Unparticles and Emergent Mottness

Thanks to: NSF, EFRC (DOE)

Kiaran dave

Charlie Kane

Brandon Langley

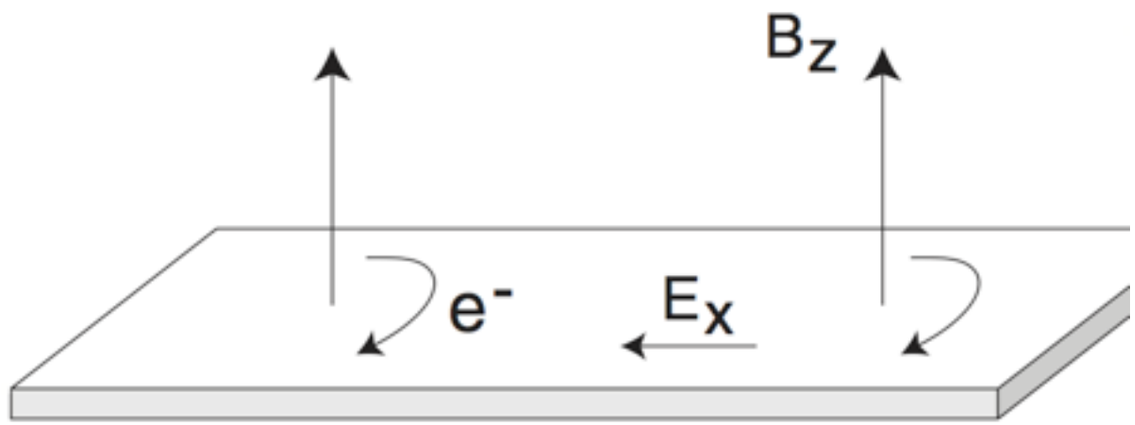
J. A. Hutasoit

Correlated Electron Matter

Correlated Electron Matter

What is carrying the current?

fractional quantum Hall effect

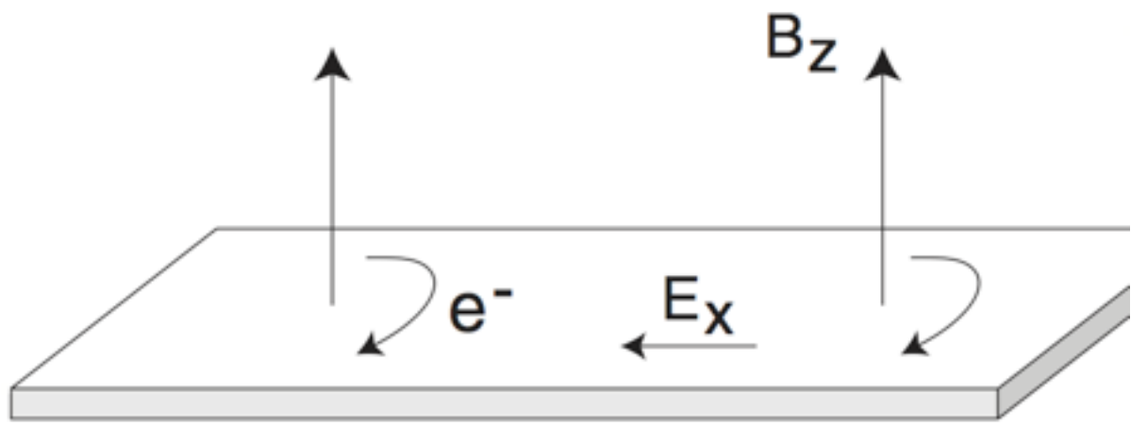


current-carrying excitations

$$e^* = \frac{e}{q}$$

(q odd)

fractional quantum Hall effect

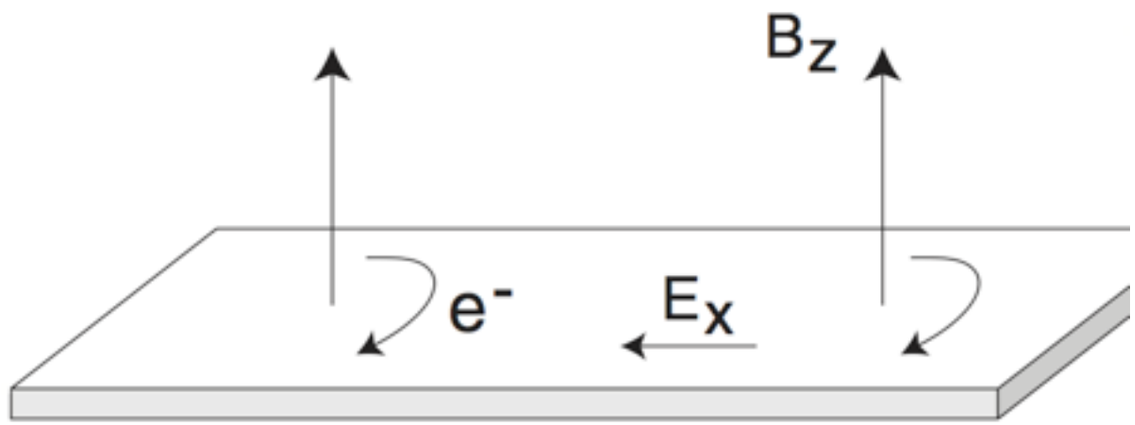


current-carrying excitations

$$e^* = \frac{e}{q}$$

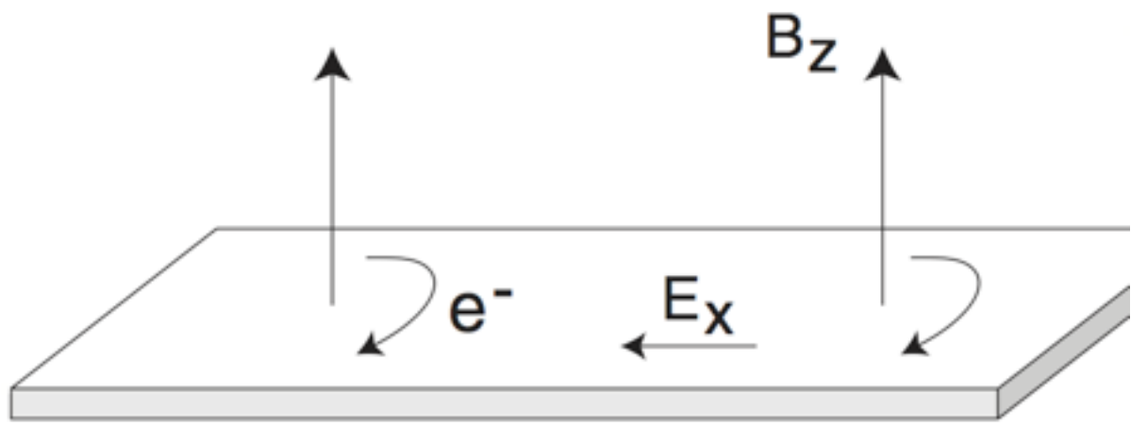
(q odd)

fractional quantum Hall effect



current-carrying excitations

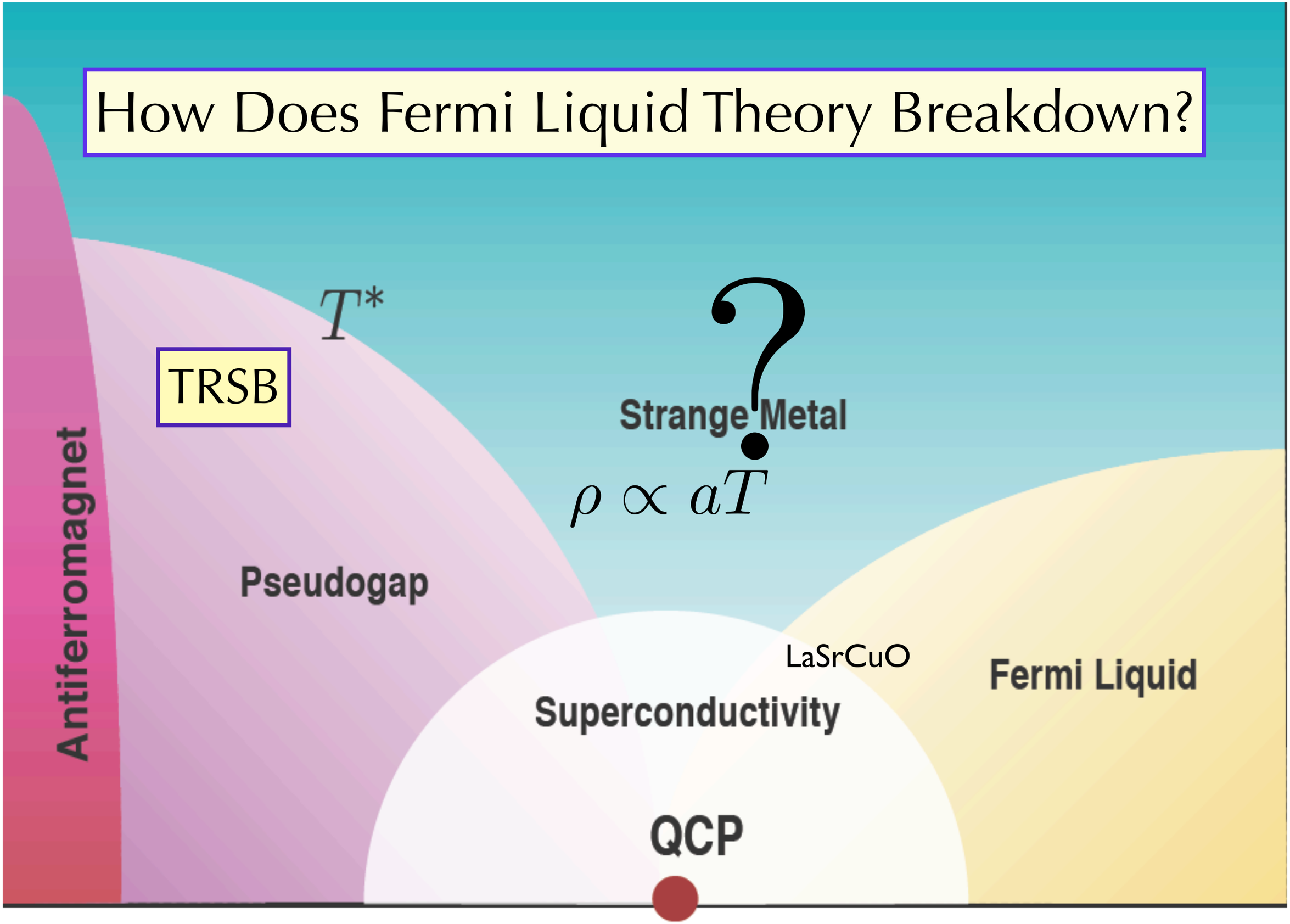
fractional quantum Hall effect



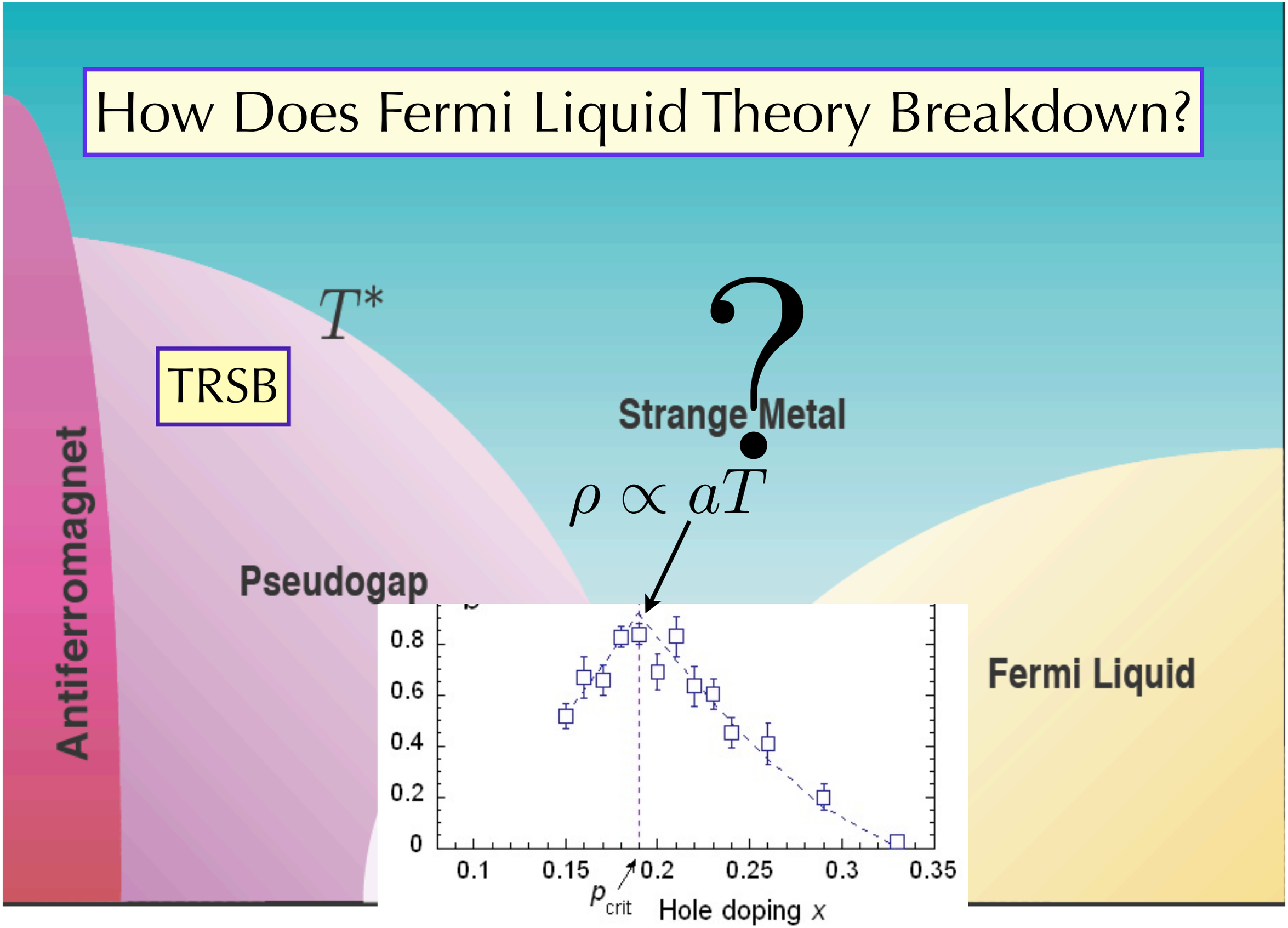
current-carrying excitations

$$e^* = ?$$

How Does Fermi Liquid Theory Breakdown?



How Does Fermi Liquid Theory Breakdown?



can the charge density be given a particle interpretation?

can the charge density be given a particle interpretation?

does charge density= total number of electrons?

if not?

if not?

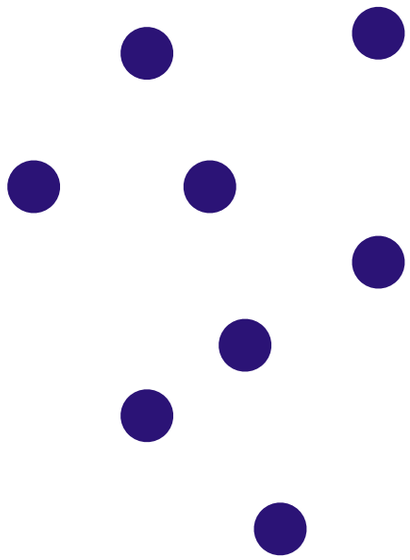
charge stuff= particles + other stuff

if not?

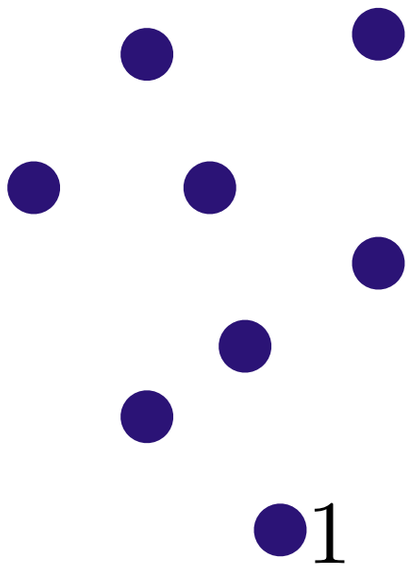
charge stuff= particles + other stuff

unparticles!

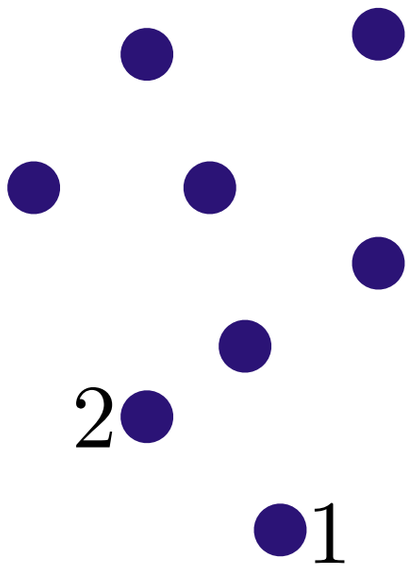
counting particles



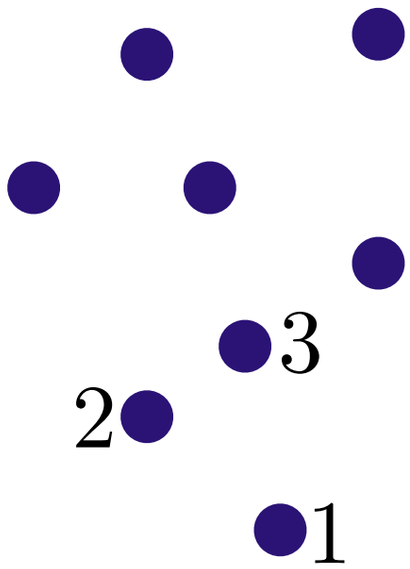
counting particles



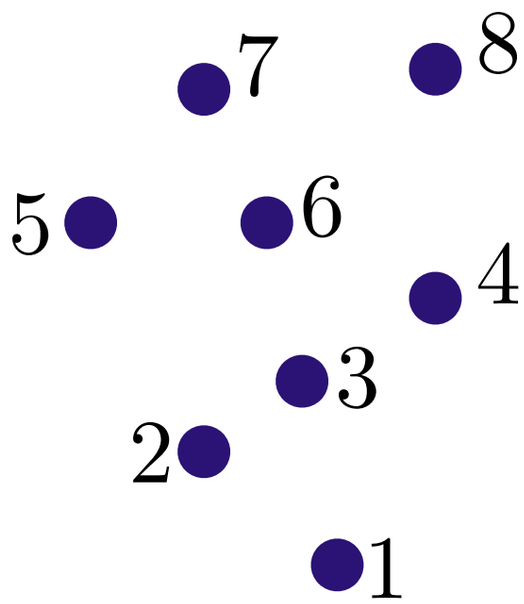
counting particles



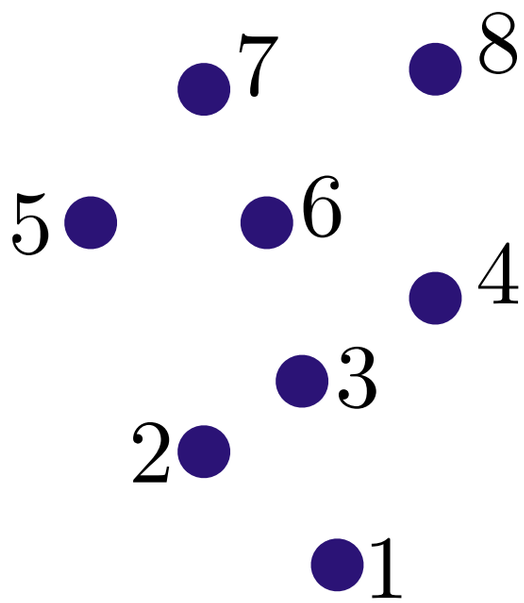
counting particles



counting particles

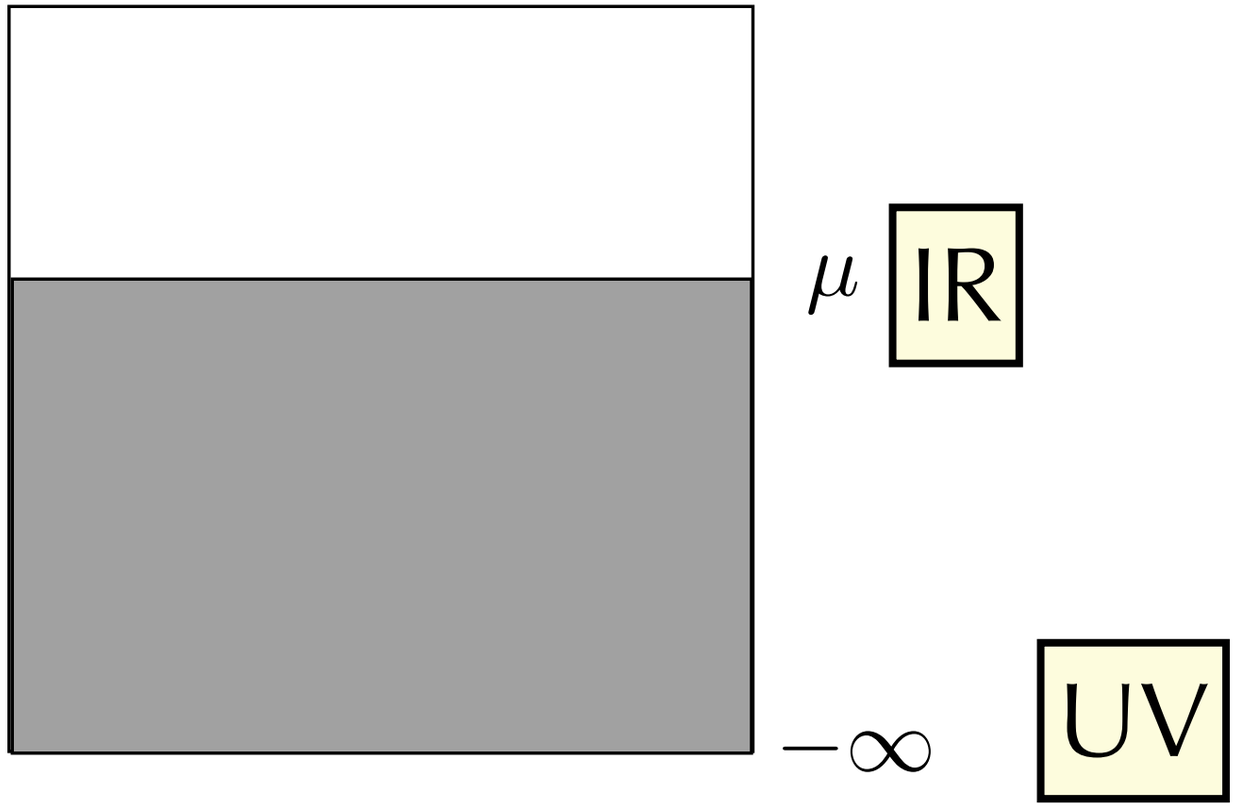


counting particles

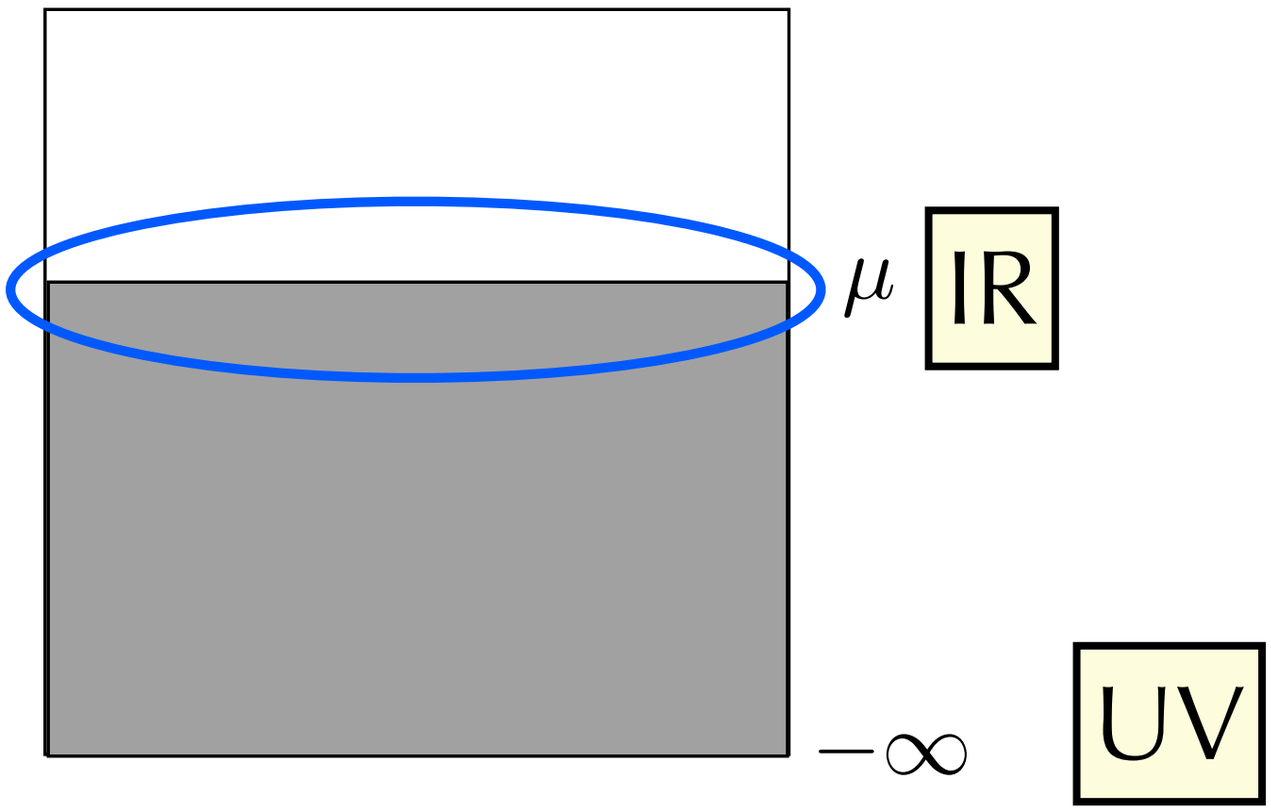


is there a more efficient way?

$$n = \int_{-\infty}^{\mu} N(\omega) d\omega$$



$$n = \int_{-\infty}^{\mu} N(\omega) d\omega$$



can n be deduced entirely from the IR(low-energy) scale?

