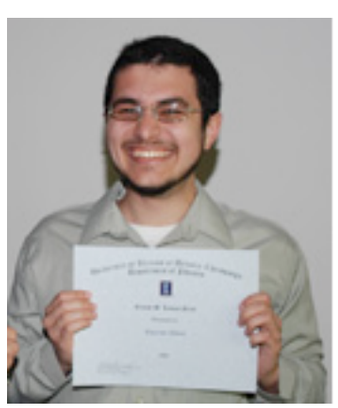
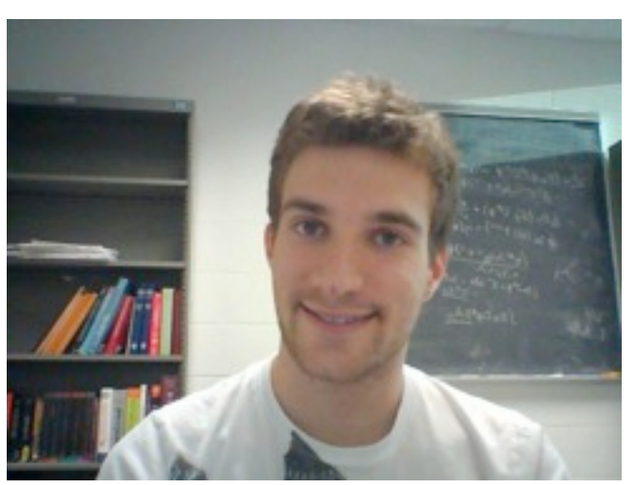
Failure of Luttinger's Theorem: Emergence of Unparticles

Thanks to: NSF, EFRC (DOE)



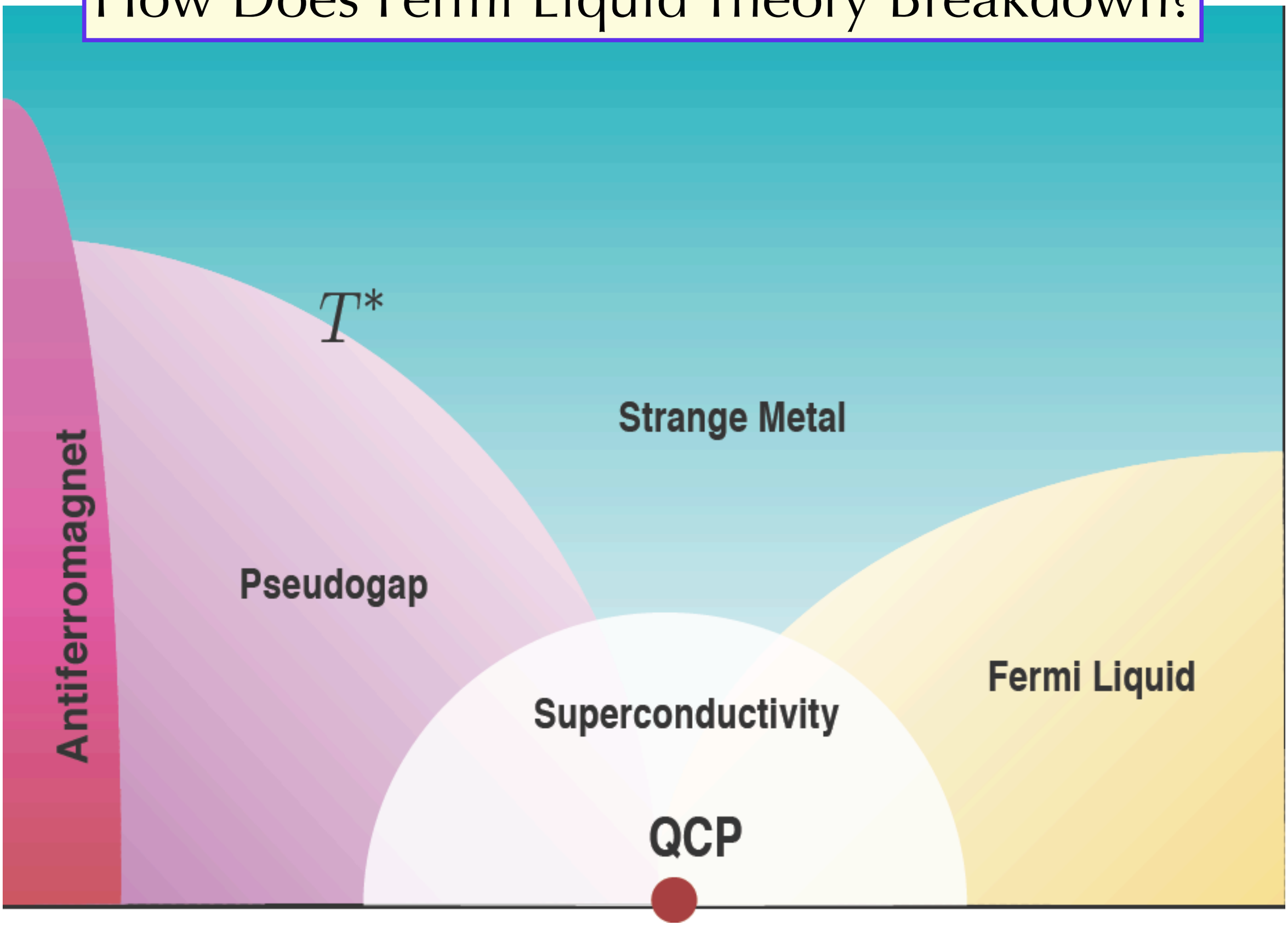


Kiaran Dave Charlie Kane Brandon Langley J. A. Hutasoit

PRL, 110, 090403 (2013)

arxiv:1305.0006 (PRB,xxx)

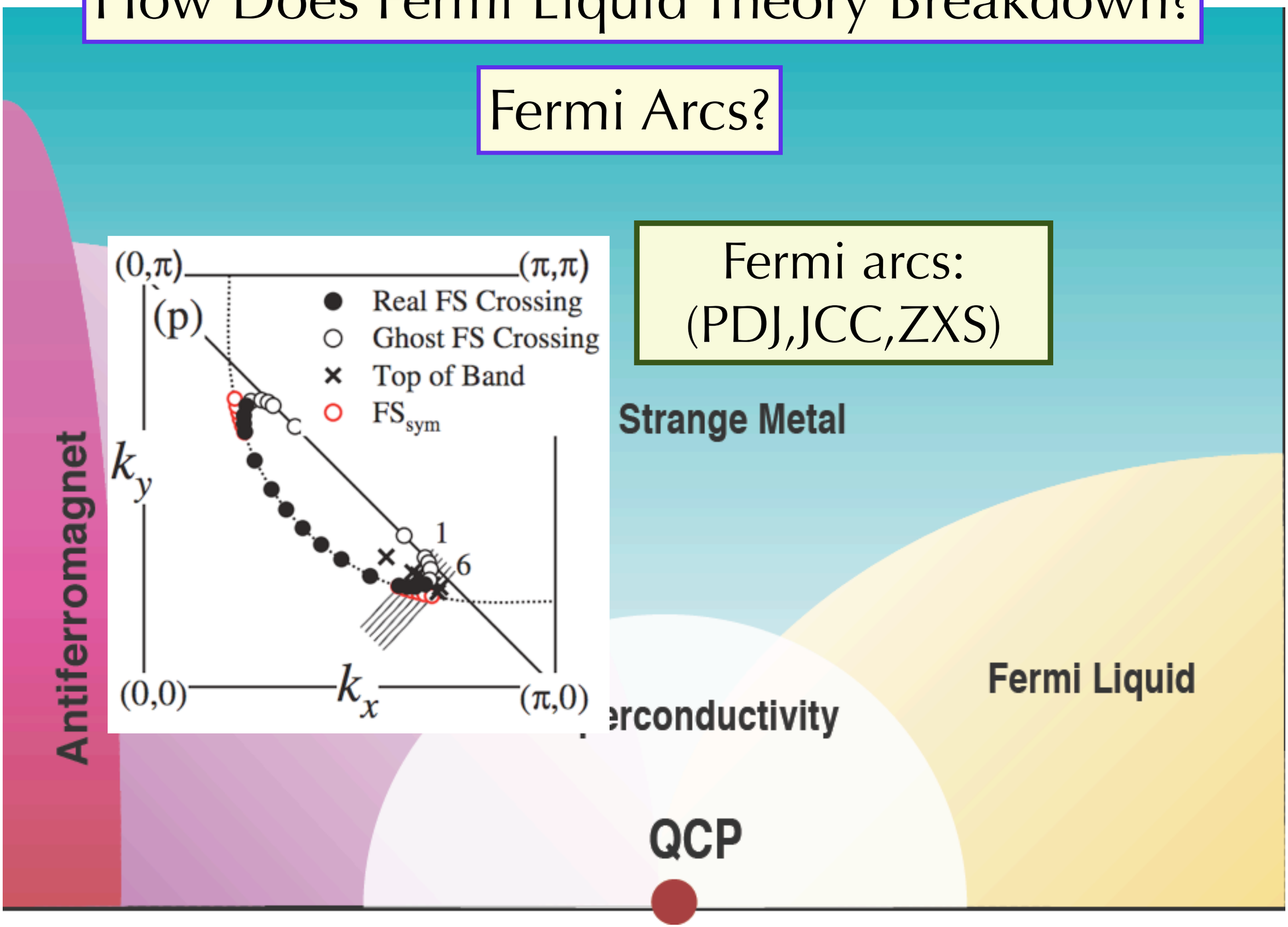
How Does Fermi Liquid Theory Breakdown?



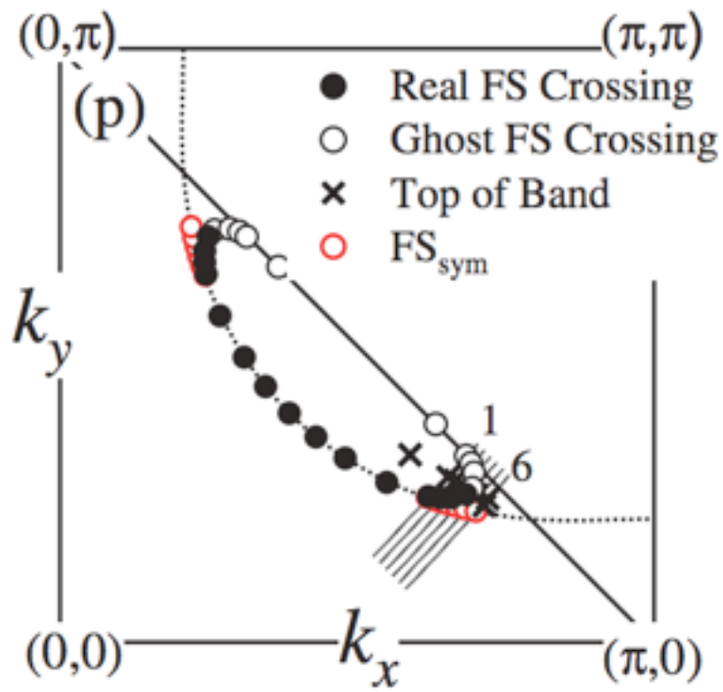
How Does Fermi Liquid Theory Breakdown?

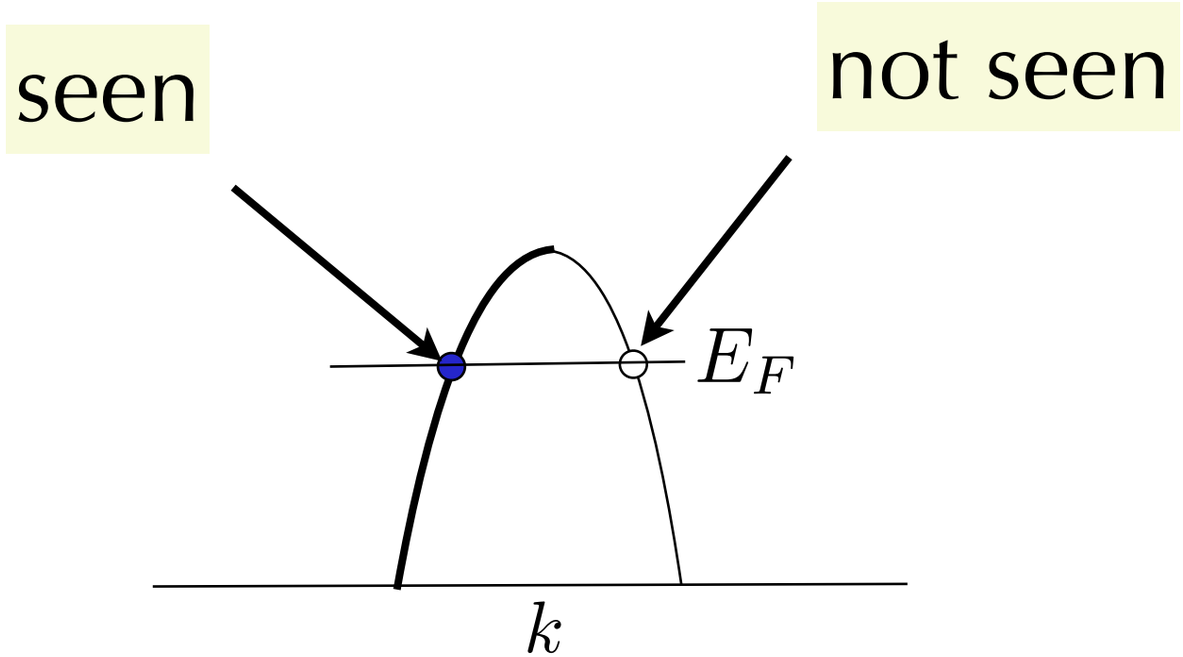
Fermi Arcs?

Fermi arcs:
(PDJ,JCC,ZXS)

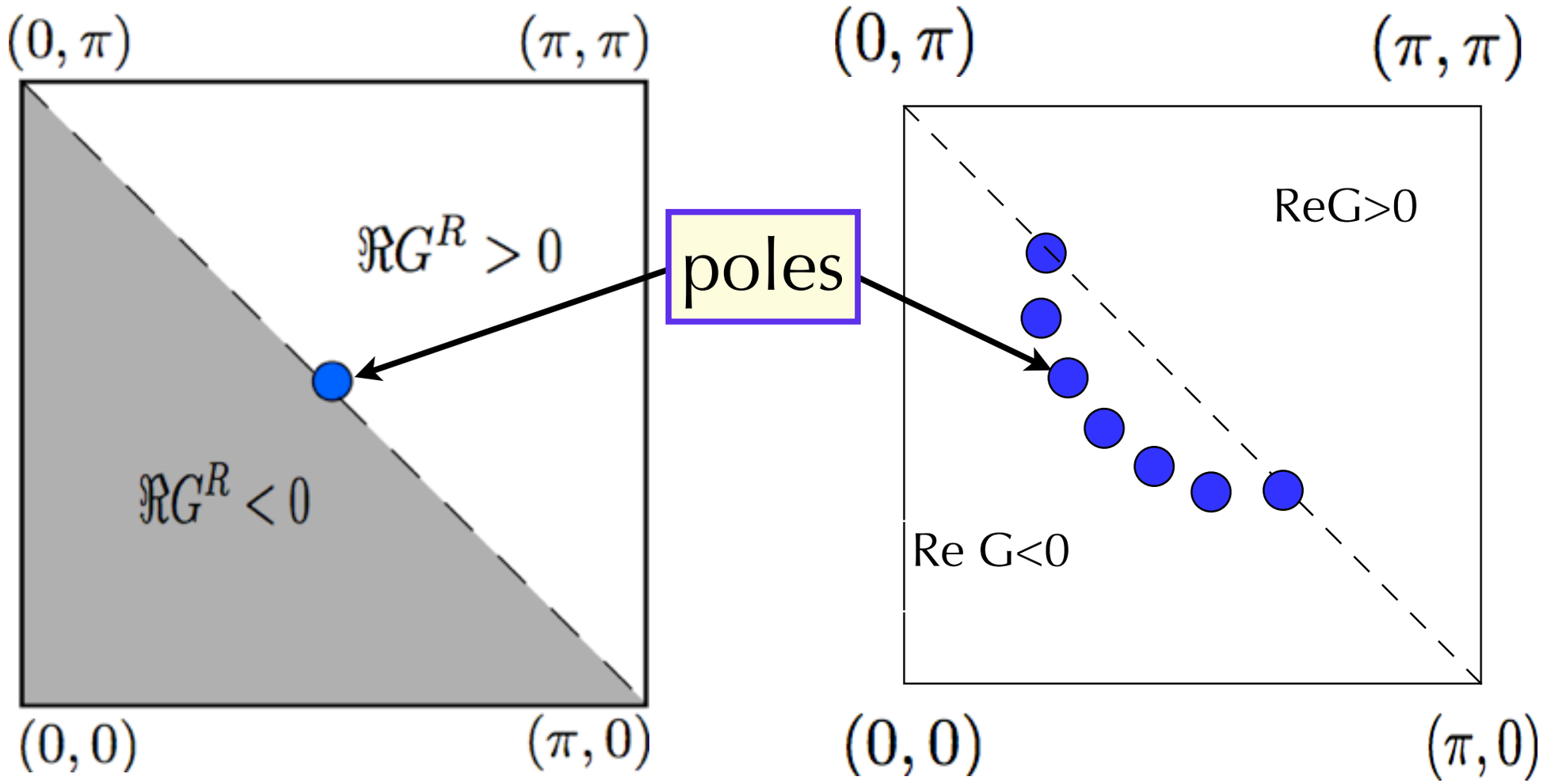


Why are Fermi Arcs Strange?





Problem



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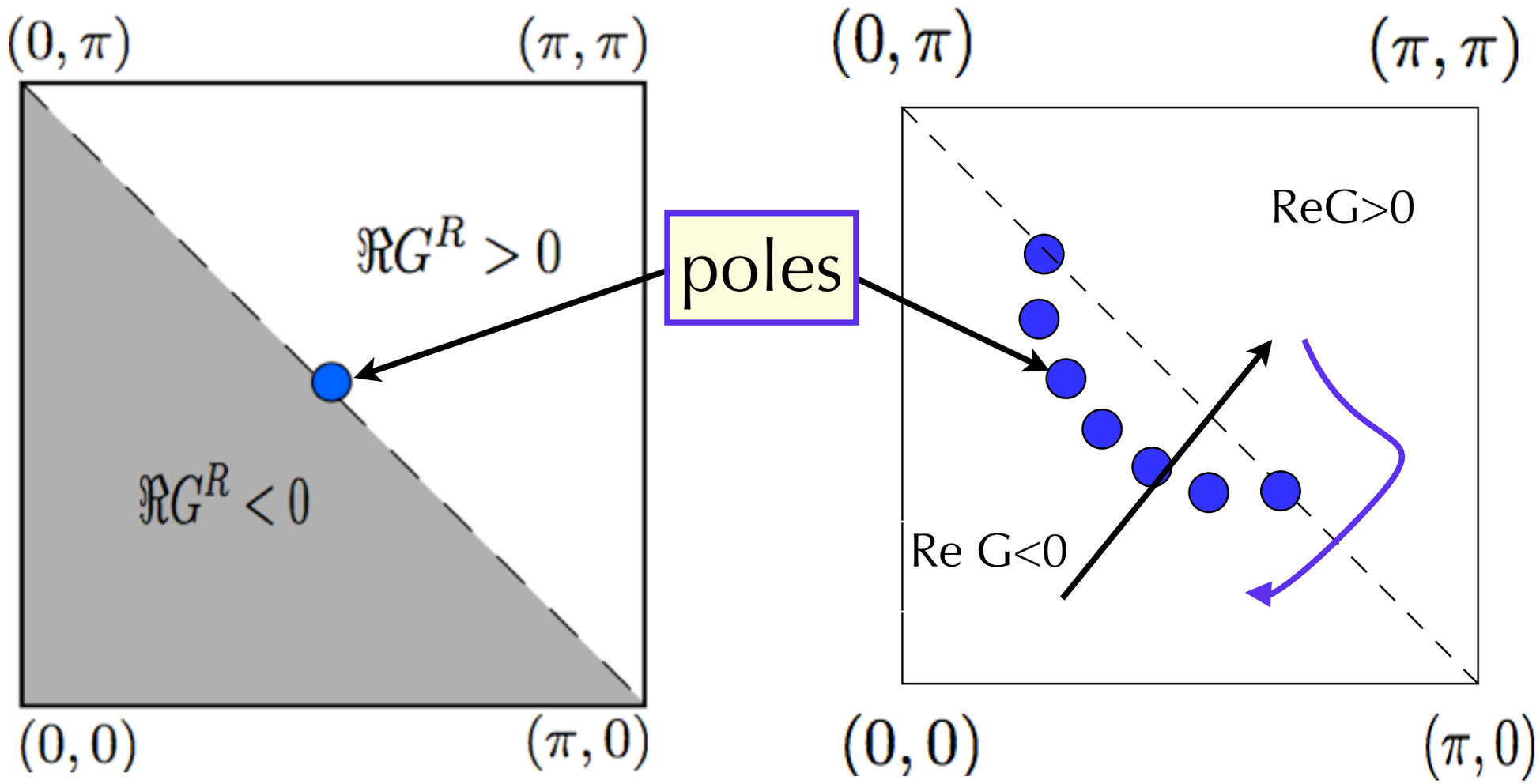
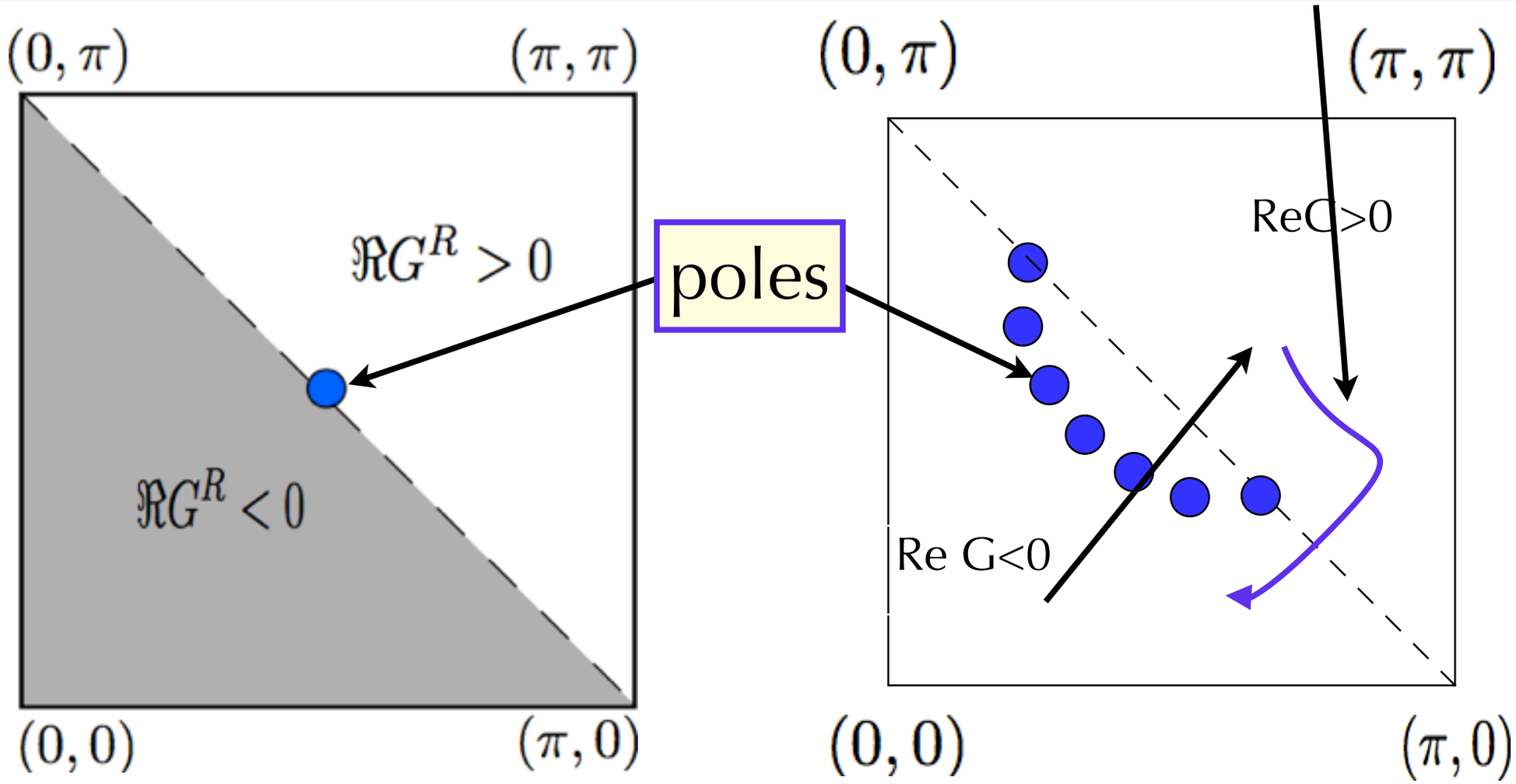


Problem

How to account for the sign change without poles?



