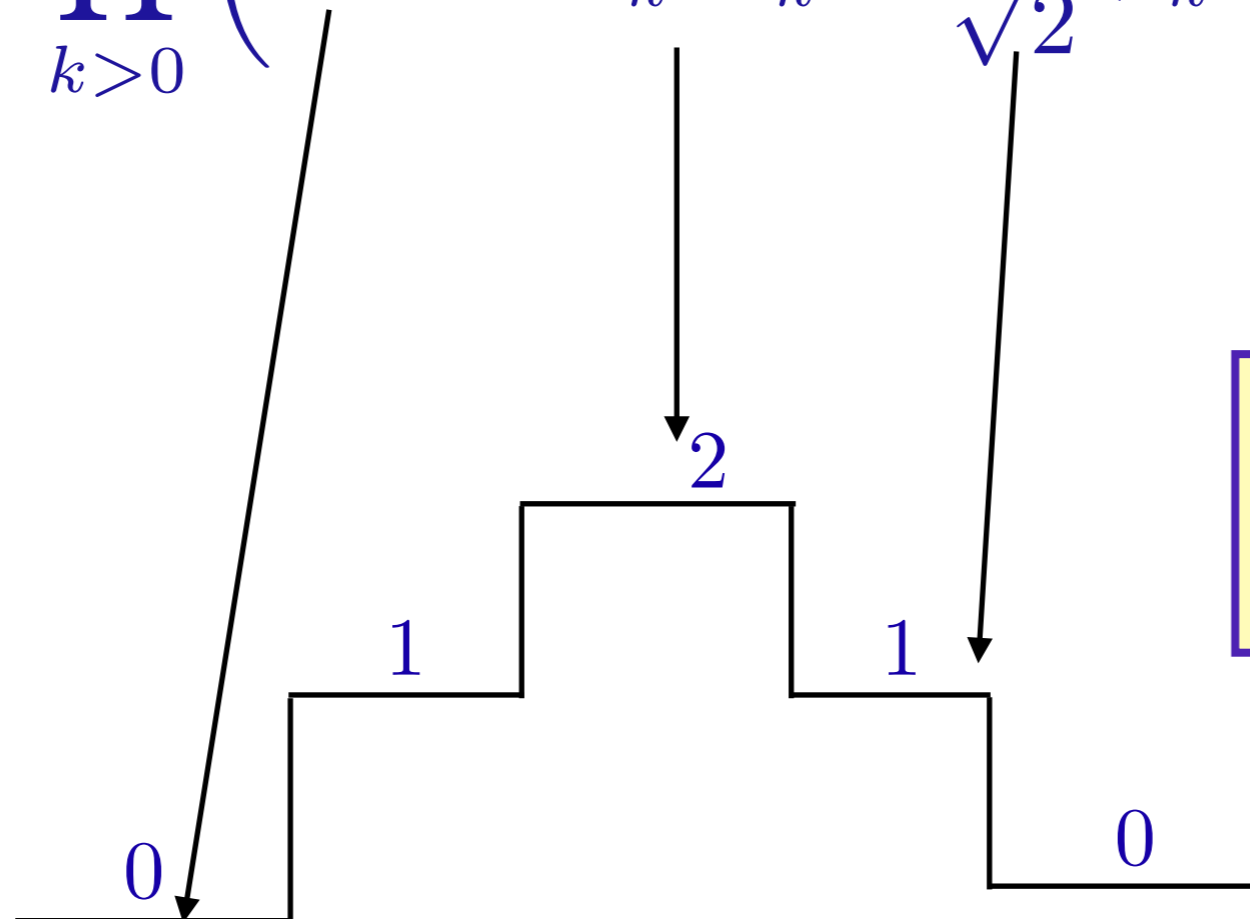
solve for T_c



$$T_c = (W - U)^{4/5} U^{1/5} \frac{e^\gamma}{\pi} e^{-\frac{4}{5} \frac{W}{g}}.$$

variational MF wave function

$$|\psi\rangle = \prod_{k>0} \left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}} (b_k^\dagger + b_{-k}^\dagger) \right) |0\rangle$$