Luttinger's `theorem'

Luttinger's theorem

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

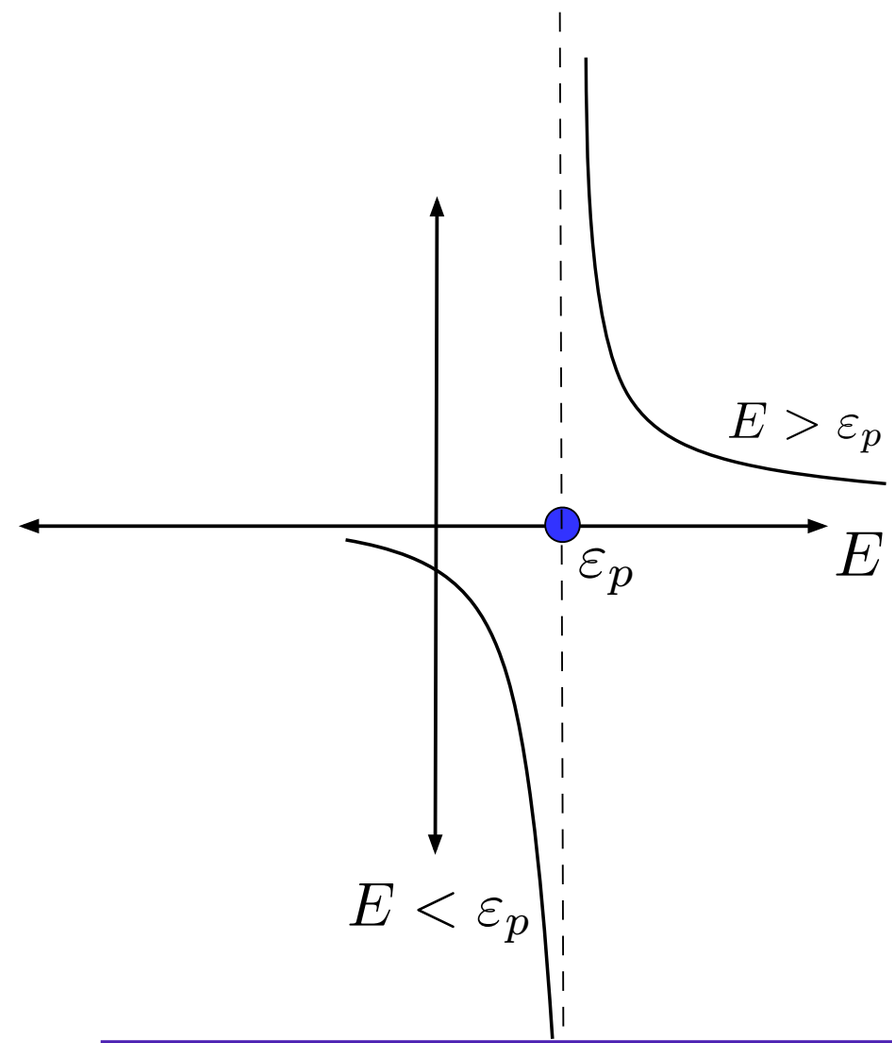
Luttinger's theorem

$$n = 2\sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

Luttinger's theorem

$$G(E) = \frac{1}{E - \varepsilon_p}$$

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$



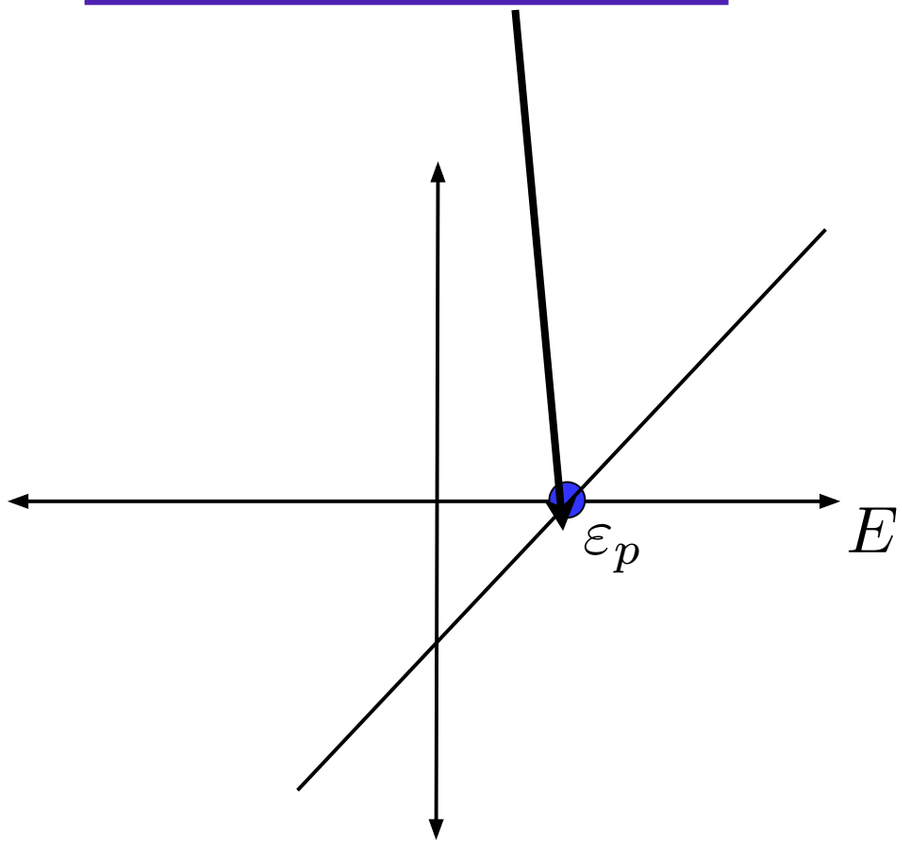
counting poles (qp)

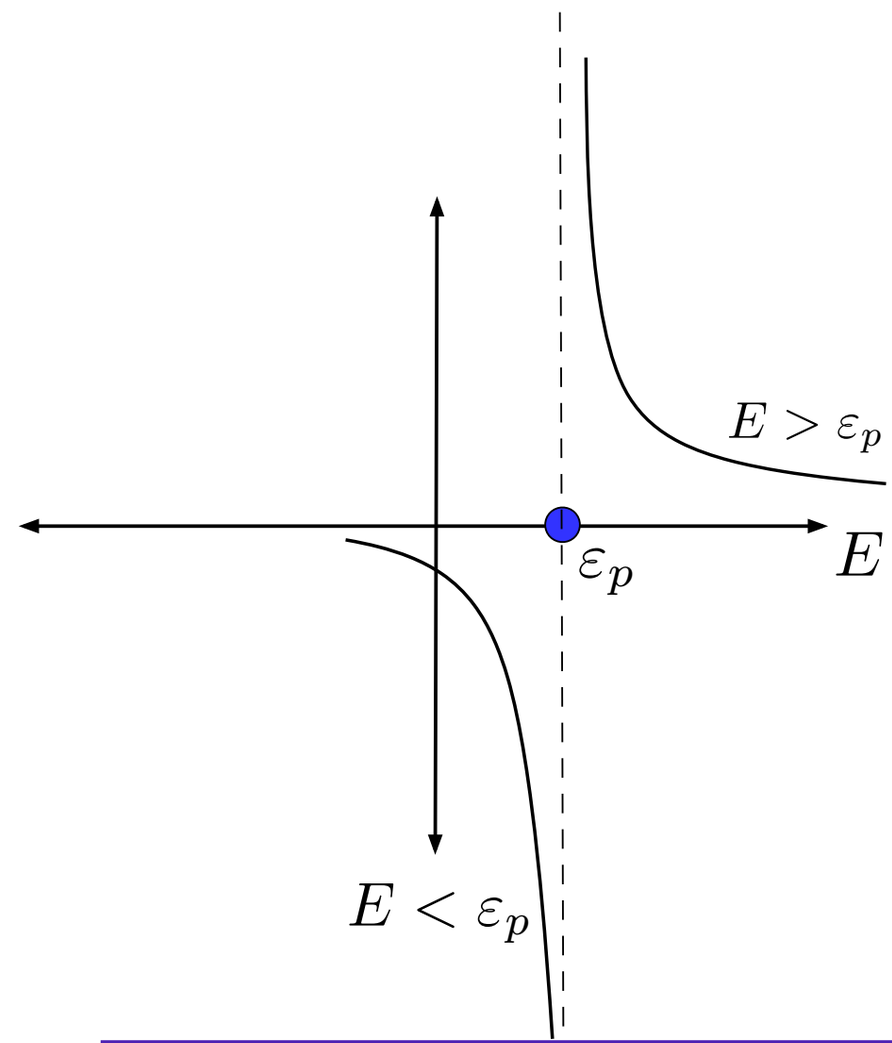
Luttinger's theorem

$$G(E) = \frac{1}{E - \varepsilon_p}$$

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

zero-crossing





counting poles (qp)

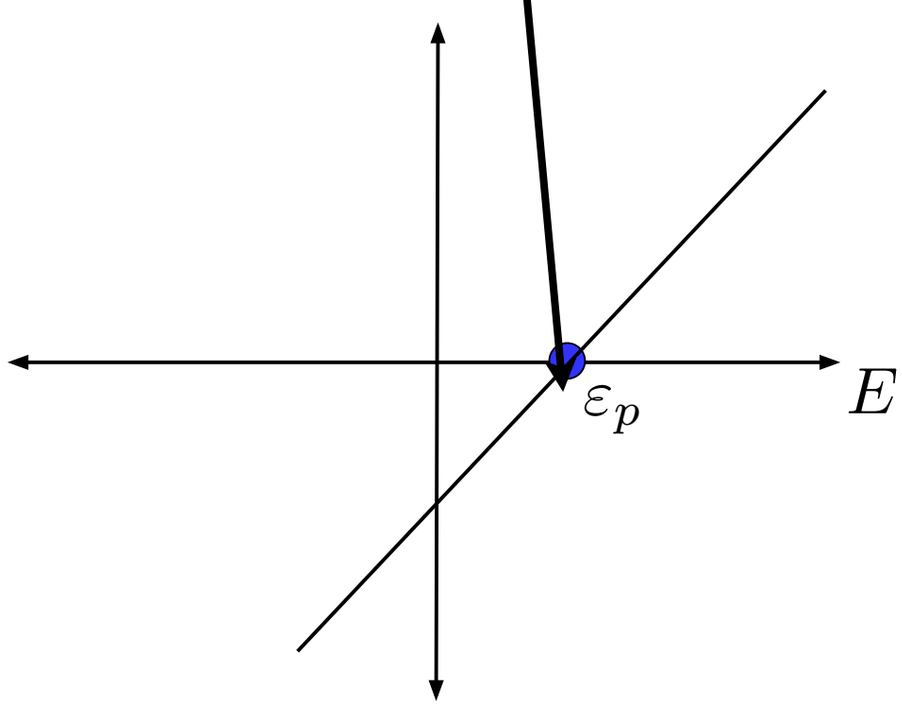
Luttinger's theorem

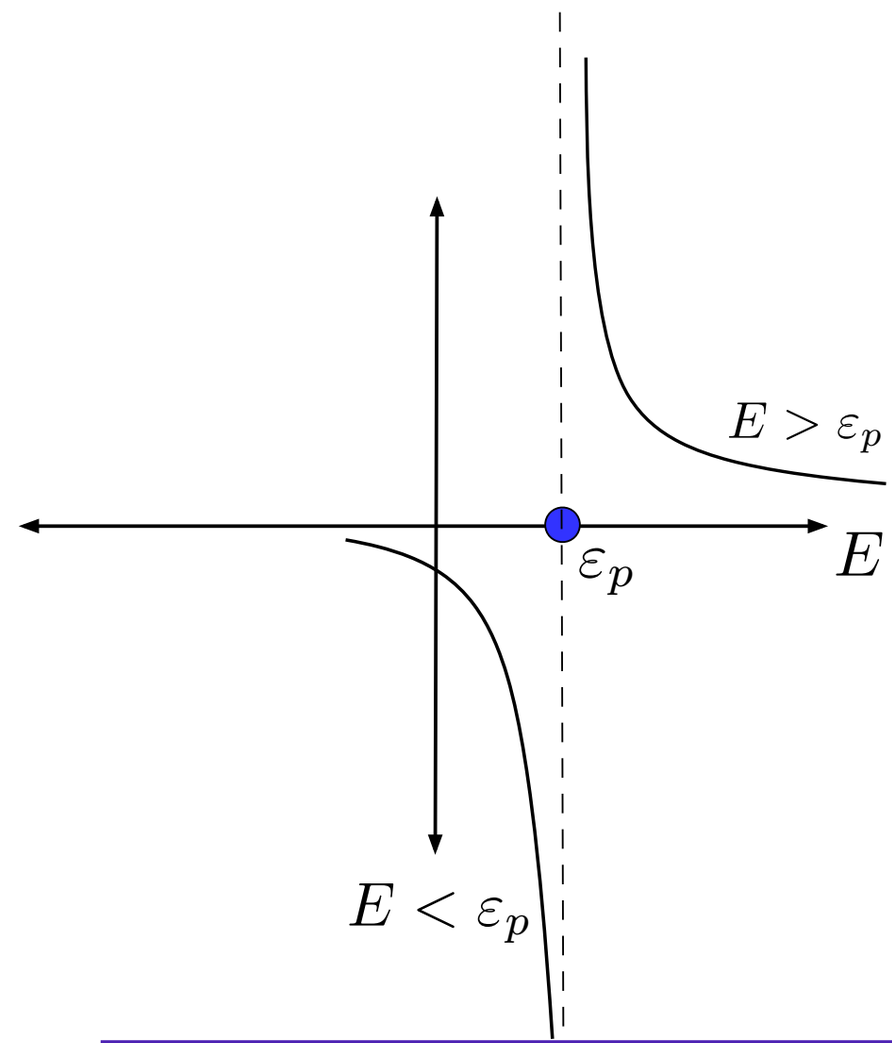
$$G(E) = \frac{1}{E - \varepsilon_p}$$

$$n = 2\sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

zero-crossing

$$\mathrm{Det}G(\omega = 0, \vec{p}) = 0$$





counting poles (qp)

singularities of $\ln G$

$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d\mathbf{p} \int_{-\infty}^{0} d\xi \ln \frac{G^R(\xi, \mathbf{p})}{G_R^*(\xi, \mathbf{p})}$$

poles+zeros
(all sign changes)

singularities of $\ln G$

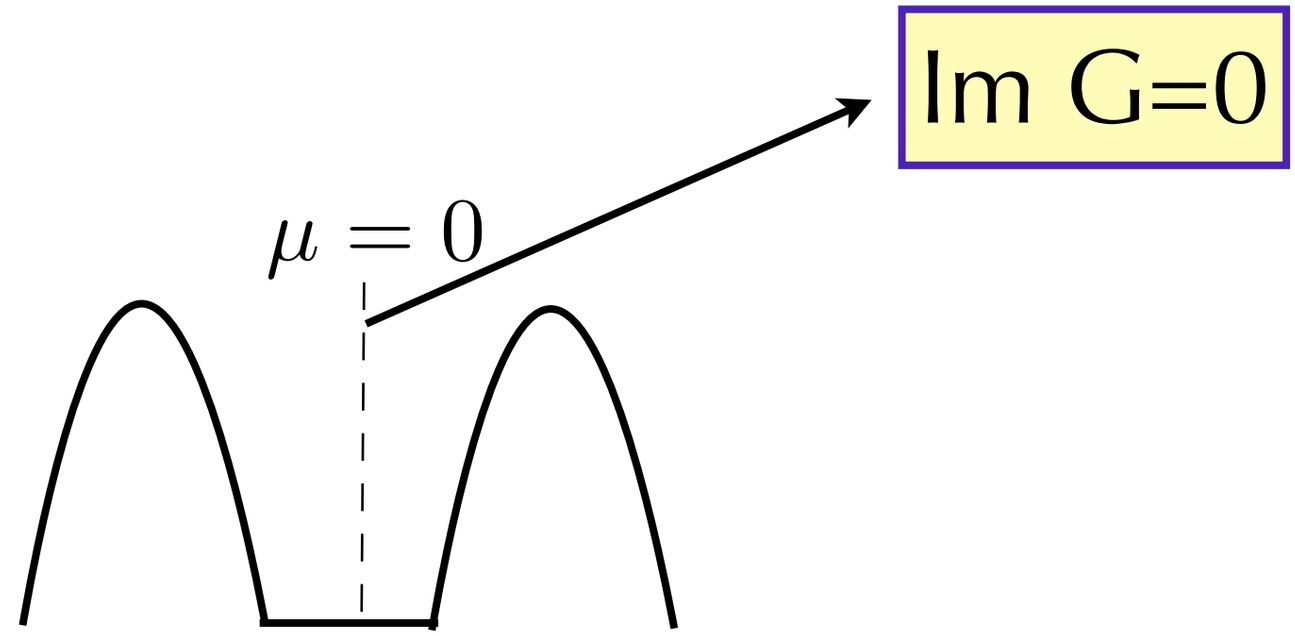
$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d\mathbf{p} \int_{-\infty}^{0} d\xi \ln \frac{G^R(\xi, \mathbf{p})}{G^*_R(\xi, \mathbf{p})}$$

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

poles+zeros
(all sign changes)

what are zeros?

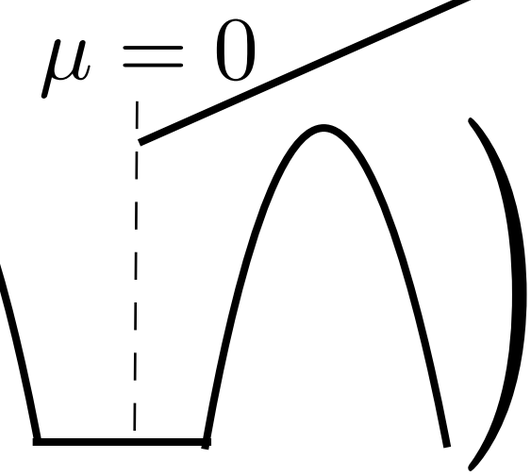
what are zeros?



what are zeros?

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{\left(\quad \right)}{\omega} d\omega$$

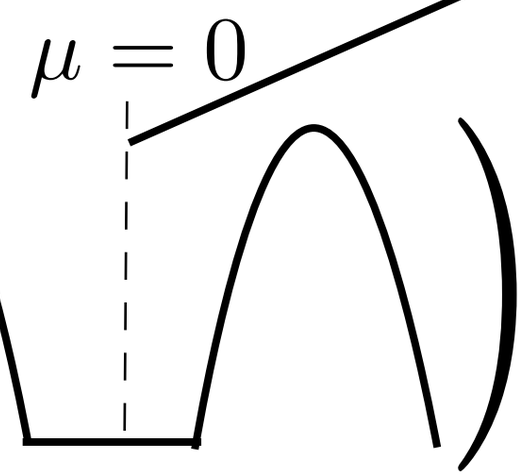


Kramers -Kronig

what are zeros?

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{\left(\quad \right)}{\omega} d\omega$$



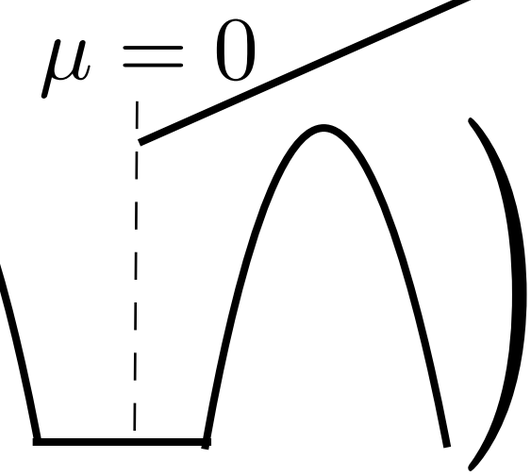
Kramers -Kronig

= below gap+above gap

what are zeros?

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{\left(\quad \right)}{\omega} d\omega$$



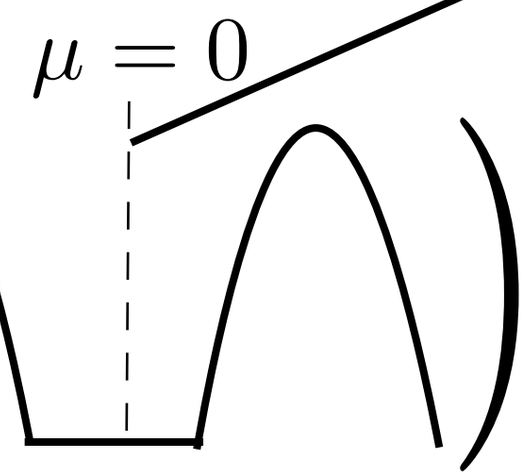
Kramers -Kronig

= below gap+above gap = 0

what are zeros?

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{\left(\quad \right)}{\omega} d\omega$$



Kramers -Kronig

= below gap+above gap = 0

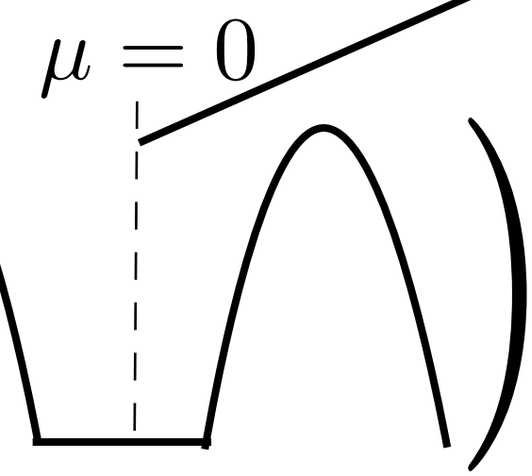
$$\mathrm{Det}G(\mathbf{k}, \omega = \mathbf{0}) = \mathbf{0}$$

(single band)

what are zeros?

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{\left(\quad \right)}{\omega} d\omega$$



Kramers -Kronig

= below gap+above gap = 0

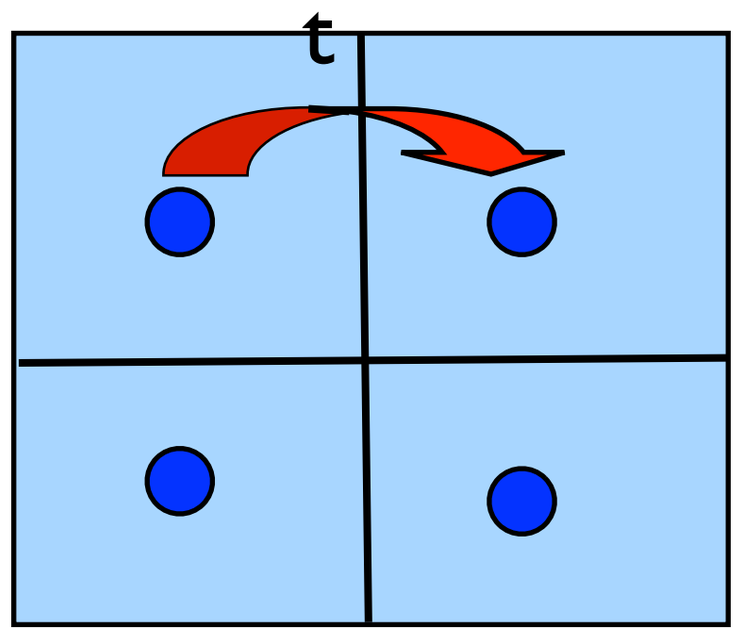
$$\mathrm{Det}G(\mathbf{k}, \omega = \mathbf{0}) = \mathbf{0}$$ (single band)

strongly correlated gapped systems

Mott mechanism (not Slater)

NiO insulates

d^8 ?

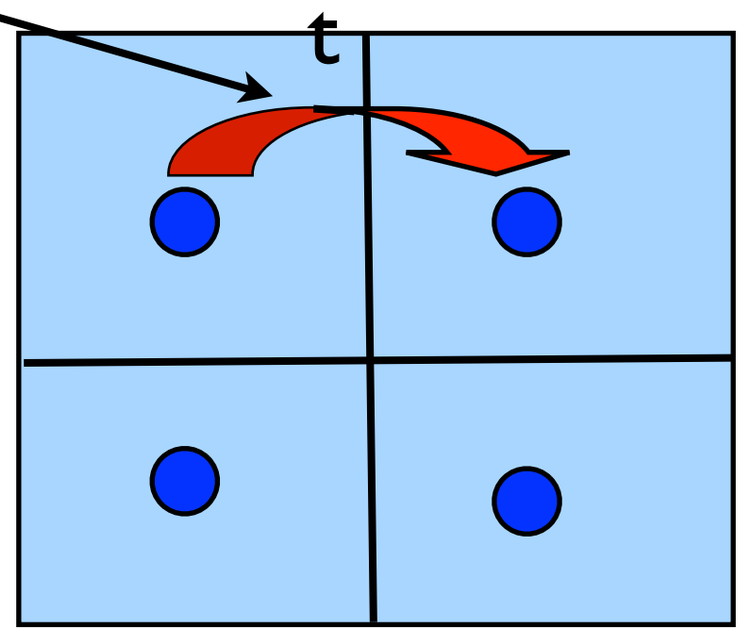


Mott mechanism (not Slater)

NiO insulates

d^8 ?

perhaps this costs energy

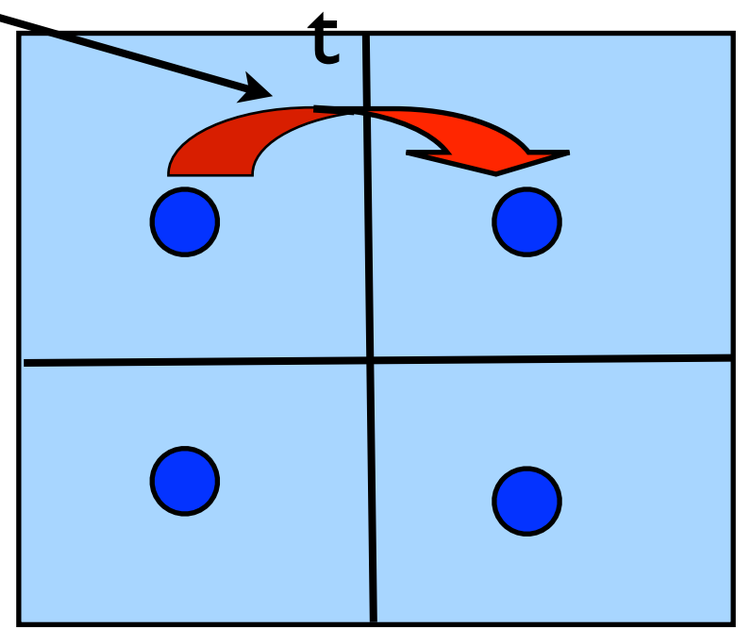


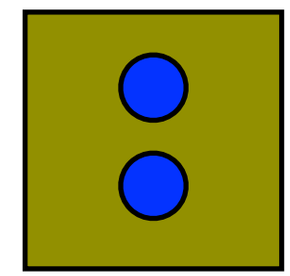
Mott mechanism (not Slater)

NiO insulates

d^8 ?

perhaps this costs energy





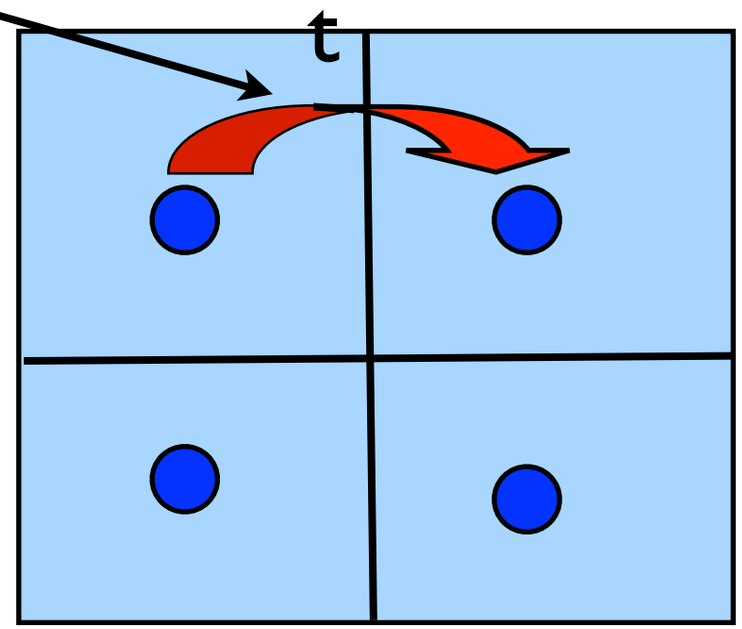
$$U \gg t$$

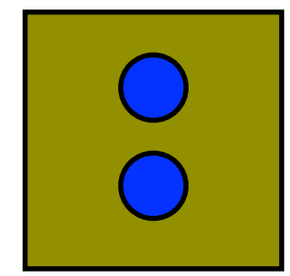
Mott mechanism (not Slater)

NiO insulates

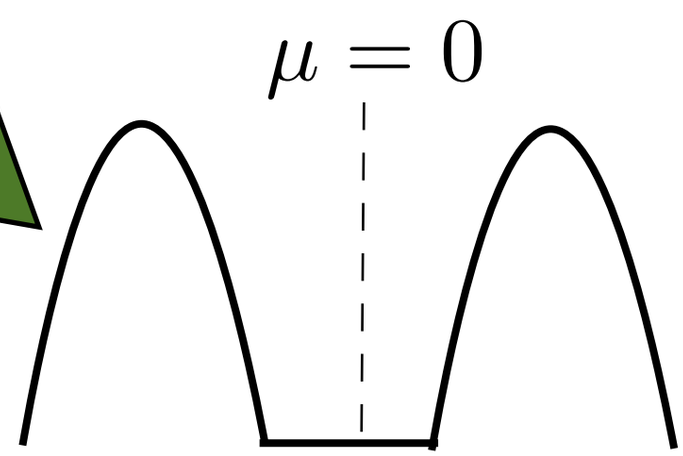
d^8 ?

perhaps this costs energy





$U \gg t$

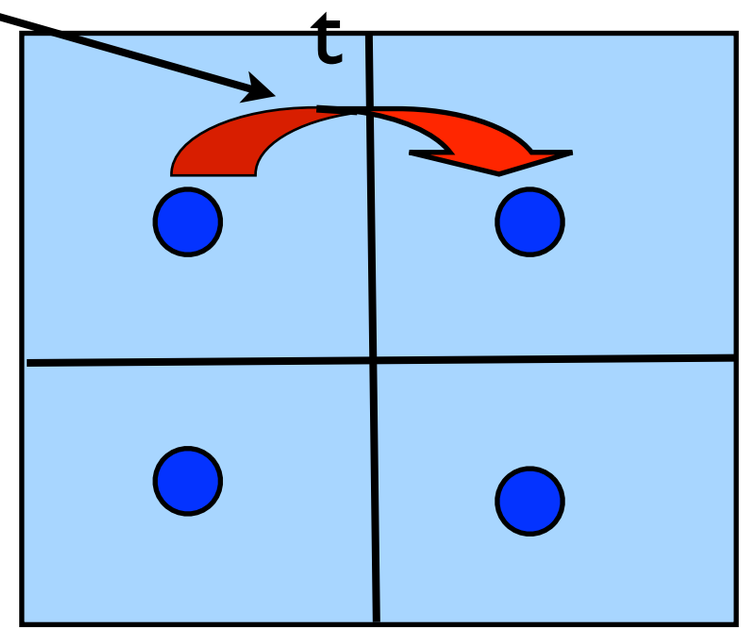


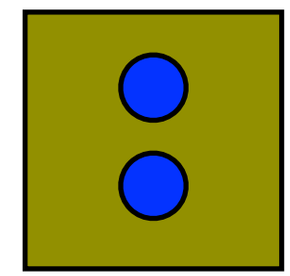
Mott mechanism (not Slater)

NiO insulates

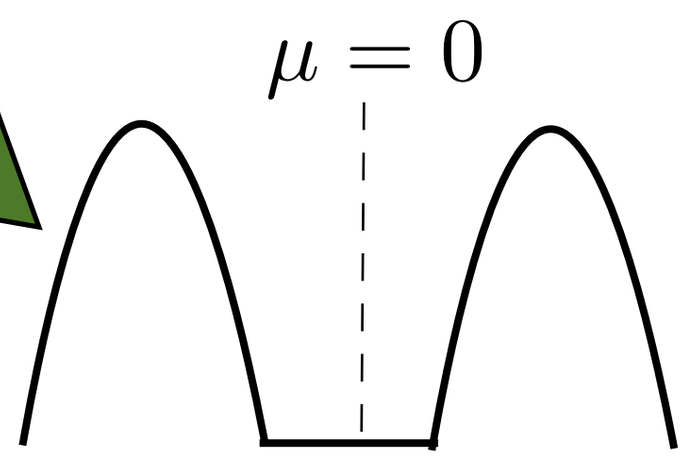
d^8 ?

perhaps this costs energy





$U \gg t$



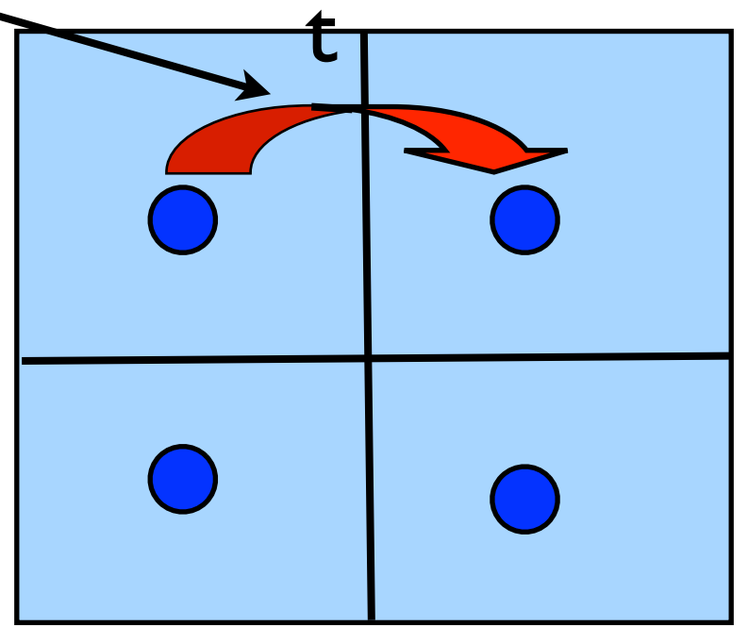
no change in size of Brillouin zone

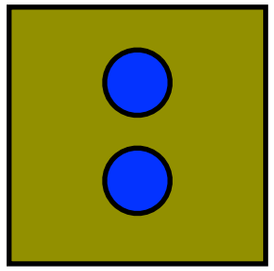
Mott mechanism (not Slater)

NiO insulates

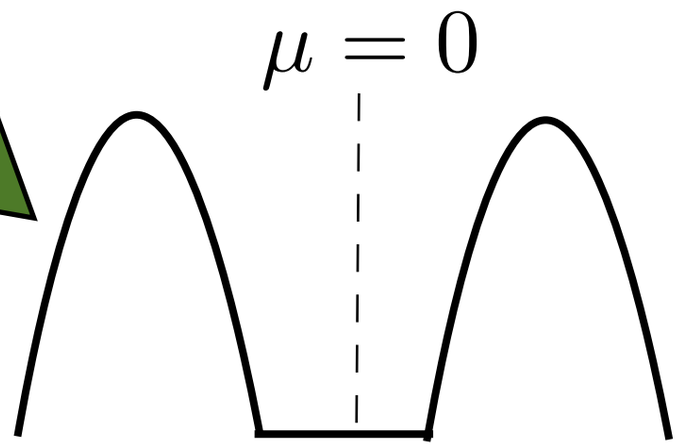
d^8 ?

