Holography and Mottness: a Discrete Marriage

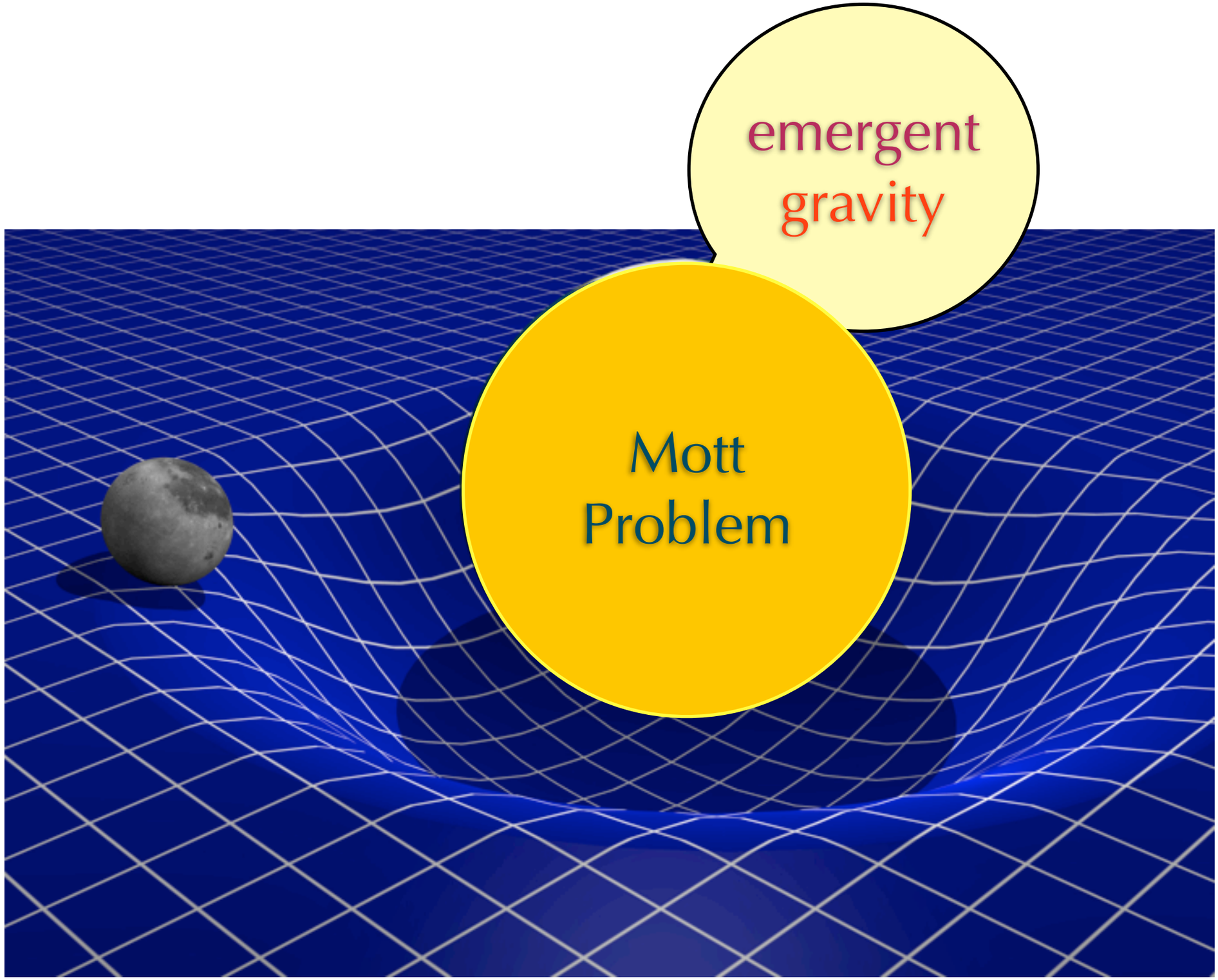
Thanks to: NSF, EFRC (DOE)

M. Edalati

Ka Wai Lo

R. G. Leigh

Mott Problem


emergent
gravity
Mott
Problem

What interacting problems
can we solve in quantum mechanics?

PHYSICS 501, FALL 2012

GRADUATE QUANTUM MECHANICS I

SYLLABUS

Chapters	Weeks	Topic
1	2.0	Linear vector spaces and bra-ket notation
2-3	1.0	Transition from classical to quantum mechanics
4	1.5	Postulates of quantum mechanics
5-7	2.0	One-dimensional problems; Harmonic oscillator
10	1.0	Higher dimensions; elementary treatment of identical particles
11	1.5	Symmetries, quantum numbers, and conserved quantities
12	2.0	Rotational invariance and angular momentum
13	1.0	The hydrogen atom
14-15	2.0	Spin and addition of angular momentum

Chapter numbers refer to text of Shankar.

Please send any comments on this page to dhv@physics.rutgers.edu.

Rutgers University Department of Physics and Astronomy

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MIT Subject 8.322 Quantum Theory II

R. L. Jaffe 2007

Syllabus

This is an **optimistic** syllabus. Material that may be omitted for lack of time is in blue.

I Relativistic Wave Equations

- A The problem: Lorentz covariance
- B The Klein-Gordon equation
- C Dirac's equation
  1. Derivation, Dirac matrices (chiral and Bjorken and Drell)
  2. Massless case: Weyl's equation
  3. Operators, angular momentum and spin
  4. Hamiltonian form and equations of motion
  5. Bispinor notation
- D Non-relativistic reduction of the Dirac equation and the g factor of the electron.
- E Lorentz covariance of Dirac's equation
  1. Meaning, example of KG equation
  2. Solution and interpretation
  3. Bilinear covariants.
  4. Generalizations
- F Single particle solutions of the Dirac equation
  1. Counting states, projection operators
  2. Momentum eigenstates and their high energy limit.
- G More applications of the Dirac equation if time permits.

II Approximation Methods

- A Stationary state perturbation theory

A. Perturbations of the harmonic oscillator
B. Fine structure of hydrogenic atoms
C. Weak and strong field Zeeman effect in hydrogen

B Variational methods

1 Basic theorem
2 Practical considerations
3 Extensions to excited states, interlacing theorem, partial diagonalization.
4 Examples: Ground state of helium, bound states in open geometies

C Semiclassical methods

1 The WKB approximation, the classical limit and the Hamilton-Jacobi equation.
2 The need for connection formulae and their derivation
3 Interpretation of the connection formulae.
4 Applications: Bohr-Sommerfield quantization, quantum tunneling, etc.
5 Reflection above the barrier.

D Adiabatic approximation

1 Discussion and "derivation"
2 Transition rate in the adiabatic limit — relation to WKB
3 Born-Oppenheimer approximation
4 Berry's phase, theory, applications, relation to Hannay's angle.

III Many particle systems

A Indistinguishability and statistics

B Exchange symmetry, spin and statistics

C Two particle systems, exchange interaction, effective spin dependence, the classical limit and the significance of statistics.

D Permutation symmetry

1 Permutations
2 The totally symmetric and totally antisymmetric representations

9 Obtaining wave functions from Young tableaux

E Second quantization

1 Classical field theory and quantum wave mechanics
2 Quantizing a classical field
3 Fock space, creation and annihilation operators
4 Statistics
5 One particle and two particle operators
6 The propagator in 2nd quantization
7 Application to the degenerate Fermi gas
A. Density of states of a Fermi gas
B. Surface energy of nuclei
8 Thomas Fermi approximation for high Z atoms
9 Thomas Fermi for a conducting surface

IV Scattering theory

A Elementary kinematics of scattering theory: Cross section, scattering amplitude, unitarity and the optical theorem

B Partial waves, spherical Bessel's functions, phase shifts, partial wave unitarity

C Calculating phase shifts: scattering length and effective range.

D Formal scattering theory:

1 Scattering states and the Lippmann Schwinger equation
2 Potential scattering
3 T-matrix, S-matrix and the Born approximation
4 Phase shifts reconsidered.

E Striking phenomena at low energies: bound states, resonances, virtual states

F Analytic properties of scattering amplitudes.

G Extended example: The separable potential: bound states, scattering resonances, convergence of the Born Approximation.

H Multichannel scattering and Fano-Feshbach Resonances.

V Time dependent perturbation theory, radiation

$$\frac{\#\text{of solvable interacting QM problems}}{\#\text{of interacting QM problems}}$$

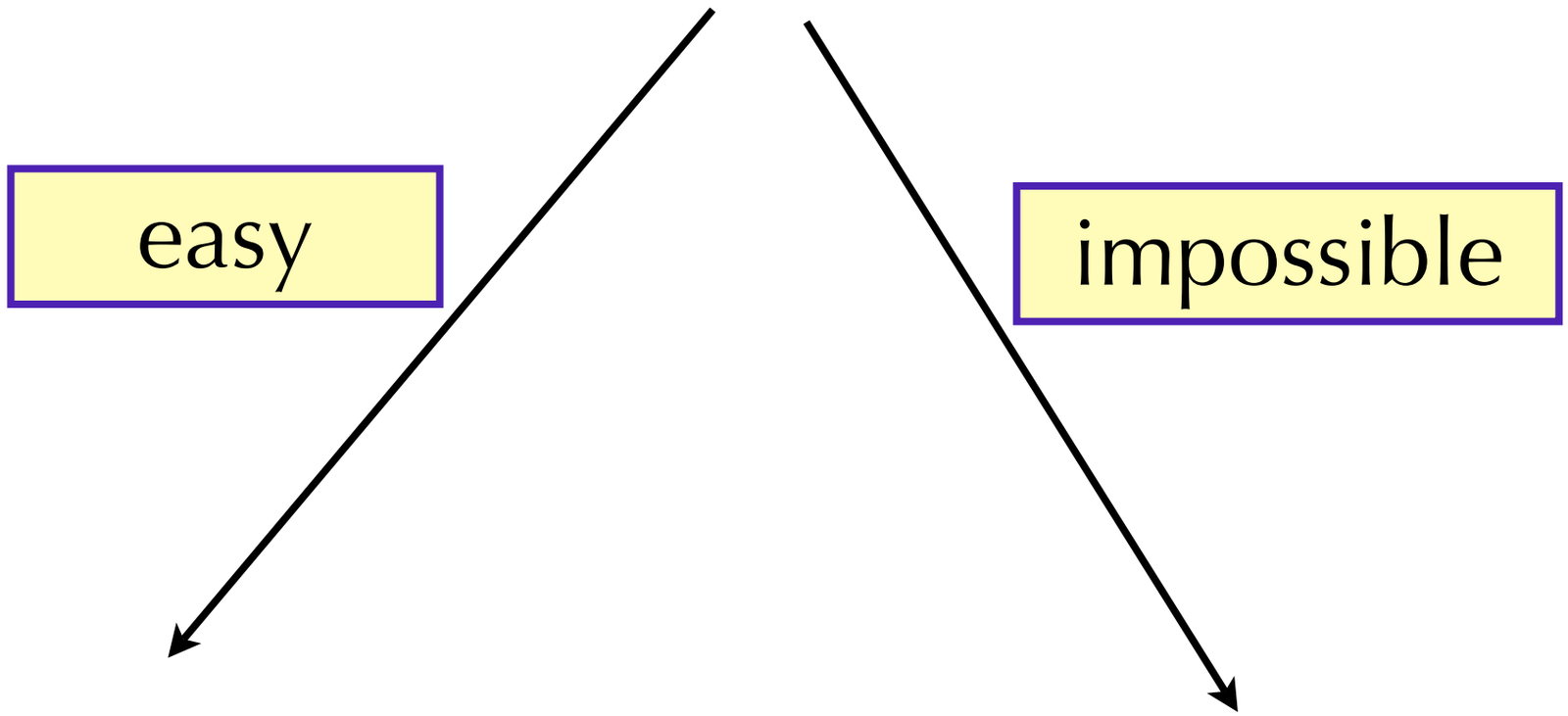
$$\frac{\#\text{of solvable interacting QM problems}}{\#\text{of interacting QM problems}} = 0$$

interacting systems $H = T + V$

interacting systems $H = T + V$

easy

interacting systems $H = T + V$



interacting systems $H = T + V$

$$r_s = \langle V \rangle / \langle T \rangle$$

$r_s < 1$

easy

impossible

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`free' systems
(perturbative interactions)

metals,Fermi liquids,...

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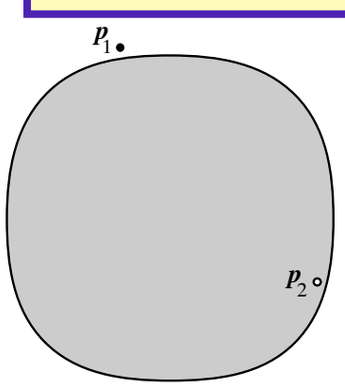

short-range
repulsions are irrelevant

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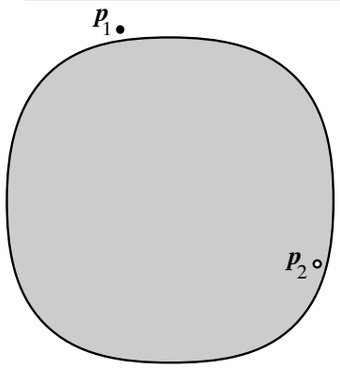
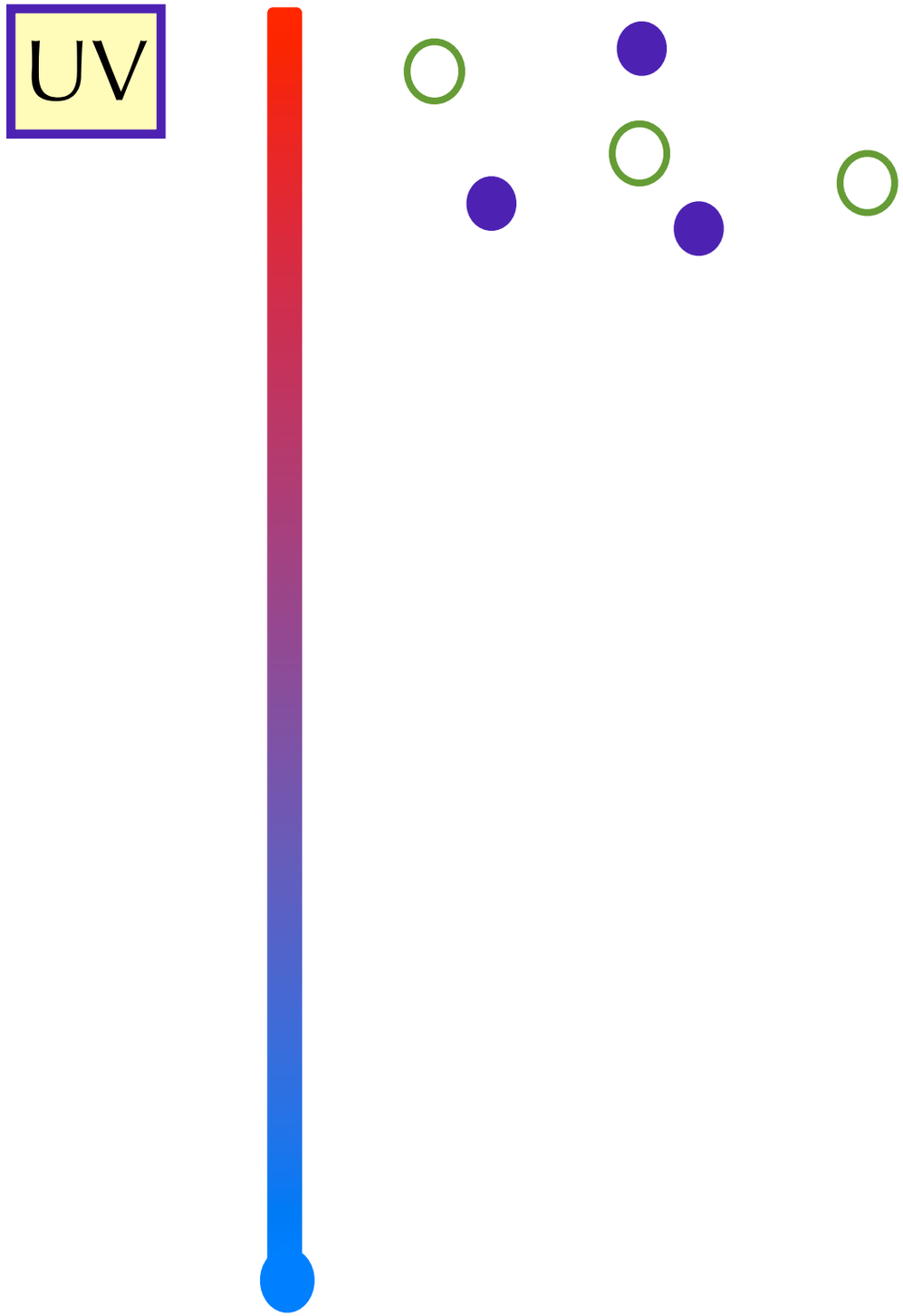


short-range
repulsions are irrelevant

strongly coupled systems
(non-perturbative)

QCD, high-temperature
superconductivity,...

why is strong coupling hard?



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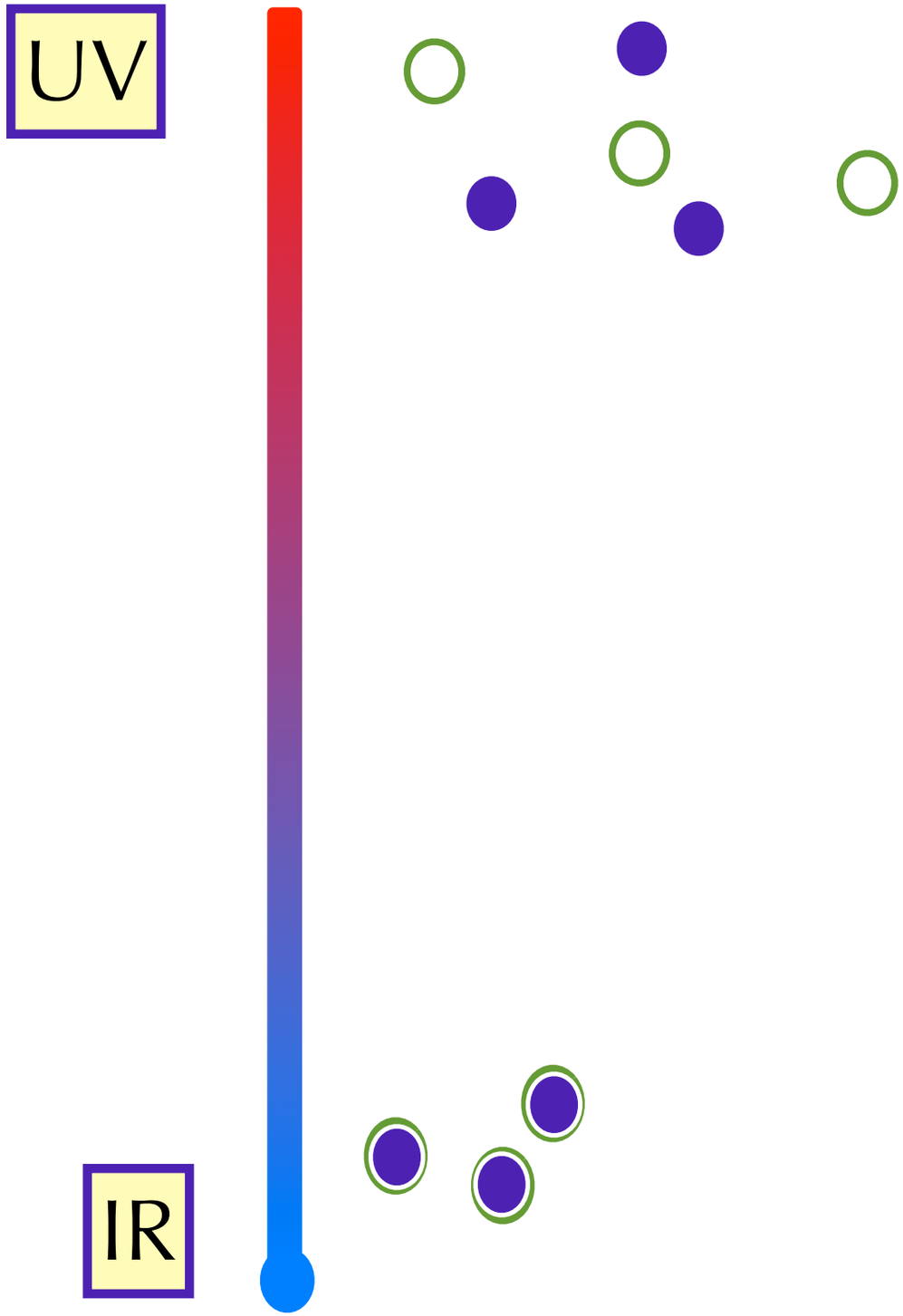

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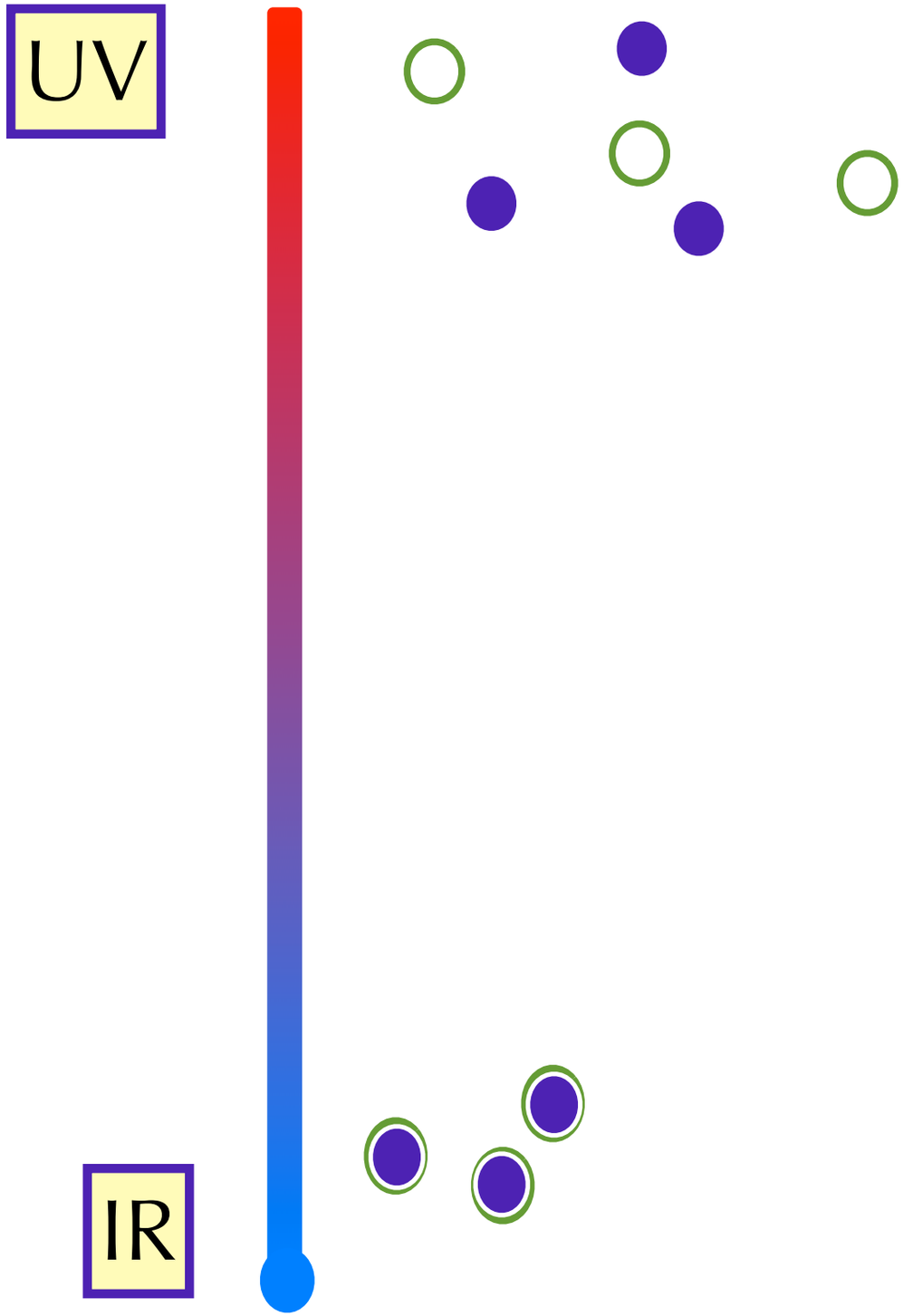

emergent
low-energy physics
(new degrees of
freedom not in UV)

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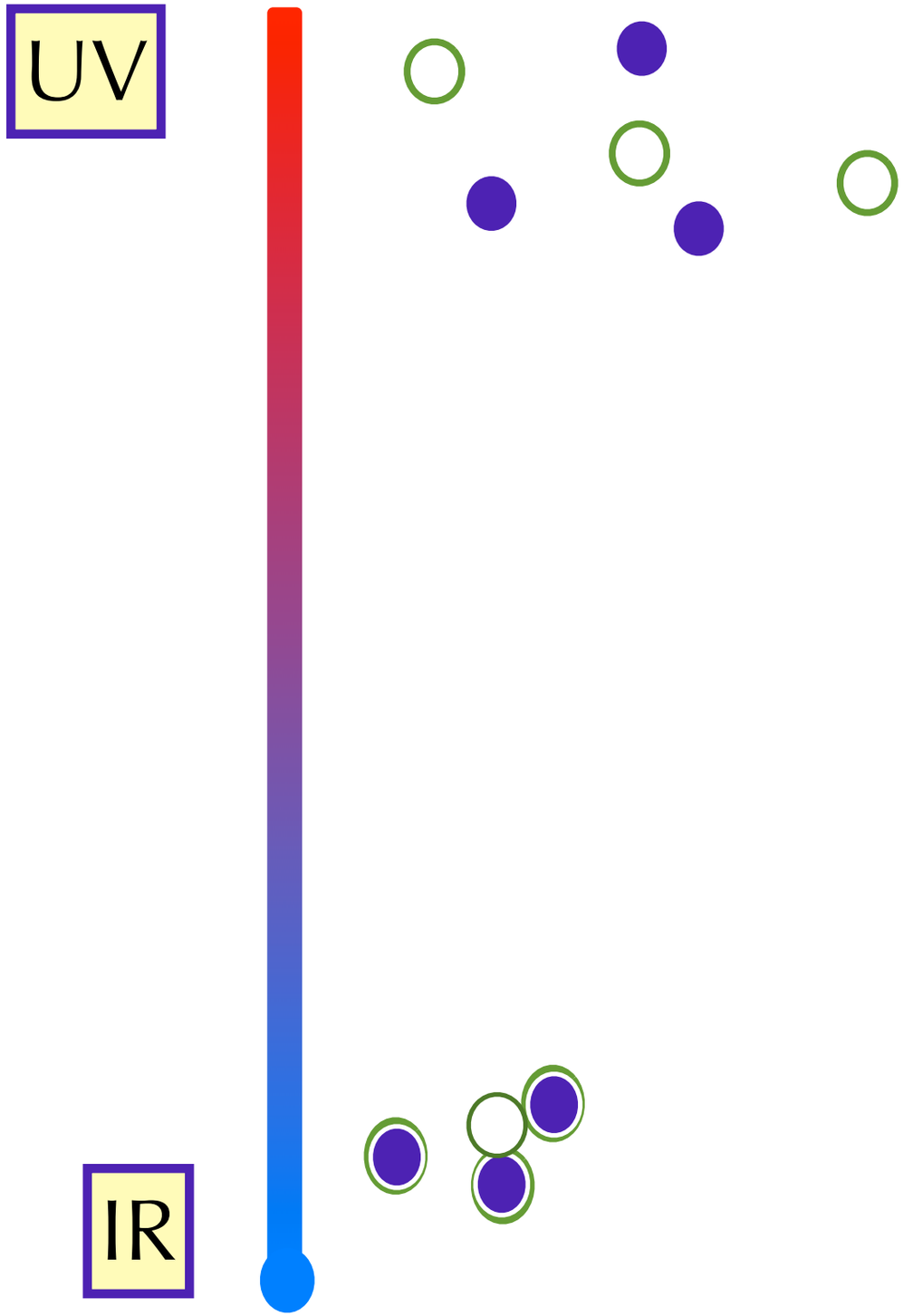

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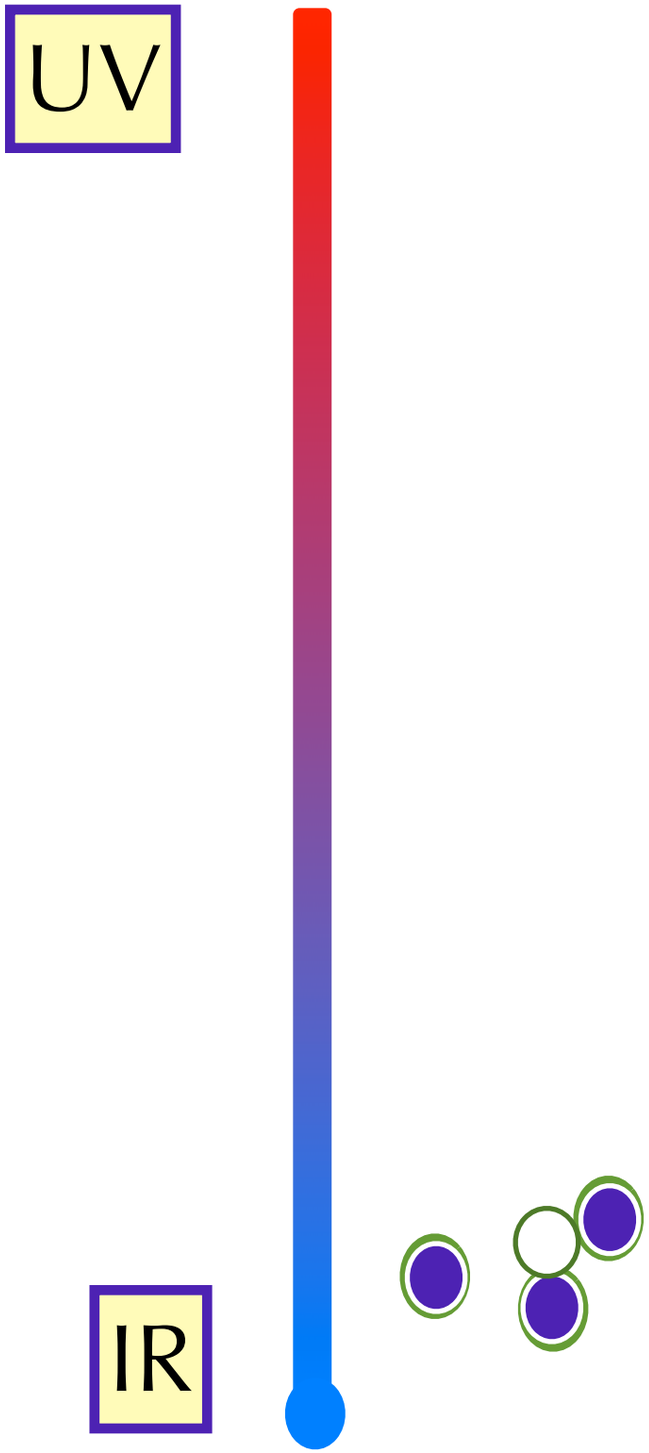

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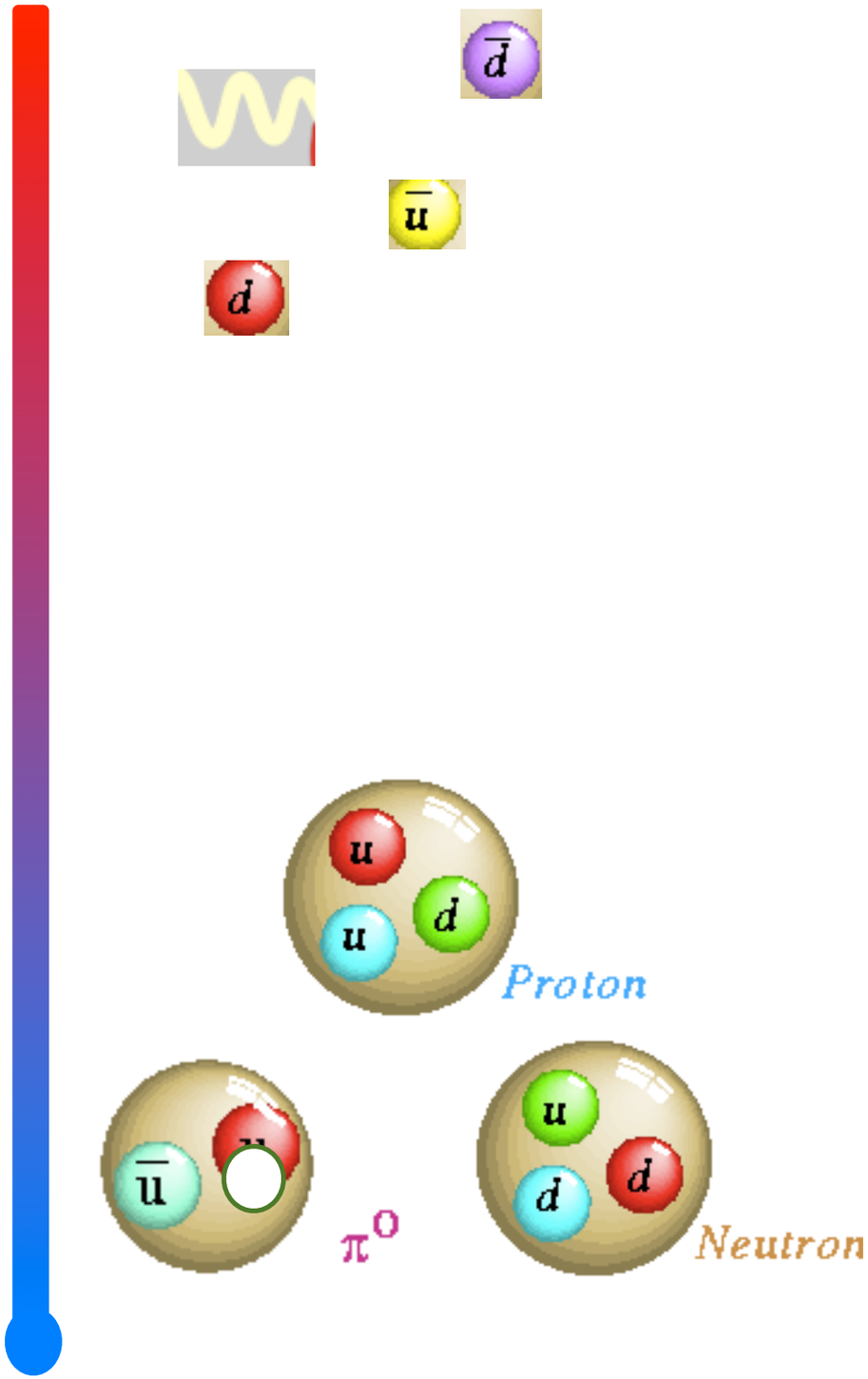

IR

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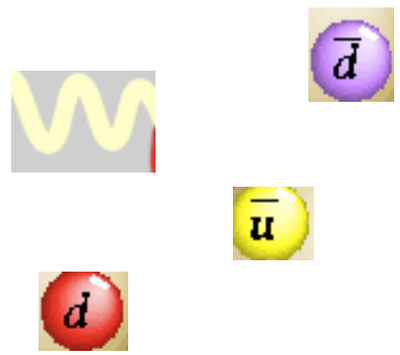
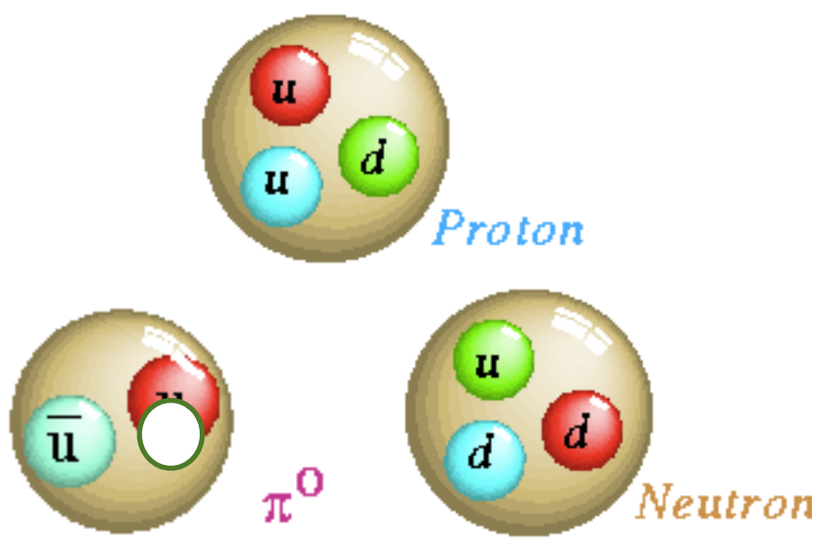
UV



?

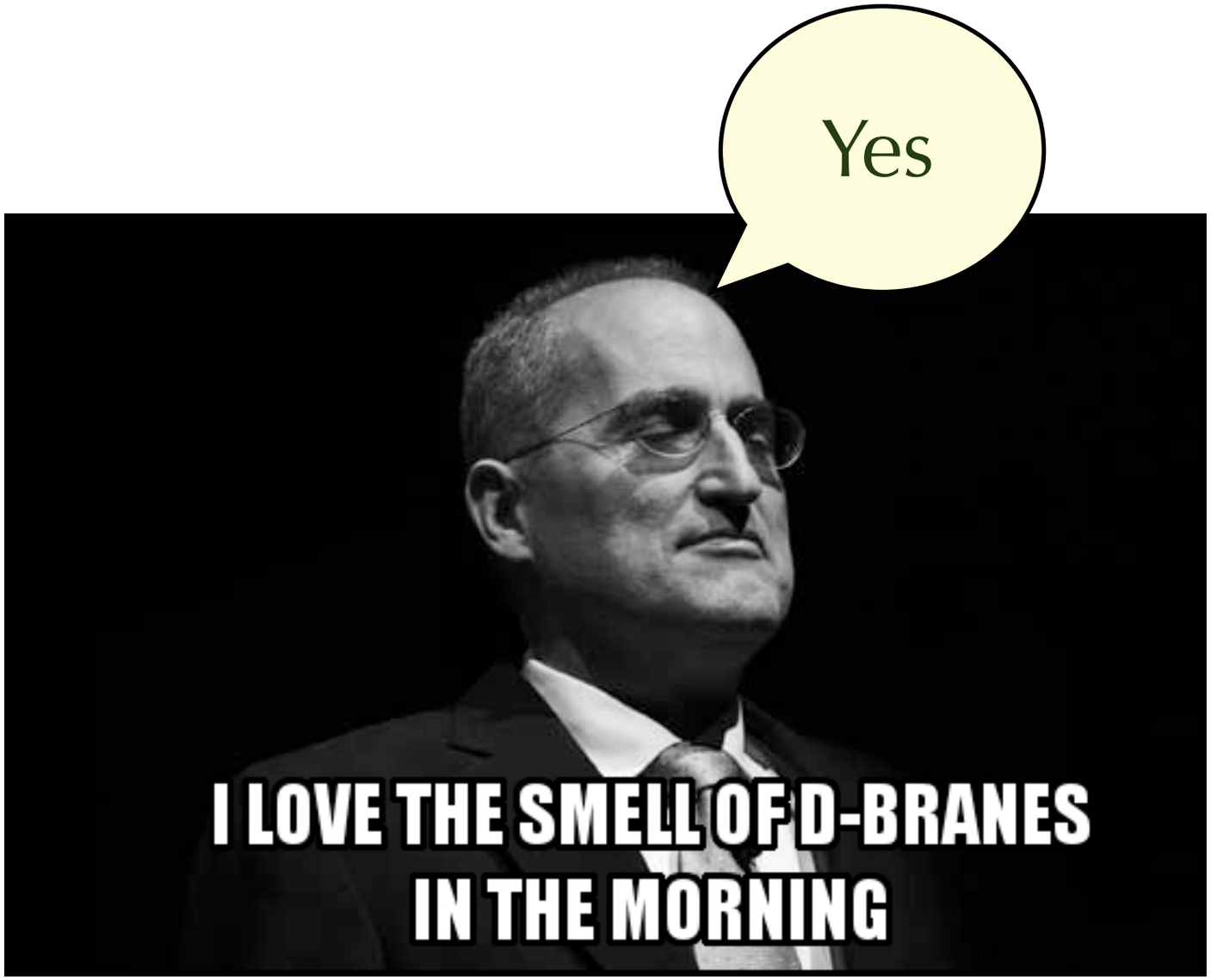


IR

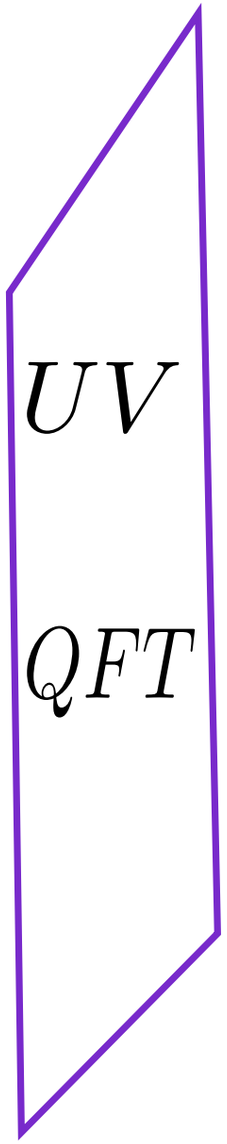
emergent
low-energy physics
(new degrees of
freedom not in UV)

What computational tools do we have for strongly correlated electron matter?

Is strong coupling hard because we are in the wrong dimension?


