Failure of Luttinger’s Theorem: Emergence of Unparticles

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How Does Fermi Liquid Theory Breakdown?

- Antiferromagnet
- Pseudogap
- Strange Metal
- Superconductivity
- Fermi Liquid

$T^*$

QCP
How Does Fermi Liquid Theory Breakdown?

Fermi Arcs?

Fermi arcs: (PDJ,JCC,ZXS)
Why are Fermi Arcs Strange?

- **Real FS Crossing**
- **Ghost FS Crossing**
- **Top of Band**
- **FS\text{sym}**

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**Figure:**

- **seen**
- **not seen**

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$E_F$
Problem

\[ \Re G^R > 0 \]

\[ \Re G^R < 0 \]

\[ \Re G < 0 \]

\[ \Re G > 0 \]
Problem

\[ \Re G^R > 0 \]

\( (0, \pi) \) \hspace{1cm} (\pi, \pi) \hspace{1cm} (0, \pi) \hspace{1cm} (\pi, \pi) \hspace{1cm} (0, 0) \hspace{1cm} (\pi, 0) \\
\Re G^R < 0 \hspace{1cm} \text{poles} \hspace{1cm} \Re G < 0 \hspace{1cm} \Re G > 0
Problem

How to account for the sign change without poles?
How to account for the sign change without poles?

Only option: \( \text{DetG}=0! \) (zeros )
Problem

How to account for the sign change without poles?

Only option: DetG=0! (zeros)
Fermi Arcs

text

Luttinger, Dzyaloshinskii, Yang, Rice, Zhang, Tsvelik...

n=zeros + poles
what are zeros?

are they (like poles) conserved?
NiO insulates $d^8$?

Mott mechanism (not Slater)
NiO insulates $d^8$? perhaps this costs energy

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$U \gg t$
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NiO insulates $d^8$?

perhaps this costs energy

$U \gg t$

$\mu = 0$
NiO insulates $d^8$? Perhaps this costs energy.

Mott mechanism (not Slater)

$t$

$U \gg t$

$\mu = 0$

No change in size of Brillouin zone

Thursday, September 12, 13
Mott Problem
Mott Problem

\[ \mu = 0 \]

Im \( G = 0 \)
\[
\text{Re} G(0, p) = \int_{-\infty}^{\infty} \frac{\mu = 0}{d\omega} \quad \text{Kramers-Kronig}
\]

Mott Problem

\[\text{Im } G = 0\]
Mott Problem

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \frac{d}{\omega} \right) \]

\[ \mu = 0 \]

\[ \text{Im} G = 0 \]

Kramers-Kronig

= below gap + above gap
Mott Problem

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \begin{array}{c} \mu = 0 \\ \omega \end{array} \right) d\omega = \text{below gap+above gap} = 0 \]

Kramers-Kronig

Im \( G = 0 \)
Mott Problem

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \begin{array}{c} \mu = 0 \\ \omega \end{array} \right) d\omega \]

= below gap + above gap

= 0

\[ \text{Det} G(k, \omega = 0) = 0 \]

(simple band)
Mott Problem

\( \text{Im } G = 0 \)

\( \text{Det } G(k, \omega = 0) = 0 \) (single band)

\( \text{DetReG}(0, p) = 0 \)  Mottness

\( \text{ReG}(0, p) = \int_{-\infty}^{\infty} \left( \frac{\mu = 0}{\omega} \right) d\omega \)

= below gap + above gap

= 0
Mott Problem

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \begin{array}{c} \mu = 0 \\ \omega \end{array} \right) d\omega \]

= below gap + above gap

\[ \text{Det} G(k, \omega = 0) = 0 \] (single band)

\[ \text{DetRe} G(0, p) = 0 \] Mottness

not true in MF theories
Luttinger theorem: singularities of $\ln G$

$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d p \int_{-\infty}^0 d\xi \ln \frac{G^R(\xi, p)}{G^*_R(\xi, p)}$$

poles+zeros (all sign changes)
Luttinger theorem:
singularities of $\ln G$

$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d p \int_{-\infty}^{0} d\xi \ln \frac{G_R^R(\xi, p)}{G^*_R(\xi, p)}$$

$$n = 2 \sum_k \Theta(\Re G(k, \omega = 0))$$

poles+zeros
(all sign changes)
Luttinger theorem: 
singularities of $\ln G$

$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d p \int_0^0 d\xi \ln \frac{G_R(\xi, p)}{G_R^*(\xi, p)}$$

$$n = 2 \sum_k \Theta(\Re G(k, \omega = 0))$$

poles+zeros 
(all sign changes)

