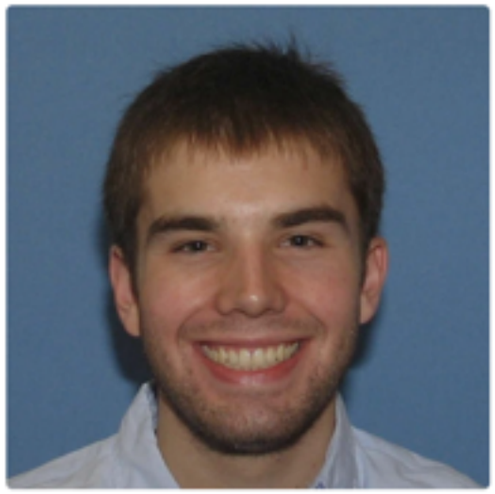
Optical Conductivity in the Cuprates from holography and unparticles

Thanks to: NSF, EFRC (DOE)

Brandon Langley



Garrett Vanacore

Kridsangaphong Limtragool

Properties all particles share?

charge

fixed mass!

Properties all particles share?

charge

fixed mass!

conserved (gauge invariance)

Properties all particles share?

charge

fixed mass!

mass sets a scale

conserved (gauge invariance)

Properties all particles share?

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mass sets a scale

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not conserved

Properties all particles share?

charge

fixed mass!

s sets a s

conserved (gauge invariance)

not conserved

optical conductivity in the cuprates

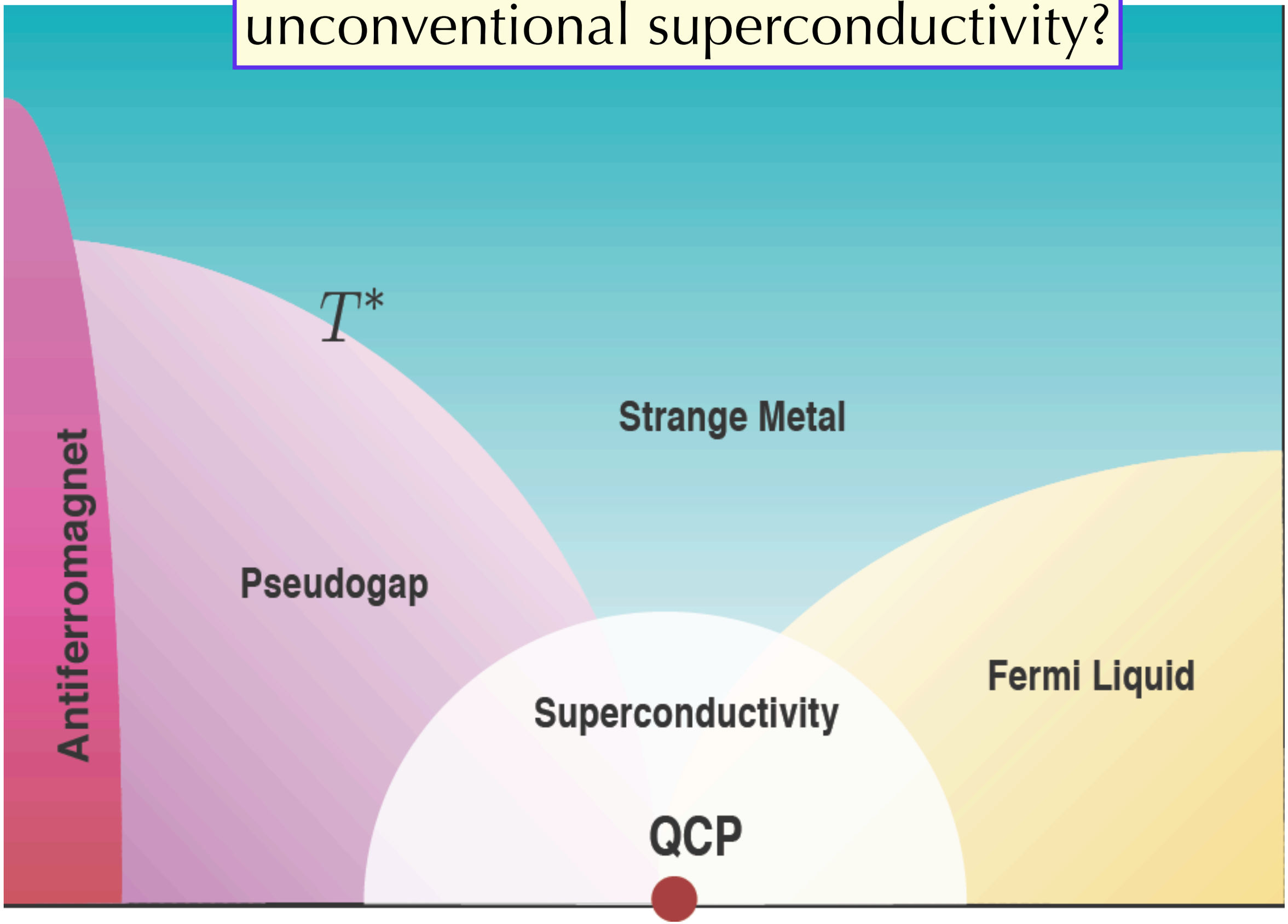
scale invariant sector
`unparticles'
(no definite mass)

optical conductivity in the cuprates

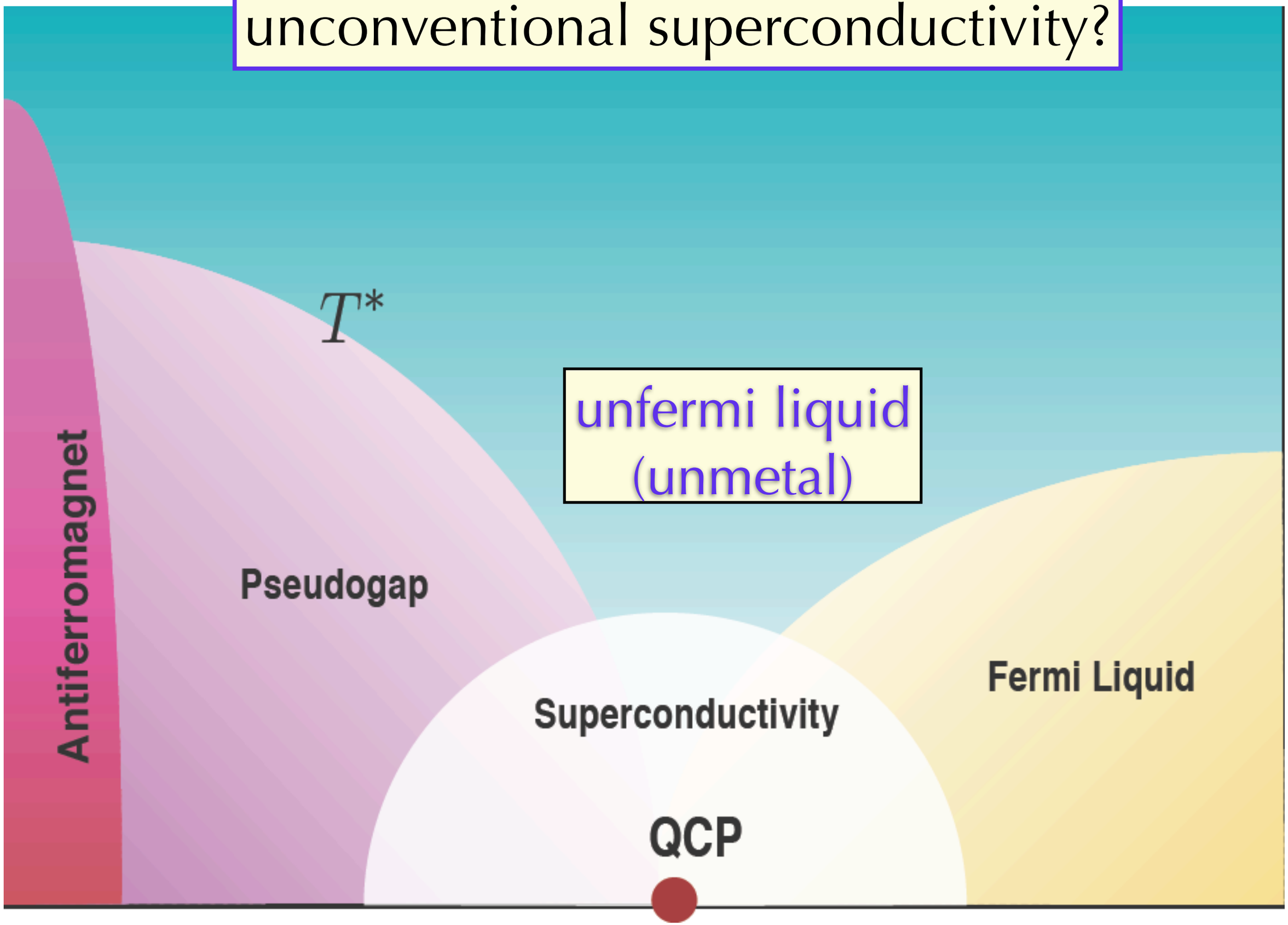
incoherent metal

scale invariant sector
`unparticles'
(no definite mass)

unconventional superconductivity?



unconventional superconductivity?



PHYSICAL REVIEW B VOLUME 43, NUMBER 10 1 APRIL 1991

Optical spectra of $La_{2-x}Sr_xCuO_4$: Effect of carrier doping on the electronic structure of the CuO_2 plane

S. Uchida
Engineering Research Institute, University of Tokyo, Yayoi, Tokyo 113, Japan

T. Ido and H. Takagi
Department of Applied Physics, University of Tokyo, Hongo, Tokyo 113, Japan

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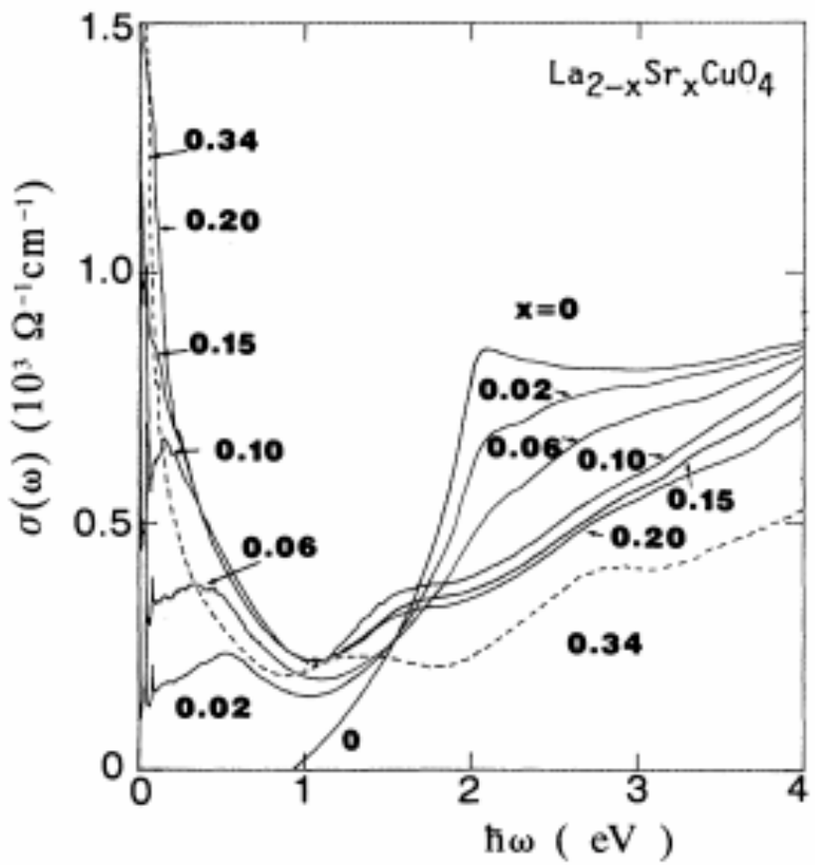
S. Tajima
Superconducting Research Laboratory, International Superconductivity Technology Center, Shinonome, Tokyo, Japan
(Received 30 August 1990)



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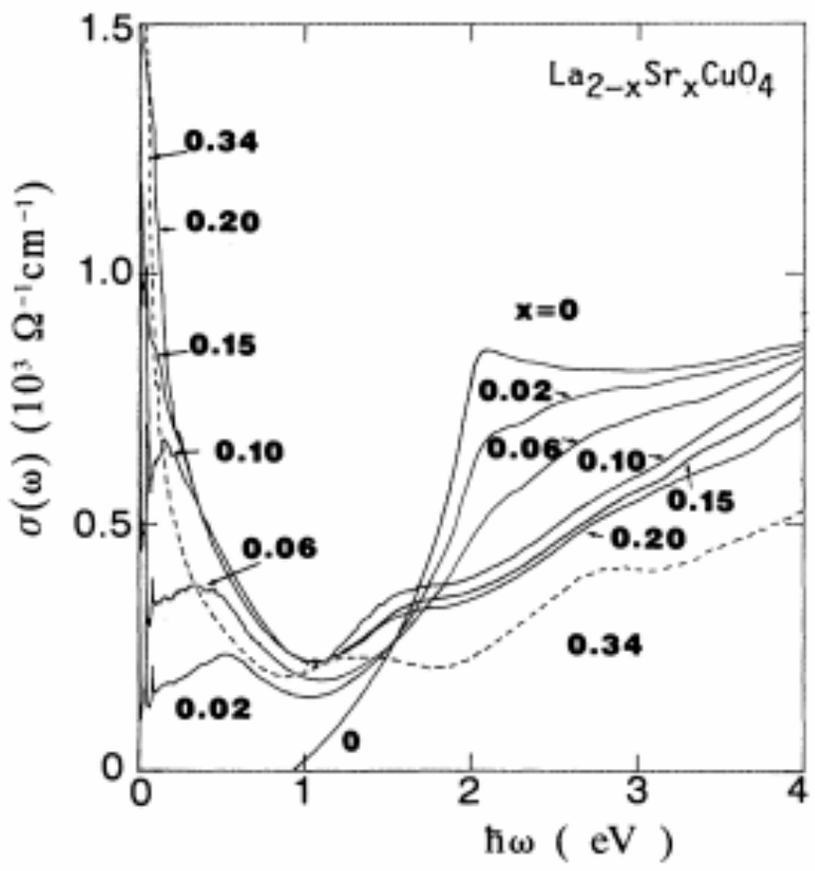
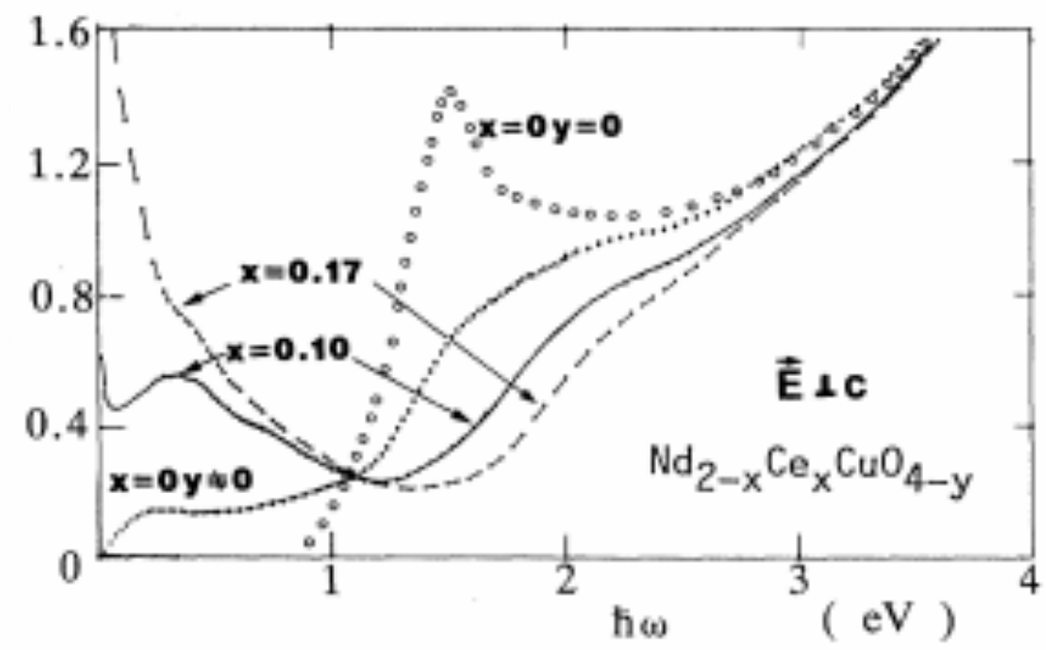
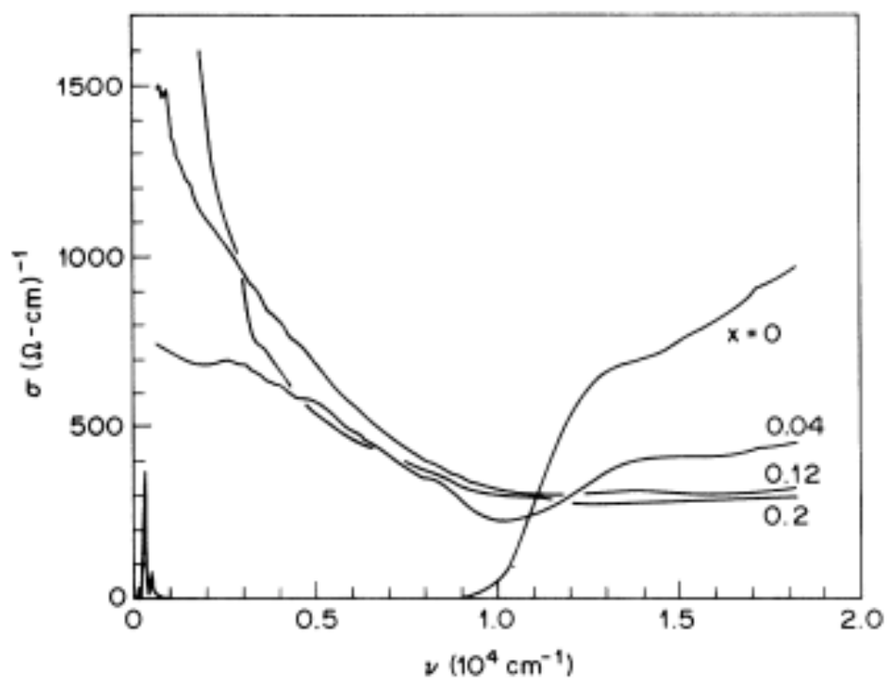




Growth of the optical conductivity in the Cu-O planes

S. L. Cooper, G. A. Thomas, J. Orenstein, D. H. Rapkine, A. J. Millis, S-W. Cheong, and A. S. Cooper
AT&T Bell Laboratories, Murray Hill, New Jersey 07974

Z. Fisk
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 7 March 1990)



$$N_{\mathrm{eff}}(\Omega) = \frac{2mV_{\mathrm{cell}}}{\pi e^2} \int_0^{\Omega} \sigma(\omega) d\omega$$

optical gap

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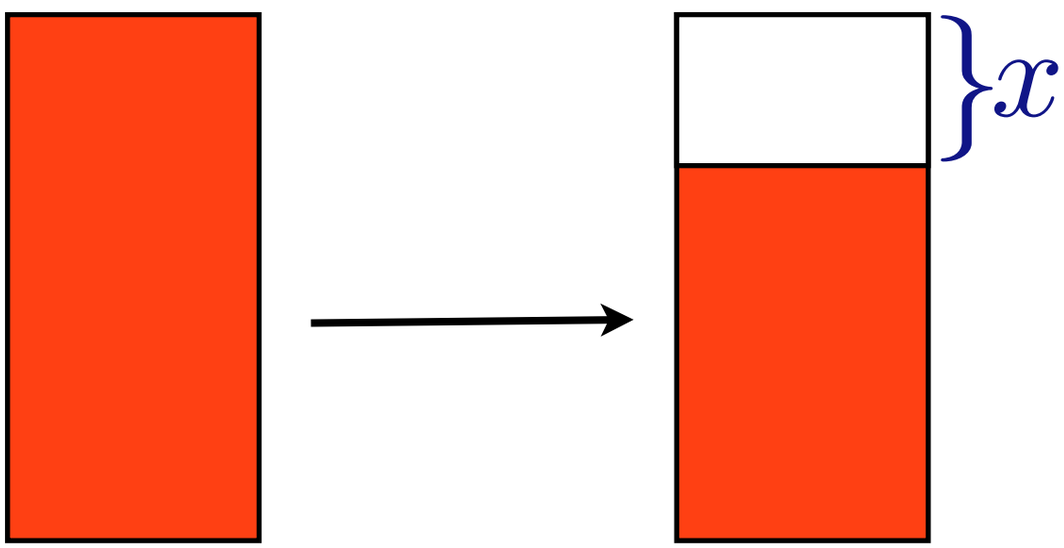

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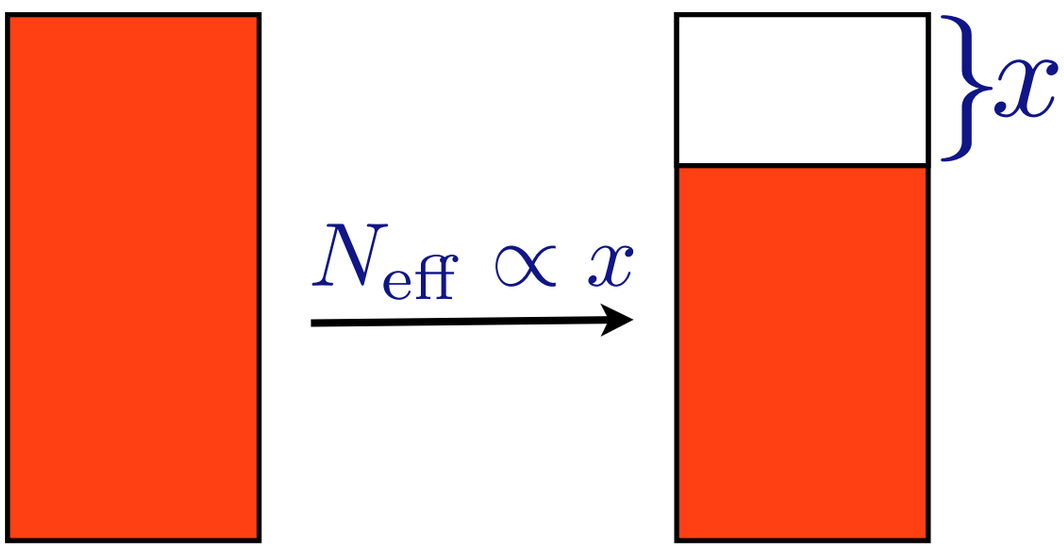
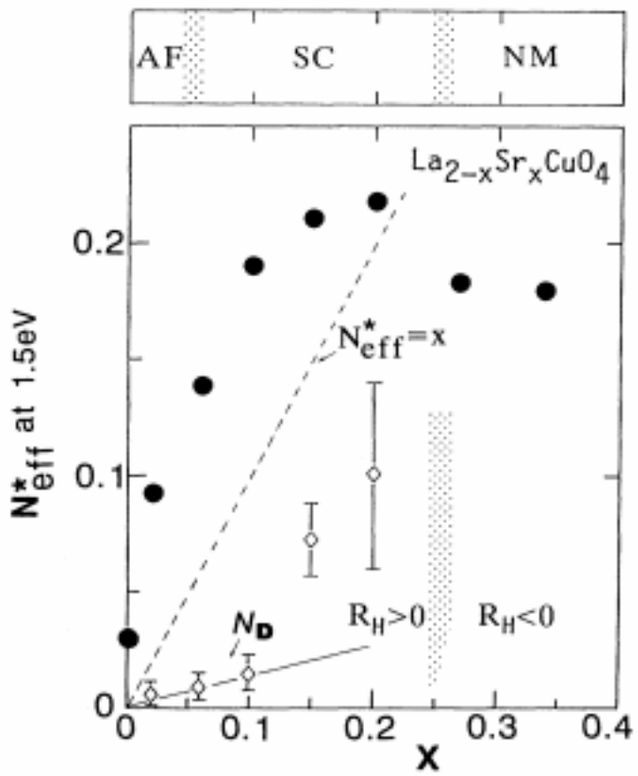
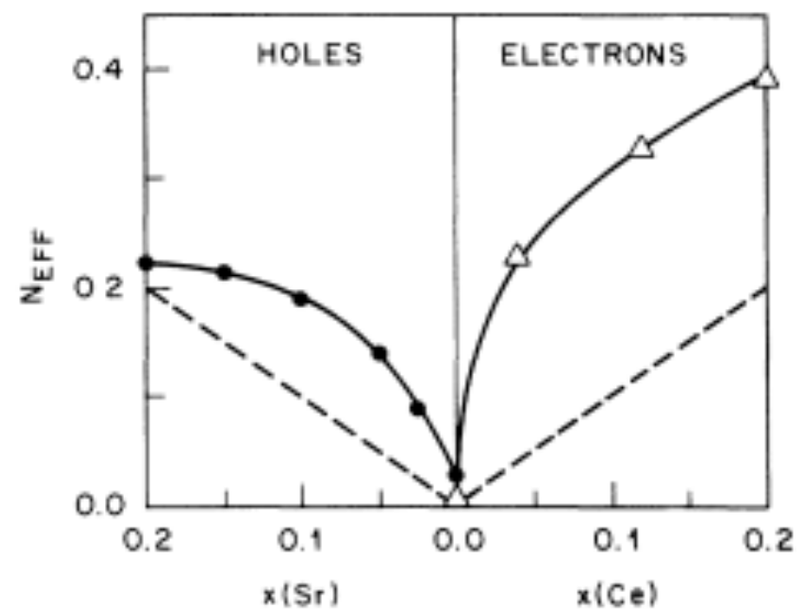


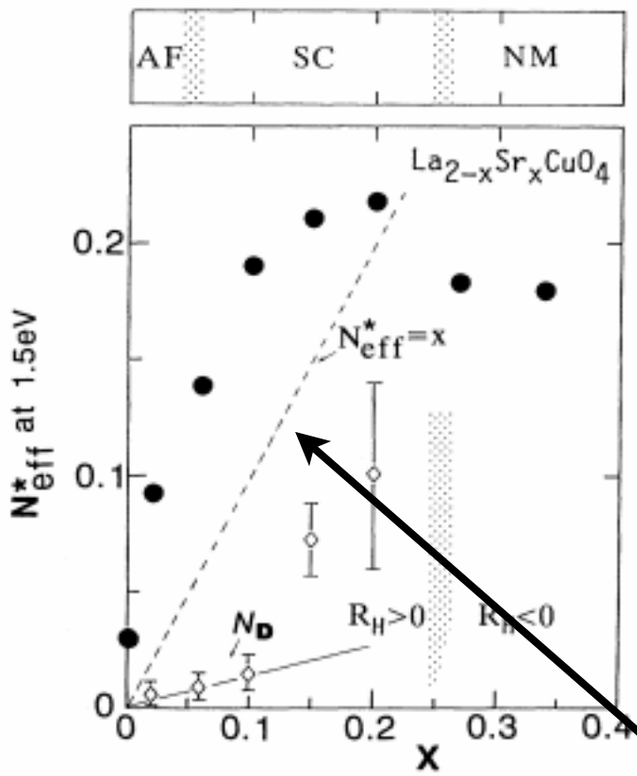
Uchida, et al.



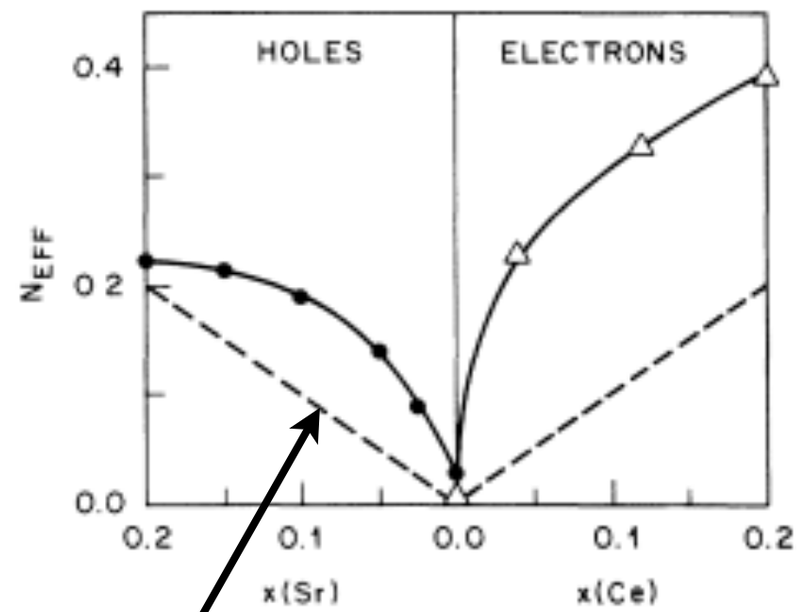
Cooper, et al.



Uchida, et al.



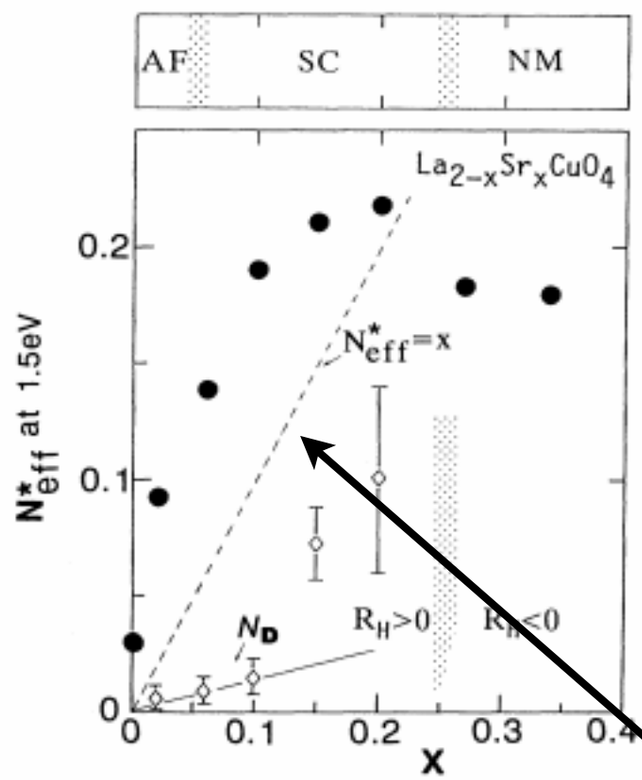
Cooper, et al.

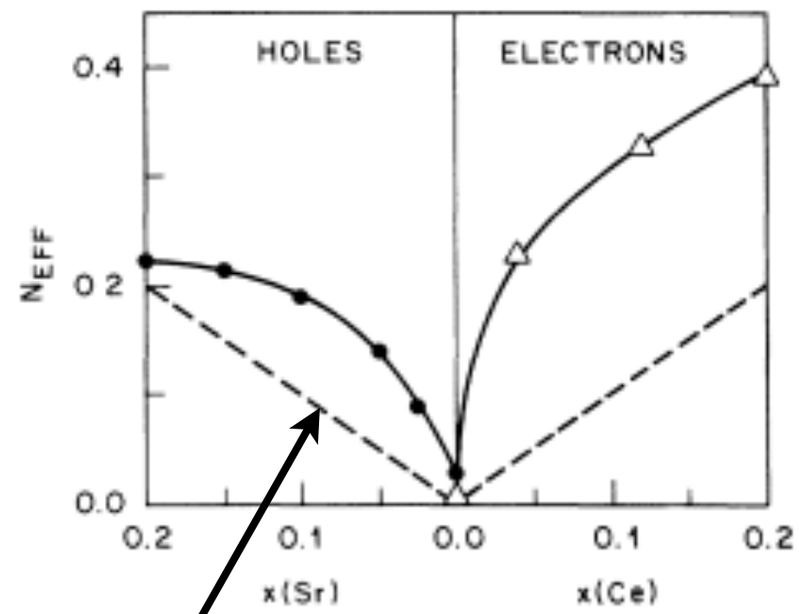


x

Uchida, et al.

