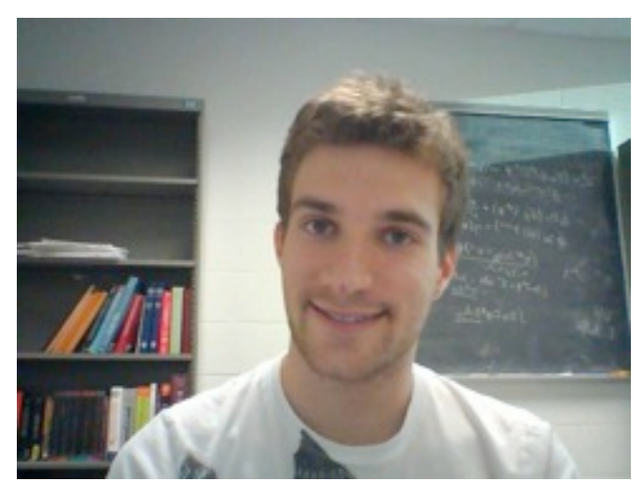
Is Strongly Correlated Electron Matter Full of Unparticles?

Thanks to: NSF, EFRC (DOE)

Kiaran Dave

Charlie Kane

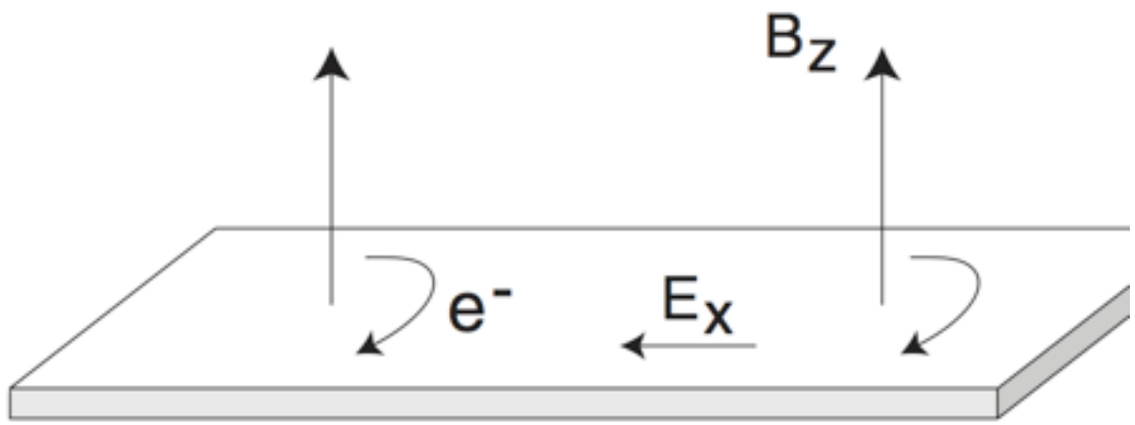


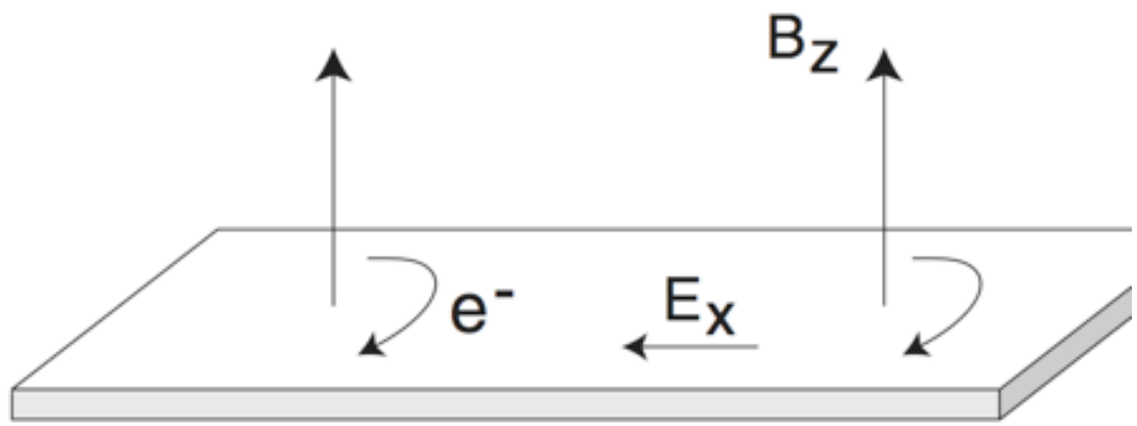
Brandon Langley

Correlated Matter

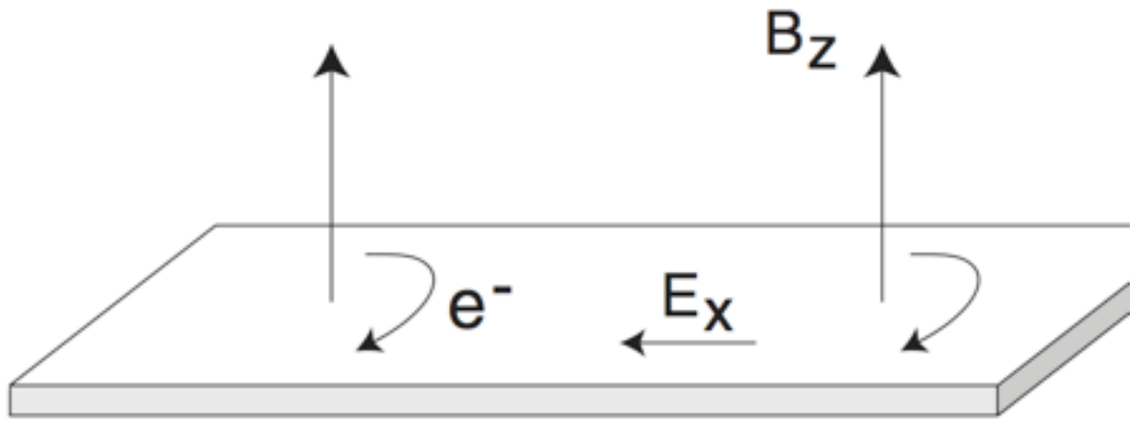