Fermi Liquids  
Mott Insulators
The proof just presented is good for any state of our system: FL, NFL, or MI [in other words, either of poles or zeros can be used to change the sign of $G_r$ in Eq. (1)]. The only way to incapacitate the Luttinger theorem in form (1) is to assume that the limit $T \to 0$ is discontinuous. Actually, one has to require that the whole line $T=0$ is a line of phase transitions.
Is this famous theorem from 1960 correct?
simple problem: $n=1$

$SU(2)$

$U$
simple problem: $n=1$

$SU(2)$

$\mu$

$U$
simple problem: $n=1$

$SU(2)$

- $\mu$
- $-\frac{U}{2}$
- $U$
simple problem: $n=1$
simple problem: n=1

\[ G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} \]
simple problem: $n=1$

\[ G = \frac{1}{\omega + \frac{U}{2}} + \frac{1}{\omega - \frac{U}{2}} = 0 \quad \text{if} \quad \omega = 0 \]
simple problem: $n=1$

\[ SU(2) \]

\[ G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0 \]

\[ n = 2\theta(0) = 1 \]
\[ G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2} \]
\[ n = 2\theta \left( \frac{2\mu}{\mu^2 - \left( \frac{U}{2} \right)^2} \right) \]

\[ G(\omega = 0) = \frac{2\mu}{\mu^2 - \left( \frac{U}{2} \right)^2} \]

A. Rosch, 2007
\[ n = 2\theta \left( \frac{2\mu}{\mu^2 - \left( \frac{U}{2} \right)^2} \right) \]

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\[ G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2} \]

A. Rosch, 2007

\[ \frac{\partial n}{\partial \mu} \neq 0 \]
fix chemical potential

\[ \lim_{T \to 0} \mu(T) \]
fix chemical potential

\[ \lim_{T \to 0} \mu(T) \]

n=1
fix chemical potential

\[ \lim_{T \to 0} \mu(T) \]

n=1

does this fix all the problems?
No
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails

no hopping => no propagation (zeros)
A model with zeros but Luttinger fails

no hopping => no propagation (zeros)
$SU(N)$

\[ H = \frac{U}{2} (n_1 + \cdots n_N)^2 \]
no particle-hole symmetry

a) \( \frac{9}{2} U \)  

b) \( \frac{3}{2} U \)  

2U  

\( \frac{1}{2} U \)  

\( \frac{5}{2} U \)  

\( \omega \)
no particle-hole symmetry

\[ \lim_{T \to 0} \mu(T) \]
\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]
\[ G_{\alpha \beta} (\omega = 0) = \frac{\delta_{\alpha \beta}}{K(n+1) - K(n)} \left( \frac{2n - N}{N} \right) \]

\[ > 0 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\( 0, 1, 1/2 \)
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ \begin{align*}
  n &= 2 \\
  N &= 3 \\
  &\quad 0, 1, 1/2
\end{align*} \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[
\begin{align*}
  n & = 2 \\
  N & = 3 \\
  2 & = 3
\end{align*}
\]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \quad N = 3 \quad 0, 1, 1/2 \]
Luttinger’s theorem

\[ n = N \Theta (2n - N) \]

even

\[ n = 2 \]

\[ N = 3 \]

\[ 0, 1, 1/2 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\( n = 2 \)

\( N = 3 \)

0, 1, 1/2

\text{even}

\text{odd}
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

- even
- odd

no solution
Problem

G=0
Problem

$G = 0$

$$G = \frac{1}{E - \varepsilon_p - \Sigma}$$
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

lifetime of a particle vanishes
Problem

G = 0

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

\[ \sum \Sigma < \varepsilon_p \]

\[ \therefore \text{lifetime of a particle vanishes} \]
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

\[ \sum \sum < \varepsilon_p \]

Lifetime of a particle vanishes

\[ \infty \]

No particle
what went wrong?
what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$
what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$

if $\Sigma \to \infty$
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)

integral does not exist
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)

integral does not exist

No Luttinger theorem!
Luttinger’s theorem
experimental confirmation of violation?
zeros do not affect the particle density
zeros do not affect the particle density

Experimental data (LSCO)

