Superconductivity and Mottness: Exact Results

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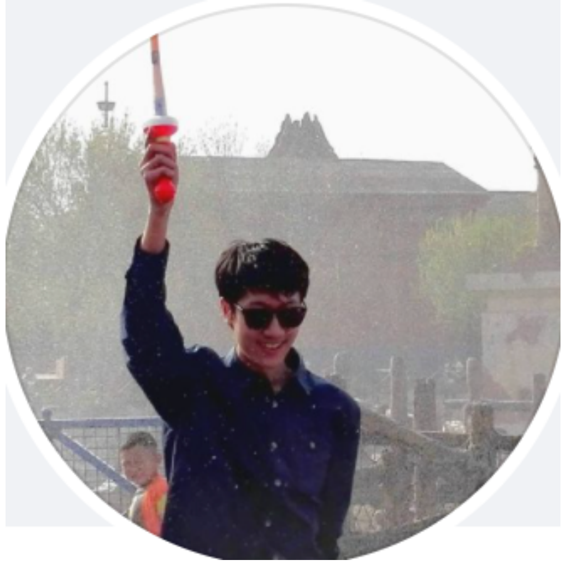
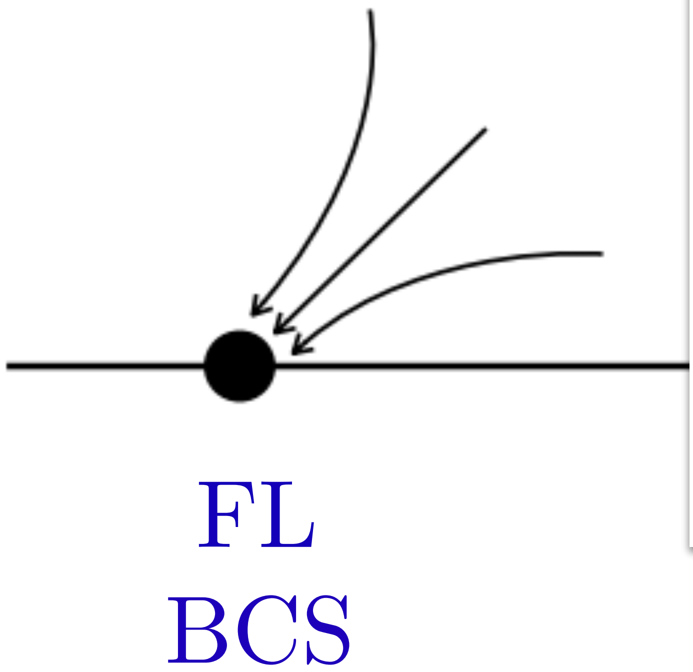




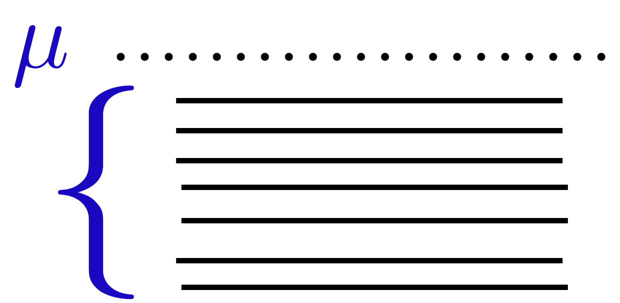
fixed
point beyond
FL? cuprates

quartic
interacting
theory?

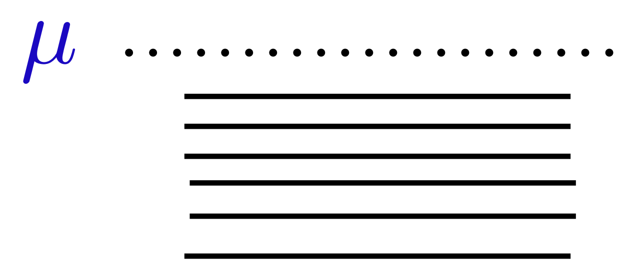
Fermi liquids

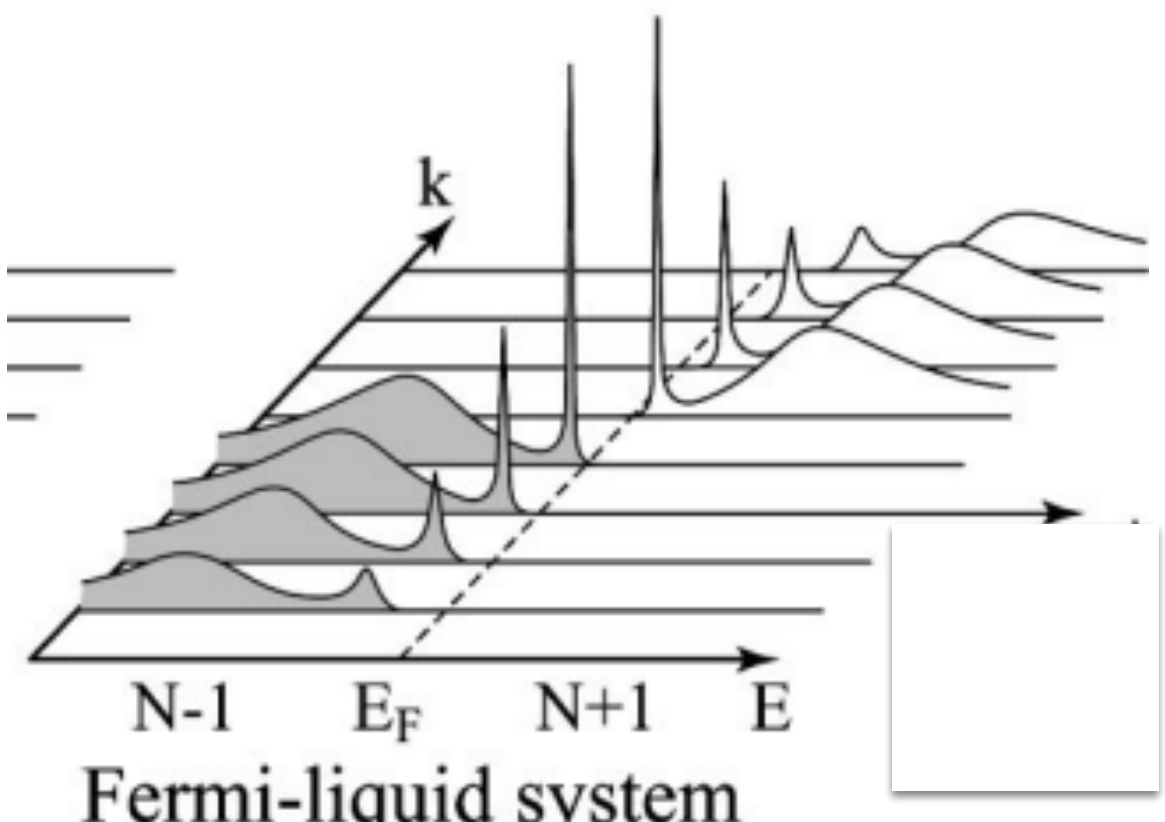
NFL

doubly occupied



Is single occupancy below chemical potential possible?



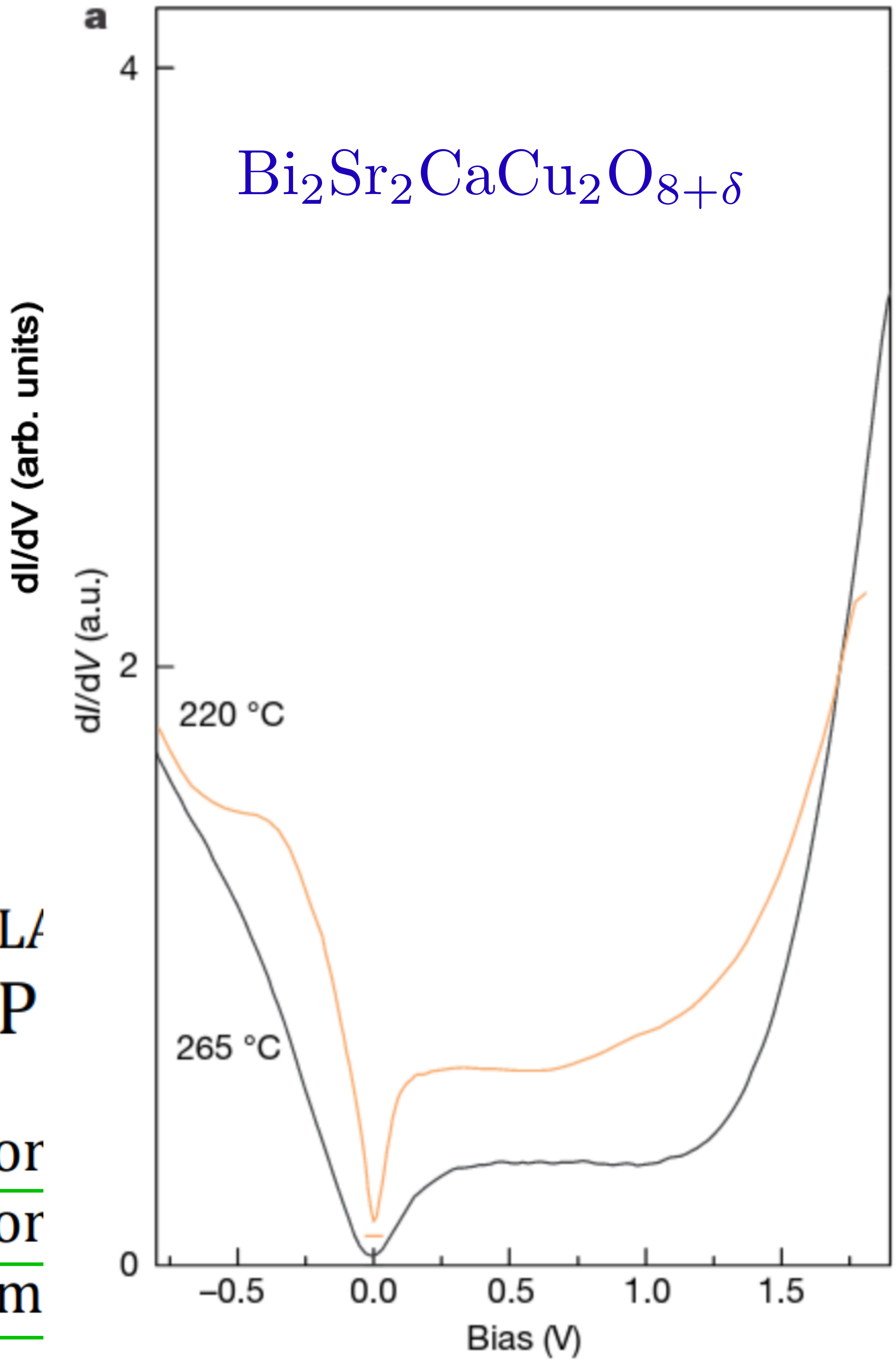


with time-reversal symmetry in tact?

