Unparticles and Emergent Mottness

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Correlated Electron Matter
Correlated Electron Matter

What is carrying the current?
current-carrying excitations

\[ e^* = \frac{e}{q} \] (q odd)
fractional quantum Hall effect

current-carrying excitations

\[ e^* = \frac{e}{q} \quad (q \text{ odd}) \]
fractional quantum Hall effect

current-carrying excitations
fractional quantum Hall effect

current-carrying excitations

\[ e^* = ? \]
How Does Fermi Liquid Theory Breakdown?

$\rho \propto aT$

周二，四月 30 日，2013
How Does Fermi Liquid Theory Breakdown?

\[ \rho \propto aT \]

TRSB

Antiferromagnet

Pseudogap

Strange Metal

Fermi Liquid

Antiferromagnet

Pseudogap

Strange Metal

Fermi Liquid

Tuesday, April 30, 2013
can the charge density be given a particle interpretation?
can the charge density be given a particle interpretation?

does charge density = total number of electrons?
if not?
if not?

charge stuff = particles + other stuff
if not?

charge stuff = particles + other stuff

unparticles!
counting particles
counting particles

1
counting particles

1

2

1
counting particles

2

3

1
counting particles
counting particles

is there a more efficient way?
\[ n = \int_{-\infty}^{\mu} N(\omega) d\omega \]
can $n$ be deduced entirely from the IR(low-energy) scale?
Luttinger’s `theorem'
Luttinger’s theorem

\[ n = 2 \sum_k \Theta(\mathcal{R}G(k, \omega = 0)) \]
Luttinger’s theorem

\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]
Luttinger’s theorem

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]

counting poles (qp)
Luttinger’s theorem

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]

zero-crossing

counting poles (qp)
Luttinger’s theorem

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

\[ n = 2 \sum_{k} \Theta(\Re G(k, \omega = 0)) \]

\[ \text{Det} G(\omega = 0, \bar{\rho}) = 0 \]

\[ E > \varepsilon_p \]

\[ E < \varepsilon_p \]

zero-crossing

counting poles (qp)
singularities of $\ln G$

\[ n = \frac{2i}{(2\pi)^{d+1}} \int d^d p \int_{-\infty}^{0} d\xi \ln \frac{G^R(\xi, p)}{G^*_R(\xi, p)} \]

poles+zeros
(all sign changes)
singularities of $\ln G$

\[ n = \frac{2i}{(2\pi)^{d+1}} \int d^d p \int_0^0 d\xi \ln \frac{G^R(\xi, p)}{G^*_R(\xi, p)} \]

\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]

poles+zeros
(all sign changes)
what are zeros?
what are zeros?

Im $G=0$

$\mu = 0$
what are zeros?

Re\(G(0, p) = \int_{-\infty}^{\infty} \left( \mu = 0 \right) d\omega \)

Im \(G=0\)

Kramers-Kronig
what are zeros?

\[ \text{Re}G(0, p) = \int_{-\infty}^{\infty} \left( \begin{array}{c} \mu = 0 \\ \omega \end{array} \right) d\omega = \text{below gap+above gap} \]

\[ \text{Im} G = 0 \]

Kramers-Kronig
what are zeros?

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \begin{array}{c} \mu = 0 \\ \omega \end{array} \right) d\omega \]

= below gap + above gap = 0

Im \(G\) = 0

Kramers-Kronig

Tuesday, April 30, 2013
what are zeros?

$\text{Im } G = 0$

$\text{Kramers-Kronig}$

$\text{Det} G(k, \omega = 0) = 0 \ (\text{single band})$
what are zeros?

Re$G(0, p) = \int_{-\infty}^{\infty} \left( \begin{array}{c} \omega \
\mu = 0
\end{array} \right) d\omega$

= below gap + above gap

$\text{Det} G(k, \omega = 0) = 0$ (single band)

strongly correlated gapped systems
NiO insulates $d^8$?

Mott mechanism (not Slater)
NiO insulates $d^8$?

perhaps this costs energy

Mott mechanism (not Slater)
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Mott mechanism (not Slater)

$U \gg t$
NiO insulates $d^8$? Perhaps this costs energy.

Mott mechanism (not Slater)

$U \gg t$

$\mu = 0$
NiO insulates $d^8$? Perhaps this costs energy.