HK
generalization

three variational parameters

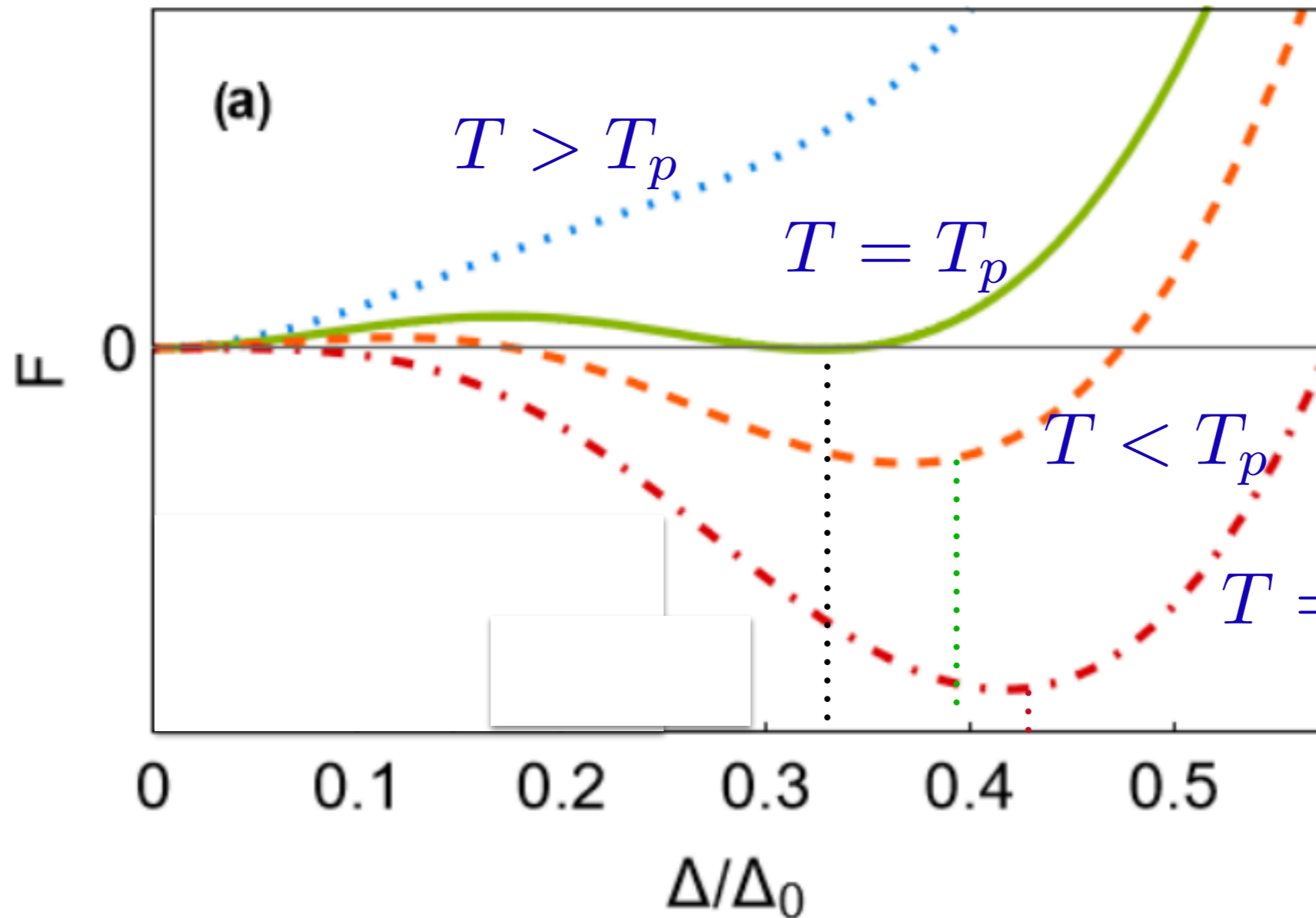
$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation


$$\Delta \ll U, W$$

$$\Delta = (W - U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

compute free energy



$t_G \approx 10^{-11}$

MF theory
is accurate!

$$\lim_{g \rightarrow 0} \frac{\Delta}{T_c} \rightarrow \infty$$

Two-stage superconductivity in the Hatsugai-Kohomoto-BCS model

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¹Kavli Institute for Theoretical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China

