Optical Conductivity in the Cuprates from holography and unparticles

Thanks to: NSF, EFRC (DOE)

Brandon Langley
Garrett Vanacore
Kridsangaphong Limtragool
Properties all particles share?

charge

fixed mass!
Properties all particles share?

charge

fixed mass!

conserved
(gauge invariance)
Properties all particles share?

- charge
- fixed mass!
- mass sets a scale
- conserved (gauge invariance)
Properties all particles share?

- charge
  - conserved (gauge invariance)

- fixed mass!
  - mass sets a scale
  - not conserved
Properties all particles share?

- charge
  - conserved (gauge invariance)

- fixed mass!
  - not conserved sets a scale
optical conductivity in the cuprates

scale invariant sector `unparticles'
(no definite mass)
optical conductivity in the cuprates

incoherent metal

scale invariant sector `unparticles'
(no definite mass)
unconventional superconductivity?
unconventional superconductivity?
Optical spectra of La$_{2-x}$Sr$_x$CuO$_4$: Effect of carrier doping on the electronic structure of the CuO$_2$ plane

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(Received 30 August 1990)
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(Received 30 August 1990)
Growth of the optical conductivity in the Cu-O planes


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(Received 7 March 1990)
\[ N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2} \int_{0}^{\Omega} \sigma(\omega)d\omega \]
\[ N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2} \int_{0}^{\Omega} \sigma(\omega) d\omega \]
$N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2} \int_0^\Omega \sigma(\omega) d\omega$
optical gap

\[ N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2} \int_0^{\Omega} \sigma(\omega) d\omega \]
Uchida, et al.

Cooper, et al.

La$_{2-x}$Sr$_x$CuO$_4$

$N^*_\text{eff}$ at 1.5eV

$N^*_\text{eff}$

$N_D$

$R_H > 0$

$R_H < 0$

$X$

Holes

Electrons

$x(Sr)$

$x(Ce)$

$N_{\text{EFF}}$

$0.2$

$0.4$

$0.0$

$0.1$

$0.2$

$0.3$

$0.4$

$0.5$
low-energy model for $N_{\text{eff}} > x$??
excess carriers?
excess carriers?

charge carriers have no particle content?
Mott insulator

\[ 1-x \]
Mott insulator

\[ U = \infty \]

\[ 1-x \]
Mott insulator

\[ U = \infty \]

\[ 1 - x \]

PES \[ \varepsilon F \] IPES
Mott insulator

\[ U = \infty \]

\[ 1 - x \]

no hopping:
Mott insulator

$\begin{align*}
PES & \quad \mathcal{E}_F \quad IPES \\
N & \quad U \quad N \\
\text{no hopping:} & \quad 1-x \\
PES & \quad N-1
\end{align*}$

$U = \infty$
Mott insulator

PES \[ \epsilon F \] IPES

\[ U = \infty \]

no hopping:

PES \[ N-1 \] IPES

1-x
Mott insulator

$U = \infty$

no hopping:

$1-x$
Mott insulator

\[ U = \infty \]

\[ 1-x \]

no hopping:

\[ \varepsilon_F \]

Sawatzky

Monday, November 9, 15
counting electron states

need to know: $N$ (number of sites)
counting electron states

\[ x = \frac{n_h}{N} \]

need to know: N (number of sites)
counting electron states

removal states

\[ x = \frac{n_h}{N} \]

\[ 1 - x \]

need to know: \( N \) (number of sites)
counting electron states

removal states $1 - x$

addition states $1 + x$

$x = n_h / N$

need to know: $N$ (number of sites)
counting electron states

\[ 2x \]

\[ x = \frac{n_h}{N} \]

removal states

\[ 1 - x \]

addition states

\[ 1 + x \]

need to know: \( N \) (number of sites)
counting electron states

\[ 2x \]

removal states \( 1 - x \)  
addition states \( 1 + x \)

low-energy electron states

\[ x = \frac{n_h}{N} \]

need to know: \( N \) (number of sites)
counting electron states

\[ x = \frac{n_h}{N} \]

removal states

\[ 1 - x \]

addition states

\[ 1 + x \]

low-energy electron states

\[ 1 - x + 2x = 1 + x \]

need to know: \( N \) (number of sites)
counting electron states

removal states

addition states

$$2x$$

$$x = \frac{n_h}{N}$$

need to know:  \( N \) (number of sites)
why is this a problem?
spectral function (dynamics)
spectral function (dynamics)
spectral function (dynamics)

$U \gg t$

density of states

$1 + x + \alpha > 1 + x$

Harris and Lange (1967)
spectral function (dynamics)

\[ U \gg t \]

\[ 1 + x + \alpha > 1 + x \]

\[ \alpha \propto \frac{t}{U} \]

