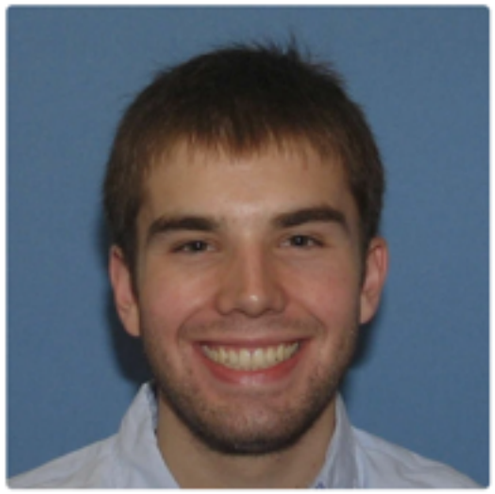
Optical Conductivity in the Cuprates from Holography and Unparticles

Thanks to: NSF, EFRC (DOE)

Brandon Langley



Garrett Vanacore

Kridsangaphong Limtragool

Quantum critical behaviour in a high-T_c superconductor

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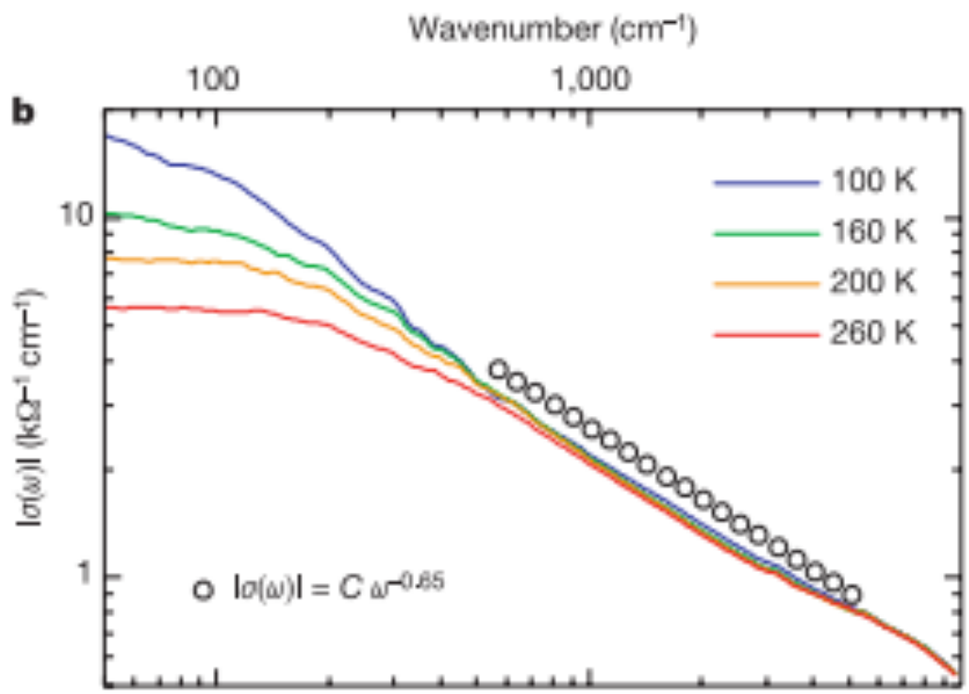
[3]Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA

Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$



$$\sigma(\omega) = C\omega^{-\frac{2}{3}}$$

scale-invariant propagators

$$\left(\frac{1}{p^2}\right)^{\alpha}$$

scale-invariant propagators

$$\left(\frac{1}{p^2}\right)^{\alpha}$$

Anderson: use Luttinger Liquid propagators

$$G^R \propto \frac{1}{(\omega - v_s k)^{\eta}}$$

compute conductivity without vertex corrections (PWA)

is flawed. In fact, in the Luttinger liquid such direct calculations are not to be trusted very firmly, since it is the nature of the Luttinger liquid that vertex corrections, if they must be included, will be singular; conventional transport theory is not applicable, and special methods such as the above are necessary.

$$\sigma(\omega) \propto \frac{1}{\omega} \int dx \int dt G^e(x,t) G^h(x,t) e^{i\omega t} \propto (i\omega)^{-1+2\eta}$$

problems

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1.) cuprates are not 1-dimensional

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2.) vertex corrections matter

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$$\left.\begin{aligned} \sigma &\propto G\left(G(\Gamma^{\mu})^2, \Gamma^{\mu\nu}\right) \\ [G] &= L^{d+1-2d_U} \\ [\Gamma^{\mu}] &= L^{2d_U-d} \\ [\Gamma^{\mu\nu}] &= L^{2d_U-d+1} \end{aligned}\right\}$$

problems

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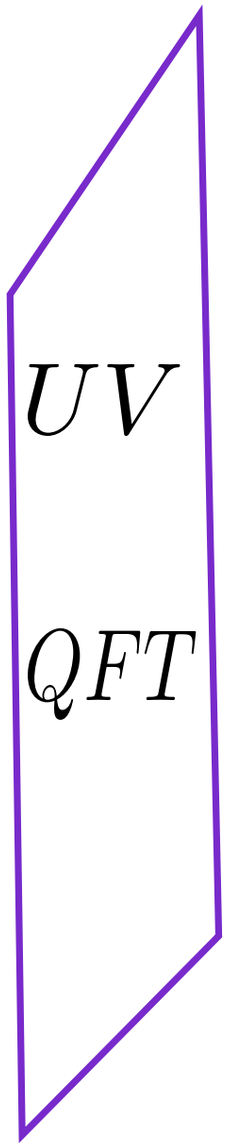
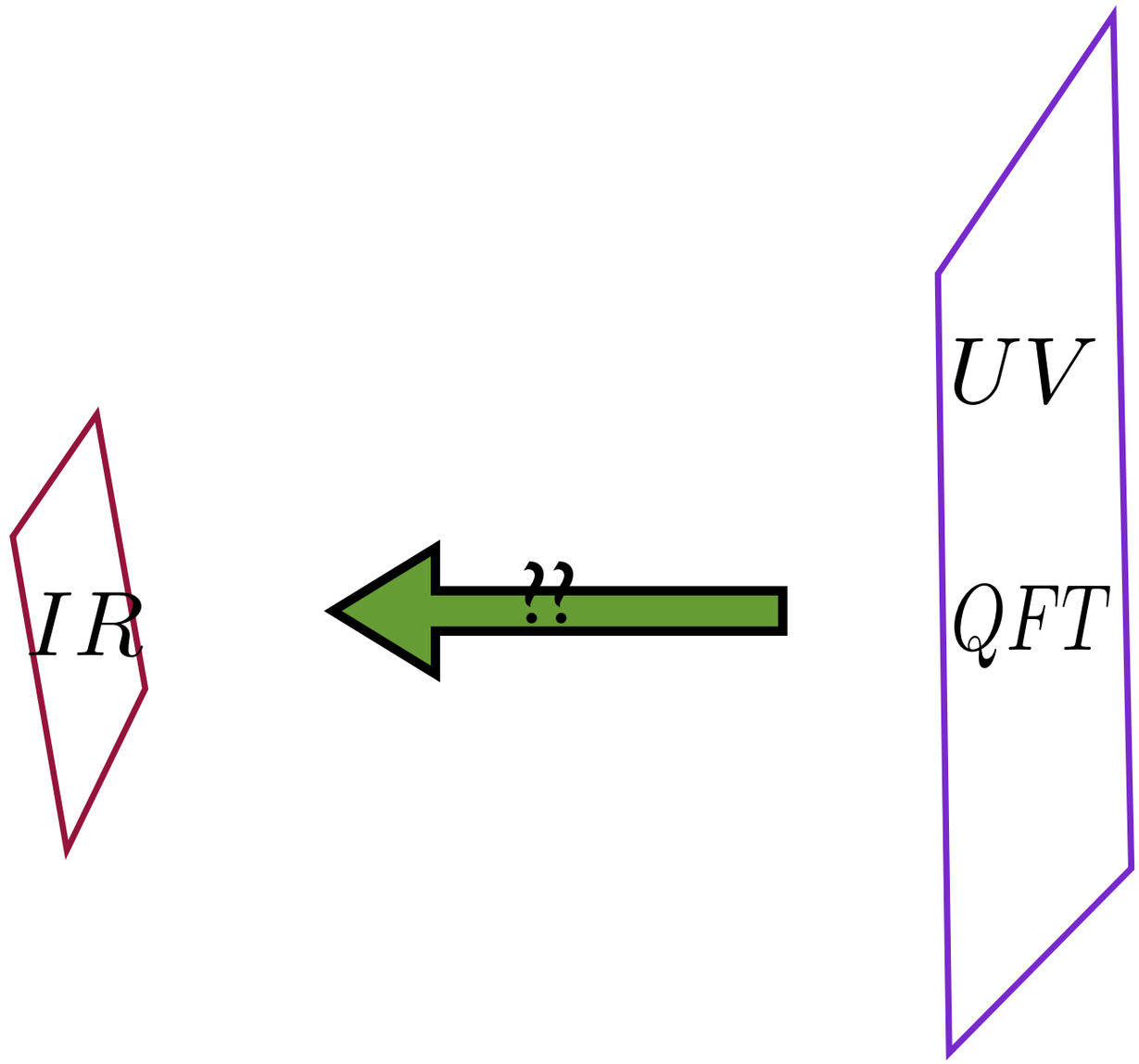
independent of d_U

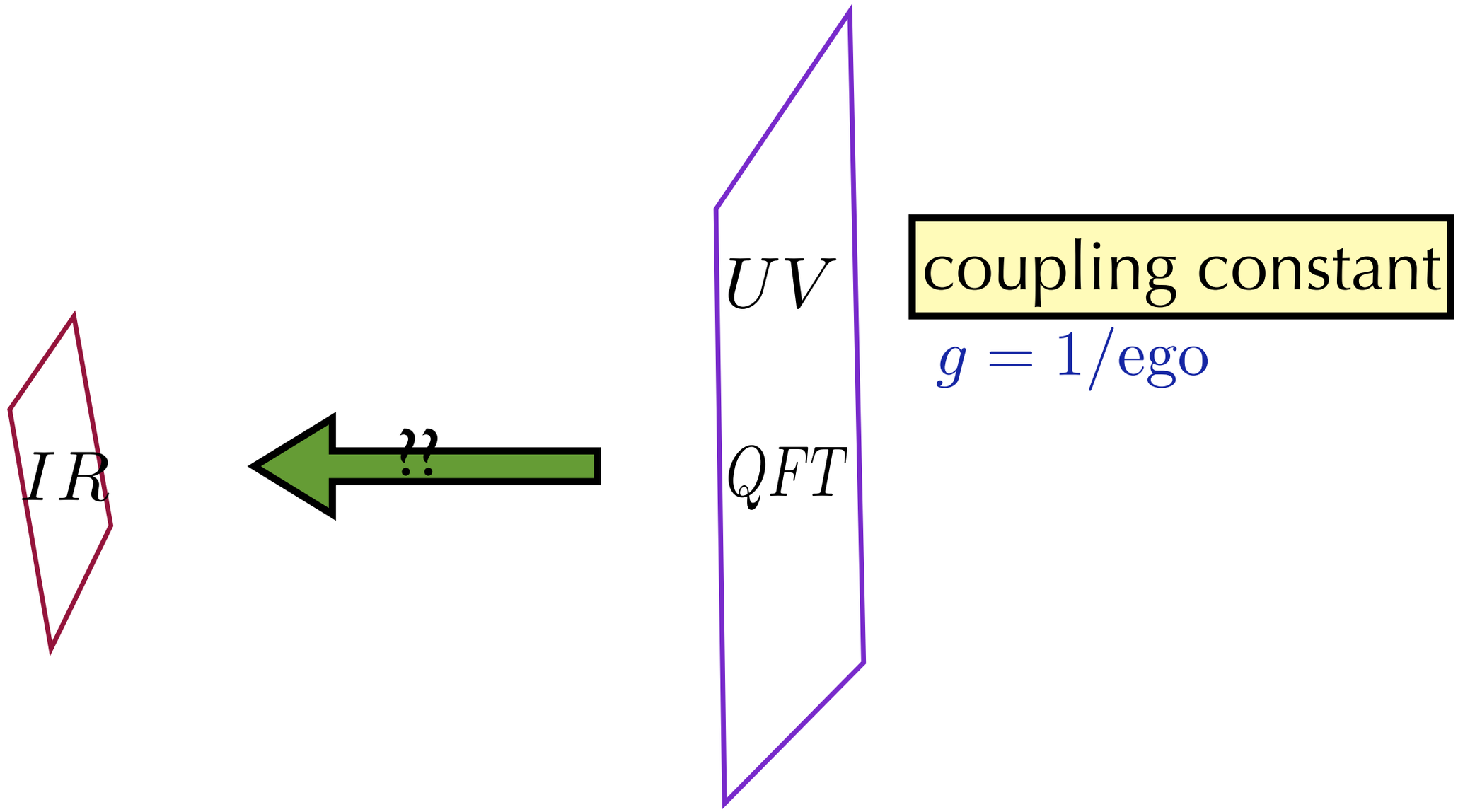
power law?