Mott mechanism (not Slater)

$\mu = 0$

$U \gg t$

No change in size of Brillouin zone.

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NiO insulates $d^8$?

perhaps this costs energy

Mott mechanism (not Slater)

$U \gg t$

no change in size of Brillouin zone

$\mu = 0$

$\text{DetReG}(0,p)=0$

$U - \mu = \Delta E - E_F$

$U - \mu = E_F - E_F$

$U - \mu = 0$

$\Delta E = 0$

$\mu = 0$

$\text{DetReG}(0,p)=0$
NiO insulates $d^8$? perhaps this costs energy

Mott mechanism (not Slater)

$U \gg t$

$\mu = 0$

no change in size of Brillouin zone

$\text{DetReG}(0,p)=0 = \text{Mottness}$
simple problem: $n=1$

$SU(2)$

$U$
simple problem: $n=1$

$SU(2)$

$\mu$

$U$
simple problem: $n=1$

$SU(2)$

\[ \mu \]

\[ -\frac{U}{2} \]
simple problem: $n=1$
simple problem: $n=1$

$$G = \frac{1}{\omega + \frac{U}{2}} + \frac{1}{\omega - \frac{U}{2}}$$

$SU(2)$
simple problem: $n=1$

\[ G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0 \]
simple problem: $n=1$

$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$

$$n = 2\theta(0) = 1$$
Fermi Liquids

Luttinger’s theorem

Mott Insulators
The proof just presented is good for any state of our system: FL, NFL, or MI [in other words, either of poles or zeros can be used to change the sign of \( G_r \) in Eq. (1)]. The only way to incapacitate the Luttinger theorem in form (1) is to assume that the limit \( T \to 0 \) is discontinuous. Actually, one has to require that the whole line \( T = 0 \) is a line of phase transitions.
Is this famous theorem from 1960 correct?
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails

no hopping => no propagation (zeros)
A model with zeros but Luttinger fails

N flavors of e-

no hopping => no propagation (zeros)
$SU(N)$

$$H = \frac{U}{2} (n_1 + \cdots n_N)^2$$
SU(N) for N=3: no particle-hole symmetry

\begin{enumerate}
  \item \( \frac{9}{2} U \) \\
  \item \( 2U \) \\
  \item \( \frac{1}{2} U \)
\end{enumerate}
SU(N) for N=3: no particle-hole symmetry

\[
\lim_{T \to 0} \mu(T)
\]
compute Green function exactly (Lehman formula)

\[ G_{\alpha\beta}(\omega) = \frac{1}{Z} \sum_{ab} \exp^{-\beta K_a} Q_{\alpha\beta}^{ab} \]

\[ Q_{\alpha\beta}^{ab} = \frac{\langle a|c_\alpha|b\rangle\langle b|c_\beta^\dagger|a\rangle}{\omega - K_b + K_a} + \frac{\langle a|c_\beta^\dagger|b\rangle\langle b|c_\alpha|a\rangle}{\omega - K_a + K_b} \]

do sum explicitly
\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]
\(G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \)} 

\( \geq 0 \)
\[ G_{\alpha\beta}(\omega = 0) = \left\{ \begin{array}{l} 0 \\ \delta_{\alpha\beta} \frac{(2n - N)}{N} (n+1) - K(n) \end{array} \right\} \wedge \]
\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]
\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} (\frac{2n - N}{N}) \]
\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]

A zero must exist if < 0 or > 0.
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ 0, 1, \frac{1}{2} \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \quad 0, 1, 1/2 \]
\[ N = 3 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \quad 0, 1, 1/2 \]

\[ N = 3 \]

\[ 2 = 3 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ \{0, 1, 1/2\} \]

\[ p = 2 \]

\[ N = 3 \]
$n = N \Theta(2n - N)$

$Luttinger's$ $theorem$

$even$

$n = 2$

$N = 3$

$0, 1, 1/2$
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\( n = 2 \)

\( N = 3 \)

\( 0, 1, 1/2 \)

even

odd
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ N = 3 \]

\[ n = 2 \]

0, 1, 1/2

even

odd

no solution
does the degeneracy matter?