p-h symmetry

LA

P iversity



theor universal refusal of
theor f hole-particle
asym t the first step of any

single occupancy

?

particle-hole asymmetry

Anderson
Haldane
2000

3 citations

Fermi liquids

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + \cdots \quad (\cdots \to 0)$$

$(n_{p\uparrow}, n_{p\downarrow})$ conserved currents

$(c_{p\uparrow}, c_{p\downarrow}, \text{h.c.})$ 4 objects

$O(4)$

$\text{DetM} = 1$ → SO(4) proper rotations

$\text{DetM} = -1$ → improper rotations

$$\text{Det}M = \pm 1 \implies Z_2 = O(4) \div SO(4)$$

Improper Rotations

Majorana basis

$$\begin{pmatrix} c_{p\uparrow} \\ c^\dagger_{p\uparrow} \\ c_{p\downarrow} \\ c^\dagger_{p\downarrow} \end{pmatrix} \longrightarrow \begin{pmatrix} c_{p\uparrow} + c^\dagger_{p\uparrow} \\ i(c_{p\uparrow} - c^\dagger_{p\uparrow}) \\ c_{p\downarrow} + c^\dagger_{p\downarrow} \\ i(c_{p\downarrow} - c^\dagger_{p\downarrow}) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} c_{p\uparrow} + c^\dagger_{p\uparrow} \\ i(c_{p\uparrow} - c^\dagger_{p\uparrow}) \\ c_{p\downarrow} + c^\dagger_{p\downarrow} \\ i(c_{p\downarrow} - c^\dagger_{p\downarrow}) \end{pmatrix} \Longrightarrow \quad c_{p\downarrow} \rightarrow c^\dagger_{p\downarrow}$$

p-h transformation

$$\epsilon(p) = \epsilon_F$$

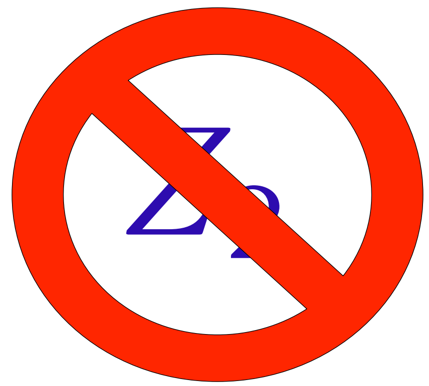
Fermi Surface

$$H = 0$$

$$\left.\begin{aligned} n_{p\uparrow} &\to 1 - n_{p\uparrow} \\ n_{p\downarrow} &\to n_{p\downarrow} \end{aligned}\right\} Z_2$$

at Fermi surface only

How to destroy Fermi liquids?



$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + U n_{p\uparrow} n_{p\downarrow}$$

odd under Z_2

scaling dimension

$[n_{p\uparrow} n_{p\downarrow}] = -2$

relevant interaction

New fixed point!

Hatsugai-Kohmoto or Baskaran model

Hubbard not necessary!

General HK Model

$$\sum_k (\xi_k(n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow})$$

Solvable Mott transition:U>W

$$G_{k\sigma}(i\omega_n \to z) = \frac{1 - \langle n_{k\bar{\sigma}} \rangle}{z - \xi_k} + \frac{\langle n_{k\bar{\sigma}} \rangle}{z - (\xi_k + U)} \neq \frac{1}{z - \omega_k}$$

lower Hubbard band

upper Hubbard band

$$\zeta_{k\sigma} = c_{k\sigma}(1 - n_{k\bar{\sigma}})$$

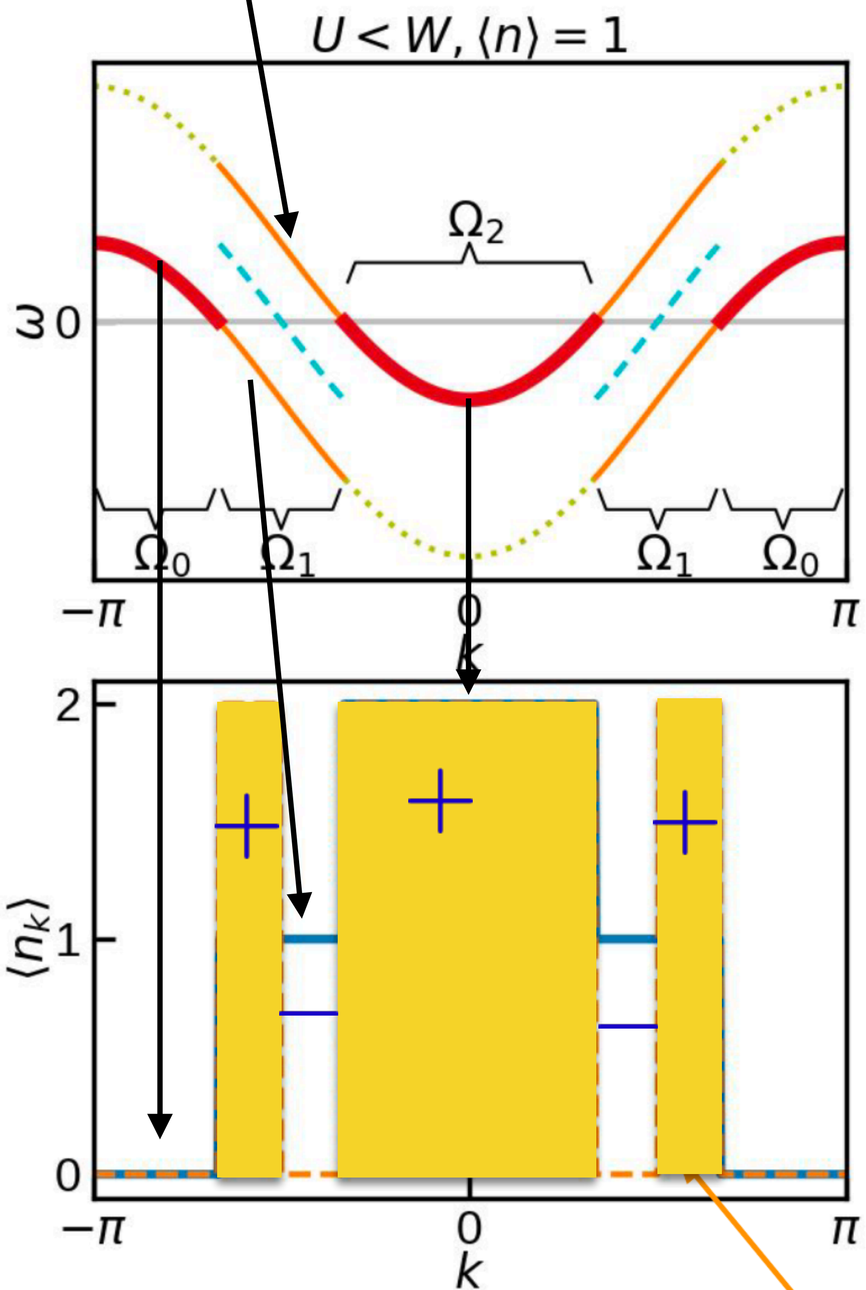
$$\eta_{k\sigma} = c_{k\sigma} n_{k\bar{\sigma}}$$

