Is Strongly Correlated Electron Matter Full of Unparticles?

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Correlated Matter
Correlated Matter

What is carrying the current?
current-carrying excitations?
current-carrying excitations?

\[ e^* = \frac{e}{q} \quad (q \text{ odd}) \]
current-carrying excitations?

\[ e^* = \frac{e}{q} \quad (q \text{ odd}) \]
fractional quantum Hall effect

current-carrying excitations?
fractional quantum Hall effect

current-carrying excitations?

\[ e^* = ? \]
High $T_c$: How Does Fermi Liquid Theory Breakdown?

$\rho \propto \alpha T$
High $T_c$: How Does Fermi Liquid Theory Breakdown?

$\rho \propto \alpha T$
can the charge density be given a particle interpretation?
if not?
if not?

charge stuff = particles + other stuff
if not?

charge stuff = particles + other stuff

unparticles!
counting particles
counting particles
counting particles
counting particles
counting particles
counting particles

is there a more efficient way?
$$n = \int_{-\infty}^{\mu} N(\omega) \, d\omega$$
\[ n = \int_{-\infty}^{\mu} N(\omega) d\omega \]
\[ n = \int_{-\infty}^{\mu} N(\omega) d\omega \]

Can n be deduced entirely from the IR (low-energy) scale?
Luttinger’s `theorem'
Each particle has an energy
Each particle has an energy

particles

\[ E_{\text{photon}} = h\nu \]

\[ E = \varepsilon_p \]
Each particle has an energy

\[ E_{\text{photons}} = h\nu \]

Photoelectric effect

energy \[ E = \varepsilon_p \]

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle = infinity
Each particle has an energy

\[ E = \varepsilon_p \]

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle count is deducible from Green function
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]
Luttinger’s Theorem

Green function (propagator)

$G(E) = \frac{1}{E - \varepsilon_p}$

particle density = number of sign changes of $G$
Counting sign changes?
Counting sign changes?

\[ \Theta(x) \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 0 & x < 0 \end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 
\end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 \\
1/2 & x = 0 
\end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 \\
1/2 & x = 0 
\end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \\ 1/2 & x = 0 \end{cases} \]

counts sign changes

\[ n = 2 \sum_{p} \Theta(G(E = 0, p)) \]

Luttinger Theorem for electrons

Monday, February 4, 2013
How do functions change sign?
How do functions change sign?

\[ E > \varepsilon_p \]

\[ E < \varepsilon_p \]

divergence
Is there another way?
zero-crossing

\[ \varepsilon_p \]

\[ \text{Det} G = 0 \]
zero-crossing

\[ \epsilon_p \]

\[ \text{Det} G = 0 \]

no divergence is necessary
closer look at Luttinger’s theorem

\[ n = 2 \sum_{\mathbf{p}} \Theta(G(E = 0, \mathbf{p})) \]
closer look at Luttinger’s theorem

\[ n = 2 \sum_p \Theta(G(E = 0, p)) \]

divergences + zeros
closer look at Luttinger’s theorem

\[ n = 2 \sum_{\mathbf{p}} \Theta(G(E = 0, \mathbf{p})) \]

divergences + zeros

how can zeros affect the particle count?
Some consequences of the Luttinger theorem: The Luttinger surfaces in non-Fermi liquids and Mott insulators

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The proof just presented is good for any state of our system: FL, NFL, or MI [in other words, either of poles or zeros can be used to change the sign of $G_r$ in Eq. (1)]. The only way to incapacitate the Luttinger theorem in form (1) is to assume that the limit $T \rightarrow 0$ is discontinuous. Actually, one has to require that the whole line $T=0$ is a line of phase transitions.
Is this famous theorem from 1960 correct?
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails

no hopping => no propagation (zeros)
A model with zeros but Luttinger fails

N flavors of e-

no hopping=> no propagation (zeros)
$e^{-}$

Spin $SU(2)$

generalization

$N$ flavors of spin $SU(N)$
$e^-$

$N = 5$

$v$ flavors of spin $SU(N)$

$SU(2)$

Generalization
$N = 5$

$SU(2)$

generalization

$N$ flavors of spin $SU(N)$

$e^-$

spin

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$\frac{1}{2} E_1 4 9 16 25 36 49 N = 5$