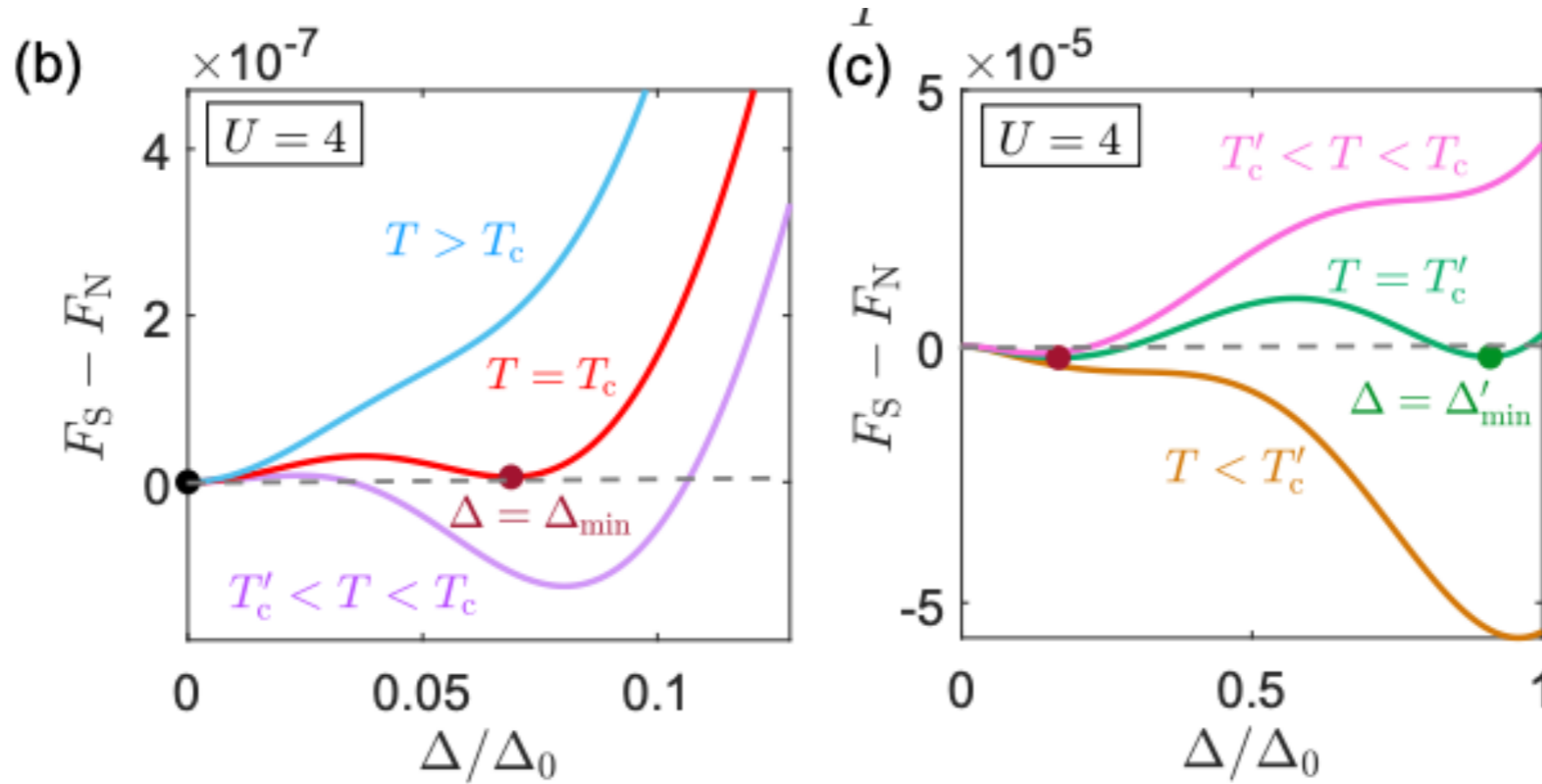
²Beijing National Laboratory for Condensed Matter Physics & Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China