Harris and Lange (1967)
The spectral function (dynamics) is not exhausted by counting electrons alone.

\[ U \gg t \]

\[ 1 + x + \alpha > 1 + x \]

density of states

\[ \alpha \propto t/U \]

\[ 1 - x - \alpha \]

Harris and Lange (1967)

not exhausted by counting electrons alone
spectral weight transfer

Hubbard Model

optical conductivity (mid-IR)

$N_{\text{eff}} > \#x$
is there anything else?
Quantum critical behaviour in a high-$T_c$ superconductor

D. van der Marel$^1$, H. J. A. Molegraaf$^1$, J. Zaanen$^2$, Z. Nussinov$^3$, F. Carbone$^4$, A. Damascelli$^5$, H. Eisaki$^1$, M. Greven$,^1$, P. H. Kes$^2$ & M. Li$^2$

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$^3$Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA

Drude conductivity

\[
\frac{n\tau e^2}{m} \left( \frac{1}{1 - i\omega\tau} \right)
\]
Quantum critical behaviour in a high-$T_c$ superconductor

D. van der Marel$^{1,2}$, H. J. A. Molegraaf$^{1,2}$, J. Zaanen$^2$, Z. Nussinov$^{2,3}$, F. Carbone$^{1,2}$, A. Damascelli$^{1,2}$, H. Elsak$^{1,2}$, M. Greven, P. H. Kes$^1$ and M. Li$^2$

$^1$Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands
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$^3$Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA

Drude conductivity

\[
\sigma(\omega) = \frac{n\tau e^2}{m} \frac{1}{1 - i\omega\tau}
\]
TABLE II. Values of the exponent $\alpha$ describing the best fit $\sigma \sim \omega^{-\alpha}$ for the various films labeled $A$, $B$, $D$, $E$, and $F$ films. $\sigma_a$ and $\sigma_b$ stand for the conductivities measured along the $a$ and $b$ directions in an untwinned single crystal (Ref. 14). $\sigma_{MFL}$ stands for the computed conductivity at 100 and 300 K for the marginal-Fermi-liquid model (MFL) (Ref. 16). The exponent is given within $\pm 0.05$.

<table>
<thead>
<tr>
<th>$\sigma_A$ (300 K)</th>
<th>$\sigma_B$ (300 K)</th>
<th>$\sigma_D$ (300 K)</th>
<th>$\sigma_E$ (300 K)</th>
<th>$\sigma_F$ (300 K)</th>
<th>$\sigma_a$ (100 K)</th>
<th>$\sigma_b$ (100 K)</th>
<th>$\sigma_{MFL}$ (300 K)</th>
<th>$\sigma_{MFL}$ (100 K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.77</td>
<td>0.70</td>
<td>0.77</td>
<td>0.76</td>
<td>0.76</td>
<td>0.67</td>
<td>none</td>
<td>0.74</td>
<td>0.68</td>
</tr>
</tbody>
</table>
Criticality

Scale Invariance

Power Law Correlations
criticality

scale invariance

power law correlations
scale-invariant propagators

\[ \left( \frac{1}{p^2} \right)^{\alpha} \]
scale-invariant propagators

\( \left( \frac{1}{p^2} \right)^\alpha \)

Anderson: use Luttinger Liquid propagators

\[ G^R \propto \frac{1}{(\omega - v_s k)^\eta} \]
compute conductivity without vertex corrections (PWA)

is flawed. In fact, in the Luttinger liquid such direct calculations are not to be trusted very firmly, since it is the nature of the Luttinger liquid that vertex corrections, if they must be included, will be singular; conventional transport theory is not applicable, and special methods such as the above are necessary.

\[ \sigma(\omega) \propto \frac{1}{\omega} \int dx \int dt G^e(x, t) G^h(x, t) e^{i\omega t} \propto (i\omega)^{-1+2\eta} \]
problems
problems

1.) cuprates are not 1-dimensional
problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter
problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter

\[
\sigma \propto \int d^d p G \left( G(\Gamma^\mu)^2, \Gamma^{\mu\nu} \right) \\
[G] = L^{d+1-2d_U} \\
[\Gamma^\mu] = L^{2d_U-d} \\
[\Gamma^{\mu\nu}] = L^{2d_U-d+1}
\]
problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter

\[ \sigma \propto \int d^d p G \left( G(\Gamma^\mu)^2, \Gamma^{\mu \nu} \right) \]

\[ [G] = L^{d+1-2d_U} \]

\[ [\Gamma^\mu] = L^{2d_U-d} \]

\[ [\Gamma^{\mu \nu}] = L^{2d_U-d+1} \]

\[ [\sigma] = L^{2-d} \]

independent of \( d_U \)
optical conductivity from a gravitational lattice

log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega \tau \lesssim 0.8$

G. Horowitz et al., Journal of High Energy Physics, 2012
optical conductivity from a gravitational lattice

log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega \tau \lesssim 0.8$

a remarkable claim!
replicates features of the strange metal? how?