$SU(2)$

generalization

$N$ flavors of spin $SU(N)$

$2E$

$N = 5$

$4 9 16 25 36 49 64 81 100$
\[ e^{-} \quad SU(2) \]

**generalization**

**N flavors of spin**

\[ SU(N) \]

\[ H = \frac{U}{2} (n_1 + \cdots n_N)^2 \]
$SU(2)$
$SU(2)$

$\mu$

$- \frac{U}{2}$

$U$
$SU(2)$

\[
\begin{align*}
\frac{U}{2} & \quad \mu \\
-\frac{U}{2} & \quad U
\end{align*}
\]
\[ SU(2) \]

\[ G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} \]
$$SU(2)$$

$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$
\[ G = \frac{1}{\omega + \frac{U}{2}} + \frac{1}{\omega - \frac{U}{2}} = 0 \quad \text{if} \quad \omega = 0 \]

\[ n = 2\theta(0) = 1 \]
SU(N) for N=3: no particle-hole symmetry

\begin{itemize}
  \item[(a)] $\frac{9}{2}U$
  \item[(b)] $\frac{1}{2}U$
  \item $2U$
  \item $\frac{1}{2}U$
  \item $0$
\end{itemize}
SU(N) for N=3: no particle-hole symmetry

\[ \lim_{T \to 0} \mu(T) \]
\[
G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right)
\]
\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]
\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]

\[ > 0 \]
\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]
\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]

a zero must exist

\(< 0 \quad > 0\)
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ 0, 1, 1/2 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

- \( n = 2 \)
- 0, 1, 1/2
- \( N = 3 \)
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \quad 0, 1, 1/2 \]

\[ N = 3 \]

\[ 2 = 3 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2, \quad N = 3, \quad 0, 1, 1/2 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \]

\[ N = 3 \]

even

\{ 0, 1, 1/2 \}
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\( n = 2 \)

\( N = 3 \)

\( 0, 1, 1/2 \)

even

odd
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\( n = 2 \)

\( N = 3 \)

\( 0, 1, 1/2 \)

no solution

even

odd
does the degeneracy matter?

\[ t = 0^+ \]
No
as long as $SU(N)$ symmetry is intact

$$\rho_{ab} = \text{Tr} \left( c_a^\dagger c_b \rho(0^+) \right) = \frac{1}{N} \text{diag}(1, 1, 1, \ldots)$$

mixed state  pure state
Luttinger’s theorem
Problem
Problem

$G = 0$
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p} = 0 \]
\[ G = \frac{1}{E - \varepsilon_p} = 0 \]

\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p} = 0 \]

\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]

must diverge
Problem

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

\[ \text{Det} G = 0 \]
\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

Problem

\[ \text{Det} G = 0 \]

self energy diverges
if $\Sigma$ is infinite
if $\Sigma$ is infinite

lifetime of a particle vanishes
If $\Sigma$ is infinite

The lifetime of a particle vanishes

$\infty \Sigma < \epsilon_p$
if $\Sigma$ is infinite

lifetime of a particle vanishes

$\therefore \Sigma < \epsilon_p$

no particle
zeros and particles are incompatible
zeros and particles are incompatible

Luttinger theorem is wrong!
how to count particles?
how to count particles?

some charged stuff has no particle interpretation
what is the extra stuff?
\[ I_0 = \frac{1}{1+r} \]

\[ G = \frac{g}{1+r} \]

\[ D = 3 \]

\[ P_{\text{MI-BG}} = (0, D/6) \]

\[ P_{\text{MI}} = (0, 0) \]

\[ P_{\text{MI-SF}} = (1, 0) \]

\[ Q_D = 4 + \varepsilon = (1 - \varepsilon/8, \varepsilon/4) \]

Fermi liquids

(particles/electrons)
Fermi liquids (particles/electrons)
I_0 = 1 / (1+r)

G = g / (1+r)

D = 3

P_{MI-BG} = (0, D/6)

P_{MI} = (0, 0)

P_{MI-SF} = (1, 0)

Q_D = 4 + \varepsilon

= (1 - \varepsilon/8, \varepsilon/4)

