Holography and Mottness: a Discrete Marriage

Thanks to: NSF, EFRC (DOE)

M. Edalati  Ka Wai Lo  R. G. Leigh
Mott Problem
emergent gravity
Mott Problem
What interacting problems can we solve in quantum mechanics?
### PHYSICS 501, FALL 2012

#### GRADUATE QUANTUM MECHANICS I

#### SYLLABUS

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Chapter numbers refer to text of Shankar.

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*Please send any comments on this page to dhy@physics.rutgers.edu.*
## Rutgers University Department of Physics and Astronomy

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Chapter numbers refer to text of Shankar.

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This is an **optimistic** syllabus. Material that may be omitted for lack of time is in blue.

I  Relativistic Wave Equations
   A  The problem: Lorentz covariance
   B  The Klein-Gordon equation
   C  Dirac’s equation
      1  Derivation, Dirac matrices (chiral and Bjorken and Drell)
      2  Massless case: Weyl’s equation
      3  Operators, angular momentum and spin
      4  Hamiltonian form and equations of motion
      5  Bispinor notation
   D  Non-relativistic reduction of the Dirac equation and the $g$ factor of the electron.
   E  Lorentz covariance of Dirac’s equation
      1  Meaning, example of KG equation
      2  Solution and interpretation
      3  Bilinear covariants.
      4  Generalizations
   F  Single particle solutions of the Dirac equation
      1  Counting states, projection operators
      2  Momentum eigenstates and their high energy limit.
   G  More applications of the Dirac equation if time permits.

II  Approximation Methods
   A  Stationary state perturbation theory
A. Perturbations of the harmonic oscillator
B. Fine structure of hydrogenic atoms
C. Weak and strong field Zeeman effect in hydrogen

B Variational methods
1 Basic theorem
2 Practical considerations
3 Extensions to excited states, interlacing theorem, partial diagonalization.
4 Examples: Ground state of helium, bound states in open geometries

C Semiclassical methods
1 The WKB approximation, the classical limit and the Hamilton-Jacobi equation.
2 The need for connection formulae and their derivation
3 Interpretation of the connection formulae.
4 Applications: Bohr-Sommerfield quantization, quantum tunneling, etc.
5 Reflection above the barrier.

D Adiabatic approximation
1 Discussion and “derivation”
2 Transition rate in the adiabatic limit — relation to WKB
3 Born-Oppenheimer approximation
4 Berry’s phase, theory, applications, relation to Hannay’s angle.

III Many particle systems
A Indistinguishability and statistics
B Exchange symmetry, spin and statistics
C Two particle systems, exchange interaction, effective spin dependence, the classical limit and the significance of statistics.
D Permutation symmetry
1 Permutations
2 The totally symmetric and totally antisymmetric representations
3 Slater determinant
9 Obtaining wave functions from Young tableaux

E Second quantization
   1 Classical field theory and quantum wave mechanics
   2 Quantizing a classical field
   3 Fock space, creation and annihilation operators
   4 Statistics
   5 One particle and two particle operators
   6 The propagator in 2nd quantization
   7 Application to the degenerate Fermi gas
      A. Density of states of a Fermi gas
      B. Surface energy of nuclei

8 Thomas Fermi approximation for high Z atoms

9 Thomas Fermi for a conducting surface

IV Scattering theory

A Elementary kinematics of scattering theory: Cross section, scattering amplitude, unitarity and the optical theorem

B Partial waves, spherical Bessel’s functions, phase shifts, partial wave unitarity

C Calculating phase shifts: scattering length and effective range.

D Formal scattering theory:
   1 Scattering states and the Lippmann Schwinger equation
   2 Potential scattering
   3 $T$-matrix, $S$-matrix and the Born approximation
   4 Phase shifts reconsidered.

E Striking phenomena at low energies: bound states, resonances, virtual states

F Analytic properties of scattering amplitudes.

G Extended example: The separable potential: bound states, scattering resonances, convergence of the Born Approximation.