Cooper, et al.





x

low-energy model for $N_{\text{eff}} > x$??

excess carriers?

excess carriers?

charge carriers have no particle content?

Mott insulator

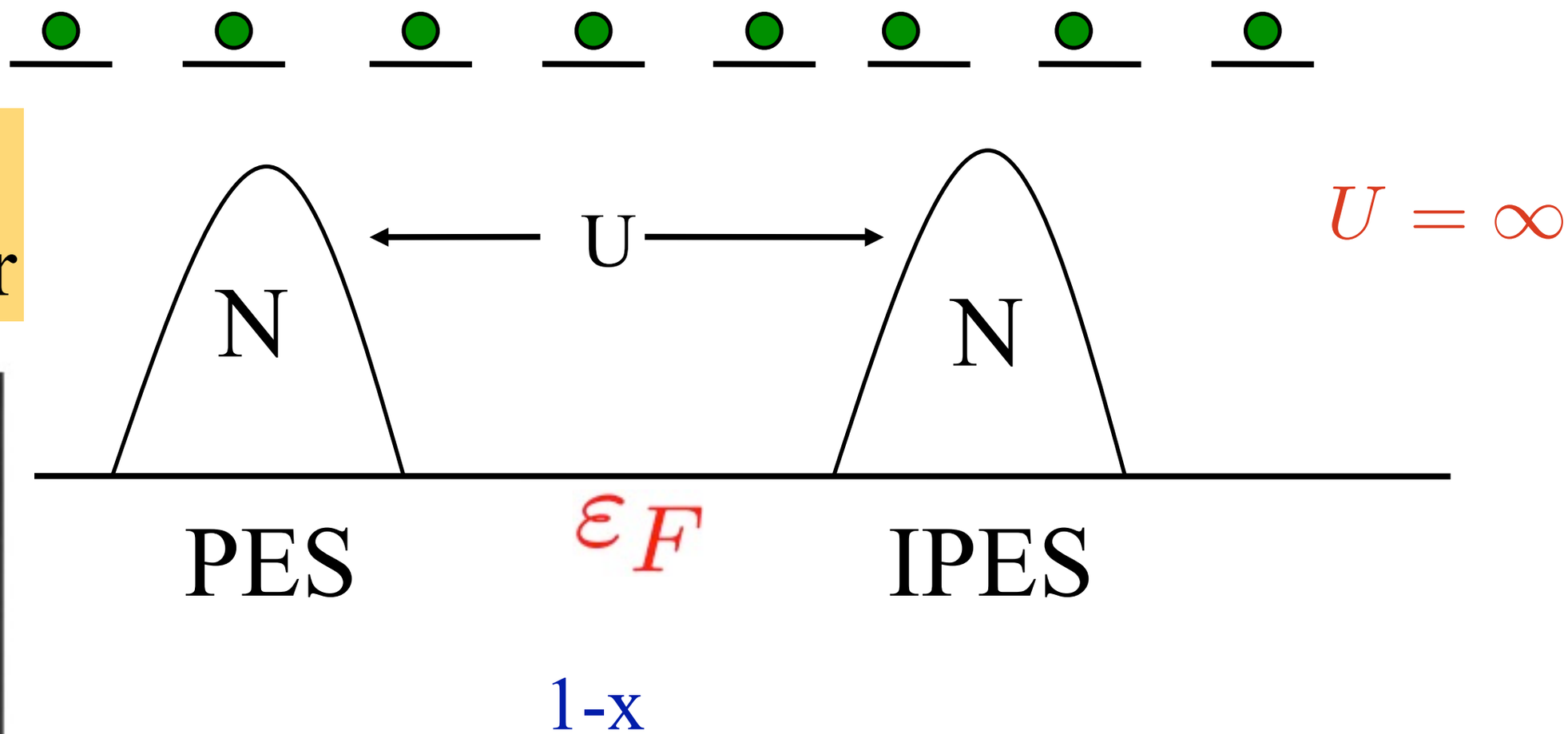
1-x

Mott insulator

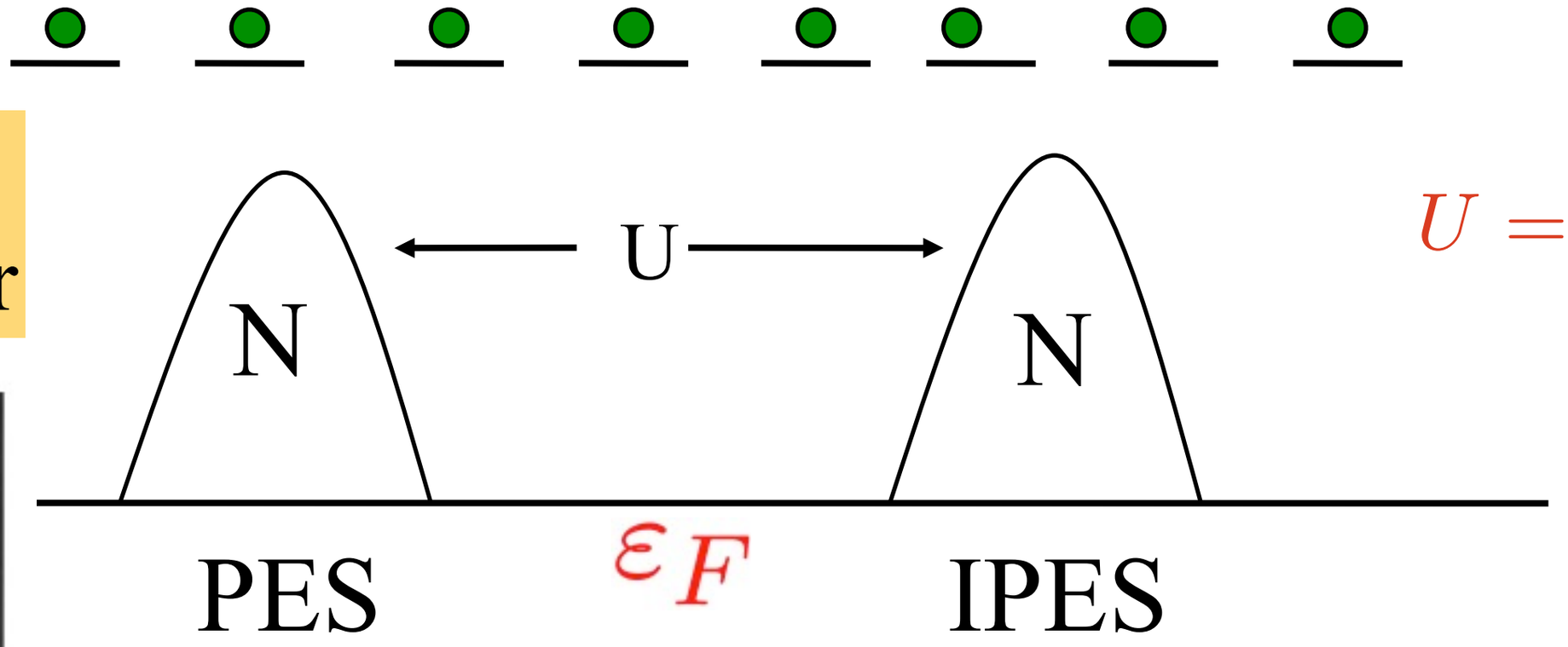
$U = \infty$

1-x

Mott insulator



Mott insulator

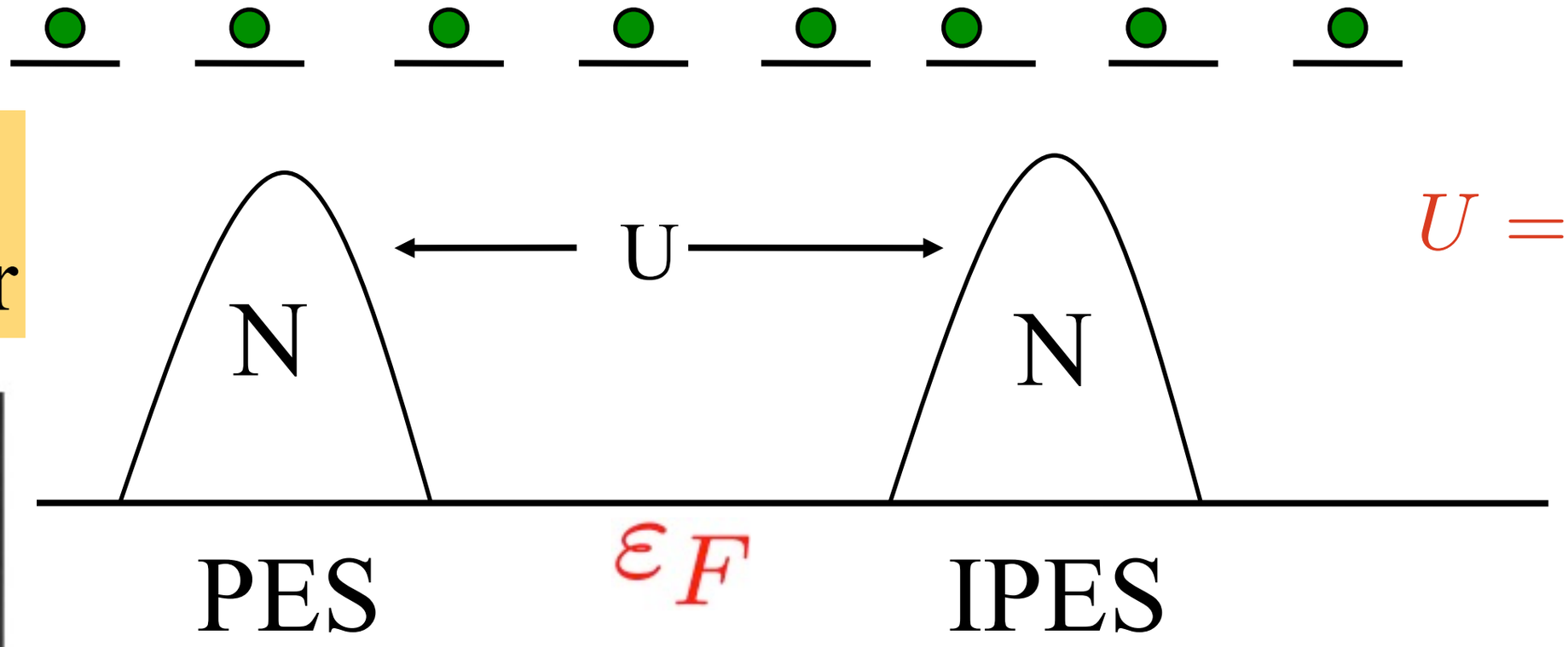


$U = \infty$

1-x

no hopping:

Mott insulator



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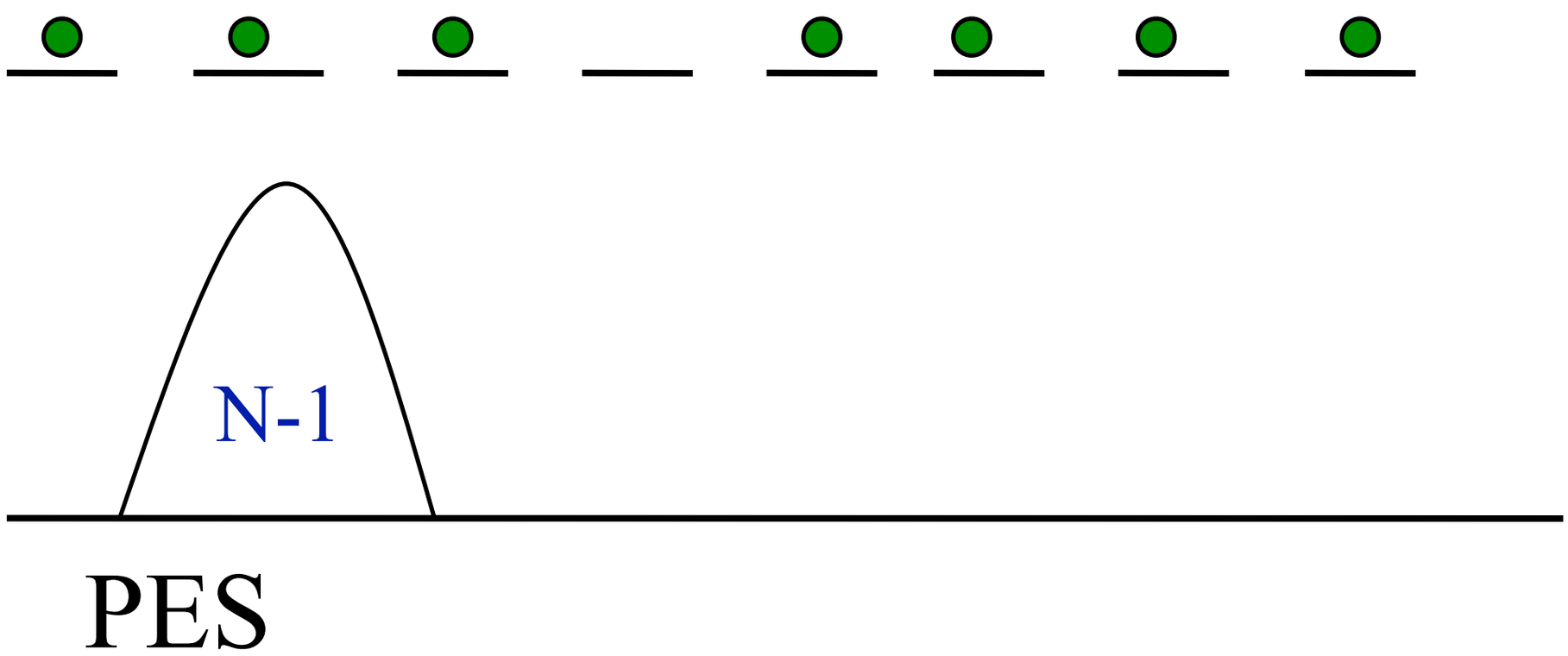
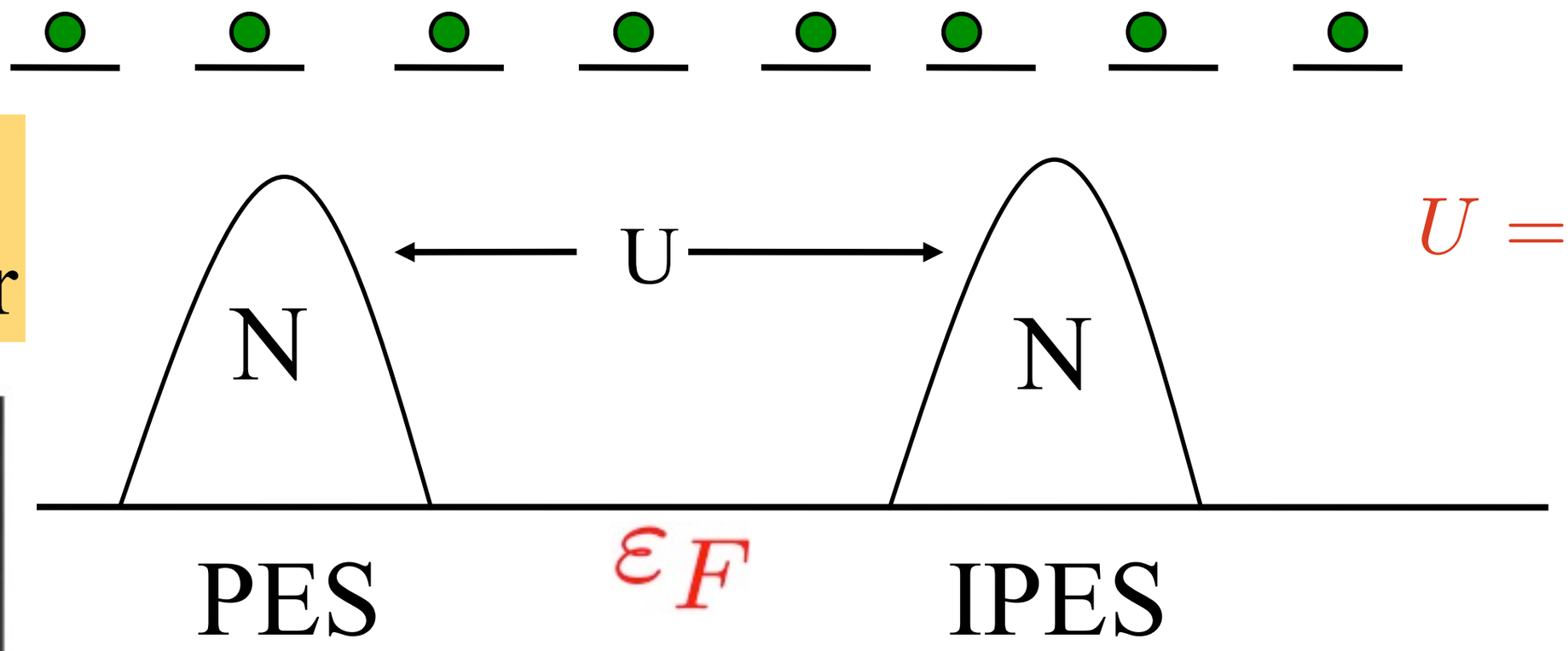


Mott insulator



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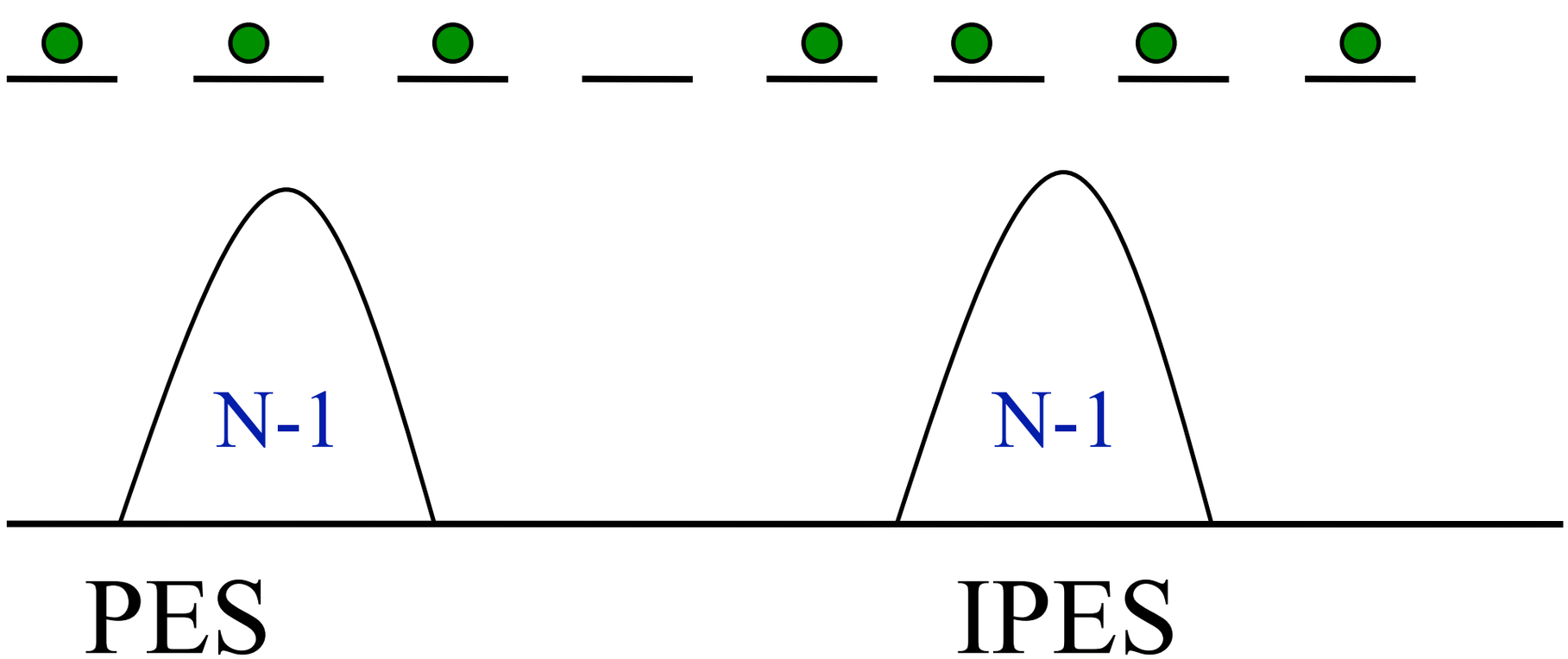
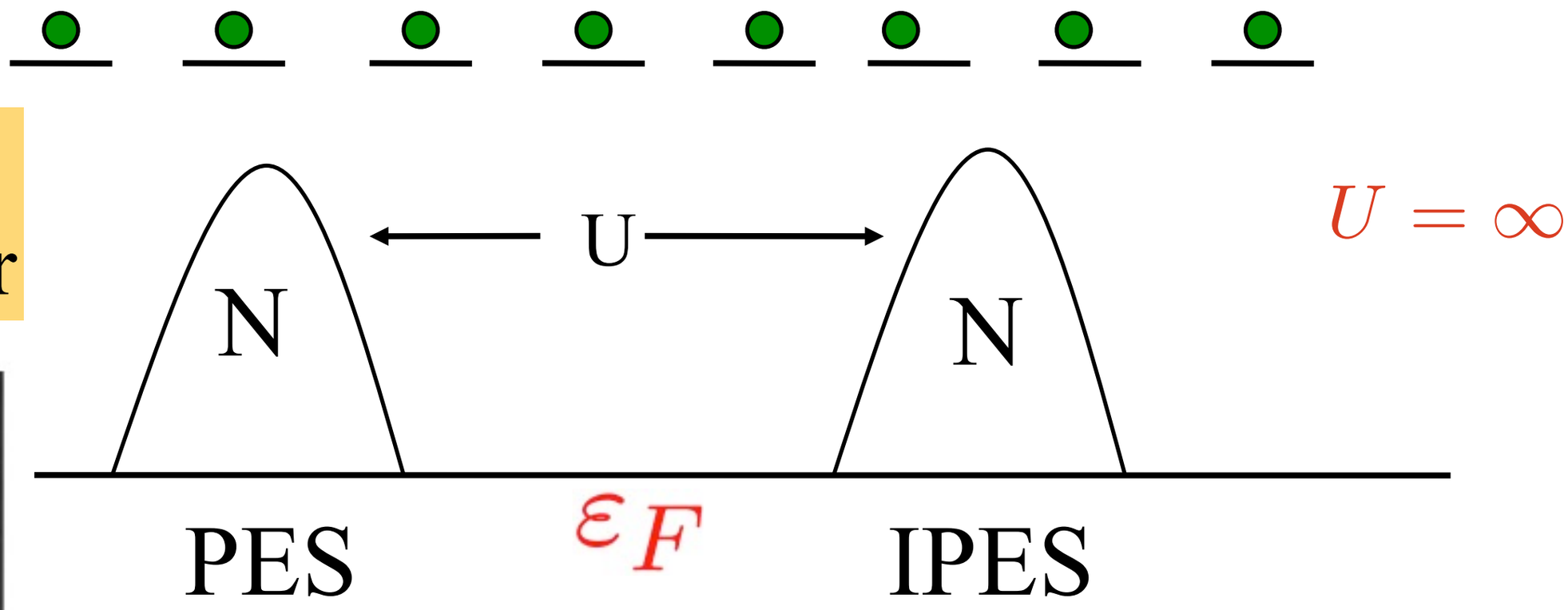


Mott insulator



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1-x

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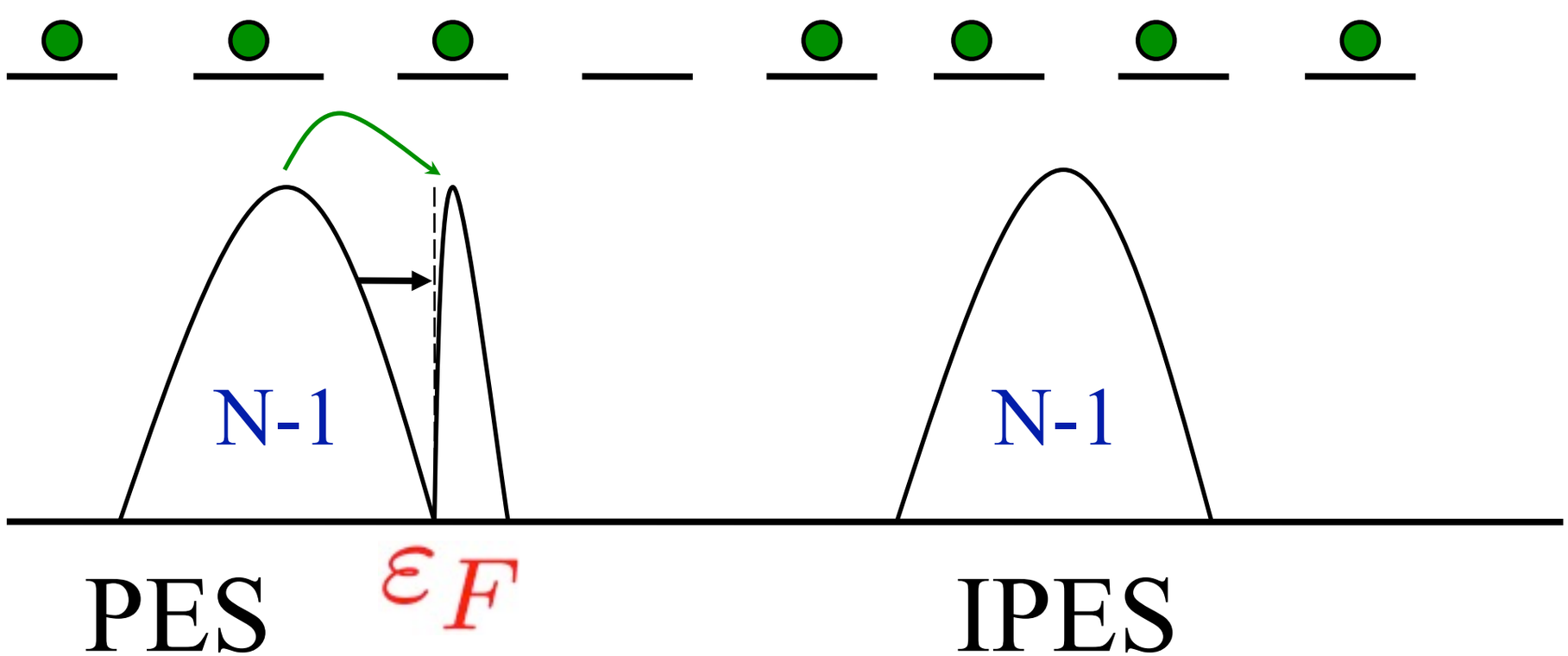
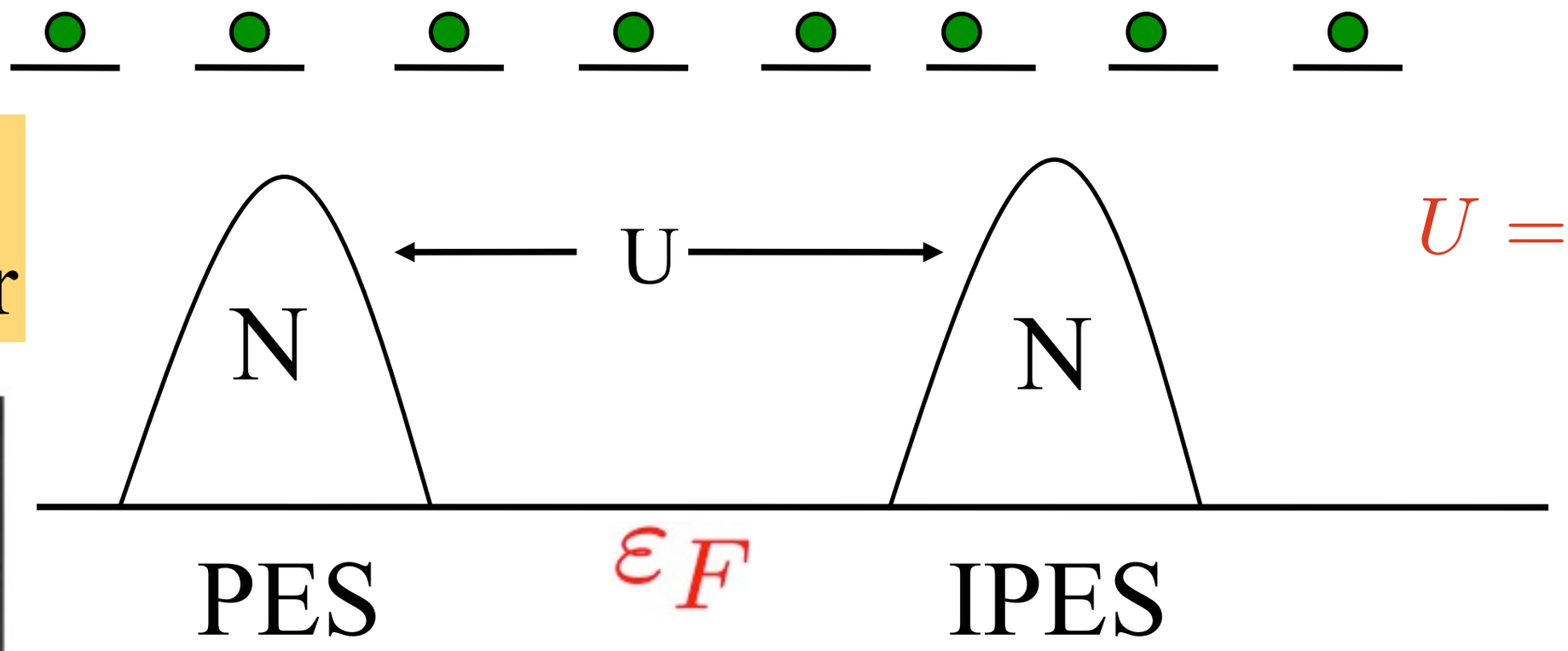