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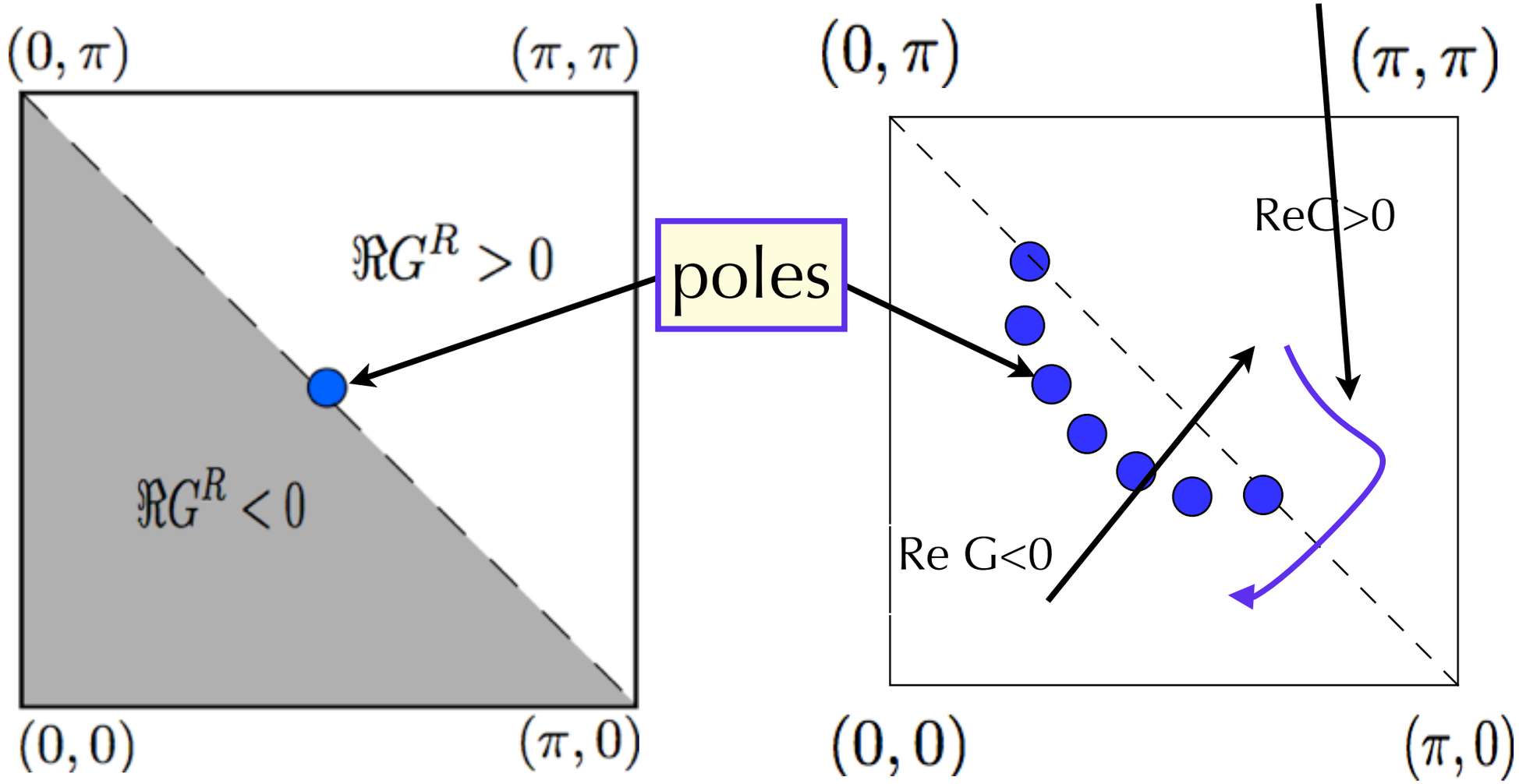

Only option: DetG=0! (zeros)

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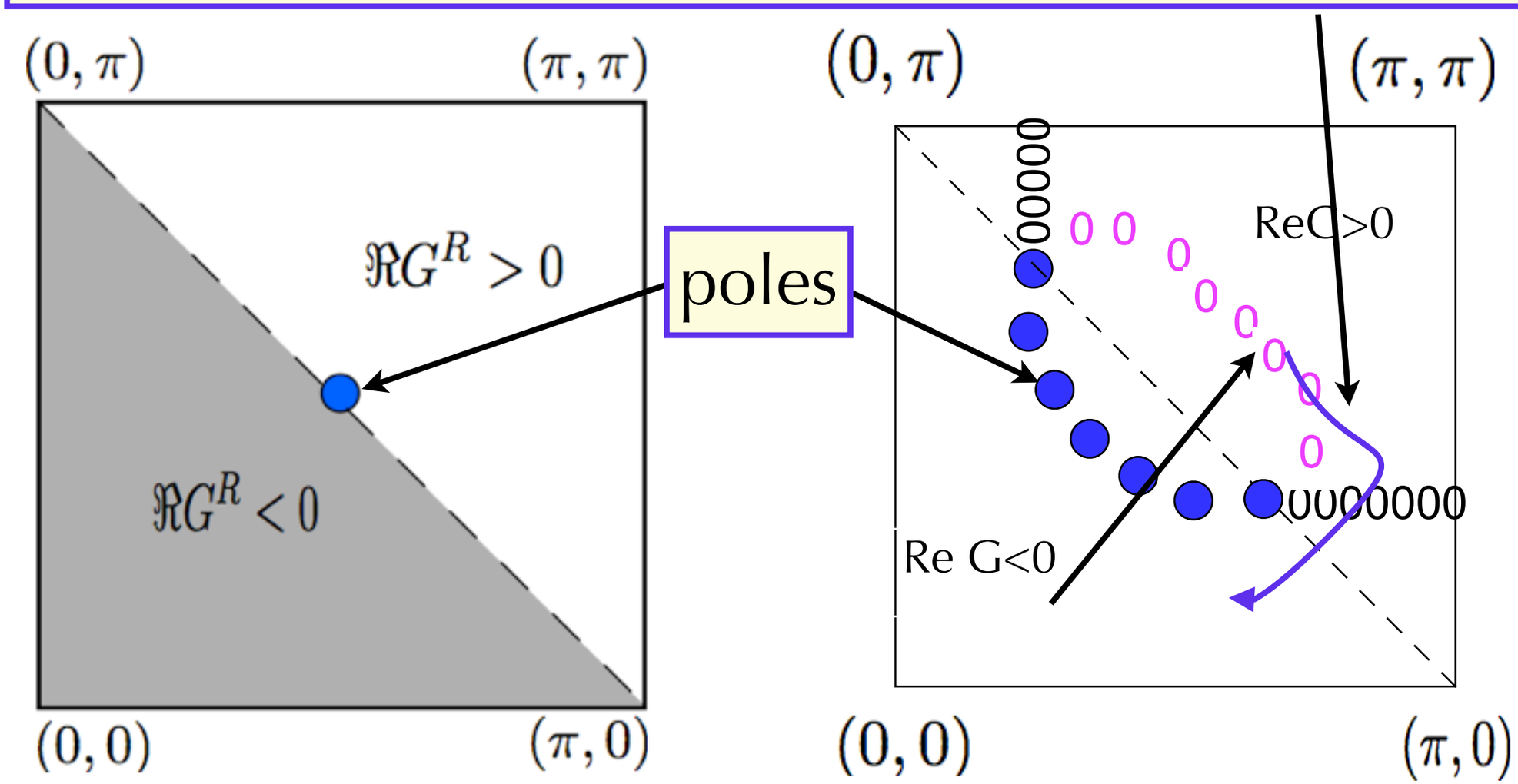


Only option: DetG=0! (zeros)

Fermi Arcs

zeros + poles

Luttinger, Dzyaloshinskii, Yang, Rice, Zhang,Tsvelik...

n=zeros + poles

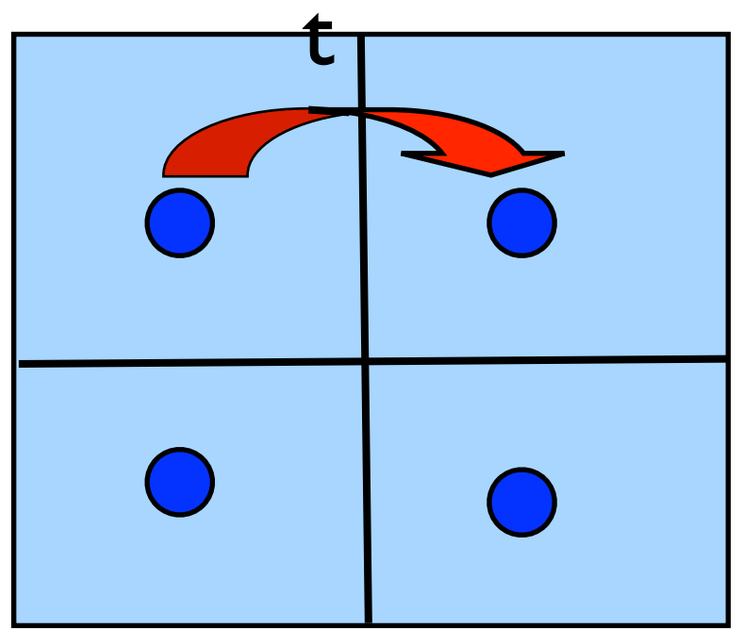
what are zeros?

are they (like poles) conserved?

Mott mechanism (not Slater)

NiO insulates

d^8 ?



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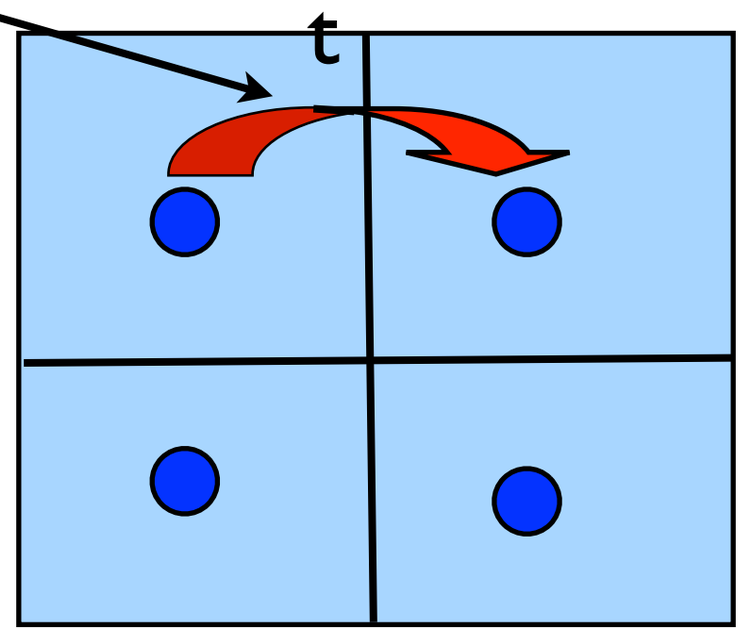

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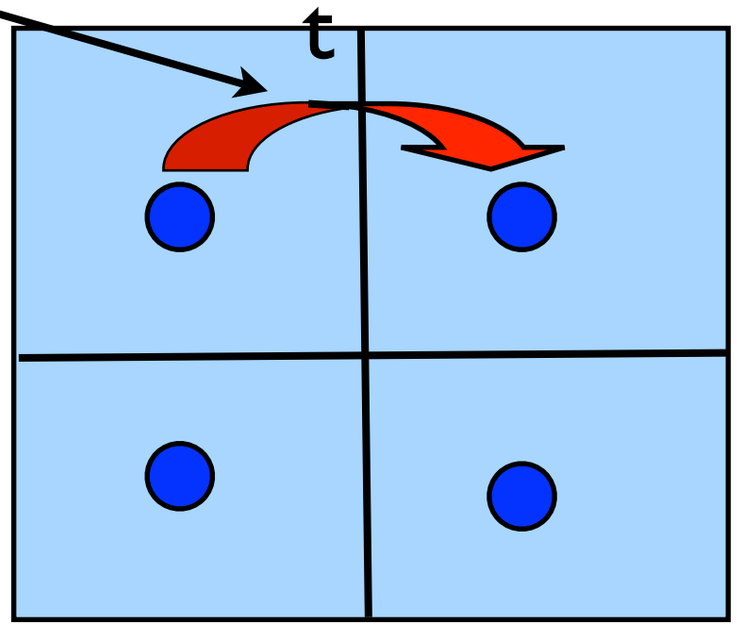

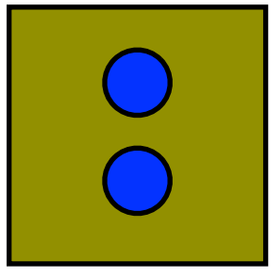

$$U \gg t$$

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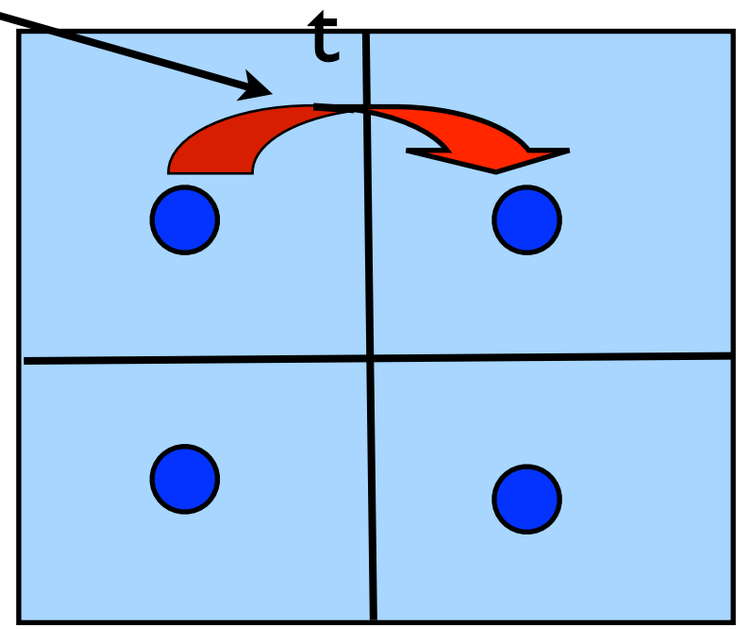

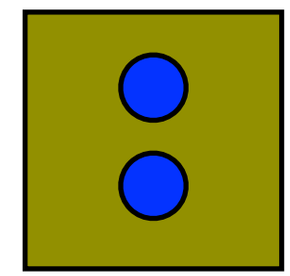

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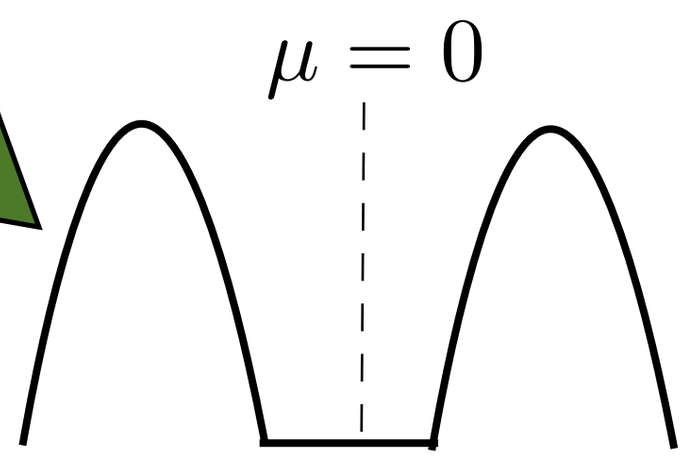

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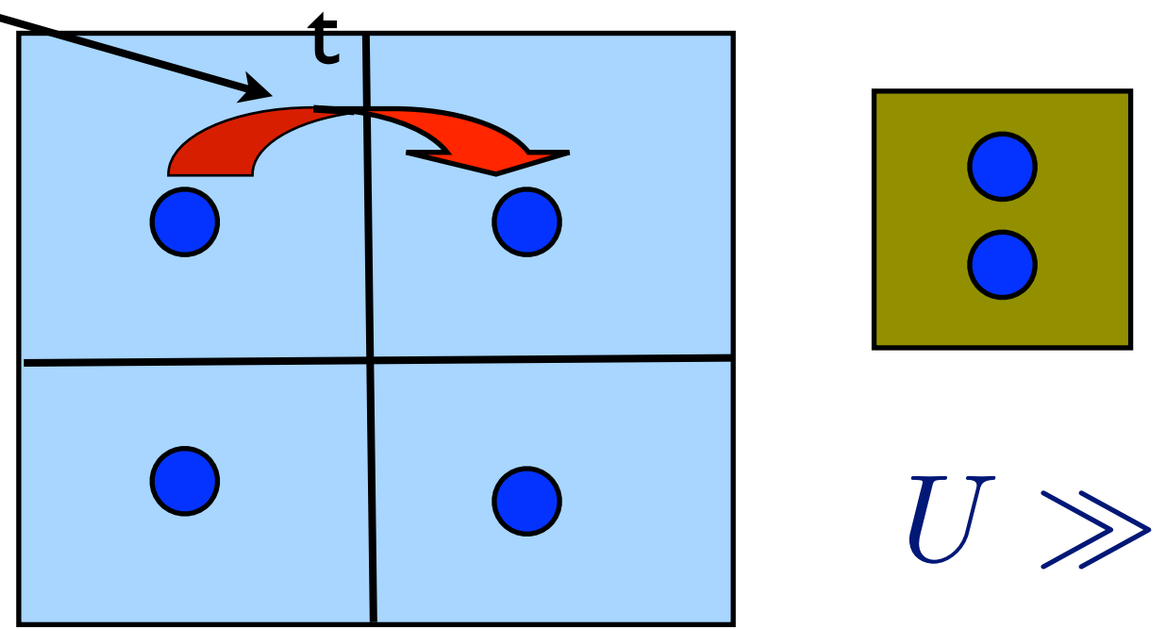

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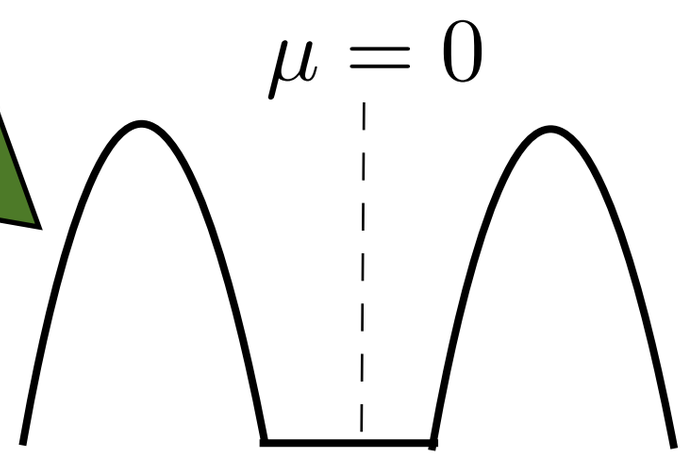
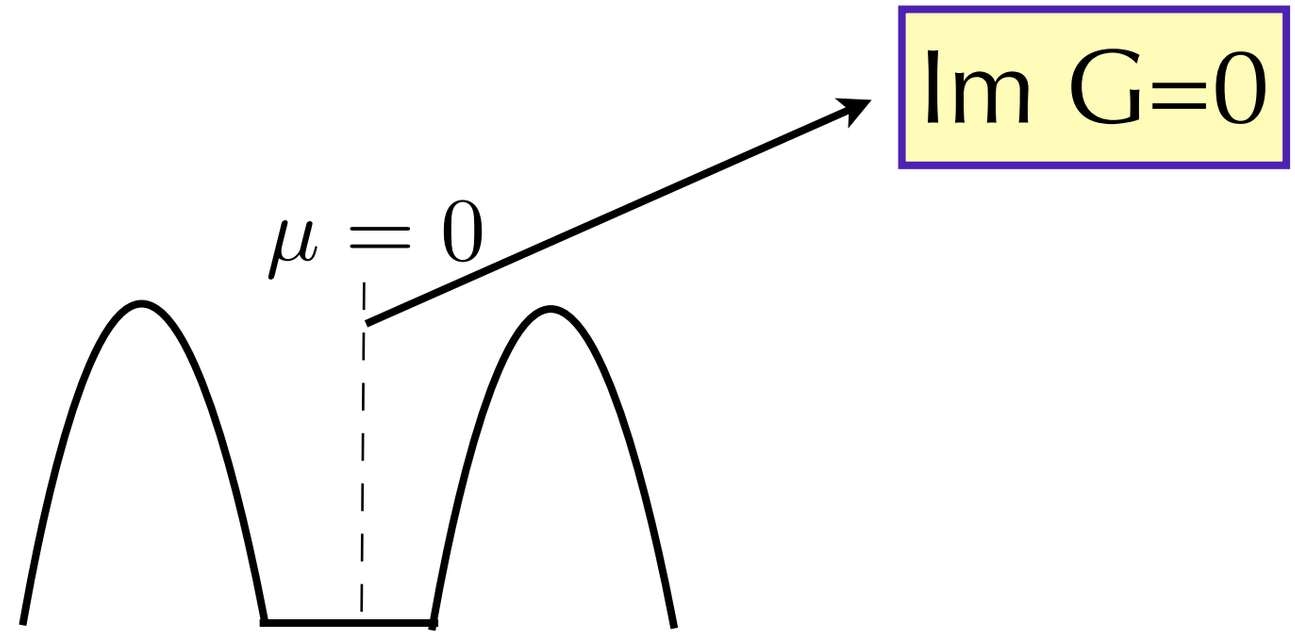


no change in size of Brillouin zone

Mott Problem

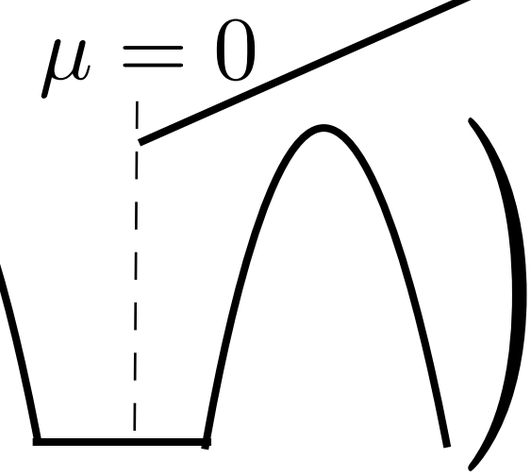
Mott Problem



Mott Problem

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{\left(\quad \right)}{\omega} d\omega$$



Kramers -Kronig

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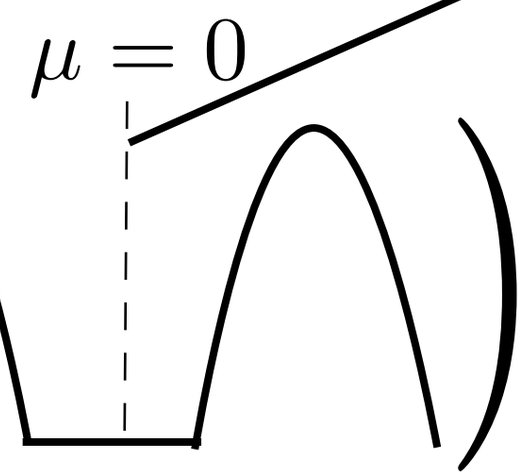

Kramers -Kronig

= below gap+above gap

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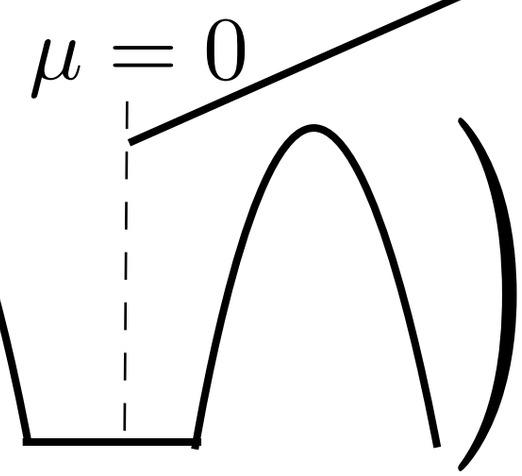

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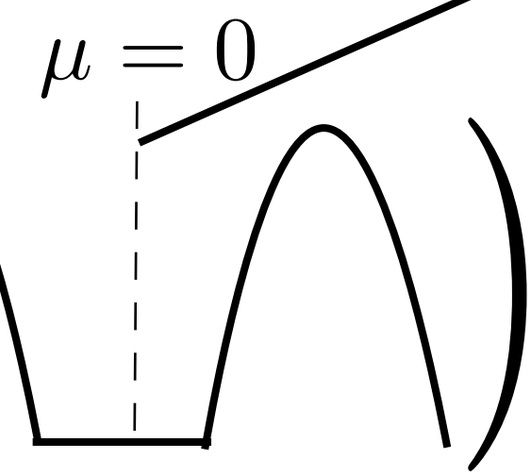

Kramers -Kronig

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$$\mathrm{Det}G(\mathbf{k}, \omega = \mathbf{0}) = \mathbf{0}$$

(single band)

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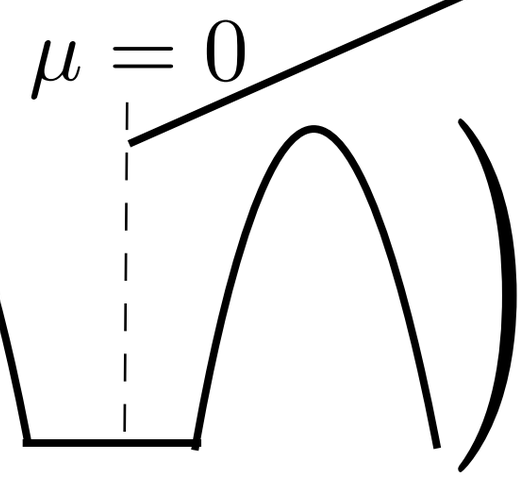

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DetReG(0,p)=0 Mottness

Mott Problem

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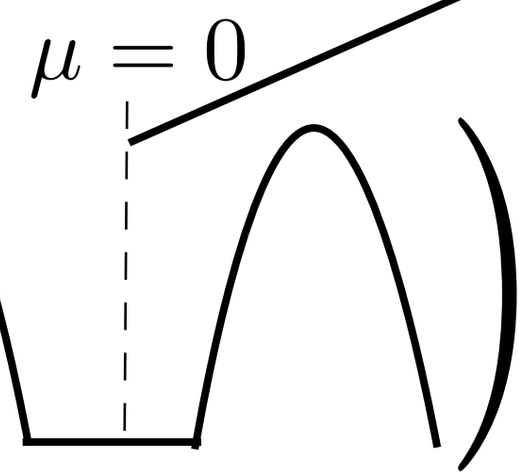

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(single band)