\[ t = 0^+ \]
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ c_a^{\dagger} \frac{1}{\omega - H} c_b + c_b \frac{1}{\omega - H} c_a^{\dagger} \right] \rho(0^+) \right) \]
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ \begin{array}{c} \frac{1}{\omega} \\ \frac{1}{\omega - H} \end{array} \begin{array}{c} c_a^\dagger \\ c_b \end{array} \right] \rho(0^+) \right) \]
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ c_a^\dagger \frac{1}{\omega} c_b + c_b \frac{1}{\omega - U} c_a^\dagger \right] \rho(0^+) \right) \]
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ \begin{array}{c}
c_b^\dagger \frac{1}{\omega} \\
c_b + c_b \frac{1}{\omega - U} c_a^\dagger
\end{array} \right] \rho(0^+) \right) \]

\[ G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega(\omega - U)} \]
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ \begin{array}{c} c_a^\dagger \\ \frac{1}{\omega} \\
 \end{array} \right] c_b + c_b \left[ \begin{array}{c} 1 \\ \frac{1}{\omega - U} c_a^\dagger \\
 \end{array} \right] \rho(0^+) \right) \]

\[ \rho_{ab} = \text{Tr} \left( c_a^\dagger c_b \rho(0^+) \right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle \]

\[ G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega(\omega - U)} \]
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ \begin{array}{c} c_a \frac{1}{\omega} \\ c_b \end{array} \right] c_b + c_b \frac{1}{\omega - U} c_a^\dagger \right) \rho(0^+) \]

\[
\rho_{ab} = \text{Tr} \left( c_a^\dagger c_b \rho(0^+) \right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle
\]

\[ G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega(\omega - U)} \]

\[ \frac{1}{N} \text{diag}(1, 1, 1, \cdots) \]

mixed state
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ c_a^{\dagger} \frac{1}{\omega} c_b + c_b \frac{1}{\omega - U} c_a^{\dagger} \right] \rho(0^+) \right) \]

\[ \rho_{ab} = \text{Tr} \left( c_a^{\dagger} c_b \rho(0^+) \right) = \langle u_0 | c_a^{\dagger} c_b | u_0 \rangle \]

\[ G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega(\omega - U)} \]

as long as SU(N) symmetry is intact

mixed state

zeros in the wrong place

\[ 1/N \text{diag}(1, 1, 1, \cdots) \]
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ c_a^\dagger \frac{1}{\omega} c_b + c_b \frac{1}{\omega - U} c_a^\dagger \right] \rho(0^+) \right) \]

\[ \rho_{ab} = \text{Tr} \left( c_a^\dagger c_b \rho(0^+) \right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle \]

\[ G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega(\omega - U)} \]

as long as SU(N) symmetry is intact

mixed state

zeros in the wrong place

limiting procedure: \( \lim_{t \to 0} \lim_{\omega \to 0} \)
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ \begin{array}{c} c_a^\dagger \\ \frac{1}{\omega} \\ c_b \\ c_b \frac{1}{\omega - U} c_a^\dagger \end{array} \right] \rho(0^+) \right) \]

\[ \rho_{ab} = \text{Tr} \left( c_a^\dagger c_b \rho(0^+) \right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle \]

\[ G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega (\omega - U)} \]

as long as SU(N) symmetry is intact

zeros in the wrong place

mixed state

limiting procedure: \( \lim_{t \to 0} \lim_{\omega \to 0} \)

limits do not commute
Problem

$\text{Det}G = 0$

$$G = \frac{1}{E - \varepsilon_p} = 0$$
Problem

\[ \text{Det} G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p} = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \rightarrow \infty \]
Problem

\[ \text{Det}G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p} = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

\[ \infty \]

\[ G = 0 \]
if $\Sigma$ is infinite
if $\Sigma$ is infinite

lifetime of a particle vanishes
if $\Sigma$ is infinite

lifetime of a particle vanishes

$\exists \Sigma < \epsilon_p$
if $\Sigma$ is infinite

lifetime of a particle vanishes

$\exists \Sigma < \epsilon_p$

no particle
what went wrong?
what went wrong?

\[ \delta I[G] = \int d\omega \sum \delta G \]
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)

integral does not exist
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)

integral does not exist

No Luttinger theorem!
zeros and particles are incompatible
Luttinger’s theorem
are zeros important?
Fermi Arcs
Fermi Arcs

Re G
Changes
Sign across
An arc
Fermi arcs necessarily imply zeros exist.

