Unparticles and Superconductivity

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Properties all particles share?
Properties all particles share?

charge

fixed mass!
Properties all particles share?

- charge
- conserved (gauge invariance)
- fixed mass!
Properties all particles share?

- charge: conserved (gauge invariance)
- fixed mass: not conserved
mass sets a scale
particles

\[ E_{\text{photon}} = h \nu \]

\[ v_{\text{max}} = 6.22 \times 10^5 \text{ m/s} \]

**Photoelectric effect**

\[ E = \varepsilon_p \]
particles

\[ E_{\text{photon}} = h\nu \]

\[ v_{\text{max}} = 6.22 \times 10^5 \text{ m/s} \]

\[ E = \varepsilon_p \]

\[ G(E) = \frac{1}{E - \varepsilon_p} \]
particles

\[ E_{\text{photon}} = h\nu \]

\[ E = \varepsilon_p \]

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle=\text{infinity}
Particles

\[ E_{\text{photon}} = h\nu \]

\[ \nu_{\text{max}} = 6.22 \times 10^5 \text{ m/s} \]

Photoelectric effect

energy \( E = \varepsilon_p \)

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle = infinity
particles = poles in Green function

\[ E_{\text{photon}} = h\nu \]

\[ v_{\text{max}} = 6.22 \times 10^5 \text{ m/s} \]

\[ \nu_{\text{max}} = 550 \text{ nm} \quad 2.25 \text{ eV} \]

\[ \nu_{\text{max}} = 400 \text{ nm} \quad 3.1 \text{ eV} \]

\[ \nu_{\text{max}} = 700 \text{ nm} \quad 1.77 \text{ eV} \]

\[ \text{Potassium - 2.0 eV needed to eject electron} \]

Photoelectric effect

energy \[ E = \varepsilon_p \]

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle = infinity
what would you see in a metal?
what would you see in a metal?
what would you see in a metal?
what would you see in a metal?

two crossings: closed surface of excitations
expect to see

$p_y$

$p_x$

closed surface
are there any exceptions?
where is this part?
where are arcs seen?
ceramics

normal state

$YBa_2Cu_3O_7$

Cuprate Superconductors

ceramics
what’s the explanation?

Two opposing Views on Fermi arcs

strange

not strange
what’s the explanation?

Two opposing Views
on Fermi arcs

strange

not strange

this dispute has an answer!
what’s the explanation?

Two opposing Views on Fermi arcs

strange

not strange

unparticles

this dispute has an answer!

Friday, March 14, 14
Right Wingers: Fermi Arcs are not Strange

intensity too small to be seen

\[ G(E) = \frac{Z_p}{\omega - \varepsilon_p} \]

pole (particle) exists but \( Z_p \to 0 \)
Fermi Arcs are Strange

seen

not seen

no pole

Friday, March 14, 14
Problem for left-wingers (me)
Problem for left-wingers (me)
Problem for left-wingers (me)

How to account for the sign change without poles?
Fermi arcs

pole

$0 \#$

no pole

$\frac{\#}{E - \epsilon_p - \infty} = 0$
do poles account for all the charged stuff?
if not?
if not?

charge stuff = particles + other stuff
if not?

charge stuff = particles + other stuff

unparticles!
counting particles
counting particles
counting particles
counting particles
counting particles
counting particles

is there a more efficient way?
Each particle has an energy

\[ E = \varepsilon_p \]

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle=\infty
Each particle has an energy $E_{\text{photon}} = h\nu$

Green function (propagator)

$$G(E) = \frac{1}{E - \varepsilon_p}$$

particle = infinity
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]
Luttinger's Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]
Luttinger's Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle density = number of sign changes of \( G \)
Counting sign changes?
Counting sign changes?

$\Theta(x)$
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 
\end{cases} \]
Counting sign changes?

$$\Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 
\end{cases}$$
Counting sign changes?

\[
\Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 \\
1/2 & x = 0 
\end{cases}
\]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 \\
1/2 & x = 0 
\end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 \\
1/2 & x = 0 
\end{cases} \]

counts sign changes

\[ n = 2 \sum_{k} \Theta(\Re G(k, \omega = 0)) \]

Luttinger Theorem for electrons
How do functions change sign?
How do functions change sign?

\[ E > \varepsilon_p \]

\[ E < \varepsilon_p \]

\textbf{divergence}
How do functions change sign?

$E > \varepsilon_p$

$E < \varepsilon_p$

pole

divergence
Is there another way?
Yes
zero-crossing
zero-crossing

$\varepsilon_p$

no divergence is necessary
closer look at Luttinger’s theorem

\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]
closer look at Luttinger’s theorem

\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]

divergences (poles) + zeros
closer look at Luttinger’s theorem

\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]

divergences (poles) + zeros

how can zeros affect the particle count?
what are zeros?

are they (like poles) conserved?
what models have zeros?
NiO insulates $d^8$?

