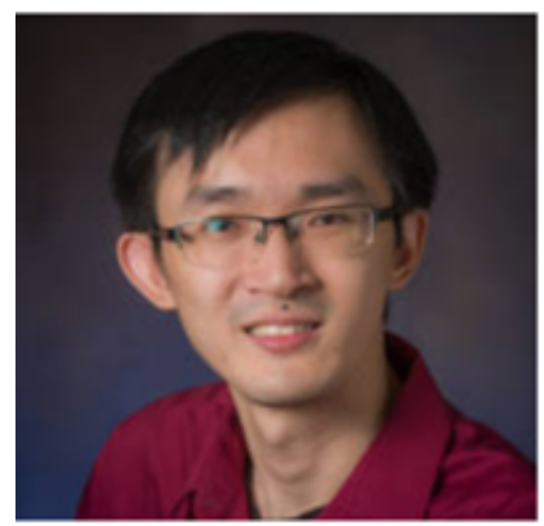
Unaprticles: MEELS, ARPES and the Strange Metal

K. Limtragool

Chandan Setty



Zhidong Leong

Bikash Padhi

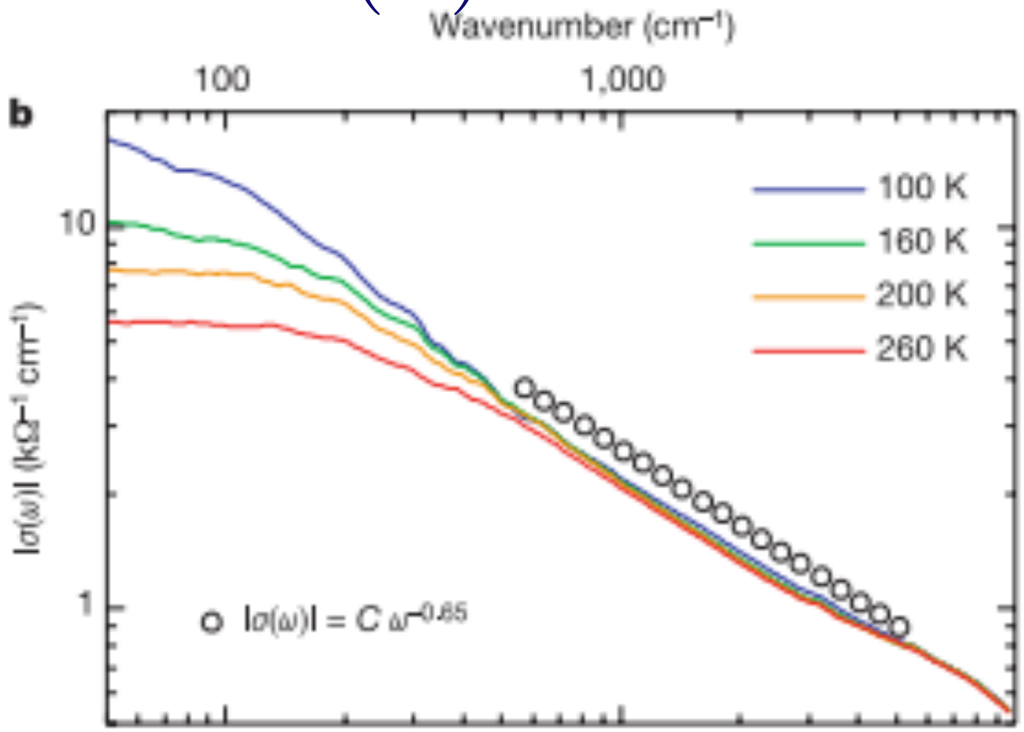
strange metal: experimental facts

Quantum critical behaviour in a high-T_c superconductor

D. van der Marel[1]*, H. J. A. Molegraaf[1]*, J. Zaanen[2], Z. Nussinov[2]*, F. Carbone[1]*, A. Damascelli[3]*, H. Eisaki[3]*, M. Greven[3], P. H. Kes[2] & M. Li[2]

[1]Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands
[2]Leiden Institute of Physics, Leiden University, 2300 RA Leiden, The Netherlands
[3]Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA

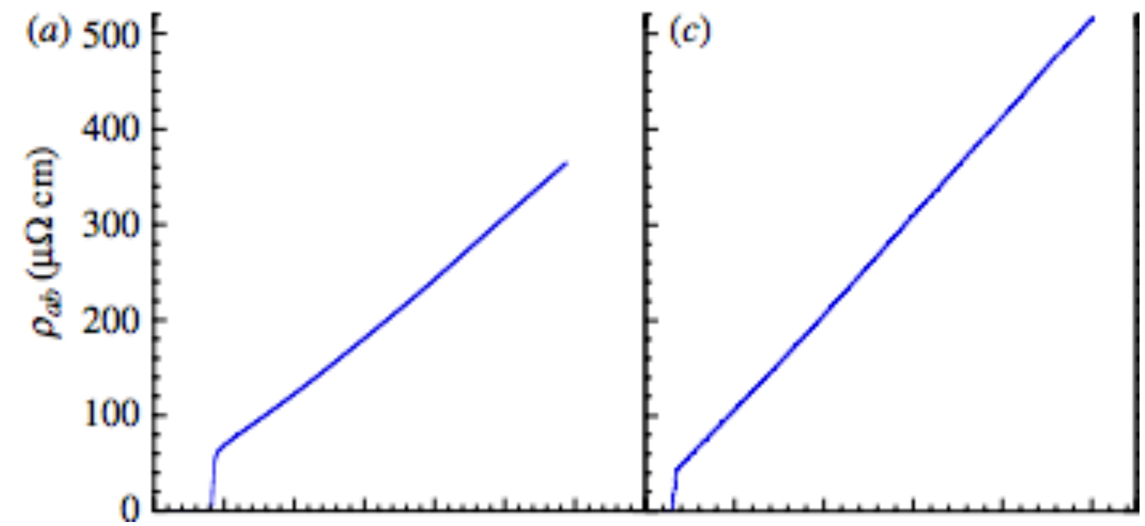
$$\sigma(\omega) = C\omega^{-\frac{2}{3}}$$



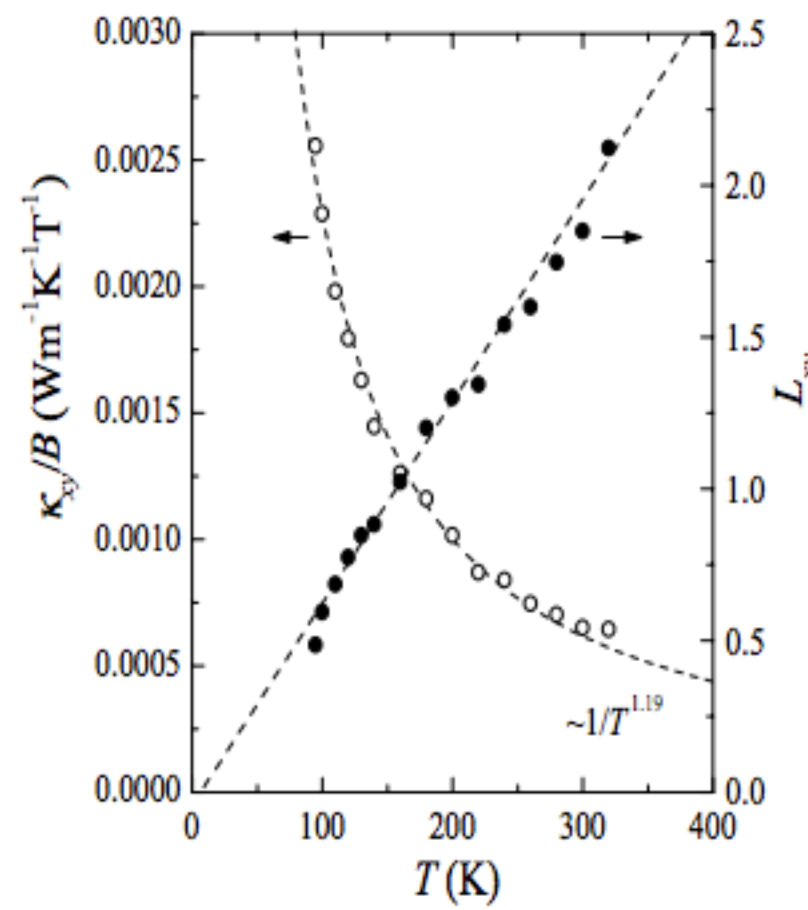
Hall Angle

$$\cot\theta_H \equiv \frac{\sigma_{xx}}{\sigma_{xy}} \approx T^2$$

T-linear resistivity



$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$



Phase Space
Restriction

$$\rho \approx T^2$$

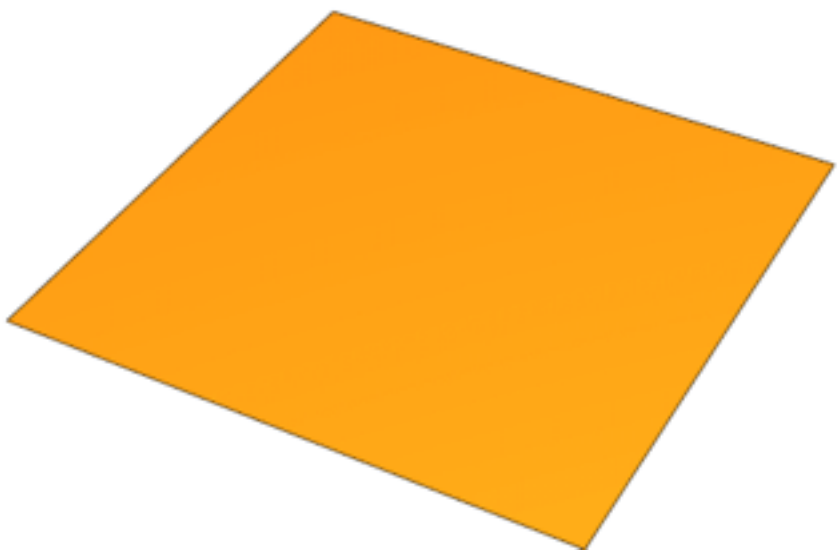
Planckian dissipation

Interactions???

$$\rho \propto T$$

M-EELS

(Momentum resolved electron energy loss spectroscopy)



$$\frac{d^2\sigma}{d\Omega dE} = \sigma_0 [V_{\text{eff}}(k_i^z, k_s^z, \mathbf{q})]^{\mathbf{2}} \cdot \mathbf{S}(\mathbf{q}, \omega)$$

$$S(\mathbf{q}, \omega) = -\frac{\mathbf{1}}{\pi} \frac{\mathbf{1}}{\mathbf{1} - \mathbf{e}^{-\hbar\omega/\mathbf{k_B T}}} \approx \Im\chi(\mathbf{q}, \omega) \approx -\Im \frac{\mathbf{1}}{\epsilon(\mathbf{q}, \omega)}$$

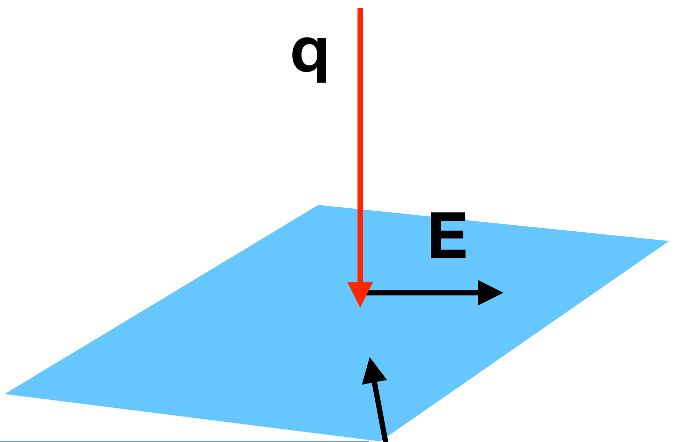
susceptibility

?

