What’s the difference?
What’s the difference?

Superconducting wires:
40% more efficient
Unparticles and Superconductivity

Thanks to: NSF, EFRC (DOE)

Kiaran dave    Charlie Kane    Brandon Langley    J. A. Hutasoit
What is a particle?
Properties all particles share?
Properties all particles share?

charge

fixed mass!
Properties all particles share?

charge

fixed mass!

conserved (gauge invariance)
Properties all particles share?

- Charge
  - Conserved (gauge invariance)

- Fixed mass!
  - Not conserved
mass sets a scale
probing particles

particles

$E_{\text{photon}} = h\nu$

700 nm 1.77 eV
550 nm 2.25 eV
no electrons

Potassium - 2.0 eV needed to eject electron

Energy

$E = \varepsilon_p$

mass = energy

Wednesday, January 15, 14
what would you see in a metal?
what would you see in a metal?
what would you see in a metal?

EMPTY STATES = METAL
Photoemission particles

\[ E_{\text{photon}} = h\nu \]

\[ v_{\text{max}} = 6.22 \times 10^5 \text{ m/s} \]

700 nm 1.77 eV
550 nm 2.25 eV
400 nm 3.1 eV

Photoelectric effect

Energy

\[ E = \varepsilon_p \propto p^2 = (-p)^2 \]
photoemission

particles

\[ E_{\text{photon}} = \hbar \nu \]

Photoelectric effect

Energy

\[ E = \varepsilon_p \propto p^2 = (-p)^2 \]
Photoelectric effect

\[ E_{\text{photon}} = h\nu \]

Energy:

\[ E = \varepsilon_p \propto p^2 = (-p)^2 \]

two crossings:

closed surface of excitations

Photoemission

Particles
expect to see

\[ p_y \]

closed surface

\[ p_x \]
are there any exceptions?
where is this part?
Fermi Arcs

where is this part?

Fermi arcs: (PDJ, JCC, ZXS)
what’s the explanation?

Two opposing Views on Fermi arcs

strange

not strange
what’s the explanation?

Two opposing Views on Fermi arcs

strange

not strange

this dispute has an answer!
what’s the explanation?

Two opposing Views on Fermi arcs

strange

not strange

unparticles

this dispute has an answer!

Wednesday, January 15, 14
where are arcs seen?
normal state

\[
Y_{2}Ba_{3}Cu_{7}O
\]

Cuprate Superconductors

ceramics
first superconductor
'Minds are like parachutes they only function when they are open'- James Dewar, Scottish physicist 1842 –1923 liquified hydrogen, 1898 (20.52K)
'Minds are like parachutes they only function when they are open'- James Dewar, Scottish physicist 1842 –1923 liquified hydrogen, 1898
(20.52K)

H. K. Onnes
1898: building lab
Race to zero

'Minds are like parachutes they only function when they are open' - James Dewar, Scottish physicist 1842 –1923 liquified hydrogen, 1898 (20.52K)

H. K. Onnes
1898: building lab
1908: liquified Helium (0.9K)
Race to zero

H. K. Onnes

1898: building lab
1908: liquified Helium (0.9K)
Race to zero

H. K. Onnes
1898: building lab
1908: liquified Helium (0.9K)
H. K. Onnes
1898: building lab
1908: liquified Helium (0.9K)
superconductivity

zero resistance: conduction without loss

repulsion of magnetic fields

$ SC = $$

$79 \text{ B economic loss (US)}$

why?
superconductivity

zero resistance: conduction without loss

$79 \text{ B economic loss (US)}$

repulsion of magnetic fields

why?
Superconductor

magnet

magnetic field is short-ranged
Superconductor

magnet

magnetic field is short-ranged

photon is massive in a superconductor
$e^- + \text{photons}$
$e^{-} + \text{photons}$
A diagram showing a cube with electrons ($e^-$) and a prohibition sign indicating that $e^- + \text{photons}$ is not possible. The text states, "photon is massive."
A photon is massive, new carriers of charge.