Correlated Matter

What is carrying the current?


B_z
e^-
E_x



current-carrying excitations?

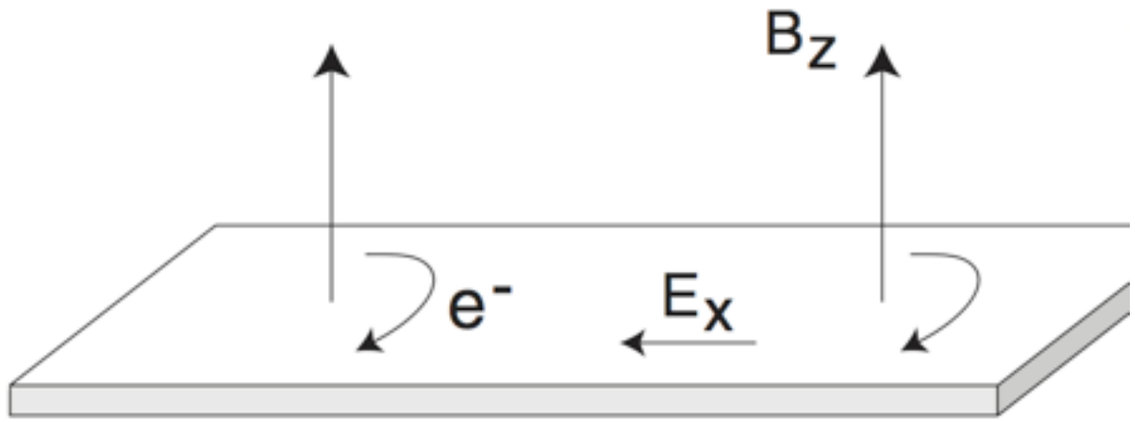


current-carrying excitations?

$$e^* = \frac{e}{q}$$

(q odd)

fractional quantum Hall effect



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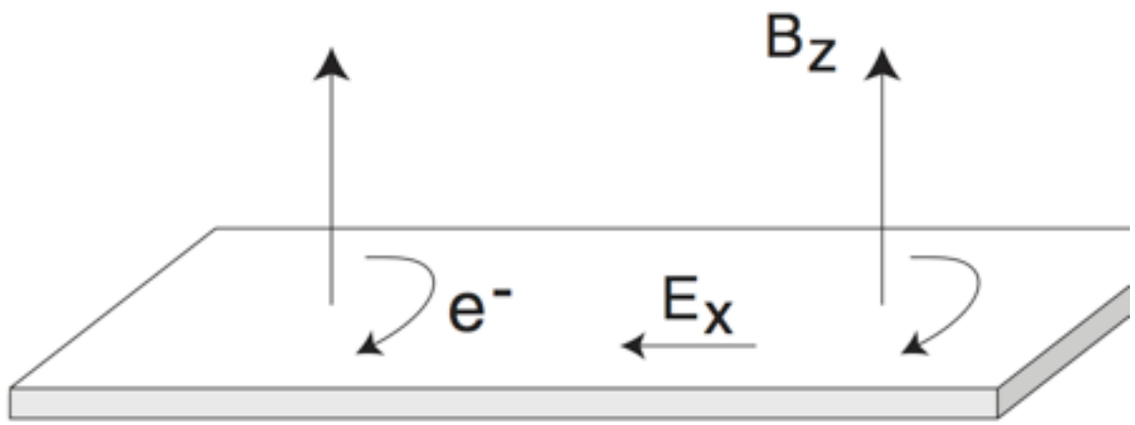