Yes
I LOVE THE SMELL OF D-BRANES
IN THE MORNING

$$\mathcal{L} = T - g\varphi^4 \cdots$$



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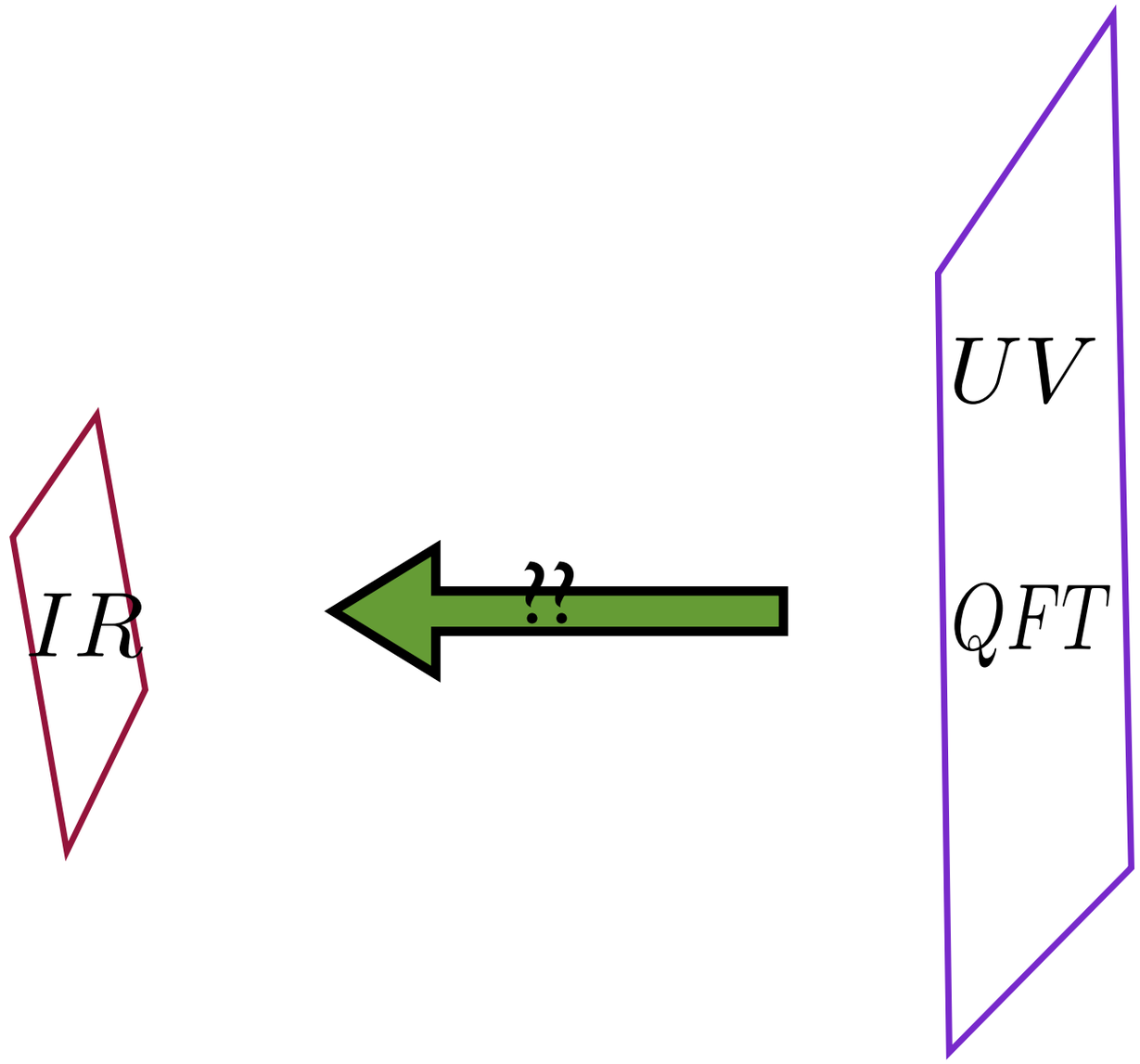

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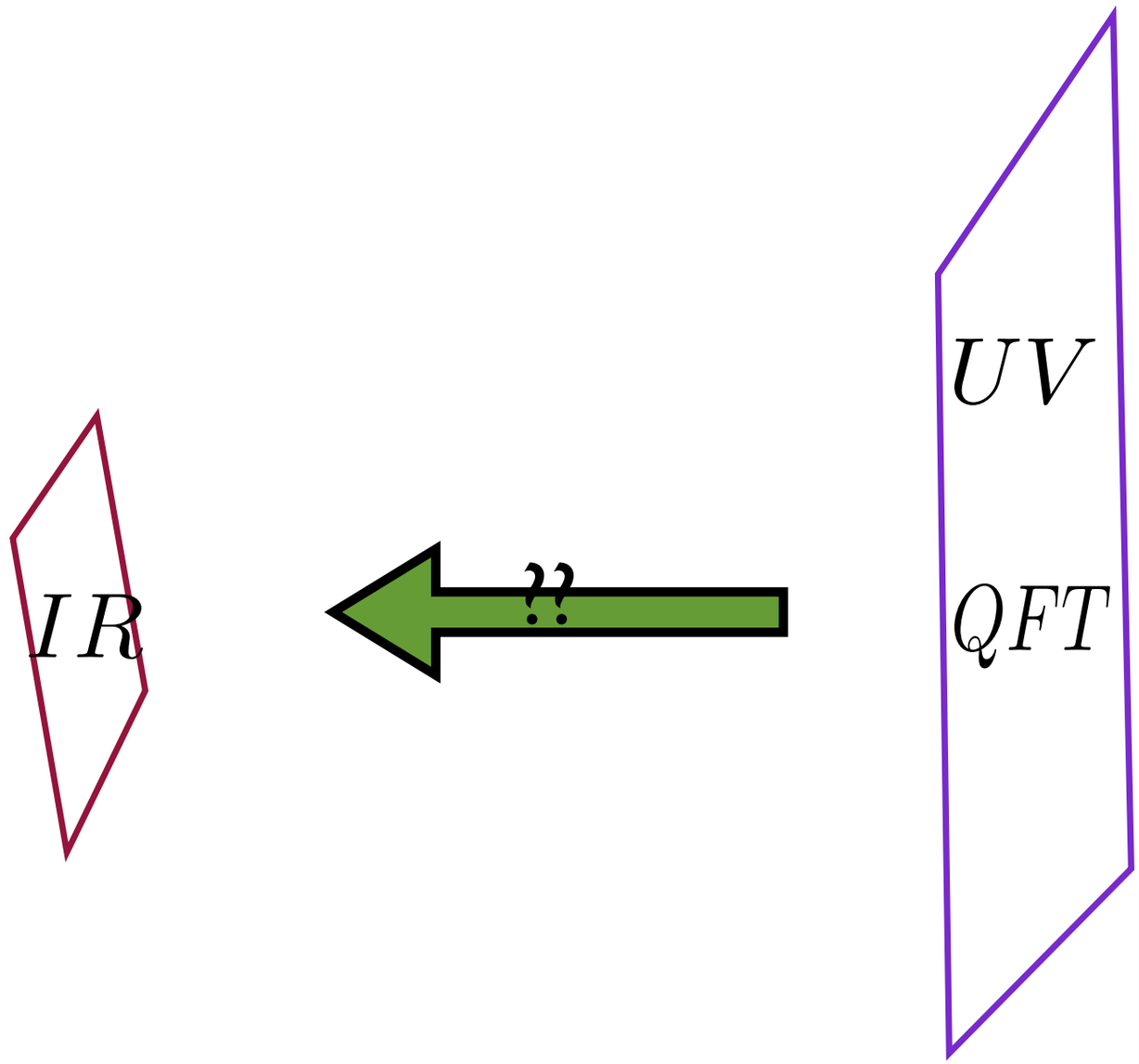
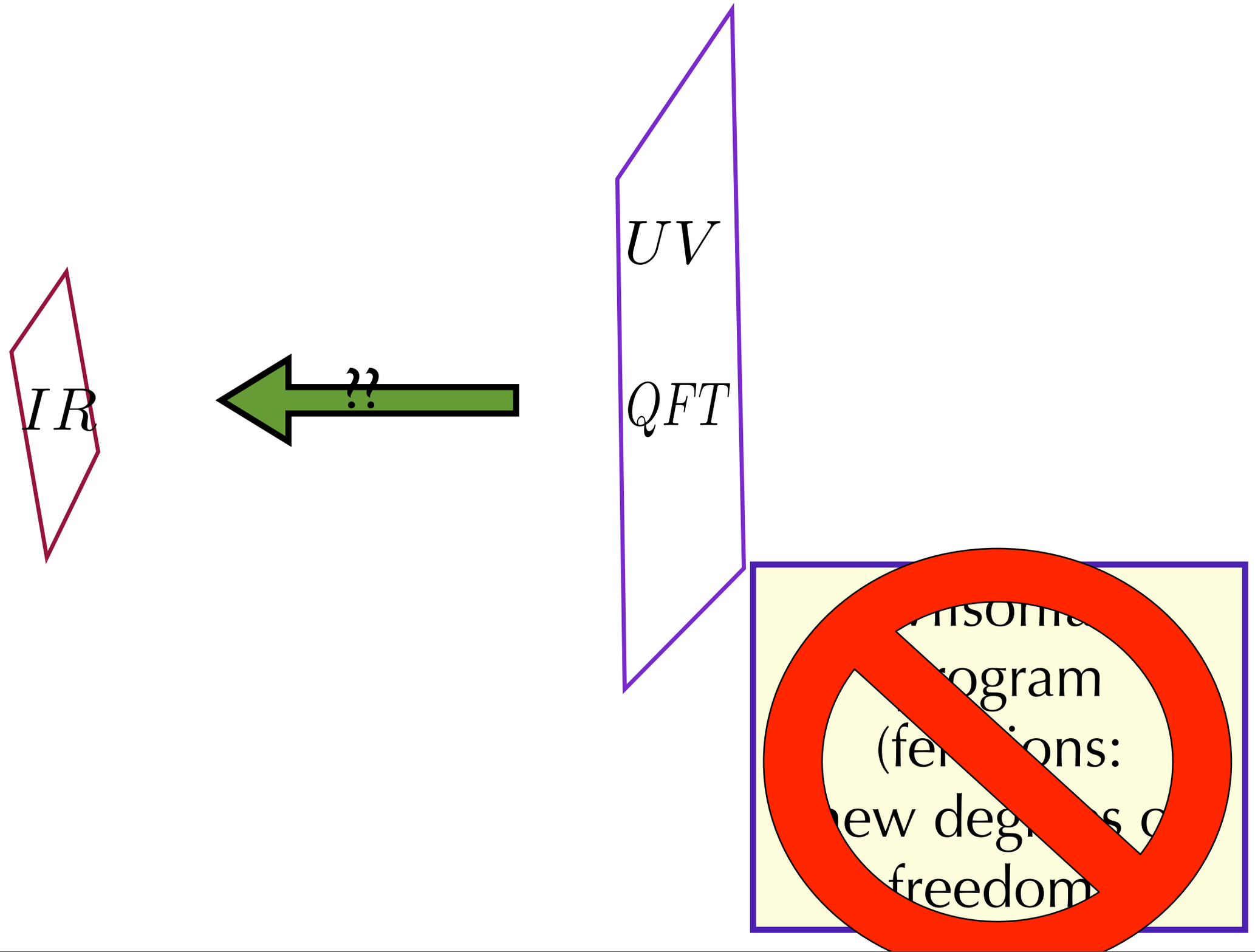


Wilsonian program (fermions: new degrees of freedom)

$$\mathcal{L} = T - g\varphi^4 \cdots$$



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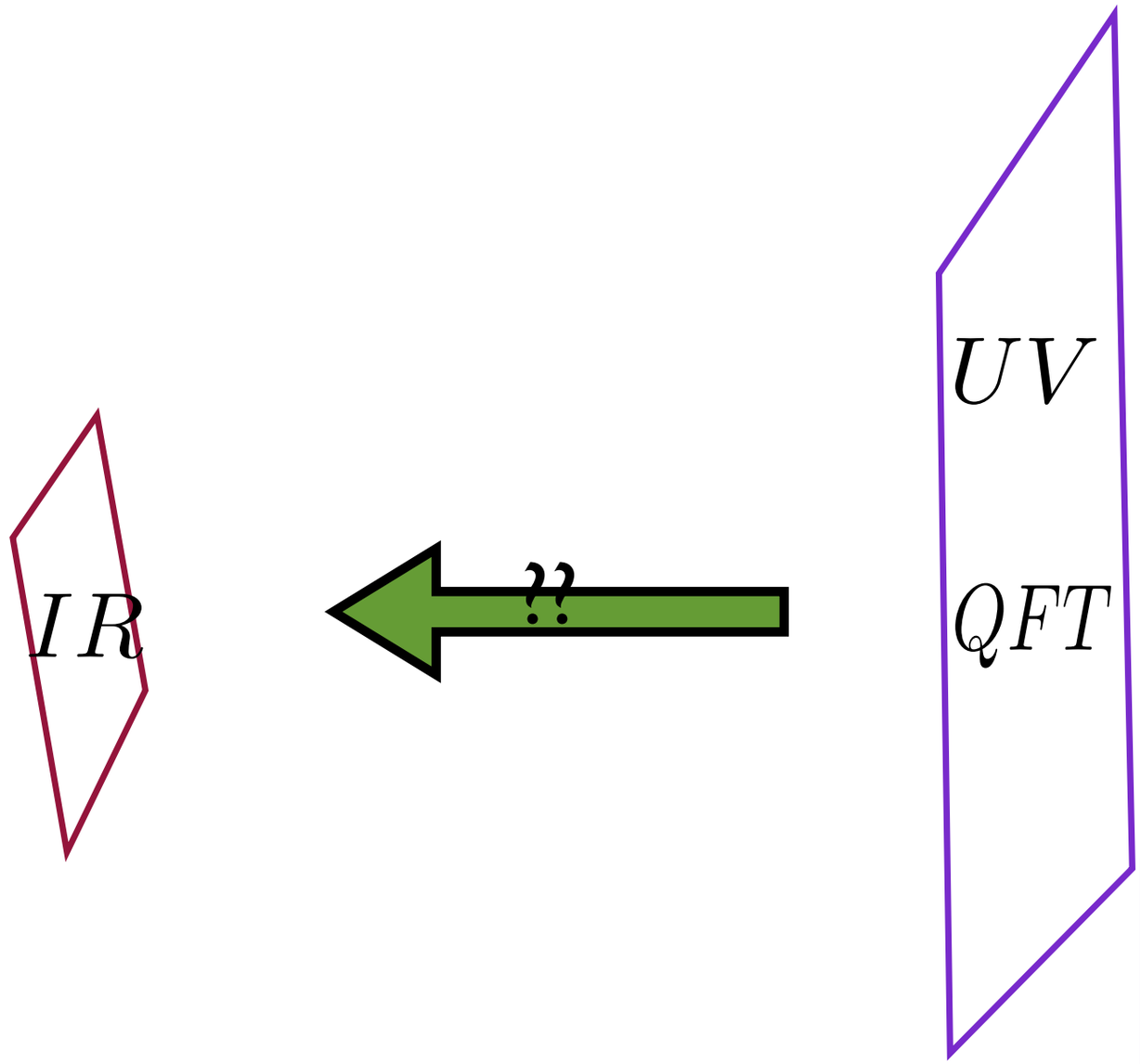

coupling constant

$g = 1/\text{ego}$

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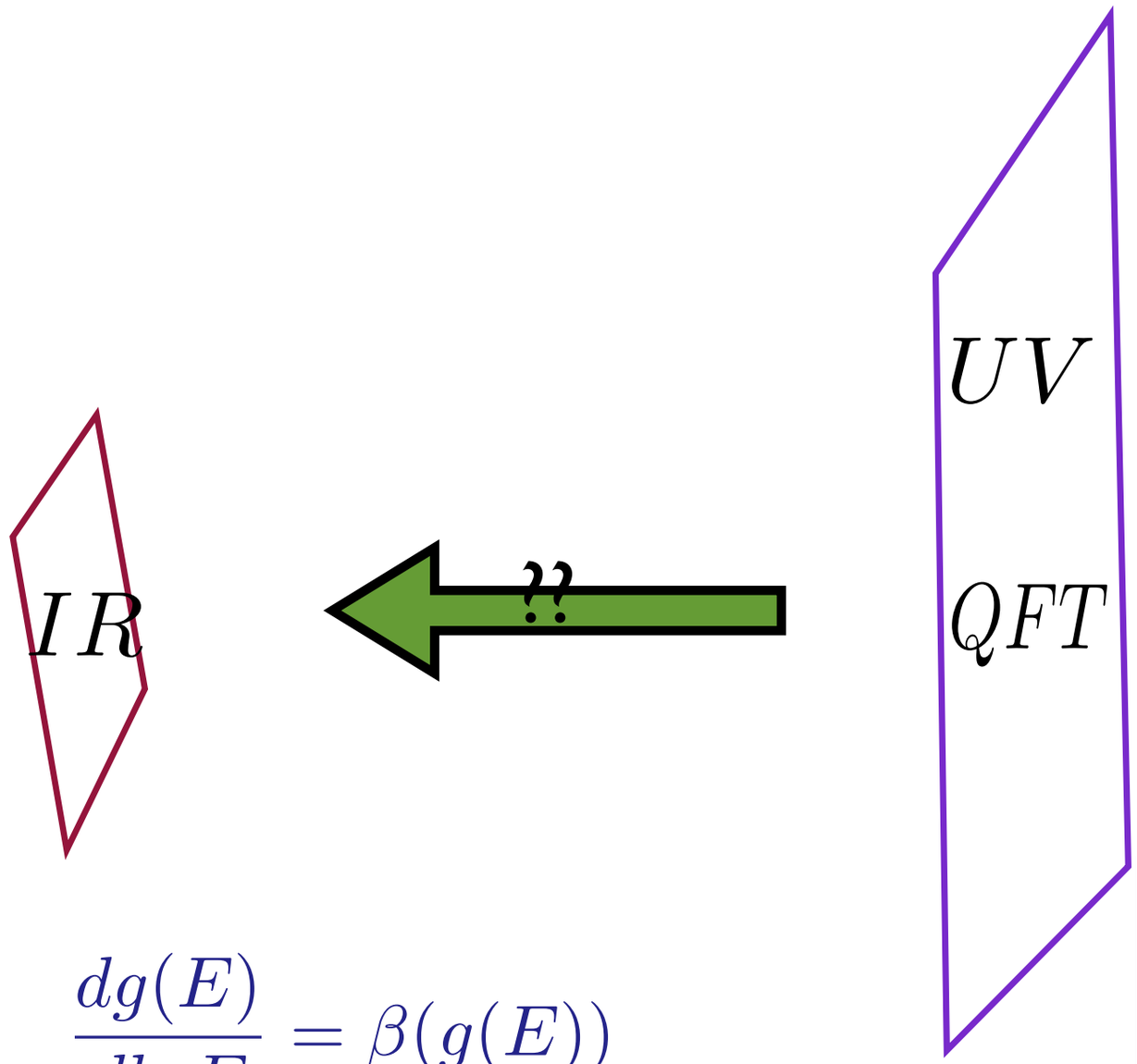

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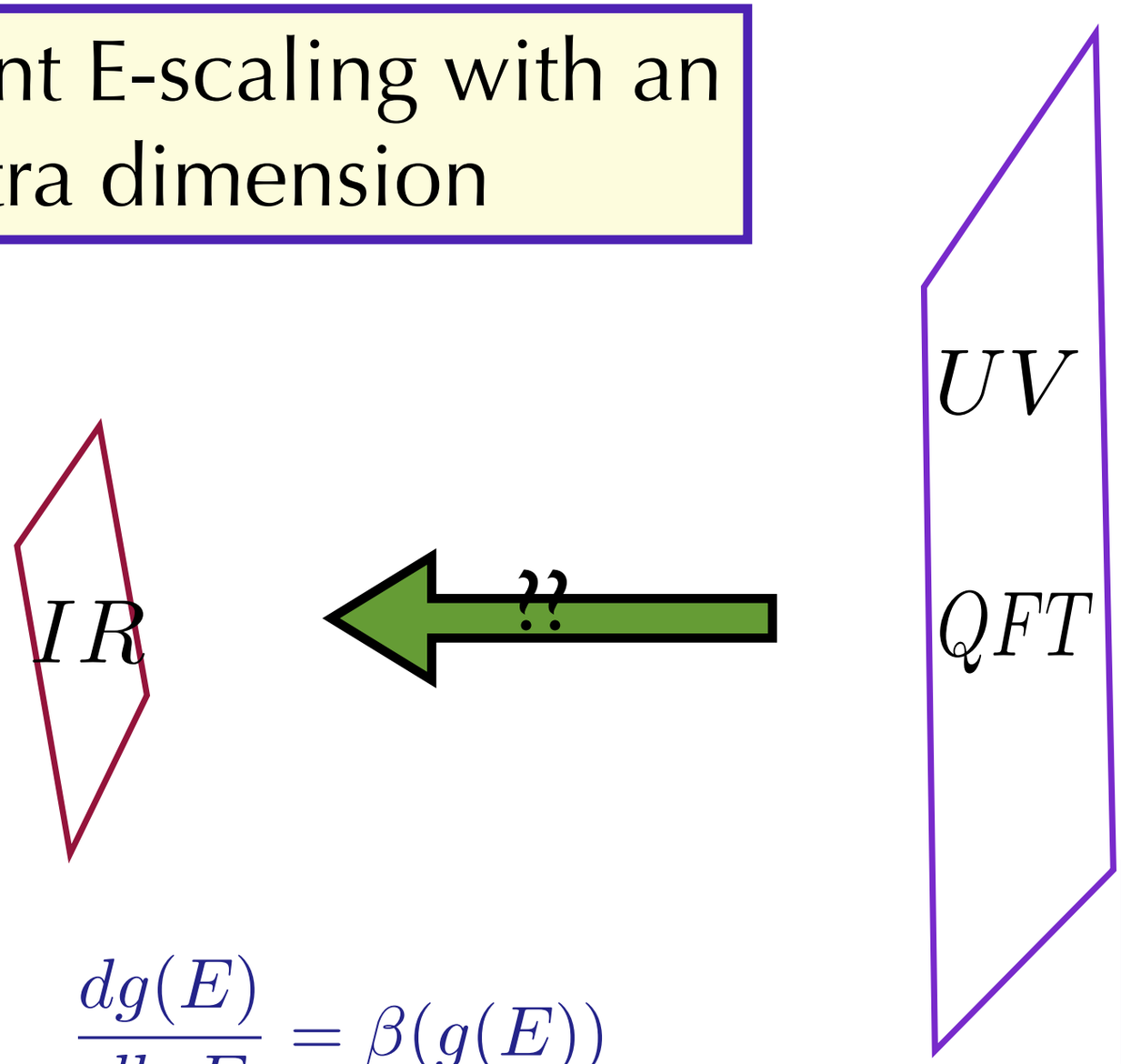
$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

$$\mathcal{L} = T - g\varphi^4 \cdots$$

implement E-scaling with an extra dimension



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locality in energy

gauge-gravity duality
(Maldacena, 1997)

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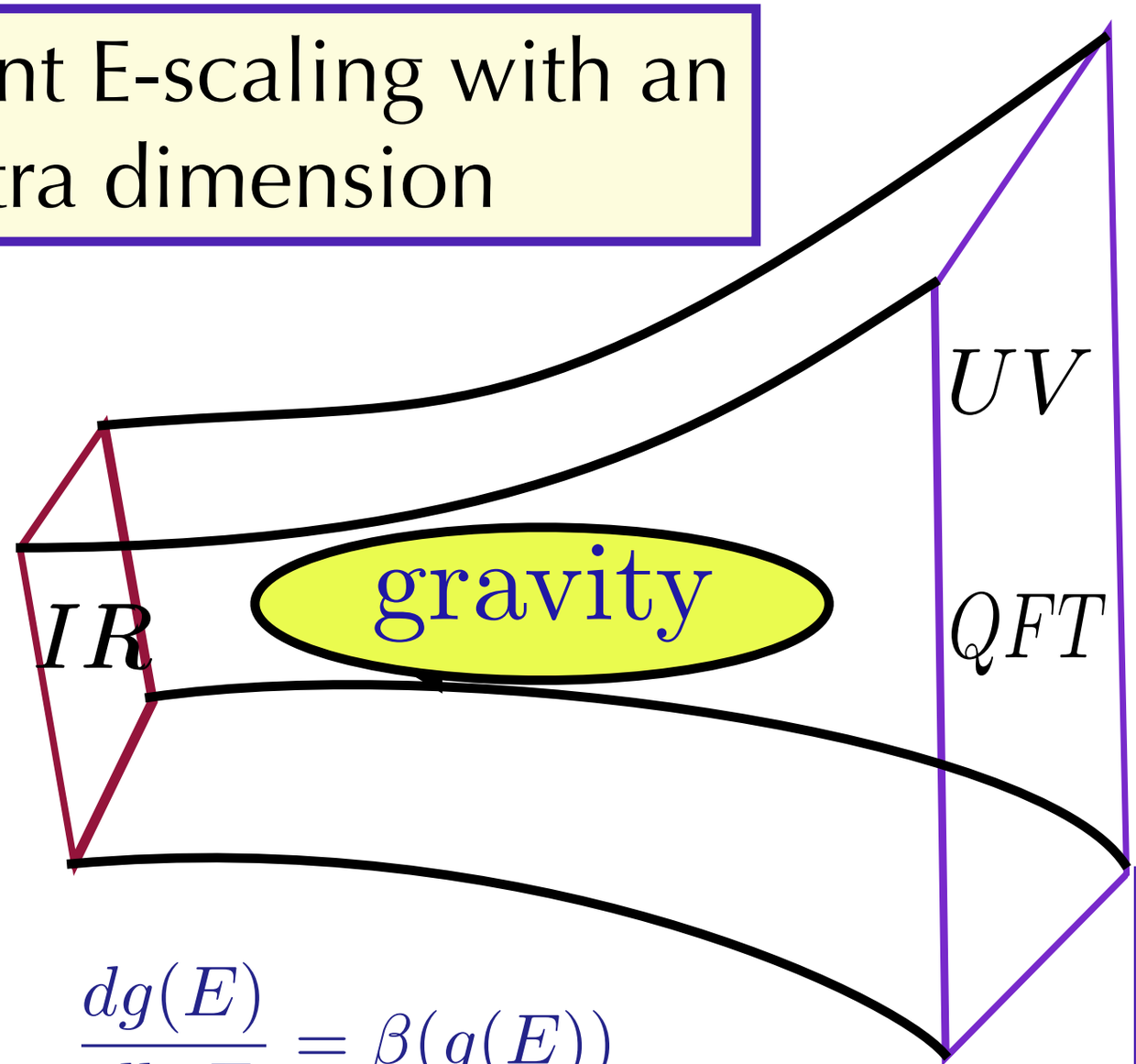

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scale invariance
(continuous)

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$$x^\mu \to x^\mu / \lambda$$

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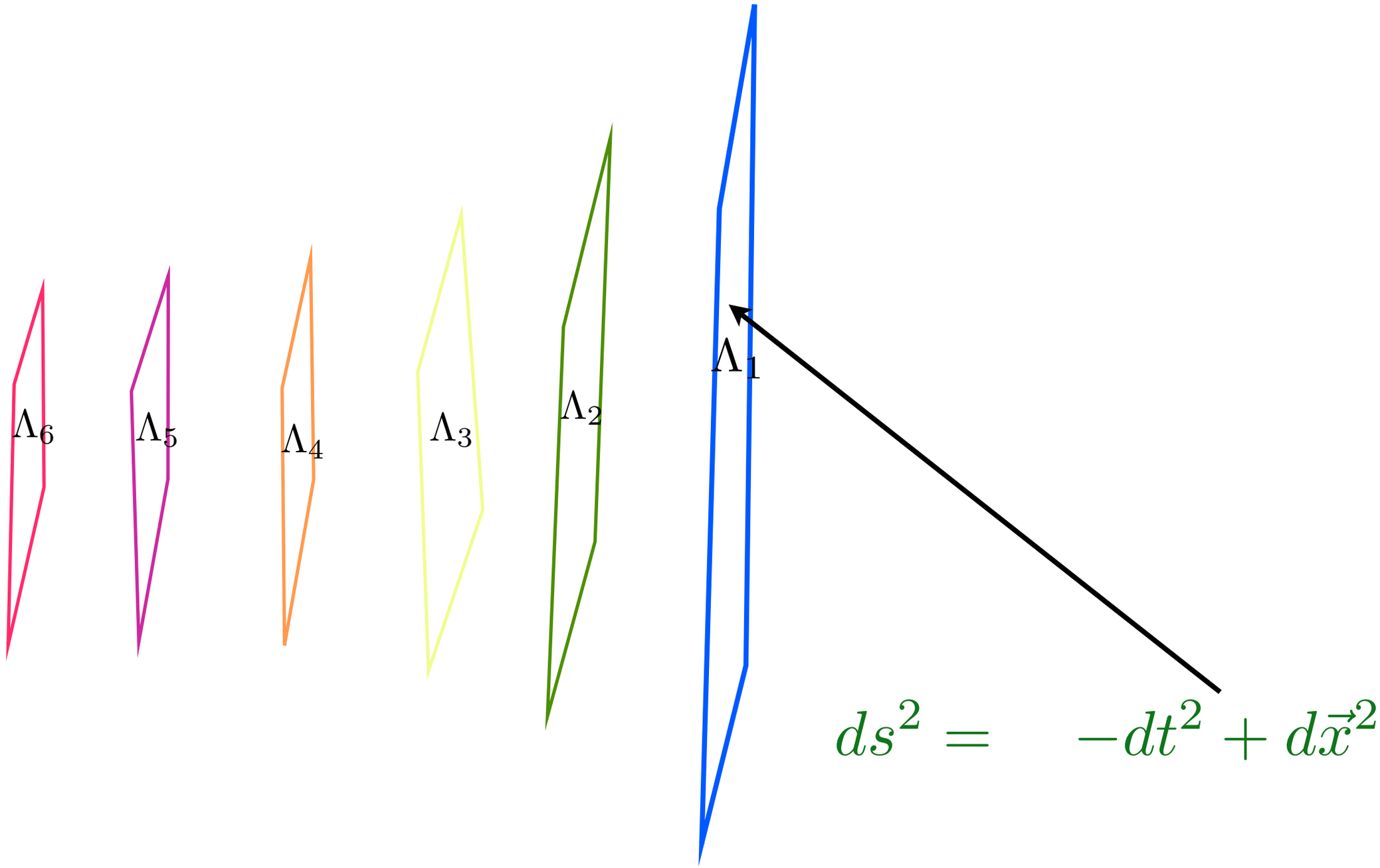
$$ds^2 = \left(\frac{u}{L}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{L}{E}\right)^2 dE^2$$

anti de-Sitter space

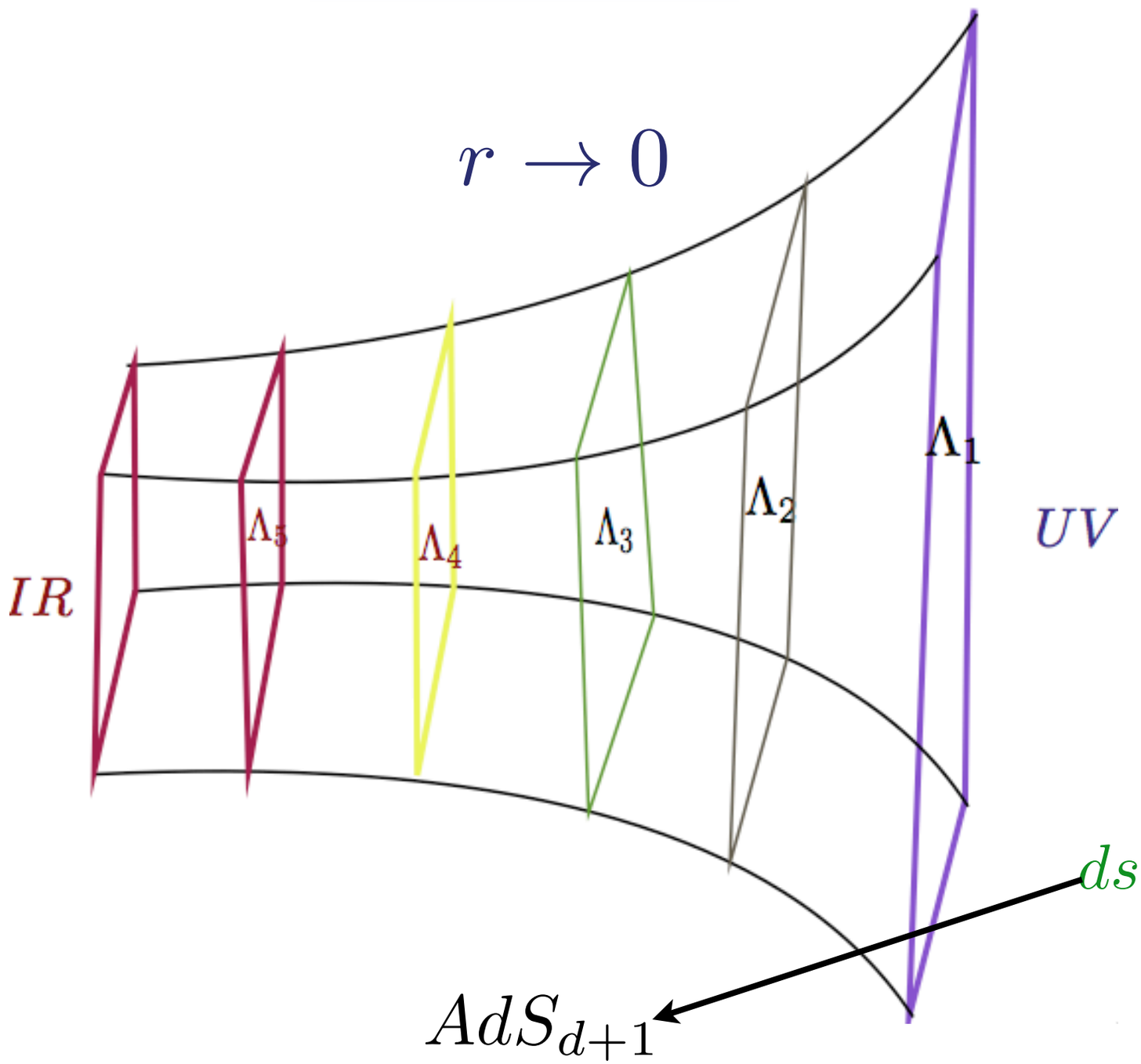
Holography



$$ds^2 = \quad -dt^2 + d\vec{x}^2$$

Holography

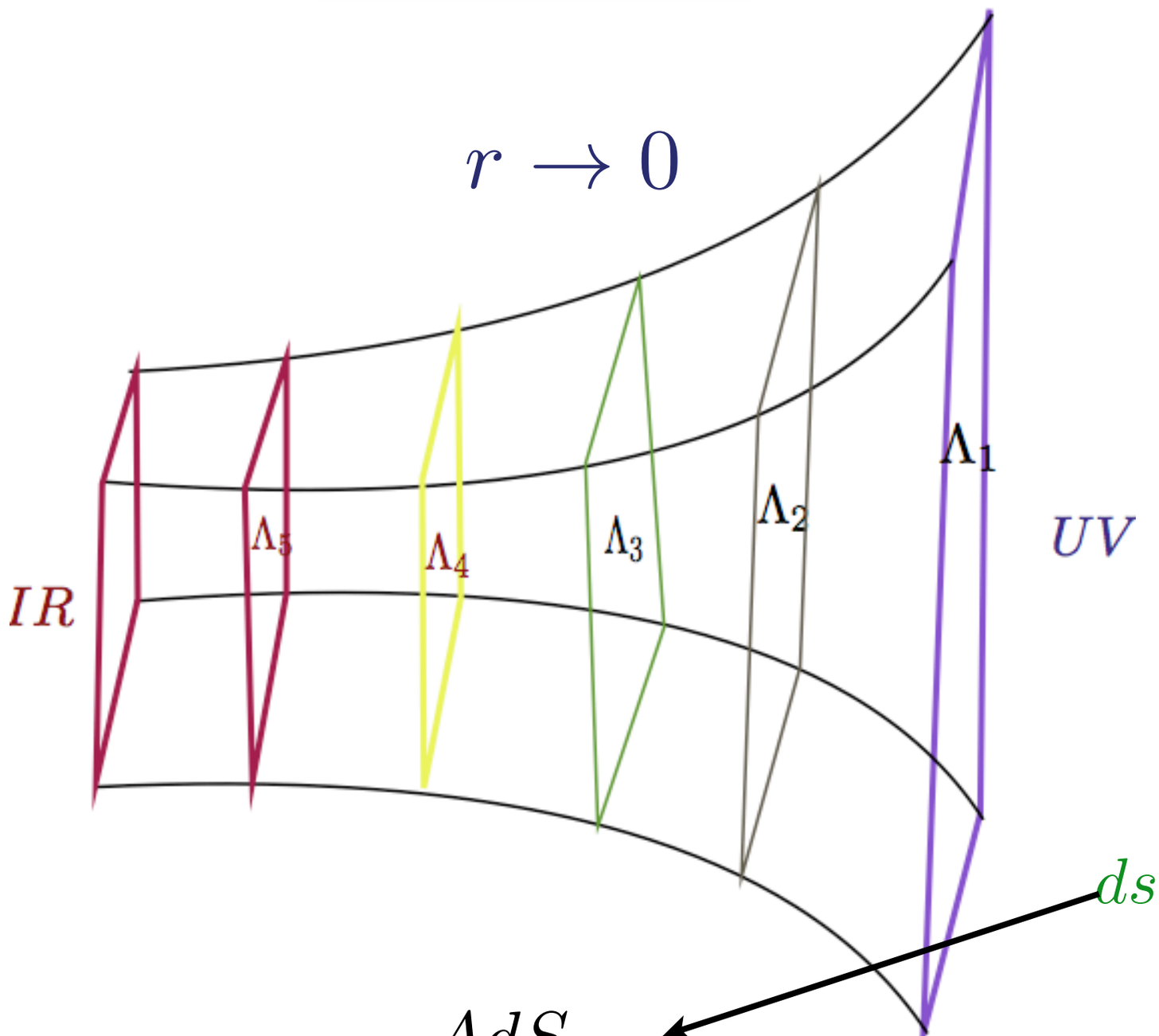
Holography



$$ds^2 = \frac{L^2}{r^2}\left(-dt^2 + d\vec{x}^2 + dr^2\right)$$

$$r = L^2/E$$

Holography



$$ds^2 = \frac{L^2}{r^2}\left(-dt^2 + d\vec{x}^2 + dr^2\right)$$

AdS_{d+1}

$\beta(g)$ is local

geometrize RG flow

$r = L^2/E$

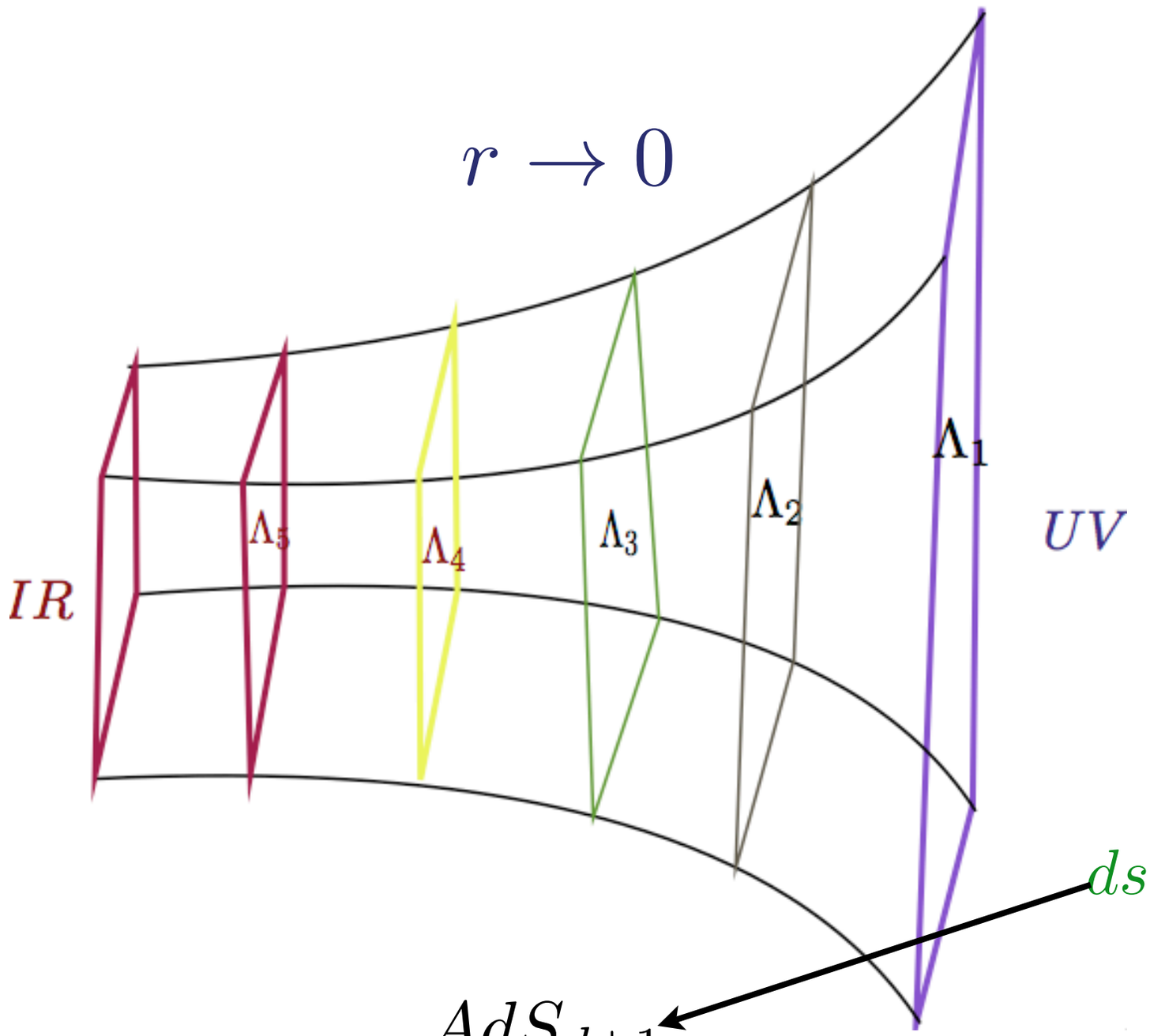
symmetry:

$$\{t, \vec{x}, r\} \to \{\lambda t, \lambda \vec{x}, \lambda r\}$$

Holography

QFT in d-dimensions

weakly-coupled classical gravity in d+1



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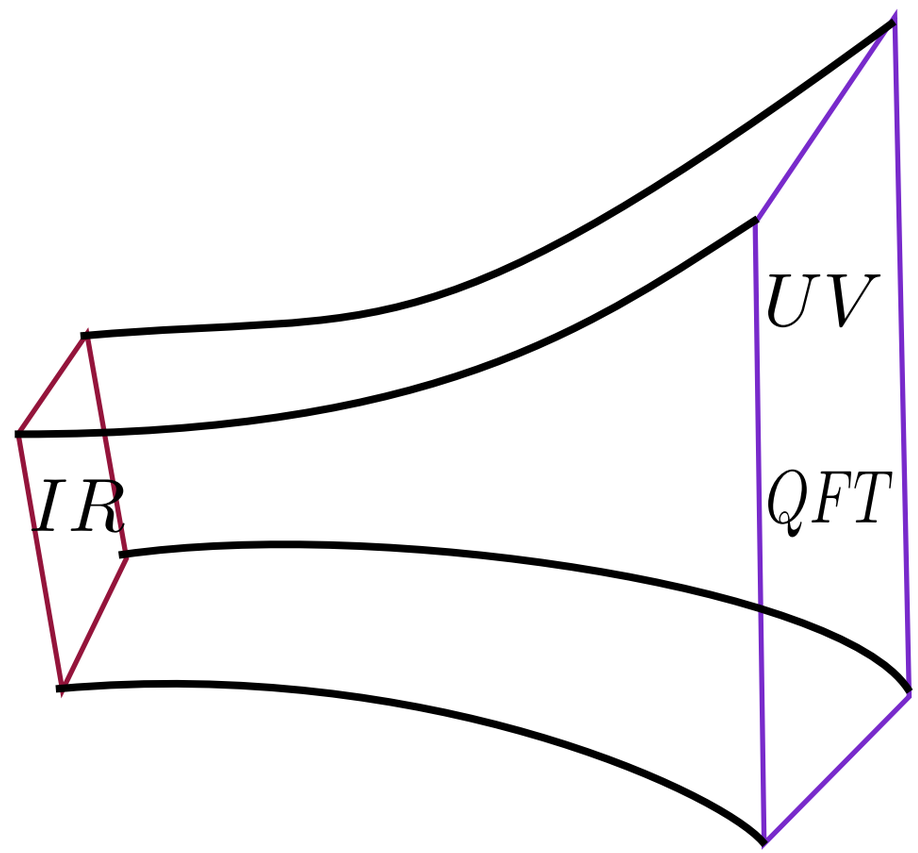
dual construction



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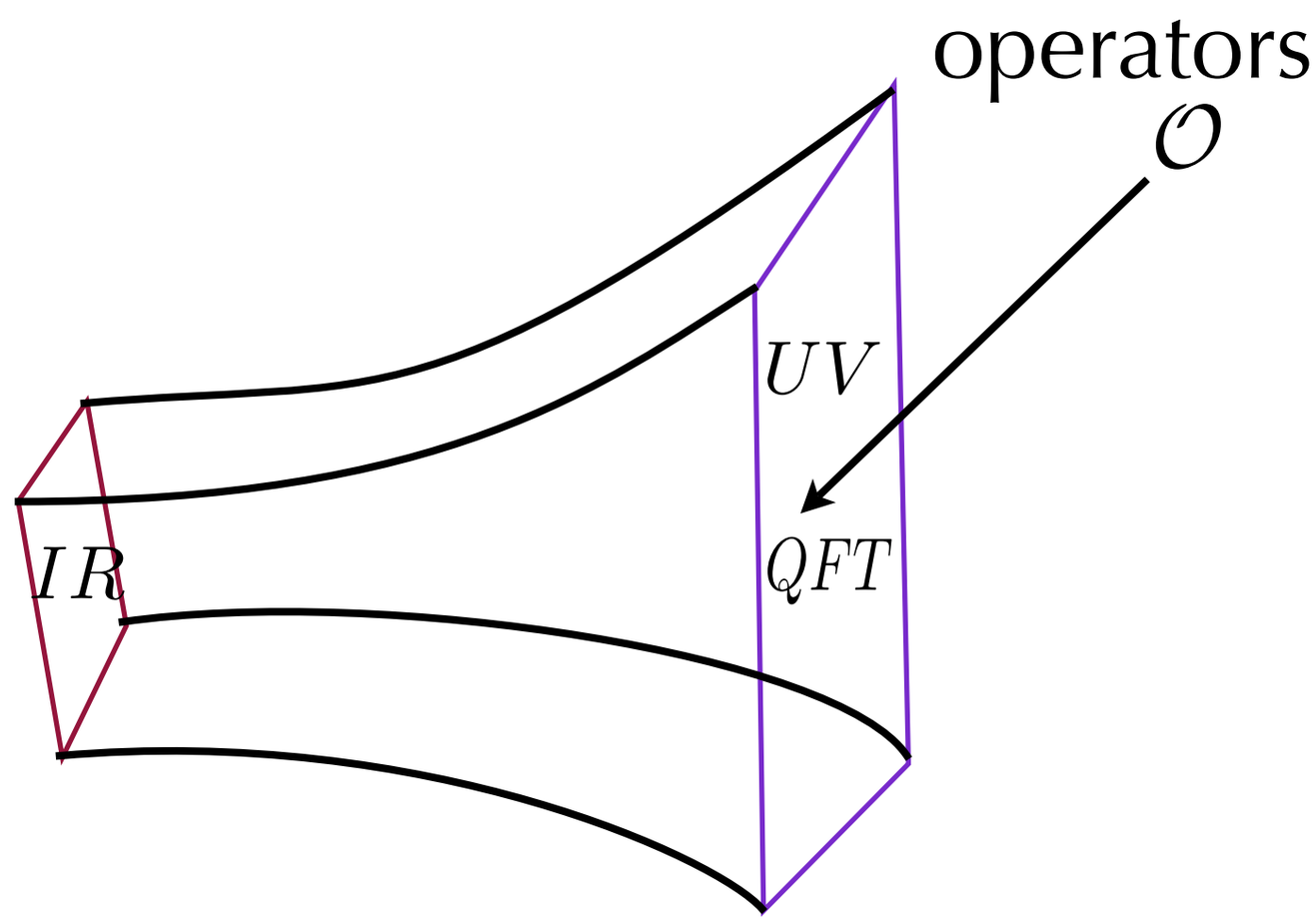
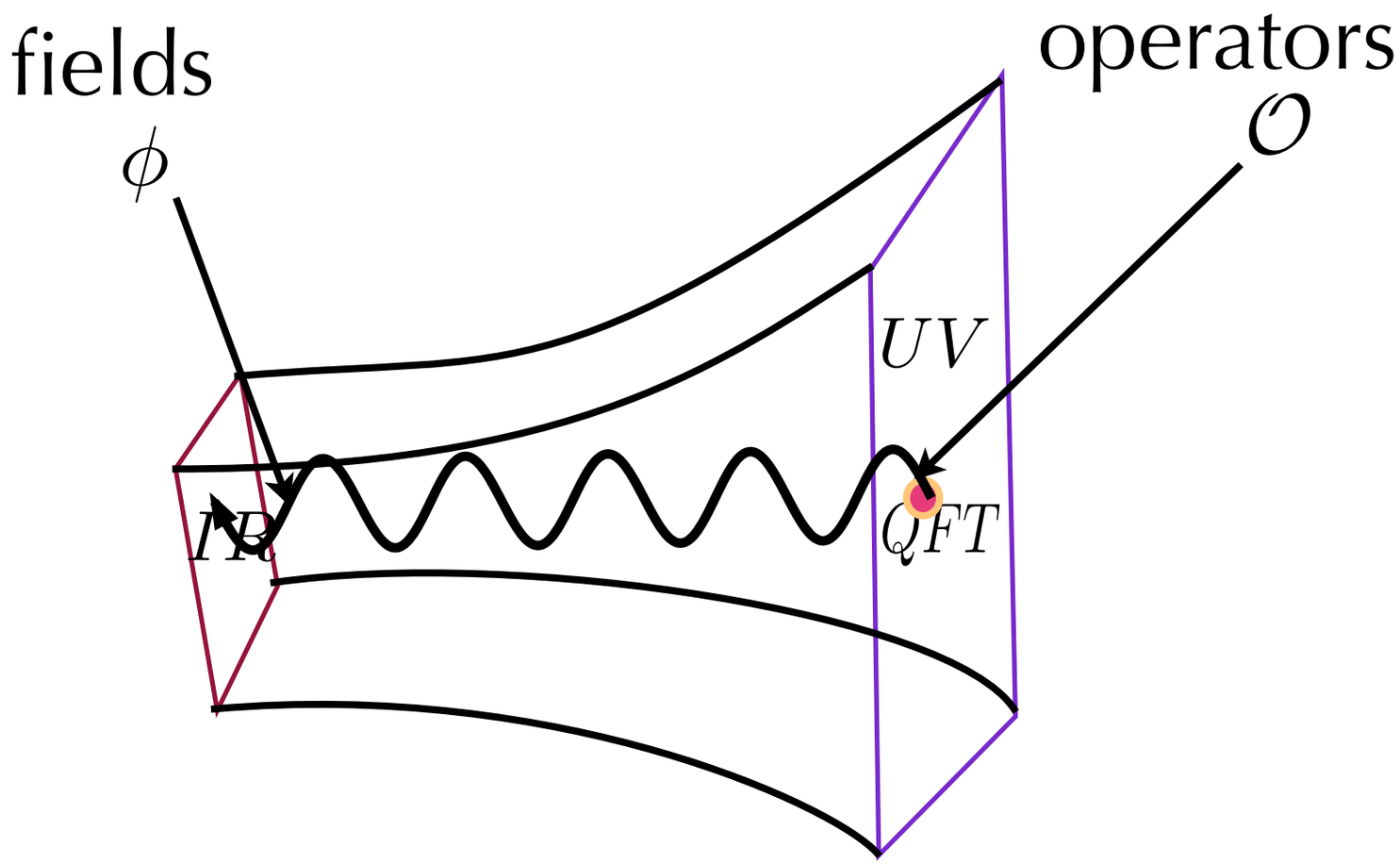
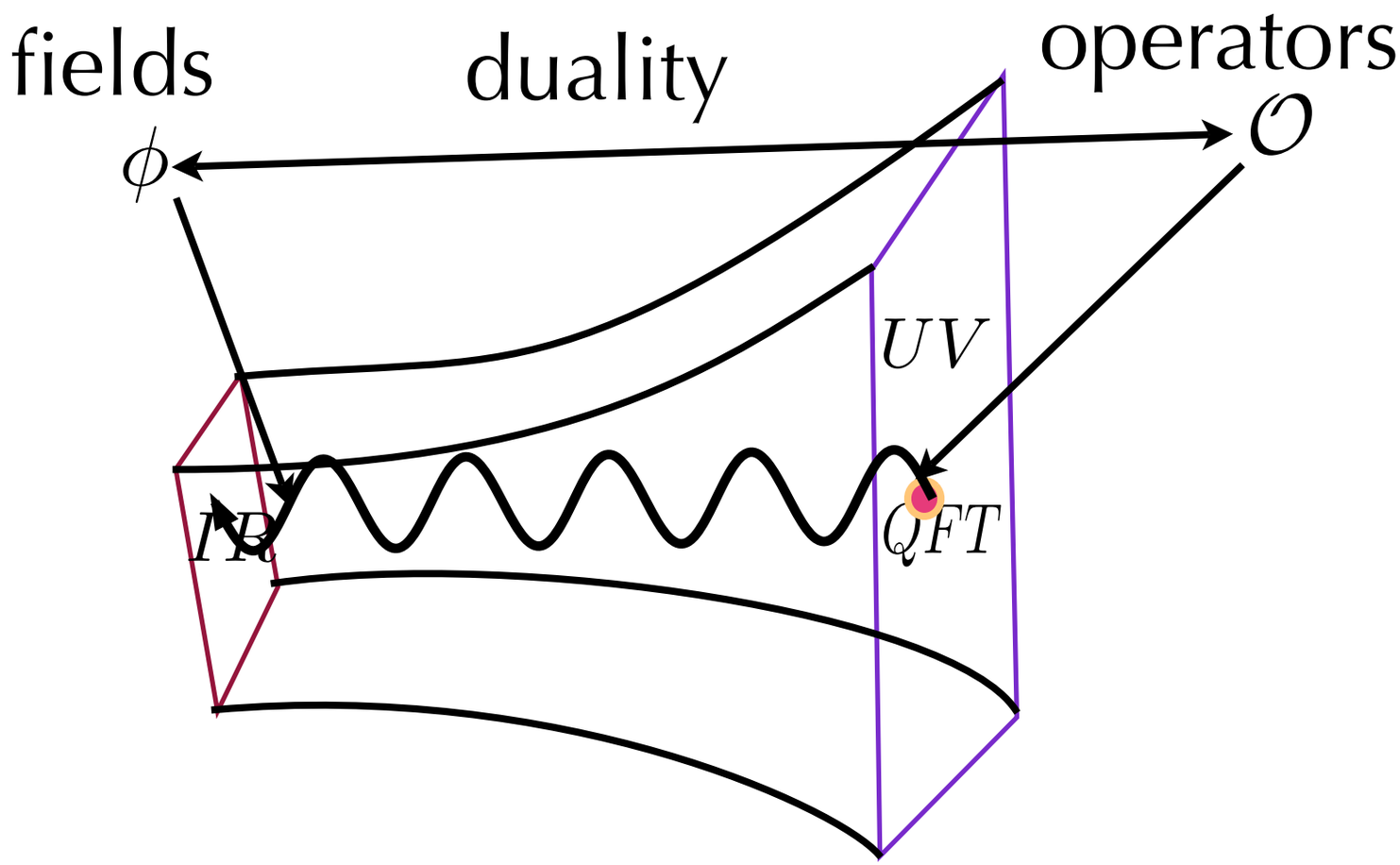