\[ e^- + \text{photons} \rightarrow 2e^- \]

\[ \text{photon is massive} \]

\[ \text{new carriers of charge} \]
$e^- + \text{photons}$

$2e^-$

photon is massive

new carriers of charge
photons

photons

$e^- +$ photons

$2e^-$

photon is massive

new carriers of charge

new carriers of charge

$e^- +$ photons
electrons are free
electrons are free
electrons are free

1. massive photon
2. zero resistance
3. energy to pull electrons apart (gap)
electrons are free

1. massive photon
2. zero resistance
3. energy to pull electrons apart (gap)

BCS theory: 1957
Nobel: 1972
electrons are free

1. massive photon
2. zero resistance
3. energy to pull electrons apart (gap)

BCS theory: 1957
Nobel: 1972

U. Illinois
The diagram illustrates the discovery of superconducting materials over time, with key years and compounds labeled. The horizontal axis represents the years, and the vertical axis represents the temperature in Kelvin (K). Key compounds include:

- HgBa$_2$Ca$_2$Cu$_3$O$_8$ at 1987
- TlCaBaCuO at 1987
- YBCO at 1987
- Nb$_3$Ge at 1976
- V$_3$Si at 1954
- Ni at 1932
- Hg at 1910

Cuprates are highlighted with a box, indicating their significant role in superconductivity research.

The text mentions Fe-based compounds at 2010.
Nitrogen liquifies at 77K.

- HgBa$_2$Ca$_2$Cu$_3$O$_8$
- TlCaBaCuO
- Hg$_{0.8}$Tl$_{0.2}$Ba$_2$Ca$_2$Cu$_3$O$_{8+x}$

Year and temperature landmarks:
- 1910: Hg
- 1932: Ni
- 1943: NbN
- 1954: $V_3Si$
- 1976: $Nb_3Ge$
- 1987: TlCaBaCuO
- 2010: Fe-based cuprates
Nitrogen liquifies at 77K, which is the temperature at which nitrogen changes from a gas to a liquid. This is important for cryogenic applications.

$100K/machine per year$ is a significant cost factor in the production of superconducting materials.

The diagram shows the timeline of superconducting materials, starting from Hg in 1910, which has a critical temperature of 0.84K, to HgBa$_2$Ca$_2$Cu$_3$O$_{8+x}$, which is one of the high-temperature superconductors discovered in the 1980s.

There are different categories of superconducting materials, including

- **Cuprates** (YBCO, HgBa$_2$Ca$_2$Cu$_3$O$_{8+x}$)
- **Fe-based**
- **Nitrogen-based**

The diagram also highlights the year of discovery for each material, such as Ni in 1932 and NbN in 1943.
Nitrogen liquifies at 77K, which is important for superconductors. The chart shows the history of superconductors, with the following compounds:

- HgBa$_2$Ca$_2$Cu$_3$O$_{8+x}$
- TlCaBaCuO
- YBCO

The year 1987 marks the discovery of the first high-temperature superconductor, which is an important milestone in the field of superconductivity. The chart also highlights the development of iron-based superconductors, which have been a significant area of research in recent years.
Does the old theory work for the cuprate superconductors?
does charge density = total number of electrons?
does charge density = total number of electrons?

Luttinger’s theorem
if not?
if not?

charge stuff = particles + other stuff
if not?

charge stuff = particles + other stuff

unparticles!
counting particles
counting particles

2

1
counting particles
counting particles
counting particles

is there a more efficient way?
Each particle has an energy
Each particle has an energy particles

\[ E_{\text{photon}} = h \nu \]

\[ v_{\text{max}} = 6.22 \times 10^5 \text{ m/s} \]

600 nm
1.77 eV
550 nm
2.25 eV
400 nm
3.1 eV

no electrons

Potassium - 2.0 eV needed to eject electron

Photoelectric effect

Energy

\[ E = \varepsilon_p \]
Each particle has an energy \( E \) given by the equation:

\[
E = \varepsilon_p
\]

The energy of a photon, \( E_{\text{photon}} \), is related to its frequency by the equation:

\[
E_{\text{photon}} = h\nu
\]

where \( h \) is Planck's constant and \( \nu \) is the frequency of the photon.