Mott insulator



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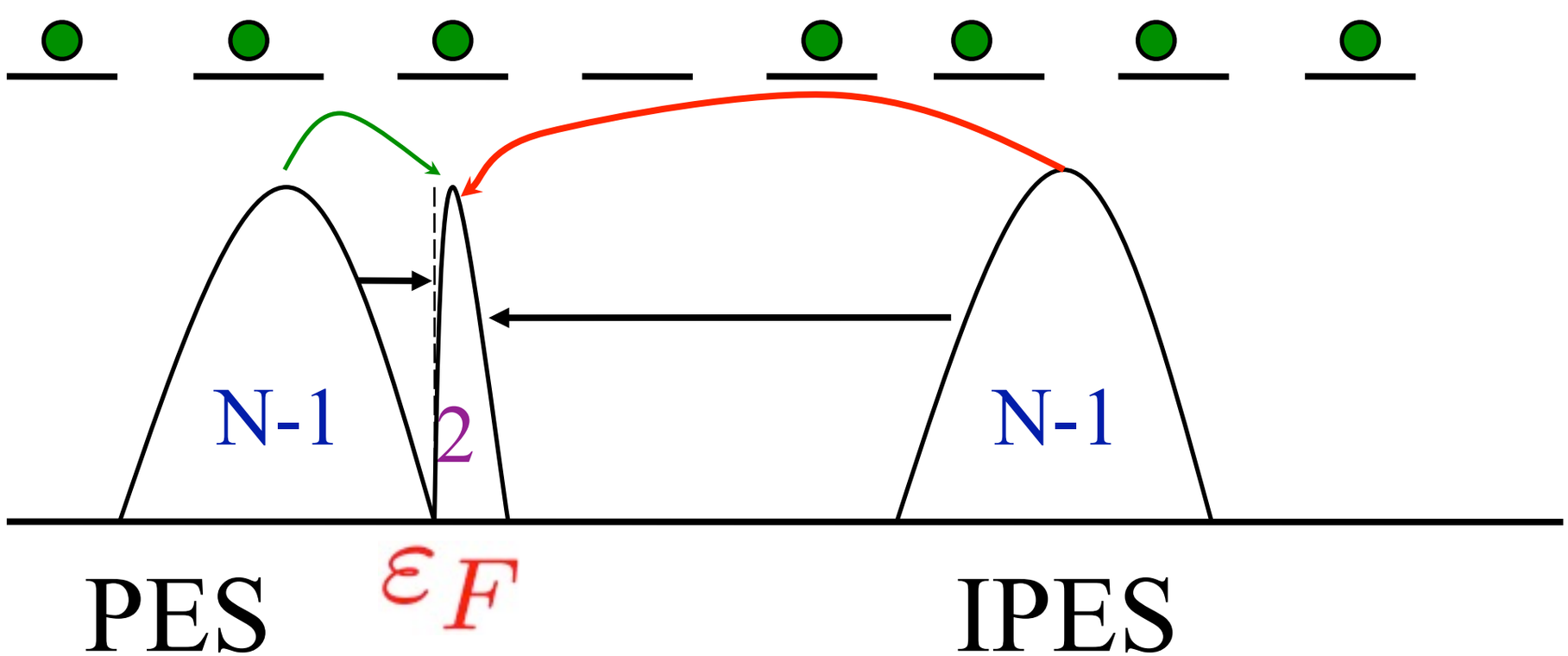
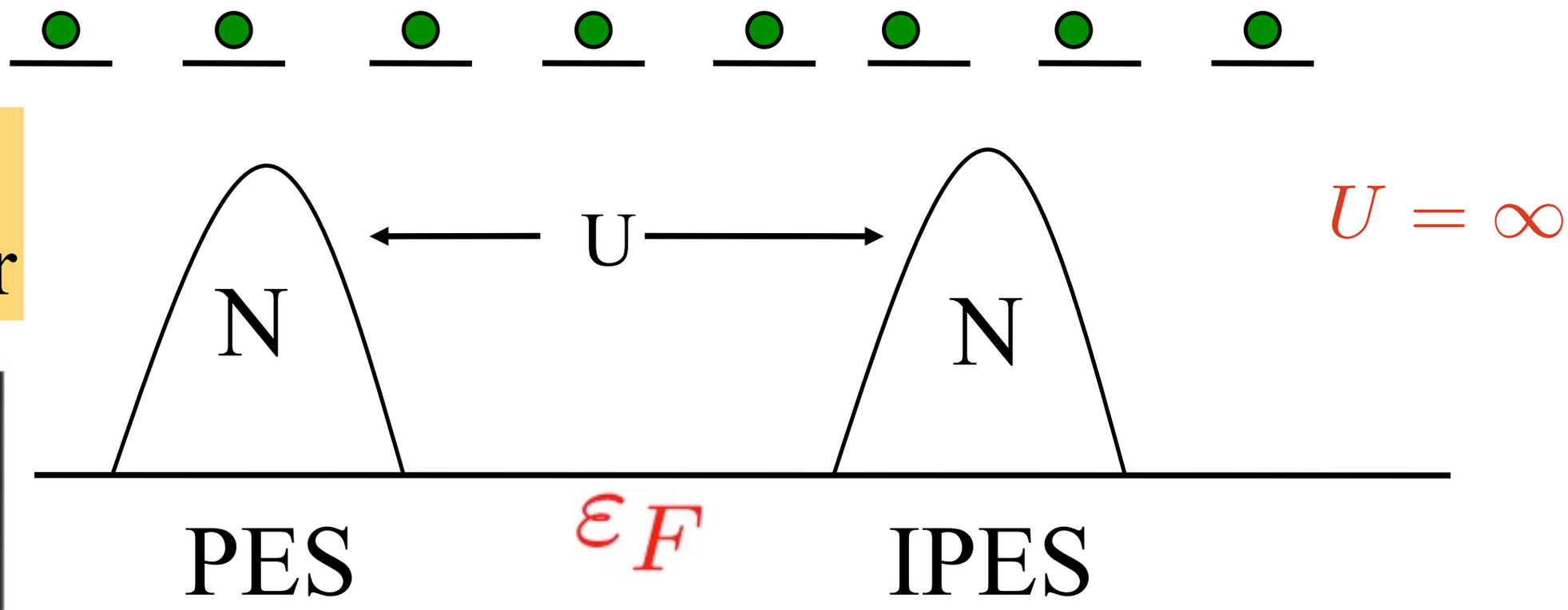


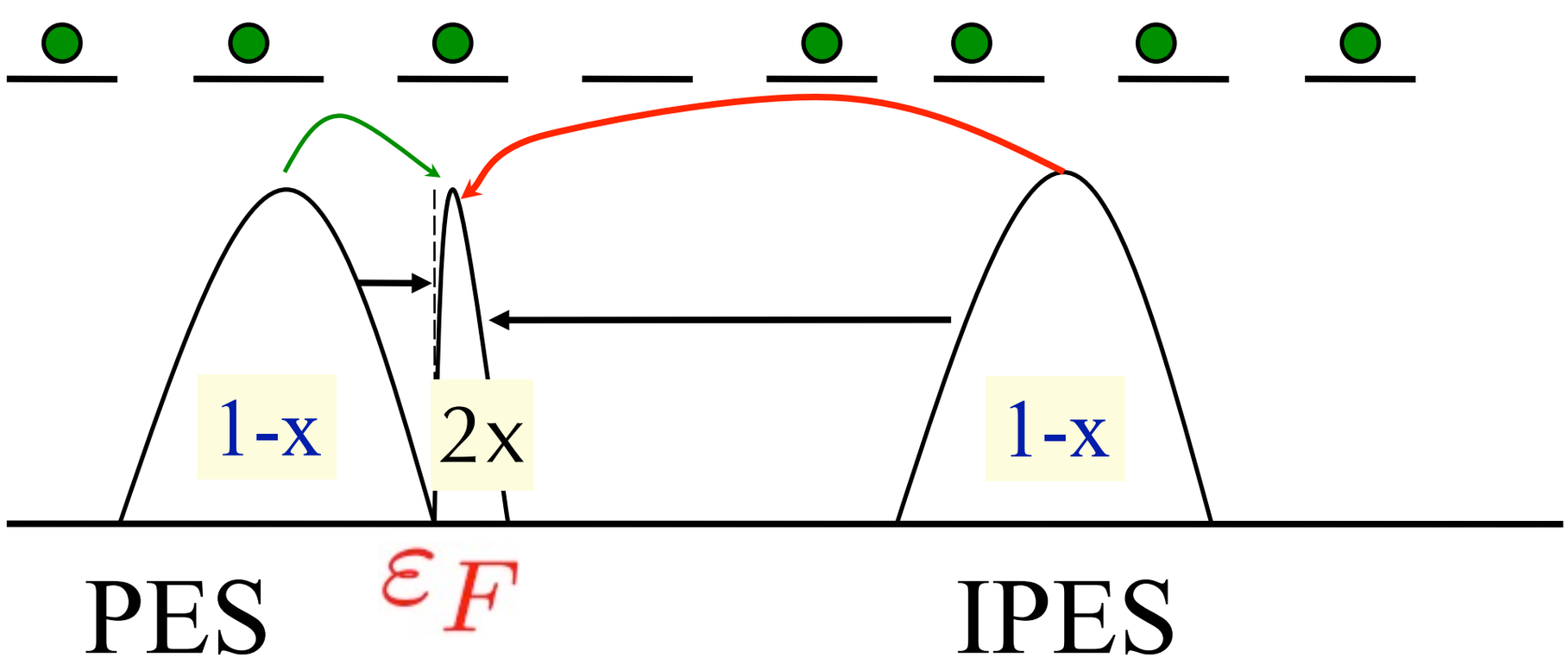
Sawatzky

Mott insulator



$U = \infty$

no hopping:



Sawatzky

counting electron states

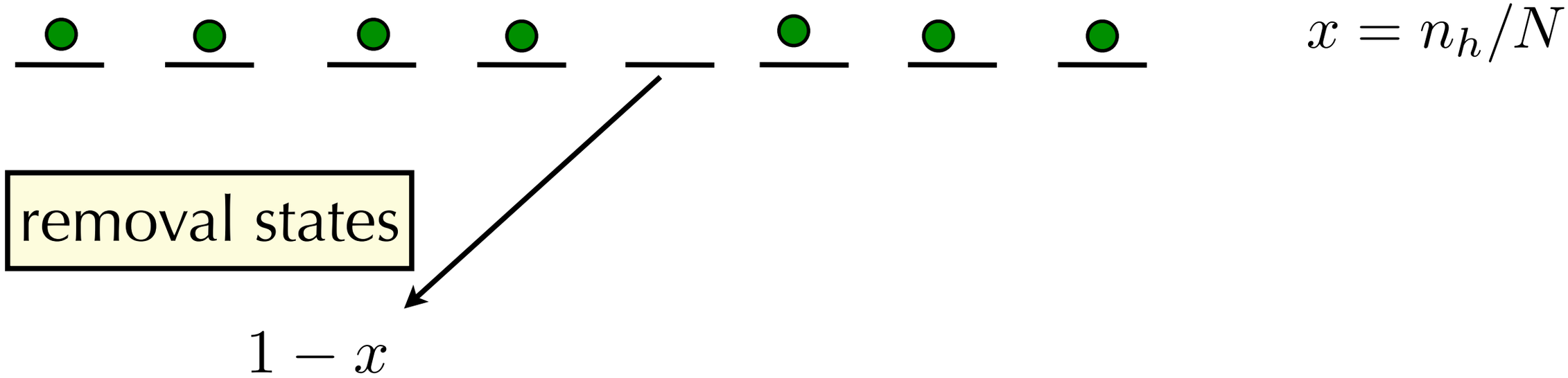
need to know: N (number of sites)

counting electron states

$x = n_h/N$

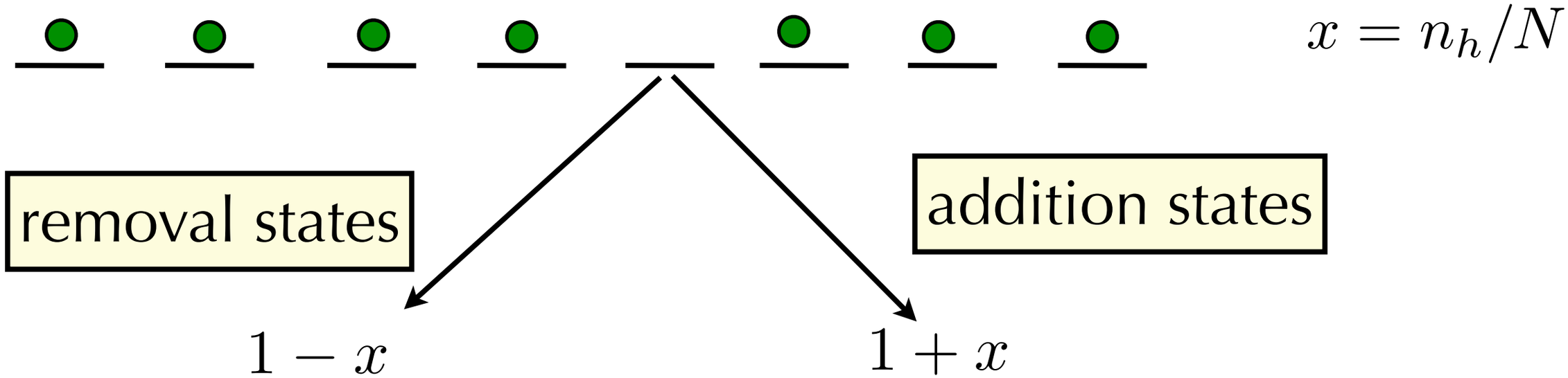
need to know: N (number of sites)

counting electron states



need to know: N (number of sites)

counting electron states



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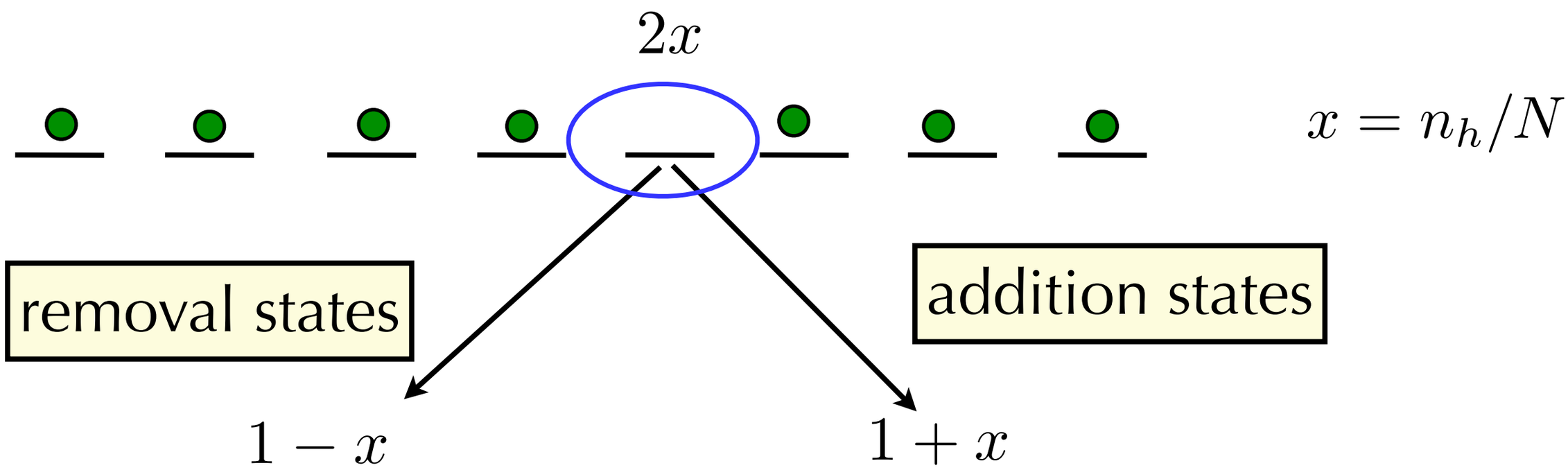
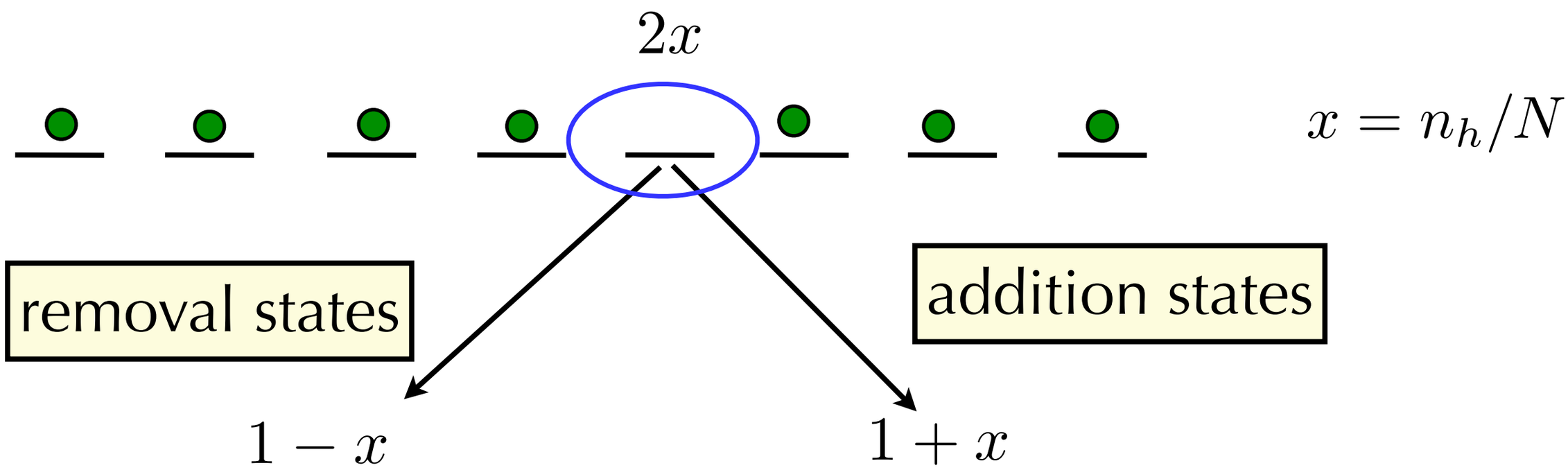


need to know: N (number of sites)

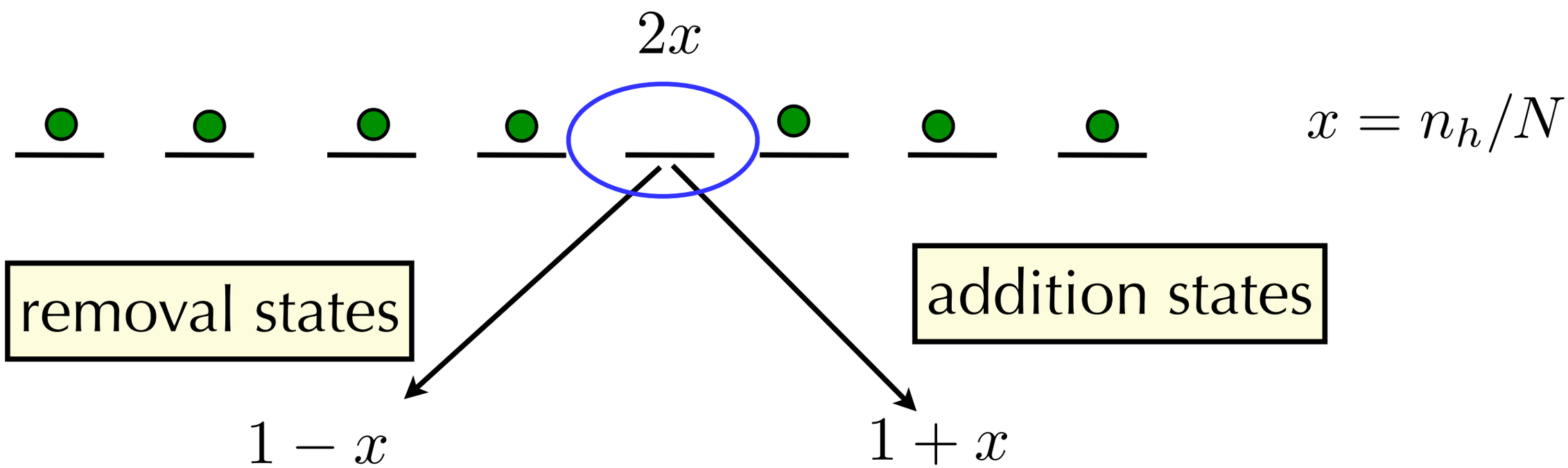
counting electron states



low-energy electron states

need to know: N (number of sites)

counting electron states

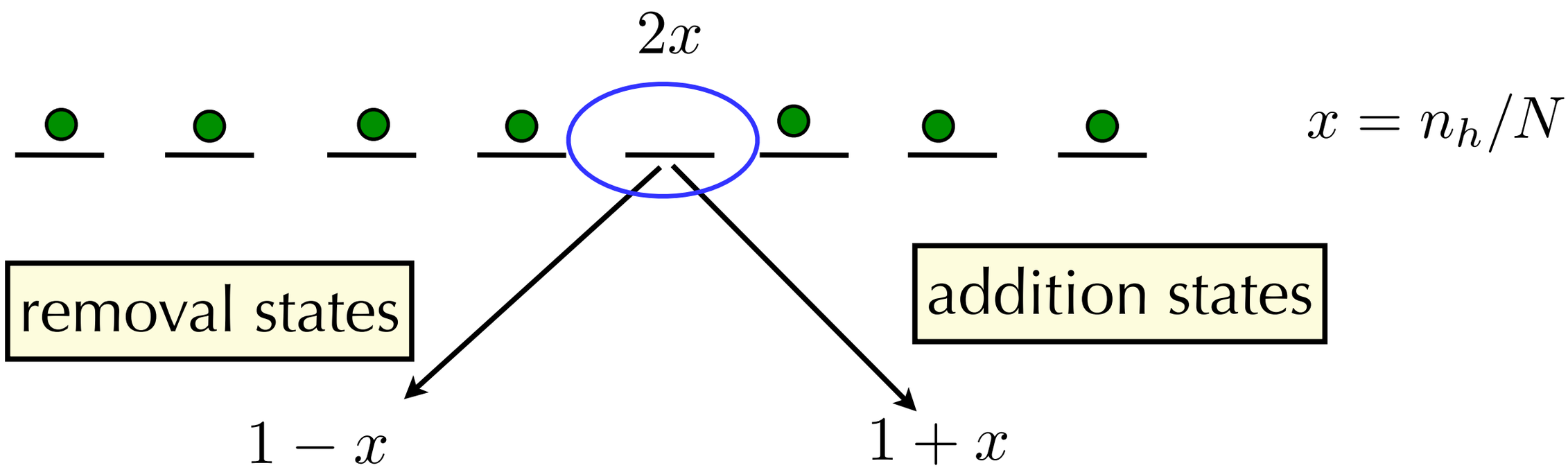


low-energy electron states

$$1 - x + 2x = 1 + x$$

need to know: N (number of sites)

counting electron states



low-energy electron states

$$1 - x + 2x = 1 + x$$

high energy

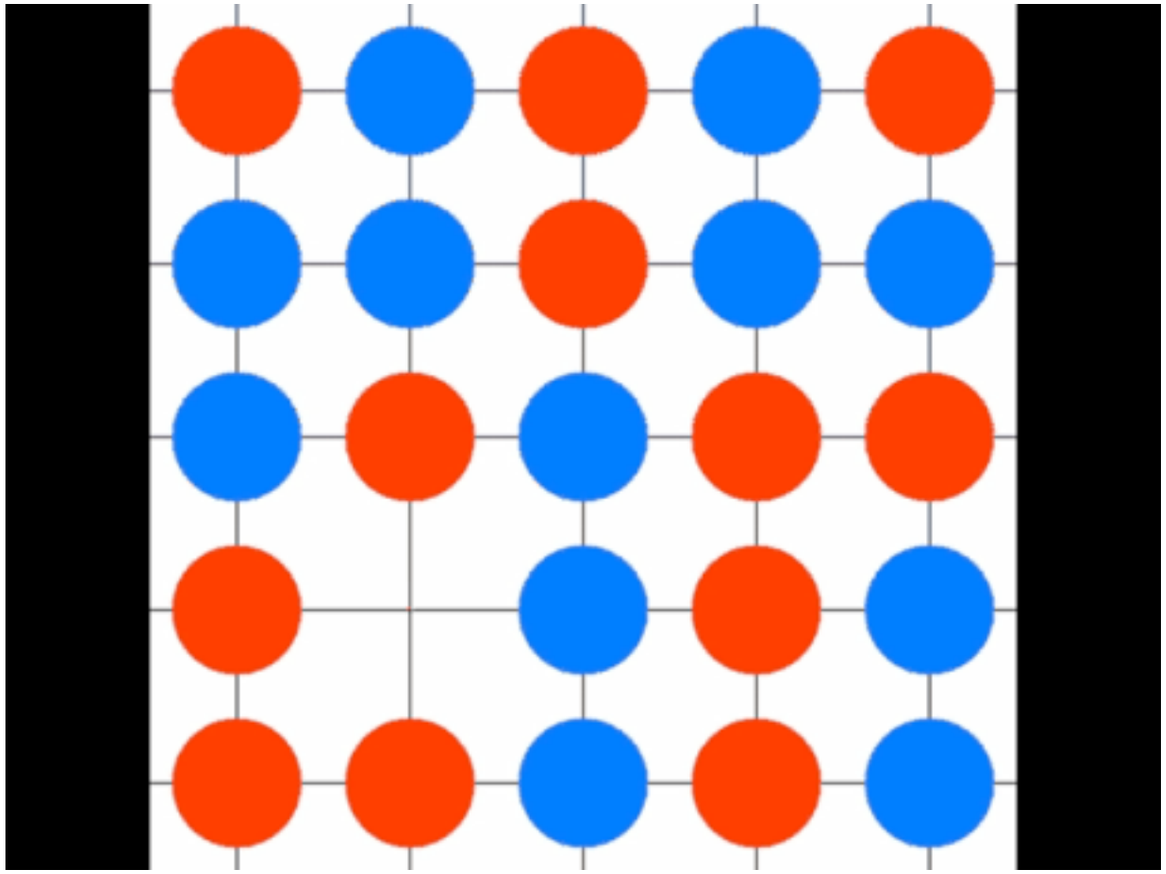
$$1 - x$$

need to know: N (number of sites)

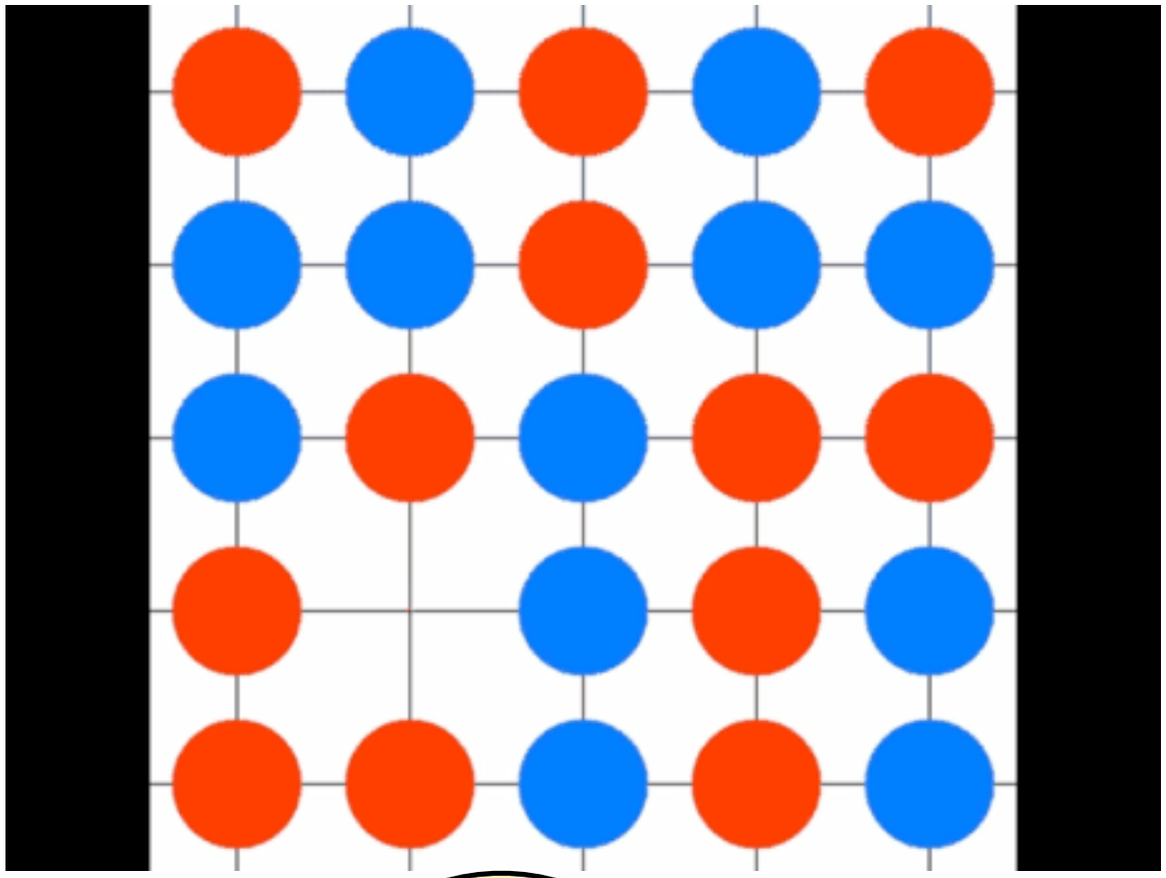
why is this a problem?

spectral function (dynamics)

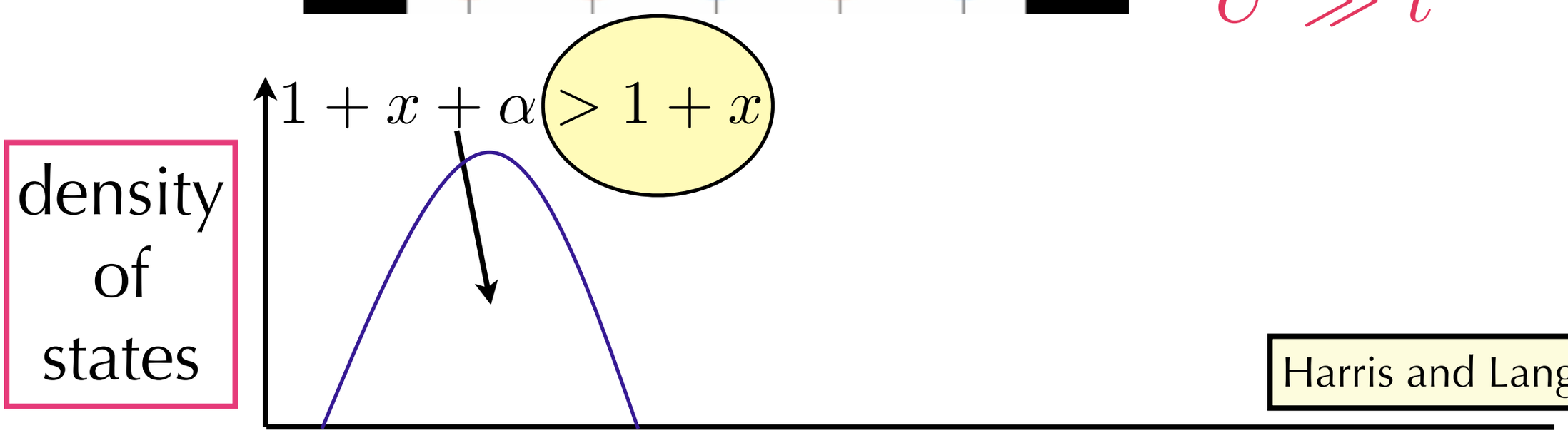
spectral function (dynamics)



spectral function (dynamics)

