Moiré is Different:

Wigner crystallization in TBLG

Bikash Padhi

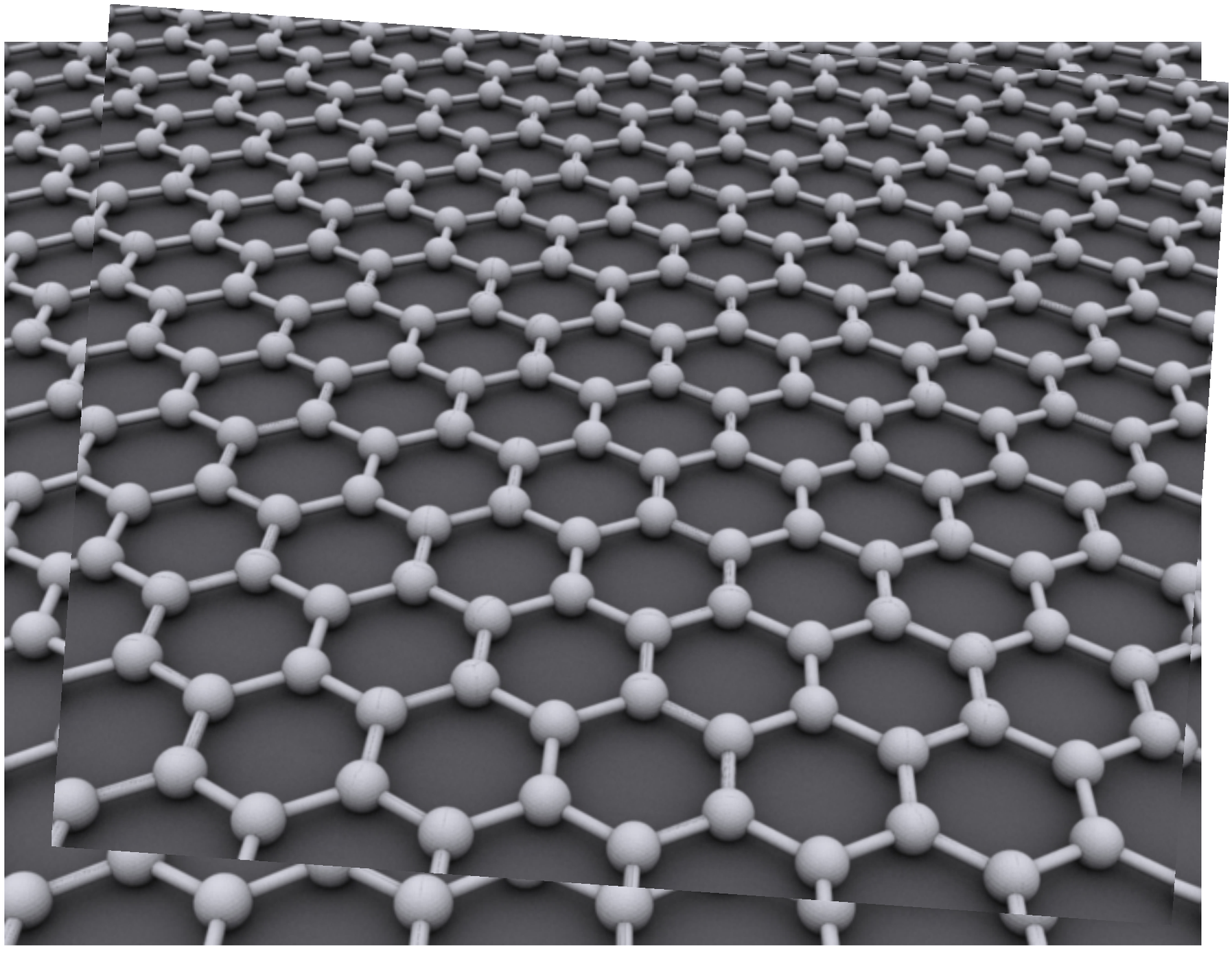
Chandan Setty

Nano. Lett. 2018+
arXiv:1810.00884

EFRC,NSF

BLG: Emergent Physics

$$1 + 1 = ?$$



4 August 1972, Volume 177, Number 4047

SCIENCE

More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

The reductionist hypothesis may still be a topic for controversy among philosophers, but among the great majority of active scientists I think it is accepted without question. The workings of our minds and bodies, and of all the animate or inanimate matter of which we have any detailed knowledge, are assumed to be controlled by the same set of fundamental laws, which except under certain extreme conditions we feel we know pretty well.

It seems inevitable to go on uncritically to what appears at first sight to be an obvious corollary of reduction-

planation of phenomena in terms of known fundamental laws. As always, distinctions of this kind are not unambiguous, but they are clear in most cases. Solid state physics, plasma physics, and perhaps also biology are extensive. High energy physics and a good part of nuclear physics are intensive. There is always much less intensive research going on than extensive. Once new fundamental laws are discovered, a large and ever increasing activity begins in order to apply the discoveries to hitherto unexplained phenomena. Thus, there are two dimensions to basic research. The frontier of science extends all along a long line from the newest and most modern intensive research, over the ex-

less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other. That is, it seems to me that one may array the sciences roughly linearly in a hierarchy, according to the idea: The elementary entities of science X obey the laws of science Y.

X	Y
solid state or many-body physics	elementary particle physics
chemistry	many-body physics
molecular biology	chemistry
cell biology	molecular biology
•	•
•	•
•	•
psychology	physiology
social sciences	psychology

single-layer graphene

$$v(p) = \pm cp \qquad c = 10^6 m/s$$

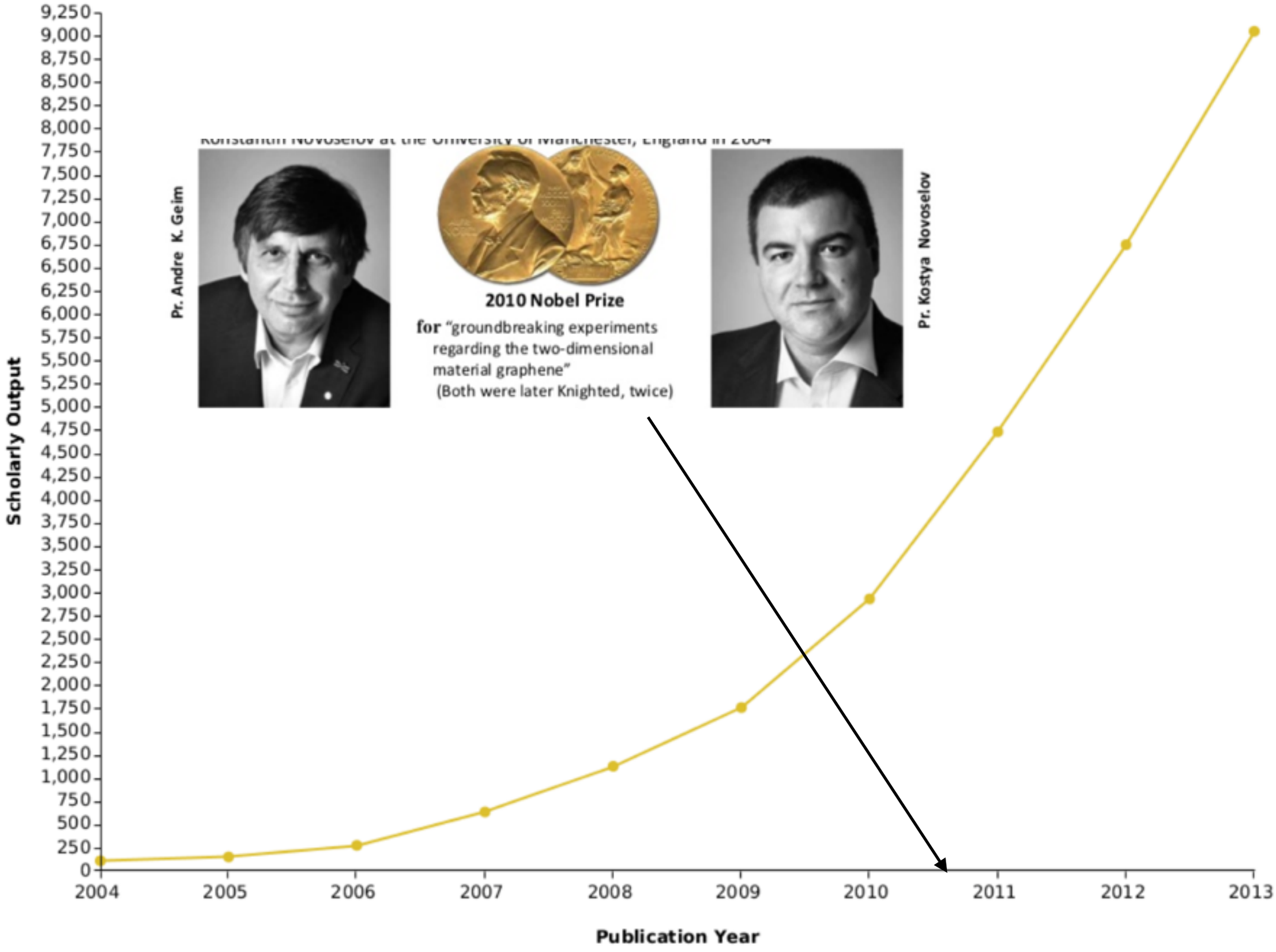
Konstantin Novoselov at the University of Manchester, England in 2004

Pr. Andre K. Geim

2010 Nobel Prize

for "groundbreaking experiments regarding the two-dimensional material graphene"
(Both were later Knighted, twice)

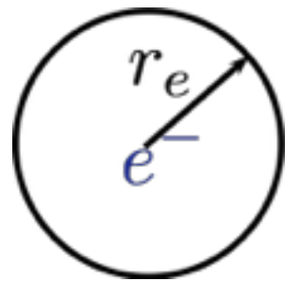
Pr. Kostya Novoselov



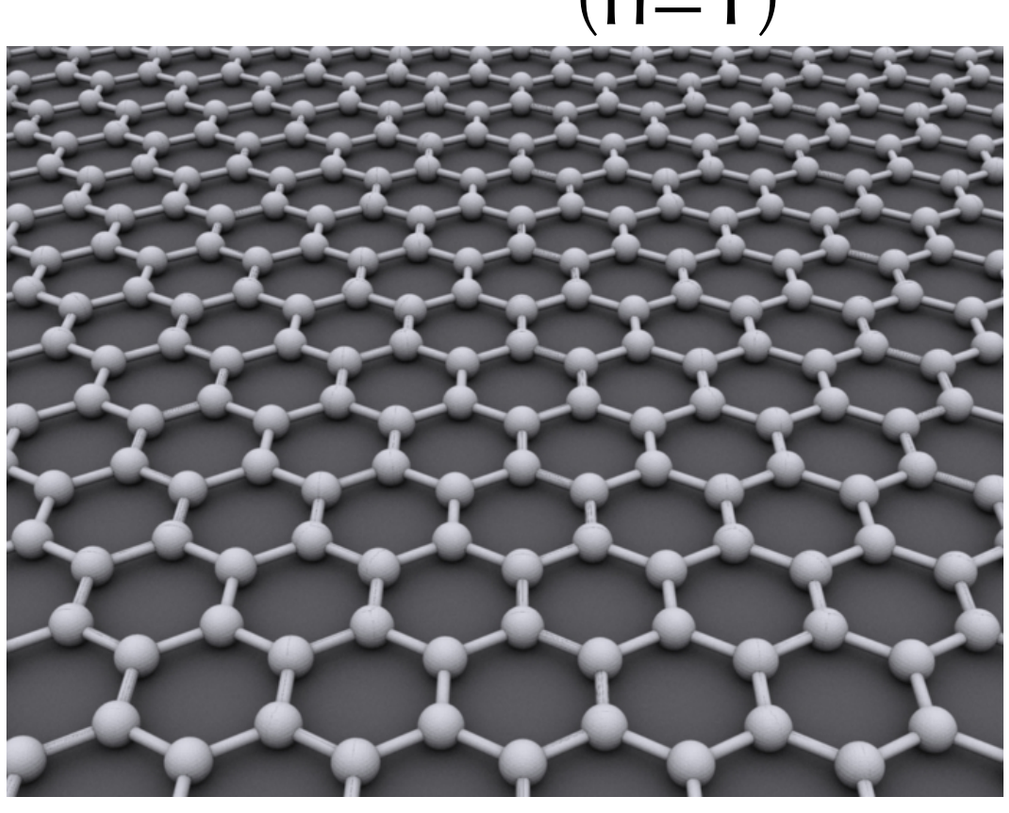
when are interactions important?

$$r_s = \frac{V_{\rm ee}}{E_{\rm kin}} = \frac{e^2}{\epsilon c_n} r_e^{n-1}$$

$c_n k^n$



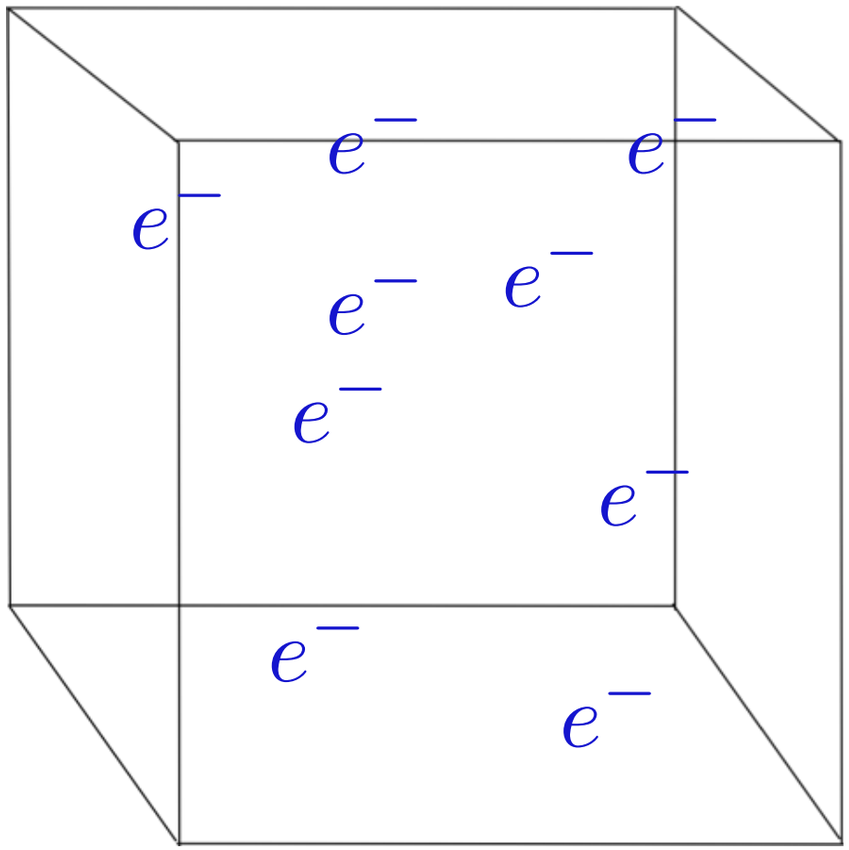
Dirac spectrum
(n=1)

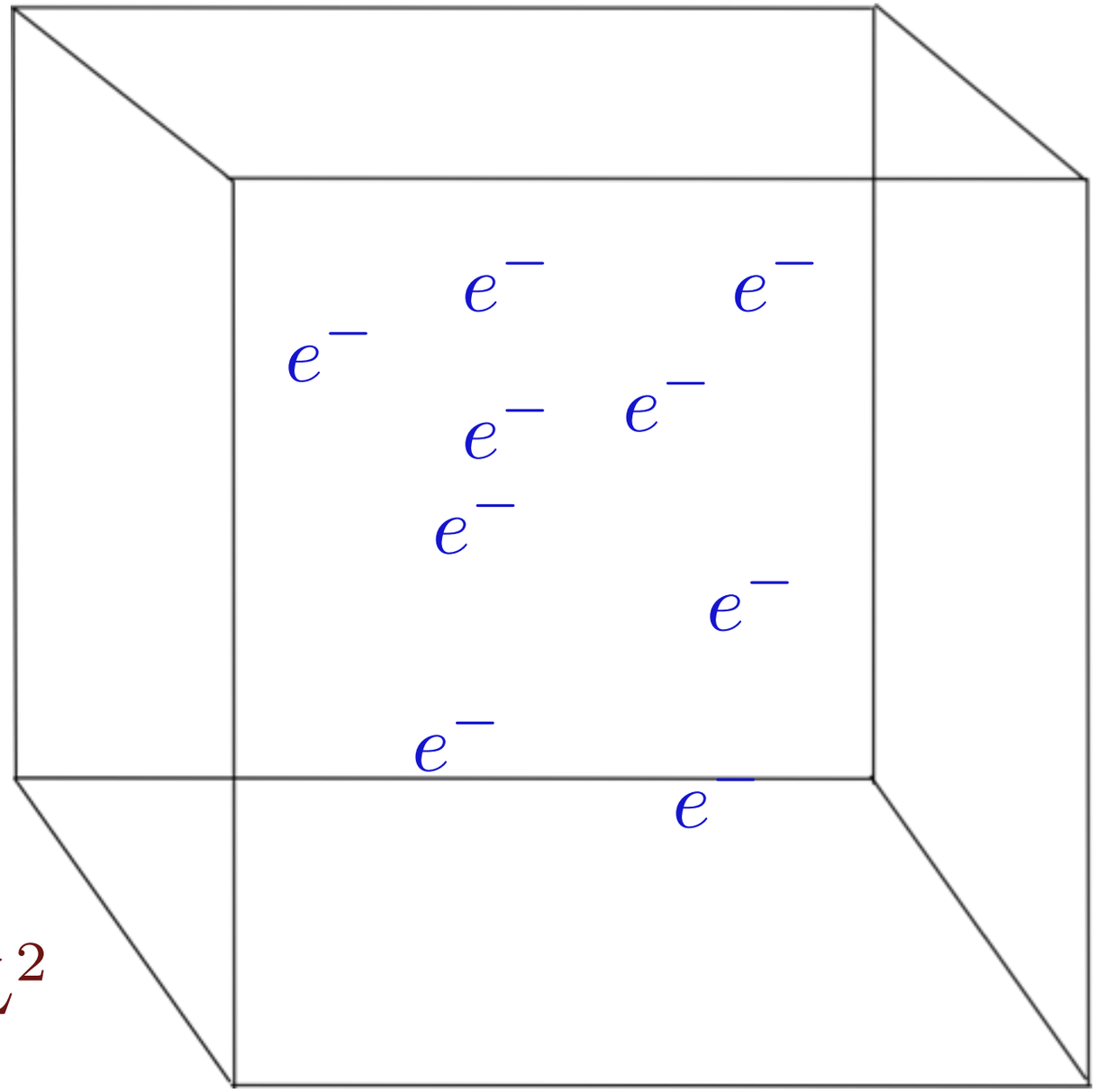


density independent

no correlated physics by changing density

Standard Case: In which case do the electrons interact more strongly?





$$V_{\rm ee} \propto 1/L \qquad \frac{p^2}{2m} \propto 1/L^2$$

$$r_s \propto 1/L \propto 1/n_e^{1/3}$$

Wigner crystallization

Wigner crystallization

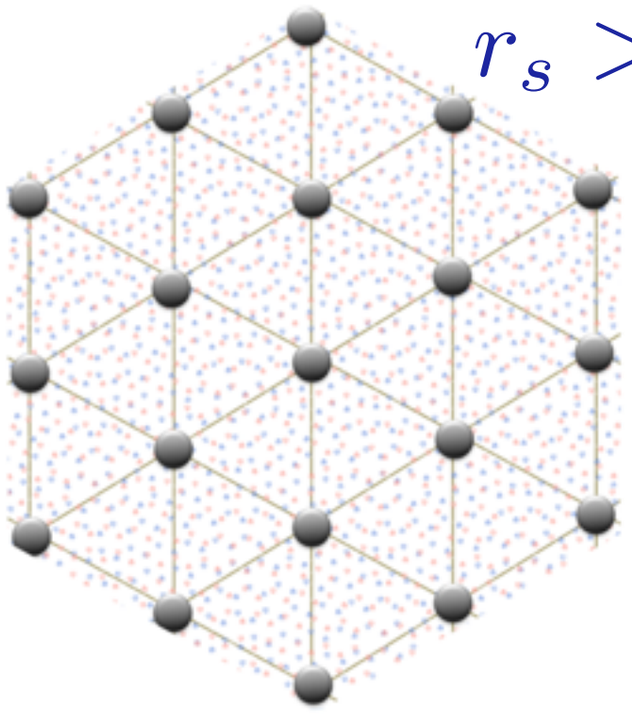
Mott Insulation

no underlying lattice

lattice needed

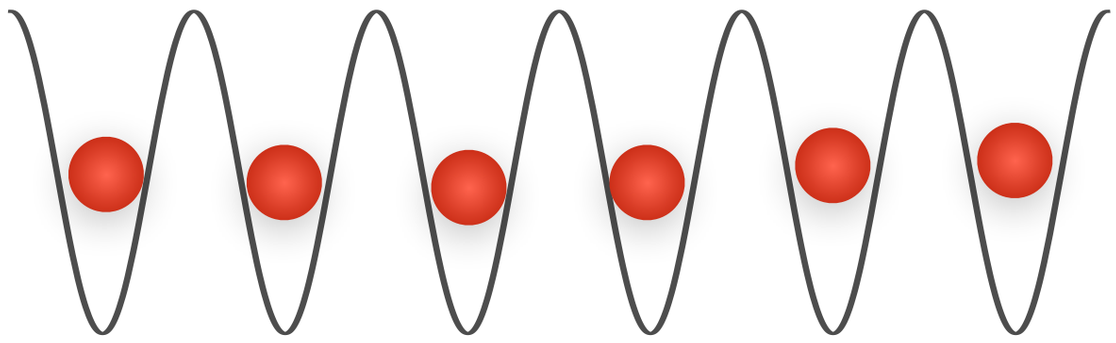
symmetry-broken state

symmetry intact

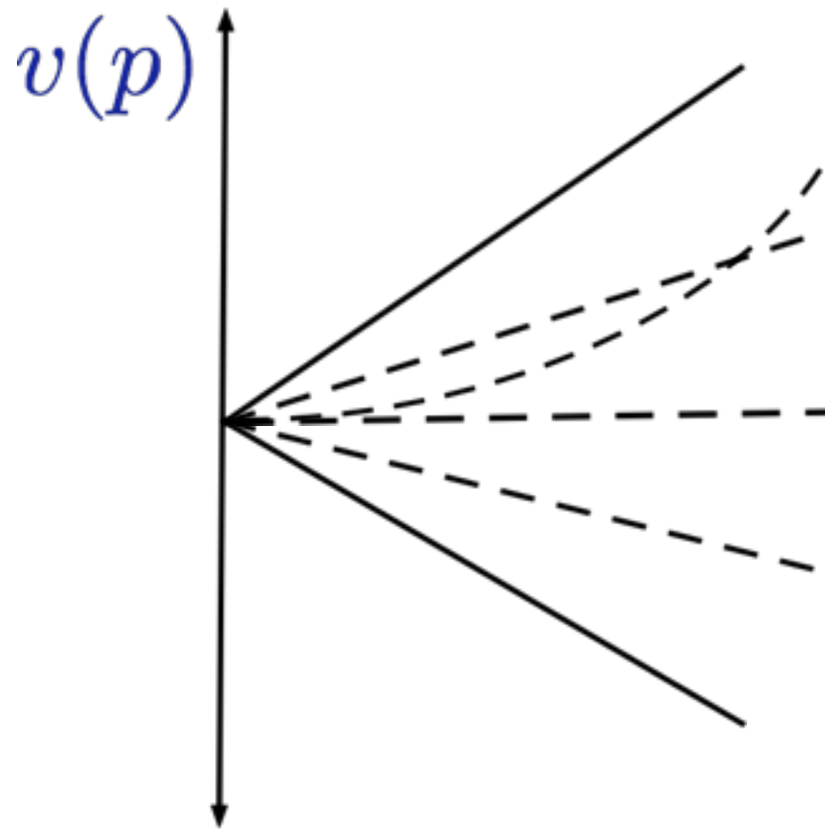


$r_s > 37$

$U \gg W$



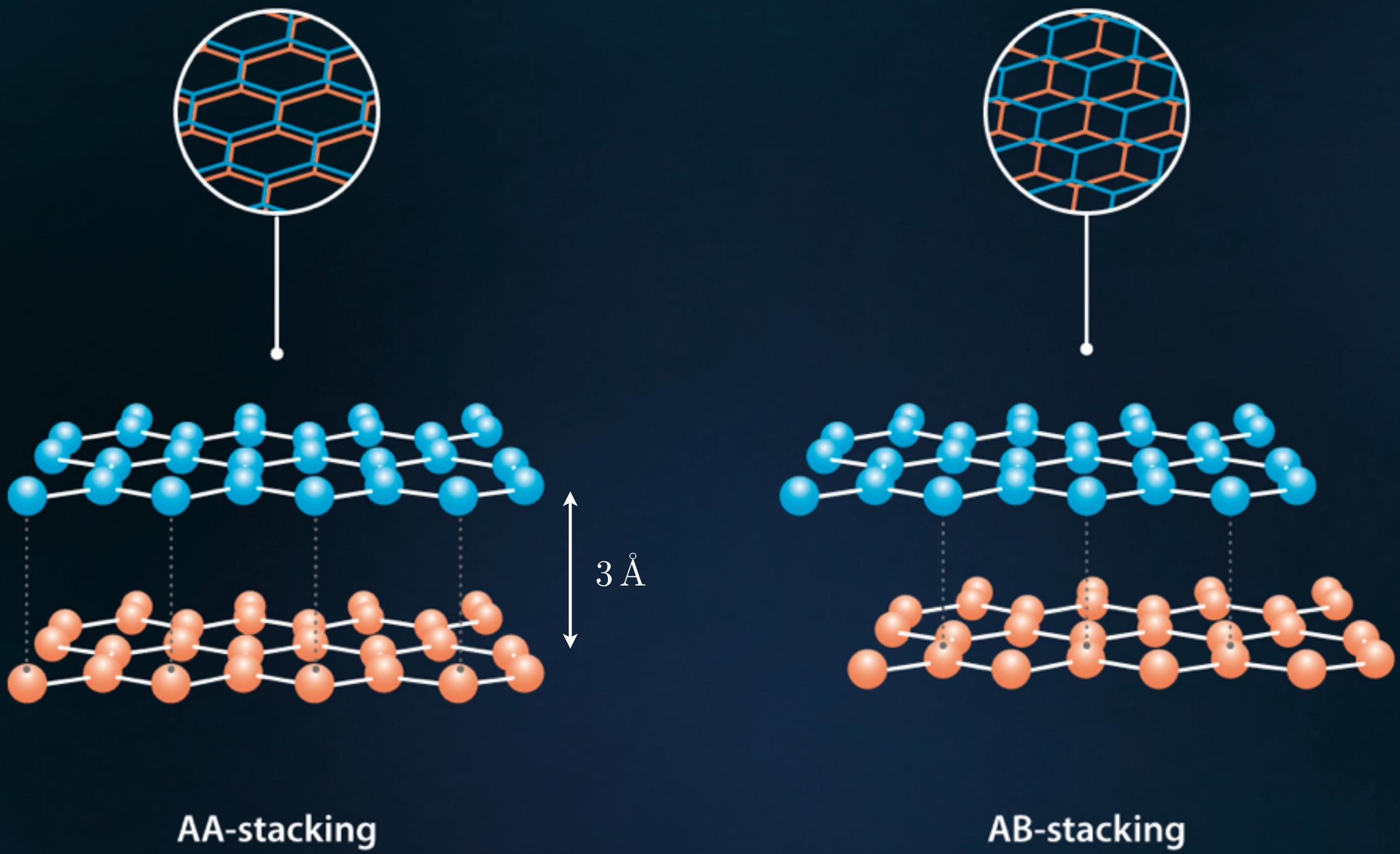
Can new physics be engineered?