DetReG(0,p)=0

Mottness

not true in MF theories

Luttinger theorem: singularities of $\ln G$

$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d\mathbf{p} \int_{-\infty}^{0} d\xi \ln \frac{G^R(\xi, \mathbf{p})}{G_R^*(\xi, \mathbf{p})}$$

poles+zeros
(all sign changes)

Luttinger theorem: singularities of $\ln G$

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$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

poles+zeros
(all sign changes)

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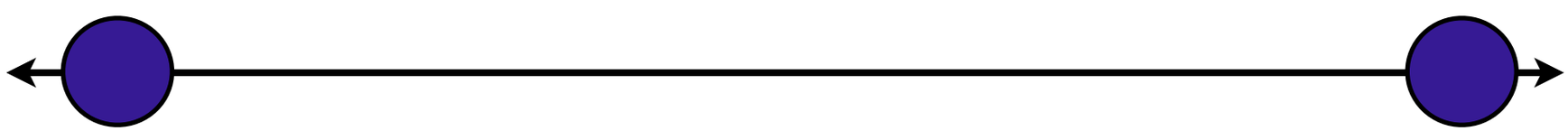

Fermi Liquids

Mott Insulators

PHYSICAL REVIEW B **68**, 085113 (2003)

Some consequences of the Luttinger theorem: The Luttinger surfaces in non-Fermi liquids and Mott insulators

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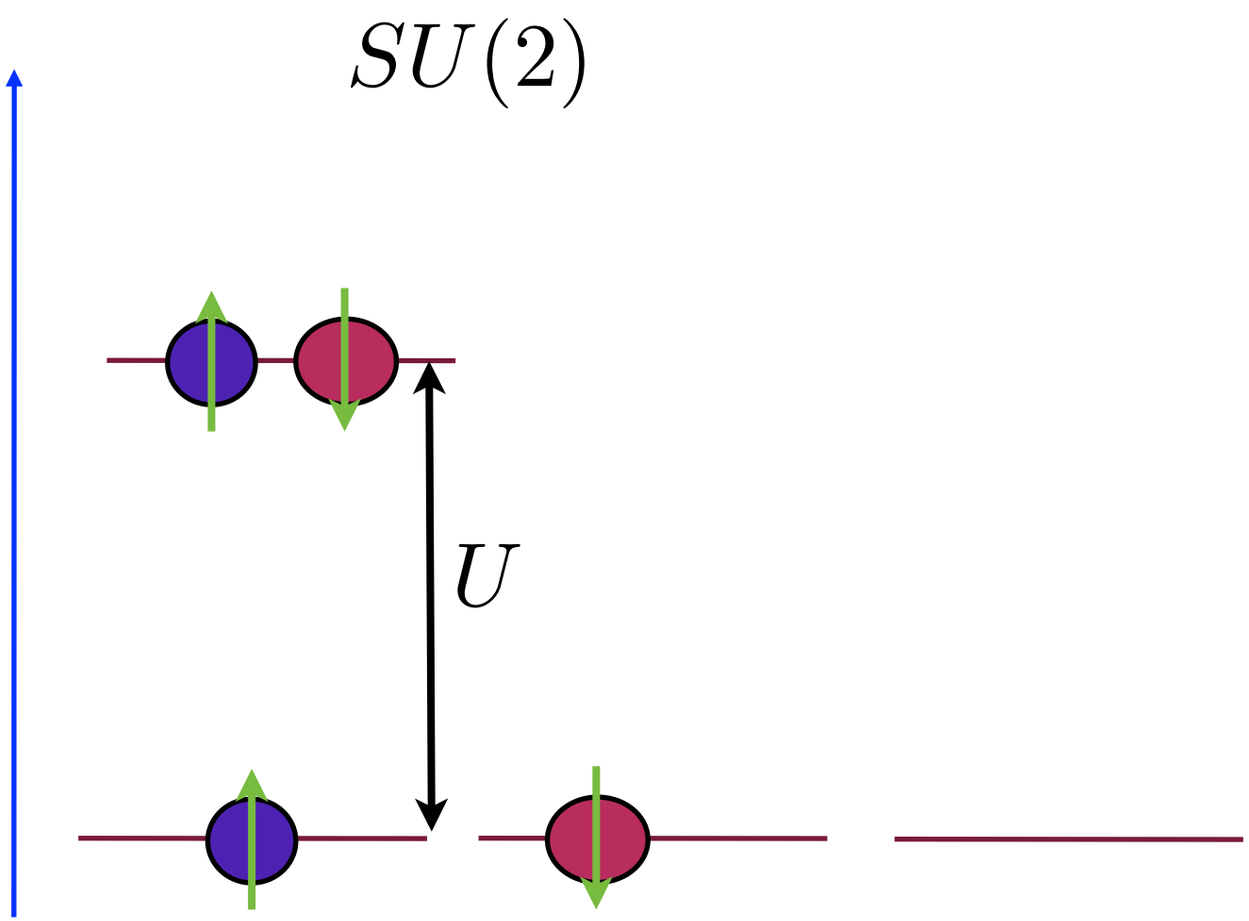
Department of Physics and Astronomy, University of California, Irvine, California 92697, USA
(Received 30 January 2003; published 27 August 2003)

The proof just presented is good for any state of our system: FL, NFL, or MI [in other words, either of poles or zeros can be used to change the sign of G_r in Eq. (1)]. The only way to incapacitate the Luttinger theorem in form (1) is to assume that the limit $T \to 0$ is discontinuous. Actually, one has to require that the whole line $T=0$ is a line of phase transitions.

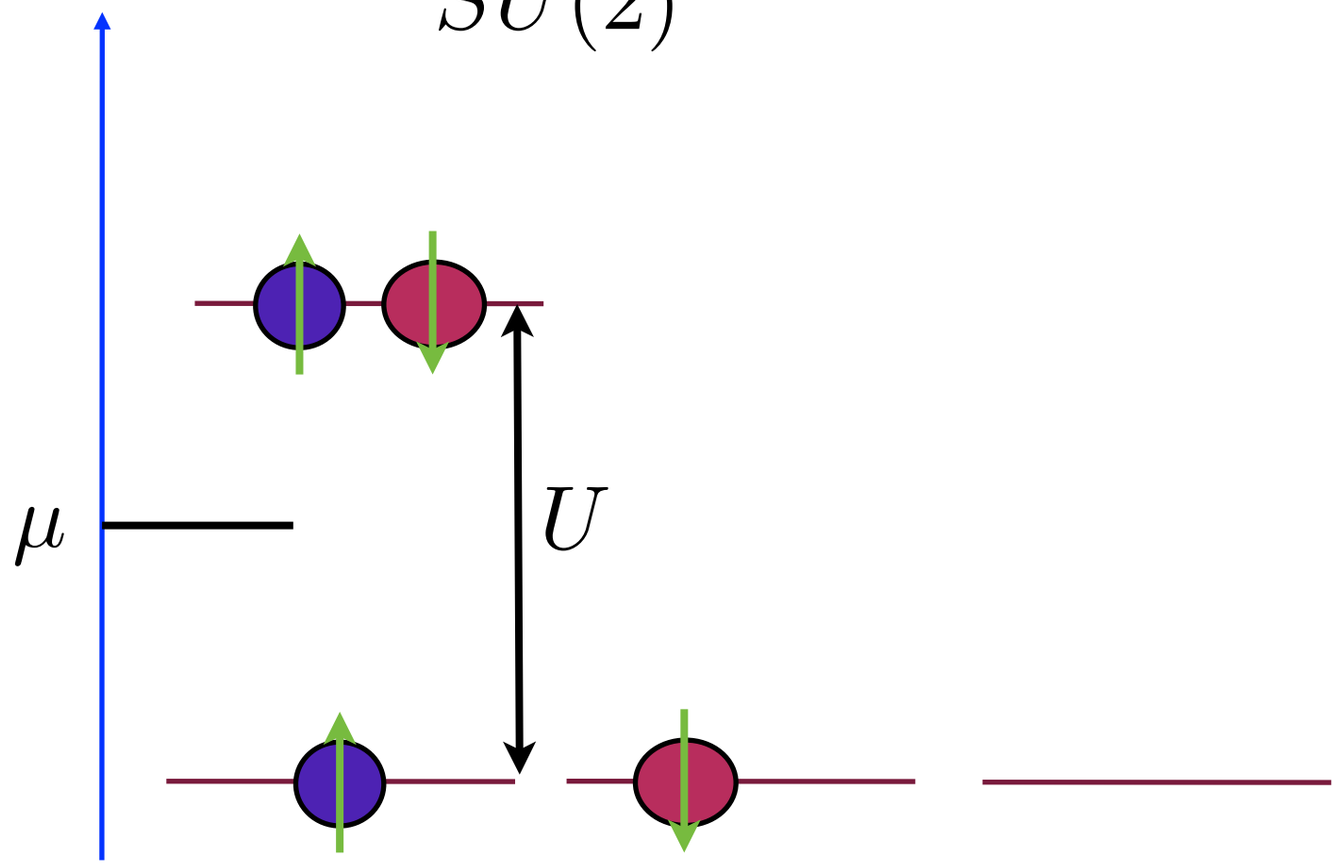
Is this famous theorem from 1960 correct?

simple problem: n=1



simple problem: n=1

$SU(2)$



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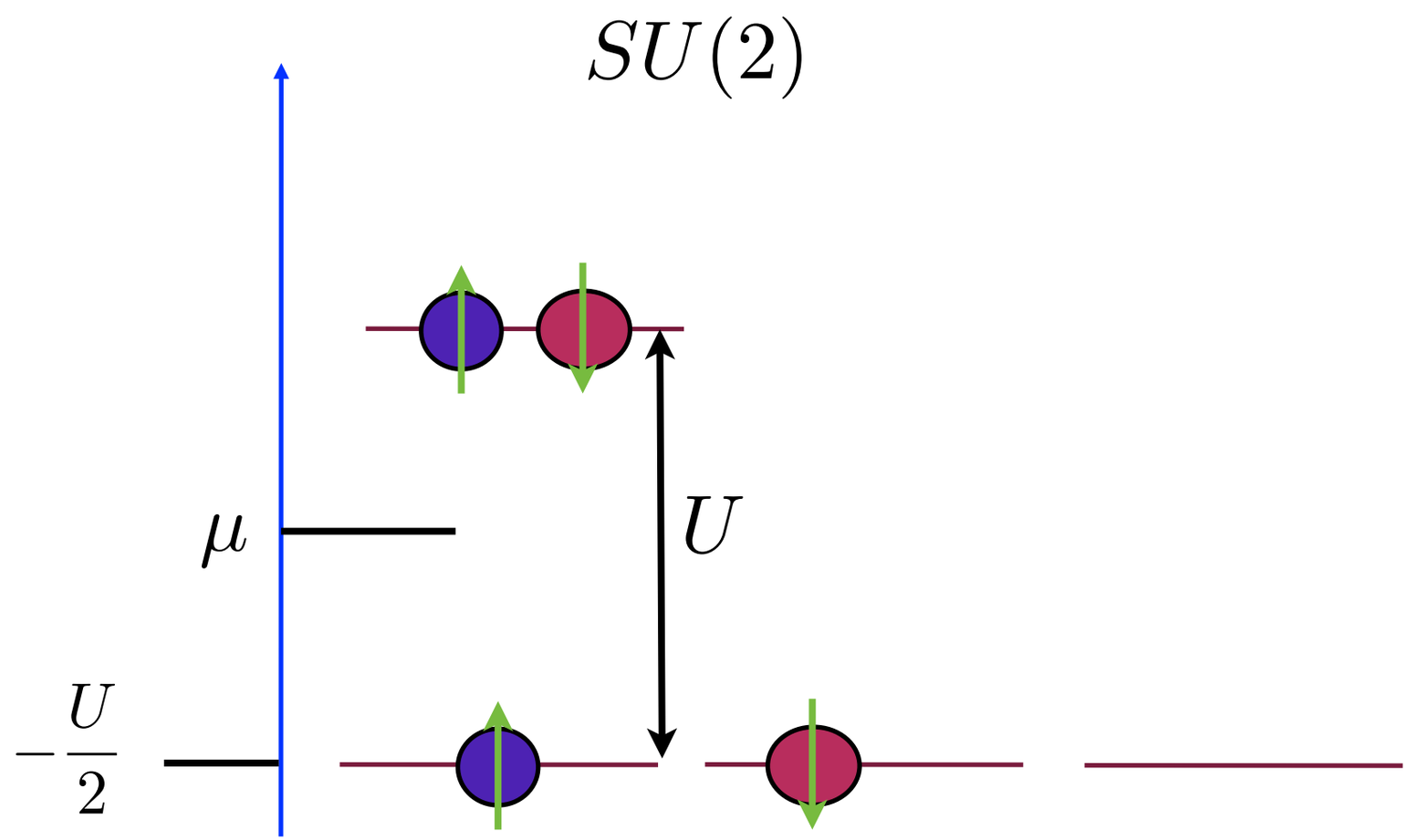

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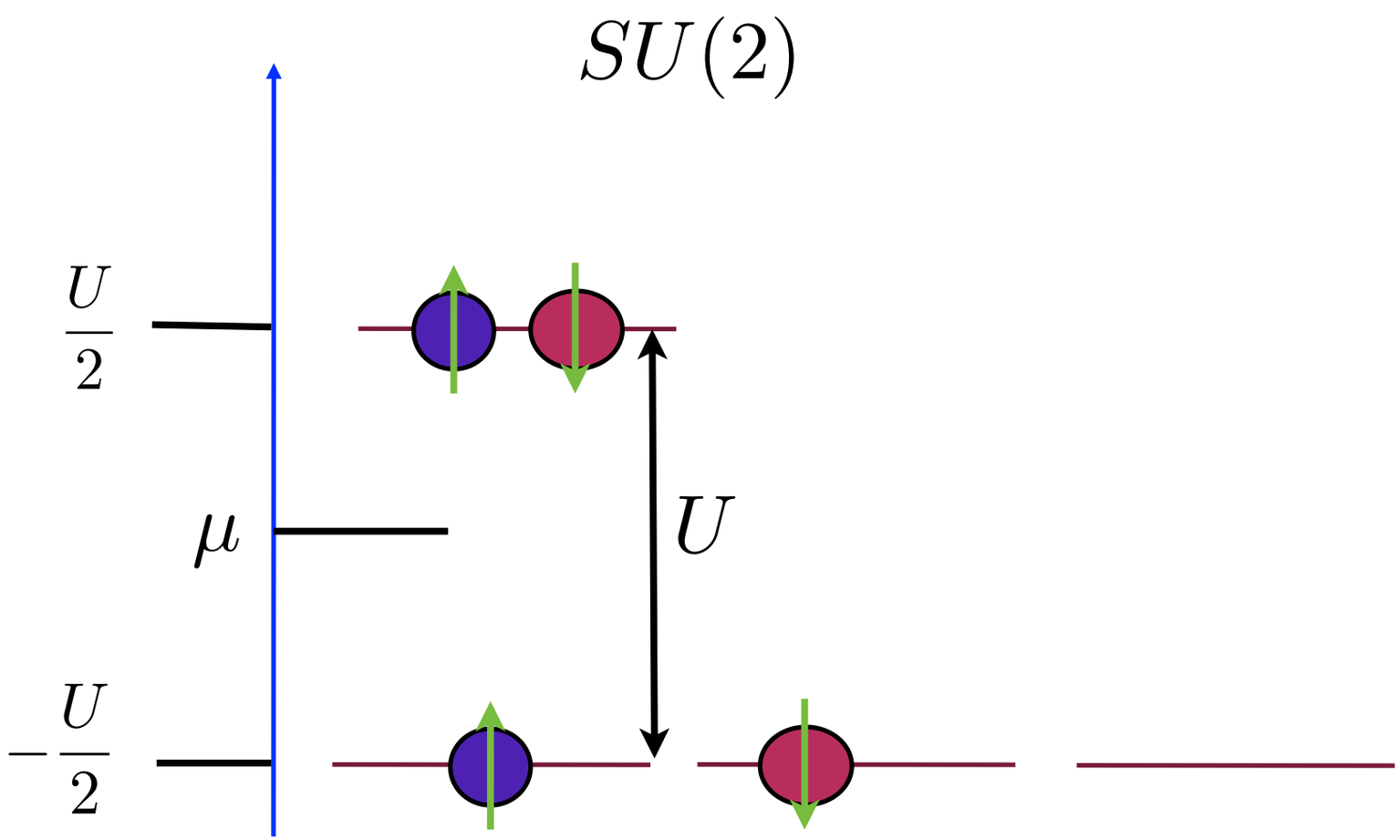

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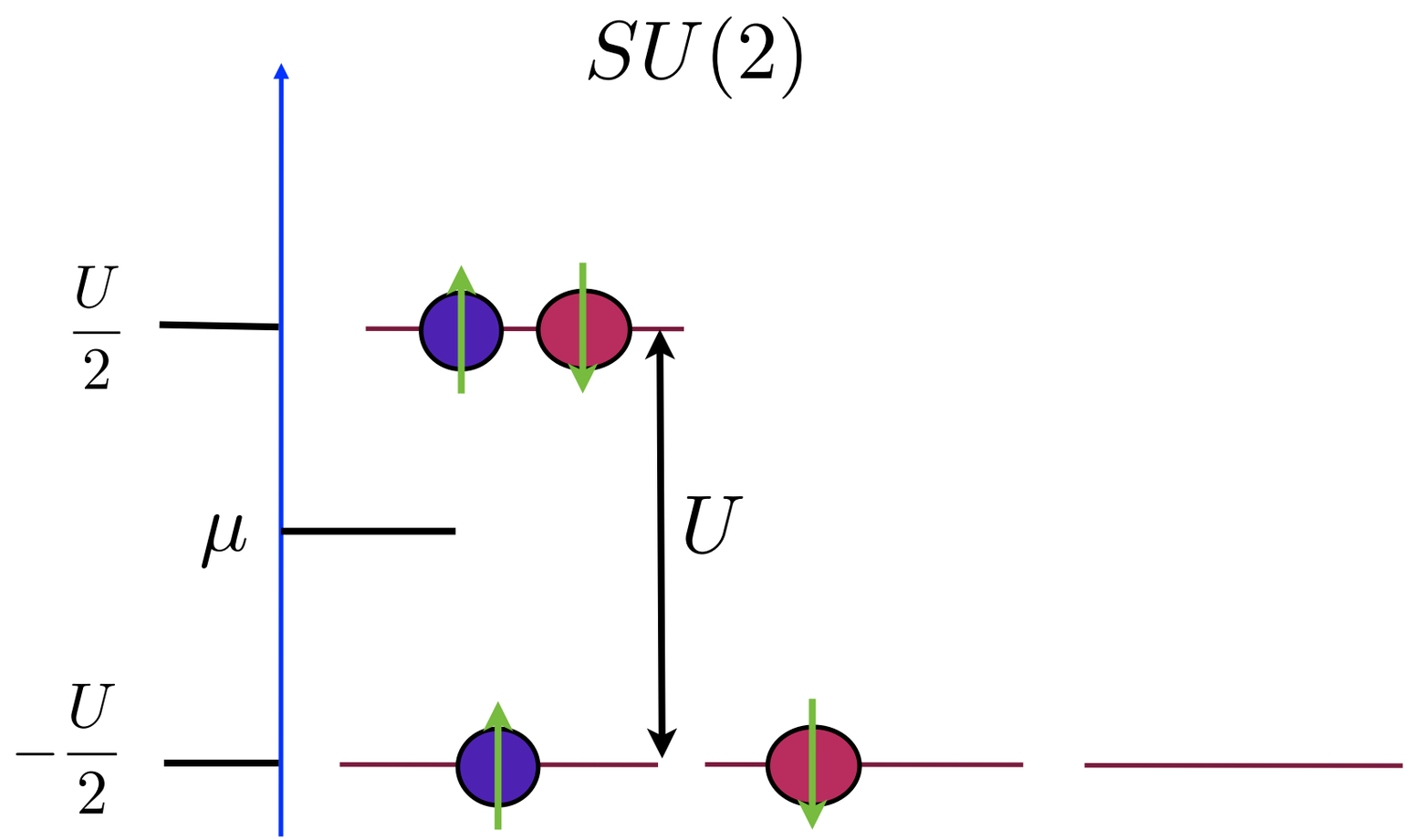
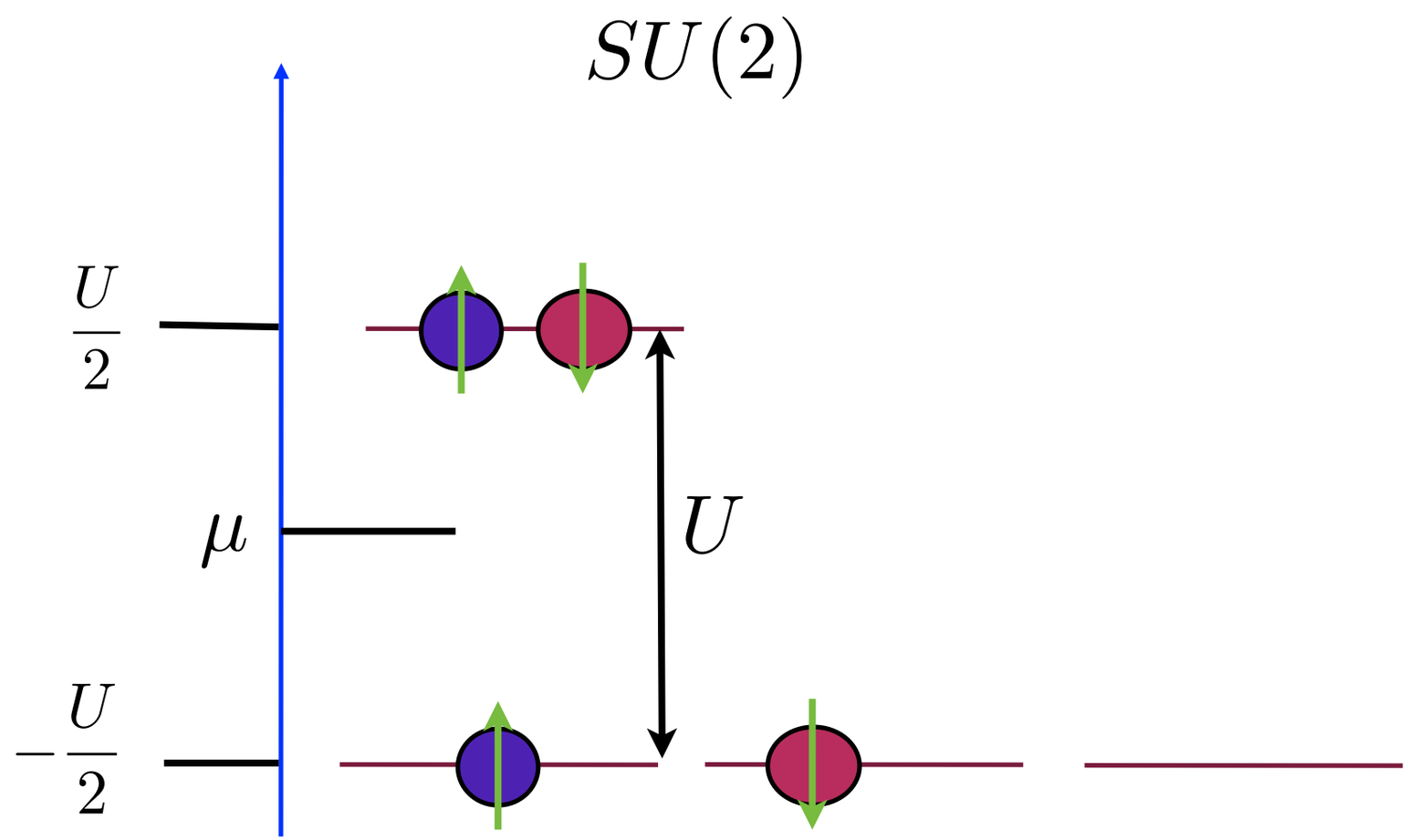


$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2}$$

simple problem: n=1



$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$

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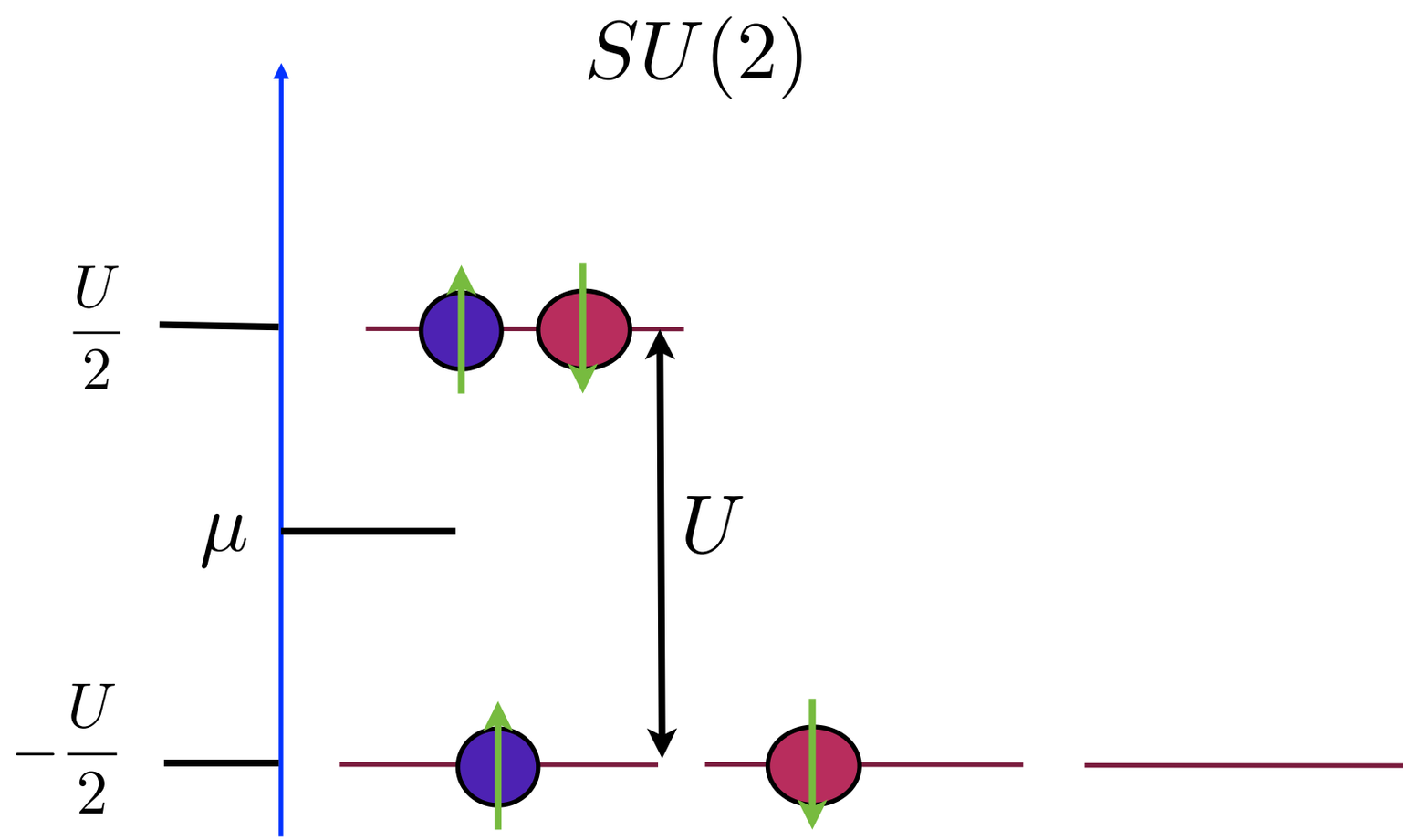