Re G
Changes
Sign across
An arc

Must cross
A zero line
(DetG=0)!!!
what is seen experimentally?

Fermi arcs: no double crossings (PDJ, JCC, ZXS)
what is seen experimentally?

Fermi arcs: no double crossings (PDJ,JCC,ZXS)

seen

not seen

infinities

zeros

$E_F$
zeros do not affect the particle density
zeros do not affect the particle density

each hole ≠ a single k-state
where do these zeros come from?
counting electron states

need to know: $N$ (number of sites)
counting electron states

\[ x = \frac{n_h}{N} \]

need to know: \( N \) (number of sites)
counting electron states

\[ x = \frac{n_h}{N} \]

removal states

\[ 1 - x \]

need to know: \( N \) (number of sites)
counting electron states

$1 - x \quad 1 + x$

removal states

addition states

$x = n_h/N$

need to know: N (number of sites)
counting electron states

\[ 2x \]

removal states

\[ 1 - x \]

addition states

\[ 1 + x \]

\[ x = \frac{n_h}{N} \]

need to know: \( N \) (number of sites)
counting electron states

removal states

\[ 1 - x \]

addition states

\[ 1 + x \]

\[ x = \frac{n_h}{N} \]

low-energy electron states

\[ 1 - x + 2x = 1 + x \]

need to know: \( N \) (number of sites)
counting electron states

removal states

addition states

low-energy electron states

high energy

need to know: N (number of sites)

\[ x = \frac{n_h}{N} \]

\[ 2x \]

\[ 1 - x \]

\[ 1 + x \]

\[ 1 - x + 2x = 1 + x \]
spectral function (dynamics)
spectral function (dynamics)
spectral function (dynamics)

$U \gg t$

$1 + x + \alpha > 1 + x$

Harris and Lange (1967)
spectral function (dynamics)

$\alpha \propto t/U$

$1 + x + \alpha > 1 + x$

$\alpha \propto t/U$

$U \gg t$

$1 - x - \alpha$

Harris and Lange (1967)
not exhausted by counting electrons alone

\[ 1 + x + \alpha > 1 + x \]

\[ \alpha \propto t/U \]

\[ U \gg t \]

\[ 1 - x - \alpha \]

Harris and Lange (1967)
how to count particles?

some charged stuff has no particle interpretation
what is the extra stuff?
\[ \Sigma(\omega = 0, p) = 0 \]
Fermi liquid

\[ \Sigma(\omega = 0, p) = \infty \]
new fixed point

\[ \tilde{g} \]
\[ \Sigma(\omega = 0, p) = 0 \]
Fermi liquid

\[ \Sigma(\omega = 0, p) = \infty \]
new fixed point

scale invariance
strongly correlated matter

new fixed point (scale invariance)
strongly correlated matter

new fixed point (scale invariance)
strongly correlated matter

new fixed point (scale invariance)

unparticles (IR) (H. Georgi)
what is scale invariance?

invariance on all length scales
spectral function from scale invariance

\[ A(\Lambda \omega, \Lambda^{\alpha \vec{k}} \vec{k}) = \Lambda^{\alpha A} A(\omega, \vec{k}) \]
spectral function from scale invariance

\[ A(\Lambda \omega, \Lambda^{\alpha \vec{k}} \vec{k}) = \Lambda^{\alpha A} A(\omega, \vec{k}) \]

\[ A(\omega, \vec{k}) = \omega^{\alpha A} f \left( \frac{\vec{k}}{\omega^{\alpha \vec{k}}} \right) \]
what’s the underlying theory?
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]
\[ \phi(x) \rightarrow \phi(x) \]

mass

\[ m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

\[ x \to x/\Lambda \]

\[ \phi(x) \to \phi(x) \]

mass

\[ m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

mass

\( x \rightarrow x/\Lambda \)

\( \phi(x) \rightarrow \phi(x) \)

no scale invariance

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

\( m^2 \phi^2 \)
mass sets a scale
unparticles
from a massive theory?
\[ \mathcal{L} = \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \]
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial_{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!
\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