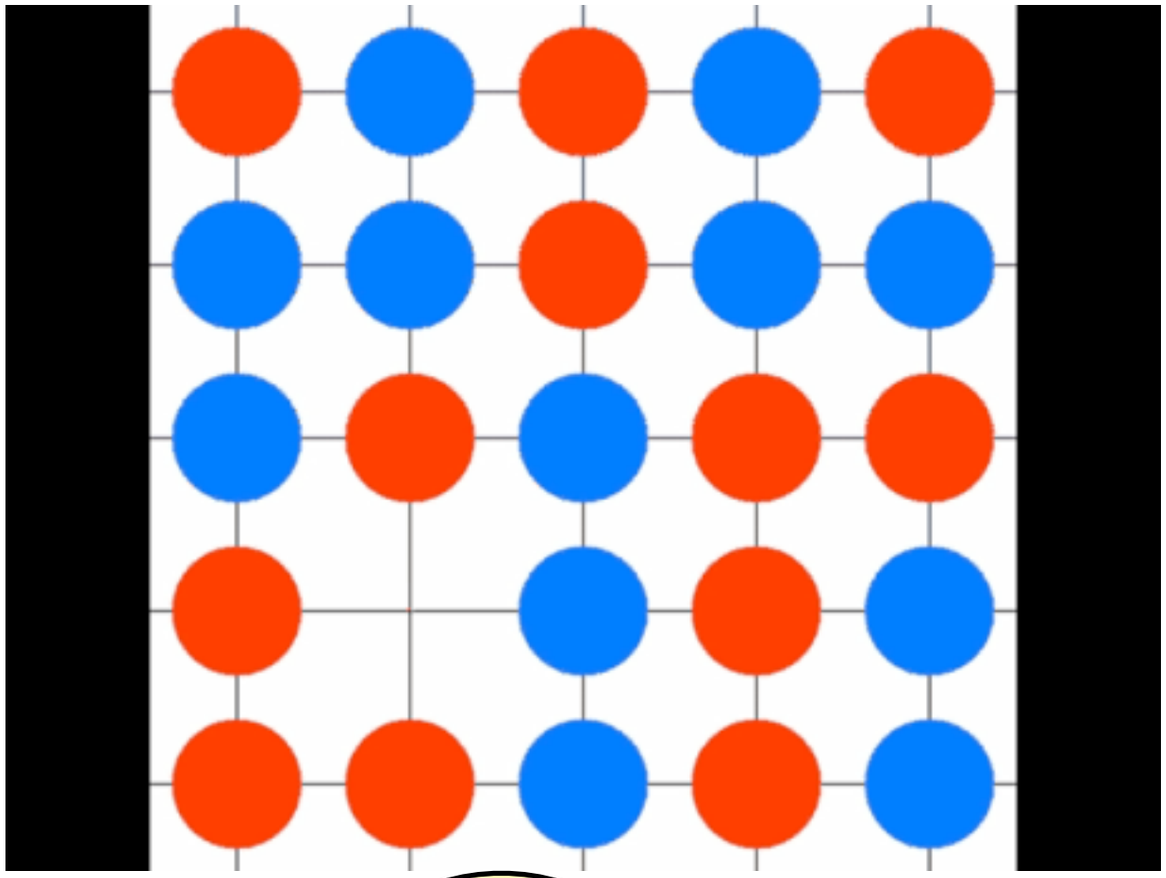


$U \gg t$



Harris and Lange (1967)

spectral function (dynamics)



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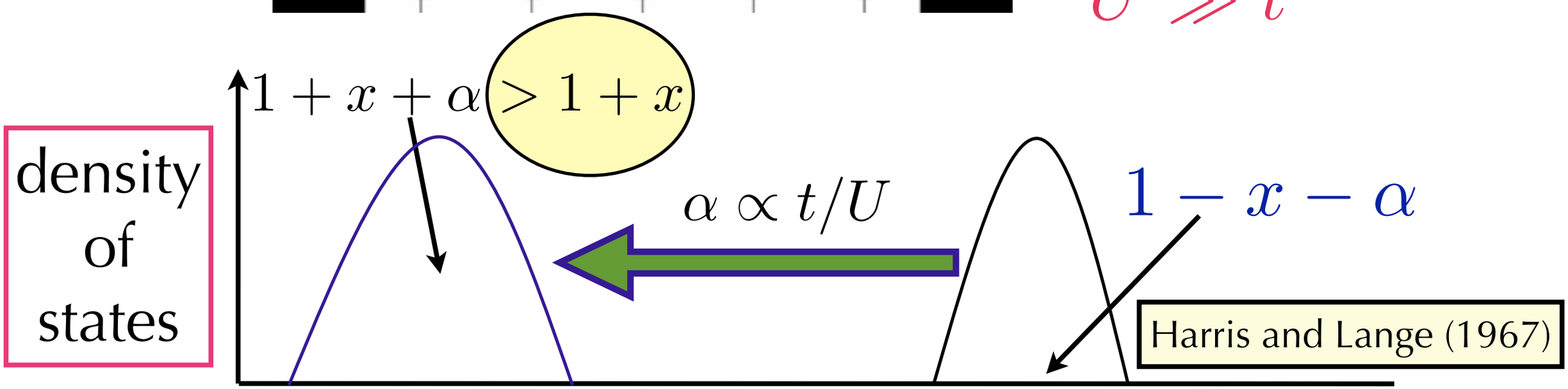

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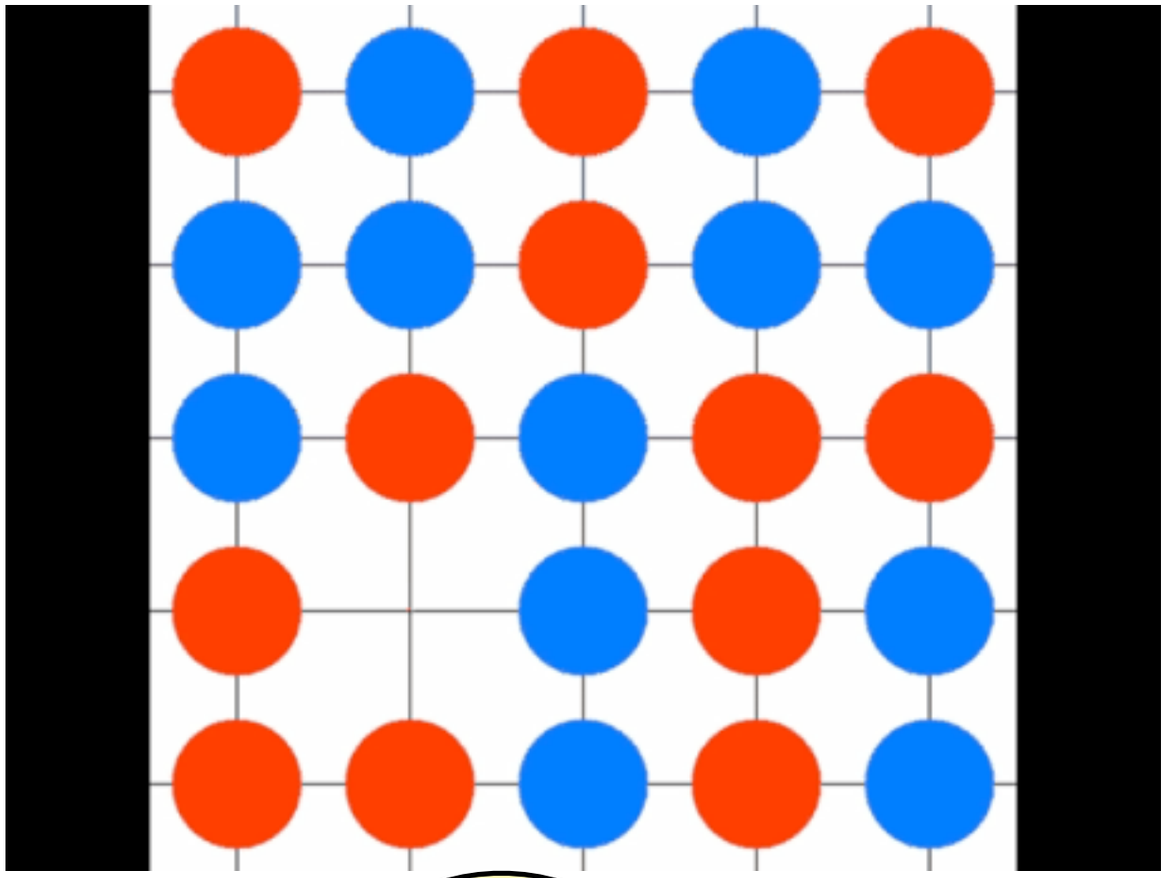

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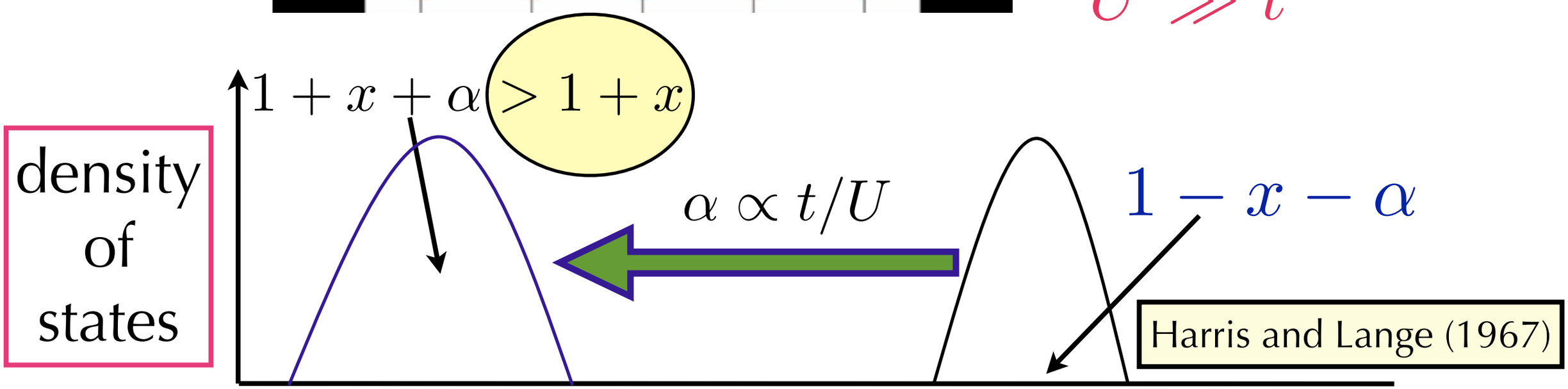
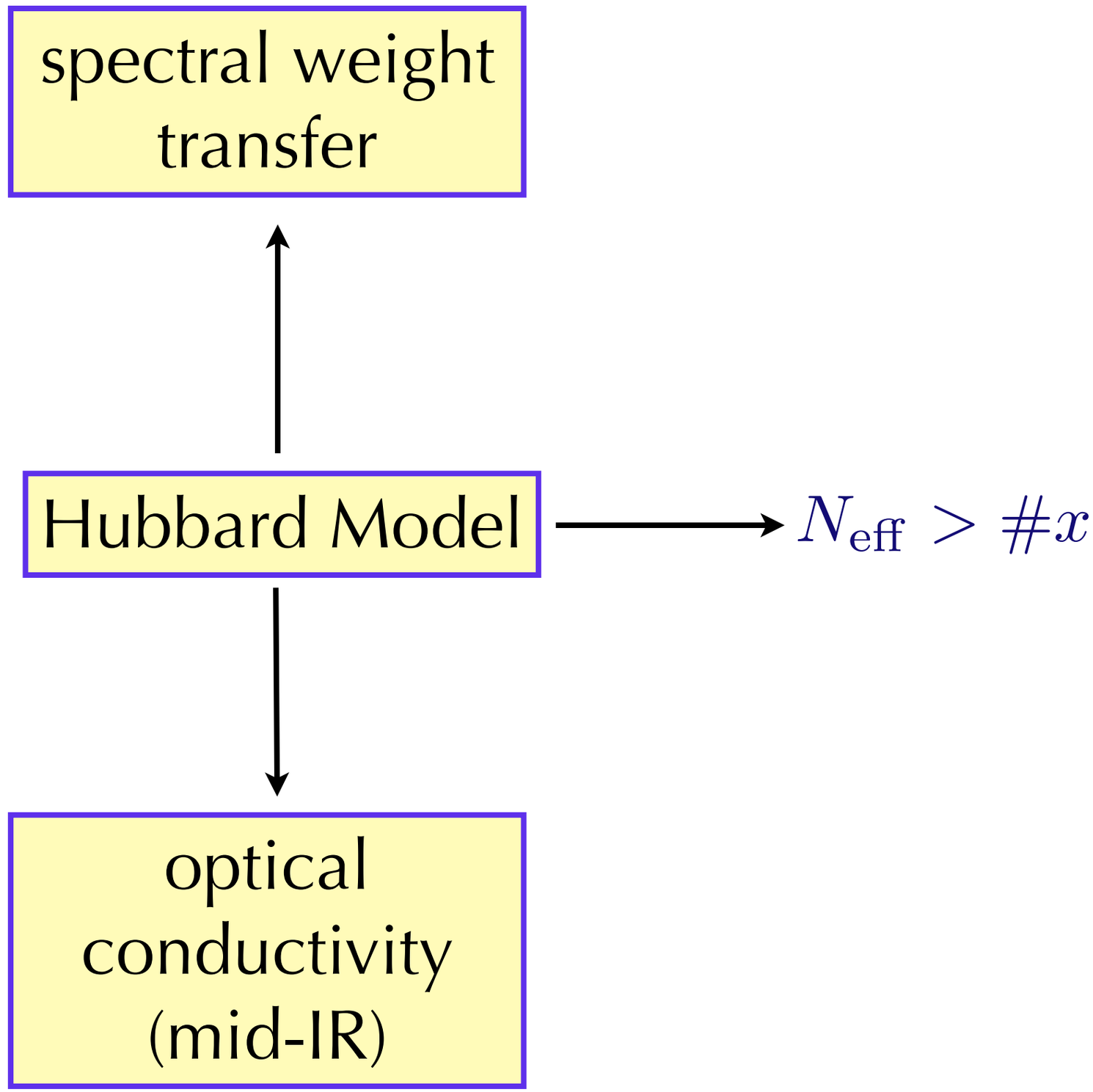


not exhausted by counting electrons alone


spectral weight transfer
Hubbard Model
$N_{\mathrm{eff}} > \#x$
optical conductivity (mid-IR)

is there
anything else?

yes

Quantum critical behaviour in a high-T_c superconductor

D. van der Marel[1*], H. J. A. Molegraaf[1*], J. Zaanen[2], Z. Nussinov[2*], F. Carbone[1*], A. Damascelli[3*], H. Eisaki[3*], M. Greven[3], P. H. Kes[2] & M. Li[2]

[1]*Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands*
[2]*Leiden Institute of Physics, Leiden University, 2300 RA Leiden, The Netherlands*
[3]*Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA*

Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

yes

Quantum critical behaviour in a high-T_c superconductor

D. van der Marel[1*], H. J. A. Molegraaf[1*], J. Zaanen[2], Z. Nussinov[2*], F. Carbone[1*], A. Damascelli[3*], H. Eisaki[3*], M. Greven[3], P. H. Kes[2] & M. Li[2]

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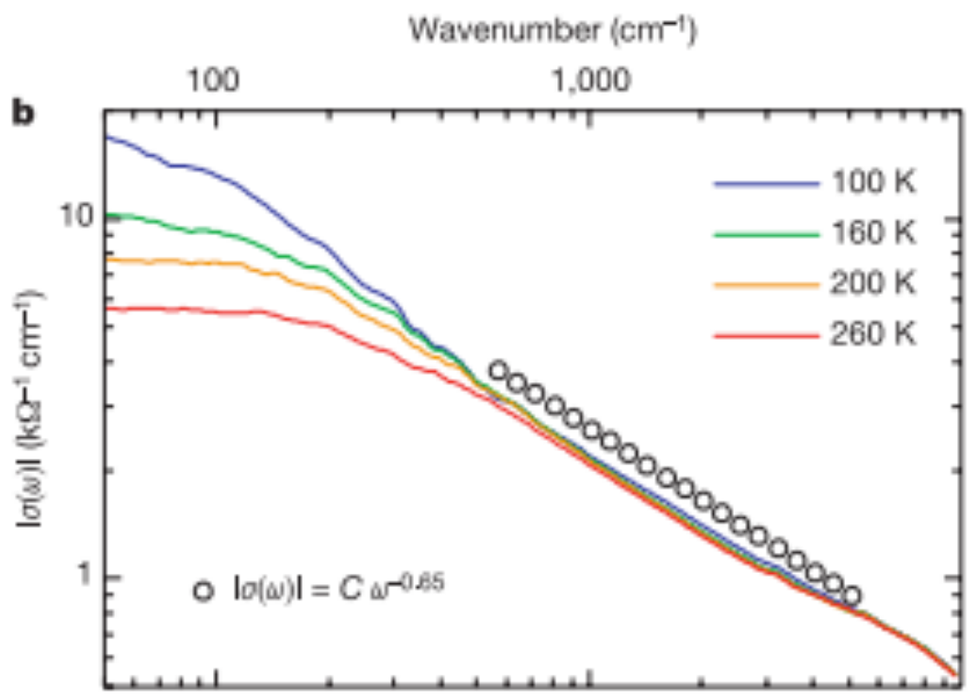
[3]*Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA*



Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

$$\sigma(\omega) = C\omega^{-\frac{2}{3}}$$

YBCO thin films

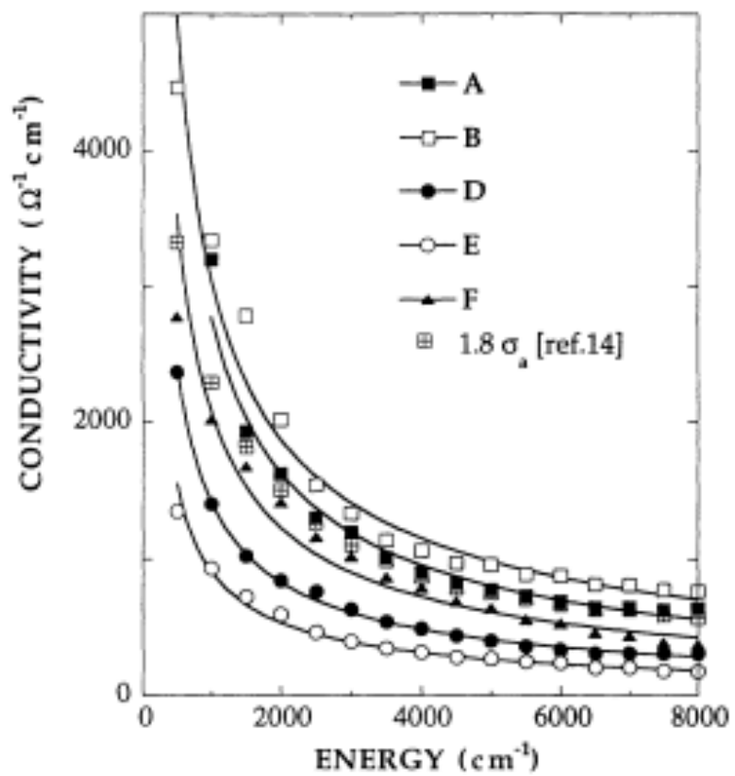


TABLE II. Values of the exponent α describing the best fit $\sigma \sim \omega^{-\alpha}$ for the various films labeled *A*, *B*, *D*, *E*, and *F* films. σ_a and σ_b stand for the conductivities measured along the *a* and *b* directions in an untwinned single crystal (Ref. 14). σ_{MFL} stands for the computed conductivity at 100 and 300 K for the marginal-Fermi-liquid model (MFL) (Ref. 16). The exponent is given within ±0.05.

σ_A (300 K)	σ_B (300 K)	σ_D (300 K)	σ_E (300 K)	σ_F (300 K)	σ_a (100 K)	σ_b (100 K)	σ_{MFL} (300 K)	σ_{MFL} (100 K)
0.77	0.70	0.77	0.76	0.76	0.67	none	0.74	0.68

criticality

↓

scale invariance

↓

power law correlations

criticality

scale invariance

power law correlations

criticality

↑

scale invariance

↑

power law correlations

scale-invariant propagators

$$\left(\frac{1}{p^2}\right)^{\alpha}$$

scale-invariant propagators

$$\left(\frac{1}{p^2}\right)^{\alpha}$$

Anderson: use Luttinger Liquid propagators

$$G^R \propto \frac{1}{(\omega - v_s k)^{\eta}}$$

compute conductivity without vertex corrections (PWA)

is flawed. In fact, in the Luttinger liquid such direct calculations are not to be trusted very firmly, since it is the nature of the Luttinger liquid that vertex corrections, if they must be included, will be singular; conventional transport theory is not applicable, and special methods such as the above are necessary.

$$\sigma(\omega) \propto \frac{1}{\omega} \int dx \int dt G^e(x,t) G^h(x,t) e^{i\omega t} \propto (i\omega)^{-1+2\eta}$$

problems

problems

1.) cuprates are not 1-dimensional

problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter

problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter

$$\sigma \propto \int d^d p G \left(G(\Gamma^\mu)^2, \Gamma^{\mu\nu} \right)$$

$$[G] = L^{d+1-2d_U}$$

$$[\Gamma^\mu] = L^{2d_U - d}$$

$$[\Gamma^{\mu\nu}] = L^{2d_U - d + 1}$$

problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter

$$\sigma \propto \int d^d p G \left(G(\Gamma^\mu)^2, \Gamma^{\mu\nu} \right)$$

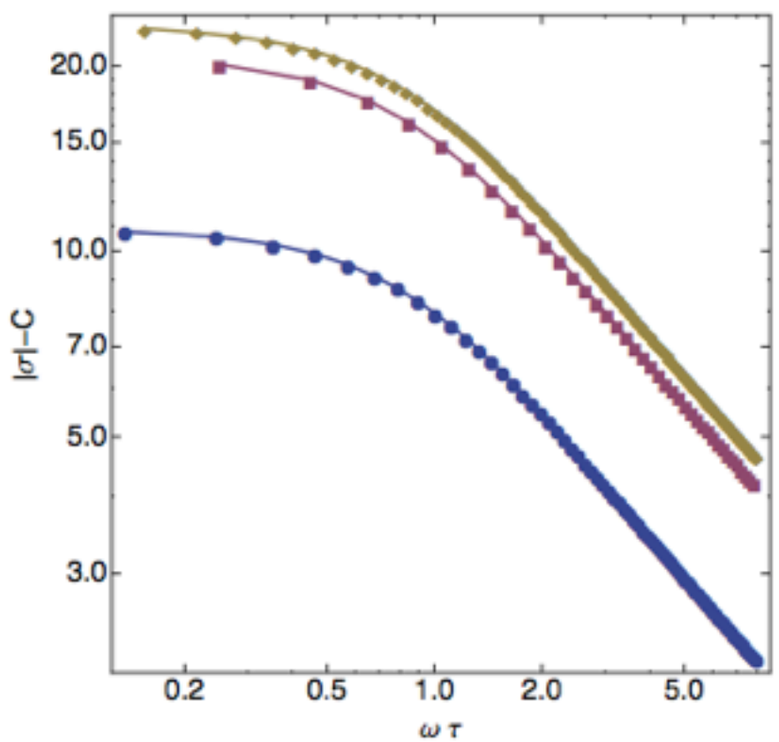
$$\left.\begin{aligned} [G] &= L^{d+1-2d_U} \\ [\Gamma^\mu] &= L^{2d_U-d} \\ [\Gamma^{\mu\nu}] &= L^{2d_U-d+1} \end{aligned}\right\} \quad [\sigma] = L^{2-d}$$

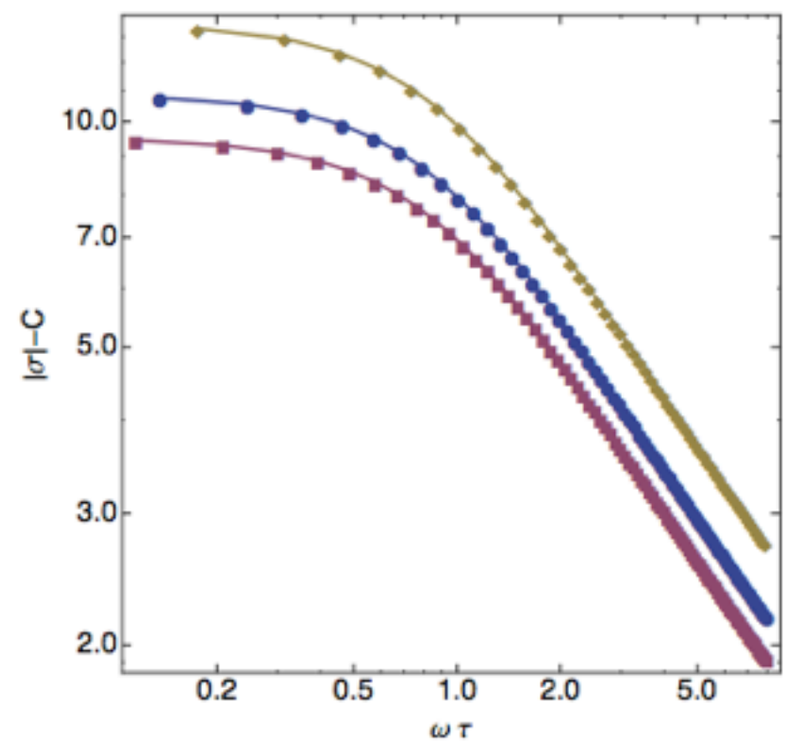
independent of d_U


1997
Anderson


1997
Anderson
2012
string theory
AdS/CFT

optical conductivity from a gravitational lattice





log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega\tau \lesssim 0.8$

G. Horowitz *et al*., Journal of High Energy Physics, 2012

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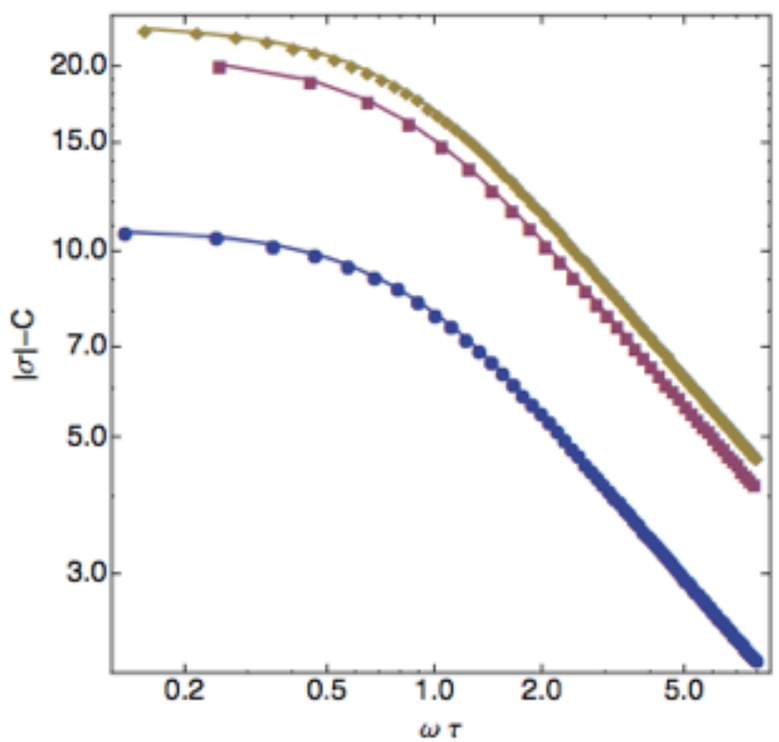

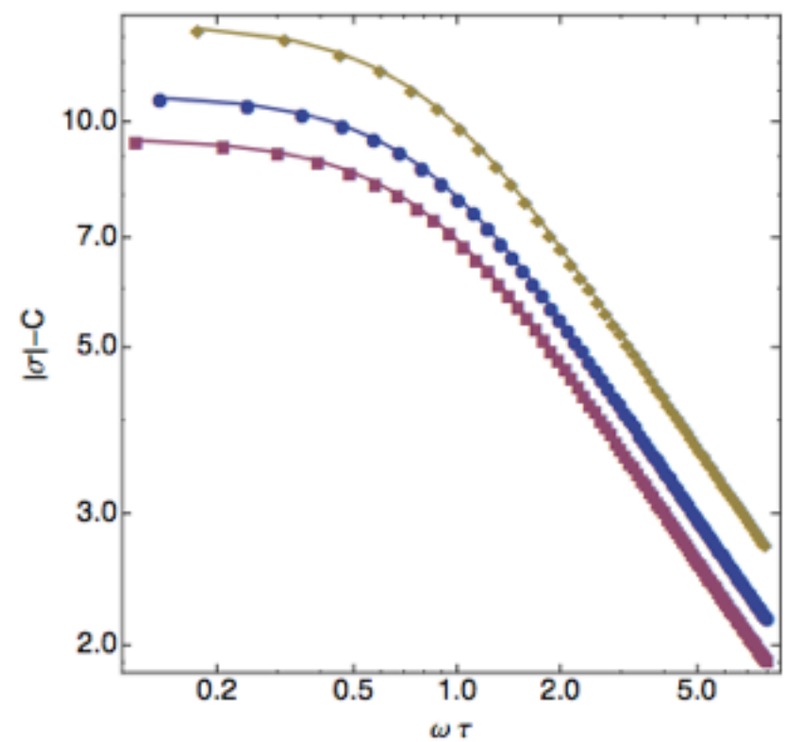

log-log plots for various parameters

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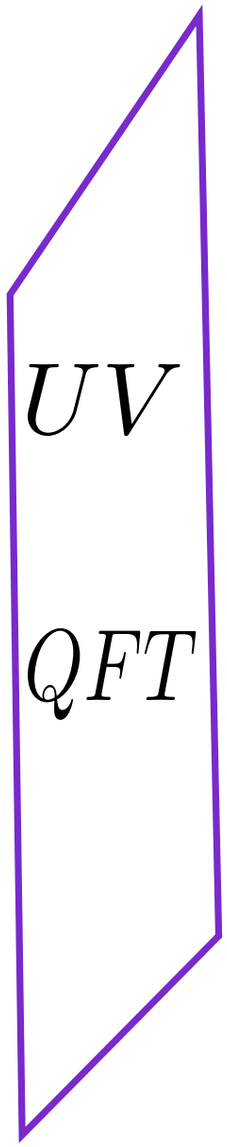
for $0.2 \lesssim \omega\tau \lesssim 0.8$

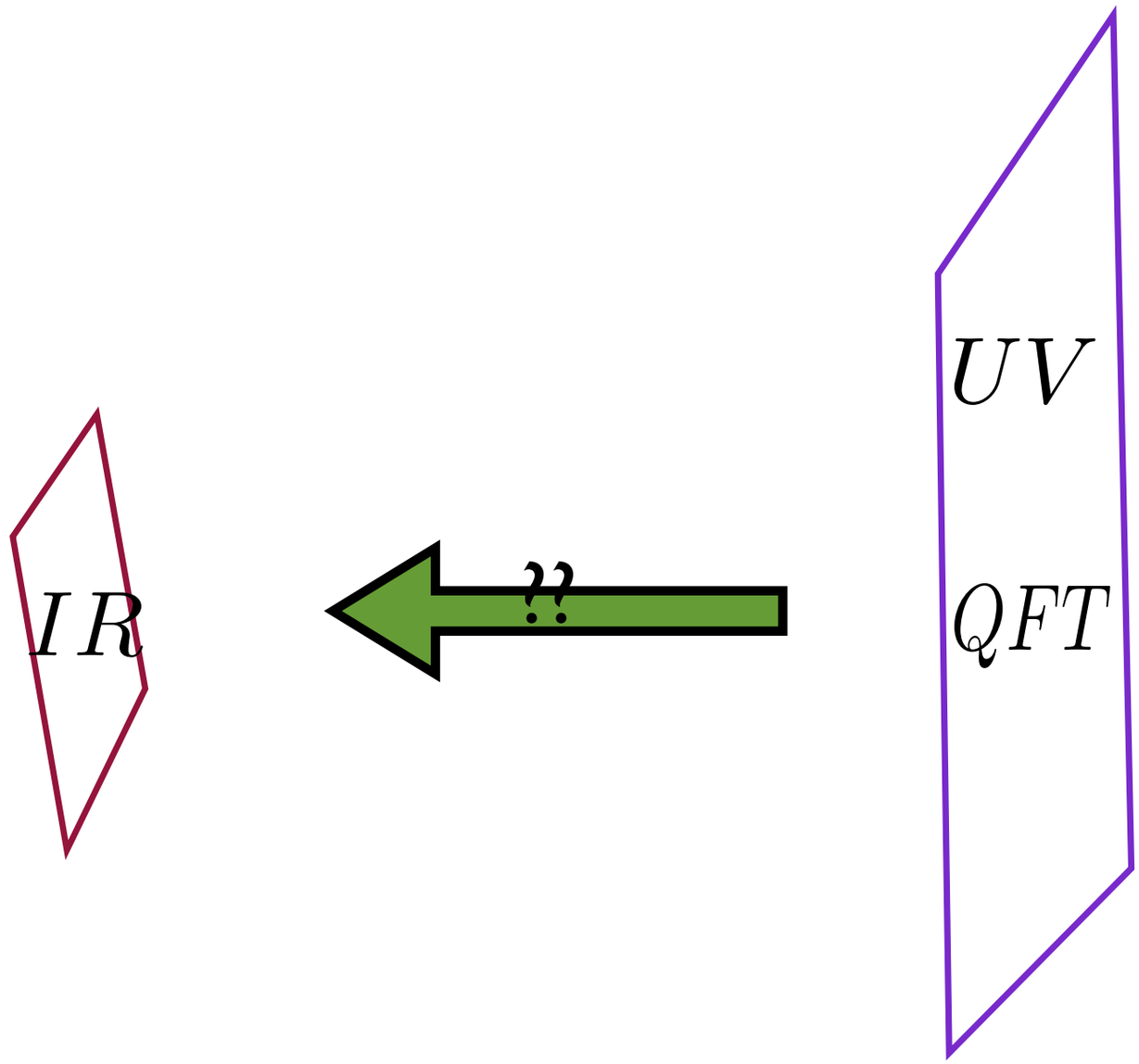
a remarkable claim!
replicates features of the strange metal? how?