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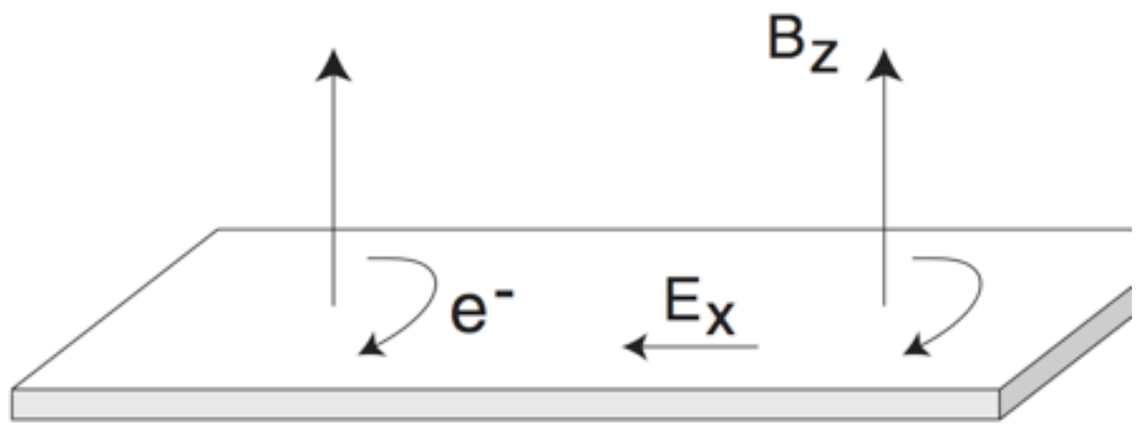
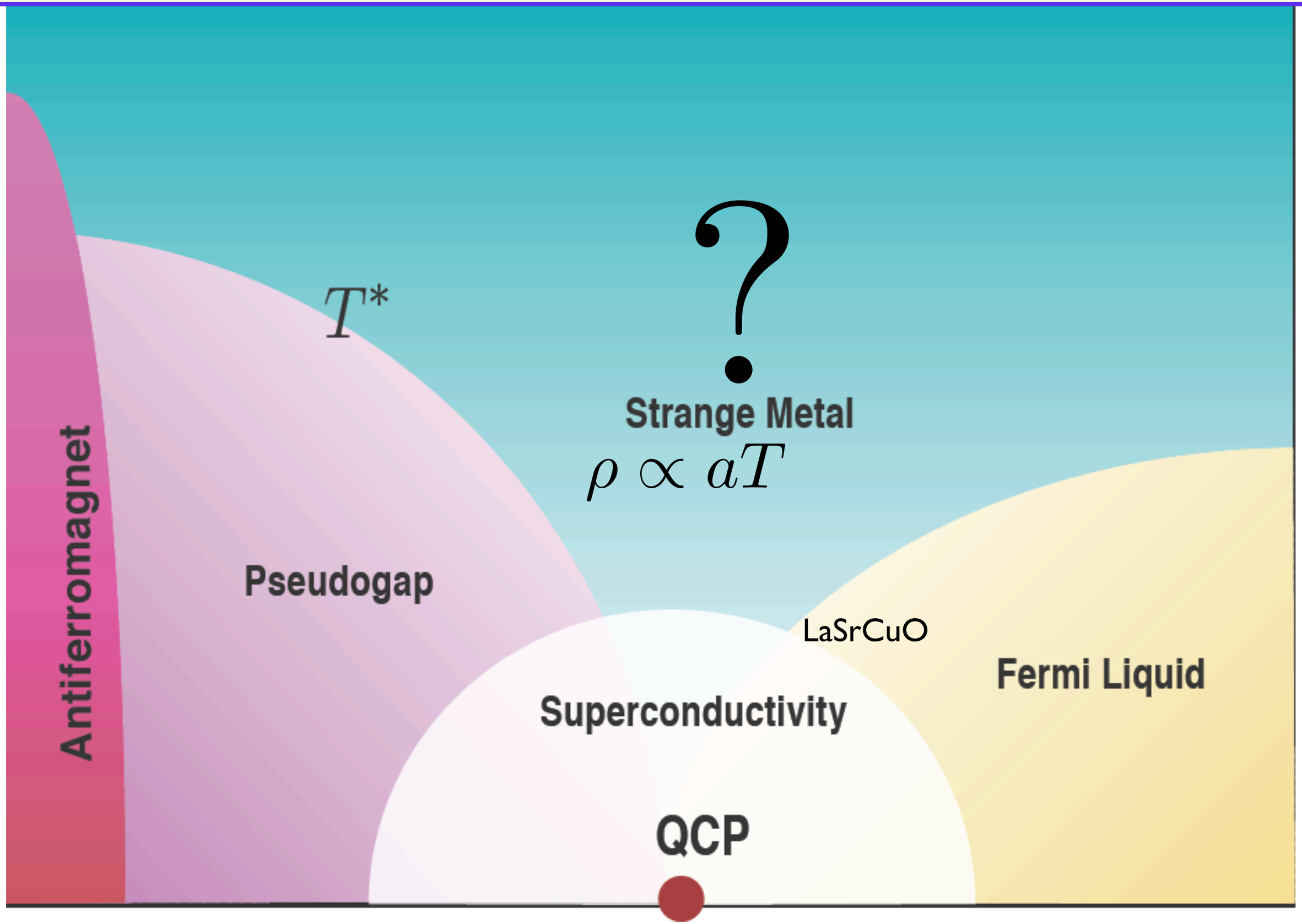