K.E.=0

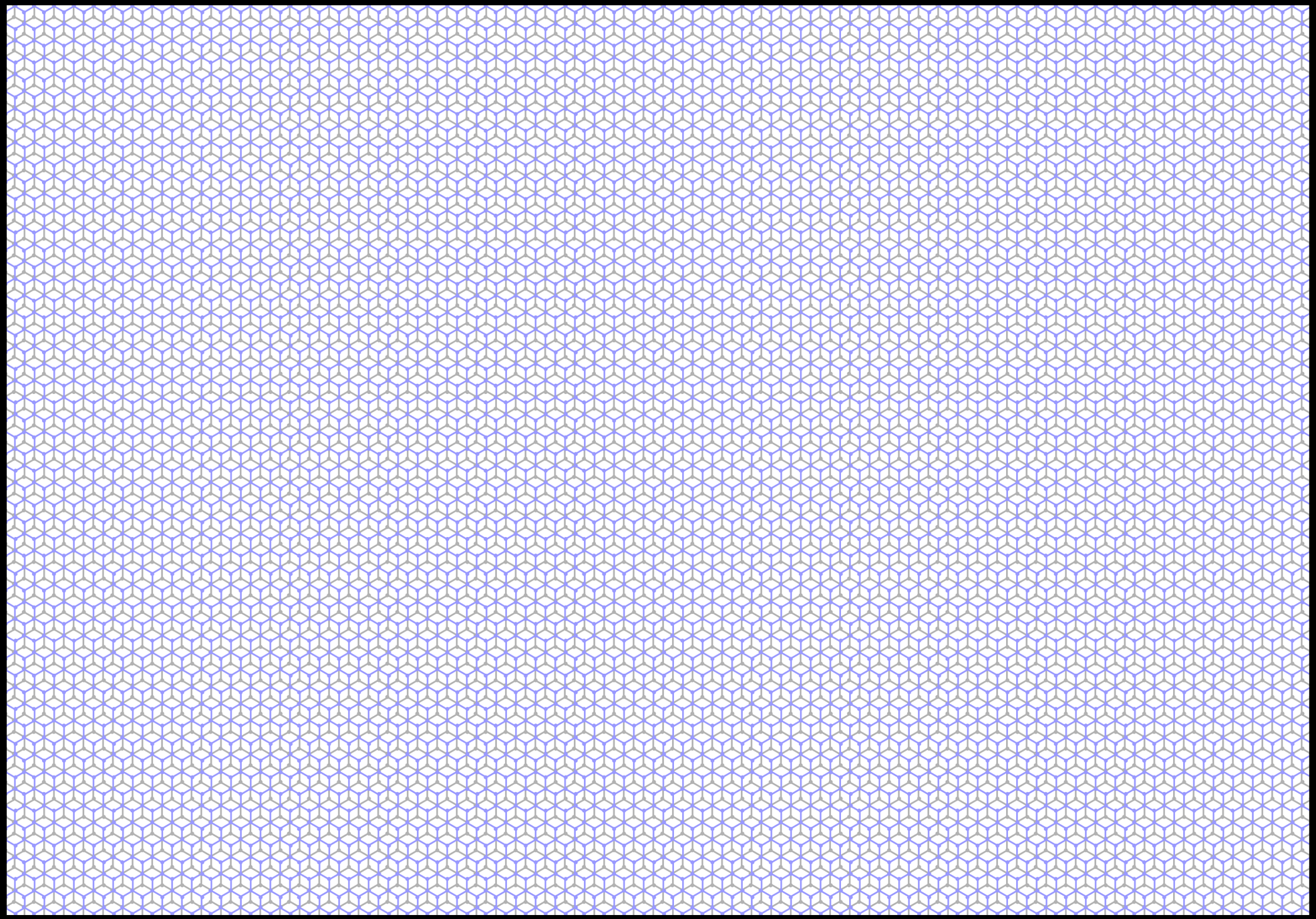
interactions dominate

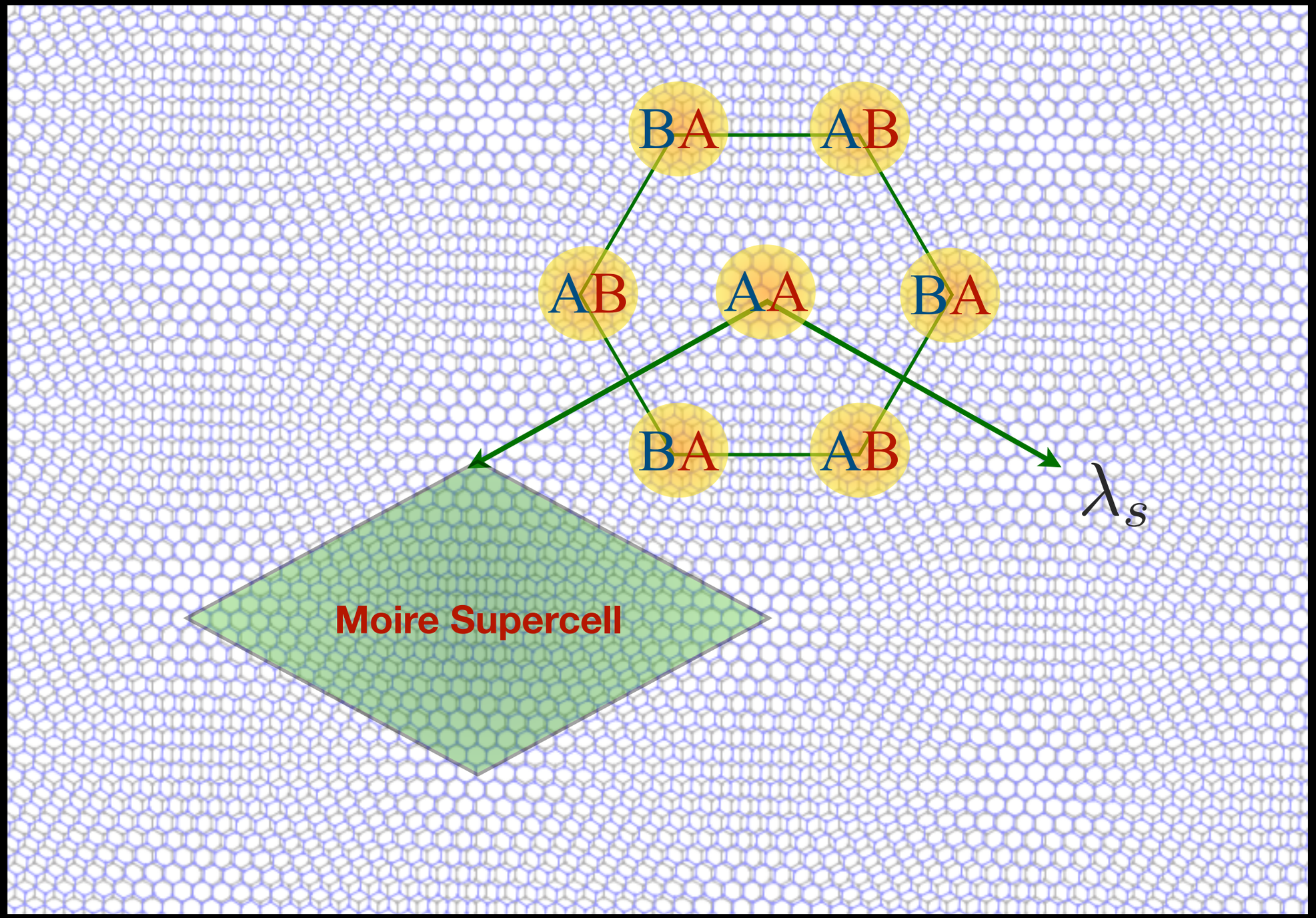
new physics



AA-stacking

AB-stacking

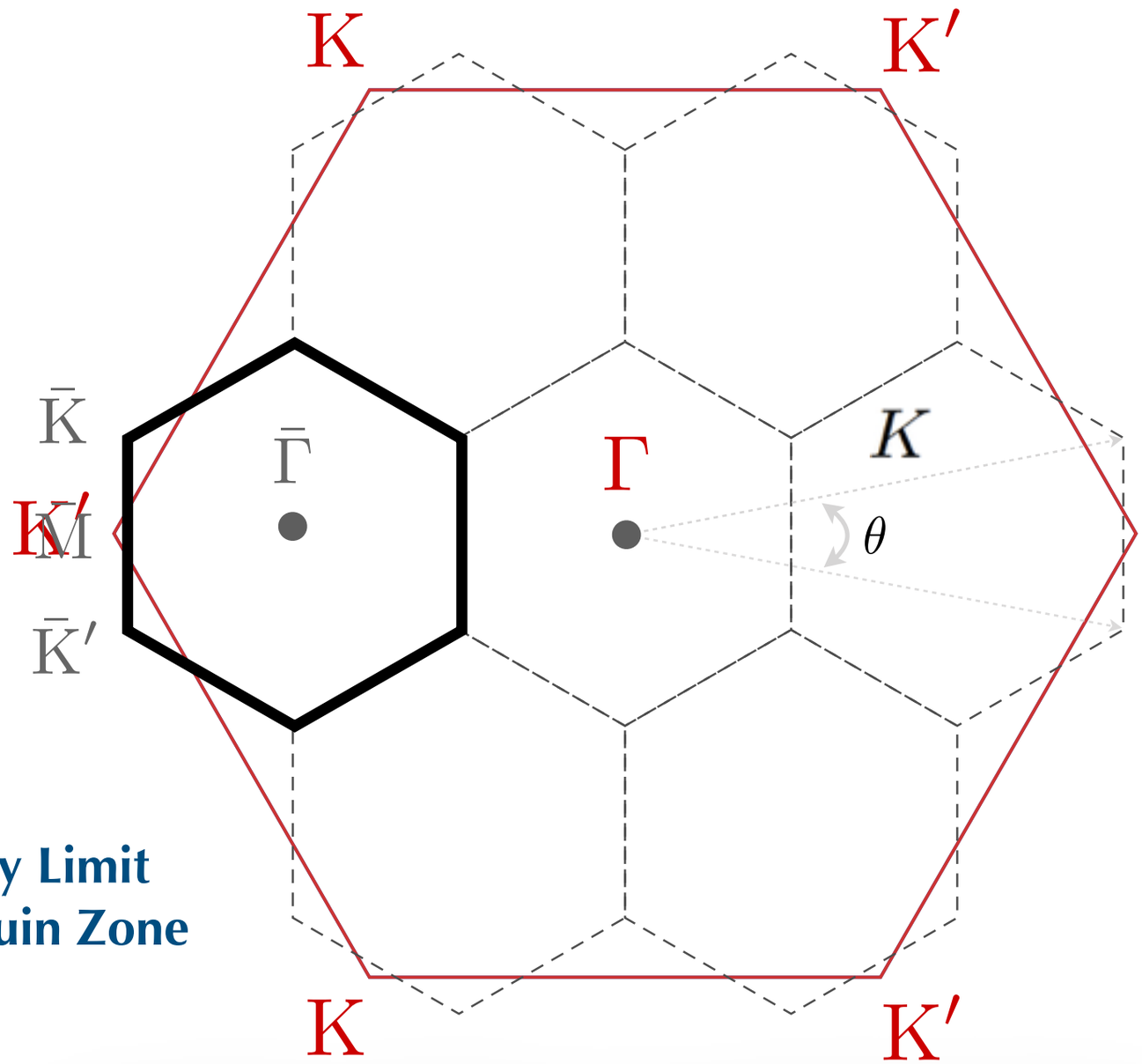



BA
AB
AB
AA
BA
BA
AB
Moire Supercell
λ_s

moiré pattern

In mathematics, physics, and art, a moiré pattern (/mwɑːrˈeɪ/; French: [mwaʁe]) or moiré fringes[1] are large-scale interference patterns that can be produced when an opaque ruled pattern with transparent gaps is overlaid on another similar pattern.

Momentum Space



$$k_\theta = 2K \sin(\theta/2)$$

Low-Energy Limit
moire Brillouin Zone

$$\text{moiré cell} \not\Rightarrow \text{moiré lattice}$$

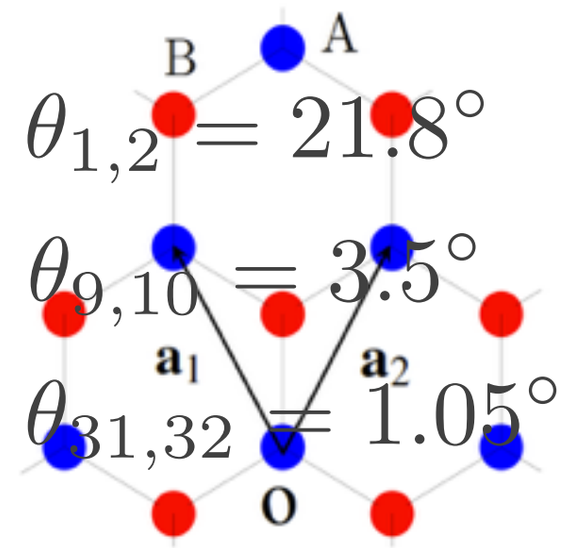
Commensuration

$$\cos\theta_{m,n} = \frac{m^2 + 4mn + n^2}{2(m^2 + mn + n^2)} \quad (m, n \in \mathbb{Z})$$

Shallcross *et al.* PRB(2010)

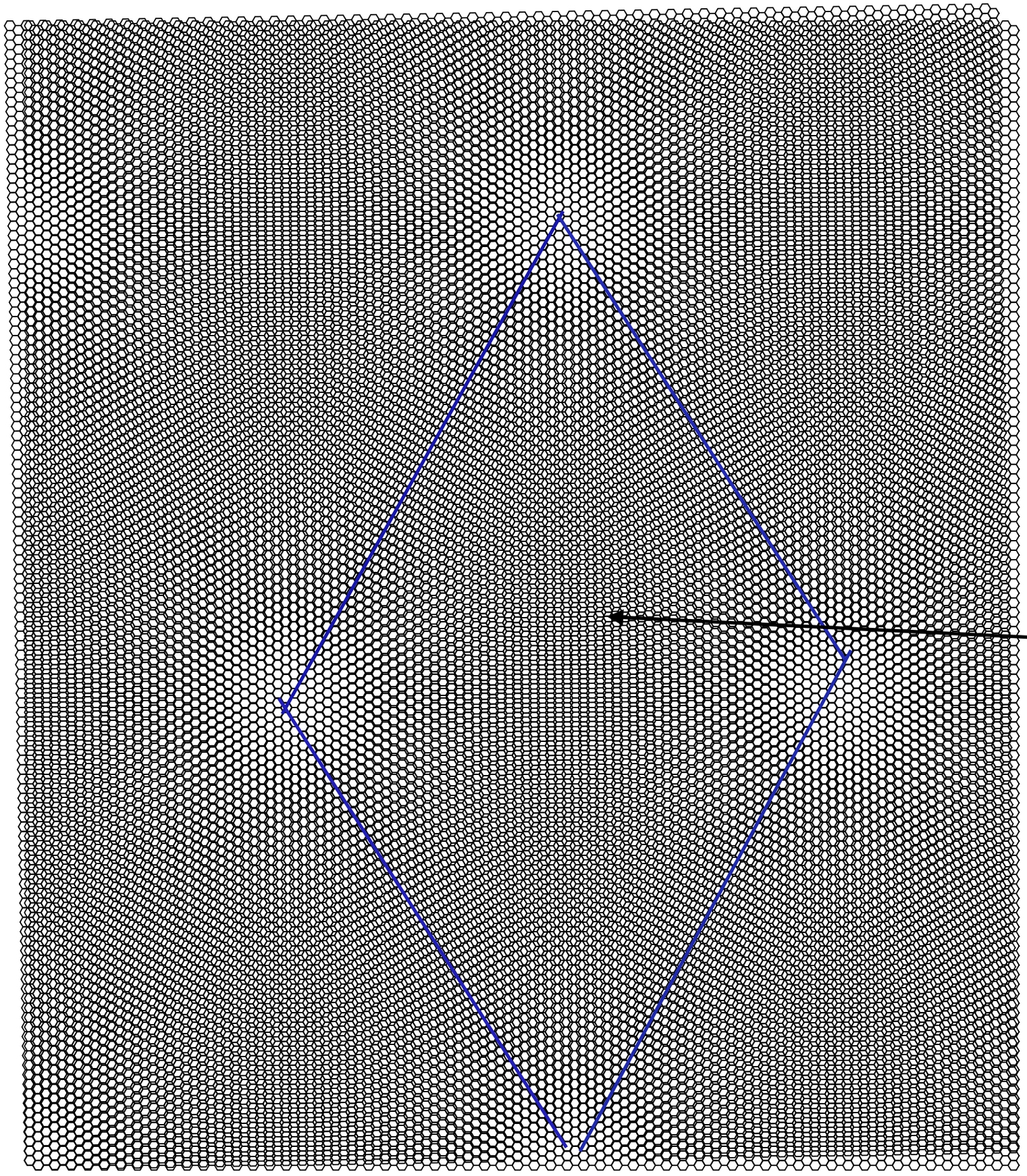
$$\boldsymbol{R}_1 = m\boldsymbol{a}_1 + n\boldsymbol{a}_2$$

$$\boldsymbol{R}_2 = -n\boldsymbol{a}_1 + (m+n)\boldsymbol{a}_2$$



$$\text{moiré lattice} = |m - n| \times \lambda_s \quad \text{(moiré cell)}$$

120 Å ← $N_a = \left[\frac{2\lambda_s}{a}\right]^2 \approx 10,000 \pm 2,000$ → 2.46 Å

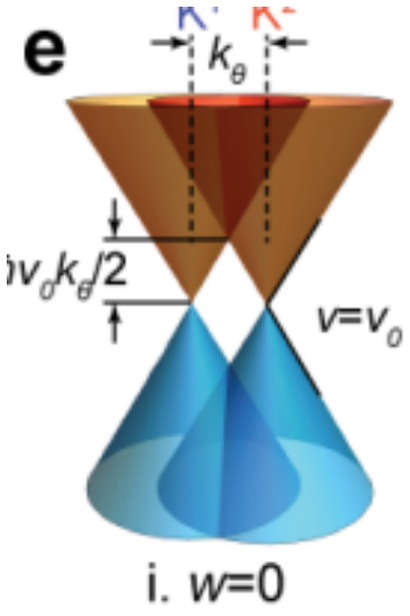


$\theta = 1.0^o$

$N = (2\lambda_s/a)^2$

≈ 130000

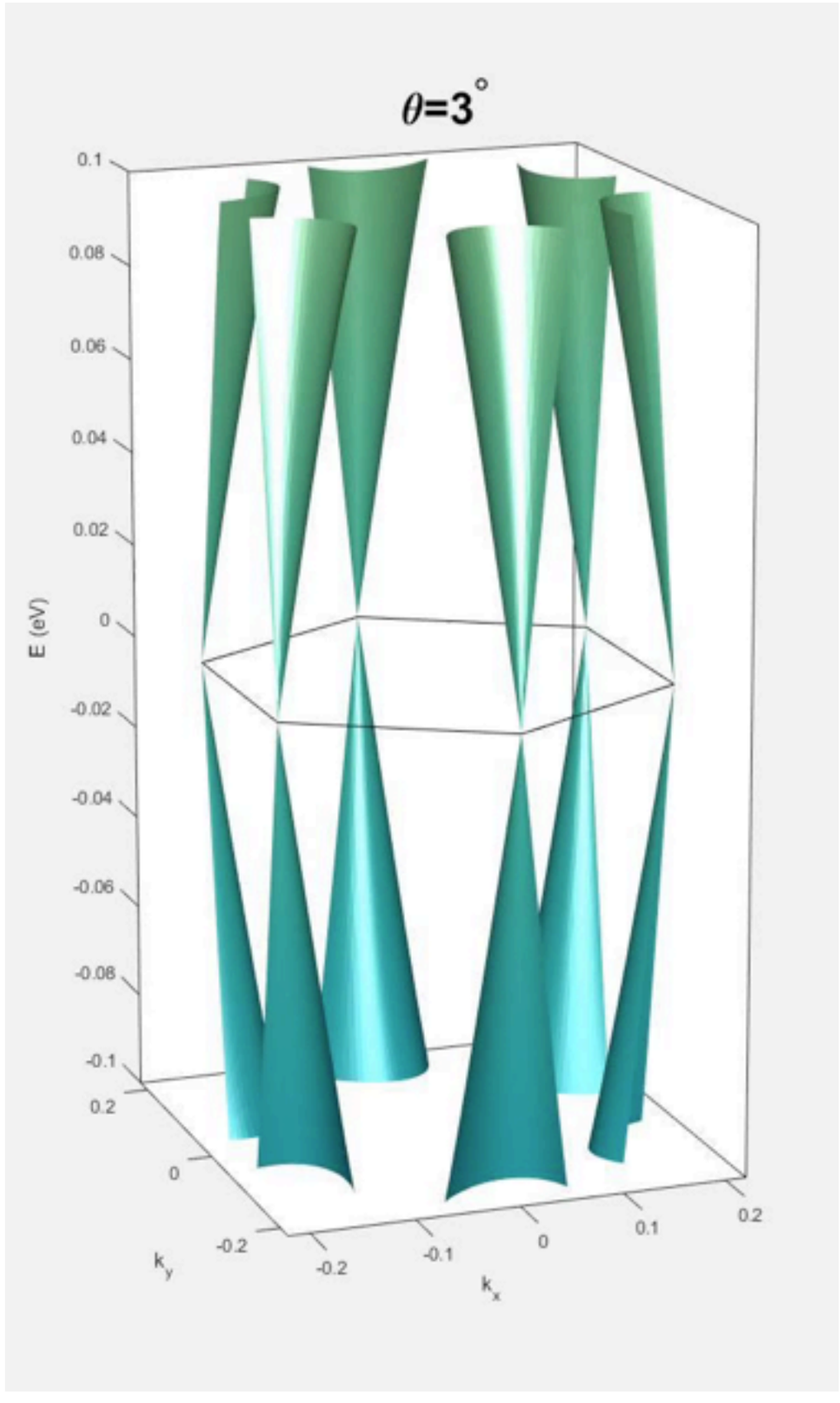
Bi-layer Graphene



i. $w=0$

$$\frac{V_{\text{ee}}}{\text{K.E.}} \ll 1$$

Evolution of Band Structure

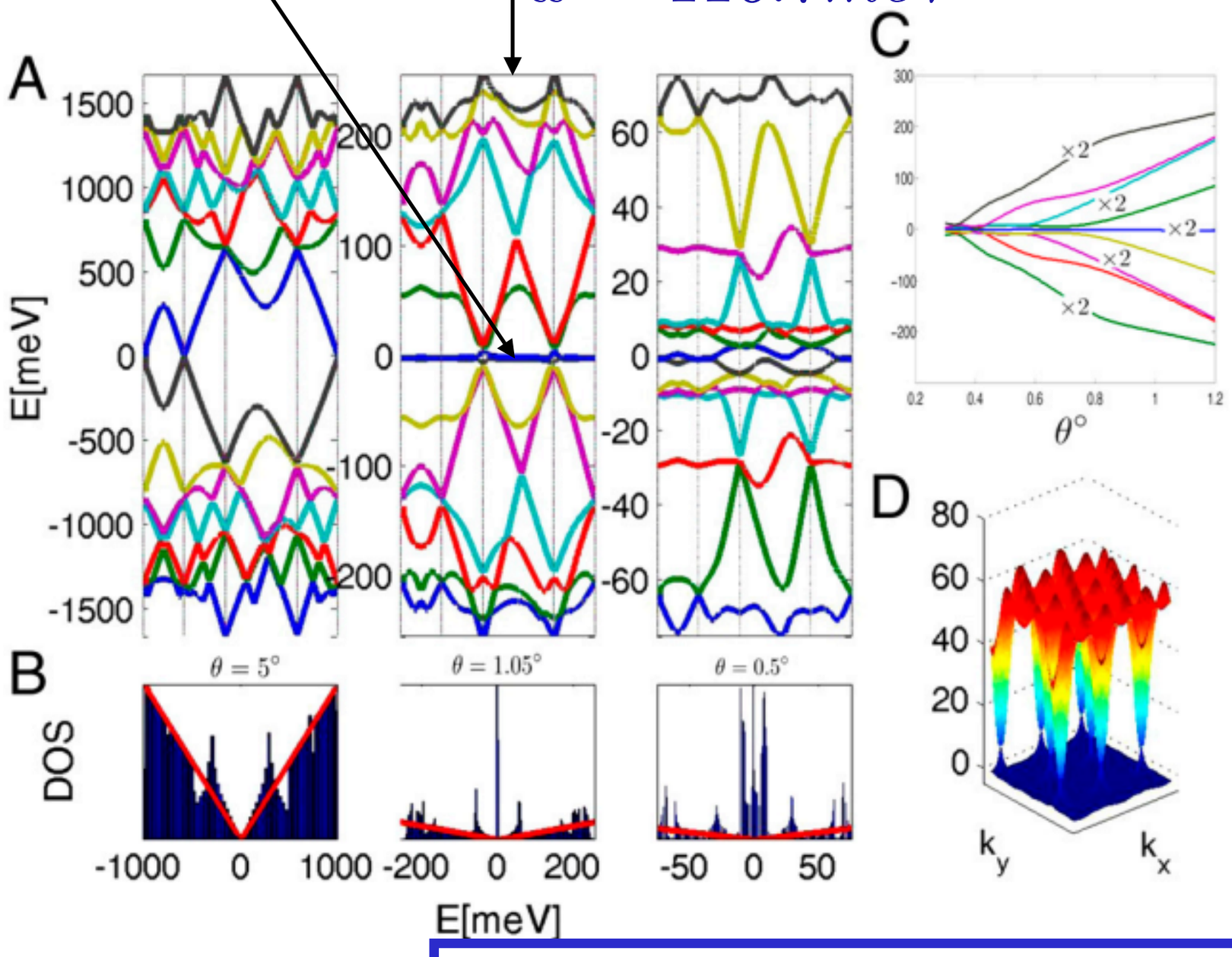


2 bands

Bi-layer Graphene

$\theta = \theta_{\text{magic}}$

$w = 116.7meV$



Bristritzer/MacDonald, 2011

Can all the other bands be ignored?