dual construction



dual construction



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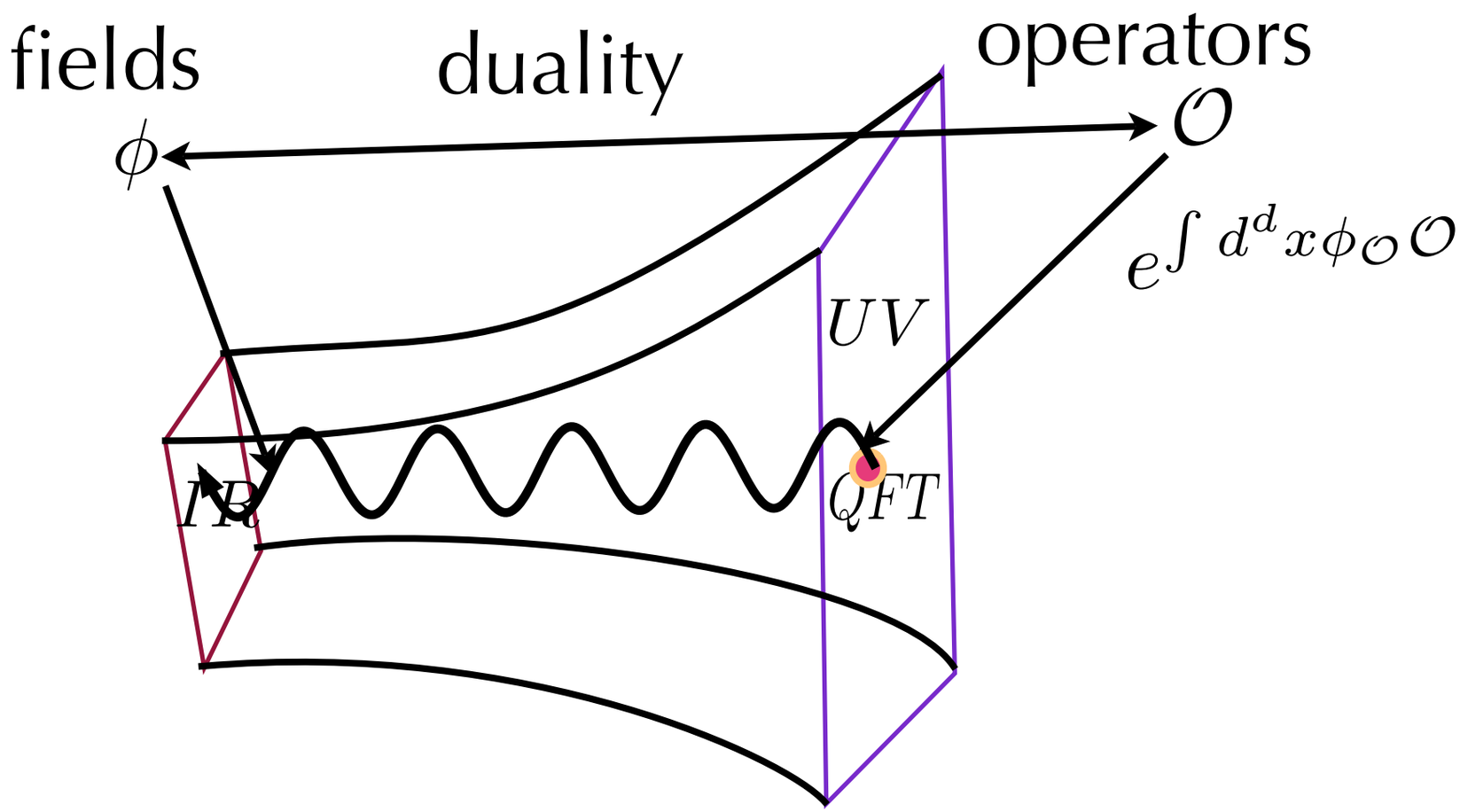

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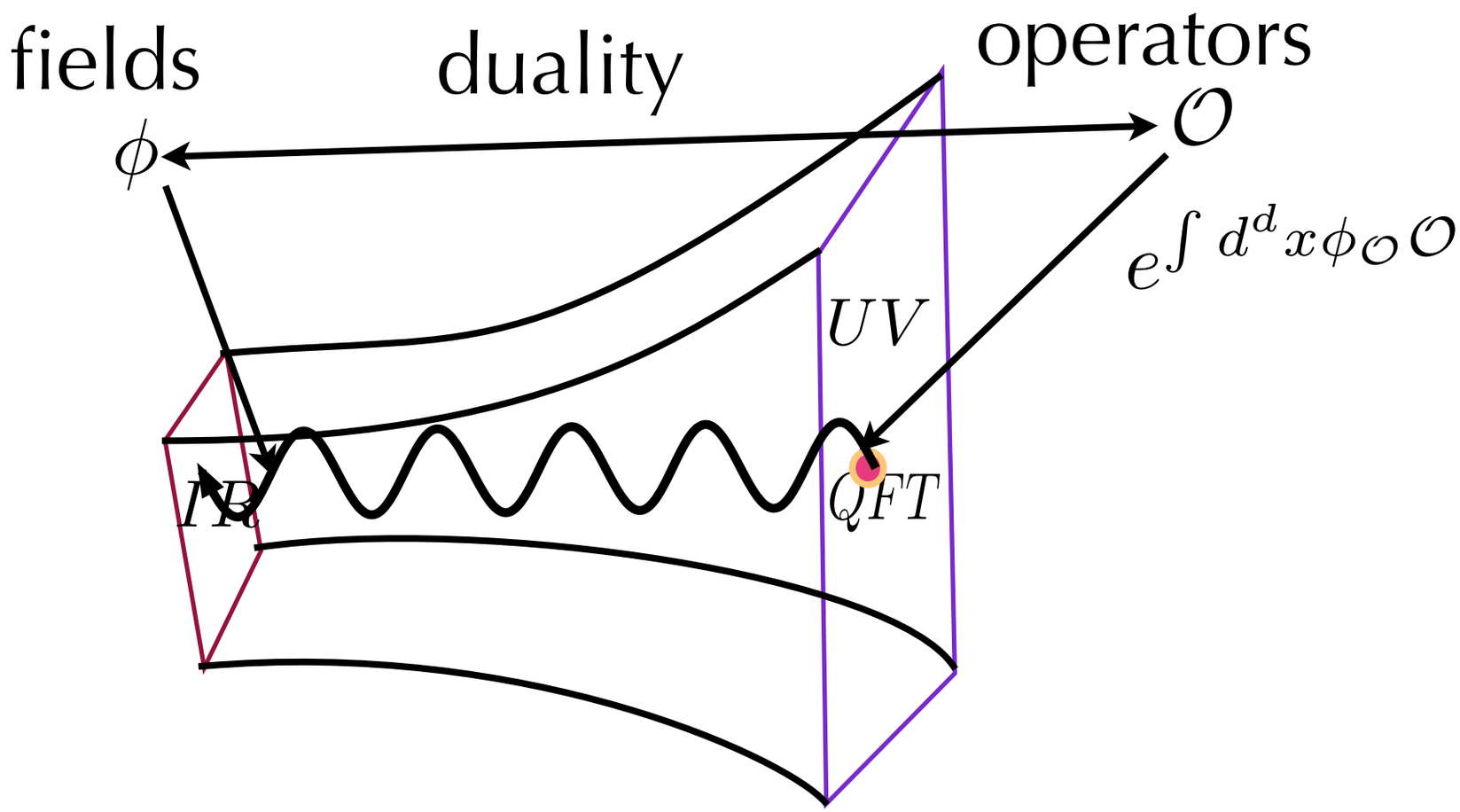
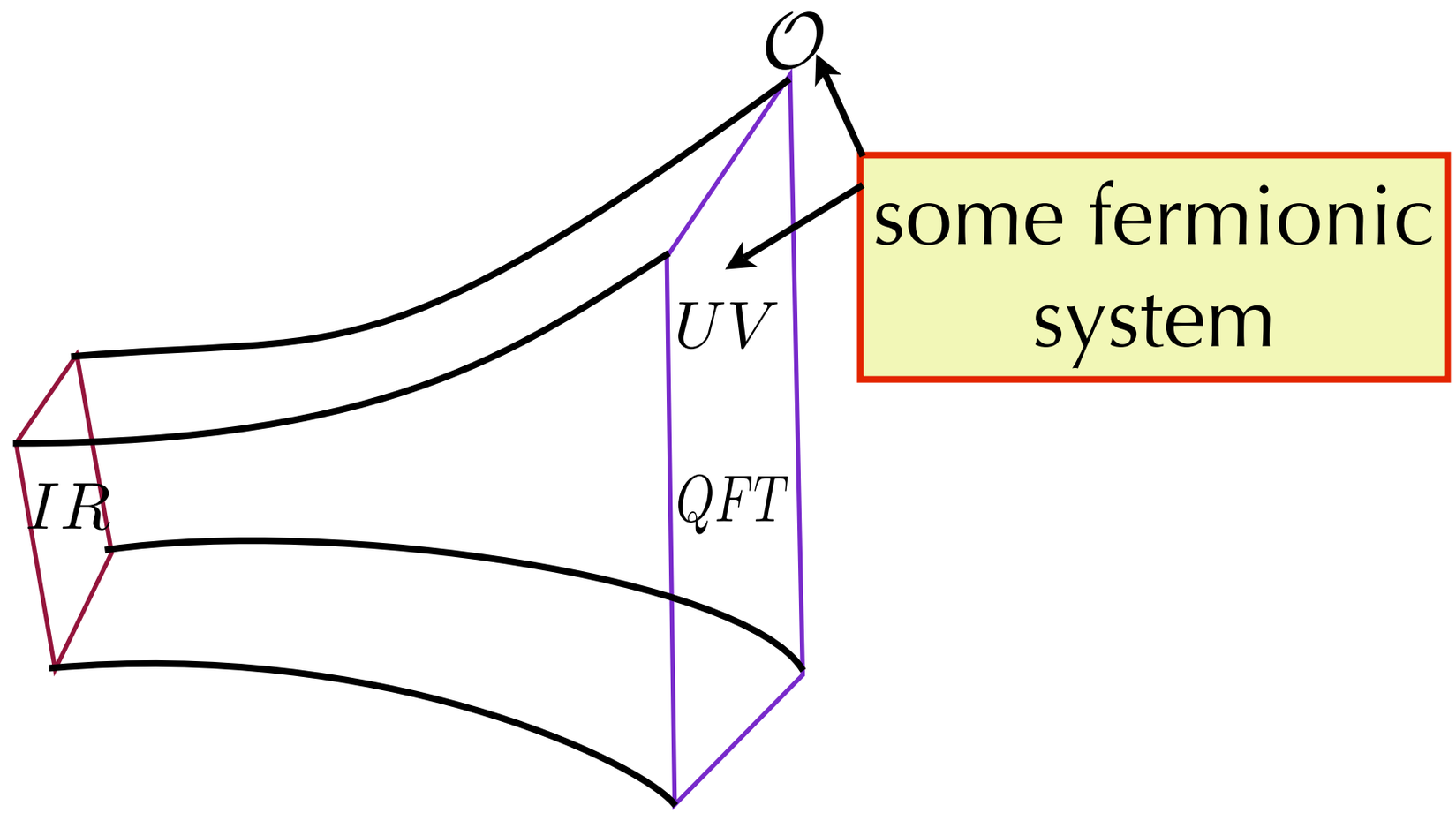


Claim: $Z_{\mathrm{QFT}} = e^{-S^{\mathrm{on-shell}}_{\mathrm{ADS}}(\phi(\phi_{\partial \mathrm{ADS}=\mathrm{J}_{\mathcal{O}}}))}$

How does it work?



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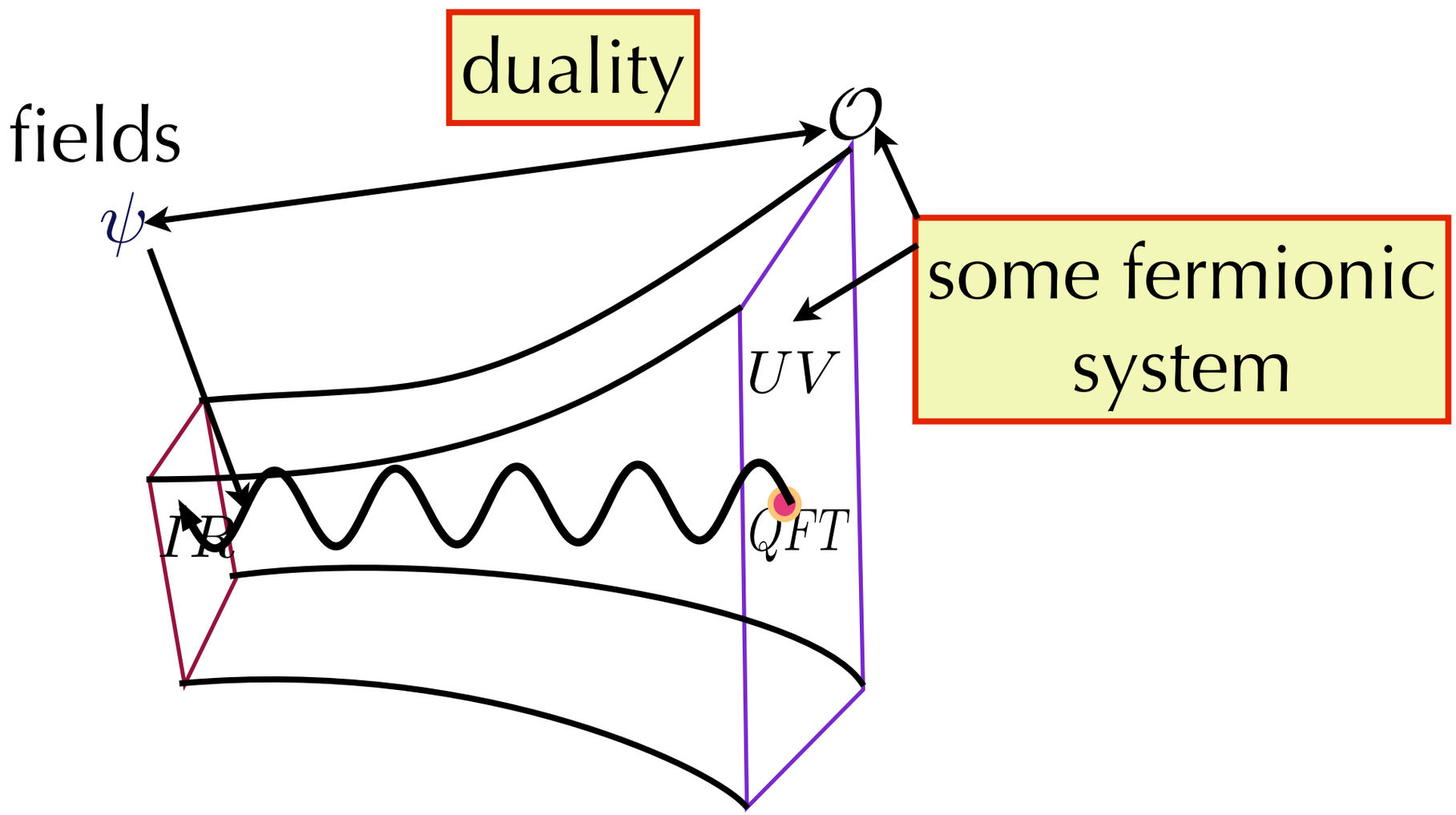

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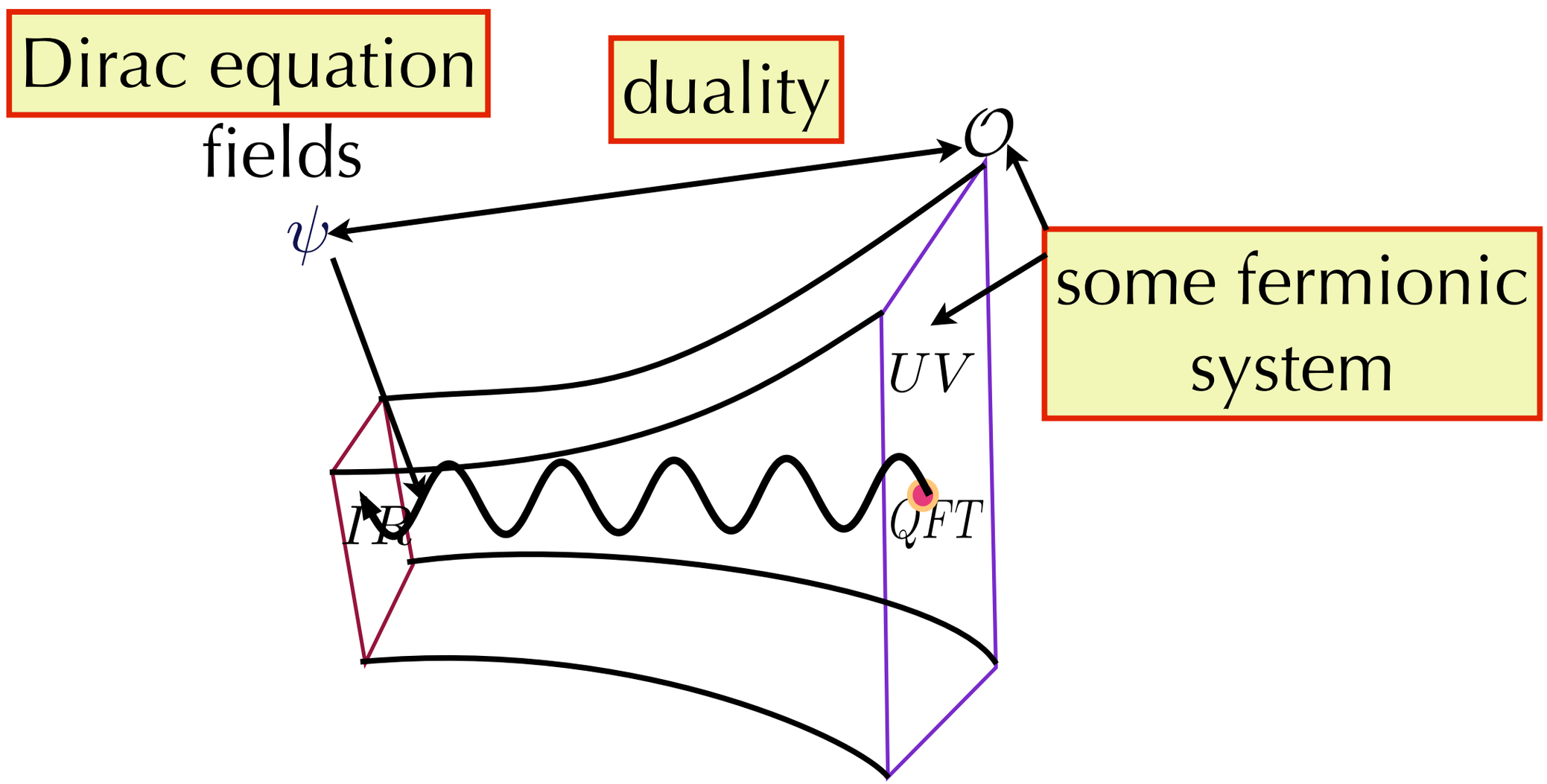

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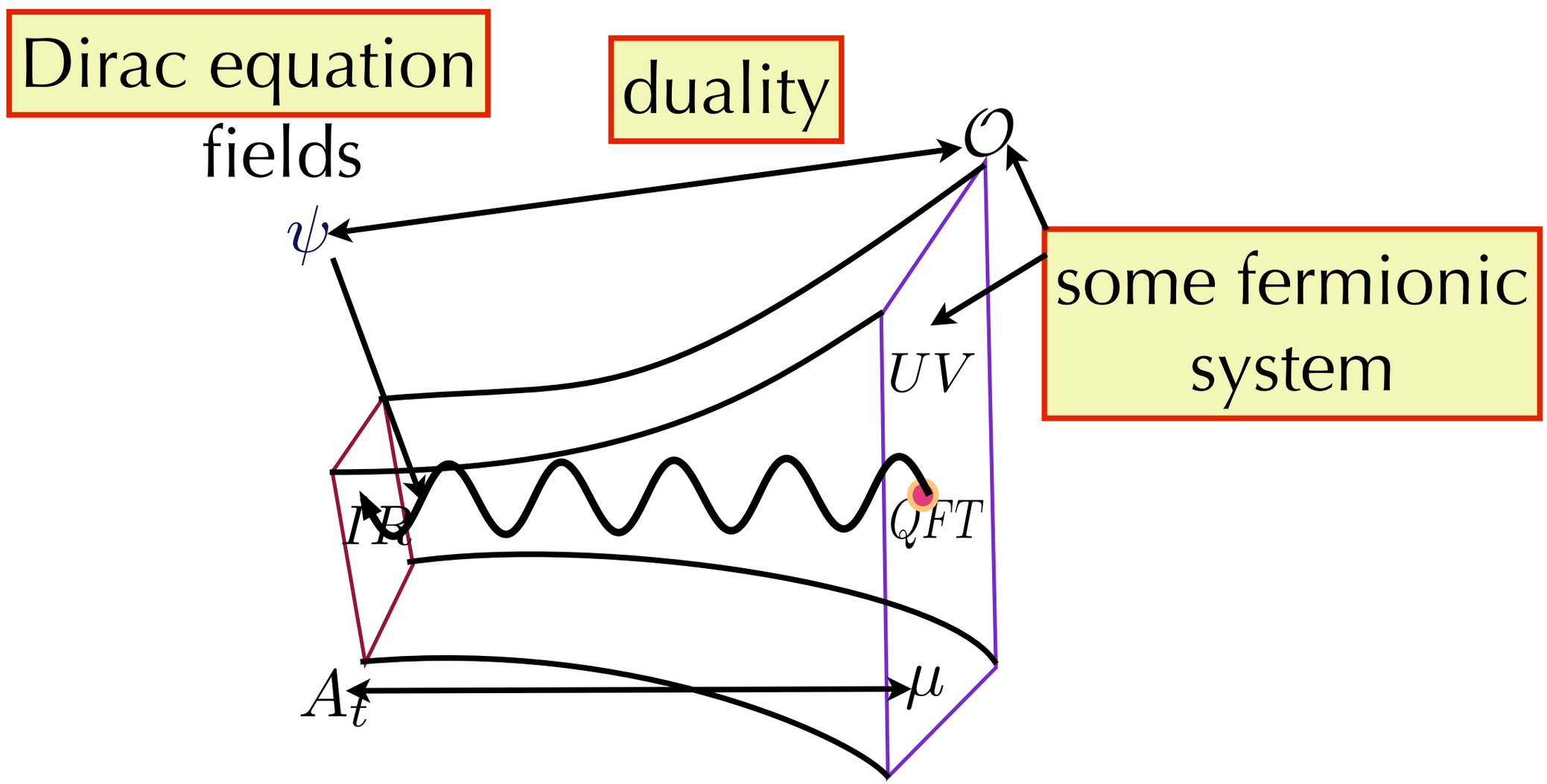

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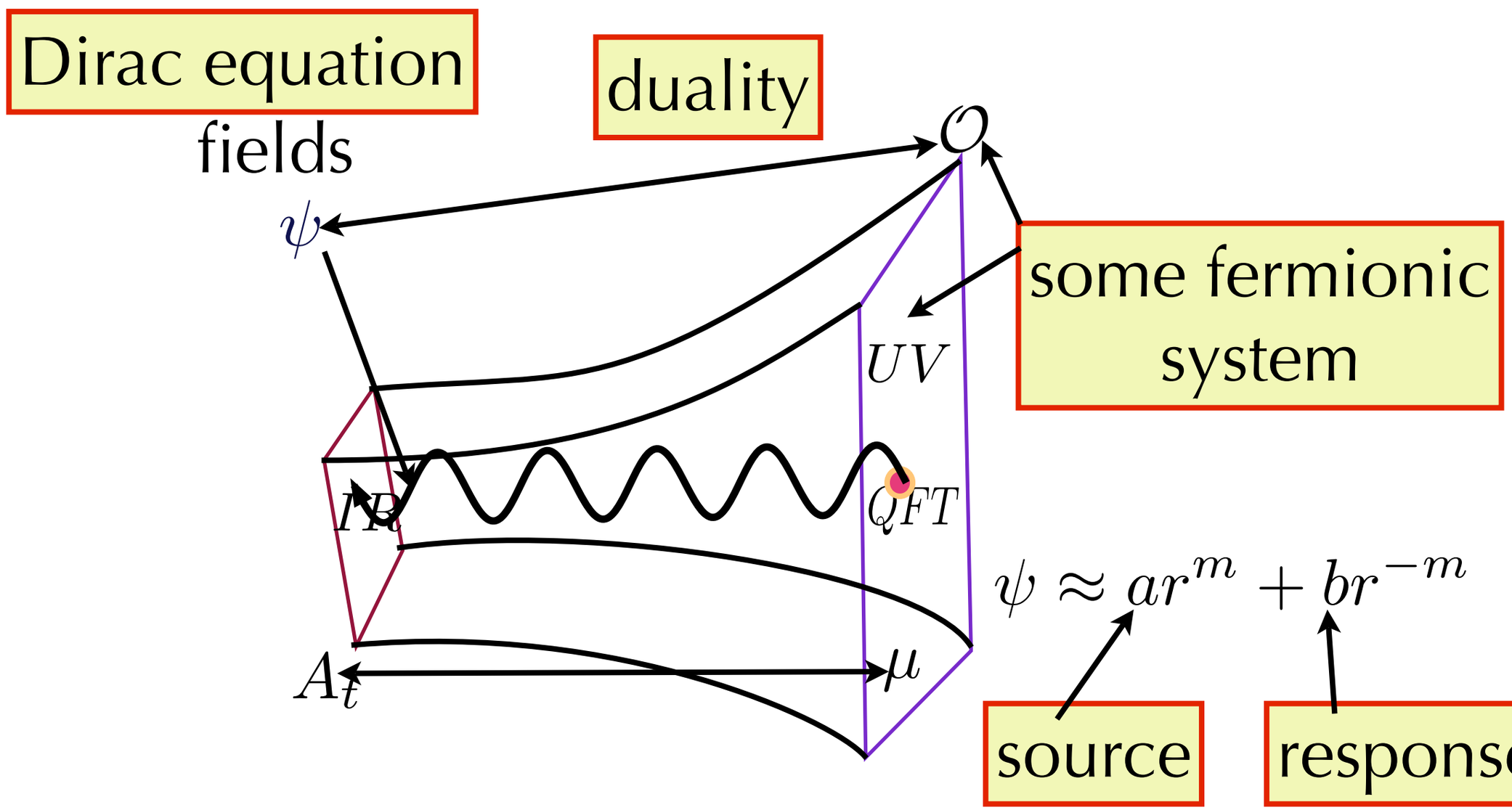
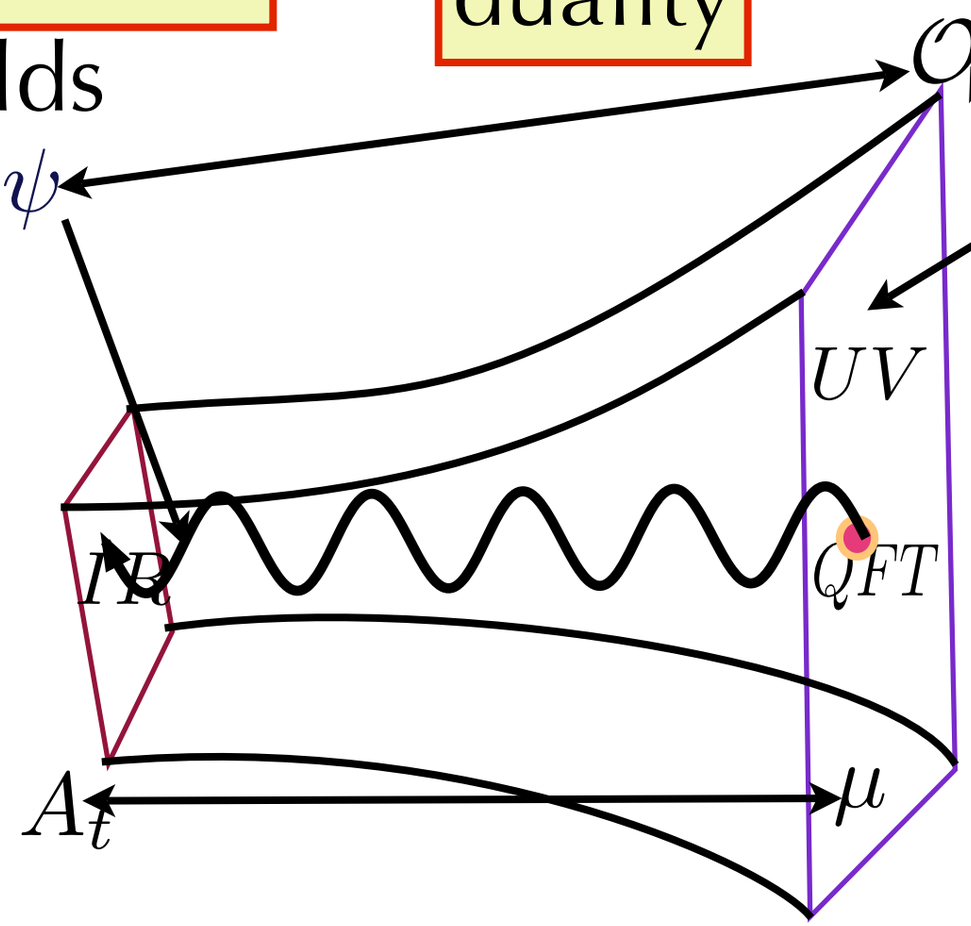


How does it work?

Dirac equation

fields

duality



some fermionic system

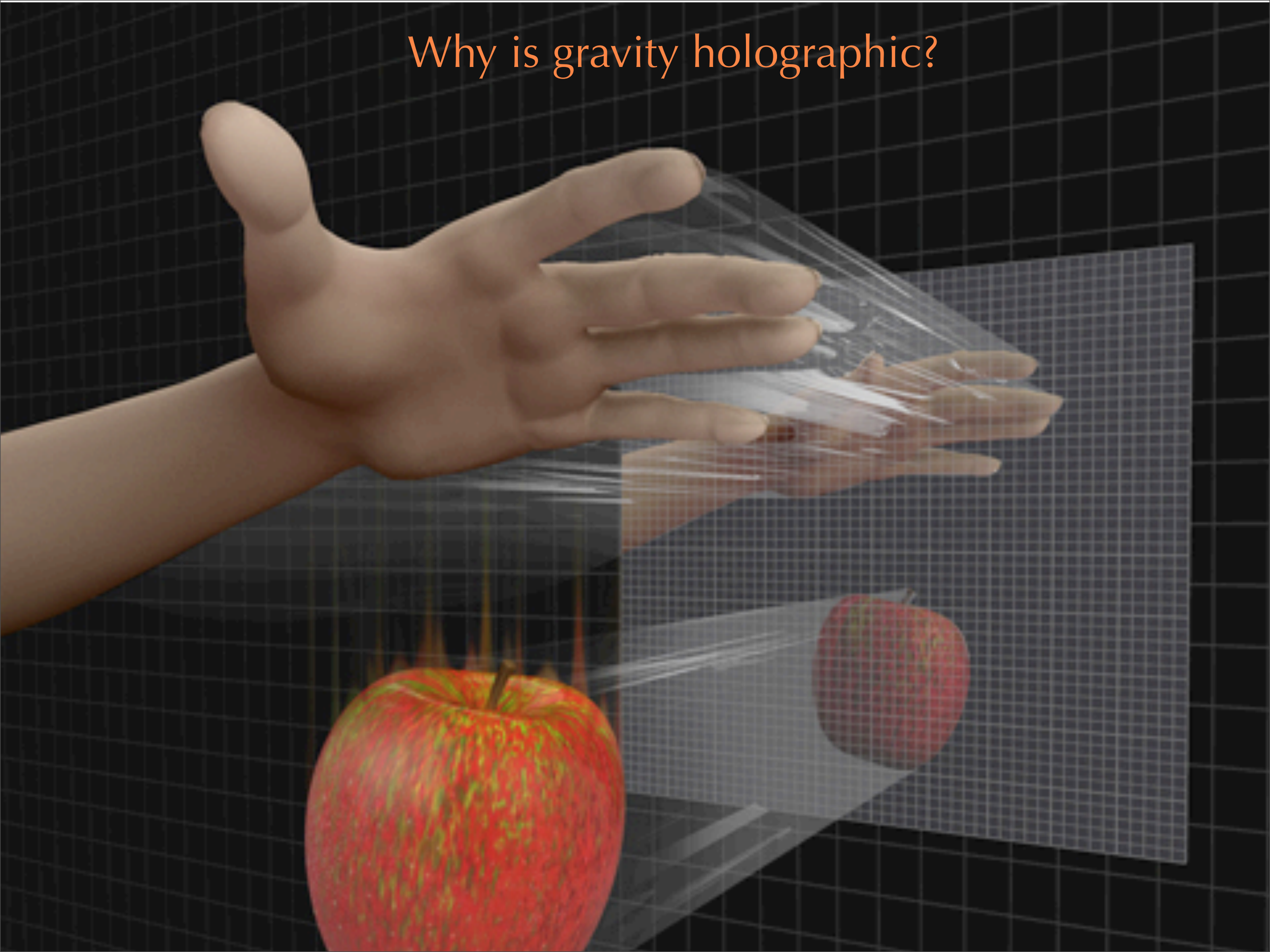
$$\psi \approx ar^m + br^{-m}$$

source

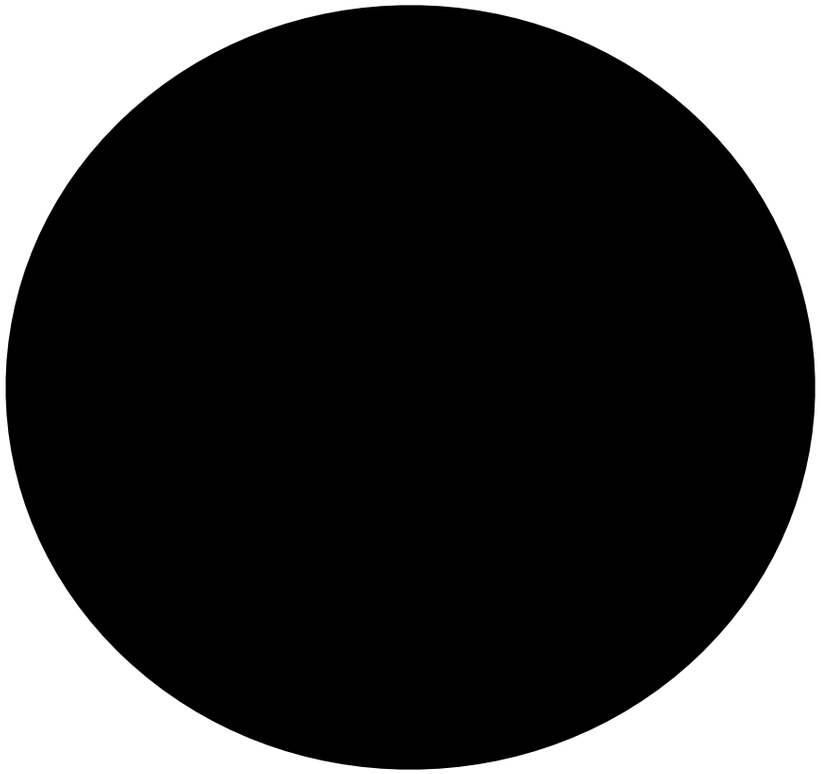
response

Green function: $G_{\mathcal{O}} = \frac{b}{a}$ =f(UV,IR)

Why is gravity holographic?



Why is gravity holographic?



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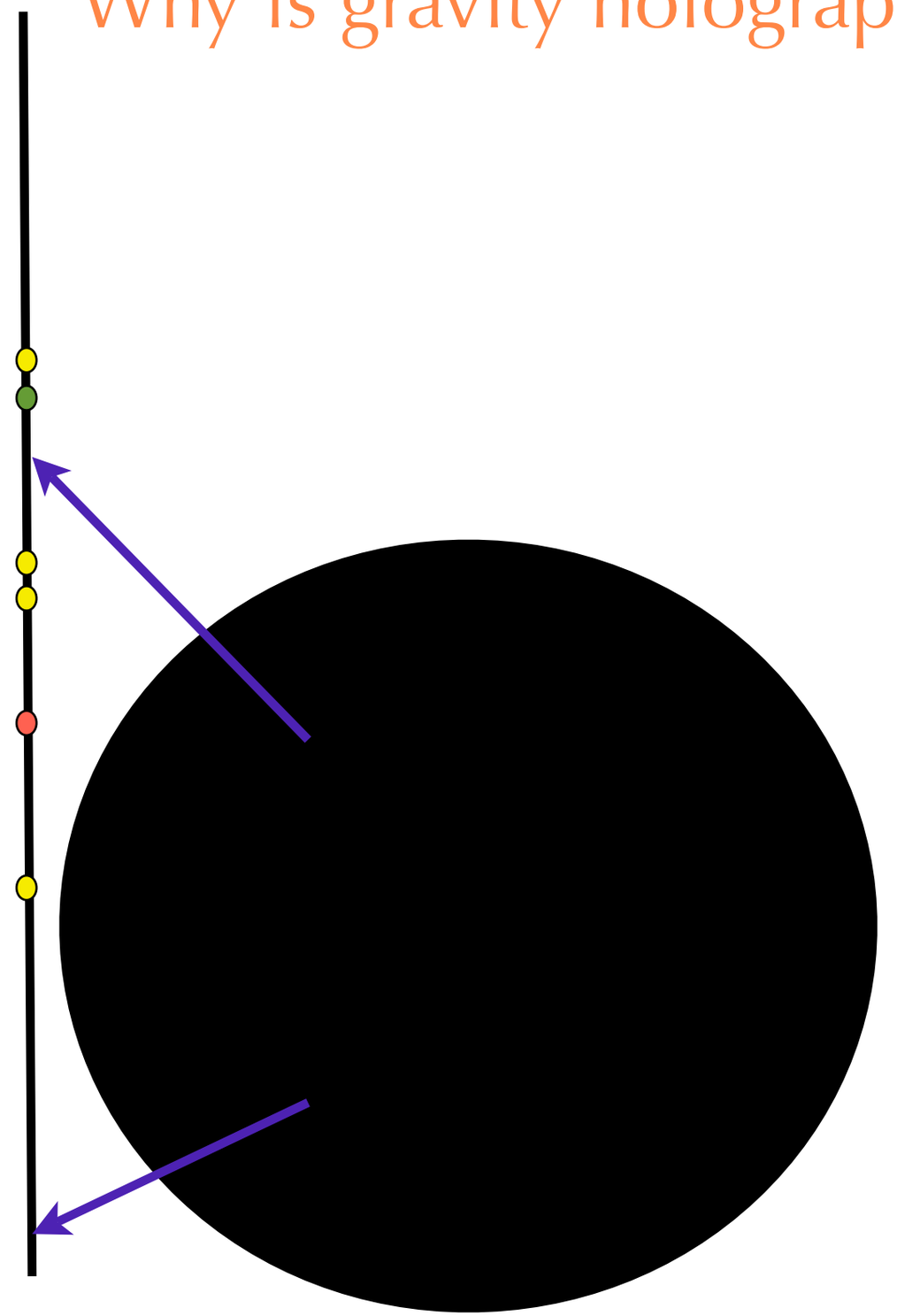

