Optical Conductivity in the Cuprates from Holography and Unparticles

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Brandon Langley  Garrett Vanacore  Kridsangaphong Limtragool
Quantum critical behaviour in a high-$T_c$ superconductor

D. van der Mare1, H. J. A. Molegraaf1, J. Zaanen2, Z. Hussinov2, F. Carbone2, A. Damascelli3, H. Elsaki4, M. Greven, P. H. Kes5 & M. Li5

1Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands
2Leiden Institute of Physics, Leiden University, 2300 RA Leiden, The Netherlands
3Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA

Drude conductivity

\[
\sigma(\omega) = C\omega^{-\frac{2}{3}}
\]

\[
\frac{n\tau e^2}{m} \frac{1}{1 - i\omega\tau}
\]
scale-invariant propagators

\[
\left( \frac{1}{p^2} \right)^\alpha
\]
scale-invariant propagators

\[ \left( \frac{1}{p^2} \right)^\alpha \]

Anderson: use Luttinger Liquid propagators

\[ G^R \propto \frac{1}{(\omega - v_s k)^\eta} \]
compute conductivity without vertex corrections (PWA)

is flawed. In fact, in the Luttinger liquid such direct calculations are not to be trusted very firmly, since it is the nature of the Luttinger liquid that vertex corrections, if they must be included, will be singular; conventional transport theory is not applicable, and special methods such as the above are necessary.

\[ \sigma(\omega) \propto \frac{1}{\omega} \int dx \int dt G^e(x, t)G^h(x, t)e^{i\omega t} \propto (i\omega)^{-1+2\eta} \]
problems
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1.) cuprates are not 1-dimensional
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2.) vertex corrections matter
problems

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2.) vertex corrections matter

\[
\sigma \propto G \left( G(\Gamma^\mu)^2, \Gamma^{\mu\nu} \right) \\
[G] = L^{d+1-2d_U} \\
[\Gamma^\mu] = L^{2d_U-d} \\
[\Gamma^{\mu\nu}] = L^{2d_U-d+1}
\]
problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter

\[ \begin{align*}
\sigma & \propto G \left( G(\Gamma^\mu)^2, \Gamma^{\mu\nu} \right) \\
[ G ] & = L^{d+1-2d_U} \\
[ \Gamma^\mu ] & = L^{2d_U-d} \\
[ \Gamma^{\mu\nu} ] & = L^{2d_U-d+1}
\end{align*} \]

\[ [\sigma] = L^{2-d} \]

independent of \( d_U \)
power law?
IR \quad ?? \quad UV 

QFT

coupling constant

\[ g = 1/\text{ego} \]
$\frac{dg(E)}{d\ln E} = \beta(g(E))$

locality in energy

coupling constant

$g = 1/\text{ego}$
replace coupled theory with geometry

scale-invariant anti de-Sitter

\[ R \propto \sqrt{g} \]

\[ g(\overline{\psi}\psi)^2 \]
replace coupled theory with geometry

scale-invariant anti de-Sitter

$AdS_2 \times R^2 \times R \rightarrow AdS_4$

$R \propto \sqrt{g}$
replace coupled theory with geometry

scale-invariant anti de-Sitter

$g(\bar{\psi}\psi)^2$

$R \propto \sqrt{g}$

$R G = G R$

$AdS_2 \times R^2$

$AdS_4$

$z = 1$

$z = 0$
replace coupled theory with geometry

scale-invariant anti de-Sitter

$\text{AdS}_2 \times R^2 \rightarrow R \rightarrow \text{AdS}_4

R \propto \sqrt{g}$

cannot describe systems at $g=0$!
optical conductivity from a gravitational lattice

log-log plots for various parameters

\[ |\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C \]

for \(0.2 \lesssim \omega \tau \lesssim 0.8\)

optical conductivity from a gravitational lattice

log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $$0.2 \lesssim \omega \tau \lesssim 0.8$$

a remarkable claim! replicates features of the strange metal? how?

new equation!

\[ \text{Einstein-Maxwell equations} + \text{non-uniform charge density} = B\omega^{-2/3} \]
not so fast!
Drude conductivity

\[ \frac{n \tau e^2}{m} \frac{1}{1 - i \omega \tau} \]
Donos and Gauntlett
(gravitational crystal)