G. Horowitz et al., Journal of High Energy Physics, 2012
Could string theory be the answer?
$g = 1/\text{ego}$

coupling constant

$\text{IR} \quad ?? \quad \text{UV QFT}$
UV QFT

$g = 1/\text{ego}$

coupling constant

$\frac{dg(E)}{dlnE} = \beta(g(E))$

locality in energy
replace coupled theory with geometry

scale-invariant anti de-Sitter

\[ R \propto \sqrt{g} \]
replace coupled theory with geometry

scale-invariant anti de-Sitter

\[ \text{AdS}_2 \times \mathbb{R}^2 \]

\[ R \propto \sqrt{g} \]
replace coupled theory with geometry

scale-invariant anti de-Sitter

$z = 1$

$AdS_2 \times R^2$

$z = 0$

$g(\bar{\psi}\psi)^2$

$R \propto \sqrt{g}$
replace coupled theory with geometry

scale-invariant anti de-Sitter

$\mathcal{R} \propto \sqrt{g}$
cannot describe systems at $g=0$!
Einstein-Maxwell equations + non-uniform charge density = $B\omega^{-2/3}$
not so fast!
Drude conductivity

\[
\frac{n \tau e^2}{m} \frac{1}{1 - i\omega \tau}
\]
Drude conductivity

\[
\frac{n\tau e^2}{m} \frac{1}{1 - i\omega \tau}
\]

\[-\frac{2}{3} = 1 + \omega \frac{|\sigma|''}{|\sigma|'}\]
Drude conductivity

\[ \frac{n \tau e^2}{m} \frac{1}{1 - i\omega \tau} \]

\[-\frac{2}{3} = 1 + \omega \frac{|\sigma|''}{|\sigma|'} \]

\[ B \omega^{-2/3} \]
Donos and Gauntlett
(gravitational crystal)

Drude conductivity

\[ \frac{n \tau e^2}{m} \left( \frac{1}{1 - i \omega \tau} \right) \]

\[ -\frac{2}{3} = 1 + \omega \frac{|\sigma|''}{|\sigma|'} \]

no power law!!

\[ B \omega^{-2/3} \]
who is correct?
who is correct?

let’s redo the calculation
model
model

\[ \text{action} = \text{gravity} + \text{EM} + \text{lattice} \]
\[ S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} F^2 \right), \]
model

action = gravity + EM + lattice

\[ S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} F^2 \right), \]
The model is given by the action:

$$S = \frac{1}{16\pi G_N} \int d^4 x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} F^2 \right),$$

and the Lagrangian for the field $\phi$:

$$\mathcal{L}(\phi) = \sqrt{-g} \left[ -|\partial\phi|^2 - V(|\phi|) \right]$$
conductivity within AdS

\((g_{ab}, V(\Phi), A_t)\)
(metric, potential, gaugefield)
conductivity within AdS

\((g_{ab}, V(\Phi), A_t)\)

(metric, potential, gaugefield)
conductivity within AdS

\((g_{ab}, V(\Phi), A_t)\)

(metric, potential, gaugefield)

\[ A_t = \mu(1 - z)dt \]

\[ \rho = \lim_{z \to 0} \sqrt{-g}F^{tz} \]
conductivity within AdS

\((g_{ab}, V(\Phi), A_t)\)

(metric, potential, gaugefield)

perturb with electric field

\[A_t = \mu(1 - z) dt\]

\[\rho \equiv \lim_{z \to 0} \sqrt{-g} F^{tz} \bar{E}\]
conductivity within AdS

\( (g_{ab}, V(\Phi), A_t) \)

(metric, potential, gaugefield)

perturb with electric field

\[
\begin{align*}
g_{ab} &= \bar{g}_{ab} + h_{ab} \\
A_a &= \bar{A}_a + b_a \\
\Phi_i &= \bar{\Phi}_i + \eta_i
\end{align*}
\]

\[ A_t = \mu(1 - z) dt \]

\[ \rho = \lim_{z \to 0} \sqrt{-g} F_{tz} \]

\[ \Phi \]

\[ \vec{E} \]

\[ Q \]
(\(g_{ab}, V(\Phi), A_t\))
(metric, potential, gaugefield)

perturb with electric field

\[
g_{ab} = \bar{g}_{ab} + h_{ab}
\]
\[
A_a = \bar{A}_a + b_a
\]
\[
\Phi_i = \bar{\Phi}_i + \eta_i
\]

\[
A_t = \mu (1 - z) dt
\]

\[
\rho = \lim_{z \to 0} \sqrt{-g} F^{tz} \vec{E}
\]

\[
\delta A_x = \frac{E}{i \omega} + J_x(x, \omega) z + O(z^2)
\]

\[
\sigma = \frac{J_x(x, \omega)}{E}
\]
conductivity within AdS

\((g_{ab}, V(\Phi), A_t)\)
(metric, potential, gaugefield)

perturb with electric field

\(g_{ab} = \bar{g}_{ab} + h_{ab}\)
\(A_a = \bar{A}_a + b_a\)
\(\Phi_i = \bar{\Phi}_i + \eta_i\)