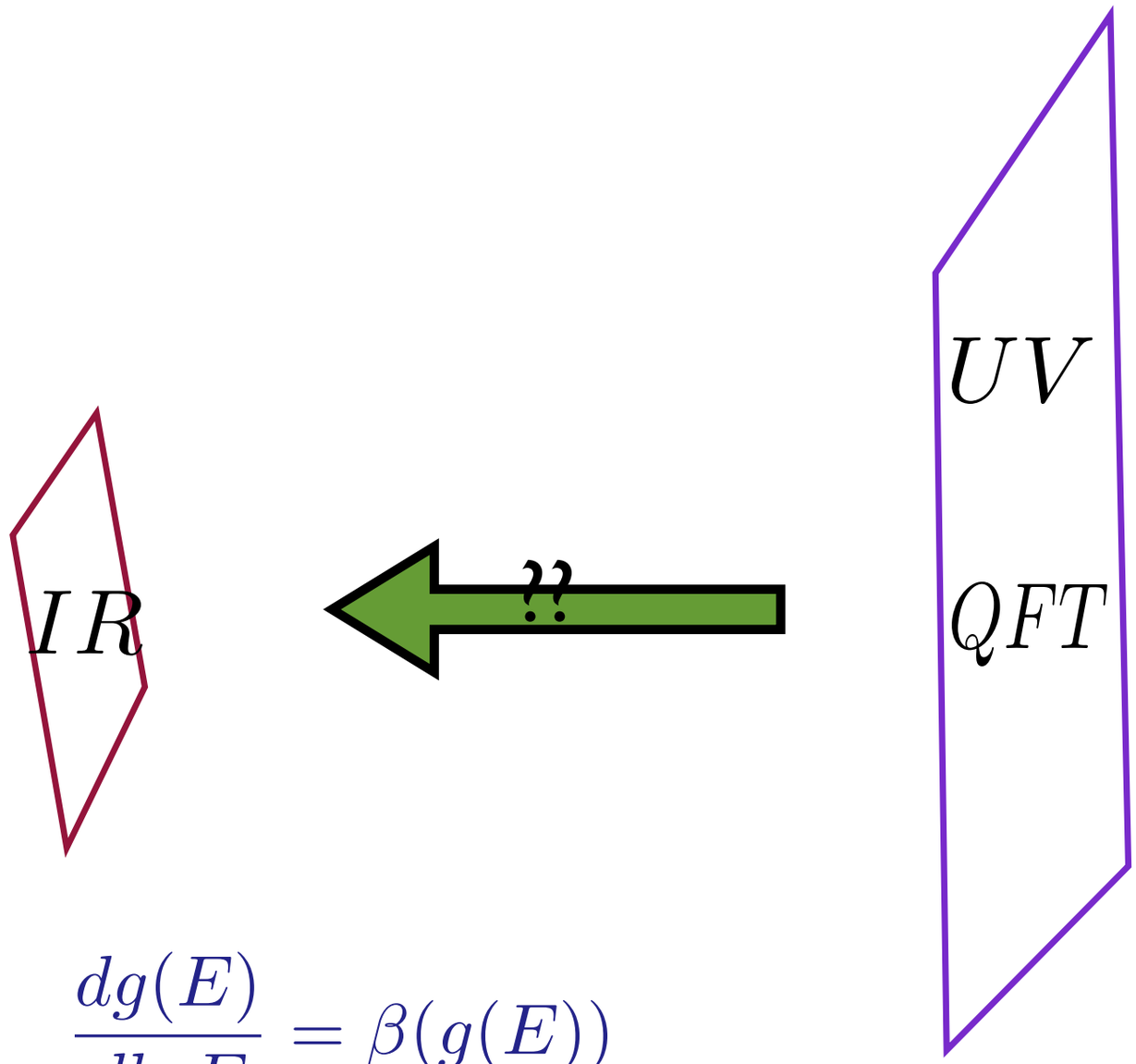
power law?

Could string theory be the answer?


UV
QFT


UV
QFT
??
IR


UV
coupling constant
$g = 1/\text{ego}$
IR
??
QFT



coupling constant

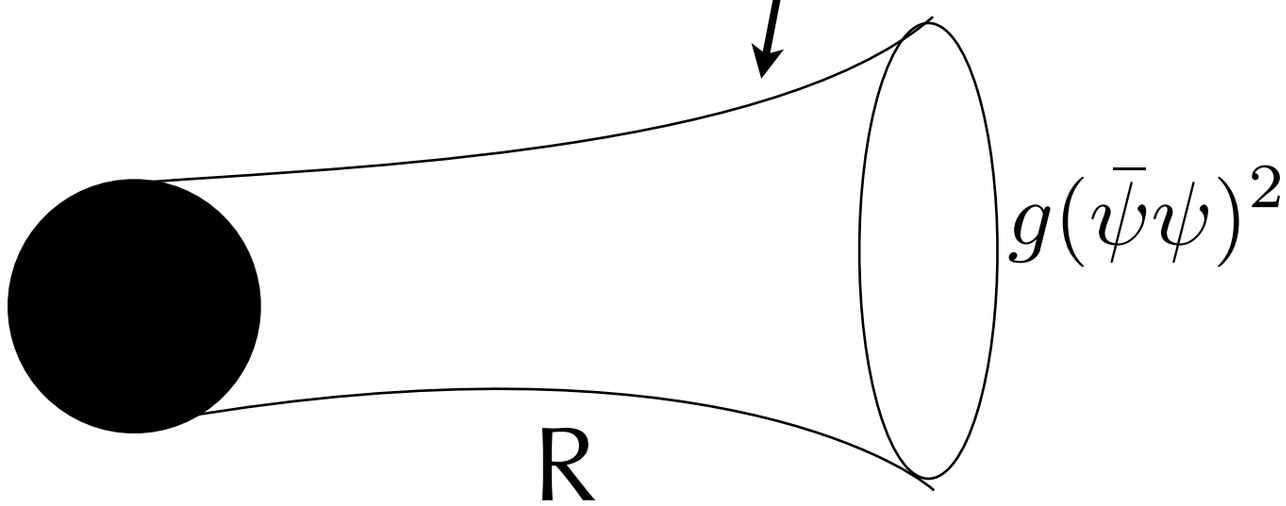
$g = 1/\text{ego}$

$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

replace coupled theory with geometry

scale-invariant anti de-Sitter



$$R \propto \sqrt{g}$$

replace coupled theory with geometry

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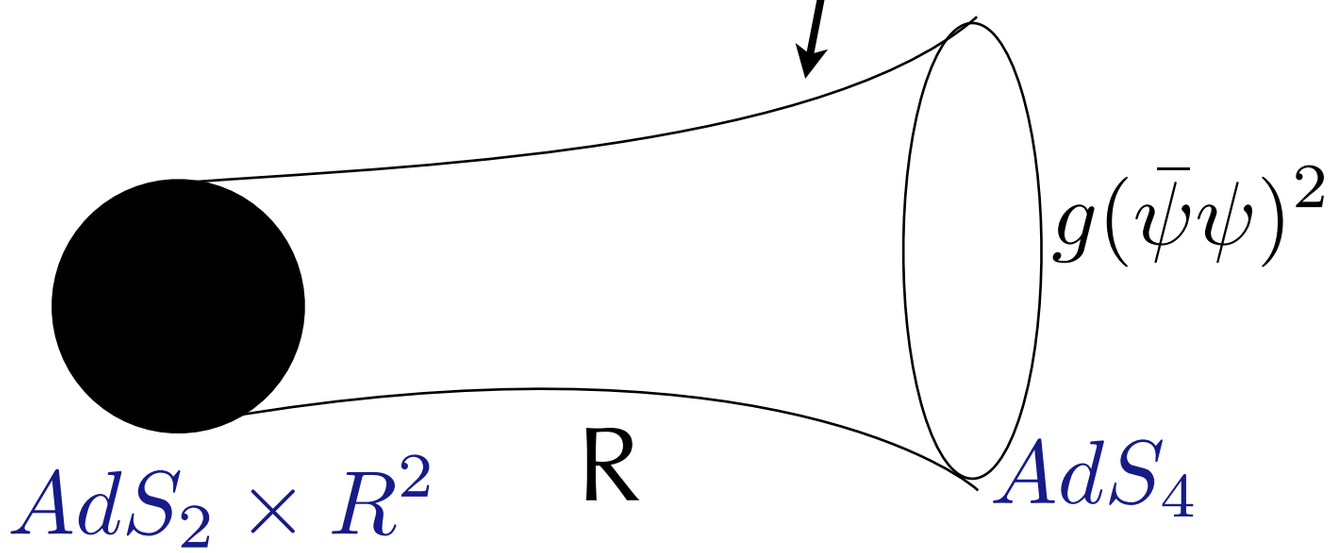

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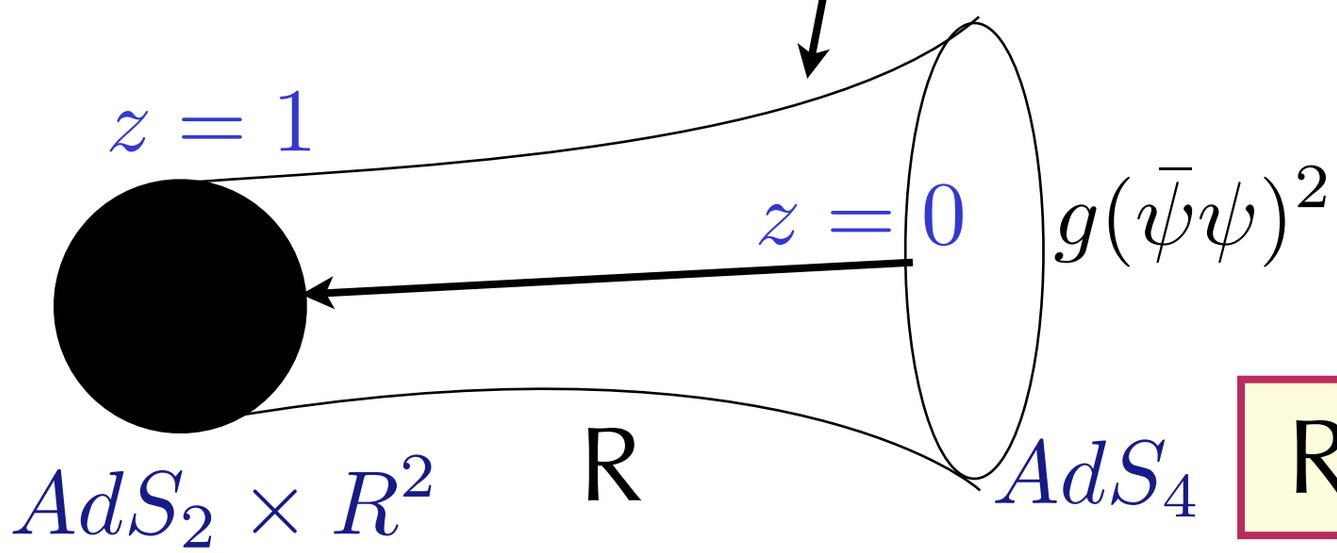

RG=GR

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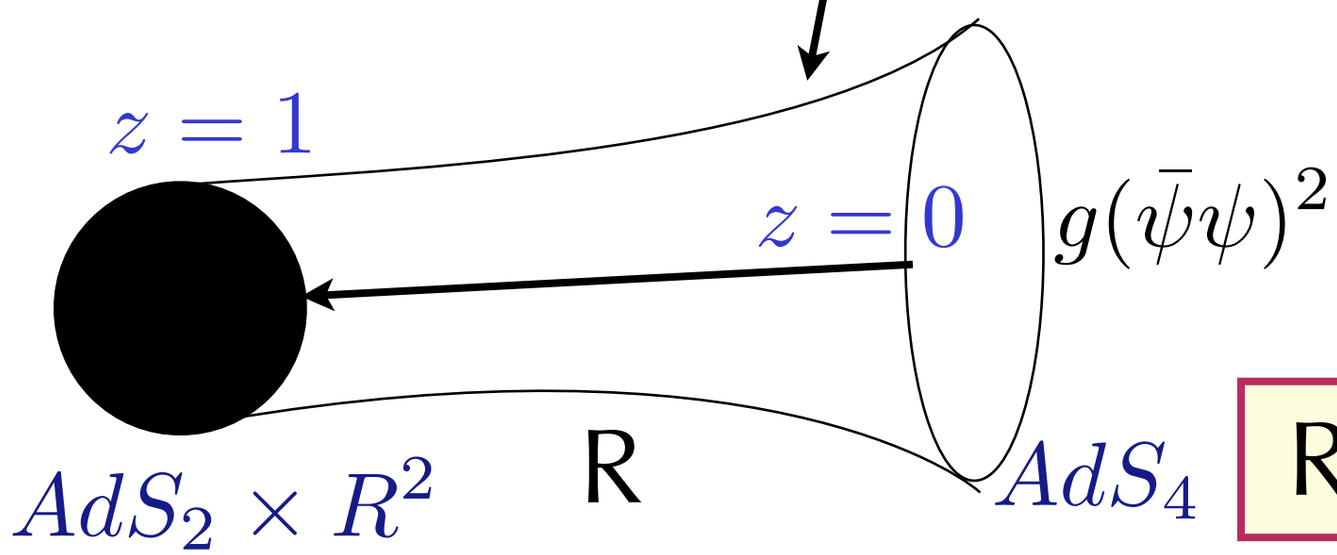
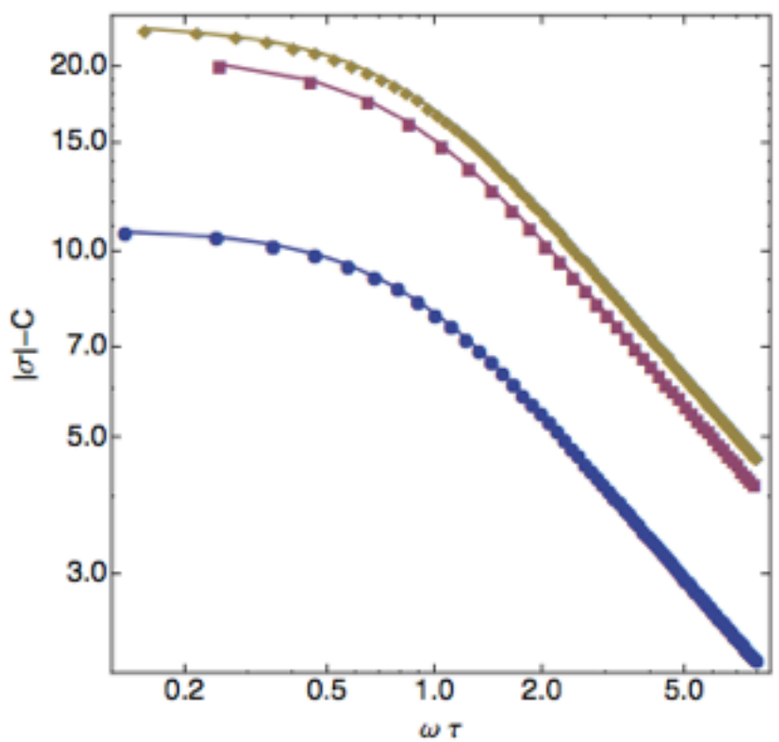