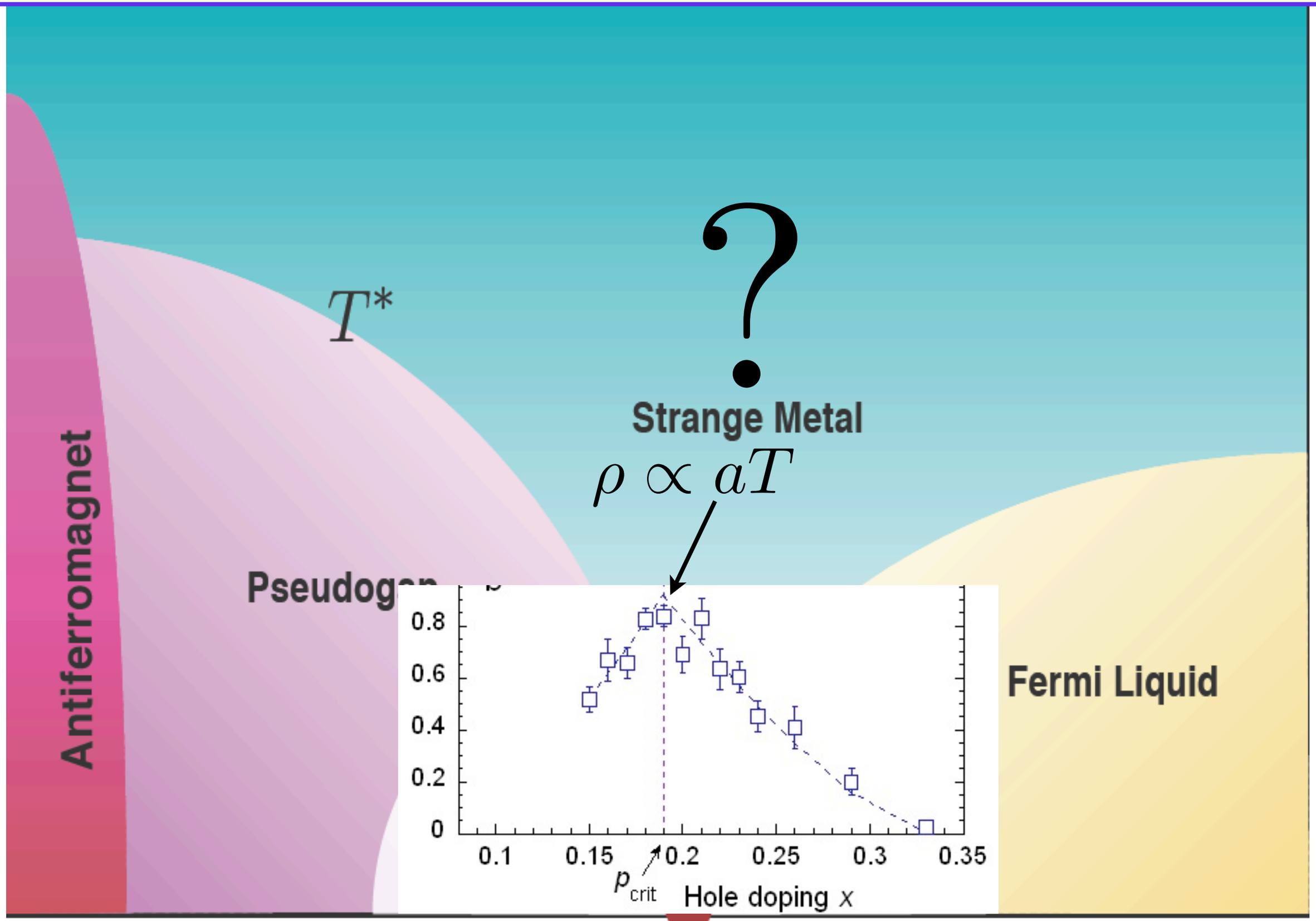
current-carrying excitations?