For the photoelectric effect, the probability distribution function is given by:

\[
G(E) = \frac{1}{E - \varepsilon_p}
\]

This function describes the probability of emission at a given energy \( E \).
Each particle has an energy

\[ E = \varepsilon_p \]

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle=infinity
Each particle has an energy

\[ E_{\text{photon}} = h\nu \]

**Photoelectric effect**

Energy

\[ E = \varepsilon_p \]

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

Particle = infinity
Each particle has an energy

\[ E = \varepsilon_p \]

particle count is deducible from Green function

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle=\infty
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

Diagram showing the behavior of the Green function $G(E)$ as a function of $E$. The equation $G(E) = \frac{1}{E - \varepsilon_p}$ is also shown.
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

- Particle density = number of sign changes of \( G \)
- \( E > \varepsilon_p \)
- \( E < \varepsilon_p \)
Counting sign changes?
Counting sign changes?

$\Theta(x)$
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 
\end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 
\end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 \\
1/2 & x = 0 
\end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 \\
1/2 & x = 0 
\end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 \\
1/2 & x = 0 
\end{cases} \]

counts sign changes

\[ n = 2 \sum_{k} \Theta(\mathcal{R}G(k, \omega = 0)) \]

Luttinger Theorem for electrons
How do functions change sign?
How do functions change sign?

\[ E > \epsilon_p \]

\[ E < \epsilon_p \]

divergence
How do functions change sign?

$E > \varepsilon_p$

$E < \varepsilon_p$

pole

divergence
Is there another way?
Yes
zero-crossing
zero-crossing

no divergence is necessary
closer look at Luttinger’s theorem

\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]
closer look at Luttinger’s theorem

\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]

divergences (poles) + zeros
closer look at Luttinger’s theorem

\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]

divergences (poles) + zeros

how can zeros affect the particle count?
how can

\[ G(E) = \frac{1}{E - \varepsilon_p} = 0? \]
how can

\[ G(E) = \frac{1}{E - \varepsilon_p} = 0? \]
how can

\[ G(E) = \frac{1}{E - \varepsilon_p} = 0? \]

\[ G(E) = \frac{Z_p}{\omega - \varepsilon_p} \]

\[ Z_p \rightarrow 0 \]
how can

\[ G(E) = \frac{1}{E - \varepsilon_p} = 0? \]

\[ G(E) = \frac{Z_p}{\omega - \varepsilon_p} \]

\[ Z_p \to 0 \]

\[ \frac{0}{\varepsilon} = 0 \]
how can

\[ G(E) = \frac{1}{E - \varepsilon_p} = 0? \]

\[ \frac{Z_p}{E - \varepsilon_p - \sum_p} \]

\[ \sum \rightarrow \infty \]

\[ G(E) = \frac{Z_p}{\omega - \varepsilon_p} \]

\[ Z_p \rightarrow 0 \]

\[ \frac{0}{\varepsilon} = 0 \]
how can

\[ G(E) = \frac{1}{E - \varepsilon_p} = 0? \]

\[ \frac{Z_p}{E - \varepsilon_p - \Sigma_p} \]

\[ \Sigma \to \infty \]

\[ \frac{1}{\infty} = 0 \]

\[ G(E) = \frac{Z_p}{\omega - \varepsilon_p} \]

\[ Z_p \to 0 \]

\[ \frac{0}{\varepsilon} = 0 \]
how can

\[ G(E) = \frac{1}{E - \varepsilon_p} = 0? \]

\[ \frac{Z_p}{E - \varepsilon_p - \Sigma_p} \]

\[ \Sigma \to \infty \]

\[ \frac{1}{\infty} = 0 \]

\[ \frac{Z_p}{\omega - \varepsilon_p} \]

\[ Z_p \to 0 \]

\[ \frac{0}{\varepsilon} = 0 \]

this debate has an answer
Right Wingers: Fermi Arcs are not Strange

intensity too small to be seen

\[ G(E) = \frac{Z_p}{\omega - \varepsilon_p} \]

pole exists but

\[ Z_p \to 0 \]
Fermi Arcs are Strange

\begin{align*}
(0, \pi) & \quad (\pi, \pi) \\
(0, 0) & \quad (\pi, 0) \\
\end{align*}