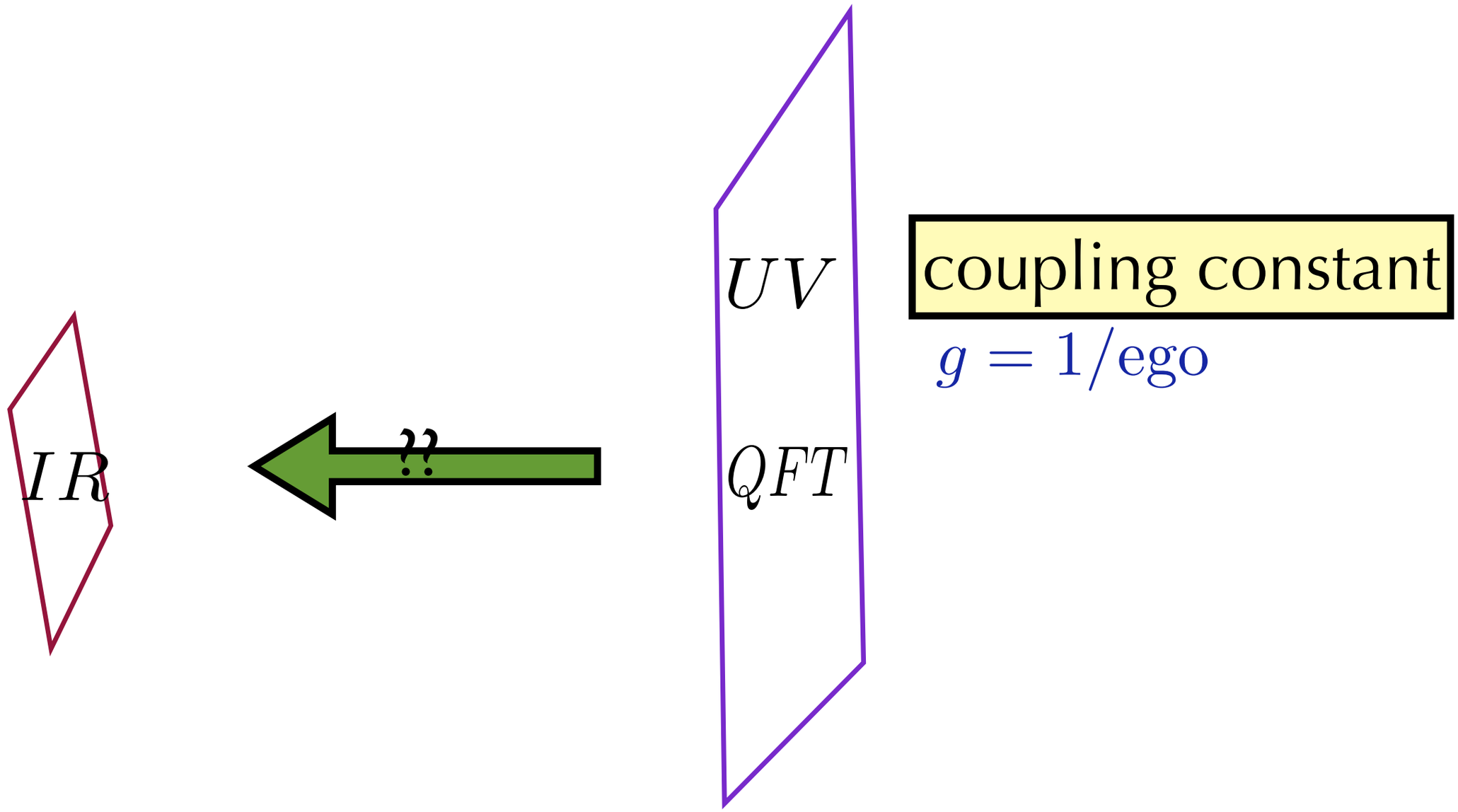
G. Horowitz *et al*., Journal of High Energy Physics, 2012

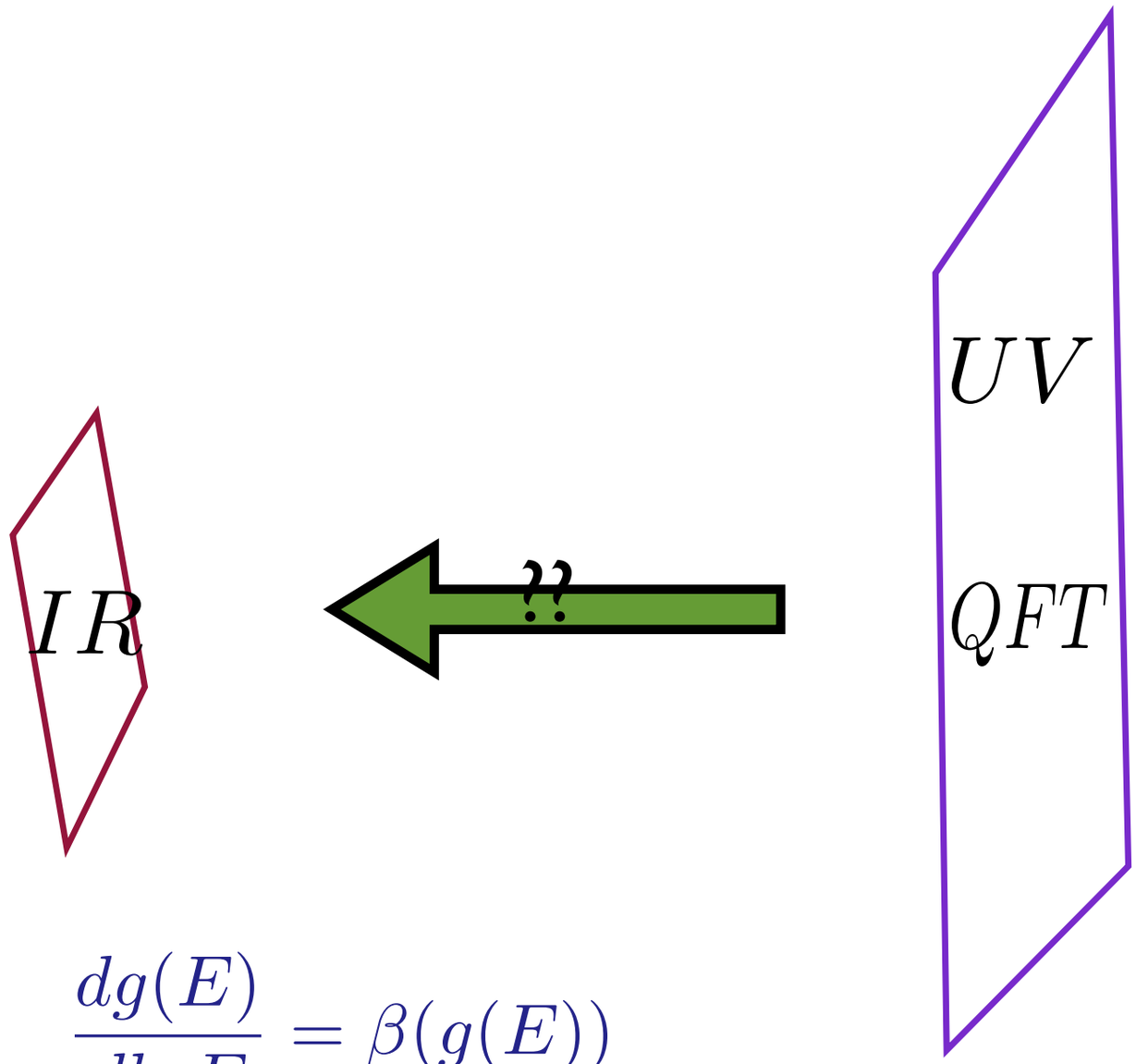
power law?

Could string theory be the answer?


UV
QFT


UV
QFT
??
IR


UV
coupling constant
$g = 1/\text{ego}$
IR
??
QFT



coupling constant

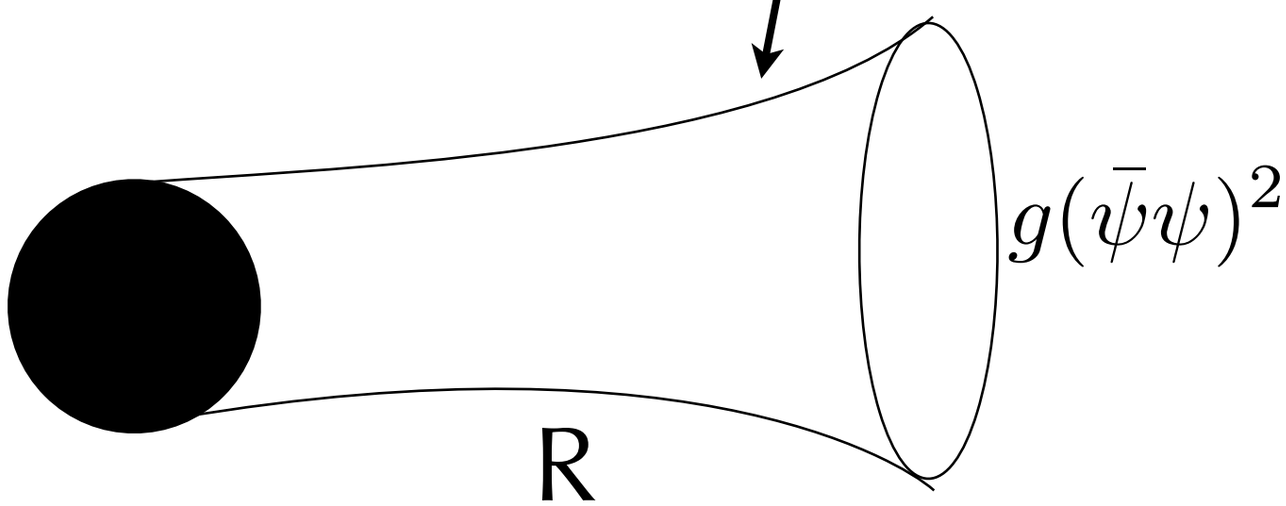
$g = 1/\text{ego}$

$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

replace coupled theory with geometry

scale-invariant anti de-Sitter



$$R \propto \sqrt{g}$$

replace coupled theory with geometry

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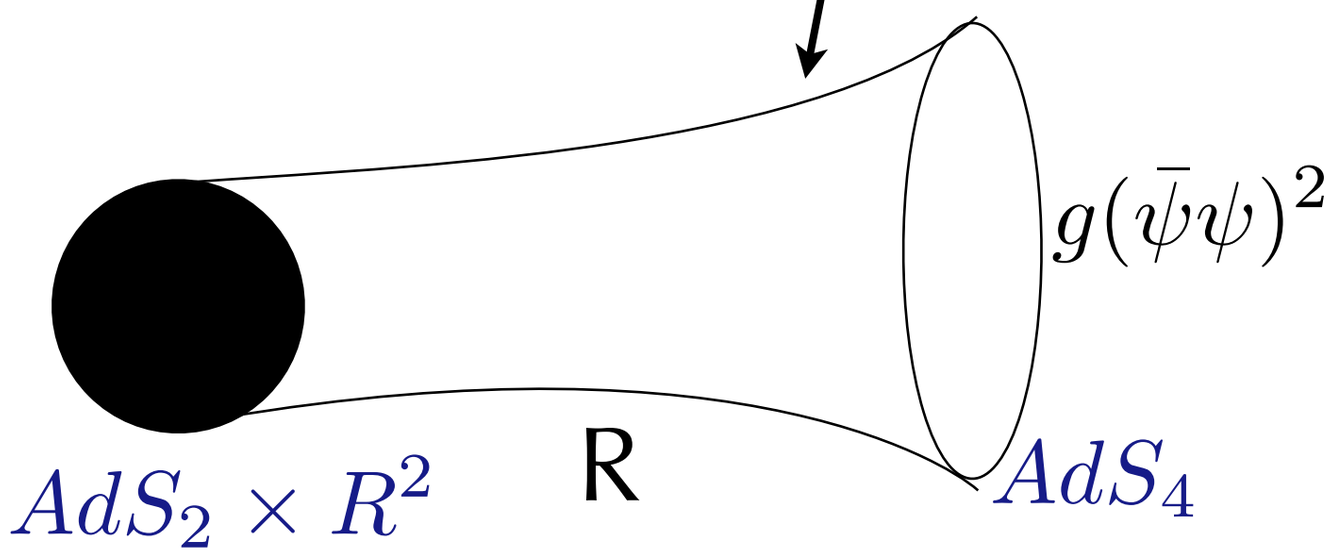

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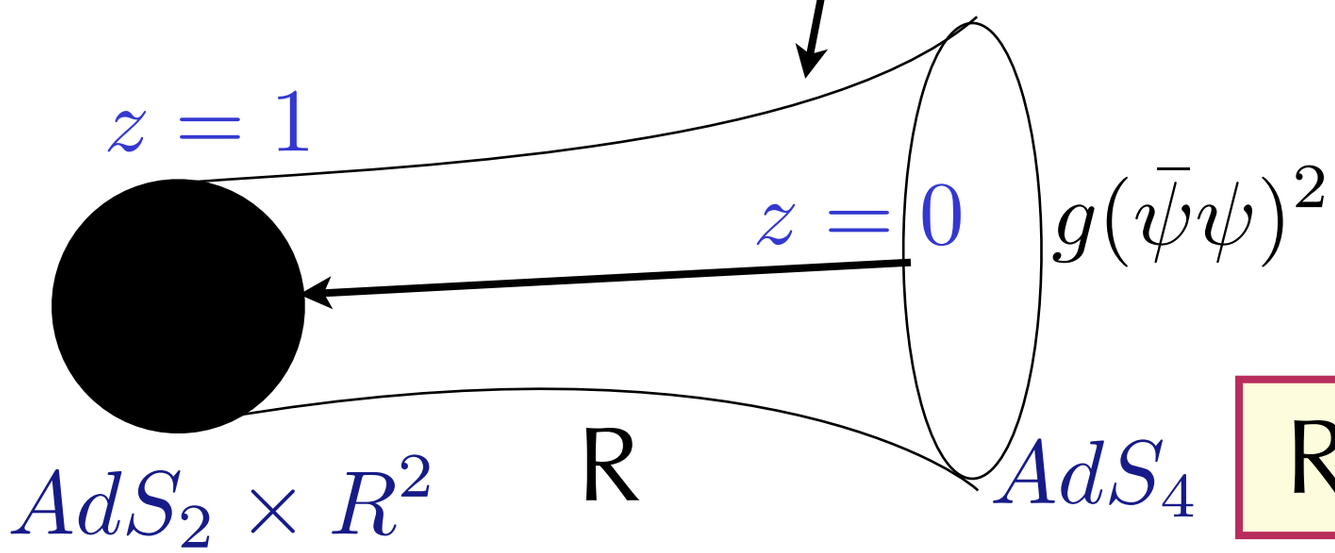

RG=GR

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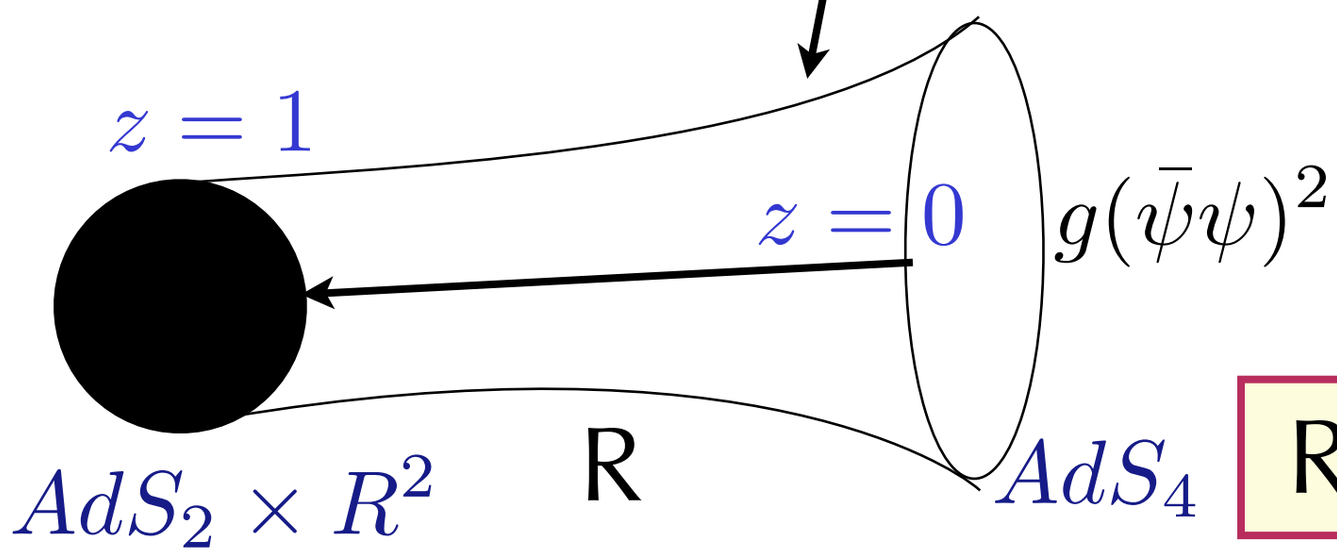


RG=GR

$$R \propto \sqrt{g}$$

cannot describe systems at g=0!

new equation!

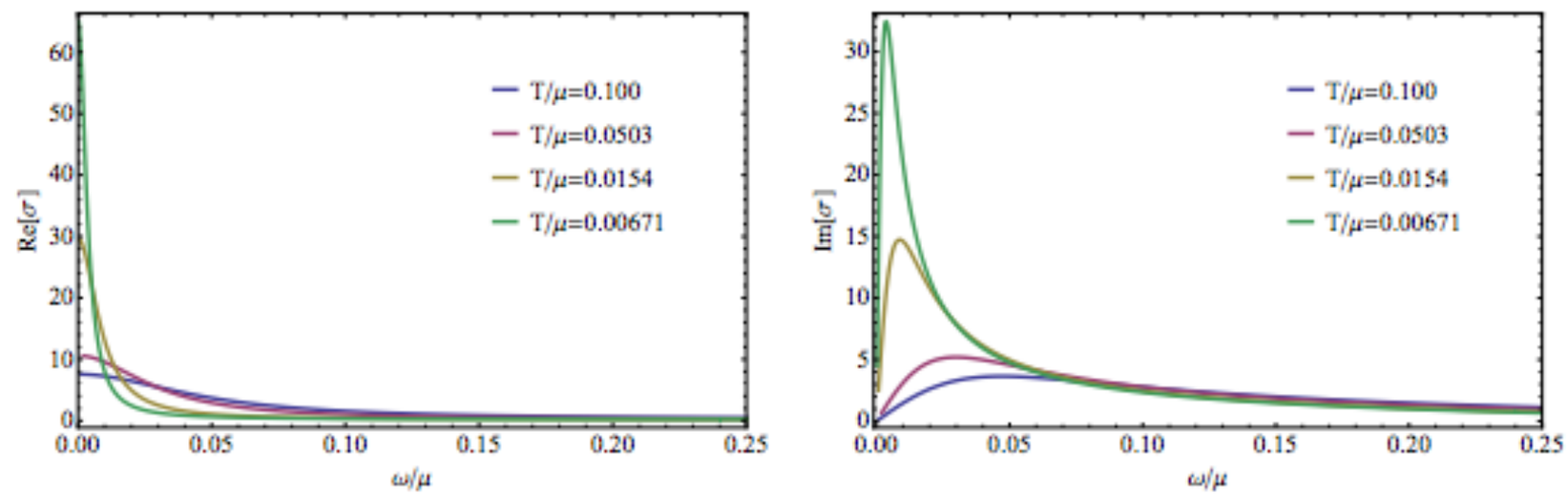
Einstein-Maxwell equations + non-uniform charge density = $B\omega^{-2/3}$

not so fast!

Donos and Gauntlett
(gravitational crystal)

Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$



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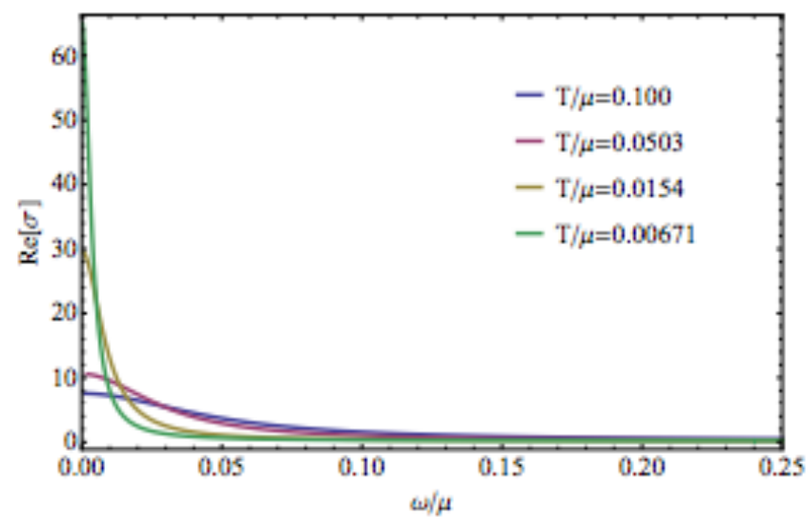

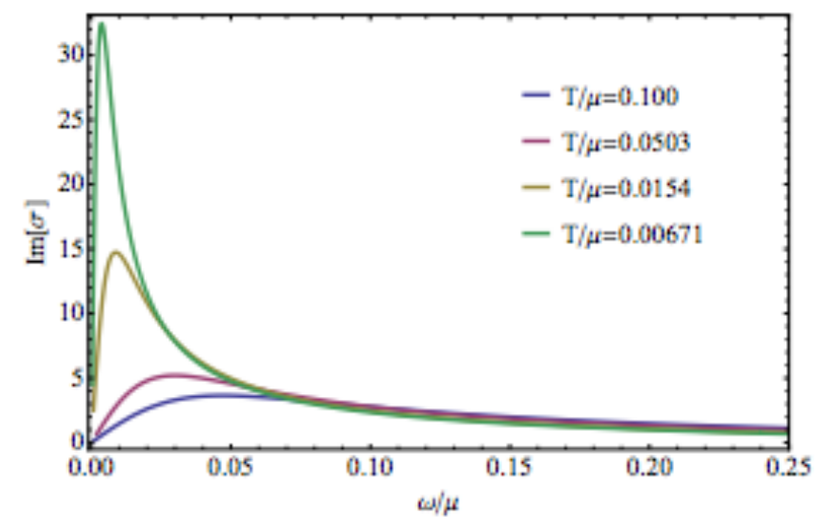

$$-\frac{2}{3} = 1 + \omega\frac{|\sigma|''}{|\sigma|'}$$

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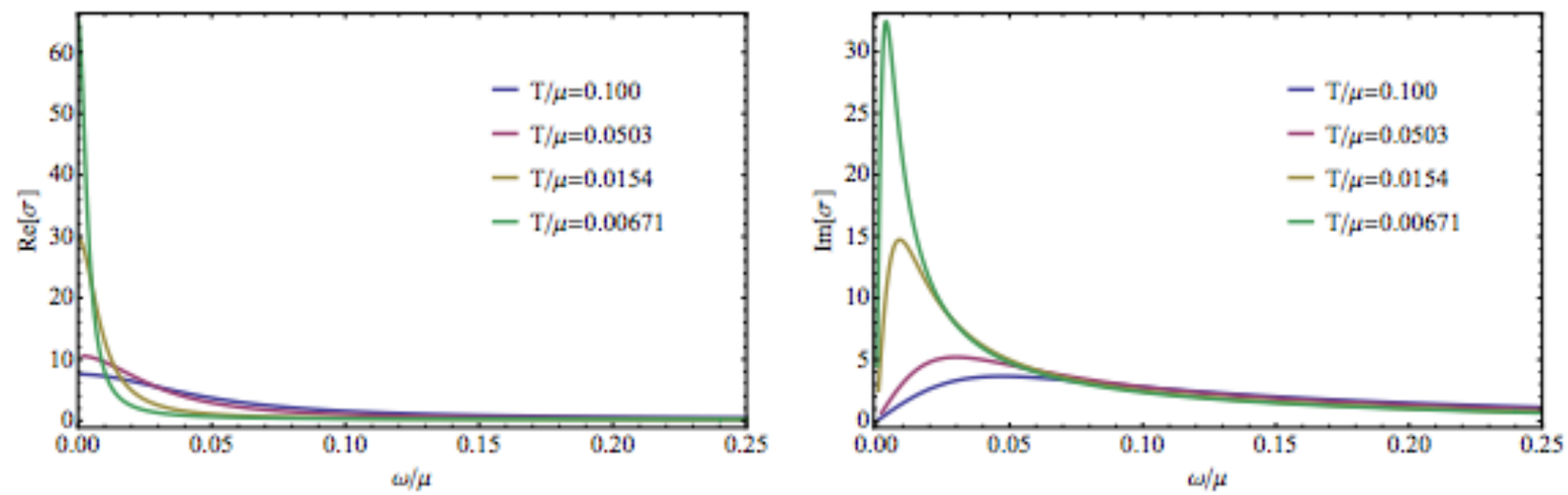

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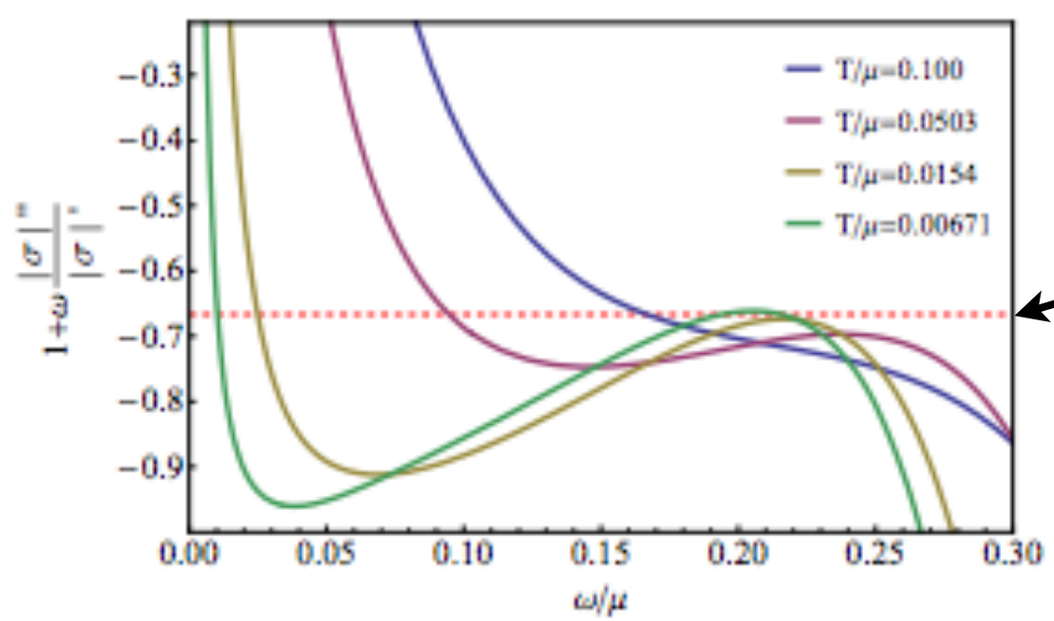

$$B\omega^{-2/3}$$

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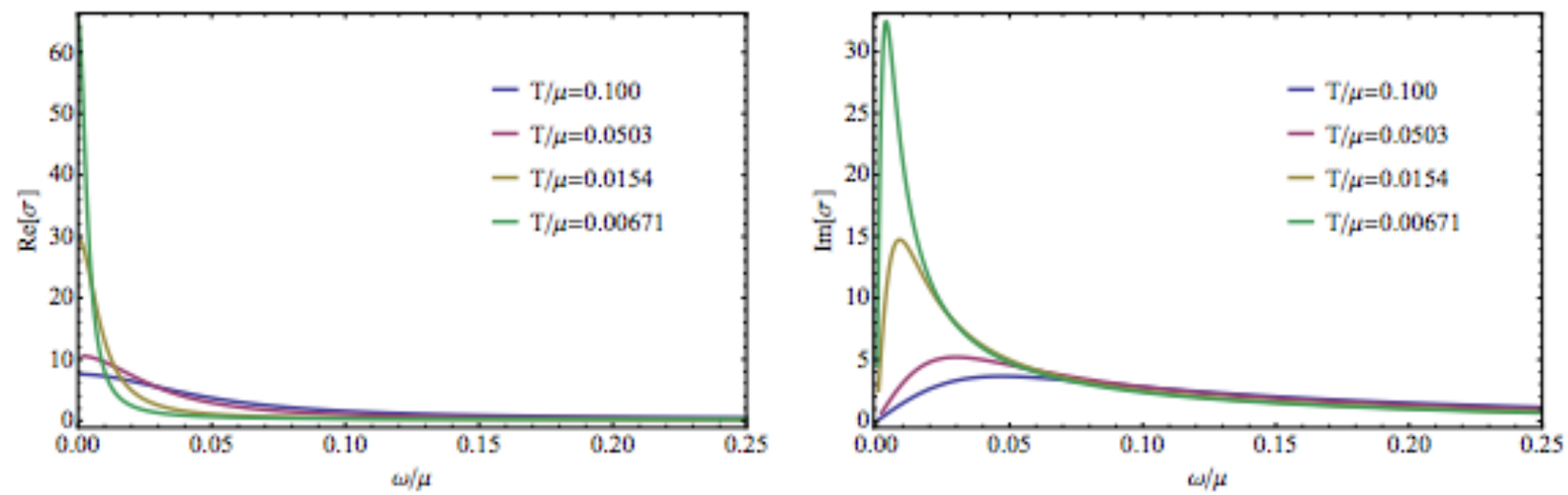