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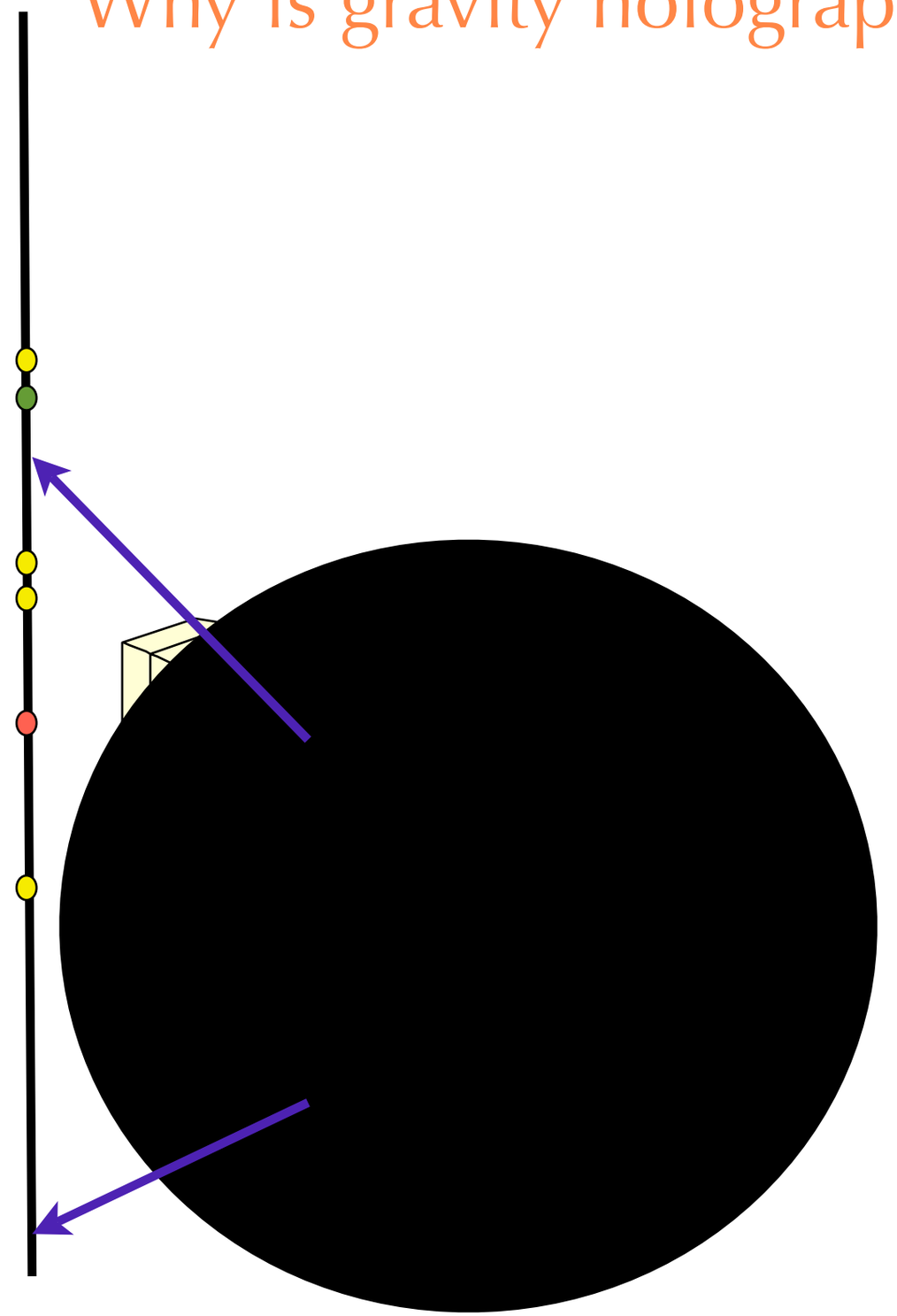
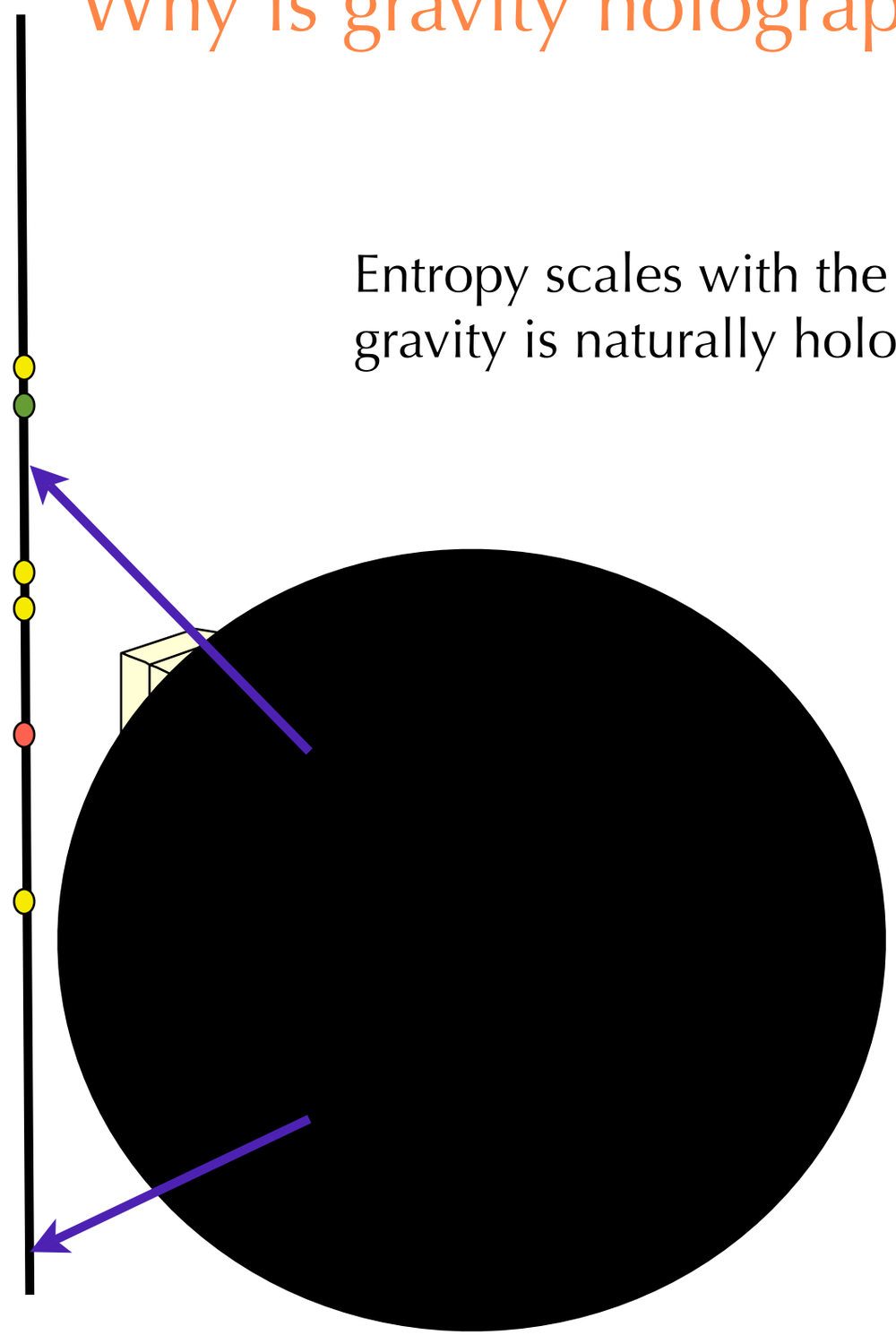
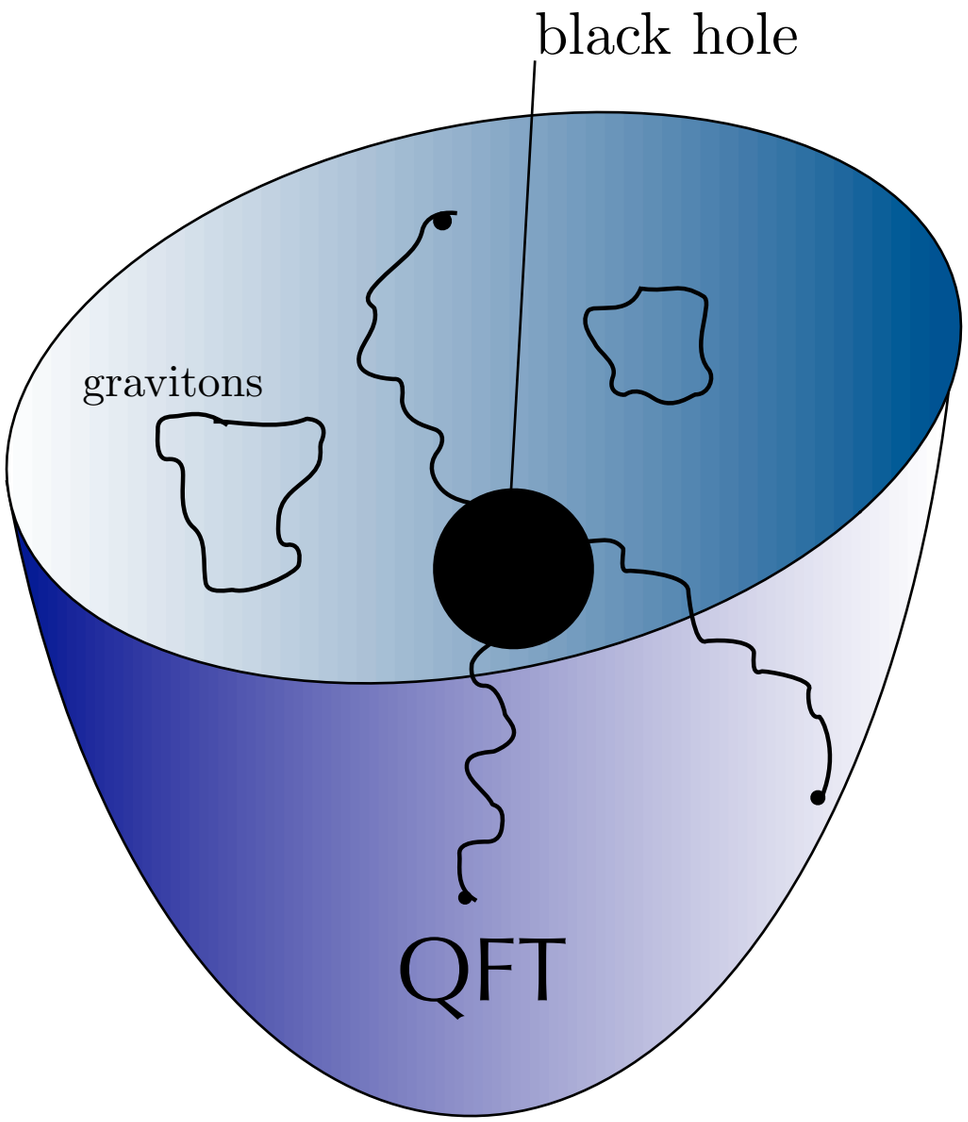


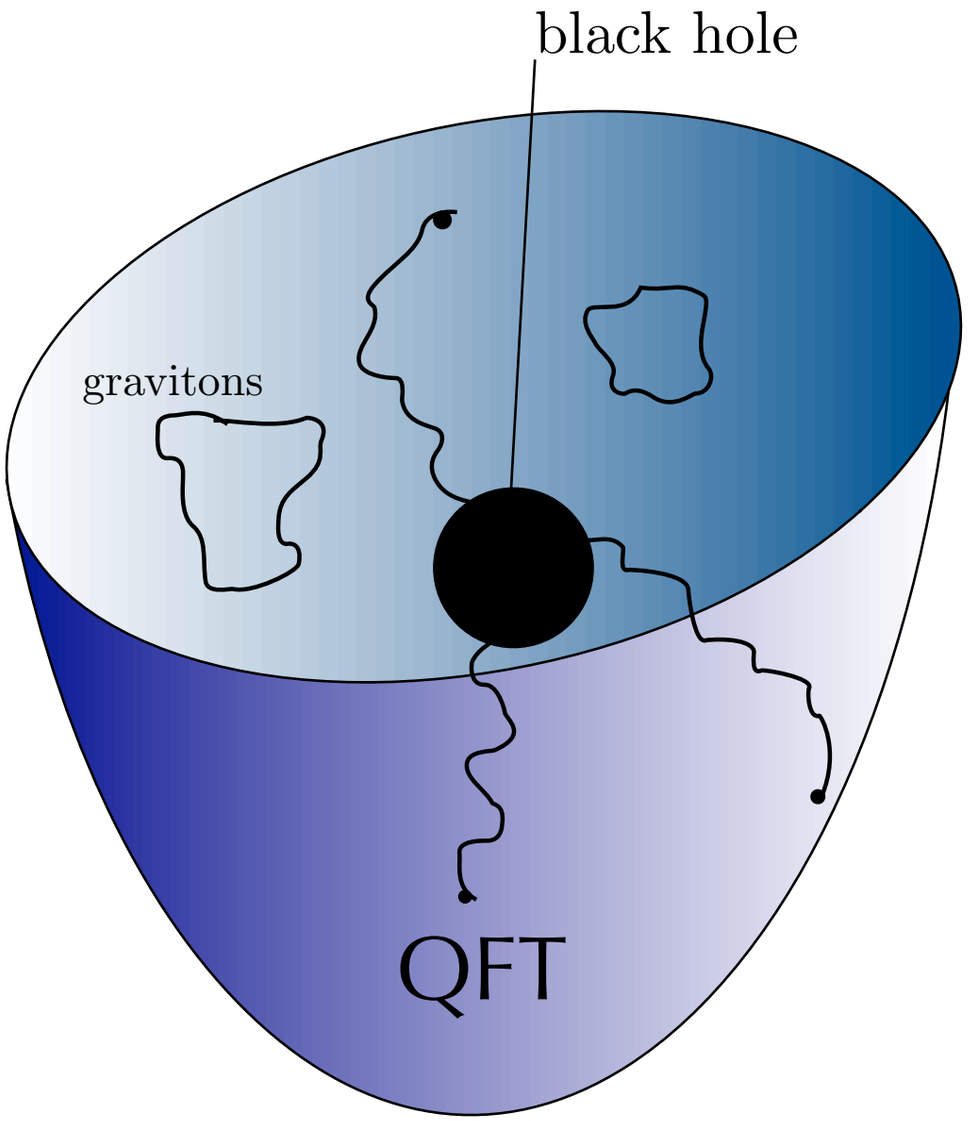
Why is gravity holographic?

Entropy scales with the area not the volume:
gravity is naturally holographic




black hole
gravitons
QFT

$$Z_{\mathrm{QFT}} = e^{-S_{\mathrm{ADS}}^{\mathrm{on-shell}}(\phi(\phi_{\partial \mathrm{ADS}=\mathrm{J}_{\mathcal{O}}}))}$$



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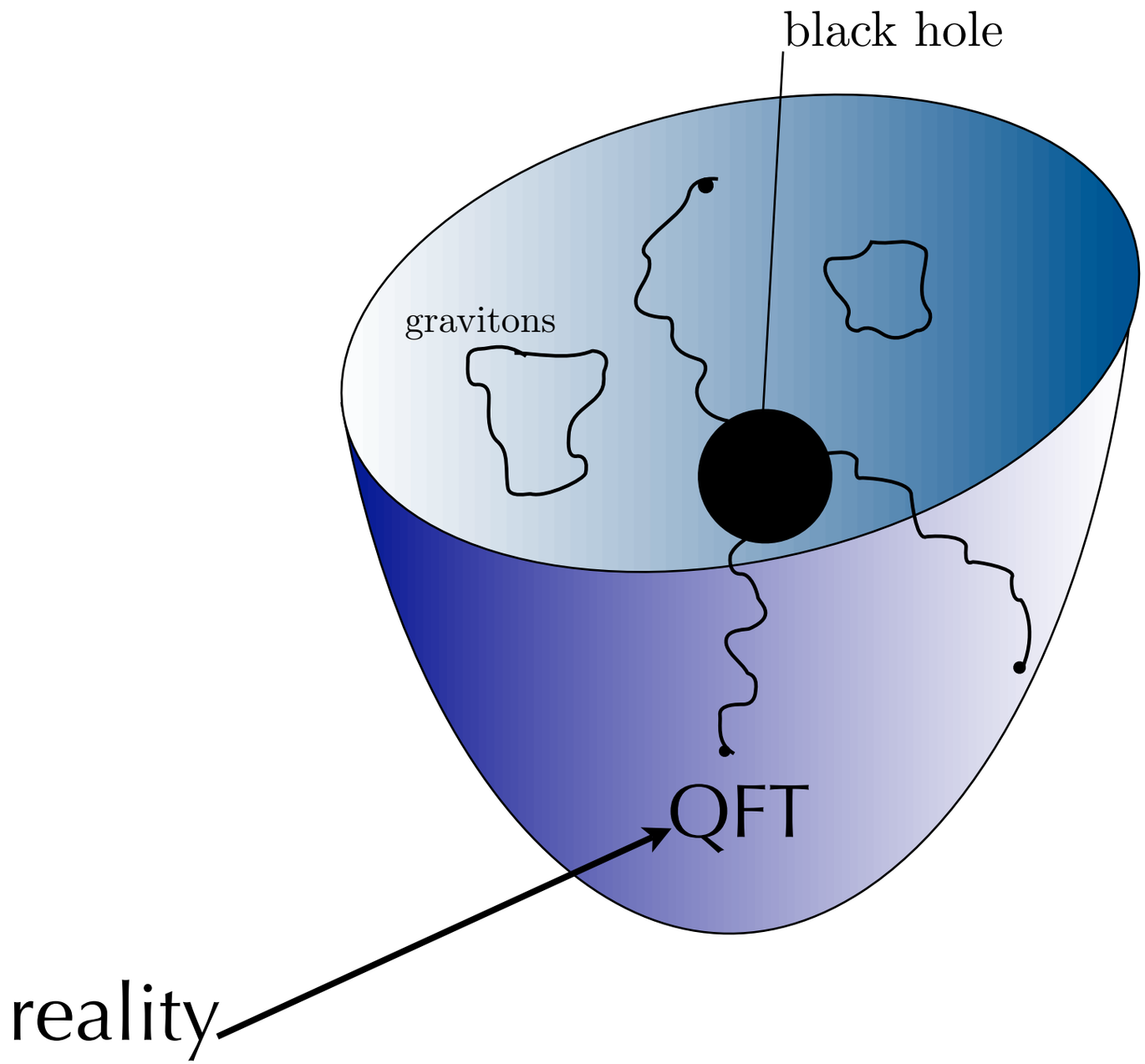

What holography does for you?

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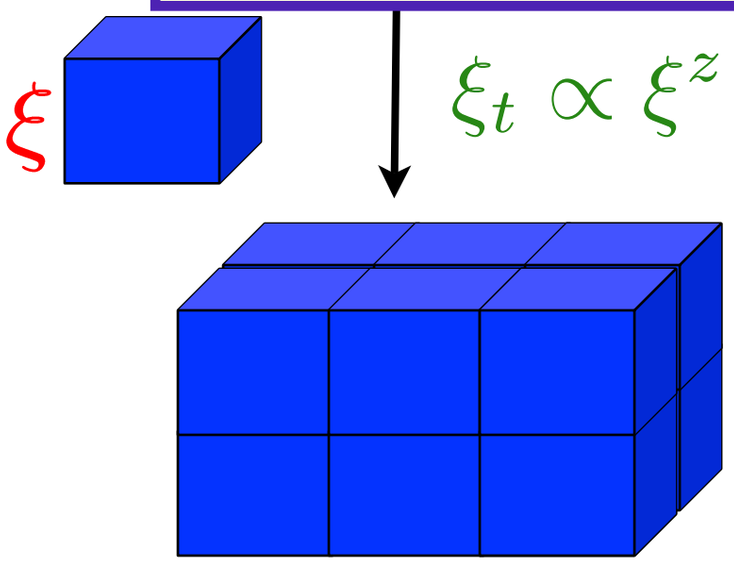
Landau-Wilson

Hamiltonian



long-wavelengths

RG equations

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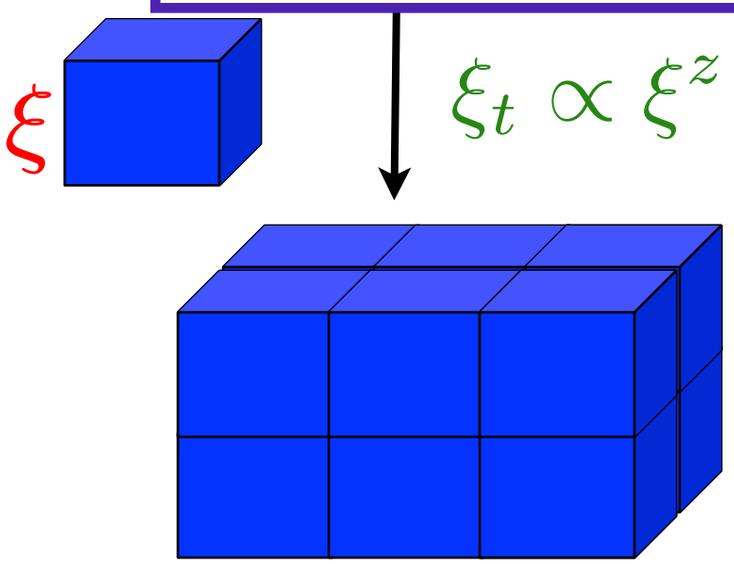

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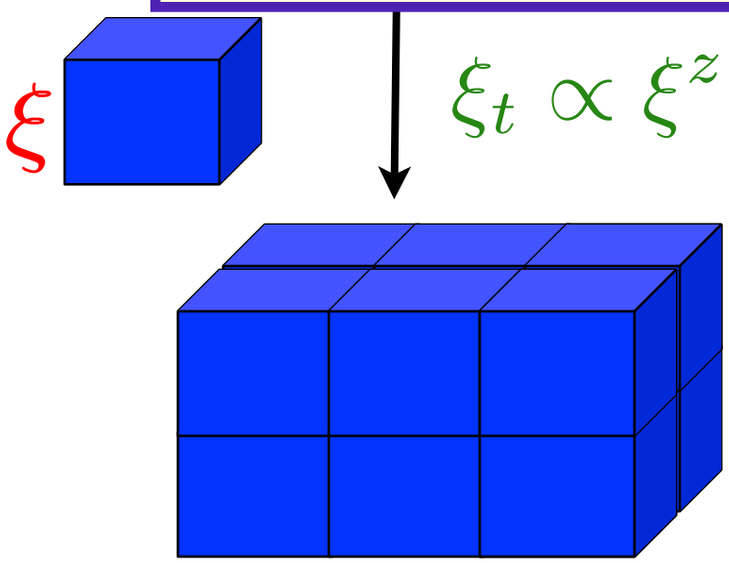

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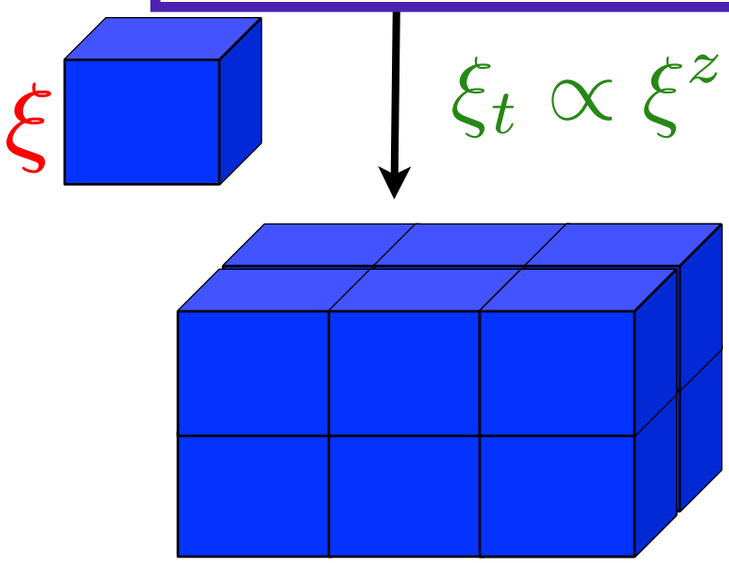

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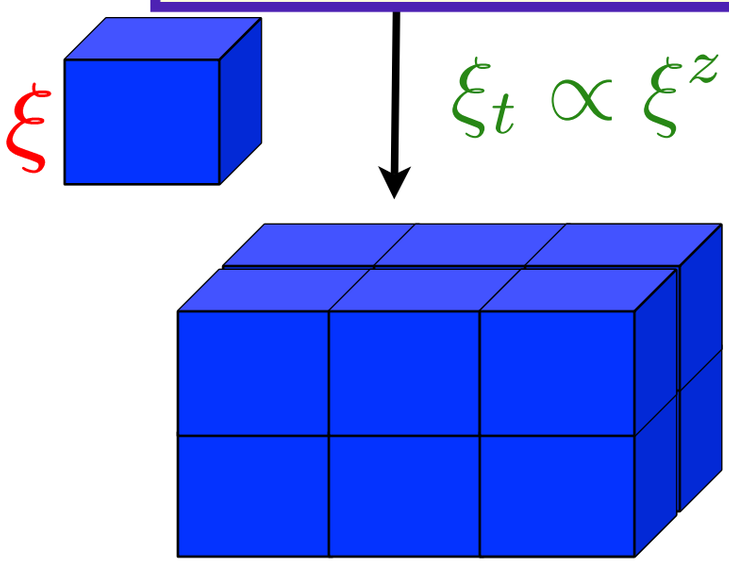

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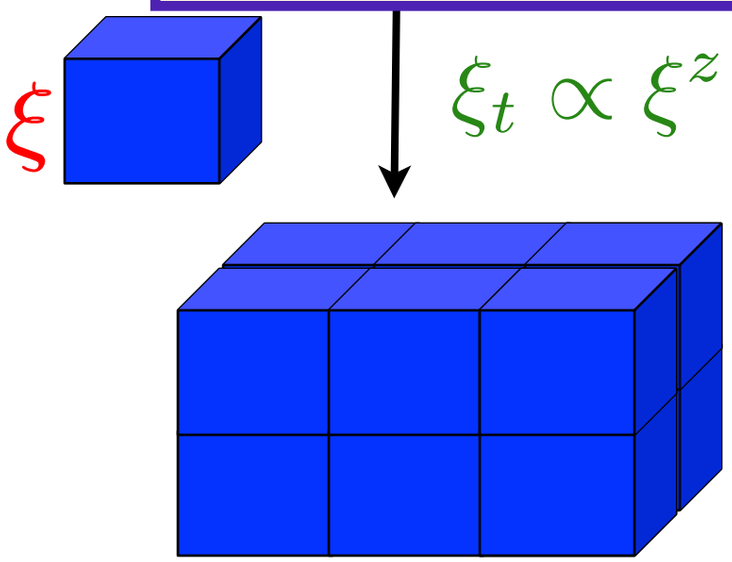

long-wavelengths

RG equations

holography

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strong-coupling is easy

microscopic UV model not easy (need M-theory)

so what (currents, symmetries)

Can holography solve the Mott problem?

What is a Mott Insulator?

NiO insulates

d^8 ?

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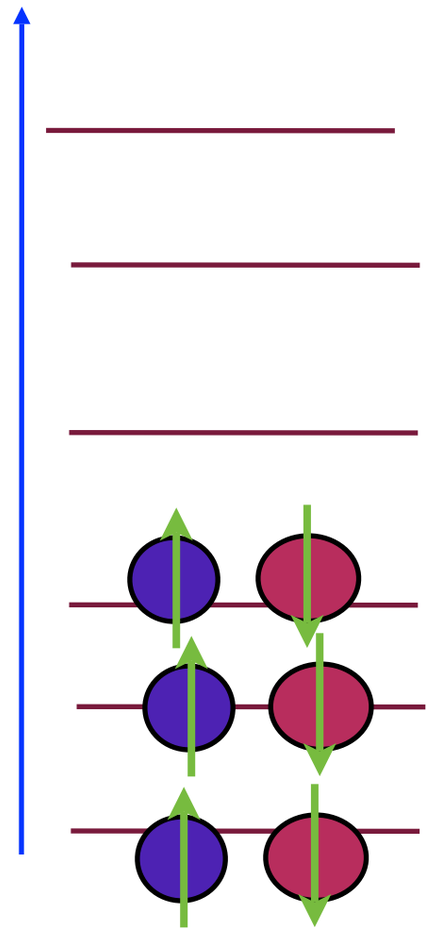
d^8 ?



EMPTY STATES=
METAL

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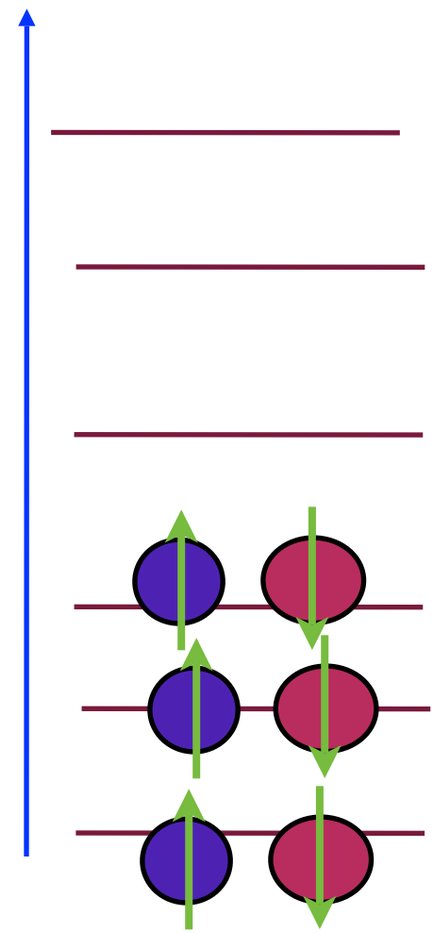


EMPTY STATES=
METAL

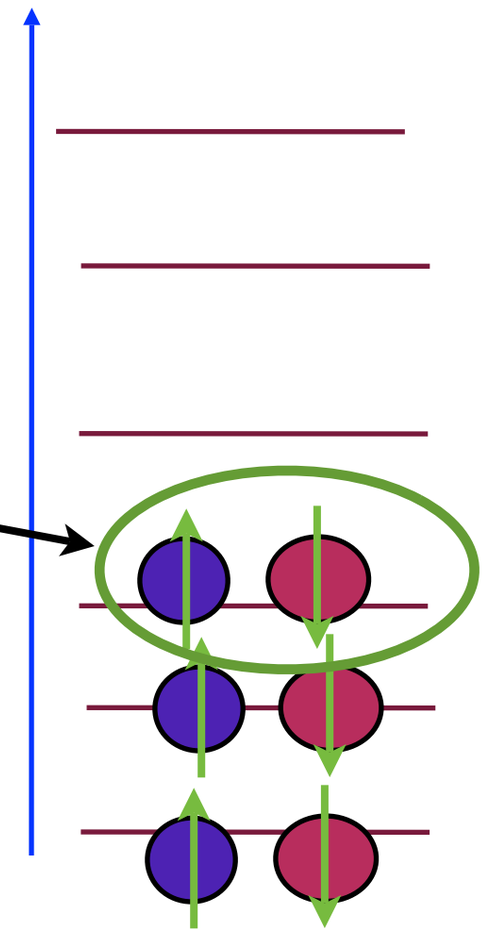
band theory fails!

What is a Mott Insulator?

NiO insulates

d^8 ?

perhaps this costs energy

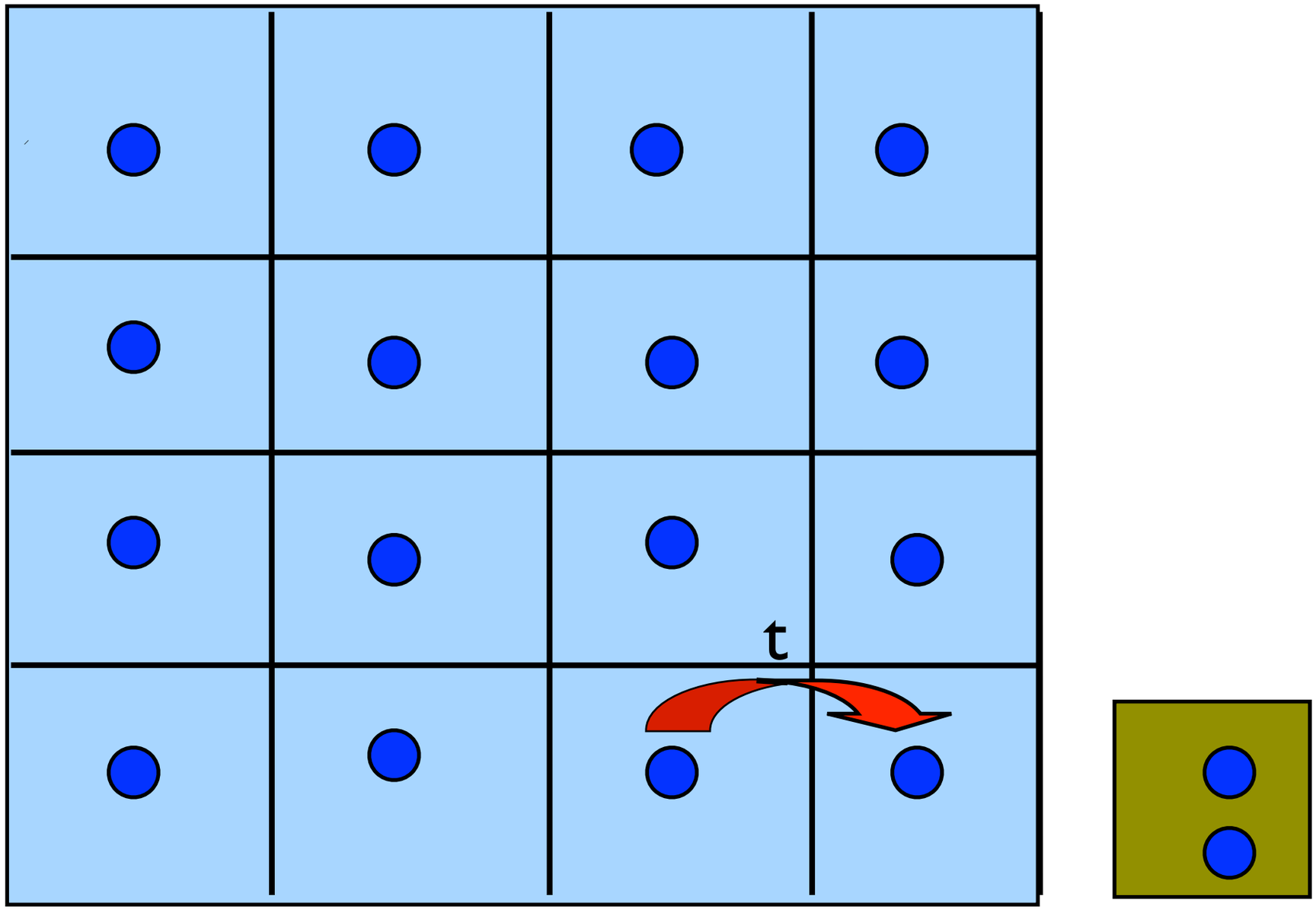


EMPTY STATES= METAL

band theory fails!

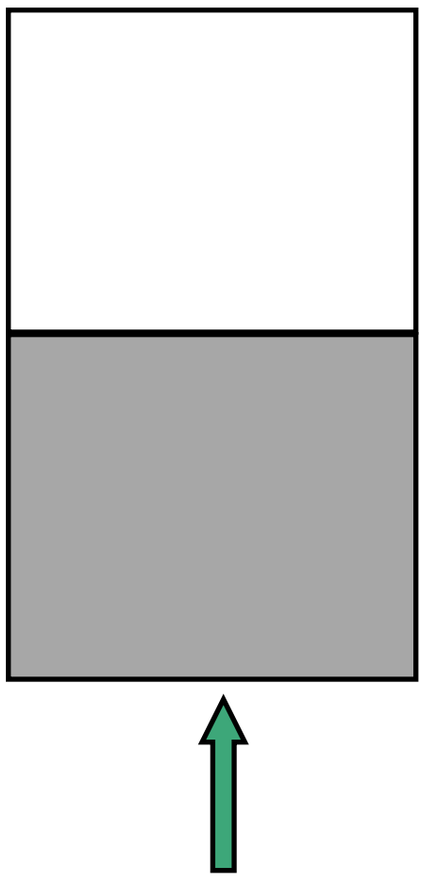
Mott Problem: NiO (Band theory failure)

(N rooms N occupants)

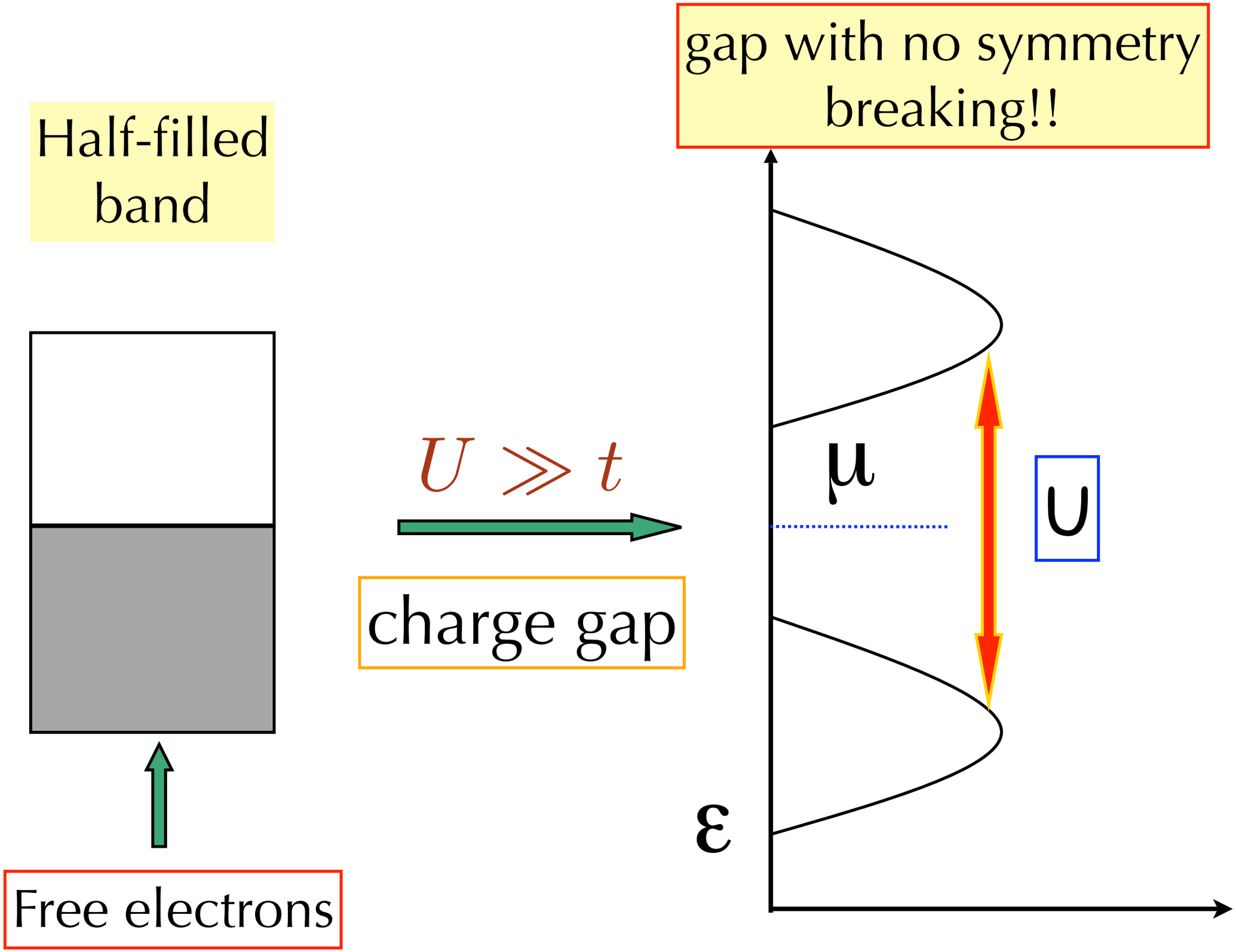


$U \gg t$

Half-filled band



Free electrons

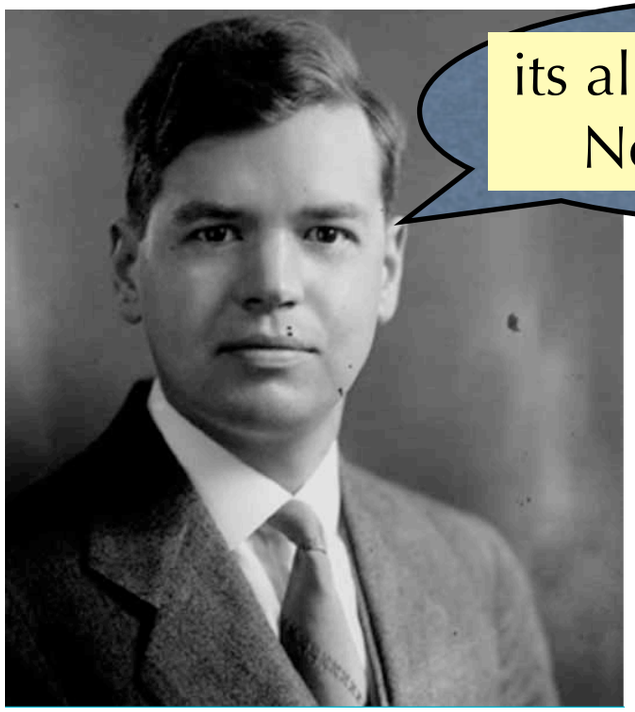

gap with no symmetry breaking!!
Half-filled band
$U \gg t$
charge gap
μ
U
ε
Free electrons

Mott Insulator-Ordering

Mott Insulator-Ordering =Mottness

Slater

Mott Insulator-Ordering =Mottness



Slater

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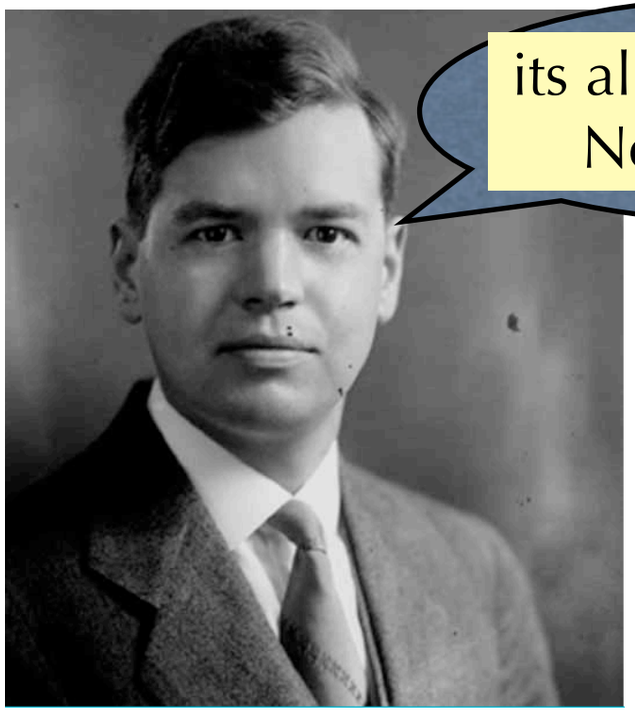


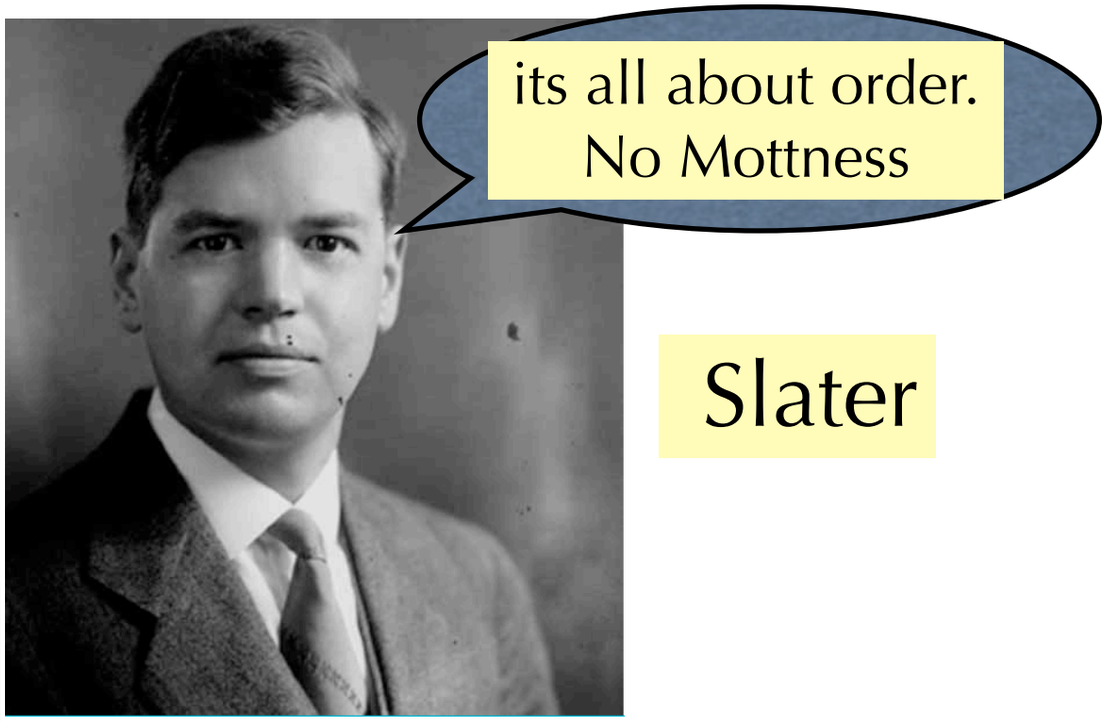
its all about order.
No Mottness

Slater

Mott Insulator-Ordering =Mottness

of course it does!!
order is secondary

Anderson



Slater

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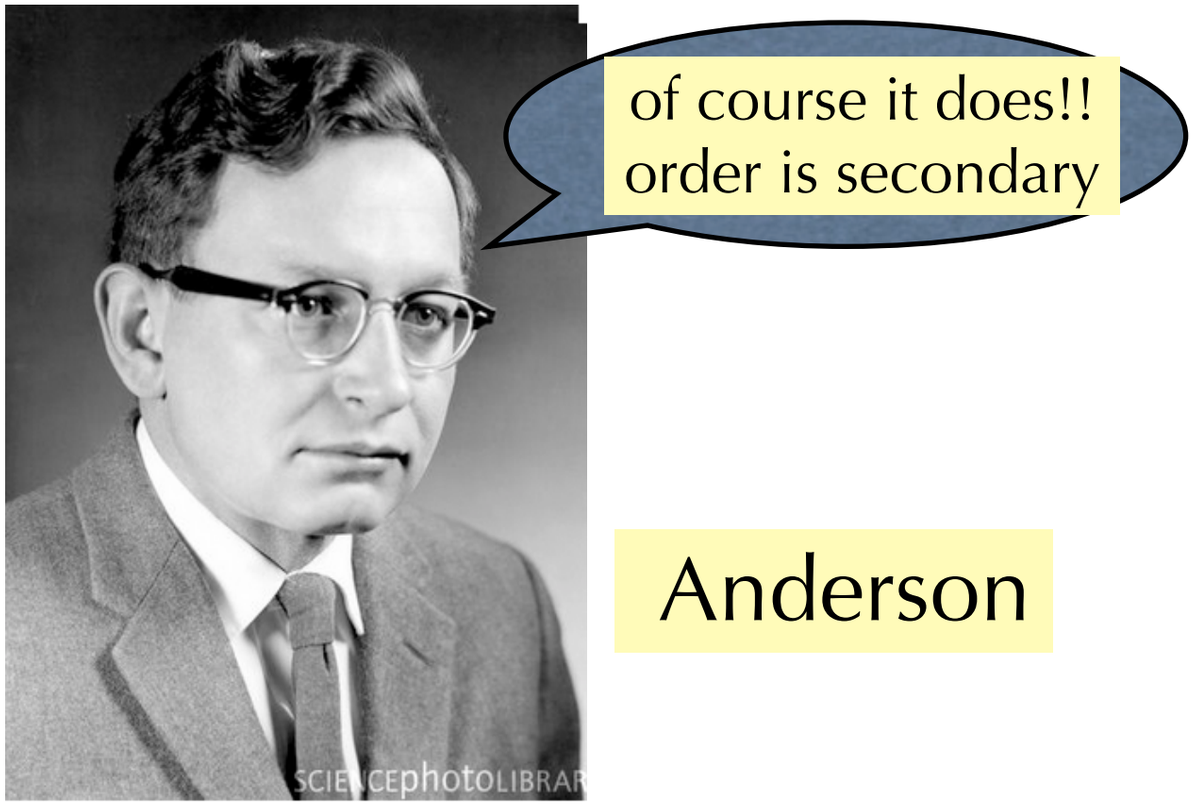
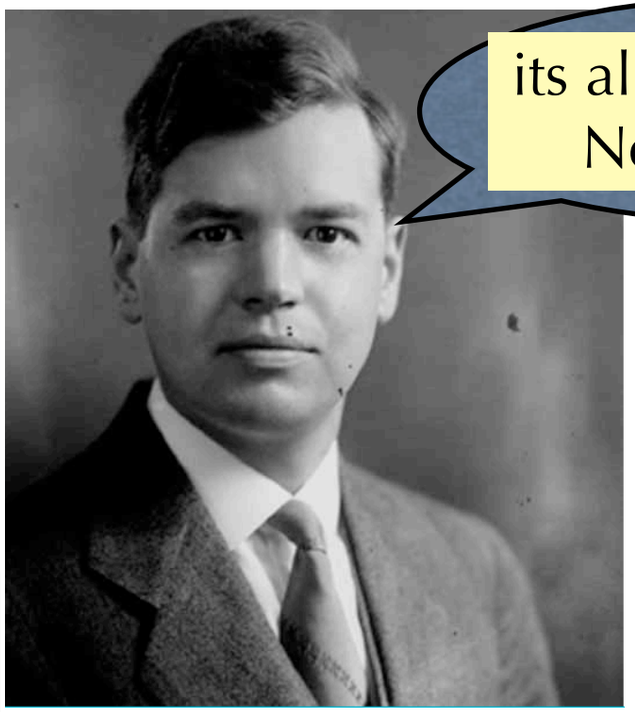


Anderson



Slater

Mott Insulator-Ordering =Mottness

Anderson

Why is the Mott problem important?