Fermi liquids
(particles/electrons)

new fixed point
(?-particle stuff)
scale invariance

new fixed point (?-particle stuff)

Fermi liquids (particles/electrons)
strongly correlated matter

scale invariance
what is scale invariance?

invariance on all length scales
$f(x) = x^2$
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \]

scale change
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \]

scale change

scale invariance
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \]  
**scale change**

**scale invariance**

\[ f(x) = x^2 \lambda^{-2} g(\lambda) \]

1
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \quad \text{scale change} \]

\[ f(x) = x^2 \lambda^{-2} g(\lambda) \]

\[ g(\lambda) = \lambda^2 \]

\[ 1 \]
\[ f(x) = ax^2 + bx^3 \]

\[ x \rightarrow x/\Lambda \]
\[ f(x) = ax^2 + bx^3 \]

\[ x \rightarrow x/\Lambda \]

\[ f(x) \rightarrow \Lambda^{-2}(ax^2 + bx/\Lambda) \]
\[ f(x) = ax^2 + bx^3 \]

\[ x \rightarrow x/\Lambda \]

\[ f(x) \rightarrow \Lambda^{-2}(ax^2 + bx/\Lambda) \]

not scale invariant
free field theory

\[ L = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ L \rightarrow \Lambda^2 L \]
free field theory

\[ \mathcal{L} = \frac{1}{2} \partial^{\mu} \phi \partial_{\mu} \phi \]

\[ x \rightarrow x / \Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ \mathcal{L} \rightarrow \Lambda^2 \mathcal{L} \quad \text{scale invariant} \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \to x/\Lambda \]
\[ \phi(x) \to \phi(x) \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]
\[ \phi(x) \rightarrow \phi(x) \]

mass
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^{\mu} \phi \partial_{\mu} \phi + m^2 \phi^2 \]

\[ \Lambda^2 \left( \frac{1}{2} \partial^{\mu} \phi \partial_{\mu} \phi \right) \]

mass

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

\[ m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

x → x/Λ

\( \phi(x) \rightarrow \phi(x) \)

\( \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \)

mass

no scale invariance

m^2 \phi^2
how should we think about field theory?
how should we think about field theory?
how should we think about field theory?
what if $m$ depends on the scale?
what if $m$ depends on the scale?
what if $m$ depends on the scale?

\[ \mathcal{L}_{m_i+7} \mathcal{L}_{m_i+6} \mathcal{L}_{m_i+5} \mathcal{L}_{m_i+4} \mathcal{L}_{m_i+3} \mathcal{L}_{m_i+2} \mathcal{L}_{m_i+1} \mathcal{L}_{m_i} \]
\[ \mathcal{L} = \sum_i \mathcal{L}(m_i) \]

sum over all mass
\[ \mathcal{L} = \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!
\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, \frac{m^2}{\Lambda^2}) \]
\[ x \rightarrow \frac{x}{\Lambda} \]
\[ m^2 / \Lambda^2 \rightarrow m^2 \]
\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]
\[ x \rightarrow x / \Lambda \]
\[ m^2 / \Lambda^2 \rightarrow m^2 \]

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2/\Lambda^2) \]
\[ x \rightarrow x/\Lambda \]
\[ m^2/\Lambda^2 \rightarrow m^2 \]

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
L = \int_0^\infty (\partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m)) \, dm^2

scale invariance is restored!!

not particles
scale-invariant stuff is weird

particles

\[ E_{\text{photon}} = h\nu \]

\[ v_{\text{max}} = 6.22 \times 10^5 \text{ m/s} \]

\[ E = \varepsilon_p \]

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle=\infty

Monday, February 4, 2013
scale-invariant stuff is weird
scale-invariant stuff is weird

unparticles
scale-invariant stuff is weird

unparticles

\[ G \propto (E - \varepsilon_p)^{d_U - 2} \]

\[ d_U > 2 \]

unparticles=zero

propagator
no particle interpretation

\[ \phi_U \neq \int B(m^2) \phi(m^2) dm^2 \]

mass-distribution

unparticle field (non-canonical)

particle field
n massless particles

\[ G \propto (E - \epsilon_p)^{n-2} \]
n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

$$G \propto (E - \varepsilon_p)^{dU-2}$$
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

unparticles=fractional number of massless particles

\[ G \propto (E - \varepsilon_p)^{dU-2} \]
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

unparticles=fractional number of massless particles

\[ G \propto (E - \varepsilon_p)^{d_U - 2} \]

d\_U < 2

almost Luttinger liquid
no pole at p\_F
\( G \propto (E - \varepsilon_p)^{n-2} \)