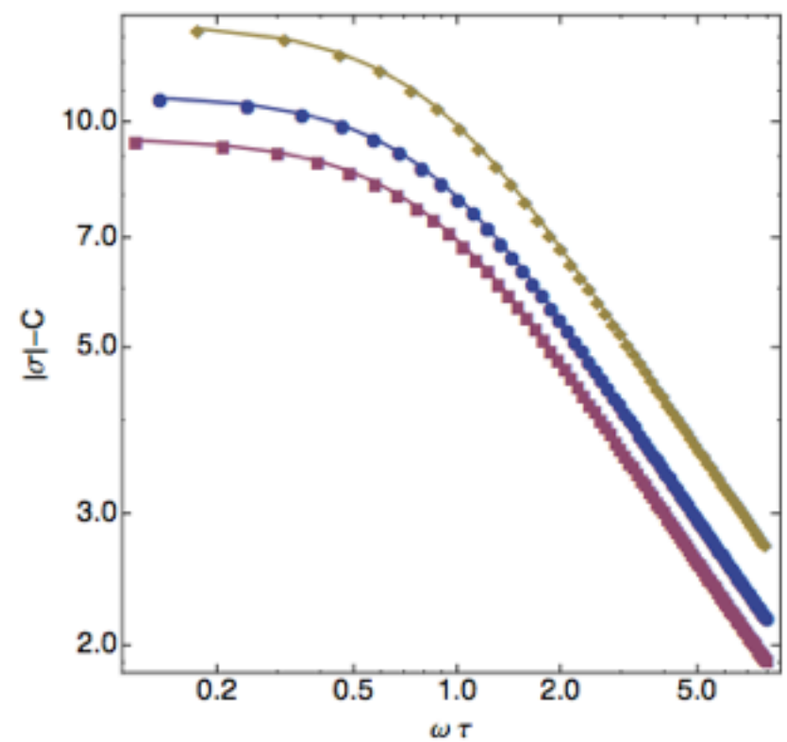
RG=GR

$$R \propto \sqrt{g}$$

cannot describe systems at g=0!

optical conductivity from a gravitational lattice





log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega\tau \lesssim 0.8$

G. Horowitz, Santos, Tong, Journal of High Energy Physics, 2012

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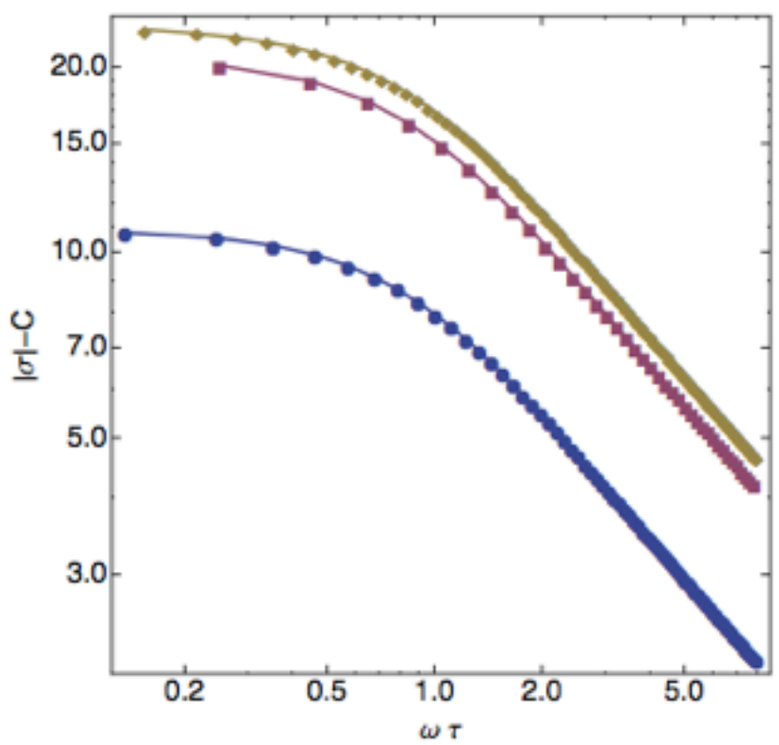

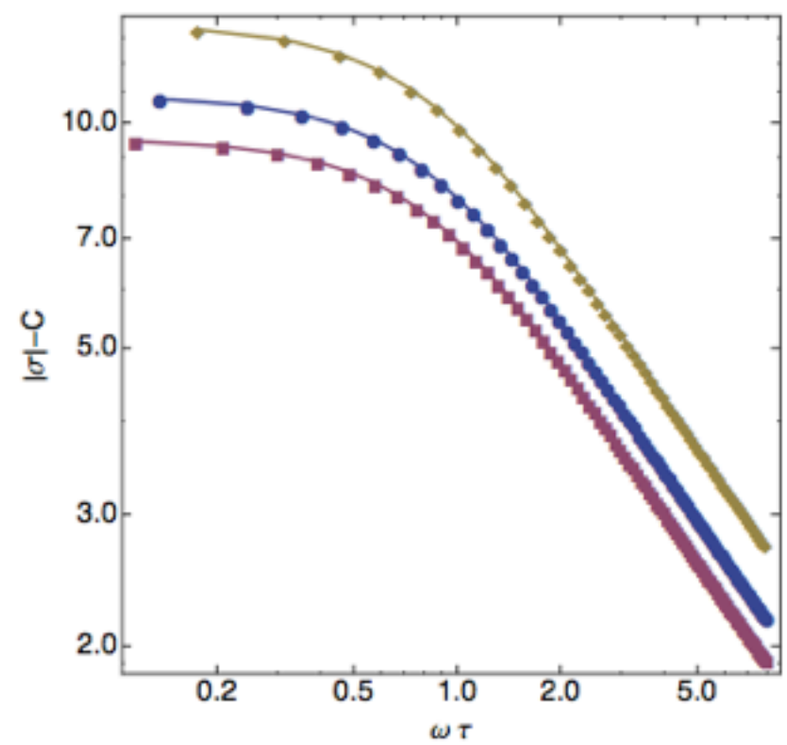

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a remarkable claim!
replicates features of the strange metal? how?

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new equation!

Einstein-Maxwell equations + non-uniform charge density = $B\omega^{-2/3}$

not so fast!

Donos and Gauntlett
(gravitational crystal)

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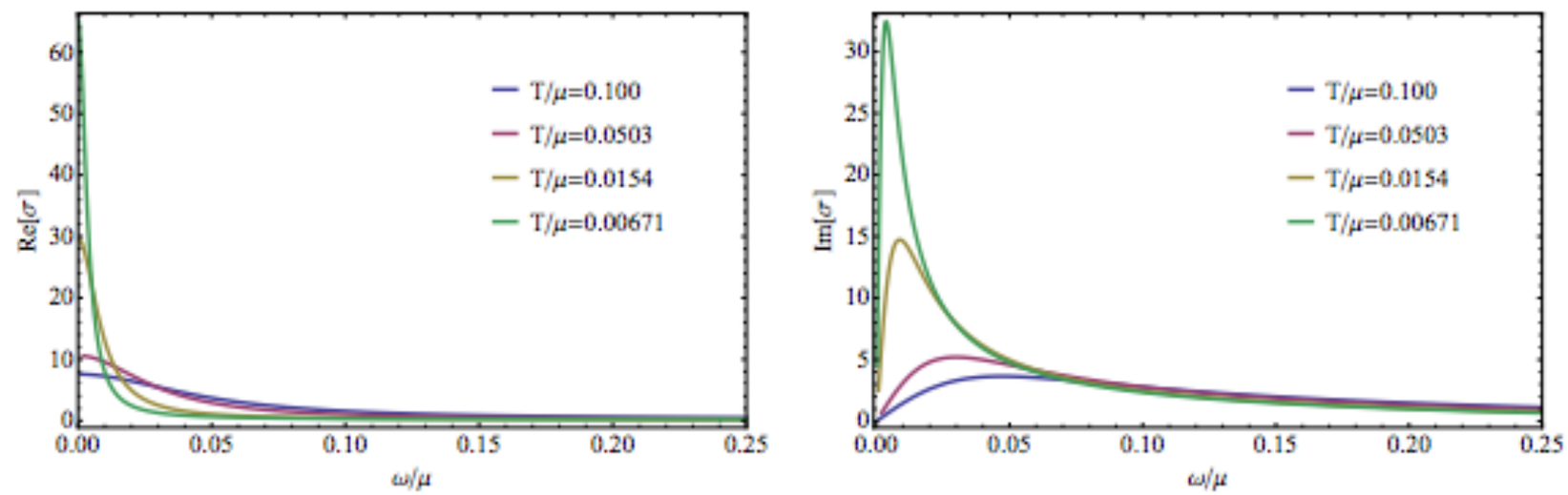

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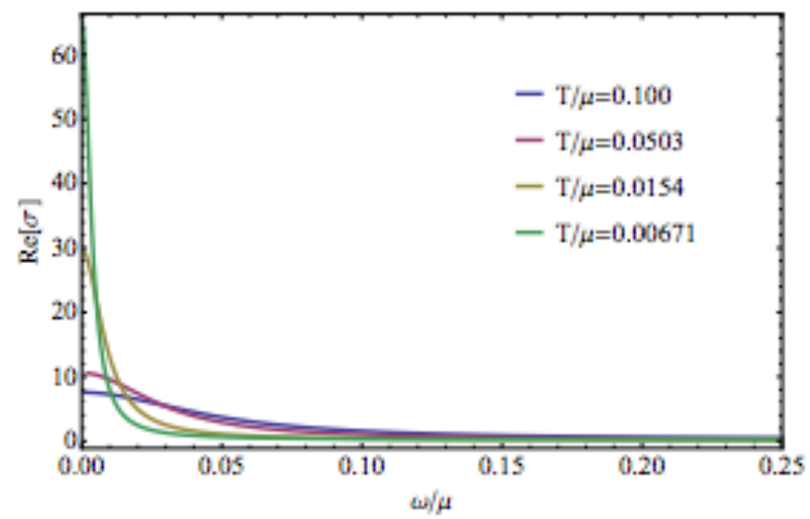

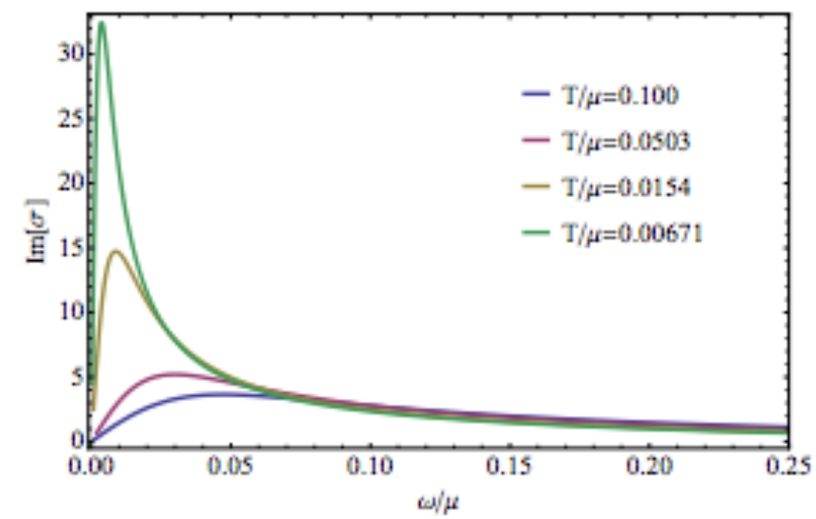

$$-\frac{2}{3} = 1 + \omega\frac{|\sigma|''}{|\sigma|'}$$

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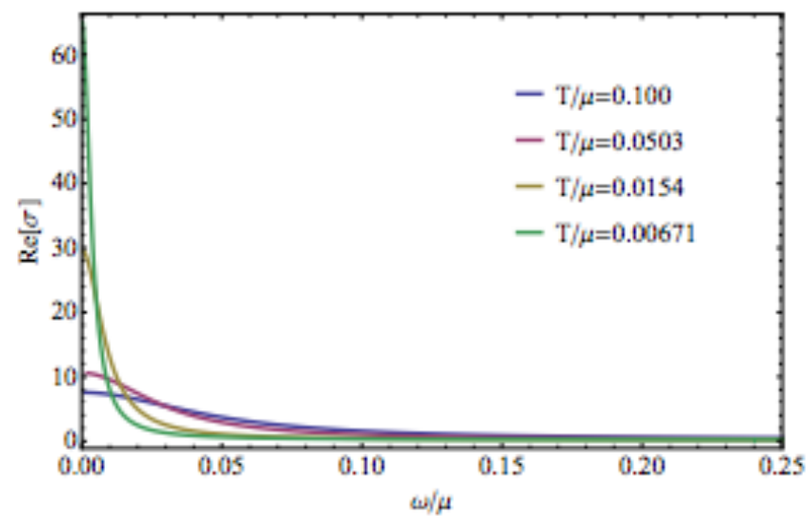

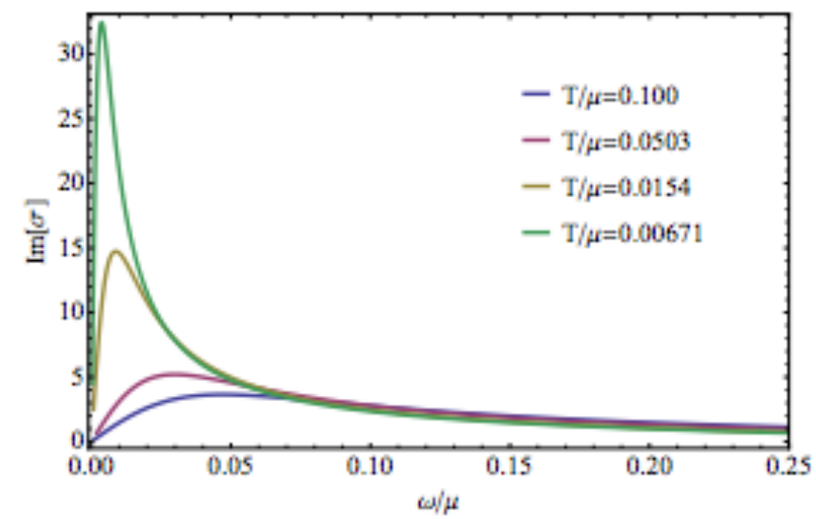

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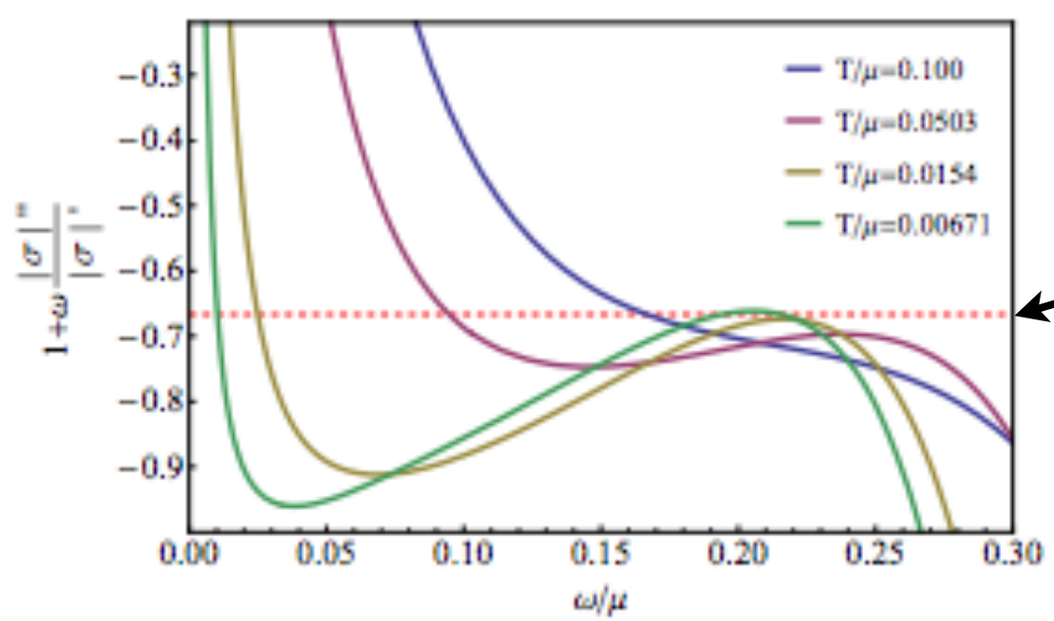

$$B\omega^{-2/3}$$

Donos and Gauntlett
(gravitational crystal)