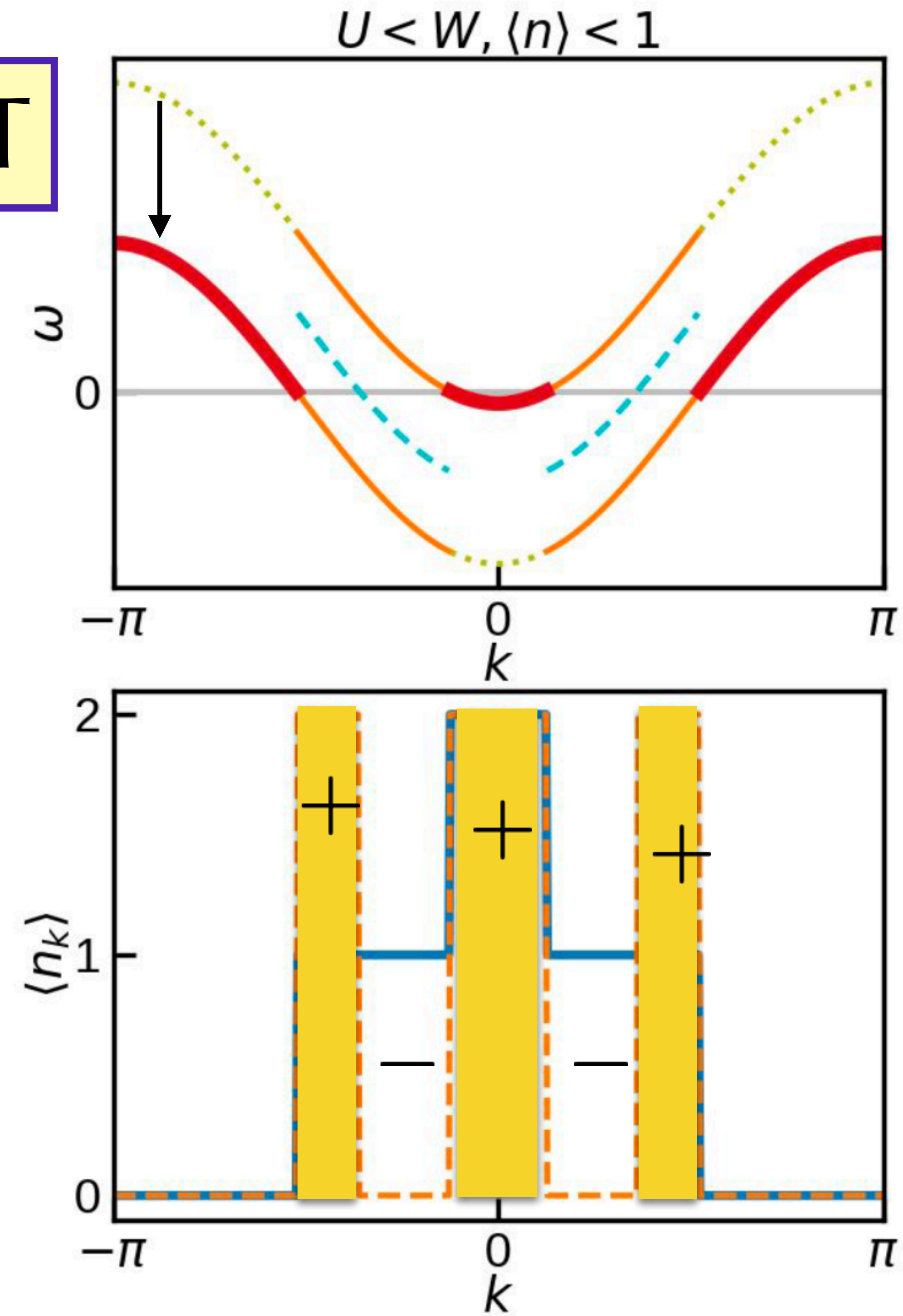
zeros

single occupancy

counting charges

SWT

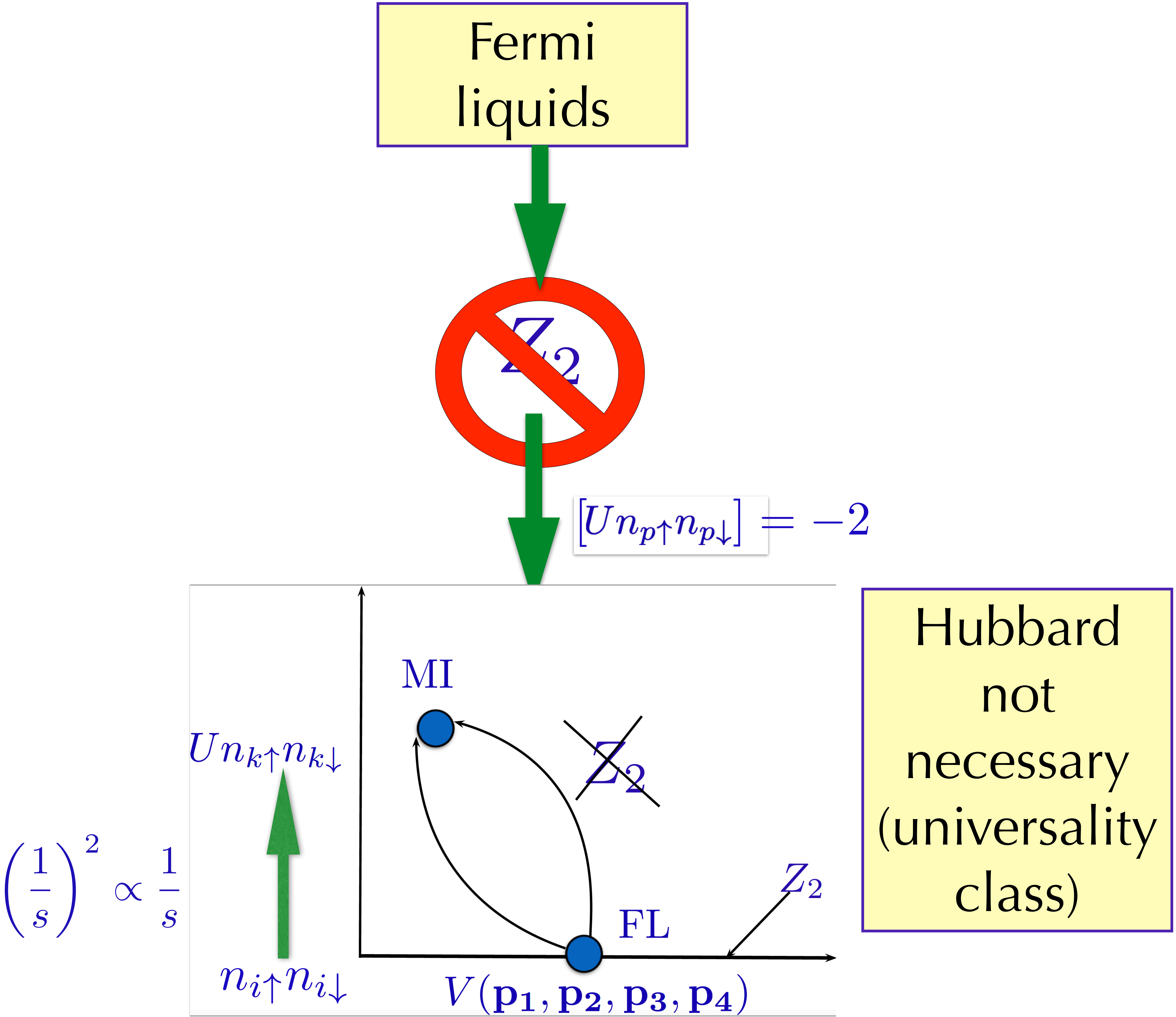




$n_{\mathrm{Lutt}} = \langle n \rangle$

$n_{\mathrm{Lutt}} \neq \langle n \rangle$

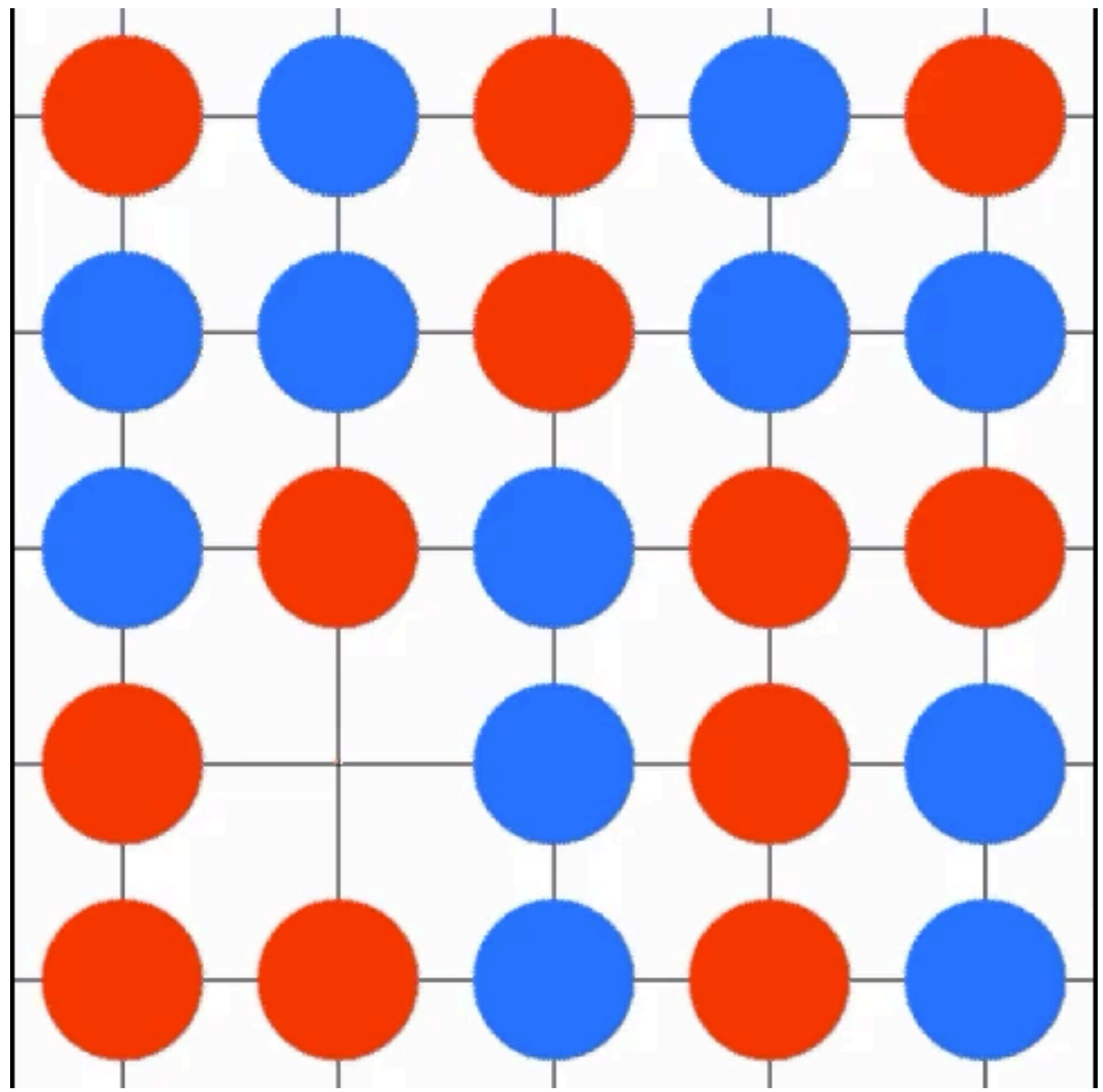
zeros ≠ particles



Fermi liquids
Z_2
$[Un_{p\uparrow}n_{p\downarrow}] = -2$
MI
$Un_{k\uparrow}n_{k\downarrow}$
Z_2
$\left(\frac{1}{s}\right)^2 \propto \frac{1}{s}$
FL
Z_2
$n_{i\uparrow}n_{i\downarrow}$
$V(\mathbf{p_1},\mathbf{p_2},\mathbf{p_3},\mathbf{p_4})$
Hubbard not necessary (universality class)