Mott mechanism

Sir Neville
NiO insulates $d^8$? perhaps this costs energy

Mott mechanism

Sir Neville
NiO insulates $d^8$? Perhaps this costs energy.

Sir Neville

Mott mechanism

$U \gg t$
NiO insulates $d^8$? Perhaps this costs energy.

Sir Neville

Mott mechanism

$U \gg t$

$\mu = 0$
NiO insulates $d^8$? perhaps this costs energy

Mott mechanism

$U \gg t$

Sir Neville

no change in size of Brillouin zone

$\mu = 0$
\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \begin{array}{c} \mu = 0 \\ \omega \end{array} \right) d\omega \]
Mott Problem

\[ \Re G(0, p) = \left( \mu = 0 \right) \int_{-\infty}^{\infty} d\omega \]

\[ = \text{below gap} + \text{above gap} \]

\[ \text{Im } G = 0 \]

Kramers-Kronig
Mott Problem

\[ \text{Im } G = 0 \]

\[ \text{Kramers-Kronig} \]

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \mu = 0 \right) d\omega \]

\[ = \text{below gap} + \text{above gap} = 0 \]
\[ \text{Mott Problem} \]

\[ \text{Re} G(0, \omega) = \int_{-\infty}^{\infty} \left( \mu = 0, \int d\omega \right) \]

\[ = \text{below gap} + \text{above gap} = 0 \]

\[ \text{Det} G(k, \omega = 0) = 0 \quad \text{(single band)} \]

\[ \text{Im} G = 0 \quad \text{Kramers-Kronig} \]
Mott Problem

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \begin{array}{c|c} \mu = 0 \\ \hline \omega \end{array} \right) d\omega \]

\[ = \text{below gap} + \text{above gap} = 0 \]

\[ \text{Det} G(k, \omega = 0) = 0 \] (single band)

\[ \text{Det} \text{Re} G(0, p) = 0 \] Mottness
Y Ba$_2$Cu$_3$O$_7$
Cuprate Superconductors
Interactions dominate: Strong Coupling Physics

\[ U/t = 10 \gg 1 \]
how do zeros show up?
Problem for left-wingers (me)

How to account for the sign change without poles?

\( (0, \pi) \quad (\pi, \pi) \)

\( (0, 0) \quad (\pi, 0) \)

\( \Re G^R < 0 \)

\( \Re G^R > 0 \)

poles

(Re G<0)

(Re G>0)
How to account for the sign change without poles?
Problem for left-wingers (me)

How to account for the sign change without poles?

Only option: $\text{Det} G = 0!$ (zeros)
Fermi Arcs

zeros + poles

Luttinger, Dzyaloshinskii, Yang, Rice, Zhang, Tsvelik, Anderson (lots of smart people) ...

n = zeros + poles
The proof just presented is good for any state of our system: FL, NFL, or MI [in other words, either of poles or zeros can be used to change the sign of $G_\gamma$ in Eq. (1)]. The only way to incapacitate the Luttinger theorem in form (1) is to assume that the limit $T \to 0$ is discontinuous. Actually, one has to require that the whole line $T=0$ is a line of phase transitions.
Is this famous theorem from 1960 correct?
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails

no hopping => no propagation (zeros)
A model with zeros but Luttinger fails

no hopping => no propagation (zeros)
$e^-$

spin

generalization

N flavors of spin
$e^-$

$N = 5$

N flavors of spin

genralization

spin
$e^-$

Spin

Generalization

$N$ flavors of spin

$N = 5$
$e^{-}$

Spin

Generalization

$N$ flavors of spin

$2E$

$N = 5$

1

4

9

16

25
Generalization

$N$ flavors of spin

$H = \frac{U}{2} \left( n_1 + \cdots + n_N \right)^2$

Friday, March 14, 14
\[ G_{\alpha\beta}(\omega = 0) = \# \left( \frac{2n - N}{N} \right) \]
\[ G_{\alpha\beta}(\omega = 0) = \# \left( \frac{2n - N}{N} \right) \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ \{ 0, 1, 1/2 \} \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[
\begin{align*}
n &= 2 \\
N &= 3
\end{align*}
\]

\{ 0, 1, 1/2 \}
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \]
\[ N = 3 \]
\[ 2 = 3 \]
Luttinger’s theorem

\[ n = N \Theta (2n - N) \]

\[ n = 2 \quad \quad N = 3 \]

\[ \{ 0, 1, 1/2 \} \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

even

0, 1, 1/2

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Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

- **even**: \( n = 2 \), \( N = 3 \)
- **odd**: \( n = 0, 1, 1/2 \)

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Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\( n = 2 \)
\( N = 3 \)
\( 0, 1, 1/2 \)