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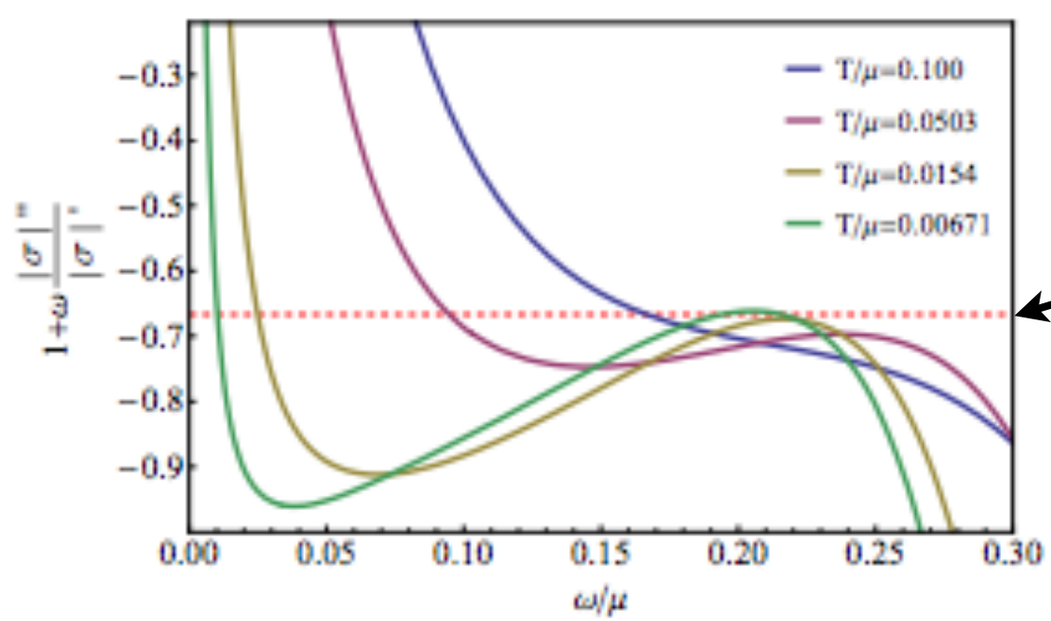

$$B\omega^{-2/3}$$

no power law!!

who is correct?

who is correct?

let's redo the calculation

model

model

action = gravity + EM + lattice

model

action = gravity + EM + lattice

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{2} F^2 \right),$$

model

action = gravity + EM + lattice

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model

action = gravity + EM + lattice

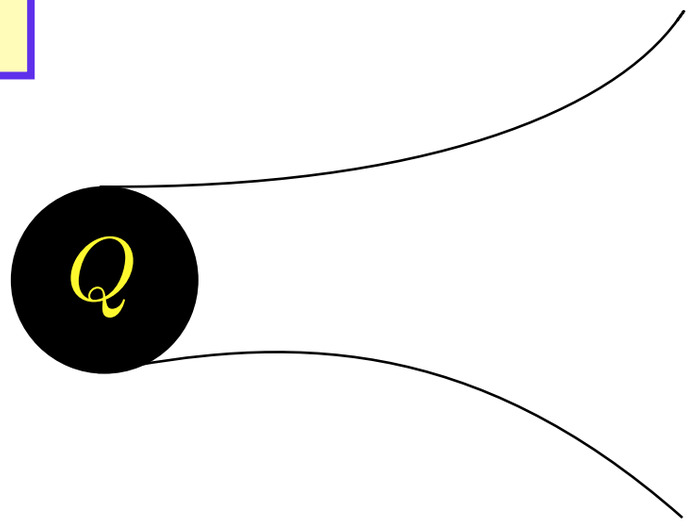
$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{2} F^2 \right),$$

$$\mathcal{L}(\phi) = \sqrt{-g}[-|\partial\phi|^2 - V(|\phi|)]$$

conductivity within AdS

$$(g_{\mathrm{ab}}, V(\Phi), A_t)$$

$$(\text{metric}\,, \text{potential}\,, \text{gaugefield})$$



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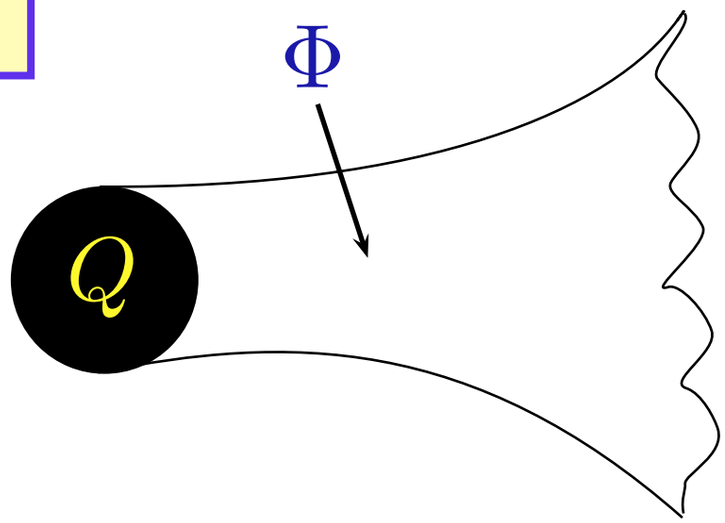

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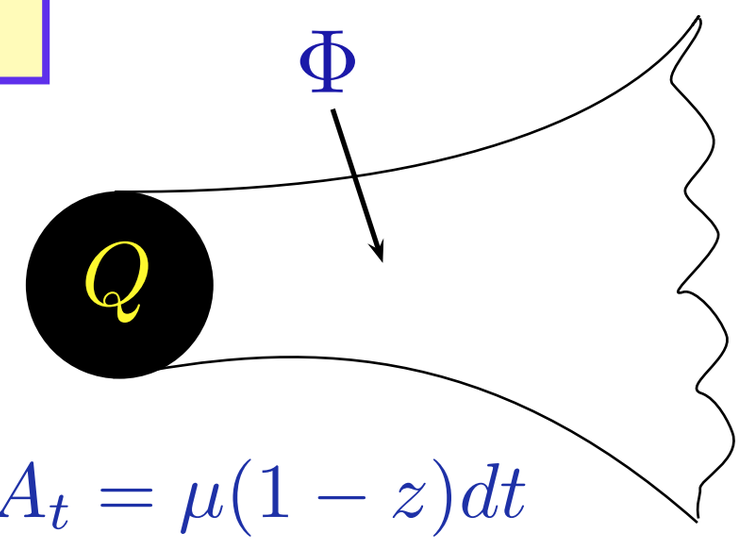


$$A_t = \mu(1-z)dt$$

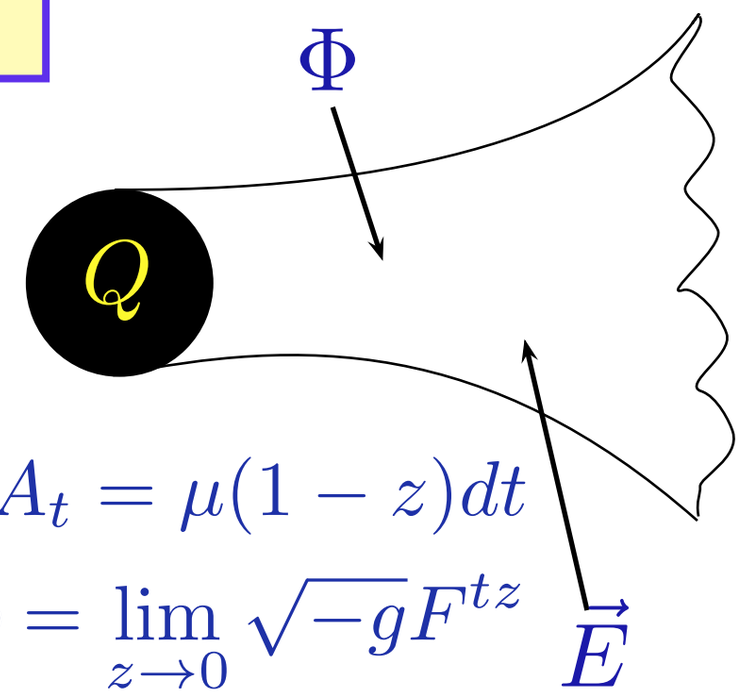
$$\rho = \lim_{z\to 0} \sqrt{-g}F^{tz}$$

conductivity within AdS

$$(g_{\mathrm{ab}}, V(\Phi), A_t)$$

(metric , potential , gaugefield)

perturb with electric field



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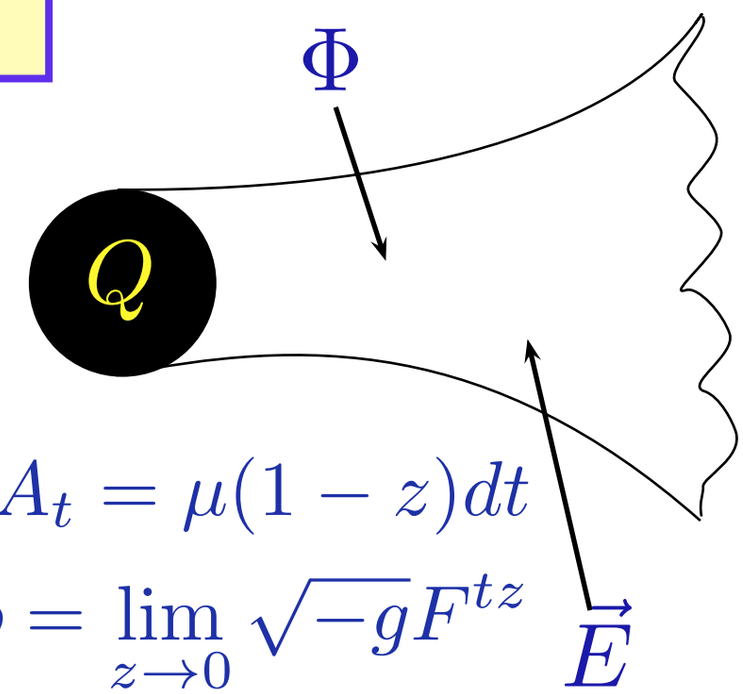

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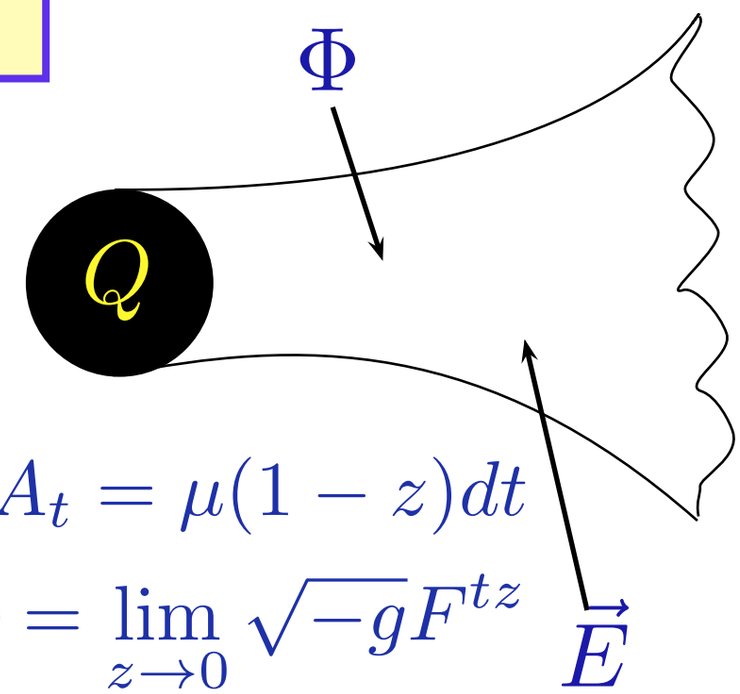

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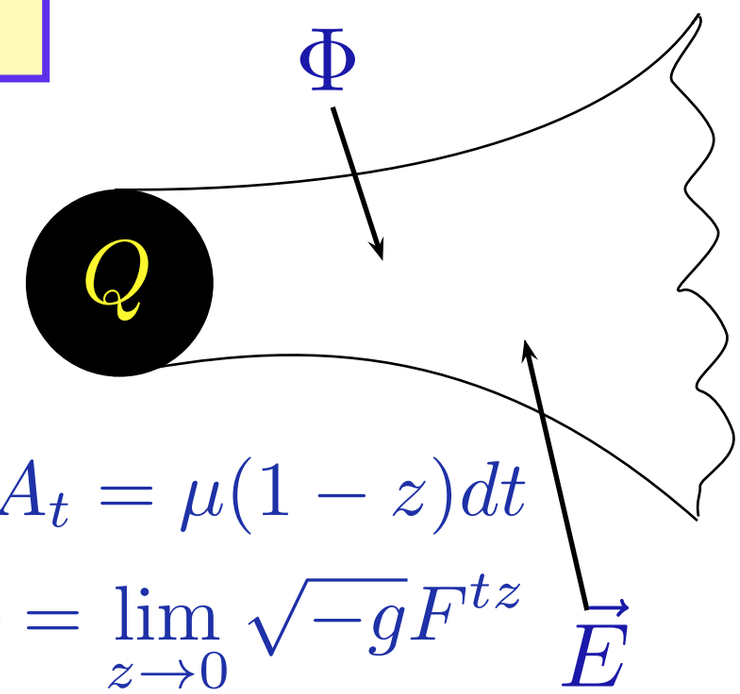

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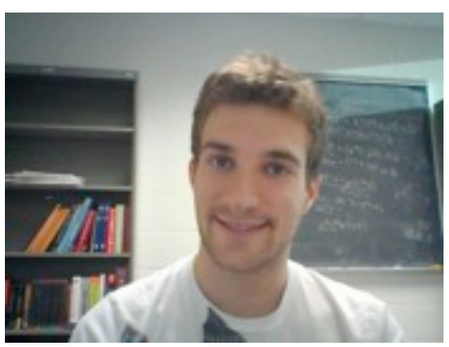

Brandon Langley

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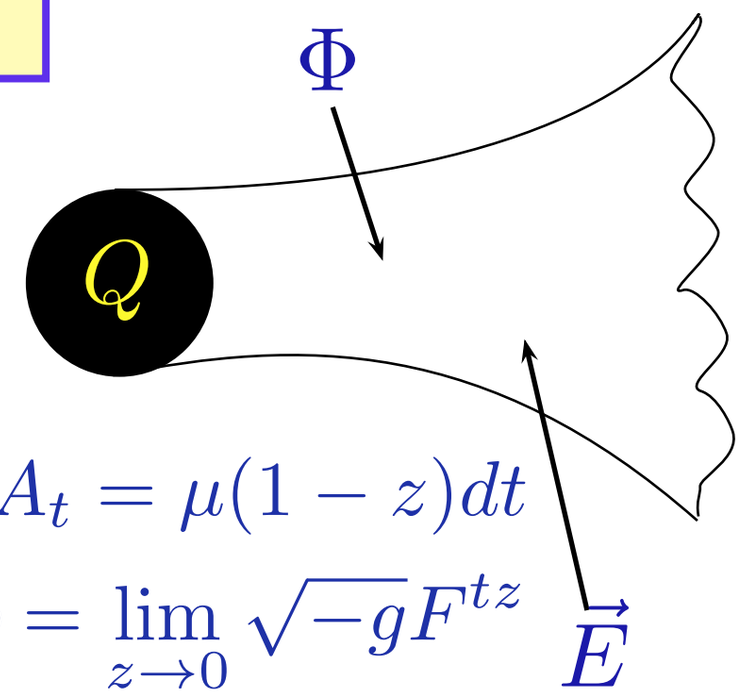

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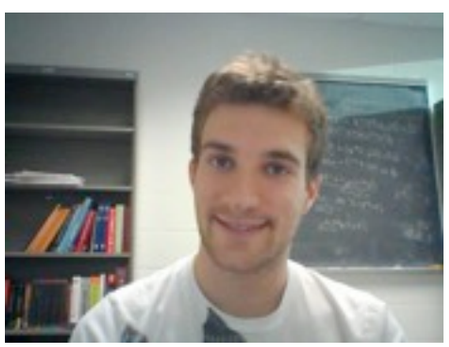

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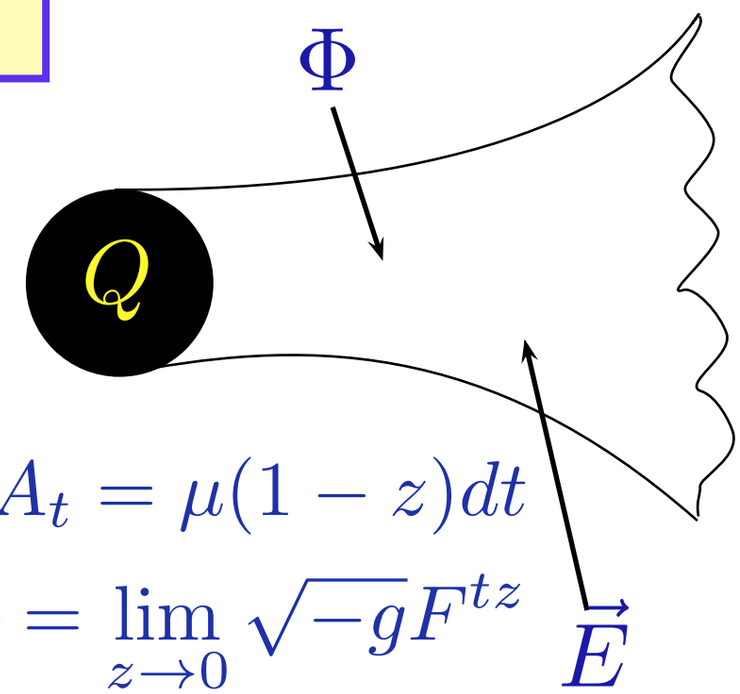

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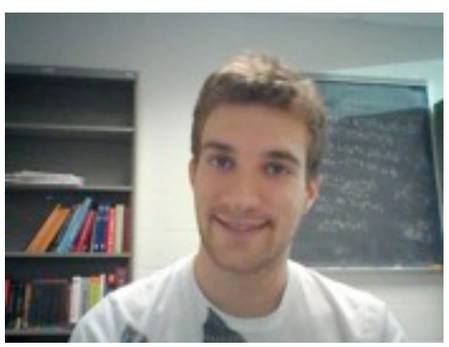

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HST vs. DG

HST vs. DG

Horowitz, Santos, Tong (HST)

$$V(\Phi) = -\Phi^2/L^2$$

$$\Phi = z\Phi^{(1)} + z^2\Phi^{(2)} + \cdots ,$$

$$\Phi^{(1)}(x) = A_0 \cos(kx)$$

inhomogeneous in x

$$m^2 = -2/L^2$$

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DG

$$V\left(|\Phi|^2\right)$$

$$\Phi(z,x) = \phi(z)e^{ikx}$$

no inhomogeneity in x

$$m^2 = -3/(2L^2)$$

HST vs. DG

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radial gauge

Our Model

$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

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$$\Phi_2 = \qquad z\Phi_2^{(1)} + z^2\Phi_2^{(2)} + \cdots\,, \quad \Phi_2^{(1)}(x) = A_0\cos\left(kx + \frac{\theta}{2}\right).$$

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$$\theta = 0$$

HST

Our Model

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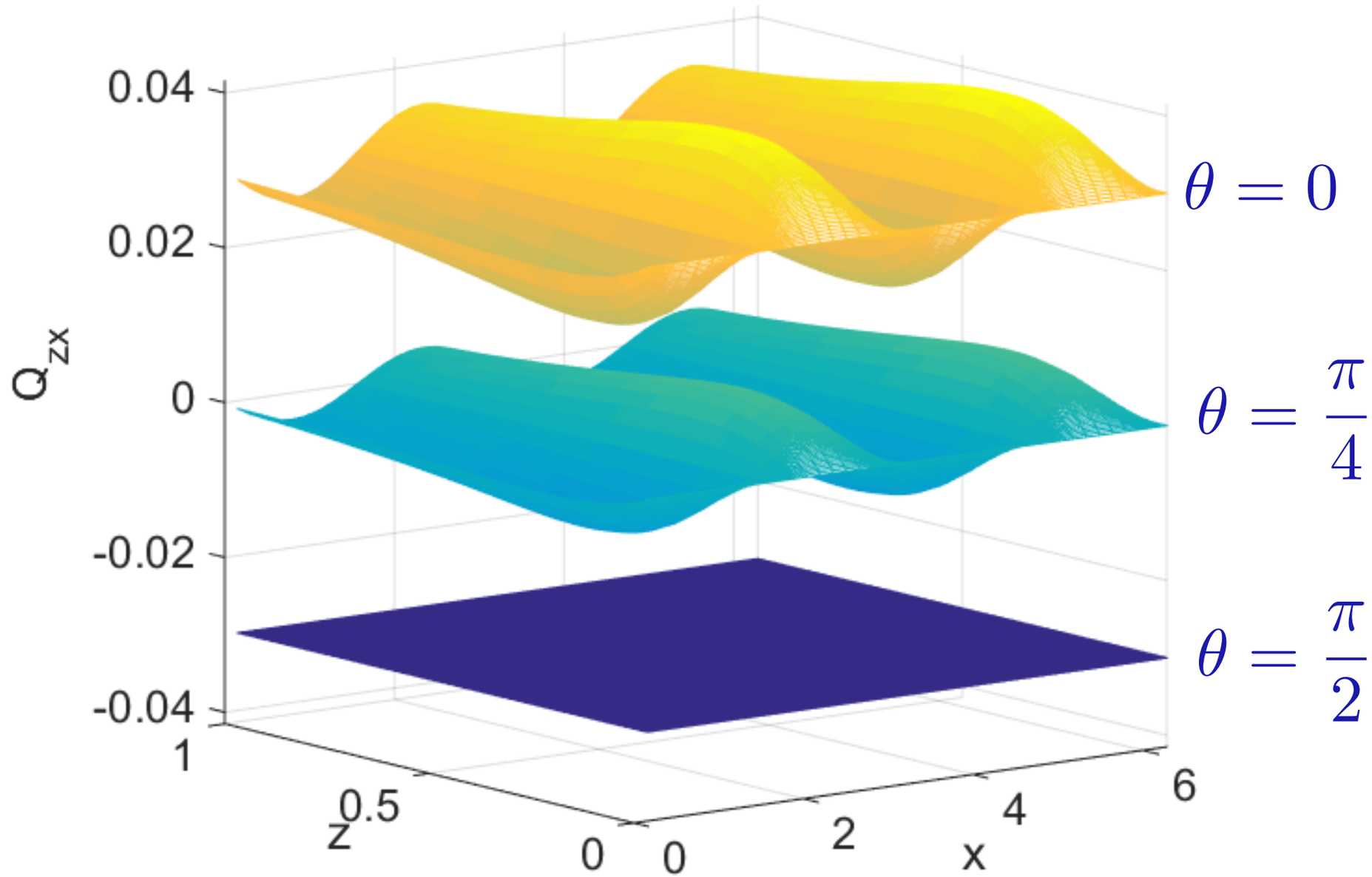
$\theta = 0$ → HST

$\theta = \frac{\pi}{2}$ → DG

$A_0 = 0.75,\ k = 1,\ \mu = 1.4,\ T/\mu = 0.115$



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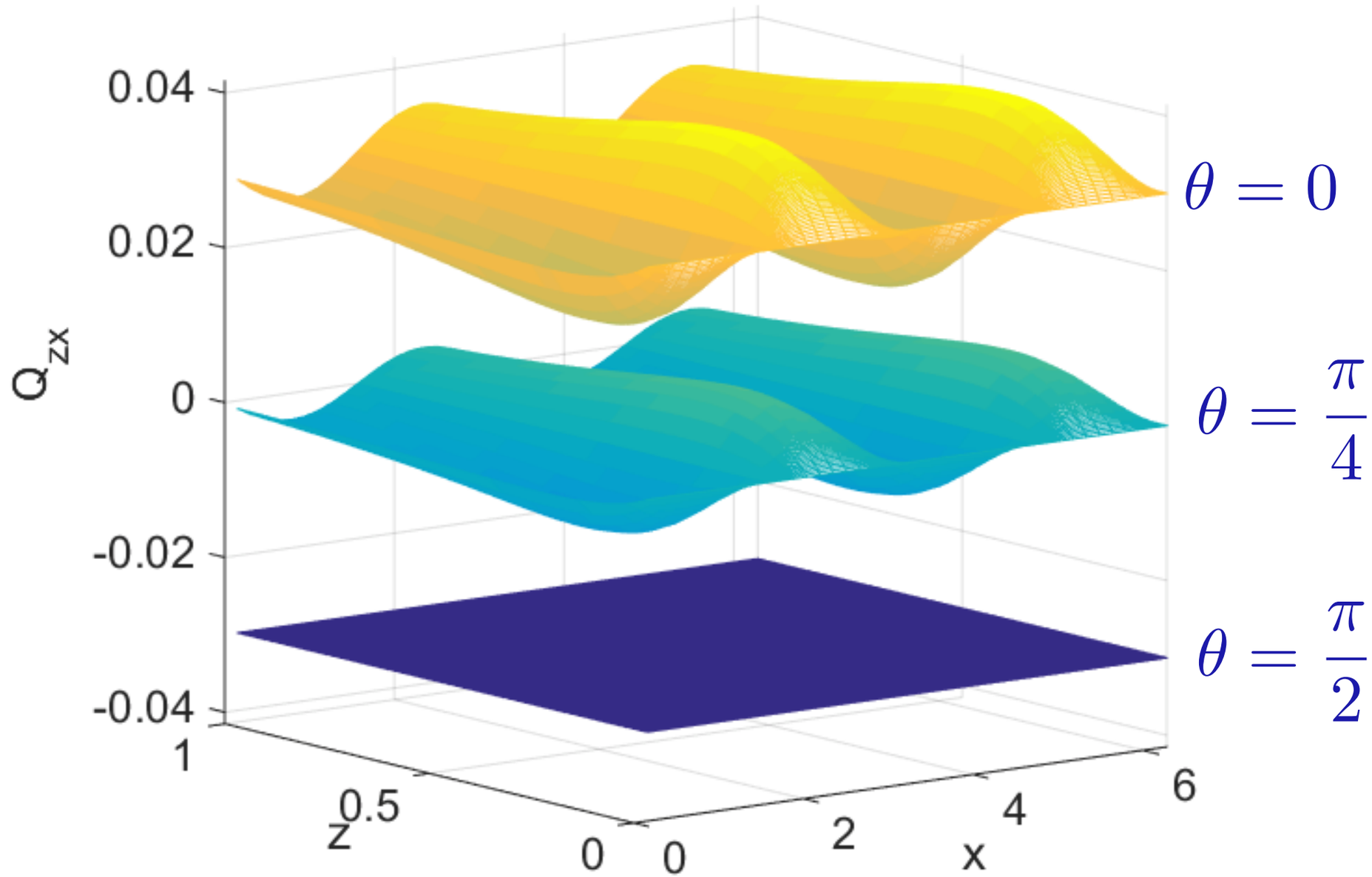
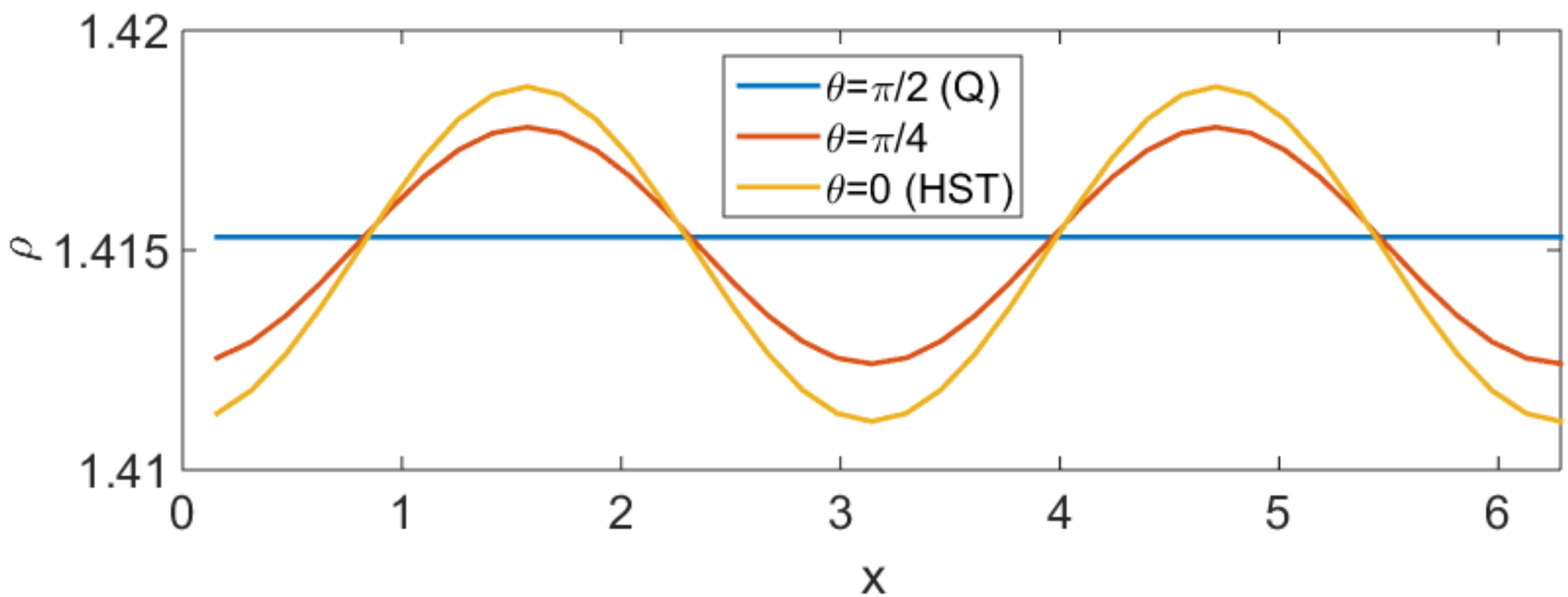


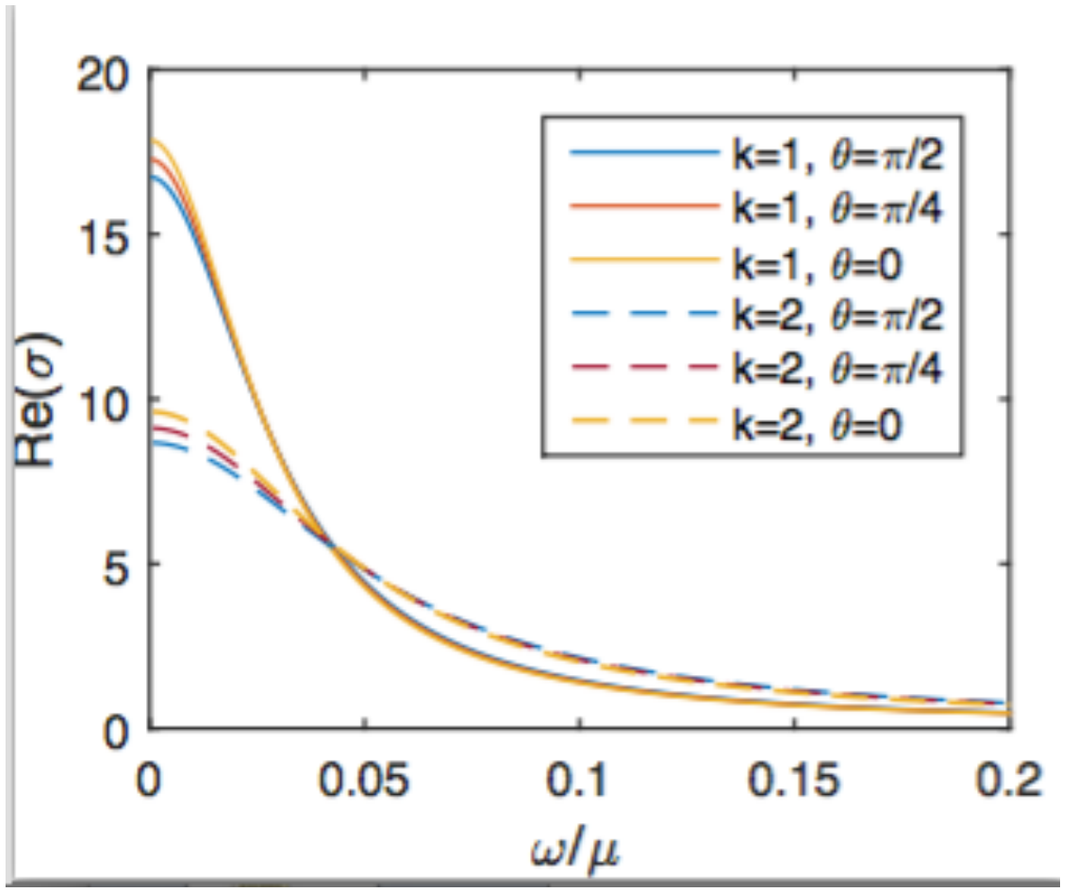
translational invariance is broken in metric in multiples of 2k