**theory with all possible mass!**

\[
\phi \to \phi(x, m^2 / \Lambda^2) \\
x \to x / \Lambda \\
m^2 / \Lambda^2 \to m^2
\]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]
\[ x \rightarrow x / \Lambda \]
\[ m^2 / \Lambda^2 \rightarrow m^2 \]

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!
\[ \mathcal{L} = \int_0^{\infty} \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[
\begin{align*}
\phi & \rightarrow \phi(x, m^2 / \Lambda^2) \\
x & \rightarrow x / \Lambda \\
m^2 / \Lambda^2 & \rightarrow m^2
\end{align*}
\]

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
unparticles

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]
\[ x \rightarrow x / \Lambda \]
\[ m^2 / \Lambda^2 \rightarrow m^2 \]
\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
\[ \left( \int_{0}^{\infty} dm^{2} m^{2\gamma} \frac{i}{p^{2} - m^{2} + i\epsilon} \right)^{-1} \propto p^{2|\gamma|} \]
\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^2|\gamma| \\
\]

propagator

\[
d_U - 2
\]
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{dU-2} \]
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{dU - 2} \]

unparticles = fractional number of massless particles
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U - 2} \]

unparticles = fractional number of massless particles

d\_U < 2

almost Luttinger liquid
no pole at p\_F
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U-2} \]

unparticles=fractional number of massless particles

\[ d_U > 2 \]

\[ d_U < 2 \]

zeros

almost Luttinger liquid no pole at p_F
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U-2} \]

unparticles = fractional number of massless particles

d\(d_U > 2\)

zeros

no simple sign change

d\(d_U < 2\)

almost Luttinger liquid

no pole at \(p_F\)
no particle interpretation

\[ \phi_U \neq \int B(m^2) \phi(m^2) dm^2 \]

mass-distribution

unparticle field (non-canonical)

particle field
\text{dU?}
what really is the summation over mass?
what really is the summation over mass?

mass=energy
summation over mass?

\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial_{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

\[ m = z^{-1} \]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

\[ m = z^{-1} \]

\[ \mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[ 1 \frac{z^2}{2R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right] \]
\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

\[ m = z^{-1} \]

\[ \mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right] \]

Can be absorbed with
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]

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can be absorbed with

\[ ds^2 = \frac{R^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right) \]

sumption over mass?
\[
L = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} dm^2
\]

\[
m = z^{-1}
\]

\[
L = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right]
\]

\[
ds^2 = \frac{R^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right)
\]

**can be absorbed with**

**anti de Sitter space!**
generating functional for unparticles

action on $AdS_{5+2\delta}$

\[ S = \frac{1}{2} \int d^{4+2\delta} x \, dz \, \sqrt{-g} \left( \partial_a \Phi \partial^a \Phi + \frac{\Phi^2}{R^2} \right) \]
generating functional for unparticles

action on $AdS_{5+2\delta}$

\[
S = \frac{1}{2} \int d^{4+2\delta} x \, dz \sqrt{-g} \left( \partial_a \Phi \partial^a \Phi + \frac{\Phi^2}{R^2} \right)
\]

\[
ds^2 = \frac{L^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right) \quad \sqrt{-g} = \left( \frac{R}{z} \right)^{5+2\delta}
\]
The generating functional for unparticles is given by

\[ S = \frac{1}{2} \int d^{4+2\delta} x \, dz \, \sqrt{-g} \left( \partial_a \Phi \partial^a \Phi + \frac{\Phi^2}{R^2} \right) \]

with the metric

\[ ds^2 = \frac{L^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right) \]

and \( \sqrt{-g} = \left( R/z \right)^{5+2\delta} \)

The unparticle lives in \( d = 4 + 2\delta \) with \( \delta \leq 0 \).
gauge-gravity duality (Maldacena, 1997)
gauge-gravity duality
(Maldacena, 1997)
gauge-gravity duality
(Maldacena, 1997)