Drude conductivity

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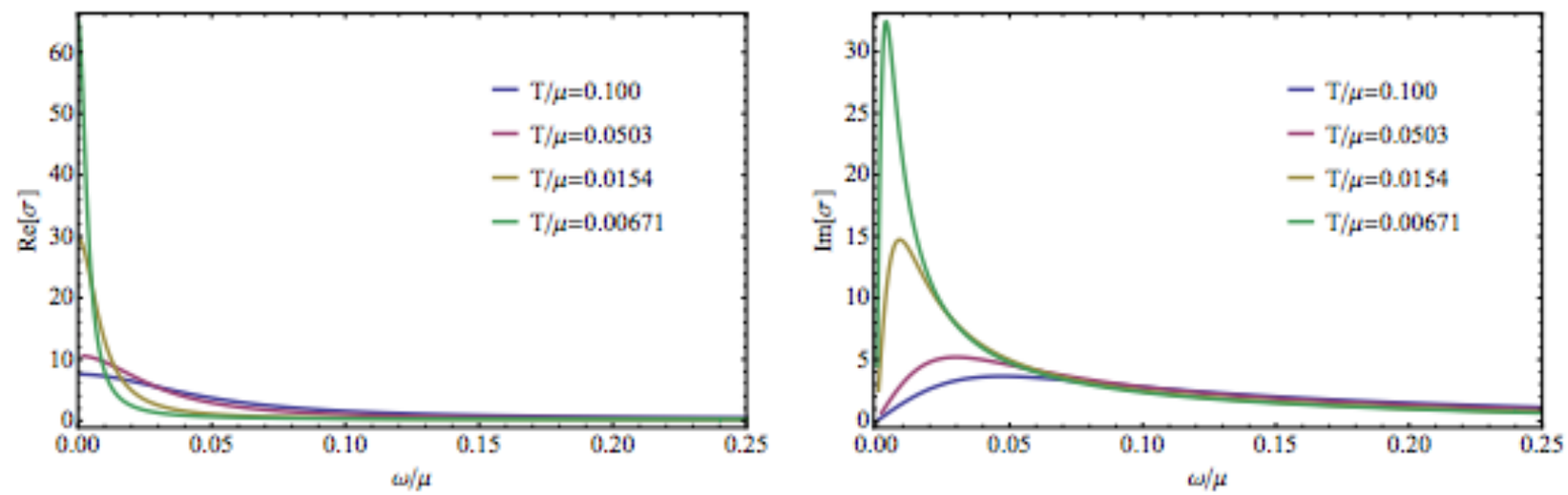

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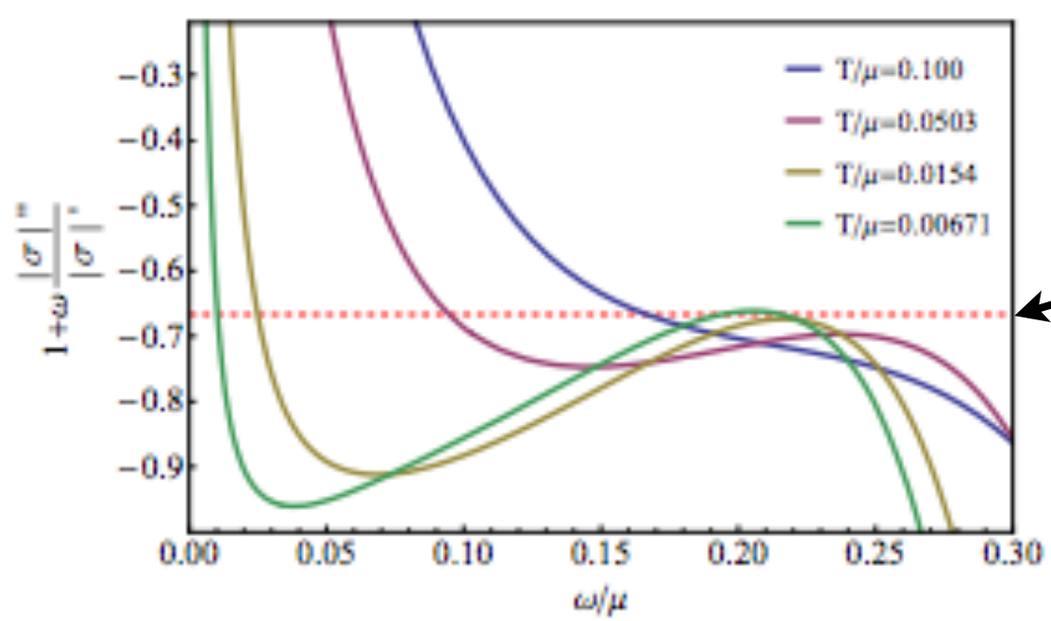


$$B\omega^{-2/3}$$

no power law!!

who is correct?

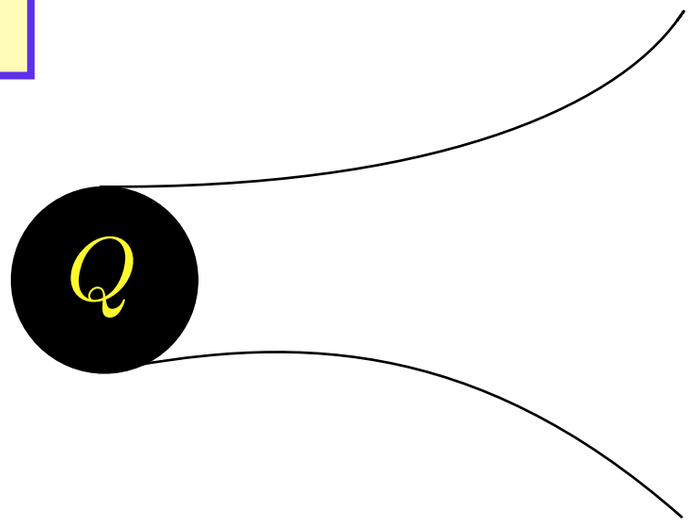
who is correct?

let's redo the calculation

conductivity within AdS

$$(g_{\mathrm{ab}}, V(\Phi), A_t)$$

$$(\text{metric}\,, \text{potential}\,, \text{gaugefield})$$



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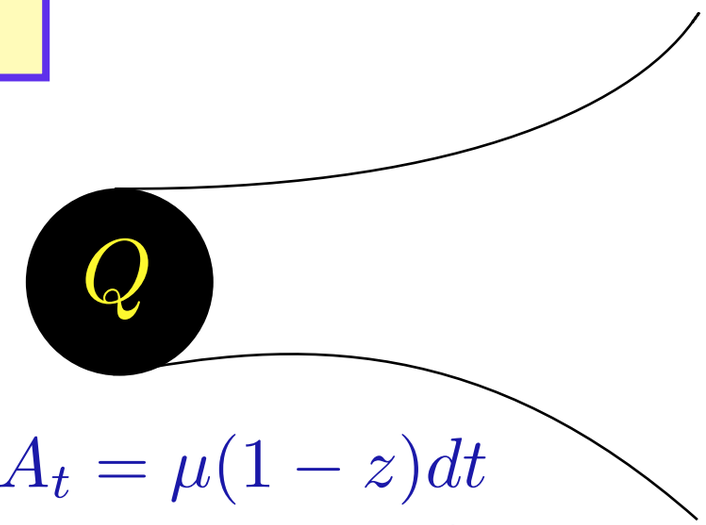

$$A_t = \mu(1-z)dt$$
$$\rho = \lim_{z\to 0} \sqrt{g} F^{tz}$$

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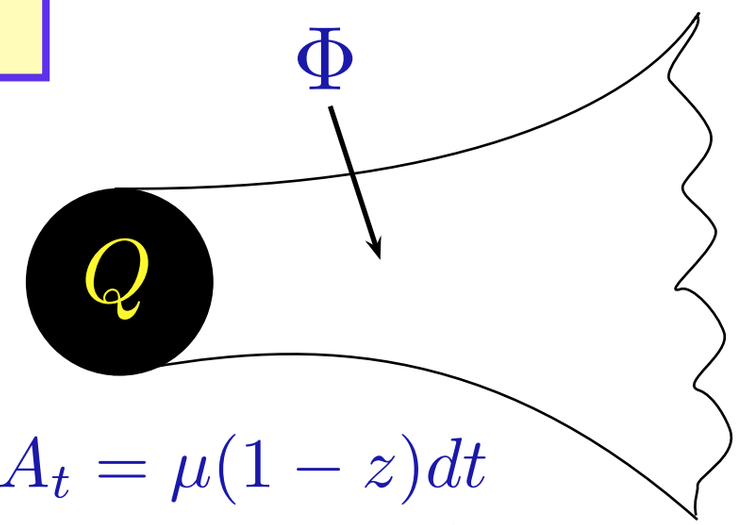

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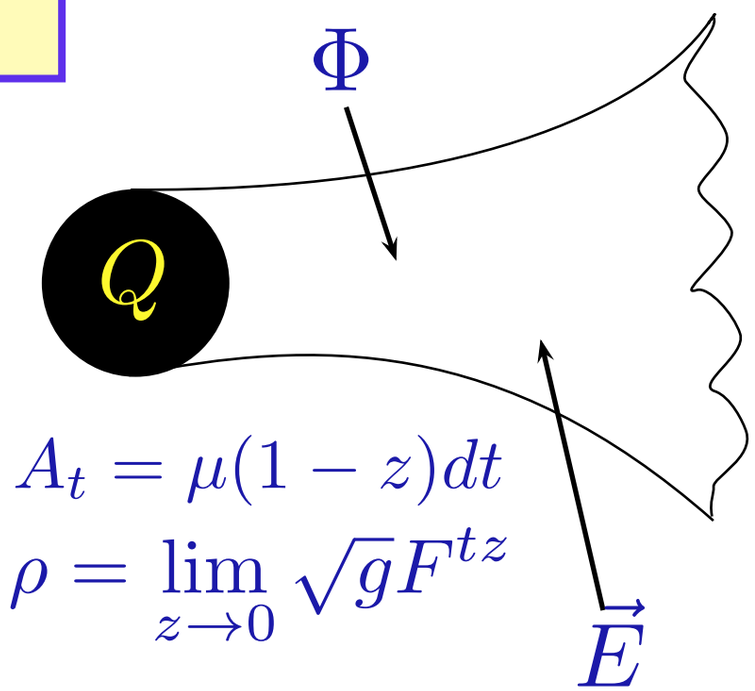
(metric , potential , gaugefield)

perturb with electric field



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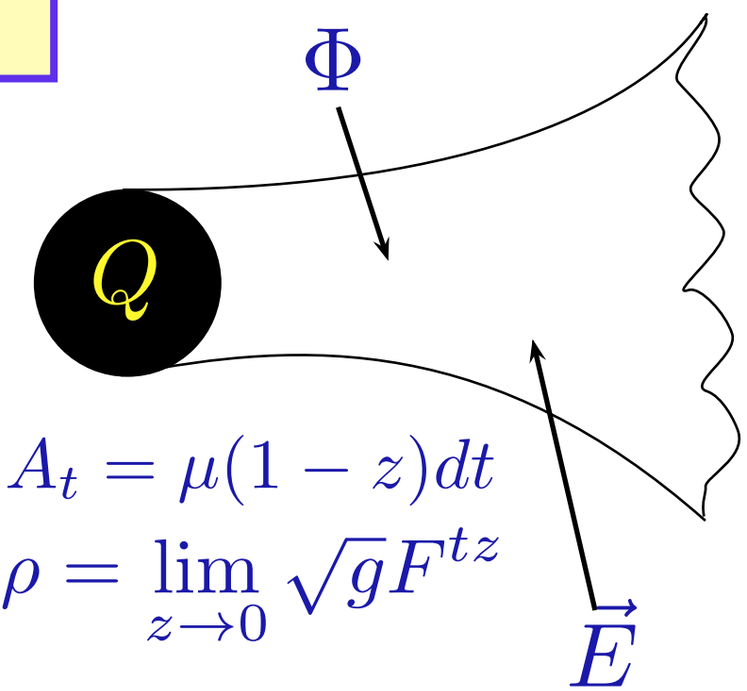

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$$g_{\rm ab} = \bar{g}_{\rm ab} + h_{\rm ab}$$
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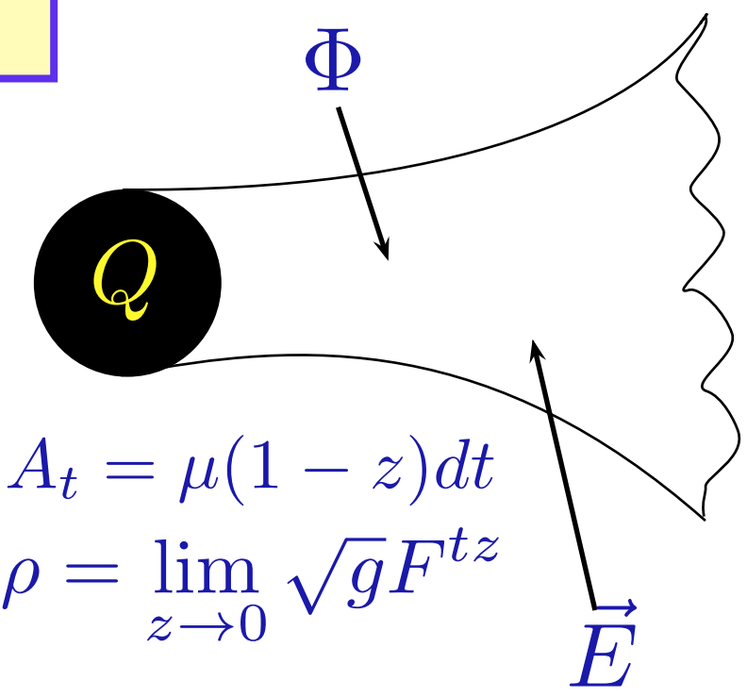

