6 Steps to Strong Coupling

Thanks to: NSF, EFRC (DOE) and

Tony Hegg

arxiv:1502.03094

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Exact Strongly Coupled Fixed Point in $g\varphi^4$ Theory

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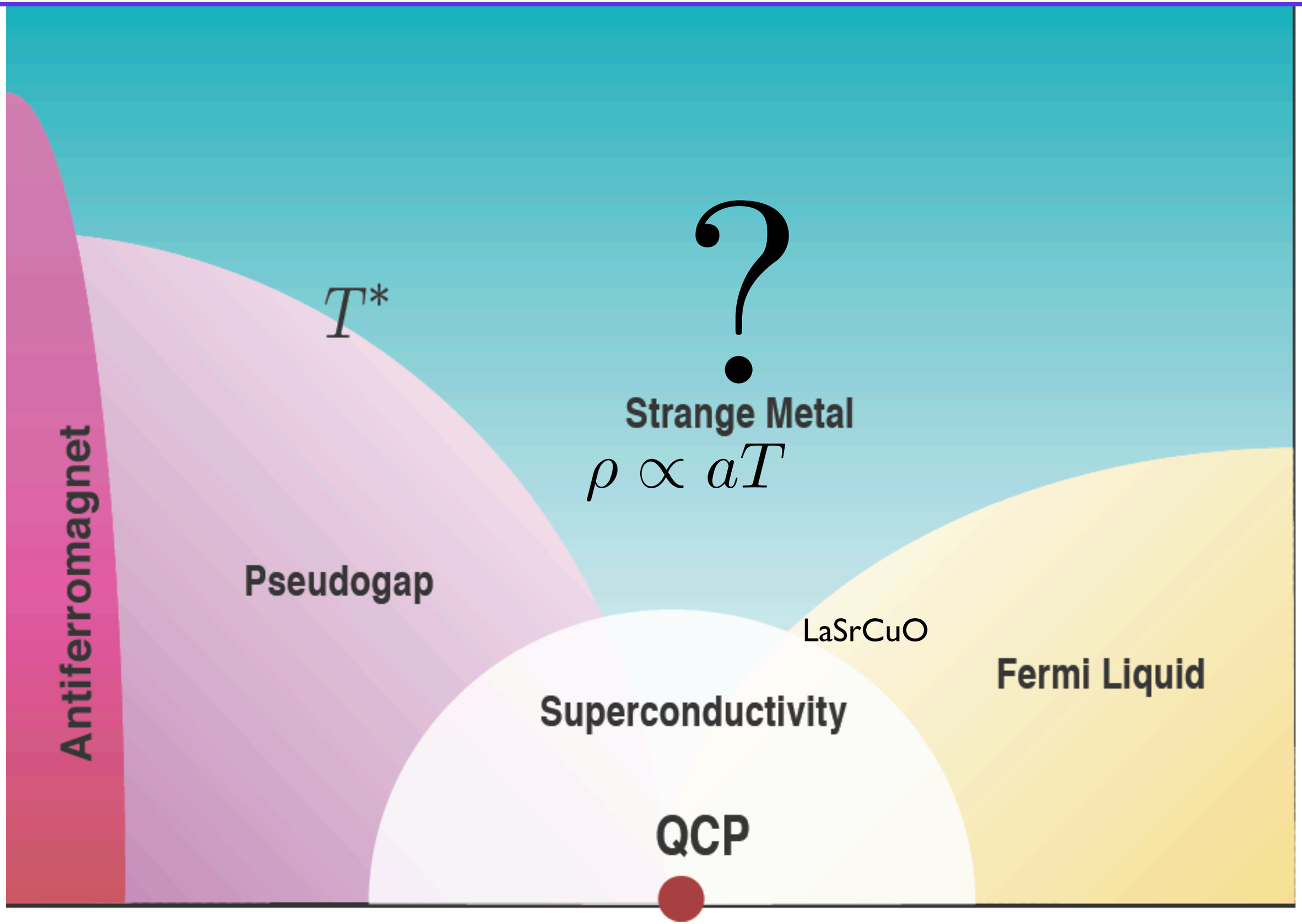
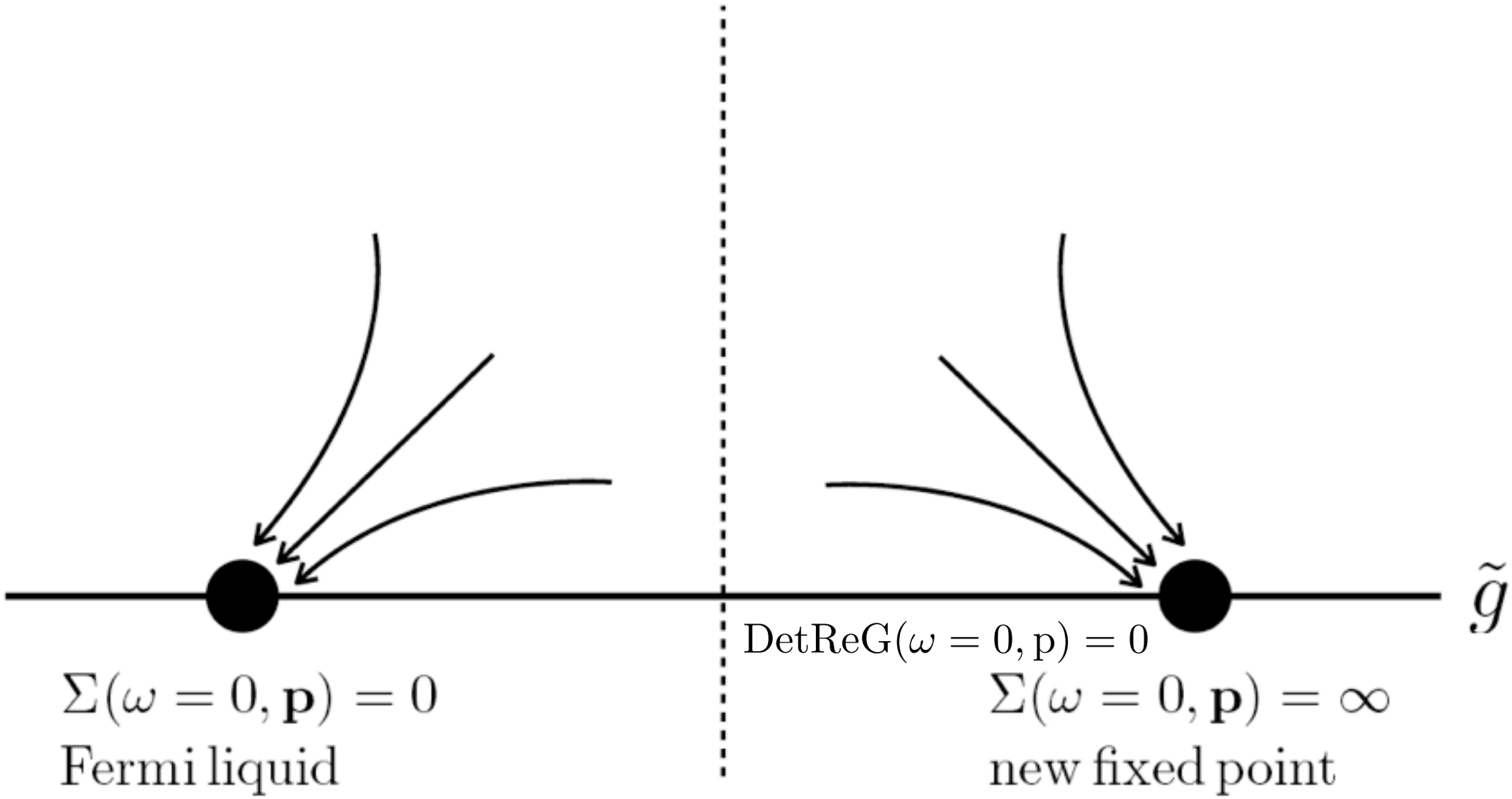
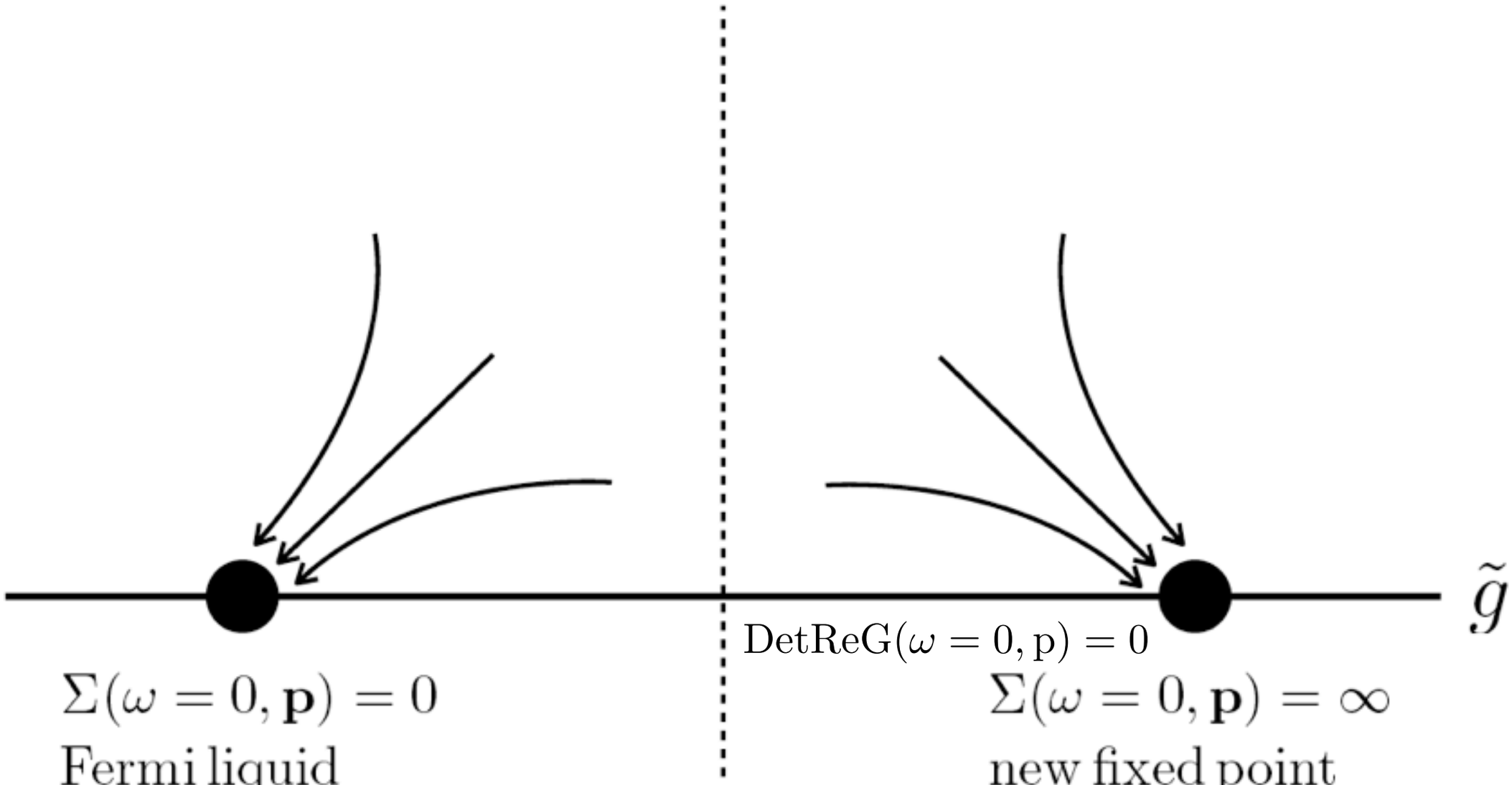
High T_c:How Does Fermi Liquid Theory Breakdown?