$$e^* = ?$$

High T_c:How Does Fermi Liquid Theory Breakdown?



High T_c:How Does Fermi Liquid Theory Breakdown?



can the charge density be given a particle interpretation?

if not?

if not?

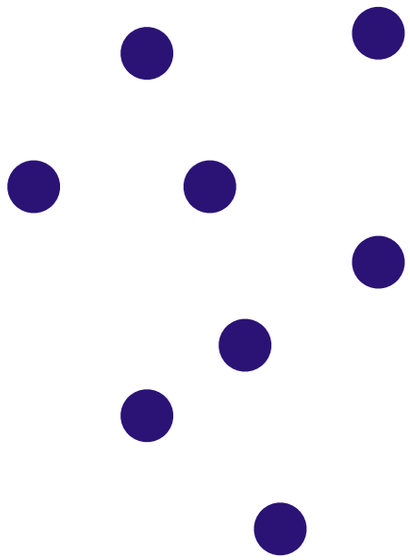
charge stuff= particles + other stuff

if not?

charge stuff= particles + other stuff

unparticles!

counting particles



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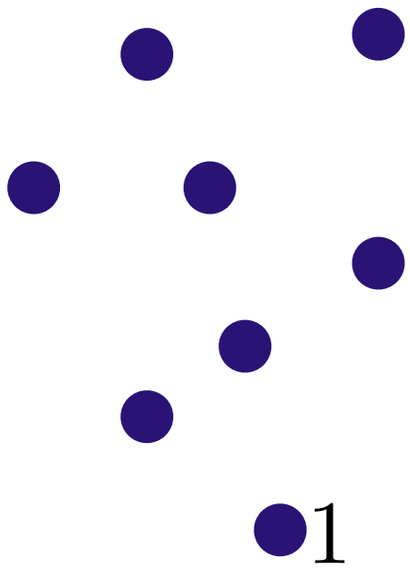

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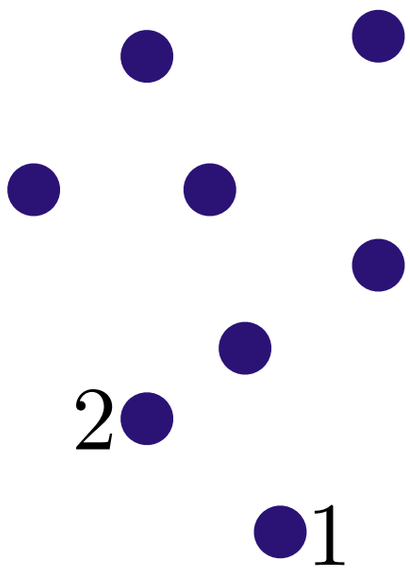

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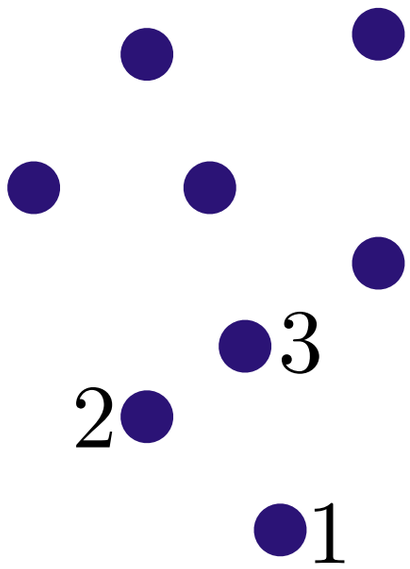