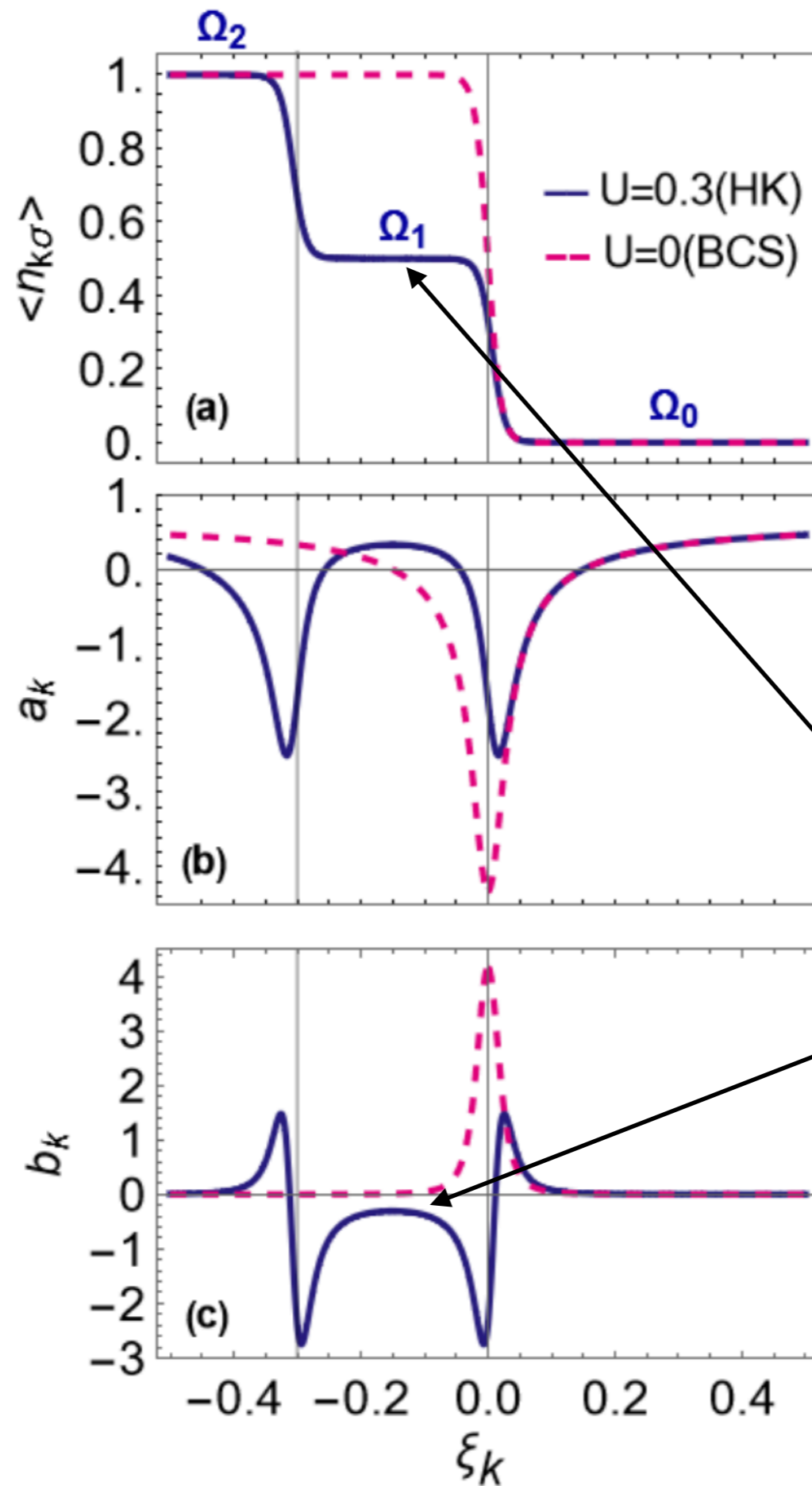
³Songshan Lake Materials Laboratory, Dongguan, Guangdong 523808, China

⁴CAS Center for Excellence in Topological Quantum Computation,
University of Chinese Academy of Sciences, Beijing 100190, China

(Dated: July 7, 2022)