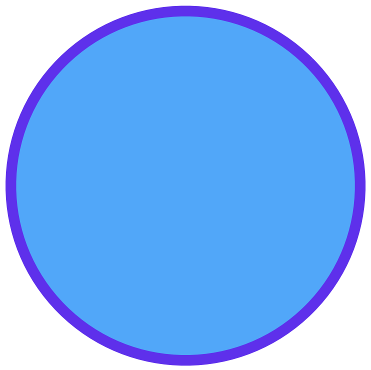
what does the HK model leave out??

$$[H_t, H_U] \neq 0$$

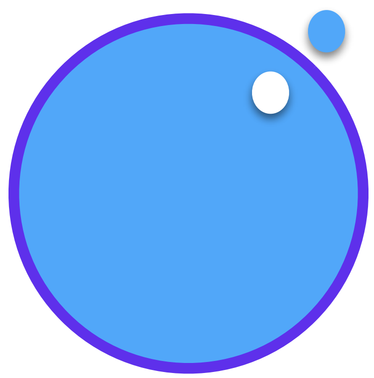


dynamical spectral weight transfer

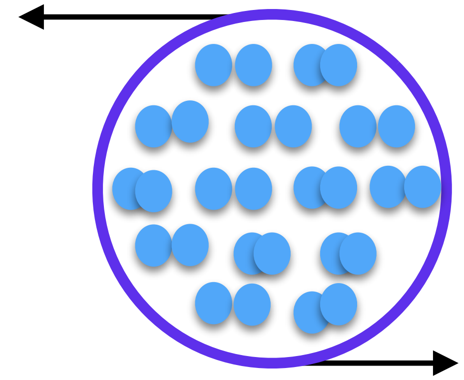
Fermi gas



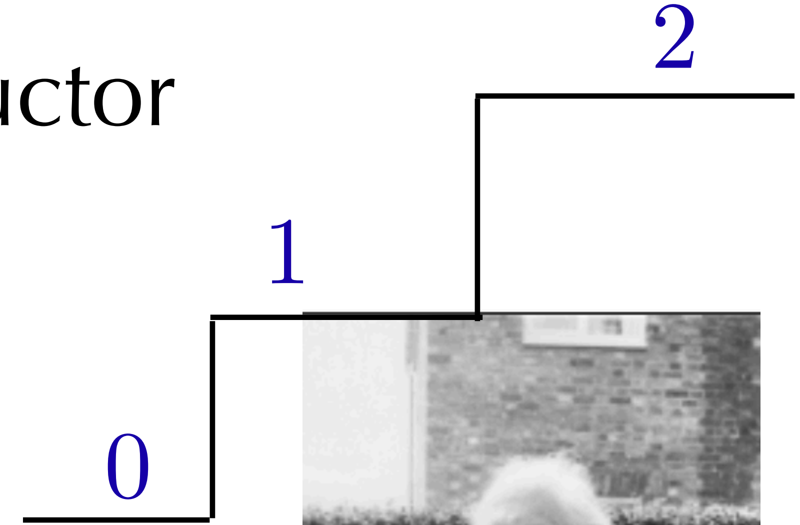
Fermi liquid



BCS superconductor

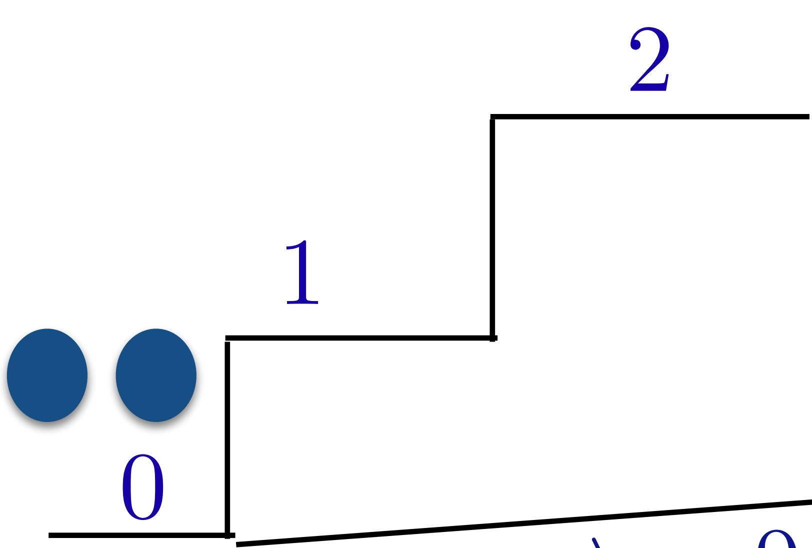


Mottness

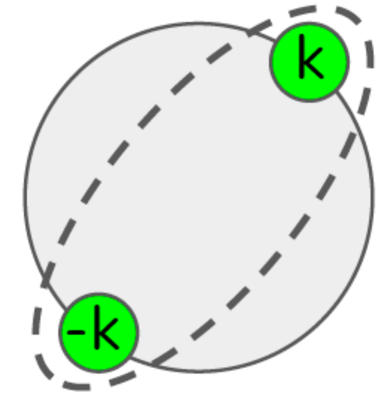


Superconductivity?

Cooper Instability



$$H = H_{\mathrm{HK}} - gH_p$$



$$|\psi\rangle = \sum_{k\in\Omega_0} \alpha_k b_k^\dagger |\mathrm{GS}\rangle$$

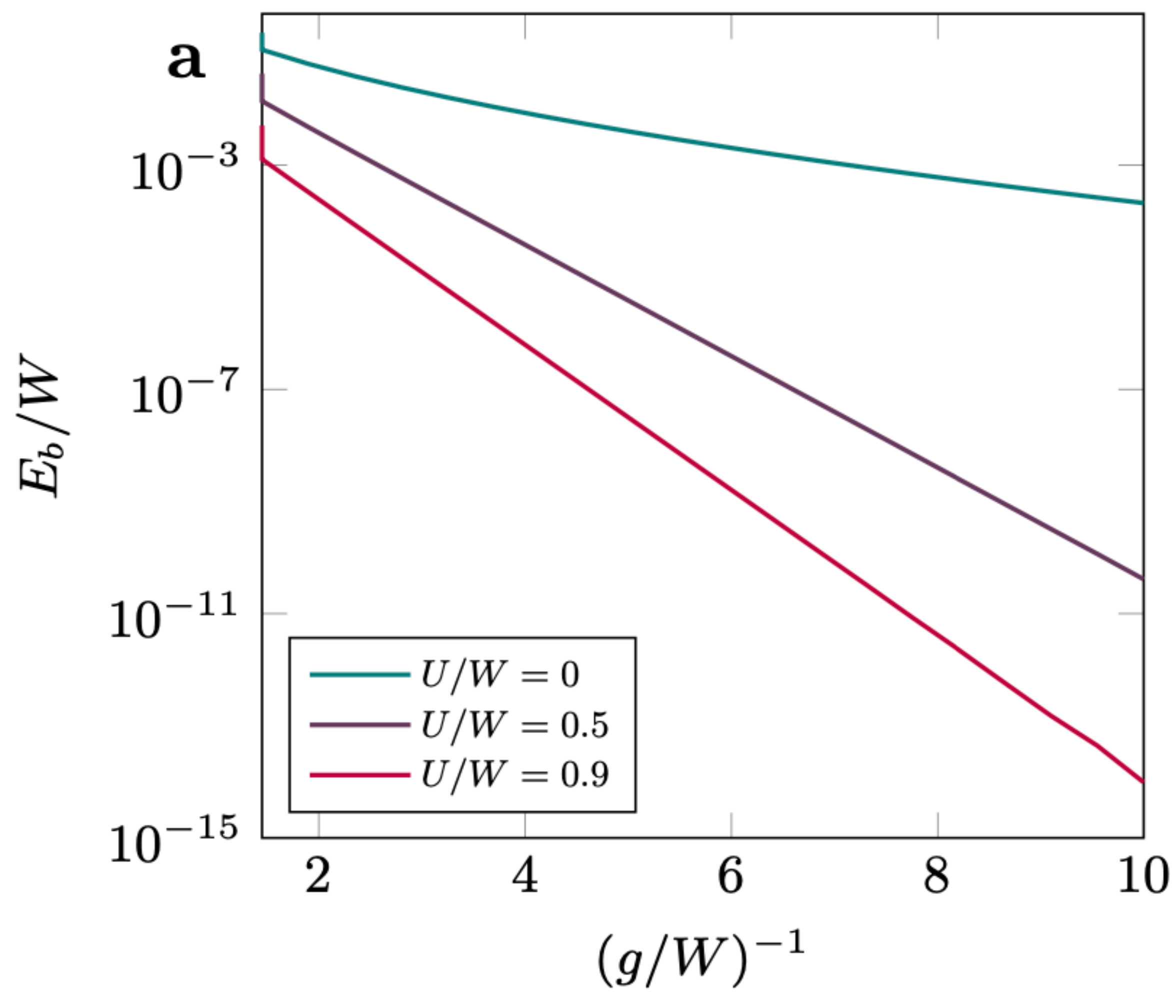
$$= \sum_{k,k'} b_k^\dagger b_{k'}$$

$$\langle n_{k\sigma}\rangle = 0$$

$$E_b = \langle \mathrm{GS}|\mathrm{H}|\mathrm{GS}\rangle - \langle\psi|\mathrm{H}|\psi\rangle \lessgtr 0$$