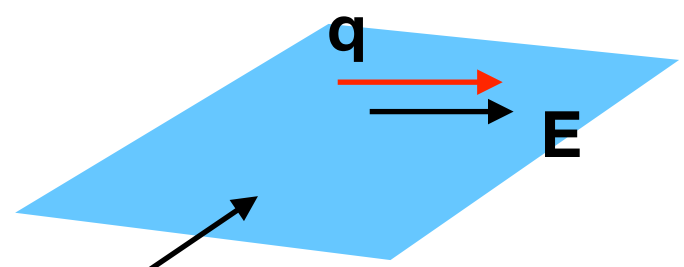
Transverse vs Longitudinal

$$\epsilon_{\perp}(\mathbf{q},\omega) \qquad \epsilon_{||}(\mathbf{q},\omega)$$

M-EELS



optics: Ellipsometry



$$\epsilon_{\perp}(\mathbf{q}\rightarrow\mathbf{0},\omega) = \epsilon_{||}(\mathbf{q}\rightarrow\mathbf{0},\omega)$$

The two dielectric functions must be the same in the zero momentum limit

The Bible: Nozieres and Pines (1999)

$$H'(t) = \int d\mathbf{r}\,\mathbf{n}(\mathbf{r},\mathbf{t})\mathbf{W}(\mathbf{r},\mathbf{t})$$

$$\langle \delta n(\mathbf{k},\omega)\rangle = \boxed{\chi(\mathbf{k},\omega)}\mathbf{W}(\mathbf{q},\omega)$$

density-density response

marginal Fermi liquids

$$\chi''(\mathbf{k},\omega) = -\mathbf{N}(\epsilon_\mathbf{F})\omega/\mathbf{T} \quad \omega \ll \mathbf{T}$$

$$\chi''(\mathbf{k},\omega) = -\mathbf{N}(\epsilon_\mathbf{F})(\mathrm{sgn}\omega), \quad \mathrm{T} \ll \omega \ll \omega_\mathrm{c}$$

is this seen experimentally?

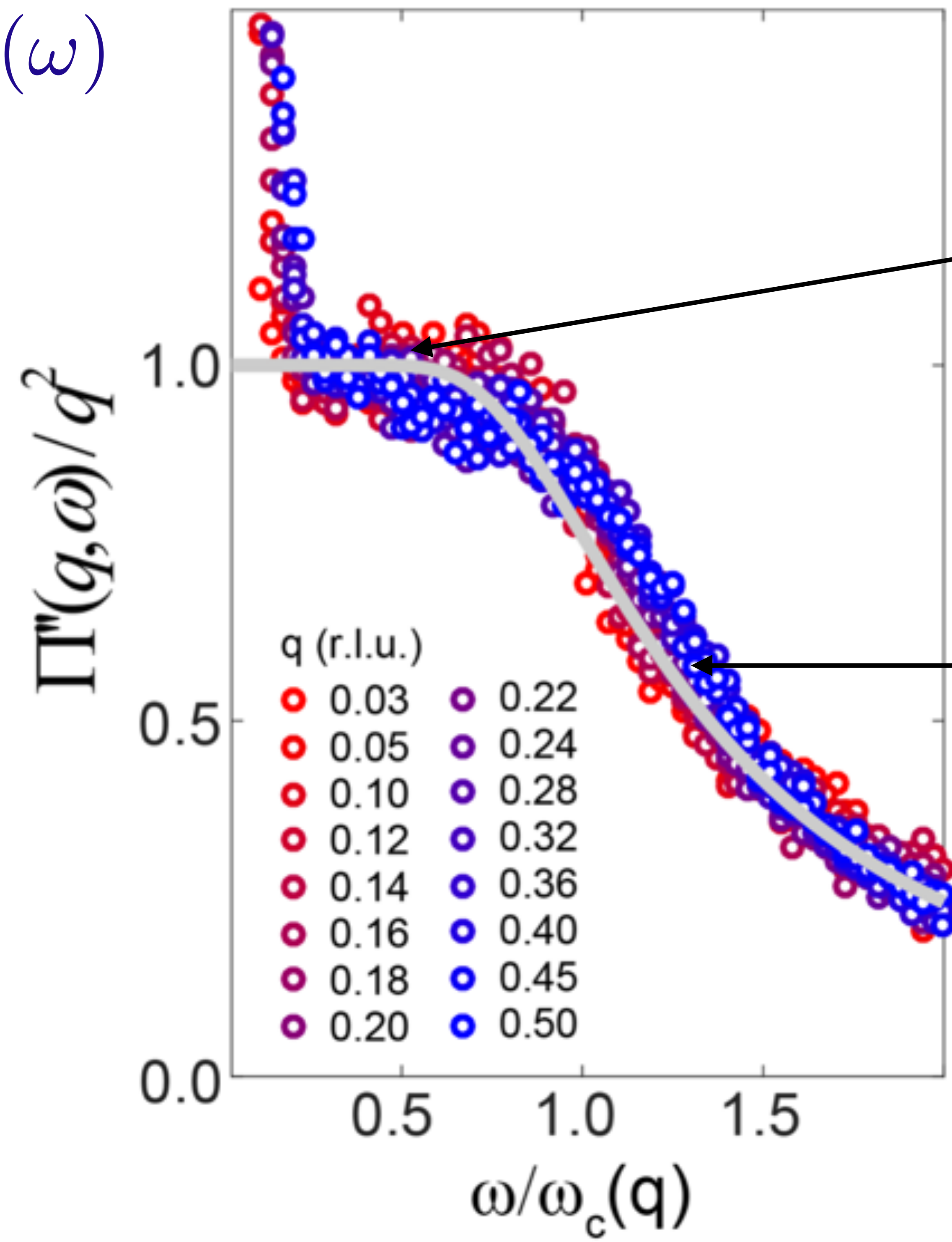
polarizability

$$\chi = \frac{\Pi(\mathbf{k},\omega)}{1 - V(\mathbf{k})\mathbf{\Pi}(\mathbf{k},\omega)}$$

1.) separable

$$\Pi'' = f(q)g(\omega)$$

2.) scale-invariance



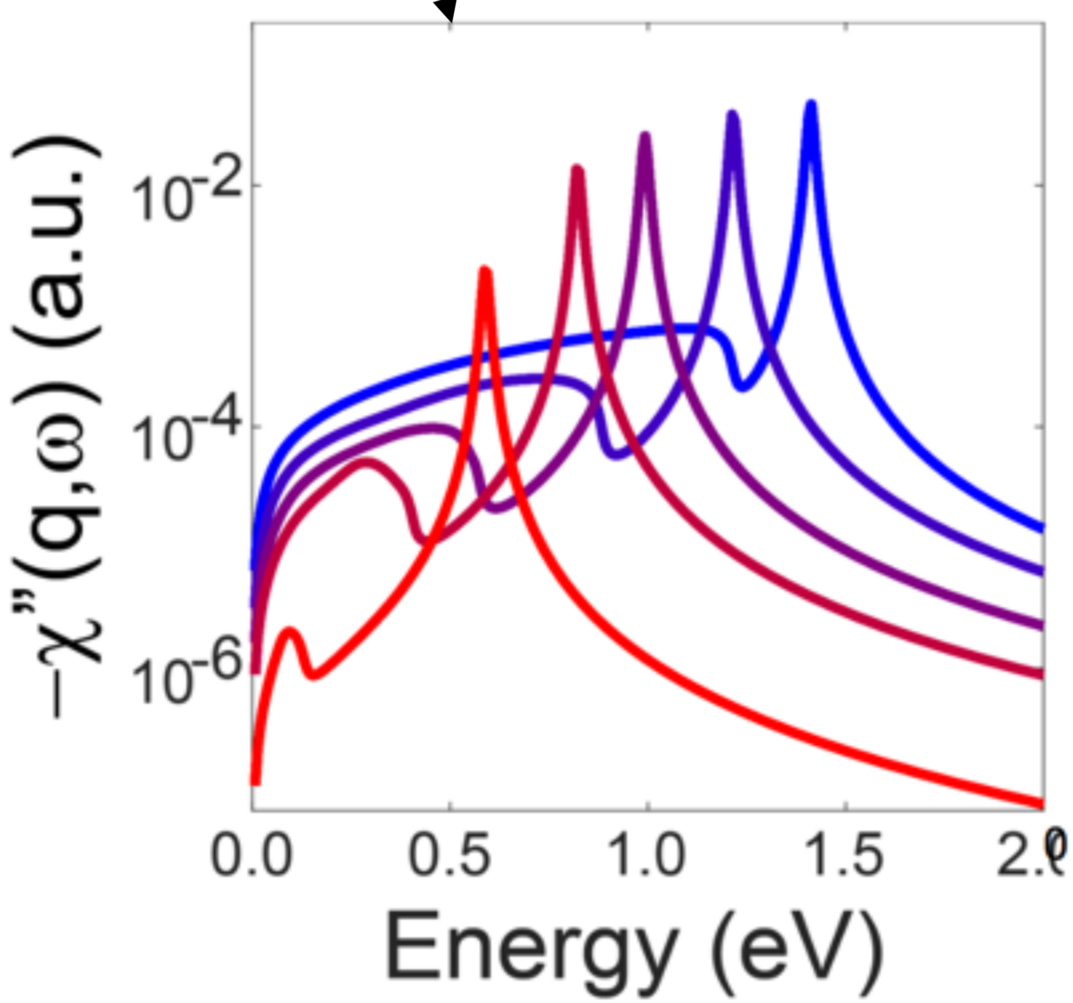
$$\Pi''(q,\omega) = q^2,\ \omega < \omega_c$$

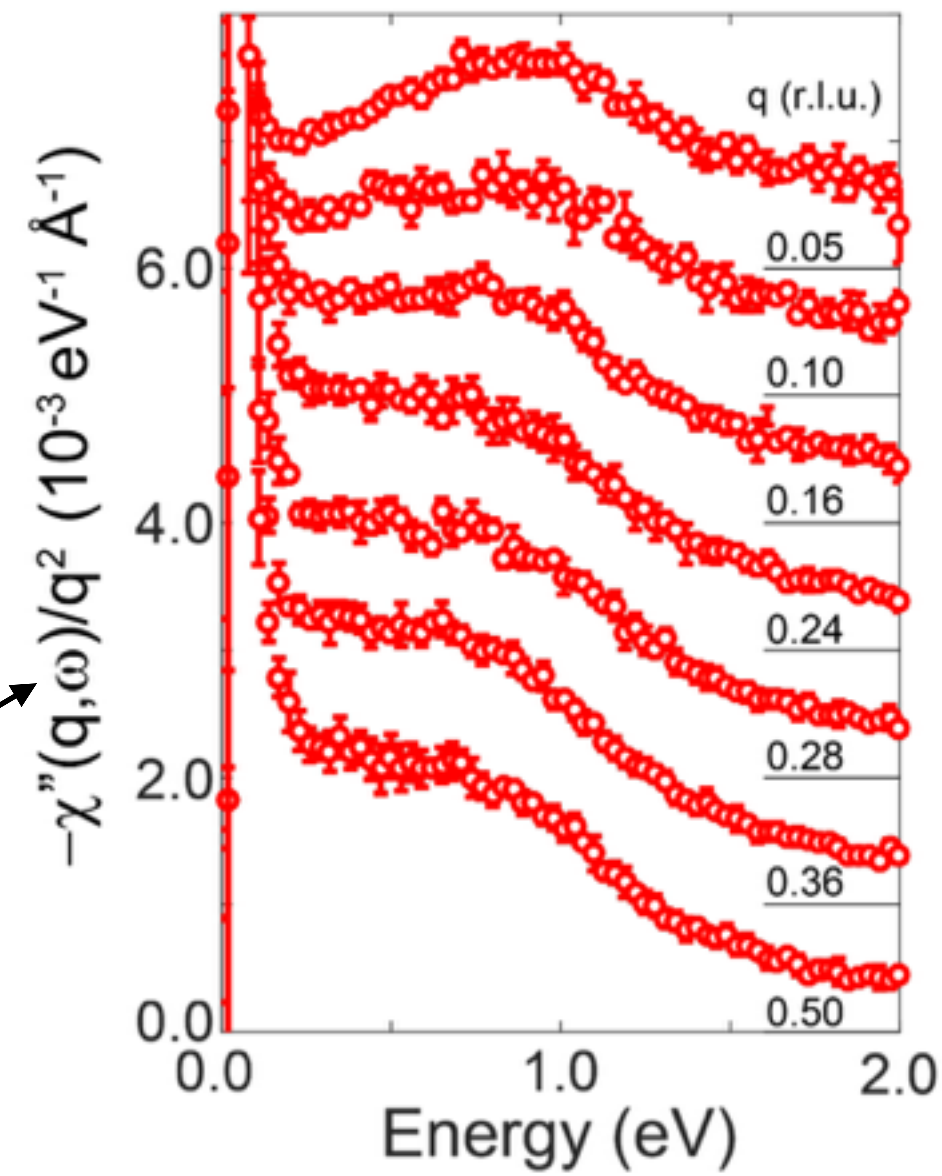
$$\Pi''(q,\omega) = \frac{q^2}{\omega^2},\ \omega > \omega_c$$