<https://arxiv.org/pdf/2207.01904.pdf>



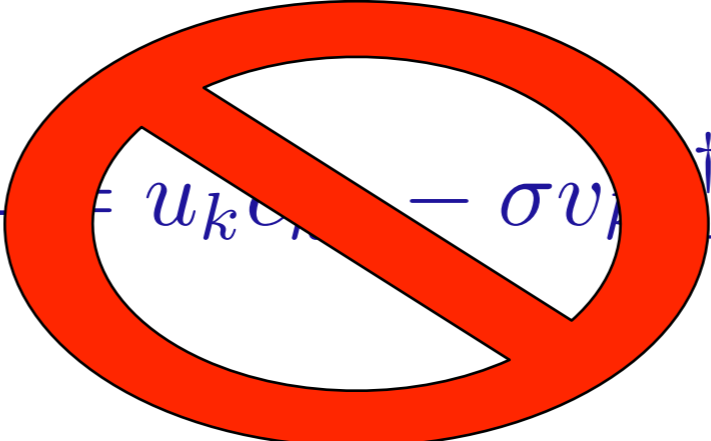


Landau parameters

Mottness

Bogoliubov excitations

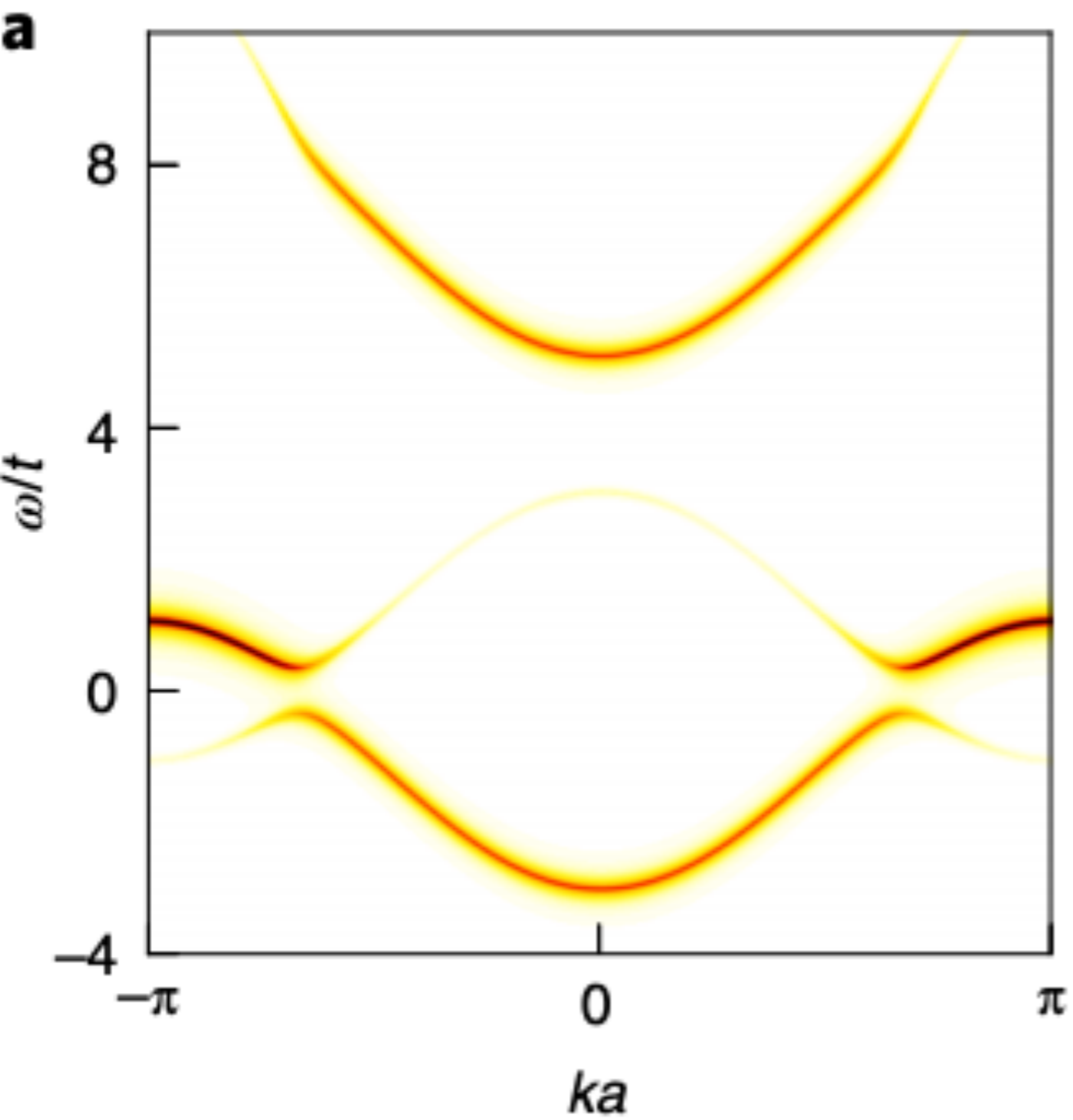
$$\gamma_{k\sigma} |\psi_{\text{BCS}}\rangle = 0$$

$$\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c_{-k\bar{\sigma}}^\dagger$$


PYHons excitations

$$\gamma_{k\sigma}^l \propto \sqrt{2} x_k \zeta_{k\sigma}^\dagger - \sigma z_k \zeta_{-k\bar{\sigma}}$$

$$\gamma_{k\sigma}^u \propto z_k \eta_{k\sigma}^\dagger - \sigma \sqrt{2} y_k \eta_{-k\bar{\sigma}}$$