perhaps this costs energy





$U \gg t$



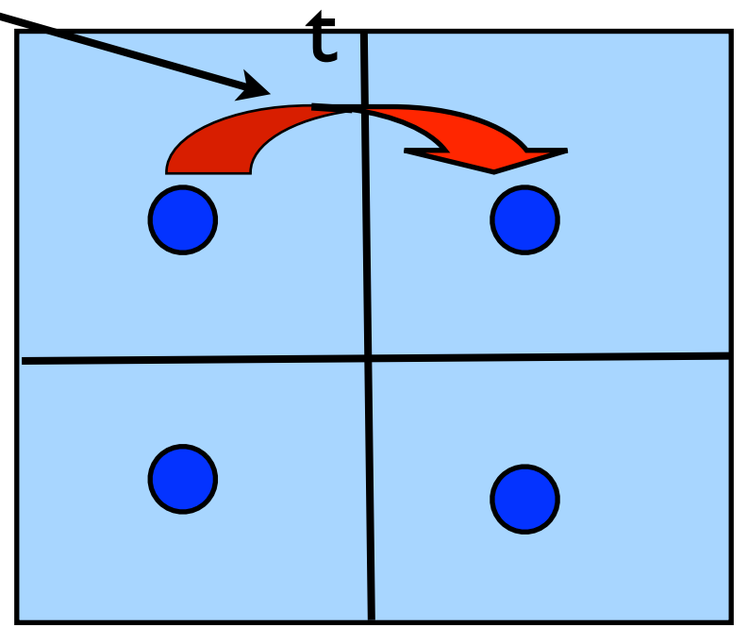
no change in size of Brillouin zone

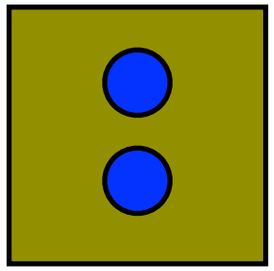
DetReG(0,p)=0

Mott mechanism (not Slater)

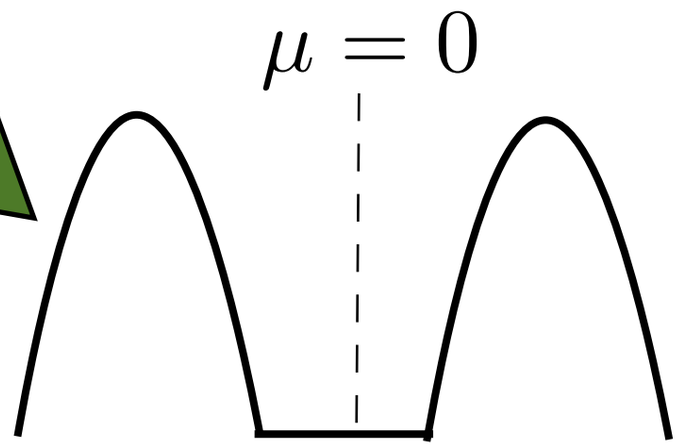
NiO insulates
d^8 ?

perhaps this costs energy





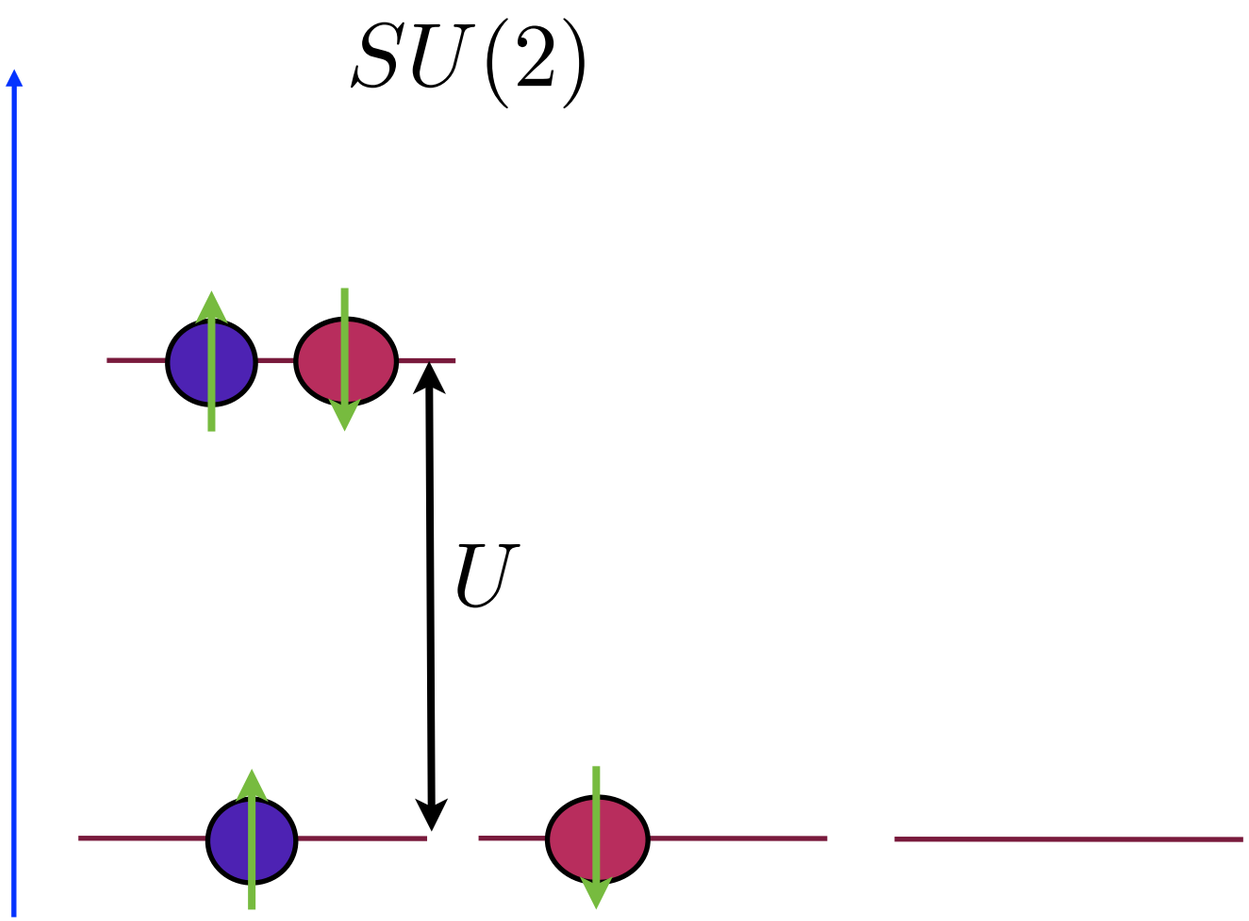
$U \gg t$



no change in size of Brillouin zone

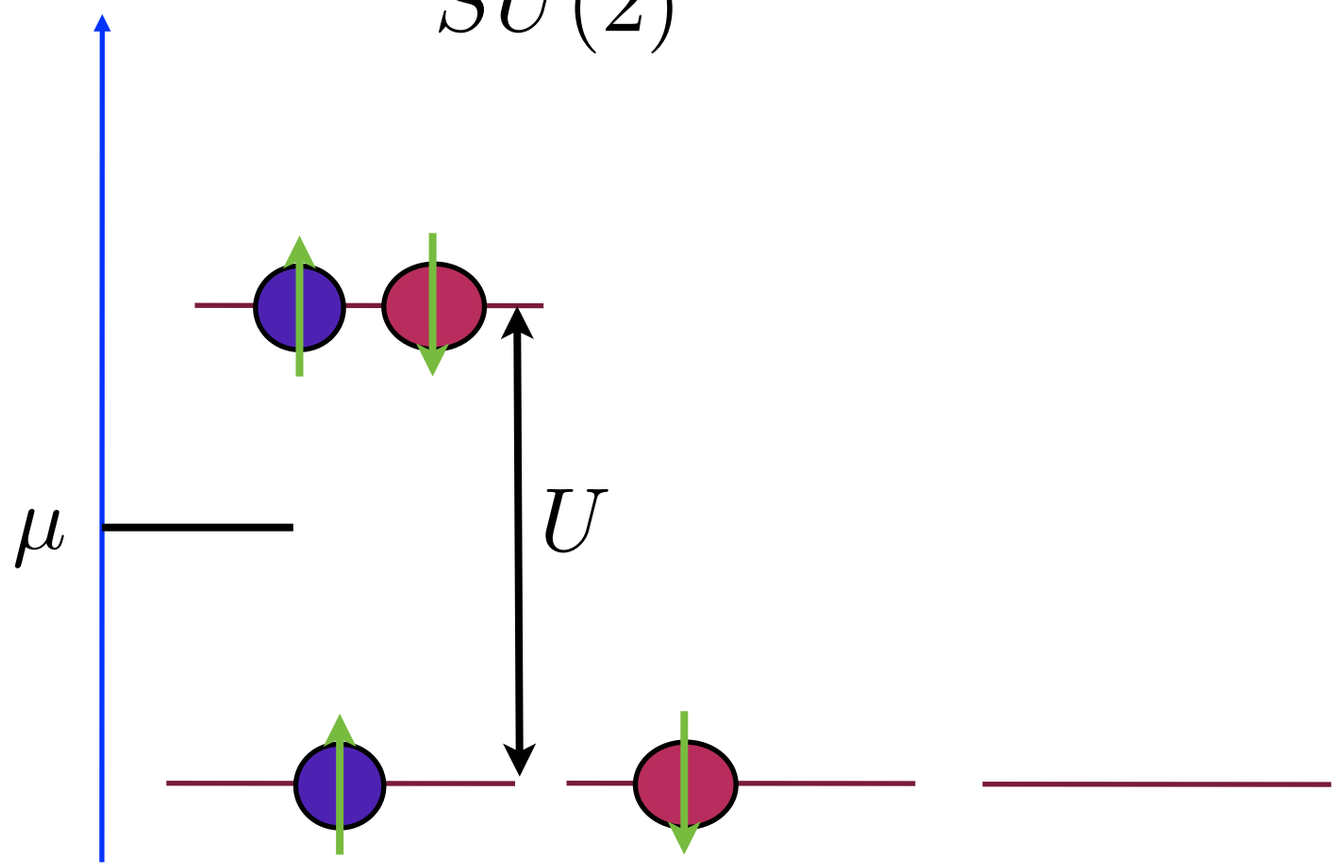
DetReG(0,p)=0 =Mottness

simple problem: n=1

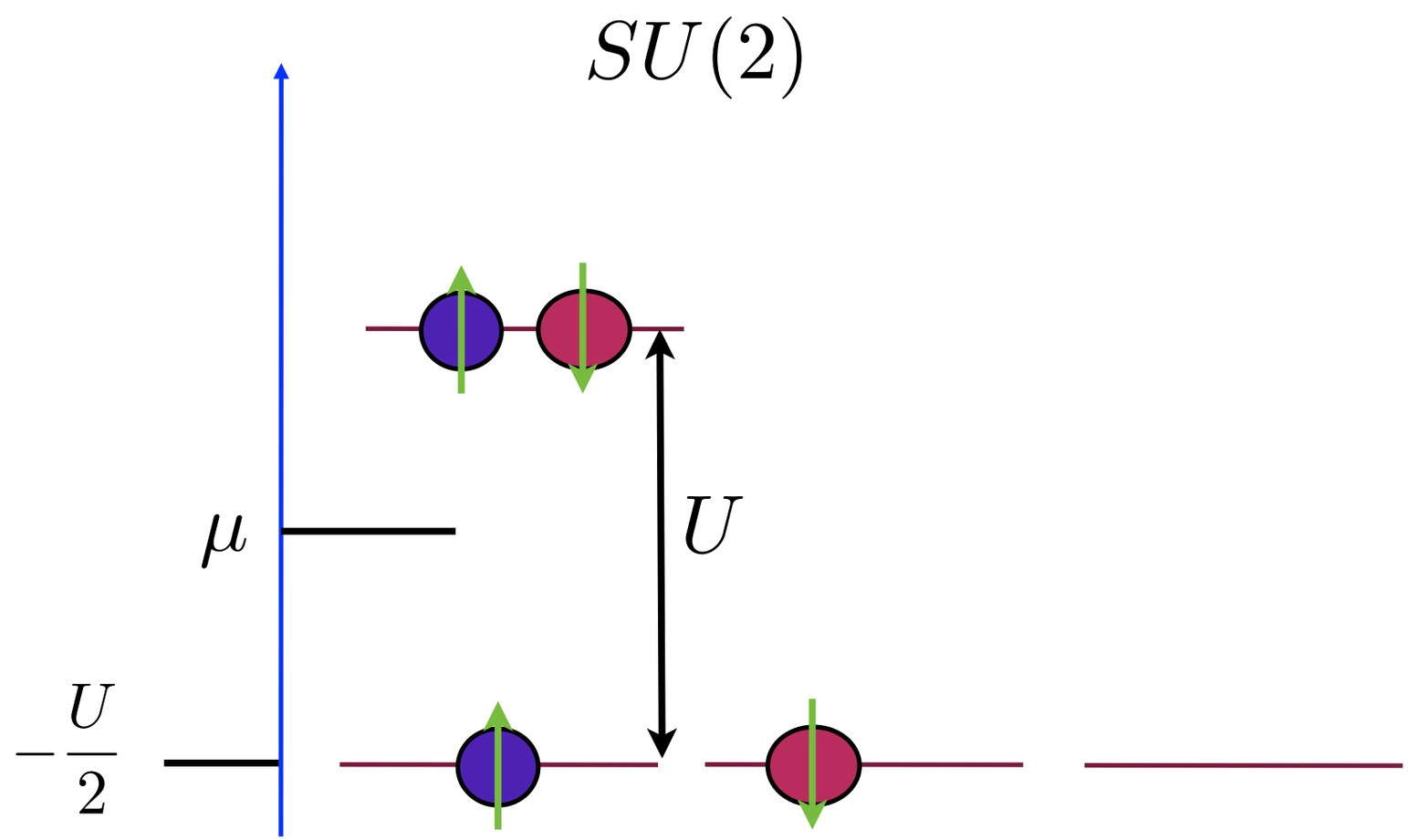


simple problem: n=1

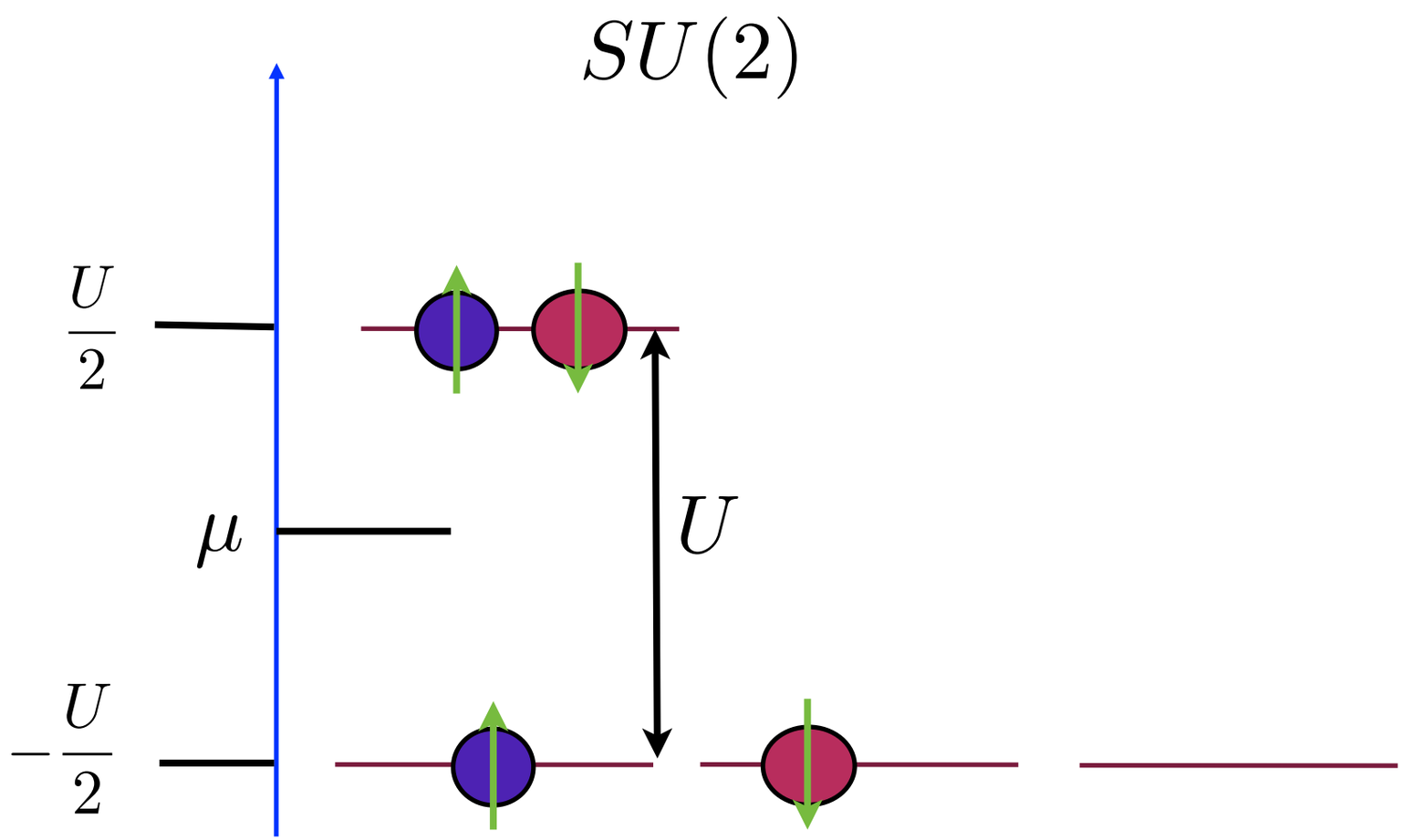
$SU(2)$



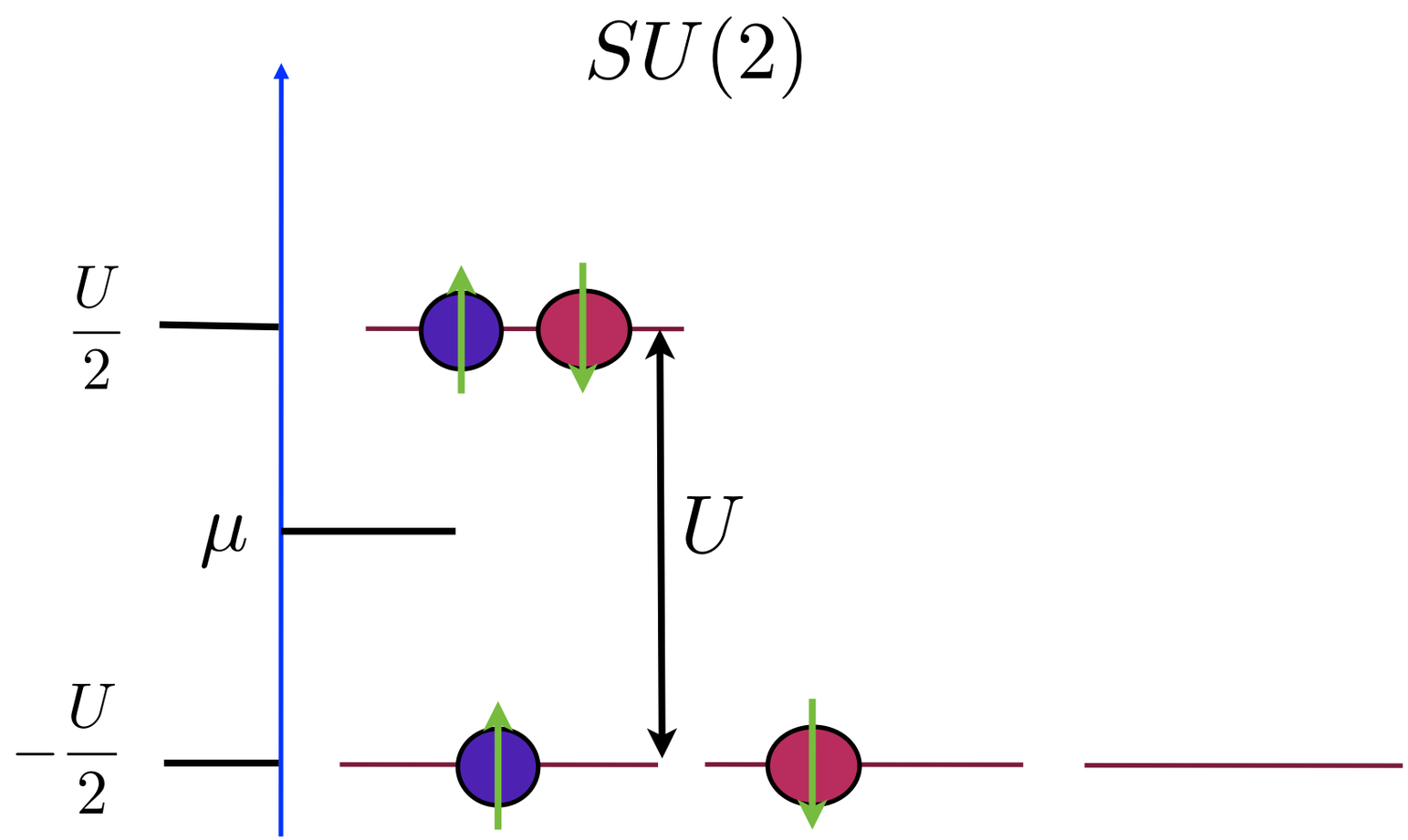
simple problem: n=1



simple problem: n=1

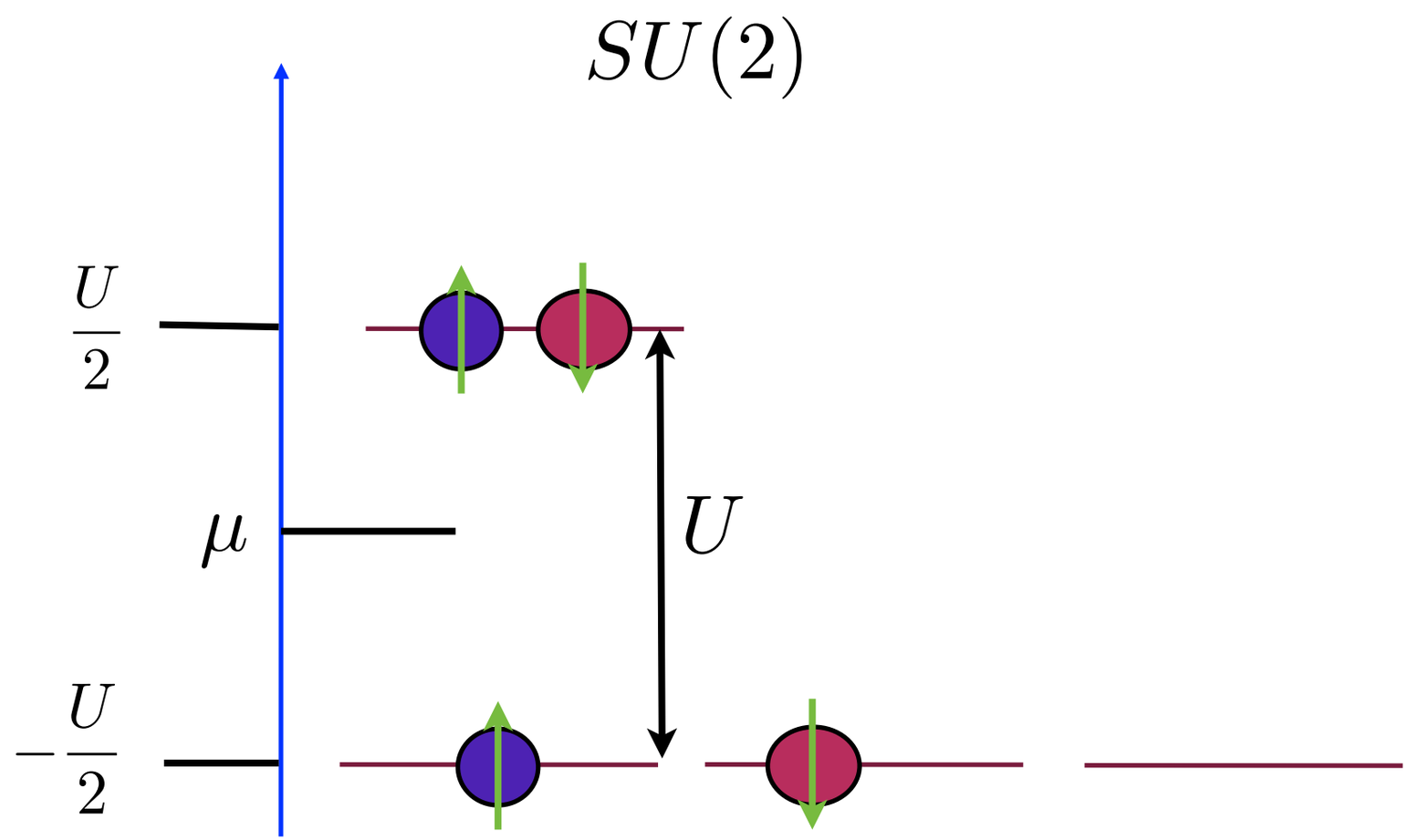


simple problem: n=1



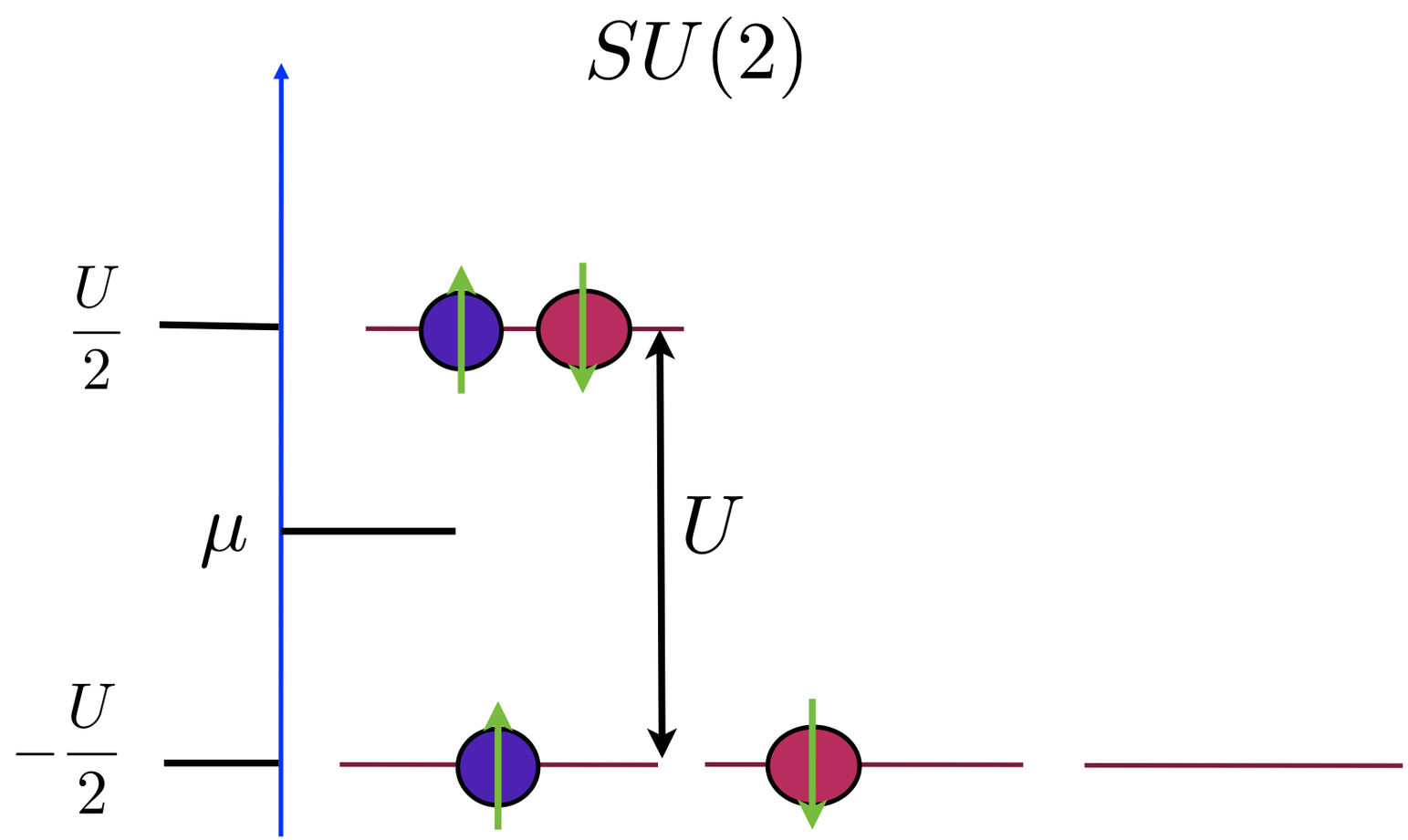
$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2}$$

simple problem: n=1



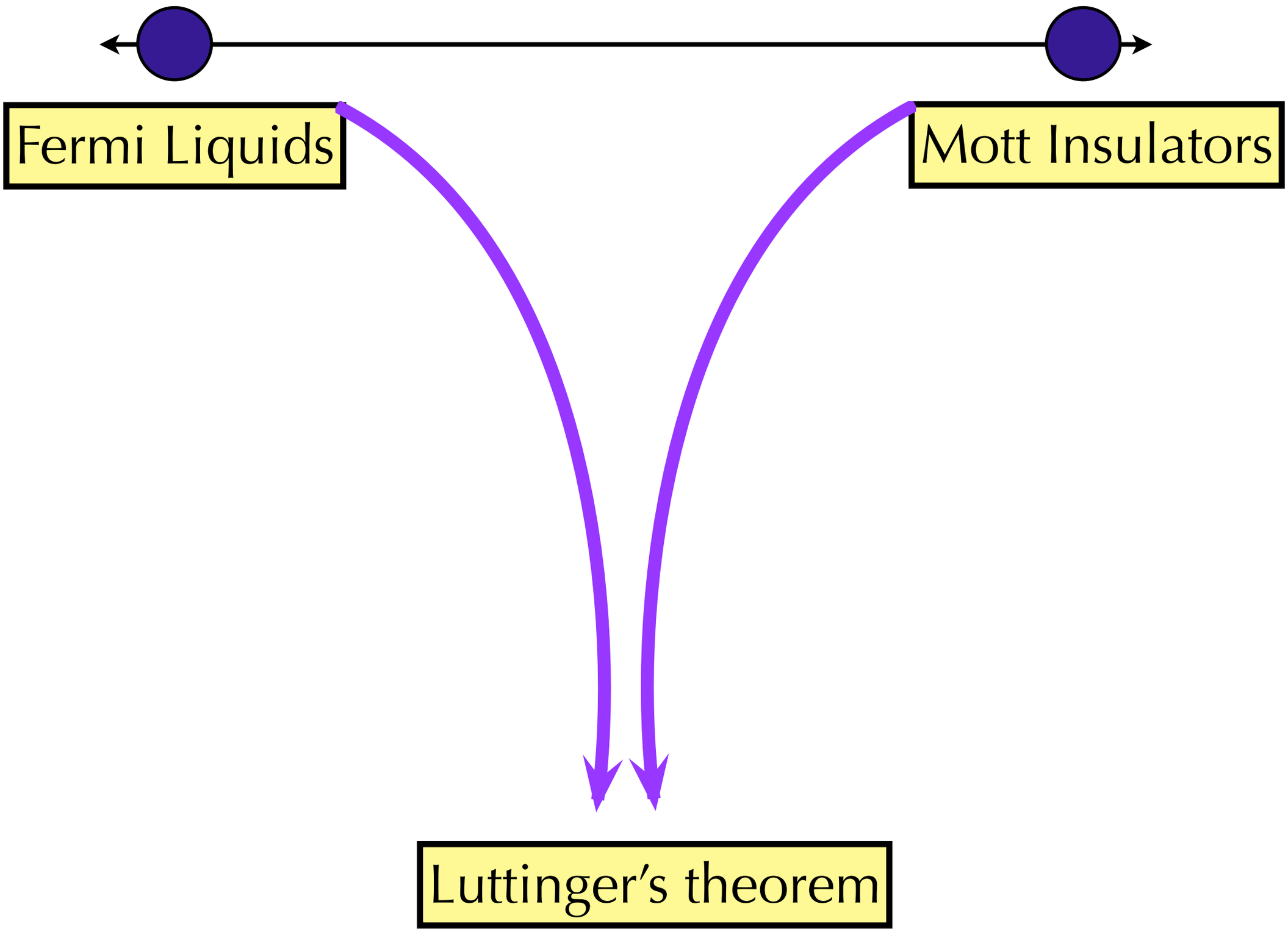
$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$

simple problem: n=1



$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$

$$n = 2\theta(0) = 1$$


Fermi Liquids
Mott Insulators
Luttinger's theorem

PHYSICAL REVIEW B **68**, 085113 (2003)

Some consequences of the Luttinger theorem: The Luttinger surfaces in non-Fermi liquids and Mott insulators

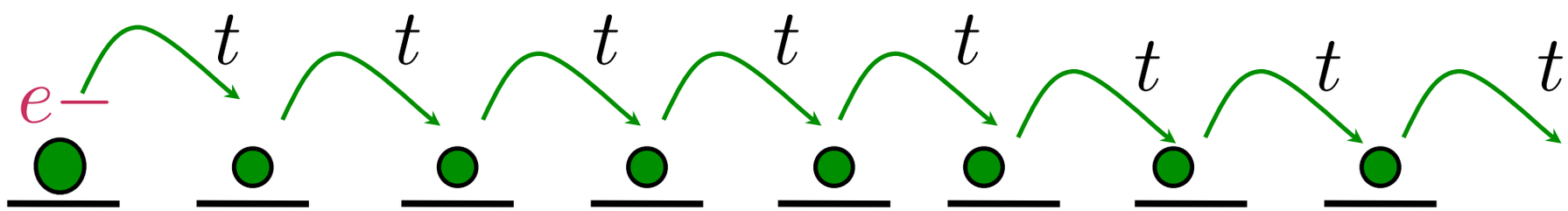
Igor Dzyaloshinskii
Department of Physics and Astronomy, University of California, Irvine, California 92697, USA
(Received 30 January 2003; published 27 August 2003)

The proof just presented is good for any state of our system: FL, NFL, or MI [in other words, either of poles or zeros can be used to change the sign of G_r in Eq. (1)]. The only way to incapacitate the Luttinger theorem in form (1) is to assume that the limit $T \to 0$ is discontinuous. Actually, one has to require that the whole line $T=0$ is a line of phase transitions.