No

chirality

Senthil, Vishwanath, Po

Minimal TBM not possible

something else

superlattice density

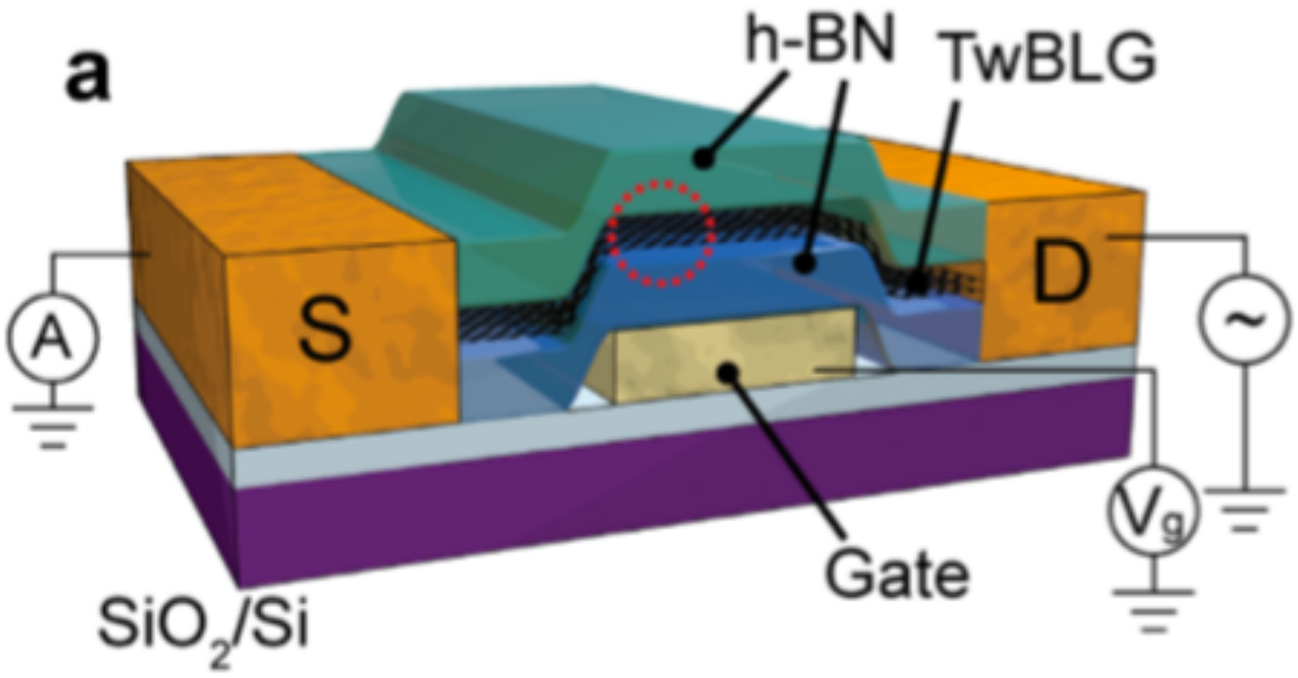
$$n_s A_s = 4$$

filled 2-lowest bands

band filling

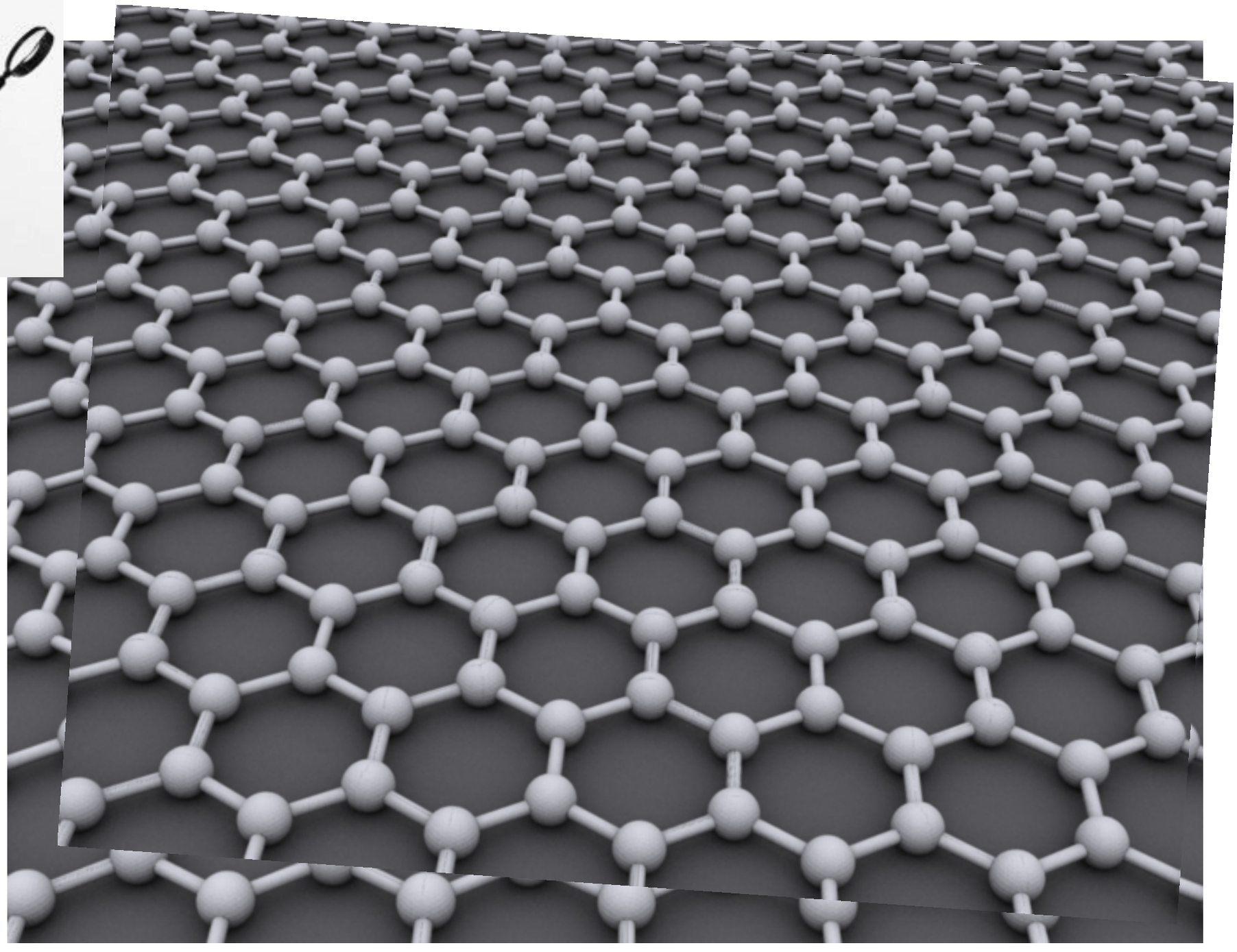
$$\nu = n_e A_s$$

experiment

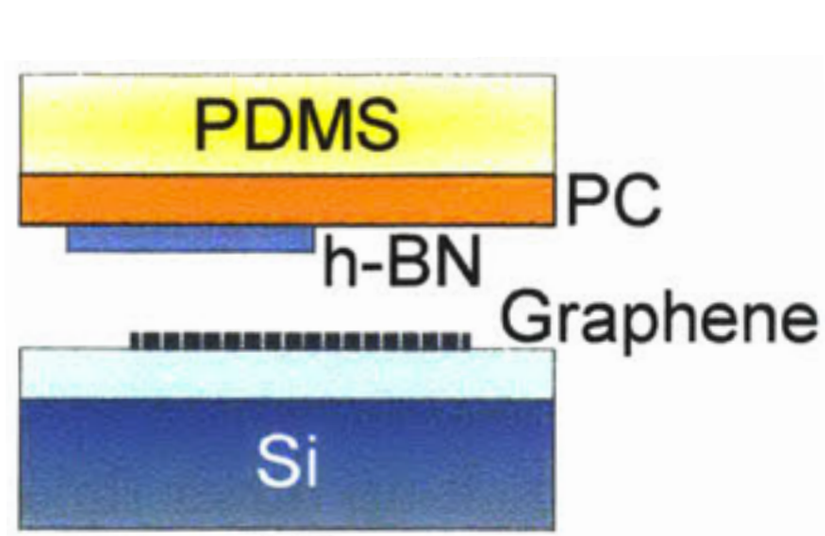


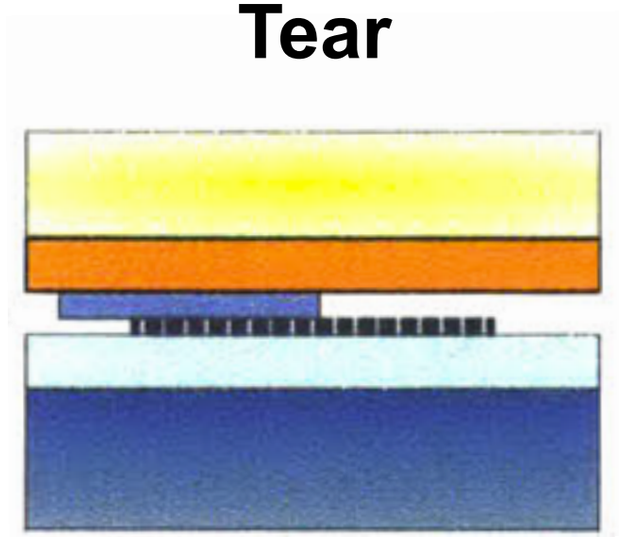
probe physics of lowest moire band

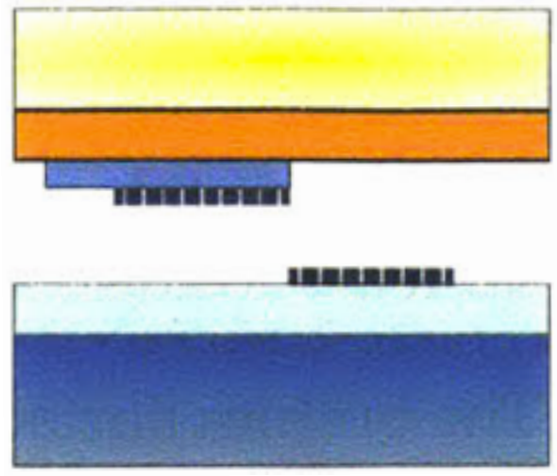
how is the angle determined?