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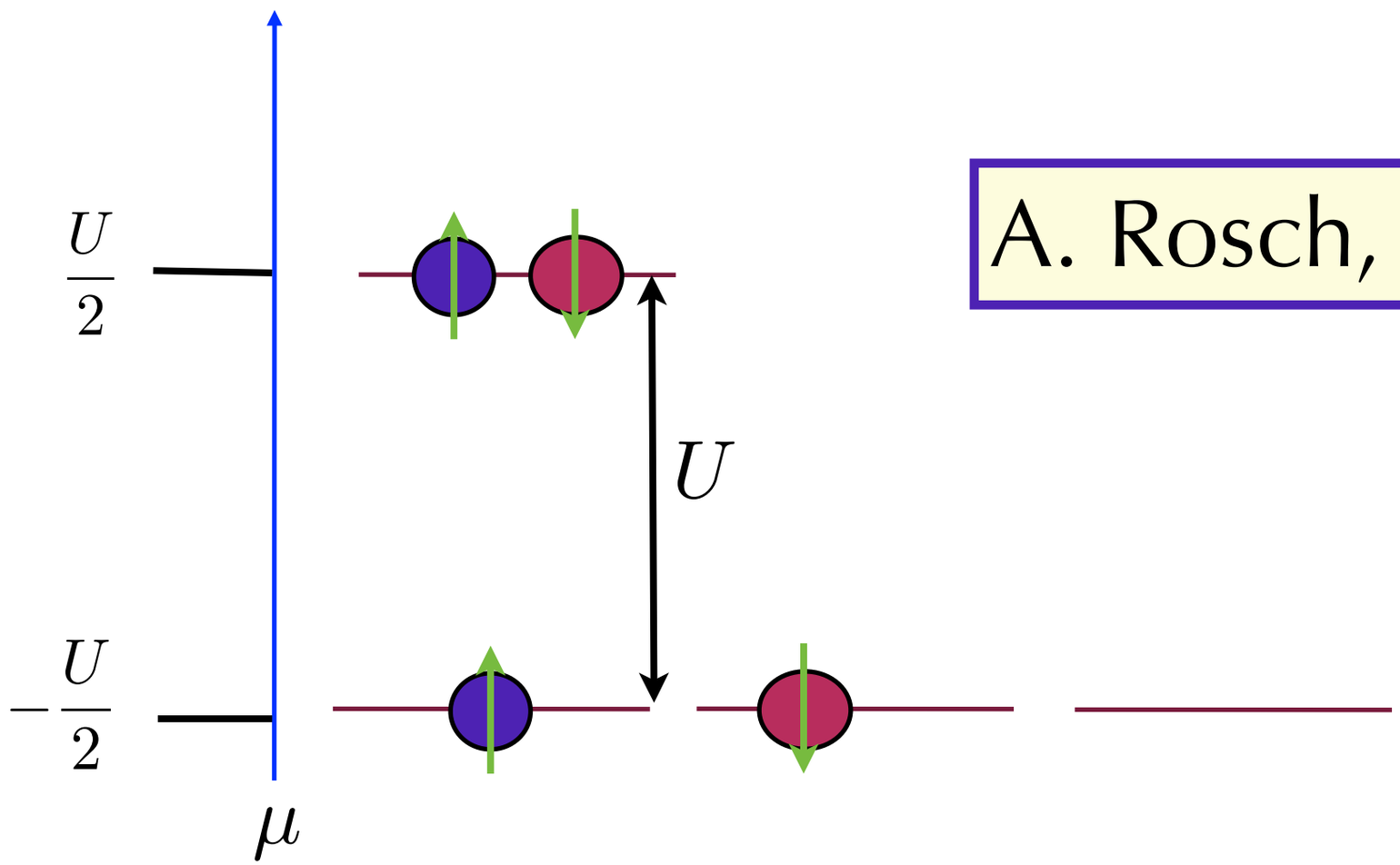
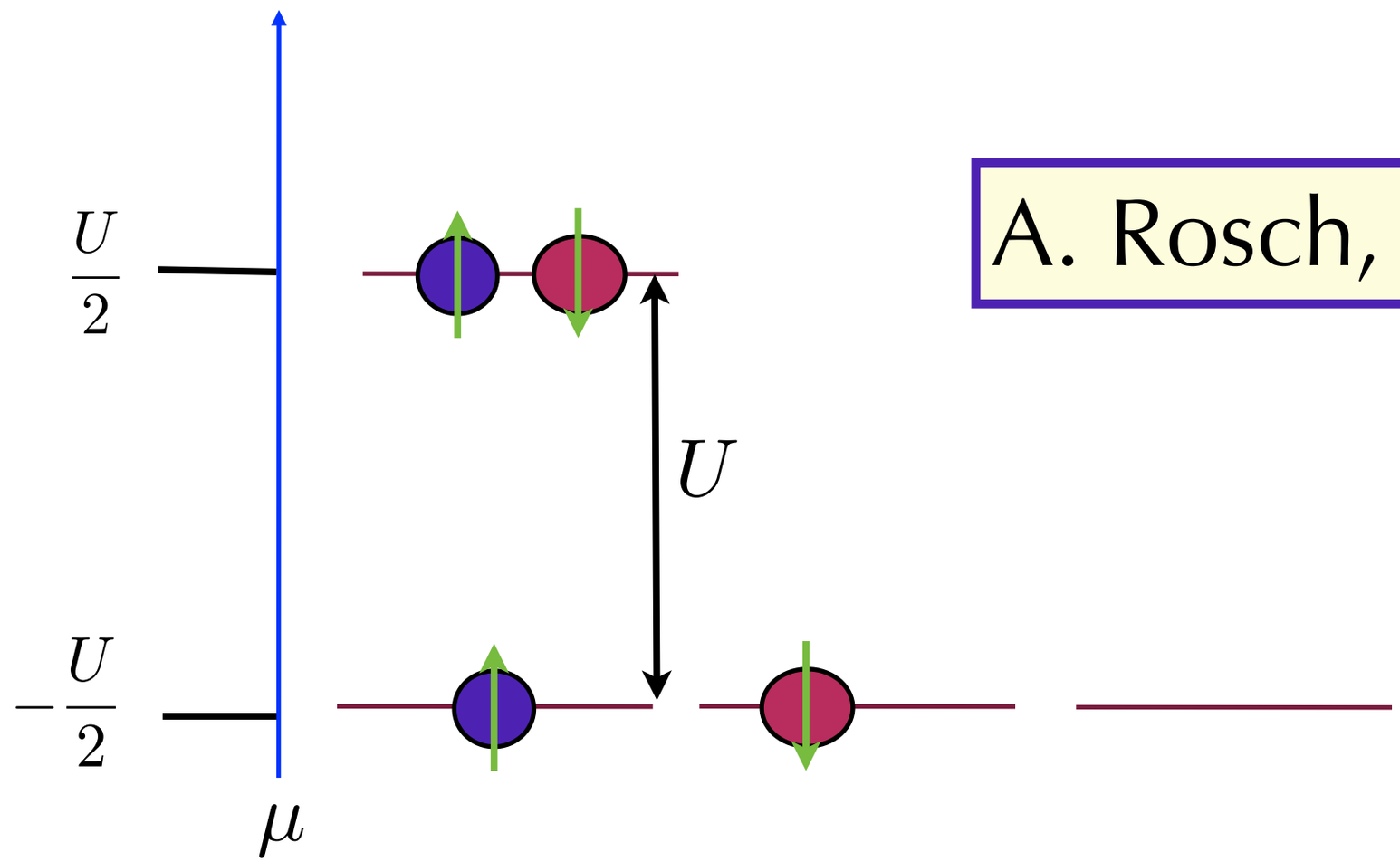
$$n = 2\theta(0) = 1$$



A. Rosch, 2007

$$G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



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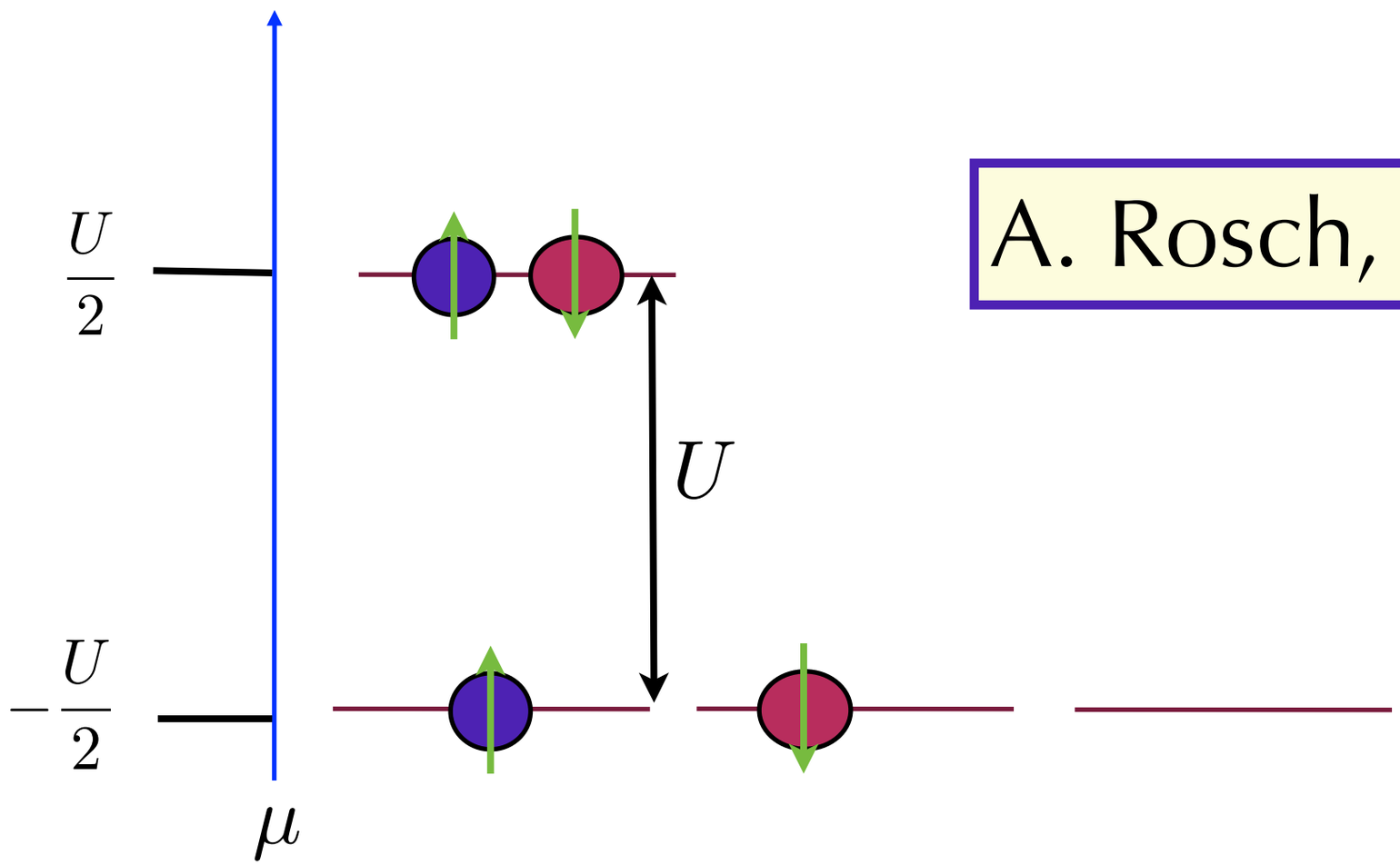
$$n = 2\theta\left(\frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}\right) \Leftarrow G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



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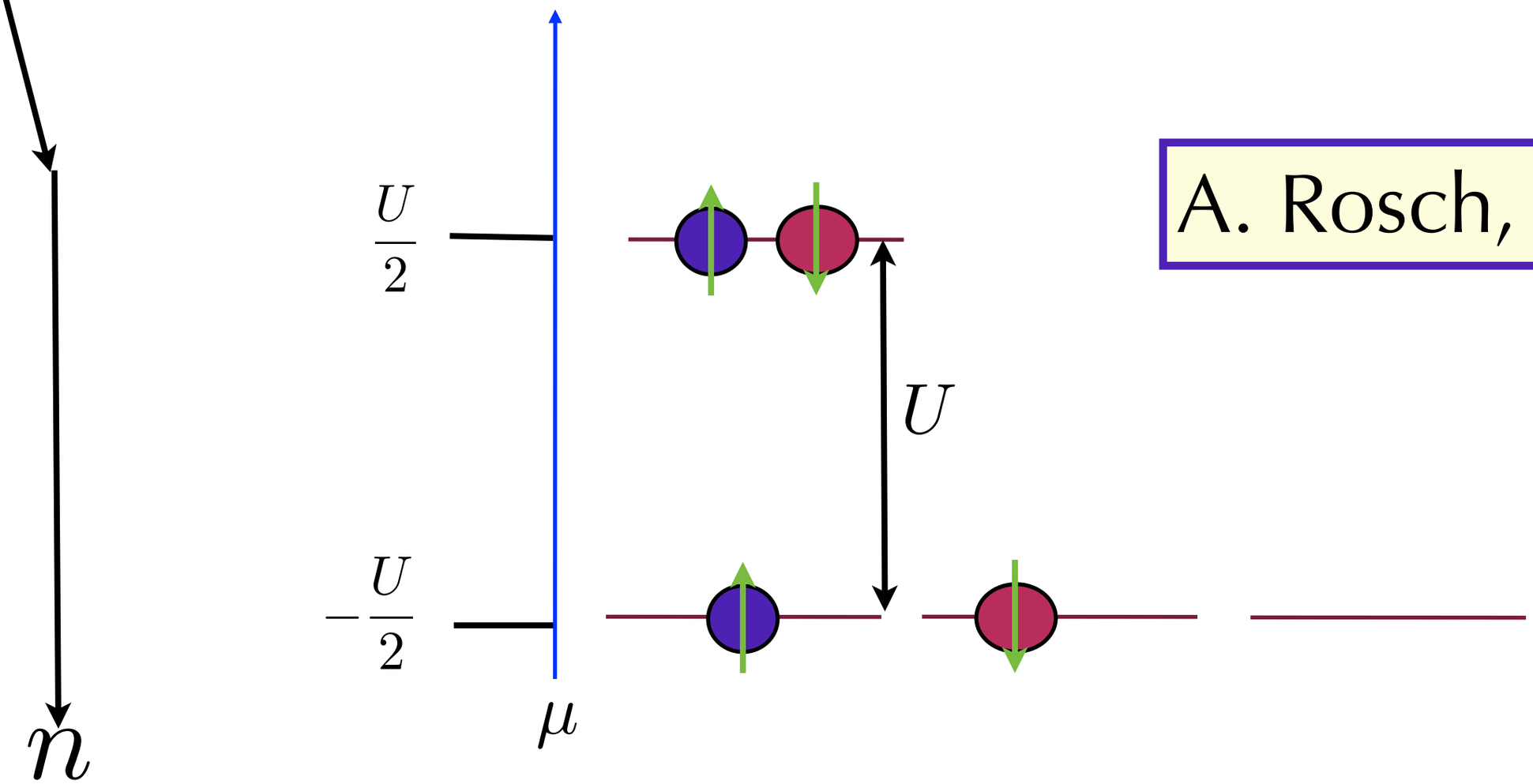

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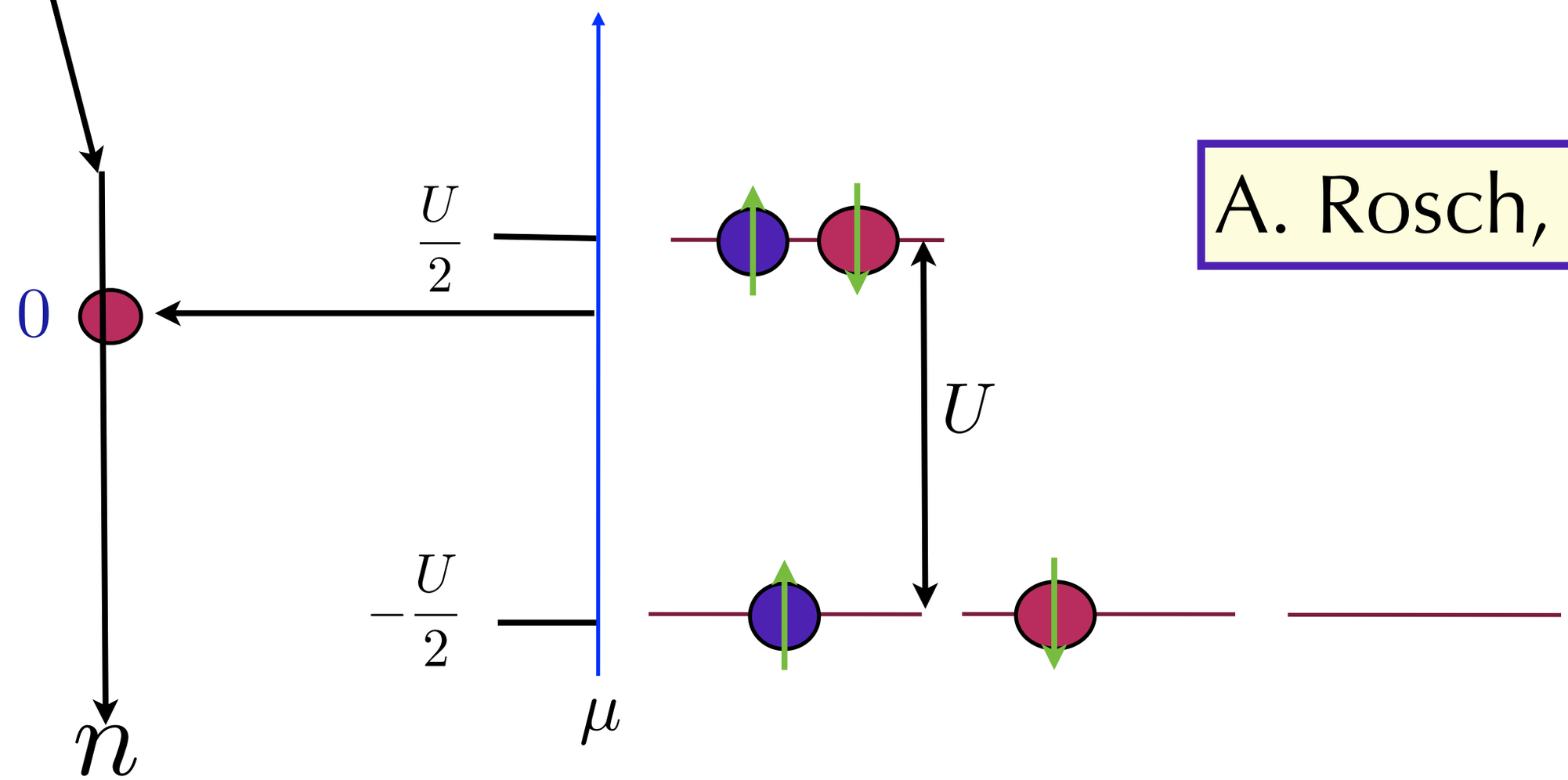

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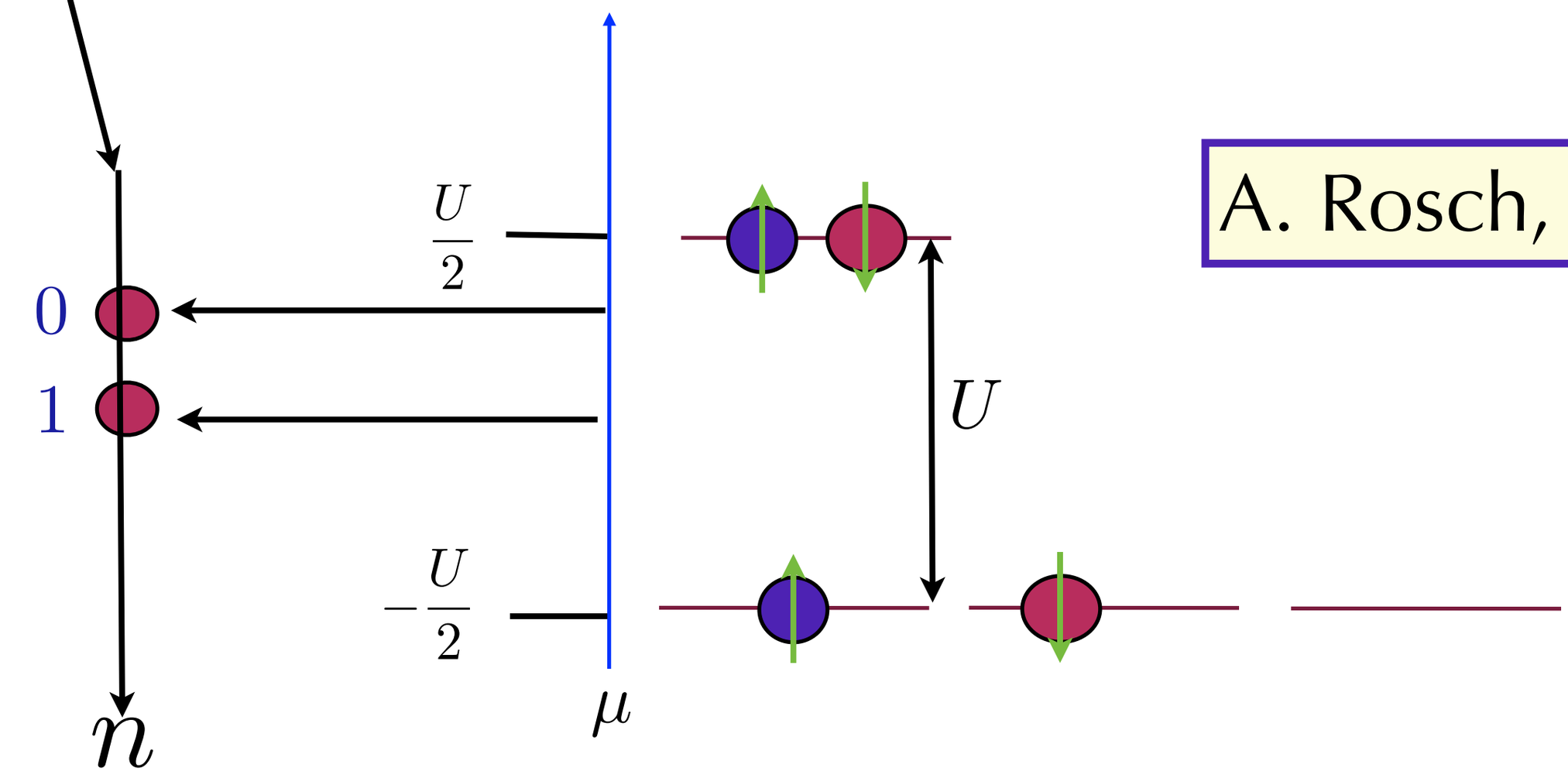