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\(\delta A_x = \frac{E}{i\omega} + J_x(x, \omega)z + O(z^2)\)

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conductivity within AdS

\[(g_{ab}, V(\Phi), A_t)\]
(metric, potential, gaugefield)

perturb with electric field

\[g_{ab} = \bar{g}_{ab} + h_{ab}\]
\[A_a = \bar{A}_a + b_a\]
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solve equations of motion with gauge invariance (without mistakes)

\[A_t = \mu(1 - z)dt\]
\[\rho = \lim_{z \to 0} \sqrt{-g} F^{tz} \vec{E}\]
\[\delta A_x = \frac{E}{i\omega} + J_x(x, \omega)z + O(z^2)\]
\[\sigma = \frac{J_x(x, \omega)}{E}\]
conductivity within AdS

\((g_{ab}, V(\Phi), A_t)\)
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perturb with electric field

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\begin{align*}
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\end{align*}
\]

solve equations of motion with gauge invariance (without mistakes)

\[
\begin{align*}
  \delta A_x &= \frac{E}{i\omega} + J_x(x, \omega)z + O(z^2) \\
  \sigma &= \frac{J_x(x, \omega)}{E}
\end{align*}
\]
HST vs. DG
Horowitz, Santos, Tong (HST)

\[ V(\Phi) = -\frac{\Phi^2}{L^2} \]

\[ \Phi = z\Phi^{(1)} + z^2\Phi^{(2)} + \cdots, \]
\[ \Phi^{(1)}(x) = A_0 \cos(kx) \]

inhomogeneous in x

\[ m^2 = -\frac{2}{L^2} \]
Horowitz, Santos, Tong (HST)

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de Donder gauge

HST vs. DG
Horowitz, Santos, Tong (HST)

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\[ \Phi = z\Phi^{(1)} + z^2\Phi^{(2)} + \cdots , \]

\[ \Phi^{(1)}(x) = A_0 \cos(kx) \]

inhomogeneous in x

\[ m^2 = -2 / L^2 \]

de Donder gauge

HST vs. DG

DG

\[ V(|\Phi|^2) \]

\[ \Phi(z, x) = \phi(z)e^{ikx} \]

no inhomogeneity in x

\[ m^2 = -3/(2L^2) \]
Horowitz, Santos, Tong (HST)

\[ V(\Phi) = -\Phi^2 / L^2 \]
\[ \Phi = z\Phi^{(1)} + z^2\Phi^{(2)} + \cdots , \]
\[ \Phi^{(1)}(x) = A_0 \cos(kx) \]

inhomogeneous in x

\[ m^2 = -2 / L^2 \]

de Donder gauge

HST vs. DG

DG

\[ V(|\Phi|^2) \]
\[ \Phi(z, x) = \phi(z)e^{ikx} \]

no inhomogeneity in x

\[ m^2 = -3 / (2L^2) \]

radial gauge
Our Model

\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]
Our Model

\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]

\[ \Phi_1 = z\Phi_1^{(1)} + z^2\Phi_1^{(2)} + \cdots, \quad \Phi_1^{(1)}(x) = A_0 \cos \left( kx - \frac{\theta}{2} \right), \]

\[ \Phi_2 = z\Phi_2^{(1)} + z^2\Phi_2^{(2)} + \cdots, \quad \Phi_2^{(1)}(x) = A_0 \cos \left( kx + \frac{\theta}{2} \right). \]
\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]

\[ \Phi_1 = z\Phi_1^{(1)} + z^2\Phi_1^{(2)} + \cdots, \quad \Phi_1^{(1)}(x) = A_0 \cos \left( kx - \frac{\theta}{2} \right), \]

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\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]

\begin{align*}
\Phi_1 &= z\Phi_1^{(1)} + z^2\Phi_1^{(2)} + \cdots, & \Phi_1^{(1)}(x) &= A_0 \cos \left( kx - \frac{\theta}{2} \right), \\
\Phi_2 &= z\Phi_2^{(1)} + z^2\Phi_2^{(2)} + \cdots, & \Phi_2^{(1)}(x) &= A_0 \cos \left( kx + \frac{\theta}{2} \right). 
\end{align*}

\[ \theta = 0 \quad \text{HST} \quad \theta = \frac{\pi}{2} \quad \text{DG} \]
$A_0 = 0.75, \ k = 1, \ \mu = 1.4, \ T/\mu = 0.115$