PYHon band

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

H. J. A. Molegraaf¹, C. Presura¹, D. van der Marel^{1,*}, P. H. Kes², M. Li²

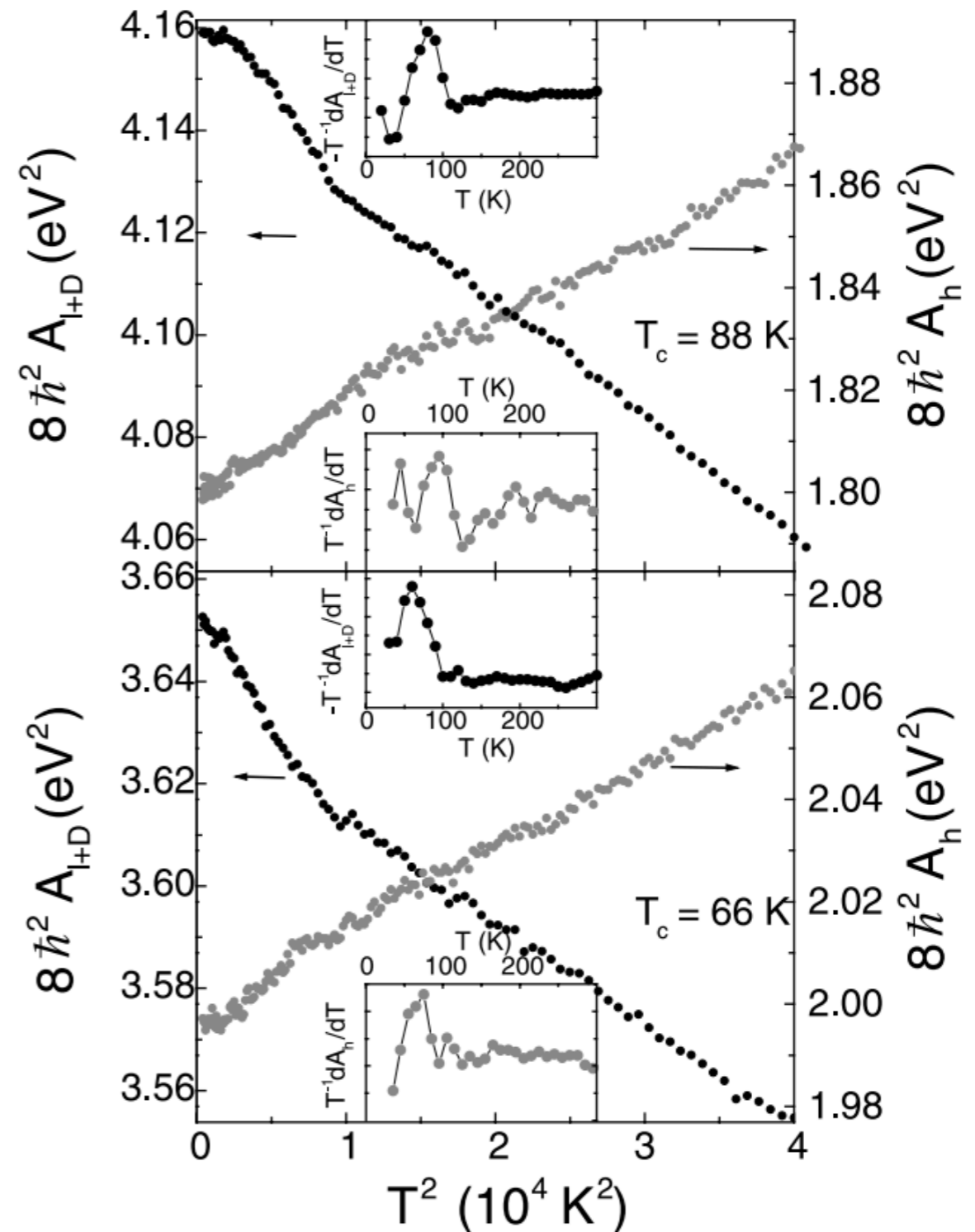
+ See all authors and affiliations

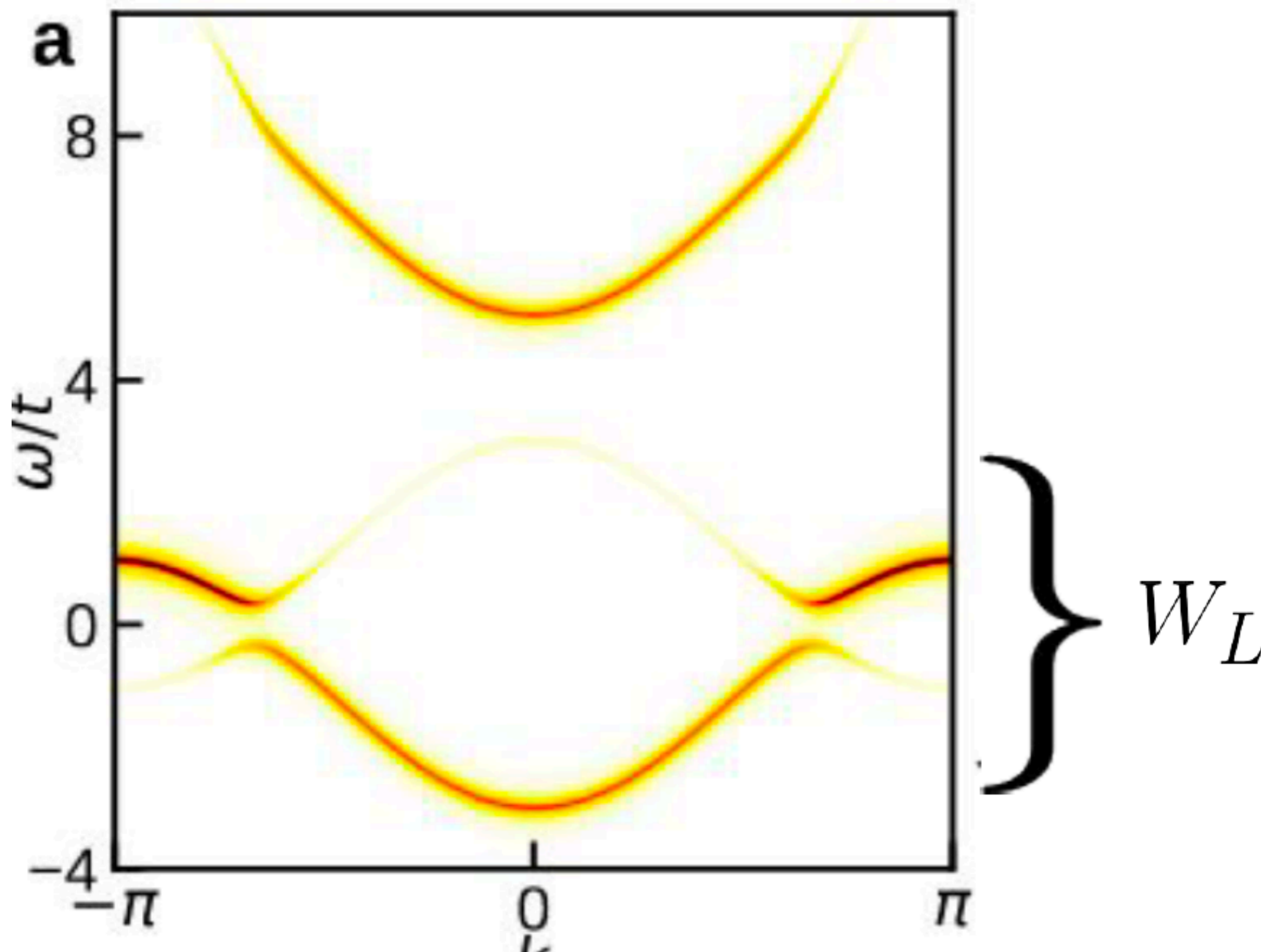
Science 22 Mar 2002:
Vol. 295, Issue 5563, pp. 2239-2241
DOI: 10.1126/science.1069947

$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 \text{ cm}^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 \text{ cm}^{-1}$$

$$\frac{\Delta A_l}{A_l} \propto 3\%$$





why?