Drude conductivity

\[
\frac{n \tau e^2}{m} \frac{1}{1 - i\omega \tau}
\]

\[
-\frac{2}{3} = 1 + \omega \frac{|\sigma|''}{|\sigma|'}
\]
Donos and Gauntlett
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Drude conductivity

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Donos and Gauntlett
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Drude conductivity

\[ \frac{n \tau e^2}{m} \frac{1}{1 - i \omega \tau} \]

\[ -\frac{2}{3} = 1 + \omega \left| \frac{\sigma''}{\sigma'} \right| \]

no power law!!

\[ B \omega^{-2/3} \]
who is correct?
who is correct?

let's redo the calculation
conductivity within AdS

\((g_{ab}, V(\Phi), A_t)\)

(metric, potential, gaugefield)
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(metric, potential, gaugefield)

\[ A_t = \mu(1 - z)dt \]
\[ \rho = \lim_{z \to 0} \sqrt{g} F^{tz} \]
conductivity within AdS

\( (g_{ab}, V(\Phi), A_t) \)

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conductivity within AdS

$$(g_{ab}, V(\Phi), A_t)$$

(metric, potential, gaugefield)

perturb with electric field

$A_t = \mu (1 - z) dt$

$$\rho = \lim_{z \to 0} \sqrt{g} F_{tz}$$
conductivity within AdS

\((g_{ab}, V(\Phi), A_t)\) (metric, potential, gaugefield)

perturb with electric field

\[ g_{ab} = \bar{g}_{ab} + h_{ab} \]
\[ A_a = \bar{A}_a + b_a \]
\[ \Phi_i = \bar{\Phi}_i + \eta_i \]
conductivity within AdS

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perturb with electric field

\[ A_t = \mu (1 - z) dt \]
\[ \rho = \lim_{z \to 0} \sqrt{g} F^{tz} \]

\[ \delta A_x = \frac{E}{i\omega} + J_x(x, \omega) z + O(z^2) \]
conductivity within AdS

\[(g_{ab}, V(\Phi), A_t) \quad \text{(metric, potential, gaugefield)}\]

perturb with electric field

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g_{ab} = \bar{g}_{ab} + h_{ab}
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\rho = \lim_{z \to 0} \sqrt{g} F_{tz}
\delta A_x = \frac{E}{i\omega} + J_x(x, \omega) z + O(z^2)
\sigma = J_x(x, \omega) / E
\]
conductivity within AdS

\((g_{ab}, V(\Phi), A_t)\)

(metric, potential, gaugefield)

perturb with electric field

\[
\begin{align*}
g_{ab} &= \bar{g}_{ab} + h_{ab} \\
A_a &= \bar{A}_a + b_a \\
\Phi_i &= \bar{\Phi}_i + \eta_i
\end{align*}
\]

solve equations of motion with gauge invariance (without mistakes)

\[
\delta A_x = \frac{E}{i\omega} + J_x(x, \omega)z + O(z^2)
\]

\[
\sigma = J_x(x, \omega)/E
\]
HST vs. DG
Horowitz, Santos, Tong (HST)

\[ V(\Phi) = -\frac{\Phi^2}{L^2} \]

\[ \Phi = z\Phi^{(1)} + z^2\Phi^{(2)} + \cdots, \]

\[ \Phi^{(1)}(x) = A_0 \cos(kx) \]

inhomogeneous in x

\[ m^2 = -\frac{2}{L^2} \]
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de Donder gauge
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HST vs. DG

DG

\[ V(|\Phi|^2) \]

\[ \Phi(z, x) = \phi(z)e^{ikx} \]

no inhomogeneity in \( x \)

\[ m^2 = -\frac{3}{(2L^2)} \]
Horowitz, Santos, Tong (HST)

\[ V(\Phi) = -\frac{\Phi^2}{L^2} \]

\[ \Phi = z\Phi^{(1)} + z^2\Phi^{(2)} + \cdots, \]
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radial gauge
Our Model