H Multichannel scattering and Fano-Feshbach Resonances.
# of solvable interacting QM problems

# of interacting QM problems
\[
\frac{\text{# of solvable interacting QM problems}}{\text{# of interacting QM problems}} = 0
\]
interacting systems \[ H = T + V \]
interacting systems \[ H = T + V \]
interacting systems $H = T + V$

easy

impossible
interacting systems

\[ H = T + V \]

\[ r_s = \frac{\langle V \rangle}{\langle T \rangle} \]

- easy
  \[ r_s < 1 \]

- impossible
interacting systems

\[ H = T + V \]

\[ r_s = \langle V \rangle / \langle T \rangle \]

- easy
  - \[ r_s < 1 \]
  - \textquote{free} systems (perturbative interactions)
  - metals, Fermi liquids, ...

- impossible
interacting systems \[ H = T + V \]

\[ r_s = \frac{\langle V \rangle}{\langle T \rangle} \]

- **easy**
  - \( r_s < 1 \)
  - "free" systems
  - (perturbative interactions)
  - metals, Fermi liquids, ...

- **impossible**
  - short-range repulsions are irrelevant
interacting systems

\[ H = T + V \]

\[ r_s = \langle V \rangle / \langle T \rangle \]

easy

\[ r_s < 1 \]

`free' systems
(perturbative interactions)

metals, Fermi liquids,...

short-range repulsions are irrelevant

impossible

\[ r_s \gg 1 \]

strongly coupled systems
(non-perturbative)

QCD, high-temperature superconductivity,...
why is strong coupling hard?
why is strong coupling hard?
why is strong coupling hard?

emergent low-energy physics (new degrees of freedom not in UV)
why is strong coupling hard?

emergent low-energy physics (new degrees of freedom not in UV)
why is strong coupling hard?

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why is strong coupling hard?

emergent low-energy physics (new degrees of freedom not in UV)
why is strong coupling hard?

UV

emergent low-energy physics (new degrees of freedom not in UV)

IR

Proton

Neutron

\( \pi^0 \)
What computational tools do we have for strongly correlated electron matter?
Is strong coupling hard because we are in the wrong dimension?
Yes

I LOVE THE SMELL OF D-BRANES IN THE MORNING
\[ \mathcal{L} = T - g\varphi^4 \cdots \]
\[ \mathcal{L} = T - g \varphi^4 \ldots \]
\[ \mathcal{L} = T - g\phi^4 \ldots \]

Wilsonian program (fermions: new degrees of freedom)
\[ \mathcal{L} = T - g\phi^4 \cdots \]

Wilsonian program (fermions: new degrees of freedom)
The Wilsonian program involves new degrees of freedom.

The Lagrangian is given by:

\[ \mathcal{L} = T - g \phi^4 \cdots \]

The coupling constant is defined as:

\[ g = 1/\text{ego} \]
\[ \mathcal{L} = T - g \varphi^4 \cdots \]

- **Wilsonian program** (fermions: new degrees of freedom)
- **IR** → **UV QFT**
- **Locality in energy**
- **Coupling constant**
  \[ g = 1/\text{ego} \]

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]
Implement E-scaling with an extra dimension.

$$\frac{dg(E)}{d\ln E} = \beta(g(E))$$

Locality in energy.

$$\mathcal{L} = T - g\varphi^4 \cdots$$

coupling constant

$$g = 1/\text{ego}$$

Wilsonian program (fermions: new degrees of freedom)
gauge-gravity duality (Maldacena, 1997)

Implement E-scaling with an extra dimension

\[ \mathcal{L} = T - g \varphi^4 \cdots \]

Coupling constant
\[ g = 1/\text{ego} \]

Locality in energy

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]
what’s the geometry?
what’s the geometry?

\[
\frac{d g(E)}{d \ln E} = \beta(g(E)) = 0
\]

scale invariance (continuous)
what's the geometry?