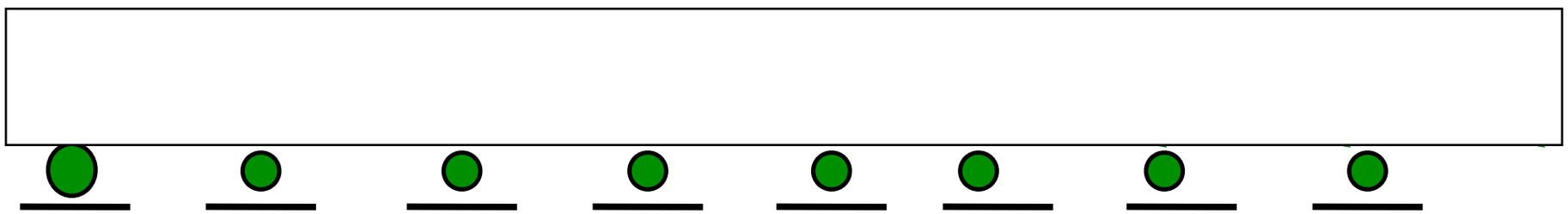
Is this famous theorem from 1960 correct?

A model with zeros but Luttinger fails

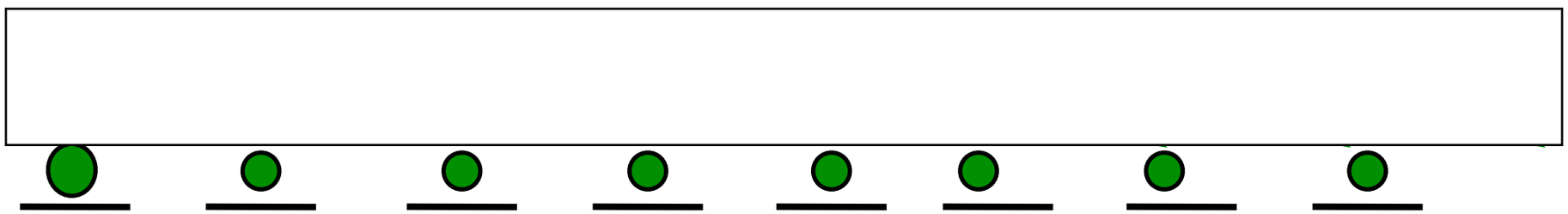
A model with zeros but Luttinger fails



A model with zeros but Luttinger fails

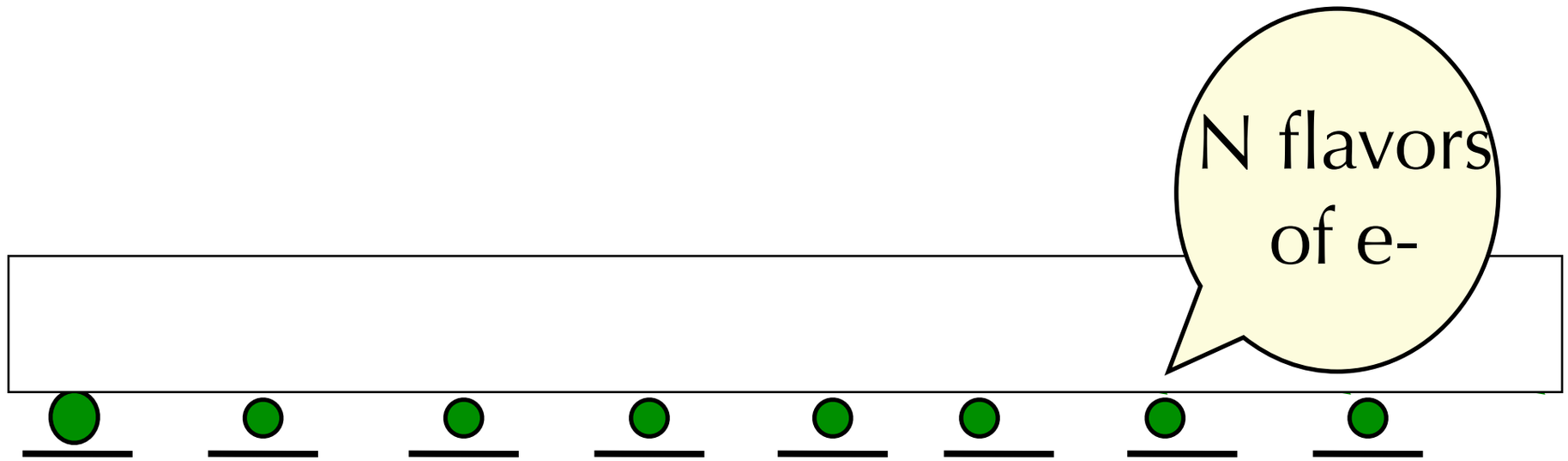


A model with zeros but Luttinger fails



no hopping=> no propagation (zeros)

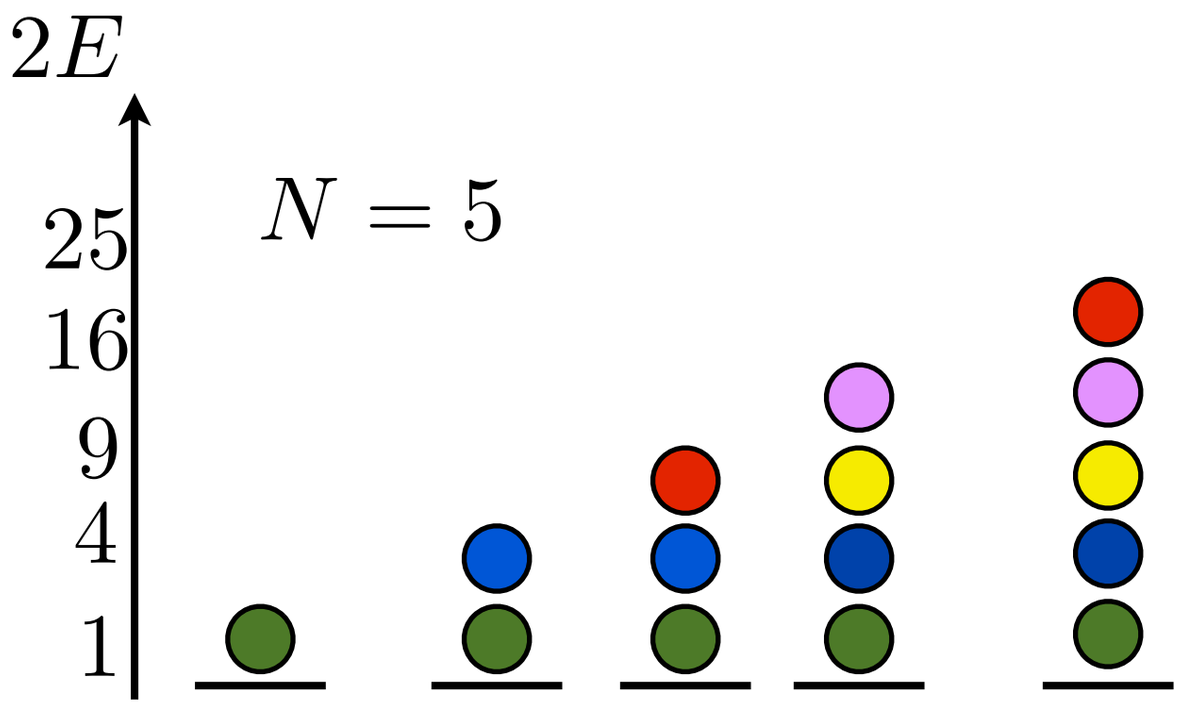
A model with zeros but Luttinger fails



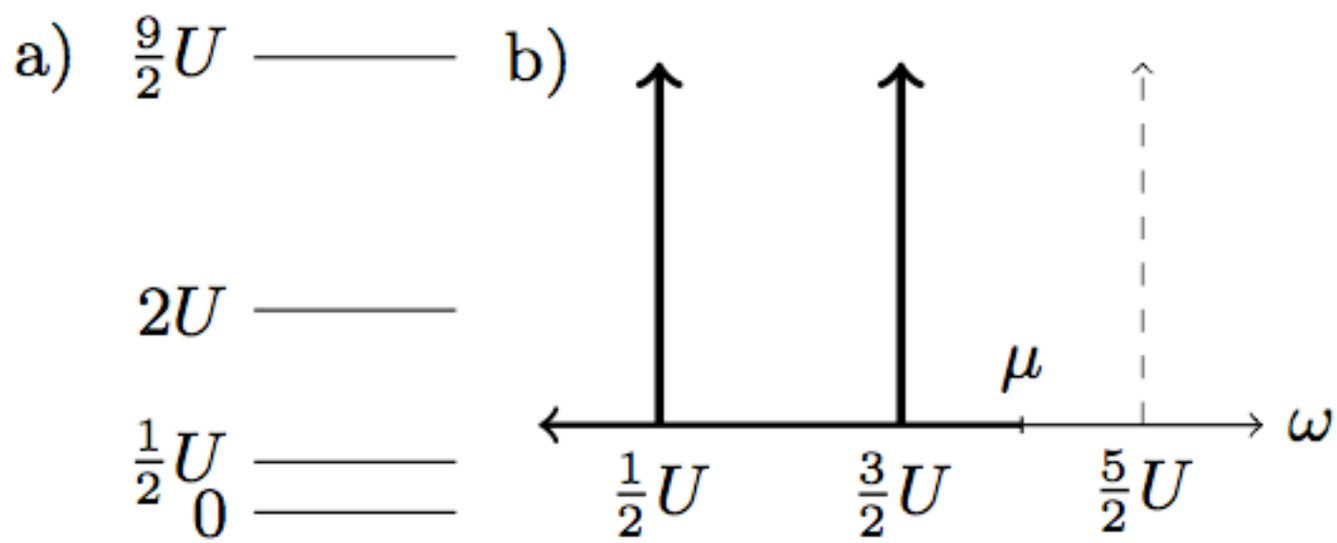
no hopping=> no propagation (zeros)

$$SU(N)$$

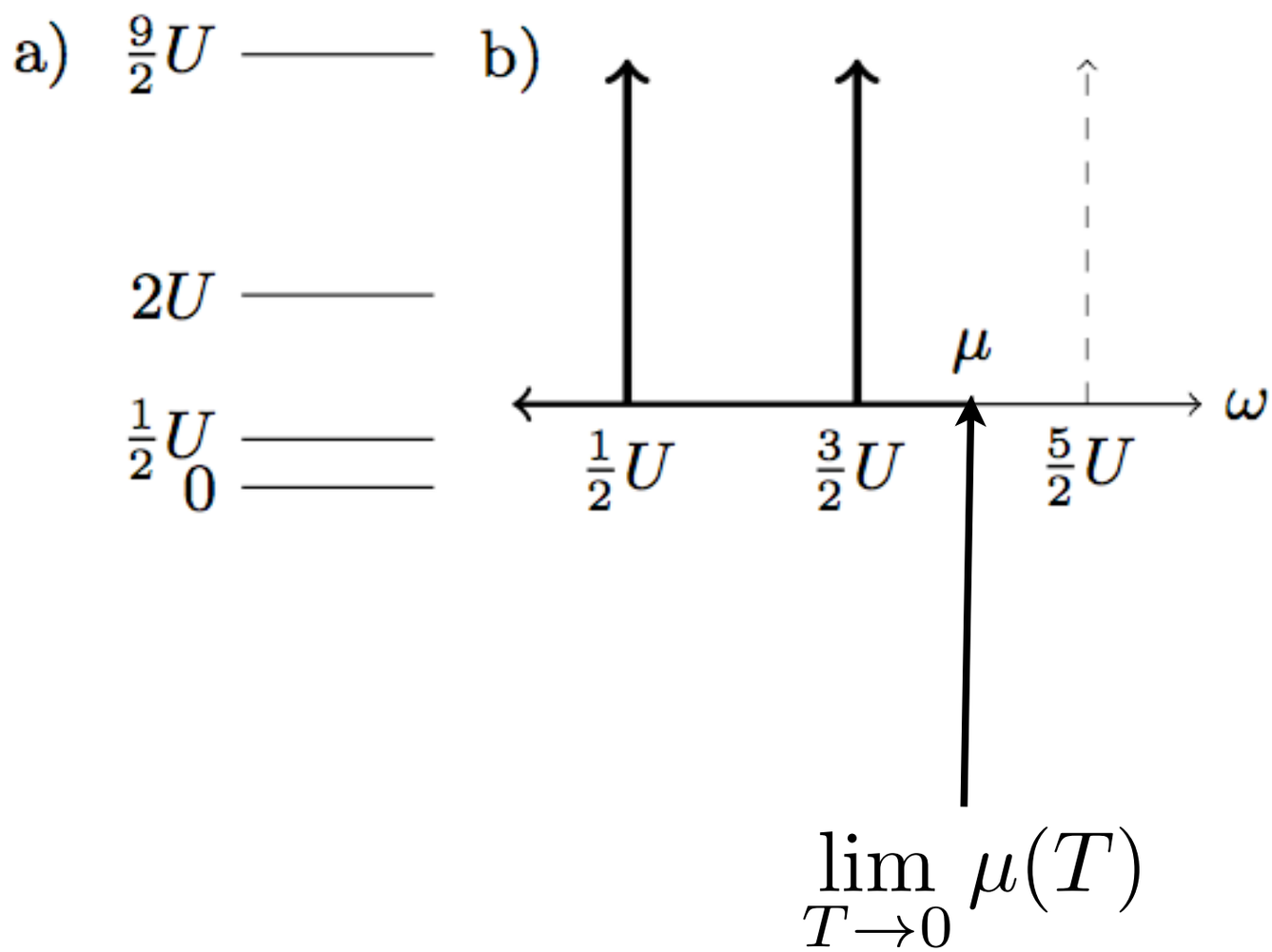
$$H = \frac{U}{2}(n_1 + \cdots n_N)^2$$



SU(N) for N=3: no particle-hole symmetry



SU(N) for N=3: no particle-hole symmetry



compute Green function exactly (Lehman formula)

$$G_{\alpha\beta}(\omega) = \frac{1}{Z}\sum_{ab} \exp^{-\beta K_a} Q^{ab}_{\alpha\beta}$$

$$Q^{ab}_{\alpha\beta} = \frac{\langle a|c_\alpha|b\rangle\langle b|c^\dagger_\beta|a\rangle}{\omega - K_b + K_a} + \frac{\langle a|c^\dagger_\beta|b\rangle\langle b|c_\alpha|a\rangle}{\omega - K_a + K_b}$$

do sum explicitly

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n+1) - K(n)} \left(\frac{2n - N}{N} \right)$$

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

< 0

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

< 0 $\qquad$ > 0

a zero must exist

< 0 $\qquad$ > 0

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

Luttinger's theorem

$$n = N\Theta(2n - N)$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

$n = 2$

$N = 3$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

$$n = 2$$

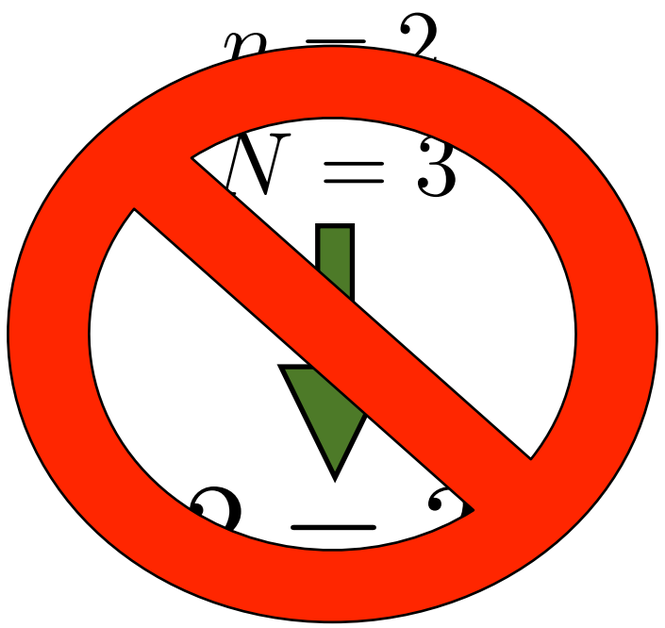
$$N = 3$$

$$\Downarrow$$

$$2 = 3$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$



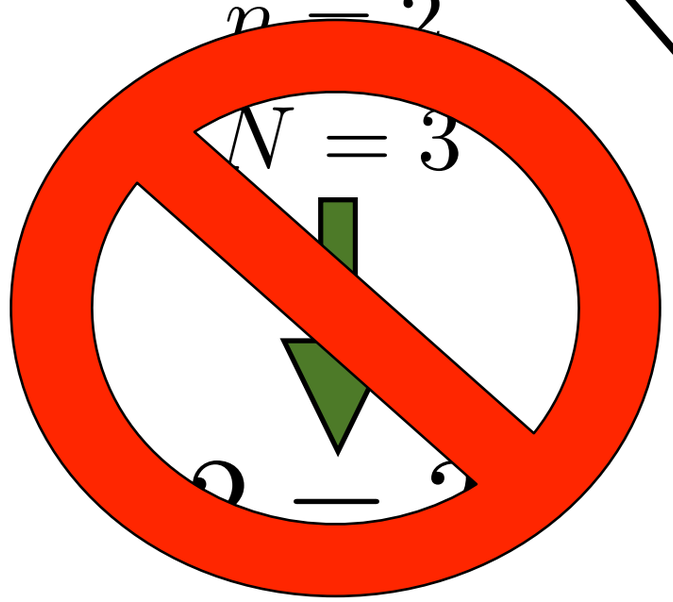
Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

even

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

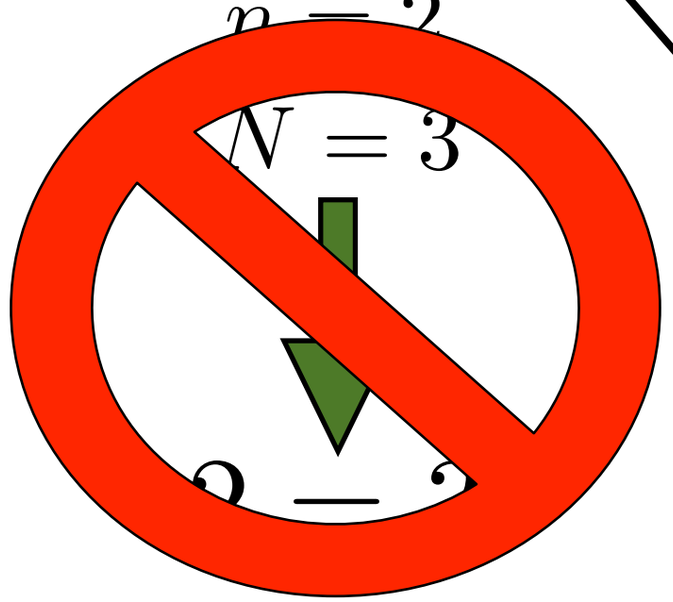


even

odd

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

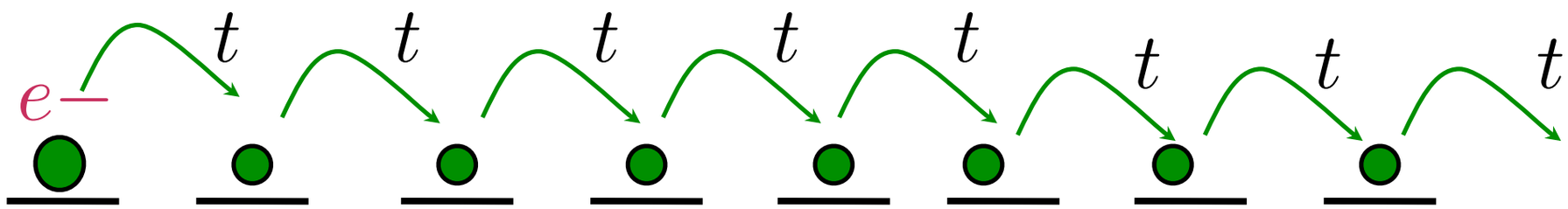


even

odd

no solution

does the degeneracy matter?



$$t = 0^+$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \frac{1}{\omega - H} c_b + c_b \frac{1}{\omega - H} c_a^\dagger\right] \rho(0^+)\right)$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \frac{1}{\omega - H} c_a^\dagger\right] \rho(0^+)\right)$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$G_{ab}(\omega) = \frac{\omega\delta_{ab} - U\rho_{ab}}{\omega(\omega - U)}$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$\rho_{ab} = \mathrm{Tr}\left(c_a^\dagger c_b \rho(0^+)\right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle$$

$$G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega(\omega - U)}$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$\rho_{ab} = \mathrm{Tr}\left(c_a^\dagger c_b \rho(0^+)\right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle$$

$$G_{ab}(\omega) = \frac{\omega\delta_{ab} - U\rho_{ab}}{\omega(\omega - U)}$$

$$1/N\mathrm{diag}(1, 1, 1, \cdots)$$

mixed state

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$\rho_{ab} = \mathrm{Tr}\left(c_a^\dagger c_b \rho(0^+)\right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle$$

$$G_{ab}(\omega) = \frac{\omega\delta_{ab} - U\rho_{ab}}{\omega(\omega - U)}$$

$$1/N\mathrm{diag}(1, 1, 1, \cdots)$$

mixed state

as long as SU(N) symmetry is intact

zeros in the wrong place

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$\rho_{ab} = \mathrm{Tr}\left(c_a^\dagger c_b \rho(0^+)\right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle$$

$$G_{ab}(\omega) = \frac{\omega\delta_{ab} - U\rho_{ab}}{\omega(\omega - U)}$$

$$1/N\mathrm{diag}(1, 1, 1, \cdots)$$

mixed state

as long as SU(N) symmetry is intact

zeros in the wrong place

limiting procedure: $\lim_{t\to 0} \lim_{\omega\to 0}$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$\rho_{ab} = \mathrm{Tr}\left(c_a^\dagger c_b \rho(0^+)\right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle$$

$$G_{ab}(\omega) = \frac{\omega\delta_{ab} - U\rho_{ab}}{\omega(\omega - U)}$$

$$1/N\mathrm{diag}(1, 1, 1, \cdots)$$

mixed state

as long as SU(N) symmetry is intact

zeros in the wrong place

limiting procedure: $\lim_{t\to 0} \lim_{\omega\to 0}$

limits do not commute

Problem

$$\mathrm{Det}G = 0$$

$$G = \frac{1}{E - \varepsilon_p} = 0$$

??

Problem

$$\mathrm{Det}G = 0$$

$$G = \frac{1}{E - \varepsilon_p} = 0$$

??

$$G = \frac{1}{E - \varepsilon_p - \Sigma} \rightarrow \infty$$

Problem

$\mathrm{Det}G = 0$

$$G = \frac{1}{E - \varepsilon_p} = 0$$

??

$$G = \frac{1}{E - \varepsilon_p - \Sigma}$$

∞

G=0

if Σ is infinite

if Σ is infinite

lifetime of a particle vanishes

if Σ is infinite

lifetime of a particle vanishes

$\Im\Sigma < \epsilon_p$

if Σ is infinite

lifetime of a particle vanishes

no particle

$\Im\Sigma < \epsilon_p$

what went wrong?

what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$

what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$

$$\Big\Downarrow \text{ if } \Sigma \to \infty$$

what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$

$$\Big\Downarrow \text{ if } \Sigma \to \infty$$

integral does not exist

what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$

$$\Big\Downarrow \text{ if } \Sigma \to \infty$$

integral does not exist

$$\Big\Downarrow$$

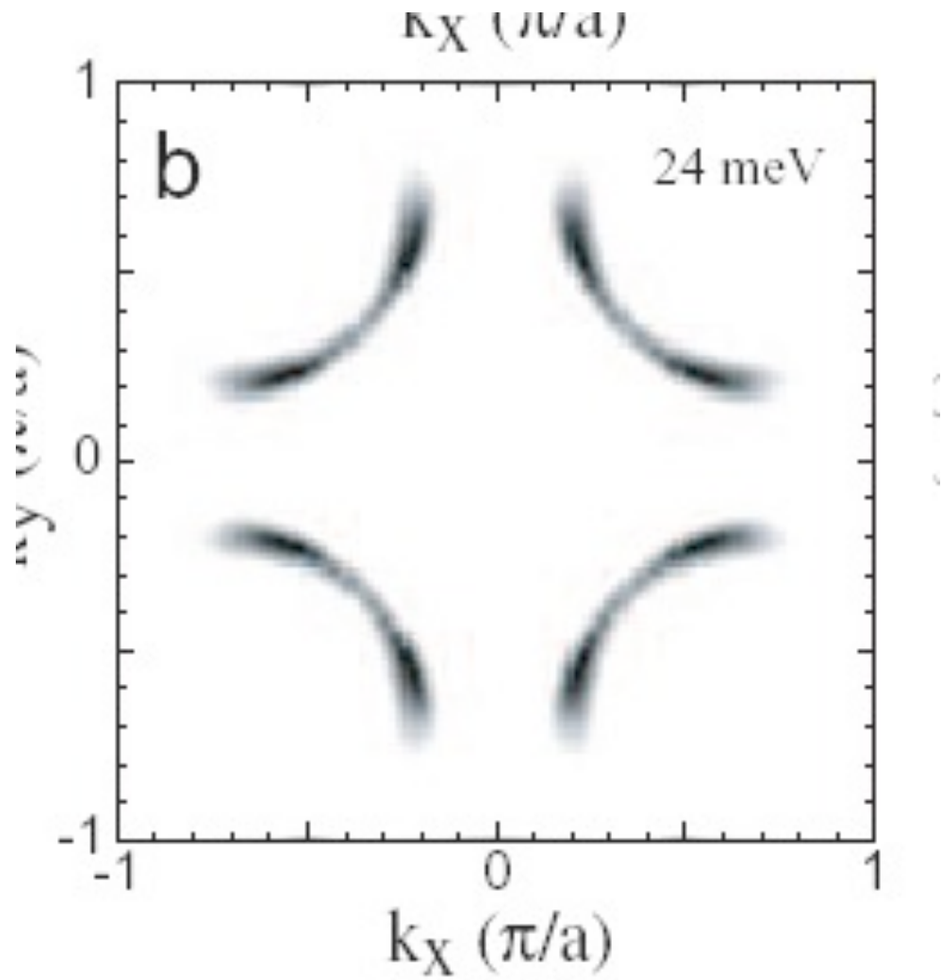
No Luttinger theorem!

zeros and particles are incompatible

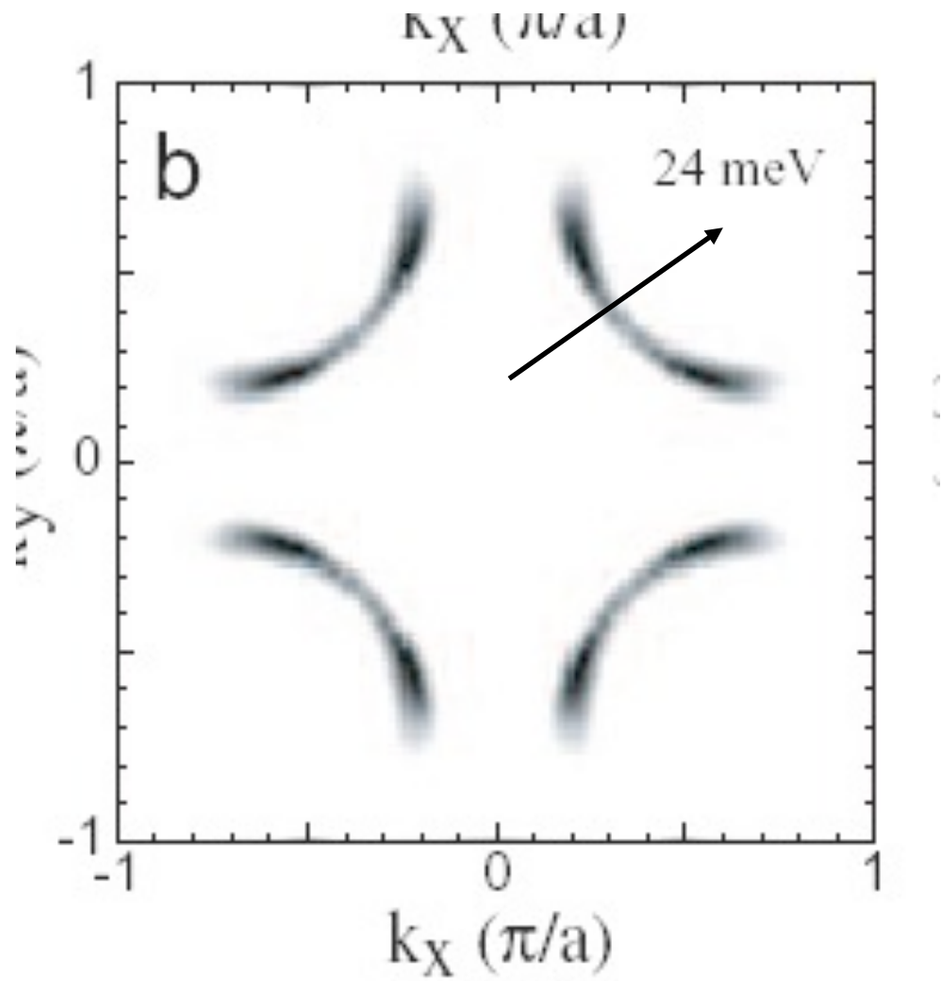
Luttinger's theorem

are zeros important?