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]

locality in energy
gauge-gravity duality
(Maldacena, 1997)

implement E-scaling with an extra dimension

\[
\frac{dg(E)}{d\ln E} = \beta(g(E))
\]

locality in energy
Claim: \[ Z_{\text{QFT}} = e^{-S_{\text{on-shell}}^{\text{on-shell}} (\phi (\phi \partial_{\text{ADS}} = J_O))} \]
Claim: \[ Z_{\text{QFT}} = e^{-S_{\text{on-shell}}^{\text{ADS}}(\phi(\phi \partial_{\text{ADS}} = J_\mathcal{O}))} \]

\[ S = \frac{1}{2} \int d^d x \, g^{zz} \sqrt{-g} \, \Phi(z, x) \partial_z \Phi(z, x) \bigg|_{z=e} \]
Claim: \( Z_{\text{QFT}} = e^{-S_{\text{on-shell}}^{\text{on-shell}}(\phi(\phi \partial_{\text{ADS}} = J^0))} \)

\[ S = \frac{1}{2} \int d^d x \, g^{zz} \sqrt{-g} \Phi(z, x) \partial_z \Phi(z, x) \bigg|_{z=\epsilon} \]

\[ \langle \Phi_U(x) \Phi_U(x') \rangle = \frac{1}{|x - x'|^{2dU}} \]
Claim: \[ Z_{QFT} = e^{-S^{\text{on-shell}}_{\text{ADS}}(\phi(\phi \partial_{\text{ADS}} = J_\mathcal{O} ))} \]

\[ S = \frac{1}{2} \int d^d x \ g^{zz} \sqrt{-g} \ \Phi(z, x) \partial_z \Phi(z, x) \bigg|_{z=\epsilon} \]

\[ \langle \Phi_{U}(x) \Phi_{U}(x') \rangle = \frac{1}{|x - x'|^{2d_U}} \]

\[ d_U = \frac{d}{2} + \frac{\sqrt{d^2 + 4}}{2} > \frac{d}{2} \]
Claim: \[ Z_{\text{QFT}} = e^{-S_{\text{on-shell}}^{\text{on-shell}}(\phi(\phi \partial_{\text{ADS}} = j_{\mathcal{O}}))} \]

\[ S = \frac{1}{2} \int d^d x \, g^{zz} \sqrt{-g} \, \Phi(z, x) \partial_z \Phi(z, x) \bigg|_{z=\epsilon} \]

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\[ d_U = \frac{d}{2} + \frac{\sqrt{d^2 + 4}}{2} > \frac{d}{2} \]

scaling dimension is fixed
$G_U(p) \propto p^2(d_U - d/2)$

\[d_U = \frac{d}{2} + \frac{\sqrt{d^2 + 4}}{2} > \frac{d}{2}\]

$G_U(0) = 0$
unparticle (AdS) propagator has zeros!

\[ G_U(p) \propto p^{2(d_U - d/2)} \]

\[ d_U = \frac{d}{2} + \frac{\sqrt{d^2 + 4}}{2} > \frac{d}{2} \]

\[ G_U(0) = 0 \]
interchanging unparticles

fractional \( (d_U) \) number of massless particles
interchanging unparticles

fractional \( (d_U) \) number of massless particles
interchanging unparticles

fractional $(d_U)$ number of massless particles
interchanging unparticles

fractional \( (d_U) \) number of massless particles
interchanging unparticles

fractional \((d_U)\) number of massless particles

\[ e^{i\pi d_U} \neq -1, 0 \]
interchanging unparticles

fractional (d_U) number of massless particles

\[ e^{i\pi d_U} \neq -1, 0 \]

fractional statistics in d=2+1
\[ d_U = \frac{d}{2} + \sqrt{d^2 + 4} > \frac{d}{2} \]

\[ e^{i\pi d_U} \neq e^{-i\pi d_U} \]

*time-reversal symmetry breaking from unparticle (zeros=\text{Fermi arcs}) matter*
The diagram shows the relationship between $T_c$, the critical temperature for pairing, and $g$, a parameter related to the coupling strength. The equation $\frac{d \ln g}{d \ln \beta} = 4d_U - d > 0$ describes the tendency towards pairing (any instability which establishes a gap). The text also mentions fermions with unparticle propagator.
breaking of scale invariance

unparticles

particles

TRSB

Antiferromagnet

Fermi Liquid

Superconductor

QCP
emergent gravity

High $T_c$
unparticles
variable mass
High $T_c$ unparticles with variable mass exhibit emergent gravity. Fractional statistics in $d=2+1$.