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) = 0 \]

scale invariance (continuous)

\[ E \rightarrow \lambda E \]
\[ x^\mu \rightarrow x^\mu / \lambda \]
what’s the geometry?

\[
\frac{dg(E)}{d\ln E} = \beta(g(E)) = 0
\]

scale invariance (continuous)

\[
E \rightarrow \lambda E
\]
\[
x^\mu \rightarrow x^\mu / \lambda
\]

solve Einstein equations
what’s the geometry?

\[
\frac{dg(E)}{d\ln E} = \beta(g(E)) = 0
\]

scale invariance (continuous)

\[E \rightarrow \lambda E\]

\[x^\mu \rightarrow x^\mu / \lambda\]

solve Einstein equations

\[ds^2 = \left(\frac{u}{L}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{L}{E}\right)^2 dE^2\]

anti de-Sitter space
Holography

\[ ds^2 = -dt^2 + d\vec{x}^2 \]
Holography
Holography

\[ ds^2 = \frac{L^2}{r^2} \left( -dt^2 + d\bar{x}^2 + dr^2 \right) \]

\[ r = \frac{L^2}{E} \]
Holography

\[ ds^2 = \frac{L^2}{r^2} \left( -dt^2 + d\vec{x}^2 + dr^2 \right) \]

\[ r \to 0 \]

\( \text{AdS}_{d+1} \)

\[ r = \frac{L^2}{E} \]

\( \beta(g) \) is local

geometrize RG flow

\{ t, \vec{x}, r \} \to \{ \lambda t, \lambda \vec{x}, \lambda r \}
Holography

AdS

$ds^2 = \frac{L^2}{r^2} (-dt^2 + d\vec{x}^2 + dr^2)$

$r \rightarrow 0$

$\beta(g)$ is local geometrize RG flow

$\Lambda_1 \Lambda_2 \Lambda_3 \Lambda_4 \Lambda_5$

$UV$

$IR$

weakly-coupled classical gravity in d+1

$\{t, \vec{x}, r\} \rightarrow \{\lambda t, \lambda \vec{x}, \lambda r\}$

symmetry:

QFT in d-dimensions

weakly-coupled classical gravity
dual construction
dual construction
dual construction

fields $\phi$

operators $O$

$UV$ $QFT$
dual construction

fields \rightarrow \text{duality} \rightarrow \text{operators}

\phi

UV

QFT
dual construction

\[ e^{\int d^d x \phi \circ \mathcal{O}} \]

fields \[ \phi \]
duality
operators \[ \mathcal{O} \]

UV

QFT
Claim: $Z_{QFT} = e^{-S_{\text{on-shell}}^{\text{on-shell}}(\phi(\phi \partial_{\text{ADS}} = J_{\phi}))}$
How does it work?

some fermionic system
How does it work?

fields

UV

QFT

some fermionic system

duality

ψ
How does it work?

- Dirac equation
- Fields
- Duality
- Some fermionic system
How does it work?

Dirac equation fields

duality

some fermionic system
Dirac equation fields

some fermionic system

How does it work?

\[ \psi \approx ar^m + br^{-m} \]

source response

UV QFT

Dirac equation

duality
How does it work?

Dirac equation

Green function: \[ G_\mathcal{O} = \frac{b}{a} = f(UV,IR) \]

Source

Response

Some fermionic system

Duality

Fields

\[ \psi \approx ar^m + br^{-m} \]
Why is gravity holographic?
Why is gravity holographic?
Why is gravity holographic?
Why is gravity holographic?
Why is gravity holographic?

Entropy scales with the area not the volume: gravity is naturally holographic.
\[ Z_{QFT} = e^{-S_{\text{on-shell}}^{\text{on-shell}}(\phi(\phi\partial_{ADS}=J_\mathcal{O}))} \]
$Z_{QFT} = e^{-S_{\text{on-shell}}^{\text{on-shell}}} (\phi (\phi_{\partial ADS} = J_{\phi}))$
What holography does for you?
What holography does for you?

Landau-Wilson

Hamiltonian

\[ \xi_t \propto \xi^z \]

long-wavelengths

RG equations
What holography does for you?

Landau-Wilson

Hamiltonian

$\xi_t \propto \xi^z$

long-wavelengths

RG equations
What holography does for you?

Landau-Wilson

Hamiltonian

\[ \xi_t \propto \xi^z \]

long-wavelengths

RG equations

holography

RG=GR
What holography does for you?

- Landau-Wilson Hamiltonian
- \( \xi_t \propto \xi^z \)
- long-wavelengths
- RG equations

holography

RG=GR

strong-coupling is easy
What holography does for you?