$\tilde{g}$
$\mathrm{DetReG}(\omega = 0, \mathrm{p}) = 0$
$\Sigma(\omega = 0, \mathbf{p}) = 0$
Fermi liquid
$\Sigma(\omega = 0, \mathbf{p}) = \infty$
new fixed point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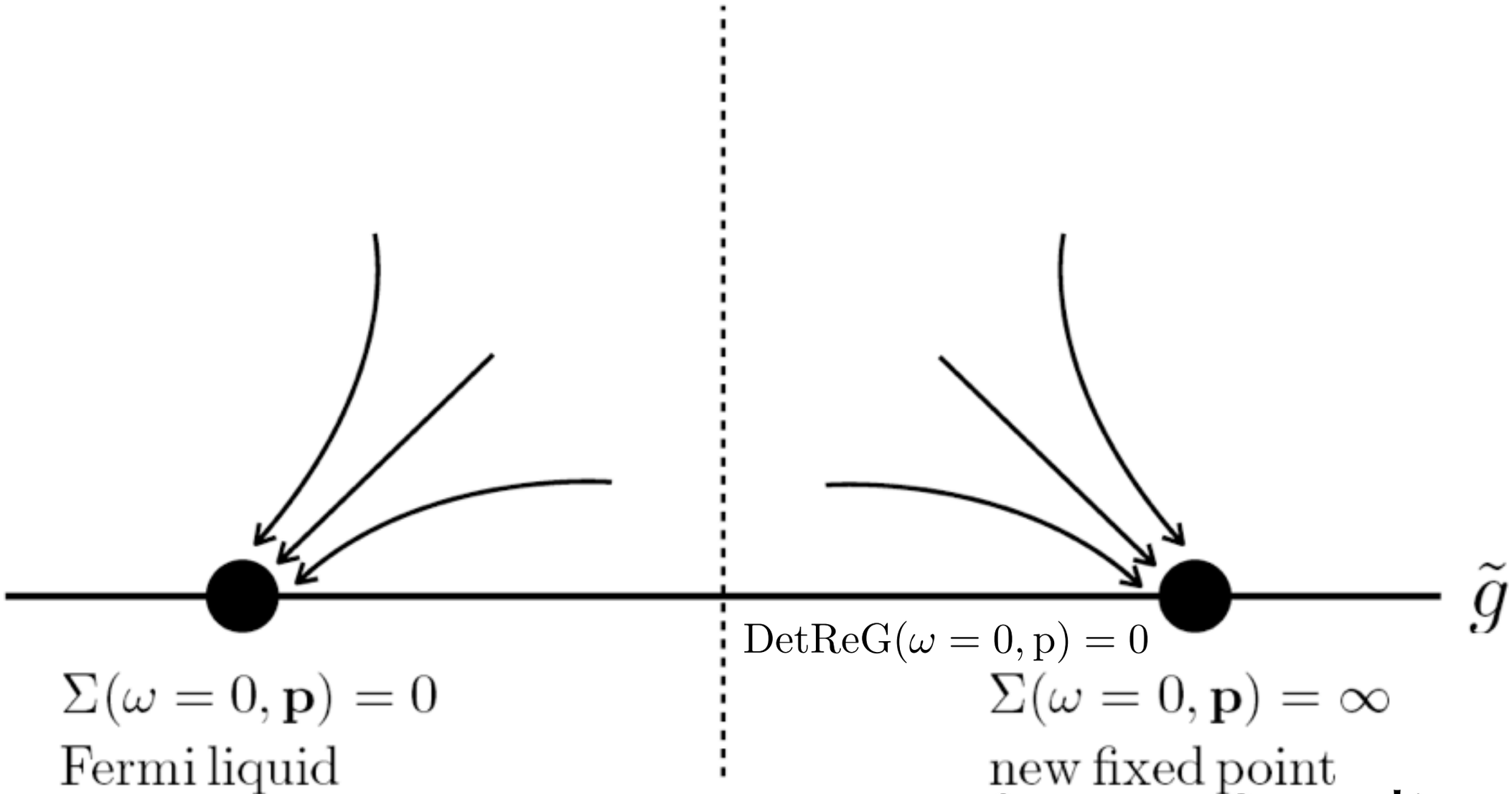
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Evidence for new Fixed Point

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Hubbard Model $g = U/t \gg 1$

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DMFT

Local Quantum Criticality, zeros

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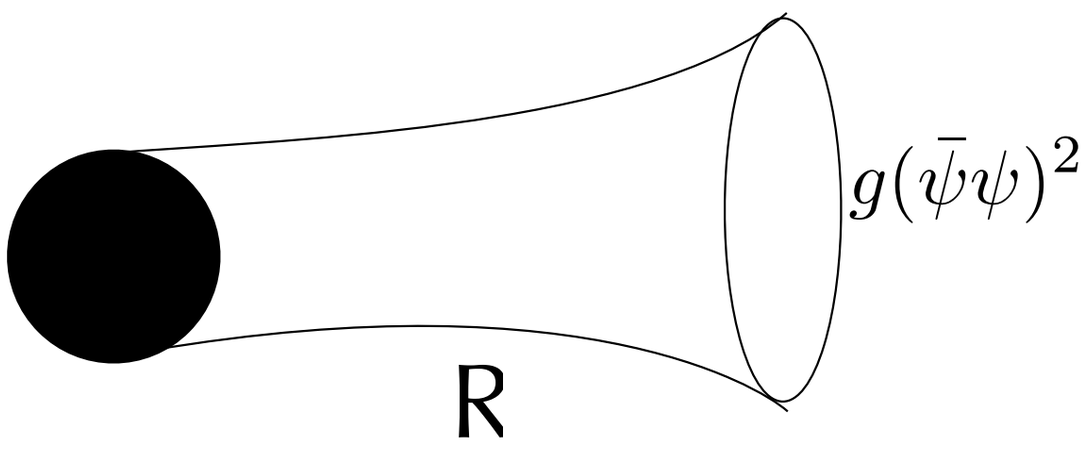
Local Quantum Criticality, zeros