Cooper Instability

$$E_b = -E \sim W(1-(U/W)^2)e^{-\pi W\sqrt{1-(U/W)^2}/g}$$



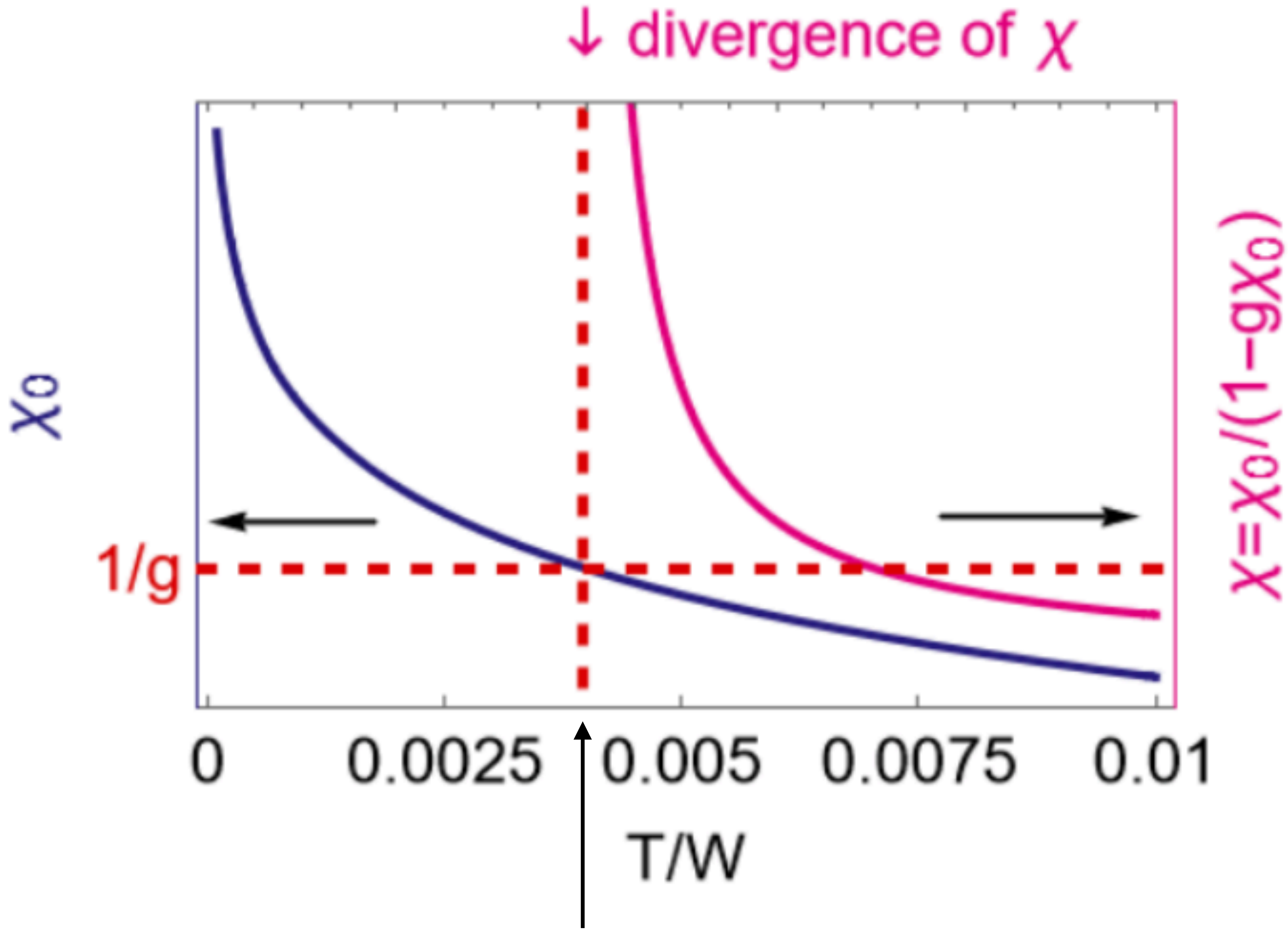
Pair Susceptibility

$$\chi(i\nu_n) \equiv \frac{1}{L^d}\int_0^\beta d\tau e^{i\nu_n\tau}\langle T\Delta(\tau)\Delta^\dagger\rangle_g$$

$$= \frac{\chi_0}{1-g\chi_0}$$

$g\chi_0 = 1$

solve for T_c

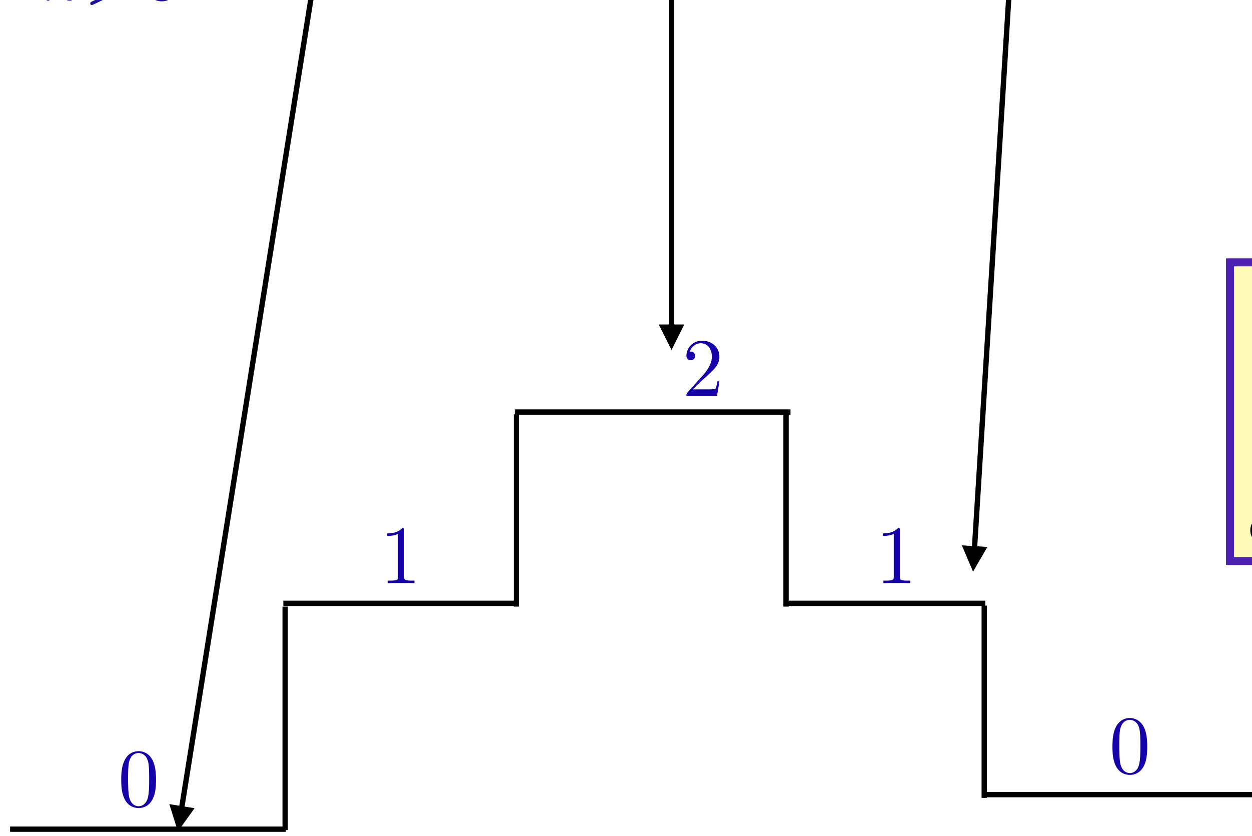


T_c

$$T_c = (W - U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}}.$$

variational MF wave function

$$|\psi\rangle = \prod_{k>0} \left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}} (b_k^\dagger + b_{-k}^\dagger) \right) |0\rangle$$



HK generalization

three variational parameters

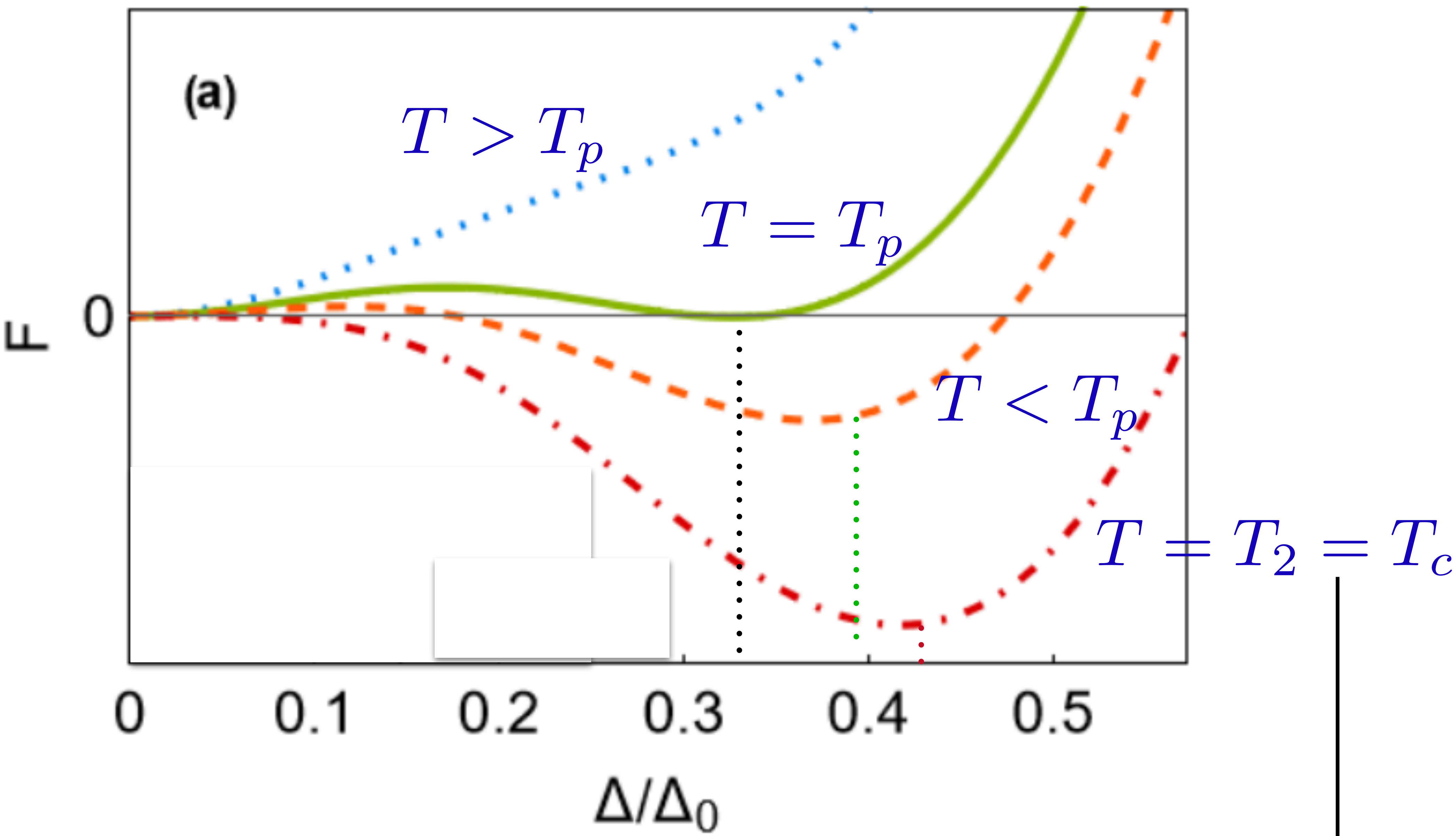
$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation

$$\Delta \ll U, W$$

$$\Delta = (W - U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

compute free energy



$t_G \approx 10^{-11}$

MF theory
is accurate!