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conductivity within AdS

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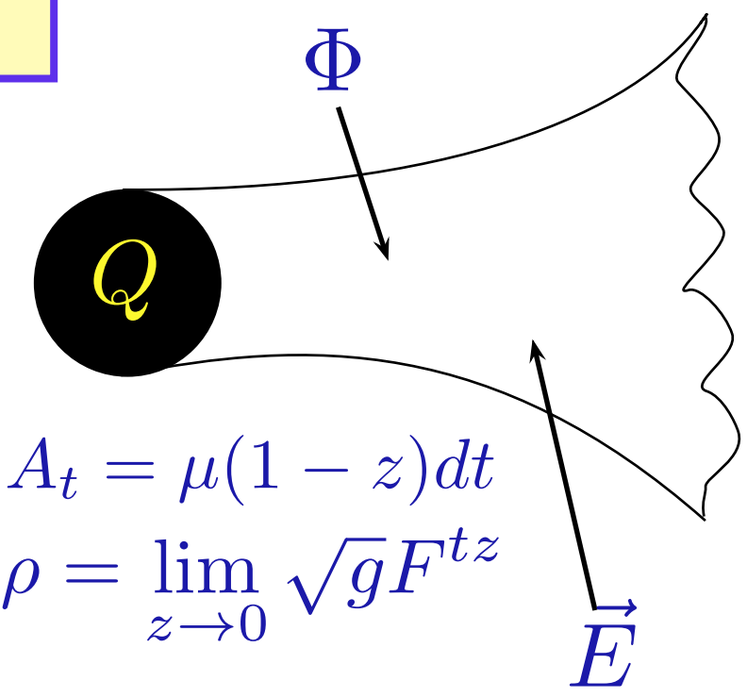

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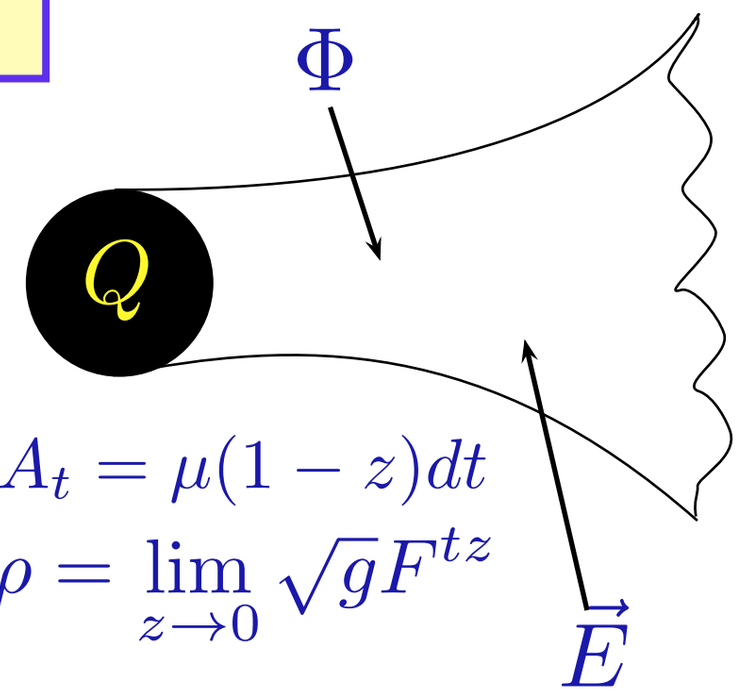

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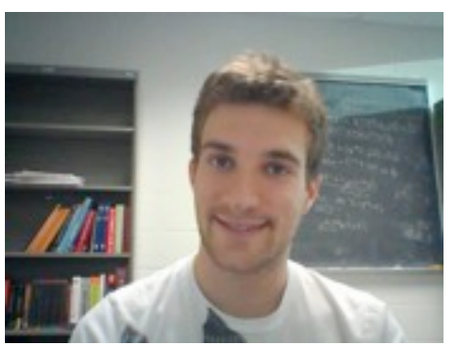

Brandon Langley

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solve equations of motion with gauge invariance (without mistakes)

$$\sigma = J_x(x,\omega)/E$$

HST vs. DG

HST vs. DG

Horowitz, Santos, Tong (HST)

$$V(\Phi) = -\Phi^2/L^2$$

$$\Phi = z\Phi^{(1)} + z^2\Phi^{(2)} + \cdots ,$$

$$\Phi^{(1)}(x) = A_0 \cos(kx)$$

inhomogeneous in x

$$m^2 = -2/L^2$$

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de Donder gauge

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DG

$$V\left(|\Phi|^2\right)$$

$$\Phi(z,x) = \phi(z)e^{ikx}$$

no inhomogeneity in x

$$m^2 = -3/(2L^2)$$

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radial gauge

Our Model

$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

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$$\Phi_1 = \qquad z\Phi_1^{(1)} + z^2\Phi_1^{(2)} + \cdots\,, \quad \Phi_1^{(1)}(x) = A_0\cos\left(kx - \frac{\theta}{2}\right),$$

$$\Phi_2 = \qquad z\Phi_2^{(1)} + z^2\Phi_2^{(2)} + \cdots\,, \quad \Phi_2^{(1)}(x) = A_0\cos\left(kx + \frac{\theta}{2}\right).$$

Our Model

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$$\theta = 0$$

HST

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$\theta = 0$ → HST

$\theta = \frac{\pi}{2}$ → DG

Einstein-De Turck EOM

$$G^H_{ab} = G_{ab} - \nabla_{(a}\xi_{b)},$$

$$\xi^a = g^{cd}\left(\Gamma^a_{cd}(g) - \Gamma^a_{cd}(\overline{g})\right).$$

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reference metric

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metric ansatz

reference metric

$$ds^2 = \frac{L^2}{z^2}\left[-(1-z)P(z)Q_{tt}dt^2 + \frac{Q_{zz}dz^2}{(1-z)P(z)} + Q_{xx}(dx + z^2 Q_{zx}dz)^2 + Q_{yy}dy^2\right],$$

$$P(z) = 1 + z + z^2 - \frac{\mu_1^2}{2}z^3.$$

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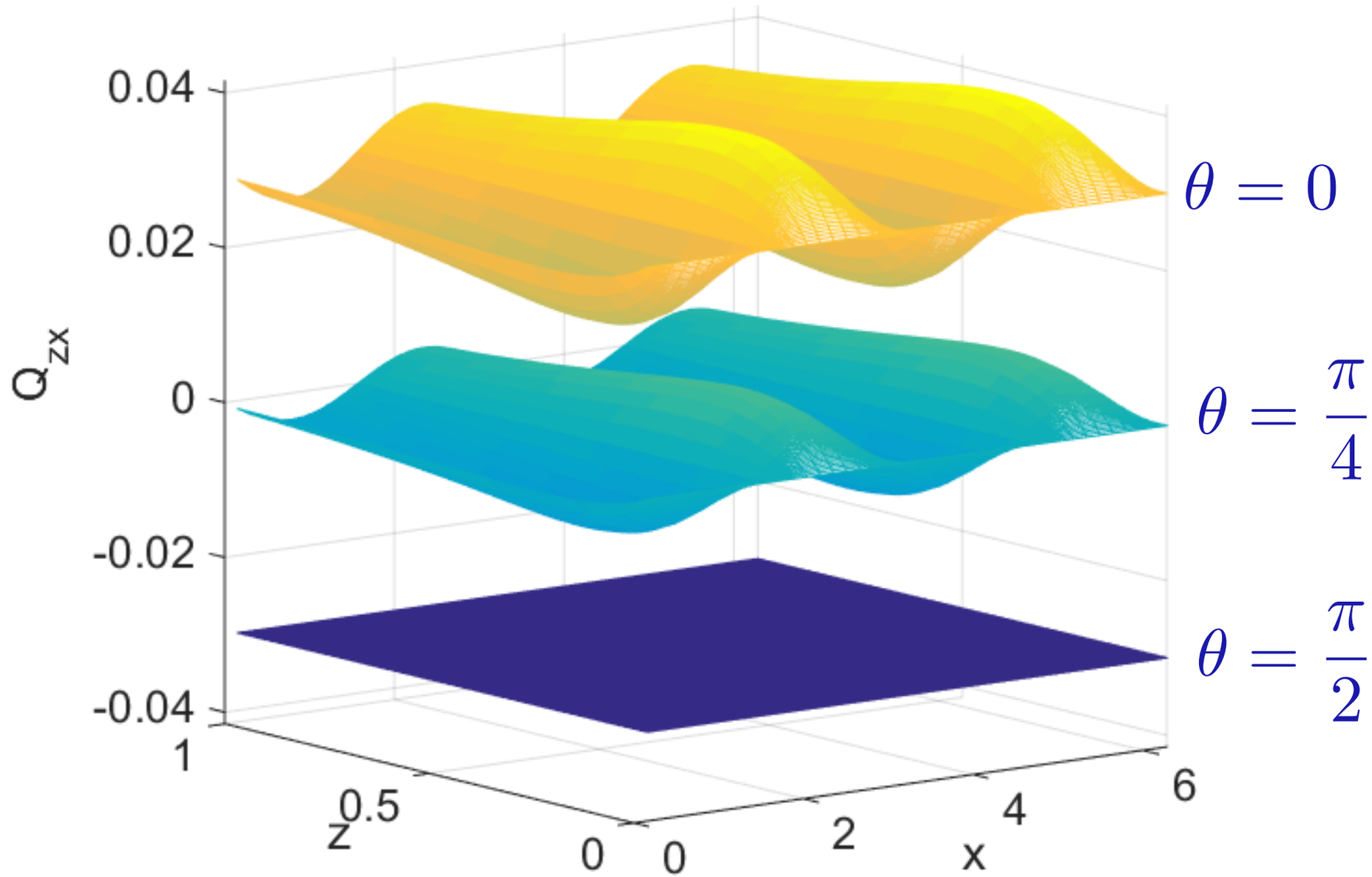
$$P(z) = 1 + z + z^2 - \frac{\mu_1^2}{2}z^3.$$

RN-AdS when

$$Q_{tt} = Q_{zz} = Q_{yy} = 1 \qquad \Phi = 0 \qquad a_t = \mu_1 = \mu$$

$A_0 = 0.75,\ k = 1,\ \mu = 1.4,\ T/\mu = 0.115$



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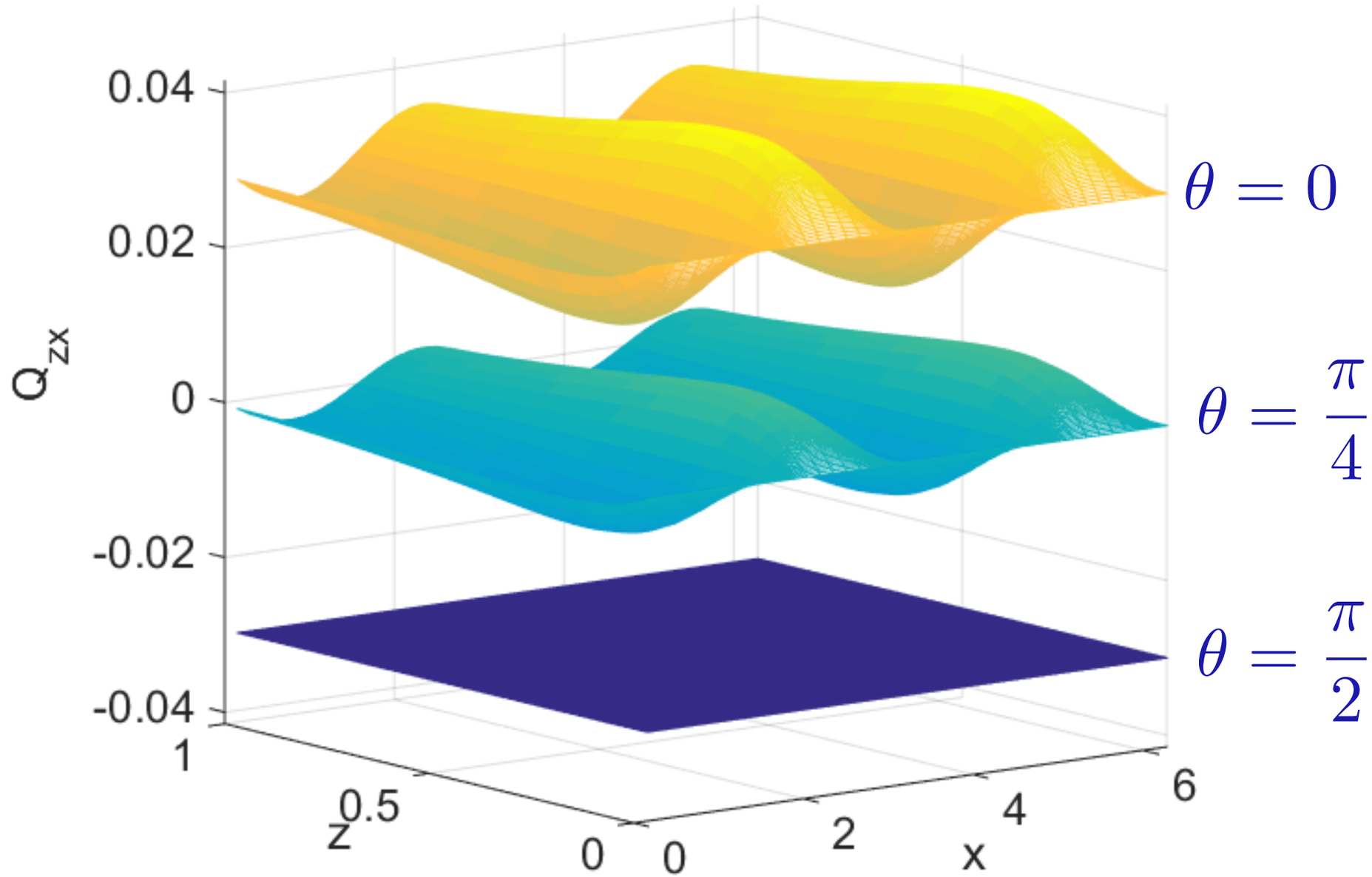
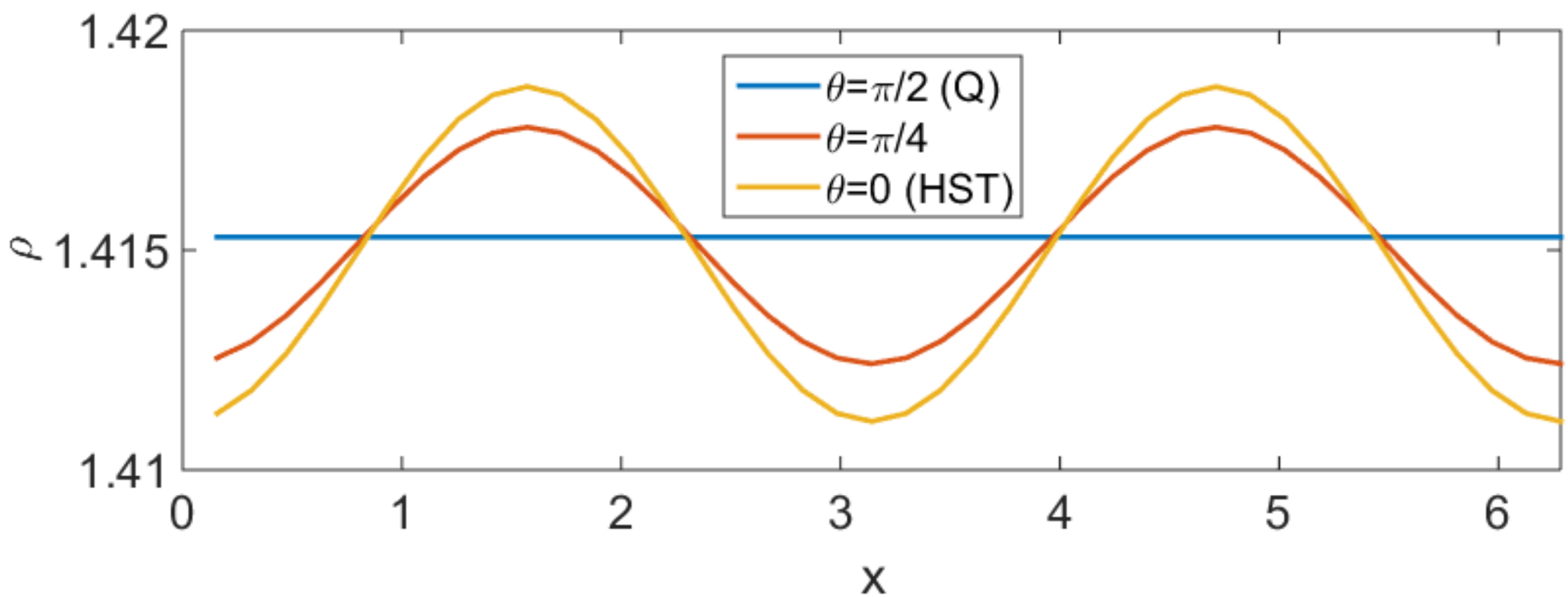