\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]
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\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]

\[ \Phi_1 = z\Phi_1^{(1)} + z^2\Phi_1^{(2)} + \cdots, \quad \Phi_1^{(1)}(x) = A_0 \cos \left( kx - \frac{\theta}{2} \right), \]

\[ \Phi_2 = z\Phi_2^{(1)} + z^2\Phi_2^{(2)} + \cdots, \quad \Phi_2^{(1)}(x) = A_0 \cos \left( kx + \frac{\theta}{2} \right). \]
Our Model

\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]

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\[ \theta = 0 \]
Our Model

\[ \mathcal{L}_\Phi = (\nabla \Phi_1)^2 + (\nabla \Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2) \]

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\[ \theta = 0 \quad \text{HST} \quad \theta = \frac{\pi}{2} \quad \text{DG} \]
Einstein-De Turck EOM

\[ G_{ab}^H = G_{ab} - \nabla_{(a} \xi_{b)}, \]

\[ \xi^a = g^{cd} (\Gamma^a_{cd}(g) - \Gamma^a_{cd}(\bar{g})). \]
\[ G^{H}_{ab} = G_{ab} - \nabla_{(a}\xi_{b)}, \]

\[ \xi^{a} = g^{cd} \left( \Gamma^{a}_{cd}(g) - \Gamma^{a}_{cd}(\bar{g}) \right). \]
\[
G^H_{ab} = G_{ab} - \nabla_{(a} \xi_{b)}, \\
\xi^a = g^{cd} (\Gamma^a_{cd}(g) - \Gamma^a_{cd}(\bar{g})).
\]

**Einstein-De Turck EOM**

\[
ds^2 = \frac{L^2}{z^2} \left[ -(1 - z)P(z)Q_{tt}dt^2 + \frac{Q_{zz}dz^2}{(1 - z)P(z)} + Q_{xx}(dx + z^2 Q_{xx}dz)^2 + Q_{yy}dy^2 \right], \\
P(z) = 1 + z + z^2 - \frac{\mu_1^2}{2} z^3.
\]
Einstein-De Turck EOM

\[ G^H_{ab} = G_{ab} - \nabla_{(a} \xi_{b)}, \]
\[ \xi^a = g^{cd} (\Gamma^a_{cd}(g) - \Gamma^a_{cd}(\overline{g})). \]

metric ansatz

reference metric

\[ ds^2 = \frac{L^2}{z^2} \left[ -(1 - z)P(z)Q_{tt} dt^2 + \frac{Q_{zz}dz^2}{(1 - z)P(z)} + Q_{xx}(dx + z^2Q_{zx}dz)^2 + Q_{yy}dy^2 \right], \]
\[ P(z) = 1 + z + z^2 - \frac{\mu_1^2}{2} z^3. \]

RN-AdS when

\[ Q_{tt} = Q_{zz} = Q_{yy} = 1 \quad \Phi = 0 \quad a_t = \mu_1 = \mu \]
$A_0 = 0.75, \ k = 1, \ \mu = 1.4, \ T/\mu = 0.115$

$\theta = 0$

$\theta = \frac{\pi}{4}$

$\theta = \frac{\pi}{2}$
translational invariance is broken in metric in multiples of $2k$
charge density

\[ \rho = \lim_{z \to 0} \sqrt{-g} F^{tz} \]
- high-frequency behavior is identical
- low-frequency RN has \( \text{Re}(\sigma) \sim \delta(\omega) \), \( \text{Im}(\sigma) \sim 1/\omega \)
- low-frequency lattice has Drude form
is there a power law?
is there a power law?

Results

HST

DG

$-\frac{2}{3}$
Results
\[ A_1 = 0.75, \ k_1 = 2, \ k_2 = 2, \ \theta = 0, \ \mu = 1.4, \ T/\mu = 0.115 \]
No!
origin of power law?
origin of power law?

phenomenology
origin of power law?

phenomenology

no well-defined mass

\[ \mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2 \]
origin of power law?

phenomenology

no well-defined mass

\[ \mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2 \]

incoherent stuff (all energies)
\[ \mathcal{L} = \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \rho(m^2) dm^2 \]
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial^{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^{2} \phi^{2}(x, m) \right) \rho(m^{2}) dm^{2} \]

theory with all possible mass!
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \rho(m^2) dm^2 \]

theory with all possible mass!