\( \text{even} \)
\( \text{odd} \)

no solution
Problem

$G=0$
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \rightarrow \infty \]
Problem

G=0

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

\[ \infty \]

lifetime of a particle vanishes
Problem

$G = 0$

$$G = \frac{1}{E - \varepsilon_p - \Sigma}$$

$\exists \Sigma < \varepsilon_p$

lifetime of a particle vanishes
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

\( \exists \Sigma < \varepsilon_p \) lifetime of a particle vanishes

no particle
what went wrong?
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)

integral does not exist
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)

integral does not exist

No Luttinger theorem!
Luttinger’s theorem
experimental confirmation of violation?
experimental data (LSCO)

\[ k_F \]

\[ 1 - x_{FS} \]

\[ Z_p \rightarrow 0 \]

`Luttinger' count

\[ \text{Bi2212} \]

\[ \times \text{ Yang et al. (2011)} \]

\[ + \text{ He et al. (2011)} \]
experimental data (LSCO)

\[ k_F \]

\[ 1 - x_{FS} \]

\[ Z_p \rightarrow 0 \]

`Luttinger’ count

Bi2212

violation=> zeros are present
`Luttinger' count

experimental data (LSCO)

$k_F$

$1 - x_{FS}$

$Z_p \to 0$

Bi2212

violation => zeros are present

each hole ≠ a single k-state
Two opposing Views on Fermi arcs

strange

not strange

this dispute has an answer!
Two opposing Views on Fermi arcs

strange

this dispute has an answer!
how to count particles?
how to count particles?

some charged stuff has no particle interpretation
what is the extra stuff?
$\Sigma(\omega = 0, p) = 0$
Fermi liquid

$\Sigma(\omega = 0, p) = \infty$
new fixed point
\[ \Sigma(\omega = 0, p) = 0 \]

Fermi liquid

\[ \Sigma(\omega = 0, p) = \infty \]

new fixed point

scale invariance
new fixed point (scale invariance)

strongly correlated matter
strongly correlated matter

new fixed point (scale invariance)
strongly correlated matter

new fixed point
(scale invariance)

unparticles (IR)
(H. Georgi)
what is scale invariance?

invariance on all length scales
\[ f(x) = x^2 \]
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \]

scale change
$$f(x) = x^2$$

$$f(x/\lambda) = (x/\lambda)^2$$

scale change

scale invariance
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \] scale change

scale invariance

\[ f(x) = x^2 \lambda^{-2} g(\lambda) \]
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \]

**scale invariance**

\[ f(x) = x^2 \lambda^{-2} g(\lambda) \]

\[ g(\lambda) = \lambda^2 \]

1
what’s the underlying theory?
free field theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ \mathcal{L} \rightarrow \Lambda^2 \mathcal{L} \]
free field theory

\[ \mathcal{L} = \frac{1}{2} \partial^{\mu} \phi \partial_{\mu} \phi \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ \mathcal{L} \rightarrow \Lambda^2 \mathcal{L} \text{ scale invariant} \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial_{\mu} \phi + m^2 \phi^2 \]

\[
x \rightarrow x/\Lambda \\
\phi(x) \rightarrow \phi(x)
\]

mass
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

- \( \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \)
- \( x \rightarrow x/\Lambda \)
- \( \phi(x) \rightarrow \phi(x) \)
- \( m^2 \phi^2 \)
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

\[ x \to \frac{x}{\Lambda} \]

\[ \phi(x) \to \phi(x) \]

\[ m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\( x \rightarrow x/\Lambda \)
\( \phi(x) \rightarrow \phi(x) \)

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

mass

no scale invariance

(\( \Rightarrow \))
unparticles
from a massive theory?
\[ \mathcal{L} = \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \]
\L = \int_0^\infty (\partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m)) \, dm^2
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial_{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

ty_theory with all possible mass!
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[
\begin{align*}
\phi &\rightarrow \phi(x, m^2 / \Lambda^2) \\
x &\rightarrow x / \Lambda \\
m^2 / \Lambda^2 &\rightarrow m^2
\end{align*}
\]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]

\[ x \rightarrow x / \Lambda \]

\[ m^2 / \Lambda^2 \rightarrow m^2 \]

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]
\[ x \rightarrow x / \Lambda \]
\[ m^2 / \Lambda^2 \rightarrow m^2 \]

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
unparticles

\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]
\[ x \rightarrow x / \Lambda \]
\[ m^2 / \Lambda^2 \rightarrow m^2 \]
\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^2 |\gamma|
\]
\[
\left( \int_{0}^{\infty} dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|}
\]

\[
d_U - 2
\]
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{dU-2} \]
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U-2} \]

unparticles=fractional number of massless particles
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U-2} \]

unparticles=fractional number of massless particles

\[ d_U > 2 \]

zeros
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U - 2} \]

unparticles=fractional number of massless particles

\[ d_U > 2 \]

zeros

no simple sign change
\textit{dU?}
what really is the summation over mass?
what really is the summation over mass?