$R \propto \sqrt{g}$

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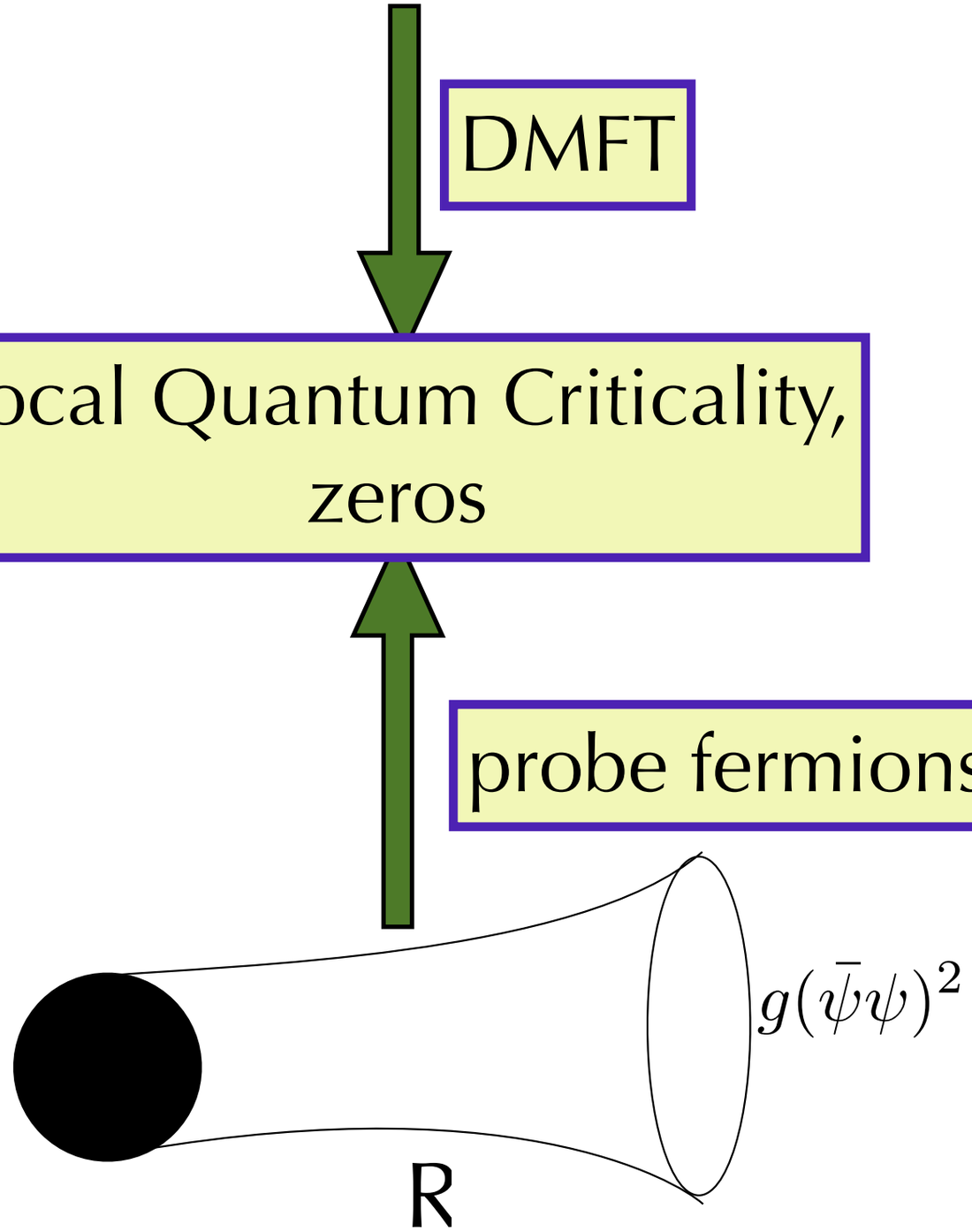
probe fermions



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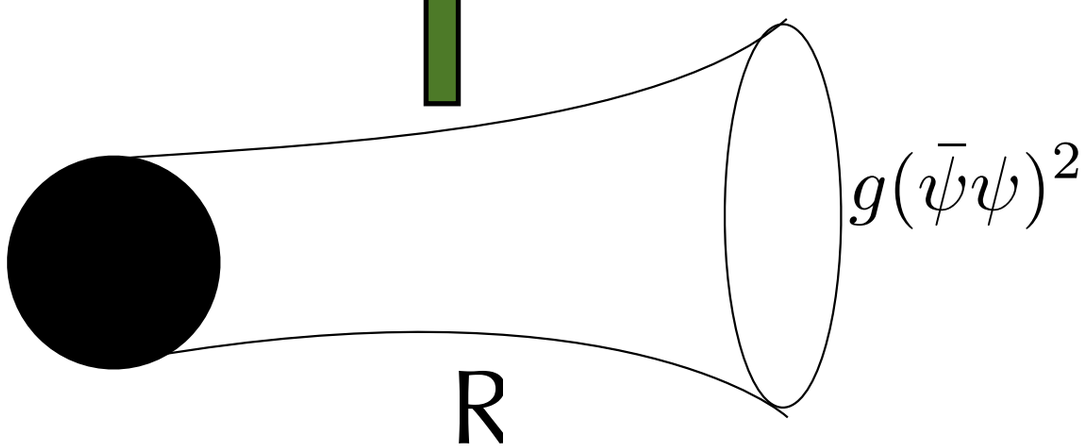

$R \propto \sqrt{g}$

How to demonstrate a strongly coupled fixed point?

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simple
but mindful argument

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$$S = -\int d^d x \left\{ -\varphi \nabla^2 \varphi + r\varphi^2 + g\varphi^4 \right\}$$

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zero
scaling
dimension

$$[\varphi] = (d-2)/2$$

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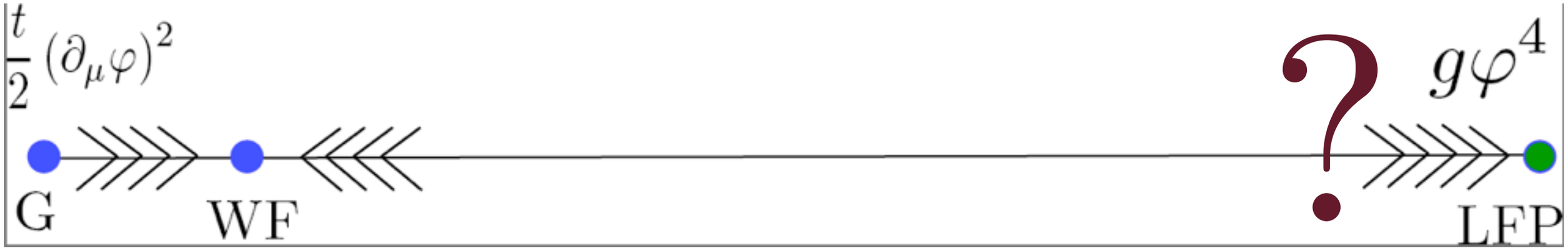
irrelevant for d<4!