not MFL

Drude metal

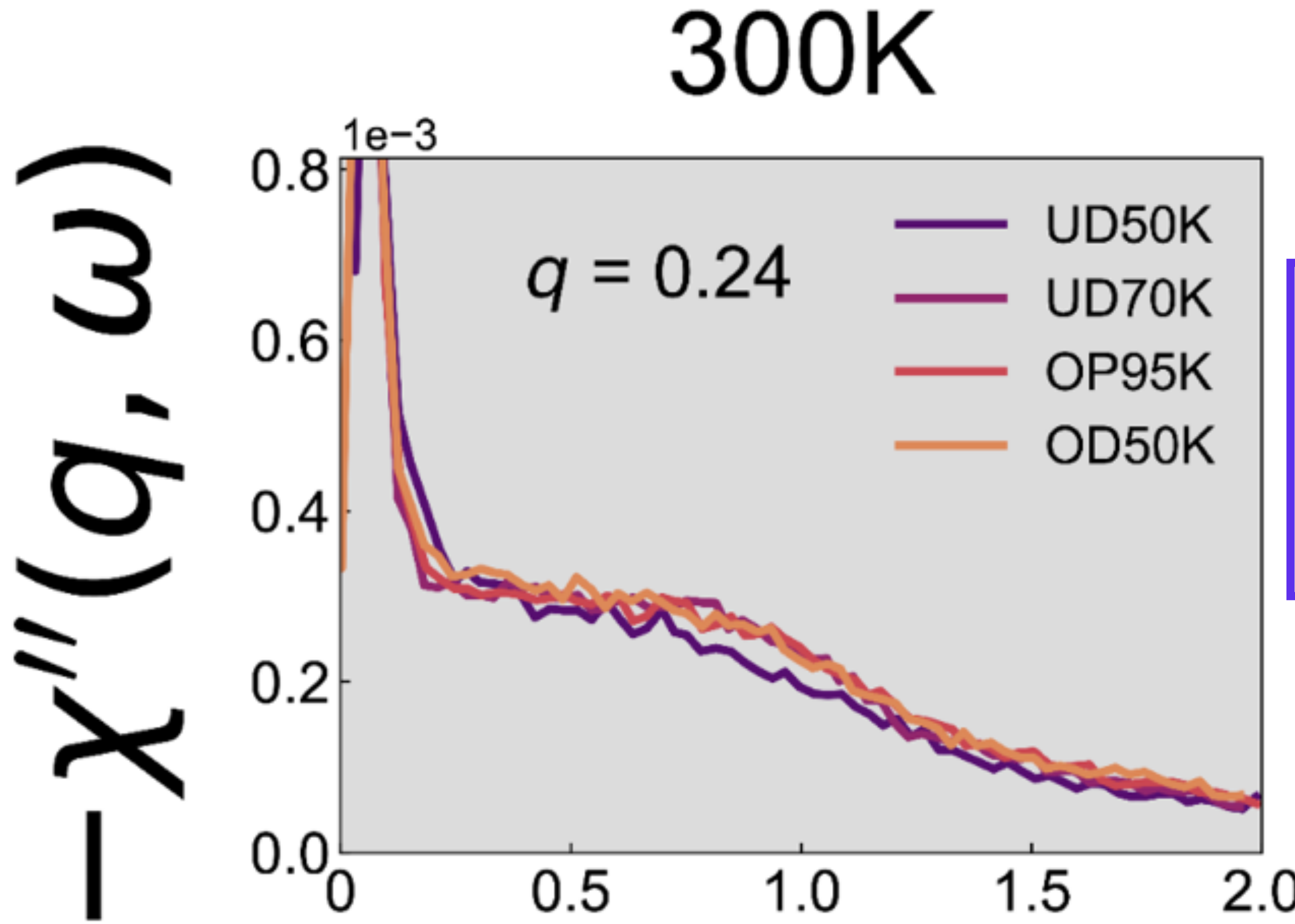
New Data (Abbamonte, et al.





strange metal

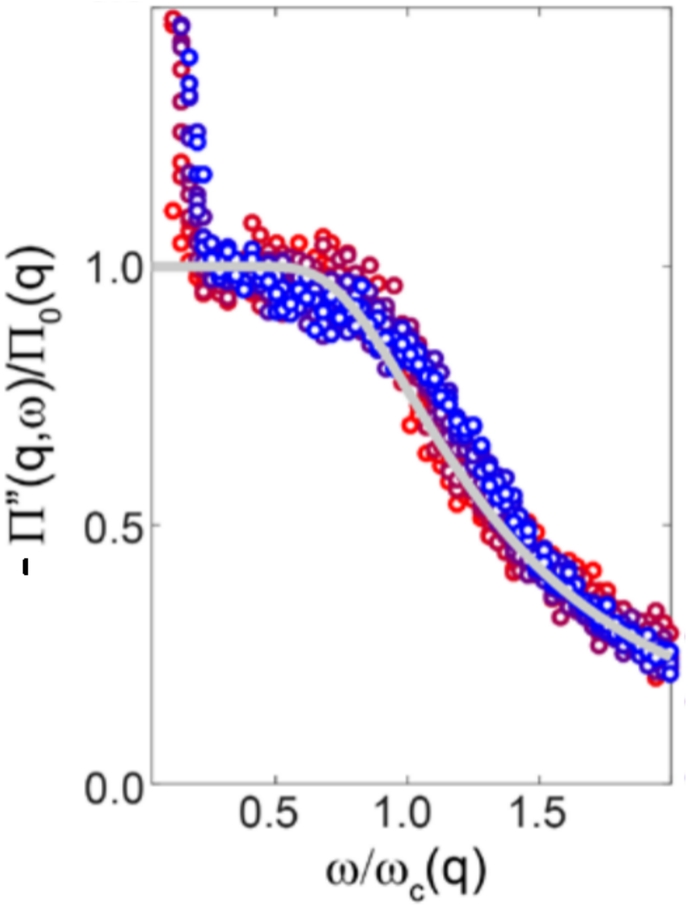
doping (in)dependence

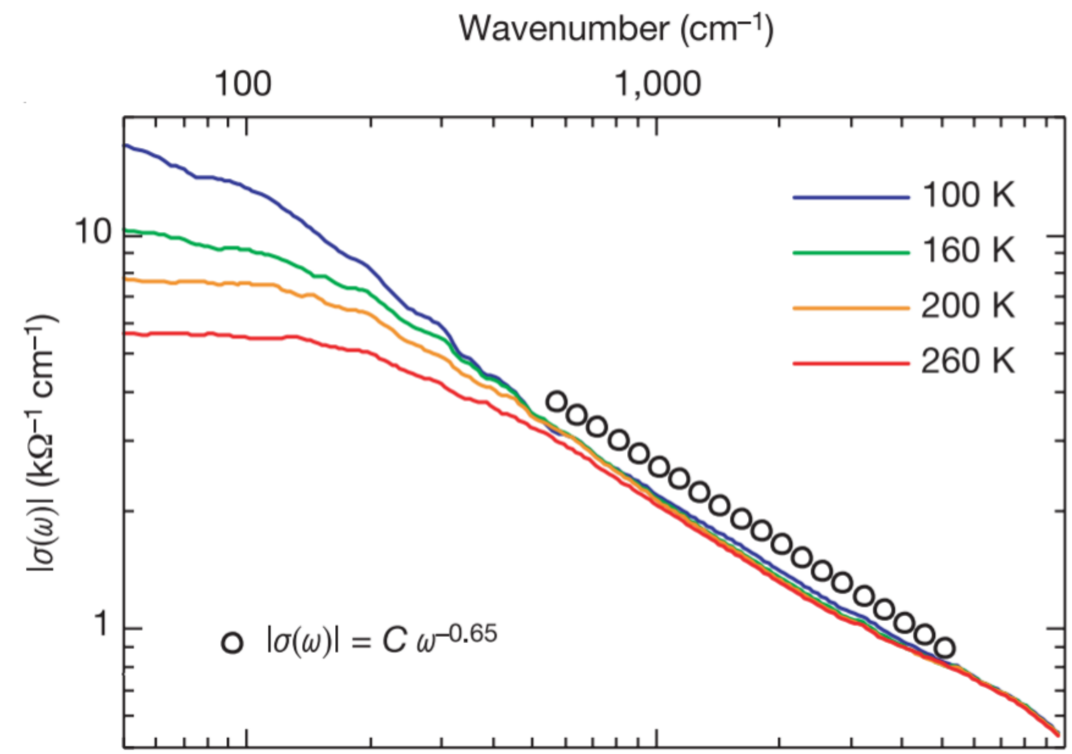


no momentum dispersion!!

are the dielectric functions equal?

$$\varepsilon_{||}(\mathbf{q},\omega) = \mathbf{1} - \mathbf{V}(\mathbf{k})\mathbf{\Pi}(\mathbf{k},\omega)$$





$$\varepsilon_{\perp}^{\rm opt}(\mathbf{q},\omega) = \varepsilon_{\infty} + \mathbf{4}\pi\mathbf{i}\sigma(\mathbf{q},\omega)/\omega$$
$$\approx \omega^{-5/3}$$

Cuprates

$$\varepsilon_{||}(\mathbf{q} \ldots, \omega) = \varepsilon_{\perp}(\ldots = \mathbf{0}, \omega)$$

$$\ldots \propto \mathrm{con} \ldots$$

$$\Pi \propto \omega^{-5 \ldots}$$

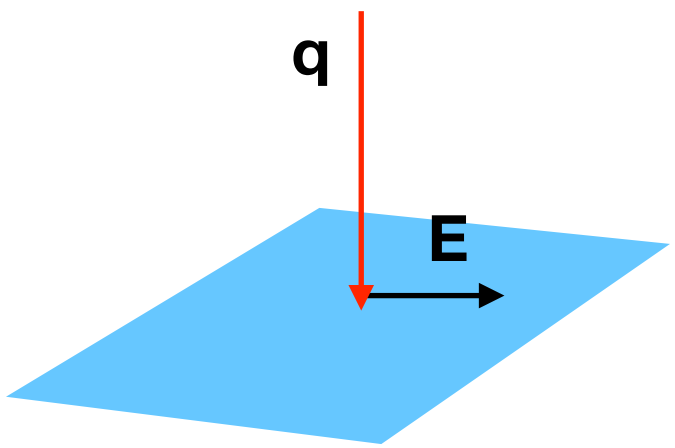
$$\varepsilon_{\text{opt}} \neq \varepsilon_{\rho\rho}$$

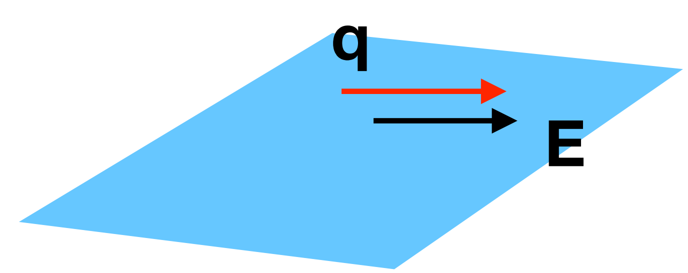
This equality doesn't hold even in limit of zero q in BSCCO!!
What's the discrepancy in Cuprates?

Does this discrepancy offer a window into the strange metal?

$\boxed{\text{Yes}}$

Order of limits





$$\lim_{\mathbf{q}_\perp \to \mathbf{0}} \lim_{q_\parallel \to 0} \qquad\qquad \lim_{q_\parallel \to 0} \; \lim_{\mathbf{q}_\perp \to \mathbf{0}}$$

??

$$\left[\lim_{q_\parallel \to 0}, \lim_{\mathbf{q}_\perp \to 0}\right] K_i(\mathbf{q}_\perp, q_\parallel, \omega) = 0$$

e answer to this question lies in
order in which the momentum
nsfer q is taken to zero, i.e.,
ether the parallel component is
en to zero first or the
rpendicular.
the case of optics: parallel first
d then perpendicular and
posite for MEELS
pically these two are the same i.e.,
commutator is zero.