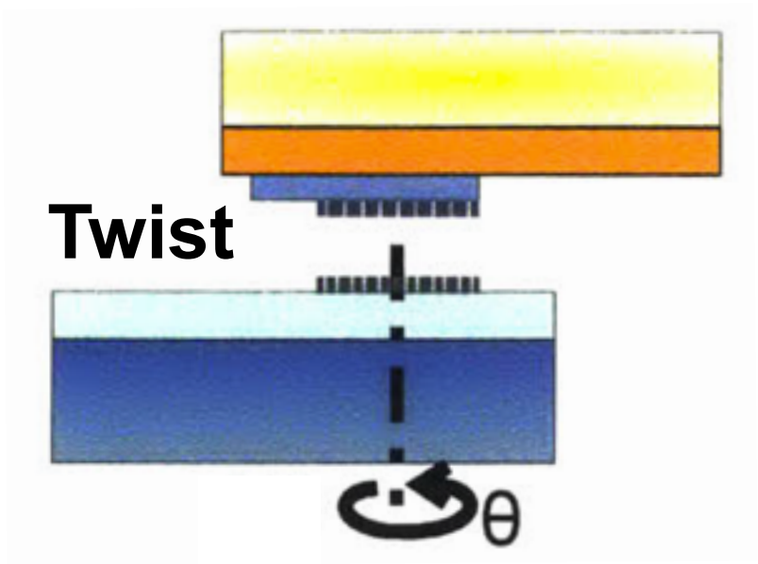


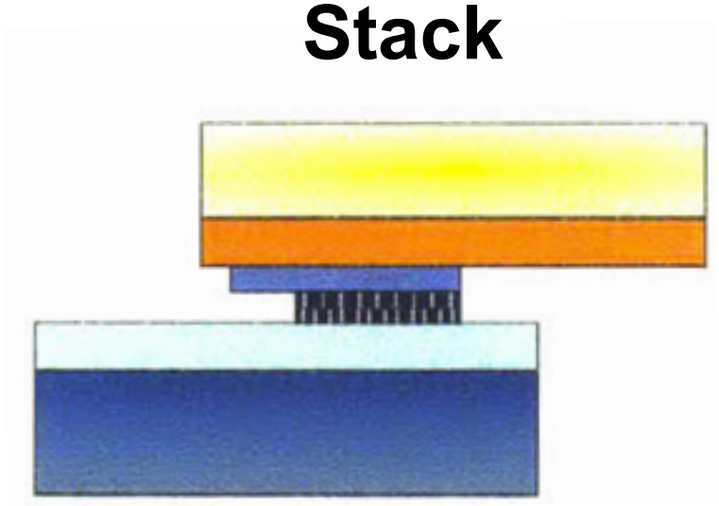
Tear-Twist-Stack

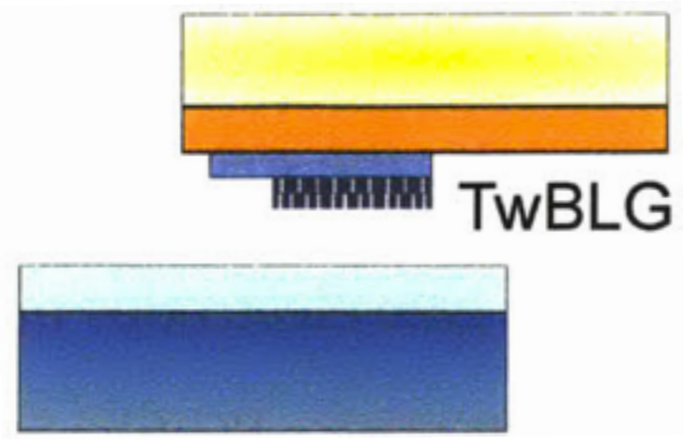








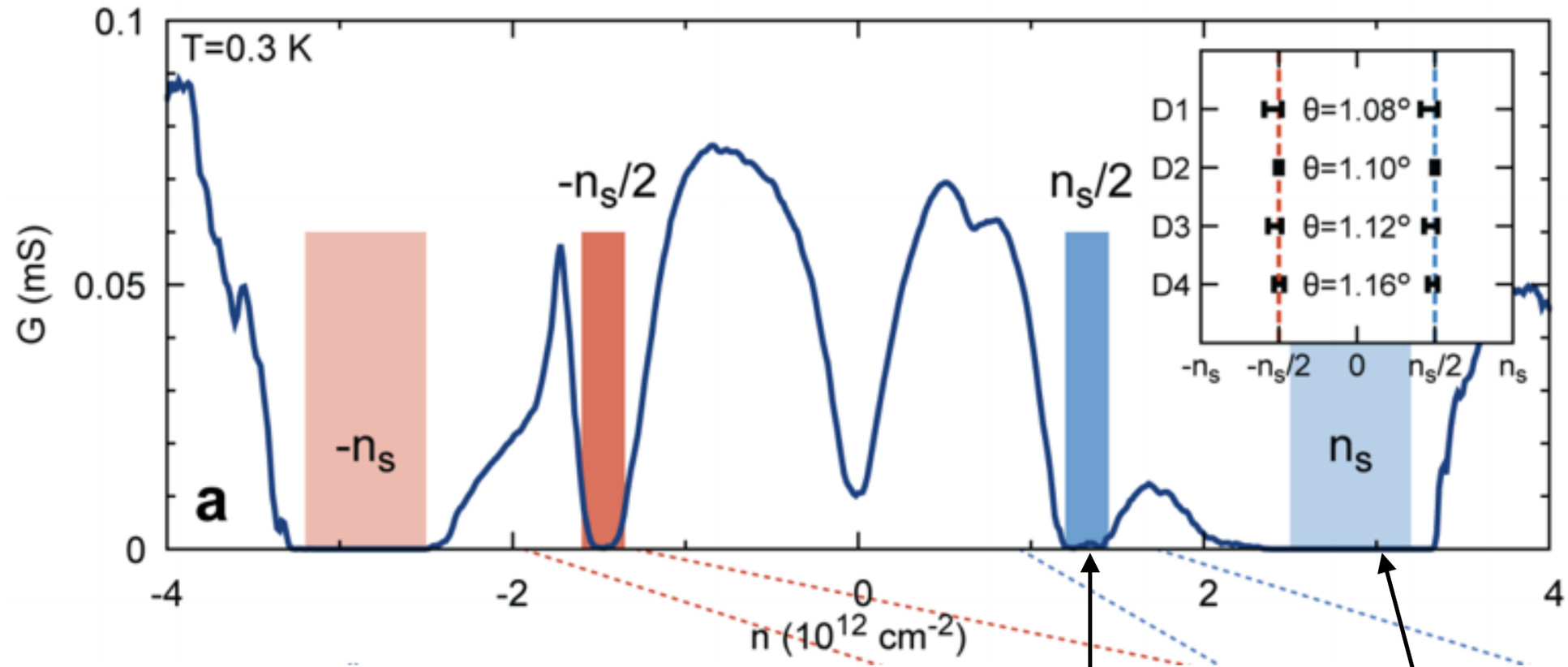




E. Tutuc *et. al.*

filling

$$\nu = n_e A_s$$

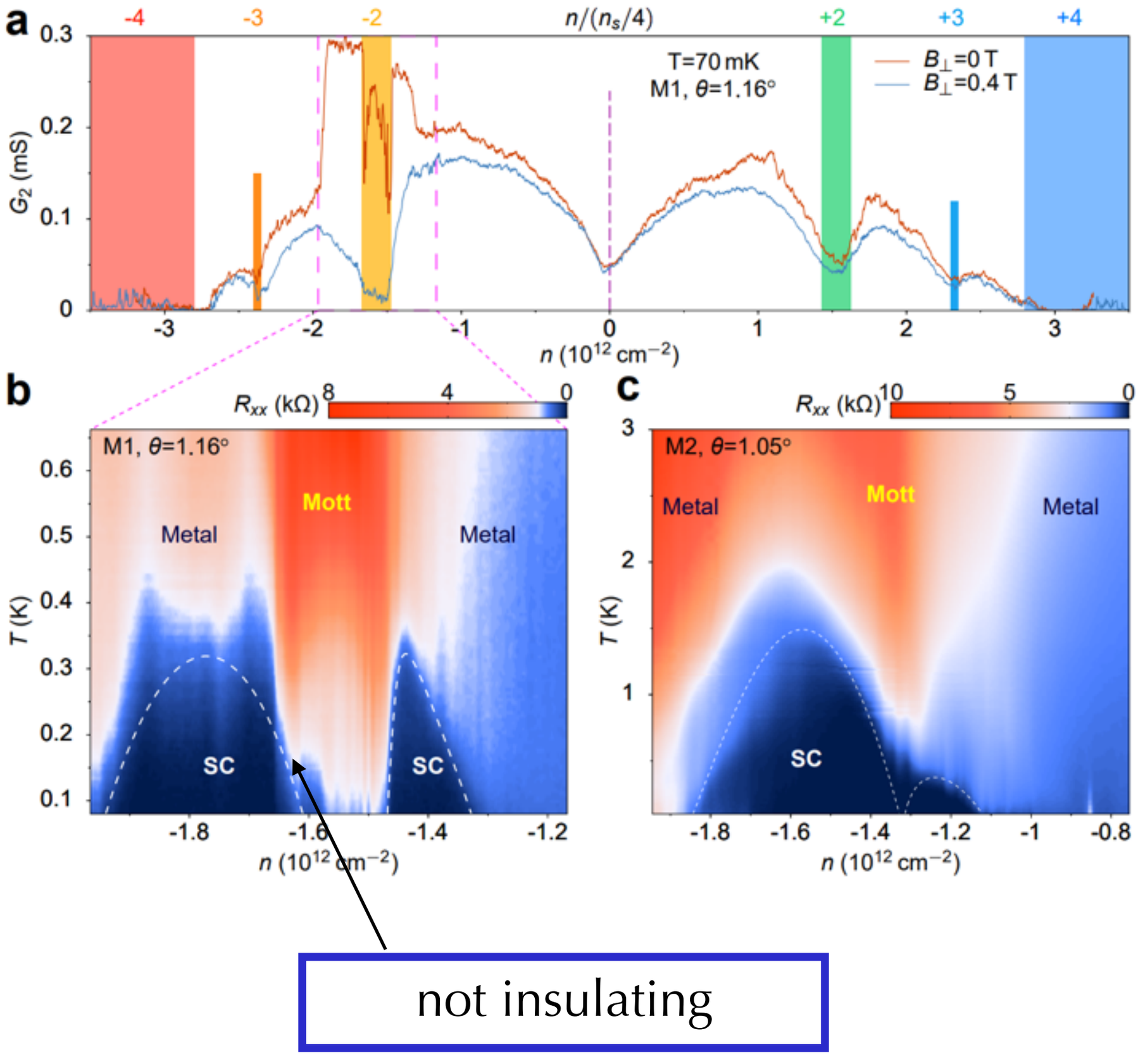


$\nu = 2$

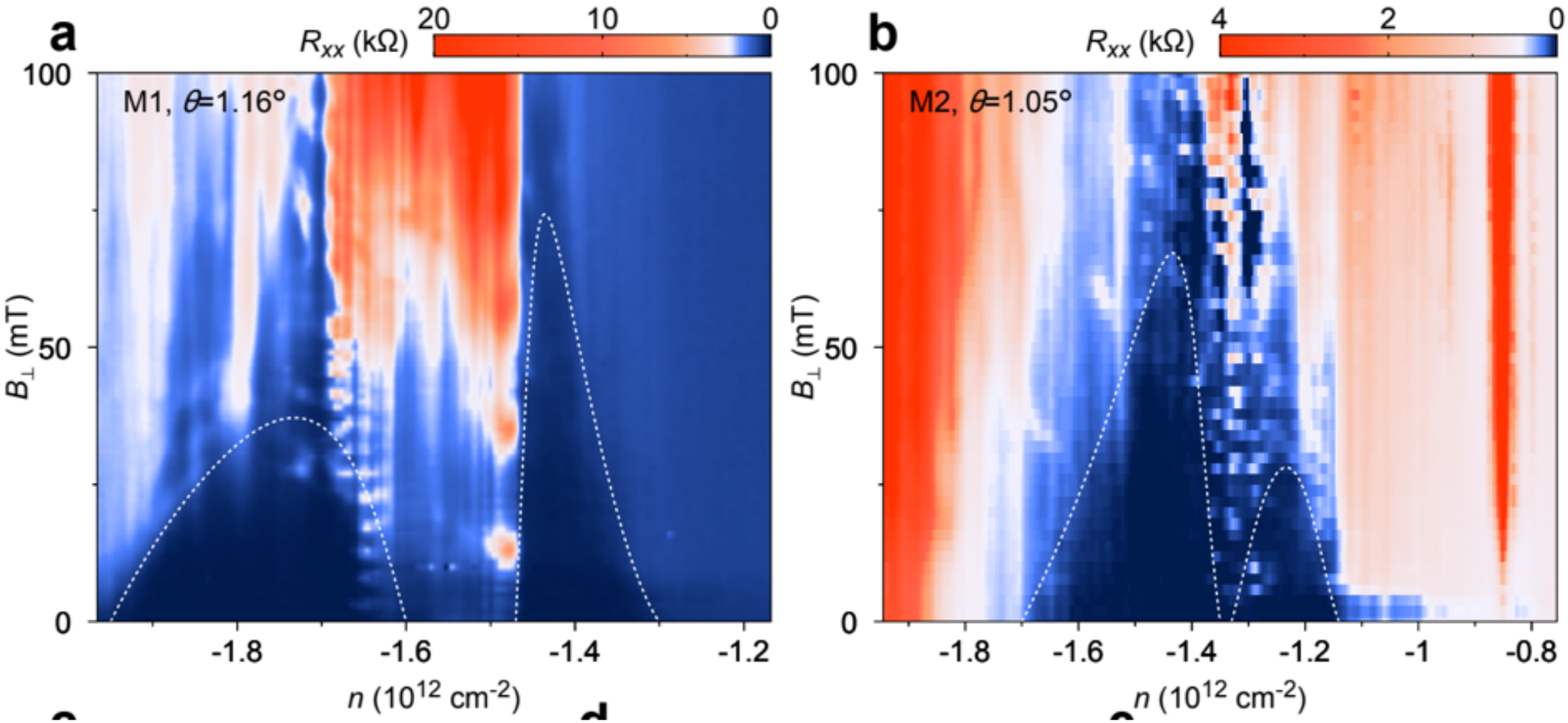
Mott Insulator

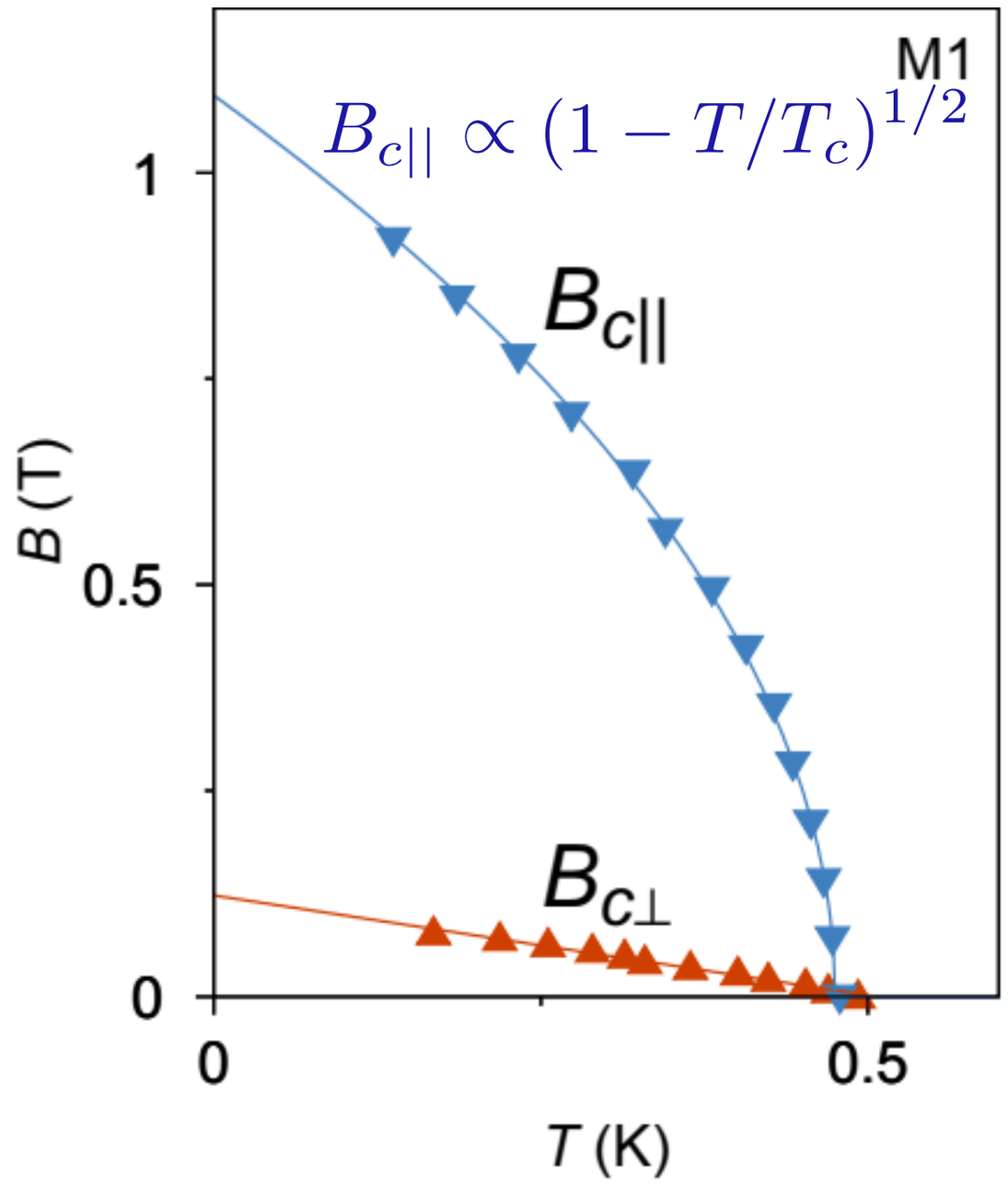
$\nu = 4$

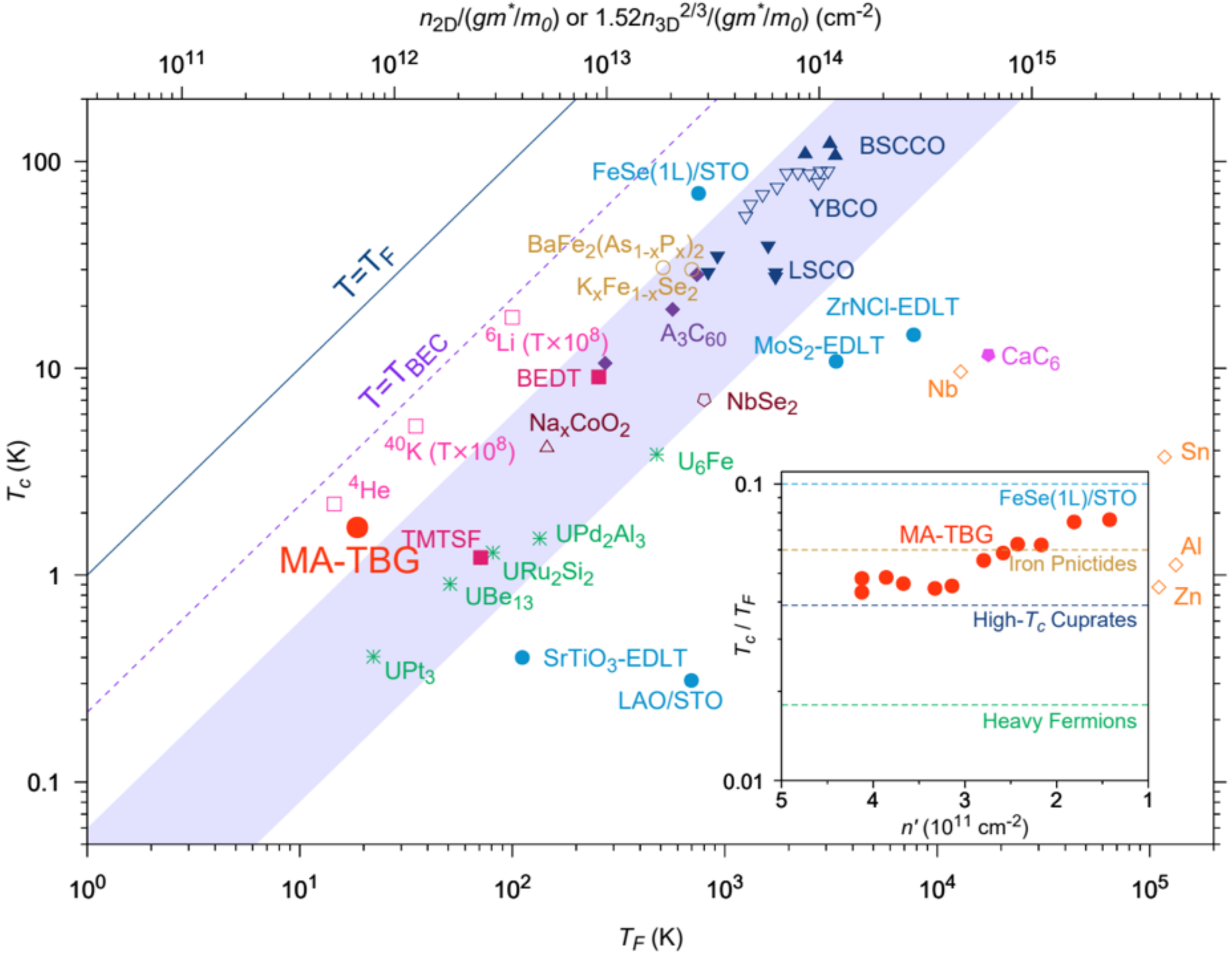
band insulator

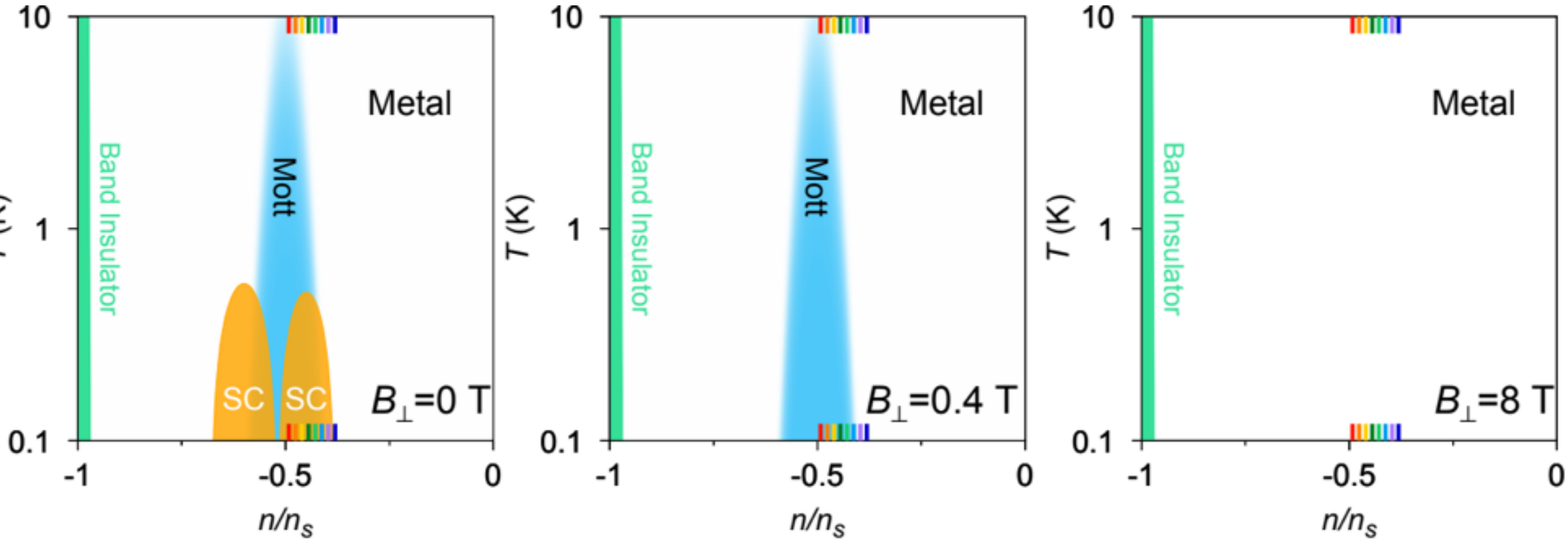

a
-4
-3
-2
n/(n_s/4)
+2
+3
+4
T=70 mK
M1, θ=1.16°
B⊥=0 T
B⊥=0.4 T
G2 (mS)
n (10^12 cm^−2)
b
Rxx (kΩ)
M1, θ=1.16°
Mott
Metal
Metal
SC
SC
T (K)
n (10^12 cm^−2)
c
Rxx (kΩ)
M2, θ=1.05°
Metal
Mott
Metal
SC
T (K)
n (10^12 cm^−2)
not insulating

$$40mT < H_c < 70mT$$


a
R_{xx} (kΩ)
20
10
0
M1, θ=1.16°
100
50
0
$B_\perp$ (mT)
-1.8
-1.6
-1.4
-1.2
n (10^{12} cm^{-2})
b
R_{xx} (kΩ)
4
2
0
M2, θ=1.05°
100
50
0
$B_\perp$ (mT)
-1.8
-1.6
-1.4
-1.2
-1
-0.8
n (10^{12} cm^{-2})


M1
$B_{c||} \propto (1 - T/T_c)^{1/2}$
$B_{c||}$
$B_{c\perp}$
$B_{c\perp} \propto (1 - T/T_c)$
B (T)
T (K)
1
0.5
0
0
0.5


$n_{2D}/(gm^*/m_0)$ or $1.52n_{3D}^{2/3}/(gm^*/m_0)$ (cm^{-2})
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
100
10
1
0.1
T_c (K)
$T=T_F$
$T=T_{BEC}$
BSCCO
FeSe(1L)/STO
YBCO
$BaFe_2(As_{1-x}P_x)_2$
LSCO
$K_xFe_{1-x}Se_2$
A_3C_{60}
ZrNCl-EDLT
^{6}Li ($T\times10^8$)
MoS_2-EDLT
CaC_6
BEDT
Nb
$NbSe_2$
Na_xCoO_2
Sn
^{40}K ($T\times10^8$)
U_6Fe
^{4}He
UPd_2Al_3
TMTSF
MA-TBG
URu_2Si_2
Al
UBe_{13}
Zn
UPt_3
$SrTiO_3$-EDLT
LAO/STO
T_c / T_F
0.1
0.01
FeSe(1L)/STO
MA-TBG
Iron Pnictides
High-T_c Cuprates
Heavy Fermions
5
4
3
2
1
n' (10^{11} cm^{-2})
10^0
10^1
10^2
10^3
10^4
10^5
T_F (K)

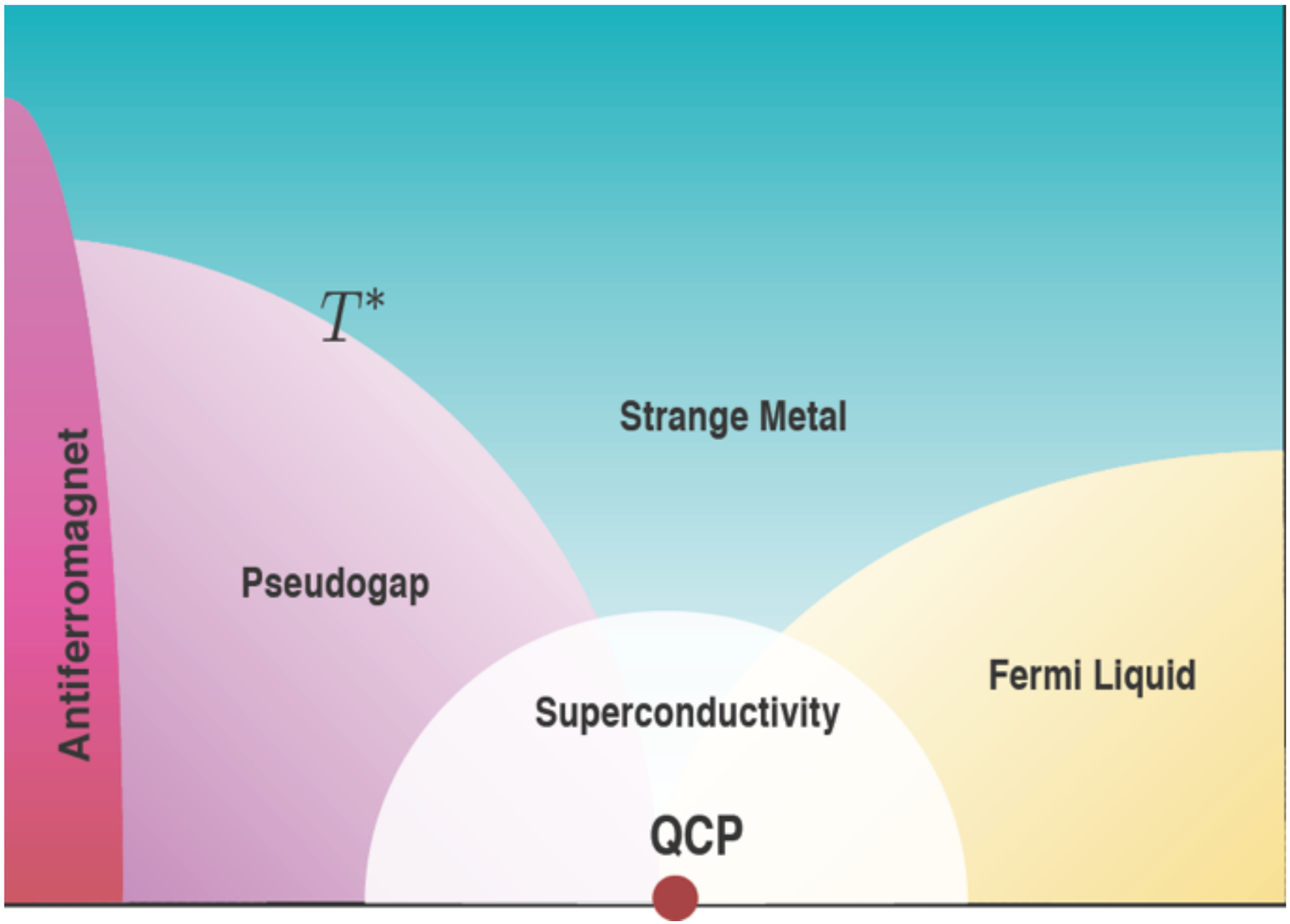

10
1
0.1
T (K)
Band Insulator
Mott
Metal
SC
SC
$B_\perp$=0 T
-1
-0.5
0
n/n_s
10
1
0.1
T (K)
Band Insulator
Mott
Metal
$B_\perp$=0.4 T
-1
-0.5
0
n/n_s
10
1
0.1
T (K)
Band Insulator
Metal
$B_\perp$=8 T
-1
-0.5
0
n/n_s

?

Bi-layer Graphene

?

Cuprates



Is this true?

Mottness in TBLG?

$$\nu = n_e A_s \quad \{\equiv\} \quad \nu_{\max} = 4$$

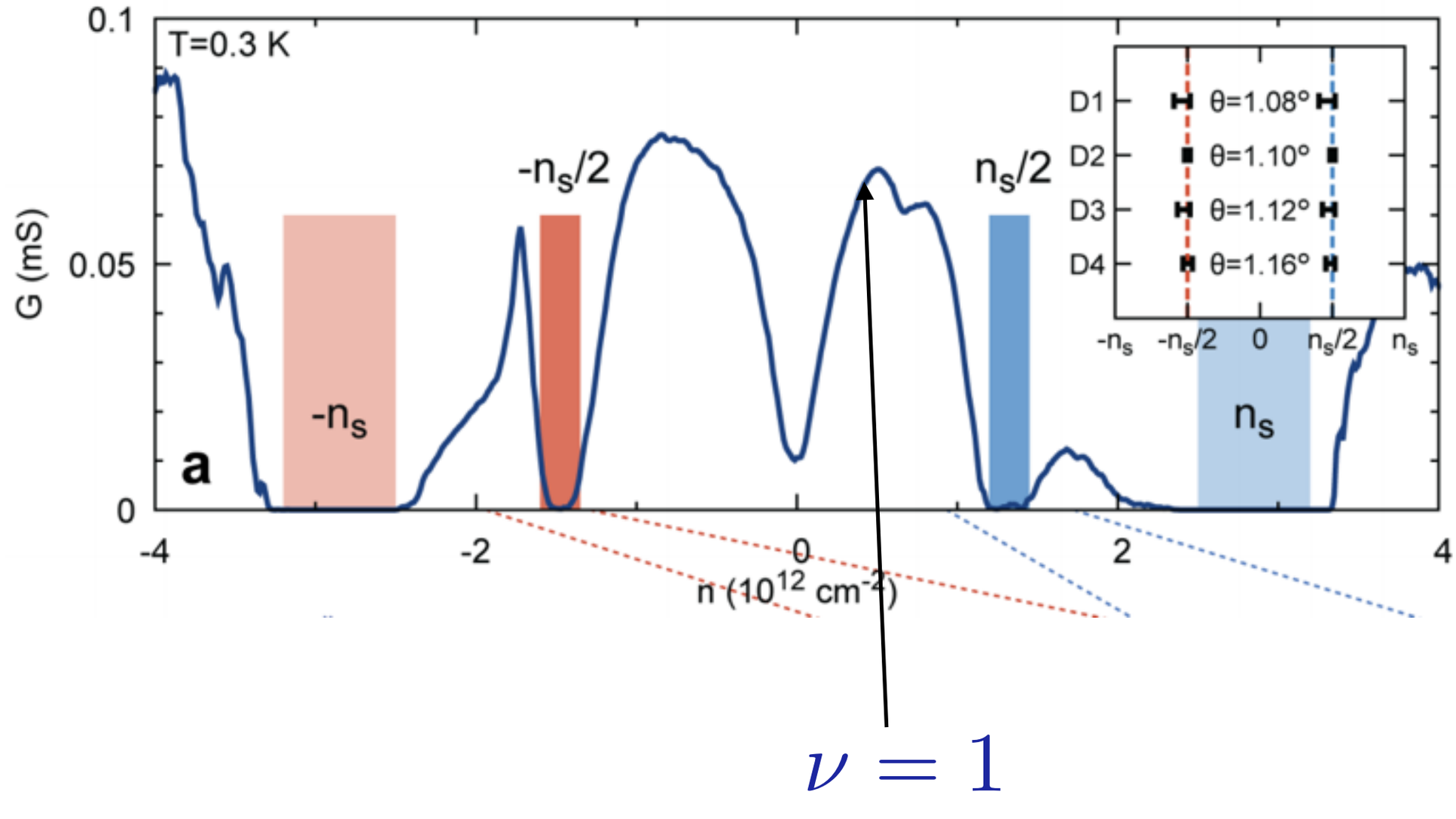
$$U = E^{\nu+1} + E^{\nu-1} - 2E^{\nu}$$

If Mott

$$\forall\, \nu < 4$$

insulator

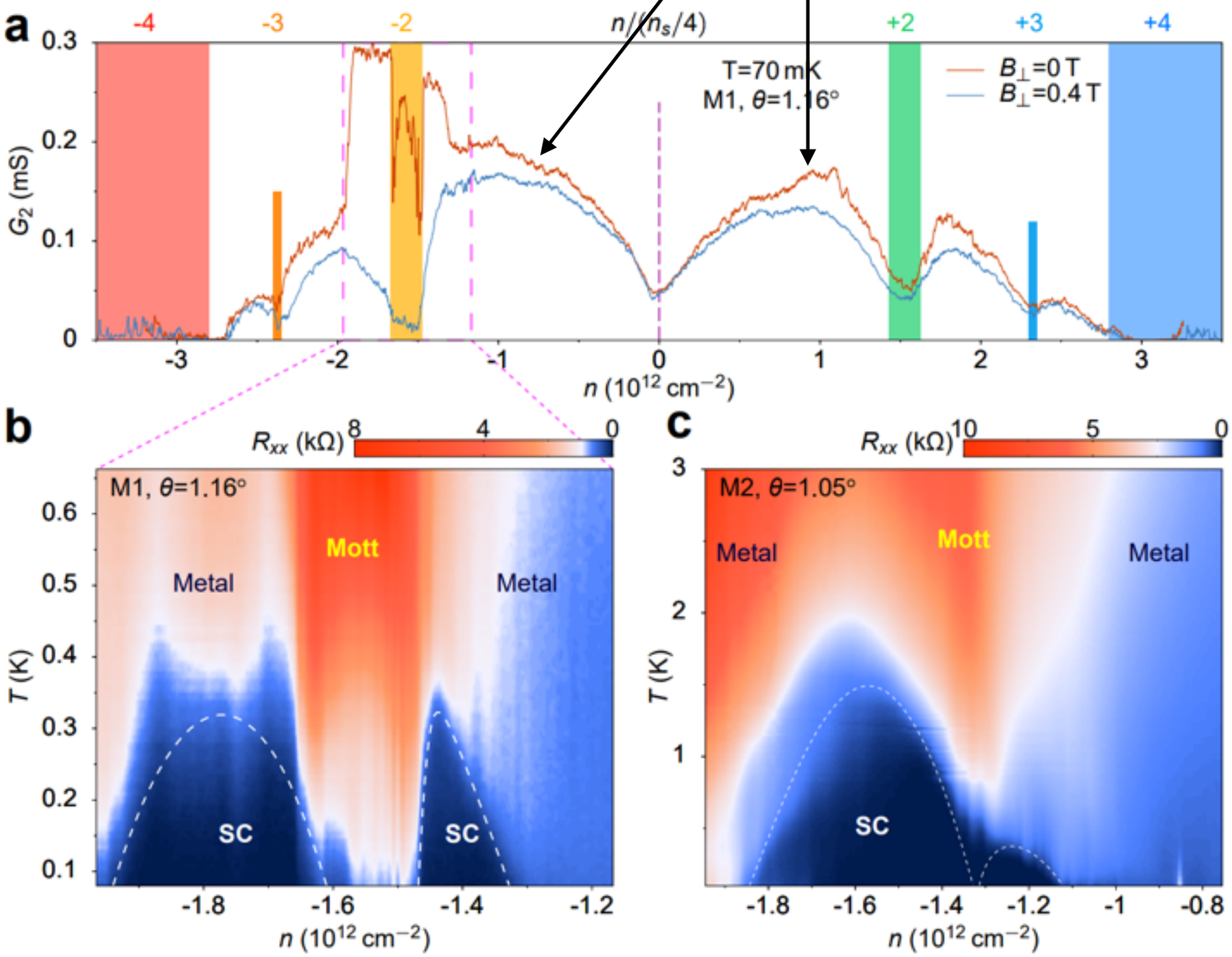
Where's the insulator at $\nu = 1$?



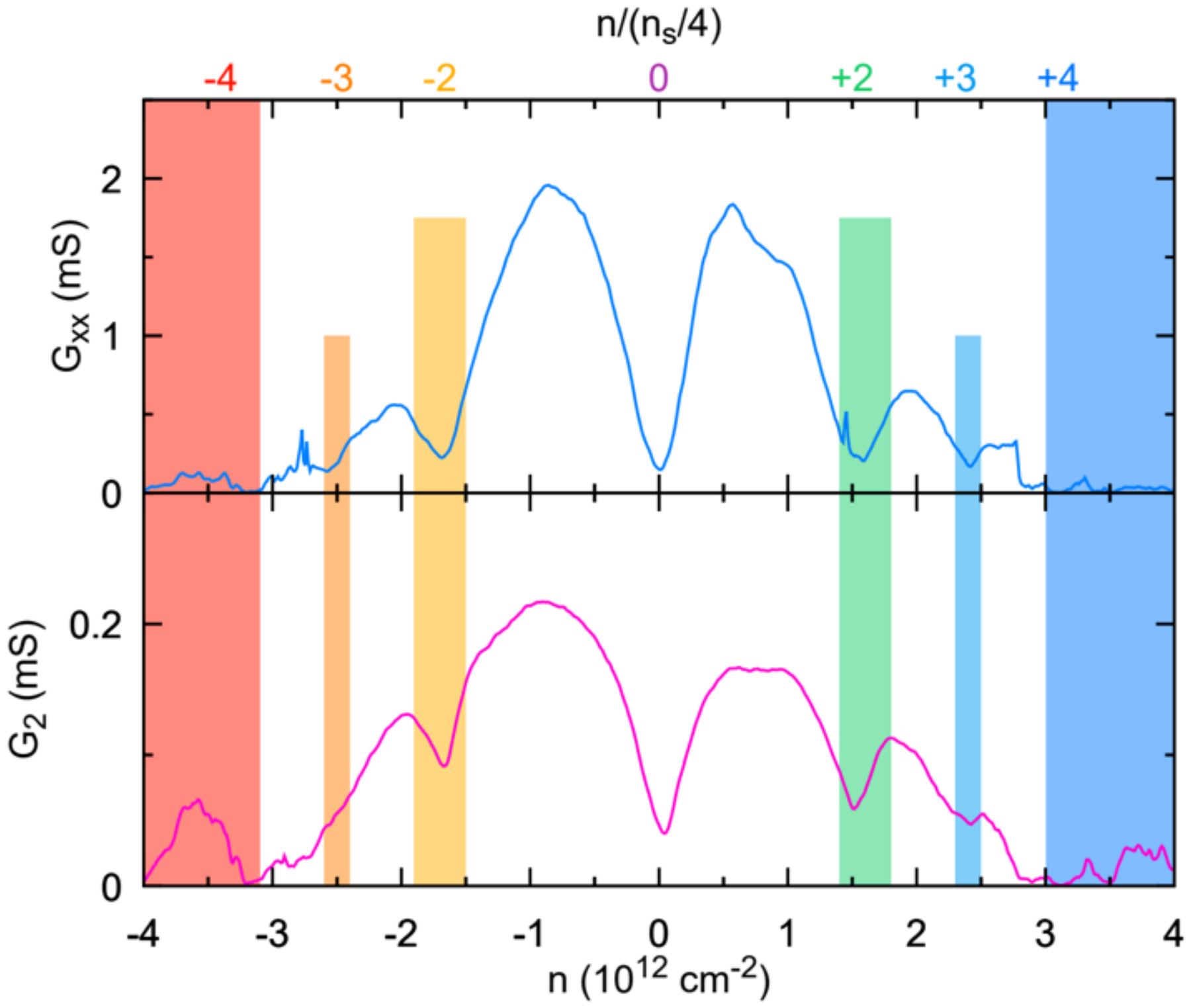
$\nu = 1$

not an insulator

$\nu = 1$



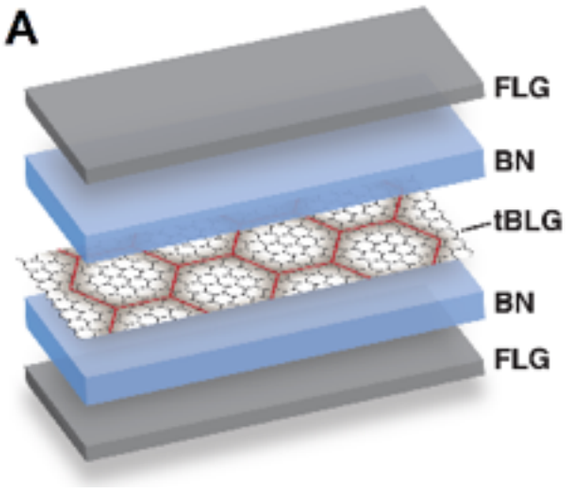
$\nu = 1 \neq$ insulator

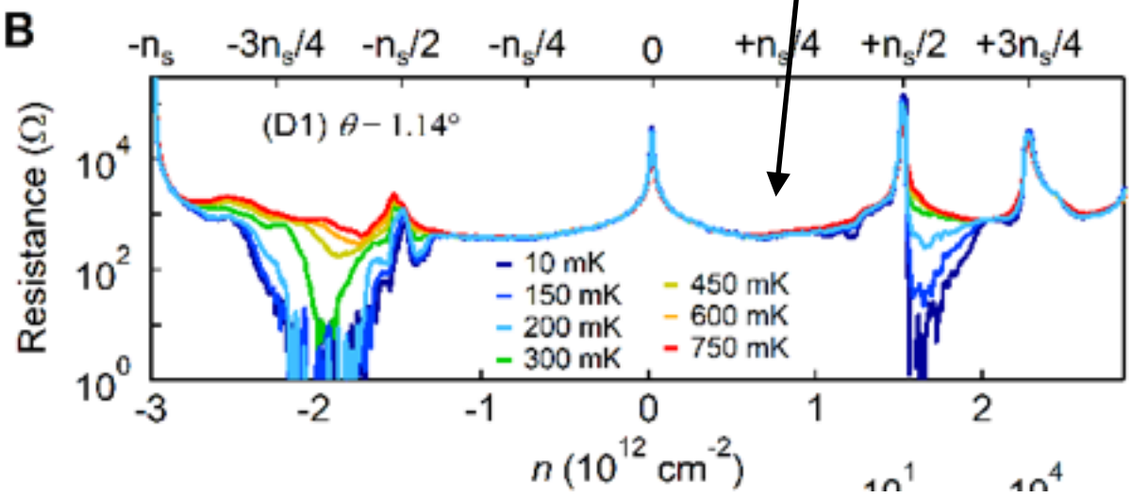


Where's the insulator at $\nu = 1$?