engineering
dimension arguments
not really
valid at strong coupling

key idea:
solve problem exactly

isolate fixed point at

$r \to 0$ $\quad$ $g \to \infty$

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isolate fixed point at

$r \to 0 \qquad g \to \infty$

if one exists

step 1:

solve non-linear eigenvalue problem

$$-\nabla^2\varphi + r\varphi + g\varphi^3 = \lambda\varphi$$

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periodic BC

$$\varphi_n = c_n \mathrm{sn}(p_n \cdot x + \theta, m_n)$$

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Jacobi-Elliptic functions

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energy

$$\lambda_n = p_n^2 + r + \frac{g c_n^2}{2}$$

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Jacobi-Elliptic functions

$$\varphi_n = c_n \mathrm{sn}(p_n \cdot x + \theta, m_n)$$

energy → $\lambda_n = p_n^2 + r + \frac{gc_n^2}{2}$

modulus $0 < m_n < 1$ → $m_n = \frac{gc_n^2}{2p_n^2}$

step 1:

solve non-linear eigenvalue problem

$$-\nabla^2\varphi + r\varphi + g\varphi^3 = \lambda\varphi$$

periodic BC

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$$\lambda_n = p_n^2 + r + \frac{gc_n^2}{2}$$

modulus

$0 < m_n < 1$

$$m_n = \frac{gc_n^2}{2p_n^2}$$

Elliptic Integral of the first kind

boundary constraint

$$p_n = \frac{4K(m_n)n}{L}$$

more definitions

$$n \equiv \vec{n}$$

$$p_n^2 = \sum_{j=1}^{d} p_{n_{x_j}}^2$$

$\theta = 0$ even solutions

$\theta = 2K(m_n)$ odd solutions

aside:

$$\begin{aligned} \dot{y} &= x \\ \dot{x} &= -y \end{aligned}$$

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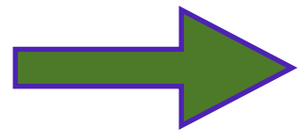

$$\begin{aligned} x &= \cos\theta \\ y &= \sin\theta \end{aligned}$$

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Jacobi Elliptics

$$\begin{aligned} \dot{x} &= yz \\ \dot{y} &= -zx \\ \dot{z} &= -k^2 xy \end{aligned} \quad \Rightarrow \quad \begin{aligned} &\mathrm{sn}(t,k) \\ &\mathrm{cn}(t,k) \\ &\mathrm{dn}(t,k) \end{aligned}$$

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Invariants

$$\begin{aligned} I &= x^2 + y^2 \\ J &= kx^2 + z^2 \end{aligned}$$

step 2:

$$S = -\int d^d x \varphi \{ \underbrace{-\nabla^2 \varphi + r\varphi + g\varphi^3}_{\lambda_n \varphi_n} \}$$

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$$S_n = -\int_0^L d^d x \, \lambda_n \varphi_n^2$$

expand diagonal elements of action in JEF

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step 3:

rescale momentum

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expand diagonal elements of action in JEF

step 3:

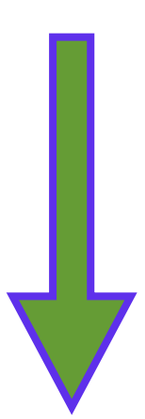
rescale momentum

$$k' = bk$$
$$b > 1$$
$$k = \frac{2\pi n}{L}$$

step 4:

energy shell RG

$$S_n \longleftrightarrow S_{n-1=n'}$$



find relations between (r′,r) and (g′,g)

step 5:

impose power-law scaling

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$$p' = b^{d_p} p$$

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kinetic term

$$\int d^d x \, p^2 c^2 \mathrm{sn}^2(px, m)$$

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$$\int d^d x \, p^2 c^2 \mathrm{sn}^2(px, m)$$

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$$c' = b^{\frac{d-2d_p}{2}} c$$

analogue of field scaling

step 5
continued:

modulus scaling

$$m = gc^2/2dp^2$$

step 5
continued:

modulus scaling

$$m = gc^2/2dp^2$$

$$\frac{g'c'^2}{2dp'^2} = \frac{g'c^2}{2dp^2} b^{d-4d_p}$$

$$\equiv m'b^{d-4d_p}$$

m′ and g′ scale jointly

step 5 continued:

modulus scaling

$$m = gc^2/2dp^2$$

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m′ and g′ scale jointly

$$\begin{aligned} p' &= \frac{2K(m'b^{d-4d_p})}{\pi} bk \\ \Rightarrow p' &= \frac{K(m'b^{d-4d_p})}{K(m)} bp \equiv b^{d_p} p \end{aligned}$$

step 5 continued:

modulus scaling

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$$m' = b^{4d_p - d} K^{-1}(b^{d_p - 1} K(m)).$$

step 5
continued:

scaling of r

$$d^d x' c'^2 (r' + g' c'^2/2)$$

step 5
continued:

scaling of r

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invariance of action

$$r' = \frac{d^d x c^2}{d^d x' c'^2}(r + g c^2/2) - g'^2 c'^2/2$$

step 5 continued:

scaling of r

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$$c'^2 = b^{d-2d_p} c^2$$
$$m d p^2 = g c^2/2$$

$$r' = b^{2d_p} r + b^{2d_p} p^2 \, (m - m')$$

summary of step 5

$$r' = b^{2d_p} r + b^{2d_p} p^2 \, (m - m')$$

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step 6:

fixed points

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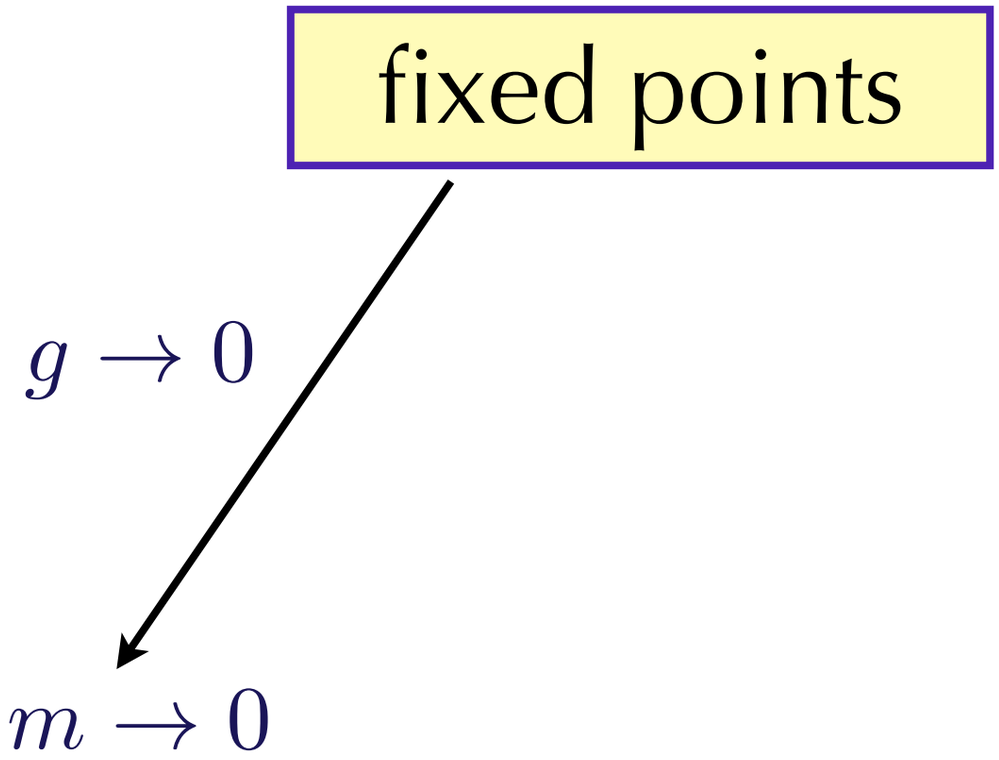

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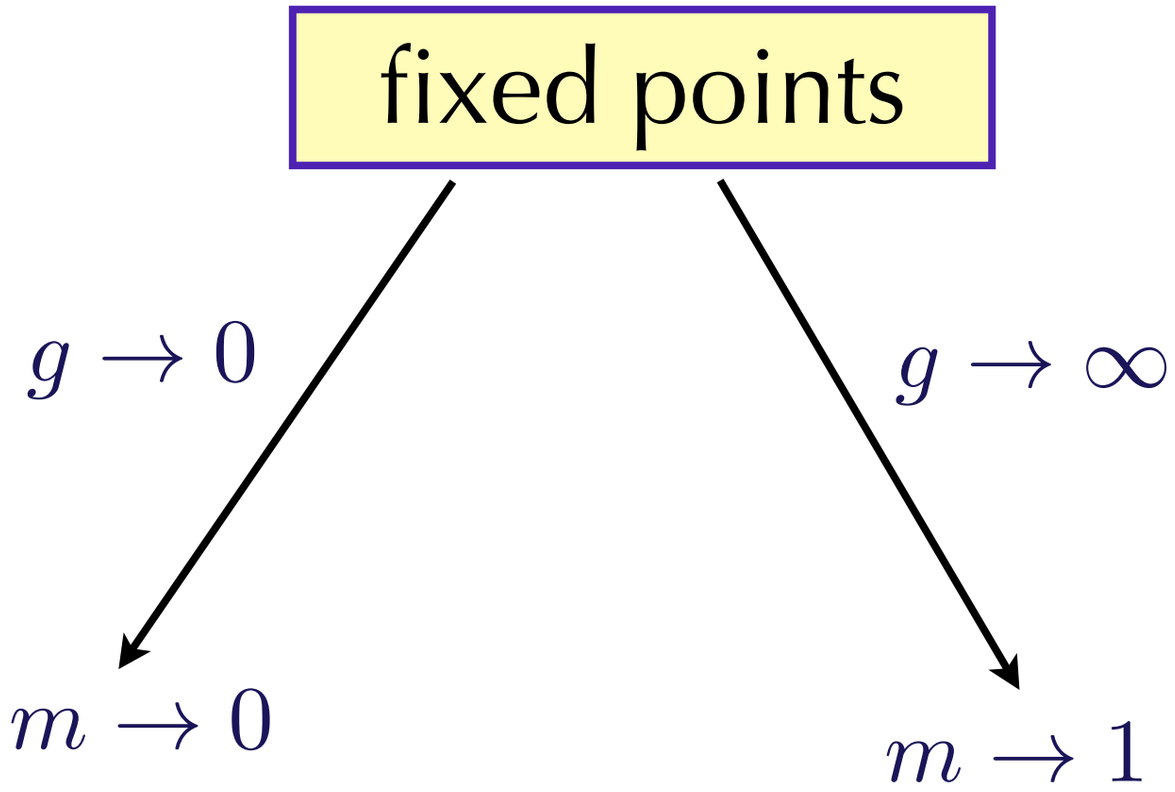