Normal metals: Mermin to Drude

$$\sigma_{||}(\mathbf{q},\omega) = \frac{i\omega}{4\pi} V_q \Pi(\mathbf{q},\omega)$$

$$\chi_M(\vec{q},\omega) = \frac{\chi_0(\vec{q},\omega+\frac{i}{\tau})}{1+(1-i\omega\tau)^{-1}\left(\frac{\chi_0(\vec{q},\omega+\frac{i}{\tau})}{\chi_0(\vec{q},0)}-1\right)}$$

$$\chi_0(\mathbf{q},\omega) = \frac{nq^2}{M\omega^2}$$

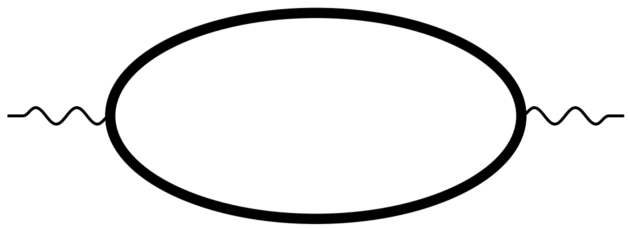
$$\sigma_{||}(\mathbf{q}=\mathbf{0},\omega) = \frac{ne^2}{m}\frac{1}{\omega+\frac{i}{\tau}}$$

$$\varepsilon_{\perp}(\vec{q}\to 0,\omega) = \varepsilon_{||}(\vec{q}\to 0,\omega)$$ (*Normal metals*)

N.D. Mermin PRB (1970)

Drude case

1) We can quickly check to see if this holds in the case of normal metals microscopically.
2) This is the bubble diagram that contributes to the conductivity, with vertices given by the momenta
3) For finite q and T this is the form of the conductivity. Describe the equation.
4) Typically q is simply set to zero here and the order doesn't matter.. show that in words.

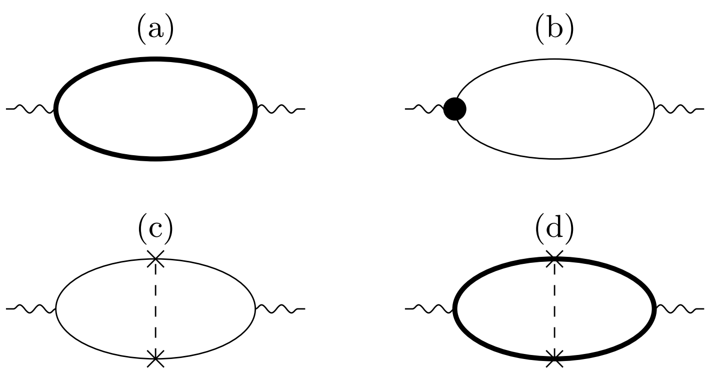


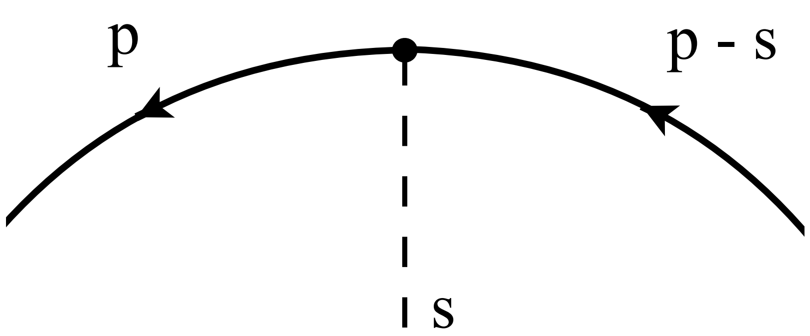
$$\sigma(\boldsymbol{q},\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} d\nu\, F(\nu,\omega) \int d\boldsymbol{p}\, \frac{(2\boldsymbol{p}+\boldsymbol{q})_i^2}{(2m)^2} G^-_{\boldsymbol{p}}(\nu) G^+_{\boldsymbol{p}+\boldsymbol{q}}(\nu+\omega)\,.$$

Finite q and T

$$[G^{\pm}_{\mathbf{p}}(\nu)]^{-1} = \nu - \xi_{\mathbf{p}} \pm \frac{i}{2\tau}$$

Reconsidering interactions: Angular Vertex





$$\mathcal{L} = \mathcal{L}_0 + \int d\mathbf{s}\, \boldsymbol{\Theta}(\mathbf{p},\mathbf{s})^2\, \bar{\psi}(\mathbf{p}-\mathbf{s})\phi(\mathbf{s})\psi(\mathbf{p})$$

$$\boldsymbol{\Theta}_{\mathbf{p},\,\mathbf{p}-\mathbf{s}} \;=\; \hat{\mathbf{p}}.(\hat{\mathbf{p}} - \hat{\mathbf{s}})$$

$$\boxed{\text{scale-invariant interaction}}$$

Angular Vertex

$$\bar{V}^{i}_{\mathbf{p},\mathbf{q},\mathbf{s}} = 4(\hat{\mathbf{q}} \cdot \hat{\mathbf{s}})^2 \left[(\mathbf{p})_i^2 + 2\mathbf{s}_i(\mathbf{p})_i(\hat{\mathbf{p}} \cdot \hat{\mathbf{s}}) + (\mathbf{p})_i^2(\hat{\mathbf{p}} \cdot \hat{\mathbf{s}})^2\right]$$

$$(\hat{\mathbf{q}}.\hat{\mathbf{s}})^2 = \frac{(\mathbf{q}.\mathbf{s})^2}{|\mathbf{q}|^2|\mathbf{s}|^2} = \frac{\left(q_{||}s_{||} + \mathbf{q}_{\perp}.\mathbf{s}_{\perp}\right)^2}{\left(q_{||}^2 + |\mathbf{q}|_{\perp}^2\right)|\mathbf{s}|^2}$$

$$\left[\lim_{q_{||}\to 0}, \lim_{\mathbf{q}_{\perp}\to 0}\right] K_i(\mathbf{q}_{\perp}, q_{||}, \omega) = \frac{\eta}{\omega^2} + O\left(\frac{1}{\omega^4}\right)$$

$\boxed{\text{scale-invariant interaction}}$

$$\varepsilon_{||}(\mathbf{q} \to 0, \omega) \neq \varepsilon_{\perp}(\mathbf{q} \to 0, \omega)$$

$\boxed{\text{unparticles}}$ $(p^2)^{d_U}$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$
$$\phi(x) \to \phi(x)$$

$$\Lambda^2\left(\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\right) \qquad m^2\phi^2$$

no scale invariance

unparticles

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} \to \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

not particles

propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

$$d_U - 2$$

continuous mass

$\phi(x, m^2)$ $\Longleftrightarrow$ $e^2(m)$

Karch, 2005

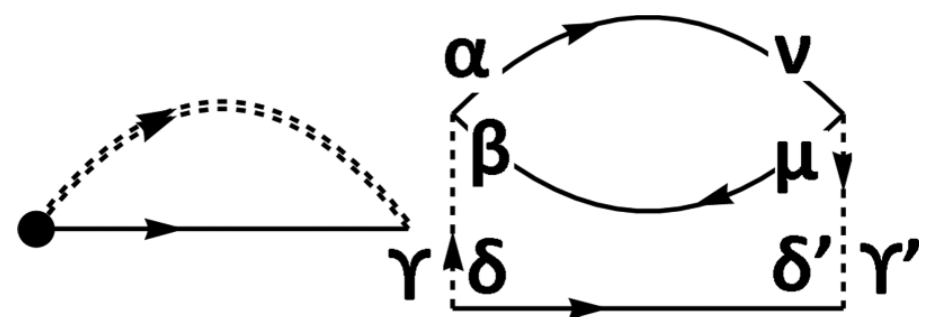
flavors

multi-bands

Does the `self-energy' exhibit a power law?

$$\Sigma \propto (\omega^2 + T^2)^\alpha$$

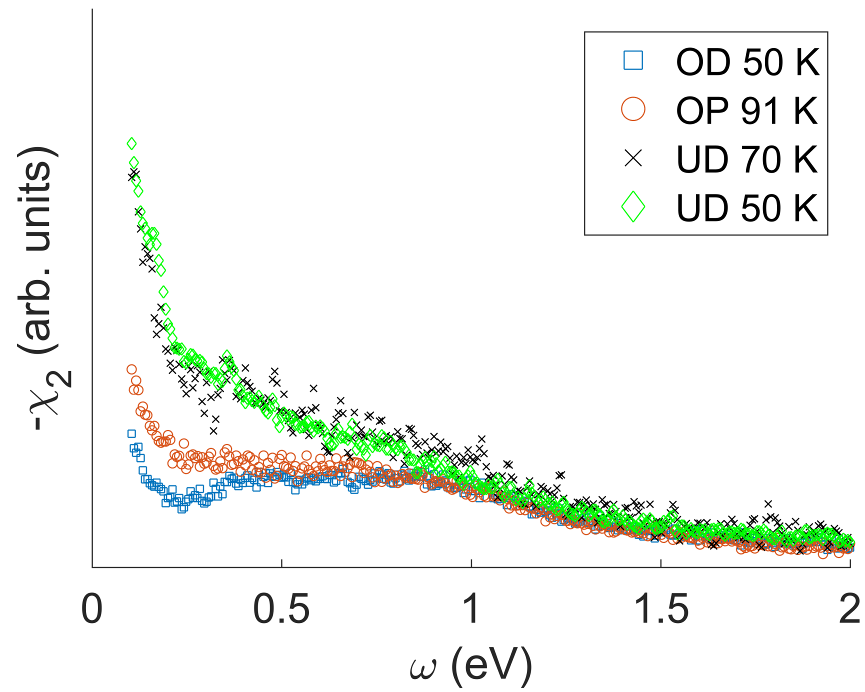
`Self-energy'



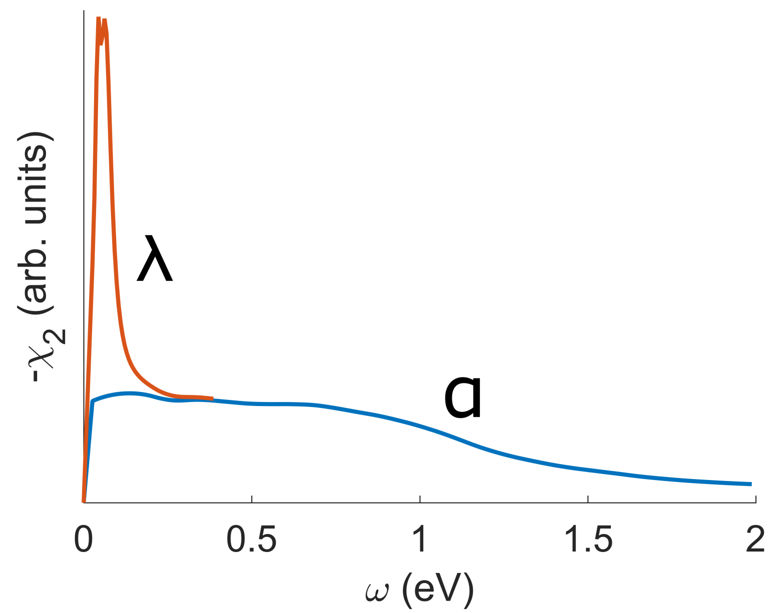
$$\Sigma_2(\mathbf{k},\omega) = \sum_{\mathbf{q}} V^2(\mathbf{q})\chi_2(\mathbf{q},\omega-\varepsilon_{\mathbf{k}-\mathbf{q}}) \times \left[n_F(-\varepsilon_{\mathbf{k}-\mathbf{q}}) + n_B(\omega-\varepsilon_{\mathbf{k}-\mathbf{q}})\right],$$

Note that the susceptibility is exact since we are obtaining it from experiment, and this goes into the self-energy calculation