Tuning superconductivity in twisted bilayer graphene

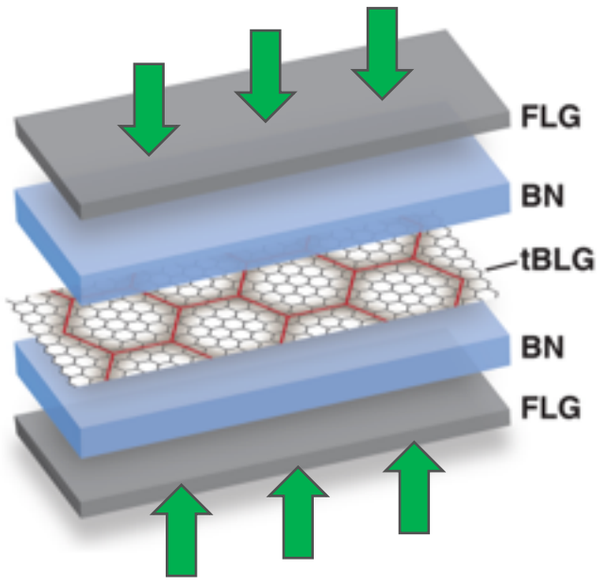
Matthew Yankowitz[1*], Shaowen Chen[1,2*], Hryhoriy Polshyn[3*], K. Watanabe[4], T. Taniguchi[4], David Graf[5], Andrea F. Young[3†], and Cory R. Dean[1†]
[1] *Department of Physics, Columbia University, New York, NY, USA*
[2] *Department of Applied Physics and Applied Mathematics, Columbia University, New York, NY, USA*
[3] *Department of Physics, University of California, Santa Barbara, CA 93106*

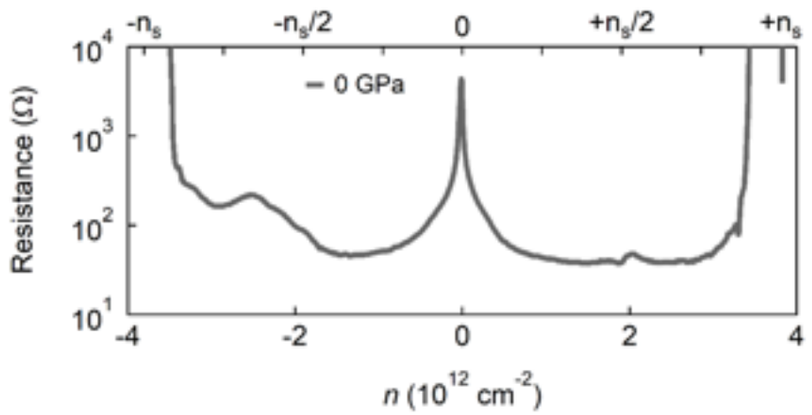


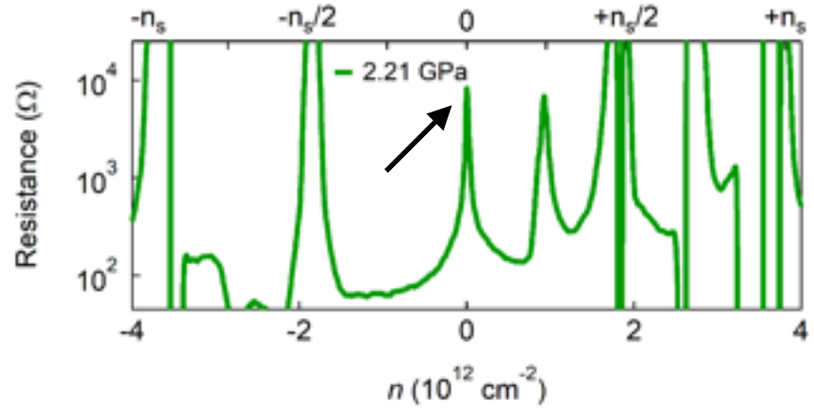


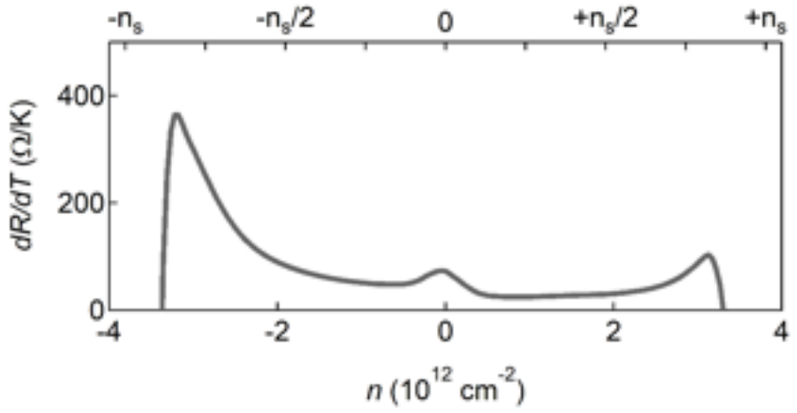
metallic

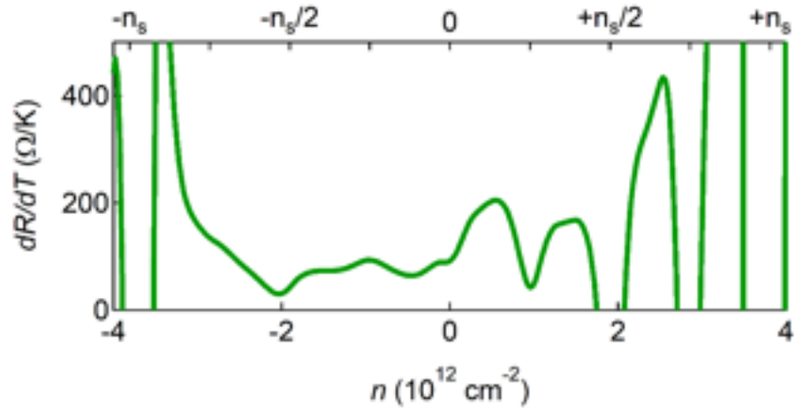
Modifying metallic-state resitivity with pressure









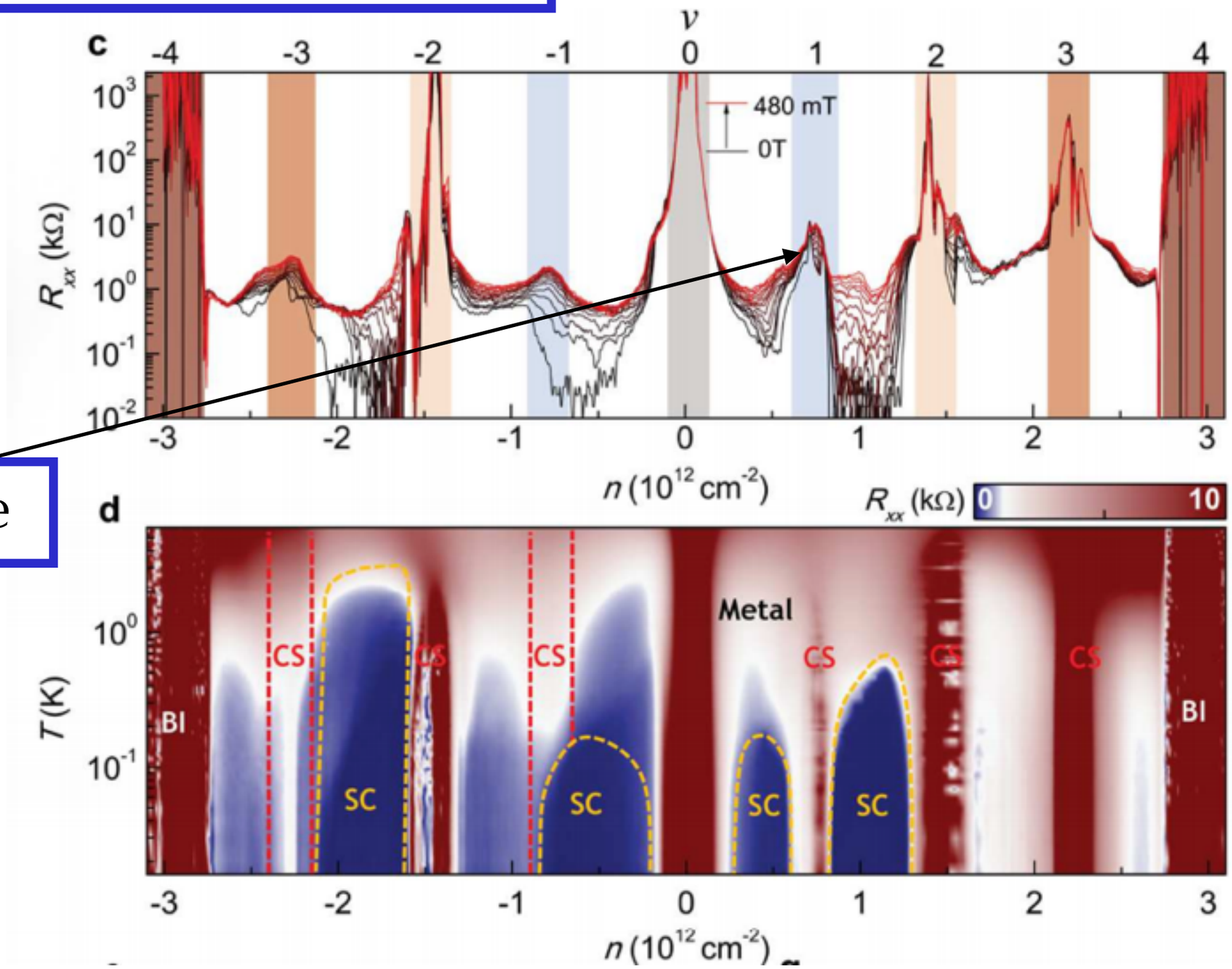


M. Yankowitz *et al.*, *Science* 10.1126/science.aav1910 (2018)

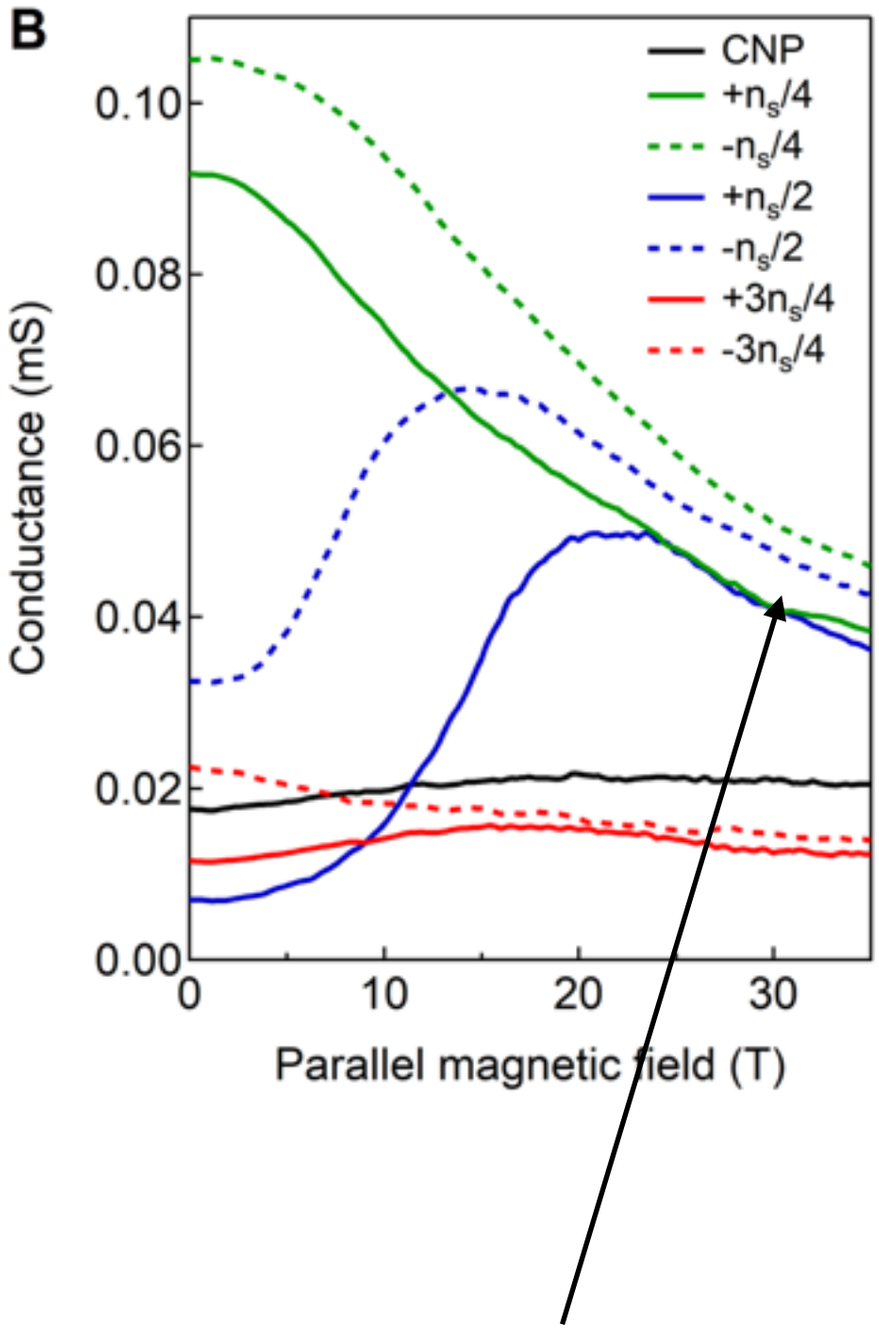
H. Polshyn*, M. Yankowitz* *et al.*, *in prep* (2019)