\( G \propto (E - \varepsilon_p)^{d_U-2} \)

unparticles = fractional number of massless particles

\( d_U > 2 \)

zeros

\( d_U < 2 \)

almost Luttinger liquid no pole at \( p_F \)
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U-2} \]

unparticles=fractional number of massless particles

d_{U} > 2

zeros

no simple sign change

d_{U} < 2

almost Luttinger liquid

no pole at p_F
scale invariance

\[ G(\Lambda p, \Lambda \omega) = \Lambda^{\alpha} G(p, \omega) \]
scale invariance

\[ G(\Lambda p, \Lambda \omega) = \Lambda^\alpha G(p, \omega) \]
scale invariance

\[ G(\Lambda p, \Lambda \omega) = \Lambda^{-\alpha} G(p, \omega) \]

\[ \alpha = -1 \]

Fermi liquids
scale invariance

\[ G(\Lambda p, \Lambda \omega) = \Lambda^\alpha G(p, \omega) \]

\[ \alpha = -1 \]

Fermi liquids
scale invariance

\[ G(\Lambda p, \Lambda \omega) = \Lambda^{\alpha} G(p, \omega) \]

\[ \alpha > -1 \]

\[ \alpha = -1 \]

Fermi liquids

Un-Fermi liquids

\[ I_0 = \frac{1}{1+r} \]

\[ G = \frac{g}{1+r} \]

\[ D = 3 \]

\[ D = 5 \]

\[ P_{\text{MI-BG}} = (0, D/6) \]

\[ P_{\text{MI}} = (0, 0) \]

\[ P_{\text{MI-SF}} = (1, 0) \]

\[ Q_{D=4+\epsilon} = (1-\epsilon/8, \epsilon/4) \]

Monday, February 4, 2013
Where do we expect to find zeros?

UV

IR
Where do we expect to find zeros?
Where do we expect to find zeros?
Where do we expect to find zeros?

not particles
$G(p, \omega) \neq 0$

$G(p_L, \omega = 0) = 0$

$G_{\text{BCS}}(\omega, p) = \frac{\omega + v(|p| - p_F)}{\omega^2 - \Delta^2 - v^2(|p| - p_F)^2}$

$G_{1+1\text{QCD}} = \frac{-ik_-}{m^2 - g^2/\pi + 2k_+k_- + g^2|k_-|/\pi\lambda - i\epsilon}$

UV

no physical quark or electron states

IR
where else do we expect to see zeros?
where else do we expect to see zeros?

strongly correlated systems
where else do we expect to see zeros?

strongly correlated systems

Mott insulators
What is a Mott Insulator?

NiO insulates $d^8$?
What is a Mott Insulator?

NiO insulates $d^8$?

EMPTY STATES = METAL
What is a Mott Insulator?

NiO insulates $d^8$?

EMPTY STATES = METAL

band theory fails!

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What is a Mott Insulator?

NiO insulates $d^8$? perhaps this costs energy

EMPTY STATES = METAL

band theory fails!
Mott Problem: NiO (Band theory failure)

(N rooms N occupants)

$U \gg t$
Mott Problem: NiO (Band theory failure)

(N rooms N occupants)

\[ \text{Det} G(\omega = 0, p) = 0 \]

\[ U \gg t \]

Monday, February 4, 2013
$\text{Y Ba}_2\text{Cu}_3\text{O}_7$

Cuprate Superconductors
interactions dominate:
Strong Coupling Physics

\[ \frac{U}{t} = 10 \gg 1 \]
photoemission
photoemission

particles

\[ E_{\text{photon}} = h\nu \]

\[ E = \varepsilon_p \]

- Energy
- \( E = \varepsilon_p \)
- Photoelectric effect
- Potassium - 2.0 eV needed to eject electron
- 700 nm 1.77 eV
- 550 nm 2.25 eV
- 400 nm 3.1 eV
- \( \nu_{\text{max}} = 2.96 \times 10^5 \) m/s
- \( \nu_{\text{max}} = 6.22 \times 10^5 \) m/s
- 3.1 eV
- 2.25 eV
- 1.77 eV
- no electrons
Photoemission