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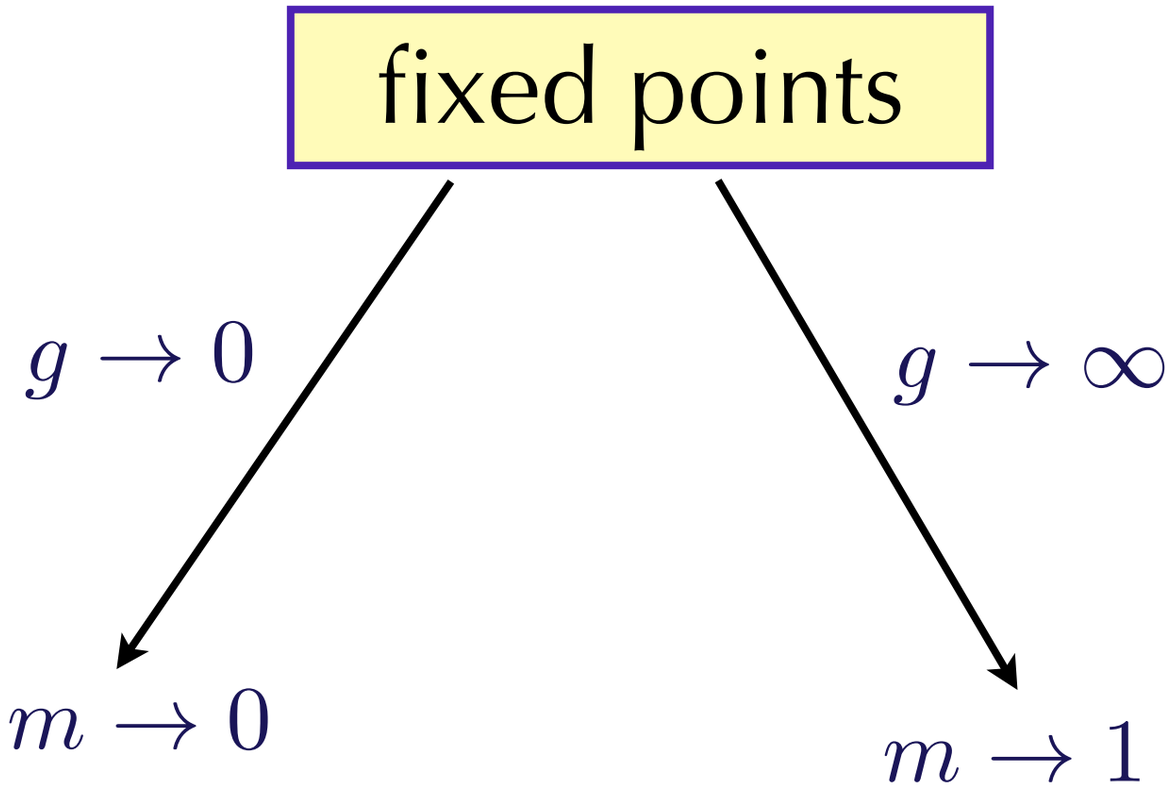

$$r' = b^{2d_p} r + b^{2d_p} p^2 \, (m - m')$$

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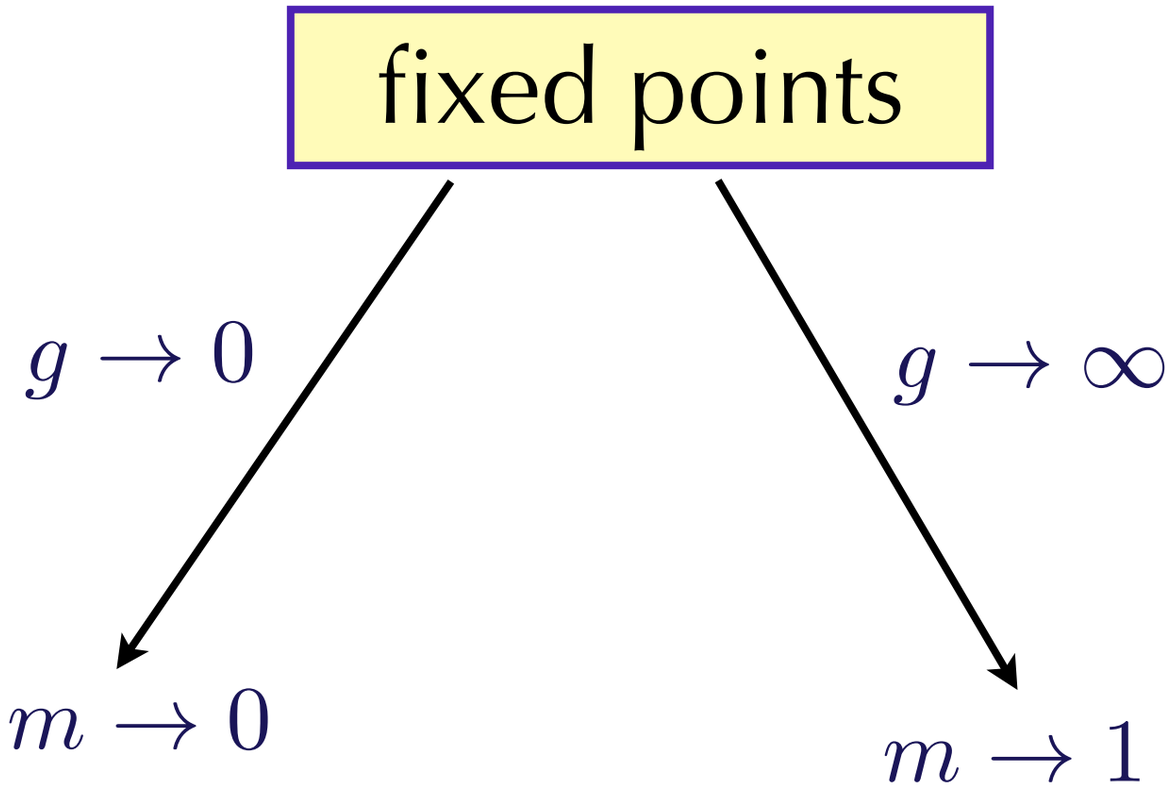
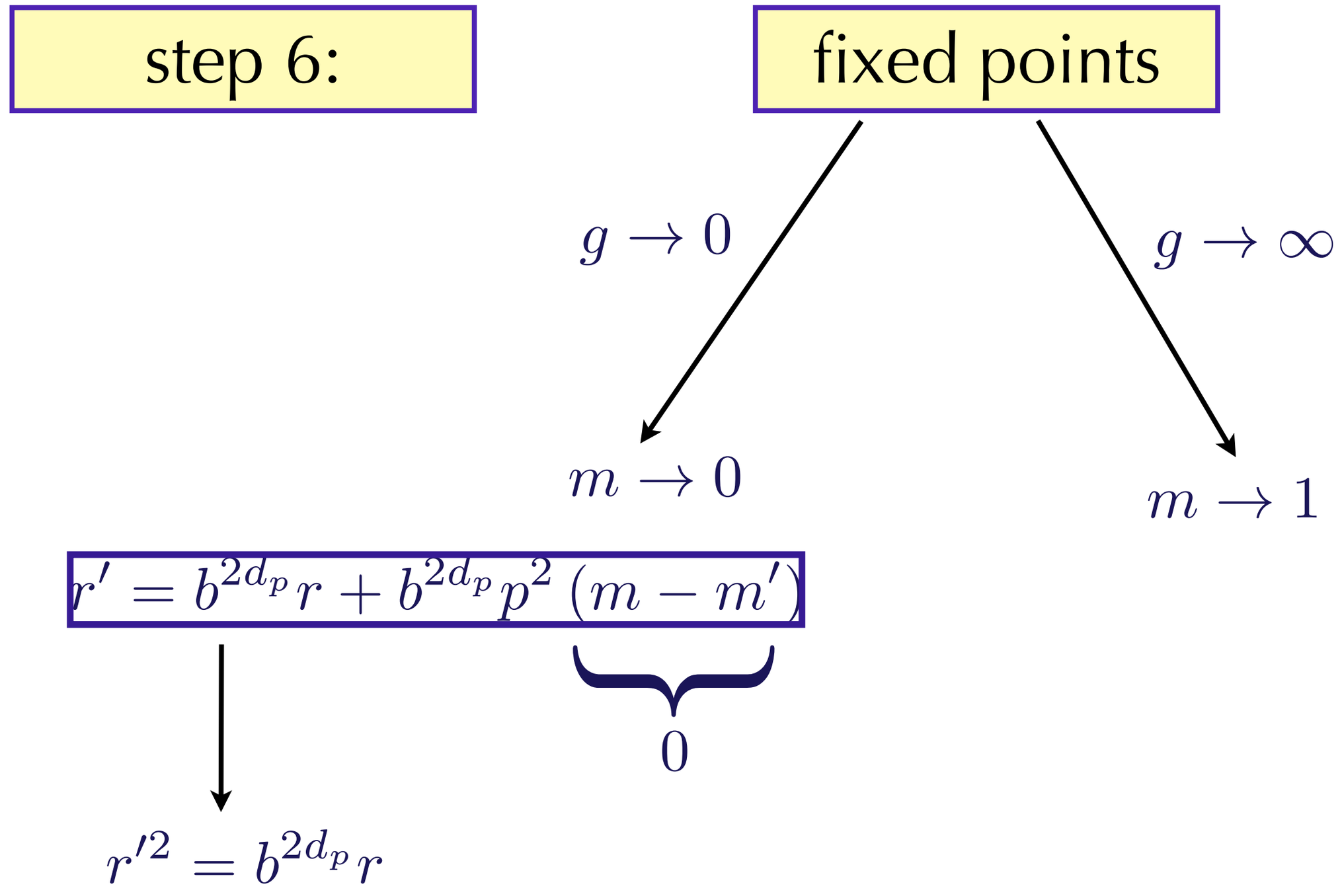