Modeling the data



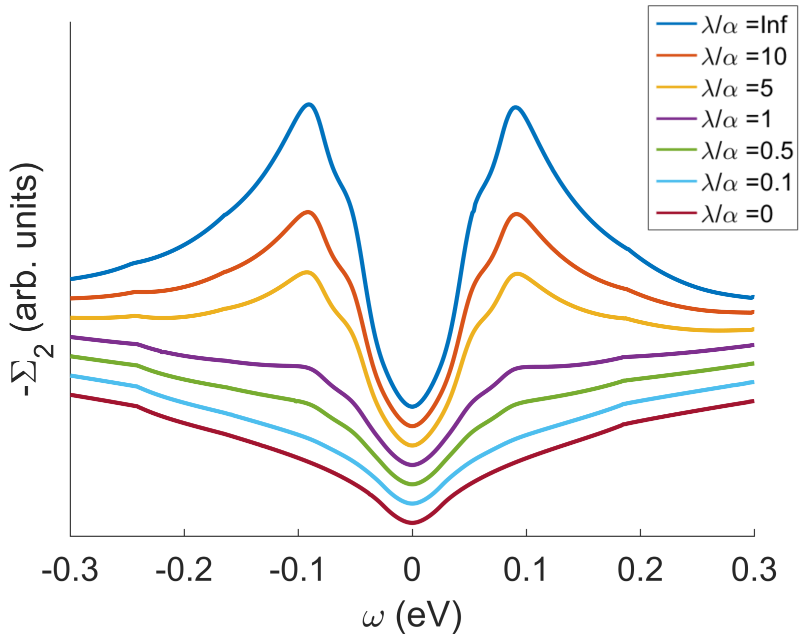
(a)



(b)

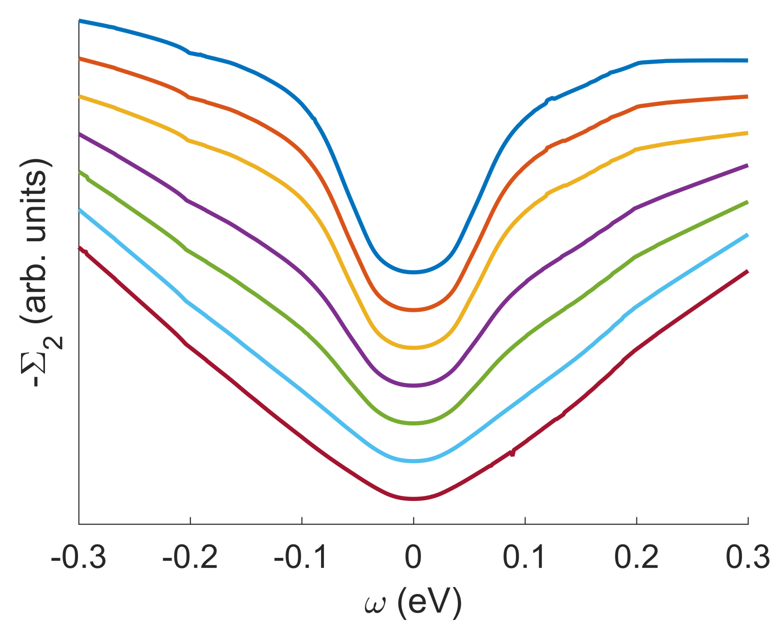
Self-energy: Optimal doping

V(q)= exp[-q z]/q



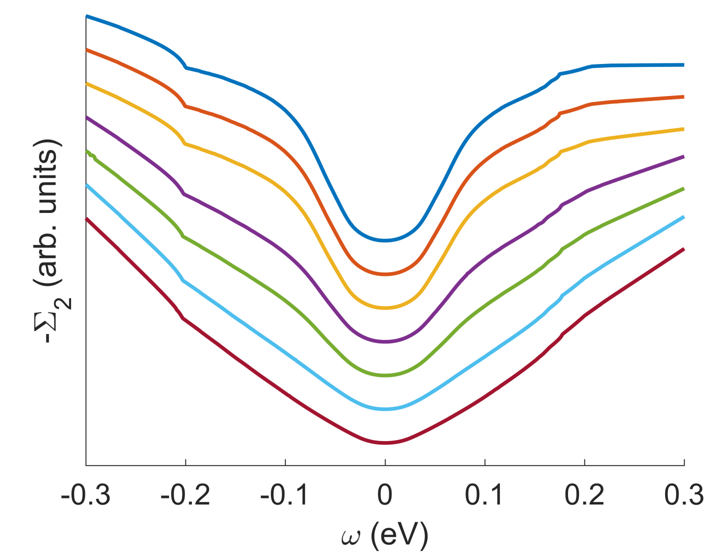
(a)

V(q)= const



(b)

V(q)=1/q

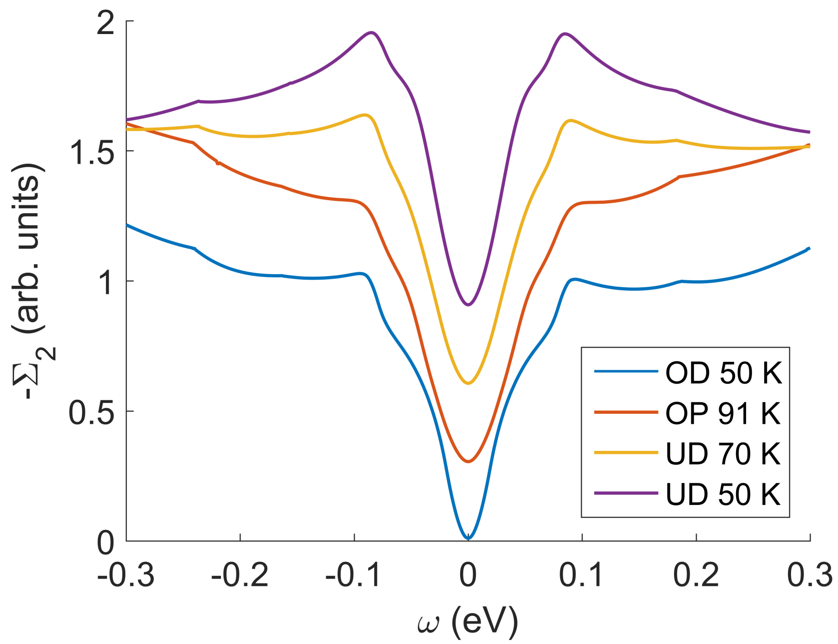


(c)

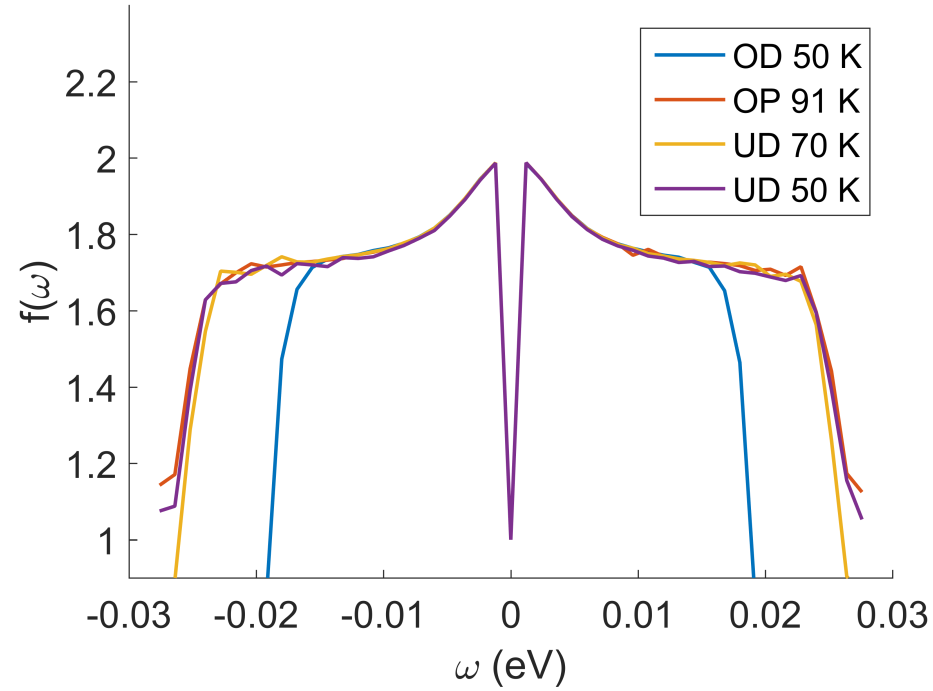
Require both phonons and an interaction with a scale for the kink and peak like feature

Self energy: doping dependence

$$f(\omega) = \omega \frac{\Sigma_2''(\omega)}{\Sigma_2'(\omega)} + 1$$



(a)

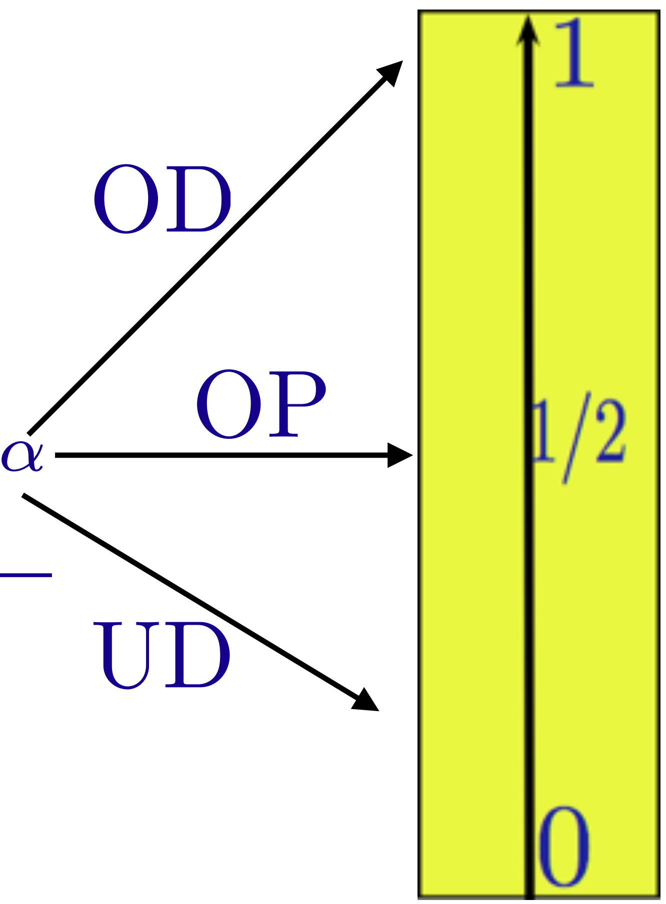


(b)

There exist important similarities and some differences with ARPES (Dessau's power-law liquid)

Dessau's Power-law liquid

$$-\Sigma''(\omega) = \Gamma_0 + \lambda \frac{\left(\omega^2 + \pi^2 T^2\right)^{\alpha}}{\omega_N^{2\alpha - 1}}$$



QC region $0 < \alpha < 1/2$

Real part of Σ

$$\Sigma^{'}(\omega) = \frac{1}{\pi}\mathcal{P}\int d\omega^{'}\,\frac{\Sigma^{''}\left(\omega^{'}\right)}{\omega^{'}-\omega}$$