Particles

\[ E_{\text{photon}} = h\nu \]

\[ E = \varepsilon_p \approx p^2 \]

Photoelectric effect

Energy

- 700 nm, 1.77 eV
- 550 nm, 2.25 eV
- 400 nm, 3.1 eV

Potassium - 2.0 eV needed to eject electron

\[ v_{\text{max}} = 6.22 \times 10^5 \text{ m/s} \]
Photoemission

Particles

$E_{\text{photon}} = h\nu$

$E = \varepsilon_p \approx p^2$

Photoelectric effect

Energy

Potassium - 2.0 eV needed to eject electron

700 nm 1.77 eV
550 nm 2.25 eV
400 nm 3.1 eV

$\nu_{\text{max}} = 6.22 \times 10^5 \text{ m/s}$

no electrons
photoemission

particles

\[ E_{\text{photon}} = h\nu \]

\[ E = \varepsilon_p \approx p^2 \]

two crossings:
closed surface of excitations
what is seen experimentally?

Fermi arcs: no double crossings (PDJ, JCC, ZXS)
what is seen experimentally?

Fermi arcs: no double crossings (PDJ, JCC, ZXS)

seen
infinities

not seen
zeros

\( E_F \)
zeros do not affect the particle density
zeros do not affect the particle density

no Luttinger theorem
zeros do not affect the particle density

no Luttinger theorem

experimental data
where do these zeros come from?
classical (atomic) limit
warm-up

no hopping: $x$ holes

$1 + x$

$1 - x$

density of states
warm-up

no hopping: $x$ holes

\[
\begin{align*}
\text{density of states} & = 1 + x \\
1 - x & = 2x
\end{align*}
\]
no hopping: $x$ holes

density of states

$1 + x$

$1 - x$

$\varepsilon_F$
no hopping: $x$ holes

density of states

spectral weight: $x$-dependent

$$G = \frac{1 + x}{\omega + \mu + U/2} + \frac{1 - x}{\omega + \mu - U/2}$$
quantum Mottness: $U$ finite

\[ U \gg t \]
quantum Mottness: \( U \) finite

\[ U \gg t \]

double occupancy in ground state!!
quantum Mottness: $U$ finite

$U \gg t$

double occupancy in ground state!!

$W_{PES} > 1 + x$
density of states

\[ 1 + x + \alpha(t/U, x) \]

\[ \alpha = \frac{t}{U} \sum_{i,j} \langle c_{i\sigma}^{\dagger} c_{j\sigma} \rangle > 0 \]

beyond the atomic limit: any real system

Monday, February 4, 2013
beyond the atomic limit: any real system

$1 + x + \alpha(t/U, x)$

$\alpha = \frac{t}{U} \sum_{ij} \langle c_i^{\dagger} c_j \rangle > 0$

$1 - x - \alpha$

Harris & Lange, 1967

dynamical spectral weight transfer
Beyond the atomic limit: any real system

Density of states

Integrity > 1 + x

Harris & Lange, 1967

Dynamical spectral weight transfer

\[ 1 + x + \alpha(t/U, x) \]

\[ \alpha = \frac{t}{U} \sum_{ij} \langle c_{i\sigma}^{\dagger} c_{j\sigma} \rangle > 0 \]
beyond the atomic limit: any real system

\[ 1 + x + \alpha(t/U, x) \]

\[ \alpha = \frac{t}{U} \sum_{ij} \langle c^\dagger_{i\sigma} c_{j\sigma} \rangle > 0 \]

Harris & Lange, 1967

dynamical spectral weight transfer

Intensity > 1 + x

# of charge e states

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beyond the atomic limit: any real system

density of states

\[ 1 + x + \alpha(t/U, x) \]

\[ \alpha = \frac{t}{U} \sum_{ij} \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle > 0 \]

Harris & Lange, 1967

dynamical spectral weight transfer

Intensity \(>1+x\)