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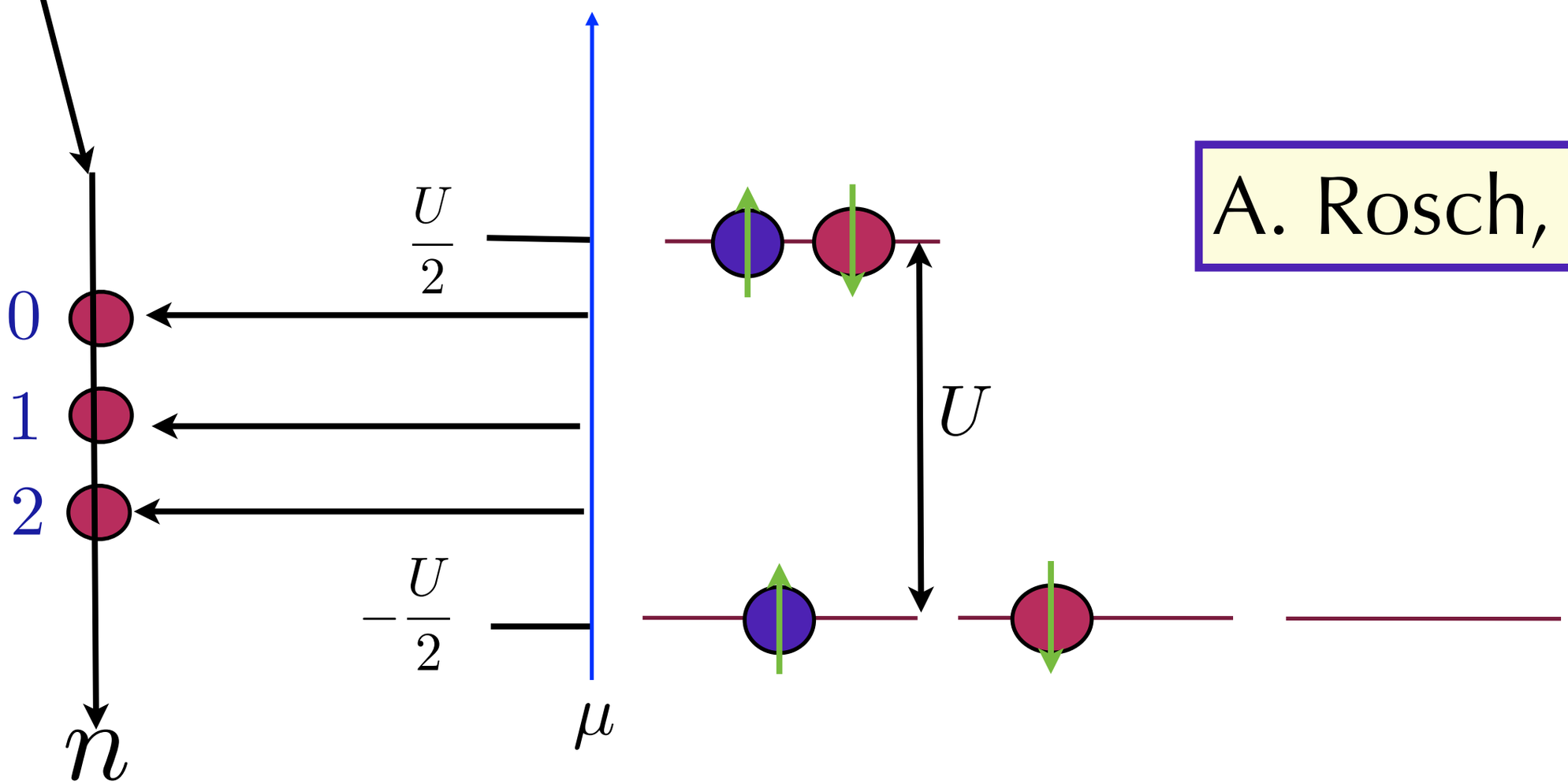

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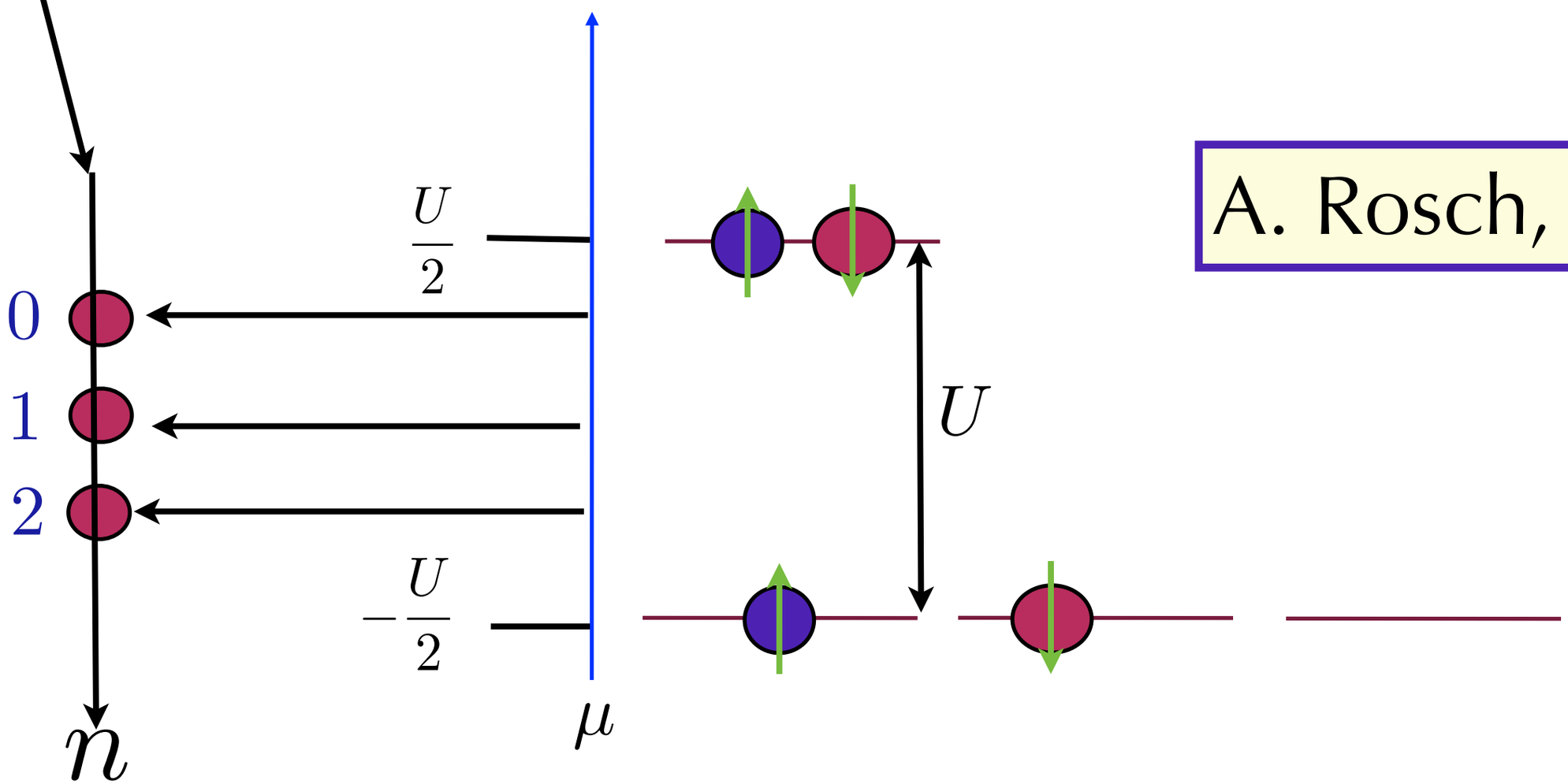


A. Rosch, 2007

incompressible but $\frac{\partial n}{\partial \mu} \neq 0$

fix chemical potential

$$\lim_{T \to 0} \mu(T)$$

fix chemical potential

$$\lim_{T \to 0} \mu(T)$$

n=1

fix chemical potential

$$\lim_{T \to 0} \mu(T)$$

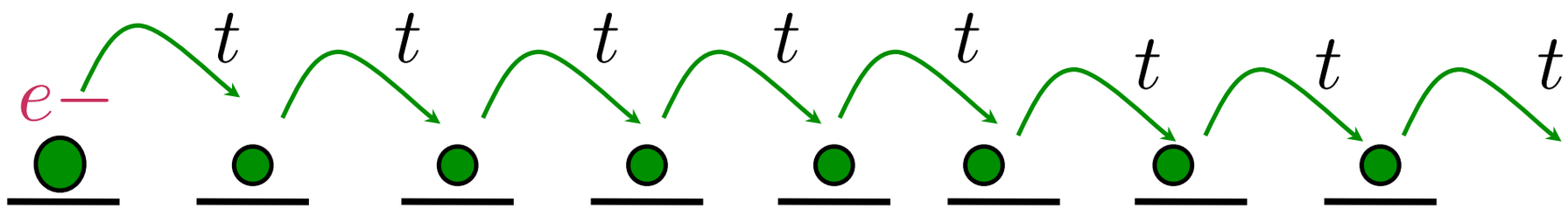
n=1

does this fix all the problems?

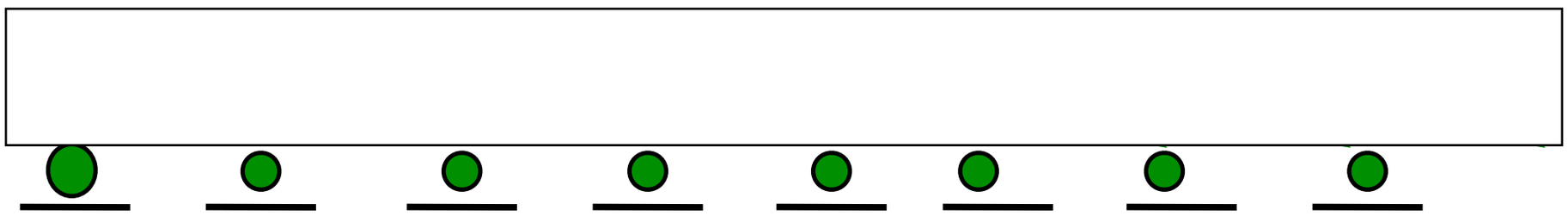
No

A model with zeros but Luttinger fails

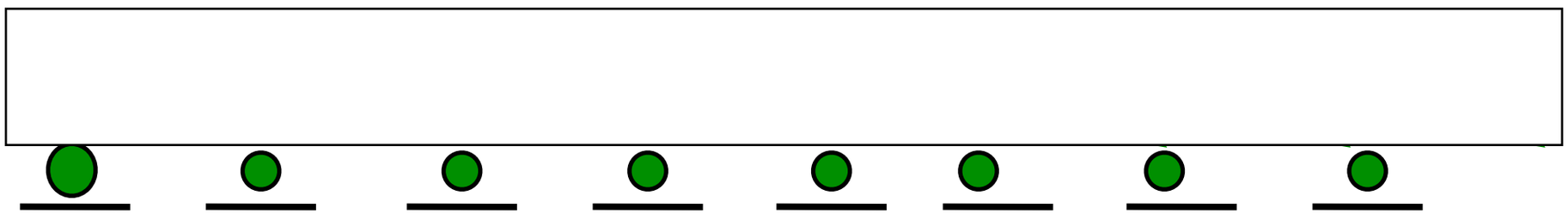
A model with zeros but Luttinger fails



A model with zeros but Luttinger fails

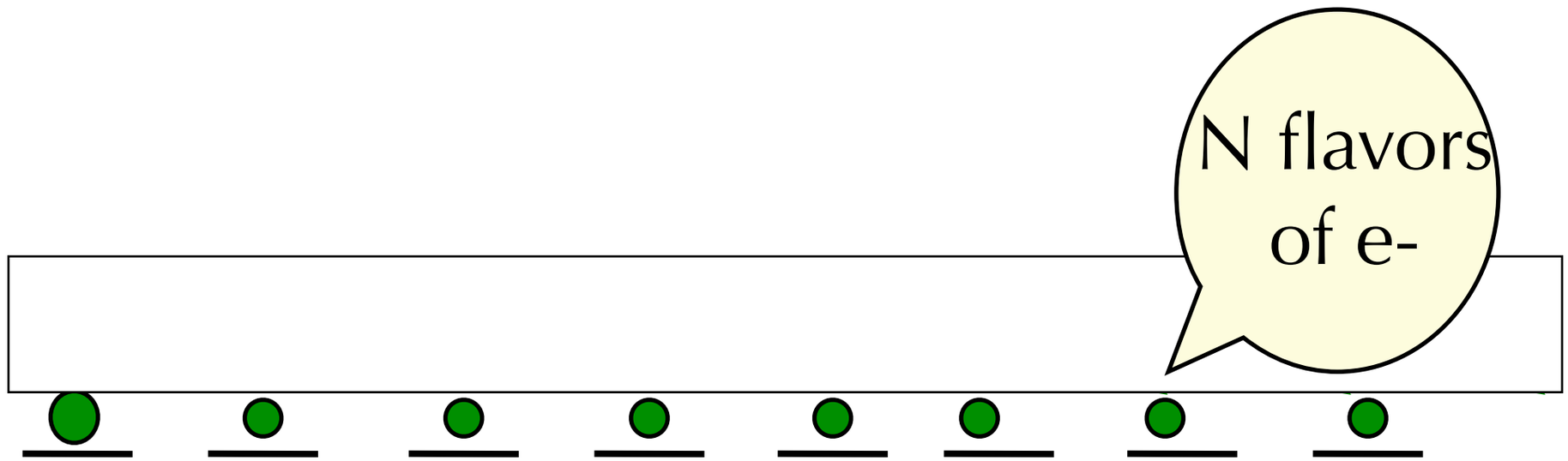


A model with zeros but Luttinger fails



no hopping=> no propagation (zeros)

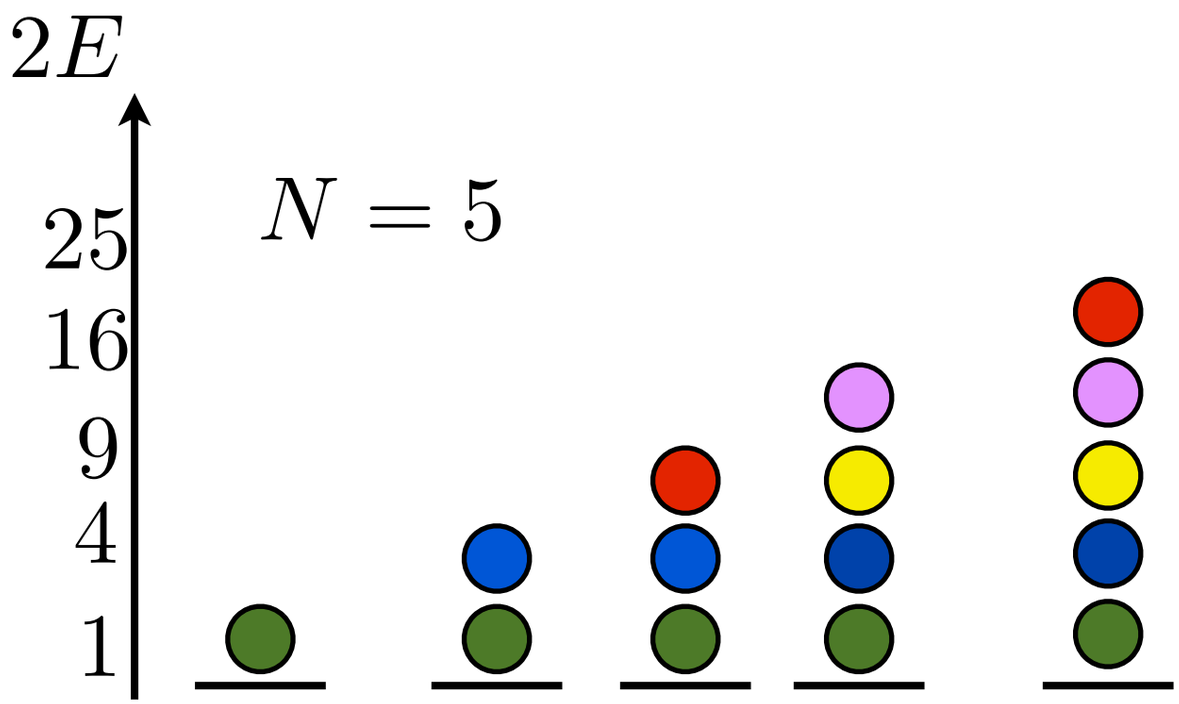
A model with zeros but Luttinger fails



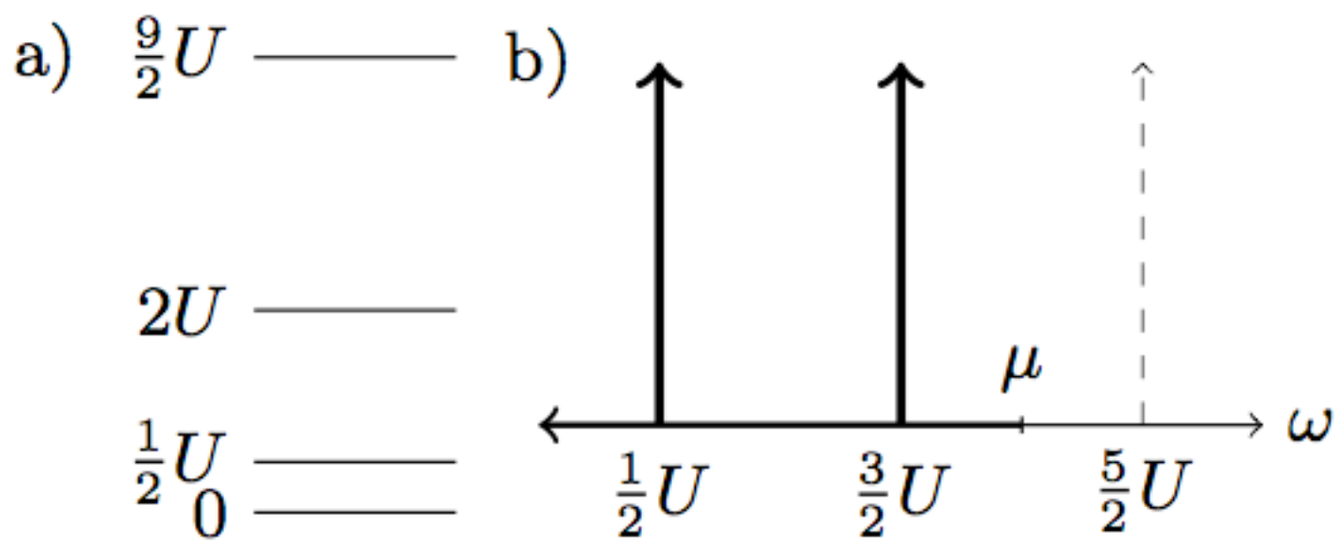
no hopping=> no propagation (zeros)

$$SU(N)$$

$$H = \frac{U}{2}(n_1 + \cdots n_N)^2$$



no particle-hole symmetry



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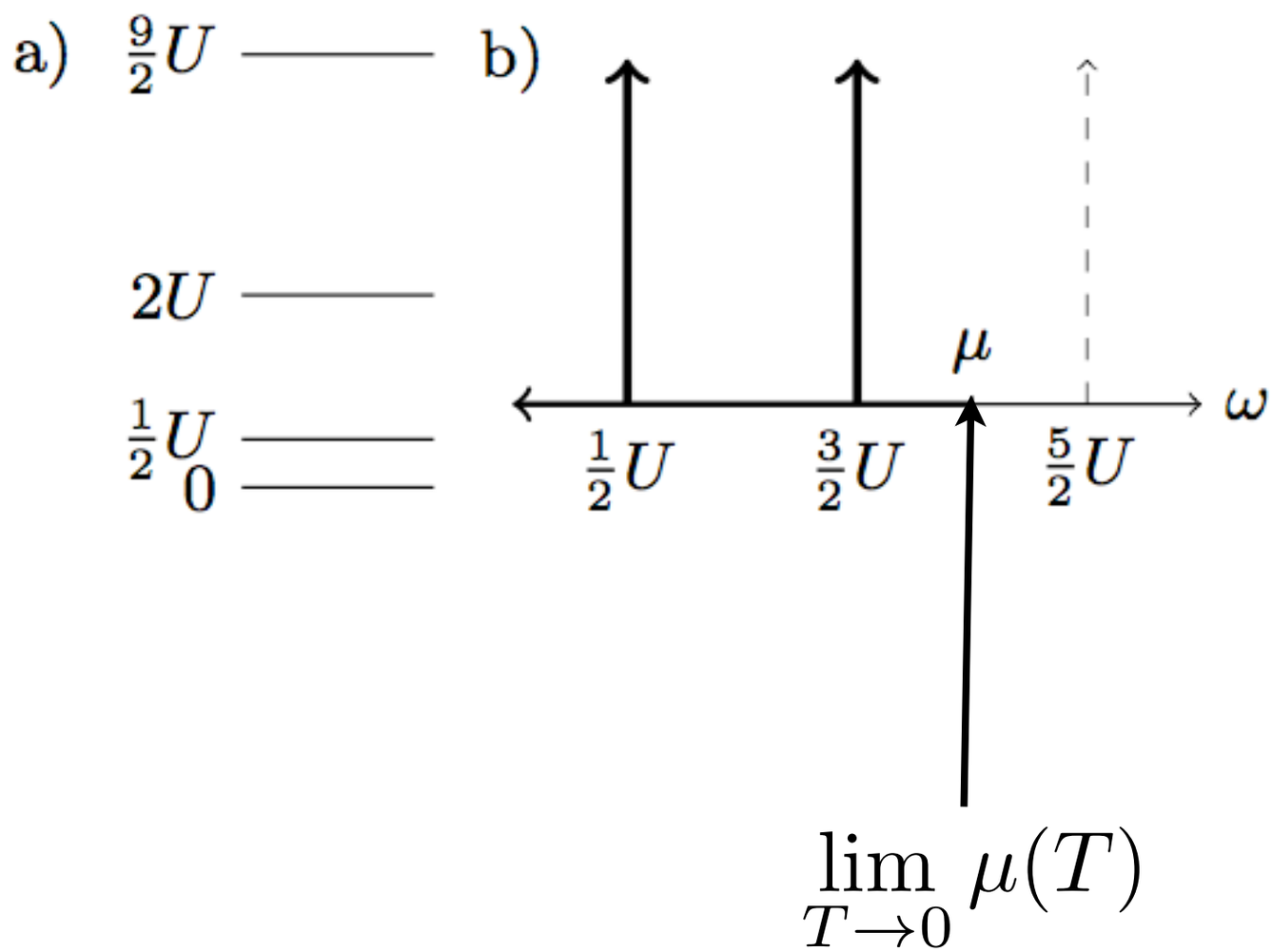


$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n+1) - K(n)} \left(\frac{2n - N}{N} \right)$$

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

Luttinger's theorem

$$n = N\Theta(2n - N)$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

$n = 2$

$N = 3$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

$$n = 2$$

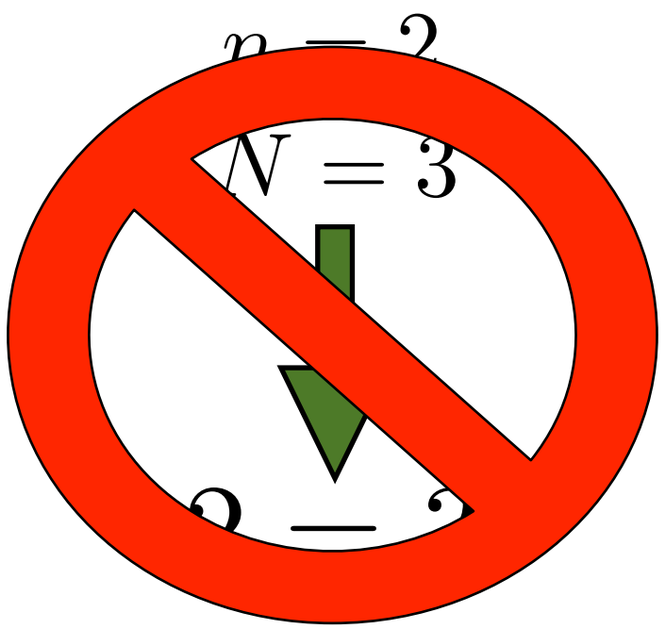
$$N = 3$$

$$\Downarrow$$

$$2 = 3$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$



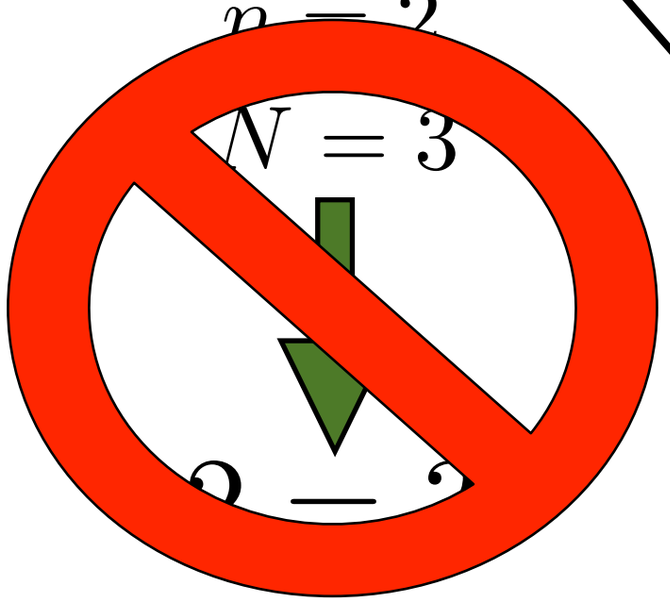
Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

even

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

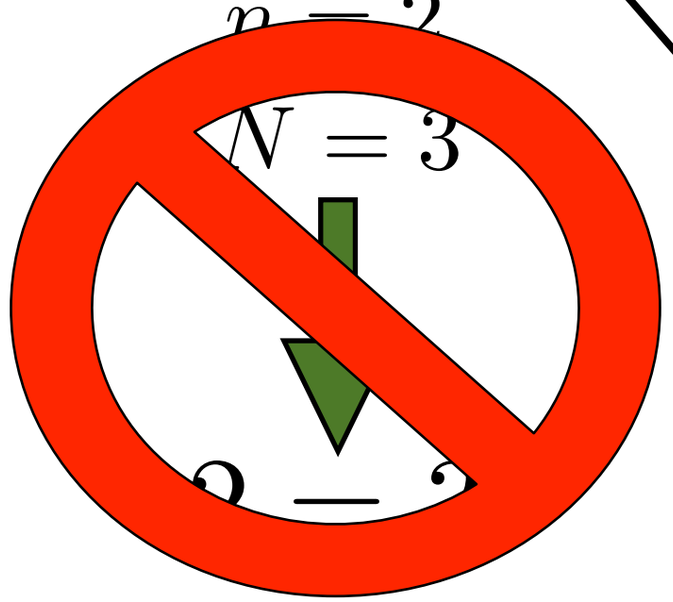


even

odd

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$



even

odd

no solution

Problem

G=0

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p - \Sigma} \rightarrow \infty$$

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p - \Sigma} \rightarrow \infty$$

lifetime of a particle vanishes

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p - \Sigma}$$

∞

$\Im\Sigma < \epsilon_p$

lifetime of a particle vanishes

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p - \Sigma}$$

$\Sigma \to \infty$

$\Im\Sigma < \epsilon_p$

lifetime of a particle vanishes

no particle

what went wrong?

what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$

what went wrong?