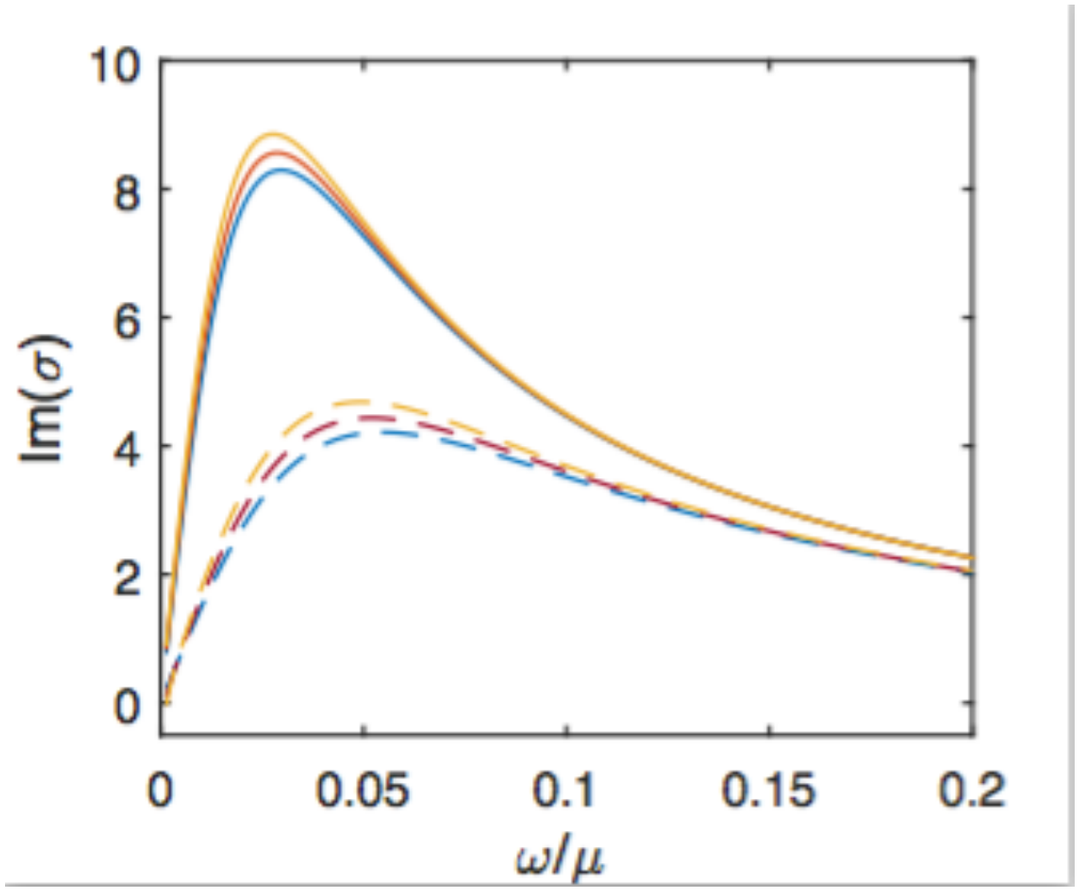
charge density

$$\rho = \lim_{z \to 0} \sqrt{-g} F^{tz}$$



conductivity



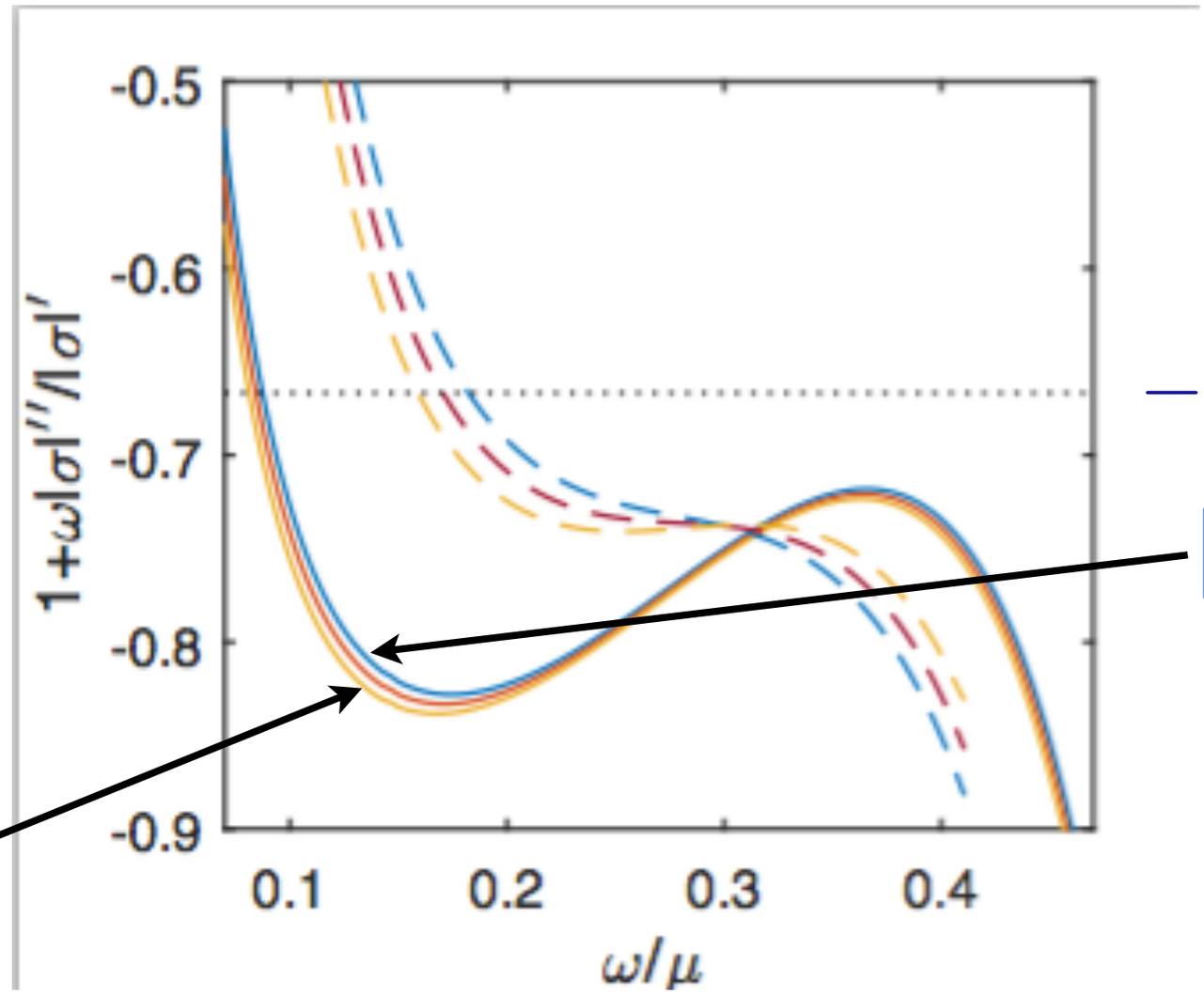


- high-frequency behavior is identical
- low-frequency RN has $\mathrm{Re}(\sigma) \sim \delta(\omega),\ \mathrm{Im}(\sigma) \sim 1/\omega$
- low-frequency lattice has Drude form

is there a power law?

is there a power law?

Results

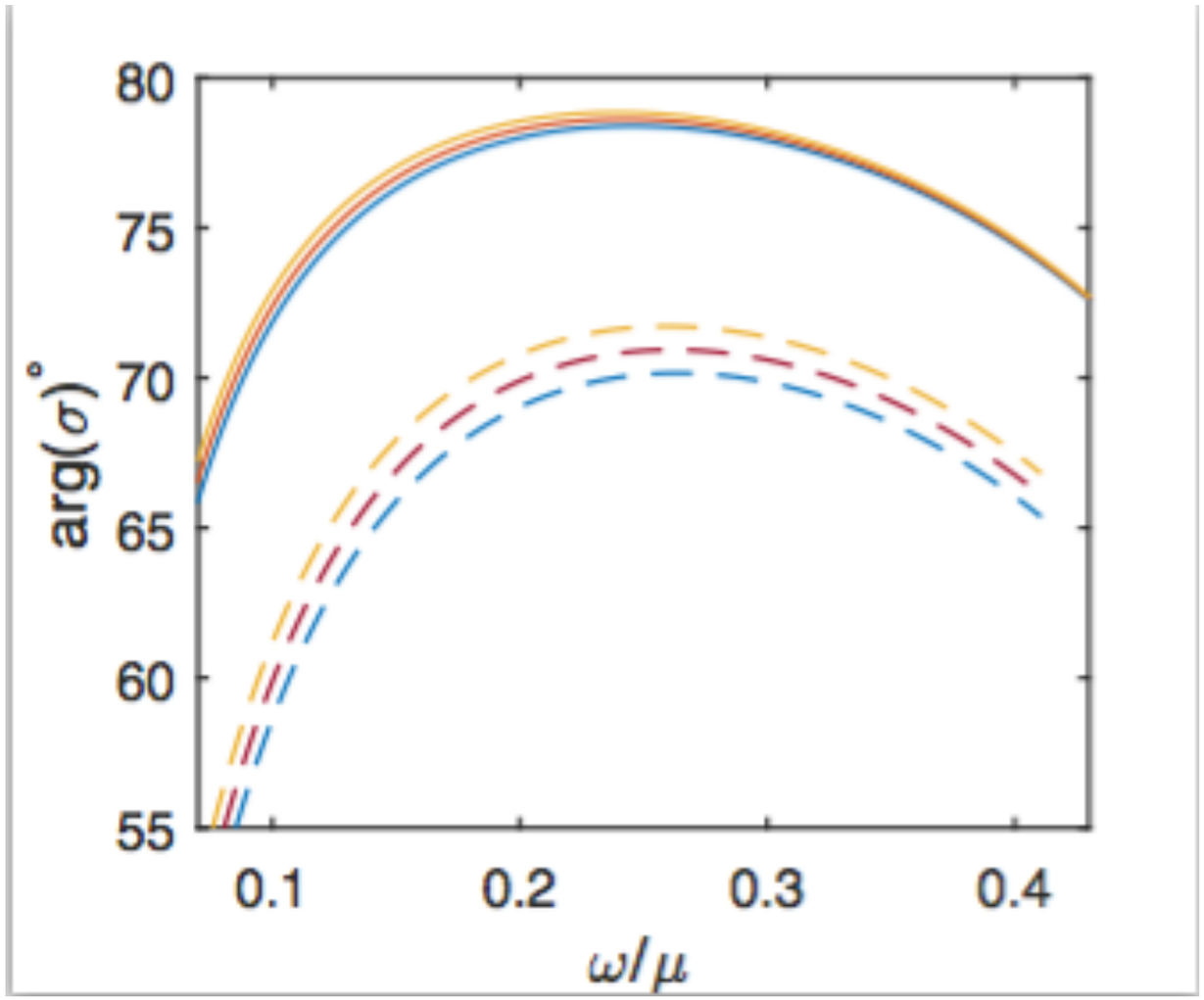


$-2/3$

DG

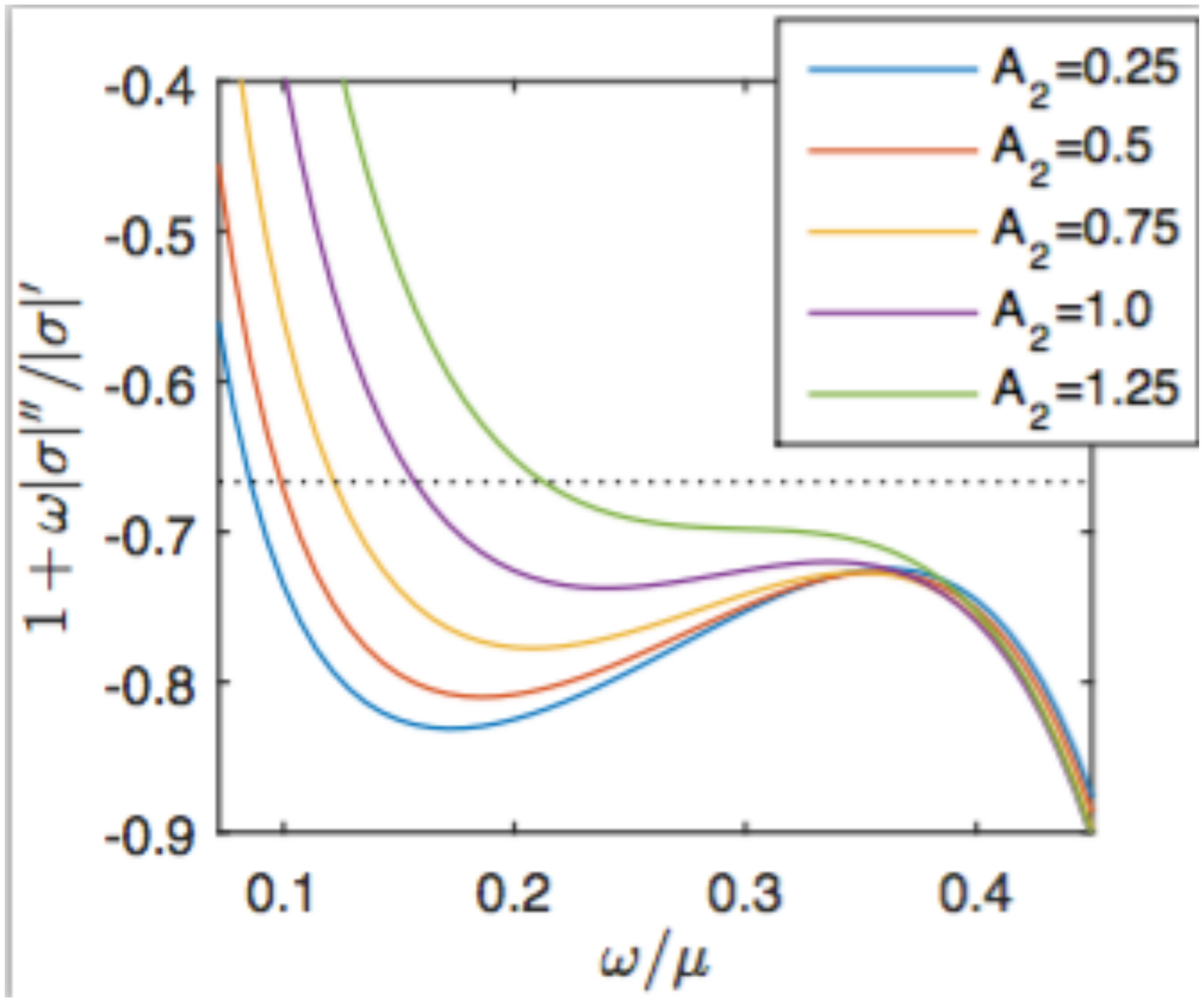
HST

Results

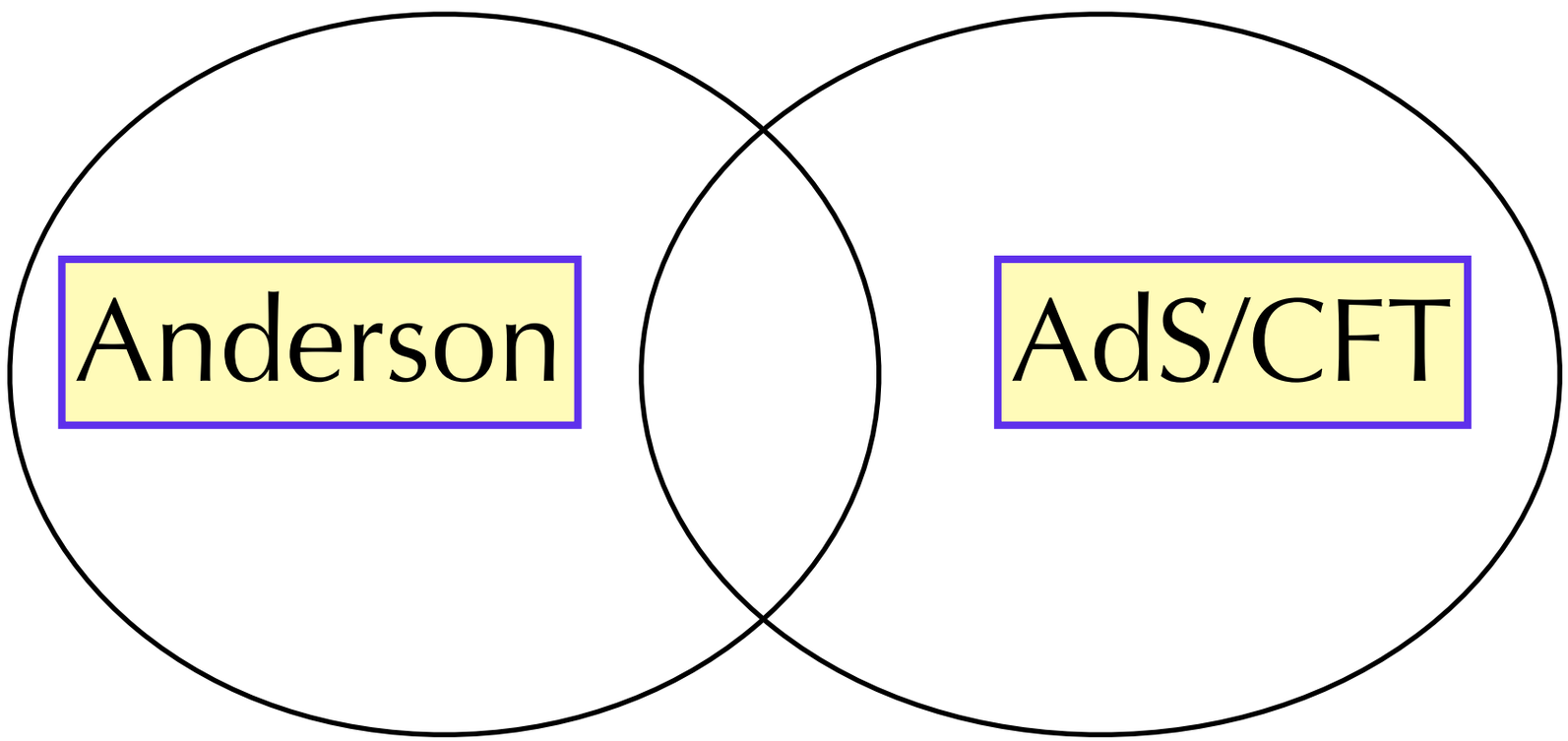


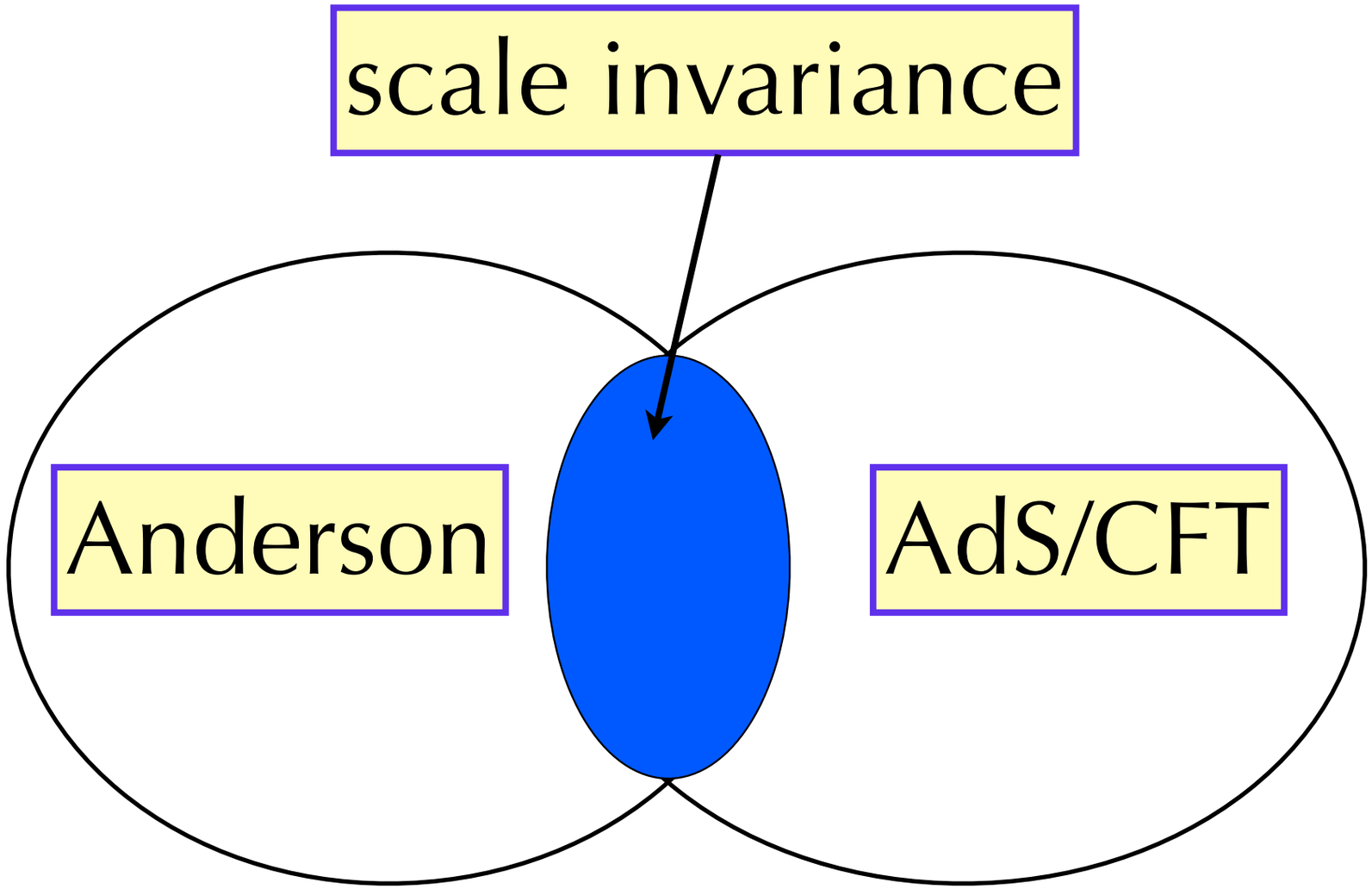
Results

$$A_1 = 0.75, k_1 = 2, k_2 = 2, \theta = 0, \mu = 1.4, T/\mu = 0.115$$



No!


Anderson
AdS/CFT


scale invariance
Anderson
AdS/CFT

origin of power law?

phenomenology

scale-invariant propagators

$$(p^2)^{d_U - d/2}$$

origin of power law?

phenomenology

scale-invariant propagators

$$(p^2)^{d_U - d/2}$$

no well-defined mass

$$\mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2$$

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scale-invariant propagators

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incoherent stuff (all energies)

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi + m^2 \phi^2$$

massive free theory

mass

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$$x \to x/\Lambda$$
$$\phi(x) \to \phi(x)$$

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no scale invariance

$$\mathcal{L} = \quad \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right)$$

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theory with all possible mass!

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not particles

unparticles

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propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

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continuous mass

$\phi(x, m^2)$

flavors

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$e^2(m)$

Karch, 2005

multi-bands

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$\phi(x, m^2)$ ⟷ $e^2(m)$

Karch, 2005

flavors

multi-bands

use unparticle propagators
to calculate conductivity

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assume Gaussian action

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$$G(p) \sim \frac{i}{(-p^2 + i\epsilon)^{\frac{d+1}{2} - d_U}}$$

gauge unparticles

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Wilson line

$$S = \int d^{d+1}x d^{d+1}y \; \phi_U^\dagger(x) F(x-y) W(x,y) \phi_U(y),$$

gauge unparticles

Wilson line

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vertices

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vertices

$$g\Gamma^\mu(p,q) = \frac{\delta^3 S}{\delta A^\mu(q)\delta\phi^\dagger(p+q)\delta\phi(p)}$$

1-gauge

$$g^2\Gamma^{\mu\nu}(p,q_1,q_2) = \frac{\delta^4 S}{\delta A^\mu(q_1)\delta A^\nu(q_2)\delta\phi^\dagger(p+q_1+q_2)\delta\phi(p)}$$

2-gauge

compute conductivity

$$\sigma^{\mu\nu}(i\omega_n) = \lim_{q\to 0} \frac{1}{\omega_n} K_n^{\mu\nu}(q)!$$

compute conductivity

$$\sigma^{\mu\nu}(i\omega_n) = \lim_{q\to 0} \frac{1}{\omega_n} K_n^{\mu\nu}(q)!$$

no power law

$$\sigma(i\omega_n) = (\frac{d+1}{2} - d_U)\sigma_0(i\omega_n)$$

what went wrong?

what went wrong?

free field

$$\phi_U(x) = \int_0^\infty dm^2 \; f(m^2)\phi(x, m^2)$$

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free field

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unparticle field has particle content

what went wrong?

free field

$$\phi_U(x) \propto \int_0 d m^2 \, f(m^2) \, \phi(x, m^2)$$

unparticle field has particle content

continuous mass taken seriously

$$S = \sum_{i=1}^{N} \int d\tau \int d^d x (|D_\mu \phi_i^2| + m_i^2 |\phi_i|^2)$$

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$$\sigma(\omega) = \int_0^M dm \rho(m) e^2(m) f(\omega, m, T)$$

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$\alpha > 0$ convergence of integral

last attempt

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take experiments seriously

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$$\sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i}$$

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continuous mass $d\sigma(m) = \sigma(m) dm$

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continuous mass $d\sigma(m) = \sigma(m) dm$

$$\sigma(\omega) = \int_0^M \frac{\rho(m) e^2(m) \tau(m)}{m} \frac{1}{1 - i\omega\tau(m)} dm$$

variable masses for everything

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

$$e(m) = e_0 \frac{m^b}{M^b}$$

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Karch, 2015

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Karch, 2015

$$\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0}{M^{a+2b+c}} \int_0^M dm \frac{m^{a+2b+c-2}}{1 - i\omega\tau_0 \frac{m^c}{M^c}} \quad =$$

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perform integral

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

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$$\Big\downarrow \; \omega\tau_0 \to \infty$$

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$$\boxed{\sigma(\omega) = \frac{1}{3}(\sqrt{3}+3i)\pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M\omega^{\frac{2}{3}}}}$$

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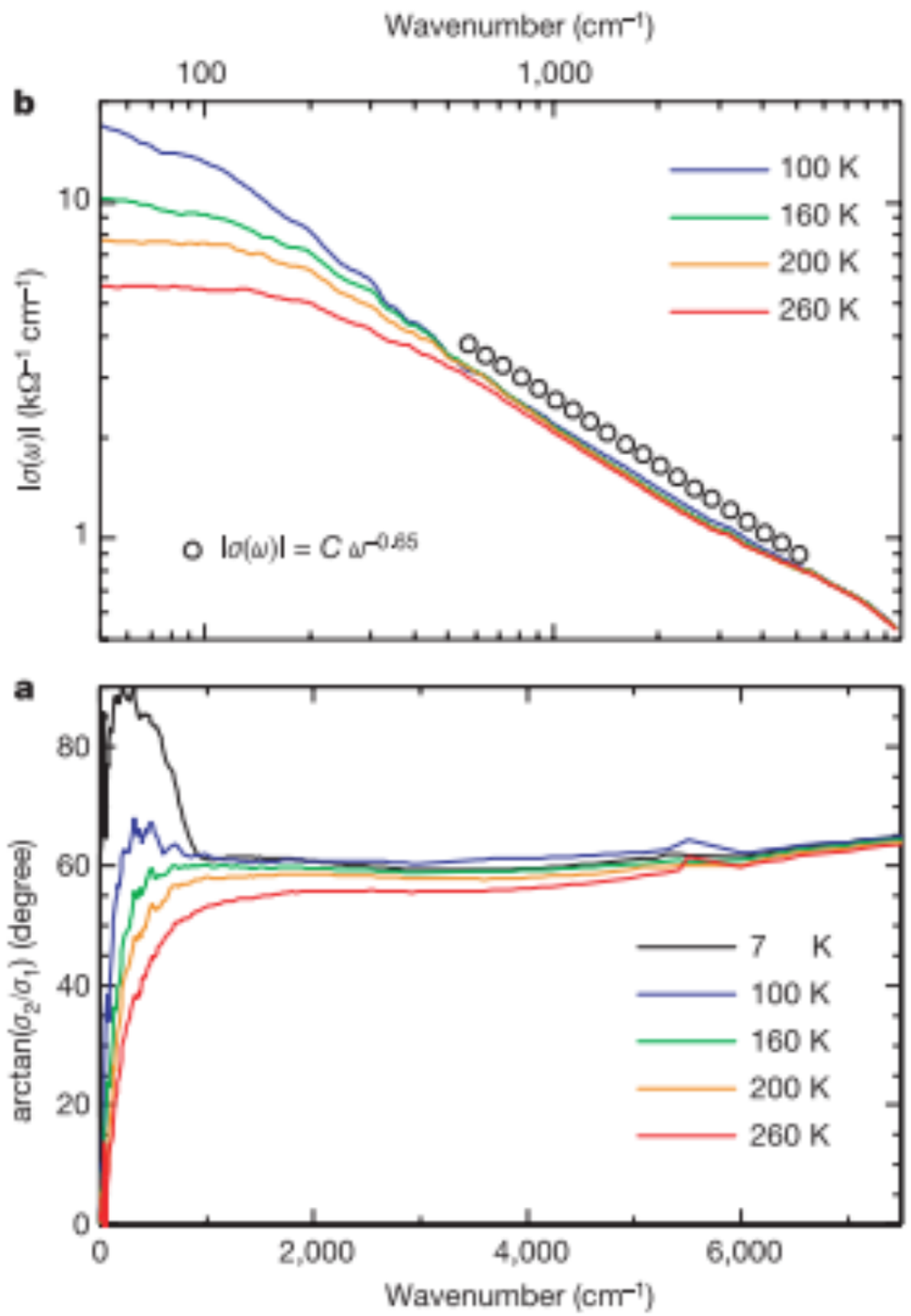
$$\boxed{\begin{array}{c} \tan\sigma = \sqrt{3} \\ 60^\circ \end{array}}$$

experiments



$$\sigma(\omega) = C\omega^{\gamma-2}e^{i\pi(1-\gamma/2)}$$

$$\gamma = 1.35$$

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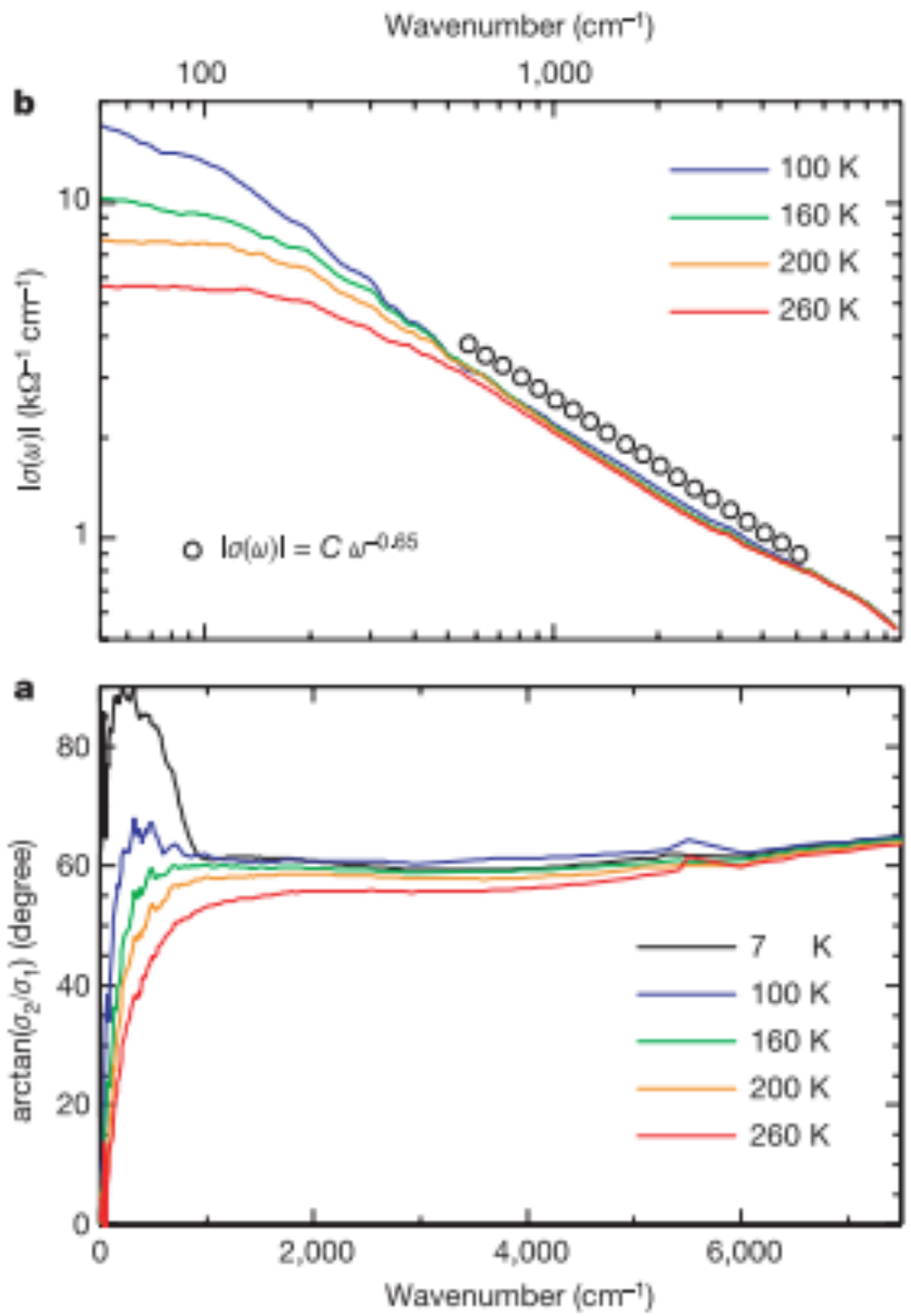