$$\lim_{g \to 0} \frac{\Delta}{T_c} \to \infty$$

Two-stage superconductivity in the Hatsugai-Kohomoto-BCS model

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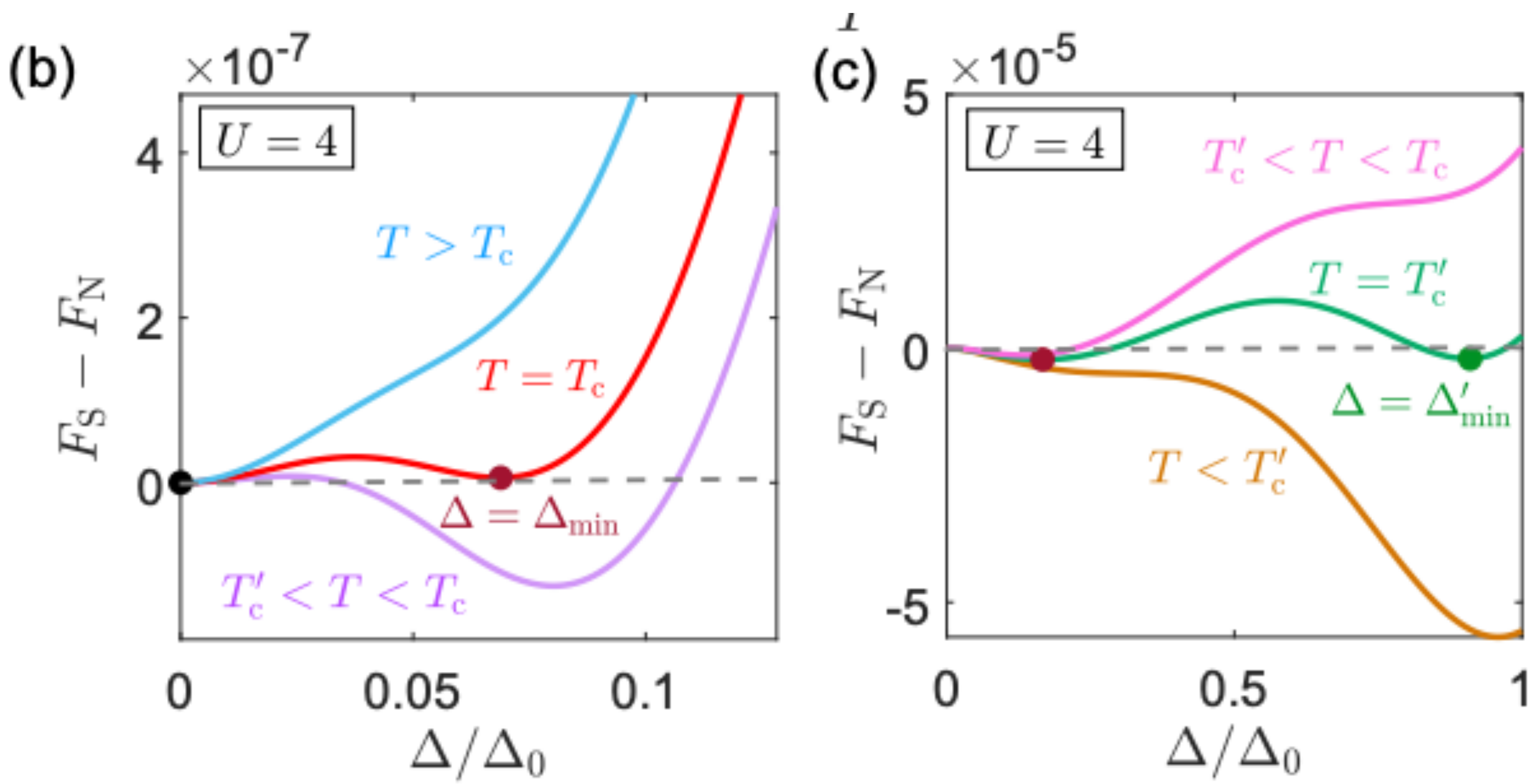
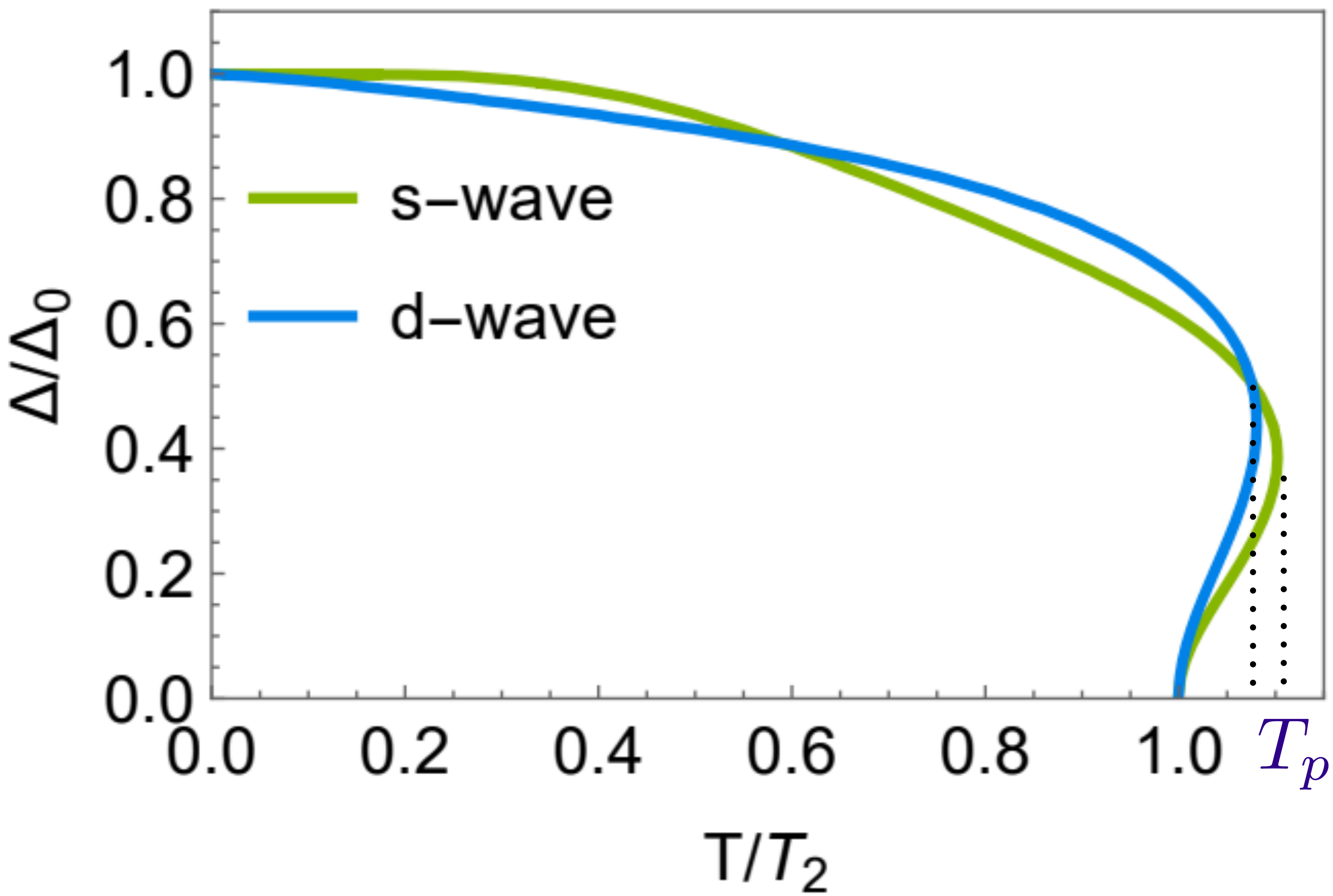
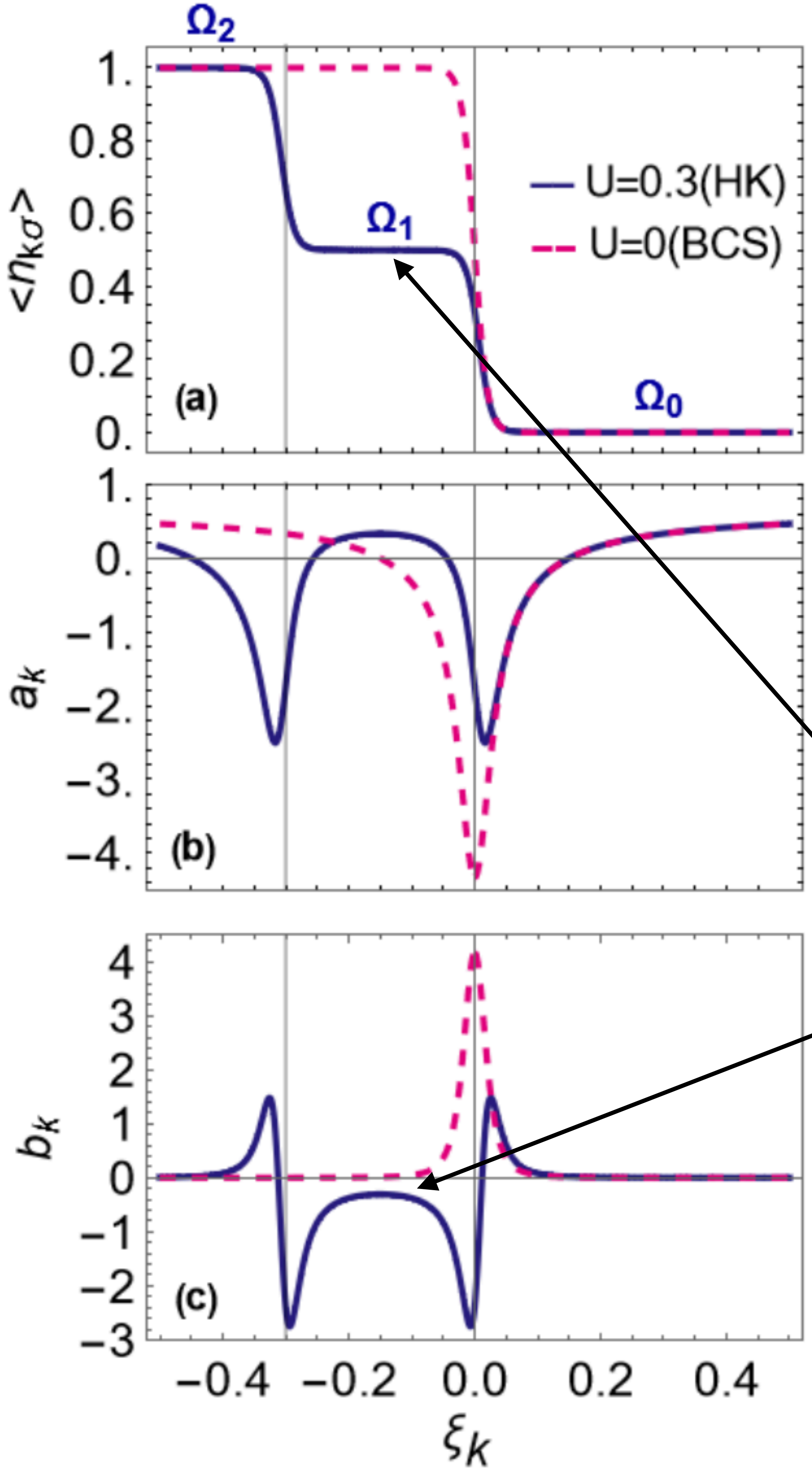
https://arxiv.org/pdf/2207.01904.pdf