Landau-Wilson

Hamiltonian

\[ \xi_t \propto \xi^z \]

long-wavelengths

RG equations

holography

RG=GR

strong-coupling is easy

microscopic UV model not easy (need M-theory)
What holography does for you?

Landau-Wilson

Hamiltonian

\[ \xi_t \propto \xi^z \]

long-wavelengths

RG equations

holography

RG=GR

strong-coupling is easy

microscopic UV model not easy (need M-theory)

so what (currents, symmetries)
Can holography solve the Mott problem?
What is a Mott Insulator?

NiO insulates $d^8$?
What is a Mott Insulator?

NiO insulates $d^8$?

EMPTY STATES = METAL
What is a Mott Insulator?

NiO insulates $d^8$?

EMPTY STATES = METAL

band theory fails!
What is a Mott Insulator?

NiO insulates $d^8$?

perhaps this costs energy

EMPTY STATES = METAL

band theory fails!
Mott Problem: NiO (Band theory failure)

(N rooms N occupants)

\[ U \gg t \]
Half-filled band

Free electrons
Half-filled band

Free electrons

$U \gg t$

charge gap

gap with no symmetry breaking!!
Mott Insulator-Ordering
Mott Insulator-Ordering = Mottness
Mott Insulator-Ordering = Mottness

its all about order. No Mottness

Slater
Mottness

Mott Insulator-Ordering = Mottness

its all about order. No Mottness

Slater

of course it does!! order is secondary

Anderson
Mott Insulator-Ordering = Mottness

its all about order. No Mottness

of course it does!! order is secondary
Mott Insulator-Ordering = Mottness

its all about order.
No Mottness

of course it does!!
order is secondary

Slater

Anderson
Why is the Mott problem important?
Why is the Mott problem important?

$\text{Y Ba}_2\text{Cu}_3\text{O}_7$

Cuprate Superconductors
Why is the Mott problem important?

\[ \frac{U}{t} = 10 \gg 1 \]

interactions dominate: Strong Coupling Physics

Y Ba Cu \(_2\) O \(_{3+7}\) Cuprate Superconductors
Experimental facts: Mottness
Experimental facts: Mottness

Experimental facts: Mottness

Transfer of spectral weight to high energies beyond any ordering scale

Recall, $eV = 10^4 K$

Experimental facts: Mottness

\[ \Delta = 0.6 \text{eV} > \Delta_{\text{dimerization}} \]  

(Mott, 1976) \[ \frac{\Delta}{T_{\text{crit}}} \approx 20 \]

\[ T_{\text{crit}} \approx 20 \]

\[ T = 360 \text{K} \quad T = 295 \text{K} \]

Transfer of spectral weight to high energies beyond any ordering scale

Recall, \( eV = 10^4 K \)

Experimental facts: Mottness

$$\Delta = 0.6 eV > \Delta_{\text{dimerization}}$$  

(Mott, 1976) \[ \frac{\Delta}{T_{\text{crit}}} \approx 20 \]

Recall, \( eV = 10^4 K \)

Transfer of spectral weight to high energies beyond any ordering scale
What gravitational theory gives rise to a gap in $\text{Im}G$ without spontaneous symmetry breaking?
What gravitational theory gives rise to a gap in ImG without spontaneous symmetry breaking?

dynamically generated gap: Mott gap (for probe fermions)
What has been done?

$$\sqrt{-g}i\bar{\psi}(D - m)\psi$$

AdS-RN
MIT, Leiden group
What has been done?

\[ \sqrt{-g} \bar{\psi} (D - m) \psi \]

AdS-RN
MIT, Leiden group
\[ \sqrt{-gi\bar{\psi}(D - m)\psi} \]

AdS-RN
MIT, Leiden group

What has been done?

\[ G(\omega, k) = \frac{Z}{v_F(k - k_F) - \omega - \Sigma} \]

Fermi peak
\[ \sqrt{-g} i \bar{\psi} (D - m) \psi \]

What has been done?

\[ G(\omega, k) = \frac{Z}{v_F (k - k_F) - \omega - \Sigma} \]

Fermi peak

MIT, Leiden group

AdS-RN

marginal Fermi liquid
\[
\sqrt{\cdot} - \overline{\psi}(D - m) \psi
\]

AdS-RN

MIT, Leiden group

What has been done?