$$H = H_{\text{HK}} + H_p$$

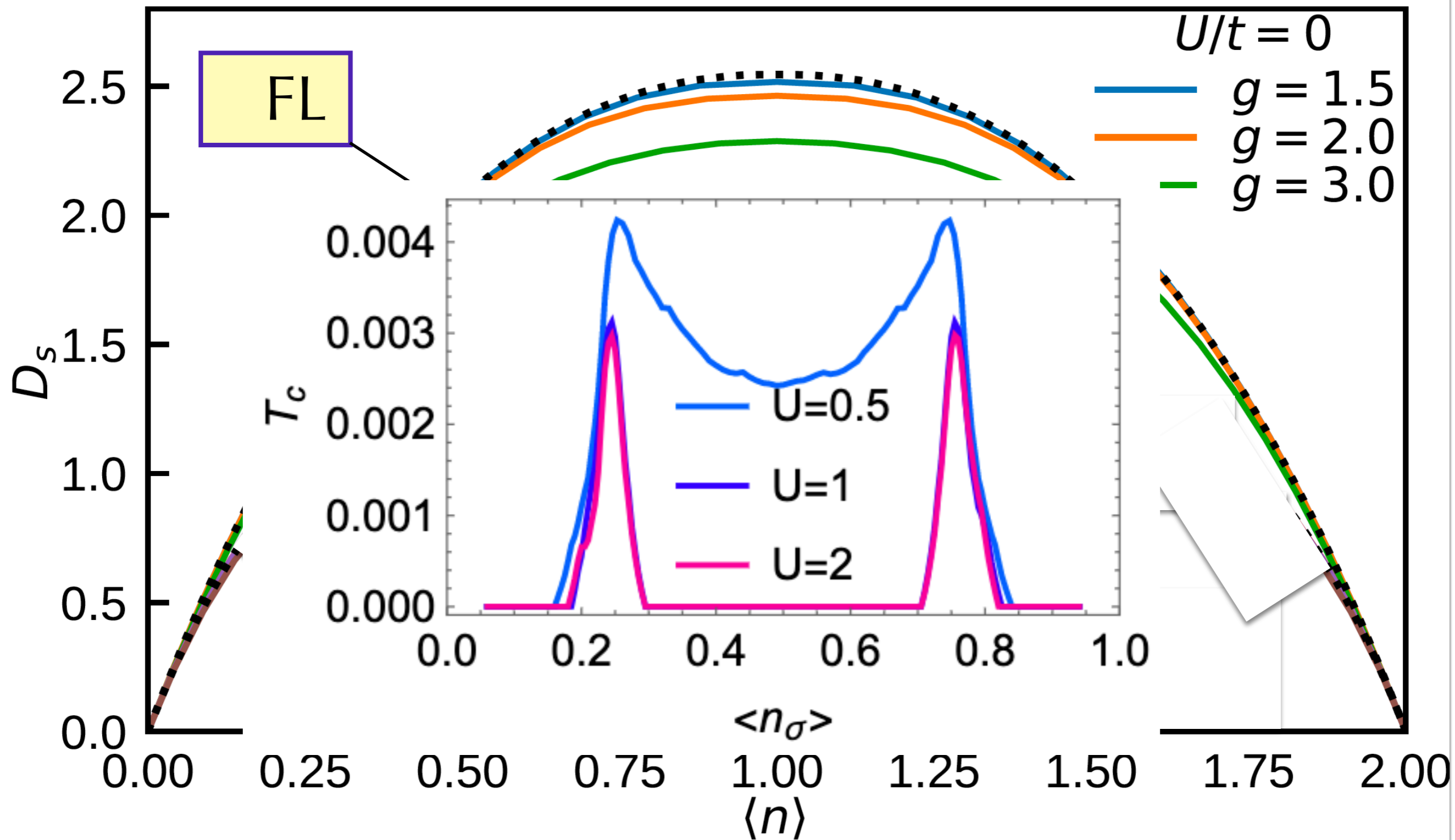
$$[H_{\text{HK}}, H_p] \neq 0$$



dynamical
spectral weight
transfer

Superfluid Density

Mottness-induced suppression



Superconductivity

Mottness

observable

$$\chi \rightarrow \infty$$

$$\Delta \neq 0$$

$$\lim_{g \rightarrow 0} 2\Delta_0/k_B T_c$$

quasi – particles

t_G (Ginzburg)

$$1/TT_1$$

Landau Expansion

$$E_{\text{cond}}/N(0)\Delta^2$$

BCS/FL

$$T_c$$

$$T_c$$

$$3.52$$

Bogoliubons

$$\approx 10^{-12}$$

HS peak

$$a = \alpha t, b > 0$$

$$-1$$

PYHZ/HK

$$T_c (= T_2)$$

$$T_p (> T_2)$$

$$\infty$$

PYHons

$$\approx 10^{-11}$$

no HS peak

$$a = \alpha t, b < 0$$

$$c > 0$$

$$[-2, -1]$$

Topology + Strong Correlations?

Are Exact Statements Possible?

Haldane + HK model



TI

1/4-filled MI