- Real FS Crossing
- Ghost FS Crossing
- Top of Band
- $F_{\text{sym}}$

$k_y$ $k_x$
Fermi Arcs are Strange

\begin{itemize}
  \item \textbf{seen}
  \item \textbf{not seen}
  \item \textbf{no pole}
\end{itemize}

$k_y$

$E_F$

$k_x$

$\pi, \pi$

$(0, \pi)$

$(0, 0)$

$(\pi, 0)$

$k_{\text{sym}}$

Real FS Crossing

Ghost FS Crossing

Top of Band
Problem for left-wingers (me)

\[ \Re G^r > 0 \]

\[ \Re G^r < 0 \]
Problem for left-wingers (me)
Problem for left-wingers (me)

How to account for the sign change without poles?
Problem for left-wingers (me)

How to account for the sign change without poles?

Only option: \( \text{Det} G = 0! \) (zeros)
Problem for left-wingers (me)

How to account for the sign change without poles?

Only option: $\text{Det}G=0$! (zeros)
Fermi Arcs

zeros + poles

Luttinger, Dzyaloshinskii, Yang, Rice, Zhang, Tsvelik, Anderson (lots of smart people)...

n=zeros + poles
what are zeros?

are they (like poles) conserved?
NiO insulates $d^8$?

Sir Neville

Mott mechanism
NiO insulates $d^8$?
Perhaps this costs energy.

Mott mechanism

Sir Neville
NiO insulates $d^8$?
perhaps this costs energy

Mott mechanism

$U \gg t$

Sir Neville
NiO insulates $d^8$?

perhaps this costs energy

Sir Neville

Mott mechanism

$U \gg t$

$\mu = 0$
NiO insulates $d^8$? perhaps this costs energy

Mott mechanism

$U \gg t$

no change in size of Brillouin zone

Sir Neville

$\mu = 0$
Mott Problem
Mott Problem

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} f(\omega) d\omega \]

\[ \mu = 0 \]

\[ \text{Im} G = 0 \]

Kramers-Kronig
\[
\text{Mott Problem}
\]

\[
\text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \begin{array}{c}
\mu = 0 \\
\omega
\end{array} \right) d\omega
\]

\[
= \text{below gap} + \text{above gap}
\]
Mott Problem

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \begin{array}{c} \mu = 0 \\ \omega \end{array} \right) d\omega \]

= below gap + above gap = 0
Mott Problem

\[ \text{Re}G(0, p) = \int_{-\infty}^{\infty} \left( \frac{1}{\omega - \mu} \right) d\omega \]

\[ = \text{below gap} + \text{above gap} = 0 \]

\[ \text{Im} G = 0 \]

Kramers-Kronig

\[ \text{Det}G(k, \omega = 0) = 0 \quad \text{(single band)} \]
Mott Problem

\[ \text{Im } G = 0 \]

\[ \text{Det } G(k, \omega = 0) = 0 \] (single band)

\[ \text{DetReG}(0, p) = 0 \] Mottnes
YBa$_2$Cu$_3$O$_7$

Cuprate Superconductors
$U/t = 10 \gg 1$

interactions dominate:
Strong Coupling Physics

Y Ba$_2$Cu$_3$O$_7$
Cuprate Superconductors
The proof just presented is good for any state of our system: FL, NFL, or MI [in other words, either of poles or zeros can be used to change the sign of $G_\gamma$ in Eq. (1)]. The only way to incapacitate the Luttinger theorem in form (1) is to assume that the limit $T\to 0$ is discontinuous. Actually, one has to require that the whole line $T=0$ is a line of phase transitions.
Is this famous theorem from 1960 correct?
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails

no hopping => no propagation (zeros)
A model with zeros but Luttinger fails

N flavors of e-

no hopping => no propagation (zeros)
$e^-$

generalization

$N$ flavors of spin
$e^-$

Generalization

\[ N = 5 \]