$L_k F$

$1 - x_{FS}$

`Luttinger` count

Bi2212

$0.3$

$0.2$

$0.1$

$0.0$

$0.3$

$0.2$

$0.1$

$0.0$

$x_{FS}$

$x$

Yang et al. (2011)

He et al. (2011)
zeros do not affect the particle density

`Luttinger’ count

Bi2212

experimental data (LSCO)

\( k_F \)

1 \(- x_{FS} \)

\( x = x_{FS} \)

Each hole \( \neq \) a single k-state
how to count particles?
how to count particles?

some charged stuff has no particle interpretation
what is the extra stuff?
\[ \Sigma(\omega = 0, p) = 0 \]
Fermi liquid

\[ \Sigma(\omega = 0, p) = \infty \]
new fixed point
$\Sigma(\omega = 0, \mathbf{p}) = 0$
Fermi liquid

$\Sigma(\omega = 0, \mathbf{p}) = \infty$
new fixed point

scale invariance
strongly correlated matter

new fixed point (scale invariance)
strongly correlated matter

new fixed point (scale invariance)
strongly correlated matter

new fixed point (scale invariance)

unparticles (IR) (H. Georgi)
what’s the underlying theory?
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]
\[ \phi(x) \rightarrow \phi(x) \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \to x/\Lambda \]
\[ \phi(x) \to \phi(x) \]

mass
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

- \( x \rightarrow x/\Lambda \)
- \( \phi(x) \rightarrow \phi(x) \)
- \( m^2 \phi^2 \)
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\( \Lambda^2 \left( \frac{1}{2} \partial_\mu \phi \partial_\mu \phi \right) \)

mass

\( x \rightarrow x/\Lambda \)

\( \phi(x) \rightarrow \phi(x) \)

\( m^2 \phi^2 \)
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\( x \to x/\Lambda \)
\( \phi(x) \to \phi(x) \)

no scale invariance

mass
\[ \mathcal{L} = \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial^{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!
\[ L = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[
\phi \rightarrow \phi(x, m^2 / \Lambda^2) \\
x \rightarrow x / \Lambda \\
m^2 / \Lambda^2 \rightarrow m^2
\]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2/\Lambda^2) \]
\[ x \rightarrow x/\Lambda \]
\[ m^2/\Lambda^2 \rightarrow m^2 \]

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!
\[
\mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2
\]

theory with all possible mass!

\[
\phi \to \phi(x, m^2 / \Lambda^2)
\]

\[
x \to x / \Lambda
\]

\[
m^2 / \Lambda^2 \to m^2
\]

\[
\mathcal{L} \to \Lambda^4 \mathcal{L}
\]

scale invariance is restored!!

not particles
unparticles

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!

\[ \phi \to \phi(x, m^2/\Lambda^2) \]
\[ x \to x/\Lambda \]
\[ m^2/\Lambda^2 \to m^2 \]
\[ \mathcal{L} \to \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
\[ \left( \int_{0}^{\infty} dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|} \]
\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^2 |\gamma| 
\]

\[d_U - 2\]

propagator
\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|}
\]

\[d_U - 2\]
$n$ massless particles

$G \propto (E - \varepsilon_p)^{n-2}$
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U - 2} \]
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U-2} \]

unparticles = fractional number of massless particles
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U - 2} \]

unparticles = fractional number of massless particles

\( d_U < 2 \)

almost Luttinger liquid
no pole at \( p_F \)
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U - 2} \]

unparticles = fractional number of massless particles

\[ d_U > 2 \]

zeros

\[ d_U < 2 \]

almost Luttinger liquid

no pole at p_F
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U-2} \]

unparticles = fractional number of massless particles

- \( d_U > 2 \):
  - zeros
  - no simple sign change

- \( d_U < 2 \):
  - almost Luttinger liquid
  - no pole at \( p_F \)
$dU$?
what really is the summation over mass?
what really is the summation over mass?

mass=energy
high energy (UV)

low energy (IR)

related to sum over mass

QFT
high energy (UV)

low energy (IR)

\[ \beta(g(E)) = \frac{dg(E)}{dlnE} \]

locality in energy

related to sum over mass

QFT
implement E-scaling with an extra dimension

$\frac{dg(E)}{d\ln E} = \beta(g(E))$

related to sum over mass

locality in energy

high energy (UV)

QFT

low energy (IR)
implement E-scaling with an extra dimension

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]