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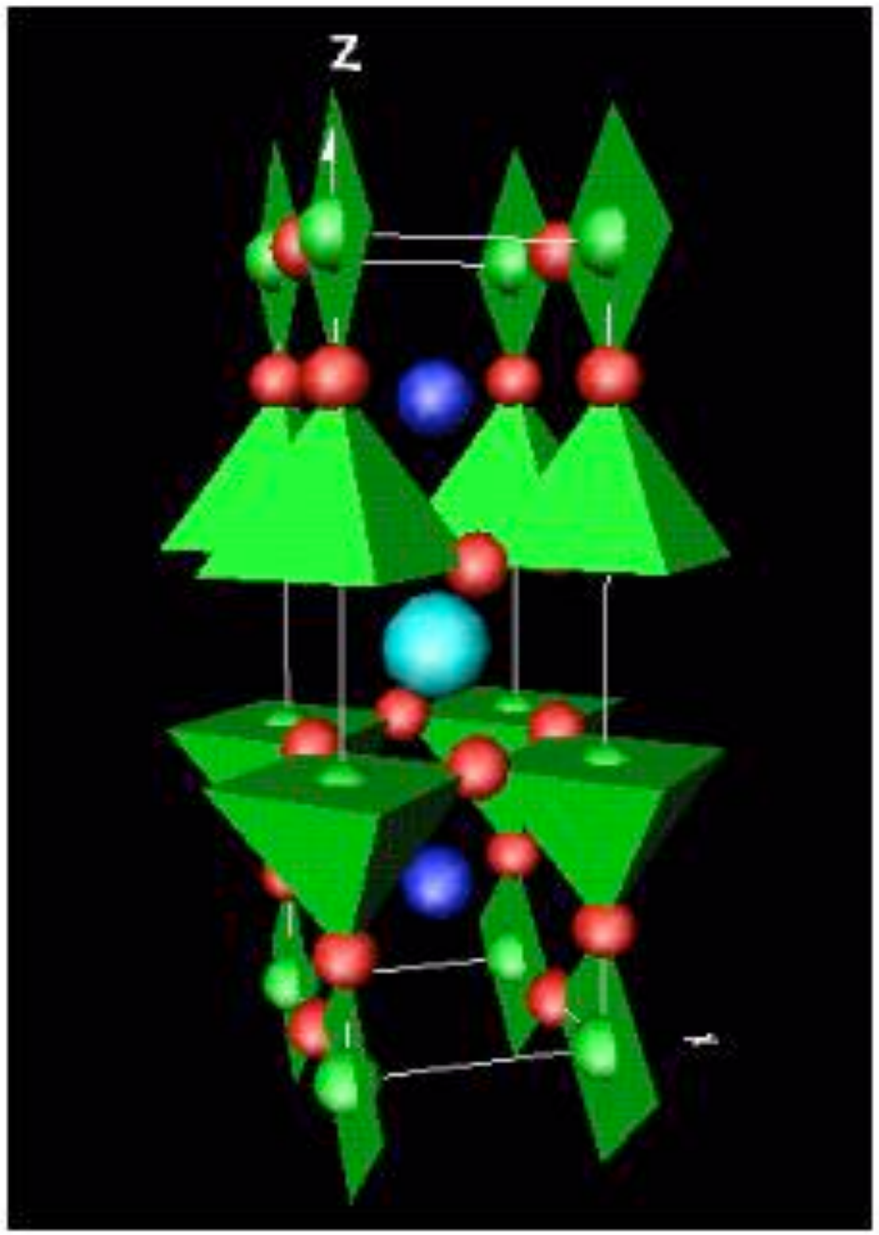

$YBa_2Cu_3O_7$

Cuprate Superconductors

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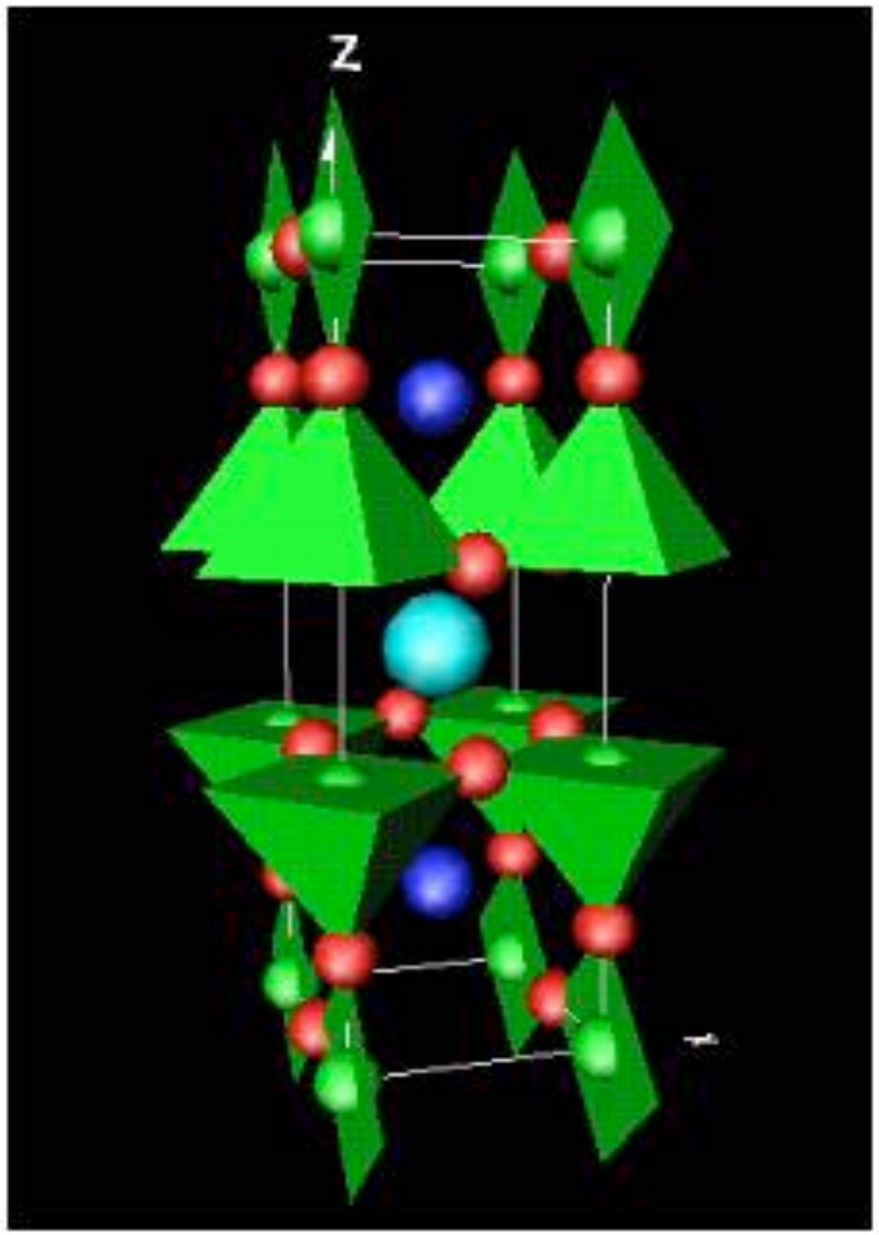

$Y\,Ba_2Cu_3O_7$
Cuprate Superconductors

$$U/t = 10 \gg 1$$

interactions dominate:
Strong Coupling Physics

Experimental facts: Mottness

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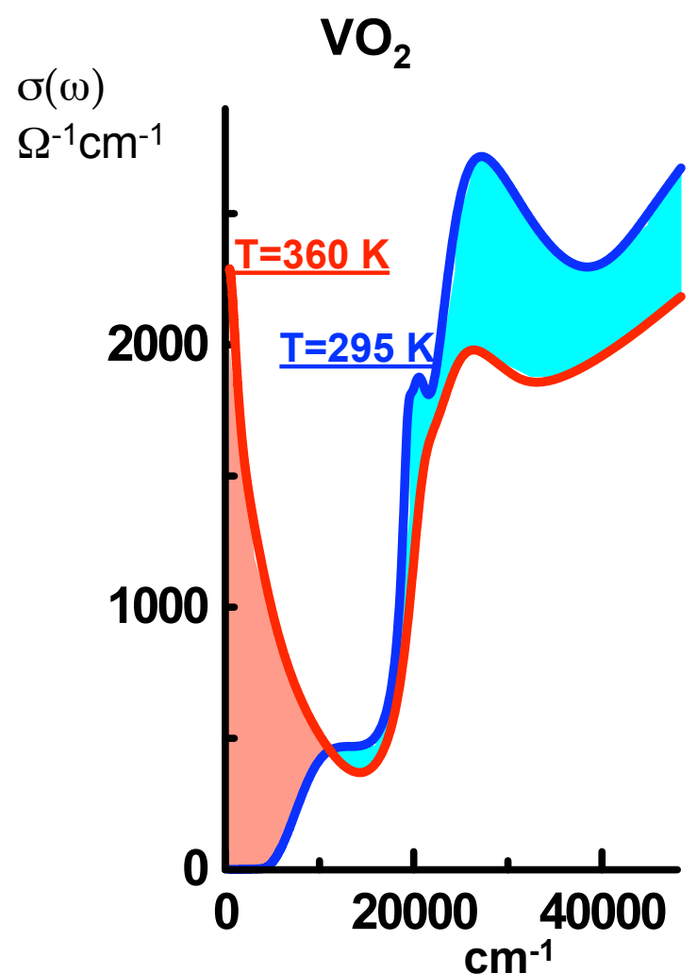

M. M. Qazilbash, K. S. Burch, D. Whisler, D. Shrekenhamer, B. G. Chae, H. T. Kim, and D. N. Basov PRB 74, 205118 (2006)

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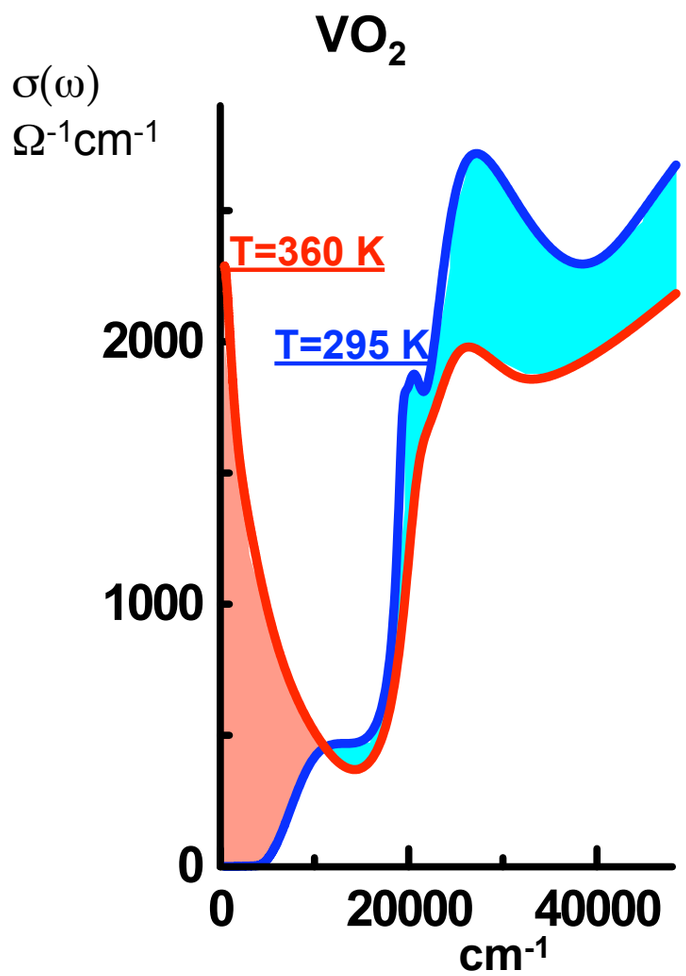

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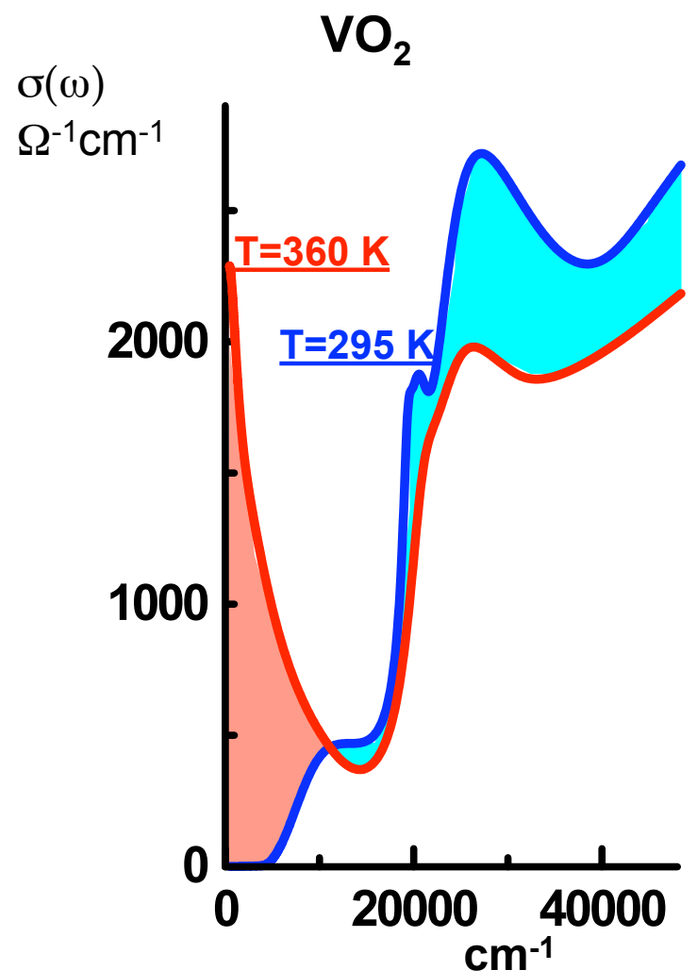
transfer of spectral weight to high energies beyond any ordering scale

Recall, $eV = 10^4 K$

Experimental facts: Mottness

$$\Delta = 0.6eV > \Delta_{\text{dimerization}}$$ (Mott, 1976) $$\frac{\Delta}{T_{\text{crit}}} \approx 20$$



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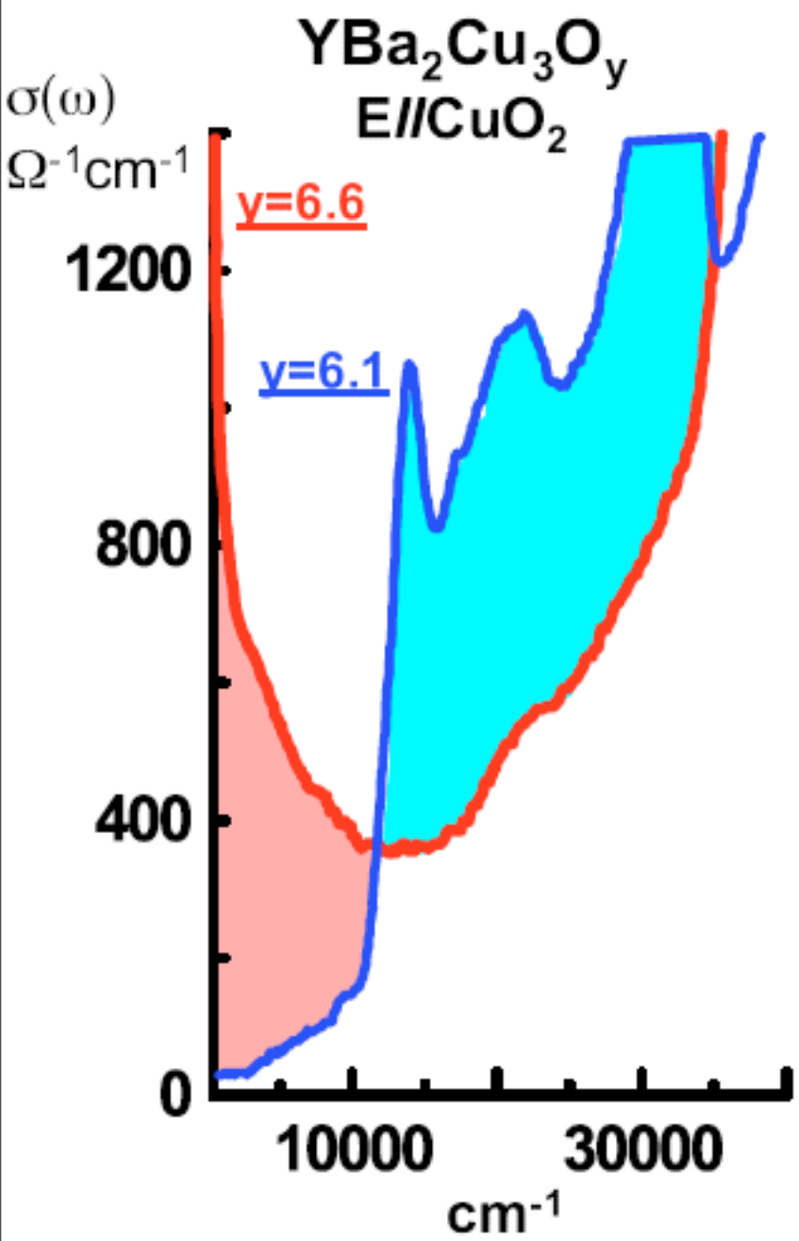

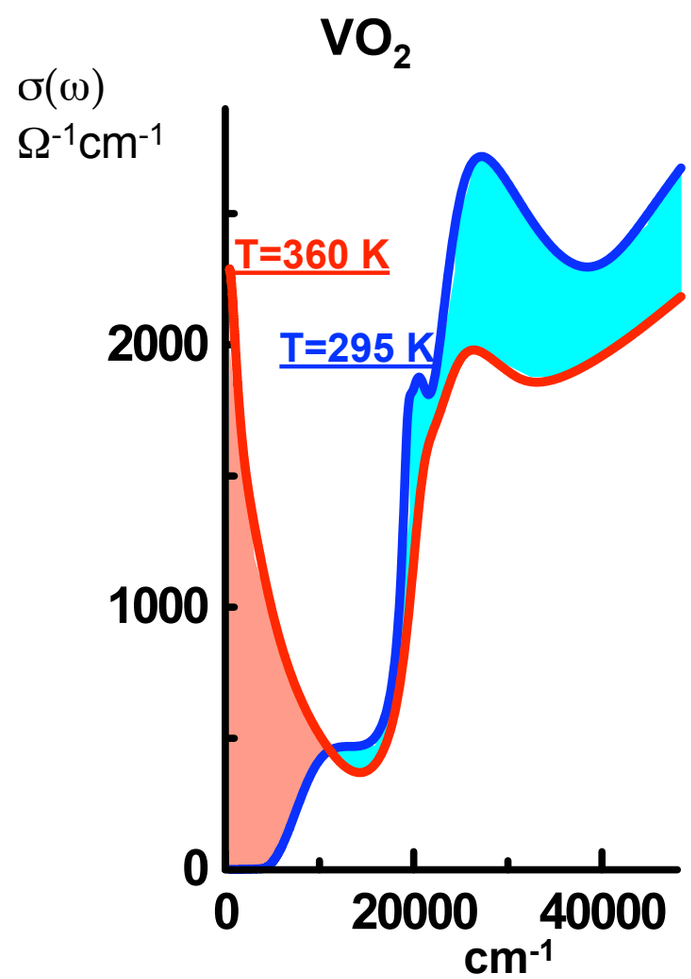

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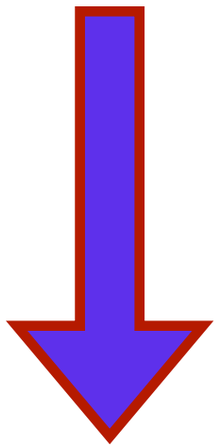

dynamically generated gap: Mott gap
(for probe fermions)

What has been done?

$$\sqrt{-g}i\bar{\psi}(D-m)\psi$$

AdS-RN
MIT, Leiden group

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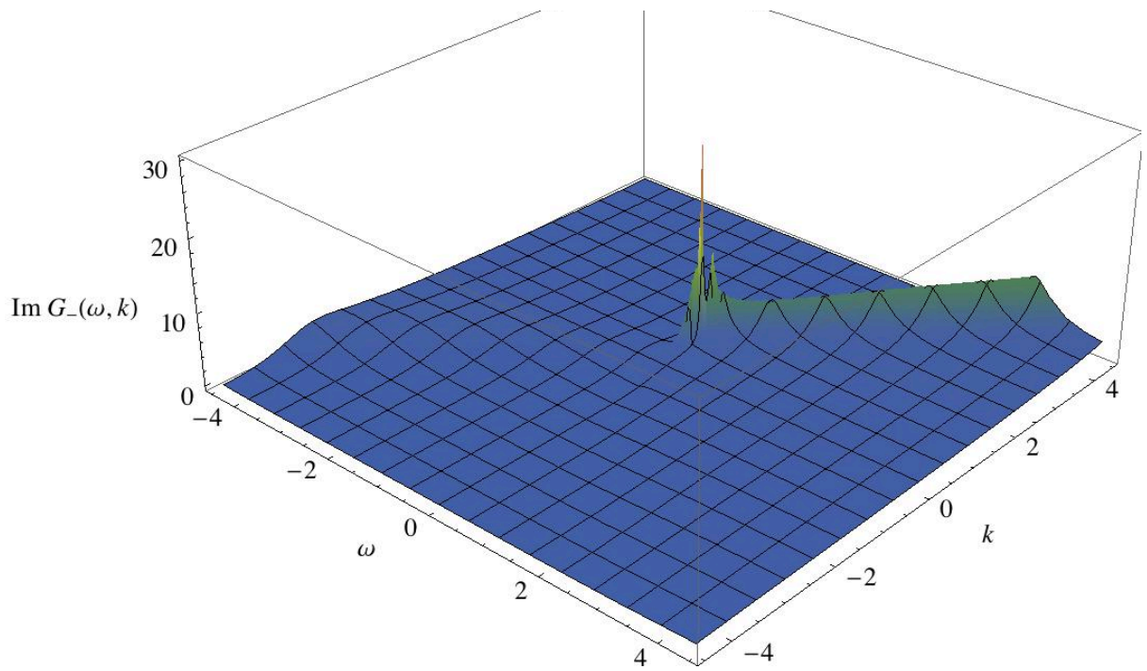

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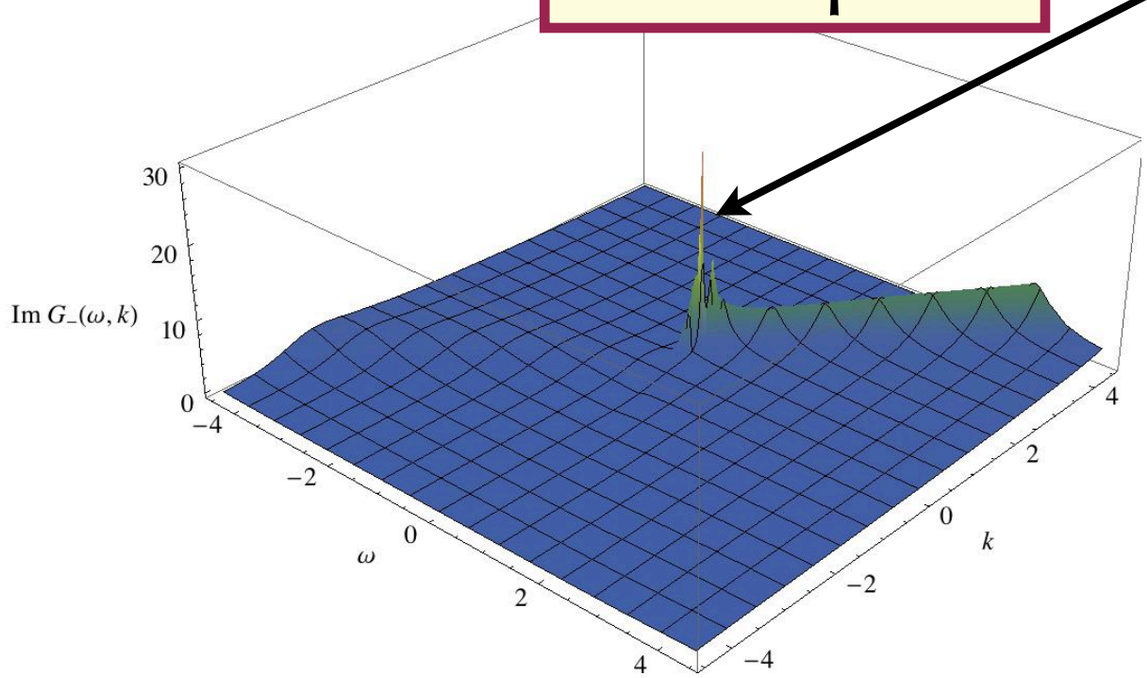
$$G(\omega,k)=\frac{Z}{v_F(k-k_F)-\omega-\Sigma}$$

Fermi peak



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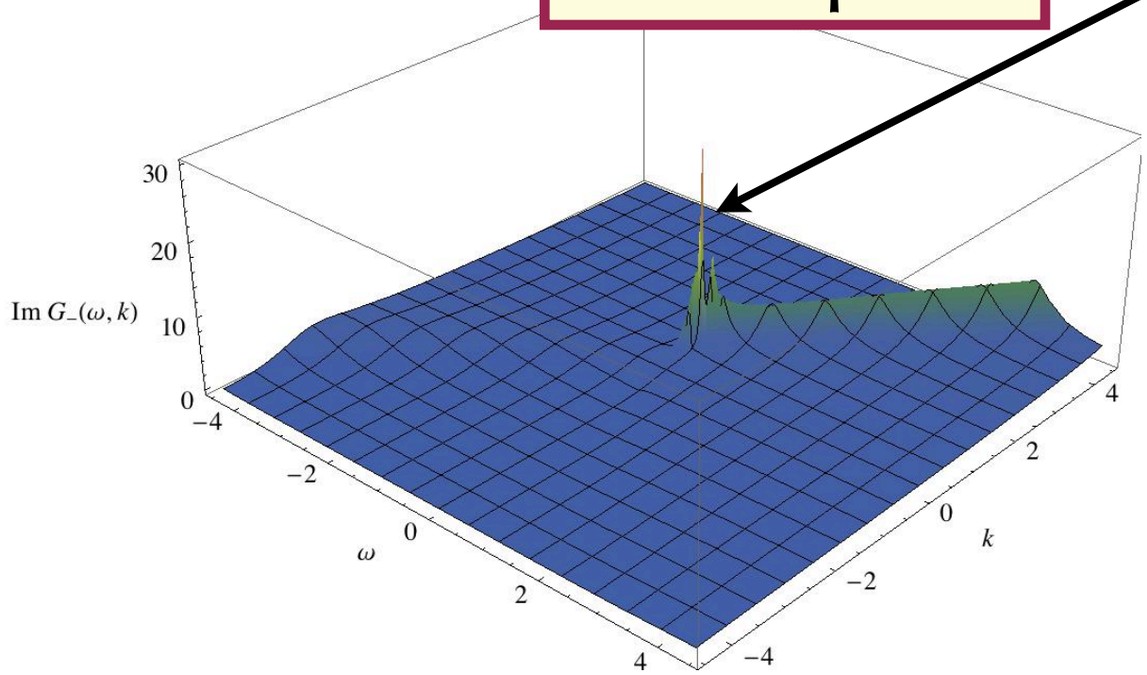

marginal Fermi liquid

?? → Mott Insulator

decoherence

$$S_{\text{probe}}(\psi, \bar{\psi}) = \int d^d x \sqrt{-g} i \bar{\psi} (\Gamma^M D_M - m + \cdots) \psi$$

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what is hidden here?

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what is hidden here?

$$F_{\mu\nu}\Gamma^{\mu\nu}$$

consider

$$\sqrt{-g} i \bar{\psi} (\not{D} - m - ip\not{F}) \psi$$

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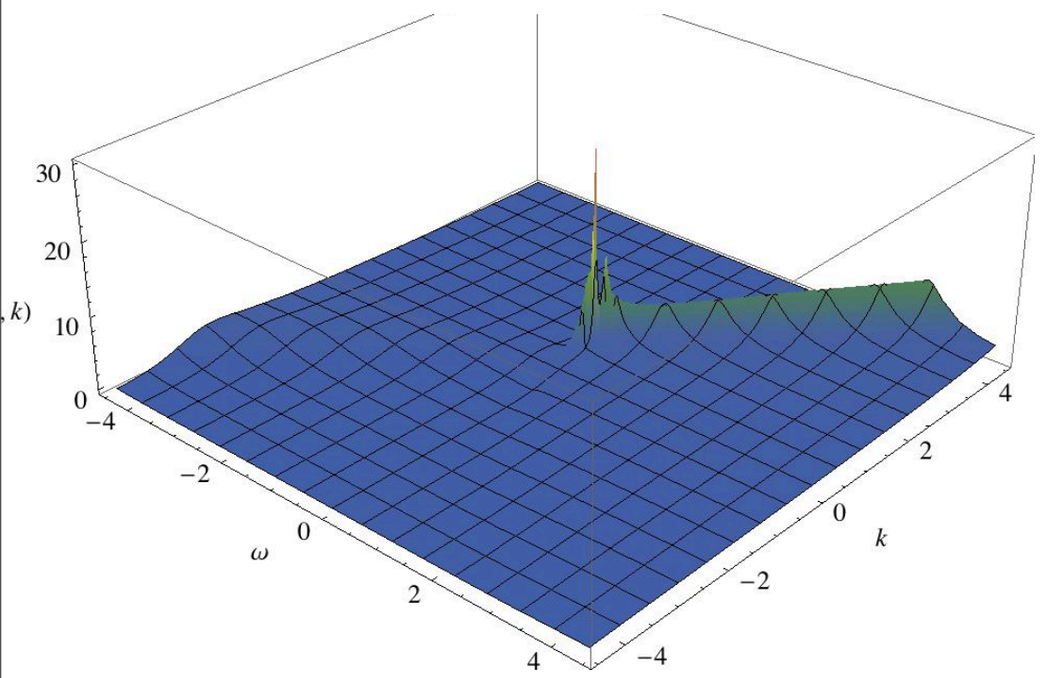
consider $\sqrt{-g} i \bar{\psi} (\not{D} - m - ip\not{F})\psi$

QED anomalous magnetic moment of an electron
(Schwinger 1949)

How is the spectrum modified?

P=0



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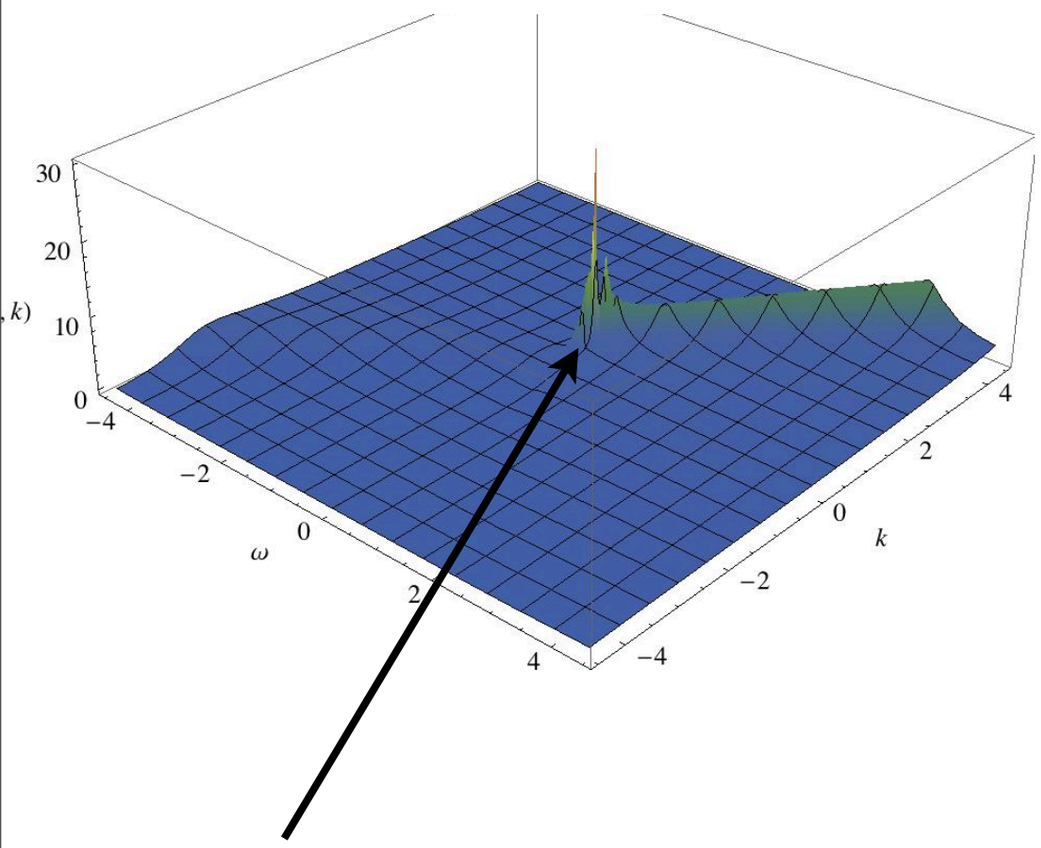
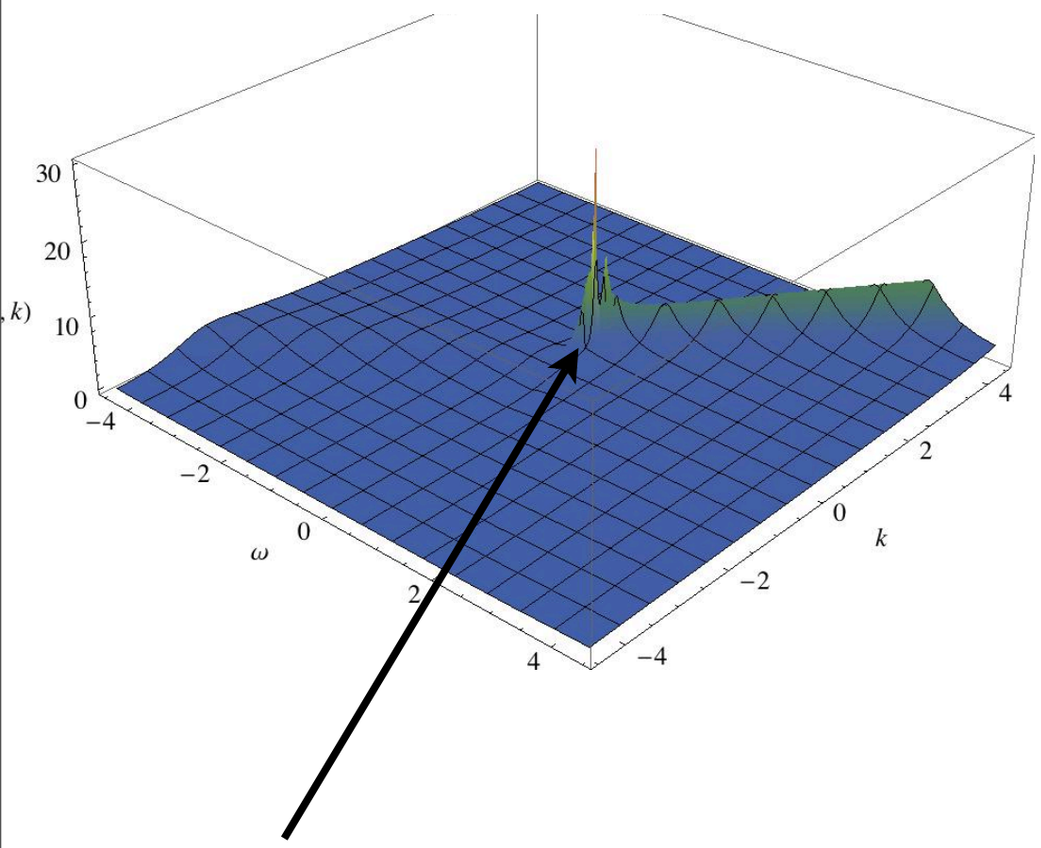


Fermi surface peak

How is the spectrum modified?

P=0

$P > 4.2$



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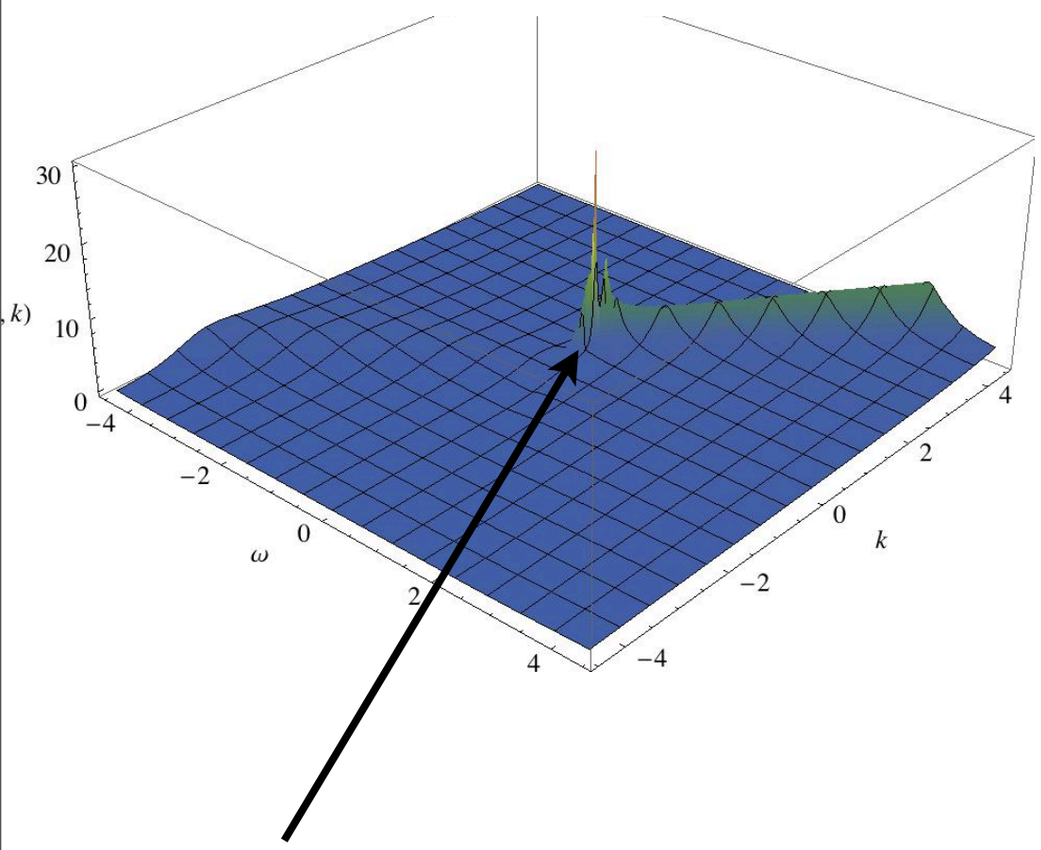

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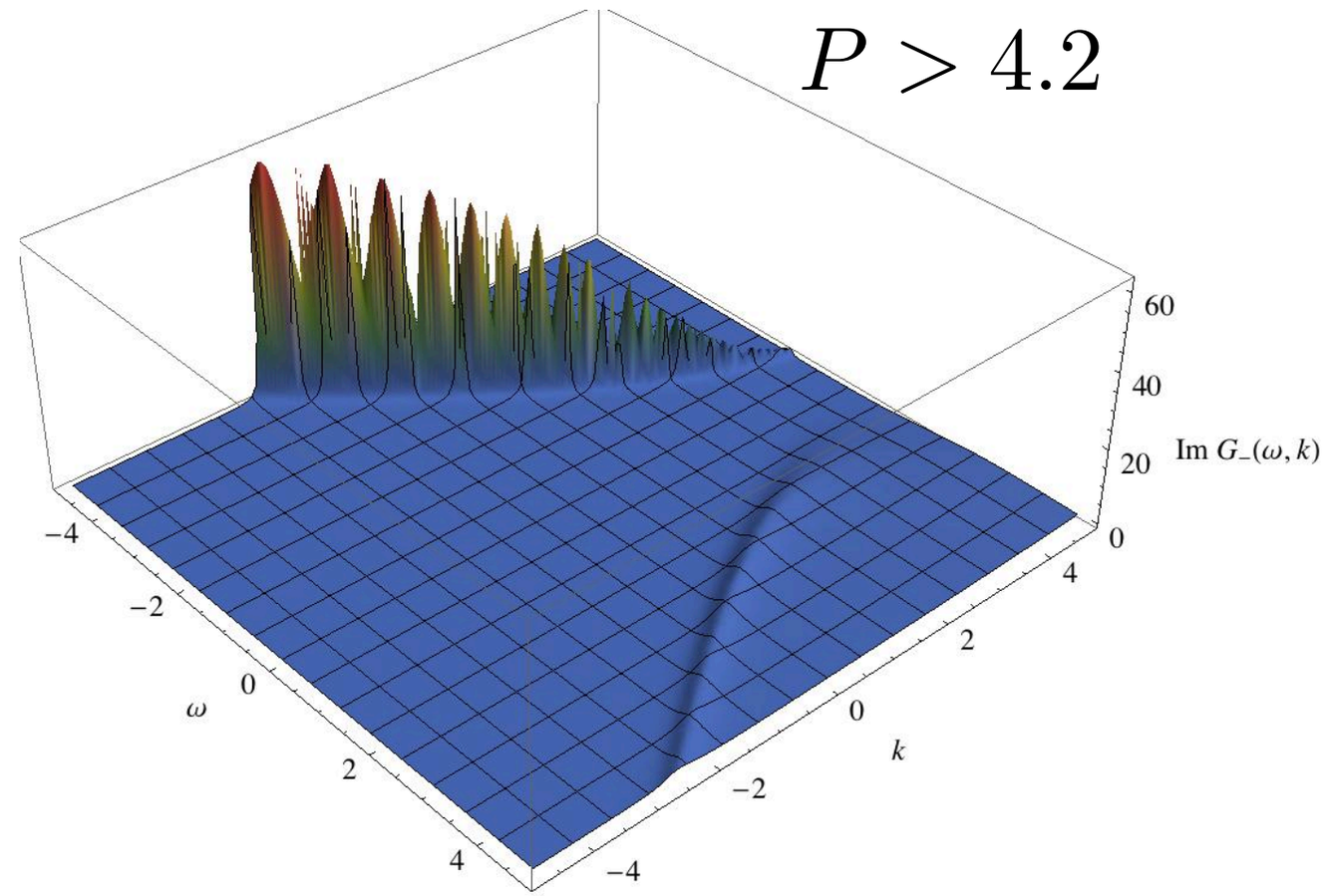

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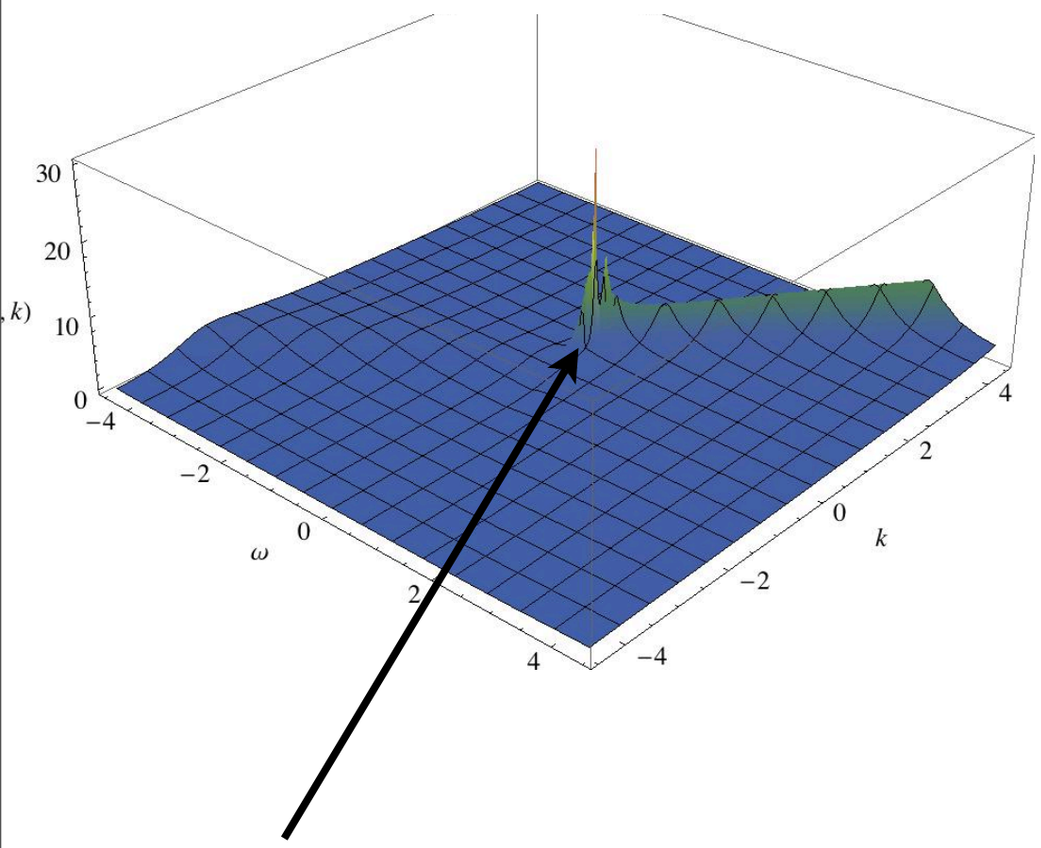