Fermi peak

\[
G(\omega, \mathbf{k}) = Z_v F(k - k_F) - \omega - \Sigma_m
\]

canonical Fermi liquid
decoherence \rightarrow \text{Mott Insulator}
\[ S_{\text{probe}}(\psi, \bar{\psi}) = \int d^d x \sqrt{-g} \bar{\psi}(\Gamma^M D_M - m + \cdots) \psi \]
\[ S_{\text{probe}}(\psi, \bar{\psi}) = \int d^d x \sqrt{-g} i \bar{\psi} (\Gamma^M D_M - m + \cdots) \psi \]
$S_{\text{probe}}(\psi, \bar{\psi}) = \int d^d x \sqrt{-g} i \bar{\psi}(\Gamma^M D_M - m + \cdots) \psi$

what is hidden here?

consider $\sqrt{-g} i \bar{\psi}(\not{D} - m - i p F) \psi$
QED anomalous magnetic moment of an electron
(Schwinger 1949)

\[ S_{\text{probe}}(\psi, \bar{\psi}) = \int d^d x \sqrt{-g} i \bar{\psi} (\Gamma^M D_M - m + \cdots) \psi \]

\[ \text{consider} \quad \sqrt{-g} i \bar{\psi} (\mathcal{D} - m - ipF) \psi \]

\[ F_{\mu\nu} \Gamma^{\mu\nu} \]

what is hidden here?
How is the spectrum modified?

$P=0$
How is the spectrum modified?

P=0

Fermi surface peak
How is the spectrum modified?

$P=0$

$P > 4.2$

Fermi surface peak
How is the spectrum modified?

$P = 0$

$P > 4.2$

Fermi surface peak
How is the spectrum modified?

$P=0$

$P > 4.2$

Fermi surface peak

dynamically generated gap:
How is the spectrum modified?

$P=0$

$P > 4.2$

Fermi surface peak

spectral weight transfer

dynamically generated gap:
How is the spectrum modified?

$P = 0$

spectral weight transfer

$P > 4.2$

dynamically generated gap:

confirmed by Gubser, Gauntlett, 2011
Mechanism?

UV

QFT
Mechanism?
emergent spacetime symmetry

$AdS_2$

$UV$

$QFT$
emergent spacetime symmetry

Mechanism?
Mechanism?

emergent spacetime symmetry

operators $\mathcal{O}$

$\psi \rightarrow \mathcal{O}_\pm$

$AdS_2$
emergent spacetime symmetry

holography within holography

Mechanism?

operators $O$

$AdS_2$

$h_{\mp}$
Mechanism?

UV

QFT

IR

where is $k_F$?

AdS

$\mathbb{S}_2$

emergent spacetime symmetry

holography within holography

holography

emergent spacetime symmetry

operators

where is $k_F$?

Mechanism?
emergent spacetime symmetry

Mechanism?

operators $\mathcal{O}$

where is $k_F$?

holoography within holoography

$k_F$ moves into log-oscillatory region: IR $\mathcal{O}_\pm$ acquires a complex dimension

$AdS_2$
What does a complex scaling dimension mean?
continuous scale invariance

\[ \mathcal{O} = \mu(\lambda)\mathcal{O}(\lambda r) \]
Continuous scale invariance

\[ \mathcal{O} = \mu(\lambda)\mathcal{O}(\lambda r) \]

\[ 1 = \mu(\lambda)\lambda^\Delta \]

\[ \Delta = -\frac{\ln \mu}{\ln \lambda} \]
$O = \mu(\lambda)O(\lambda r)$

$1 = \mu(\lambda)\lambda^\Delta$

$\Delta = -\frac{\ln \mu}{\ln \lambda}$

$\Delta$ is real, independent of scale
Continuous scale invariance

\[ O = \mu(\lambda)O(\lambda r) \]

\[ 1 = \mu(\lambda)\lambda^\Delta \]

\[ \Delta = -\frac{\ln \mu}{\ln \lambda} \]

\[ \Delta \text{ is real, independent of scale} \]