Fermi Arcs

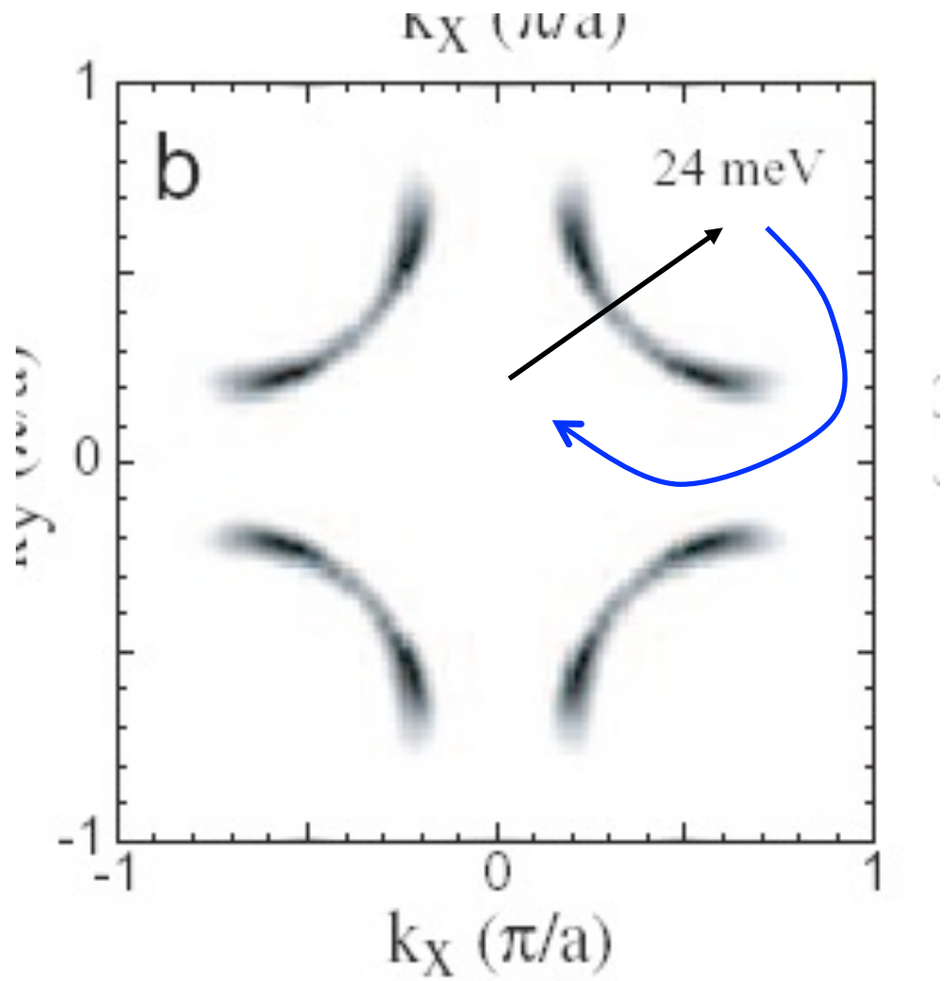


Fermi Arcs



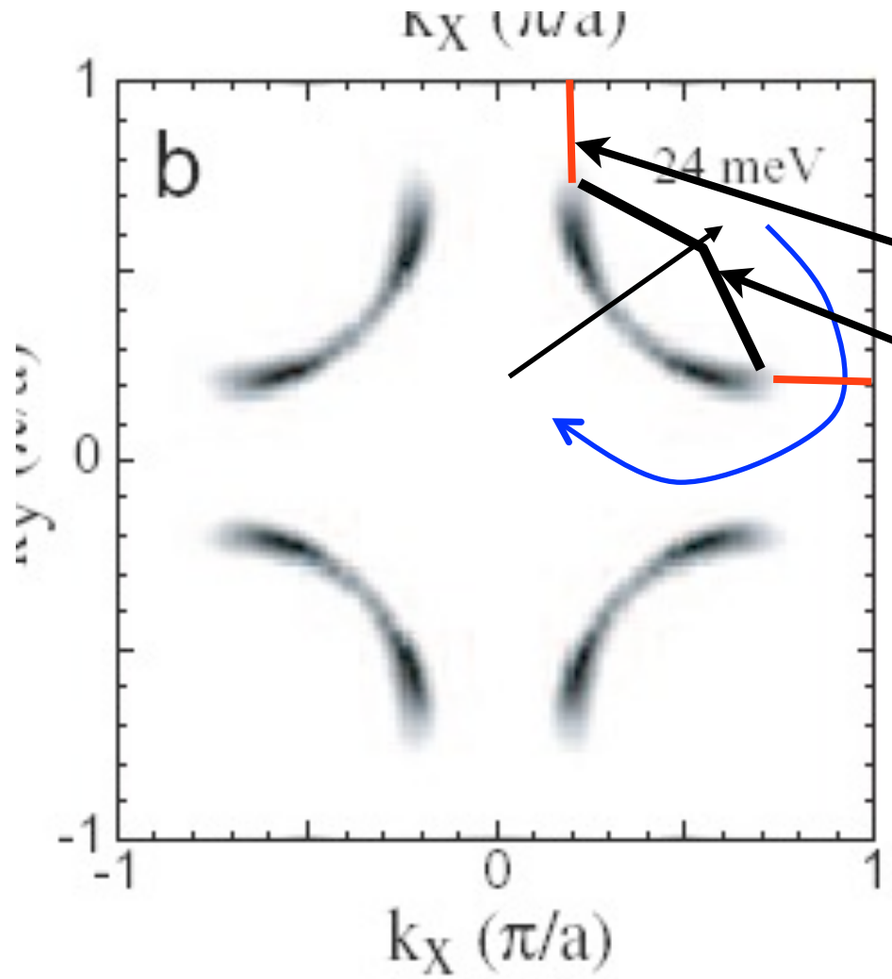
Fermi Arcs

Re G
Changes
Sign across
An arc



Fermi Arcs

Re G
Changes
Sign across
An arc

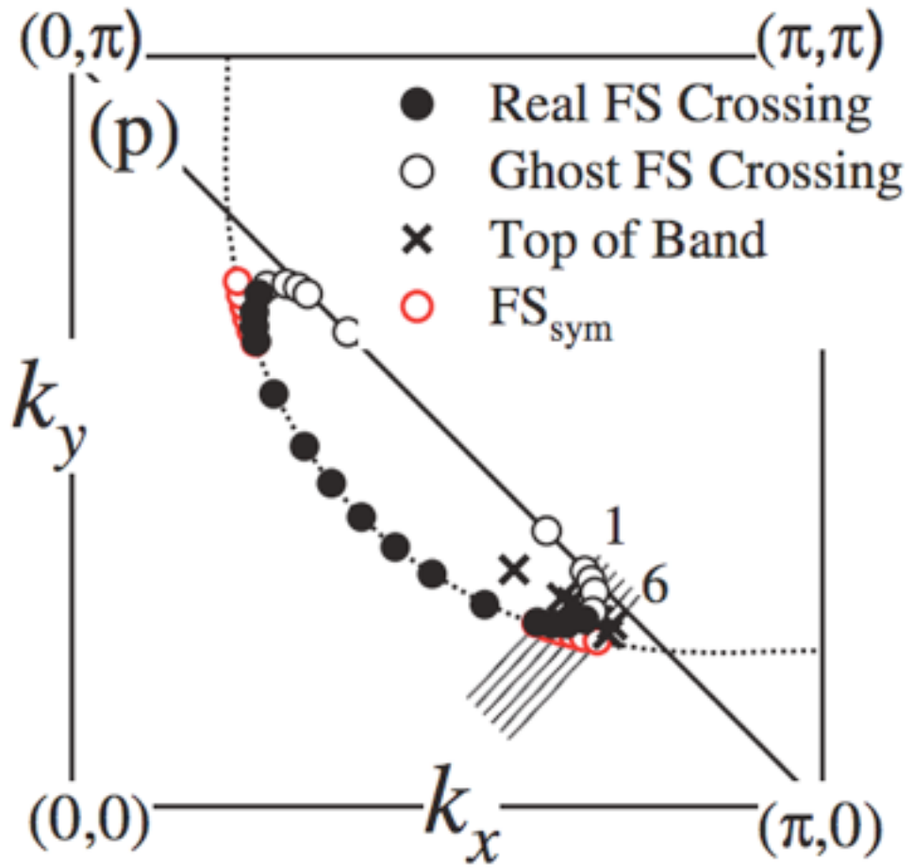


Must cross
A zero line
(DetG=0)!!!

Fermi arcs necessarily imply zeros exist.

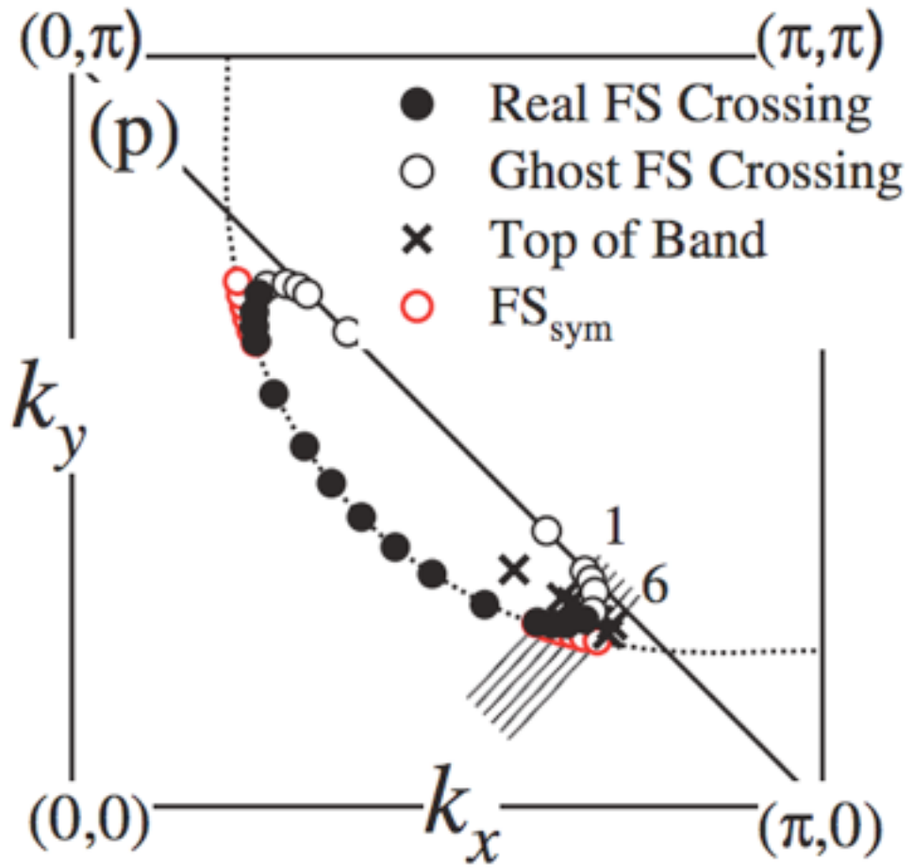
what is seen experimentally?

Fermi arcs: no double crossings (PDJ,JCC,ZXS)



what is seen experimentally?

Fermi arcs: no double crossings (PDJ,JCC,ZXS)

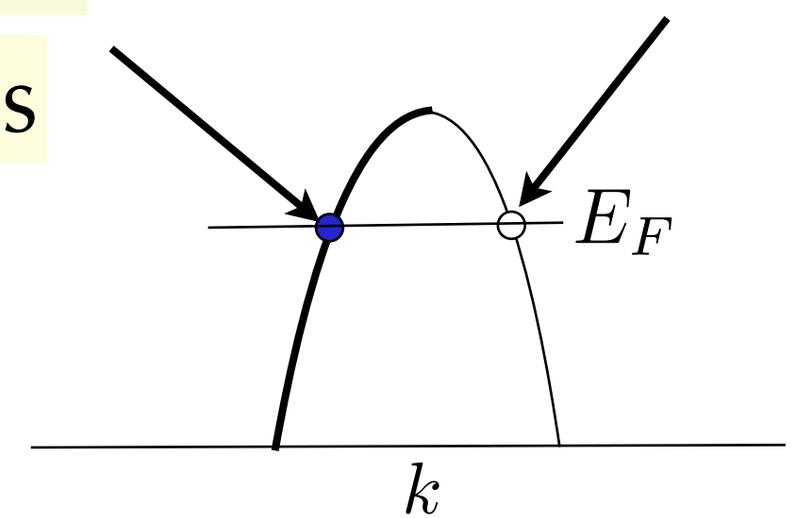


seen

infinities

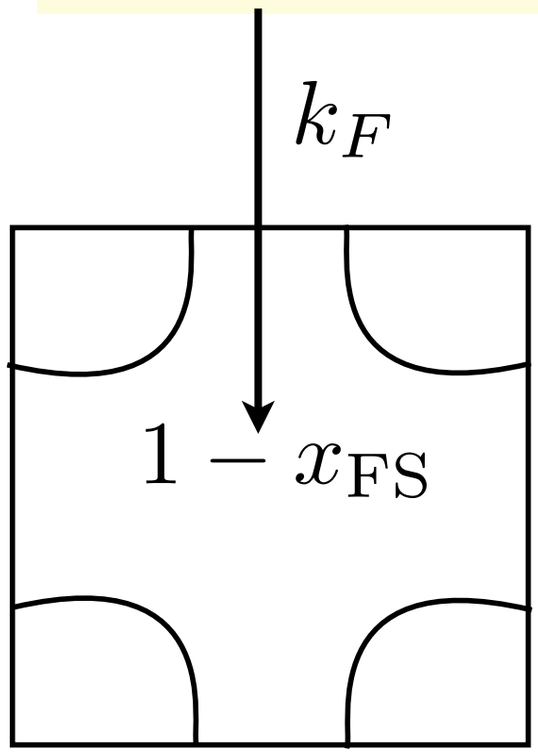
not seen

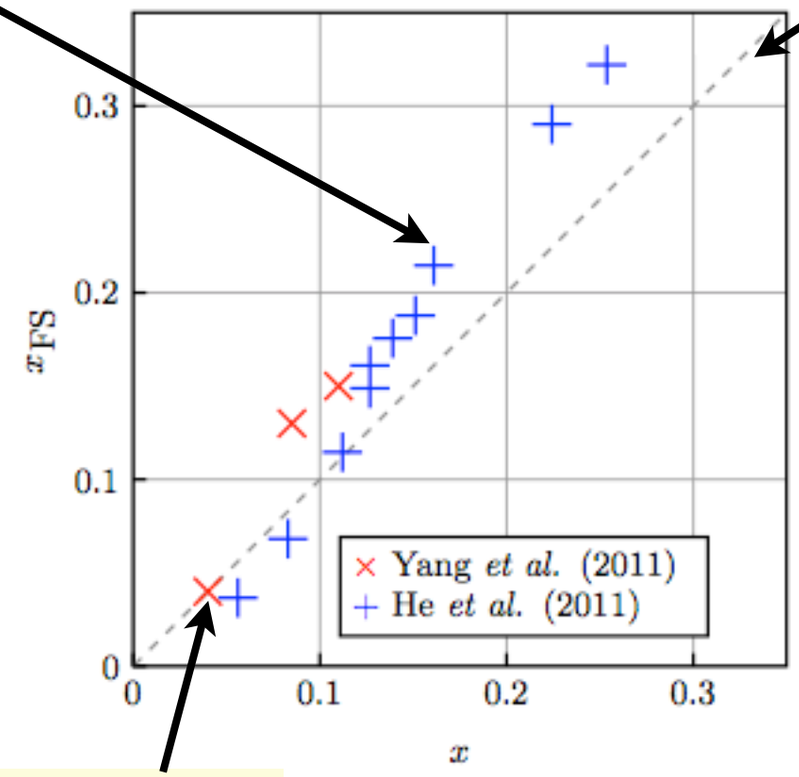
zeros



experimental data (LSCO)

`Luttinger' count



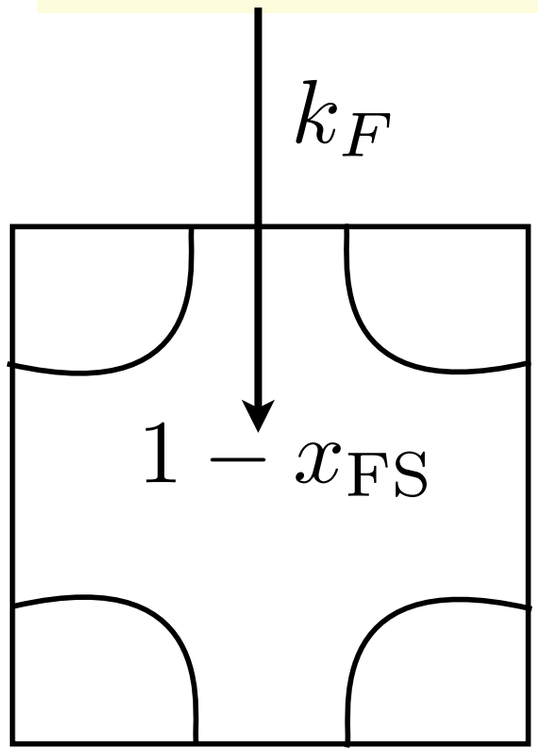


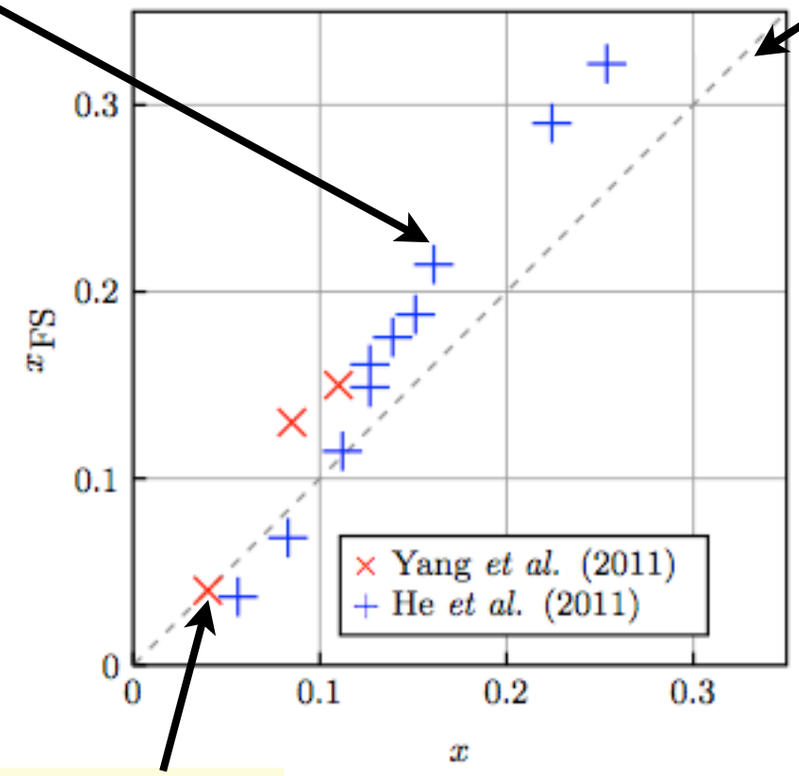
Bi2212

zeros do not affect the particle density

experimental data (LSCO)

`Luttinger' count





Bi2212

zeros do not
affect the particle density

each hole $\neq$ a single k-state

where do these zeros come from?

counting electron states

need to know: N (number of sites)

counting electron states

$$x = n_h/N$$

need to know: N (number of sites)

counting electron states

$x = n_h/N$

removal states

$1 - x$

need to know: N (number of sites)

counting electron states

$x = n_h/N$

removal states

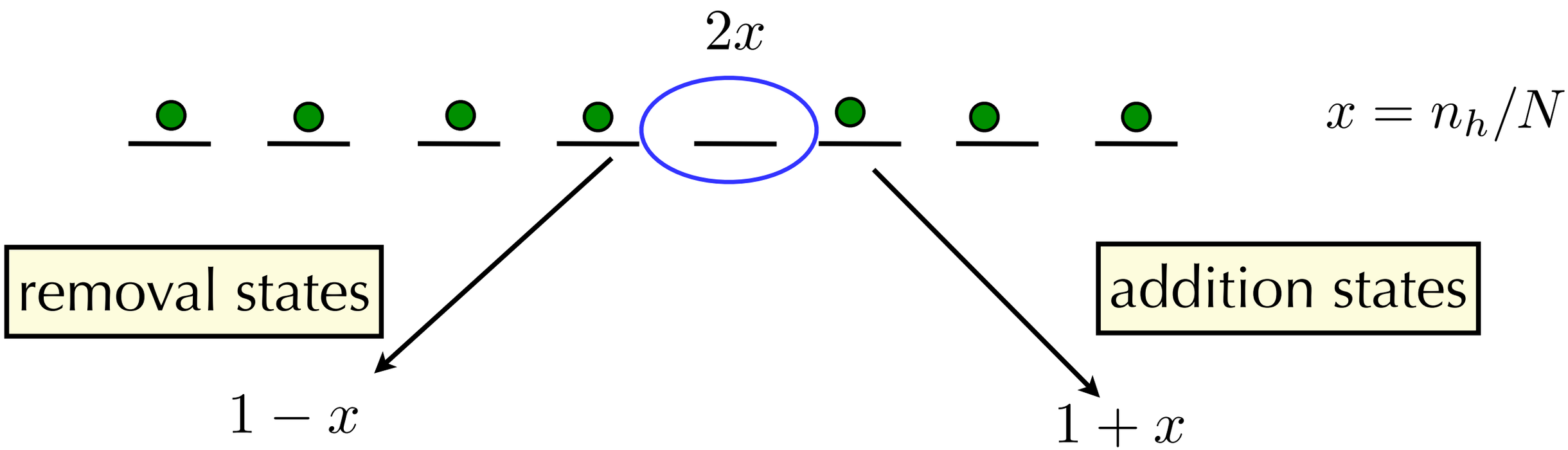
addition states

$1 - x$

$1 + x$

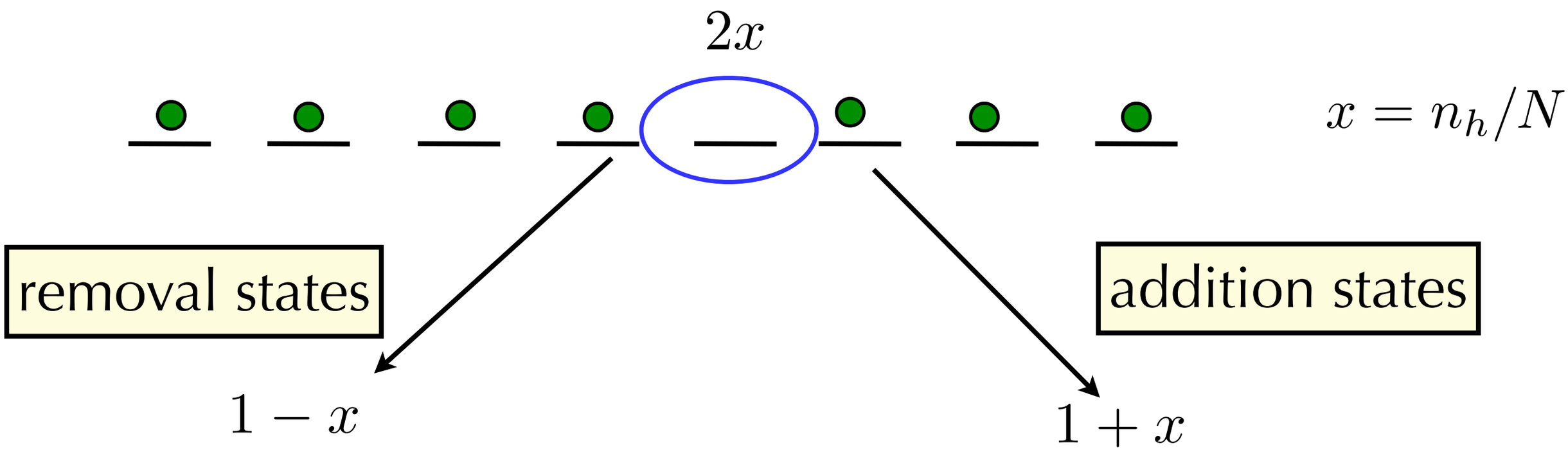
need to know: N (number of sites)

counting electron states



need to know: N (number of sites)

counting electron states

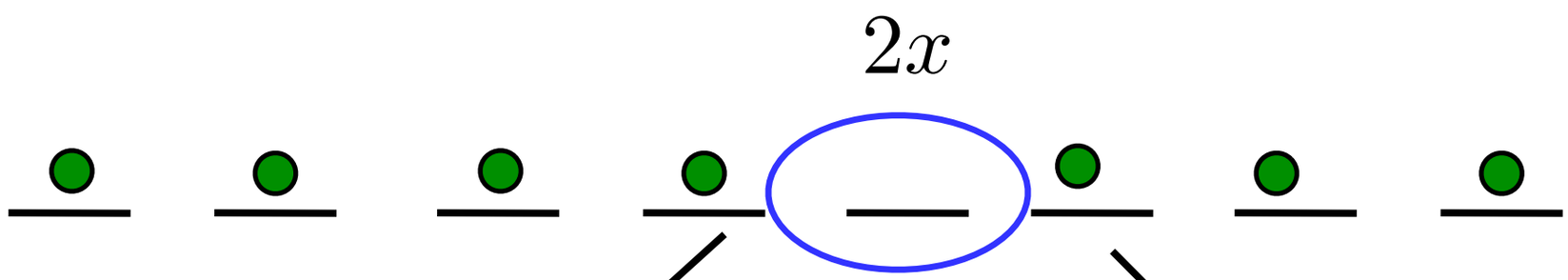


low-energy electron states

$$1 - x + 2x = 1 + x$$

need to know: N (number of sites)

counting electron states



$x = n_h/N$

removal states

addition states

$1 - x$

$1 + x$

low-energy electron states

$$1 - x + 2x = 1 + x$$

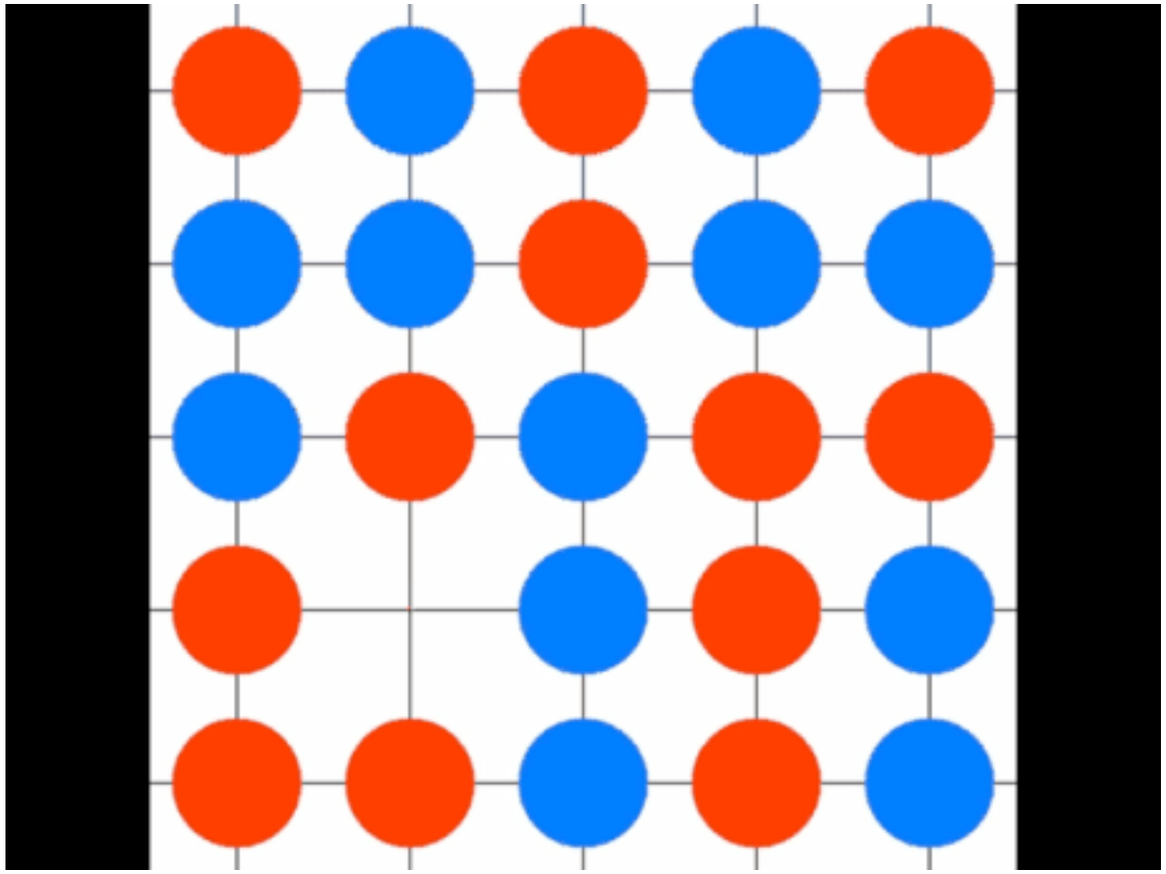
high energy

$$1 - x$$

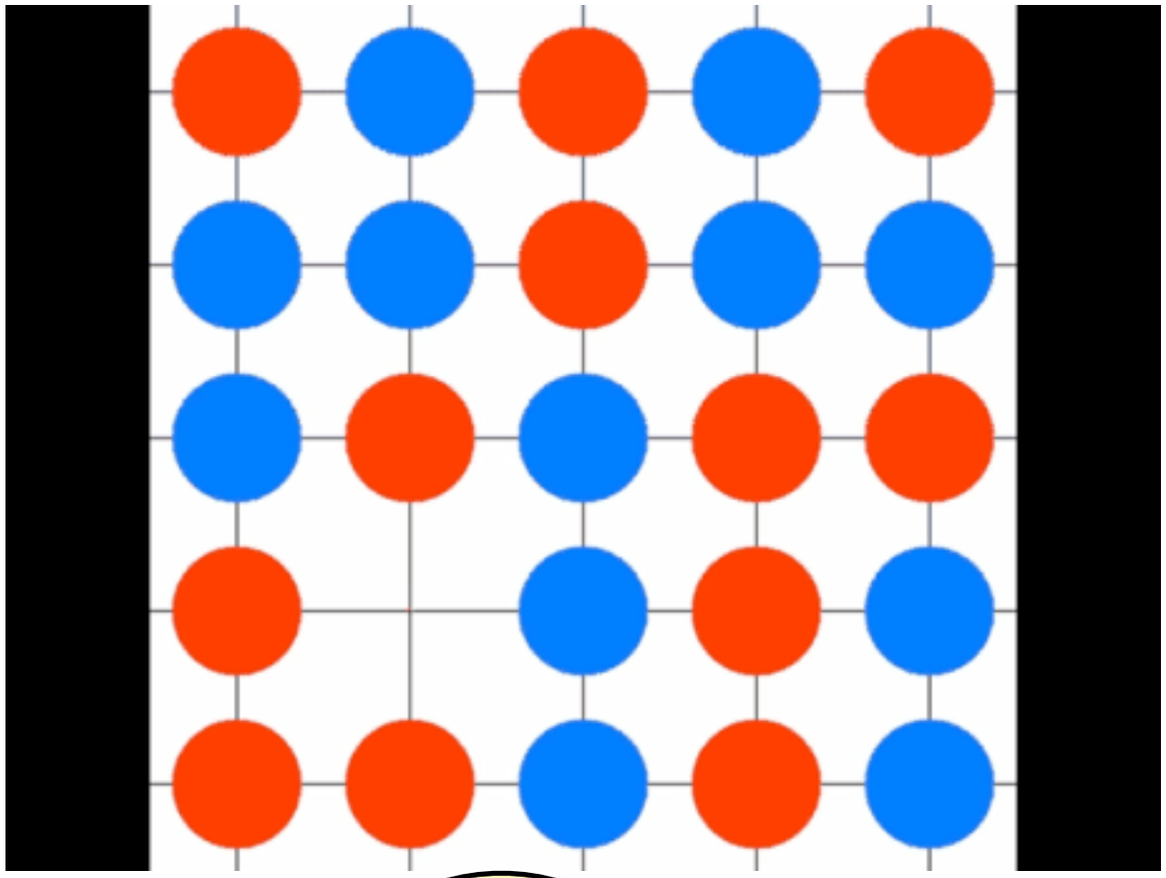
need to know: N (number of sites)

spectral function (dynamics)