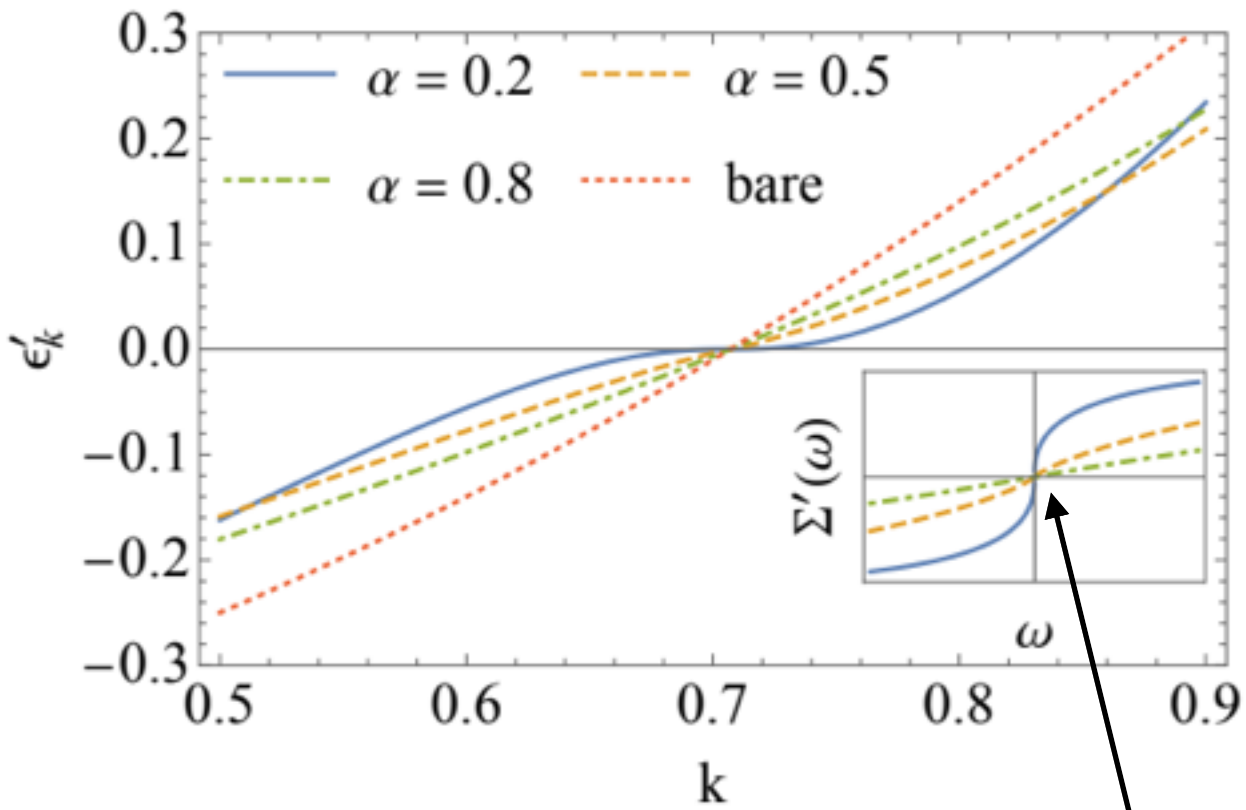
$$\Big\downarrow \; T = 0$$

$$\Sigma^{'}(x\omega_N) = -\frac{1}{\pi}\mathcal{P}\int_{-\omega_N}^{\omega_N}\frac{d\omega^{'}}{\omega^{'}-x\omega_N}\left(\Gamma_0 + \lambda\frac{\left|\omega^{'}\right|^{2\alpha}}{\omega_N^{2\alpha-1}}\right)$$

$$= \cdots + |x|^{\alpha} + \cdots$$

$$\epsilon_k' = \epsilon_k + \Sigma'$$

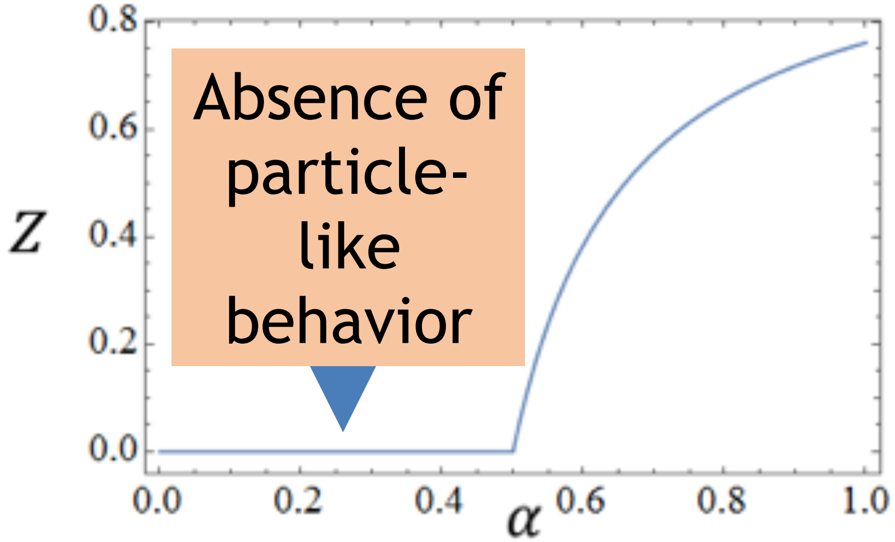
$$\left[1 - \frac{2\bar{\Gamma}_0}{\pi} + \frac{2\lambda}{(2\alpha - 1)\,\pi}\right]^{-1}, \alpha > \frac{1}{2}$$

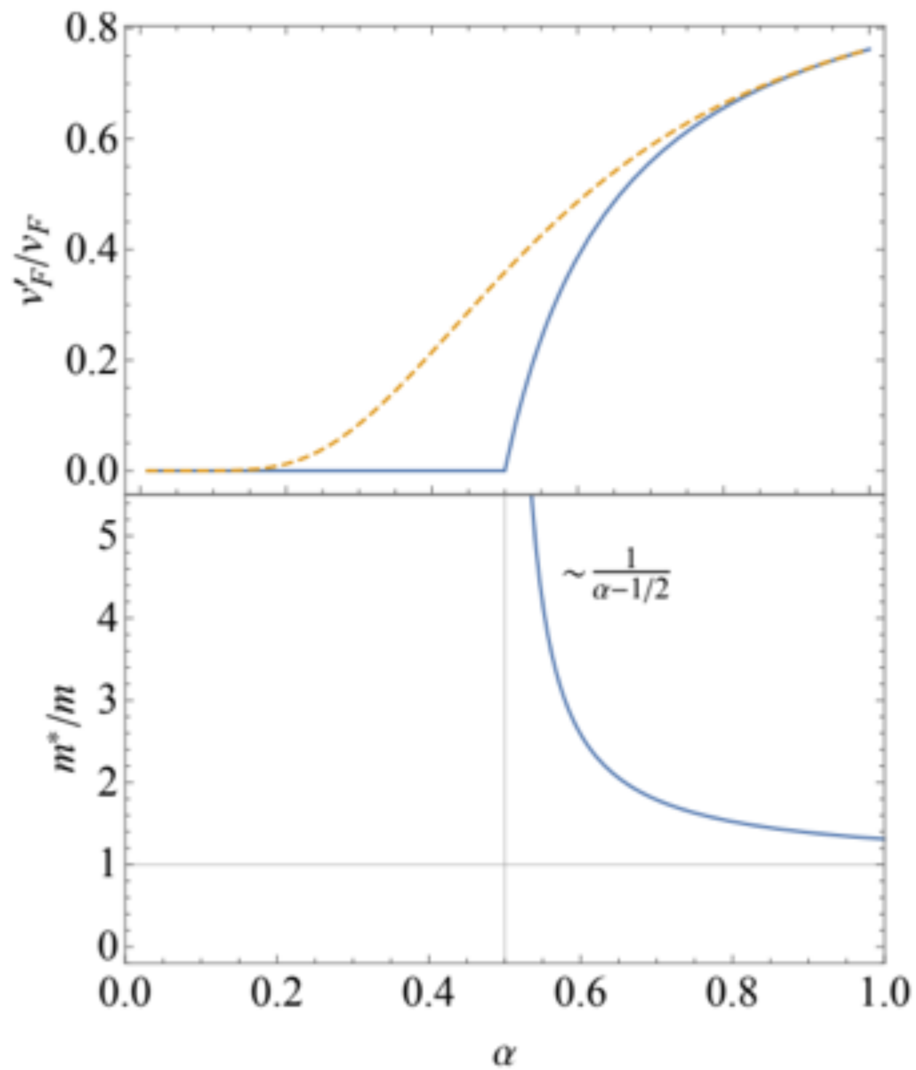


$$Z = \left(1 - \left.\frac{d\Sigma'}{d\omega}\right|_{\omega=0}\right)^{-1}$$

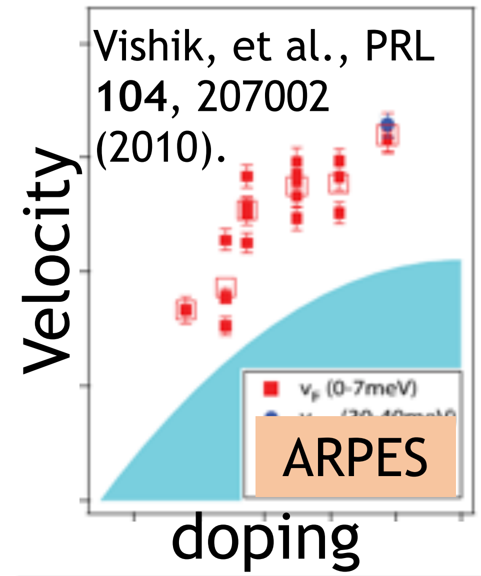
$$0 \quad \alpha \leq \frac{1}{2}$$

$\boxed{\text{non-analytic at } \omega = 0}$





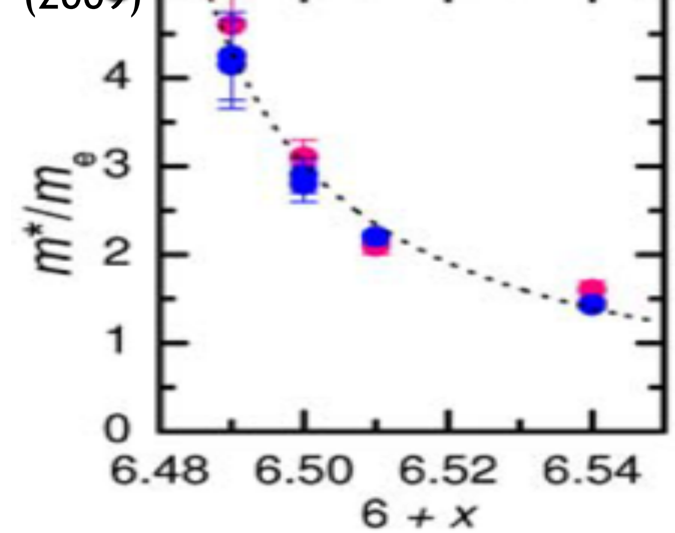
$$v'_F = Z v_F$$



Velocity near Fermi level decreases with α

Quantum oscillation measurements

S.E. Sebastian, et al, PNAS **107**, 6175 (2009)