Increasing magic-angle homogeneity

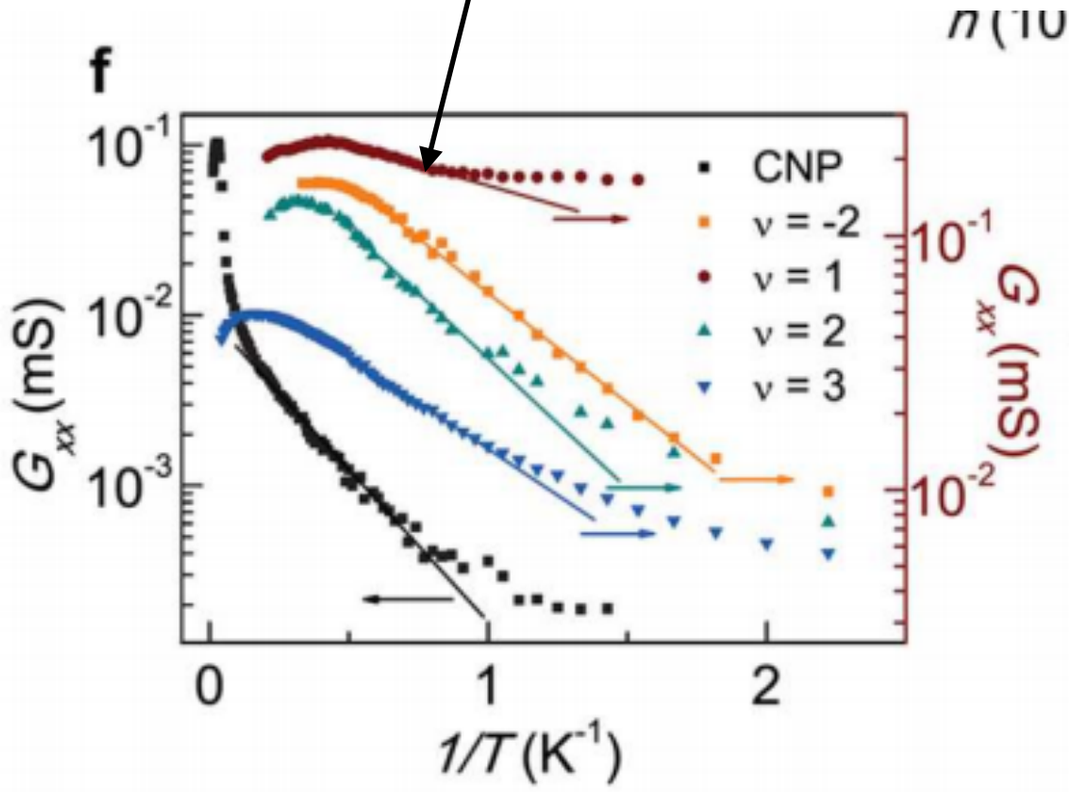
et. al., D. Efetov, 1903.06513



weak feature



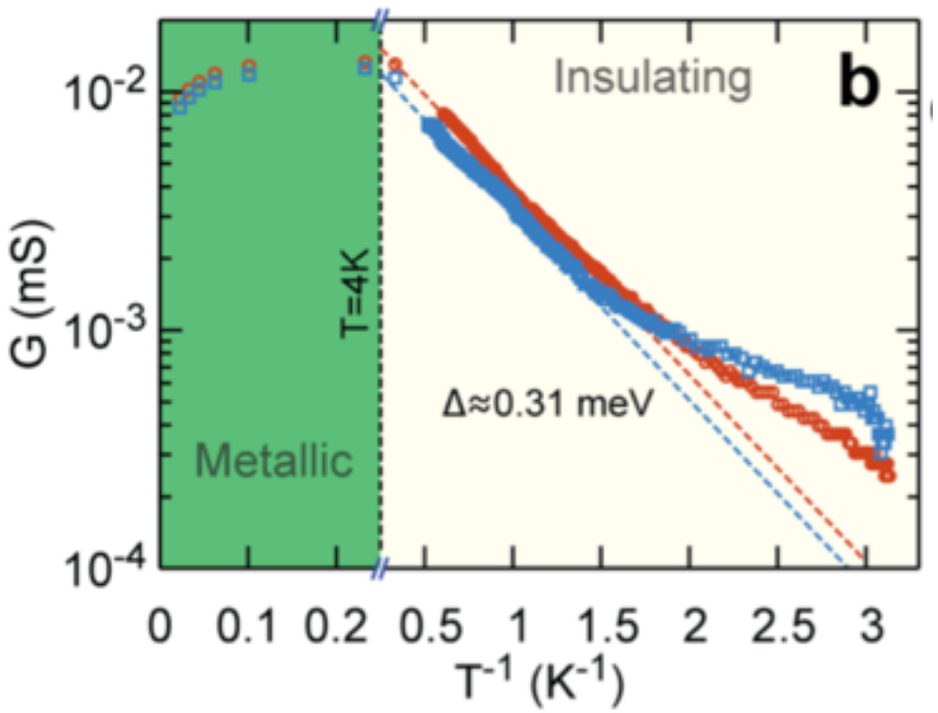
$\Delta \approx 0.13meV$

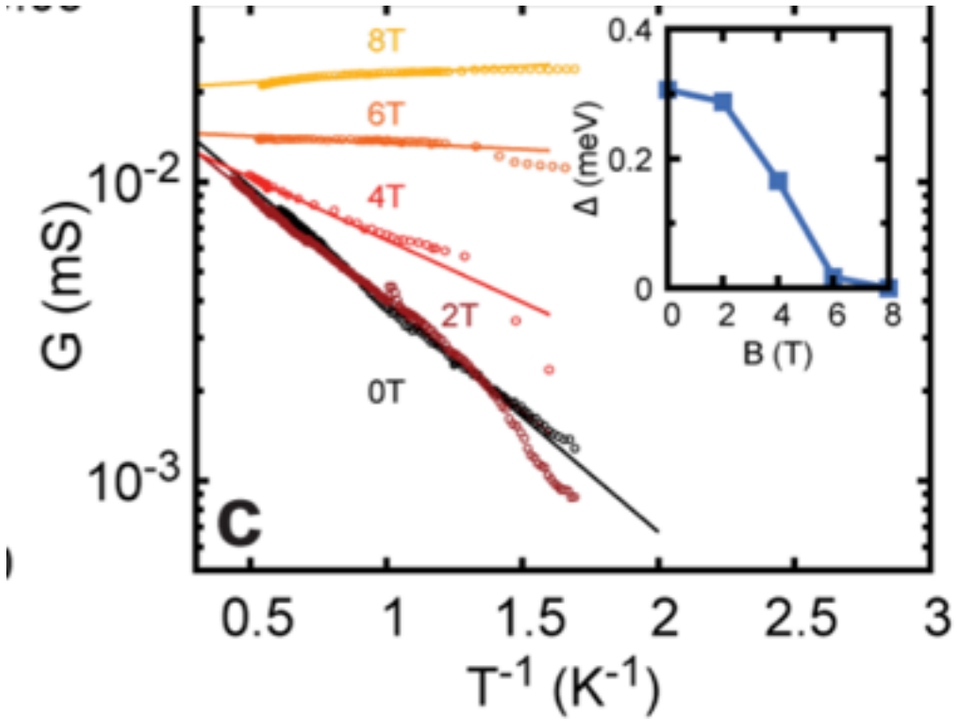


More insulating

experiment

$$\Delta = .31meV \approx 4K \approx \mu_B B_c$$

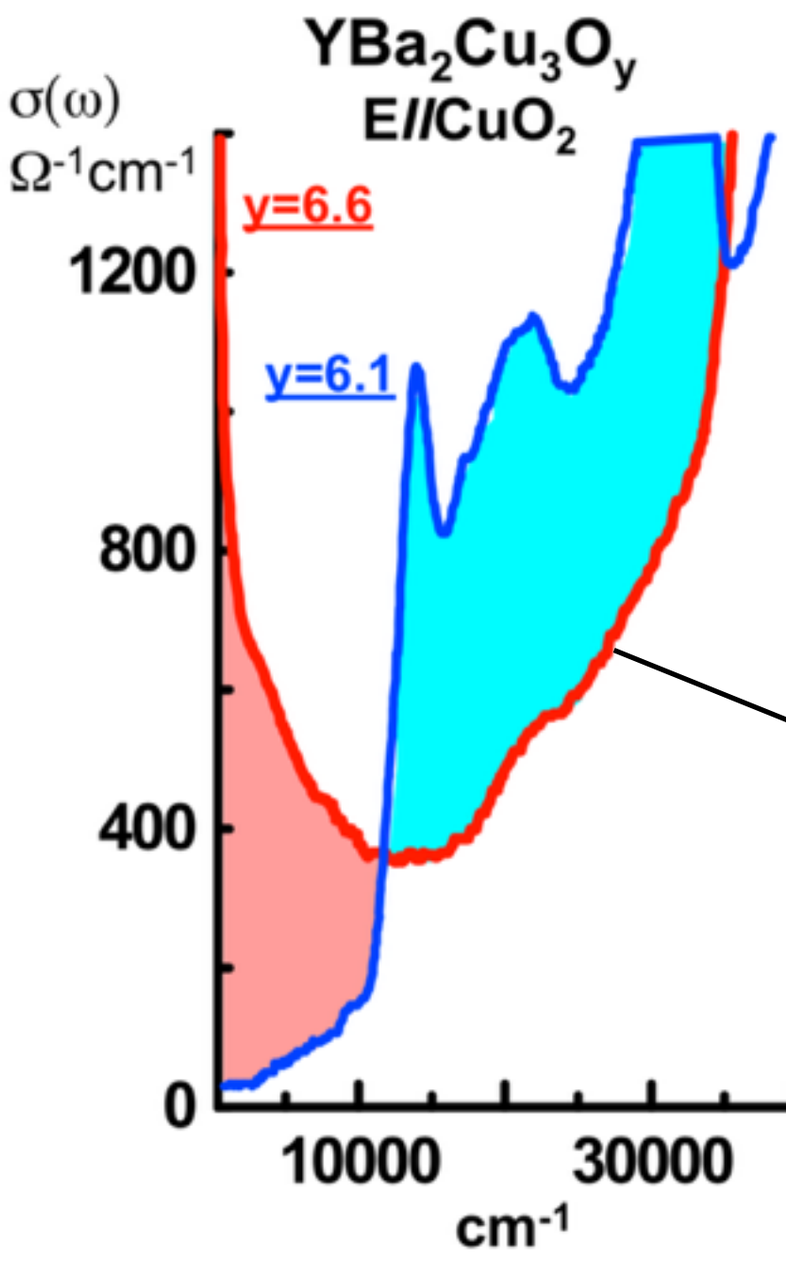


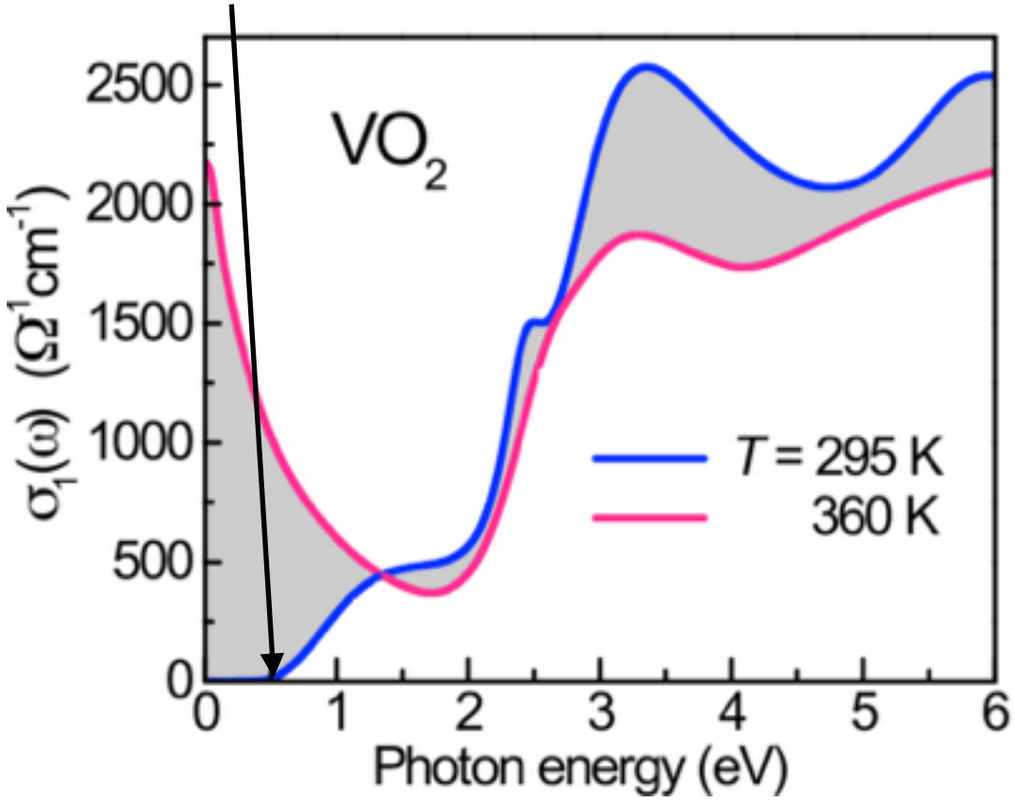


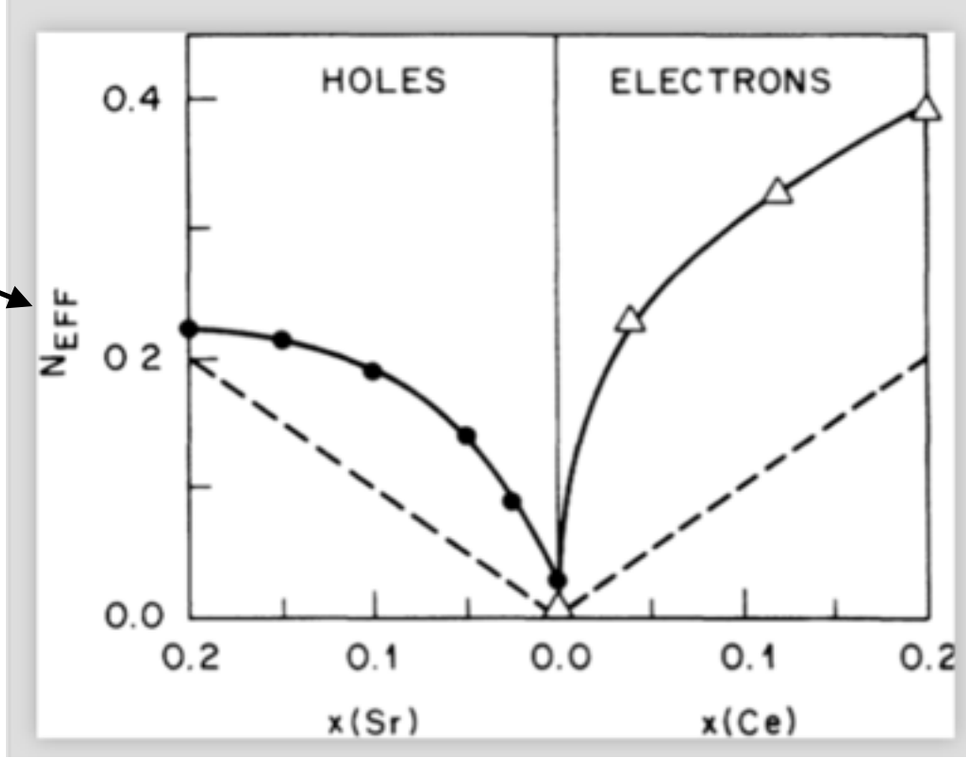
one energy scale

Mottness: 2-energy scales

$$\Delta = .5eV \gg 295K$$





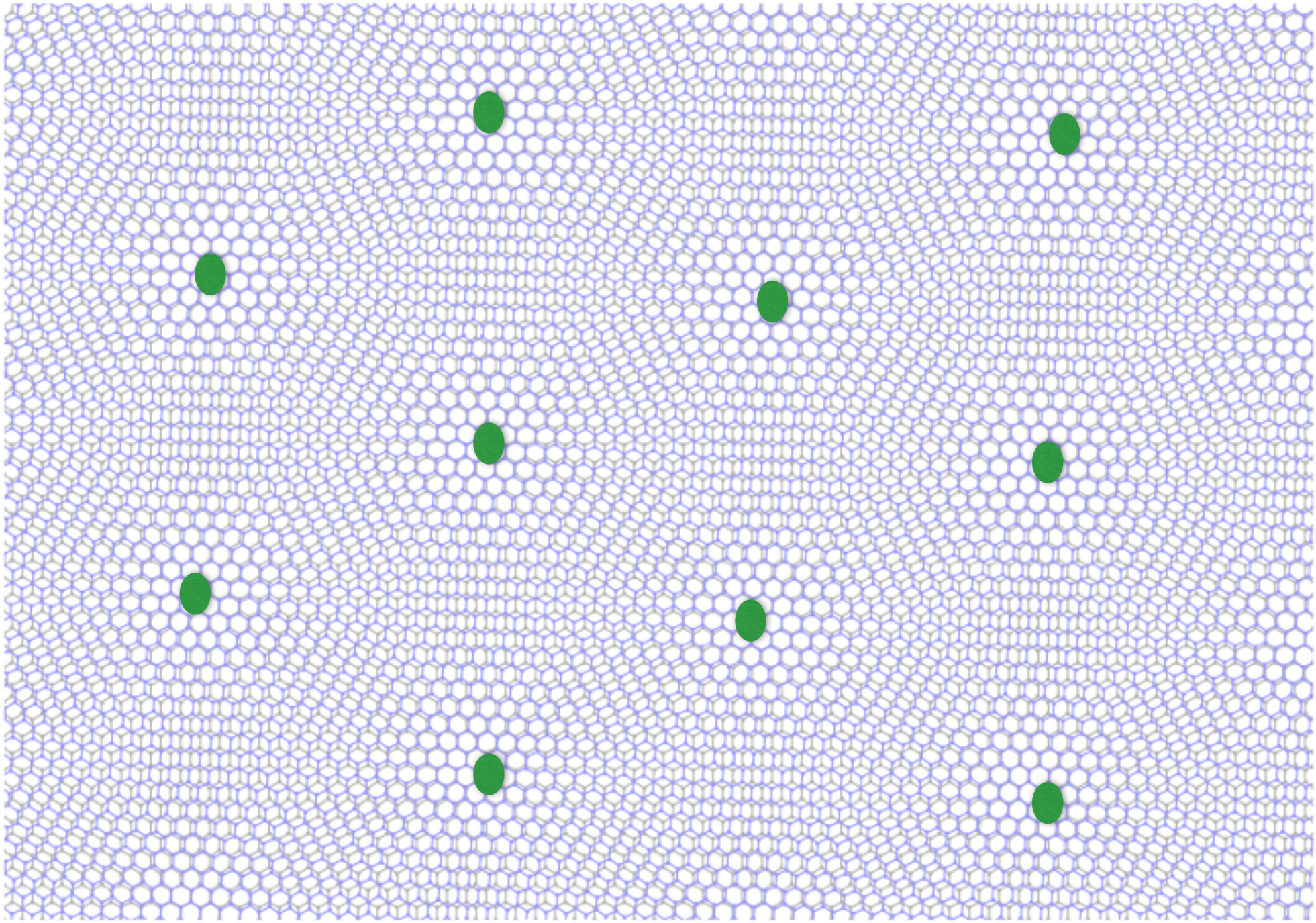


general problem for Mottness

arrange n charges without breaking triangular symmetry

$n > 1$

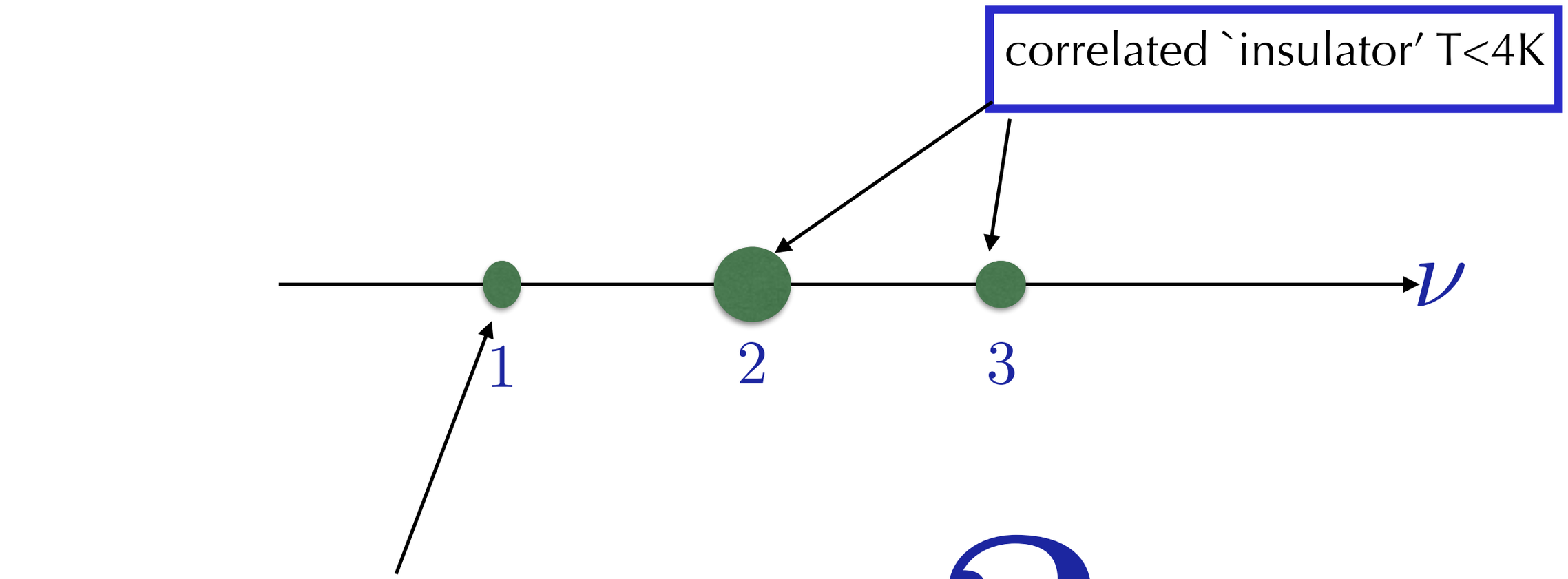
?



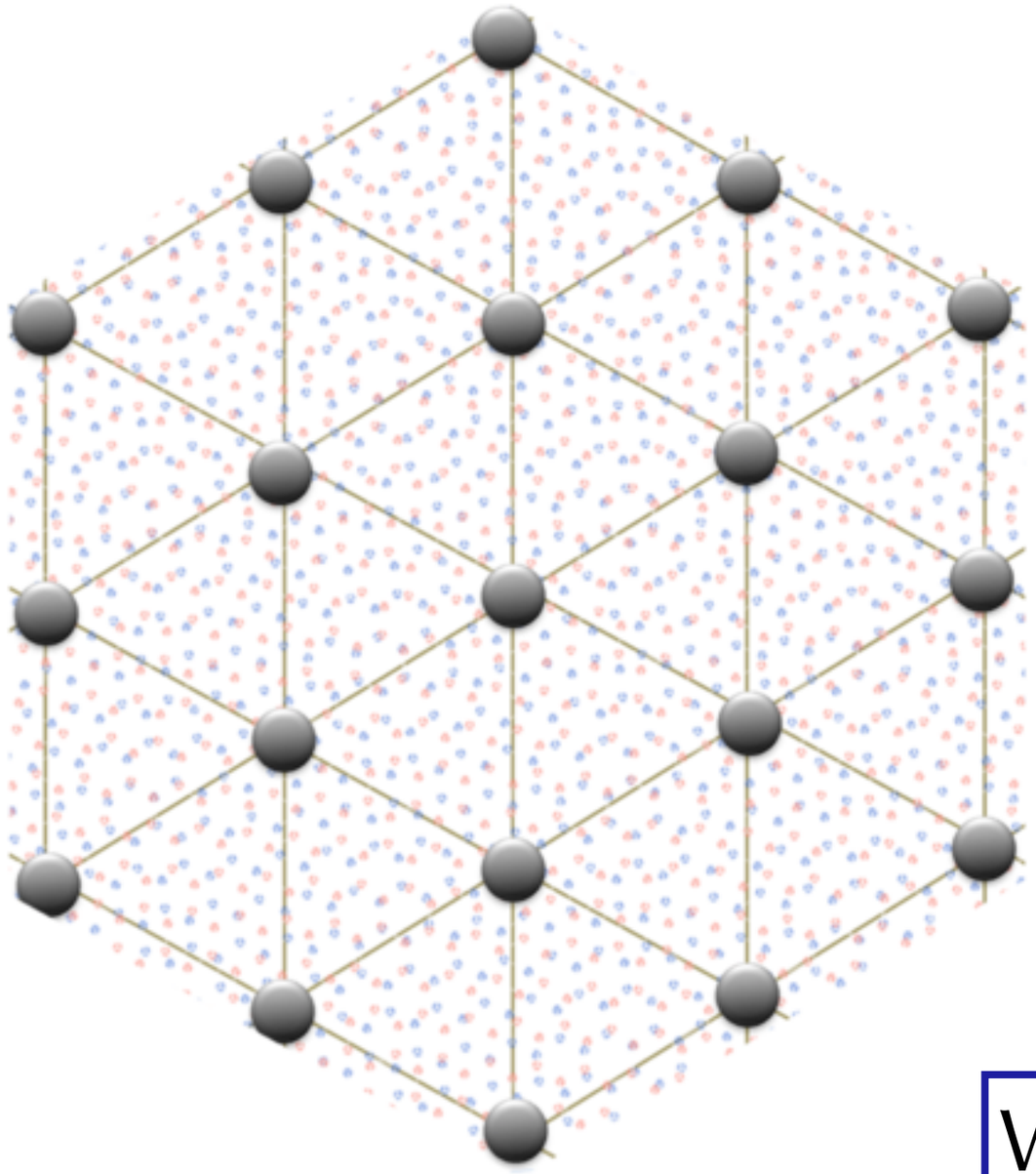
Not Mott
Insulation

dilute electron gas
in a periodic potential

Experimental puzzle: hierarchy of insulators



other correlated insulator



$r_s > 37$

Wigner crystal

recall

Dirac spectrum

$$r_s = \text{const}$$

TBLG: deviation from Dirac

$$H_\theta(\mathbf{k}) = \begin{pmatrix} 0 & v_F\mathbf{k}^\dagger + \frac{\mathbf{1}}{\mathbf{2m}}\mathbf{k^2} \\ v_F\mathbf{k} + \frac{\mathbf{1}}{\mathbf{2m}}(\mathbf{k}^\dagger)^\mathbf{2} & 0 \end{pmatrix}$$

flat-band condition

$$\tilde{v} = \frac{v_0}{v_F} = \frac{\partial_\mathbf{k} E_\mathbf{k}|_{\mathbf{k}=\mathbf{\Gamma}}}{\partial_\mathbf{k} E_\mathbf{k}|_{\mathbf{k}=\mathbf{K},\mathbf{K}'}} = \frac{1+6\kappa^2}{1-3\kappa^2}$$

$$\kappa = 1/\sqrt{3}$$

compute r_s

$r_s^{(1)}(\theta)$

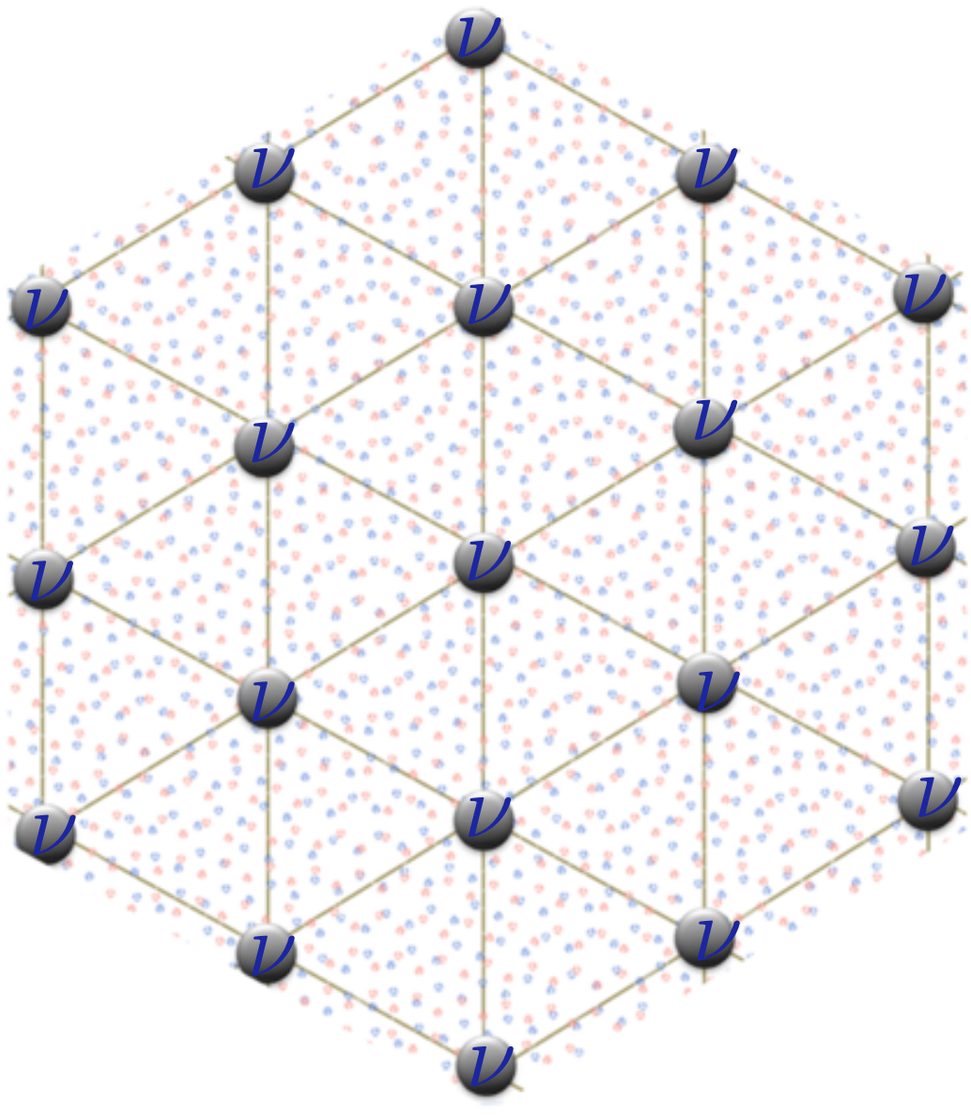
$$r_s(\theta,\nu) = \frac{\alpha}{\epsilon}\tilde{v}(\theta) \times (1-\gamma+\frac{1}{2}\gamma^2)^{-1/2}$$

$$\alpha = \frac{e^2}{\hbar v_0} \sim 2$$

$$\gamma = \frac{\sqrt{2}\hbar}{m_* v_F}\frac{1}{2r_e} = \frac{\theta}{a}\frac{\hbar}{m_* v_F}\sqrt{\frac{\pi\nu}{\sqrt{3}}} \sim \frac{\theta^\circ}{100}\frac{\tilde{v}(\theta)}{\tilde{m}(\theta,\nu)}\sqrt{\nu}$$

filling dependence

Wigner Crystals for $\nu \neq 1$

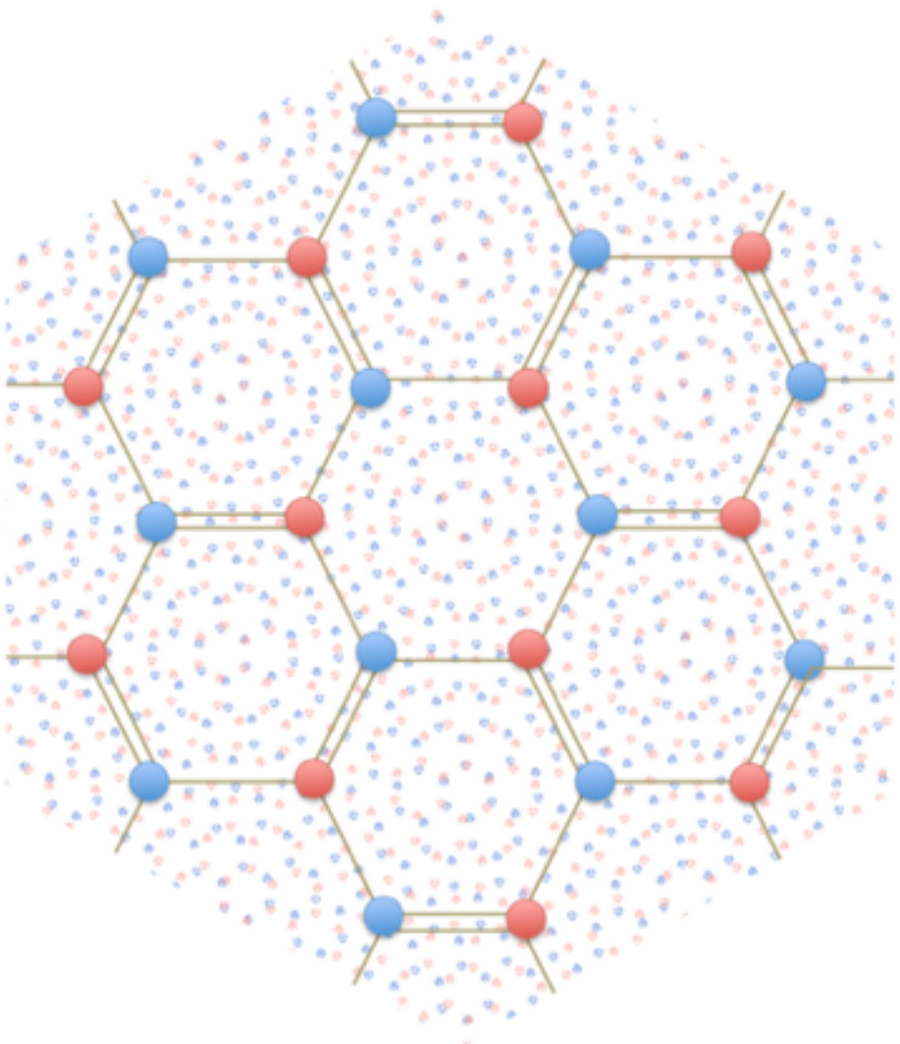


$$r_s^{\mathrm{crit}} = \frac{37}{\nu^2}$$

lower bound

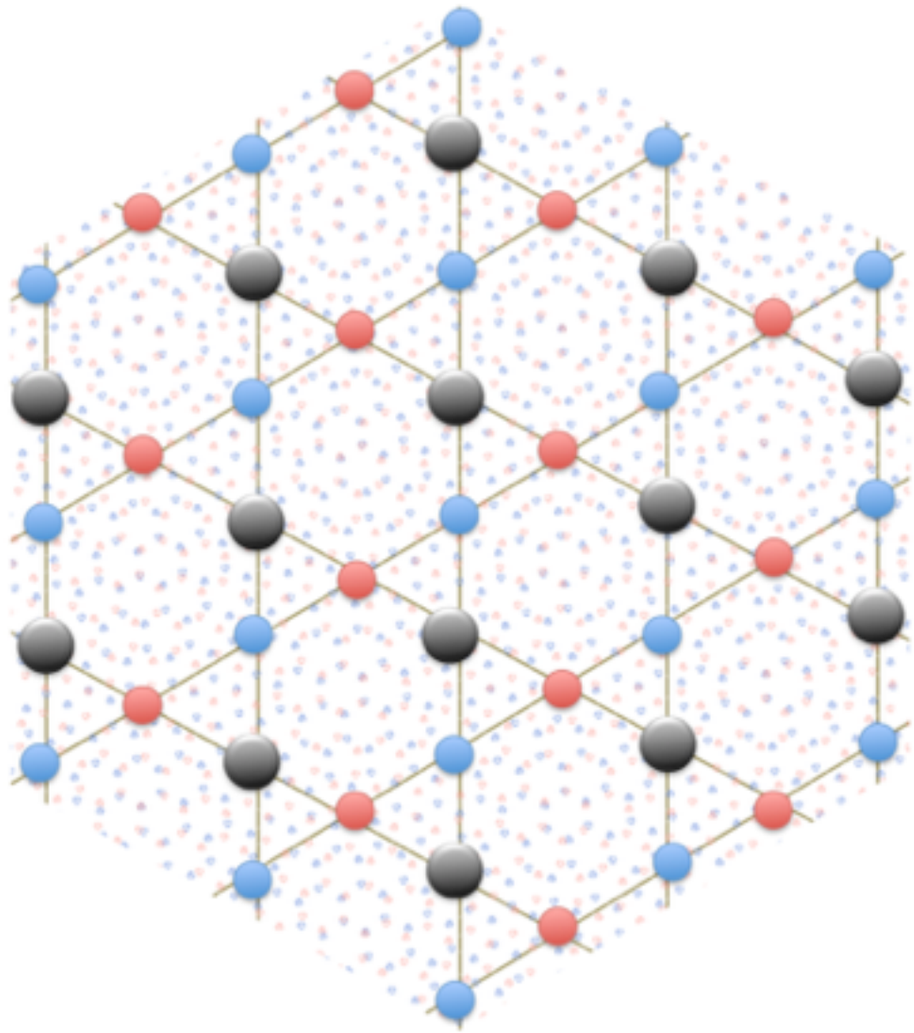
realistic lattices for $\nu > 1$

$\nu = 2$



$\lambda_s \to \lambda_s/\sqrt{3}$

$\nu = 3$



$\lambda_s \to \lambda_s/2$

$$r_s = \frac{a}{a_0^*}$$

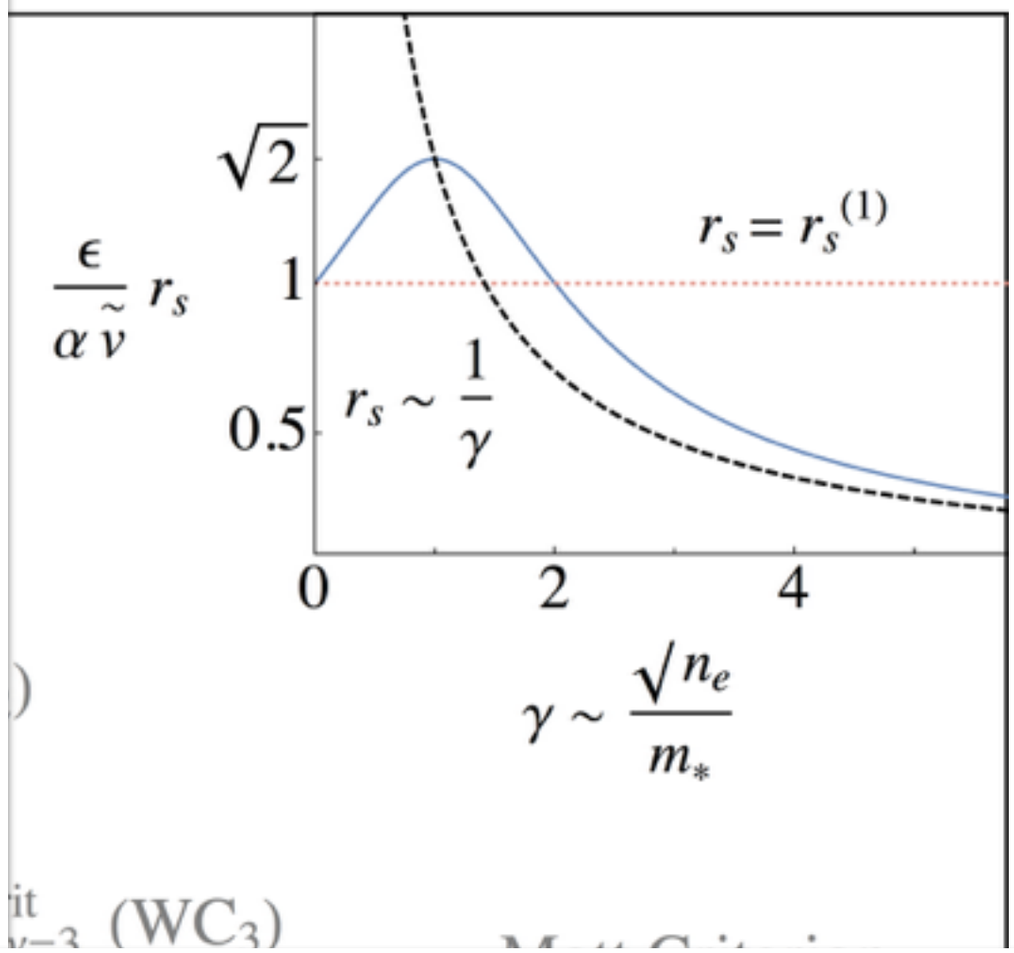
$$r_s = d_\nu 37$$

$$d_2 = 1/\sqrt{3}$$

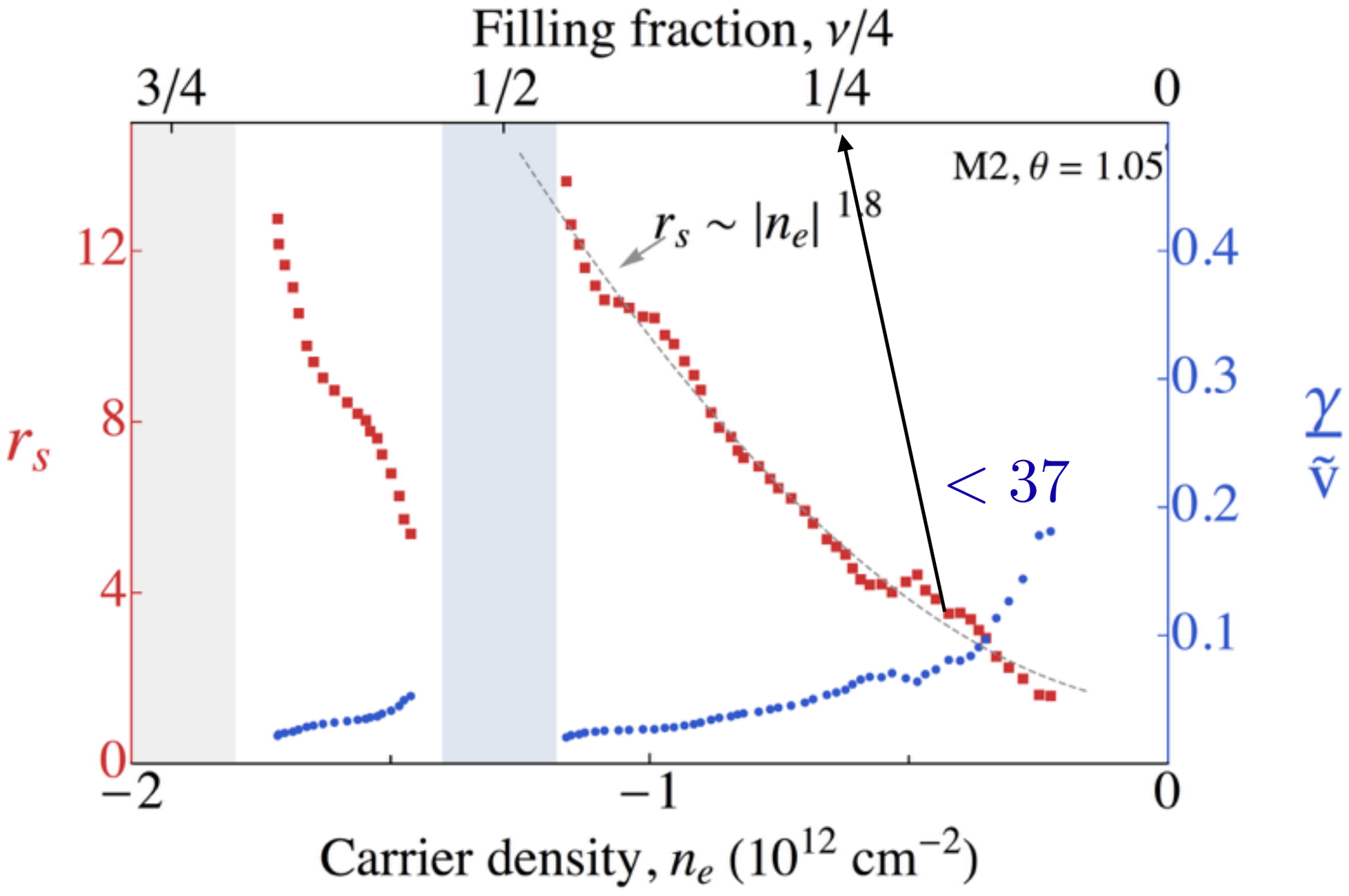
$$d_3 = 1/2$$

$$\frac{37}{\nu^2} < r_s^{\text{crit}} < d_\nu 37$$

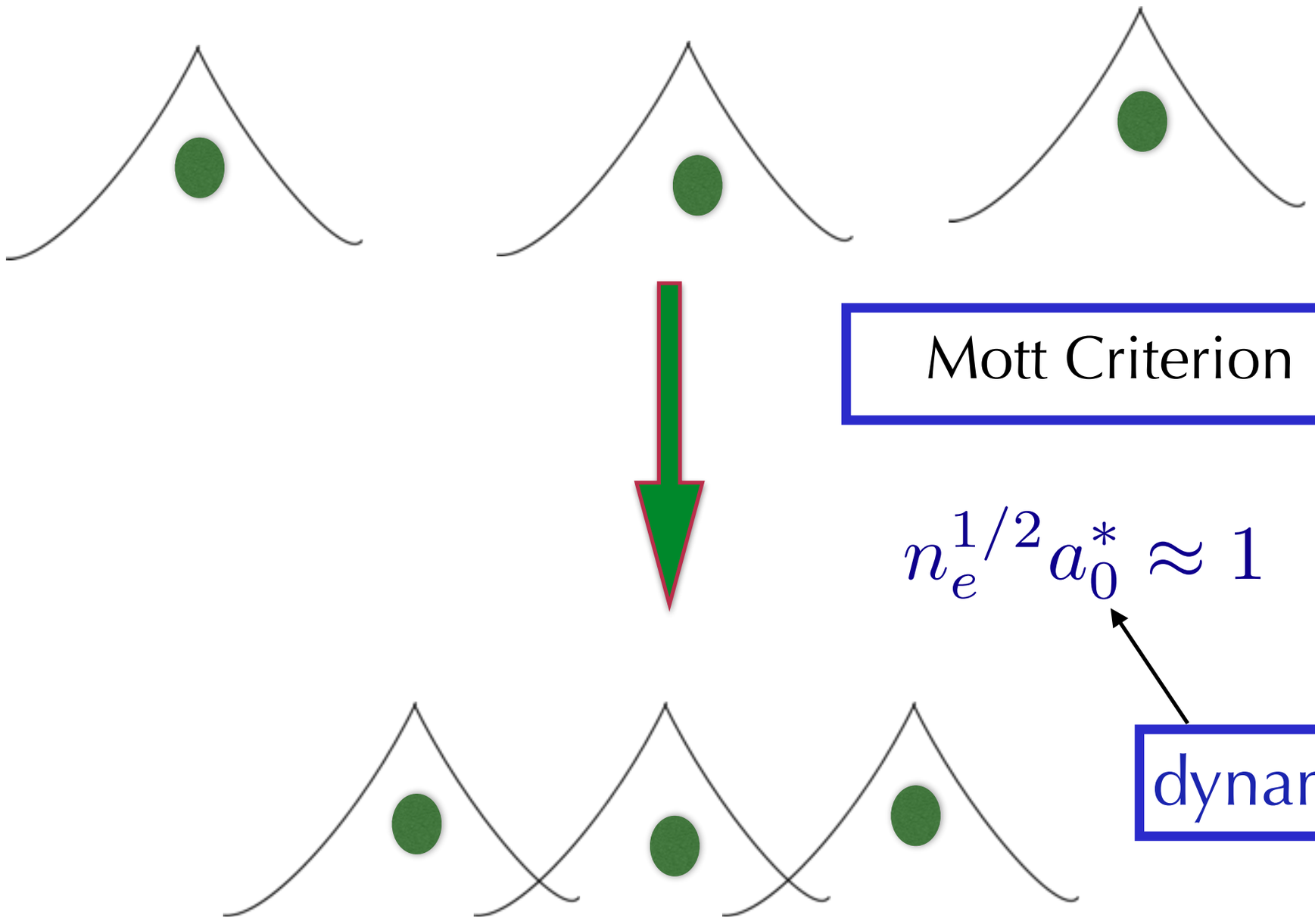
experiments



using experimental data for effective mass



What happened to Mottness?