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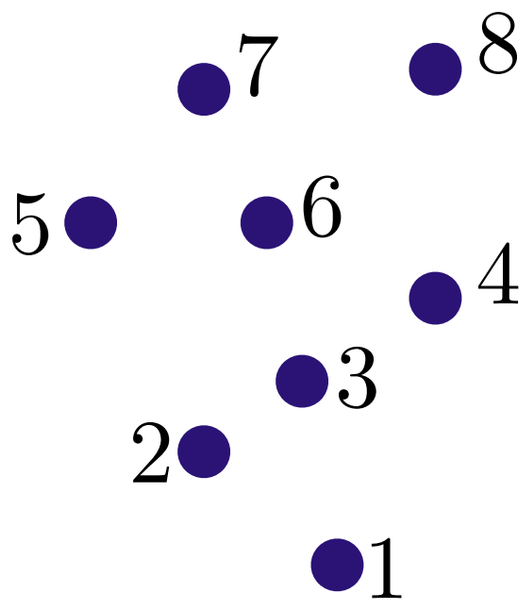

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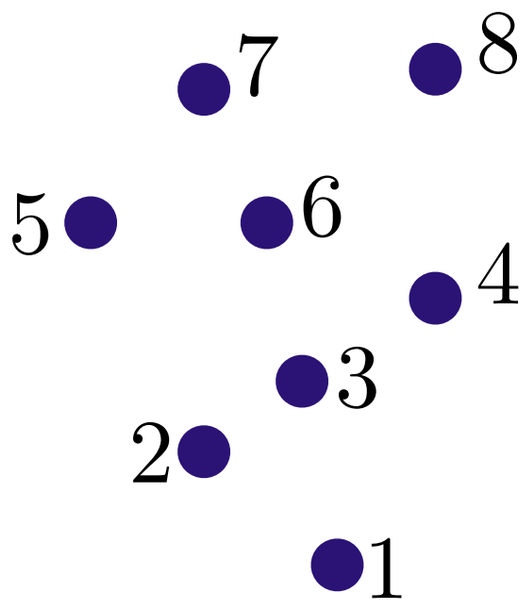
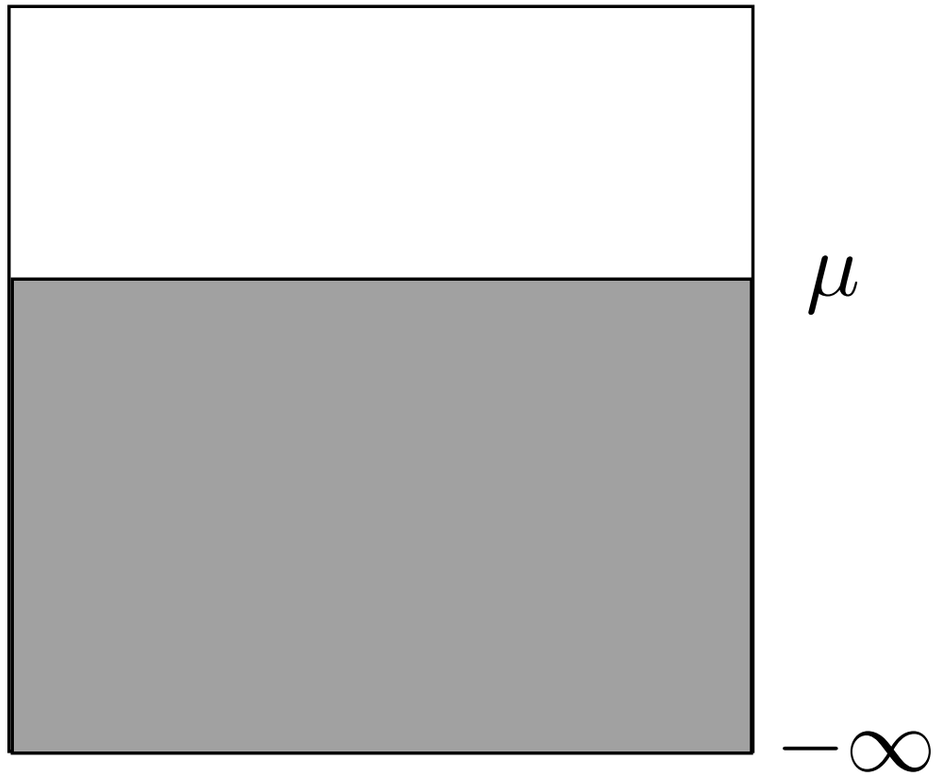
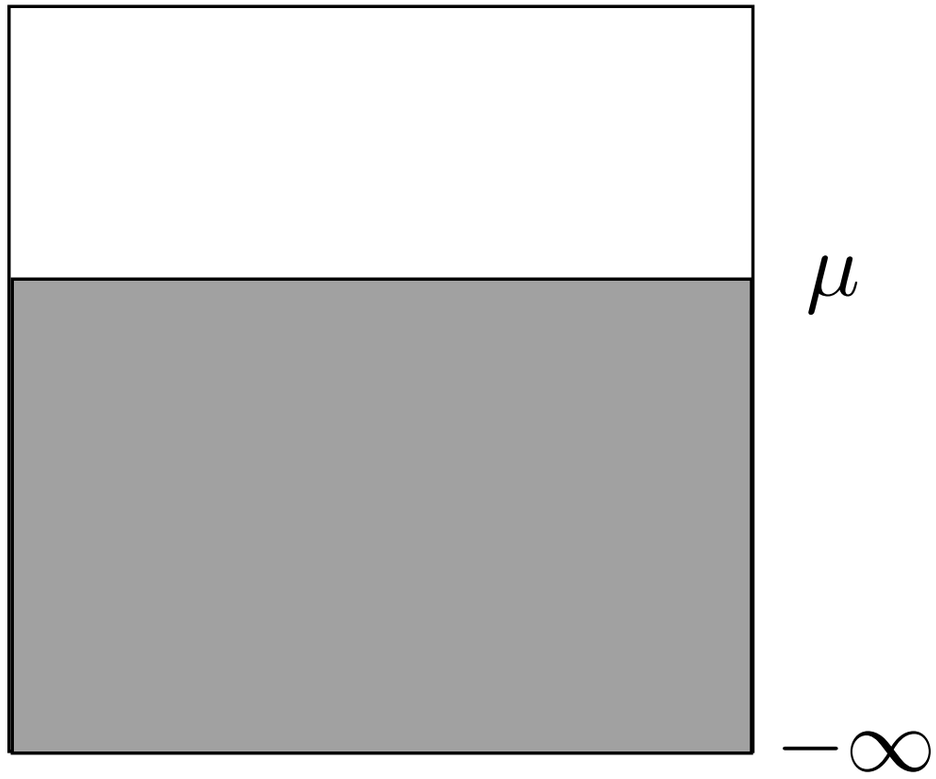