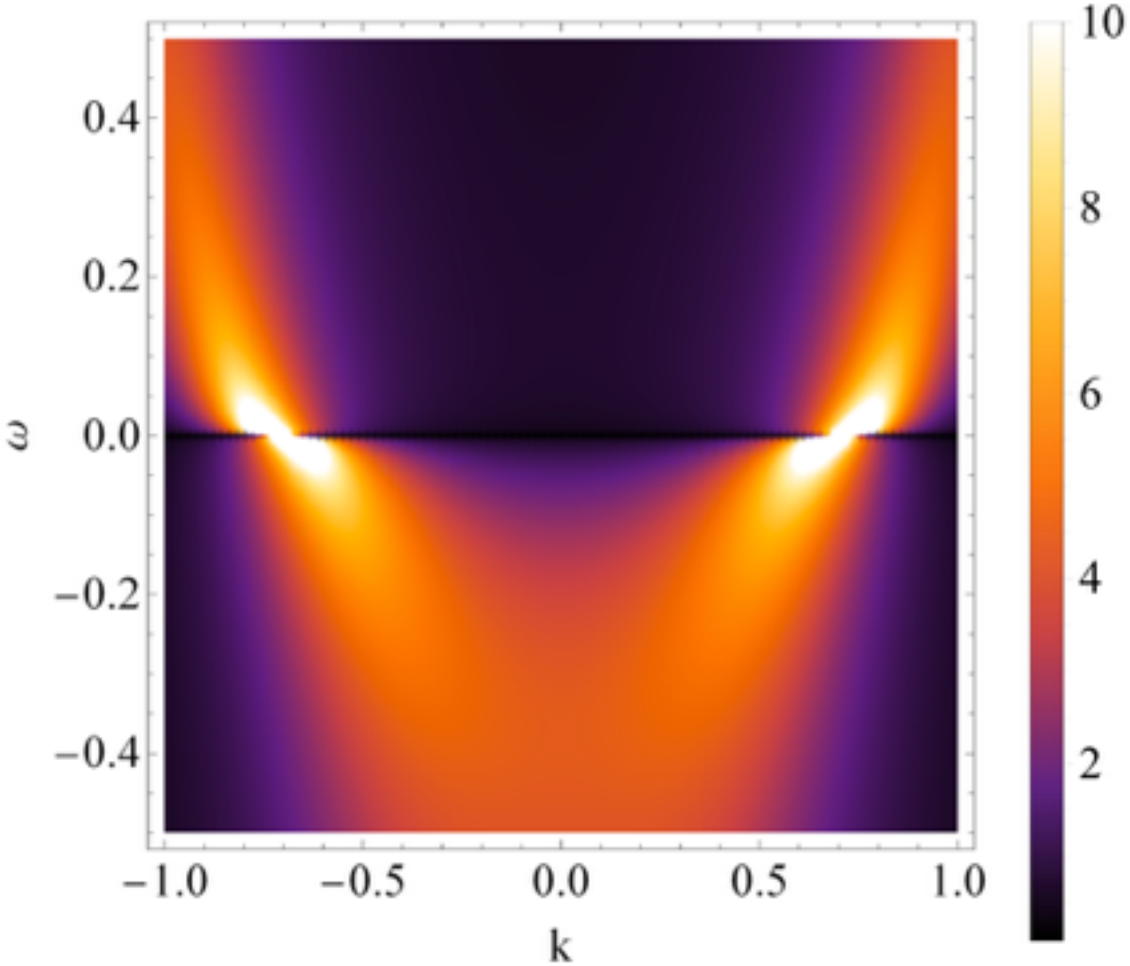


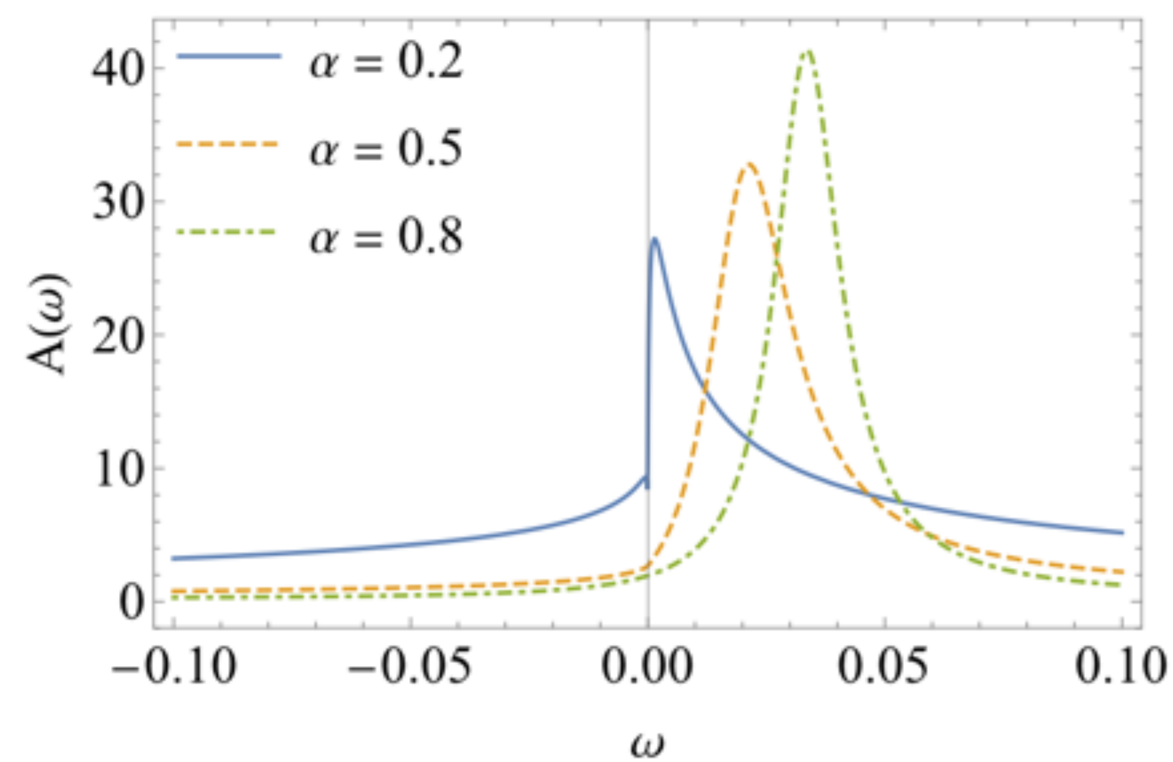
Metal-insulator transition

spectral function

$$A(k,\omega) = N \frac{-\Sigma''(\omega)}{[\omega - \epsilon_k - \Sigma'(\omega)]^2 + [\Sigma''(\omega)]^2},$$

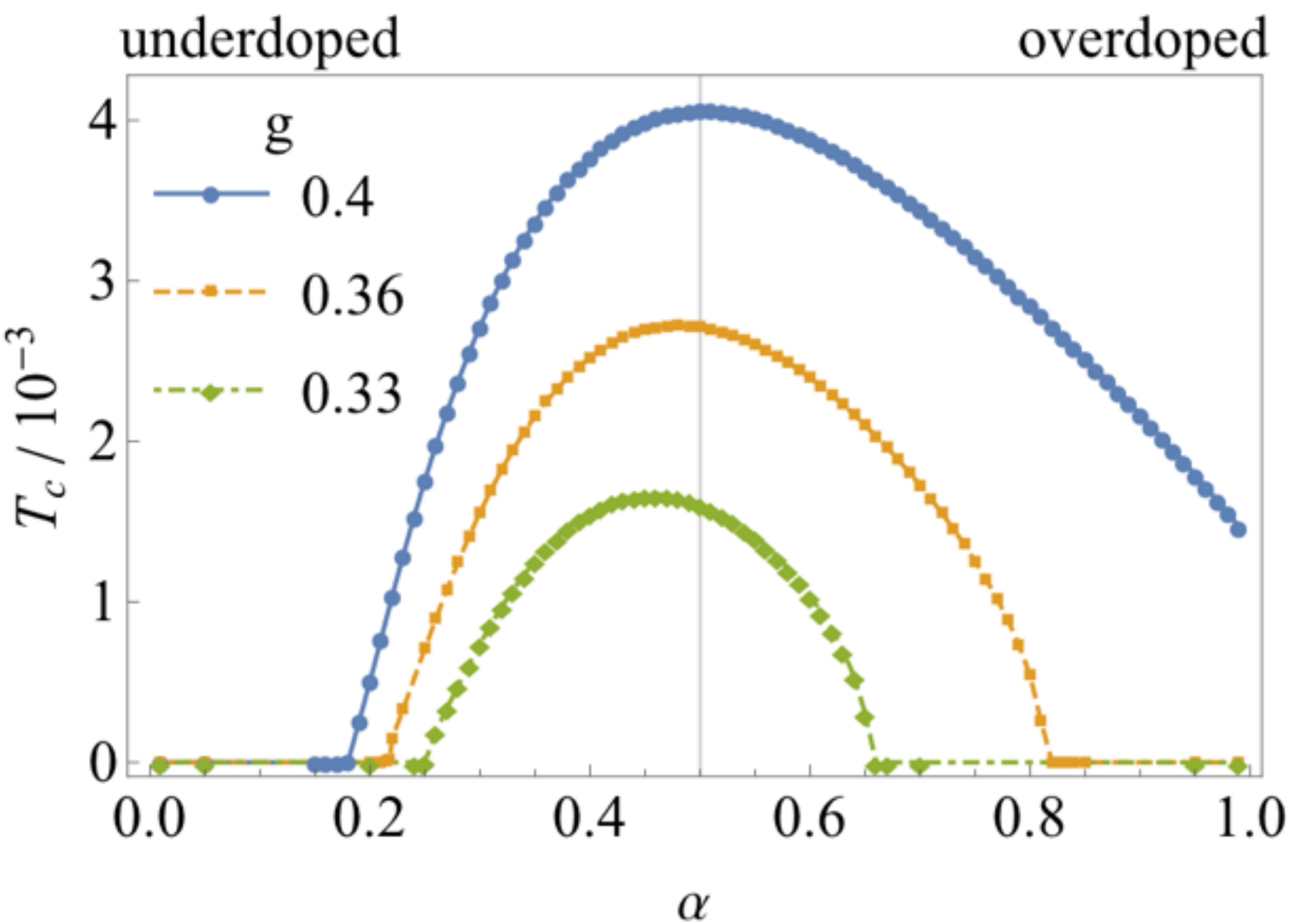
critical FS (Z=0)





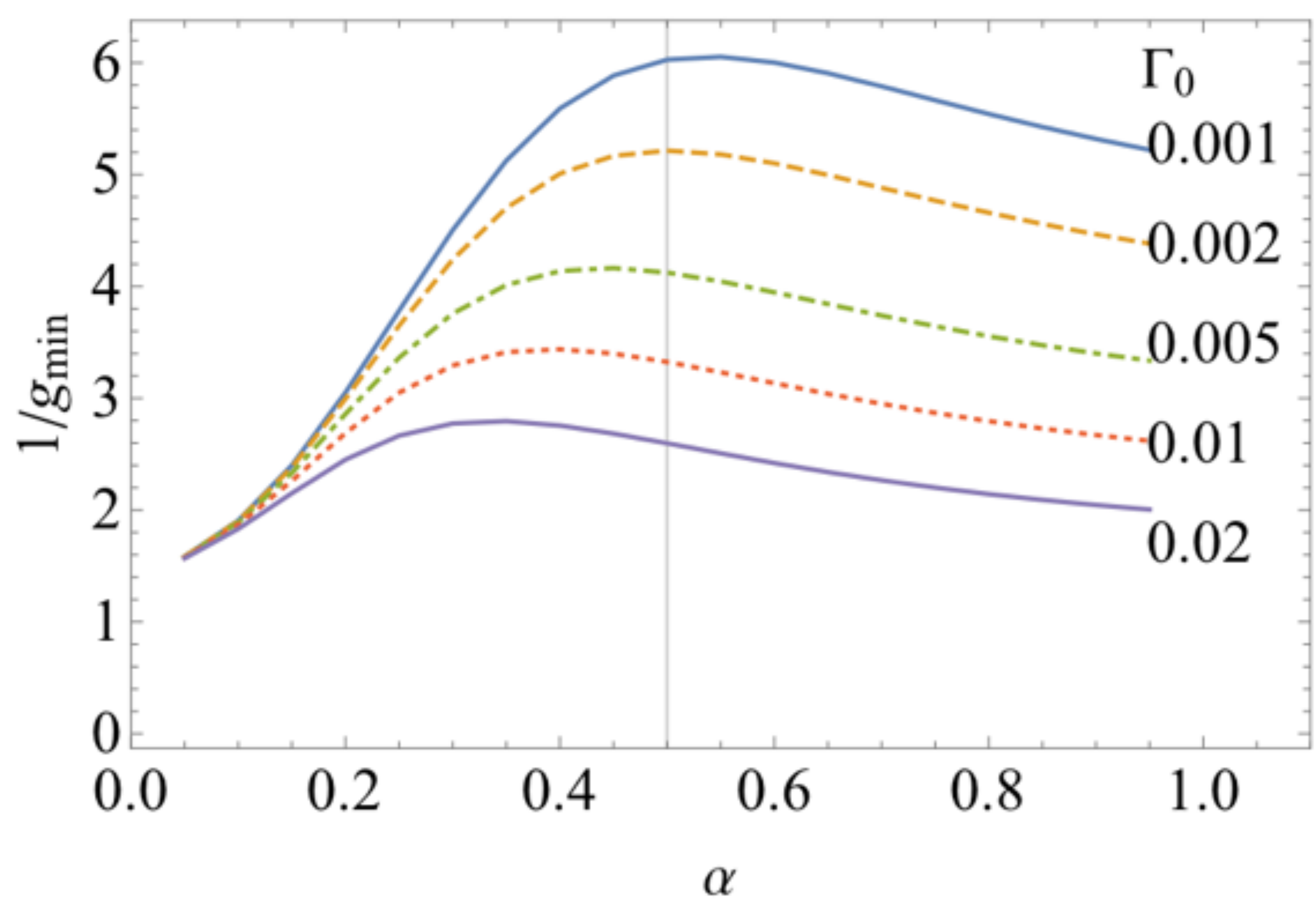
superconductivity

$$\frac{1}{g} = \sum_k \int d\omega d\omega' \frac{1}{2} \frac{\tanh \frac{\omega}{2T_c} + \tanh \frac{\omega'}{2T_c}}{\omega + \omega'} A(k,\omega) A(-k,\omega')$$



minimal coupling

$$\frac{1}{g_{\min}} = \int d\epsilon \int d\omega d\omega' \frac{1}{2} \frac{1}{\omega + \omega'} A(\epsilon, \omega) A\left(\epsilon, \omega'\right)$$



Why?