$$r' = b^{2d_p} r + b^{2d_p} p^2 \underbrace{(m - m')}_{0}$$

step 6:


fixed points
$g \to 0$
$g \to \infty$
$m \to 0$
$m \to 1$
$r' = b^{2d_p} r + b^{2d_p} p^2 \underbrace{(m - m')}_{0}$
$r'^2 = b^{2d_p} r$

step 6:

fixed points

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$m \to 0$

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$$r' = b^{2d_p} r + b^{2d_p} p^2 \underbrace{(m - m')}_{0}$$

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$$m' = b^{4d_p - d} K^{-1}(b^{d_p - 1} K(m))$$

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$g \to \infty$

$m \to 1$

$m' = b^{4d_p - d} K^{-1}(b^{d_p - 1} K(m))$

$m \to 0$

$K(0) = \pi/2$

$0 = K^{-1}(b^{d_p - 1} \pi/2)$

step 6:

fixed points

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$m \to 0$

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$m \to 1$

$\lim_{m \to 1} m'_{SC} = b^{4d_p - d} m$

summary

Gaussian

Strongly Coupled

summary

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$$0 = K^{-1}(b^{d_p - 1}\pi/2)$$

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$$0 = K^{-1}(b^{d_p - 1}\pi/2)$$

$$d_p = 1$$

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$$\lim_{m \to 1} m'_{SC} = b^{4d_p - d} m$$

summary

Gaussian

$$0 = K^{-1}(b^{d_p - 1}\pi/2)$$

$$d_p = 1$$

Strongly Coupled

$$\lim_{m \to 1} m'_{SC} = b^{4d_p - d} m$$

$m \to 1$

$$d_p = \frac{d}{4}$$

step 6:

exponents

$$r' = b^{2d_p} r$$

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$$\nu = \frac{1}{2d_p}$$

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exponents

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$$\nu = \frac{1}{2d_p}$$

Gaussian

$$\nu = \frac{1}{2}$$

SC

$$\nu = \frac{2}{d} \Rightarrow \alpha = 0$$

$$\eta$$

$$c' = b^{\frac{d-2d_p}{2}} c$$

analogue of field scaling

$$\eta$$

$$c' = b^{\frac{d-2d_p}{2}} c$$

analogue of field scaling

$$\eta = \frac{d-2d_p}{2} - \frac{d-2}{2}$$ ← free limit

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free limit

Gaussian

$$\eta = 0$$

$$\eta$$

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analogue of field scaling

$$\eta = \frac{d-2d_p}{2} - \frac{d-2}{2}$$

free limit

Gaussian

$$\eta = 0$$

SC

$$\eta = \frac{1}{2} - \frac{d}{8}$$

$$r \to 0$$

$$g \to \infty$$

FP	G	SC
ν	$\frac{1}{2}$	$\frac{2}{d}$
η	0	$\frac{1}{2} - \frac{d}{8}$
α	0	0
β	$\frac{1}{2}$	$\frac{7}{8} - \frac{3}{2d}$
γ	1	$\frac{3}{d} + \frac{1}{4}$
δ	3	$\frac{9d+12}{7d-12}$

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$d = 2$

Onsager d=2 results

$$r \to 0$$

$$g \to \infty$$

FP	G	SC
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η	0	$\frac{1}{2} - \frac{d}{8}$
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γ	1	$\frac{3}{d} + \frac{1}{4}$
δ	3	$\frac{9d+12}{7d-12}$