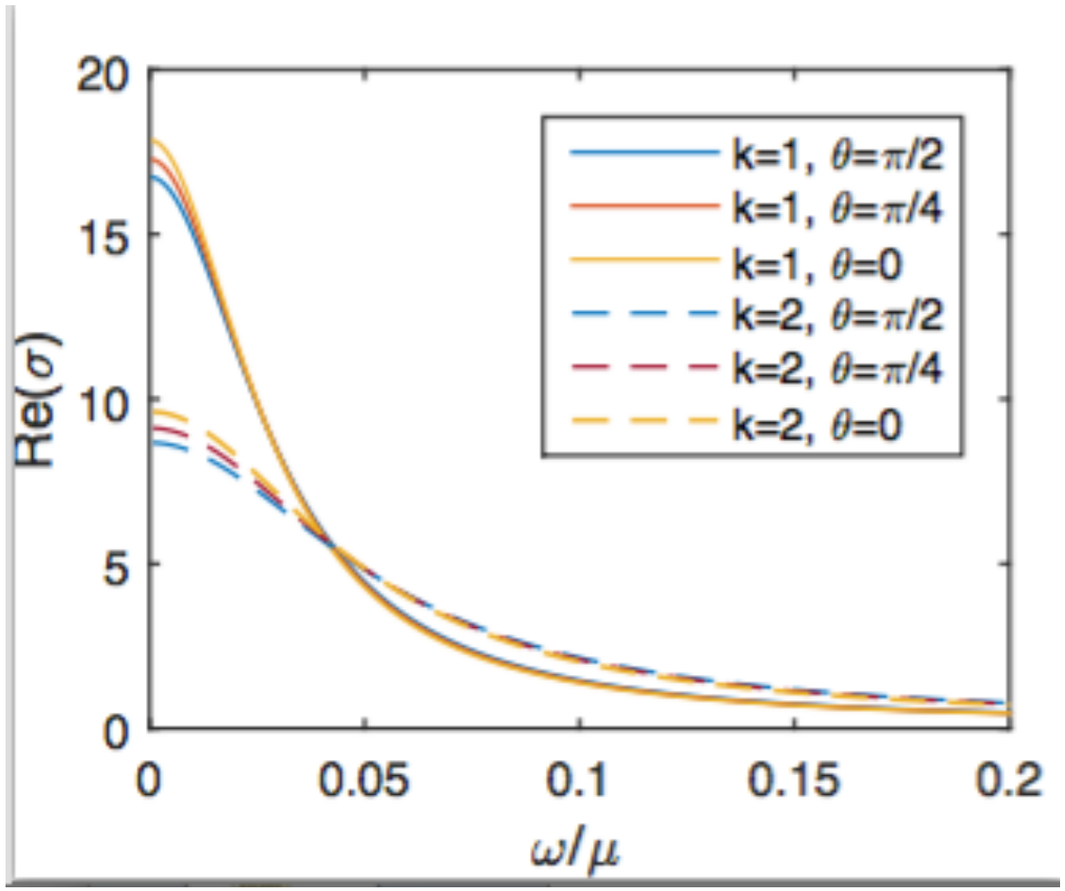
translational invariance is broken in metric in multiples of 2k

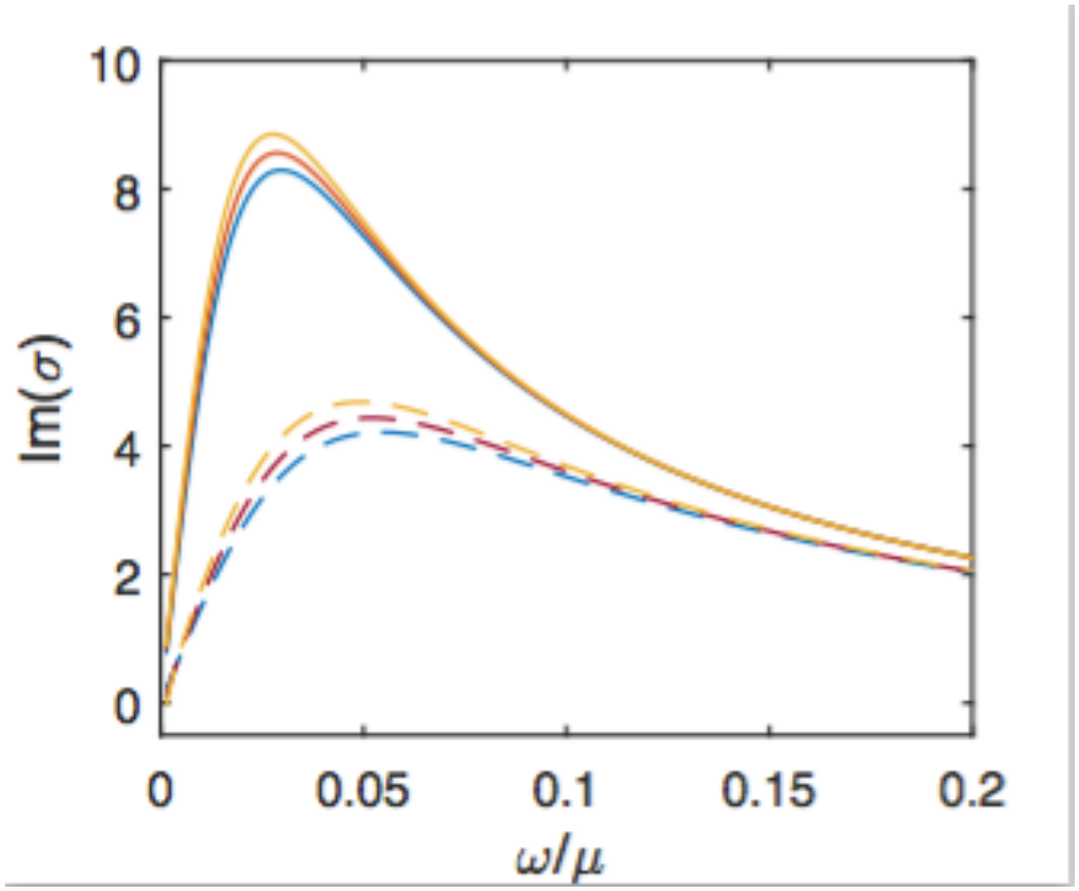
charge density

$$\rho = \lim_{z \to 0} \sqrt{-g} F^{tz}$$



conductivity



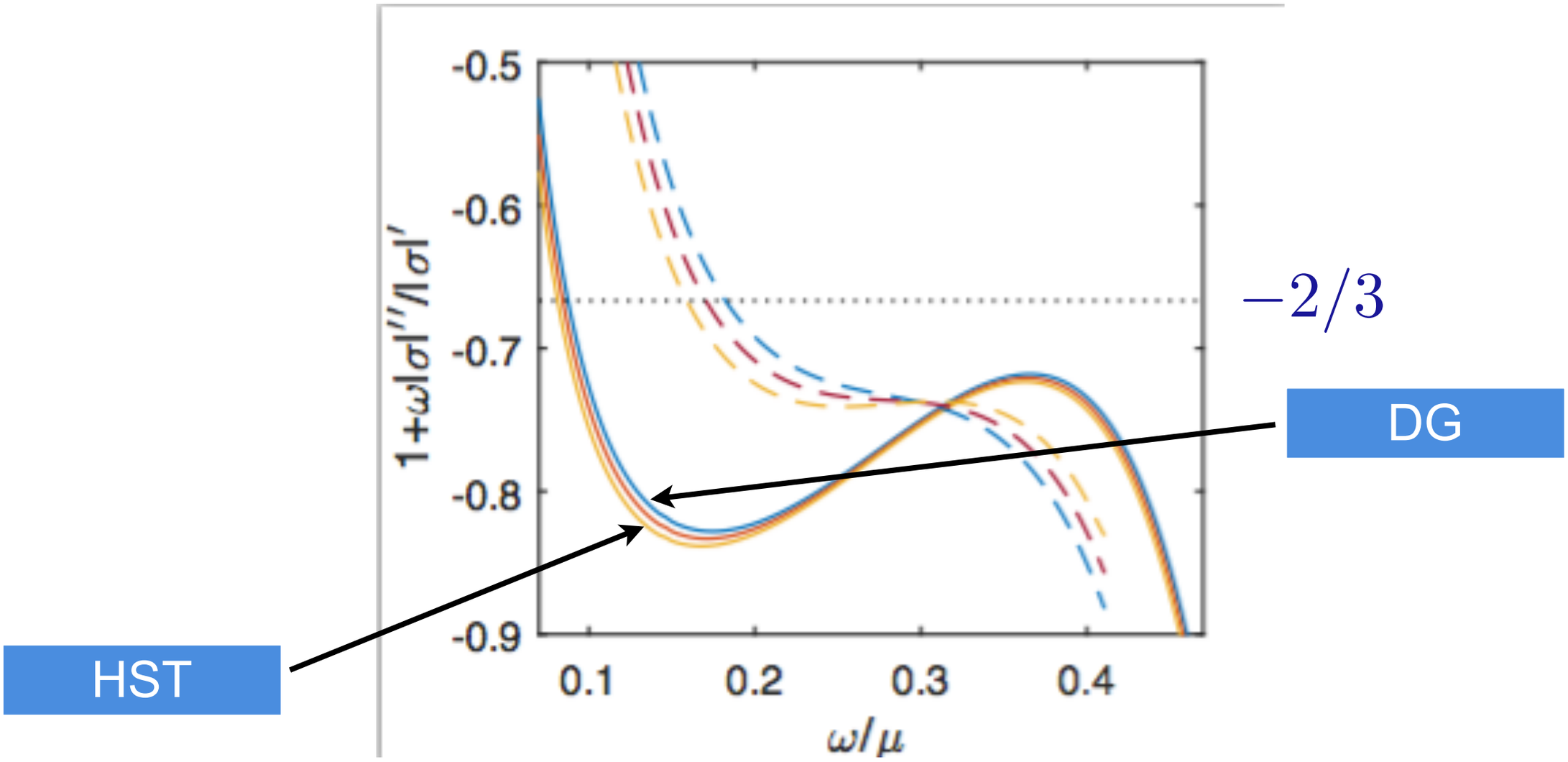


- high-frequency behavior is identical
- low-frequency RN has $\mathrm{Re}(\sigma) \sim \delta(\omega),\ \mathrm{Im}(\sigma) \sim 1/\omega$
- low-frequency lattice has Drude form