$\theta = 0$

$\theta = \frac{\pi}{4}$

$\theta = \frac{\pi}{2}$
translational invariance is broken in metric in multiples of 2k
charge density

\[ \rho = \lim_{z \to 0} \sqrt{-g} F^{tz} \]
- high-frequency behavior is identical
- low-frequency RN has \( \text{Re}(\sigma) \sim \delta(\omega) \), \( \text{Im}(\sigma) \sim 1/\omega \)
- low-frequency lattice has Drude form
is there a power law?
is there a power law?

Results

- $-2/3$

HST

DG
Results
Results

\[ A_1 = 0.75, k_1 = 2, k_2 = 2, \theta = 0, \mu = 1.4, \frac{T}{\mu} = 0.115 \]
No!
scale invariance

Anderson

AdS/CFT
origin of power law?

phenomenology

scale-invariant propagators

\[(p^2)^{d_U - d/2}\]
origin of power law?

phenomenology

scale-invariant propagators

\[ (p^2)^d U - d/2 \]

no well-defined mass

\[ \mathcal{L}_{\text{eff}} = \int_{0}^{\infty} \mathcal{L}(x, m^2) dm^2 \]
origin of power law?

phenomenology

scale-invariant propagators

\((p^2)^{d_U - d/2}\)

no well-defined mass

\[ L_{\text{eff}} = \int_0^\infty L(x, m^2) dm^2 \]

incoherent stuff (all energies)
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x / \Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ \Lambda^2 \left( \frac{1}{2} \partial_\mu \phi \partial_\mu \phi \right) \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

mass
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \to x/\Lambda \quad \phi(x) \to \phi(x) \]

\[ \Lambda^2 \left( \frac{1}{2} \partial_\mu \phi \partial_\mu \phi \right) \]

mass
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\( x \rightarrow x/\Lambda \)

\( \phi(x) \rightarrow \phi(x) \)

no scale invariance

mass

\( m^2 \phi^2 \)
\[ \mathcal{L} = \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \]
\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]
\[ L = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!
\[ \mathcal{L} = \int_0^\infty (\partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m)) \, dm^2 \]

theory with all possible mass!

\[
\phi \rightarrow \phi(x, m^2 / \Lambda^2) \\
x \rightarrow x / \Lambda \\
m^2 / \Lambda^2 \rightarrow m^2
\]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!

\[ \phi \to \phi(x, m^2 / \Lambda^2) \]
\[ x \to x / \Lambda \]
\[ m^2 / \Lambda^2 \to m^2 \]
\[ \mathcal{L} \to \Lambda^4 \mathcal{L} \]

scale invariance is restored!!
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]
\[ x \rightarrow x / \Lambda \]
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\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
unparticles

\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]
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\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{\epsilon}{p^2 - m^2 + \epsilon} \right)^{-1} \propto p^2|\gamma|
\]
propagator

\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^2 |\gamma| \\
\downarrow \quad d_U - 2
\]
\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|}
\]

\text{propagator}

\text{continuous mass}

\phi(x, m^2)

\text{flavors}

\text{dU} - 2
propagator

\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|}
\]

continuous mass

\[
\phi(x, m^2) \quad e^2(m) \quad \text{Karch, 2005}
\]

flavors multi-bands
\[
\left( \int_{0}^{\infty} \, dm^2 \, m^{2\gamma} \, \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|}
\]

\[
d_U - 2
\]

propagator

continuous mass

\[\phi(x, m^2) \quad \leftrightarrow \quad e^2(m)\]

Karch, 2005

flavors

multi-bands
use unparticle propagators to calculate conductivity
use unparticle propagators to calculate conductivity

assume Gaussian action

\[ S = \int d^{d+1}p \, \phi_U^\dagger(p)iG^{-1}(p)\phi_U(p) \]
use unparticle propagators to calculate conductivity

assume Gaussian action

\[
S = \int d^{d+1}p \, \phi_U^\dagger(p) iG^{-1}(p) \phi_U(p)
\]

\[
\phi_U(x) = \int_0^\infty dm^2 \, f(m^2) \phi(x, m^2)
\]
use unparticle propagators to calculate conductivity

assume Gaussian action

\[ S = \int d^{d+1}p \; \phi_U^\dagger(p) iG^{-1}(p)\phi_U(p) \]

\[ \phi_U(x) = \int_0^\infty \! dm^2 \; f(m^2)\phi(x, m^2) \]

\[ G(p) \sim \frac{i}{(-p^2 + i\epsilon)^{\frac{d+1}{2}} - d_U} \]
gauge unparticles
\[ S = \int d^{d+1}x d^{d+1}y \, \phi_U^\dagger(x) F(x - y) W(x, y) \phi_U(y), \]
\[ S = \int d^{d+1}x d^{d+1}y \, \phi^\dagger_U(x) F(x - y) W(x, y) \phi_U(y), \]
gauge unparticles