is there a more efficient way?


μ
$-\infty$

$$n = \int_{-\infty}^{\mu} N(\omega) d\omega$$



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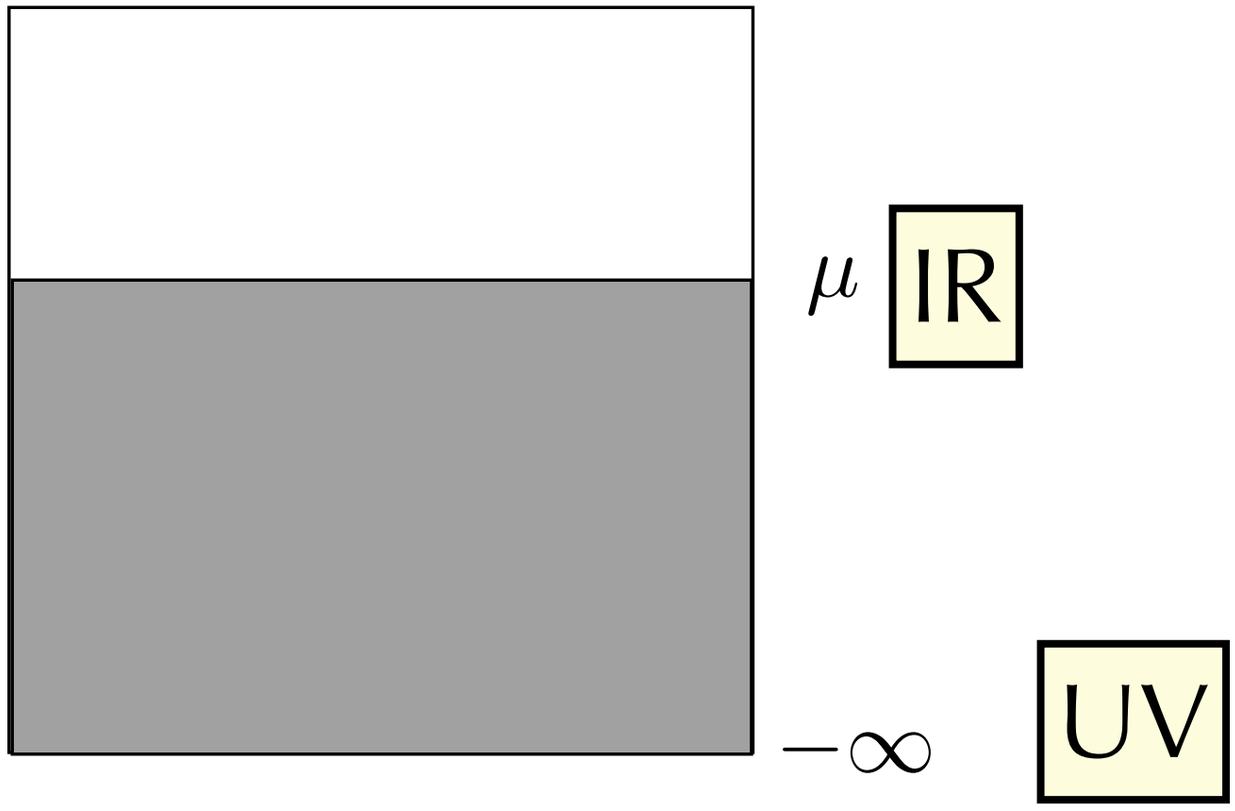