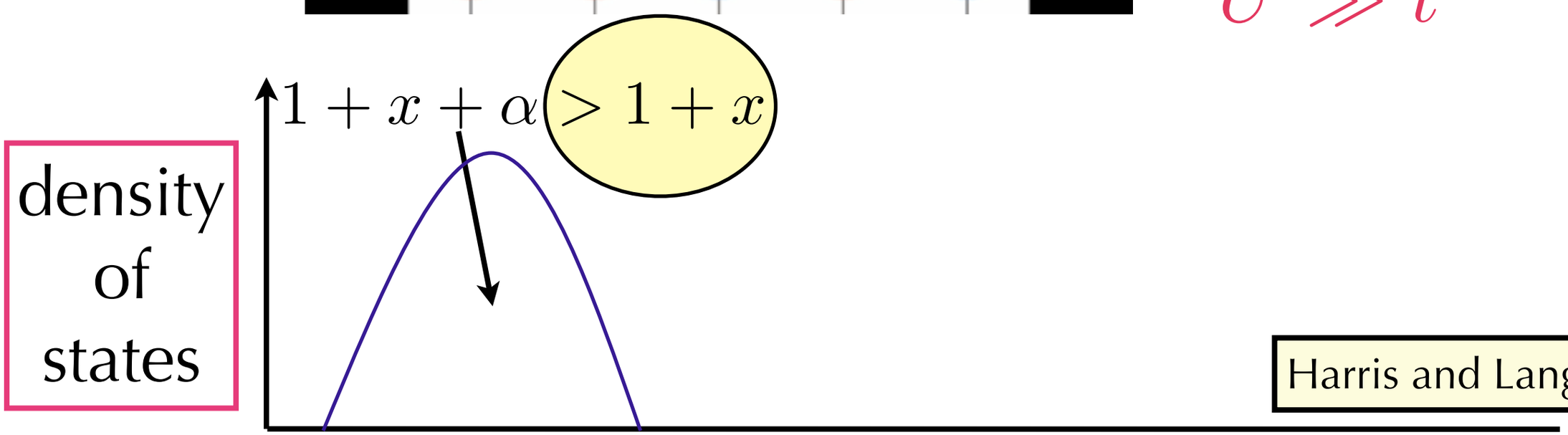
spectral function (dynamics)



spectral function (dynamics)

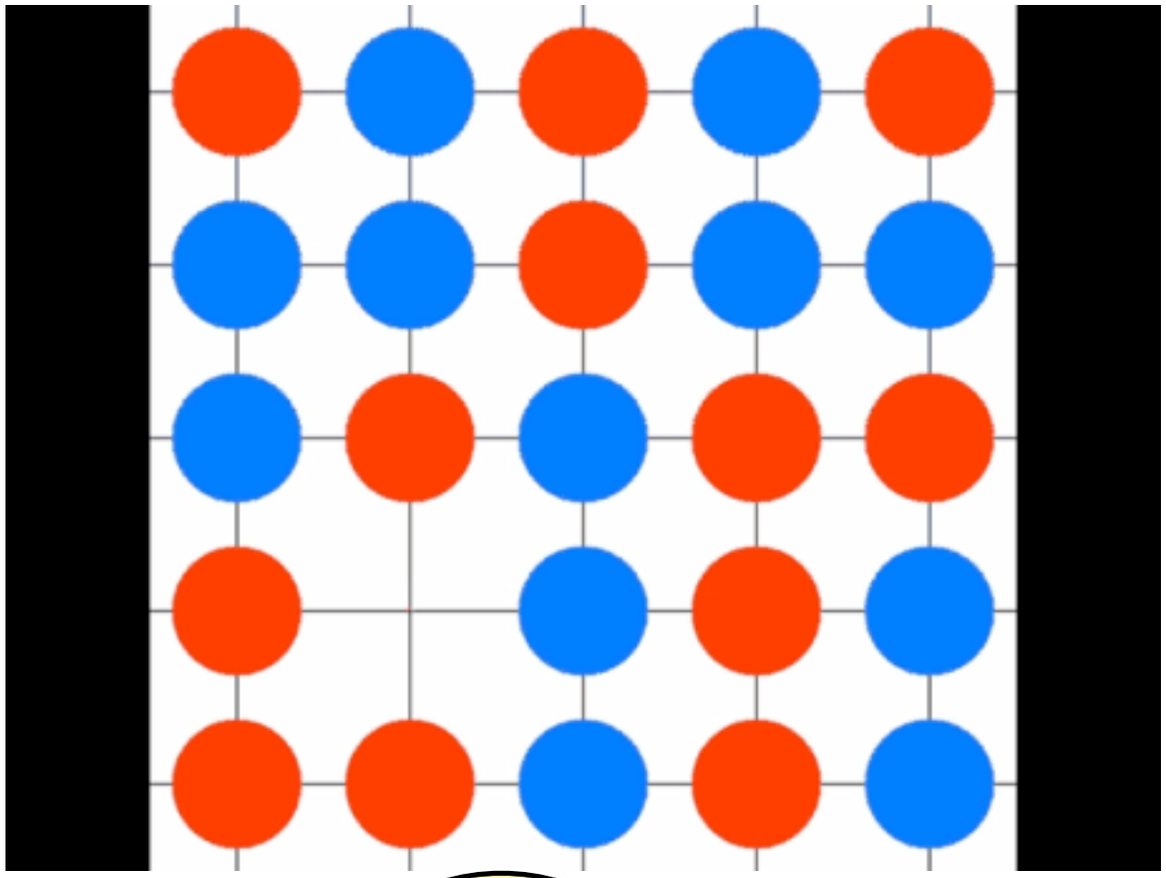


$U \gg t$

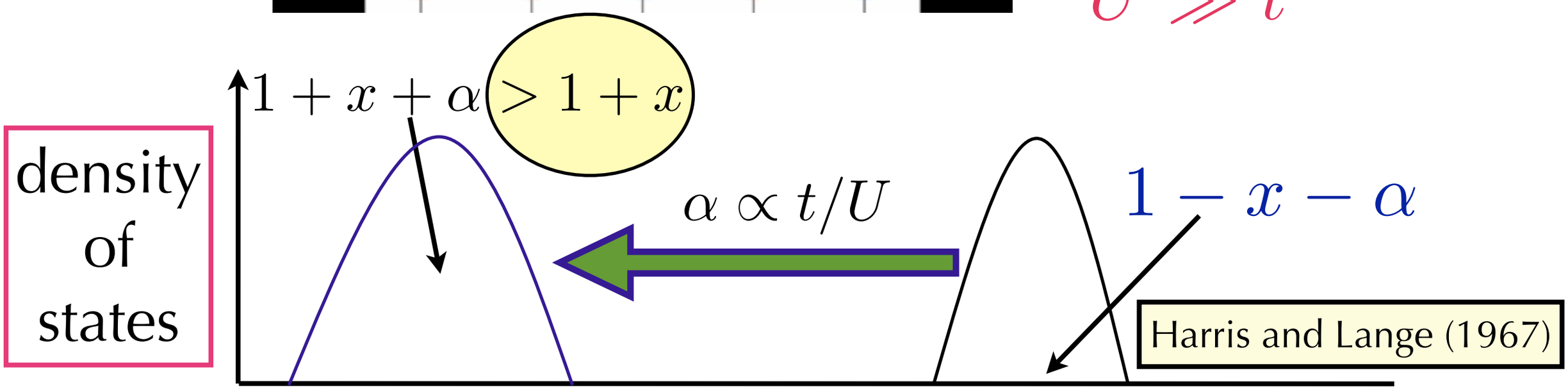


Harris and Lange (1967)

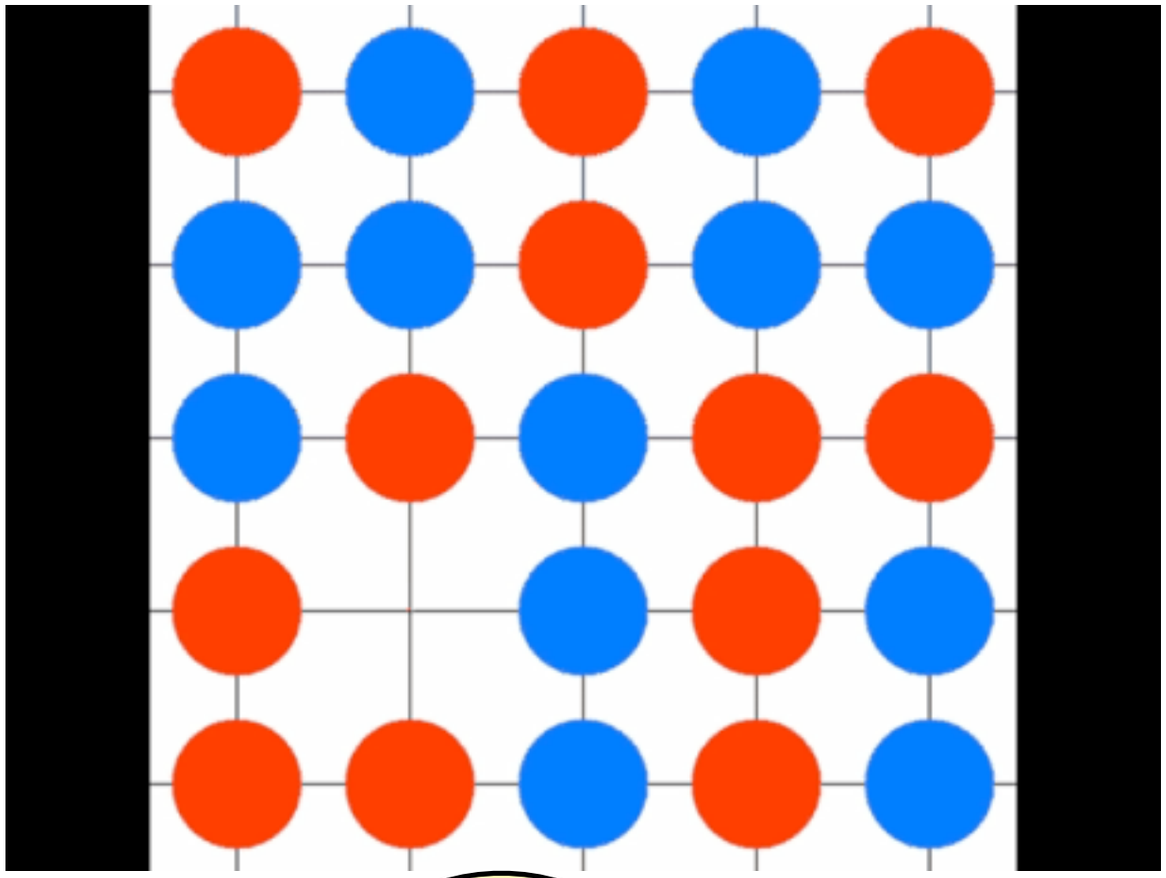
spectral function (dynamics)



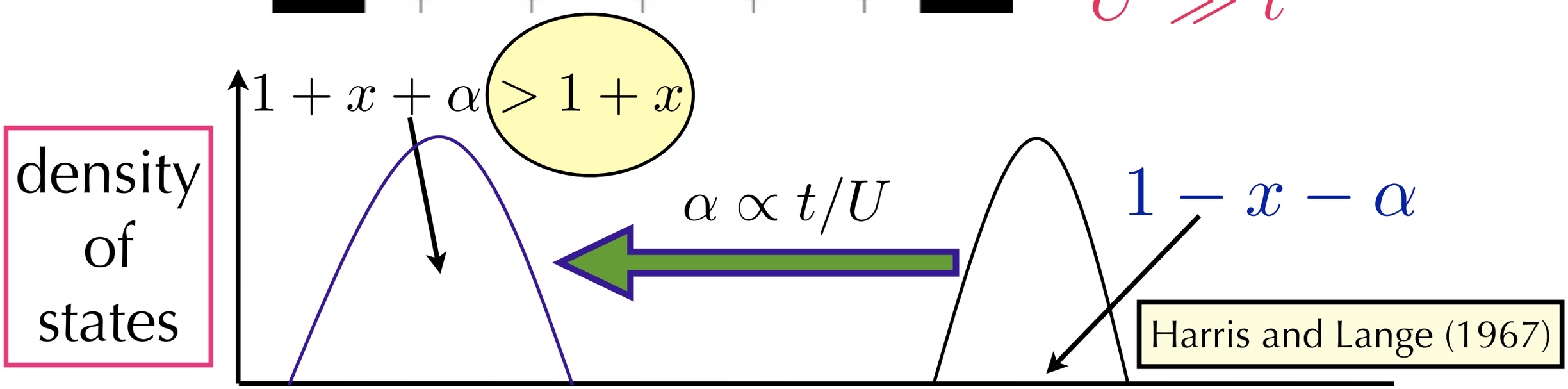
$U \gg t$



spectral function (dynamics)

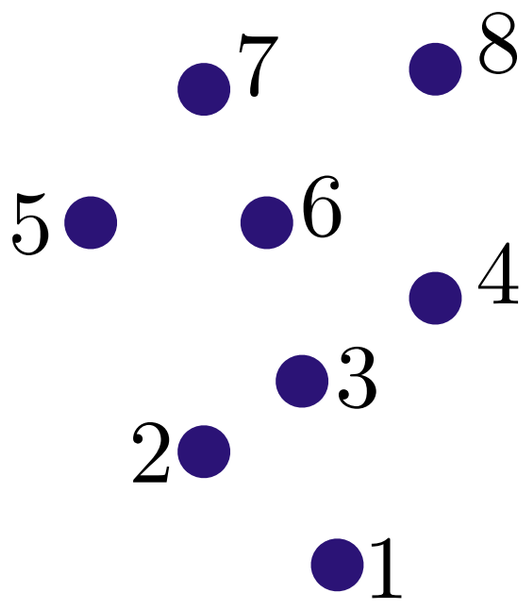


$U \gg t$



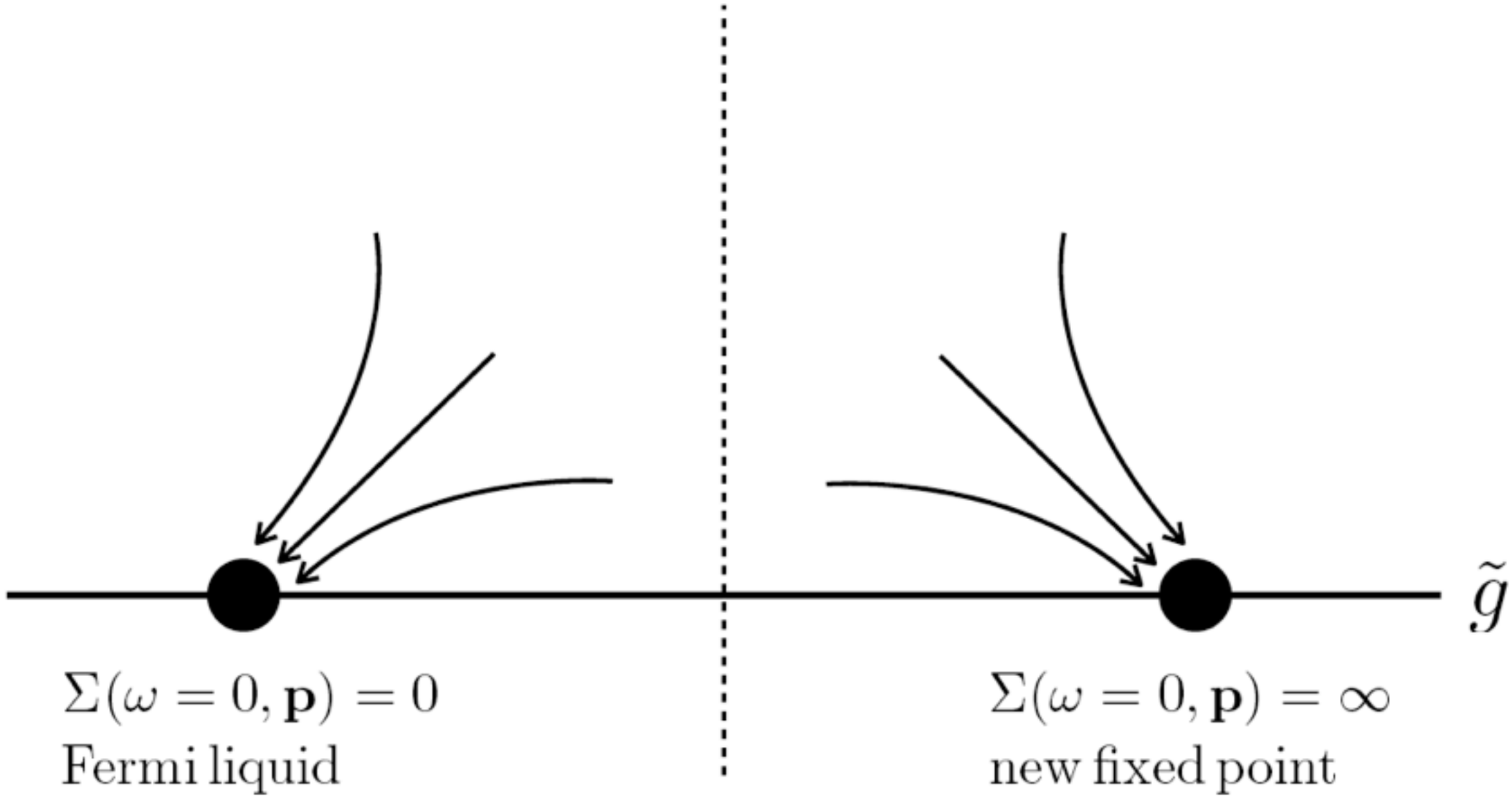
not exhausted by counting electrons alone

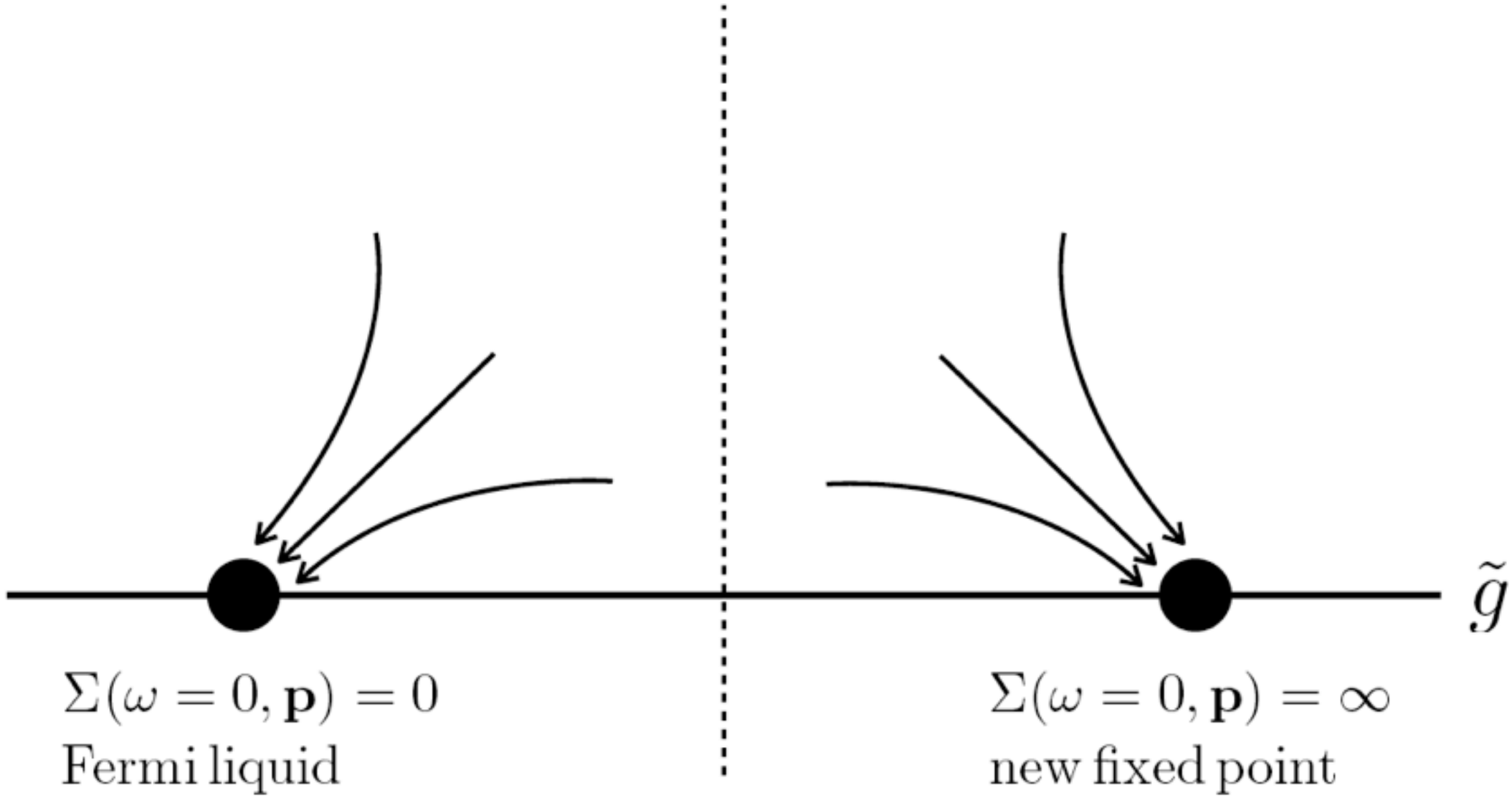
how to count particles?



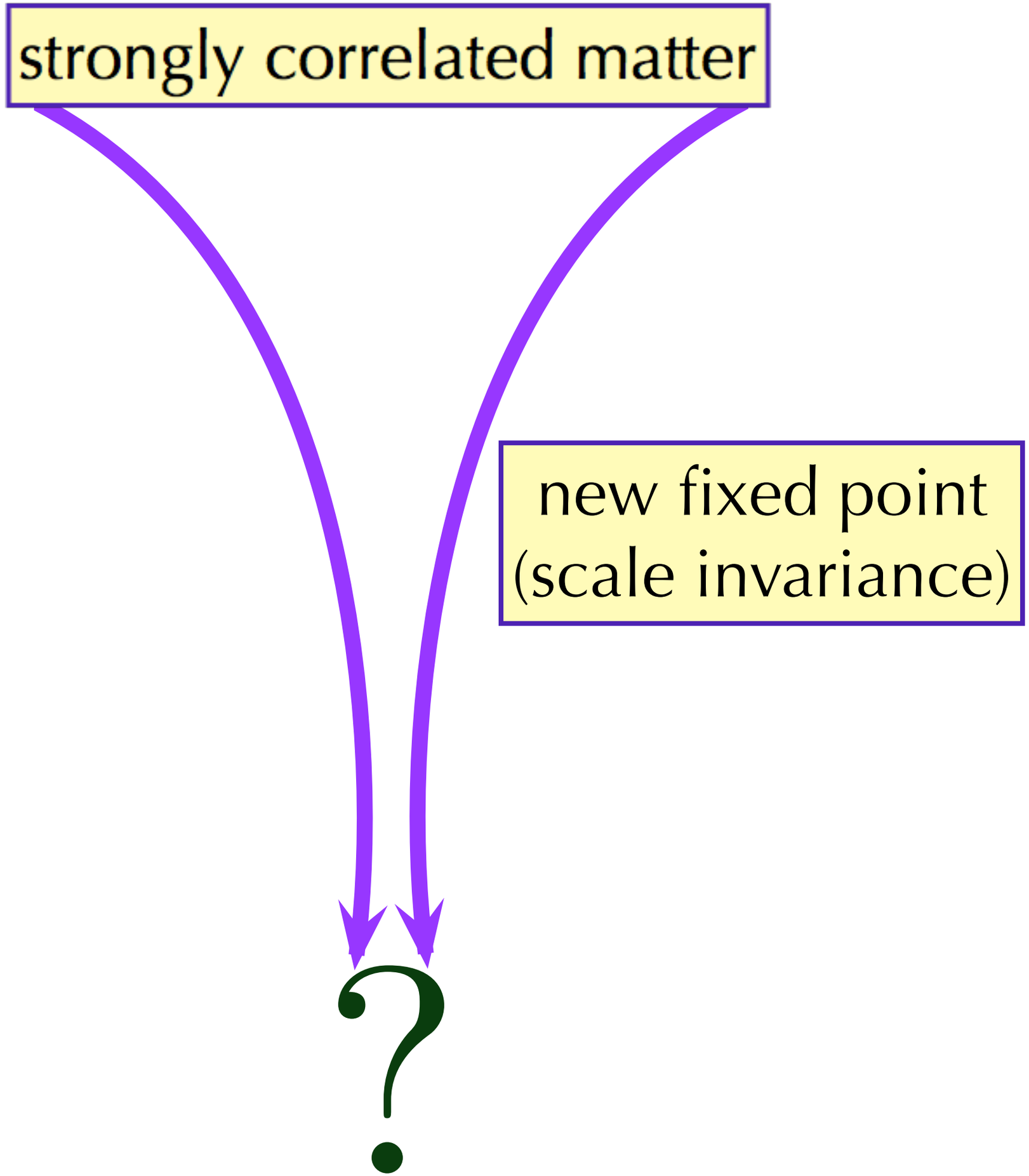
some charged stuff
has no particle interpretation

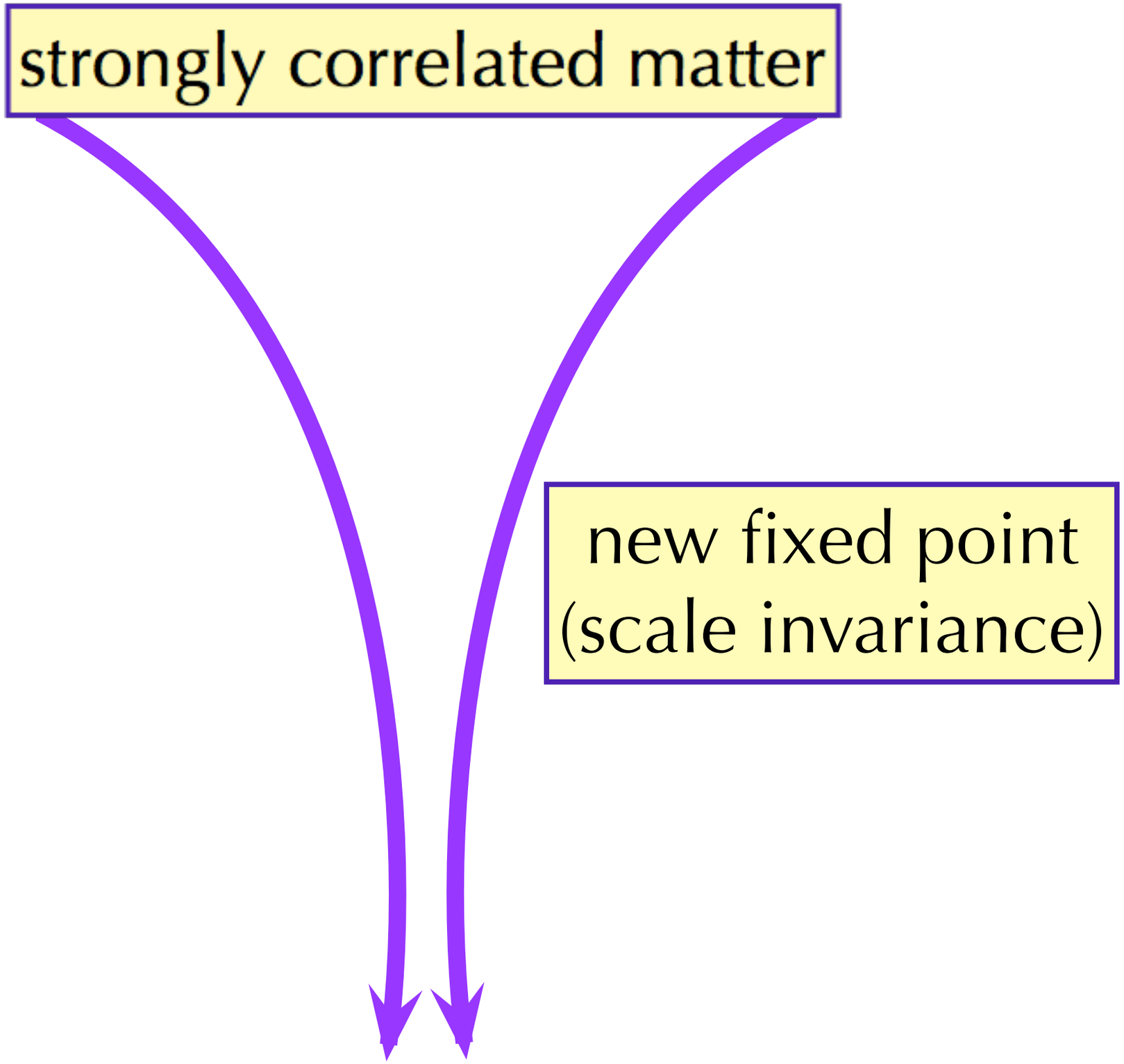
what is the extra stuff?