Wilson line

\[ S = \int d^{d+1}x d^{d+1}y \, \phi_U^\dagger(x) F(x - y) W(x, y) \phi_U(y), \]

vertices

1-gauge

\[ g \Gamma^\mu(p, q) = \frac{\delta^3 S}{\delta A^\mu(q) \delta \phi^\dagger(p + q) \delta \phi(p)} \]

2-gauge

\[ g^2 \Gamma^{\mu\nu}(p, q_1, q_2) = \frac{\delta^4 S}{\delta A^\mu(q_1) \delta A^\nu(q_2) \delta \phi^\dagger(p + q_1 + q_2) \delta \phi(p)} \]
compute conductivity

\[ \sigma^{\mu\nu}(i\omega_n) = \lim_{q\to 0} \frac{1}{\omega_n} K^{\mu\nu}_n(q)! \]
\[
\sigma_{\mu\nu}(i\omega_n) = \lim_{q \to 0} \frac{1}{\omega_n} K_{\mu\nu}^n(q)!
\]

compute conductivity

no power law

\[
\sigma(i\omega_n) = \left( \frac{d+1}{2} - d_U \right) \sigma_0(i\omega_n)
\]
what went wrong?
what went wrong?

\[ \phi_U(x) = \int_0^\infty dm^2 \, f(m^2) \phi(x, m^2) \]

free field
what went wrong?

\[ \phi_U(x) = \int_0^\infty dm^2 f(m^2) \phi(x, m^2) \]

free field

unparticle field has particle content
what went wrong?

free field

\[ \phi_U(x) = \int_0^\infty d\lambda \exp(-\lambda^2 f(m^2) + \lambda x, m^2) \]

unparticle field has particle content
continuous mass taken seriously

\[ S = \sum_{i=1}^{N} \int d\tau \int d^d x (|D_\mu \phi_i|^2 + m_i^2 |\phi_i|^2) \]
continuous mass taken seriously

\[ S = \sum_{i=1}^{N} \int d\tau \int d^d x (|D_\mu \phi_i|^2 + m_i^2 |\phi_i|^2) \]

\[ \sum_i \rightarrow \int \rho(m) dm \]
continuous mass taken seriously

\[ S = \sum_{i=1}^{N} \int d\tau \int d^{d}x \left( |D_{\mu} \phi_{i}^{2}| + m_{i}^{2} |\phi_{i}|^{2} \right) \]

\[ \sum_{i} \rightarrow \int \rho(m) dm \]

\[ \sigma(\omega) = \int_{0}^{M} dm \rho(m)e^{2}(m)f(\omega, m, T) \]
continuous mass taken seriously

\[ S = \sum_{i=1}^{N} \int d\tau \int d^d x (|D_\mu \phi_i^2| + m_i^2 |\phi_i|^2) \]

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\[ \propto \omega^\alpha \quad \alpha > 0 (\omega < 2M) \]
continuous mass taken seriously

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\[ \sum_i \rightarrow \int \rho(m) dm \]

\[ \sigma(\omega) = \int_0^M dm \rho(m) e^2(m) f(\omega, m, T) \]

\[ \propto \omega^\alpha \quad \alpha > 0 (\omega < 2M) \]

\[ \alpha > 0 \text{ convergence of integral} \]
last attempt
last attempt

take experiments seriously
last attempt

take experiments seriously

\[ \sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \cdot \frac{1}{1 - i\omega \tau_i} \]
last attempt
take experiments seriously

\[ \sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \cdot \frac{1}{1 - i\omega \tau_i} \]  

continuous mass \[ d\sigma(m) = \sigma(m)dm \]
last attempt

take experiments seriously

\[ \sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega \tau_i} \]

continuous mass

\[ d\sigma(m) = \sigma(m) dm \]

\[ \sigma(\omega) = \int_0^M \frac{\rho(m) e^2(m) \tau(m)}{m} \frac{1}{1 - i\omega \tau(m)} dm \]
variable masses for everything

\[\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}\]

\[e(m) = e_0 \frac{m^b}{M^b}\]

\[\tau(m) = \tau_0 \frac{m^c}{M^c}\]