1.0
0.8
0.6
0.4
0.2
0.0
Δ/Δ0
s–wave
d–wave
0.0
0.2
0.4
0.6
0.8
1.0
T_p
T/T_2



Landau parameters

Mottness

Bogoliubov excitations

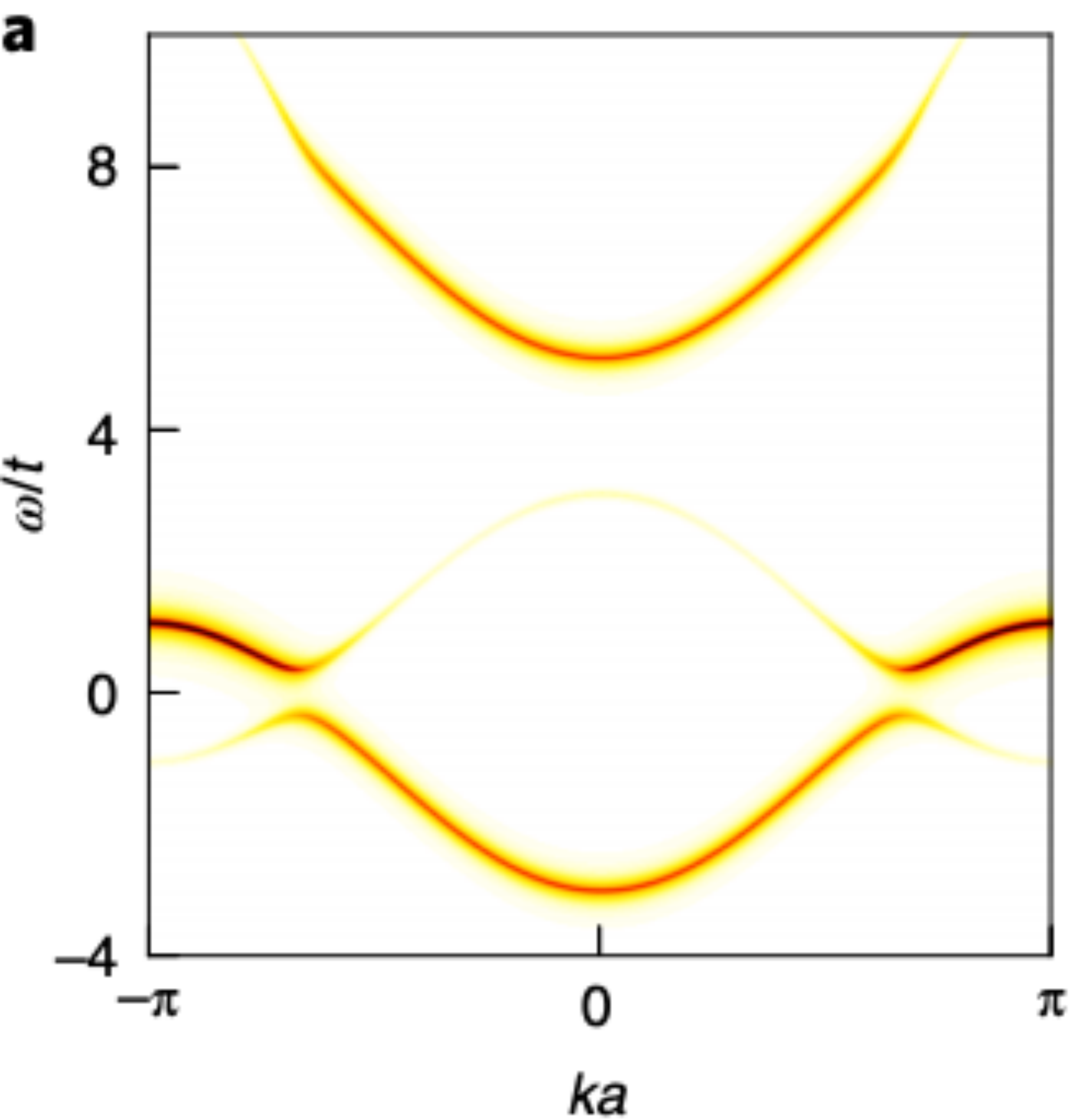
$$\gamma_{k\sigma}|\psi_{\mathrm{BCS}}\rangle = 0$$

$$\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c^\dagger_{-k\overline{\sigma}}$$

PYHons excitations

$$\gamma^l_{k\sigma} \propto \sqrt{2} x_k \zeta^\dagger_{k\sigma} - \sigma z_k \zeta_{-k\overline{\sigma}}$$

$$\gamma^u_{k\sigma} \propto z_k \eta^\dagger_{k\sigma} - \sigma \sqrt{2} y_k \eta_{-k\overline{\sigma}}$$