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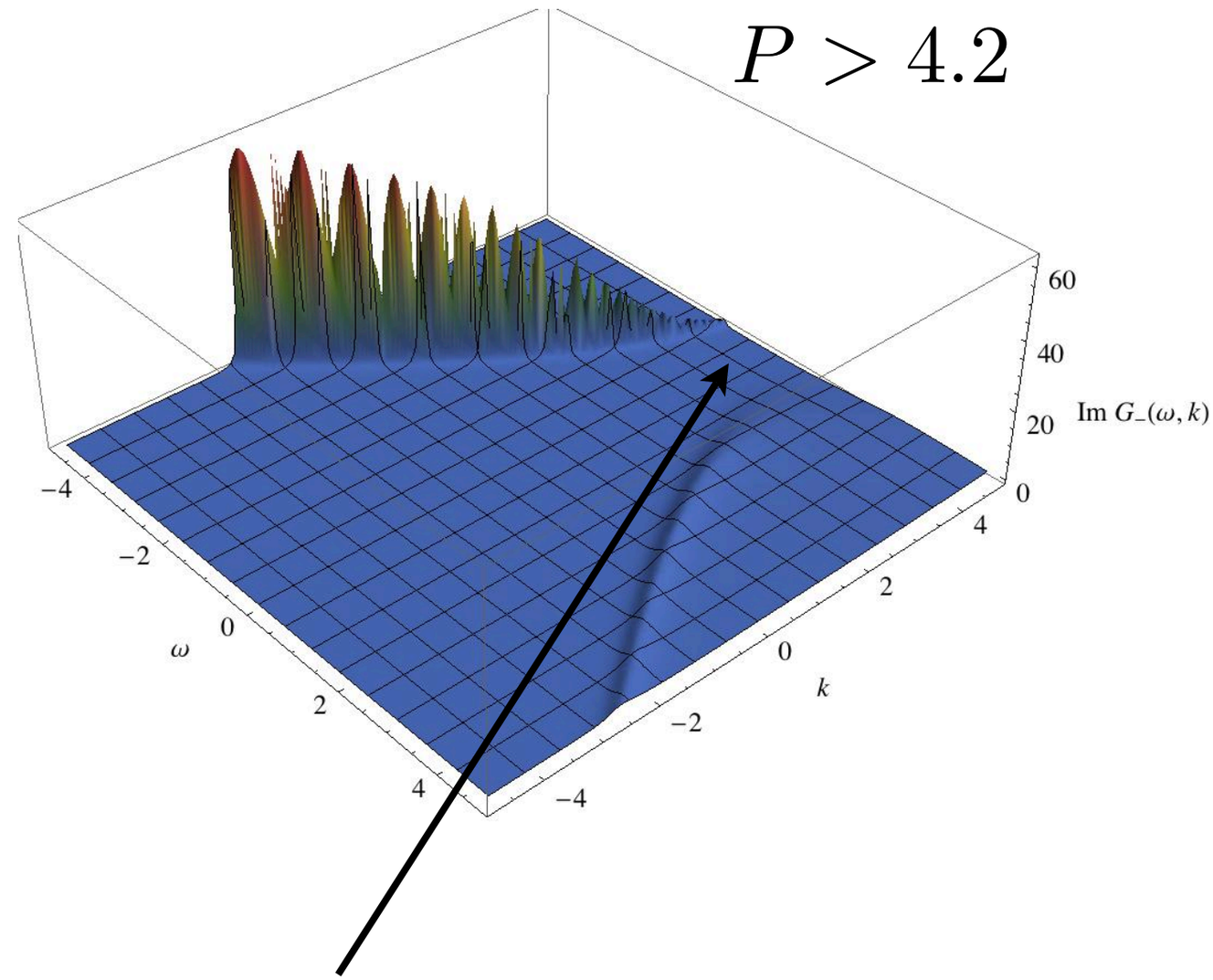

dynamically generated gap:

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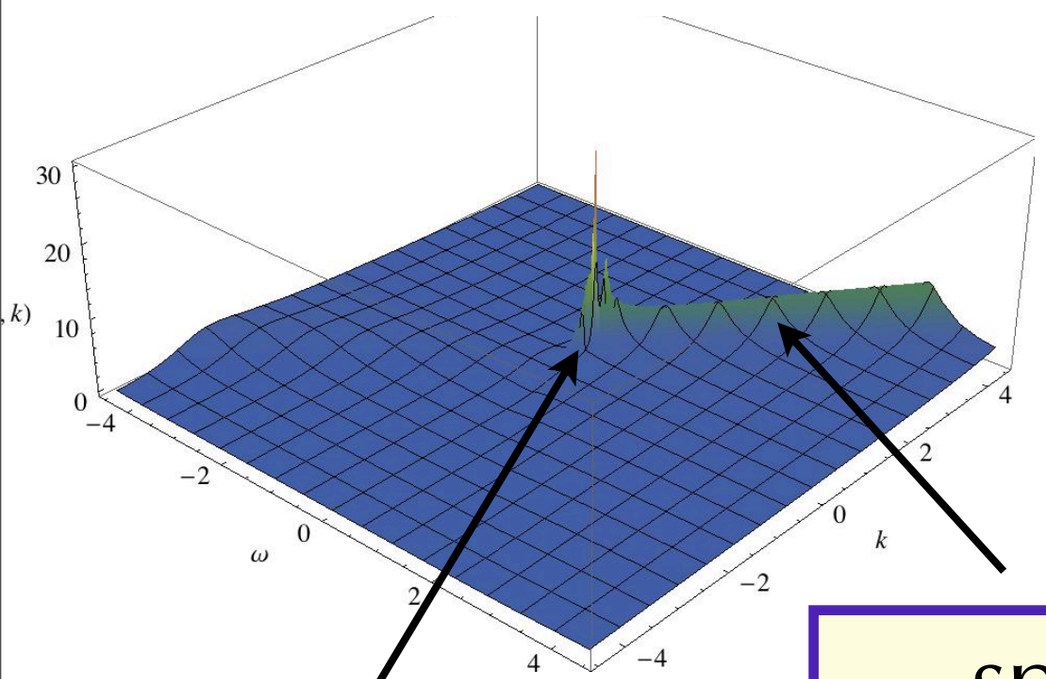

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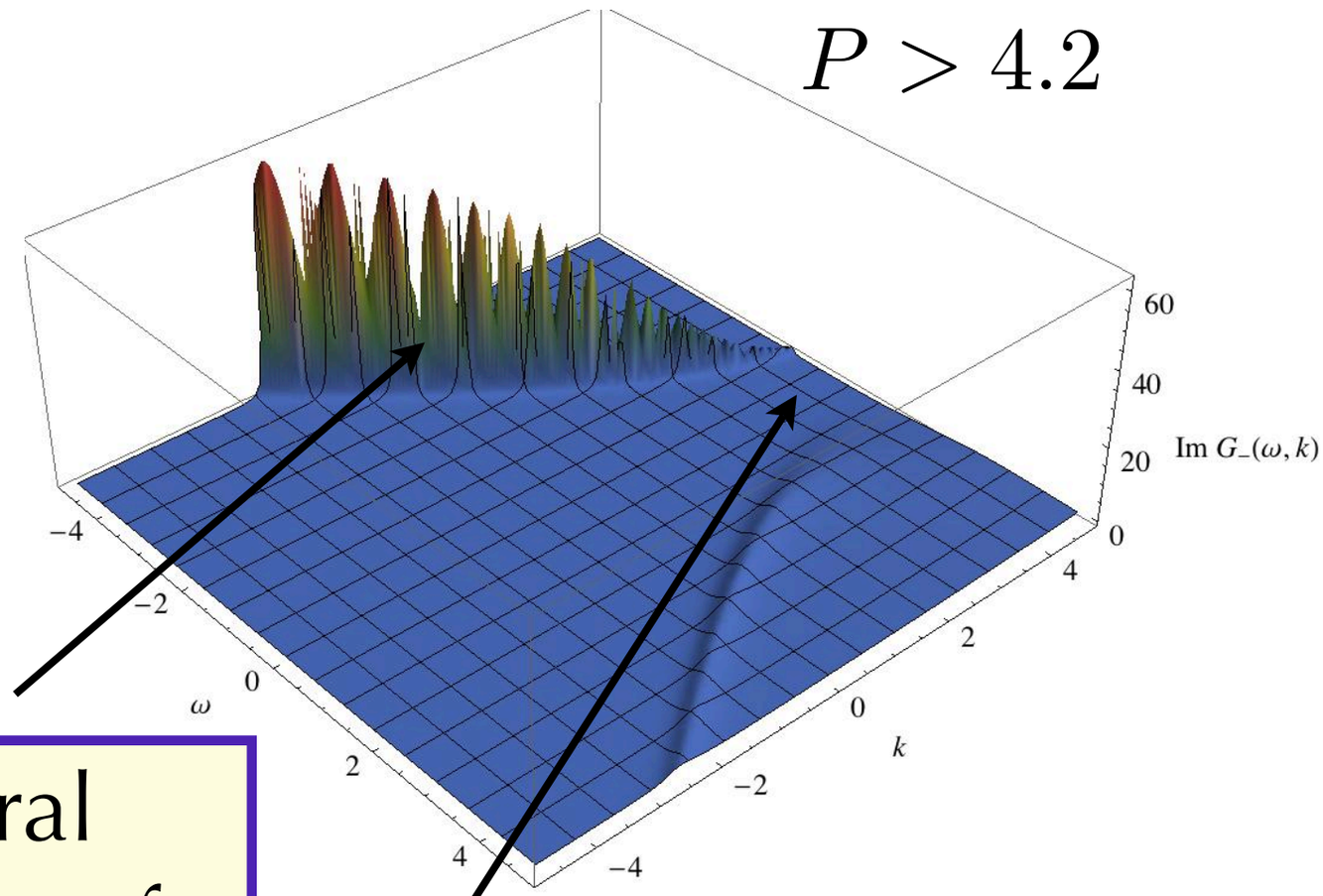


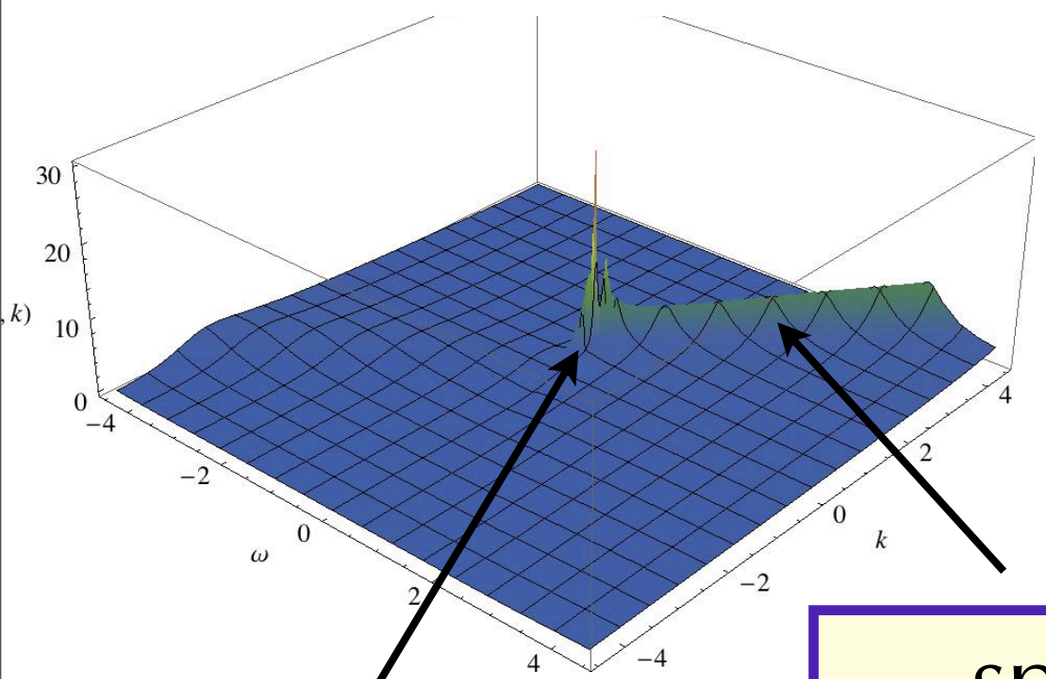
spectral weight transfer

Fermi surface peak

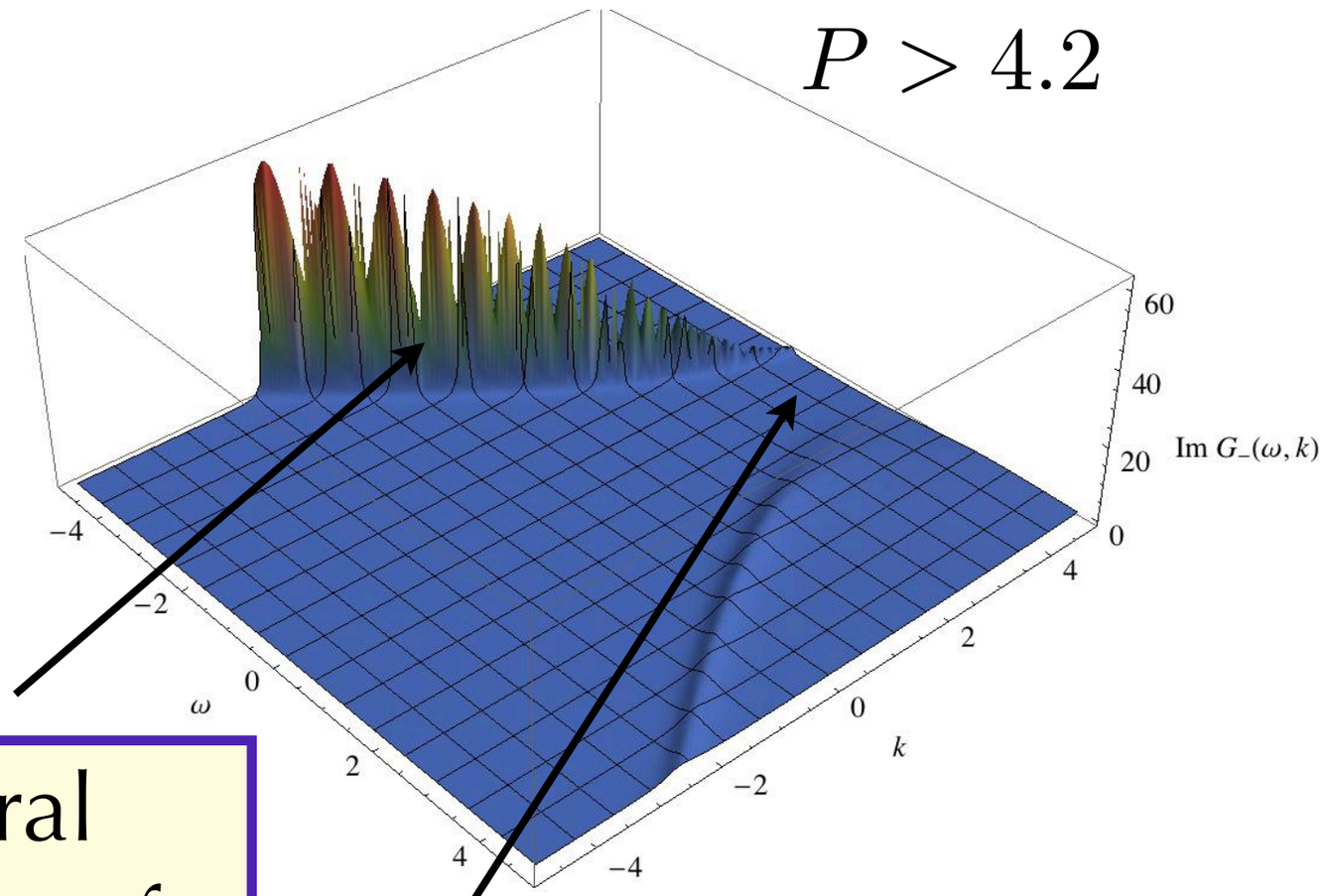
dynamically generated gap:

How is the spectrum modified?

P=0



$P > 4.2$

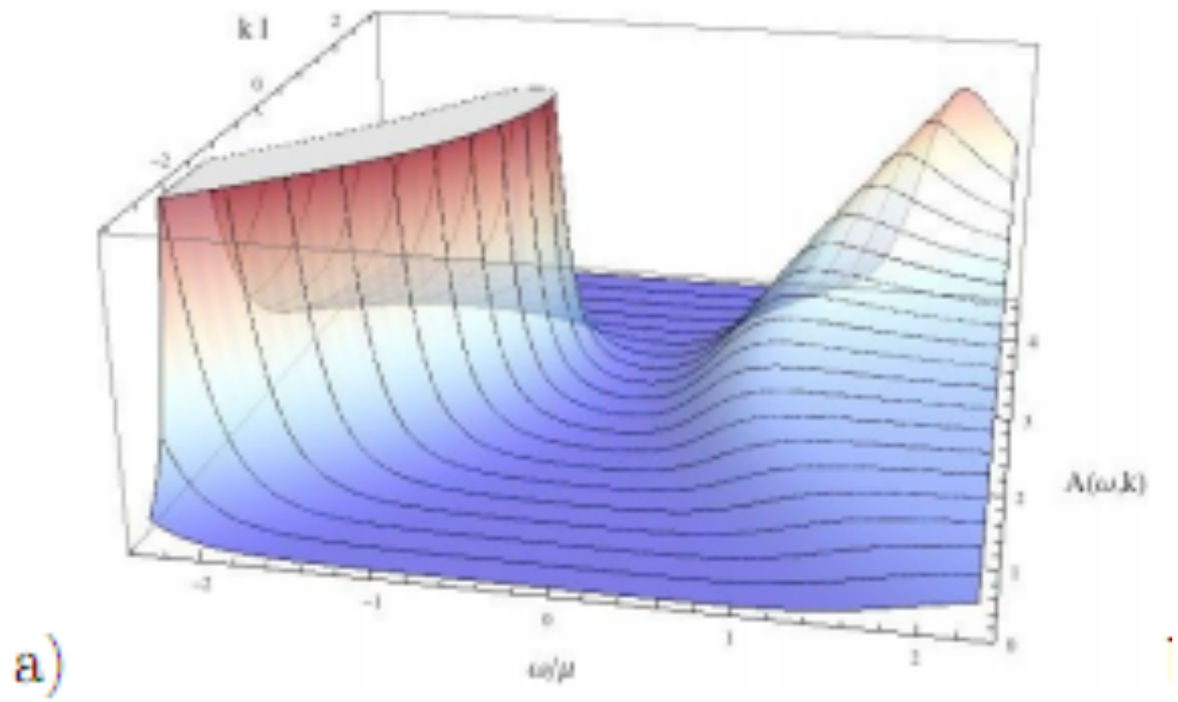


spectral weight transfer

Fermi surface peak

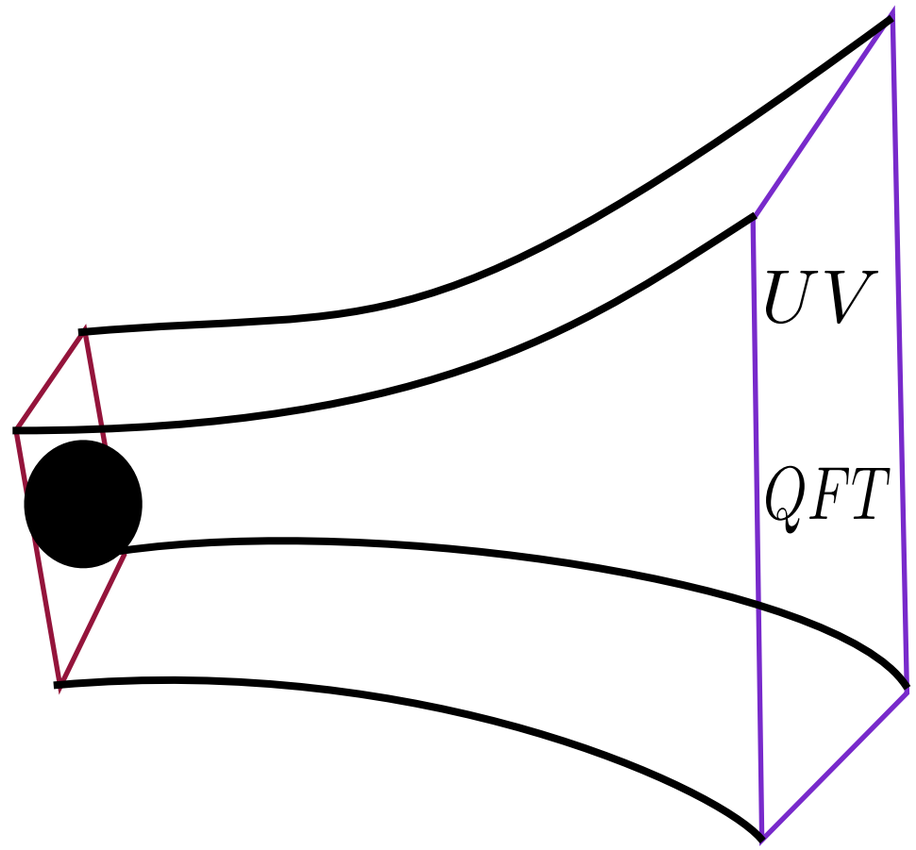
dynamically generated gap:

confirmed by Gubser, Gauntlett, 2011



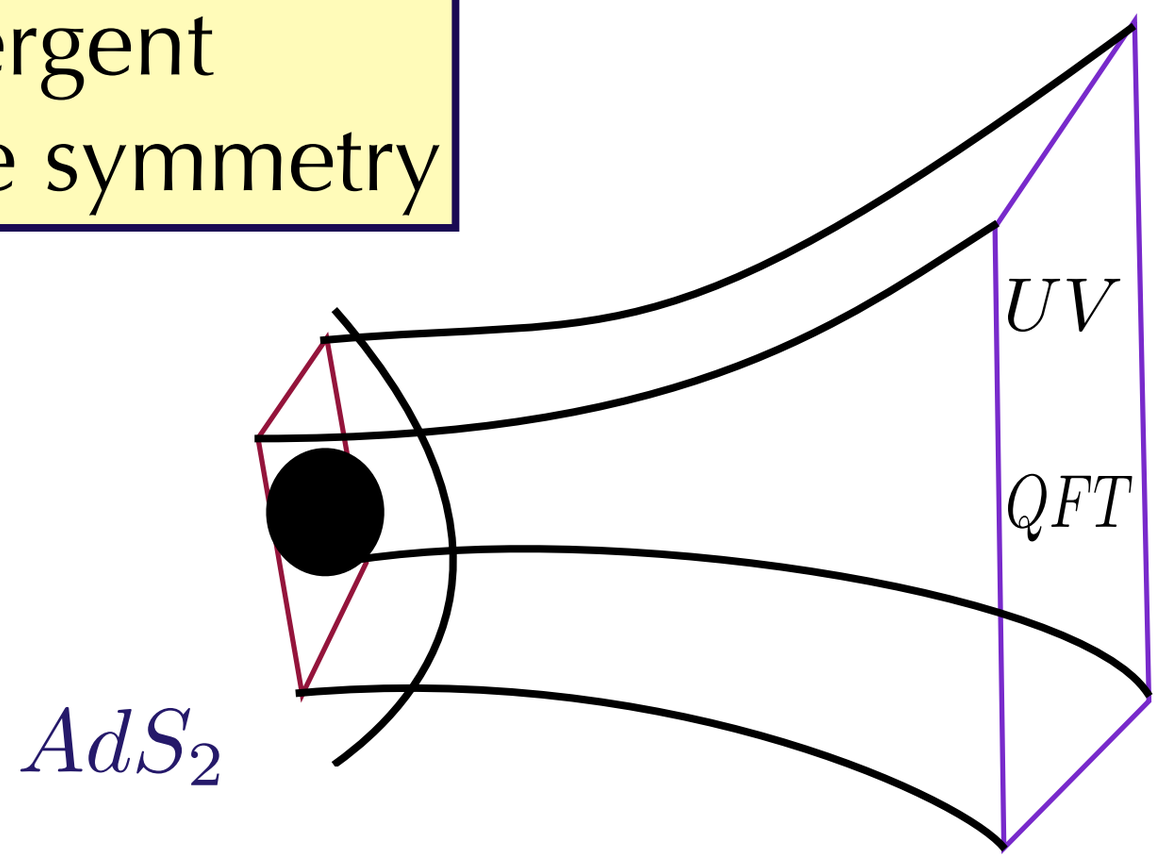
Gauntlett, 2011

Mechanism?



Mechanism?

emergent
spacetime symmetry



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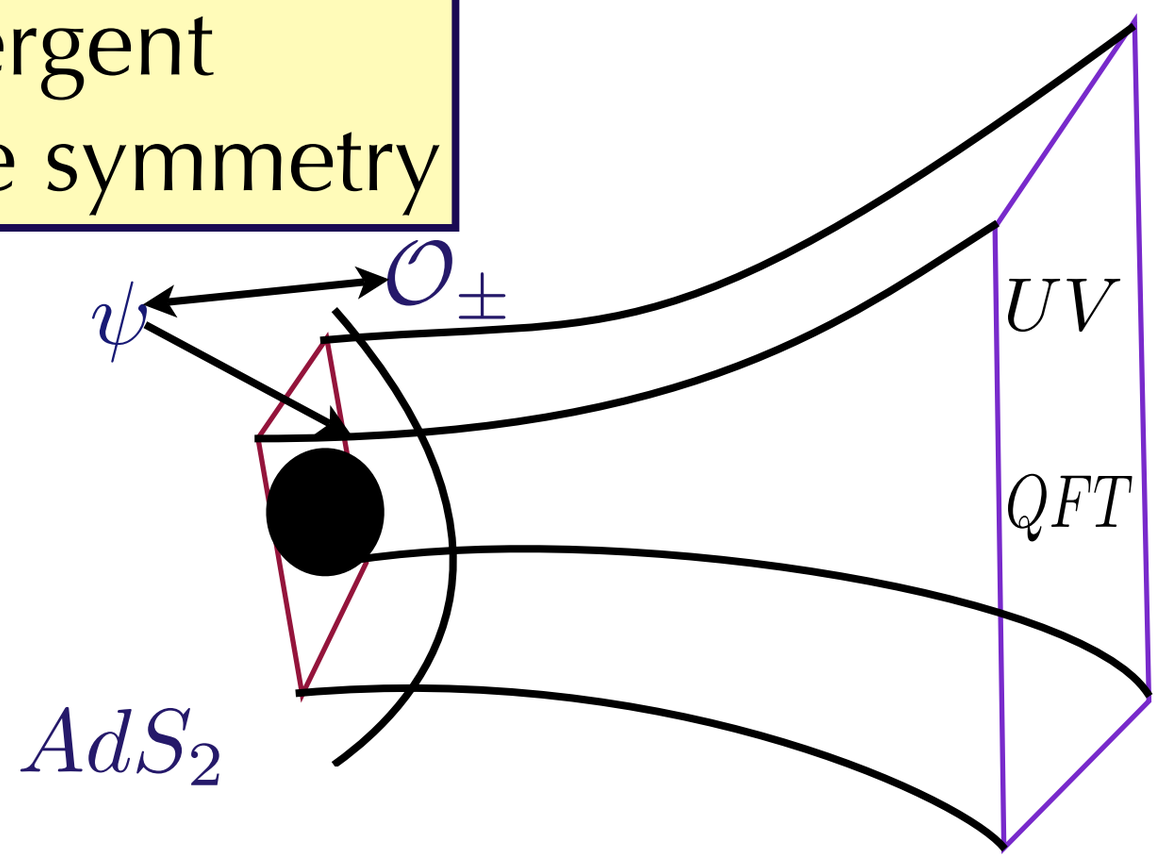
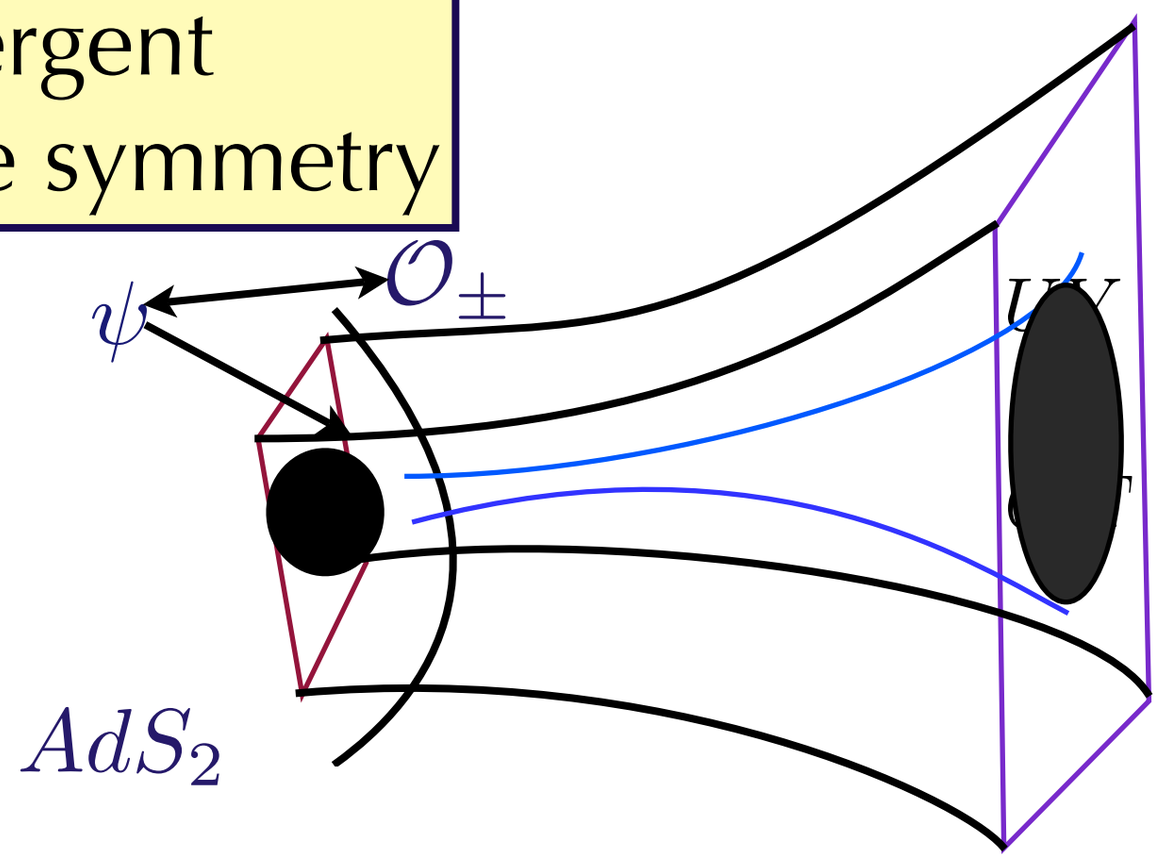


Mechanism?

operators
$\mathcal{O}$

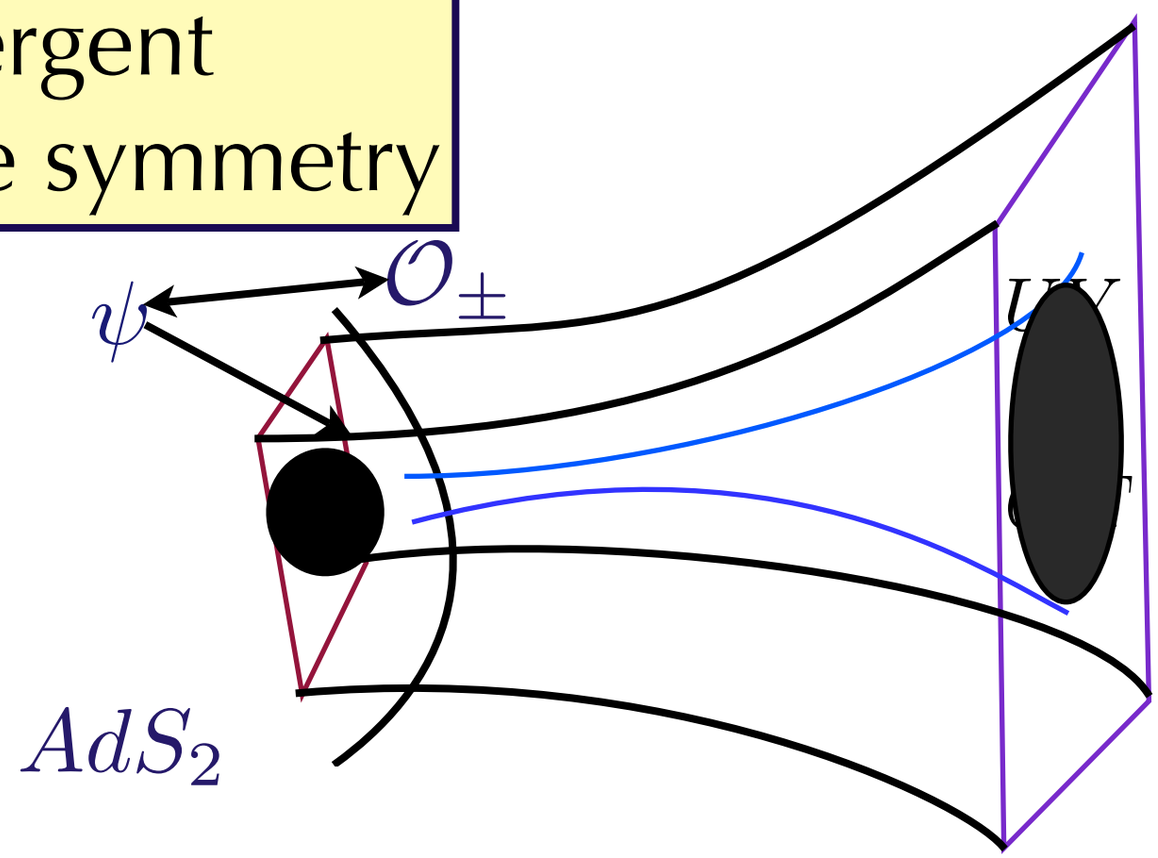
emergent
spacetime symmetry



Mechanism?

operators
$\mathcal{O}$

emergent
spacetime symmetry



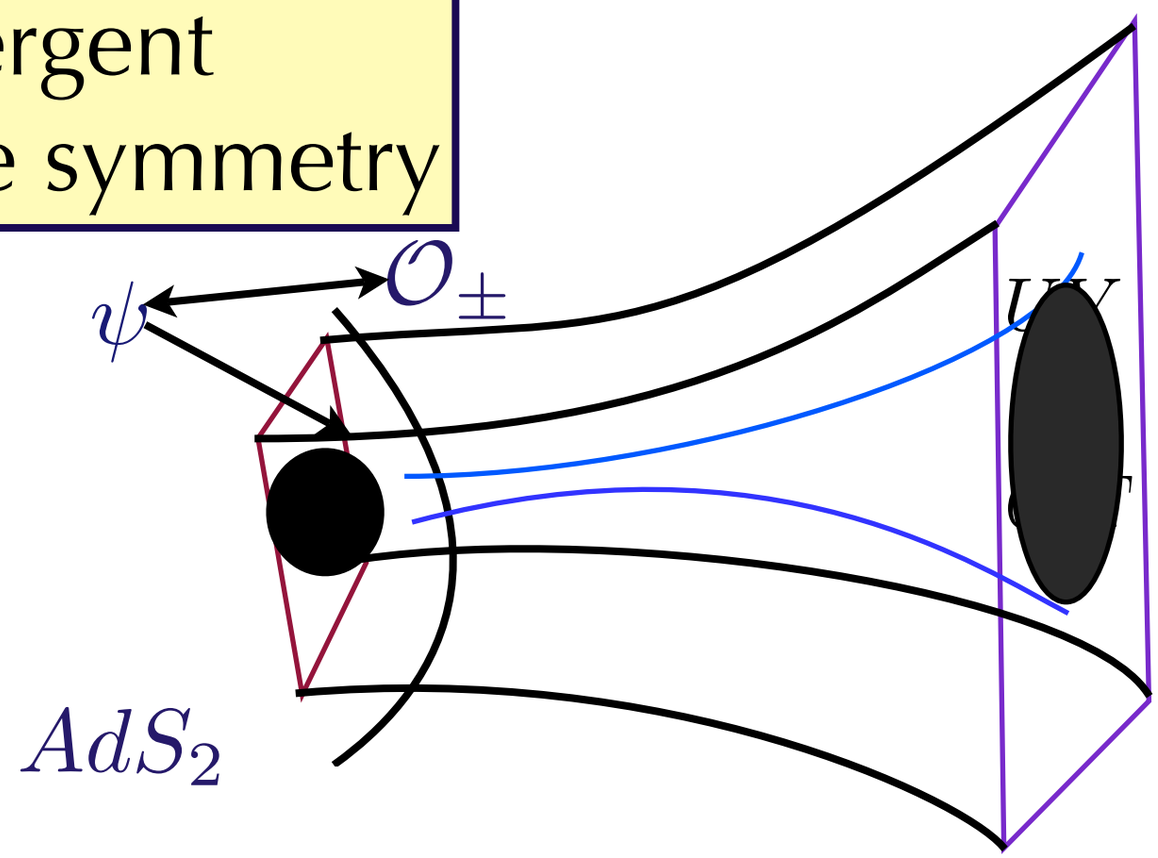
holography within
holography

Mechanism?

operators $\mathcal{O}$

emergent spacetime symmetry

where is k_F?



holography within holography

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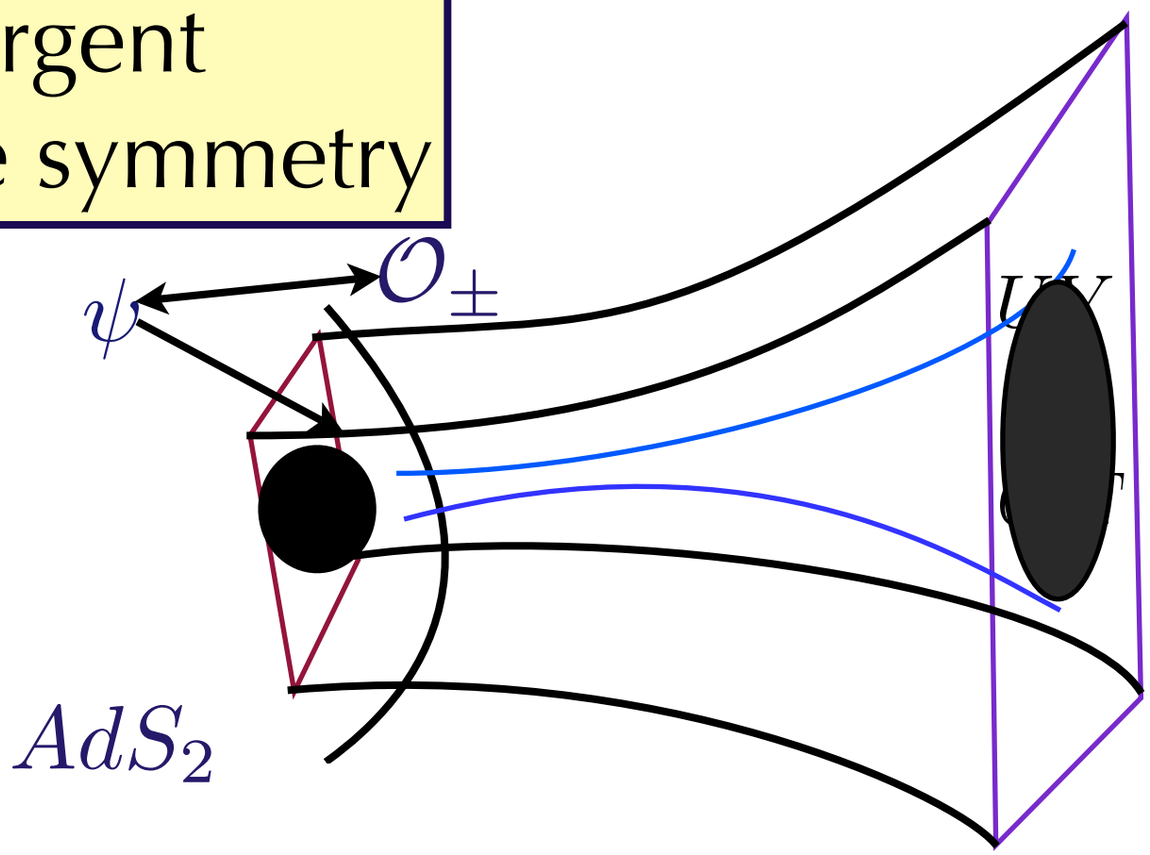

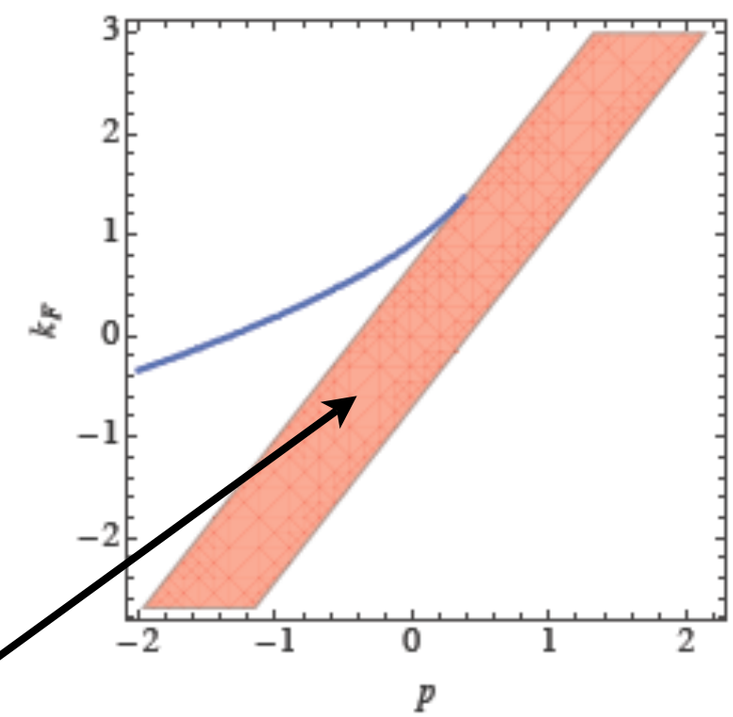

holography within holography

k_F moves into log-oscillatory region: IR
$\mathcal{O}_\pm$ acquires a complex dimension

What does a complex scaling dimension mean?

continuous
scale invariance

$$\mathcal{O} = \mu(\lambda)\mathcal{O}(\lambda r)$$

continuous
scale invariance

$$\mathcal{O} = \mu(\lambda)\mathcal{O}(\lambda r)$$

$$1 = \mu(\lambda)\lambda^{\Delta}$$

$$\Delta = -\frac{\ln \mu}{\ln \lambda}$$

continuous scale invariance

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Δ is real, independent of scale

continuous scale invariance

$$\mathcal{O} = \mu(\lambda)\mathcal{O}(\lambda r)$$

$$1 = \mu(\lambda)\lambda^{\Delta}$$

$$\Delta = -\frac{\ln \mu}{\ln \lambda}$$

Δ is real, independent of scale

what about complex Δ

$$1 = \mu\lambda^{\Delta}$$

$$e^{2\pi i n} = 1 = \mu \lambda^{\Delta}$$

Discrete scale invariance (DSI)

$$e^{2\pi i n} = 1 = \mu\lambda^{\Delta}$$

$$\Delta = -\frac{\ln\mu}{\ln\lambda} + \frac{2\pi i n}{\ln\lambda}$$

n=0:CSI

Discrete scale invariance (DSI)

$$e^{2\pi i n} = 1 = \mu \lambda^{\Delta}$$

n=0:CSI

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scaling dimension depends on scale

$$\lambda_n = \lambda^n$$

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magnification

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scaling dimension depends on scale

$$\lambda_n = \lambda^n$$

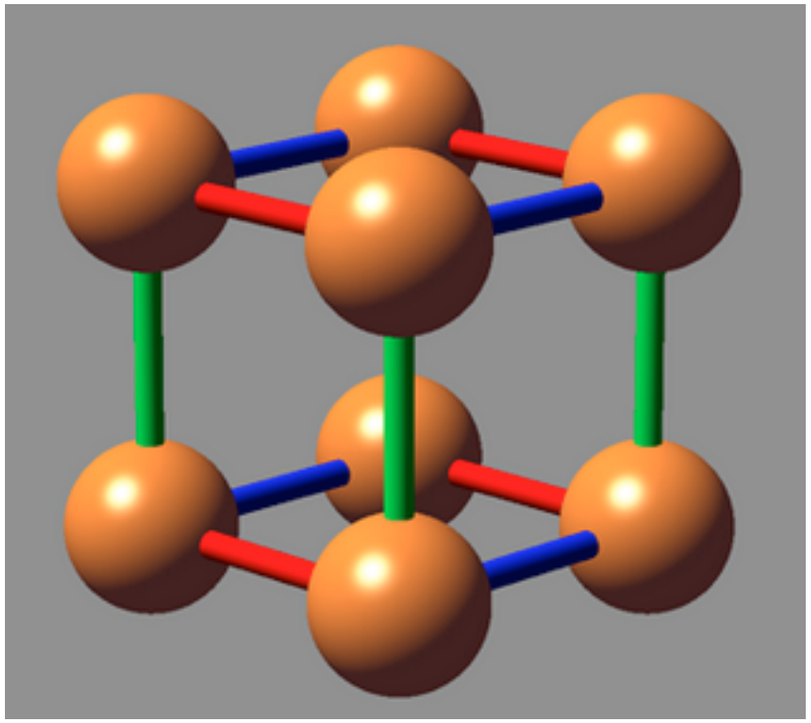
magnification

preferred scale

analogy

liquid

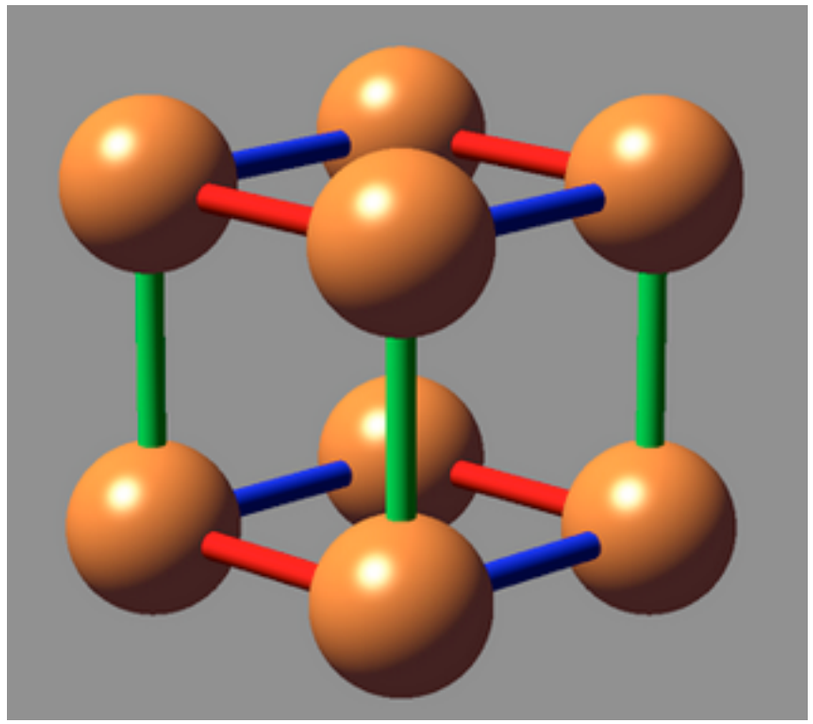
solid



analogy

liquid

solid



continuous translational invariance

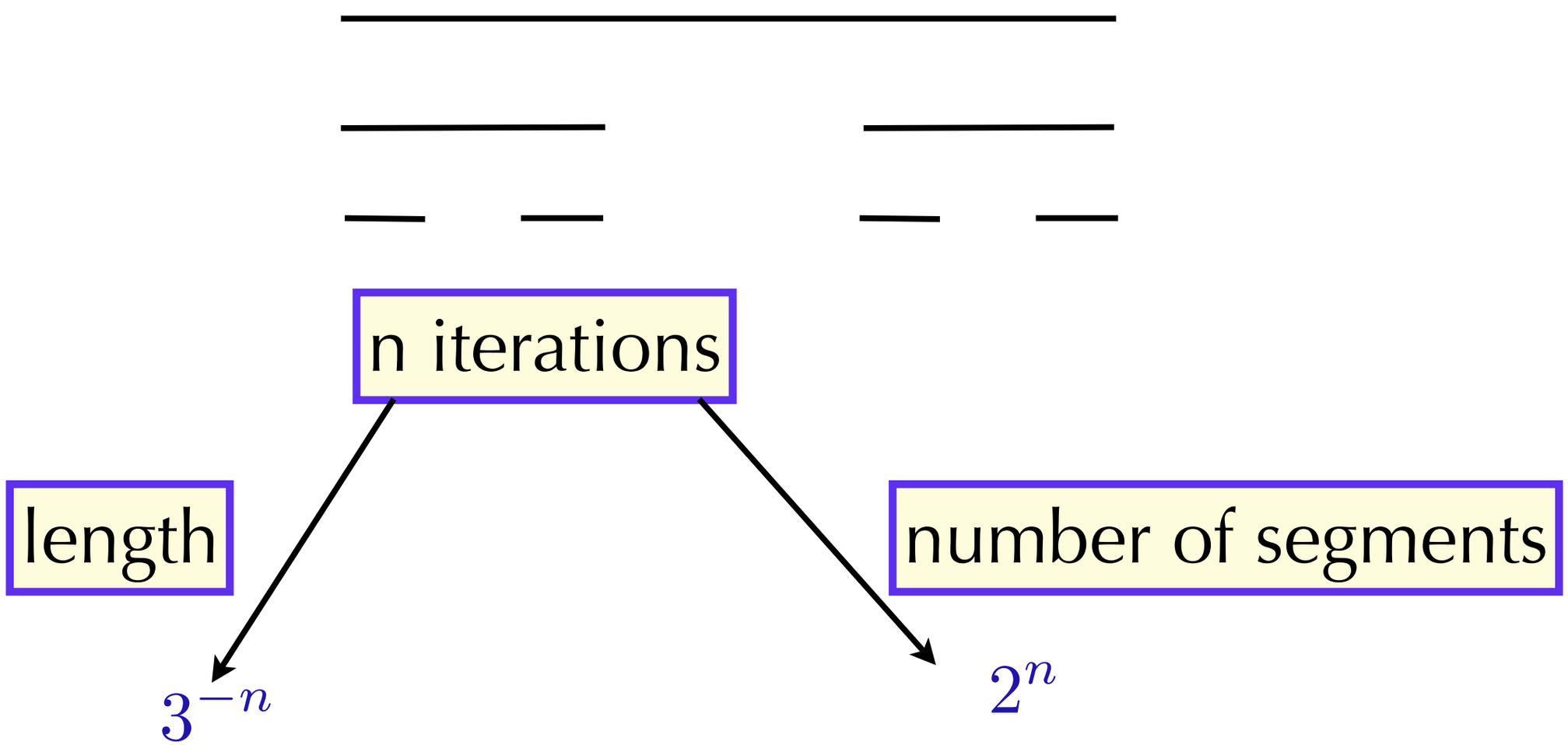
discrete translational invariance

example

example

example

example



example

n iterations

length

3^{-n}

number of segments

2^{n}

scale invariance only
for $\lambda_p = 3^p$

example

n iterations

length

3^{-n}

number of segments

2^n

scale invariance only for $\lambda_p = 3^p$

discrete scale invariance: $D = -\frac{\ln 2}{\ln 3} + \frac{2\pi i n}{\ln 3}$

more examples

stock market crashes

turbulence

DLA

earthquakes

fractals

TB Law

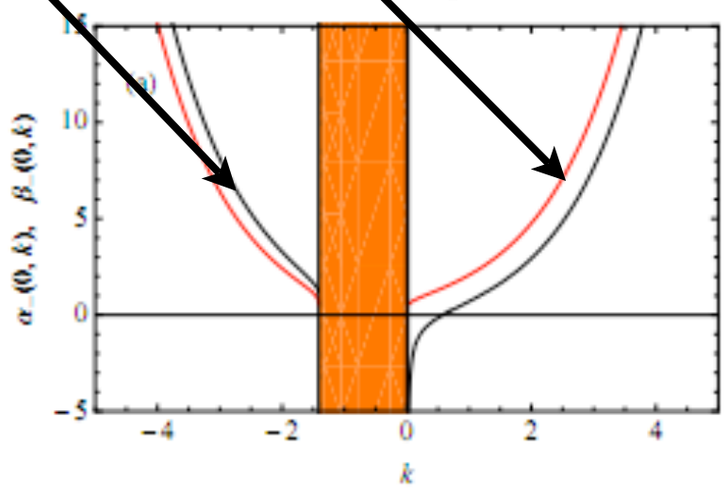
percolation

discrete scale invariance

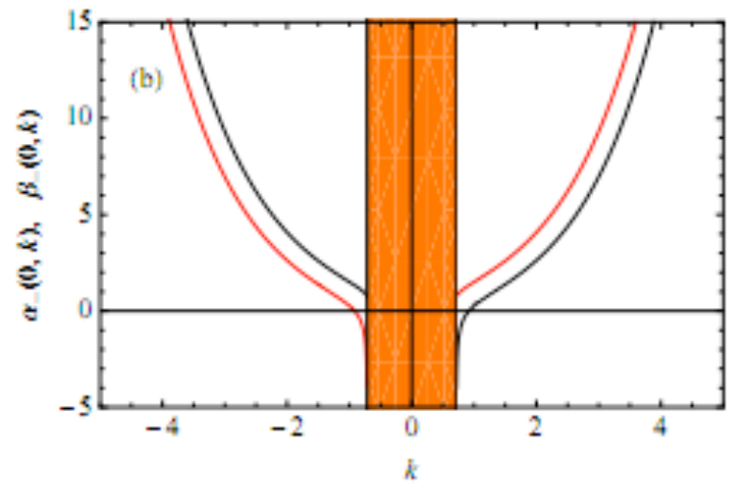
hidden scale
(length, energy,...)