\[
\begin{align*}
\phi &\rightarrow \phi(x, m^2 / \Lambda^2) \\
x &\rightarrow x / \Lambda \\
m^2 / \Lambda^2 &\rightarrow m^2
\end{align*}
\]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \rho(m^2) dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]
\[ x \rightarrow x / \Lambda \]
\[ m^2 / \Lambda^2 \rightarrow m^2 \]

\[ \mathcal{L} \rightarrow \Lambda^{4+d_\rho} \mathcal{L} \]

scale invariance is restored!!
\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \rho(m^2) \, dm^2 \]

theory with all possible mass!

\[
\begin{align*}
\phi & \to \phi(x, m^2/\Lambda^2) \\
x & \to x/\Lambda \\
m^2/\Lambda^2 & \to m^2
\end{align*}
\]

\[ \mathcal{L} \to \Lambda^{4+d_\rho} \mathcal{L} \]

scale invariance is restored!!

not particles
unparticles

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \rho(m^2) dm^2 \]

theory with all possible mass!

\[ \phi \to \phi(x, m^2 / \Lambda^2) \]
\[ x \to x / \Lambda \]
\[ m^2 / \Lambda^2 \to m^2 \]

\[ \mathcal{L} \to \Lambda^{4+d_\rho} \mathcal{L} \]

scale invariance is restored!!

not particles
\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|}
\]
\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|}
\]

\[d_U - 2\]
(\int_0^\infty dm^2 m^2 \gamma \frac{i}{p^2 - m^2 + i\epsilon})^{-1} \propto p^{2|\gamma|}

\phi(x, m^2)

\text{continuous mass}

\text{propagator}

\text{flavors}

\text{Monday, August 17, 15}
\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|} d_U - 2
\]

**Continuous mass**

\[\phi(x, m^2) \leftrightarrow e^2(m)\]

**Propagator**

**Flavors**

**Multi-bands**

Karch, 2015
take experiments seriously
take experiments seriously

\[ \sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i} \]
take experiments seriously

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continuous mass
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\[ \sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega \tau_i} \]

continuous mass

\[ \sigma(\omega) = \int_{0}^{M} \frac{\rho(m) e^2(m) \tau(m)}{m} \frac{1}{1 - i\omega \tau(m)} dm \]
variable masses for everything

\[ \rho(m) = \rho_0 \frac{m^{a-1}}{M^a} \]

\[ e(m) = e_0 \frac{m^b}{M^b} \]

\[ \tau(m) = \tau_0 \frac{m^c}{M^c} \]
variable masses for everything

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e(m) = e_0 \frac{m^b}{M^b}
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\tau(m) = \tau_0 \frac{m^c}{M^c}
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\[
\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0}{M^{a+2b+c}} \int_0^M dm \frac{m^{a+2b+c-2}}{1 - i\omega \tau_0 \frac{m^c}{M^c}}
\]
variable masses for everything

\[ \rho(m) = \rho_0 \frac{m^{a-1}}{M^a} \]
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variable masses for everything

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perform integral
\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]
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\[
\sigma(\omega) = \frac{\rho_0 e^{2\tau_0^{1/3}}}{M} \frac{1}{\omega^{2/3}} \int_0^{\omega \tau_0} dx \frac{x^{-1/3}}{1 - ix}
\]
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\sigma(\omega) = \frac{\rho_0 e^{2\tau_0 \frac{1}{3}}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_{0}^{\omega \tau_0} dx \frac{x^{-\frac{1}{3}}}{1 - ix}
\]

\[
\omega \tau_0 \to \infty
\]
\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
\]

\[
\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0 \omega^{1/3}}{M} \frac{1}{\omega^{2/3}} \int_0^{\omega \tau_0} dx \frac{x^{-1/3}}{1 - ix}
\]

\[
\omega \tau_0 \rightarrow \infty
\]

\[
\sigma(\omega) = \frac{1}{3} (\sqrt{3} + 3i) \pi \frac{\rho_0 e_0^2 \tau_0^{1/3}}{M \omega^{2/3}}
\]
\[ \frac{a + 2b - 1}{c} = -\frac{1}{3} \]