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$$\Big\Downarrow \text{ if } \Sigma \to \infty$$

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integral does not exist

what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$

$$\Big\Downarrow \text{ if } \Sigma \to \infty$$

integral does not exist

$$\Big\Downarrow$$

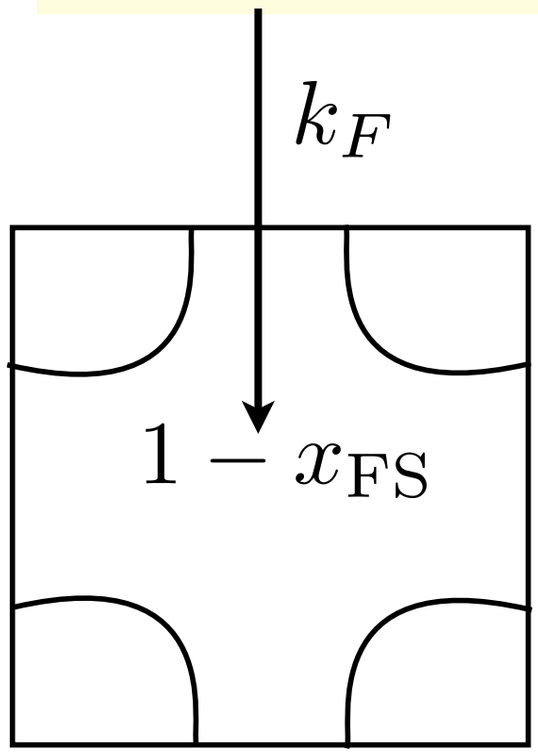
No Luttinger theorem!

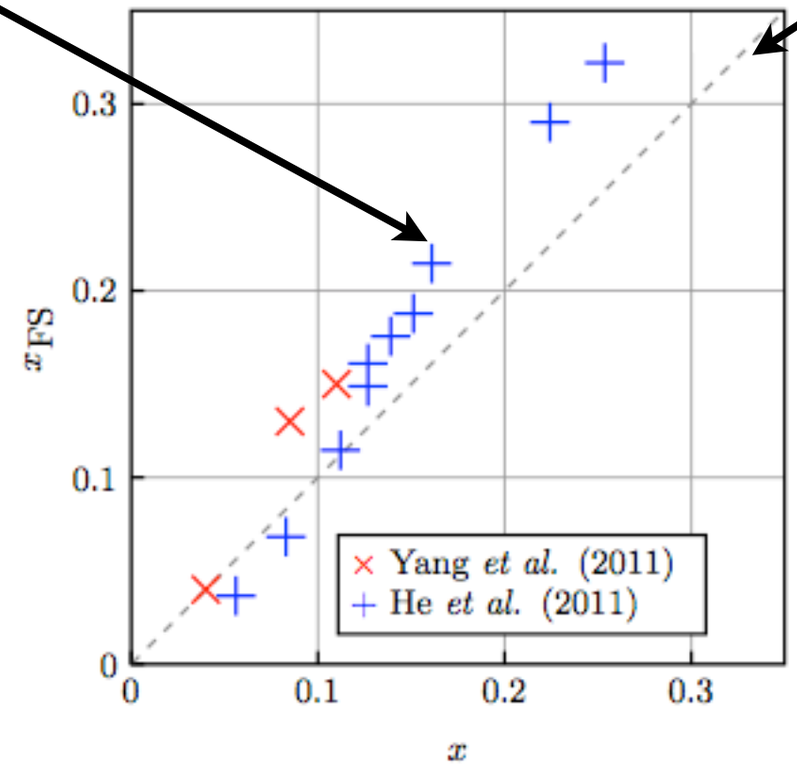
Luttinger's theorem

experimental
confirmation
of violation?

experimental data (LSCO)

`Luttinger' count

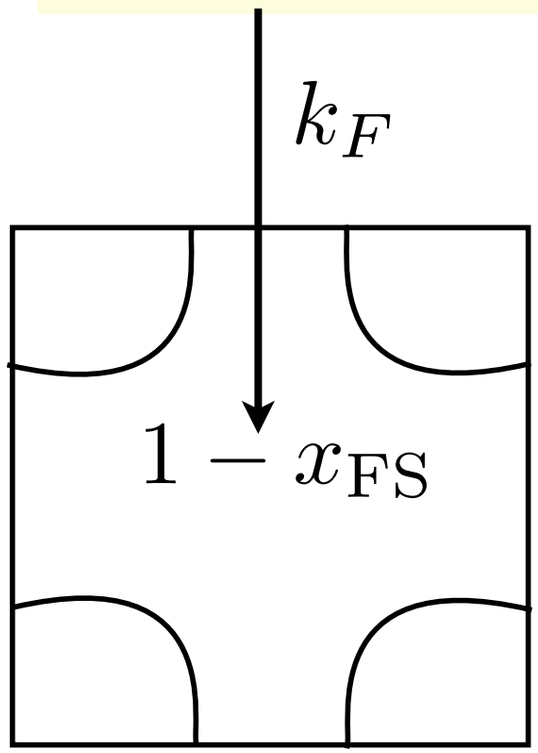


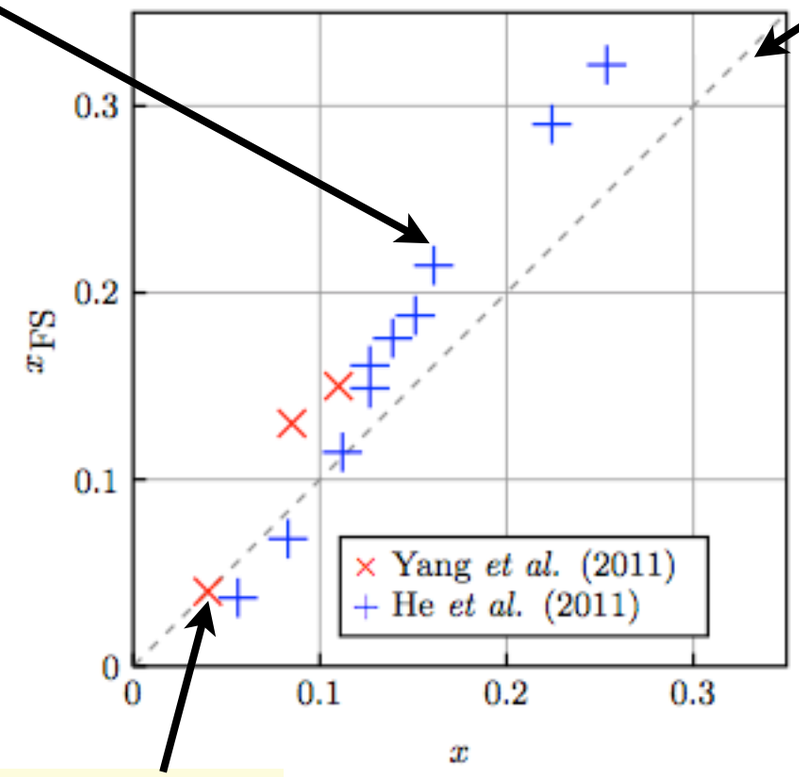


zeros do not
affect the particle density

experimental data (LSCO)

`Luttinger' count



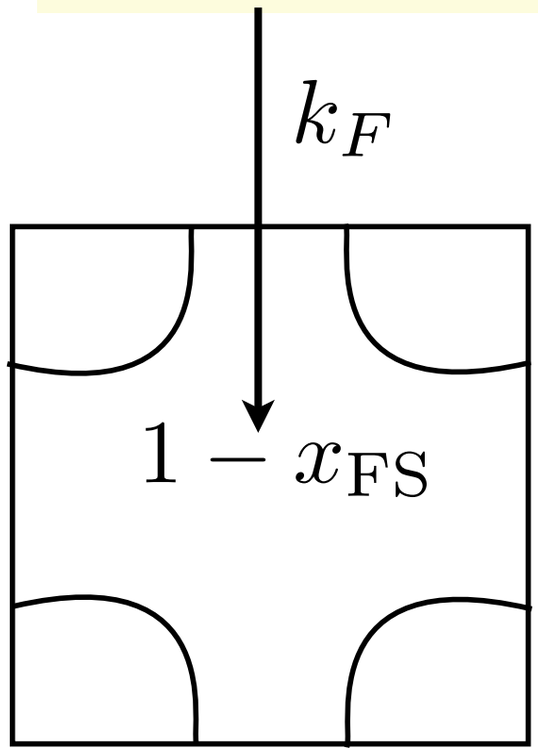


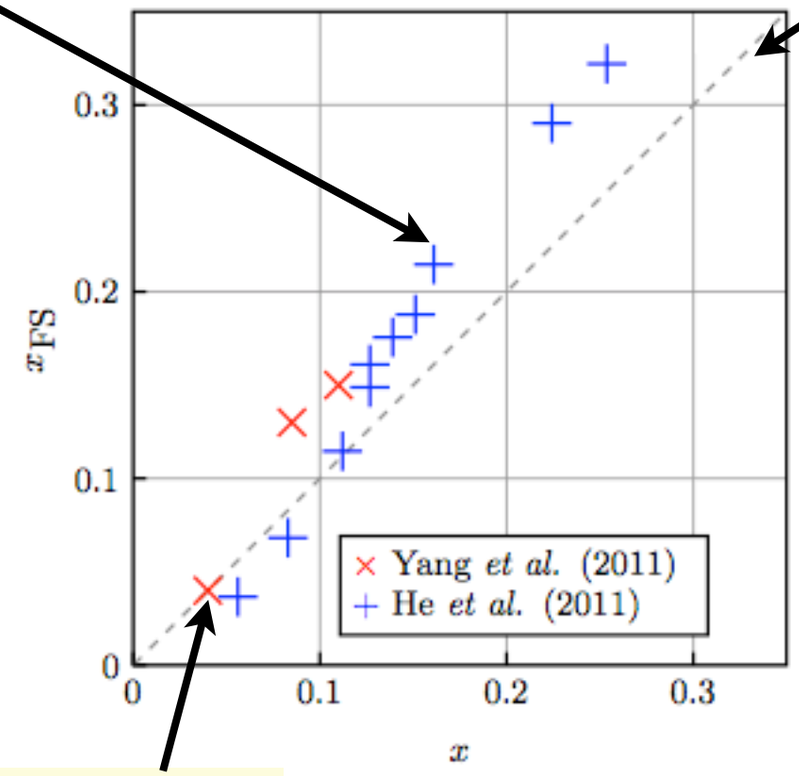
Bi2212

zeros do not affect the particle density

experimental data (LSCO)

`Luttinger' count



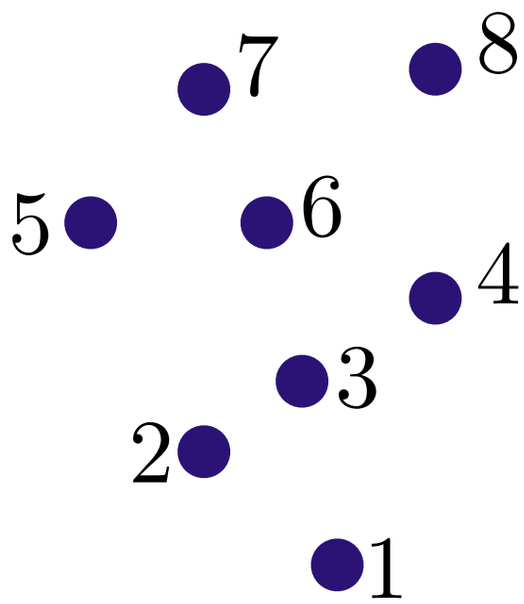


Bi2212

zeros do not
affect the particle density

each hole $\neq$ a single k-state

how to count particles?



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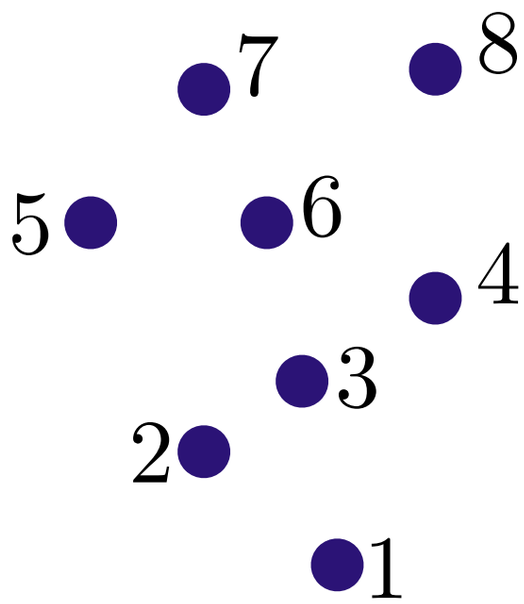
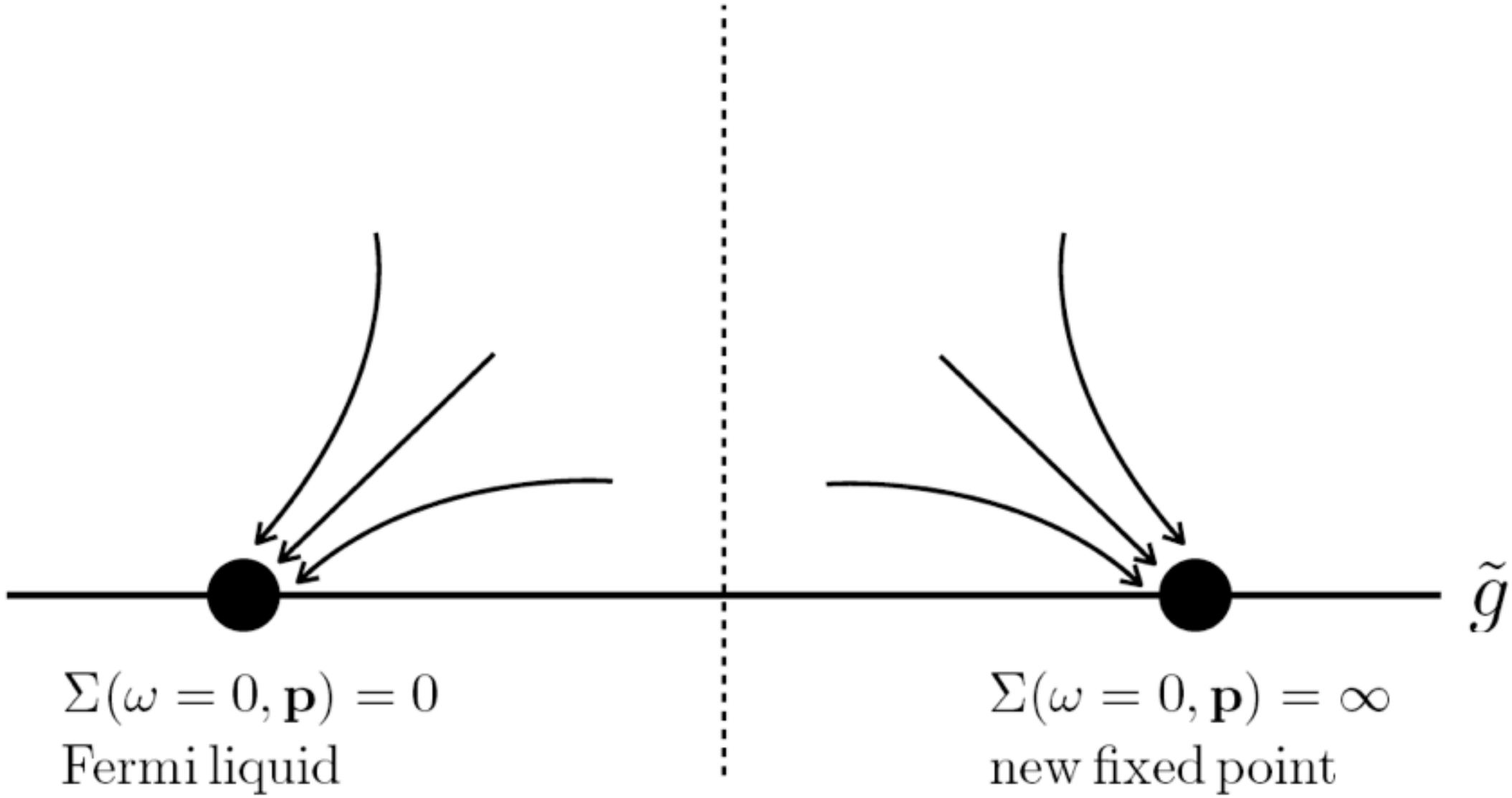
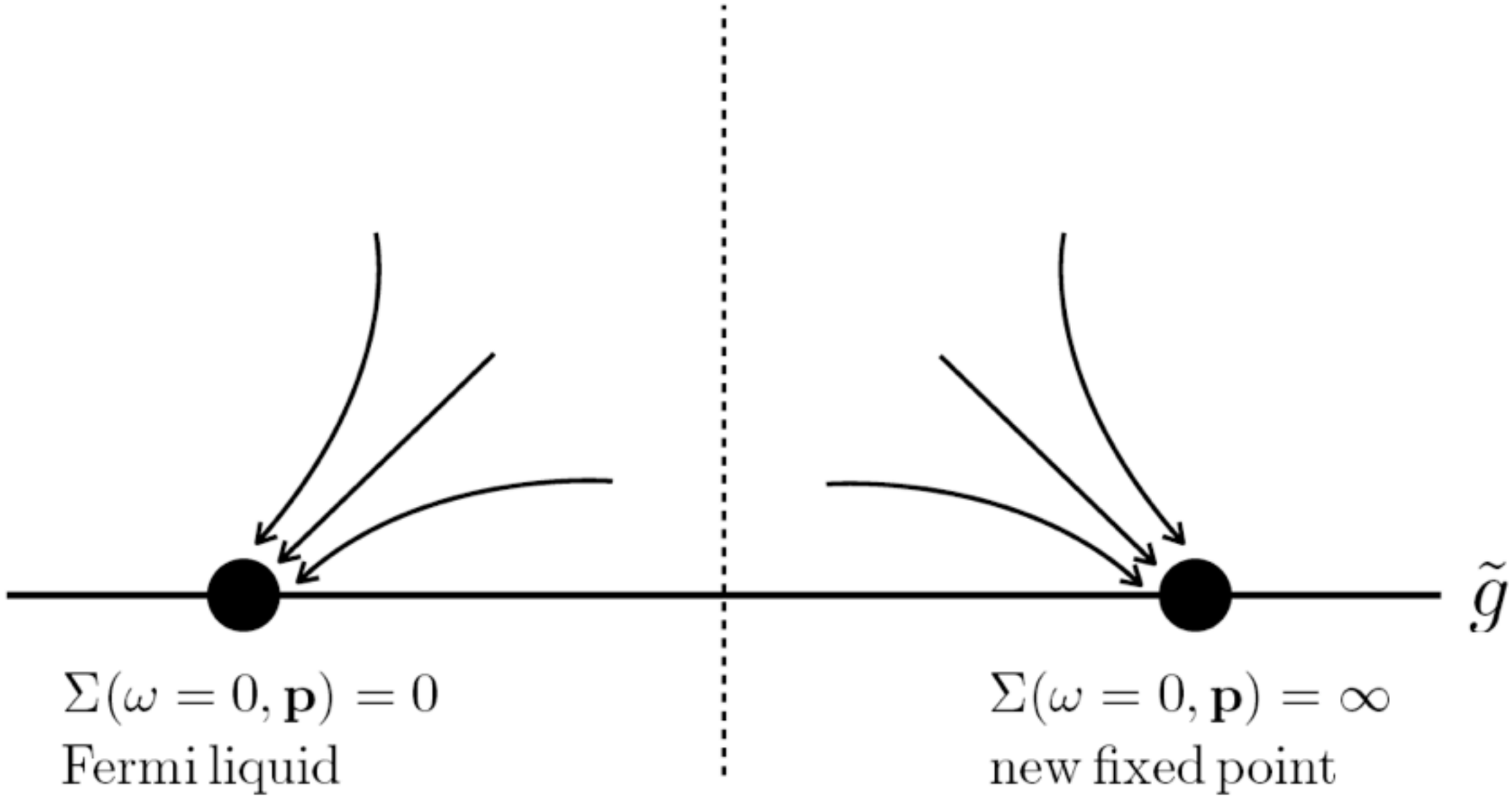


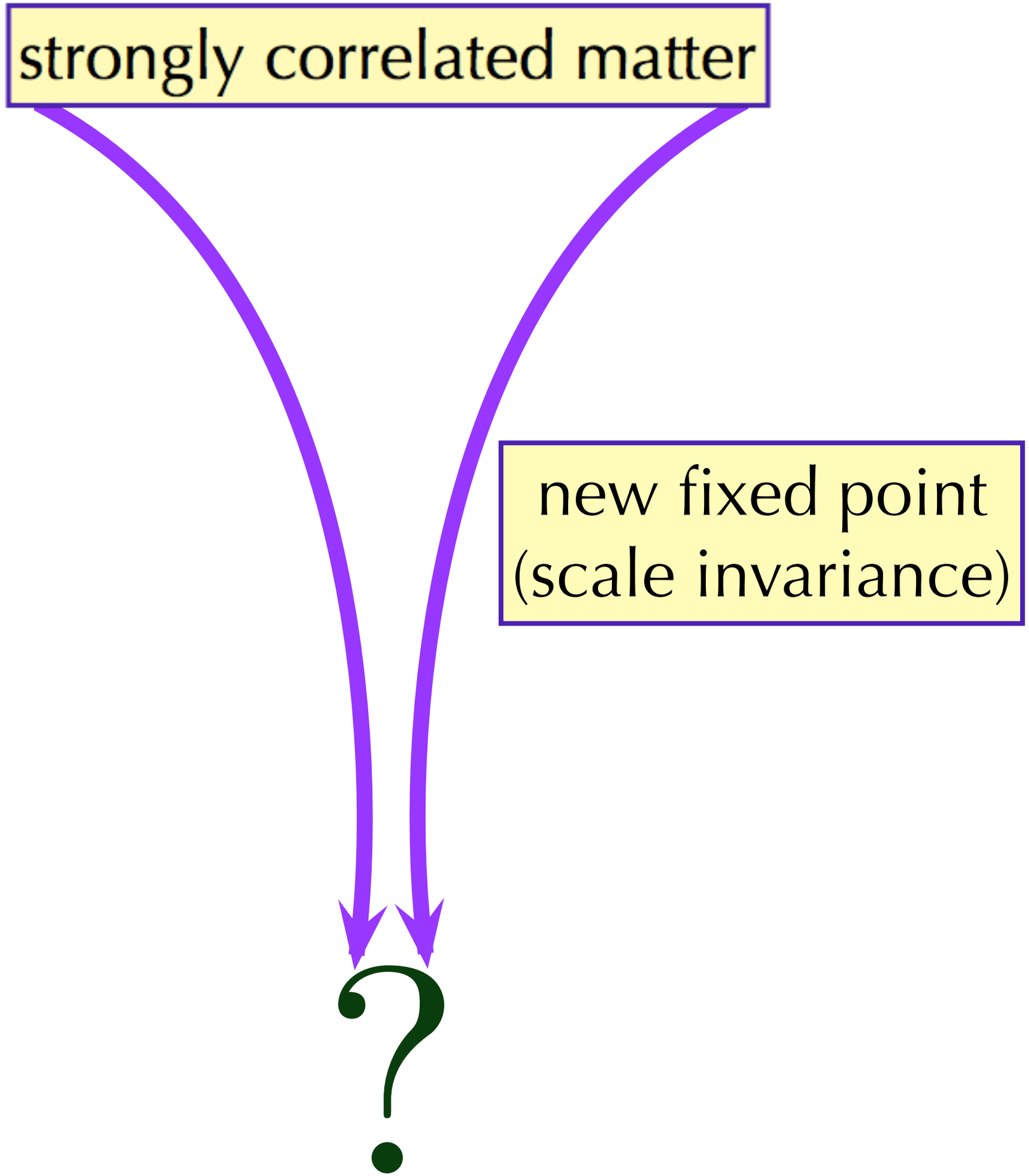
some charged stuff
has no particle interpretation

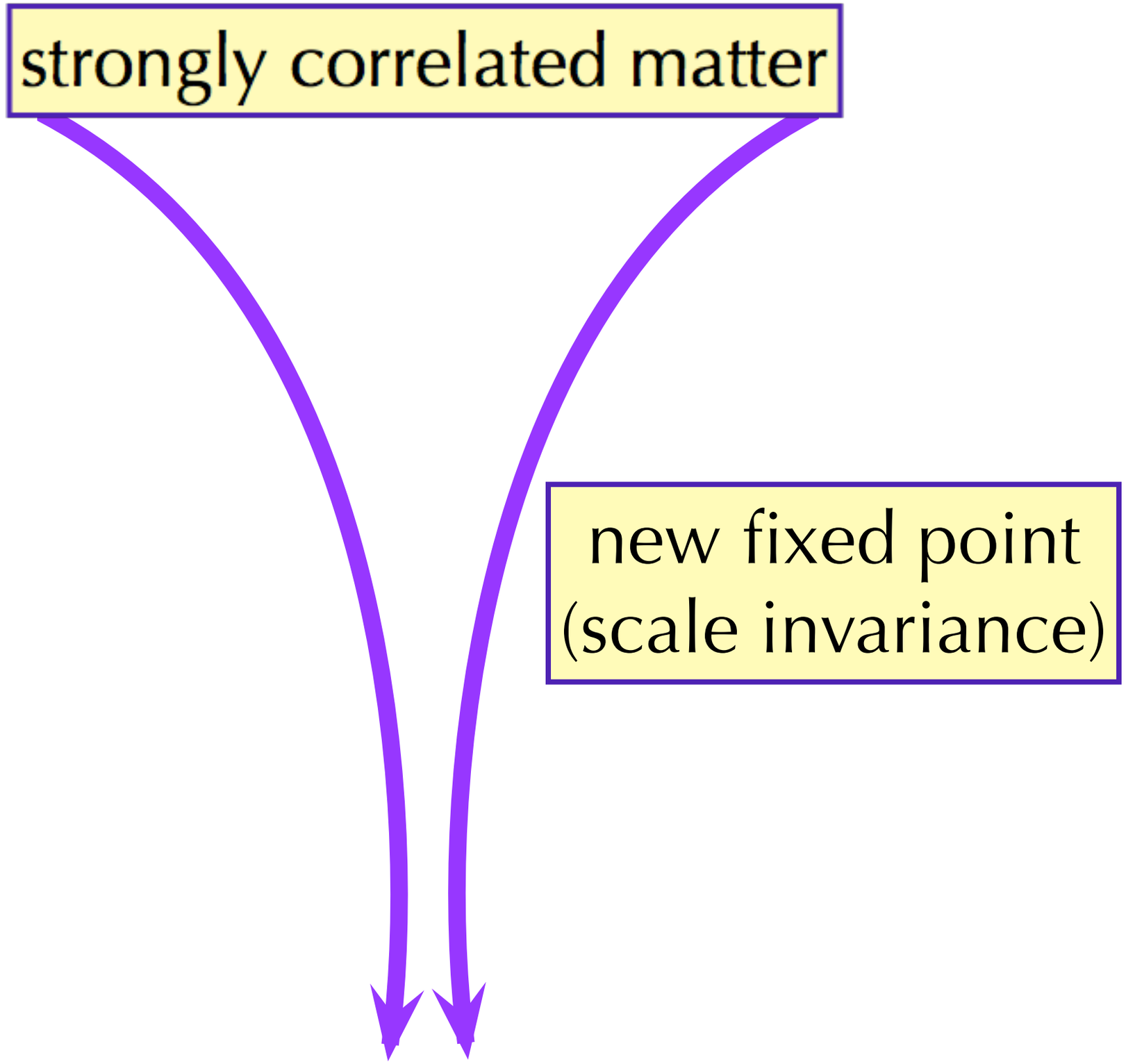
what is the extra stuff?