$d = 2$

Onsager d=2 results

$d = 4$

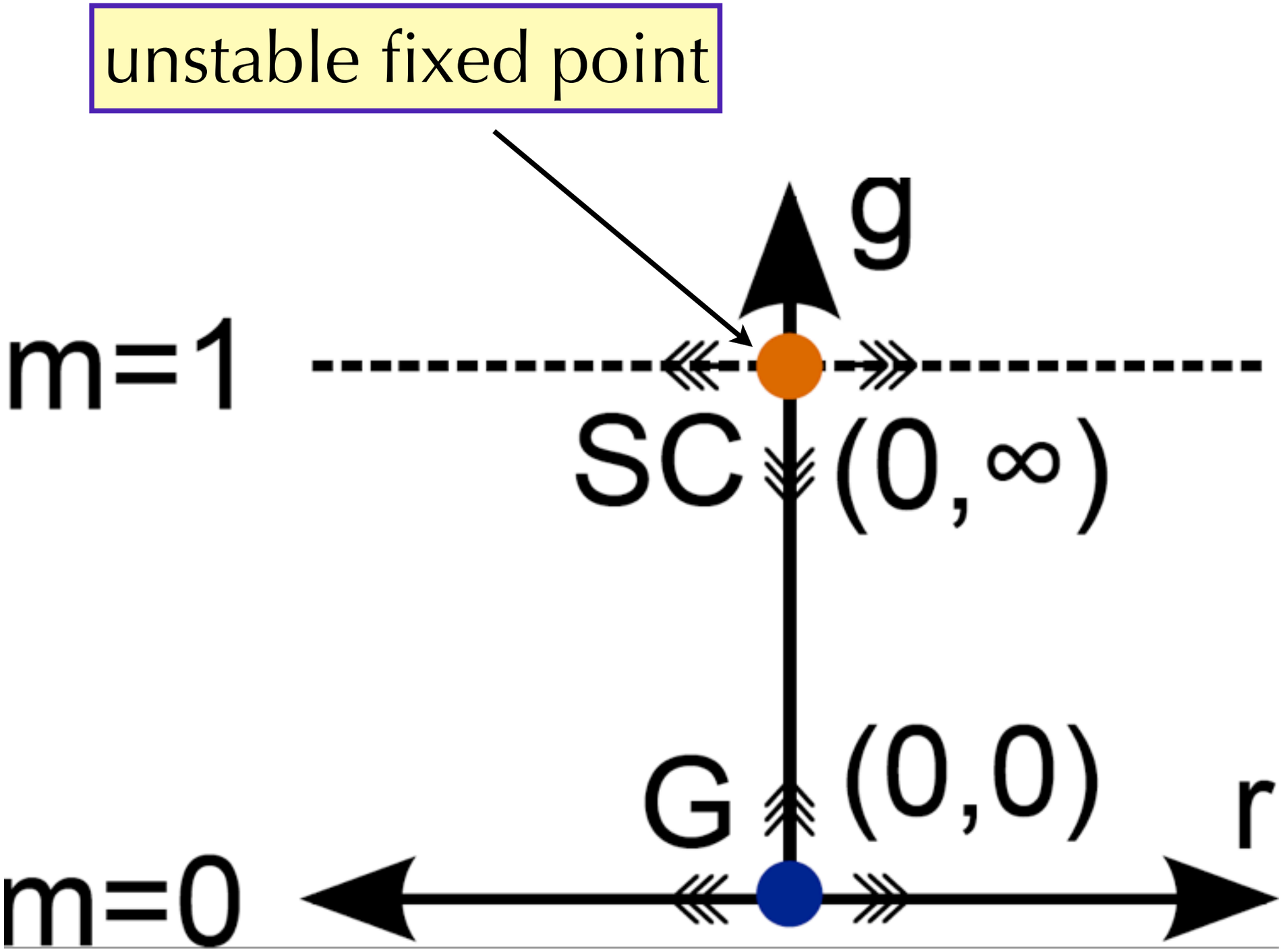
MF exponents

$$m'_{b_p=d/4} = K^{-1}(b^{d/4-1}K(m))$$

m′ decreases for d<4

$$r'_{b_p=d/4} = b^{d/2}r$$

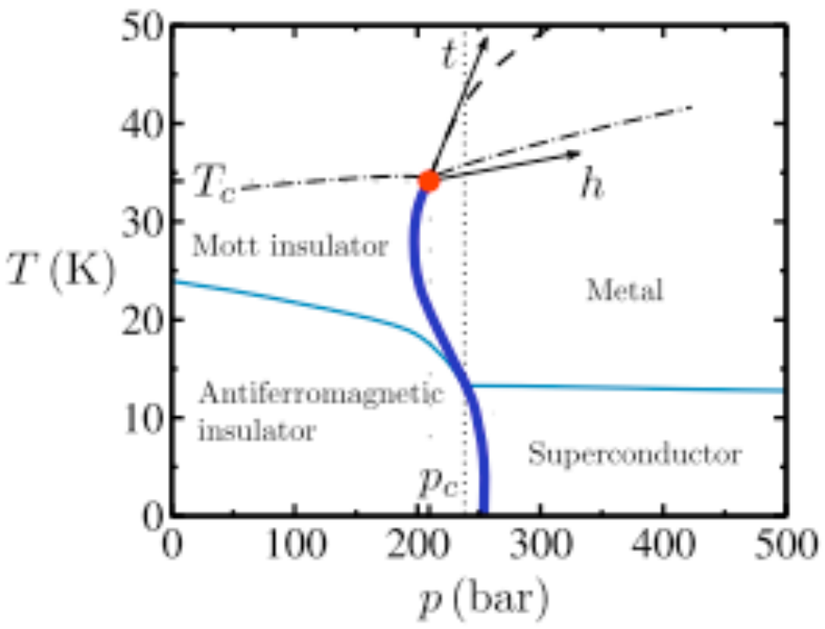
d=2, r is marginal


unstable fixed point
g
m=1
SC
(0,∞)
G
(0,0)
r
m=0

what systems exhibit $\alpha = 0$

what systems exhibit $\alpha = 0$

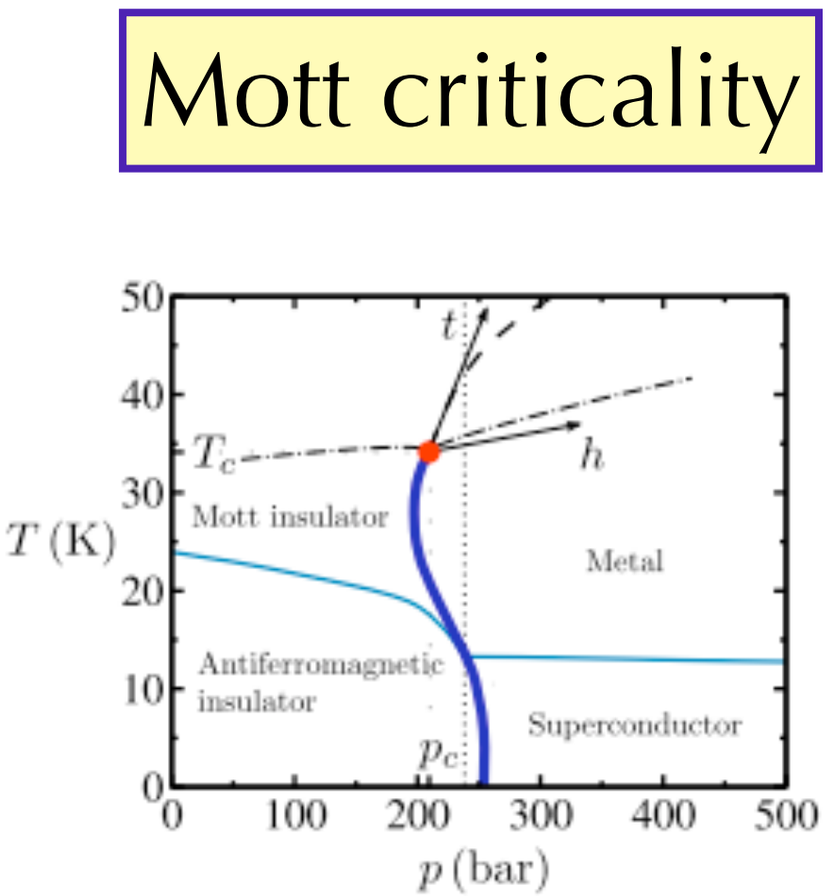
Mott criticality



organics, oxides

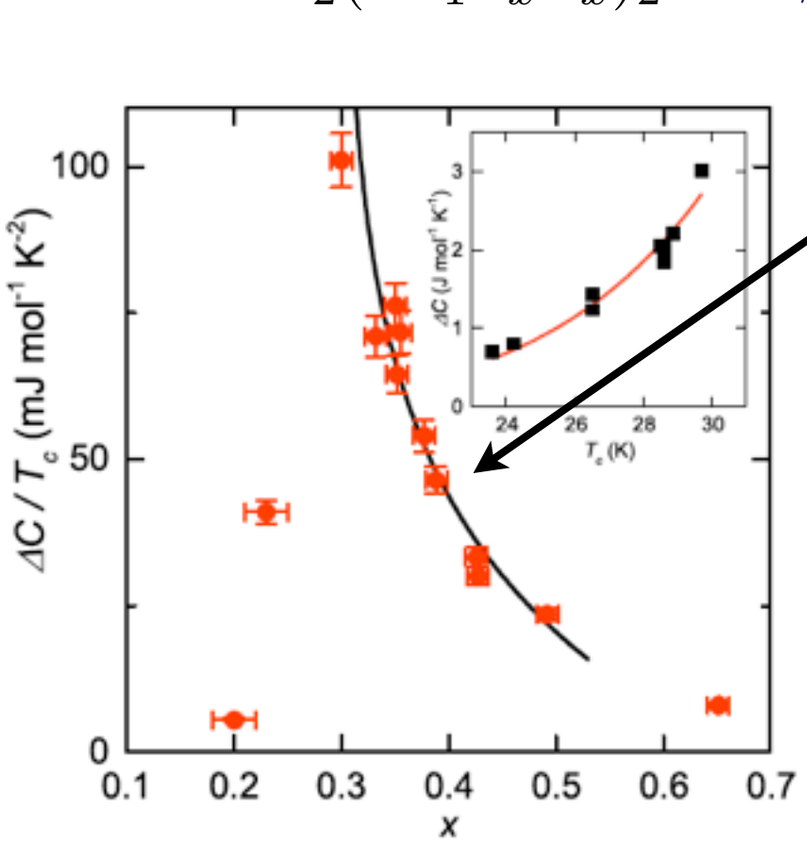
what systems exhibit $\alpha = 0$

Mott criticality



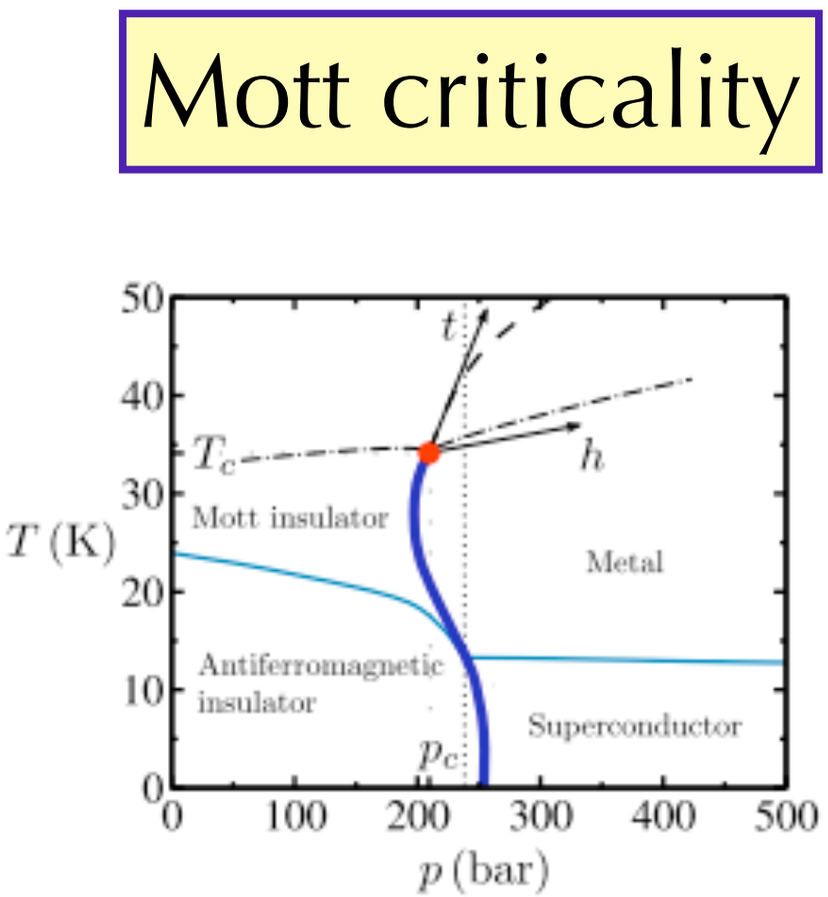
organics, oxides

$BaFe_2(As_{1-x}P_x)_2$ $\Delta C/T_c = c_0 + c_1 \ln|x - .31|$



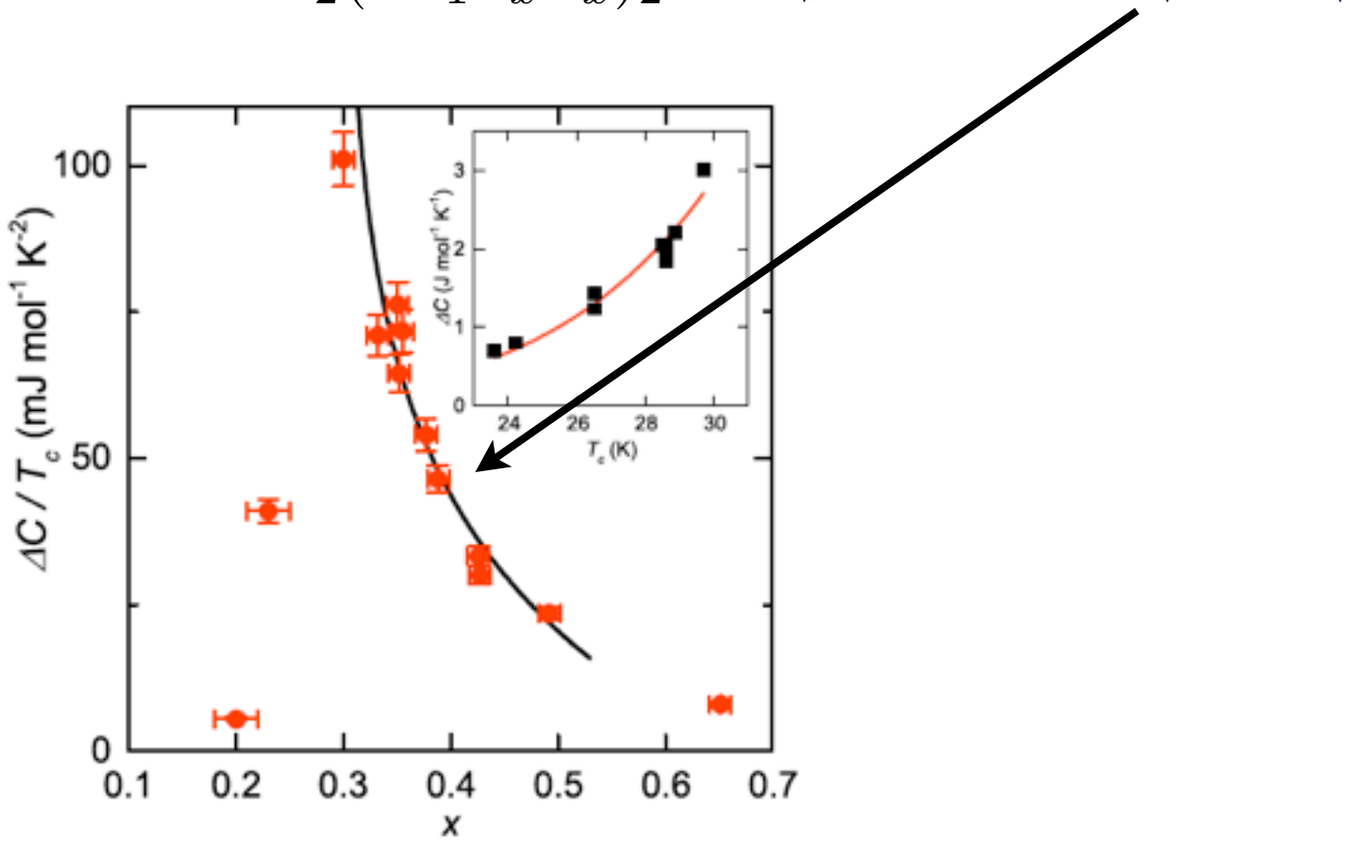
what systems exhibit $\alpha = 0$

Mott criticality



organics, oxides

$BaFe_2(As_{1-x}P_x)_2$ $\Delta C/T_c = c_0 + c_1 \ln|x - .31|$



$r \to 0$

$g \to \infty$

fixed point

step 7:

put paper on web and wait for comment by E. Fradkin

step 7:

put paper on web and wait for comment by E. Fradkin

Philip

I saw the paper you post. This paper is wrong. What you are doing is not looking a new strong coupling fixed point but simply perturbing around the infinite temperature limit of the classical Ising model.

Eduardo

step 7:

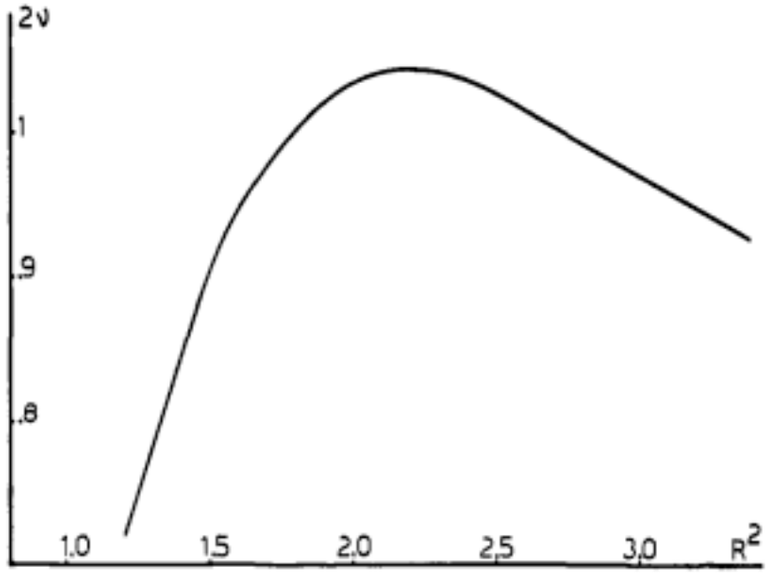


Fig. 14.

in the limit $\lambda \to \infty$ (taking $\frac{1}{2}m + \lambda/4! = 1$) the $\lambda\Phi^4$ theory becomes the Ising model. $\lambda\Phi^4$ theory differs from the Ising model in the value of the semi-invariants H_{2n}. For the Ising model

$$m \to -\infty$$
$$\lambda \to \infty$$

put paper on web and wait for comment by E. Fradkin

Philip

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Eduardo

Eduardo

we studied

$$r \to 0$$

$$g \to \infty$$

we studied

$$r \to 0$$

$$g \to \infty$$

generalise to quantum case