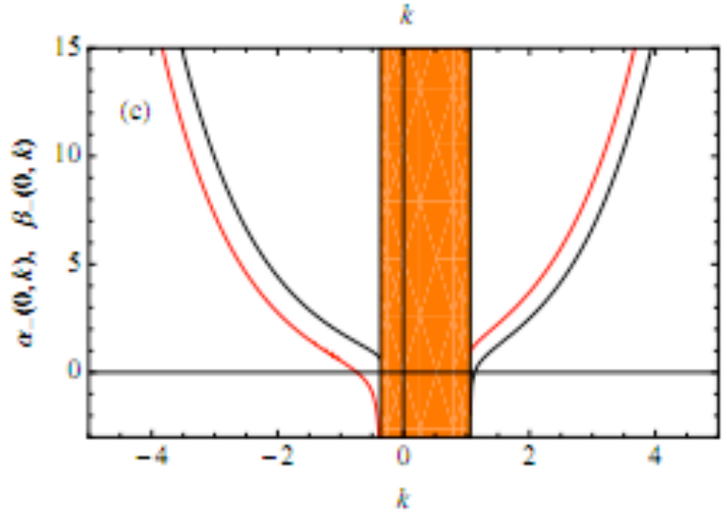
$$G = \frac{\beta_-(0,k)}{\alpha_-(0,k)}$$

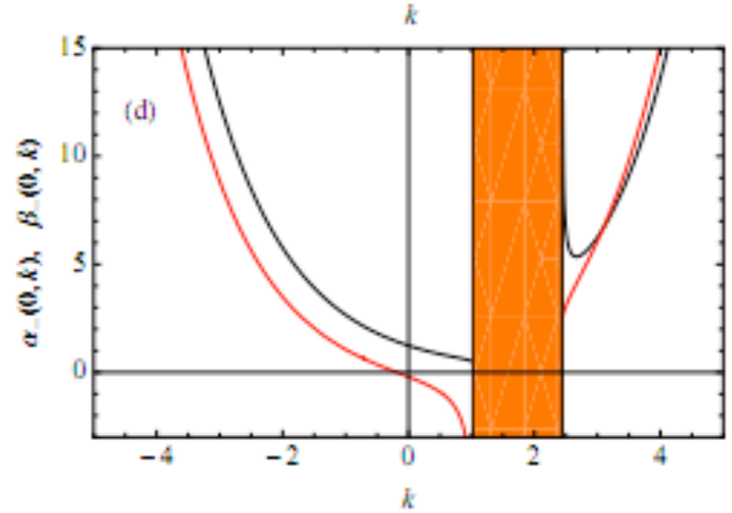
p=-0.4



p=0.0





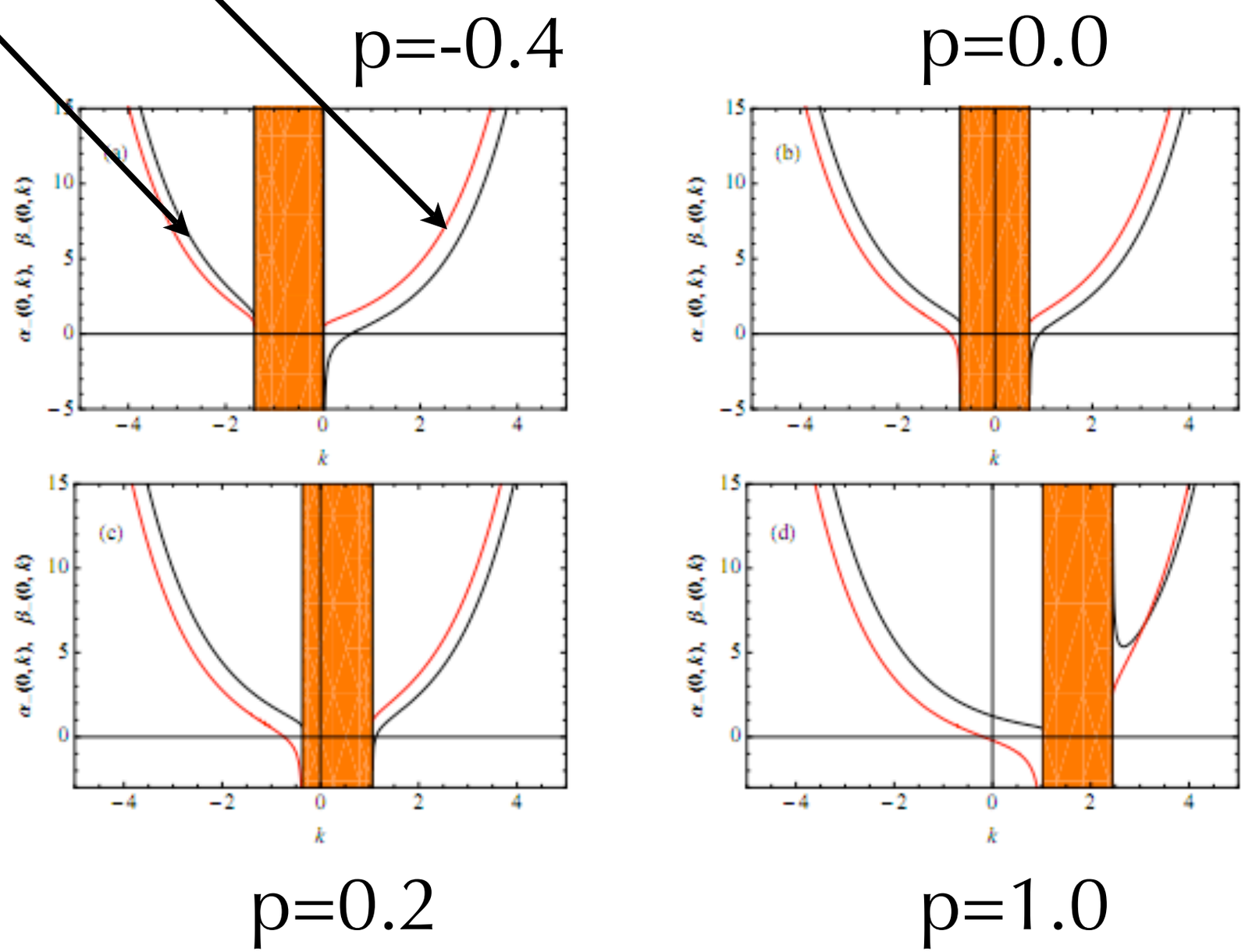


p=0.2

p=1.0

no poles outside log-oscillatory region for $p > 1/\sqrt{6}$

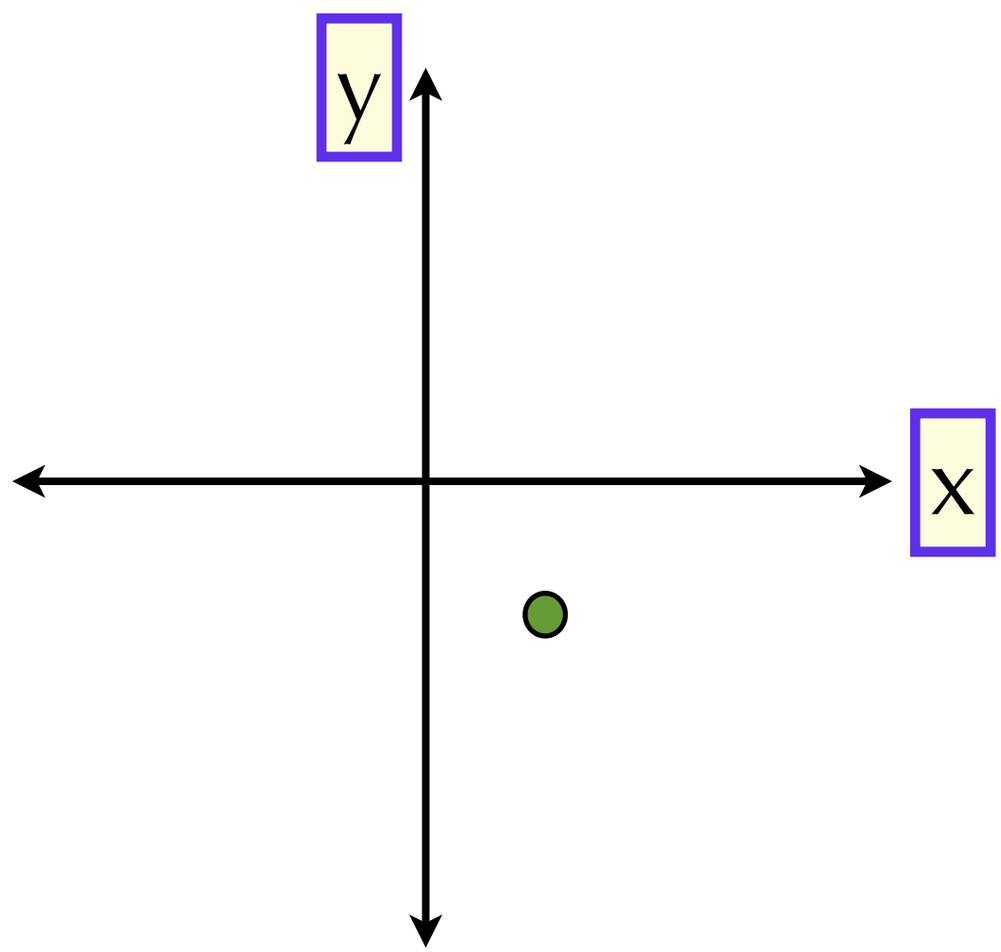
$$G = \frac{\beta_-(0,k)}{\alpha_-(0,k)}$$



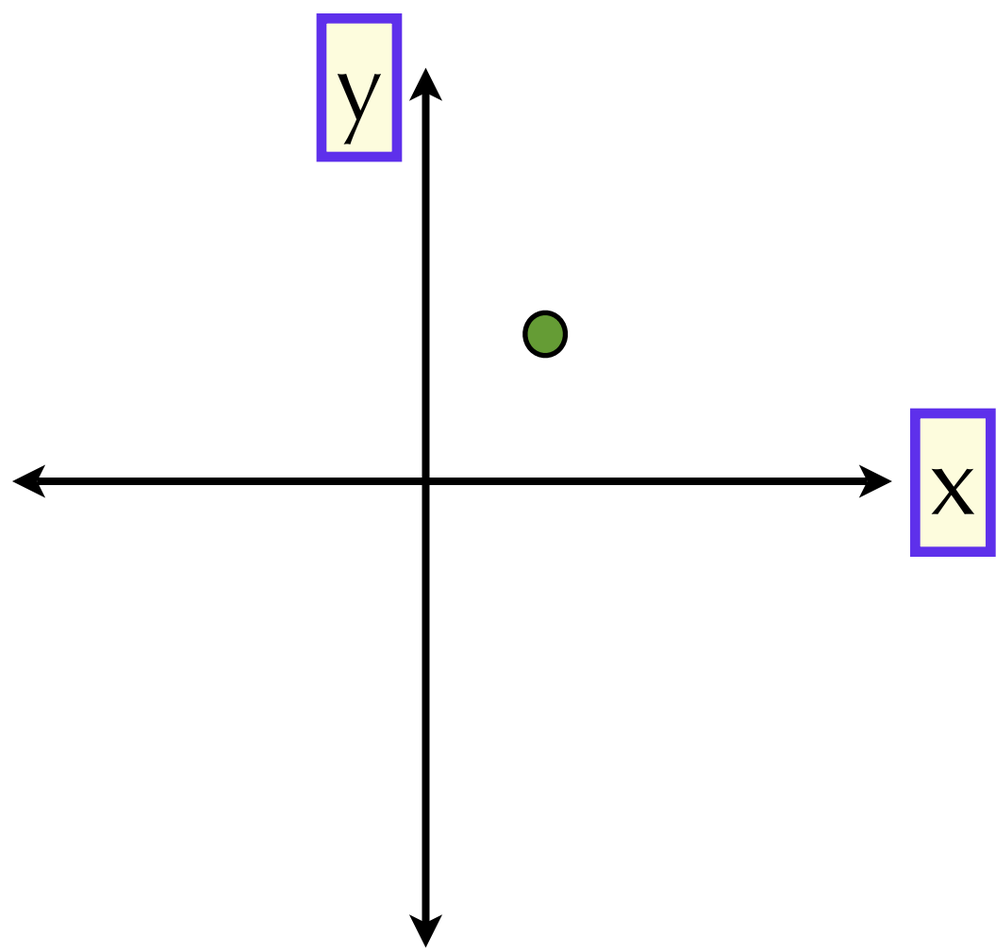
no poles outside log-oscillatory region for $p > 1/\sqrt{6}$

Mott physics and DSI are linked!

is there an instability?

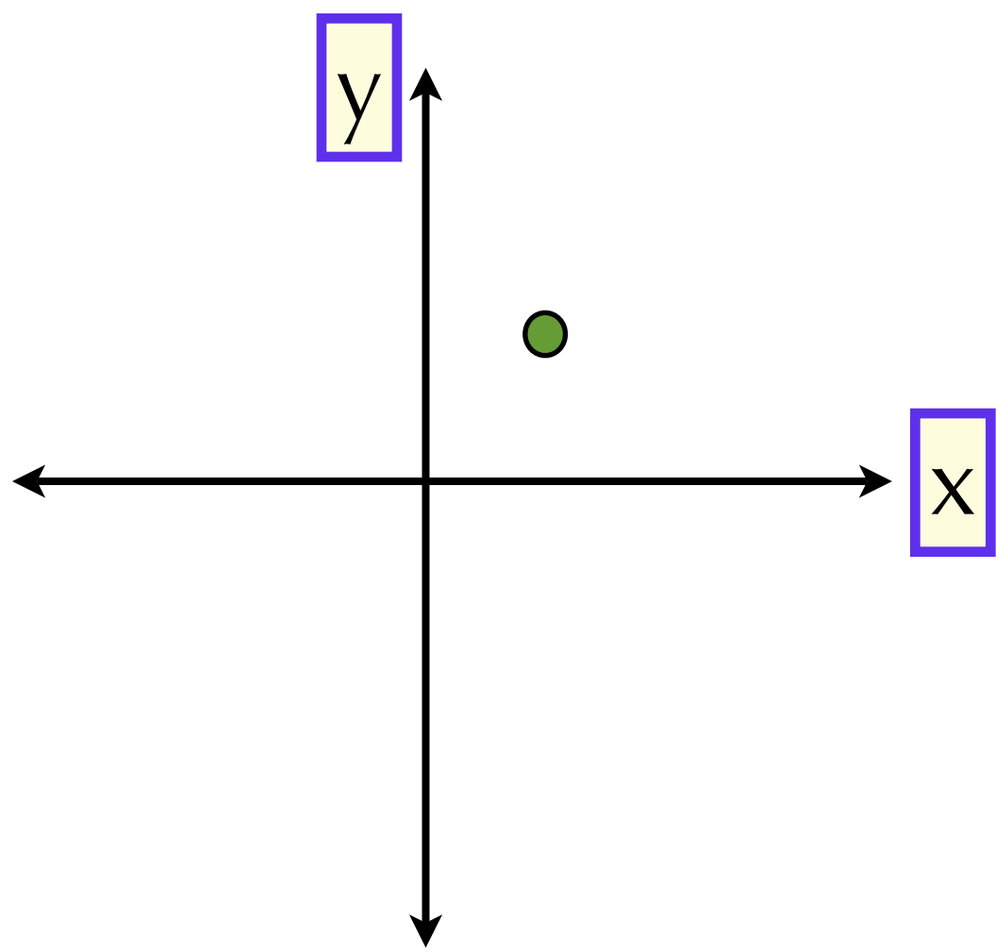


is there an instability?



is there an instability?

$$E \to E + i\gamma \quad \gamma > 0$$



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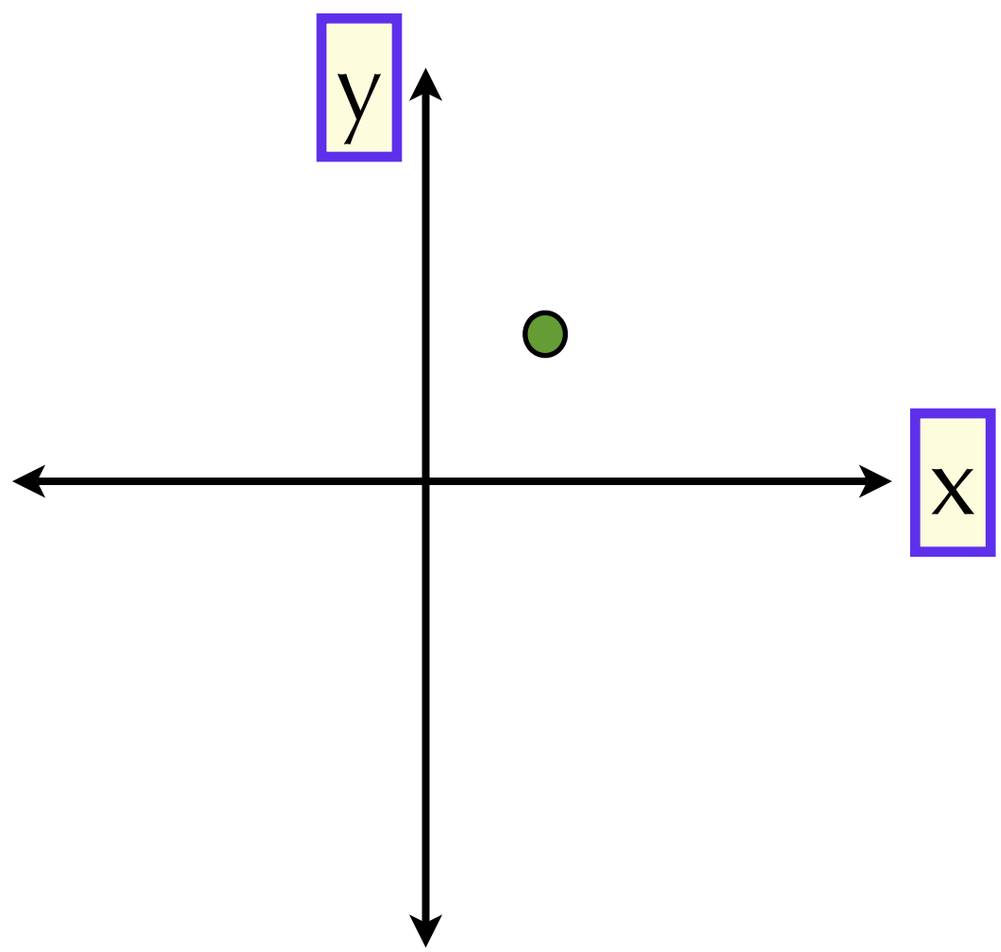
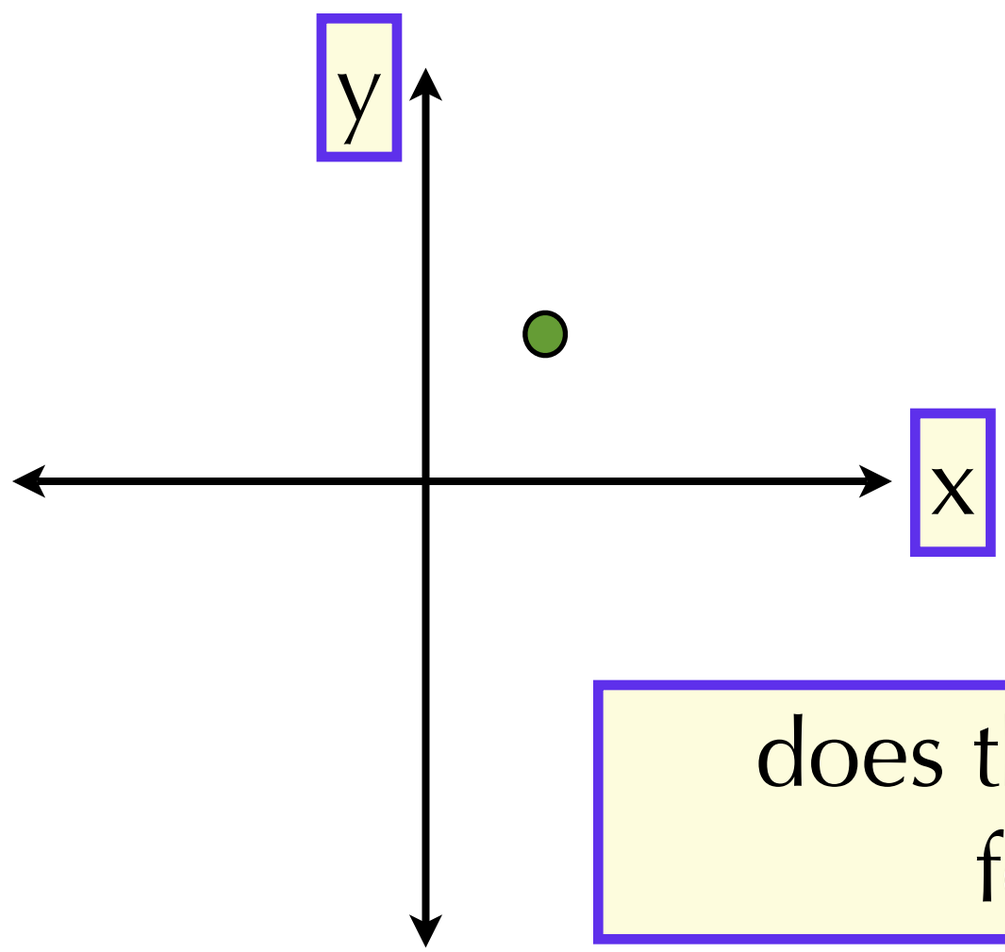


$$\langle \phi \rangle \neq 0$$

condensate
(bosonic)

is there an instability?

$$E \to E + i\gamma \quad \gamma > 0$$

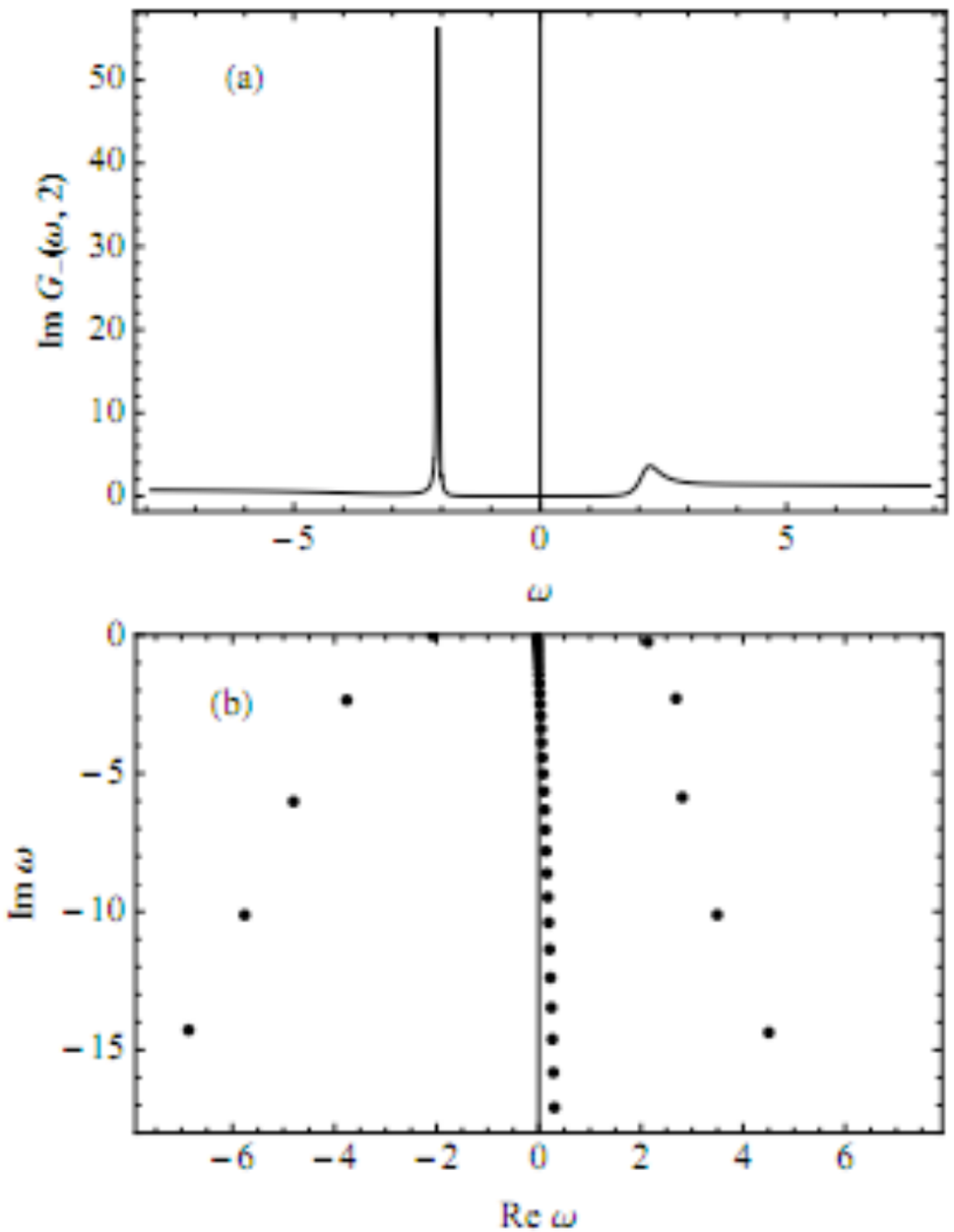


$$\langle \phi \rangle \neq 0$$

condensate (bosonic)

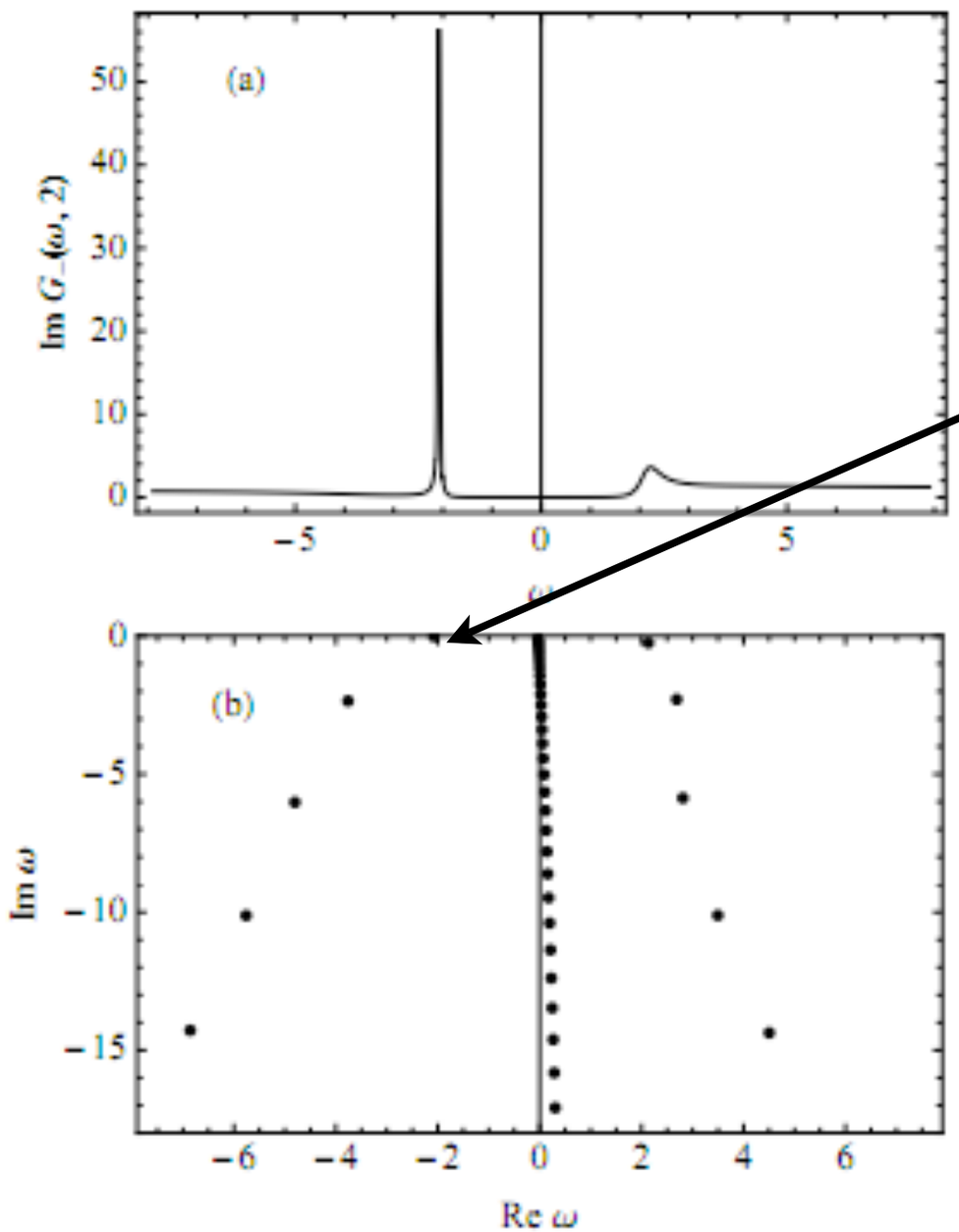
does this happen for fermions?

quasi-normal modes

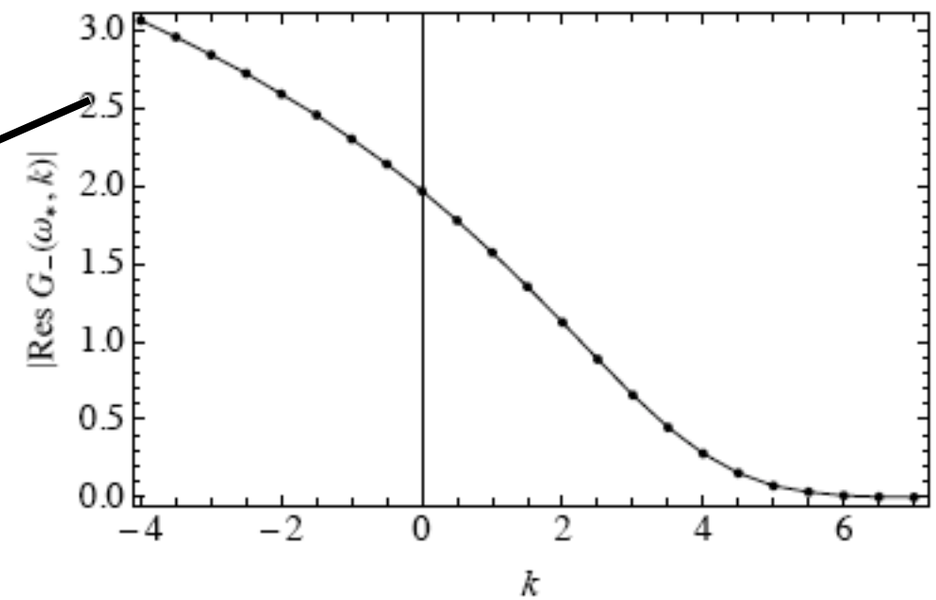


$Im\omega < 0$ no instability

quasi-normal modes



quasi-particle residue



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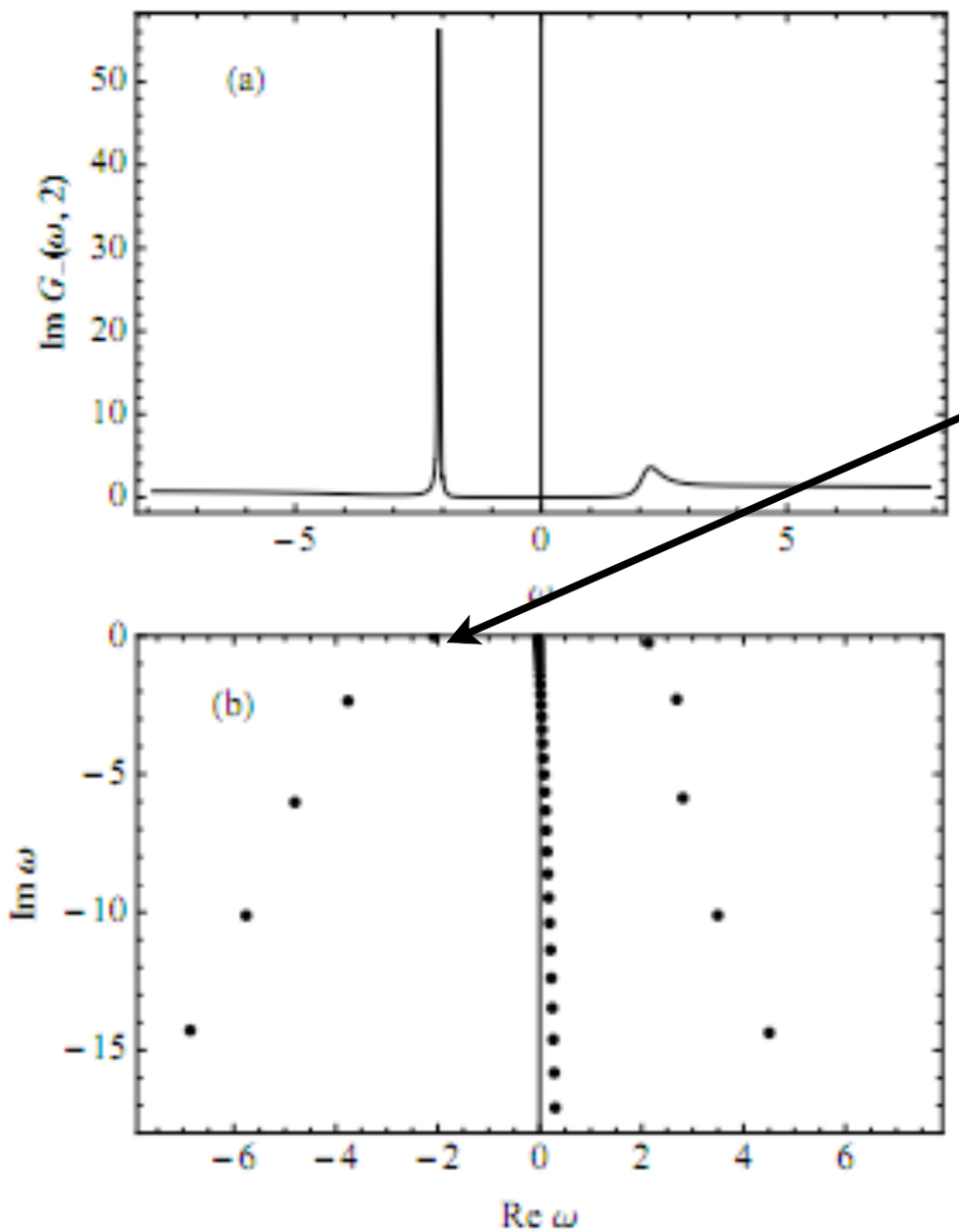
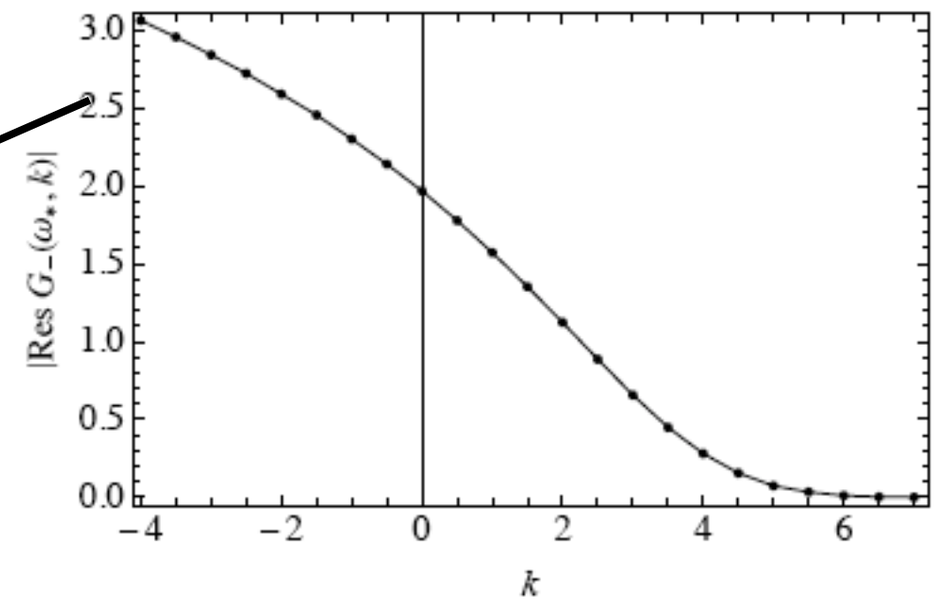


quasi-particle residue



but the residue drops to zero: opening of a gap

$Im\omega < 0$ no instability

$$G(\omega, k) = G(\omega\lambda^n, k)$$

$$G(\omega, k) = G(\omega\lambda^n, k)$$

$\Downarrow$

discrete scale invariance in energy

$$G(\omega, k) = G(\omega\lambda^n, k)$$

discrete scale invariance in energy

emergent IR scale

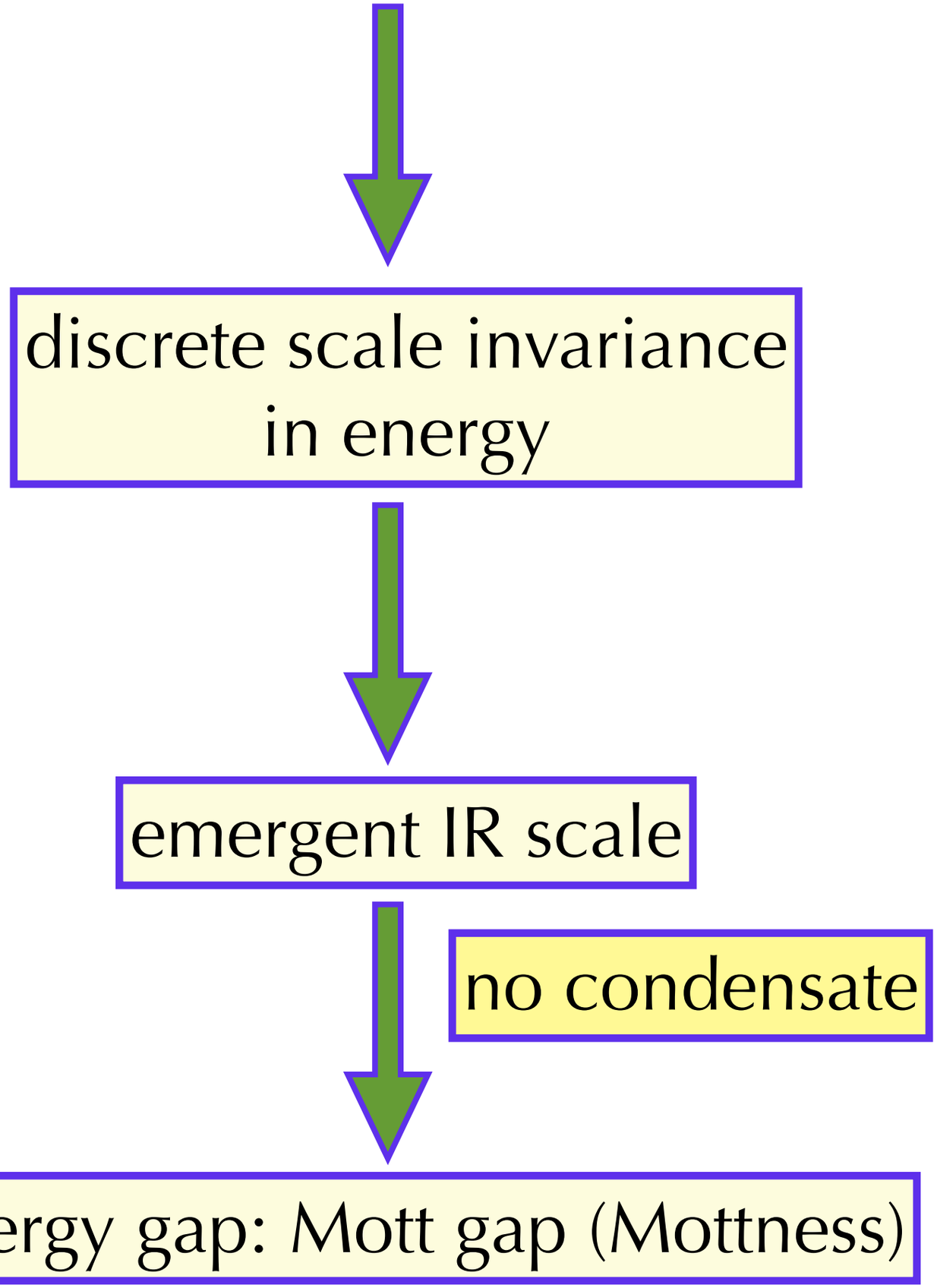
$$G(\omega, k) = G(\omega\lambda^n, k)$$

discrete scale invariance in energy

emergent IR scale

no condensate

$$G(\omega, k) = G(\omega\lambda^n, k)$$



continuous scale invariance

↓

discrete scale invariance in energy

continuous scale invariance

↓

discrete scale invariance in energy

is this the symmetry that is ultimately broken in the Mott problem?