$\tilde{g}$
$\Sigma(\omega = 0, \mathbf{p}) = 0$
Fermi liquid
$\Sigma(\omega = 0, \mathbf{p}) = \infty$
new fixed point



scale invariance


strongly correlated matter
new fixed point
(scale invariance)
?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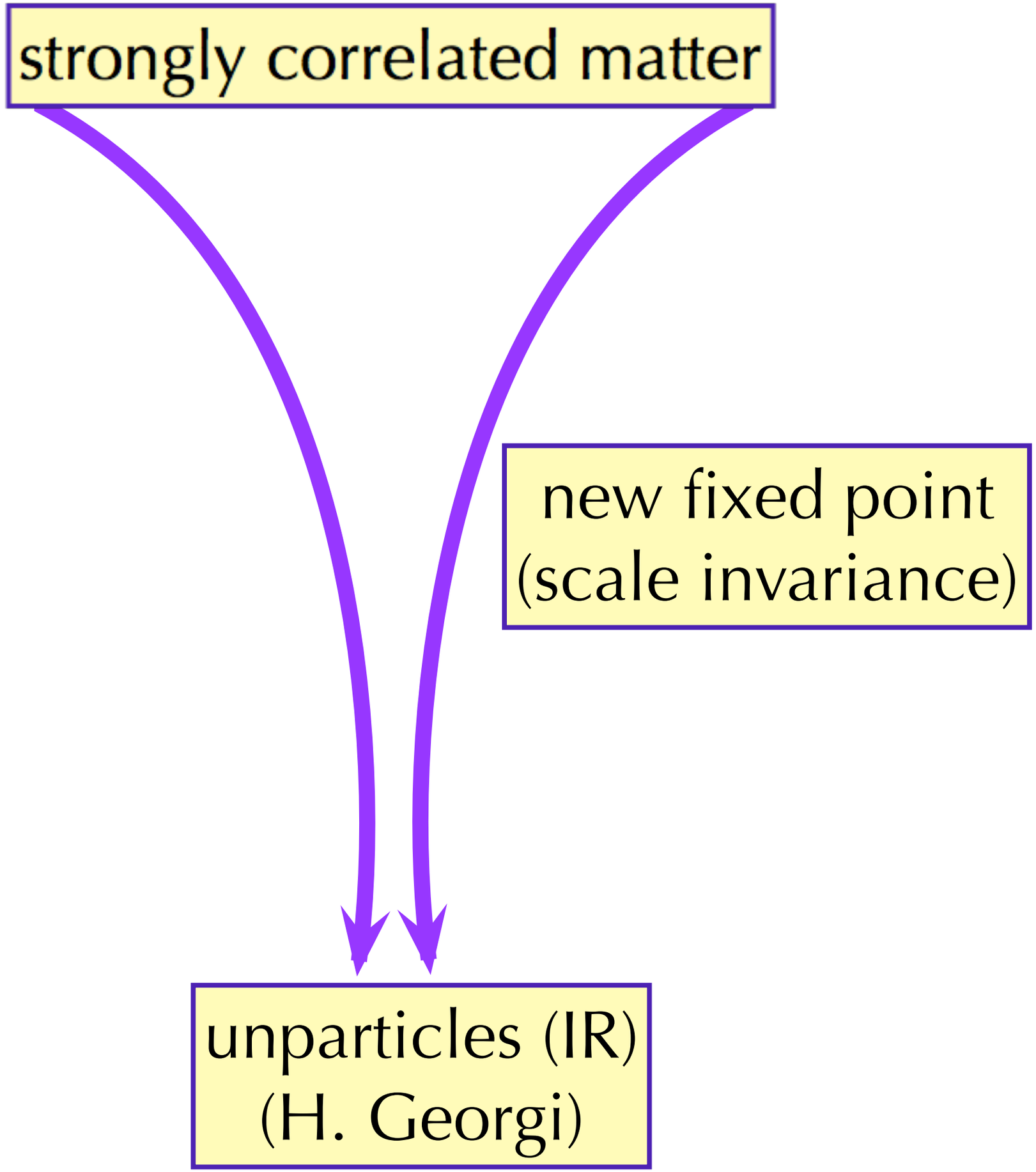
new fixed point
(scale invariance)


strongly correlated matter
new fixed point
(scale invariance)
unparticles (IR)
(H. Georgi)

what's the underlying theory?

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi + m^2 \phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$
$$\phi(x) \to \phi(x)$$

massive free theory

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$$m^2\phi^2$$

no scale invariance

$$\mathcal{L} = \quad \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right)$$

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2 \phi^2(x,m)\right) dm^2$$

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theory with all possible mass!

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theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
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$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} \to \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

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not particles

unparticles

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not particles

propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

propagator

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$$\downarrow$$

$$d_U - 2$$

propagator

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$$\downarrow$$

$$d_U - 2$$

S. Lee, S. Kachru,...

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

$$G \propto (E - \varepsilon_p)^{d_U - 2}$$

n massless particles

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unparticles=fractional number of massless particles

n massless particles

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unparticles=fractional number of massless particles

$d_U < 2$

almost Luttinger liquid
no pole at p_F

n massless particles

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unparticles=fractional number of massless particles

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$d_U < 2$

zeros

almost Luttinger liquid
no pole at p_F

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unparticles=fractional number of massless particles

$d_U > 2$

$d_U < 2$

zeros

no simple sign change

almost Luttinger liquid

no pole at p_F

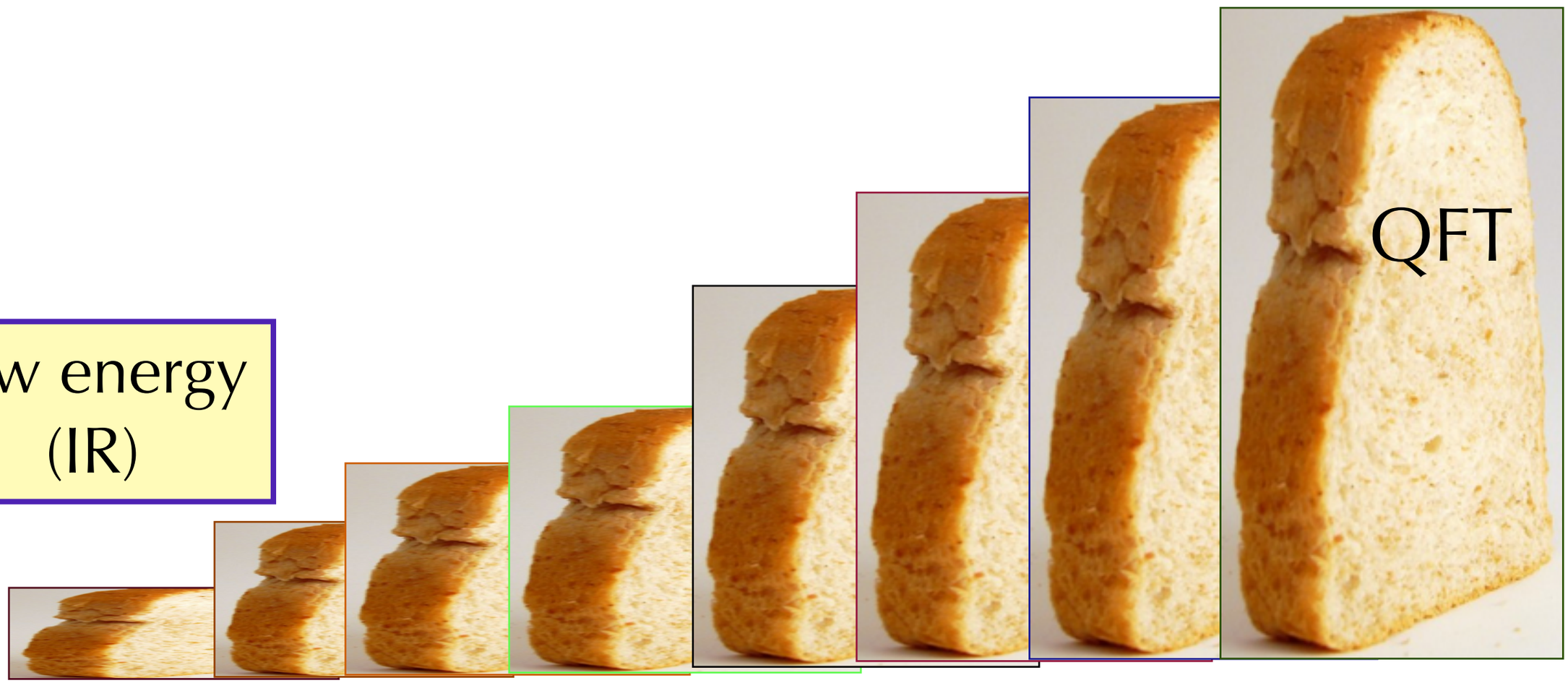
$$d_U\,?$$

what really is the summation over mass?

what really is the summation over mass?

mass=energy

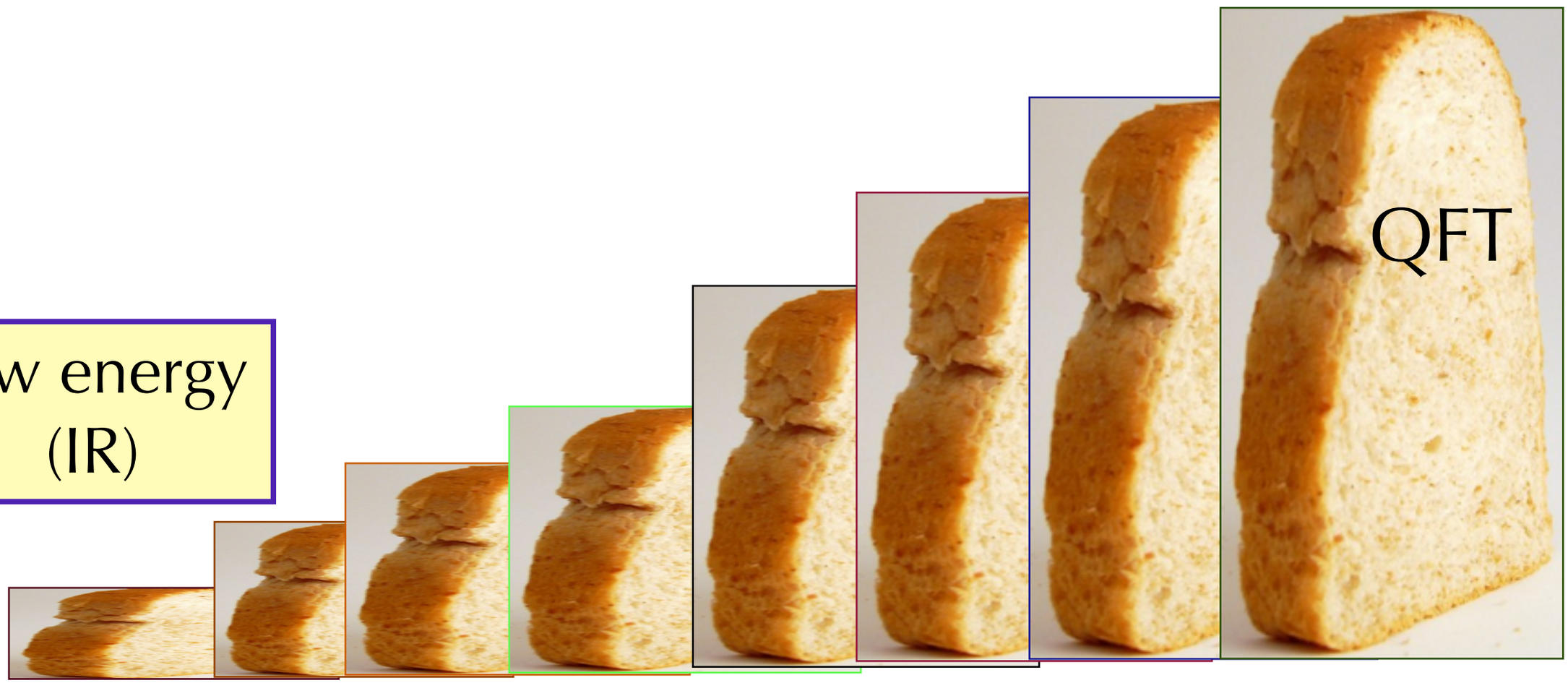
high energy
(UV)



low energy
(IR)

related to sum
over mass

high energy
(UV)



low energy
(IR)

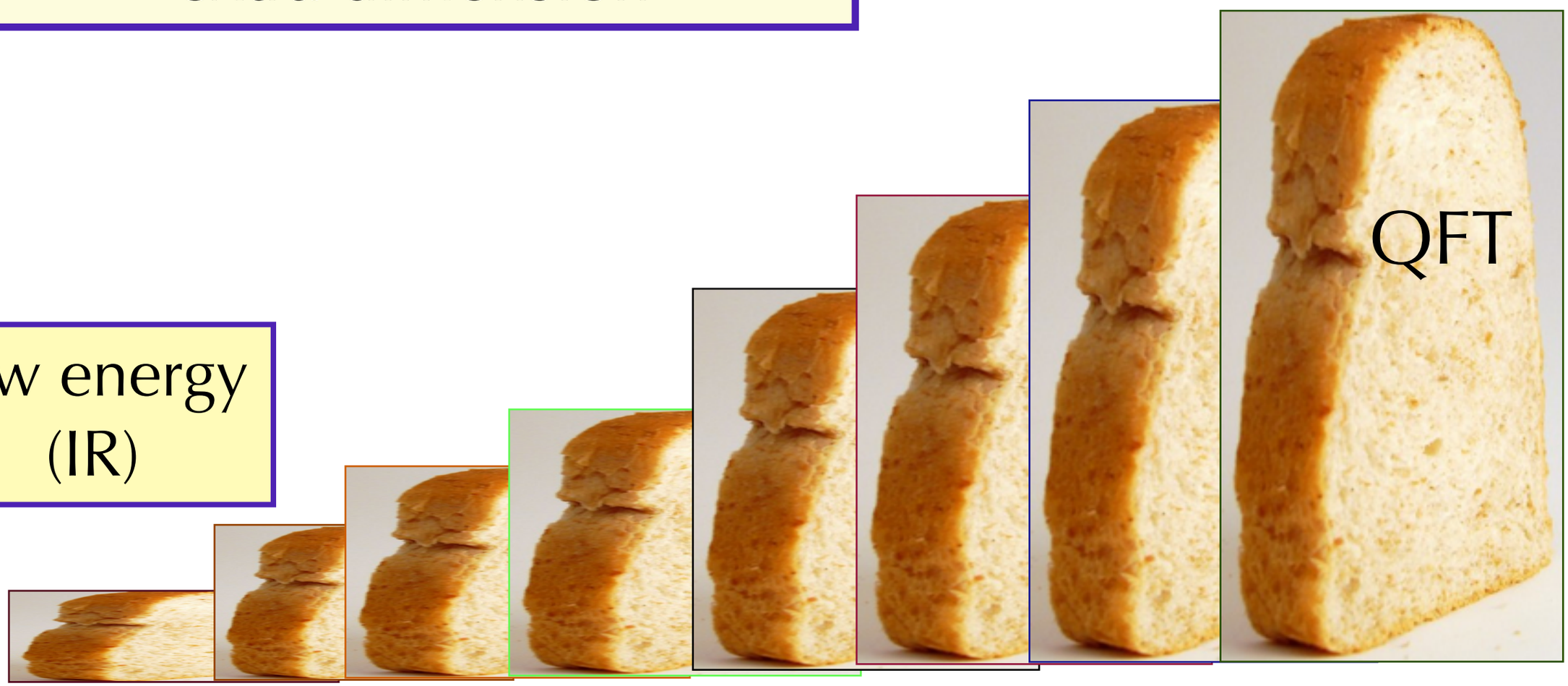
$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

related to sum
over mass

locality in energy

implement E-scaling with an extra dimension

high energy (UV)



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QFT

$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

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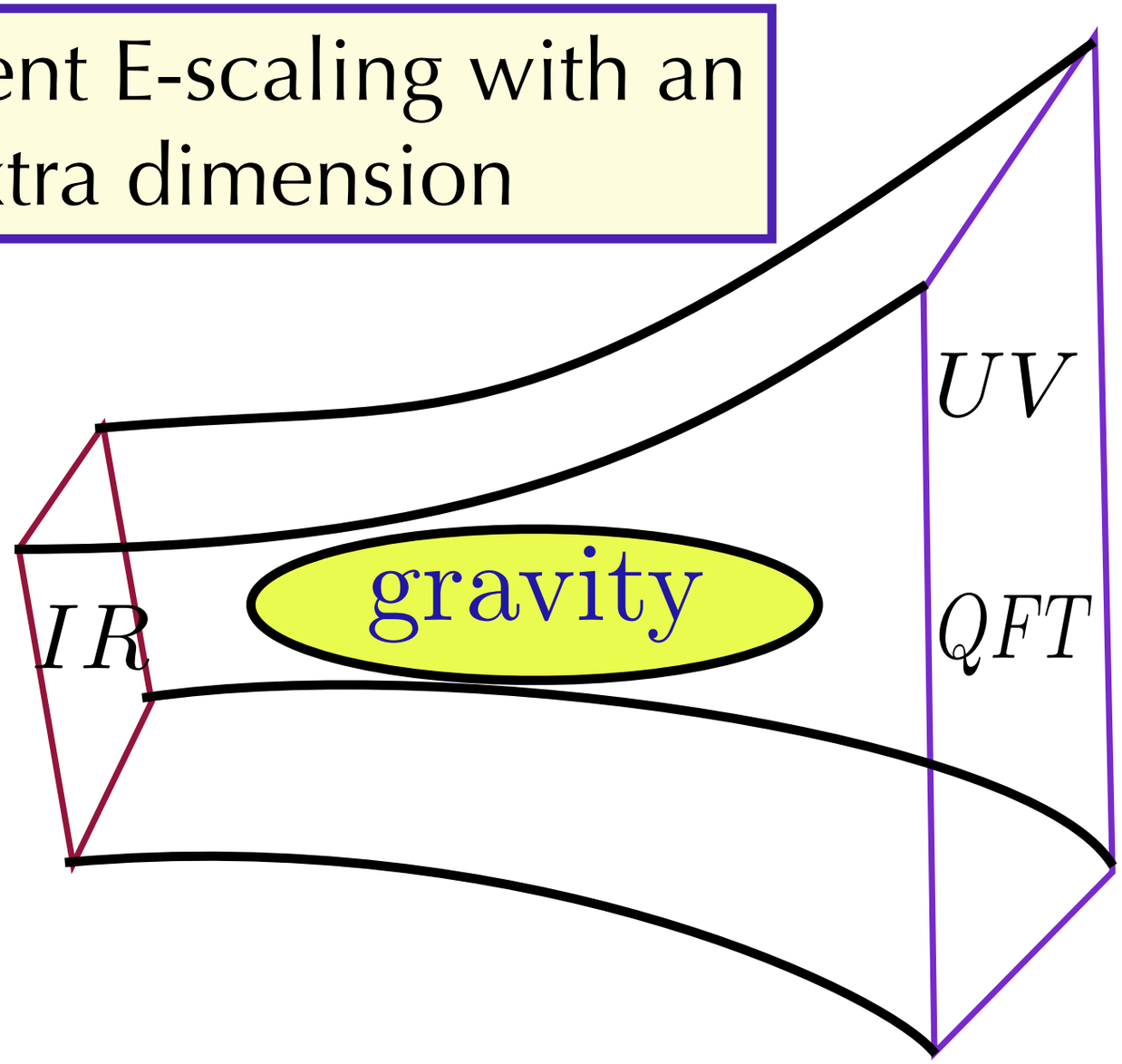
gauge-gravity duality
(Maldacena, 1997)

implement E-scaling with an extra dimension



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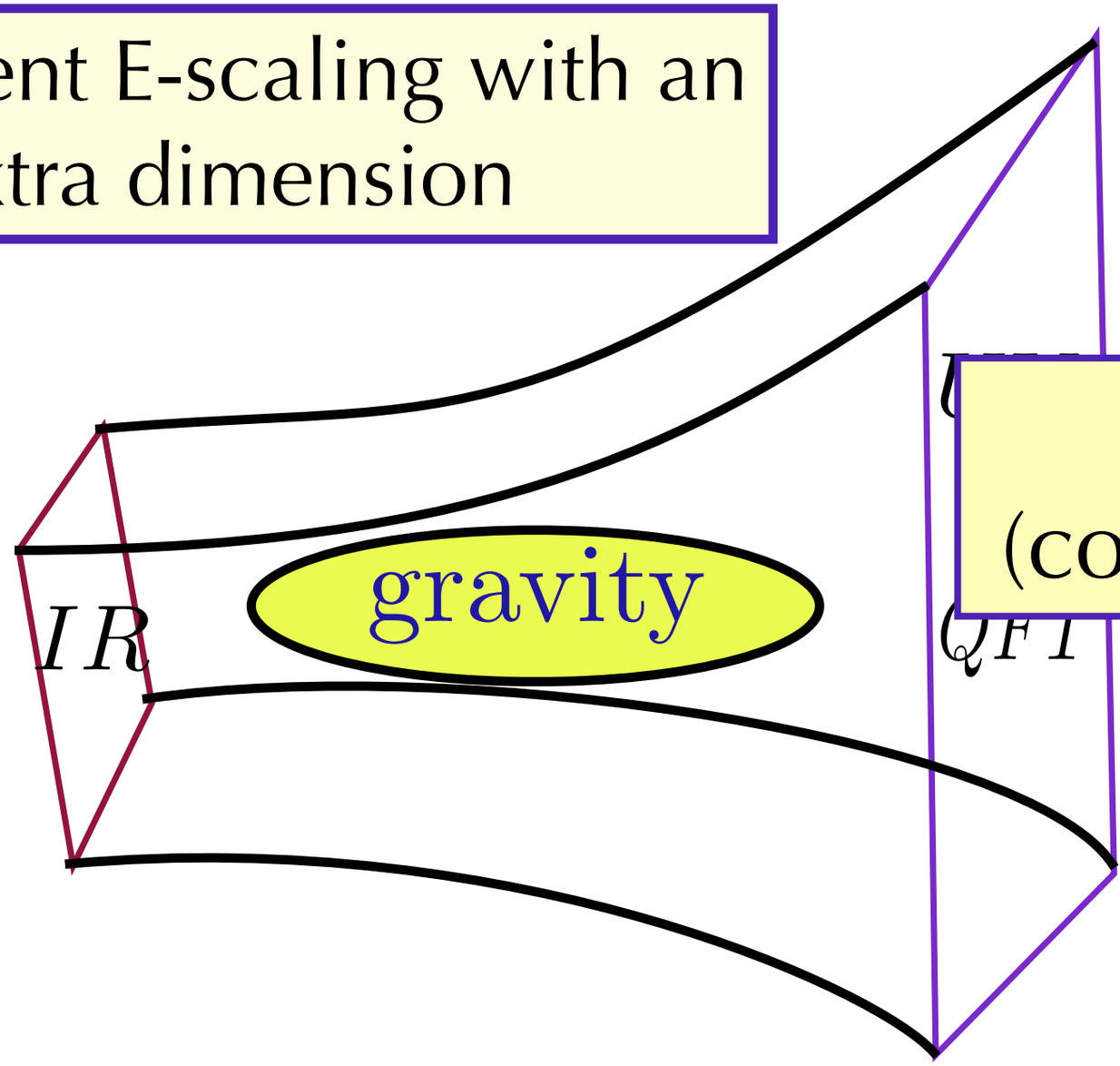


no particles
(conserved currents)

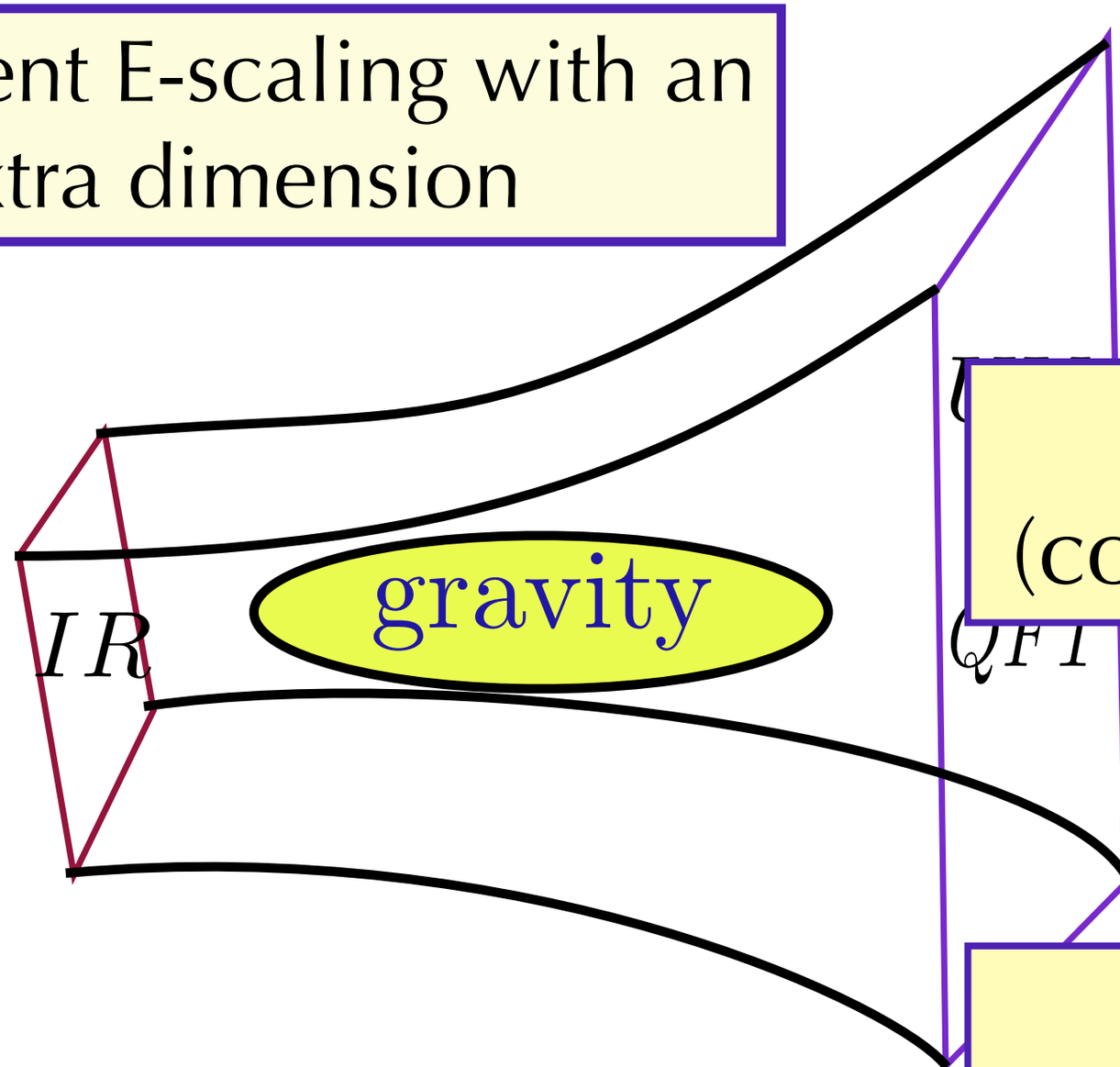
$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

related to sum over mass

locality in energy

gauge-gravity duality
(Maldacena, 1997)

implement E-scaling with an extra dimension



no particles
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unparticles?