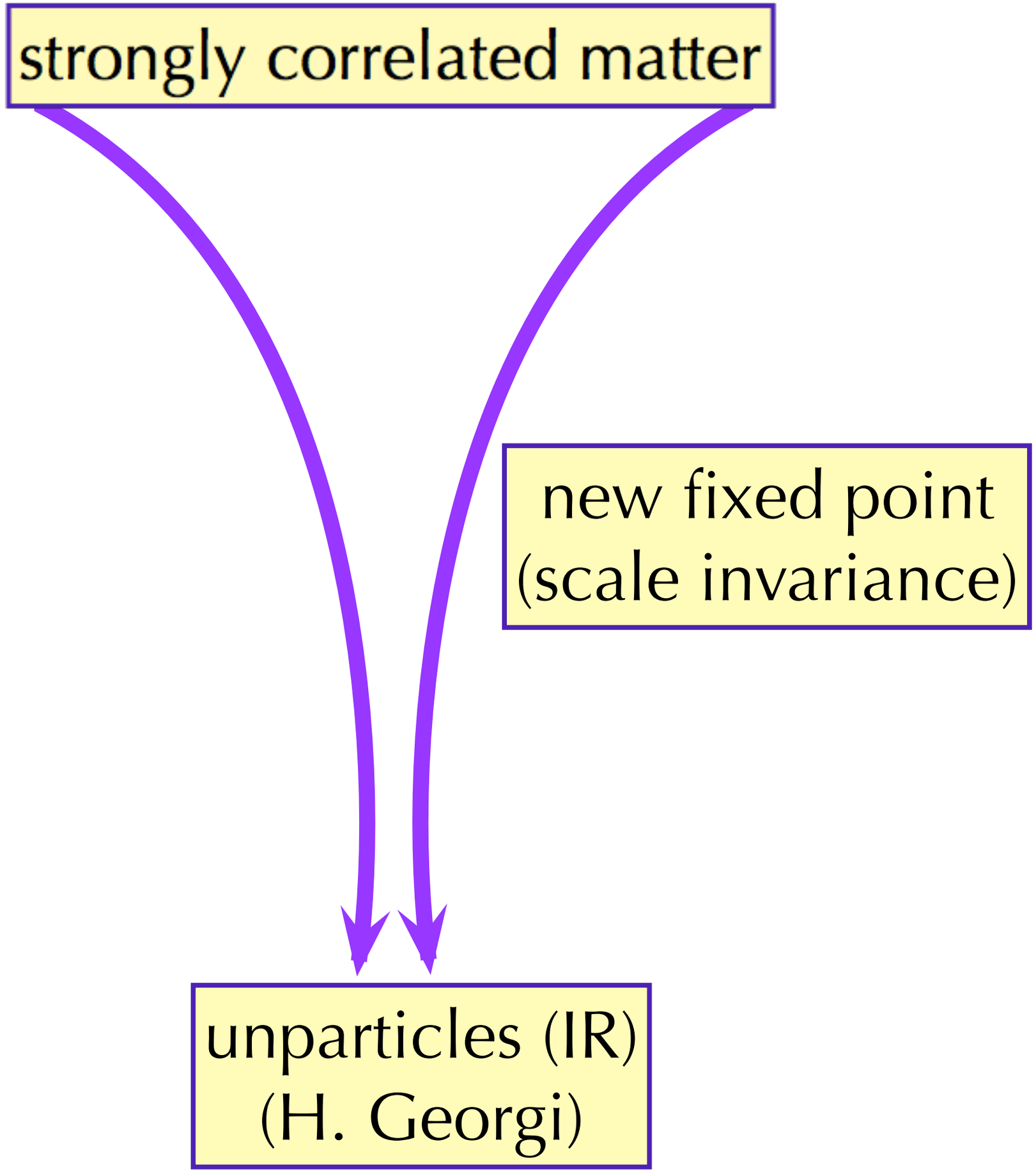

$\tilde{g}$
$\Sigma(\omega = 0, \mathbf{p}) = 0$
Fermi liquid
$\Sigma(\omega = 0, \mathbf{p}) = \infty$
new fixed point



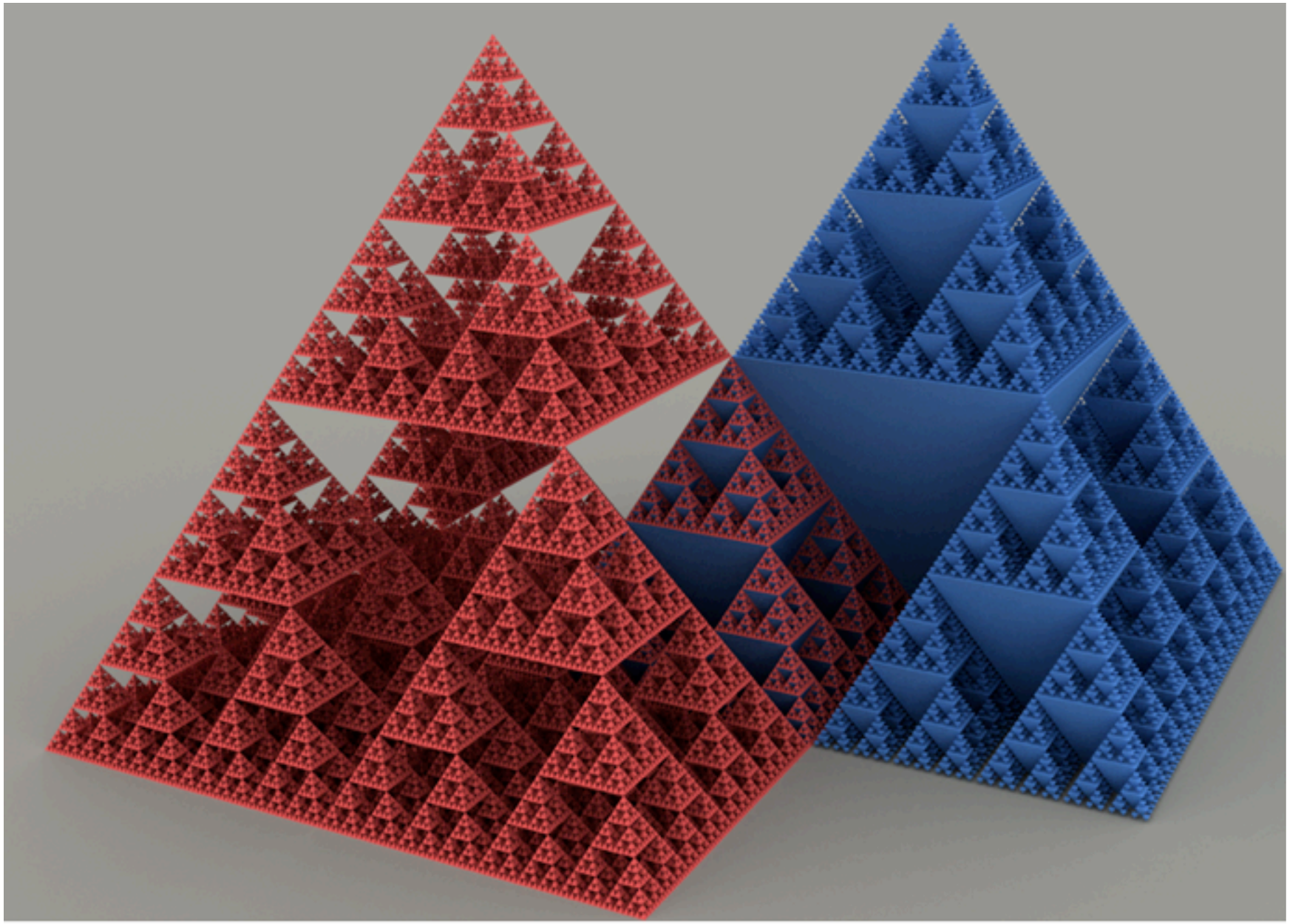
scale invariance


strongly correlated matter
new fixed point
(scale invariance)
?


strongly correlated matter
new fixed point
(scale invariance)


strongly correlated matter
new fixed point
(scale invariance)
unparticles (IR)
(H. Georgi)

what is scale invariance?



invariance on all length scales

spectral function from scale invariance

$$A(\Lambda\omega, \Lambda^{\alpha_{\vec{k}}}\vec{k}) = \Lambda^{\alpha_A} A(\omega, \vec{k})$$

spectral function from scale invariance

$$A(\Lambda\omega, \Lambda^{\alpha_{\vec{k}}}\vec{k}) = \Lambda^{\alpha_A} A(\omega, \vec{k})$$

$$A(\omega, \vec{k}) = \omega^{\alpha_A} f\left(\frac{\vec{k}}{\omega^{\alpha_{\vec{k}}}}\right)$$

what's the underlying theory?

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi + m^2 \phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$
$$\phi(x) \to \phi(x)$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

$$m^2\phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

$$\Lambda^2\left(\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\right)$$

$$m^2\phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

$$\Lambda^2\left(\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\right)$$

$$m^2\phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

$$\Lambda^2\left(\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\right)$$

$$m^2\phi^2$$

no scale invariance

mass sets a scale

unparticles
from a massive theory?

$$\mathcal{L} = \quad \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right)$$

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2 \phi^2(x,m)\right) dm^2$$

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

theory with all possible mass!

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} \to \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} \to \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

not particles

unparticles

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} \to \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

not particles

propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

$$\downarrow$$

$$d_U - 2$$

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

$$G \propto (E - \varepsilon_p)^{d_U - 2}$$

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

$$G \propto (E - \varepsilon_p)^{d_U - 2}$$

unparticles=fractional number of massless particles

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

$$G \propto (E - \varepsilon_p)^{d_U - 2}$$

unparticles=fractional number of massless particles

$d_U < 2$

almost Luttinger liquid
no pole at p_F

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

$$G \propto (E - \varepsilon_p)^{d_U - 2}$$

unparticles=fractional number of massless particles

$d_U > 2$

$d_U < 2$

zeros

almost Luttinger liquid
no pole at p_F

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

$$G \propto (E - \varepsilon_p)^{d_U - 2}$$

unparticles=fractional number of massless particles

$d_U > 2$

$d_U < 2$

zeros

no simple sign change

almost Luttinger liquid

no pole at p_F

no particle interpretation

mass-distribution

$$\phi_U \neq \int B(m^2)\phi(m^2)dm^2$$

unparticle field
(non-canonical)

particle field

$$d_U\,?$$

what really is the summation over mass?

what really is the summation over mass?

mass=energy

summation over mass?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

summation over mass?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1}$$

summation over mass?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1}$$

$$\mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[\frac{1}{2}\frac{z^2}{R^2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{\phi^2}{2R^2}\right]$$

summation over mass?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1}$$

$$\mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[\frac{1}{2}\frac{z^2}{R^2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{\phi^2}{2R^2}\right]$$

can be absorbed with

summation over mass?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1}$$

$$\mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[\frac{1}{2}\frac{z^2}{R^2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{\phi^2}{2R^2}\right]$$

can be absorbed with

$$ds^2 = \frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

summation over mass?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1}$$

$$\mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[\frac{1}{2}\frac{z^2}{R^2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{\phi^2}{2R^2}\right]$$

can be absorbed with

anti de Sitter space!

$$ds^2 = \frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

generating functional for unparticles

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2}\int d^{4+2\delta}x\, dz\, \sqrt{-g}\left(\partial_a\Phi\partial^a\Phi + \frac{\Phi^2}{R^2}\right)$$

generating functional for unparticles

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2}\int d^{4+2\delta}x\, dz\, \sqrt{-g}\left(\partial_a\Phi\partial^a\Phi + \frac{\Phi^2}{R^2}\right)$$

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \qquad \sqrt{-g} = (R/z)^{5+2\delta}$$

generating functional for unparticles

action on $AdS_{5+2\delta}$

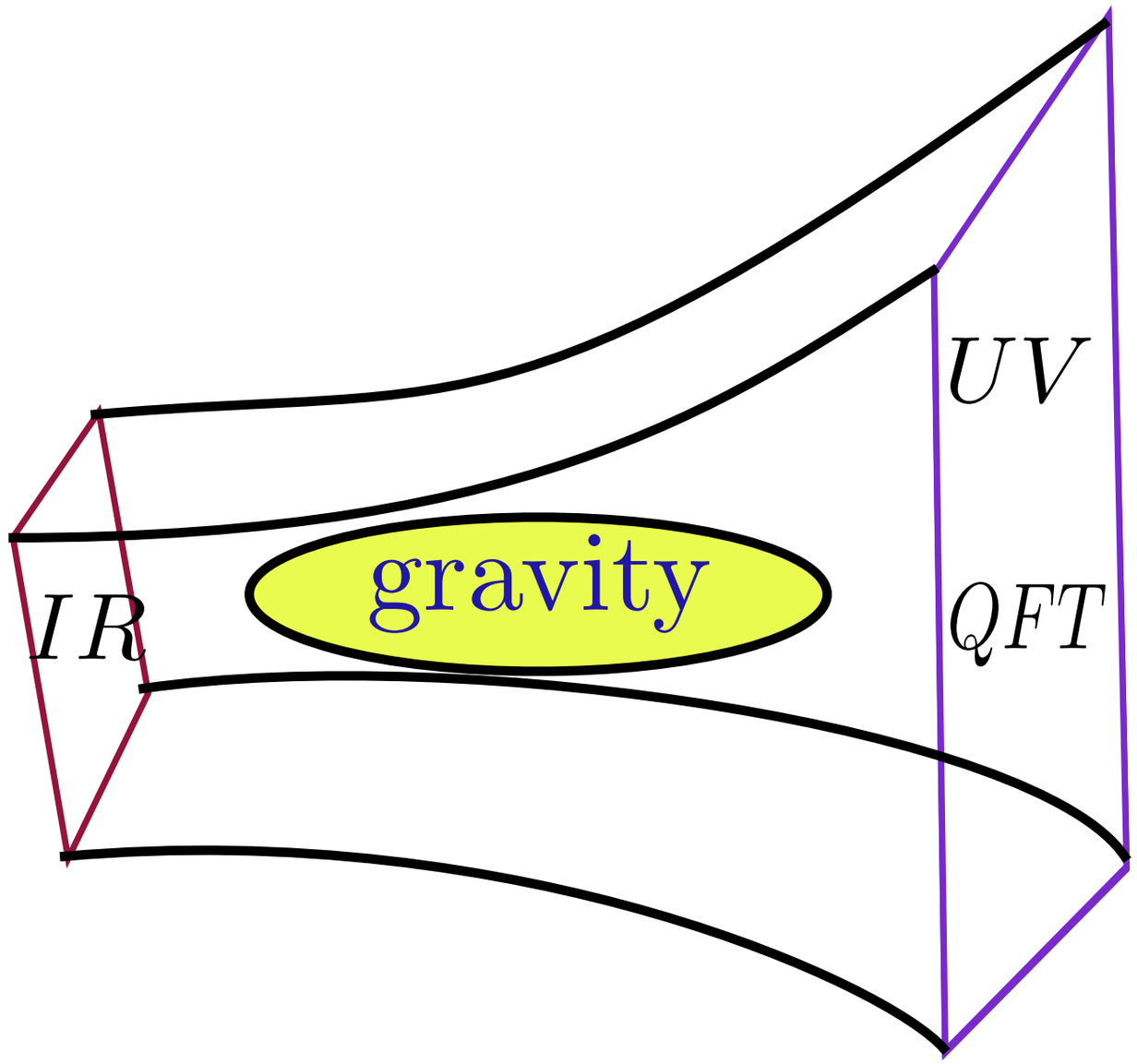
$$S = \frac{1}{2}\int d^{4+2\delta}x\, dz\, \sqrt{-g}\left(\partial_a\Phi\partial^a\Phi + \frac{\Phi^2}{R^2}\right)$$

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \qquad \sqrt{-g} = (R/z)^{5+2\delta}$$

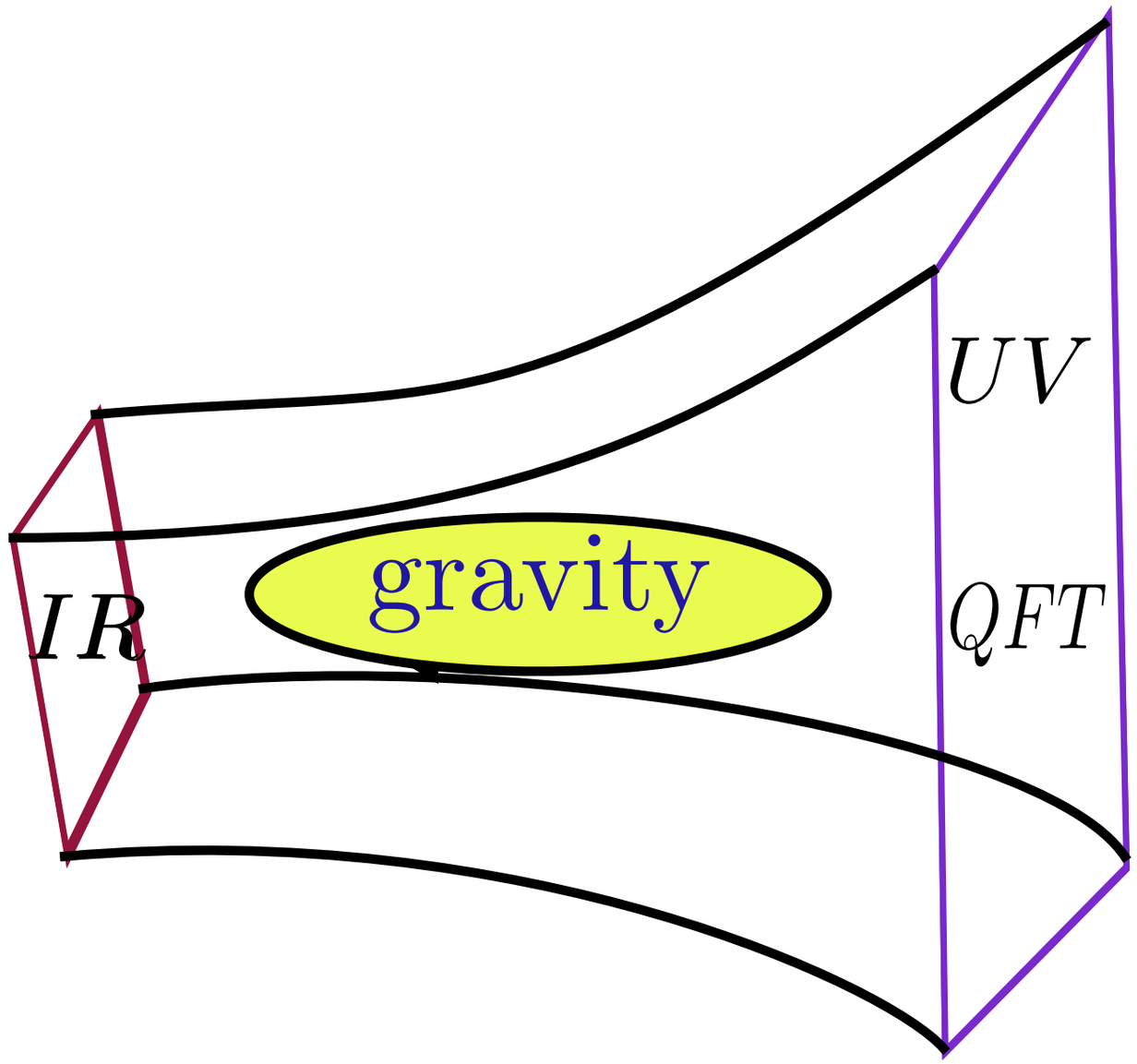
unparticle lives in

$$d = 4 + 2\delta \qquad \delta \leq 0$$

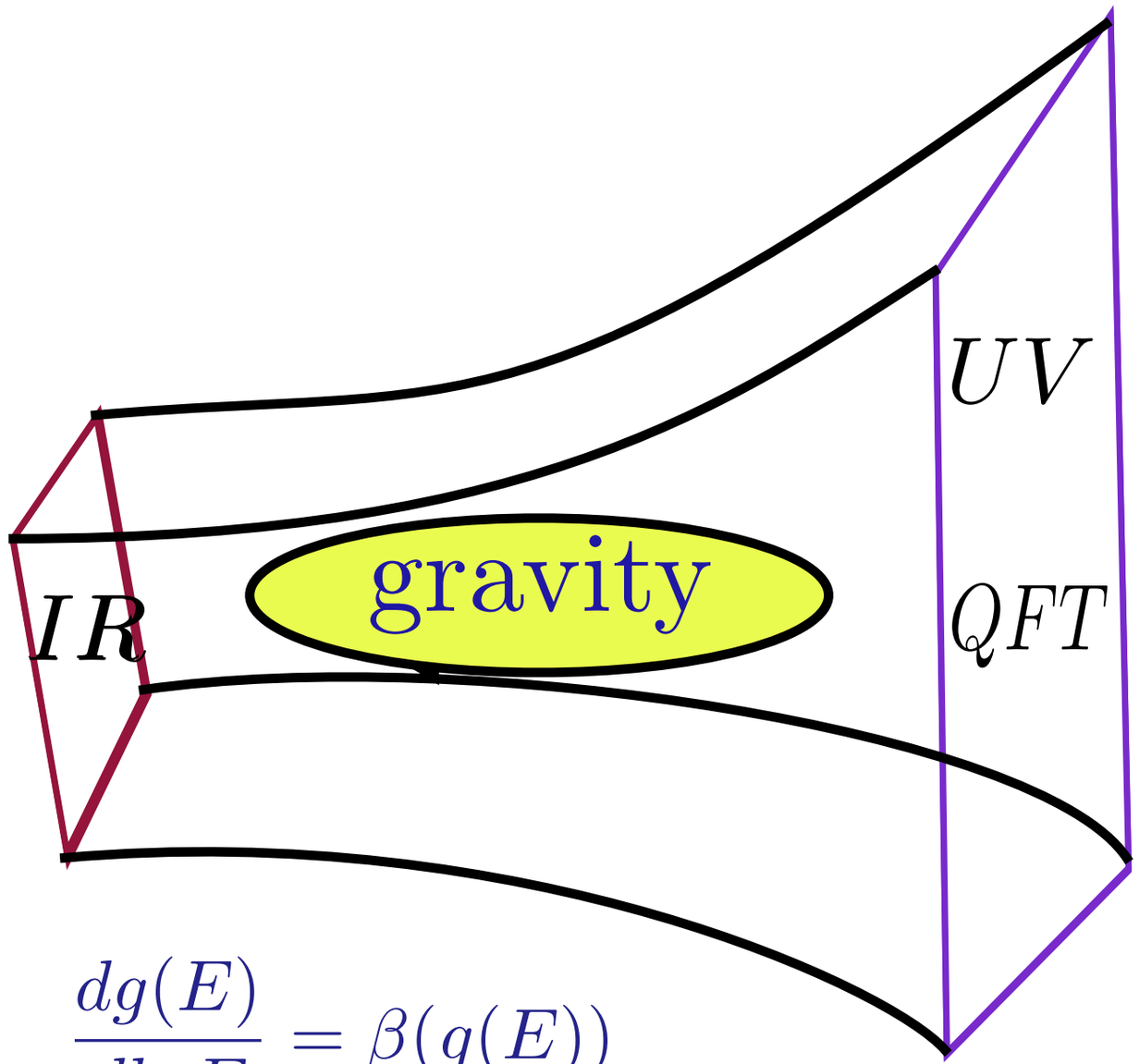
gauge-gravity duality (Maldacena, 1997)



gauge-gravity duality (Maldacena, 1997)



gauge-gravity duality
(Maldacena, 1997)

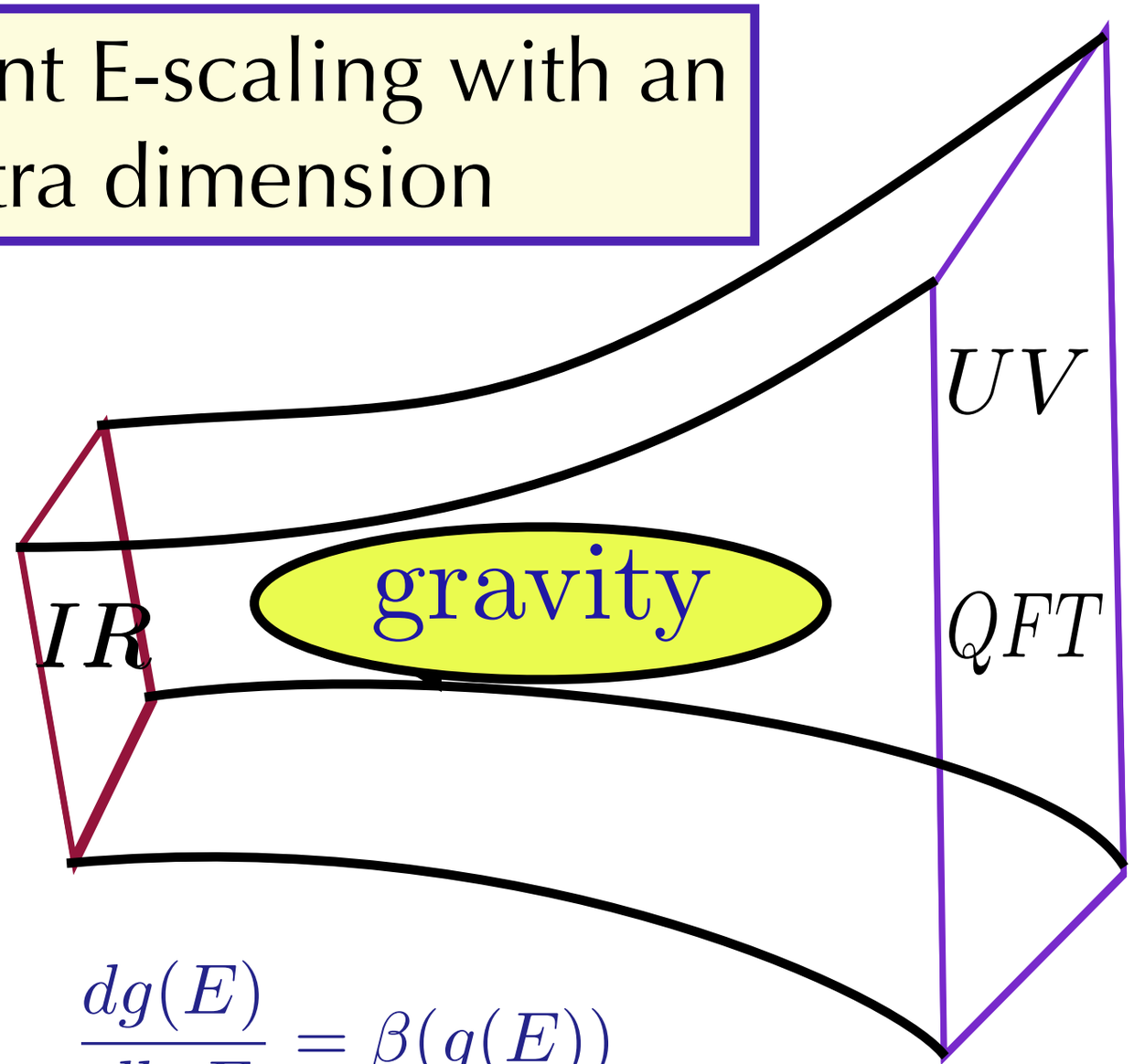


$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

gauge-gravity duality
(Maldacena, 1997)

implement E-scaling with an extra dimension



$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

Claim: $Z_{\mathrm{QFT}} = e^{-S_{\mathrm{ADS}}^{\mathrm{on-shell}}(\phi(\phi_{\partial \mathrm{ADS}=\mathrm{J}_{\mathcal{O}}}))}$

Claim: $Z_{\mathrm{QFT}} = e^{-S_{\mathrm{ADS}}^{\mathrm{on-shell}}(\phi(\phi_{\partial \mathrm{ADS}=\mathrm{J}_{\mathcal{O}}}))}$

$$S = \frac{1}{2}\int d^d x\, g^{zz}\sqrt{-g}\,\Phi(z,x)\partial_z\Phi(z,x)\Big|_{z=\epsilon}$$

Claim: $Z_{\rm QFT} = e^{-S^{\rm on-shell}_{\rm ADS}(\phi(\phi_{\partial {\rm ADS}={\rm J}_{\mathcal{O}}}))}$

$$S = \frac{1}{2}\int d^d x\, g^{zz}\sqrt{-g}\,\Phi(z,x)\partial_z\Phi(z,x)\Big|_{z=\epsilon}$$

$$\langle \Phi_U(x)\Phi_U(x')\rangle = \frac{1}{|x-x'|^{2d_U}}$$

Claim: $Z_{\rm QFT} = e^{-S^{\rm on-shell}_{\rm ADS}(\phi(\phi_{\partial {\rm ADS}={\rm J}_{\mathcal{O}}}))}$

$$S = \frac{1}{2}\int d^d x\, g^{zz}\sqrt{-g}\,\Phi(z,x)\partial_z\Phi(z,x)\Big|_{z=\epsilon}$$

$$\langle \Phi_U(x)\Phi_U(x')\rangle = \frac{1}{|x-x'|^{2d_U}}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

Claim: $Z_{\rm QFT} = e^{-S^{\rm on-shell}_{\rm ADS}(\phi(\phi_{\partial {\rm ADS}={\rm J}_{\mathcal{O}}}))}$

$$S = \frac{1}{2}\int d^d x\, g^{zz}\sqrt{-g}\,\Phi(z,x)\partial_z\Phi(z,x)\Big|_{z=\epsilon}$$

$$\langle \Phi_U(x)\Phi_U(x')\rangle = \frac{1}{|x-x'|^{2d_U}}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

scaling dimension is fixed

$$G_U(p) \propto p^{2(d_U - d/2)}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

$$G_U(0) = 0$$

unparticle (AdS) propagator has zeros!

$$G_U(p) \propto p^{2(d_U - d/2)}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