a.) yes

b.) no

holography

a.) yes	√
b.) no	

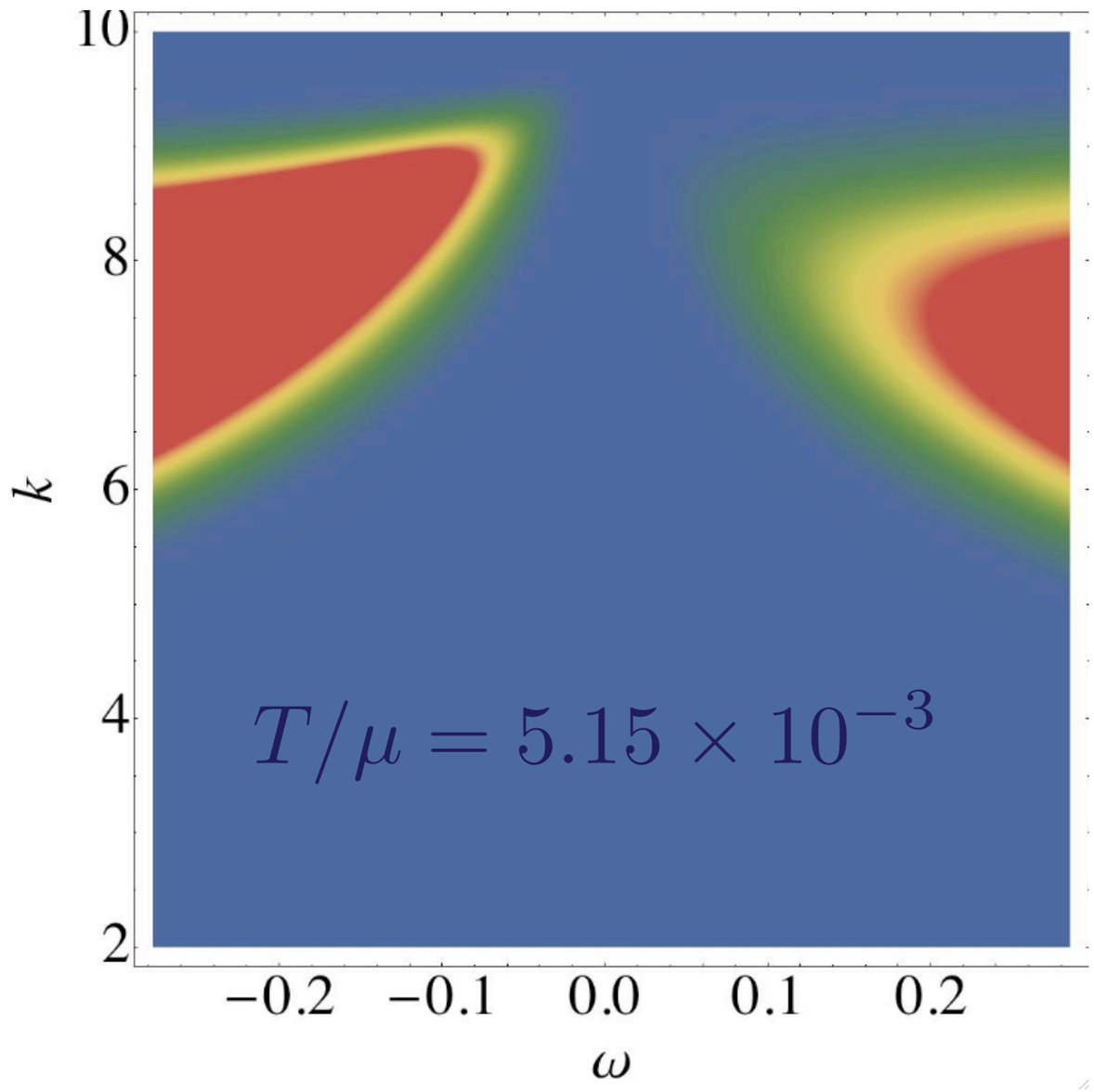
	holography	VO_2, cuprates,...
a.) yes	√	
b.) no		

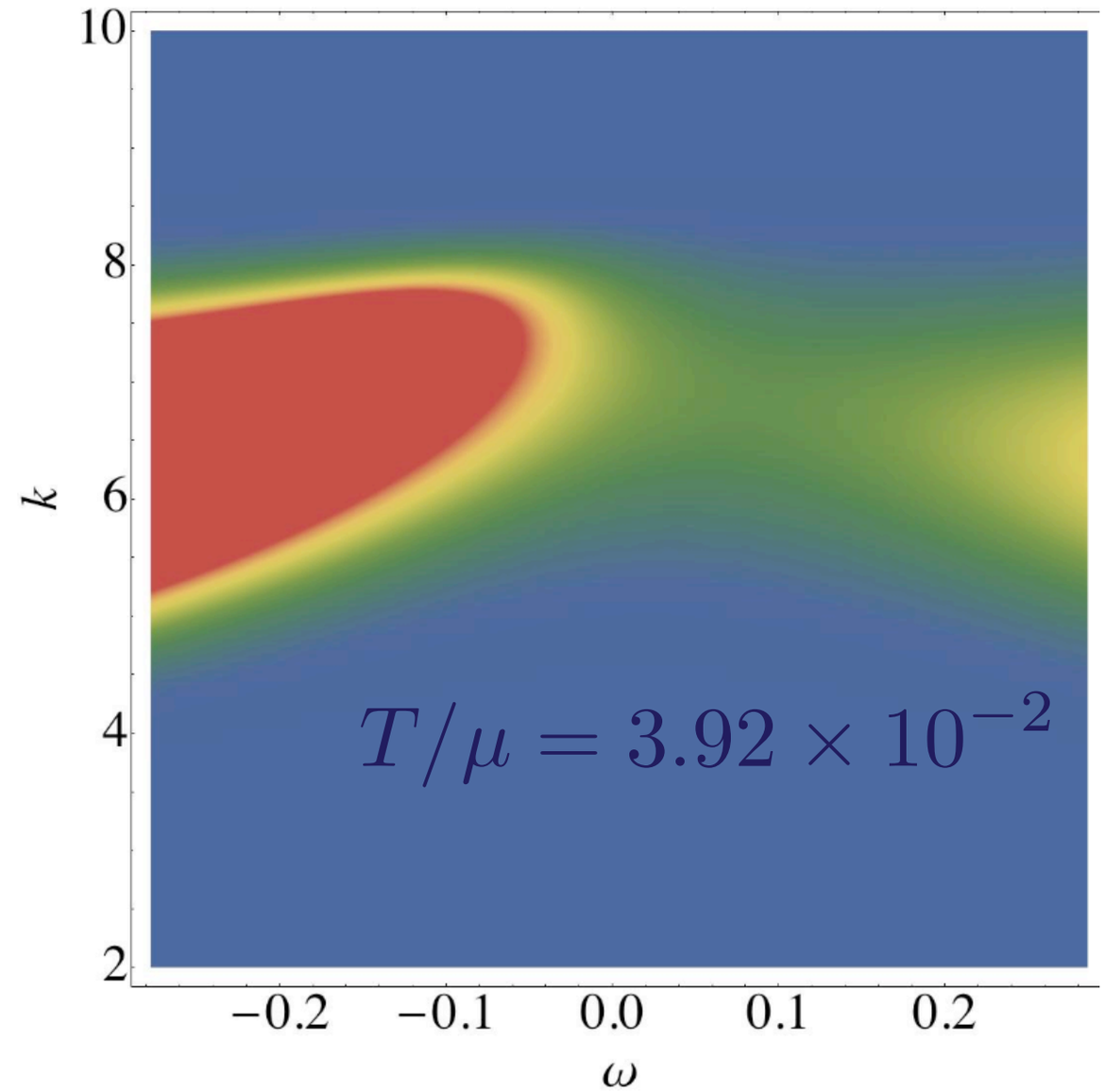
	holography	VO_2, cuprates,...
a.) yes	√	
b.) no		

if yes: holography has solved the Mott problem

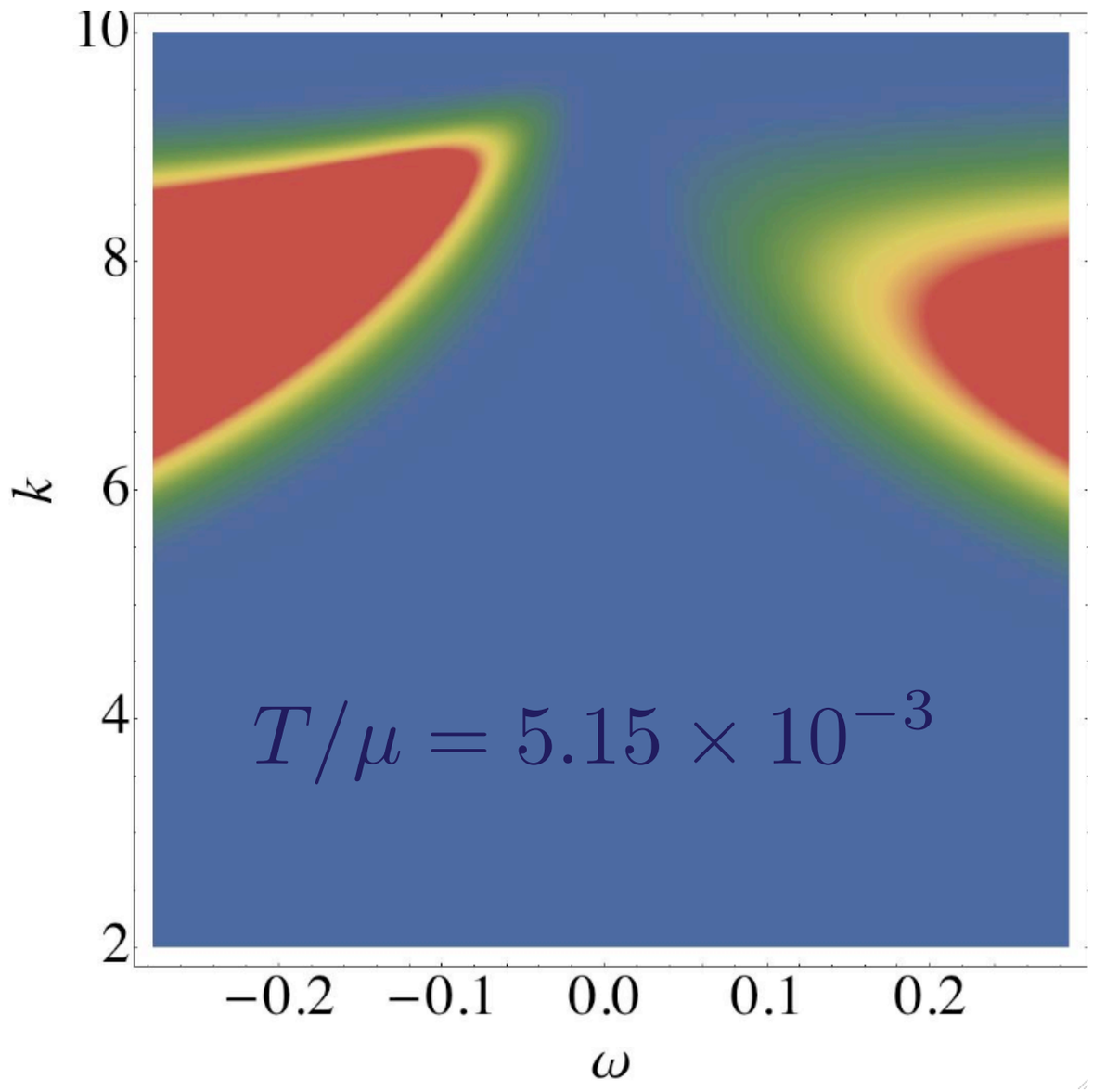
what else can holography do?

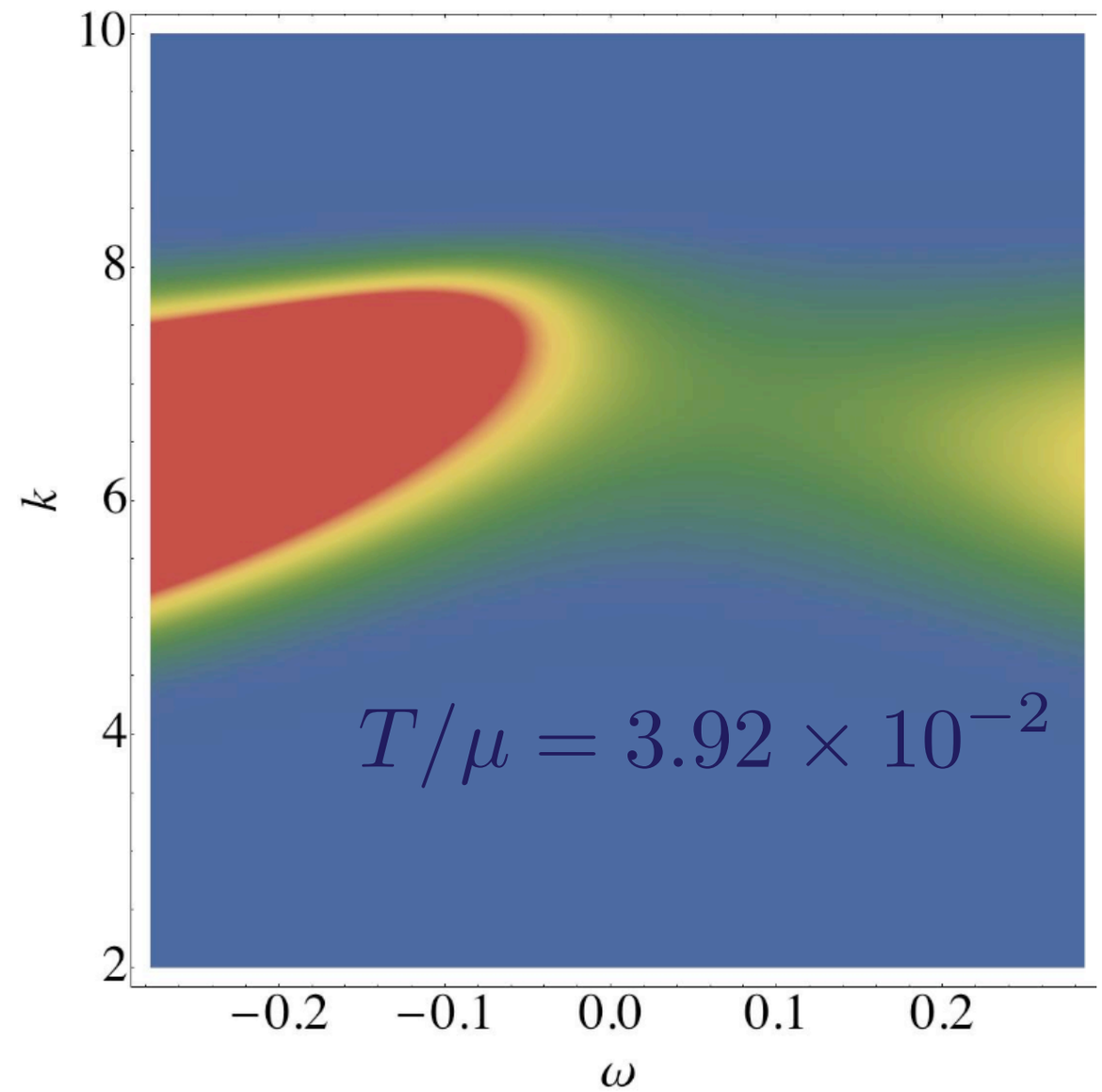
Finite Temperature Mott transition from Holography





Finite Temperature Mott transition from Holography

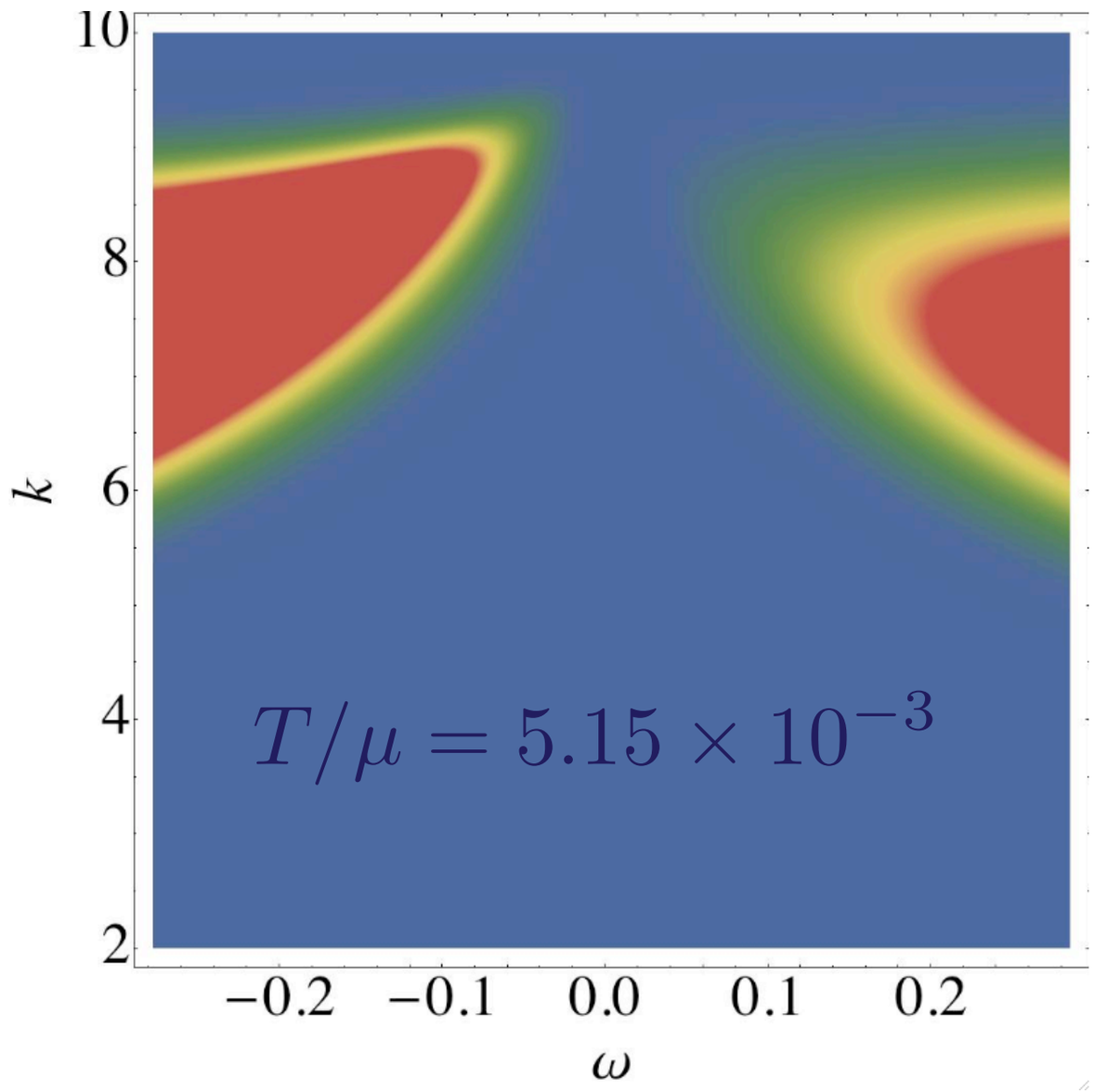


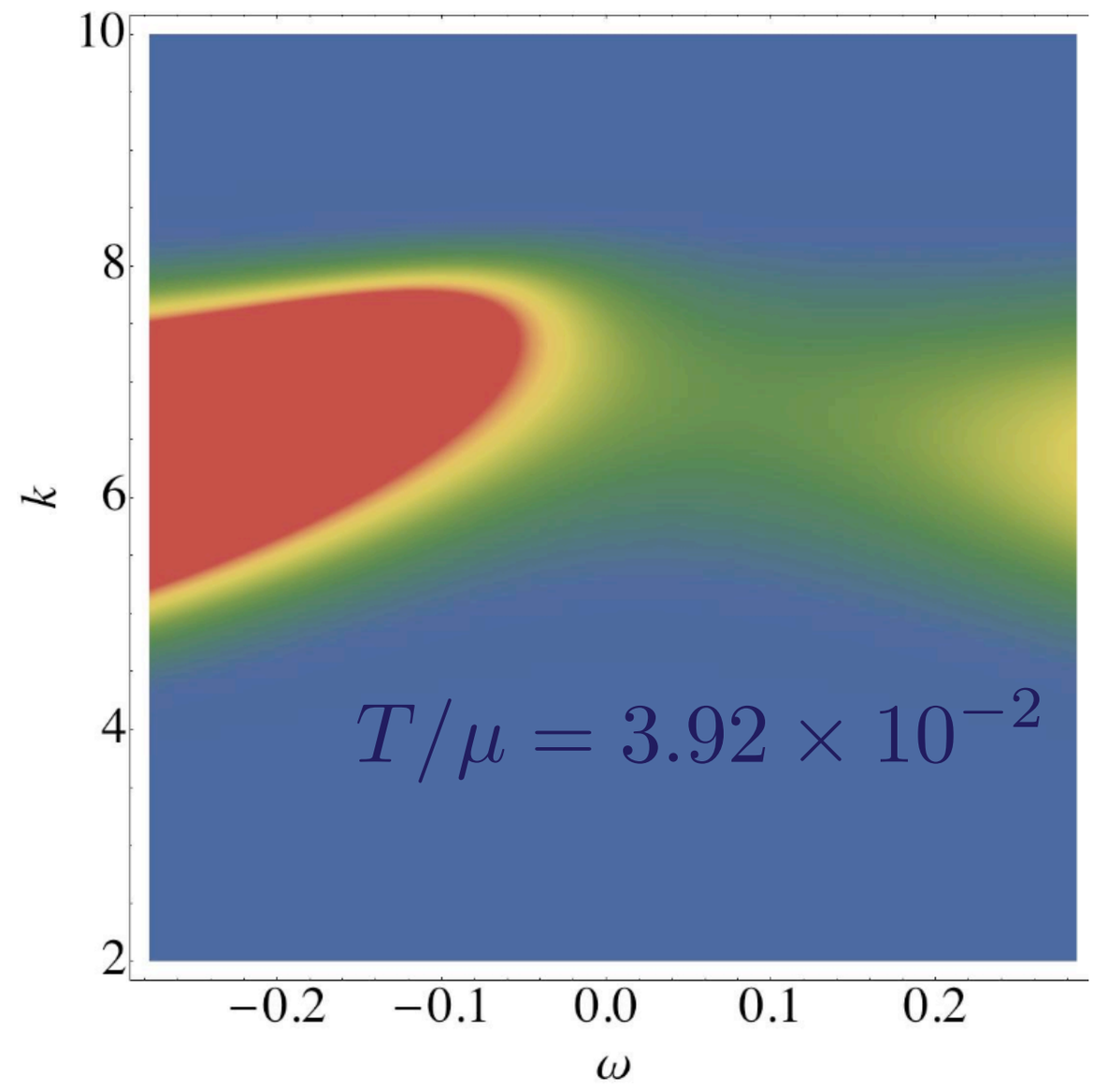


$$\frac{\Delta}{T_{\rm crit}} \approx 20$$

vanadium oxide

Finite Temperature Mott transition from Holography

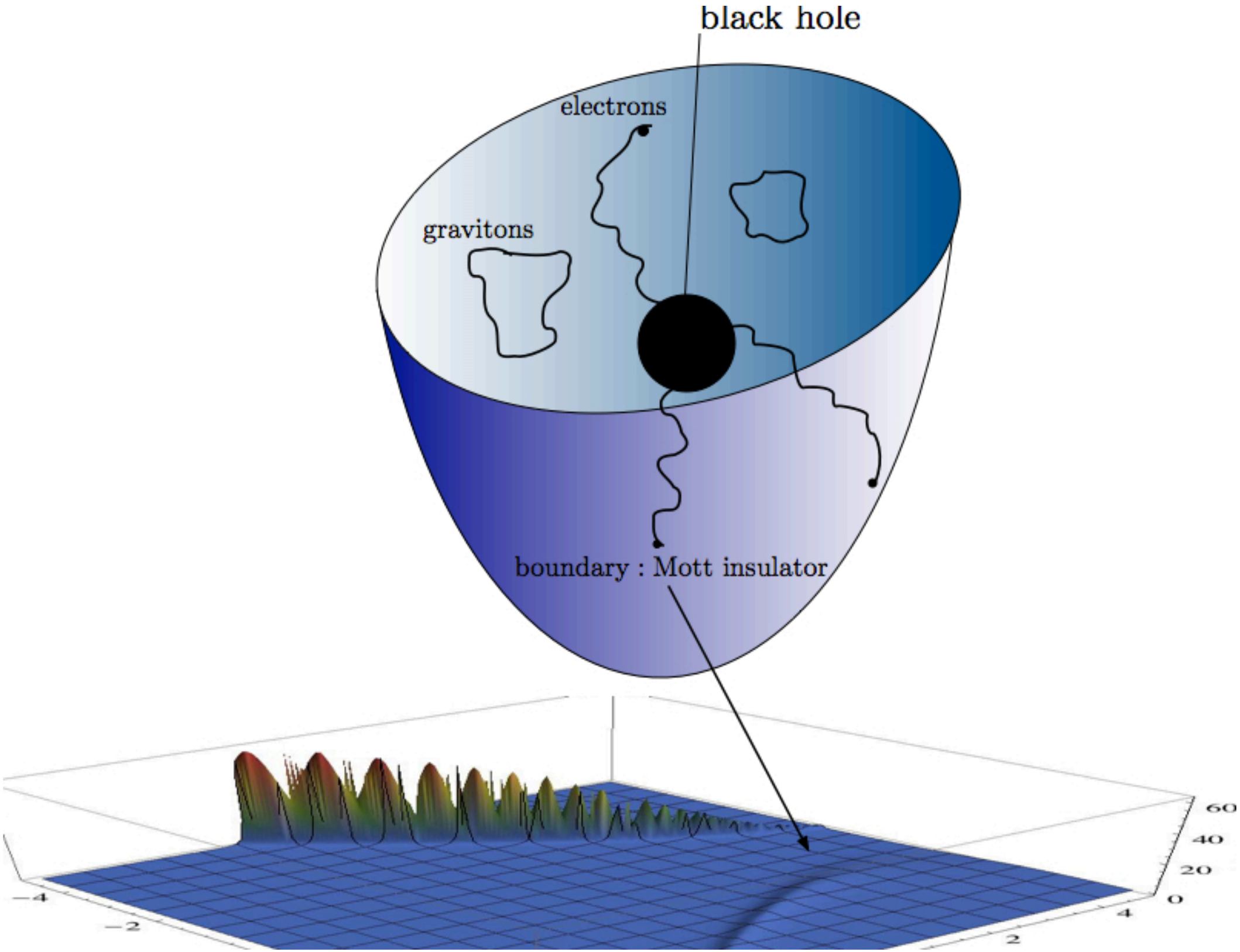




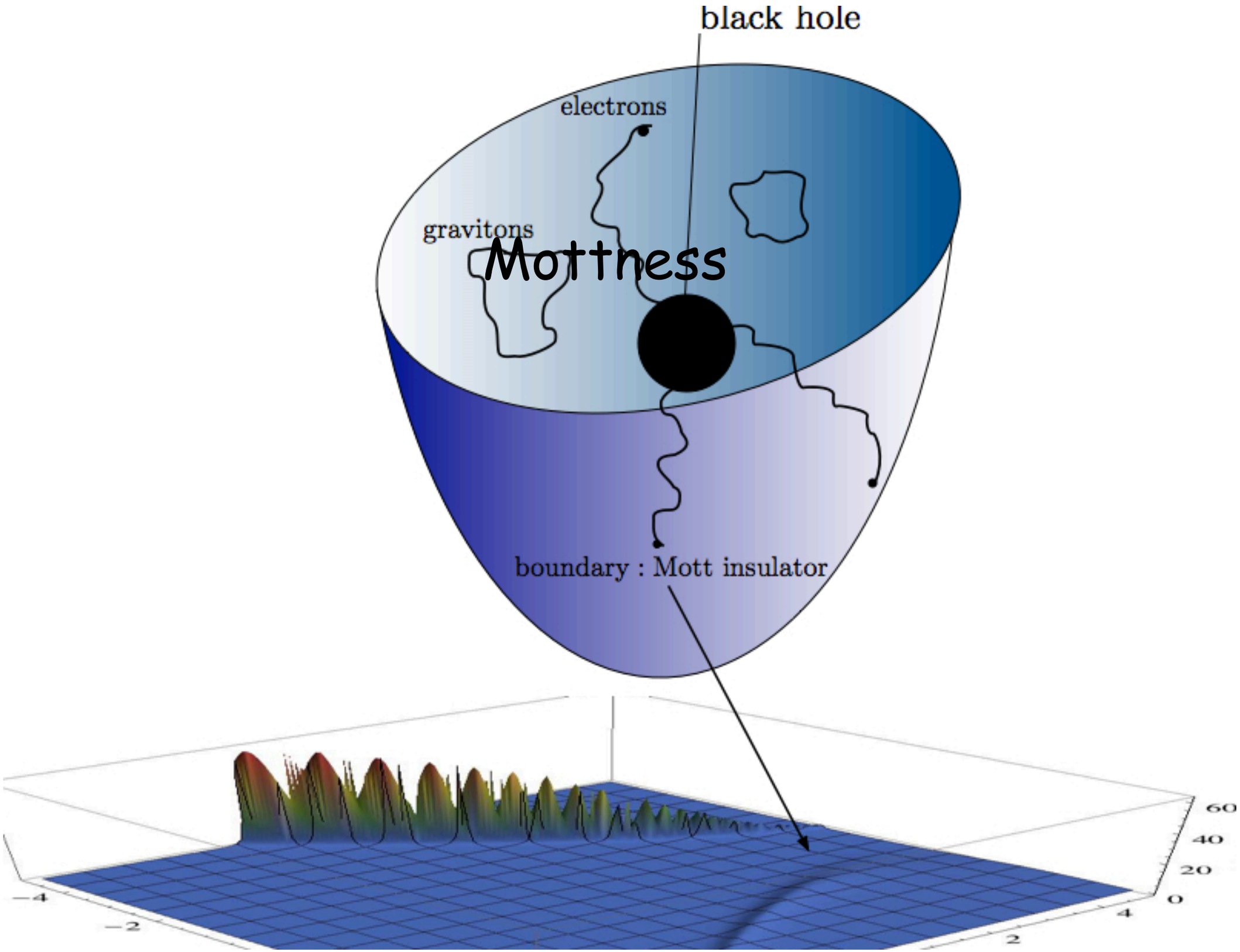
$$\frac{\Delta}{T_{\rm crit}} \approx 10$$

$$\frac{\Delta}{T_{\rm crit}} \approx 20$$

vanadium oxide



looks just like the experiments



looks just like the experiments

MIT Subject 8.322 Quantum Theory II

R. L. Jaffe 2007

Syllabus

This is an **optimistic** syllabus. Material that may be omitted for lack of time is in blue.

I Relativistic Wave Equations

- A The problem: Lorentz covariance
- B The Klein-Gordon equation
- C Dirac's equation
  1. Derivation, Dirac matrices (chiral and Bjorken and Drell)
  2. Massless case: Weyl's equation
  3. Operators, angular momentum and spin
  4. Hamiltonian form and equations of motion
  5. Bispinor notation
- D Non-relativistic reduction of the Dirac equation and the g factor of the electron.
- E Lorentz covariance of Dirac's equation
  1. Meaning, example of KG equation
  2. Solution and interpretation
  3. Bilinear covariants.
  4. Generalizations
- F Single particle solutions of the Dirac equation
  1. Counting states, projection operators
  2. Momentum eigenstates and their high energy limit.
- G More applications of the Dirac equation if time permits.

II Approximation Methods

- A Stationary state perturbation theory

toy model: merging of UV and IR fixed points

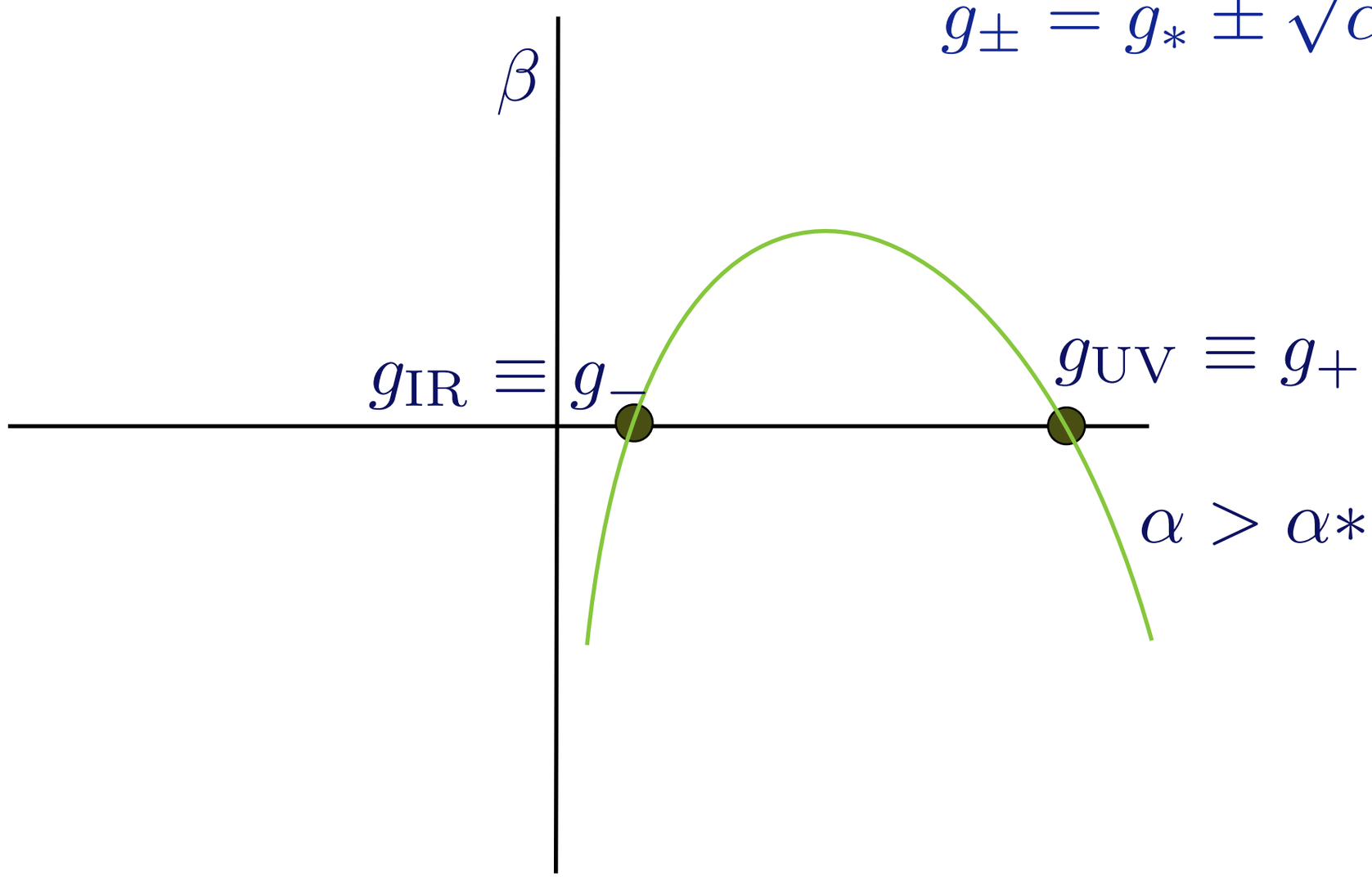
$$\beta = (\alpha - \alpha_*) - (g - g_*)^2$$

$$g_\pm = g_* \pm \sqrt{\alpha - \alpha_*}$$

toy model: merging of UV and IR fixed points

$$\beta = (\alpha - \alpha_*) - (g - g_*)^2$$

$$g_\pm = g_* \pm \sqrt{\alpha - \alpha_*}$$



toy model: merging of UV and IR fixed points

$$\beta = (\alpha - \alpha_*) - (g - g_*)^2$$

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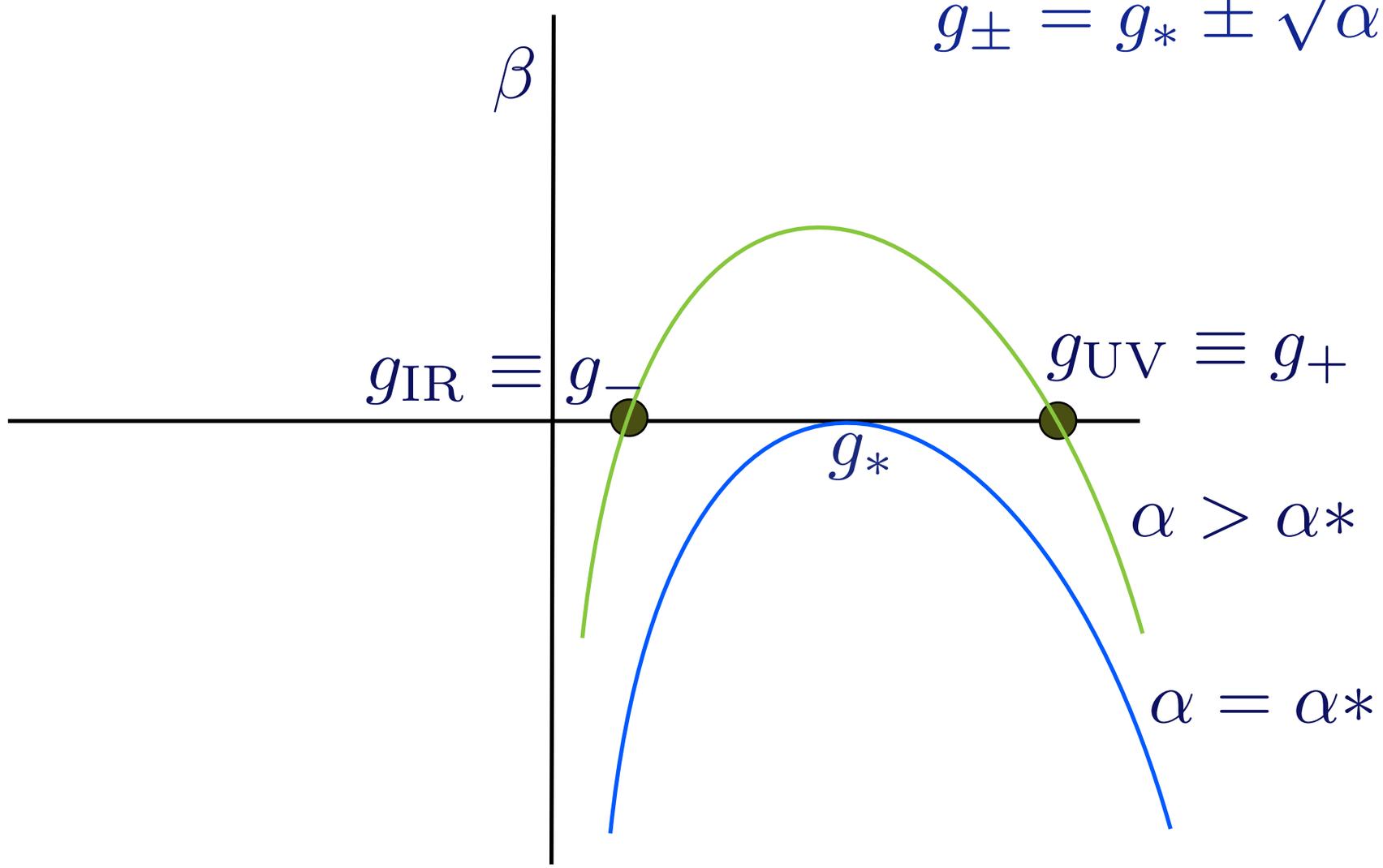
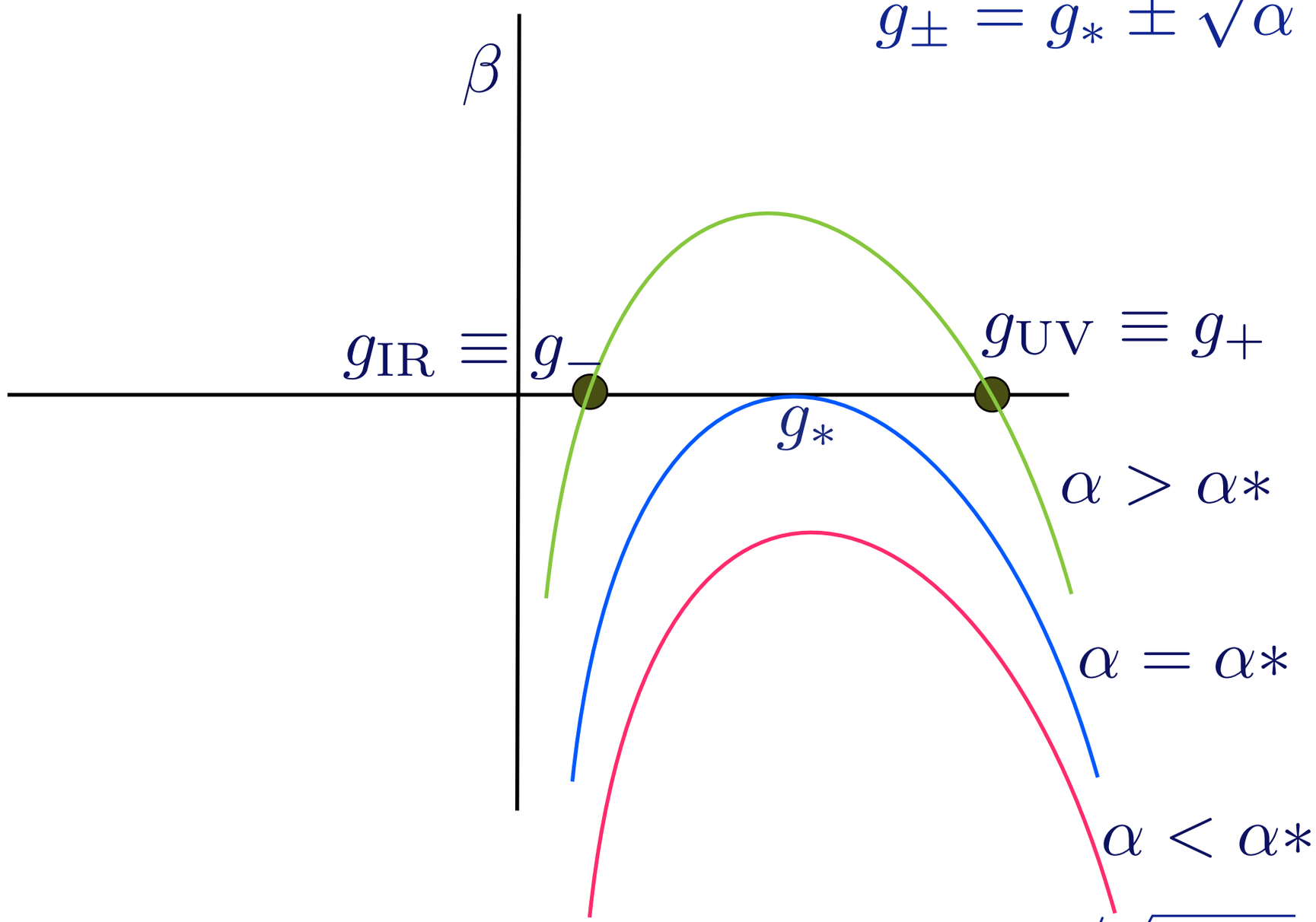


toy model: merging of UV and IR fixed points

$$\beta = (\alpha - \alpha_*) - (g - g_*)^2$$

$$g_\pm = g_* \pm \sqrt{\alpha - \alpha_*}$$

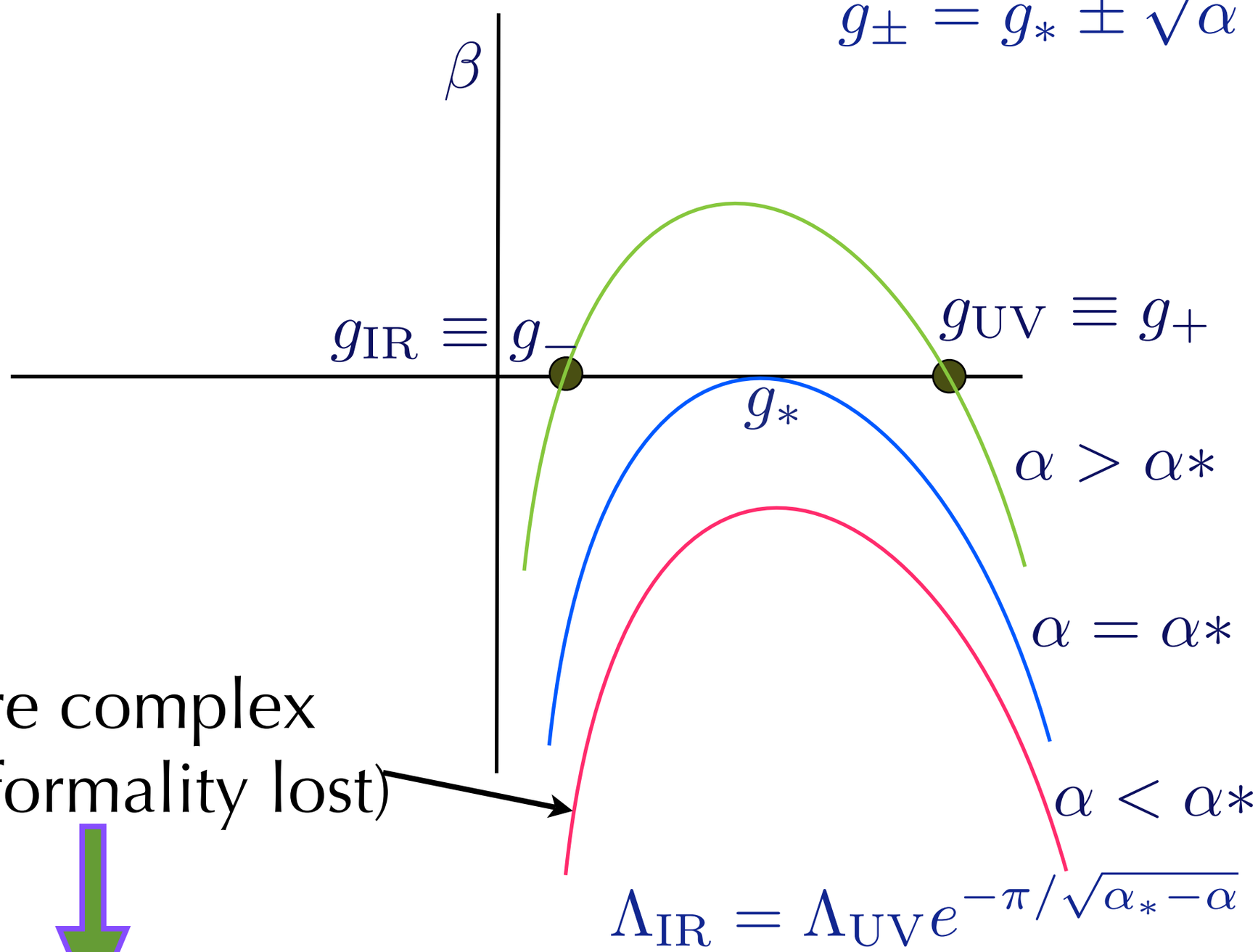


$$\Lambda_{\rm IR} = \Lambda_{\rm UV} e^{-\pi/\sqrt{\alpha_* - \alpha}}$$

toy model: merging of UV and IR fixed points

$$\beta = (\alpha - \alpha_*) - (g - g_*)^2$$

$$g_\pm = g_* \pm \sqrt{\alpha - \alpha_*}$$



$g_\pm$ are complex
(conformality lost)

DSI

BKT transition

Kaplan, arxiv:0905.475