# of charge e states

# of electron states in lower band

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beyond the atomic limit: any real system

\[ 1 + x + \alpha(t/U, x) \]

\[ \alpha = \frac{t}{U} \sum_{ij} \langle c_i^{\dagger} c_j \rangle > 0 \]

Harris & Lange, 1967

dynamical spectral weight transfer

Intensity > 1 + x

# of charge e states

# of electron states in lower band

not exhausted by counting electrons alone?
Wilsonian analysis

density of states

\[ \int d(UHB) \]
Wilsonian analysis

\[
d(UHB) = 1 + x + \alpha(t/U, x)
\]

density of states

charge 2e stuff

\[
\int \quad d(UHB)
\]
Wilsonian analysis

\[ 1 + x + \alpha(t/U, x) \]

charge 2e stuff

density of states

\[ \int d(UHB) \]

zeros

\[ n < \text{conserved charge} \]
what really is the summation over mass?

\[ \mathcal{L}_{m_i+7} \quad \mathcal{L}_{m_i+6} \quad \mathcal{L}_{m_i+5} \quad \mathcal{L}_{m_i+4} \quad \mathcal{L}_{m_i+3} \quad \mathcal{L}_{m_i+2} \quad \mathcal{L}_{m_i+1} \quad \mathcal{L}_{m_i} \]
mass=energy
high energy (UV)

low energy (IR)

related to sum over mass

QFT
\[
\frac{dg(E)}{dlnE} = \beta(g(E))
\]

Locality in energy

related to sum over mass

high energy (UV)

low energy (IR)

QFT

Monday, February 4, 2013
implement E-scaling with an extra dimension

low energy (IR)

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]

locality in energy

high energy (UV)

related to sum over mass

QFT
implement E-scaling with an extra dimension

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]

related to sum over mass

locality in energy
Implement E-scaling with an extra dimension.

gauge-gravity duality (Maldacena, 1997)

\[
dg(E) \over d\ln E = \beta(g(E))
\]

Locality in energy

Related to sum over mass

UV

QFT

IR

Gravity

QFT

Monday, February 4, 2013
what's the geometry?
what’s the geometry?

\[
\frac{dg(E)}{d\ln E} = \beta(g(E)) = 0
\]

scale invariance (continuous)
what's the geometry?

\[ \frac{dg(E)}{dlnE} = \beta(g(E)) = 0 \]

scale invariance (continuous)

\[ E \rightarrow \lambda E \]
\[ x^\mu \rightarrow x^\mu / \lambda \]
what’s the geometry?

\[
\frac{dg(E)}{d\ln E} = \beta(g(E)) = 0
\]

scale invariance (continuous)

\[
E \rightarrow \lambda E
\]

\[
x^\mu \rightarrow x^\mu / \lambda
\]

solve Einstein equations
what's the geometry?

$$\frac{dg(E)}{d\ln E} = \beta(g(E)) = 0$$

scale invariance (continuous)

$$E \rightarrow \lambda E$$

$$x^\mu \rightarrow x^\mu / \lambda$$

solve Einstein equations

$$ds^2 = \left( \frac{u}{L} \right)^2 \eta_{\mu\nu} dx^{\mu} dx^{\nu} + \left( \frac{L}{E} \right)^2 dE^2$$

anti de-Sitter space
\[ L = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

classical gravity in a d+1 curved spacetime
$\mathcal{L} = \int_{0}^{\infty} \left( \partial_{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^{2} \phi^{2}(x, m) \right) dm^{2}$

classical gravity in a d+1 curved spacetime

gauge theory in $AdS_{4}$
interchanging unparticles

fractional \( (d_U) \) number of massless particles
interchanging unparticles

fractional (d_U) number of massless particles
interchanging unparticles

fractional (d_U) number of massless particles
interchanging unparticles

fractional (d_U) number of massless particles
interchanging unparticles

fractional \( (d_U) \) number of massless particles

\[
e^{i\pi d_U} \neq -1, 0
\]
interchanging unparticles

fractional \((d_U)\) number of massless particles

\[
e^{i\pi d_U} \neq -1, 0
\]

fractional statistics in \(d=2\)
High $T_c$
unparticles
variable mass
$UPt_3$
emergent gravity

High T_c
unparticles
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High $T_c$
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fractional statistics in $d=2$