$N$ flavors of spin
Spin generalization

$N$ flavors of spin

$N = 5$
$e^-$

Spin generalization

$N$ flavors of spin

$2E$

$N = 5$

$25$

$16$

$9$

$4$

$1$

Wednesday, January 15, 14
e$^-$

N flavors of spin

$H = \frac{U}{2}(n_1 + \cdots n_N)^2$

$N = 5$

2E

- 25
- 16
- 9
- 4
- 1
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ \{0, 1, 1/2\} \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \quad 0, 1, 1/2 \]

\[ N = 3 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \quad 0, 1, 1/2 \]
\[ N = 3 \]
\[ 2 = 3 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ \begin{align*}
N &= 3 \\
n &= 2
\end{align*} \]

\[ 0, 1, 1/2 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\( n = 2 \)
\( N = 3 \)

even

\( 0, 1, 1/2 \)
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \]

\[ N = 3 \]

0, 1, 1/2

even

odd

Wednesday, January 15, 14
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

- \( n = 2 \)
- \( N = 3 \)

\( 0, 1, 1/2 \)

- even
- odd

no solution
Problem

G=0
Problem

$G = 0$

$$ G = \frac{1}{E - \varepsilon_p - \Sigma} $$

$\infty$
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

lifetime of a particle vanishes
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

\[ \exists \Sigma < \varepsilon_p \]

lifetime of a particle vanishes

\[ \infty \]
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

\[ \exists \Sigma < \varepsilon_p \]

Lifetime of a particle vanishes

\[ \infty \]

no particle
what went wrong?
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)

integral does not exist
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)

integral does not exist

No Luttinger theorem!
Luttinger’s theorem
experimental confirmation of violation?
zeros do not affect the particle density

`Luttinger' count

experimental data (LSCO)

$k_F$

$1 - x_{FS}$

$Z_p \to 0$

Bi2212

-Yang et al. (2011)
+He et al. (2011)
zeros do not affect the particle density

`Luttinger' count

experimental data (LSCO)

$k_F$

$1 - x_{FS}$

$Z_p \to 0$

Bi2212

zeros do not affect the particle density

each hole $\neq$ a single k-state
Two opposing Views on Fermi arcs

strange

not strange

this dispute has an answer!
Two opposing Views on Fermi arcs

strange

this dispute has an answer!
how to count particles?
how to count particles?

some charged stuff has no particle interpretation
what is the extra stuff?
Properties all particles share
Properties all particles share

- charge
- fixed mass!
Properties all particles share

charge

fixed mass!

extra stuff cannot have fixed mass!
extra stuff has no definite mass
extra stuff has no definite mass

scale invariant
what is scale invariance?

invariance on all length scales
\[ f(x) = x^2 \]
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \]  

scale change
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \]  \hspace{1cm} \text{scale change}

\hspace{1cm} \text{scale invariance}
\( f(x) = x^2 \)

\( f(x/\lambda) = (x/\lambda)^2 \)  

scale change

scale invariance

\( f(x) = x^2 \lambda^{-2} g(\lambda) \)

1
$f(x) = x^2$

$f(x/\lambda) = (x/\lambda)^2$

scale invariance

$g(\lambda) = \lambda^2$

$g(\lambda) = \lambda^2$
\[ f(x) = ax^2 + bx^3 \]

\[ x \rightarrow x/\Lambda \]
\[ f(x) = ax^2 + bx^3 \]

\[ x \rightarrow x/\Lambda \]

\[ f(x) \rightarrow \Lambda^{-2}(ax^2 + bx/\Lambda) \]
\[ f(x) = ax^2 + bx^3 \]

\[ x \rightarrow x/\Lambda \]

\[ f(x) \rightarrow \Lambda^{-2}(ax^2 + bx/\Lambda) \]