$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

related to sum over mass

locality in energy

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

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$$\mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[\frac{1}{2}\frac{z^2}{R^2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{\phi^2}{2R^2}\right]$$

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

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can be absorbed with AdS metric

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$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

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can be absorbed with AdS metric

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$$m^2_{\mathrm{AdS}} = \frac{d_U(d_U - d)}{R^2}$$

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$$\Big\downarrow \; m = 1/L$$

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$$\boxed{\frac{1}{R^2}} = \frac{d_U(d_U - d)}{R^2}$$

$$m^2_{\mathrm{AdS}} = \frac{d_U(d_U - d)}{R^2}$$

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$$\boxed{\frac{1}{R^2}} = \frac{d_U(d_U - d)}{R^2}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

unparticle (AdS) propagator has zeros!

$$G_U(p) \propto p^{2(d_U - d/2)}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

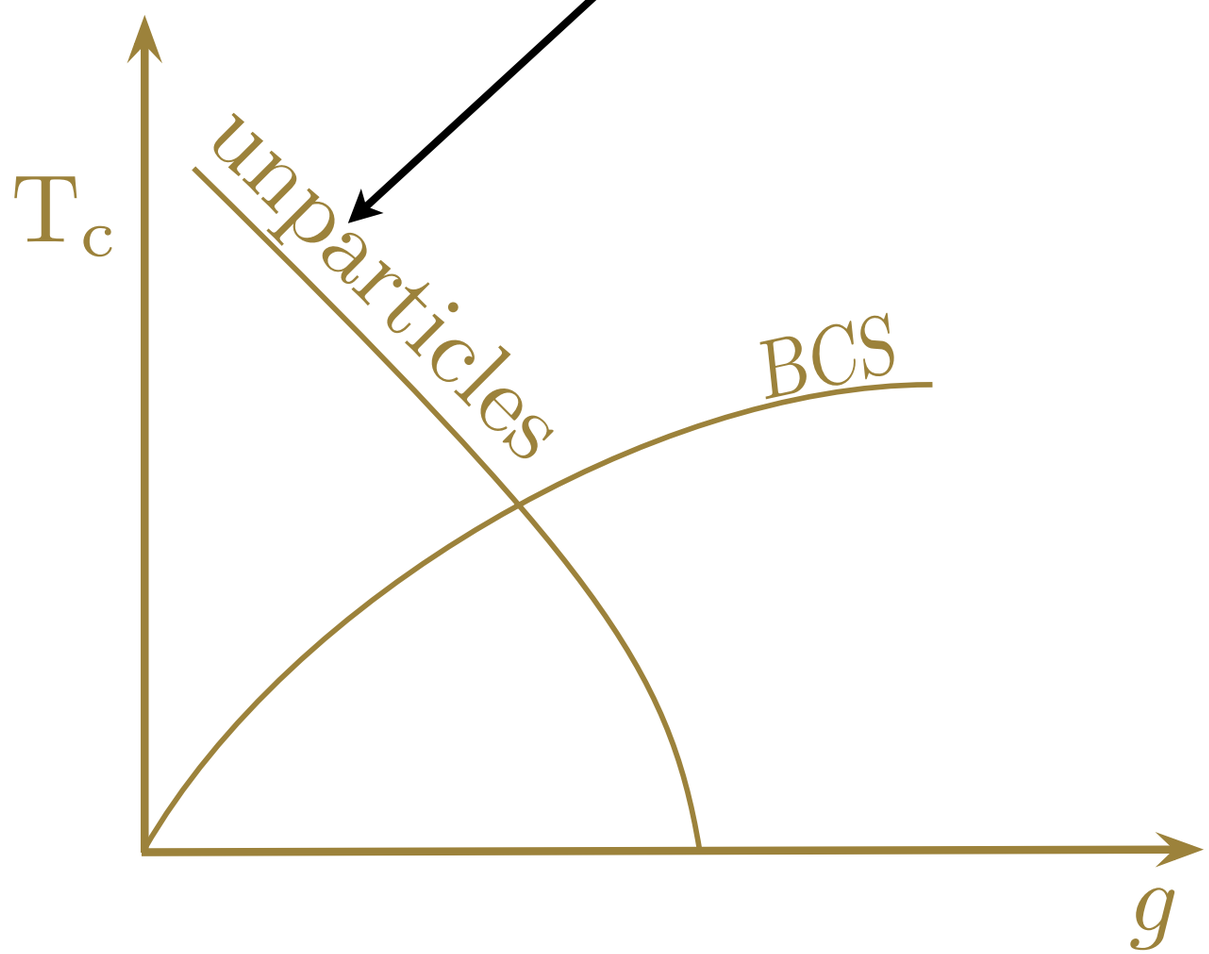
$$G_U(0) = 0$$

Superconducting Instability

ladder approximation

$$1 = \lambda T \sum_{n\vec{k}} |w_{n\vec{k}}|^2 G_U\left(\omega_n, \vec{k}\right) G_U\left(-\omega_n, -\vec{k}\right),$$

$$\frac{d \ln g}{d \ln \beta} = 4d_U - d > 0$$



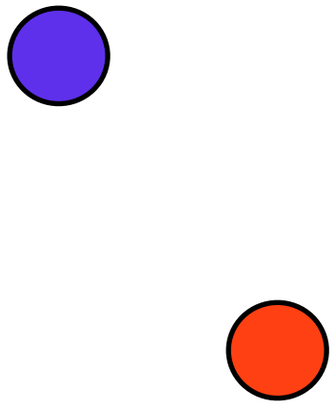
tendency towards pairing (any instability which establishes a gap)

interchanging unparticles

fractional (d_U) number of massless particles

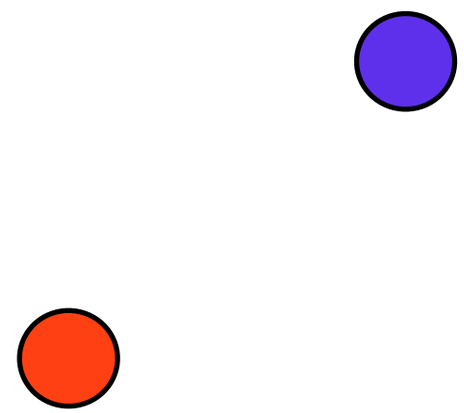
interchanging unparticles

fractional (d_U) number of massless particles



interchanging unparticles

fractional (d_U) number of massless particles



interchanging unparticles

fractional (d_U) number of massless particles

interchanging unparticles

fractional (d_U) number of massless particles

$$e^{i\pi d_U} \neq -1, 0$$

interchanging unparticles

fractional (d_U) number of massless particles

$$e^{i\pi d_U} \neq -1, 0$$

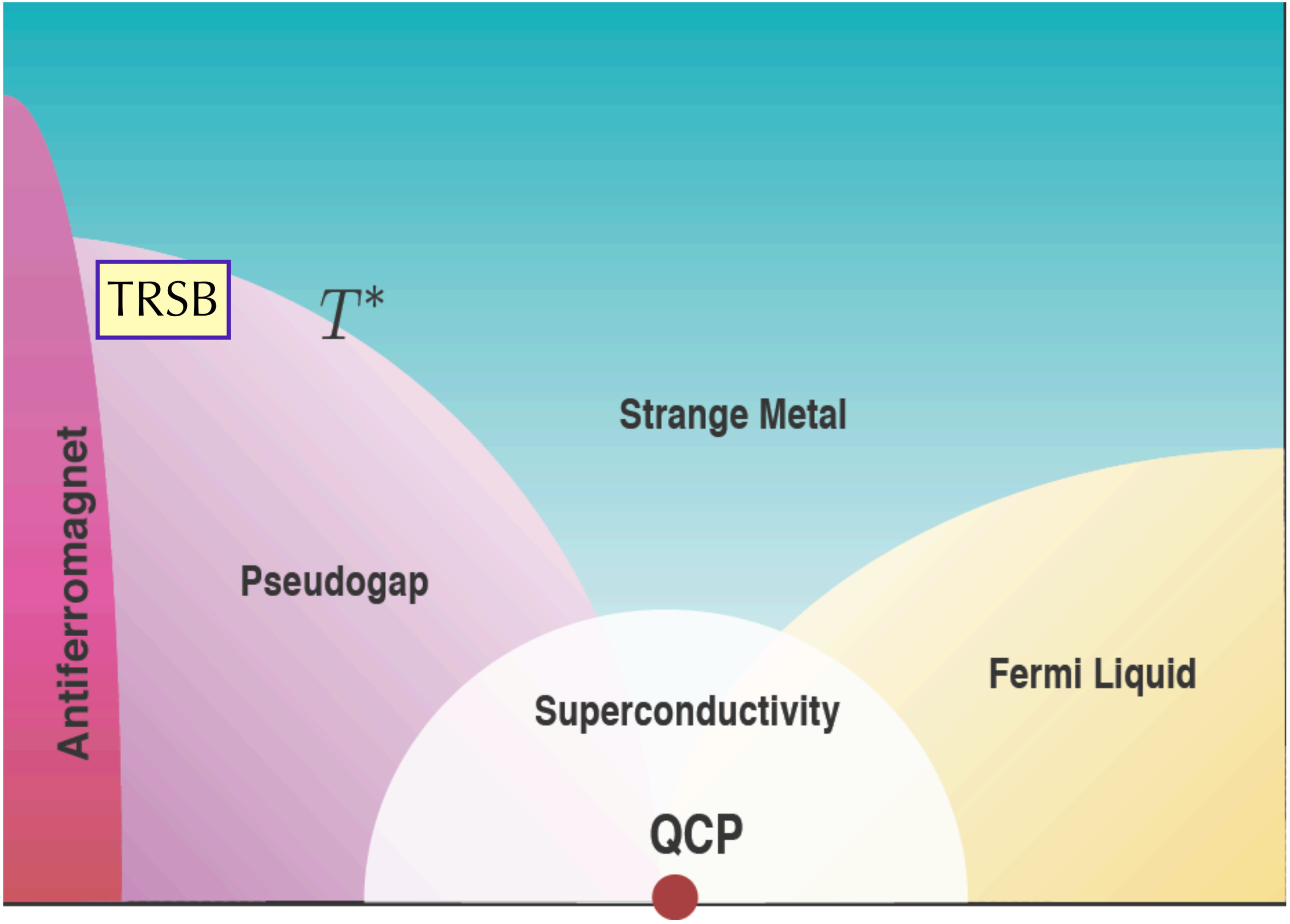
fractional statistics in d=2+1

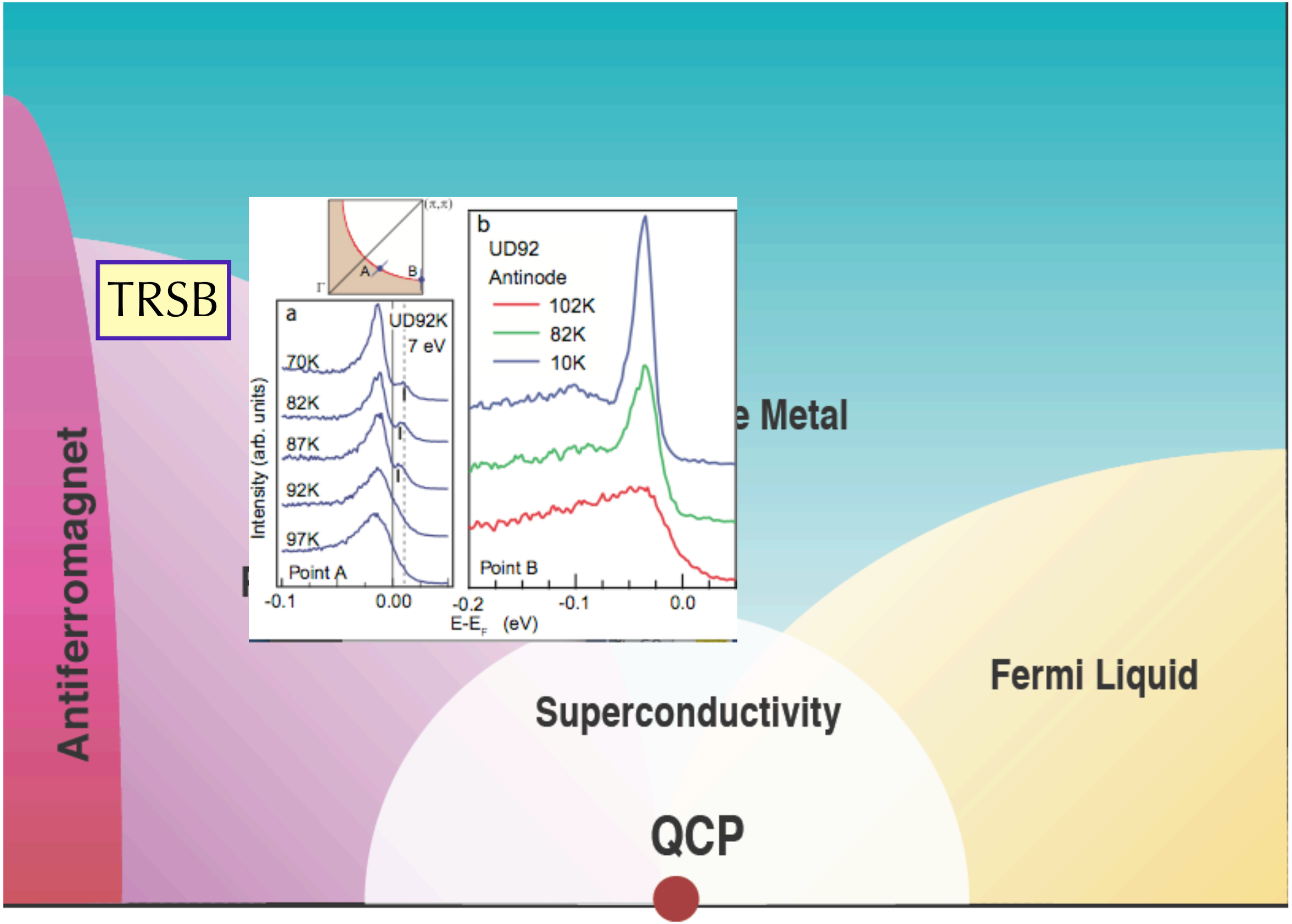
$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

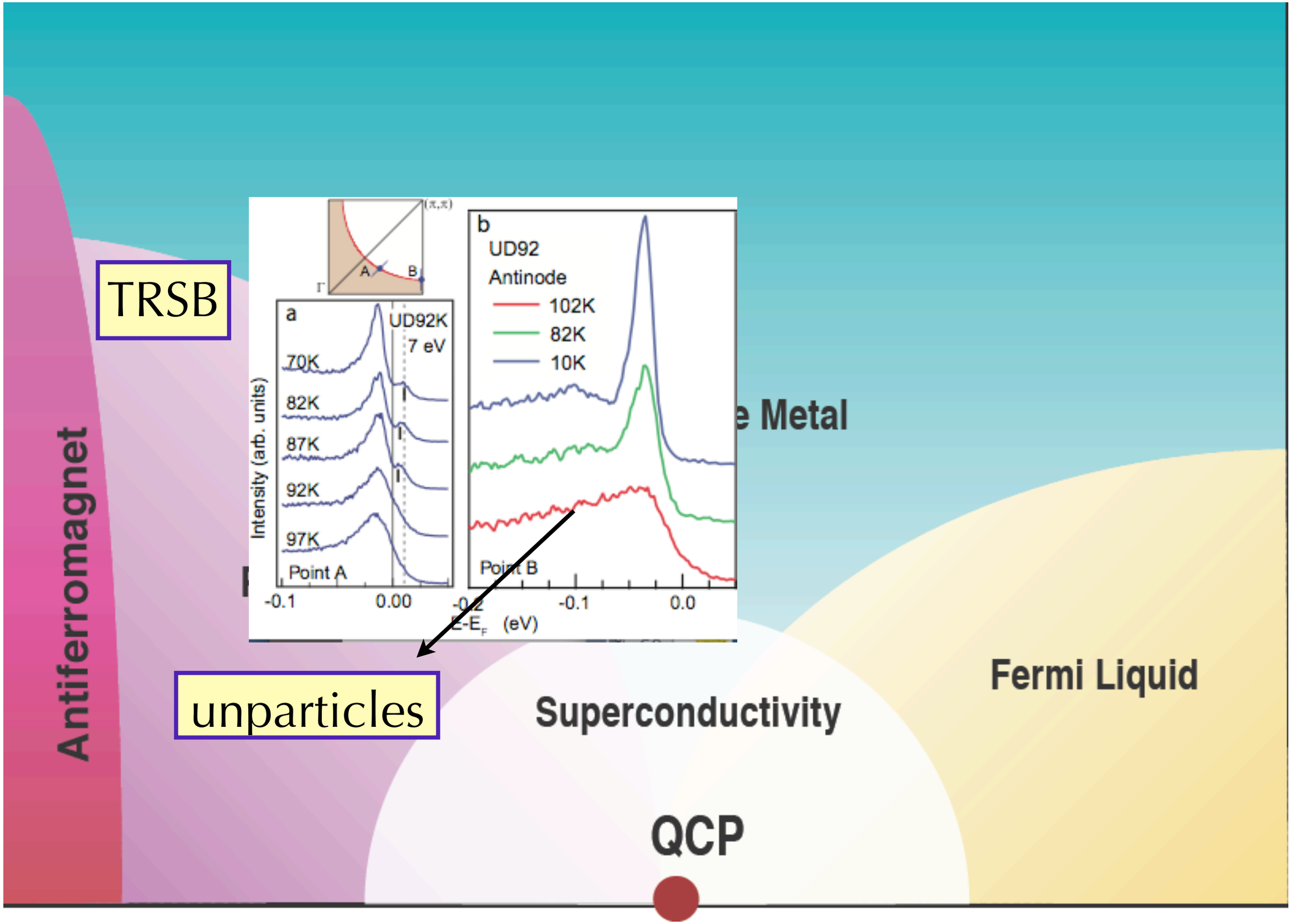
$$e^{i\pi d_U} \neq e^{-i\pi d_U}$$

$$\Downarrow$$

time-reversal symmetry breaking
from unparticle (zeros=Fermi arcs) matter


TRSB
T^*
Strange Metal
Antiferromagnet
Pseudogap
Fermi Liquid
Superconductivity
QCP


TRSB
Antiferromagnet
e Metal
Fermi Liquid
Superconductivity
QCP
(π,π)
Γ
A
B
a
UD92K
7 eV
70K
82K
87K
92K
97K
Point A
Intensity (arb. units)
-0.1
0.00
b
UD92
Antinode
102K
82K
10K
Point B
-0.2
-0.1
0.0
E-E_F (eV)


TRSB
Antiferromagnet
Metal
Fermi Liquid
Superconductivity
QCP
unparticles
(π,π)
Γ
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Antinode
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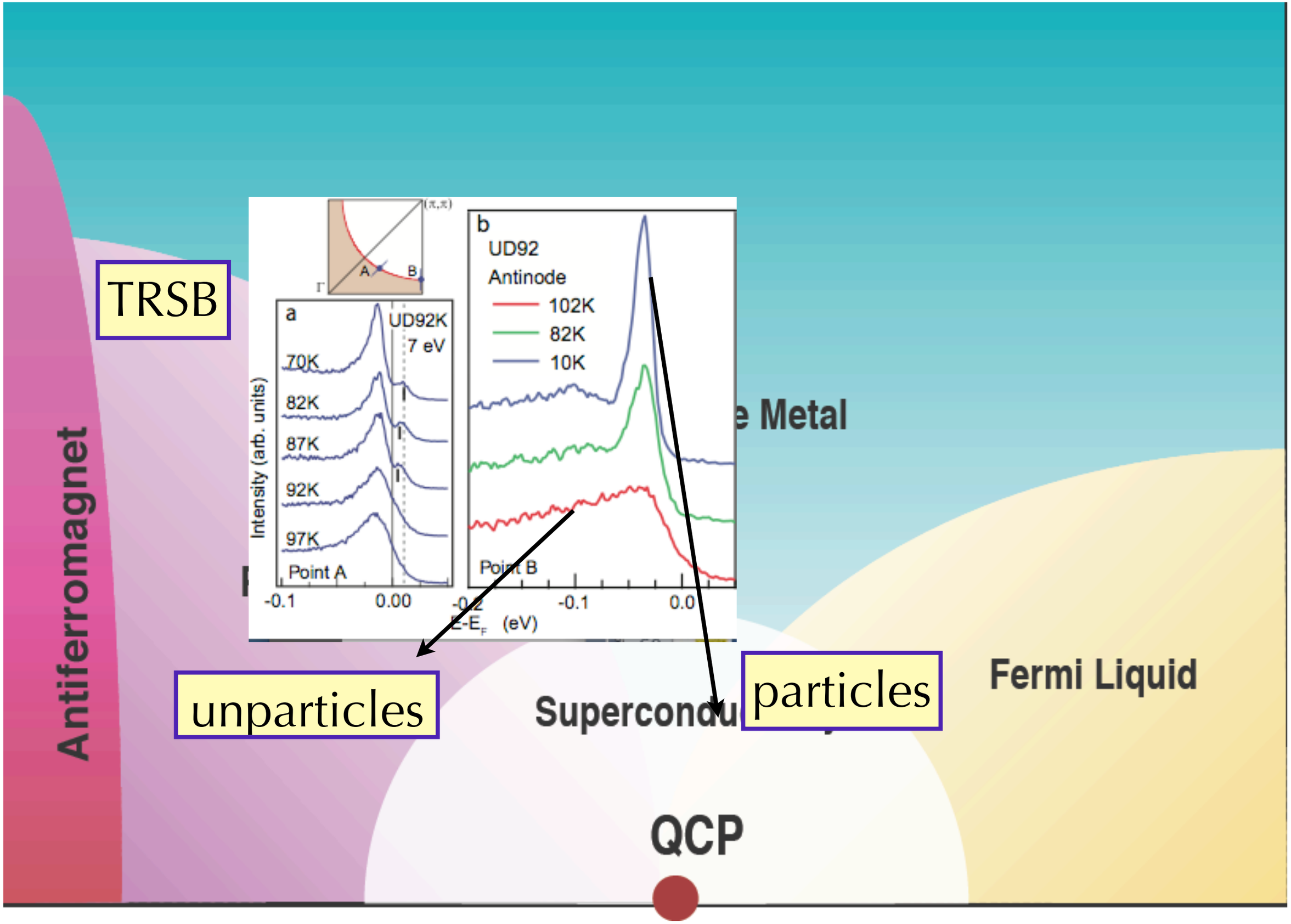
10K
Point B
-0.2
-0.1
0.0
E-E_F (eV)


TRSB
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breaking of scale invariance