QFT

related to sum over mass

locality in energy
gauge-gravity duality
(Maldacena, 1997)

implement E-scaling with an extra dimension

\[ \frac{dg(E)}{d \ln E} = \beta(g(E)) \]

locality in energy

related to sum over mass
Implement E-scaling with an extra dimension

gauge-gravity duality
(Maldacena, 1997)

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]

locality in energy

related to sum over mass

no particles (conserved currents)
gauge-gravity duality (Maldacena, 1997)

implement E-scaling with an extra dimension

$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

related to sum over mass

locality in energy

no particles (conserved currents)

unparticles?
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} dm^2 \]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

\[ m = z^{-1} \]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

\[ m = z^{-1} \]

\[ \mathcal{L} = \int_0^\infty \, dz \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right] \]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

\[ m = z^{-1} \]

\[ \mathcal{L} = \int_0^\infty \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right] \]

\text{can be absorbed with AdS metric}
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} dm^2 \]

\[ m = z^{-1} \]

\[ \mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right] \]

**can be absorbed with AdS metric**

\[ ds^2 = \frac{L^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right) \]
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial_{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^{2} \phi^{2}(x, m) \right) m^{2\delta} \, dm^{2} \]

\[ m = z^{-1} \]

\[ \mathcal{L} = \int_{0}^{\infty} \, dz \frac{2R^{2}}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^{2}}{R^{2}} \eta^{\mu\nu} (\partial_{\mu} \phi) (\partial_{\nu} \phi) + \frac{\phi^{2}}{2R^{2}} \right] \]

can be absorbed with AdS metric

\[ ds^{2} = \frac{L^{2}}{z^{2}} \left( \eta_{\mu\nu} \, dx^{\mu} \, dx^{\nu} + dz^{2} \right) \]
\[ m_{\text{AdS}}^2 = \frac{d_U (d_U - d)}{R^2} \]
\[ m_{\text{AdS}}^2 = \frac{d_U (d_U - d)}{R^2} \]

\[ m = 1/L \]
\[ m^2_{\text{AdS}} = \frac{d_\mathcal{U}(d_\mathcal{U} - d)}{R^2} \]

\[ m = 1/L \]

\[ \frac{1}{R^2} = \frac{d_\mathcal{U}(d_\mathcal{U} - d)}{R^2} \]
\[ m_{\text{AdS}}^2 = \frac{d_U(d_U - d)}{R^2} \]

\[ m = \frac{1}{L} \]

\[ \frac{1}{R^2} = \frac{d_U(d_U - d)}{R^2} \]

\[ d_U = \frac{d}{2} + \frac{\sqrt{d^2 + 4}}{2} > \frac{d}{2} \]
unparticle (AdS) propagator has zeros!

\[ G_U(p) \propto p^{2(d_U - d/2)} \]

\[ d_U = \frac{d}{2} + \frac{\sqrt{d^2 + 4}}{2} > \frac{d}{2} \]

\[ G_U(0) = 0 \]
Superconducting Instability

ladder approximation

$$1 = \lambda T \sum_{n\vec{k}} |w_{n\vec{k}}|^2 G_U (\omega_n, \vec{k}) G_U (\omega_n, -\vec{k}),$$


\[ \frac{d \ln g}{d \ln \beta} = 4d_U - d > 0 \]

\( T_c \) unparticles

\( g \)

\textit{tendency towards pairing (any instability which establishes a gap)}
interchanging unparticles

fractional (d_U) number of massless particles
interchanging unparticles

fractional \((d_{U})\) number of massless particles
interchanging unparticles

fractional (d_U) number of massless particles
interchanging unparticles

fractional \( d_U \) number of massless particles
interchanging unparticles

fractional \( (d_U) \) number of massless particles

\[ e^{i\pi d_U} \neq -1, 0 \]
interchanging unparticles

fractional \((d_U)\) number of massless particles

\[ e^{i\pi d_U} \neq -1, 0 \]

fractional statistics in \(d=2+1\)
\[ d_U = \frac{d}{2} + \frac{\sqrt{d^2 + 4}}{2} > \frac{d}{2} \]

\[ e^{i\pi d_U} \neq e^{-i\pi d_U} \]

**time-reversal symmetry breaking from unparticle (zeros=Fermi arcs) matter**
unparticles

particles
breaking of scale invariance

unparticles

particles

TRSB

Antiferromagnet

Superconductor

Fermi Liquid

QCP