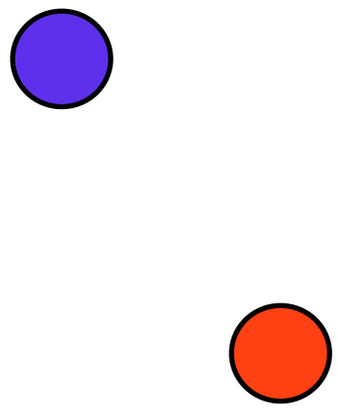
$$G_U(0) = 0$$

interchanging unparticles

fractional (d_U) number of massless particles

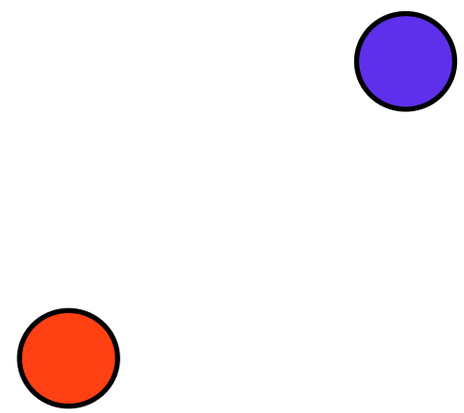
interchanging unparticles

fractional (d_U) number of massless particles



interchanging unparticles

fractional (d_U) number of massless particles



interchanging unparticles

fractional (d_U) number of massless particles

interchanging unparticles

fractional (d_U) number of massless particles

$$e^{i\pi d_U} \neq -1, 0$$

interchanging unparticles

fractional (d_U) number of massless particles

$$e^{i\pi d_U} \neq -1, 0$$

fractional statistics in d=2+1

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

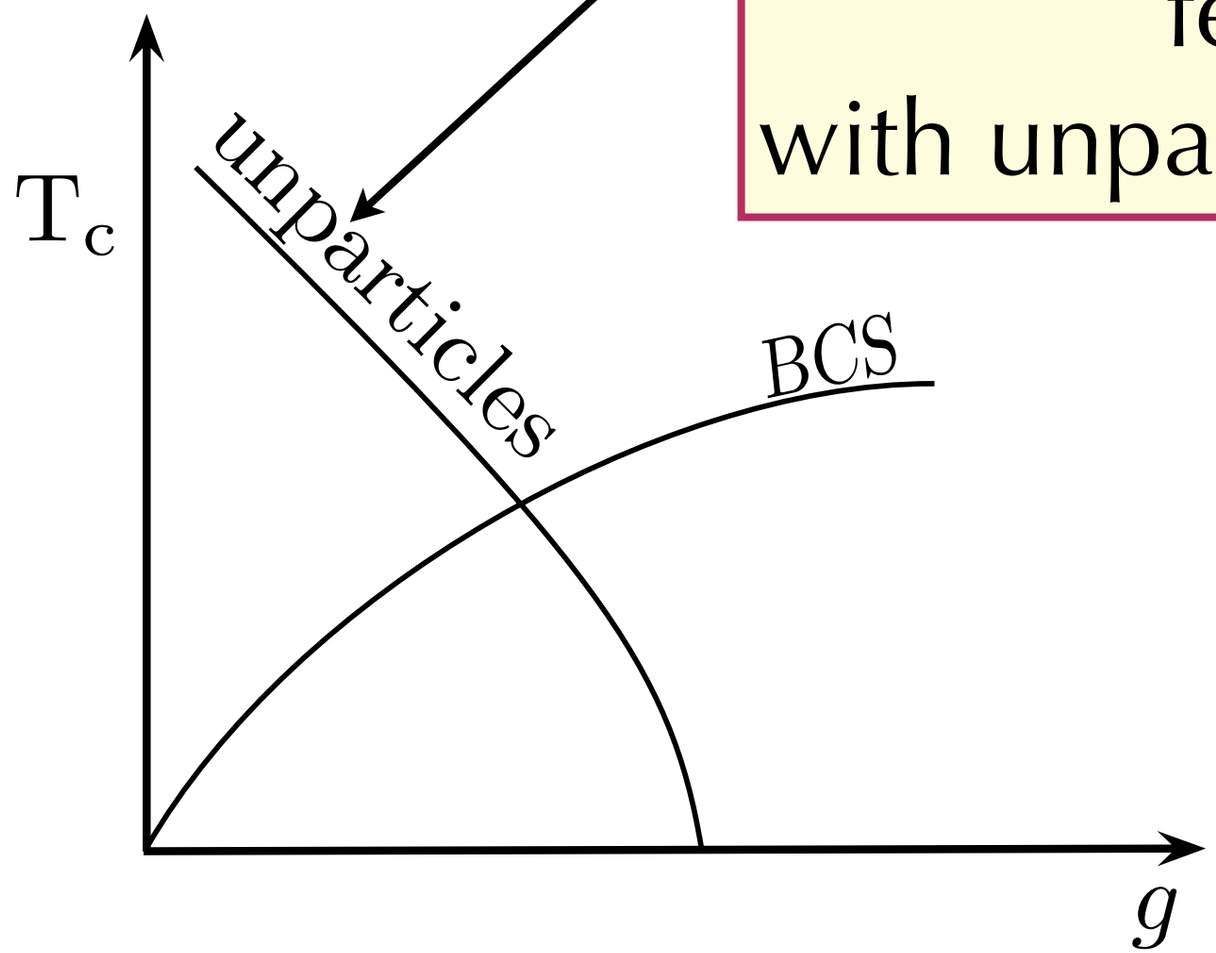
$$e^{i\pi d_U} \neq e^{-i\pi d_U}$$

$$\Downarrow$$

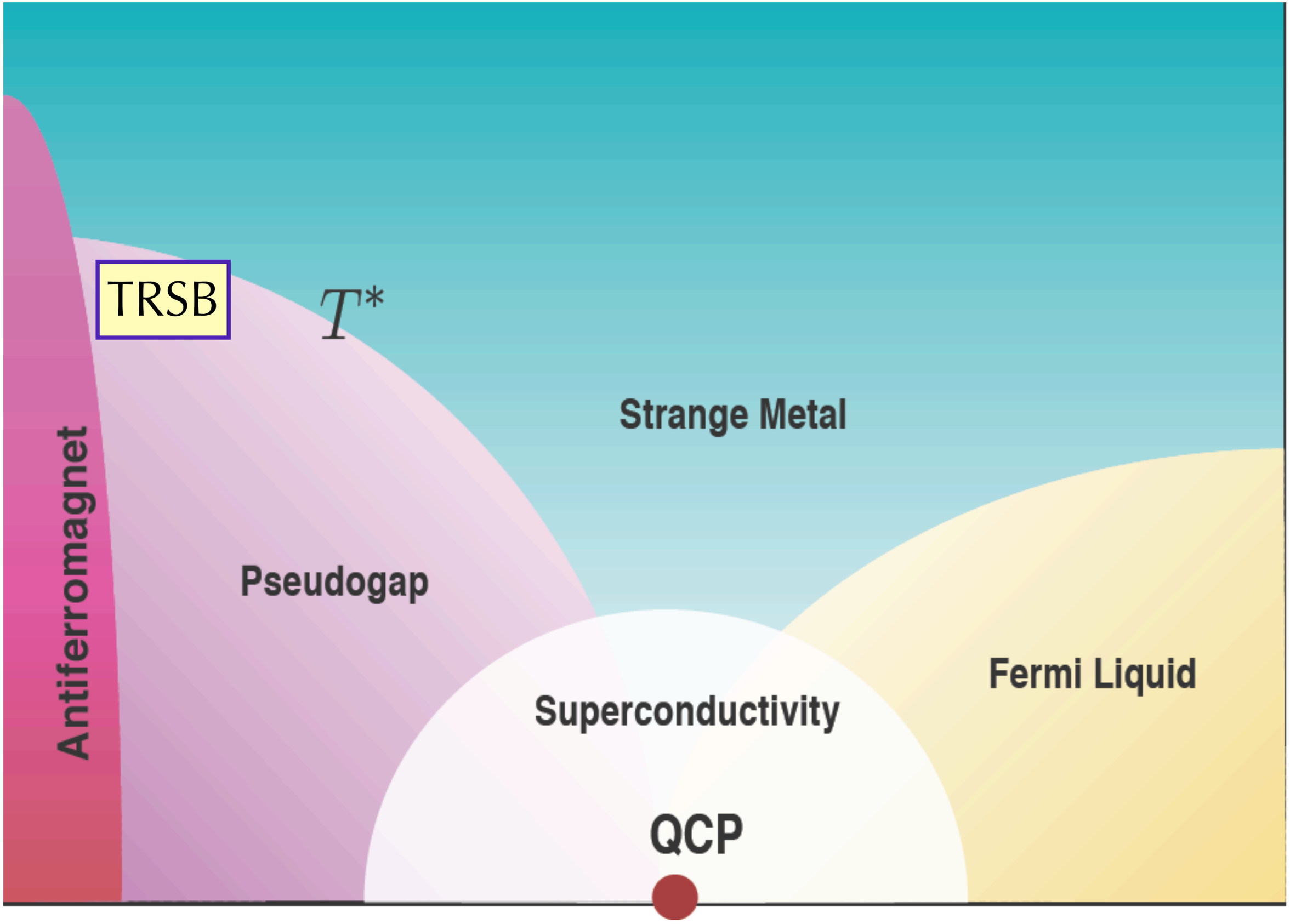
time-reversal symmetry breaking
from unparticle (zeros=Fermi arcs) matter

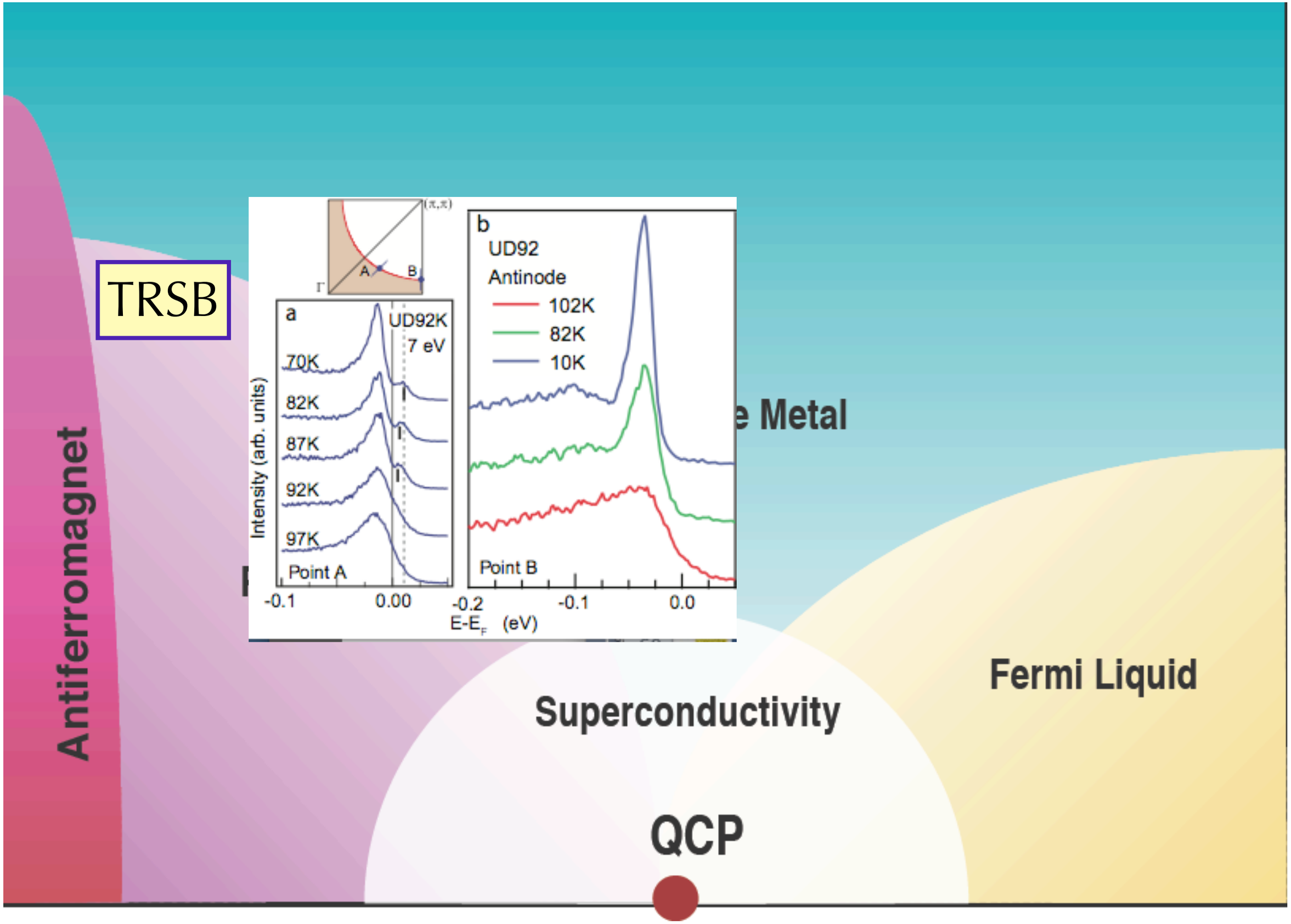
$$\frac{d \ln g}{d \ln \beta} = 4d_U - d > 0$$

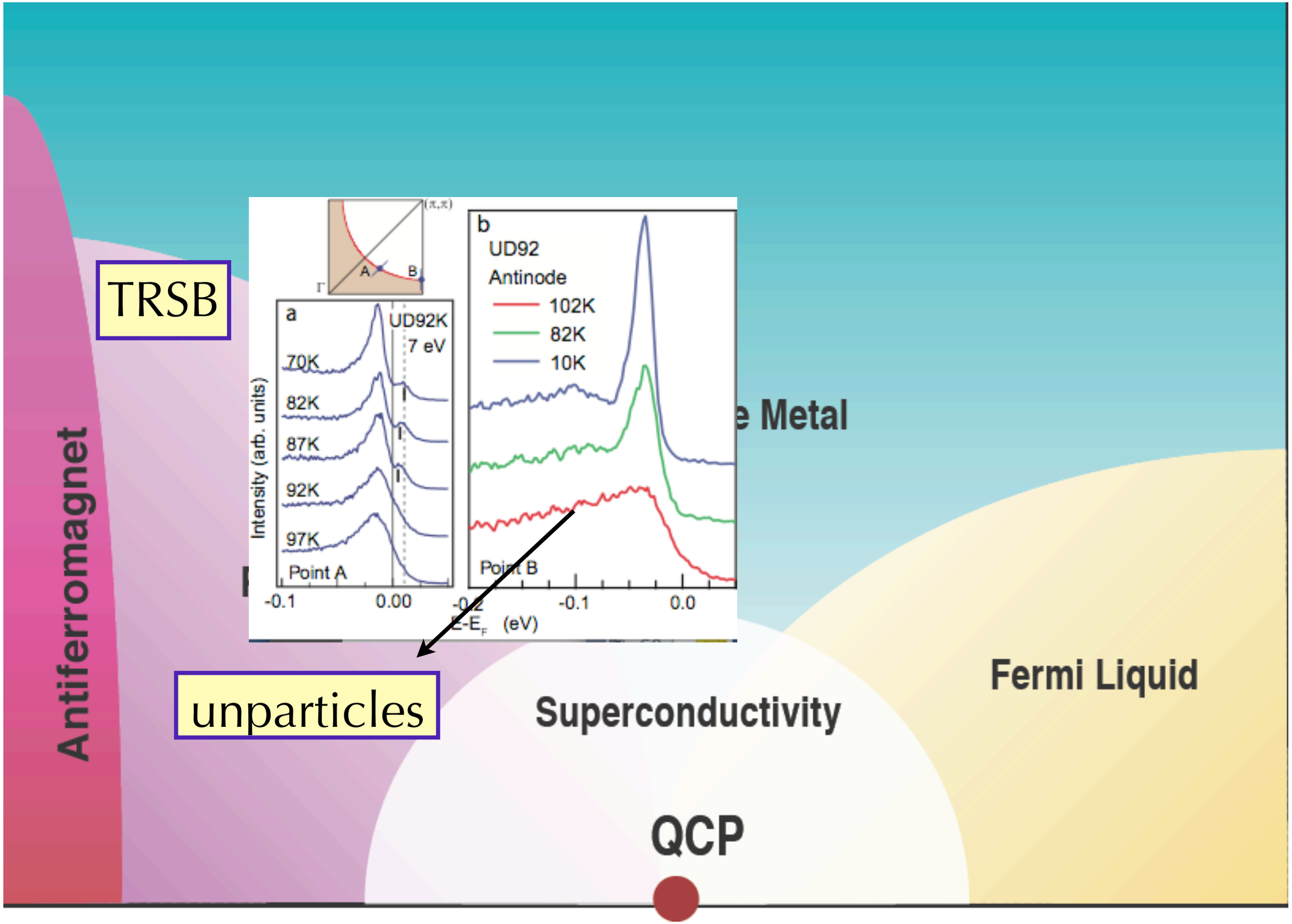
fermions
with unparticle propagator

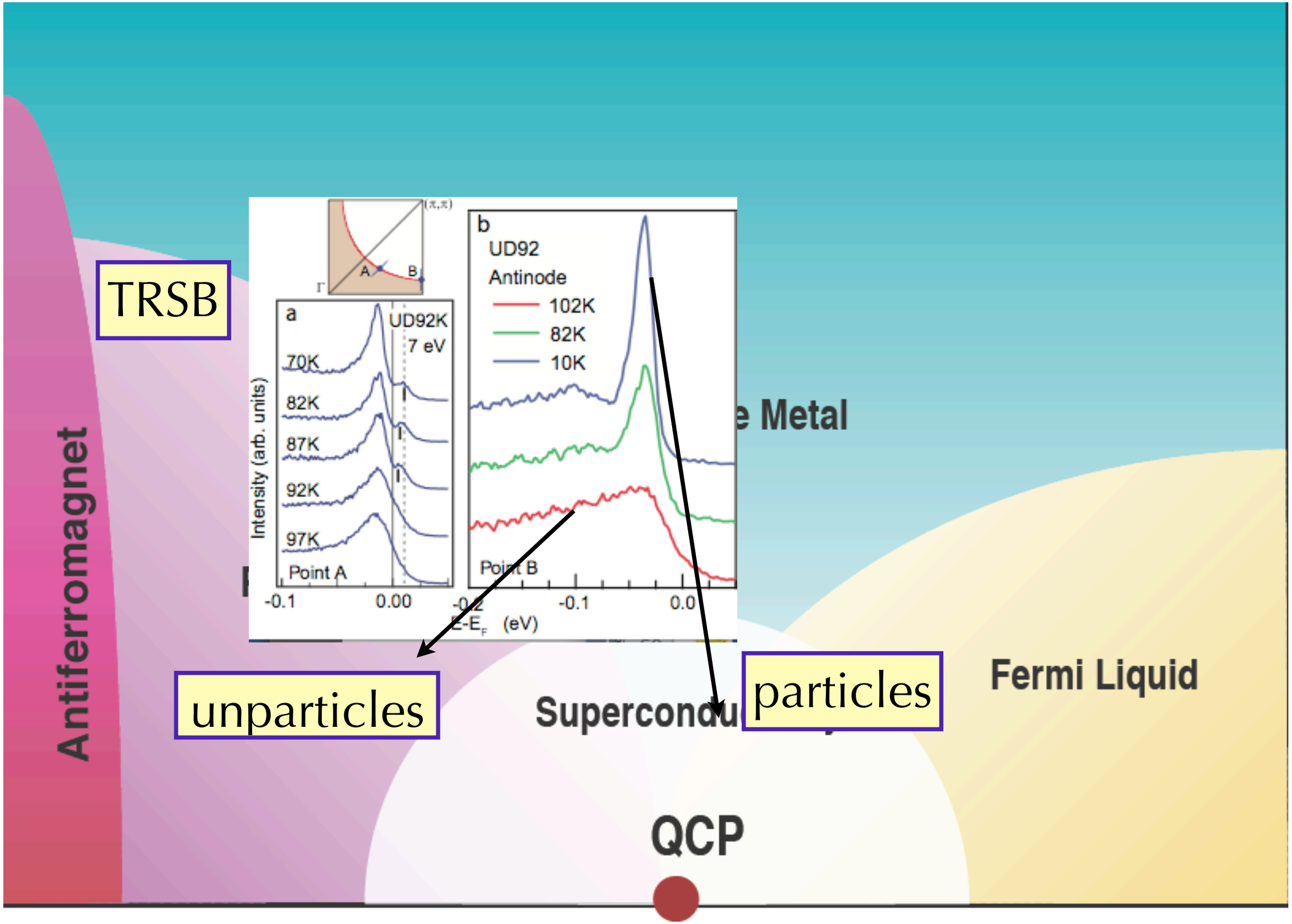


tendency towards pairing (any instability which establishes a gap)


TRSB
T^*
Strange Metal
Antiferromagnet
Pseudogap
Fermi Liquid
Superconductivity
QCP

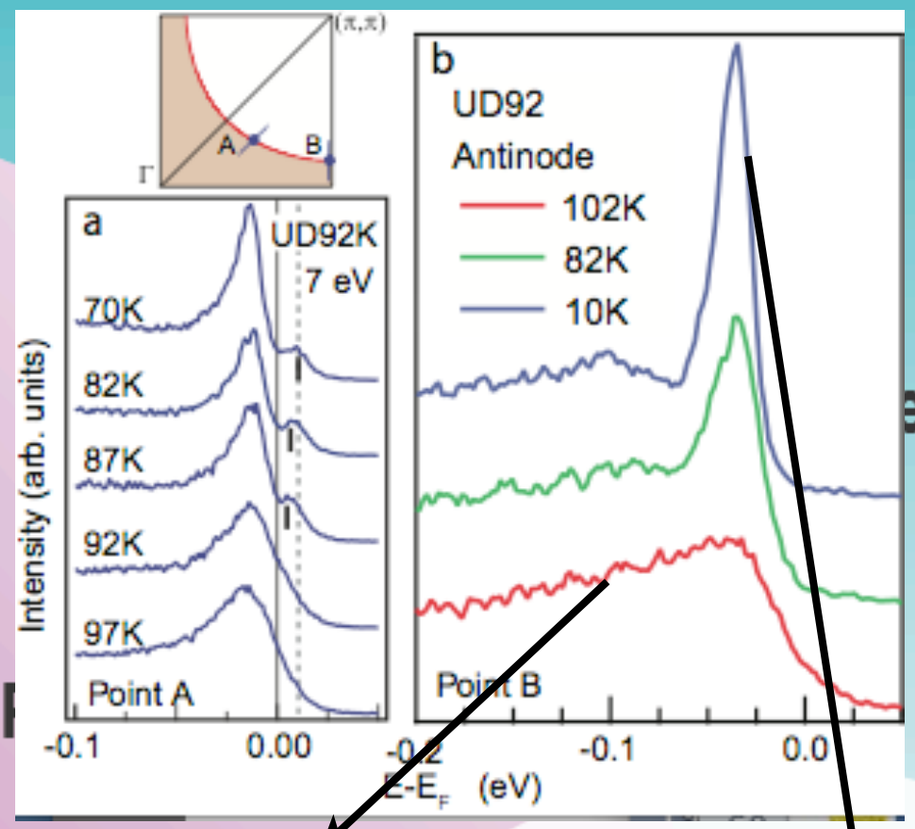

TRSB
Antiferromagnet
e Metal
Fermi Liquid
Superconductivity
QCP
(π,π)
Γ
A
B
a
UD92K
7 eV
70K
82K
87K
92K
97K
Point A
Intensity (arb. units)
-0.1
0.00
b
UD92
Antinode
102K
82K
10K
Point B
-0.2
-0.1
0.0
E-E_F (eV)


TRSB
Antiferromagnet
Metal
Fermi Liquid
Superconductivity
QCP
unparticles
(π,π)
Γ
A
B
a
UD92K
7 eV
70K
82K
87K
92K
97K
Point A
Intensity (arb. units)
-0.1
0.00
b
UD92
Antinode
102K
82K
10K
Point B
-0.2
-0.1
0.0
E-E_F (eV)


TRSB
Antiferromagnet
Metal
Fermi Liquid
Supercondu
QCP
unparticles
particles
(π,π)
Γ
A
B
a
UD92K
7 eV
70K
82K
87K
92K
97K
Point A
Intensity (arb. units)
-0.1
0.00
b
UD92
Antinode
102K
82K
10K
Point B
-0.1
0.0
E-E_F (eV)

breaking of scale invariance

TRSB



Antiferromagnet

e Metal

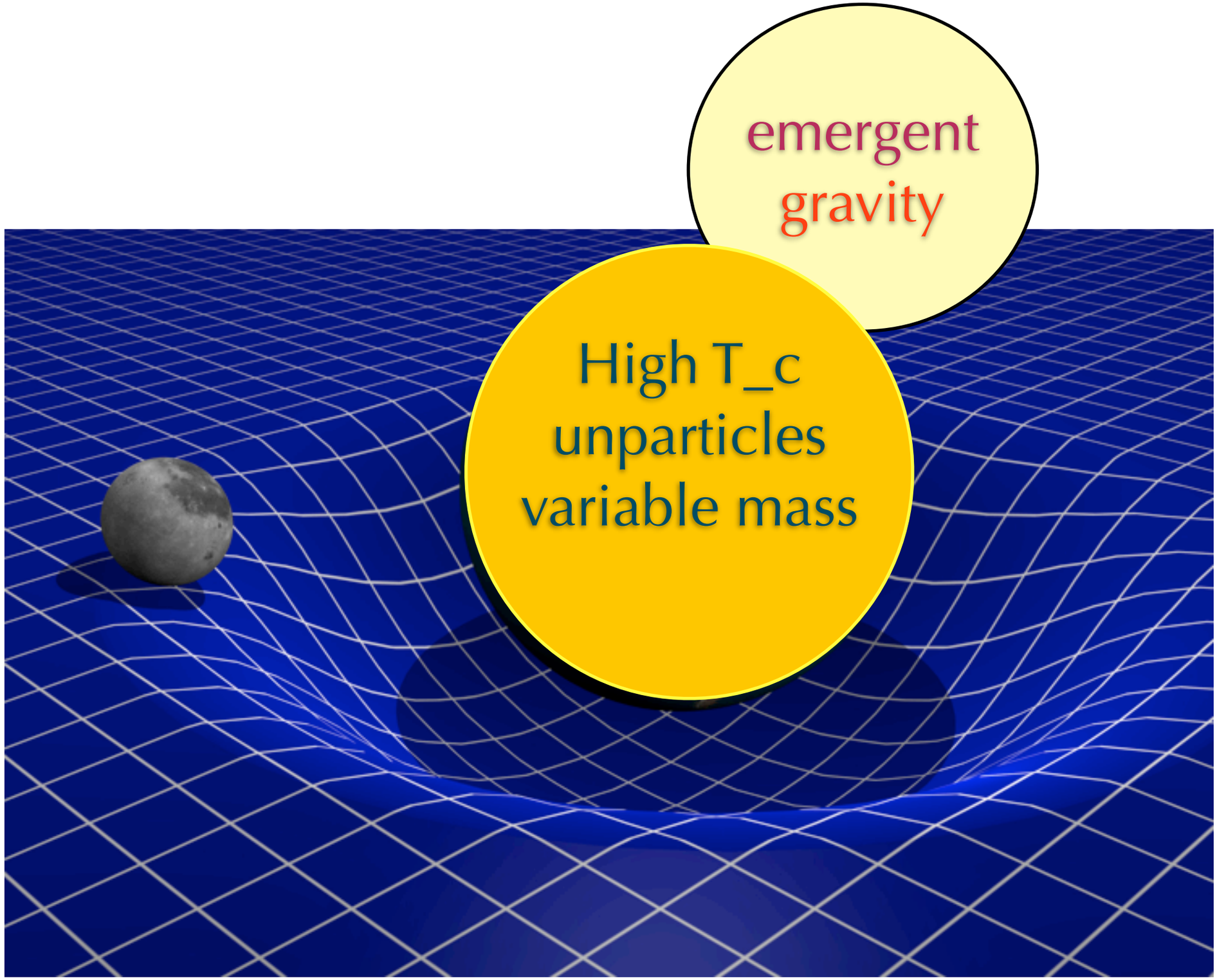
unparticles

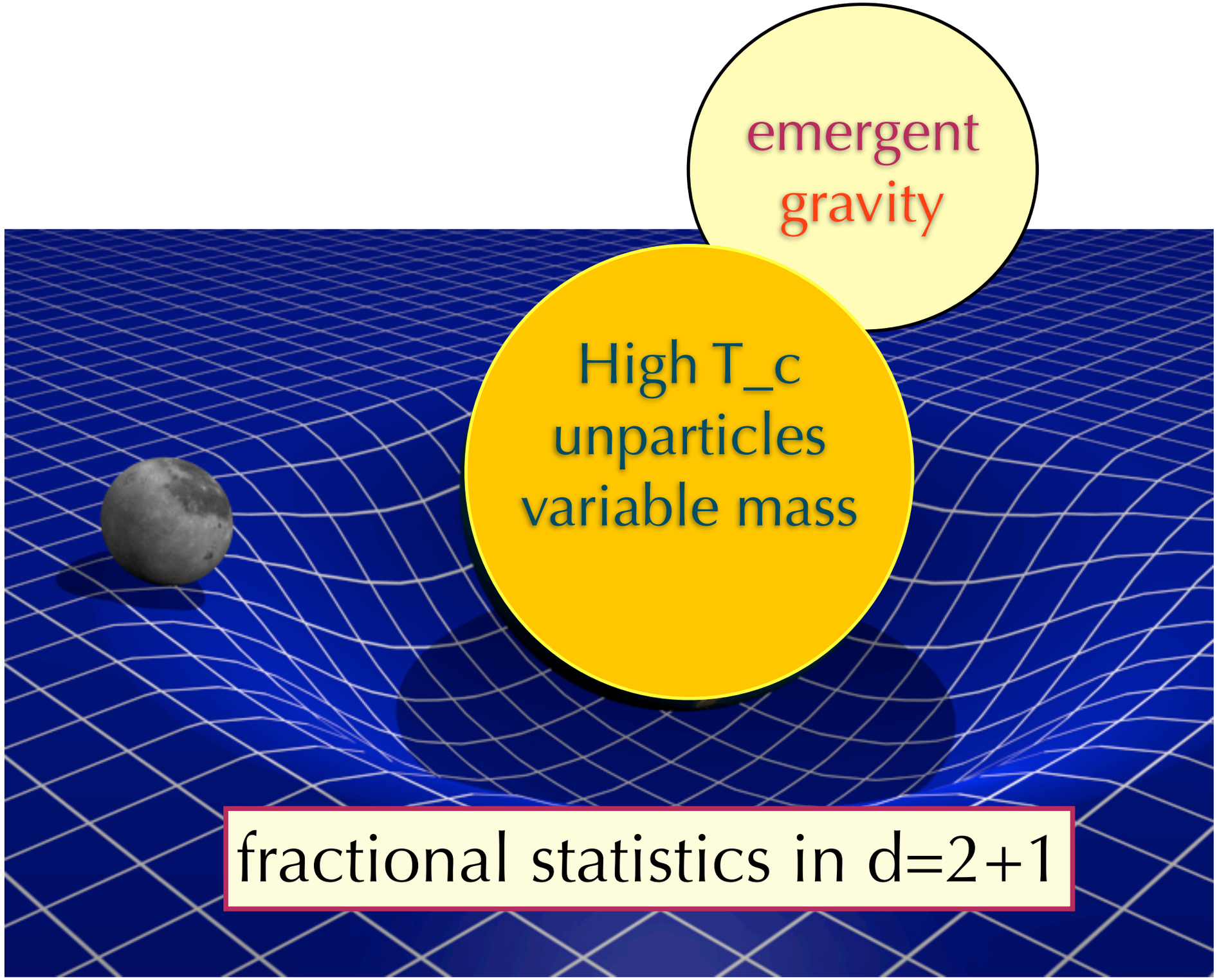
Supercondu

particles

Fermi Liquid

QCP


emergent
gravity
High T_c
unparticles
variable mass


emergent
gravity
High T_c
unparticles
variable mass
fractional statistics in d=2+1