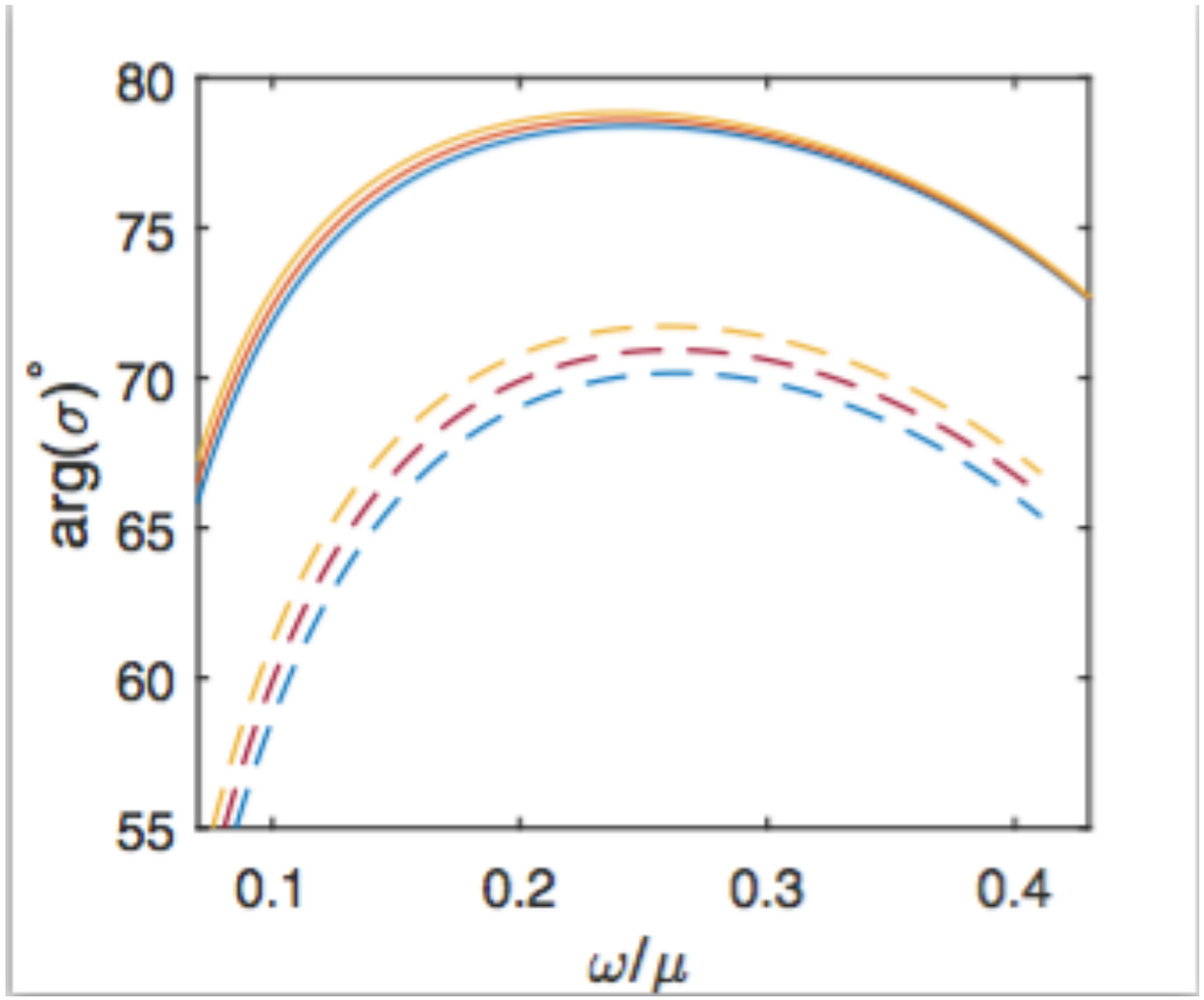
is there a power law?

is there a power law?

Results

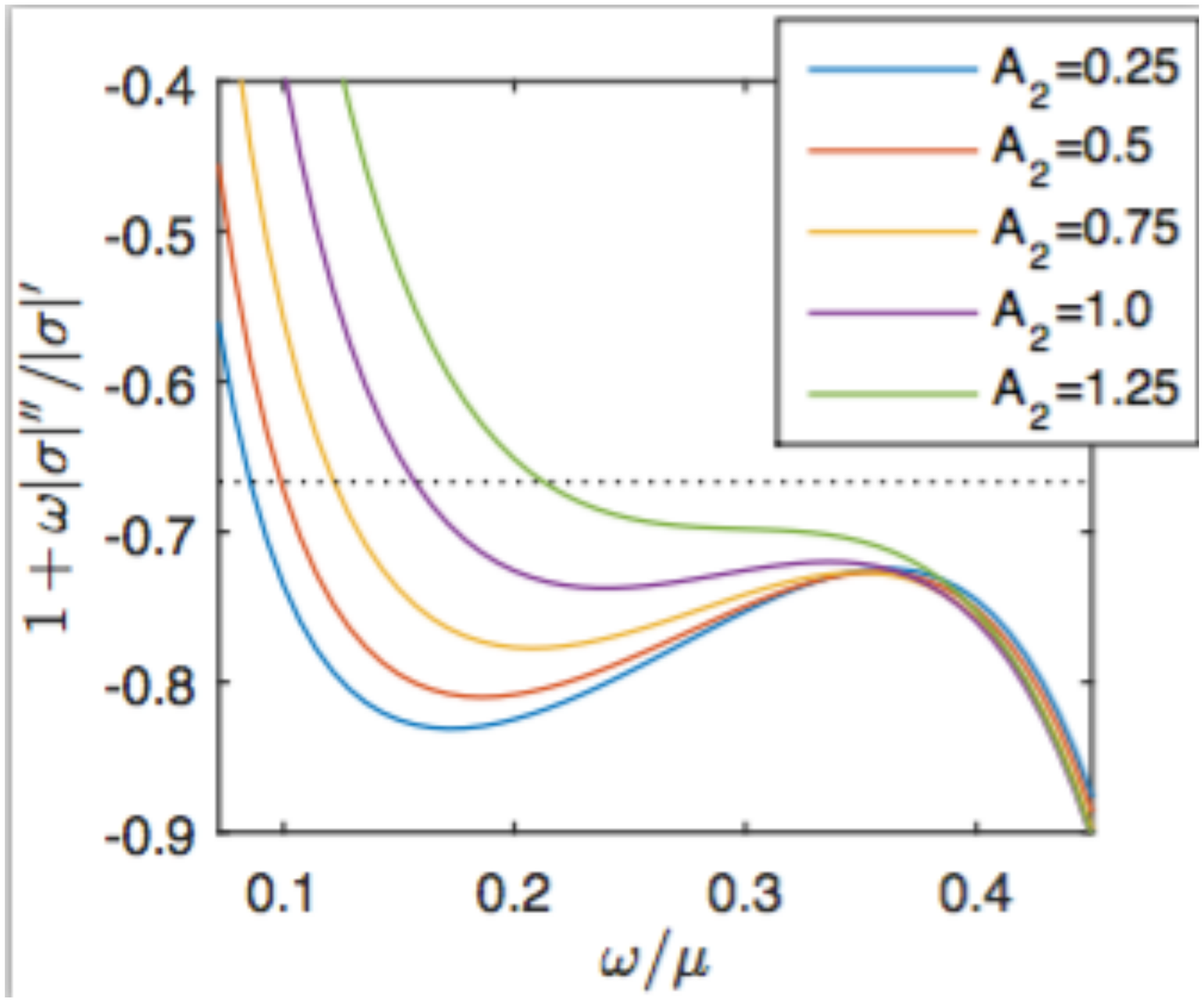


Results



Results

$$A_1 = 0.75, k_1 = 2, k_2 = 2, \theta = 0, \mu = 1.4, T/\mu = 0.115$$



No!

origin of power law?

origin of power law?

phenomenology

origin of power law?

phenomenology

no well-defined mass

$$\mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2$$

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incoherent stuff (all energies)

$$\mathcal{L} = \quad \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right)$$

$$\mathcal{L} = \int_0^\infty \big(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\big)\rho(m^2)dm^2$$

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theory with all possible mass!

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theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

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$$\mathcal{L} \to \Lambda^{4+d_\rho}\mathcal{L}$$

scale invariance is restored!!

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not particles

unparticles

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propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

propagator

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$$\downarrow$$

$$d_U - 2$$

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$$\downarrow$$

$$d_U - 2$$

continuous mass

$\phi(x, m^2)$

flavors

propagator

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$$d_U - 2$$

continuous mass

$\phi(x, m^2)$ ⟷ $e^2(m)$

Karch, 2015

flavors

multi-bands

take experiments seriously

take experiments seriously

$$\sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i}$$

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continuous mass

take experiments seriously

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continuous mass

$$\sigma(\omega) = \int_0^M \frac{\rho(m) e^2(m) \tau(m)}{m} \frac{1}{1 - i\omega\tau(m)} dm$$

variable masses for everything

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

$$e(m) = e_0 \frac{m^b}{M^b}$$

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

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perform integral

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

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$$\Big\downarrow \; \omega\tau_0 \to \infty$$

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$$\boxed{\sigma(\omega) = \frac{1}{3}(\sqrt{3}+3i)\pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M\omega^{\frac{2}{3}}}}$$

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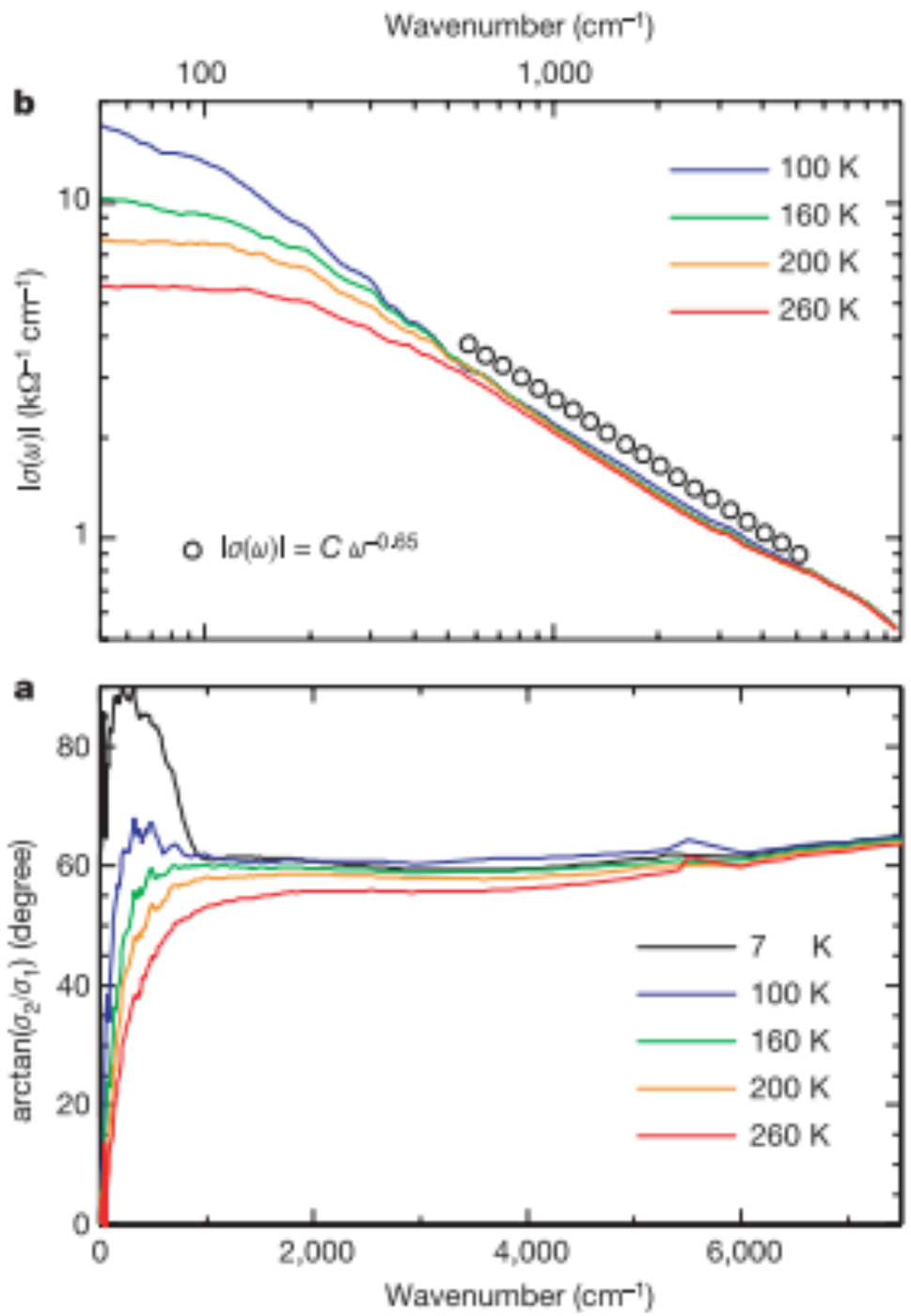
$$\boxed{\begin{array}{c} \tan\sigma = \sqrt{3} \\ 60^\circ \end{array}}$$