What about complex \( \Delta \)?
1 = \mu \lambda^\Delta
\[ e^{2\pi in} = 1 = \mu \lambda^\Delta \]
Discrete scale invariance (DSI)

\[ e^{2\pi i n} = 1 = \mu \lambda^\Delta \]

\[ \Delta = -\frac{\ln \mu}{\ln \lambda} + \frac{2\pi i n}{\ln \lambda} \]

\[ n=0: CSI \]
Discrete scale invariance (DSI)

\[ e^{2\pi i n} = 1 = \mu \lambda^n \]

\[ \Delta = -\frac{\ln \mu}{\ln \lambda} + \frac{2\pi i n}{\ln \lambda} \]

scaling dimension depends on scale

\[ \lambda_n = \lambda^n \]
Discrete scale invariance (DSI)

\[ e^{2\pi \text{i} n} = 1 = \mu \lambda^\Delta \]

\[ \Delta = -\frac{\ln \mu}{\ln \lambda} + \frac{2\pi \text{i} n}{\ln \lambda} \]

scaling dimension depends on scale

\[ \lambda_n = \lambda^n \]

magnification

n=0: CSI
Discrete scale invariance (DSI)

\[ e^{2\pi i n} = 1 = \mu \lambda^\Delta \]

\[ \Delta = -\frac{\ln \mu}{\ln \lambda} + \frac{2\pi i n}{\ln \lambda} \]

scaling dimension depends on scale

\[ \lambda_n = \lambda^n \]

magnification

preferred scale

n=0: CSI
analogy

liquid

solid
analogy

liquid

continuous translational invariance

discrete translational invariance

solid
example
example
example
example

\[ n \text{ iterations} \]

\[ 3^{-n} \quad 2^n \]

length

number of segments
Example

$n$ iterations

length: $3^{-n}$

number of segments: $2^n$

Scale invariance only for $\lambda_p = 3^p$
example

\[ D = \frac{-\ln 2}{\ln 3} + \frac{2\pi i n}{\ln 3} \]

scale invariance only for \( \lambda_p = 3^p \)

length: \( 3^{-n} \)

number of segments: \( 2^n \)

n iterations
discrete scale invariance

hidden scale (length, energy,...)
\[ G = \frac{\beta_-(0, k)}{\alpha_-(0, k)} \]

no poles outside log-oscillatory region for \( p > \frac{1}{\sqrt{6}} \)
\[ G = \frac{\beta_{-}(0, k)}{\alpha_{-}(0, k)} \]

no poles outside log-oscillatory region for \( p > 1/\sqrt{6} \)

Mott physics and DSI are linked!
is there an instability?
is there an instability?
is there an instability?

\[ E \rightarrow E + i\gamma \quad \gamma > 0 \]
is there an instability?

\[ E \rightarrow E + i\gamma \quad \gamma > 0 \]

condensate (bosonic)
is there an instability?

\[ E \rightarrow E + i\gamma \quad \gamma > 0 \]

\[ \langle \phi \rangle \neq 0 \]

condensate (bosonic)

does this happen for fermions?
quasi-normal modes

\[ \text{Im}\, \omega < 0 \quad \text{no instability} \]
quasi-normal modes

$\text{Im} \omega < 0$  no instability
quasi-normal modes

but the residue drops to zero: opening of a gap

$\text{Im} \omega < 0$ \hspace{0.5cm} \text{no instability}
\[ G(\omega, k) = G(\omega \lambda^n, k) \]
$G(\omega, k) = G(\omega \lambda^n, k)$

discrete scale invariance in energy
$G(\omega, k) = G(\omega \lambda^n, k)$

discrete scale invariance in energy

emergent IR scale
\[ G(\omega, k) = G(\omega \lambda^n, k) \]

- discrete scale invariance in energy
- emergent IR scale
- no condensate
$G(\omega, k) = G(\omega \lambda^n, k)$