Mott Criterion

$$n_e^{1/2} a_0^* \approx 1$$

dynamics

$$m_*(\theta,\nu) = m_e \, \frac{\left(\gamma^2 - 2\gamma + 1\right)^{3/2}}{\gamma^3 - 3\gamma^2 + .45\gamma - 2}$$

Mott criterion

$$\frac{\lambda_s(\theta)}{a_0} \, \frac{m_*(\theta,\nu)}{m_e\sqrt{\nu}} \lesssim \epsilon$$

$150 - 200$

const

never satisfied

$$m_*/m_e = \tilde{v}\sqrt{h^2 n_e/8\pi v_0^2 m_e^2}$$

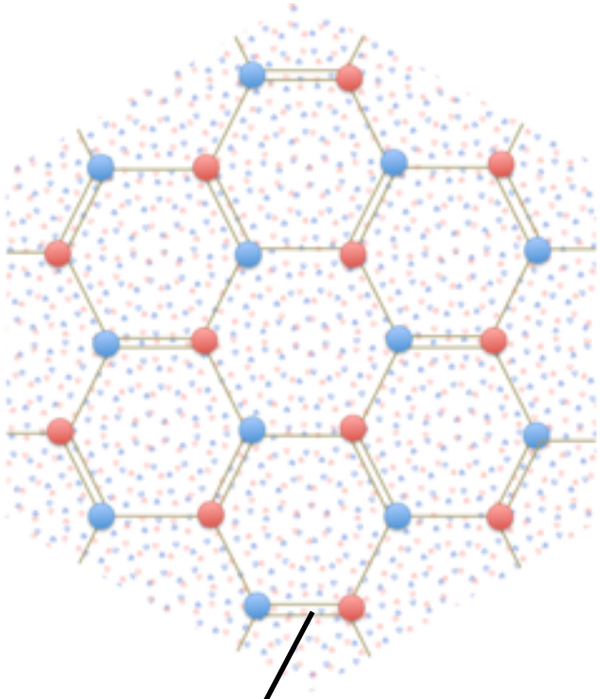
Mott criterion

$$r_s^{(1)}(\theta) \lesssim \frac{1}{\sqrt{\pi}}$$

$$\theta - \theta_{\text{magic}} = .7$$

$$n_e \approx 10^4 n_s$$

honeycomb



`metal′

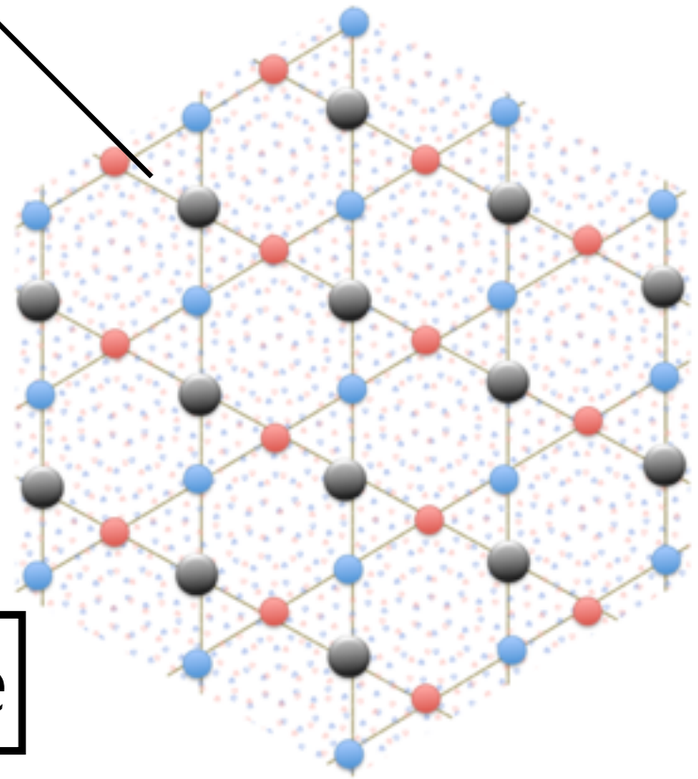
$r_s < 37$

ν

1

2

3



kagome

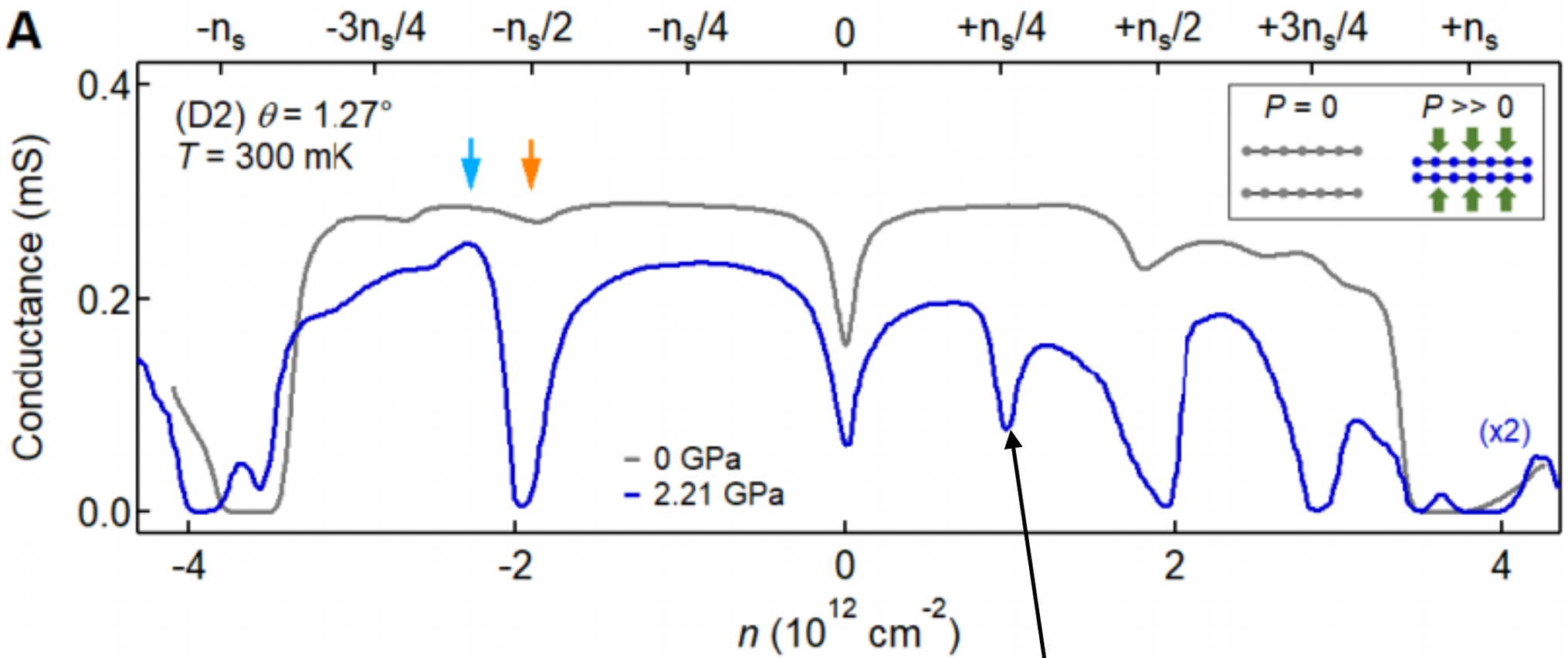
Estimate of coulomb repulsion

$$\frac{e^2}{4\pi\epsilon\lambda_s} \approx 30meV$$

$$T_{\text{melt}} \propto .01U_{\text{coul}}$$

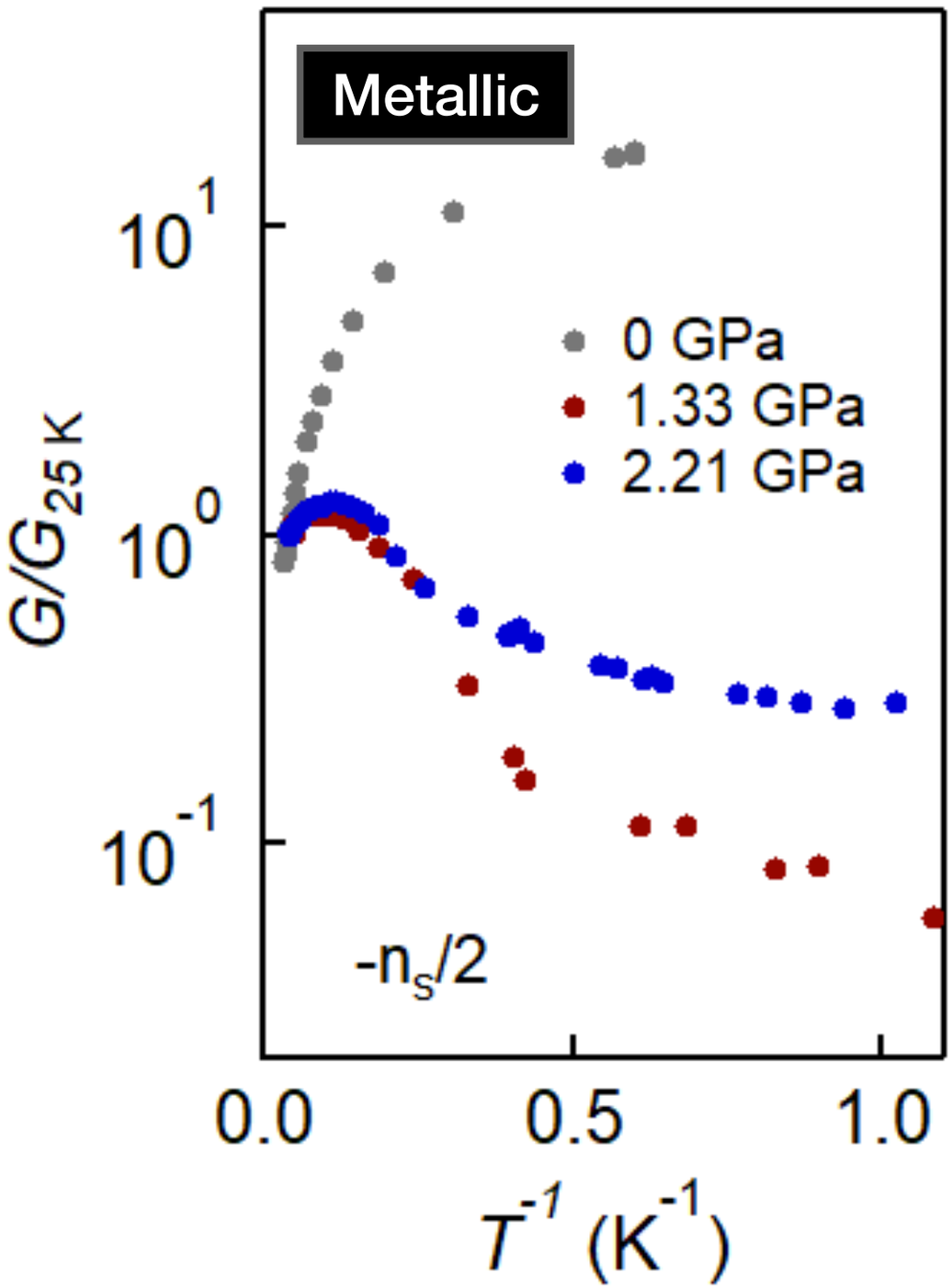
$$4K$$

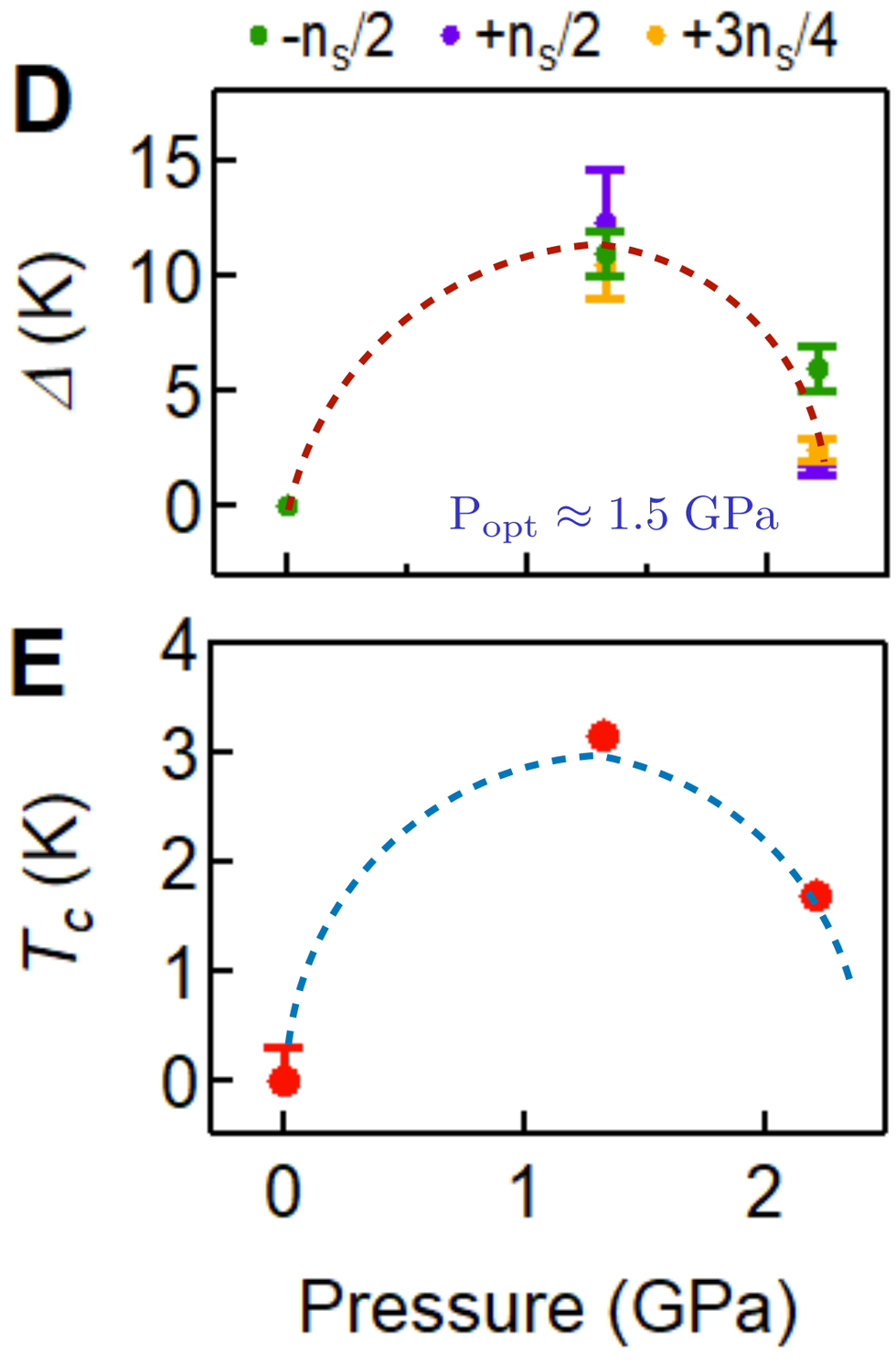
High-pressure insulator



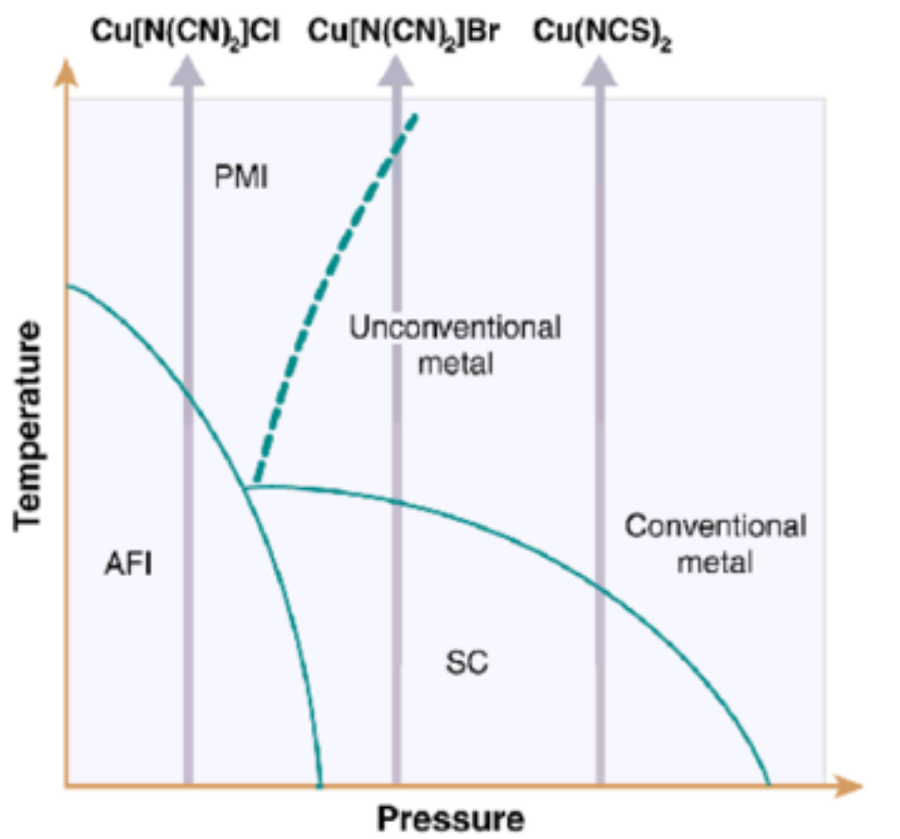
$\nu = 1$ insulator

TBLG under Pressure





Mott insulators under pressure



pressure destroys Mottness!

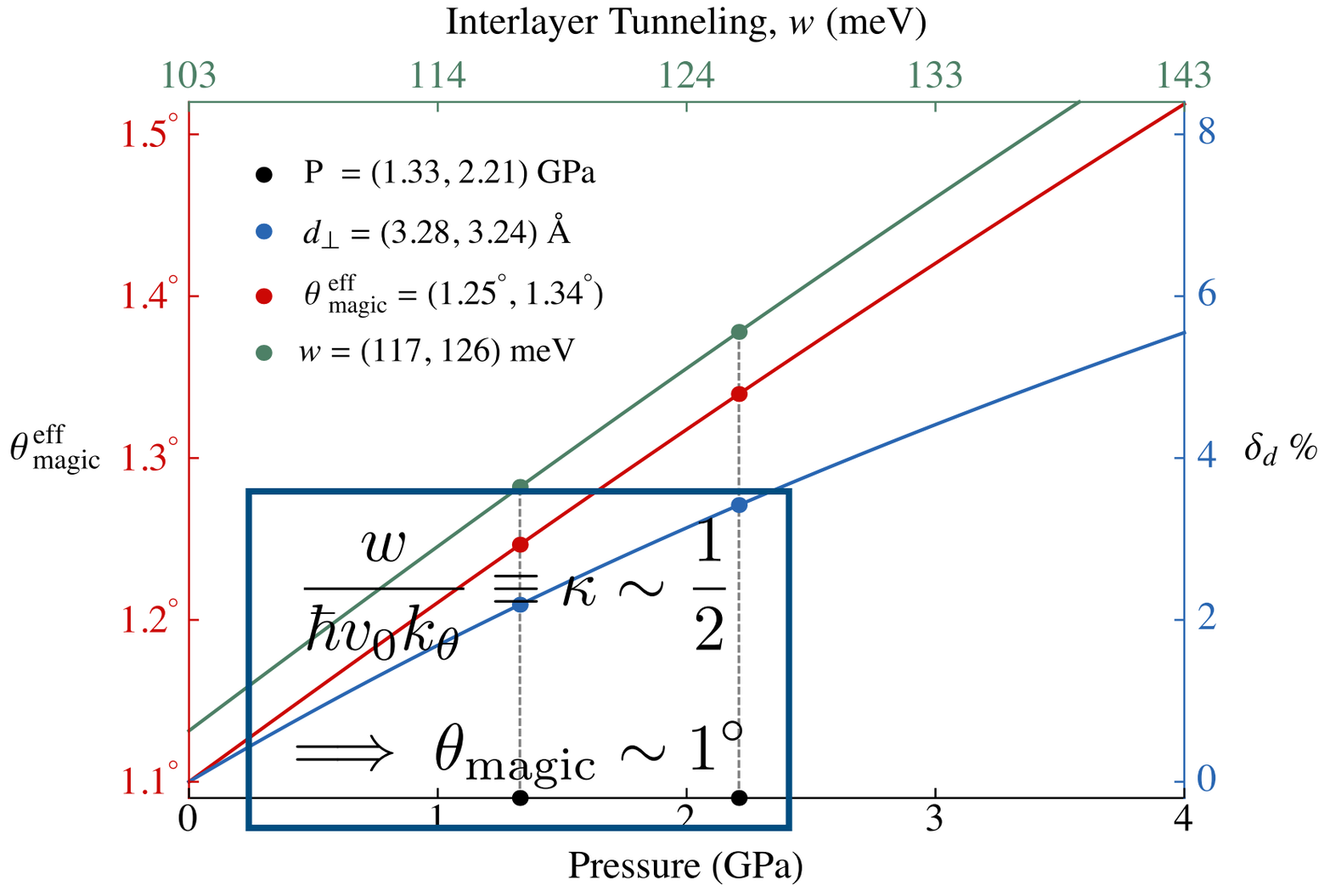
Effects of Pressure

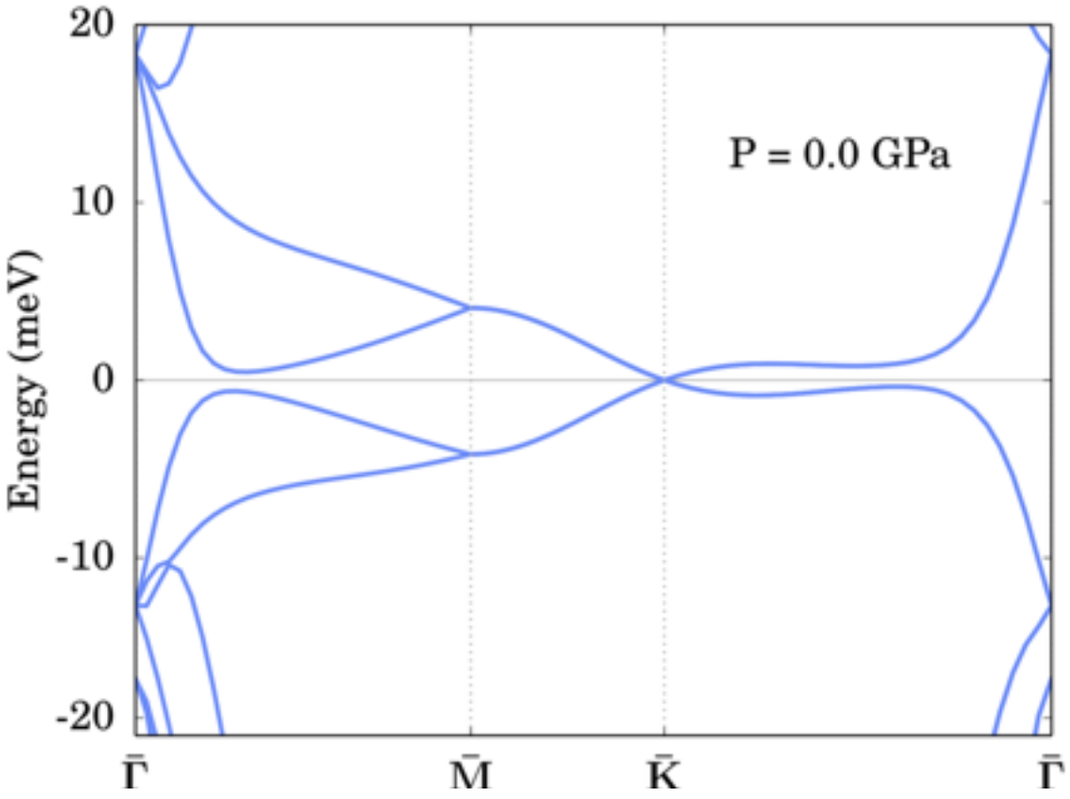
Inter-particle Distance

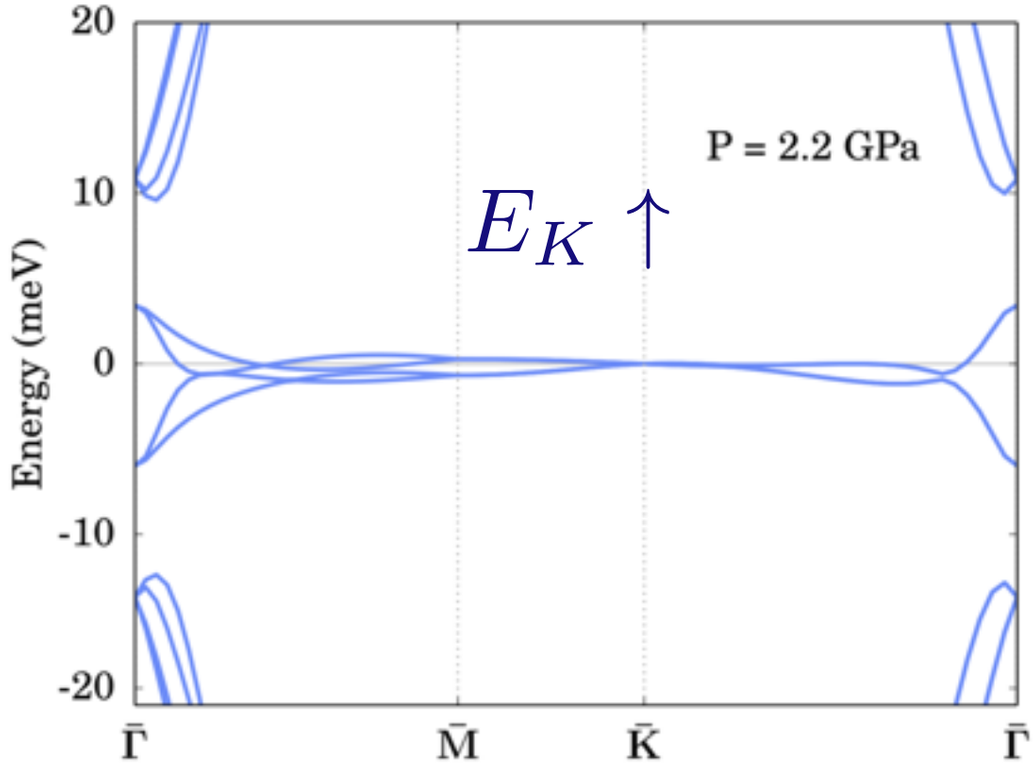
$$1 - \frac{d_\perp}{\quad} \equiv \delta_d = 10.48 \log\left(1 + \frac{\mathrm{P}}{\quad}\right)\%$$