Karch, 2015
variable masses for everything

\[ \rho(m) = \rho_0 \frac{m^{a-1}}{M^a} \]

\[ e(m) = e_0 \frac{m^b}{M^b} \]

\[ \tau(m) = \tau_0 \frac{m^c}{M^c} \]

\[ \sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0}{M^{a+2b+c}} \int_0^M dm \frac{m^{a+2b+c-2}}{1 - i\omega \tau_0 \frac{m^c}{M^c}} = \]

Karch, 2015
variable masses for everything

\[ \rho(m) = \rho_0 \frac{m^{a-1}}{M^a} \]

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\[ \sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0}{M^{a+2b+c}} \int_0^M \frac{m^{a+2b+c-2}}{1 - i\omega \tau_0 \frac{m^c}{M^c}} \, dm \]

\[ = \frac{\rho_0 e_0^2}{cM} \frac{1}{\omega(\omega \tau_0)} \int_0^{\omega \tau_0} \frac{x^{\frac{a+2b-1}{c}}}{1 - ix} \, dx \]

Karch, 2015
variable masses for everything

\[ \rho(m) = \rho_0 \frac{m^{a-1}}{M^a} \]

\[ e(m) = e_0 \frac{m^b}{M^b} \]

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\[ \sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0}{M^{a+2b+c}} \int_0^M dm \frac{m^{a+2b+c-2}}{1 - i\omega \tau_0 \frac{m^c}{M^c}} = \frac{\rho_0 e_0^2}{cM} \frac{1}{\omega (\omega \tau_0)^{\frac{a+2b-1}{c}}} \int_0^\omega \tau_0 dx \frac{x^{\frac{a+2b-1}{c}}}{1 - ix} \]

perform integral

Karch, 2015
\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]
\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]

\[
\sigma(\omega) = \frac{\rho_0 e^{2} \tau_0^{\frac{1}{3}}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_{0}^{\omega \tau_0} dx \frac{x^{-\frac{1}{3}}}{1 - ix}
\]
\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]

\[
\sigma(\omega) = \frac{\rho_0 e_0^{2\tau_0^{\frac{1}{3}}}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_0^{\omega \tau_0} dx \frac{x^{-\frac{1}{3}}}{1 - ix}
\]

\[
\omega \tau_0 \rightarrow \infty
\]
\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]

\[
\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_0^{\omega \tau_0} dx \frac{x^{-\frac{1}{3}}}{1 - ix}
\]

\[\omega \tau_0 \rightarrow \infty\]

\[
\sigma(\omega) = \frac{1}{3} (\sqrt{3} + 3i) \pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M \omega^{\frac{2}{3}}}
\]
\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]

\[
\sigma(\omega) = \frac{\rho_0 e_{0}^{2} \tau_{0}^{\frac{1}{3}}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_{0}^{\omega \tau_{0}} dx \frac{x^{-\frac{1}{3}}}{1 - ix}
\]

\[
\omega \tau_{0} \to \infty
\]

\[
\sigma(\omega) = \frac{1}{3} (\sqrt{3} + 3i) \pi \frac{\rho_0 e_{0}^{2} \tau_{0}^{\frac{1}{3}}}{M \omega^{\frac{2}{3}}}
\]

\[
\tan \sigma = \sqrt{3}
\]

\[
60^\circ
\]
experiments

\[ \sigma(\omega) = C \omega^{\gamma - 2} e^{i\pi(1 - \gamma/2)} \]

\[ \gamma = 1.35 \]
\[ \sigma(\omega) = \frac{1}{3}(\sqrt{3} + 3i)\pi \rho_0 e_0^2 \tau_0^{\frac{1}{3}} \frac{1}{M \omega^{\frac{2}{3}}} \]

\[ \gamma = 1.35 \]

\[ \sigma(\omega) = C \omega^{-2} e^{i\pi(1-\gamma/2)} \]
experiments

\[ \sigma(\omega) = \frac{1}{3}(\sqrt{3} + 3i)\pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M \omega^{\frac{2}{3}}} \]

\[ \tan \frac{\sigma_2}{\sigma_1} = \sqrt{3} \]

\[ \theta = 60^\circ \]

\[ \sigma(\omega) = C \omega^{-2} e^{i\pi(1-\gamma/2)} \]

\[ \gamma = 1.35 \]
experiments

\[ \sigma(\omega) = \frac{1}{3}(\sqrt{3} + 3i)\pi \rho e_0^2 \tau_0^{\frac{1}{3}} M \omega^{\frac{2}{3}} \]

victory!!

\[ \tan \sigma_2/\sigma_1 = \sqrt{3} \]
\[ \theta = 60^\circ \]

\[ \sigma(\omega) = C \omega^{\gamma - 2} e^{i\pi(1-\gamma/2)} \]
\[ \gamma = 1.35 \]
are anomalous dimensions necessary