not scale invariant
free field theory

$$\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi$$

$$x \rightarrow x/\Lambda$$

$$\phi(x) \rightarrow \phi(x)$$

$$\mathcal{L} \rightarrow \Lambda^2 \mathcal{L}$$
free field theory

$$\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi$$

$x \to x/\Lambda$

$$\phi(x) \to \phi(x)$$

$$\mathcal{L} \to \Lambda^2 \mathcal{L} \text{ scale invariant}$$
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial_{\mu} \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]
\[ \phi(x) \rightarrow \phi(x) \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

\[ x \to x/\Lambda \]

\[ \phi(x) \to \phi(x) \]

mass
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ \Lambda^2 \left( \frac{1}{2} \partial_\mu \phi \partial_\mu \phi \right) \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

mass

\[ m^2 \phi^2 \]

no scale invariance

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]
how should we think about field theory?
how should we think about field theory?
how should we think about field theory?
what if \( m \) depends on the scale?
what if m depends on the scale?
what if $m$ depends on the scale?

$L_{m_i+7} \quad L_{m_i+6} \quad L_{m_i+5} \quad L_{m_i+4} \quad L_{m_i+3} \quad L_{m_i+2} \quad L_{m_i+1} \quad L_{m_i}$

QFT
\[ \mathcal{L} = \sum_i \mathcal{L}(m_i) \]
\[ \mathcal{L} = \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]
\[ \mathcal{L} = \int_0^\infty (\partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m)) \, dm^2 \]

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]

\[ x \rightarrow x / \Lambda \]
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial^{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^{2} \phi^{2}(x, m) \right) \, dm^{2} \]

\[ \phi \rightarrow \phi(x, m^{2}/\Lambda^{2}) \]

\[ x \rightarrow x/\Lambda \]

theory with all possible mass!
\[ \mathcal{L} = \int_0^\infty (\partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m)) \, dm^2 \]

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]

\[ x \rightarrow x / \Lambda \]

theory with all possible mass!

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

\[ \phi \rightarrow \phi(x, m^2/\Lambda^2) \]

\[ x \rightarrow x/\Lambda \]

theory with all possible mass!

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

\[ \phi \to \phi(x, m^2/\Lambda^2) \]

\[ x \to x/\Lambda \]

theory with all possible mass!

\[ \mathcal{L} \to \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
scale-invariant stuff is weird
scale-invariant stuff is weird

particles

\[ E_{\text{photon}} = h\nu \]

**Photoelectric effect**

energy \[ E = \varepsilon_p \]

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle=\infty
scale-invariant stuff is weird

particles

\[ E = \varepsilon_p \]

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

unparticles

\[ G(E) \propto (E - \varepsilon_p)^{d_U} \]

\[ d_U > 0 \]

unparticles=zero propagator
scale-invariant stuff is weird

particles

\[ E = \varepsilon_p \]

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle=\infty

unparticles

\[ G(E) \propto (E - \varepsilon_p)^{d_U} \]

\[ d_U > 0 \]

unparticles=zero

propagator

Fermi arcs

Photoelectric effect

\[ E_{\text{photon}} = h\nu \]

700 nm 1.77 eV

550 nm 2.25 eV

v_{\text{max}} = 2.96 \times 10^5 \text{ m/s}

400 nm 3.1 eV

v_{\text{max}} = 6.22 \times 10^5 \text{ m/s}

no electrons

Potassium - 2.0 eV needed to eject electron
what really is the summation over mass?
what really is the summation over mass?

mass=energy
high energy (UV)

low energy (IR)

related to sum over mass

QFT
related to sum over mass
gauge-gravity duality
(Maldacena, 1997)

related to sum over mass
holography

quantum ↔ gravity
unparticles
unparticles

classical gravity in a d+1 curved spacetime
unparticles

\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

classical gravity in a d+1 curved spacetime
unparticles

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

classical gravity in a d+1 curved spacetime

\[ AdS_{5+2\delta} \]
High $T_c$
unparticles
variable mass
UPt$_3$
emergent gravity

High $T_c$
unparticles
variable mass
$UPt_3$
High $T_c$
unparticles
variable mass
$UPt_3$

string theory

emergent
gravity
string theory