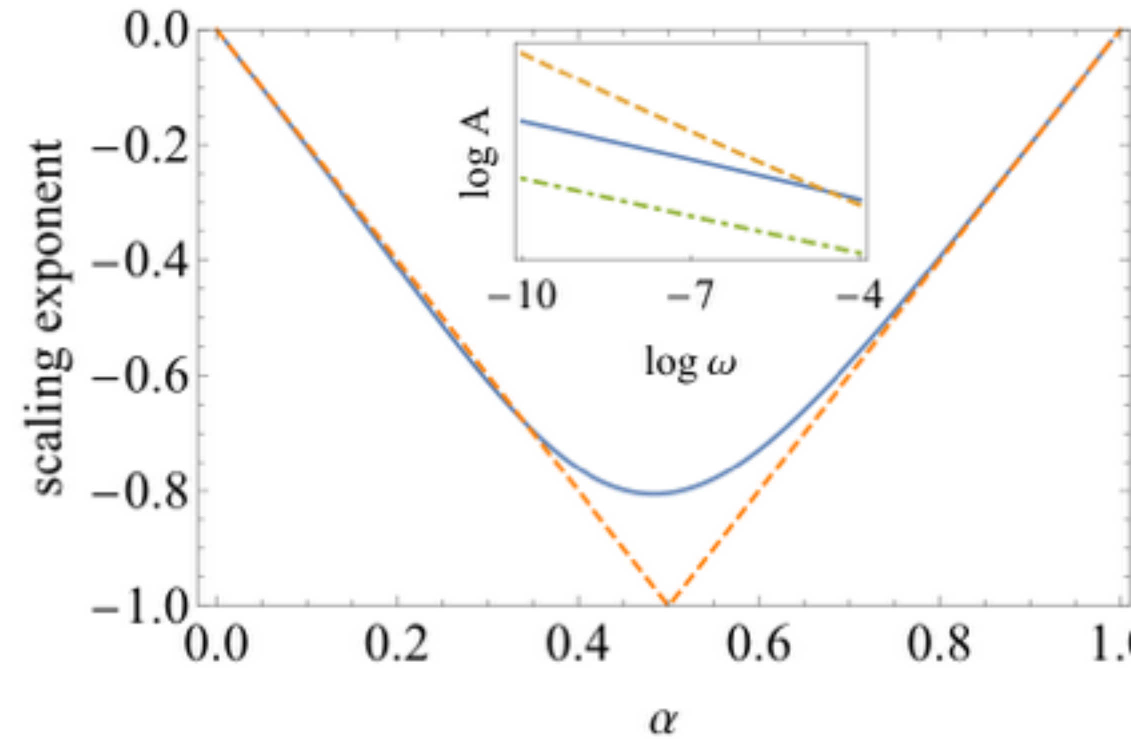
$A_{\Gamma_0=0}$

$$A\left(\epsilon = r, \omega = r\right) \approx \frac{r^{2\alpha}}{\left(r + r^{2\alpha}\right)^2 + r^{4\alpha}}$$

$\alpha > 1/2$ $\qquad$ $\alpha < 1/2$

$r^{2\alpha-2}$ $\qquad$ $r^{-2\alpha}$

$$A\left(r, r\right) \approx r^{2\left|\alpha - \frac{1}{2}\right| - 1}$$



Scaling exponent smallest at $\alpha = \frac{1}{2}$

$$\boxed{\text{origin of dome}}$$

$$\frac{1}{g_{\min}} \approx \int dr\ r^{4\left|\alpha-\frac{1}{2}\right|-1}$$

$$\Big\downarrow\ \alpha = 1/2$$

$$\boxed{\text{log divergence}}$$

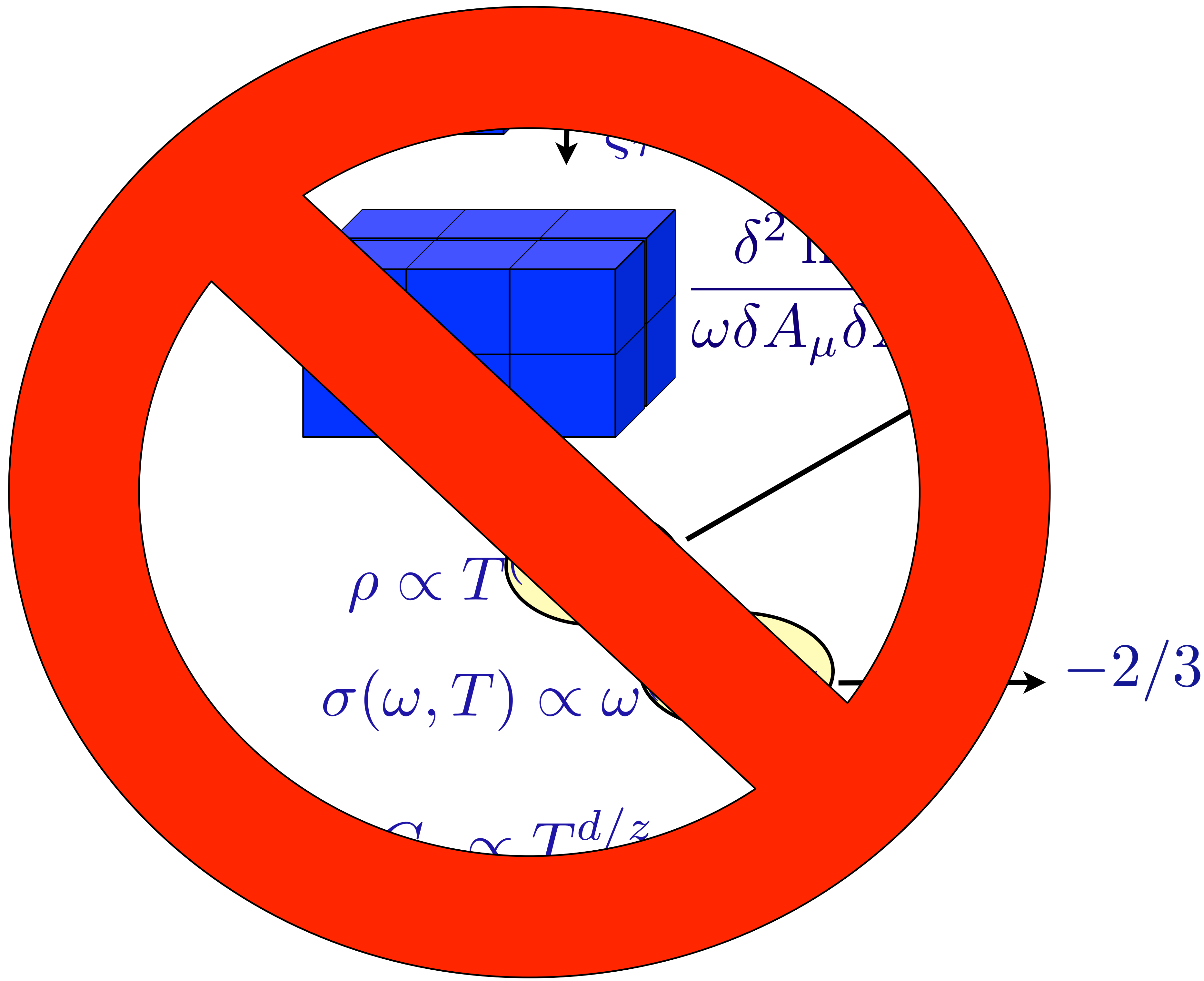
$$g_{\min} \sim 0$$

unparticles
are unstable to
superconductivity

unparticles
are unstable to
gap formation

what kind of scale-invariant theories?

single-parameter scaling



$$d_A \neq 1??$$

extra degree of freedom

$$\rho \propto T^{(2-d)/z}$$

$$\downarrow$$

$$2d_A$$

$$[A_\mu] = d_A = 1$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

$$\Big\Downarrow$$

$$\oint A \cdot d\ell = \#$$

fixes dimension of current

$$[d^d x J A] = 0$$

$$[J] = d - 1$$

strange metal

$$[J_\mu] = d - \theta + \Phi + z - 1$$

$$[A_\mu] = 1 - \Phi$$

$$\Phi = -2/3$$

$$[E] = 1 + z - \Phi$$

$$[B] = 2 - \Phi$$

BG,EK,SH,AK

$\boxed{\text{note}}$ $\pi r^2 B \neq$ flux loophole

Noether's 1st theorem

$$S \to S + \int d^d x J_\mu \partial \Lambda$$

$$\partial_\mu J^\mu = 0$$

what if

$$[\partial_\mu, \hat{Y}] = 0$$

$$\partial_\mu \hat{Y} J^\mu = \partial_\mu \tilde{J}^\mu = 0$$

new current

$$[\tilde{J}] = d - 1 - D_Y$$

$$\boxed{A_\mu \to A_\mu + \partial_\mu \Lambda + \partial_\mu \partial_\nu G^\nu + \cdots}$$

Noether's 2nd theorem

Noether's Second Theorem and Ward Identities for Gauge Symmetries

Steven G. Avery[a], Burkhard U. W. Schwab[b]

For simplicity, we focus on the case when the transformation may be written in the form[6]

$$\delta_\lambda \phi = f(\phi)\,\lambda + f^\mu(\phi)\partial_\mu\lambda, \tag{10}$$

but it is straightforward to consider transformations, as Noether did, involving arbitrarily high derivatives of λ. (Although, the authors know of no physically interesting examples.) Let us start with

arxiv:1510.07038

possible gauge transformations

$$S = -\frac{1}{4}\int d^d x F^2$$

$$S = \frac{1}{2}\int \frac{d^d k}{2\pi^d} A_\mu(k)[\underbrace{k^2\eta^{\mu\nu} - k^\mu k^\nu}_{M_{\mu\nu}k^\nu = 0}]A_\nu(k)$$

zero eigenvector

$$ik_\mu \to \partial_\mu$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

$$\boxed{\text{family of zero eigenvalues}}$$

$$M_{\mu\nu}\underbrace{f k^{\nu}}_{\boxed{\text{generator of gauge symmetry}}} = 0$$

1.) rotational invariance

2.) A is still a 1-form

3.) $[f, k_\mu] = 0$

only choice

$$f \equiv f(k^2)$$

$$(\Delta)^{\gamma}$$

$$A_\mu \to A_\mu + (\Delta)^{\frac{(\gamma-1)}{2}} \partial_\mu \Lambda \qquad [A_\mu] = \gamma$$

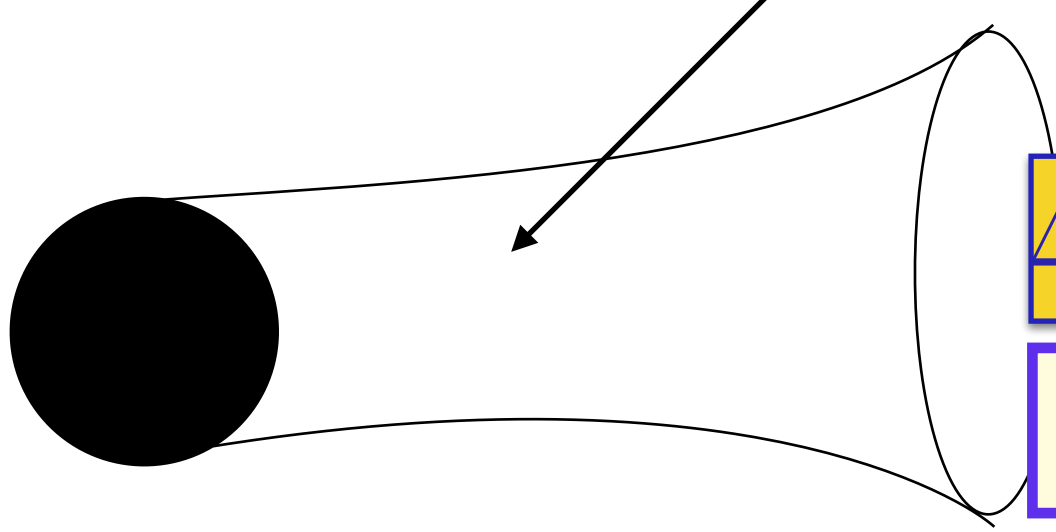
what kind of E&M has such gauge transformations?

$$S = \int dV_d dy \left(y^a F^2 + \cdots\right)$$

eom

$$d(y^a \star dA) = 0$$

$$y \neq 0 \longrightarrow A \to A + \partial\Lambda$$



$\Delta^\gamma A_\perp = J$

+GL in CIMP

family of zero eigenvalues

$$M_{\mu\nu} f k^{\nu} = 0$$

most fundamental conservation law

$$\partial^{\mu}(-\nabla^2)^{(\gamma-1)/2} J_{\mu} = 0$$

scale invariance in strange metal

MEELS, ARPES, Optics,STM

non-local actions

anomalous dimensions

running charge (no charge quantization)