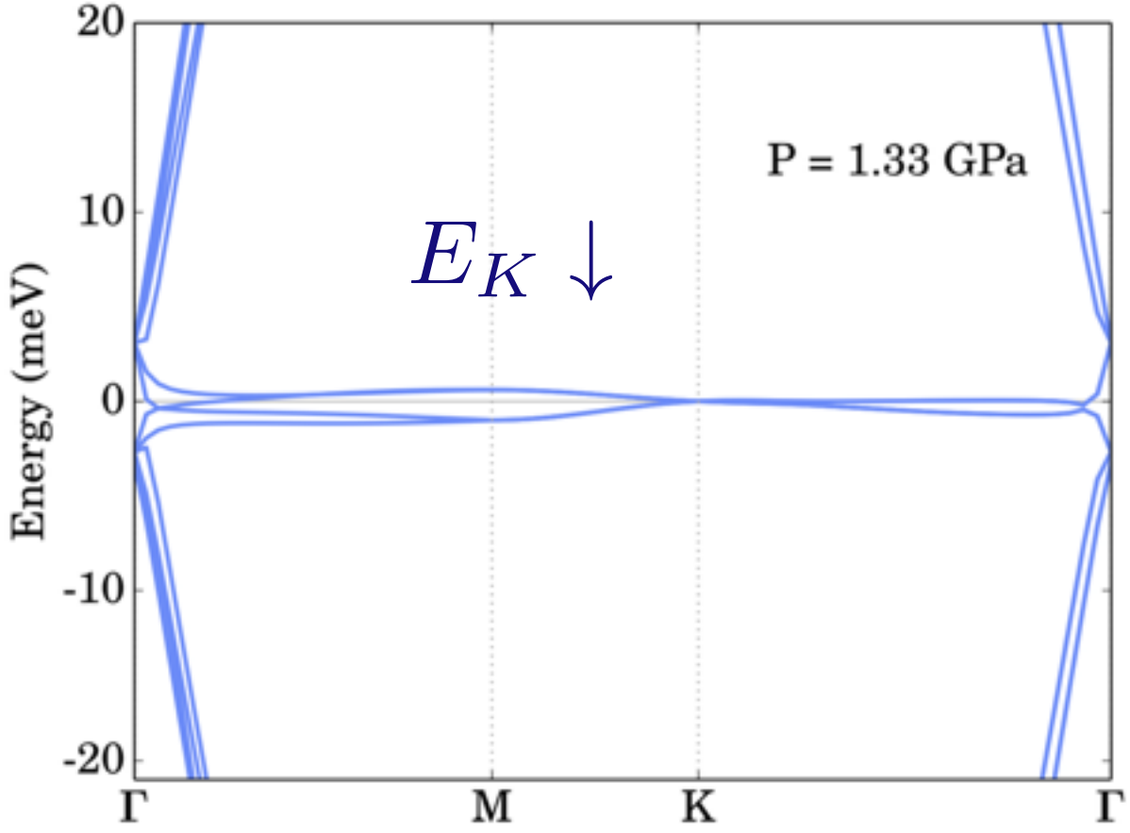
Inter-layer Tunneling

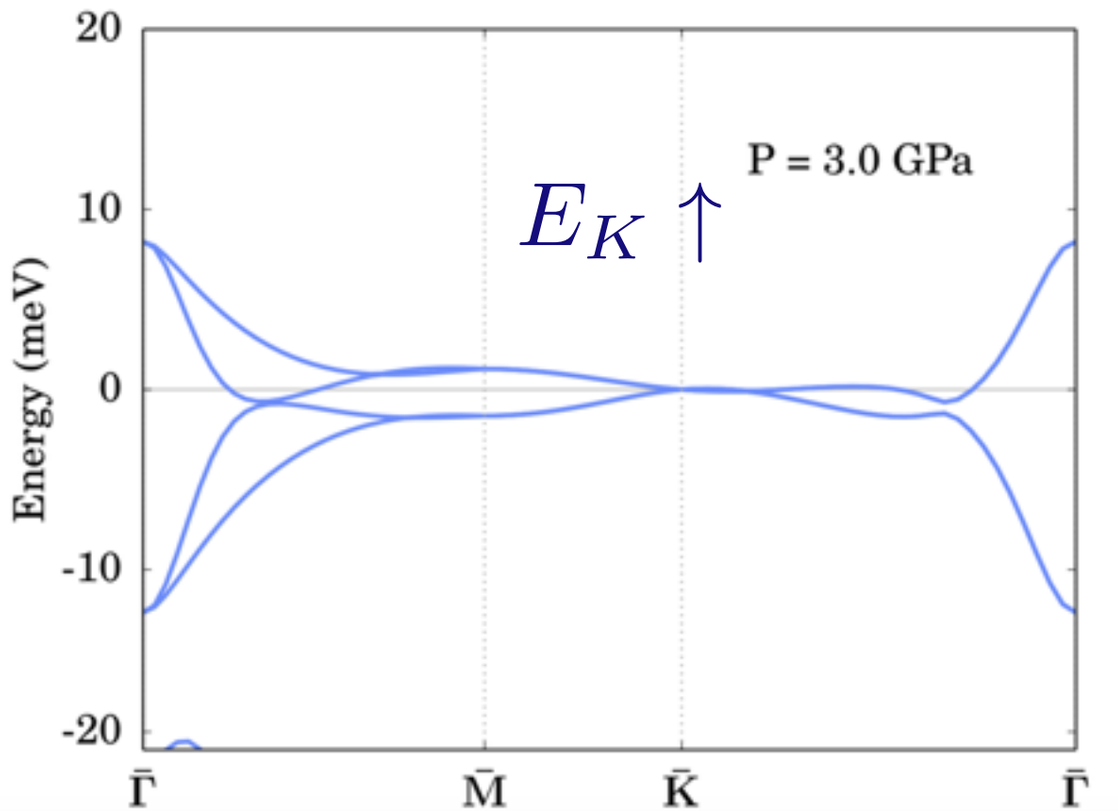
Effective Magic-Ang




P = 0.0 GPa
Energy (meV)
20
10
0
-10
-20
Γ̄
M̄
K̄
Γ̄


P = 2.2 GPa
$E_K \uparrow$
Energy (meV)
20
10
0
-10
-20
Γ̄
M̄
K̄
Γ̄


P = 1.33 GPa
$E_K \downarrow$
Energy (meV)
20
10
0
-10
-20
Γ
M
K
Γ


P = 3.0 GPa
$E_K \uparrow$
Energy (meV)
20
10
0
-10
-20
Γ̄
M̄
K̄
Γ̄

estimated

$$r_s$$

$$r_s \approx 15\,\mathrm{meV}\,\frac{\theta^\circ}{E_K}\sqrt{\nu} \xrightarrow[\mathrm{D2}]{\text{Device}} \frac{20\,\mathrm{meV}}{E_K\,\mathrm{meV}}\sqrt{\nu}$$

Charge per supercell (e^-)

4 2 0 −2 −4

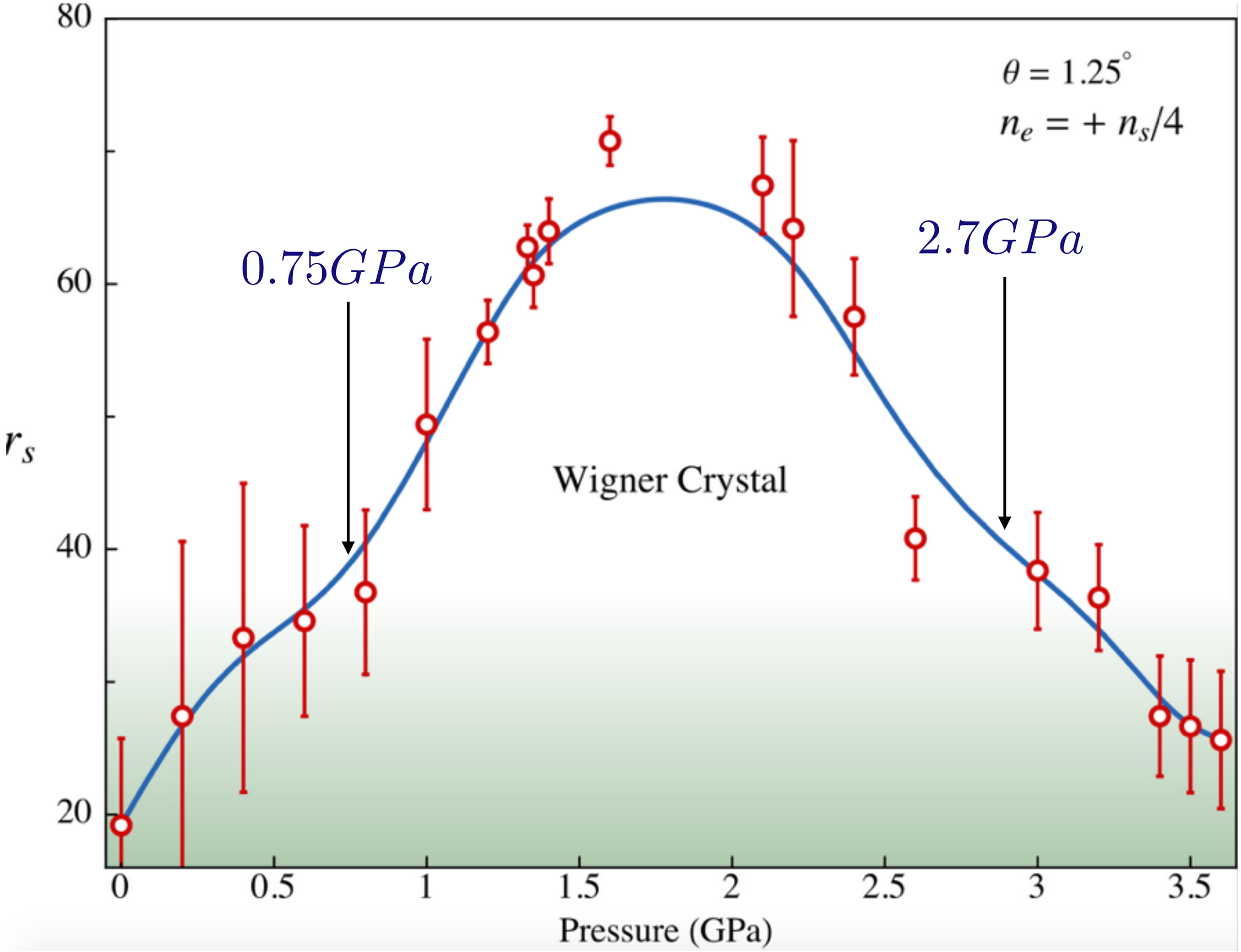
15 10 5 0 −5 −10

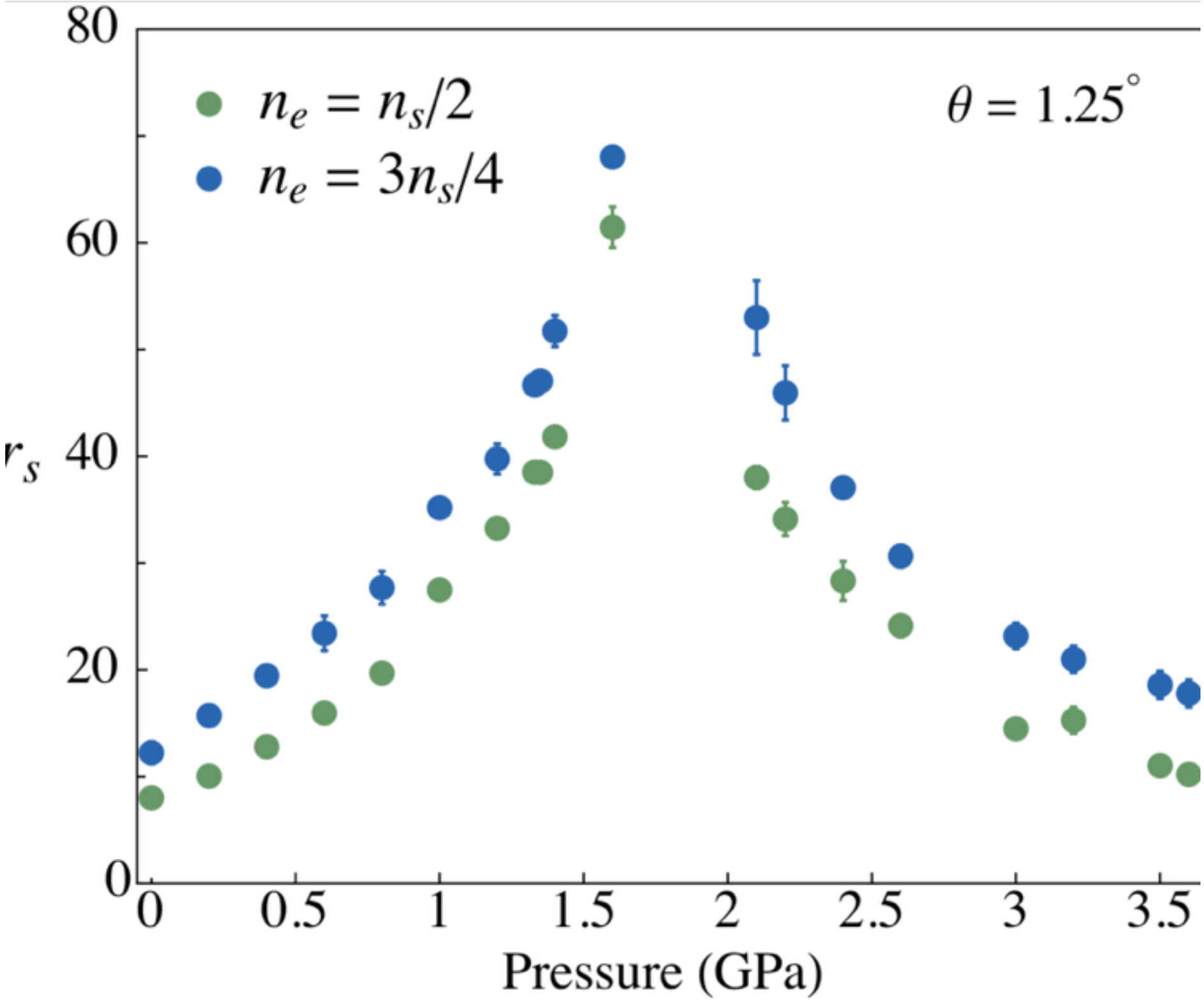
Chemical Potential, μ (meV)

0.0 GPa

1.35 GPa

2.2 GPa


$\theta = 1.25^{\circ}$
$n_e = + n_s/4$
$0.75GPa$
$2.7GPa$
Wigner Crystal
r_s
80
60
40
20
0
0.5
1
1.5
2
2.5
3
3.5
Pressure (GPa)


80
60
40
20
0
r_s
$n_e = n_s/2$
$n_e = 3n_s/4$
$\theta = 1.25^\circ$
0
0.5
1
1.5
2
2.5
3
3.5
Pressure (GPa)

Prediction

$$0.75GPa < P < 2.75GPa$$

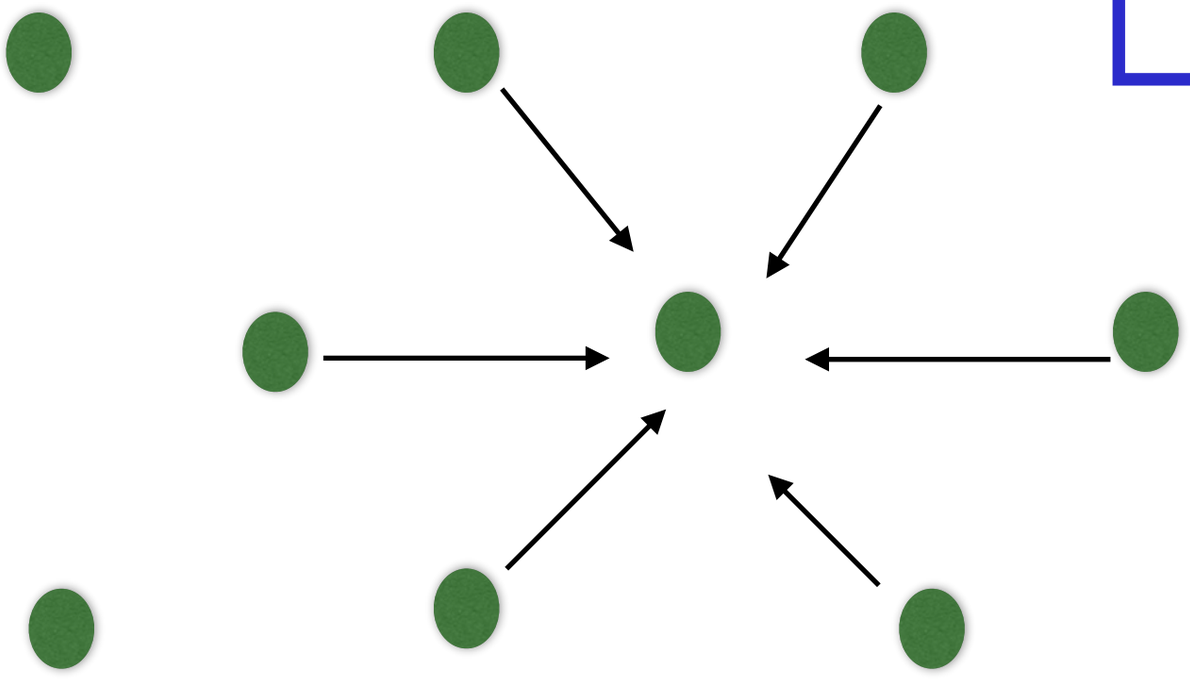
dome-shaped
phase diagram

Letter

Superconductivity in a two-dimensional electron gas

Philip Phillips ✉, Yi Wan, Ivar Martin, Sergey Knysh & Denis Dalidovich

Nature **395**, 253–257 (17 September 1998) | Received: 19 May 1998

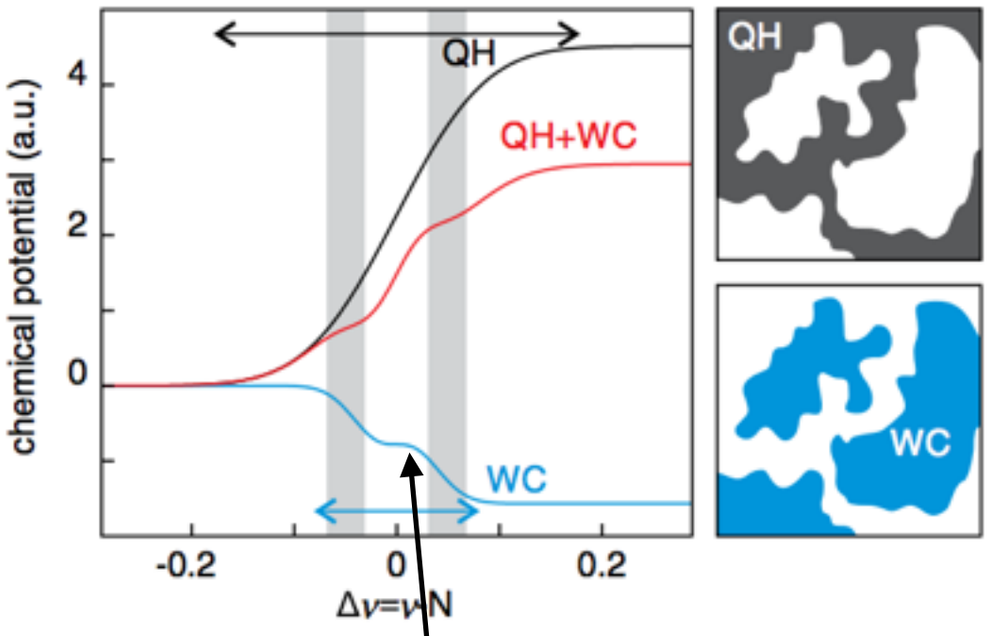


Wigner Crystal

dynamically generated attraction

compressibility

…,von Klitzing, PRL 113, 076804 (2014)



$$\kappa = \frac{\partial n}{\partial \mu} < 0$$

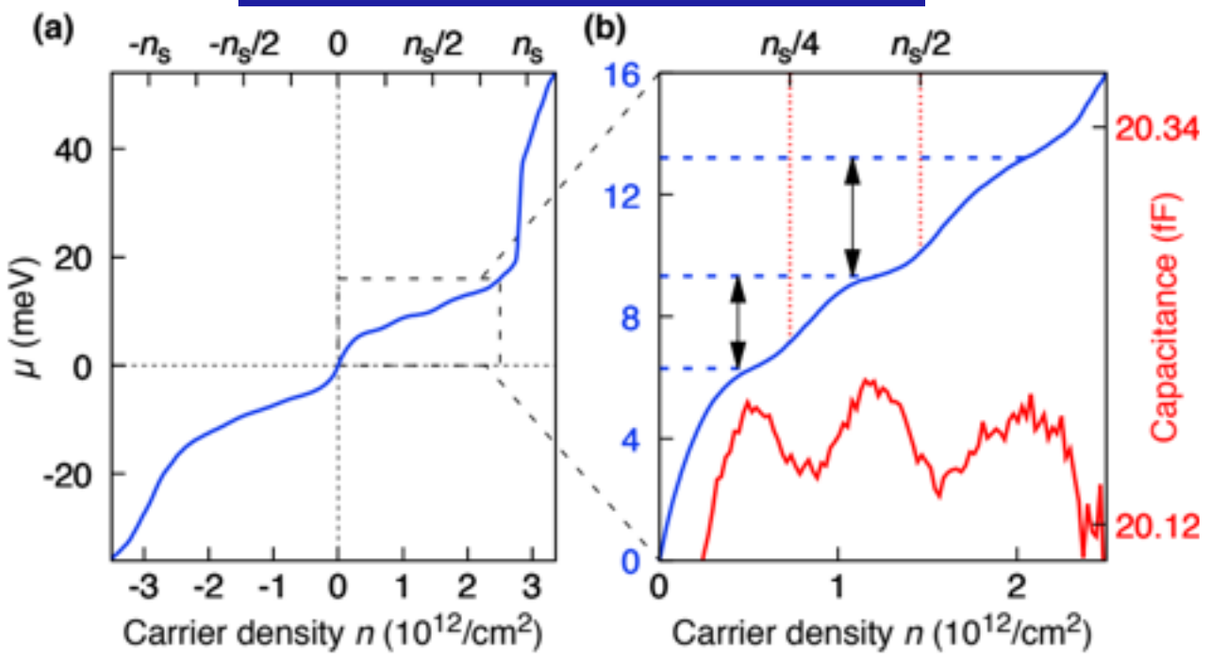
Electronic compressibility of magic angle graphene superlattices

S.L. Tomarken,[1] Y. Cao,[1] A. Demir,[1] K. Watanabe,[2] T. Taniguchi,[2] P. Jarillo-Herrero,[1,*] and R.C. Ashoori[1,†]

[1] *Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139*

[2] *National Institute of Materials Science 1-1 Namiki, Tsukuba 305-0044, Japan*

1903.10492

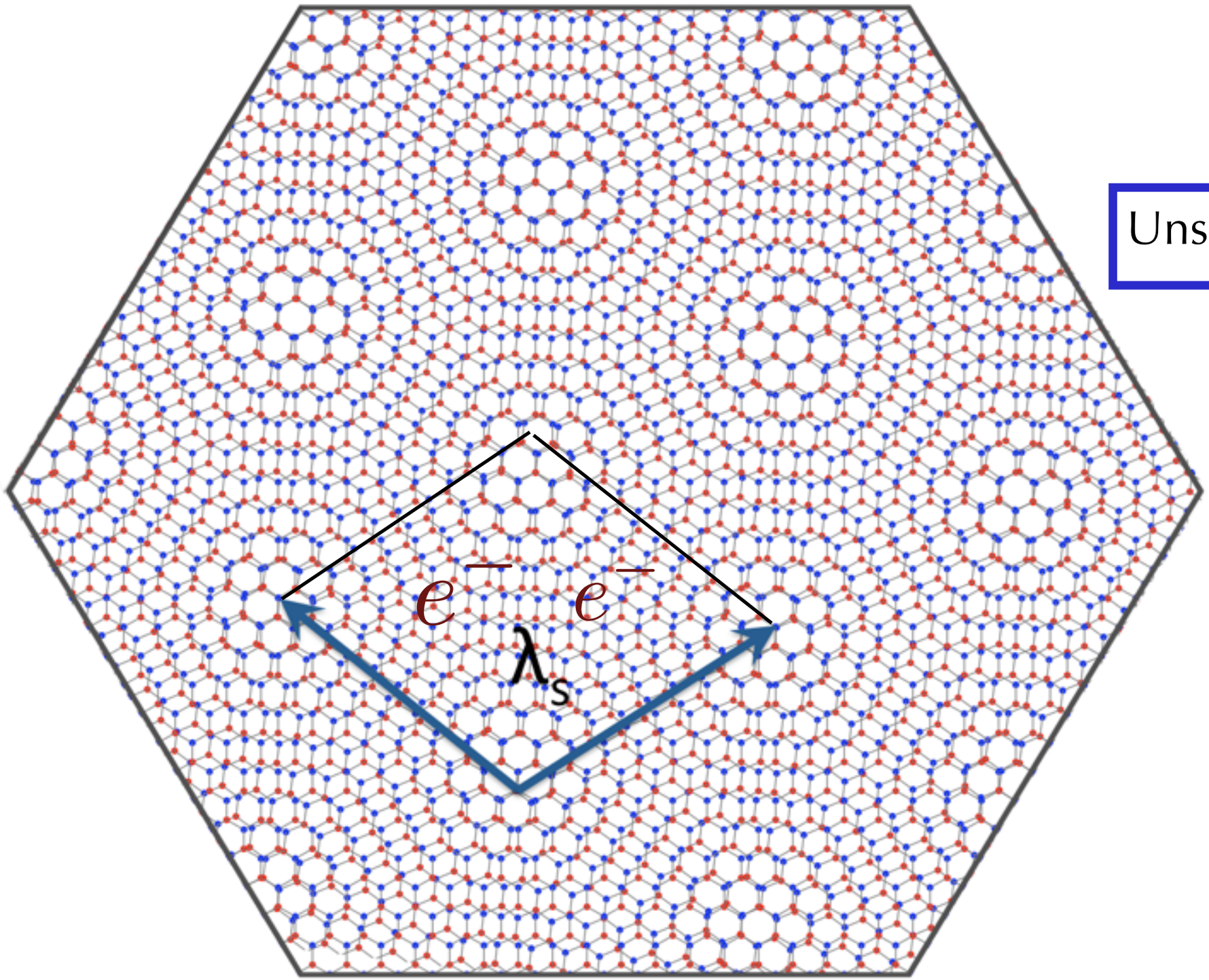


Moiré is Different

?

FL

WC



Unsolved Problem

$r_s \gg 1$