\[ \sigma(\omega) = \frac{\rho_0 e^{2/2} \tau_0^{1/3}}{M} \frac{1}{\omega^{2/3}} \int_0^{\omega \tau_0} dx \frac{x^{-1/3}}{1 - ix} \]

\[ \omega \tau_0 \rightarrow \infty \]

\[ \sigma(\omega) = \frac{1}{3} (\sqrt{3} + 3i) \pi \frac{\rho_0 e^{2/2} \tau_0^{1/3}}{M \omega^{2/3}} \]

\[ \tan \sigma = \sqrt{3} \]

\[ 60^\circ \]
\[ \sigma(\omega) = C \omega^{\gamma-2} e^{i\pi(1-\gamma/2)} \]

\[ \gamma = 1.35 \]
$\sigma(\omega) = \frac{1}{3} (\sqrt{3} + 3i) \pi \rho_0 e_0^2 \tau_0^{\frac{1}{3}} M \omega^{\frac{2}{3}}$

\[ \sigma(\omega) = C \omega^{\gamma-2} e^{i\pi(1-\gamma/2)} \]

$\gamma = 1.35$
experiments

\[ \sigma(\omega) = \frac{1}{3}(\sqrt{3} + 3i)\pi \frac{\rho_0 e_{\gamma}^2 \tau_0^{\frac{1}{3}}}{M \omega^{\frac{2}{3}}} \]

\[ \tan \sigma_2 / \sigma_1 = \sqrt{3} \]
\[ \theta = 60^\circ \]

\[ \sigma(\omega) = C \omega^{\gamma - 2} e^{i\pi(1 - \gamma/2)} \]
\[ \gamma = 1.35 \]
experiments

\[ \sigma(\omega) = \frac{1}{3}(\sqrt{3} + 3i)\pi \rho_0 e_0^2 \tau_0^{\frac{1}{3}} M \omega^{\frac{2}{3}} \]

victory!!

\[ \tan \frac{\sigma_2}{\sigma_1} = \sqrt{3} \]
\[ \theta = 60^\circ \]

\[ \sigma(\omega) = C \omega^{\gamma-2} e^{i\pi(1-\gamma/2)} \]
\[ \gamma = 1.35 \]
are anomalous dimensions necessary

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\frac{a + 2b - 1}{c} = -\frac{1}{3}
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\[ \rho(m) = \rho_0 \frac{m^{a-1}}{M^a} \]

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\[ \tau(m) = \tau_0 \frac{m^c}{M^c} \]
are anomalous dimensions necessary

\[
\frac{a + 2b - 1}{c} = -\frac{1}{3}
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\[c = 1\]

\[a + 2b = 2/3\]
are anomalous dimensions necessary

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\frac{a + 2b - 1}{c} = -\frac{1}{3}
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\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}
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e(m) = e_0 \frac{m^b}{M^b}
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\[
\tau(m) = \tau_0 \frac{m^c}{M^c}
\]

\[c = 1\]
\[a + 2b = \frac{2}{3}\]
\[b = 0\]
\[a = \frac{2}{3}\]

hyperscaling violation

anomalous dimension
not necessarily
not necessarily

but they are a possibility
what really is the summation over mass?
what really is the summation over mass?

$$\mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2$$
what really is the summation over mass?

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\[ m = z^{-1} \]
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\[ \mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right] \]
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\]

\[
ds^2 = \frac{L^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right)
\]

antide Sitter metric
what really is the summation over mass?

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can be absorbed with AdS metric

\[ ds^2 = \frac{L^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right) \]

anti de Sitter metric
unparticles

\[ \mathcal{L}_{\text{eff}} = \int_{0}^{\infty} \mathcal{L}(x, m^2) \, dm^2 \]
unparticles

\[ \mathcal{L}_{\text{eff}} = \int_{0}^{\infty} \mathcal{L}(x, m^2) \, dm^2 \]

power-law optical conductivity
unparticles (flavors, ‘incoherent’ stuff)

action on AdS
emergent gravity

Mott problem