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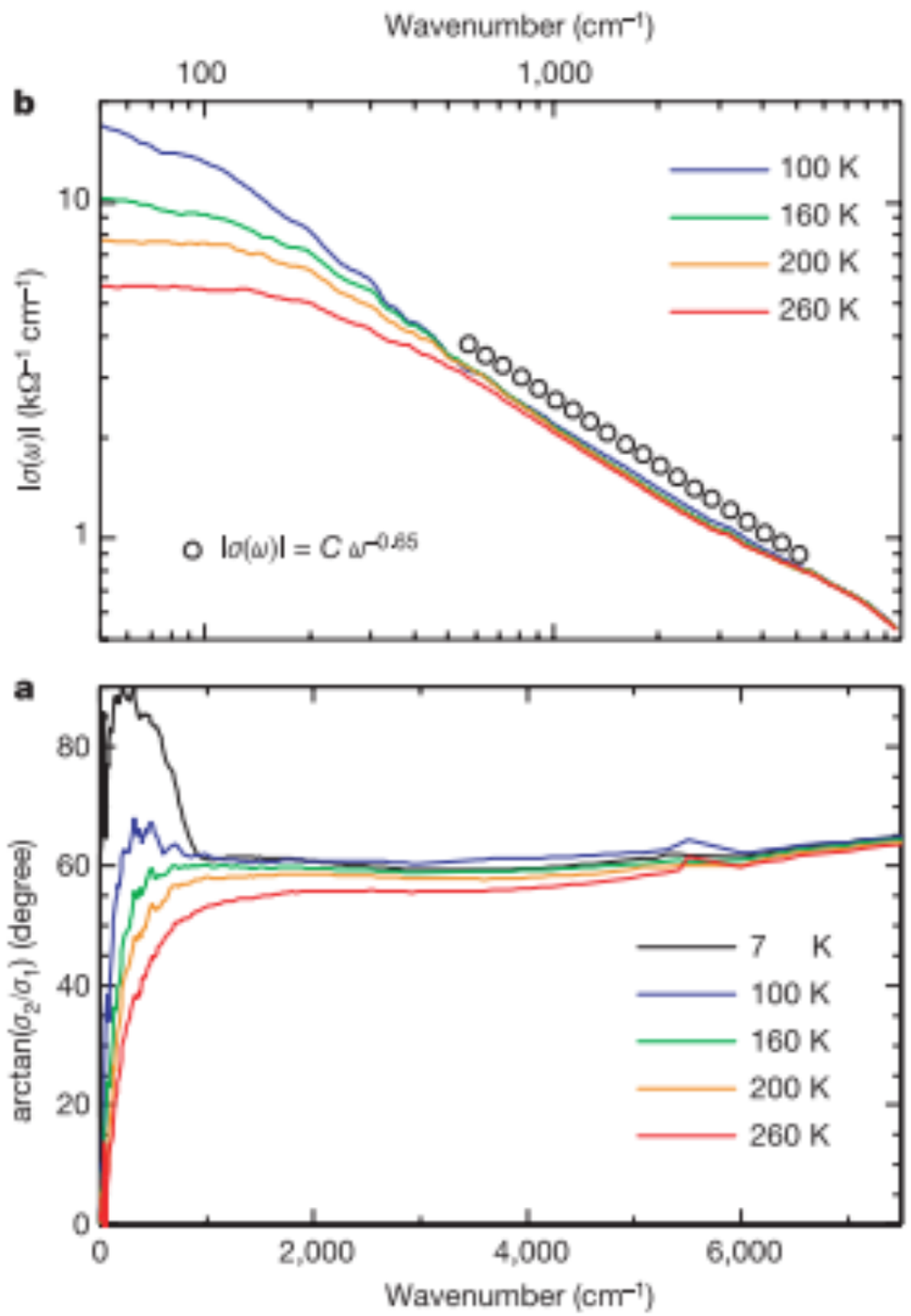

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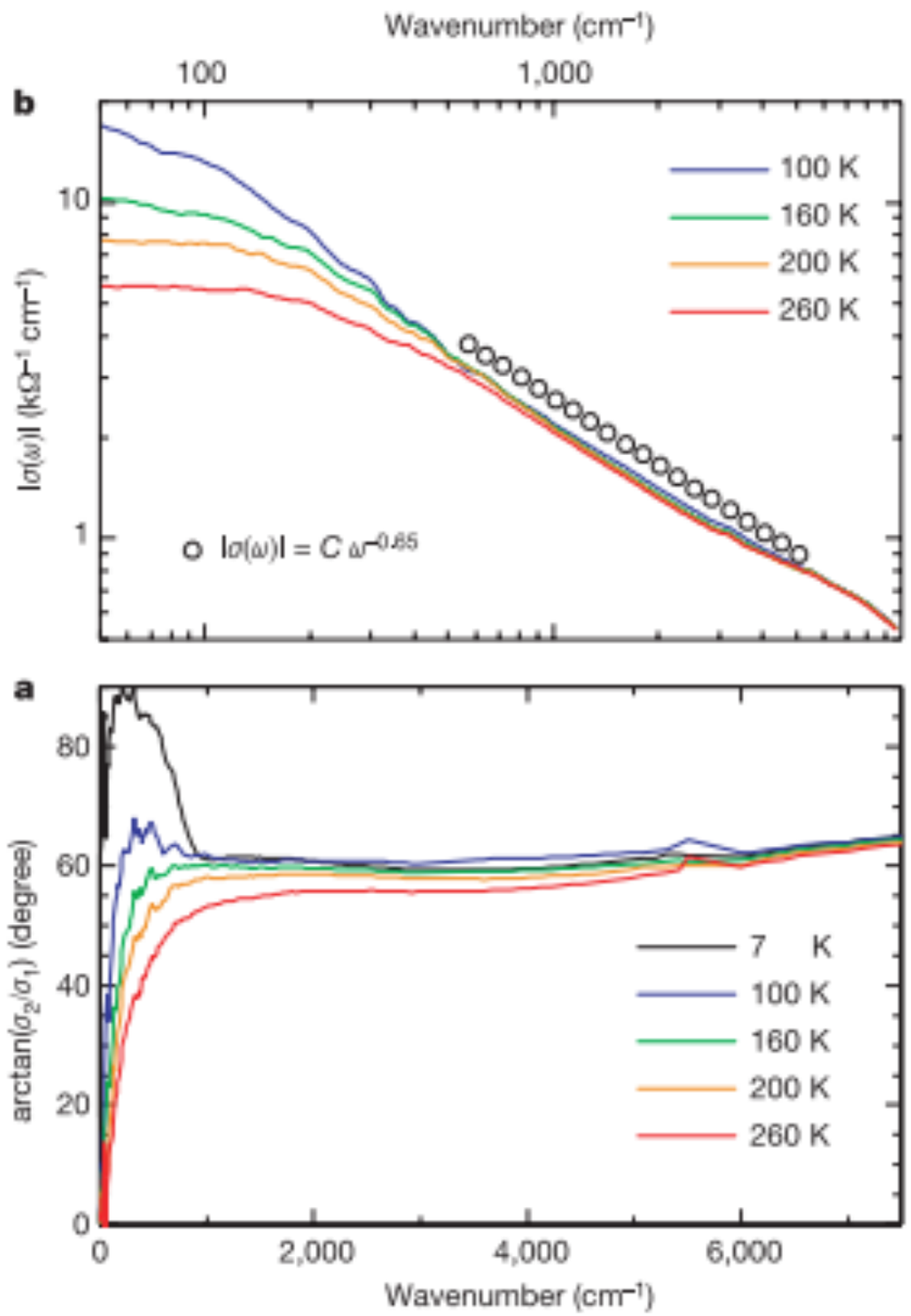

$$\sigma(\omega) = \frac{1}{3}(\sqrt{3}+3i)\pi\frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M\omega^{\frac{2}{3}}}$$

victory!!

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are anomalous dimensions necessary

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$$a + 2b = 2/3$$

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anomalous dimension

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$$c = 1 \qquad\qquad b = 0$$

$$a + 2b = 2/3 \quad \Rightarrow \quad a = 2/3$$

No

No

but the Lorenz ratio is not a constant

$$L_H = \frac{\kappa_{xy}}{T\sigma_{xy}} \sim T \equiv T^{-2\Phi/z}$$

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Hartnoll/Karch

$$\Phi = bz = -2/3$$

combine AC
+DC transport

fixes all
exponents
a,b,c

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current has anomalous dimension
(probe with noise measurements)

what really is the summation over mass?

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$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

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$$\mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[\frac{1}{2}\frac{z^2}{R^2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{\phi^2}{2R^2}\right]$$

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$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

anti de Sitter metric

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$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1}$$

$$\mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[\frac{1}{2}\frac{z^2}{R^2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{\phi^2}{2R^2}\right]$$

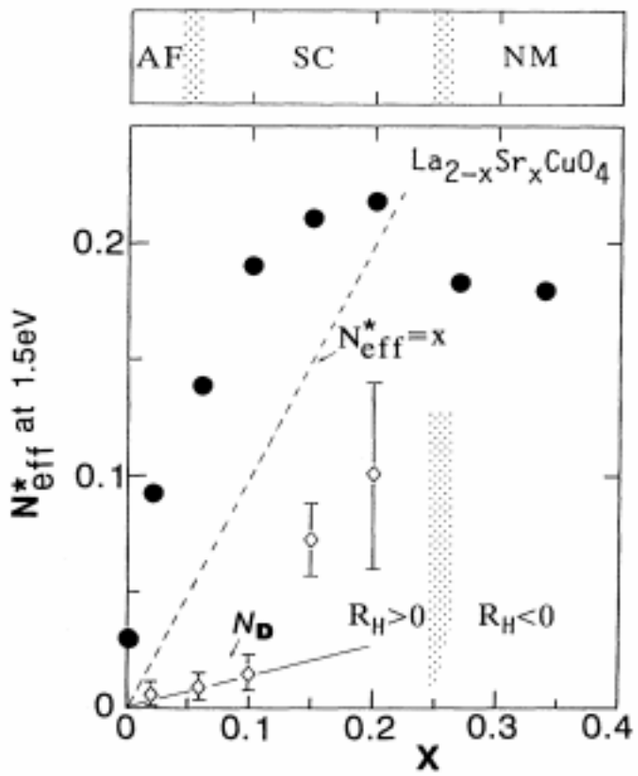
can be absorbed with AdS metric

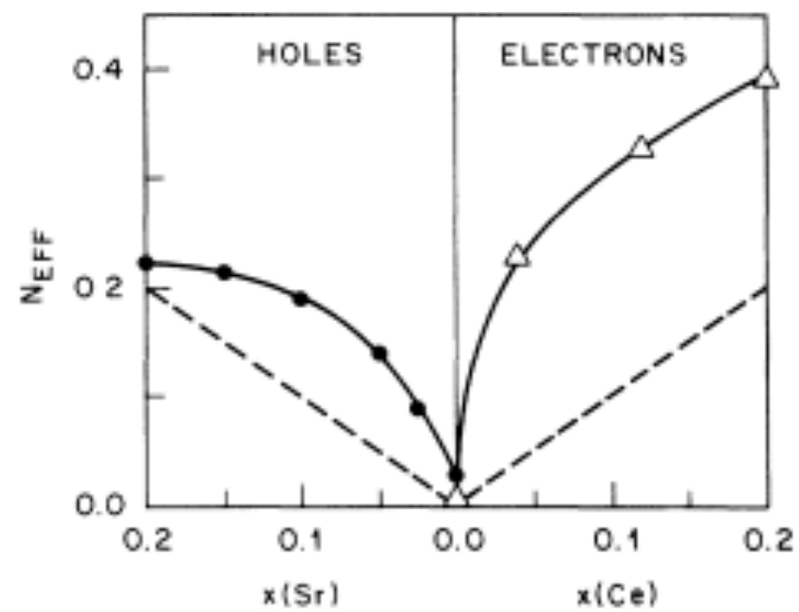
$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

anti de Sitter metric

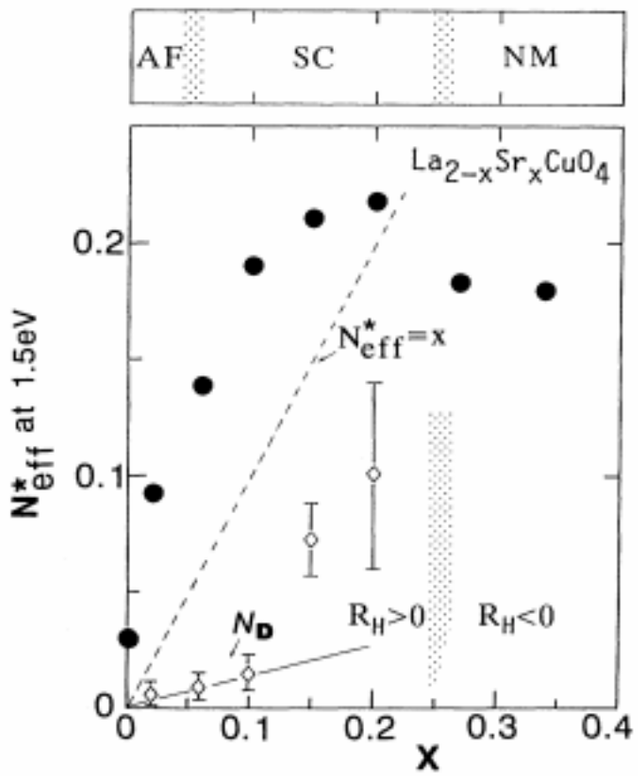
no gravity?

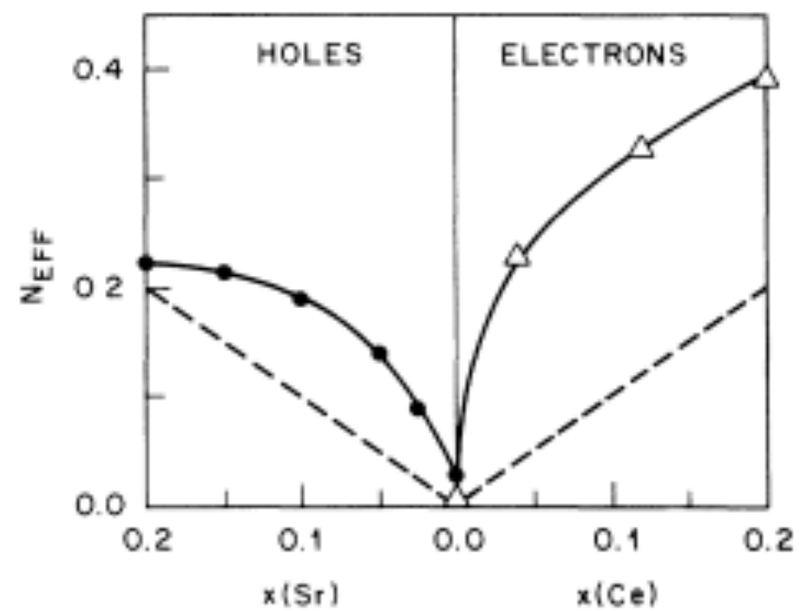
Cooper, et al.


AF
SC
NM
$La_{2-x}Sr_xCuO_4$
N^*_{eff} at 1.5eV
$N^*_{eff}=x$
N_D
$R_H>0$
$R_H<0$
0.2
0.1
0
0
0.1
0.2
0.3
0.4
x


HOLES
ELECTRONS
N_{EFF}
0.4
0.2
0.0
0.2
0.1
0.0
0.1
0.2
x(Sr)
x(Ce)

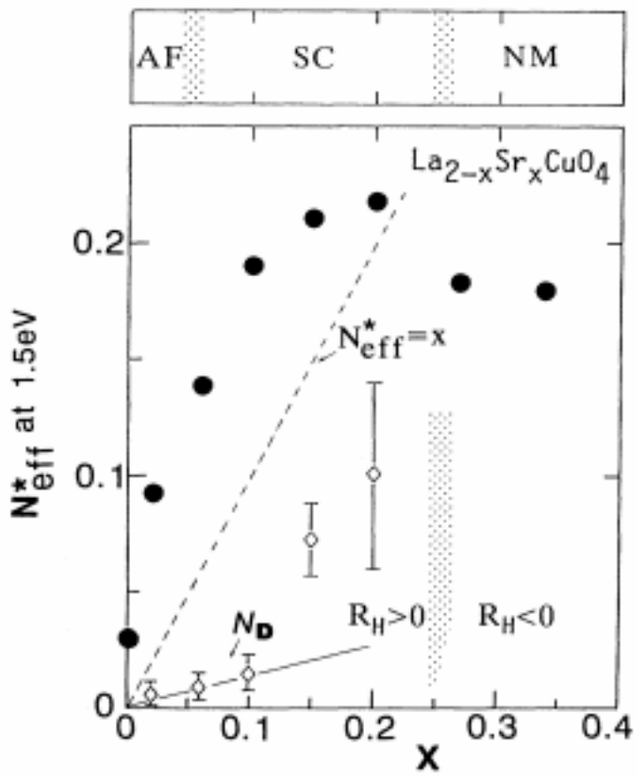
Cooper, et al.

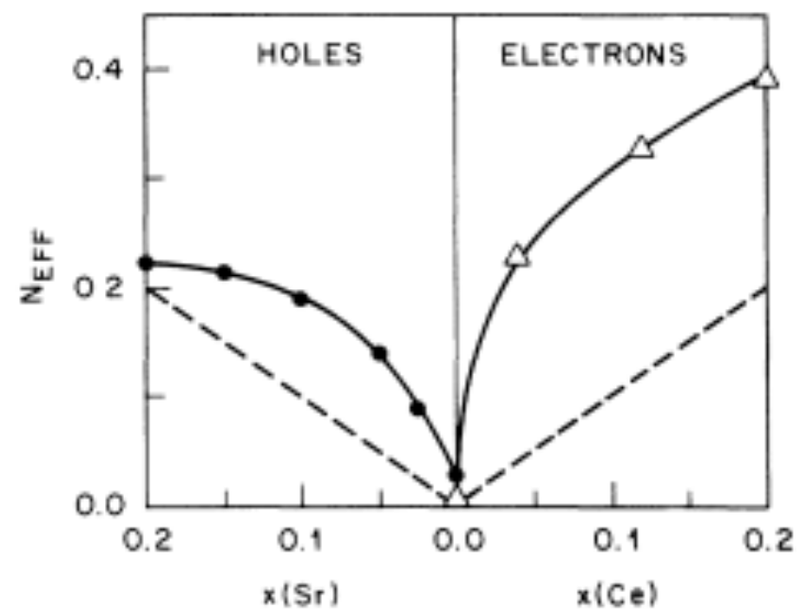




low-energy model for $N_{\text{eff}} > x$??

Cooper, et al.





low-energy model for $N_{\text{eff}} > x$??

does such a theory include gravity?

f-sum rule

$$\text{K.E.} = p^2/2m$$

$$N_{\text{eff}} = x$$

what if?

$$\text{K.E.} \propto (\partial_\mu^2)^\alpha$$

f-sum rule

$$\frac{W(n,T)}{\pi e^2} = Acn^{1+\frac{2(\alpha-1)}{d}} + B\frac{(\alpha-1)(d+2(\alpha-1))T^2}{c}n^{1-\frac{2(\alpha+1)}{d}}$$

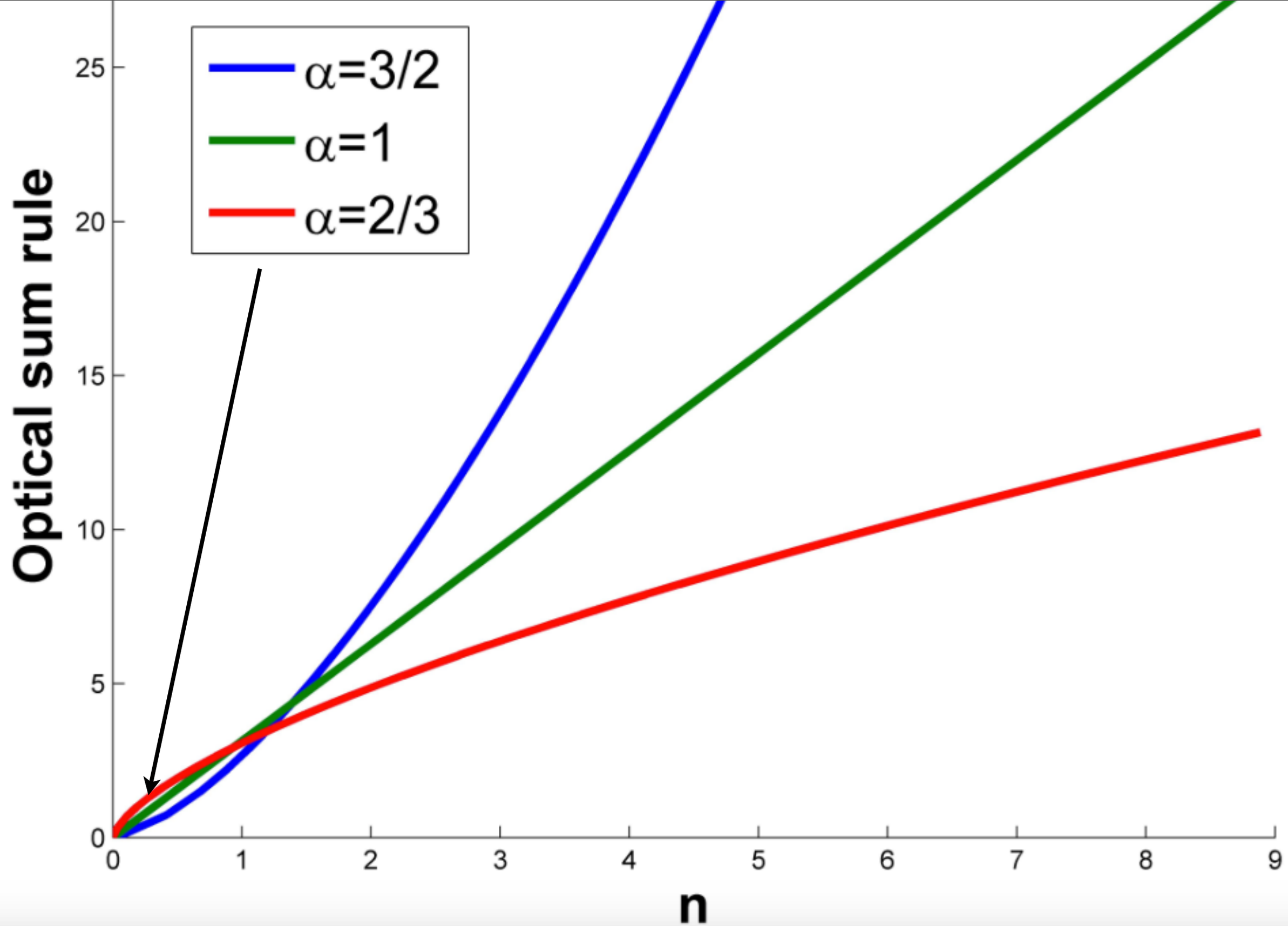
what if?

$$\text{K.E.} \propto (\partial_\mu^2)^\alpha$$

f-sum rule

$$\frac{W(n,T)}{\pi e^2} = Acn^{1+\frac{2(\alpha-1)}{d}} + B\frac{(\alpha-1)(d+2(\alpha-1))T^2}{c}n^{1-\frac{2(\alpha+1)}{d}}$$

W>n if $\alpha < 1$


α=3/2
α=1
α=2/3
Optical sum rule
n
0
5
10
15
20
25
0
1
2
3
4
5
6
7
8
9

what theories have $\alpha < 1$?

gravitational crystals

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

$$\frac{1}{2}\Box h_{\mu\nu} - 2m^2 h_{\mu\nu} + \cdots$$

$$m^2 \propto V(\Phi))$$

graviton has a mass!

Unparticles as the Holographic Dual of Gapped AdS Gravity

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New York, NY 10003, USA

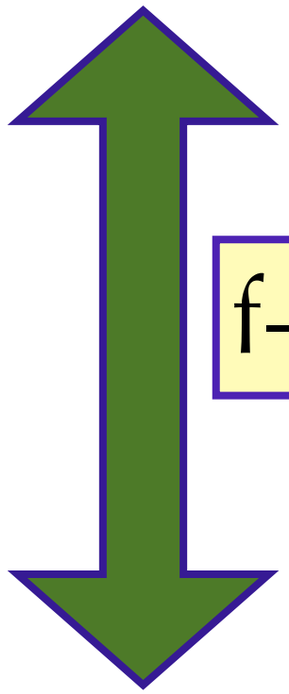
$$S_{eff} = (-1)^{-\frac{2\tilde{\Delta}+1}{\tilde{\Delta}+1}} \int d^d x \left[\Phi(-\partial^2)^{-\tilde{\Delta}} \Phi + (-1)^{-\frac{1}{(\tilde{\Delta}+1)(\tilde{\Delta}+2)}} A^{\perp}_{\mu}(-\partial^2)^{-\tilde{\Delta}-1} A^{\perp\mu} + h^{Tt}_{\mu\nu}(-\partial^2)^{-\tilde{\Delta}} h^{Tt\mu\nu} \right] . \tag{79}$$

$\tilde{\Delta}_{\Phi} < 0$

non-local action with non-canonical kinetic energy

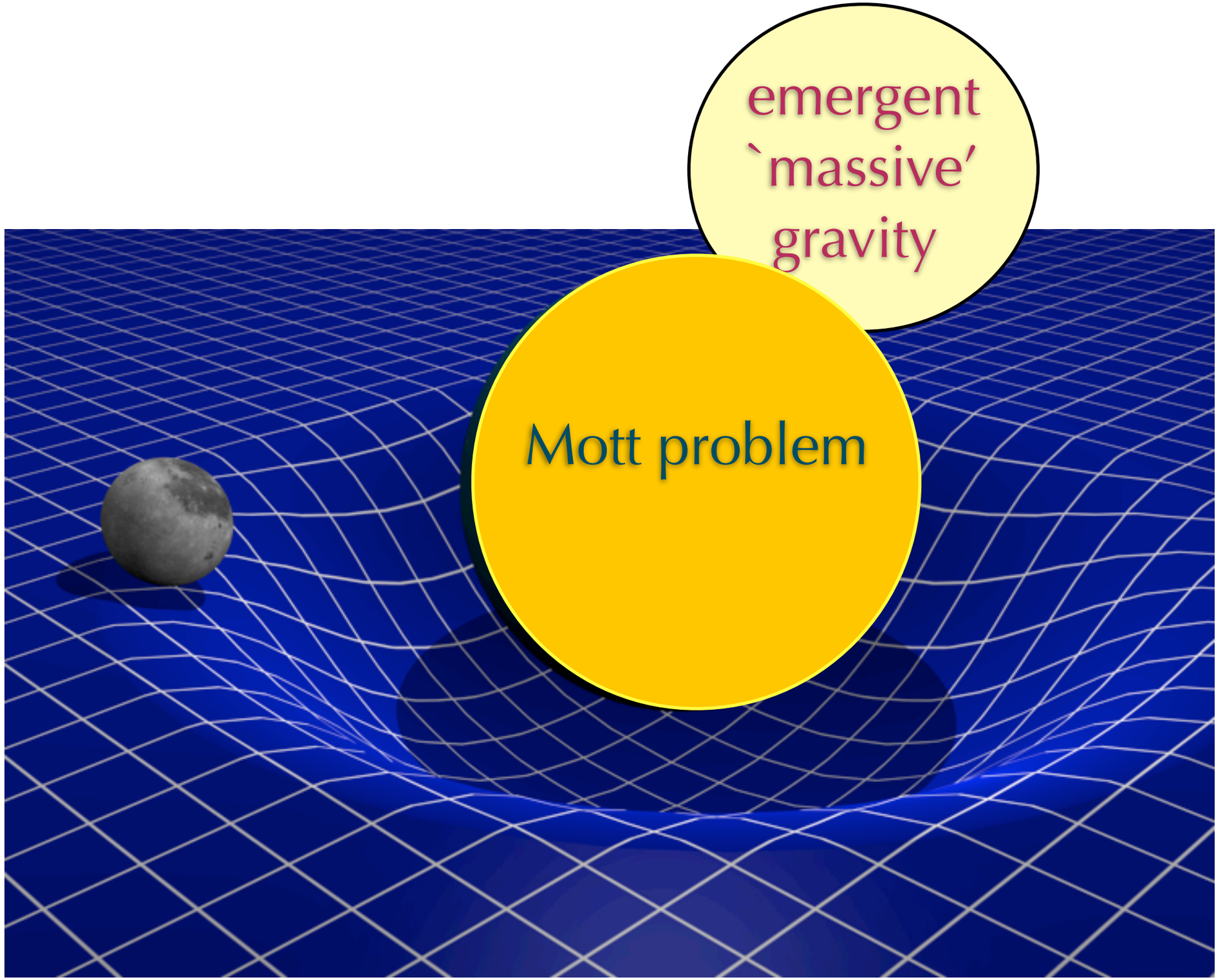
incoherent metals: Mottness

unparticles



f-sum rule violation

`massive gravity'


emergent
`massive'
gravity
Mott problem