\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]

\[
\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}
\]

\[
e(m) = e_0 \frac{m^b}{M^b}
\]

\[
\tau(m) = \tau_0 \frac{m^c}{M^c}
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are anomalous dimensions necessary

\[ \frac{a + 2b - 1}{c} = -\frac{1}{3} \]

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\[ \tau(m) = \tau_0 \frac{m^c}{M^c} \]

hyperscaling violation
are anomalous dimensions necessary

\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]

\[
\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}
\]

\[
e(m) = e_0 \frac{m^b}{M^b}
\]

\[
\tau(m) = \tau_0 \frac{m^c}{M^c}
\]
are anomalous dimensions necessary

\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]

\[
\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}
\]

\[
e(m) = e_0 \frac{m^b}{M^b}
\]

\[
\tau(m) = \tau_0 \frac{m^c}{M^c}
\]

\[
c = 1
\]

\[
a + 2b = \frac{2}{3}
\]
are anomalous dimensions necessary

\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]

\[
\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}
\]

\[
e(m) = e_0 \frac{m^b}{M^b}
\]

\[
\tau(m) = \tau_0 \frac{m^c}{M^c}
\]

\[
c = 1 \quad b = 0 \quad a + 2b = \frac{2}{3} \quad a = \frac{2}{3}
\]
No
No

but the Lorenz ratio is not a constant

\[ L_H = \frac{\kappa_{xy}}{T \sigma_{xy}} \sim T \equiv T^{-2\Phi/z} \]
No

but the Lorenz ratio is not a constant

\[ L_H = \frac{\kappa_{xy}}{T \sigma_{xy}} \sim T \equiv T^{-2\Phi/z} \]

\[ \Phi = bz = -\frac{2}{3} \]
combine AC + DC transport

fixes all exponents a, b, c
combine AC + DC transport

fixes all exponents a, b, c

current has anomalous dimension (probe with noise measurements)
what really is the summation over mass?
what really is the summation over mass?

\[ \mathcal{L} = \int_{0}^{\infty} (\partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m)) m^{2\delta} dm^2 \]
what really is the summation over mass?

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

\[ m = z^{-1} \]
what really is the summation over mass?

\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

\[ m = z^{-1} \]

\[ \mathcal{L} = \int_{0}^{\infty} dz \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right] \]
what really is the summation over mass?

\[
L = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2
\]

\[
m = z^{-1}
\]

\[
L = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right]
\]

\[
ds^2 = \frac{L^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right)
\]

anti de Sitter metric
what really is the summation over mass?

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} dm^2 \]

\[ m = z^{-1} \]

\[ \mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right] \]

can be absorbed with AdS metric

\[ ds^2 = \frac{L^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2) \]

anti de Sitter metric
no gravity?
Cooper, et al.

low-energy model for $N_{\text{eff}} > x$??
low-energy model for $N_{\text{eff}} > x$?

does such a theory include gravity?
f-sum rule

\[ K.E. = \frac{p^2}{2m} \]

\[ N_{\text{eff}} = x \]
what if?

K.E. \( \propto (\partial_\mu^2)^\alpha \)

f-sum rule

\[
\frac{W(n, T)}{\pi e^2} = Acn^{1+\frac{2(\alpha-1)}{d}} + B \frac{(\alpha - 1)(d + 2(\alpha - 1))T^2}{c} n^{1-\frac{2(\alpha+1)}{d}}
\]
what if?

K.E. \( \propto (\partial^2_\mu)^\alpha \)

f-sum rule

\[
\frac{W(n, T)}{\pi e^2} = A c n^{1+2(\alpha-1)/d} + B \frac{c}{(\alpha - 1)(d + 2(\alpha - 1))T^2} n^{1-2(\alpha+1)/d}
\]

\( W>n \) if \( \alpha < 1 \)
what theories have $\alpha < 1$?
gravitational crystals

\[ g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu} \]

\[ \frac{1}{2} \Box h_{\mu\nu} - 2m^2 h_{\mu\nu} + \cdots \]

\[ m^2 \propto V(\Phi) \]

graviton has a mass!
Unparticles as the Holographic Dual of Gapped AdS Gravity

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\[ S_{\text{eff}} = (-1)^{-\frac{2\Delta + 1}{\Delta + 1}} \int d^d x \left[ \Phi(-\partial^2)^{-\Delta} \Phi + (-1)^{-\frac{1}{(\Delta+1)(\Delta+2)}} A_\mu(\partial^2)^{-\Delta-1} A^{\perp \mu} + h_{\mu\nu}(\partial^2)^{-\Delta} h^{\mu\nu} \right] . \] (79)

\[ \tilde{\Delta}_\Phi < 0 \]

non-local action with non-canonical kinetic energy
incoherent metals: Mottness

unparticles

f-sum rule violation

`massive gravity'