PYHon band

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$

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Science 22 Mar 2002:
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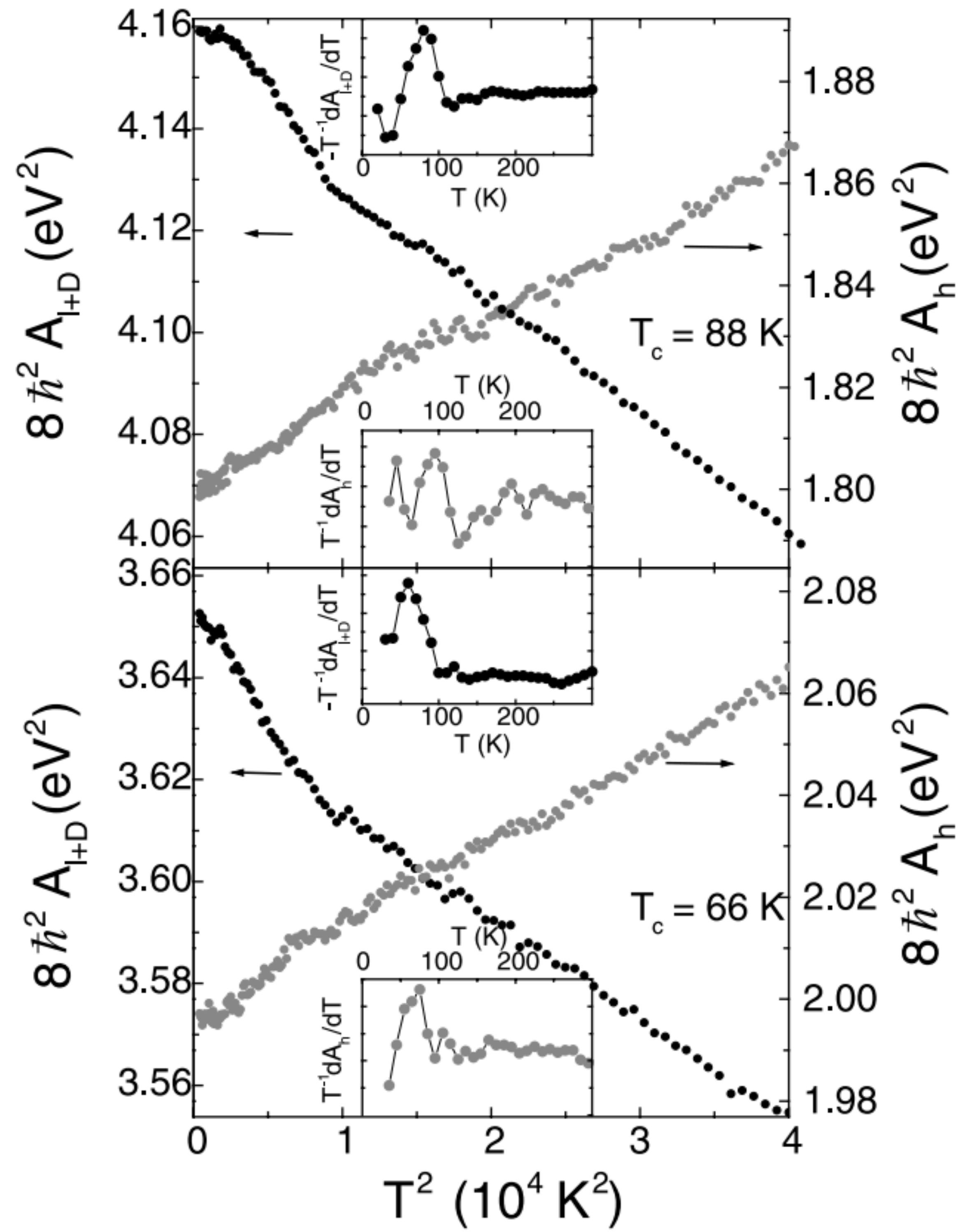
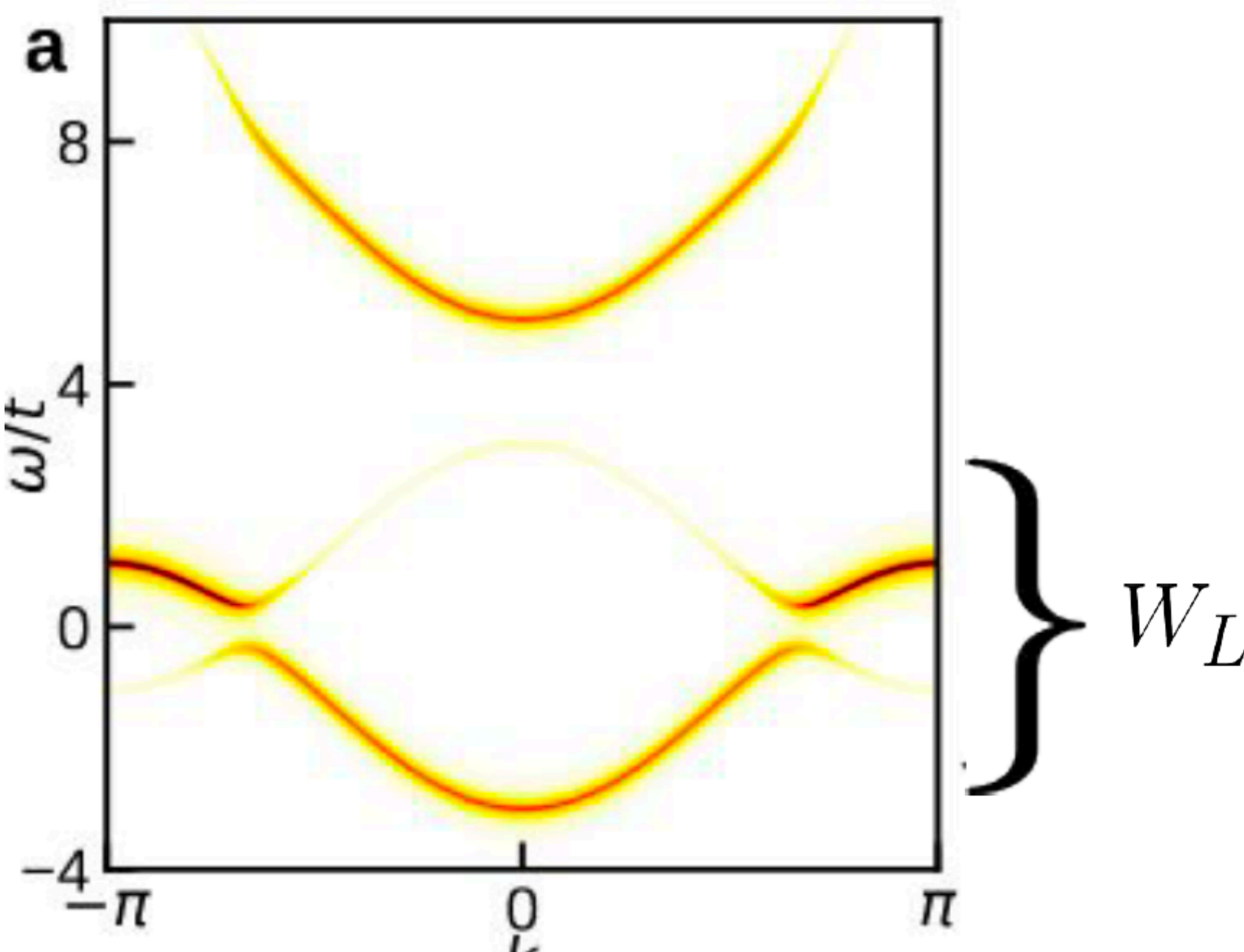
DOI: 10.1126/science.1069947



$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$\frac{\Delta A_l}{A_l} \propto 3\%$$


a
8
4
0
−4
ω/t
−π
0
π
k
W_L

why?

$$H = H_{\mathrm{HK}} + H_p$$

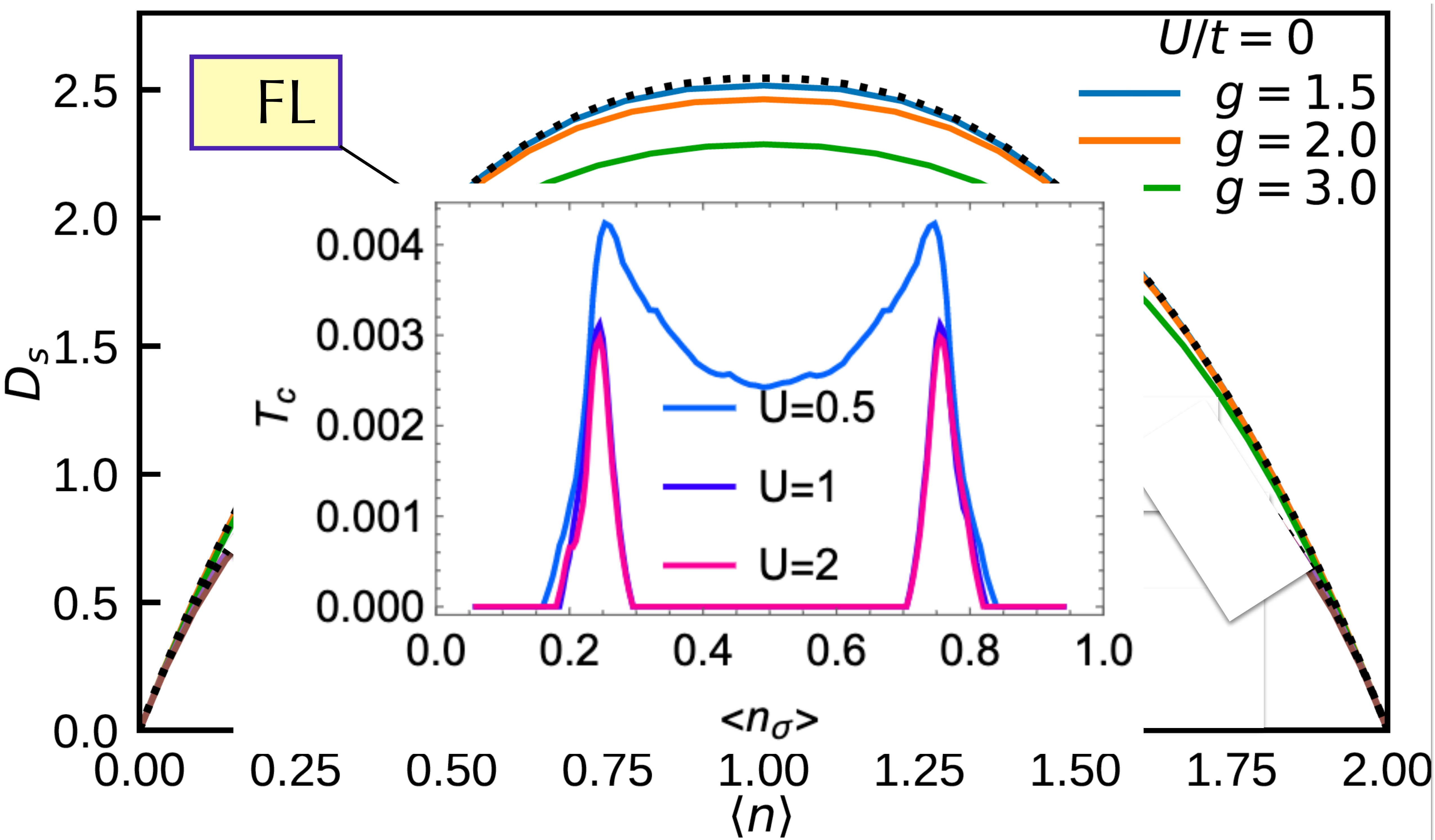
$$[H_{\mathrm{HK}}, H_p] \neq 0$$

↓

dynamical spectral weight transfer

Superfluid Density

Mottness-induced suppression



observable	Superconductivity BCS/FL	Mottness PYHZ/HK
$\chi \to \infty$	T_c	$T_c(= T_2)$
$\Delta \neq 0$	T_c	$T_p(> T_2)$
$\lim_{g\to 0} 2\Delta_0/k_B T_c$	3.52	∞
quasi − particles	Bogoliubons	PYHons
t_G(Ginzburg)	$\approx 10^{-12}$	$\approx 10^{-11}$
$1/TT_1$	HS peak	no HS peak
Landau Expansion	$a = \alpha t,\ b > 0$	$a = \alpha t,\ b < 0$ $c > 0$
$E_{\rm cond}/N(0)\Delta^2$	-1	$[-2, -1]$

Topology + Strong Correlations?

Are Exact Statements Possible?

Haldane +HK model

TI

1/4-filled MI