mass=energy
high energy (UV)

low energy (IR)

related to sum over mass

QFT
high energy (UV)

low energy (IR)

\[ \frac{dg(E)}{dlnE} = \beta(g(E)) \]

related to sum over mass

locality in energy

QFT
implement E-scaling with an extra dimension

low energy (IR)

d\frac{g(E)}{d\ln E} = \beta(g(E))

locality in energy

related to sum over mass

high energy (UV)

QFT

Friday, March 14, 14
implement E-scaling with an extra dimension

\[
\frac{dg(E)}{dlnE} = \beta(g(E))
\]

locality in energy

related to sum over mass

QFT
implement E-scaling with an extra dimension

gauge-gravity duality (Maldacena, 1997)

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]

related to sum over mass

locality in energy
implement E-scaling with an extra dimension

gauge-gravity duality (Maldacena, 1997)

UV QFT

IR QFT

no particles (conserved currents)

\[ \frac{dg(E)}{dlnE} = \beta(g(E)) \]

related to sum over mass

locality in energy
implement E-scaling with an extra dimension

gauge-gravity duality (Maldacena, 1997)

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]

locality in energy

no particles (conserved currents)

unparticles?

related to sum over mass
mass=extra dimension

fixed by metric
rewriting the action
rewriting the action

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]
rewriting the action

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

\[ m = z^{-1} \]
rewriting the action

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} dm^2 \]

\[ m = z^{-1} \]

\[ \mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right] \]
rewriting the action

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

\[ m = z^{-1} \]

\[ \mathcal{L} = \int_0^\infty \, d\!z \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right] \]

can be absorbed with AdS metric
generating functional for unparticles

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2} \int d^{4+2\delta} x \, dz \, \sqrt{-g} \left( \partial_a \Phi \partial^a \Phi + \frac{\Phi^2}{R^2} \right)$$
generating functional for unparticles

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2} \int d^{4+2\delta}x \, dz \, \sqrt{-g} \left( \partial_a \Phi \partial^a \Phi + \frac{\Phi^2}{R^2} \right)$$

$$ds^2 = \frac{L^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right) \quad \sqrt{-g} = \left( \frac{R}{z} \right)^{5+2\delta}$$
generating functional for unparticles

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2} \int d^{4+2\delta} x \, dz \, \sqrt{-g} \left( \partial_a \Phi \partial^a \Phi + \frac{\Phi^2}{R^2} \right)$$

$$ds^2 = \frac{L^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right) \quad \sqrt{-g} = \left( \frac{R}{z} \right)^{5+2\delta}$$

unparticle lives in

$$d = 4 + 2\delta \quad \delta \leq 0$$
scaling dimension is fixed

\[ m = \frac{1}{z} \]
scaling dimension is fixed

\[ m = \frac{1}{z} \]

\[ m_{\text{AdS}}^2 = \frac{d_U (d_U - d)}{R^2} \]
scaling dimension is fixed

\[ m = \frac{1}{z} \]

\[ 1 = d_U (d_U - d) \]
scaling dimension is fixed

\[ m = \frac{1}{z} \]

\[ 1 = d_U (d_U - d) \]

\[ d_U = \frac{d}{2} + \frac{\sqrt{d^2 + 4}}{2} \]
\[ G_U(p) \propto p^{2(d_U - d/2)} \]

\[ G_U(0) = 0 \]
unparticle (AdS) propagator has zeros!

\[ G_U(p) \propto p^{2(d_U - d/2)} \]

\[ G_U(0) = 0 \]
interchanging unparticles

fractional \((d_{U})\) number of massless particles
interchanging unparticles

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interchanging unparticles

fractional \( (d_U) \) number of massless particles

\[ e^{i\pi d_U} \neq -1, 0 \]
interchanging unparticles

fractional \((d_U)\) number of massless particles

\[ e^{i\pi d_U} \neq -1, 0 \]

fractional statistics in \(d=2+1\)
\[ d_U = \frac{d}{2} + \frac{\sqrt{d^2 + 4}}{2} > \frac{d}{2} \]

\[ e^{i\pi d_U} \neq e^{-i\pi d_U} \]

**time-reversal symmetry breaking from unparticle (zeros=Fermi arcs) matter**
\[
\frac{d \ln g}{d \ln \beta} = 4d_U - d > 0
\]

fermions with unparticle propagator

tendency towards pairing (any instability which establishes a gap)
High $T_c$
unparticles
variable mass
$UPt_3$
High T_c
unparticles
variable mass
UPt_3

emergent
gravity