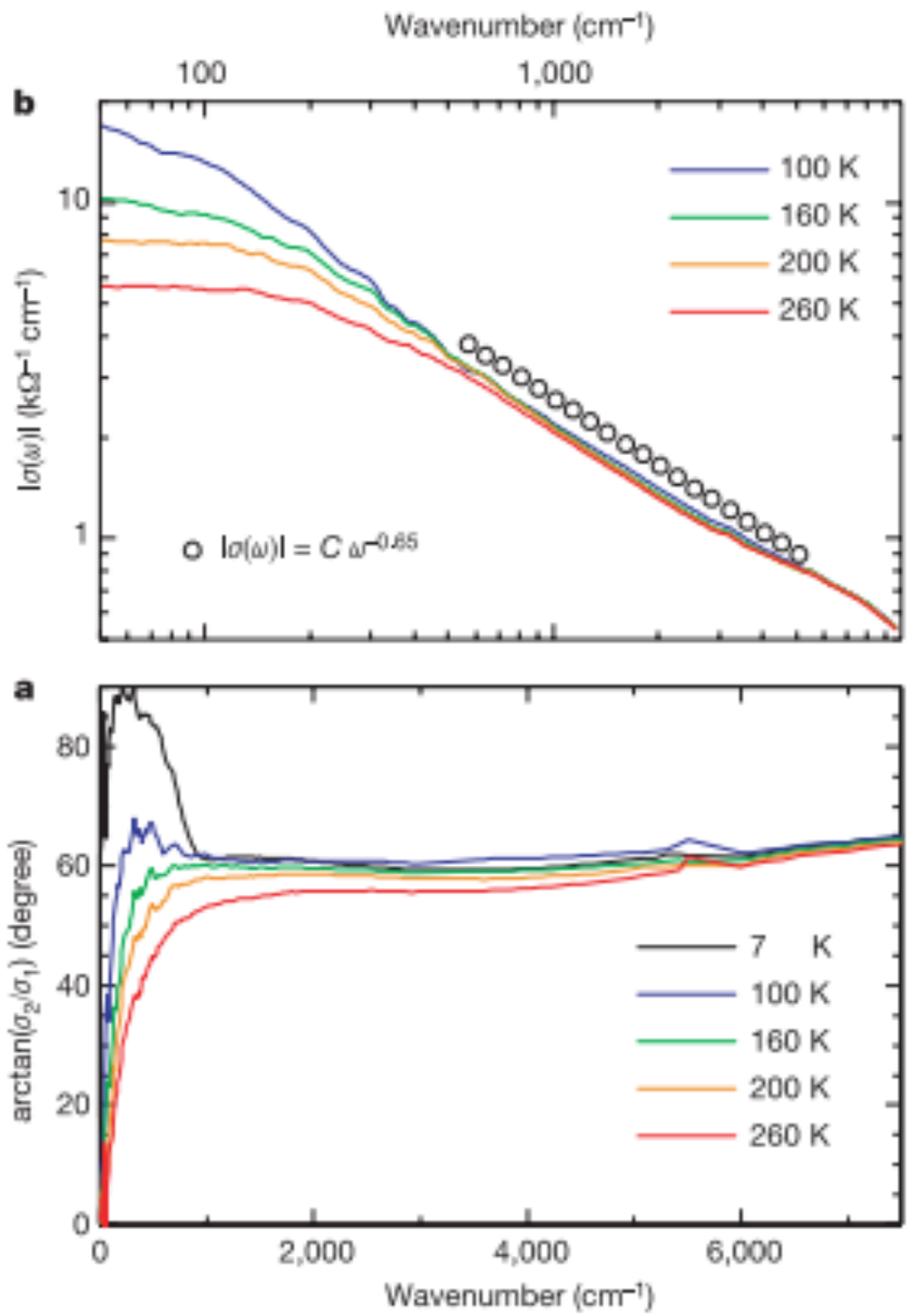
experiments



$$\sigma(\omega) = C\omega^{\gamma-2}e^{i\pi(1-\gamma/2)}$$

$$\gamma = 1.35$$

experiments



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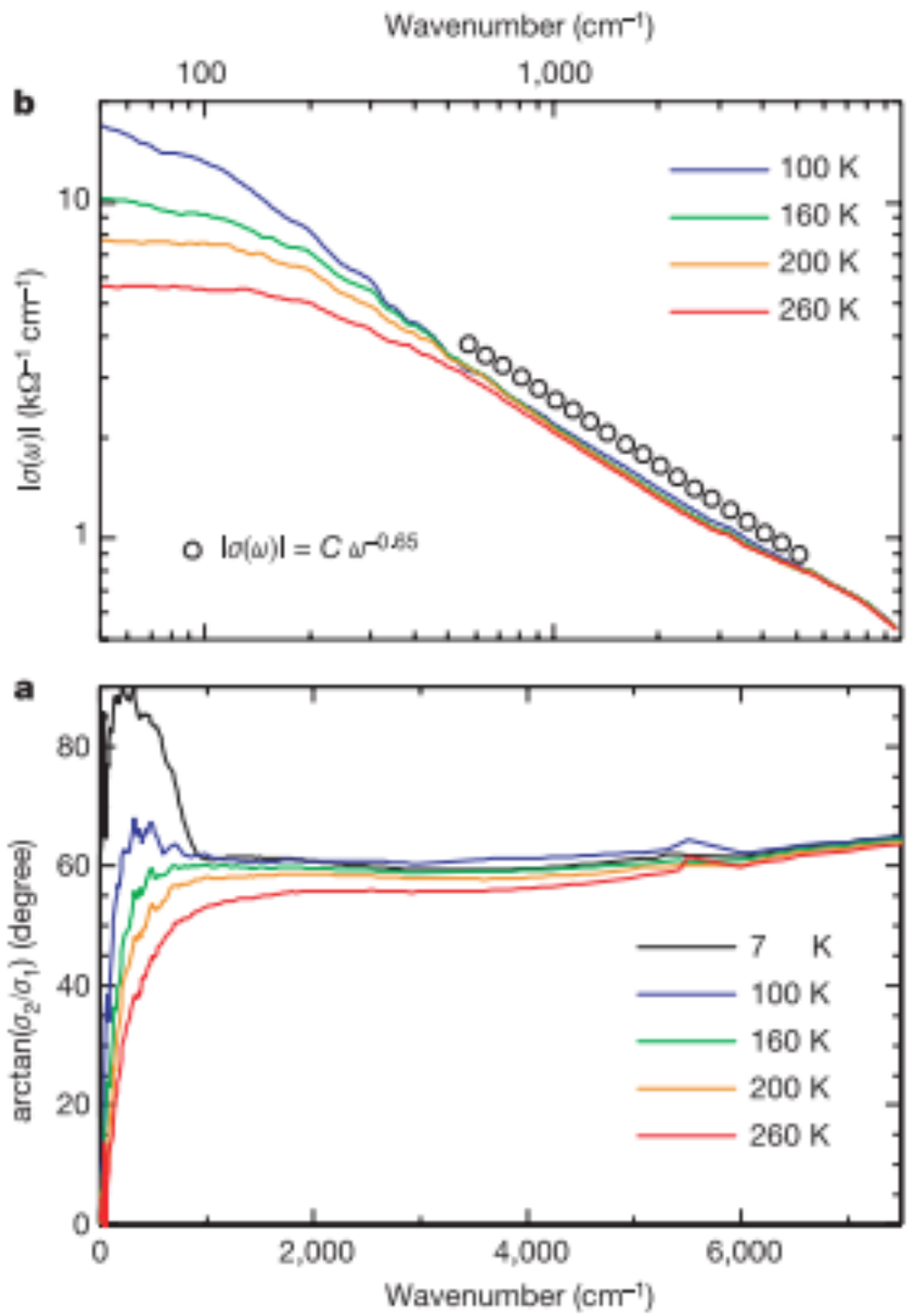

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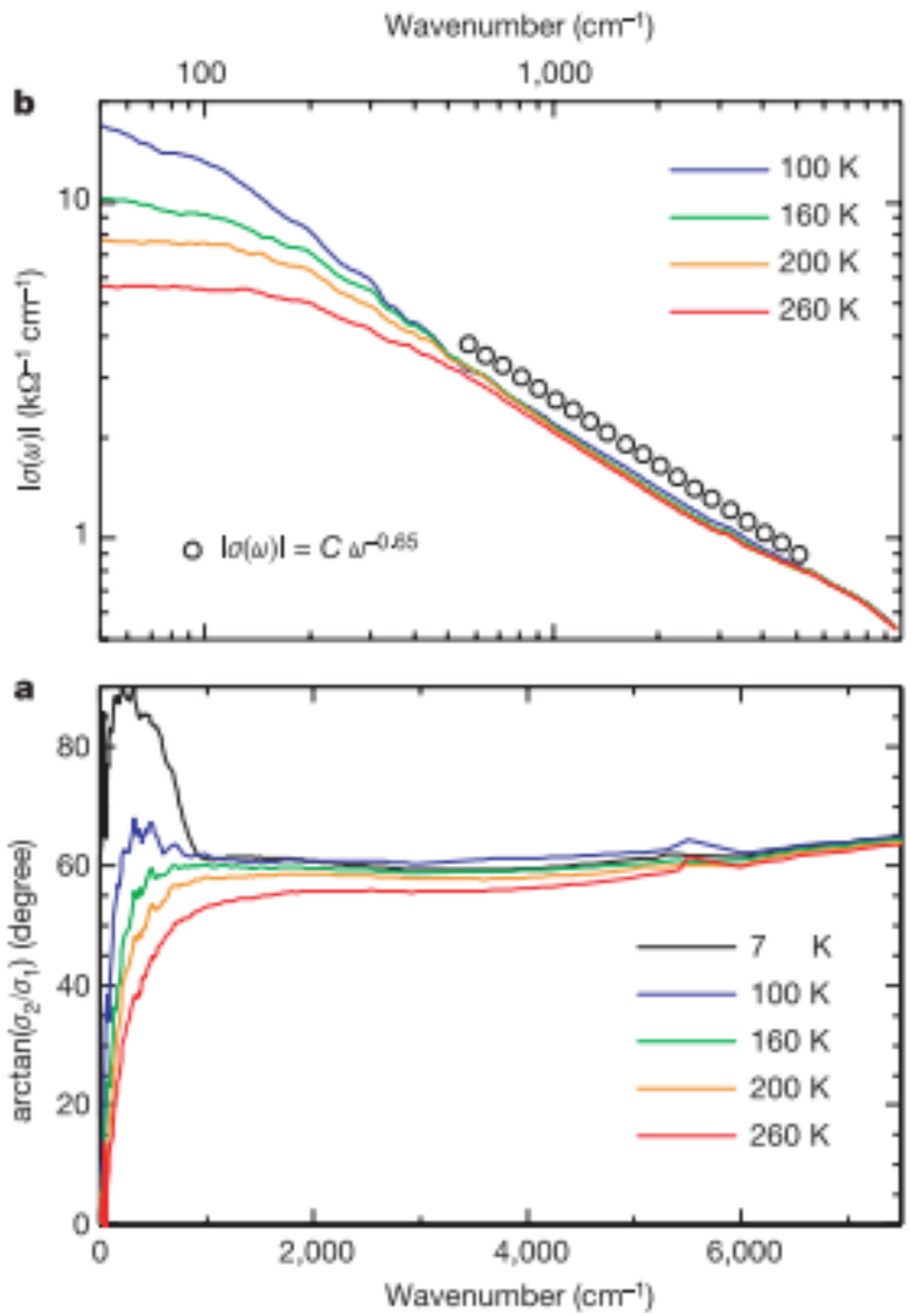

$$\sigma(\omega) = \frac{1}{3}(\sqrt{3}+3i)\pi\frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M\omega^{\frac{2}{3}}}$$

victory!!

$$\tan\sigma_2/\sigma_1 = \sqrt{3}$$
$$\theta = 60^\circ$$

$$\sigma(\omega) = C\omega^{\gamma-2}e^{i\pi(1-\gamma/2)}$$
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are anomalous dimensions necessary

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hyperscaling violation

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$$a + 2b = 2/3$$

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$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

hyperscaling violation

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

anomalous dimension

$$e(m) = e_0 \frac{m^b}{M^b}$$

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

$$c = 1 \qquad\qquad b = 0$$

$$a + 2b = 2/3 \quad \Rightarrow \quad a = 2/3$$

not
necessarily

not
necessarily

but they are
a possibility

what really is the summation over mass?

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$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

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$$m = z^{-1}$$

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$$\mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[\frac{1}{2}\frac{z^2}{R^2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{\phi^2}{2R^2}\right]$$

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$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

anti de Sitter metric

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can be absorbed with AdS metric

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

anti de Sitter metric

unparticles

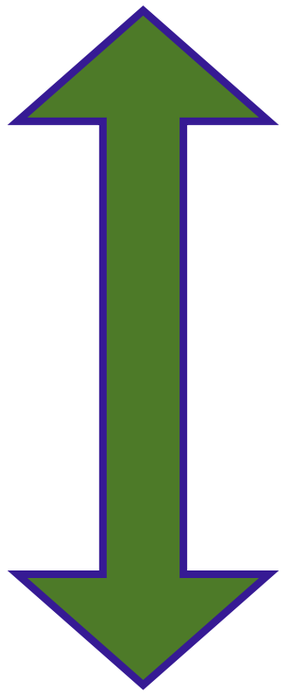
$$\mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2$$

unparticles

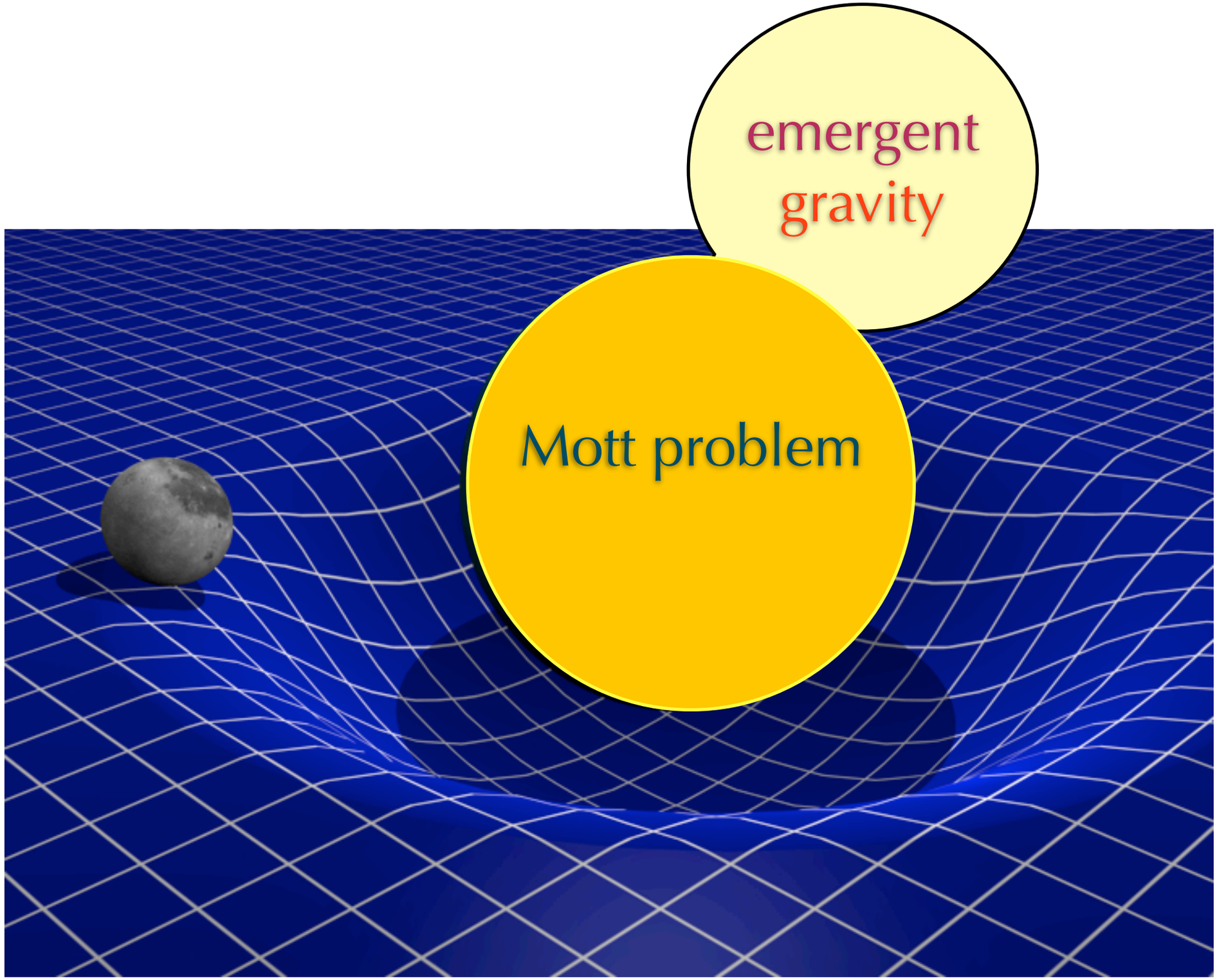
$$\mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2$$

power-law optical conductivity

unparticles (flavors, ‘incoherent’ stuff)



action on AdS


emergent
gravity
Mott problem