- discrete scale invariance in energy

- emergent IR scale

- no condensate

- energy gap: Mott gap (Mottness)
continuous scale invariance

\[
\text{discrete scale invariance in energy}
\]
continuous scale invariance

discrete scale invariance in energy

is this the symmetry that is ultimately broken in the Mott problem?
a.) yes

b.) no
a.) yes

b.) no
a.) yes ✓
b.) no
a.) yes
b.) no

if yes: holography has solved the Mott problem

holography

VO_2, cuprates,...
what else can holography do?
Finite Temperature Mott transition from Holography

\[ \frac{T}{\mu} = 5.15 \times 10^{-3} \]

\[ \frac{T}{\mu} = 3.92 \times 10^{-2} \]
Finite Temperature Mott transition from Holography

\[ \frac{\Delta}{T_{\text{crit}}} \approx 20 \] vanadium oxide

\[ T/\mu = 5.15 \times 10^{-3} \]

\[ T/\mu = 3.92 \times 10^{-2} \]
Finite Temperature Mott transition from Holography

\[ \frac{\Delta}{T_{\text{crit}}} \approx 10 \]

\[ \frac{\Delta}{T_{\text{crit}}} \approx 20 \text{ vanadium oxide} \]

\[ T/\mu = 5.15 \times 10^{-3} \]

\[ T/\mu = 3.92 \times 10^{-2} \]
looks just like the experiments
Mottness looks just like the experiments
This is an optimistic syllabus. Material that may be omitted for lack of time is in blue.

I Relativistic Wave Equations

A The problem: Lorentz covariance
B The Klein-Gordon equation
C Dirac’s equation
   1 Derivation, Dirac matrices (chiral and Bjorken and Drell)
   2 Massless case: Weyl’s equation
   3 Operators, angular momentum and spin
   4 Hamiltonian form and equations of motion
   5 Bispinor notation
D Non-relativistic reduction of the Dirac equation and the $g$ factor of the electron.
E Lorentz covariance of Dirac’s equation
   1 Meaning, example of KG equation
   2 Solution and interpretation
   3 Bilinear covariants.
   4 Generalizations
F Single particle solutions of the Dirac equation
   1 Counting states, projection operators
   2 Momentum eigenstates and their high energy limit.
G More applications of the Dirac equation if time permits.

II Approximation Methods

A Stationary state perturbation theory
toy model: merging of UV and IR fixed points

\[ \beta = (\alpha - \alpha_*) - (g - g_*)^2 \]

\[ g_\pm = g_* \pm \sqrt{\alpha - \alpha_*} \]
toy model: merging of UV and IR fixed points

\[ \beta = (\alpha - \alpha_\ast) - (g - g_\ast)^2 \]

\[ g_{\pm} = g_\ast \pm \sqrt{\alpha - \alpha_\ast} \]

\[ g_{\text{IR}} \equiv g_- \]

\[ g_{\text{UV}} \equiv g_+ \]

\[ \alpha > \alpha_\ast \]
toy model: merging of UV and IR fixed points

\[ \beta = (\alpha - \alpha_*) - (g - g_*)^2 \]

\[ g_\pm = g_* \pm \sqrt{\alpha - \alpha_*} \]
toy model: merging of UV and IR fixed points

\[ \beta = (\alpha - \alpha^*) - (g - g^*)^2 \]

\[ g_{\pm} = g^* \pm \sqrt{\alpha - \alpha^*} \]

\[ \beta \]

\[ g_{\text{IR}} \equiv g_- \quad \text{and} \quad g_{\text{UV}} \equiv g_+ \]

\[ \alpha > \alpha^* \]

\[ \alpha = \alpha^* \]

\[ \alpha < \alpha^* \]

\[ \Lambda_{\text{IR}} = \Lambda_{\text{UV}} e^{-\pi/\sqrt{\alpha^* - \alpha}} \]
toy model: merging of UV and IR fixed points

\[ \beta = (\alpha - \alpha_*) - (g - g_*)^2 \]

\[ g_\pm = g_* \pm \sqrt{\alpha - \alpha_*} \]

\[ \beta \]

\[ g_{\text{IR}} \equiv g_- \]
\[ g_{\text{UV}} \equiv g_+ \]

\[ g_\pm \text{ are complex} \]
\( \text{(conformality lost)} \)

\[ \Lambda_{\text{IR}} = \Lambda_{\text{UV}} e^{-\pi/\sqrt{\alpha_* - \alpha}} \]

BKT transition

Kaplan, arxiv:0905.475