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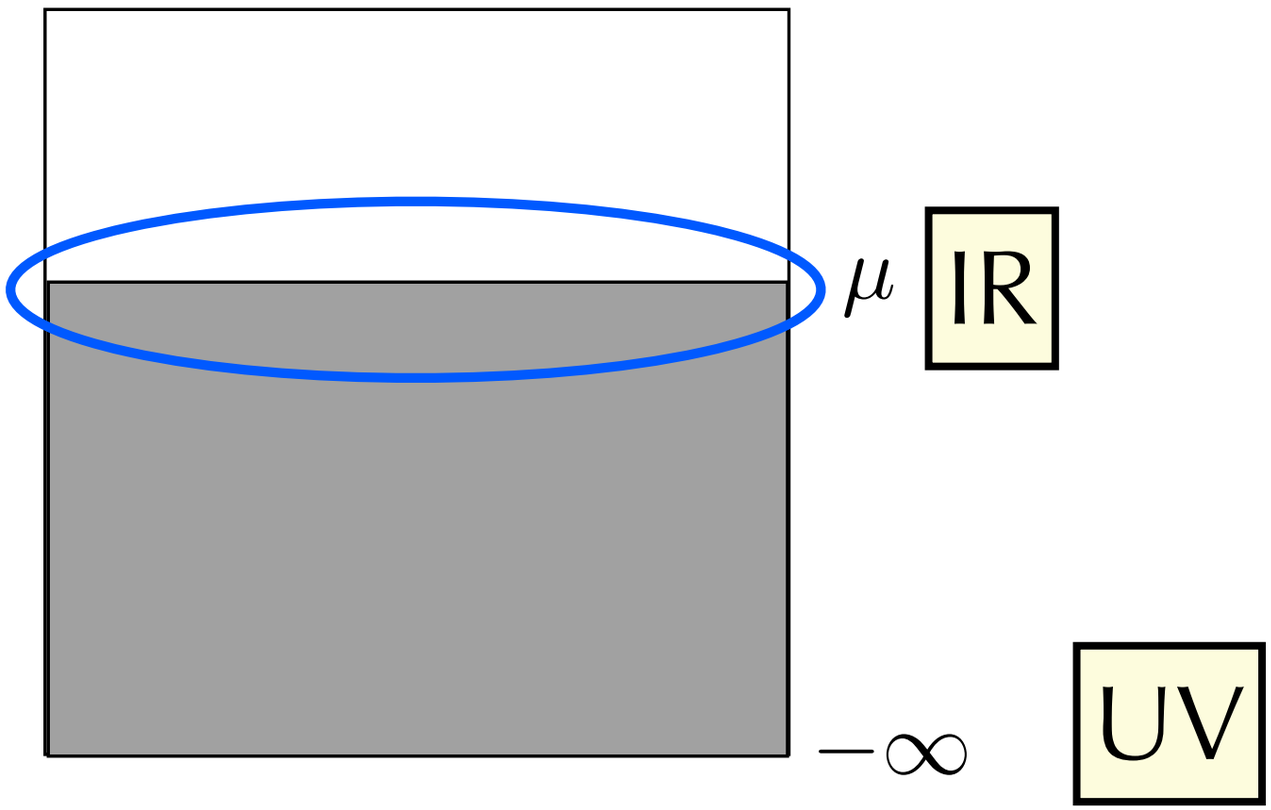


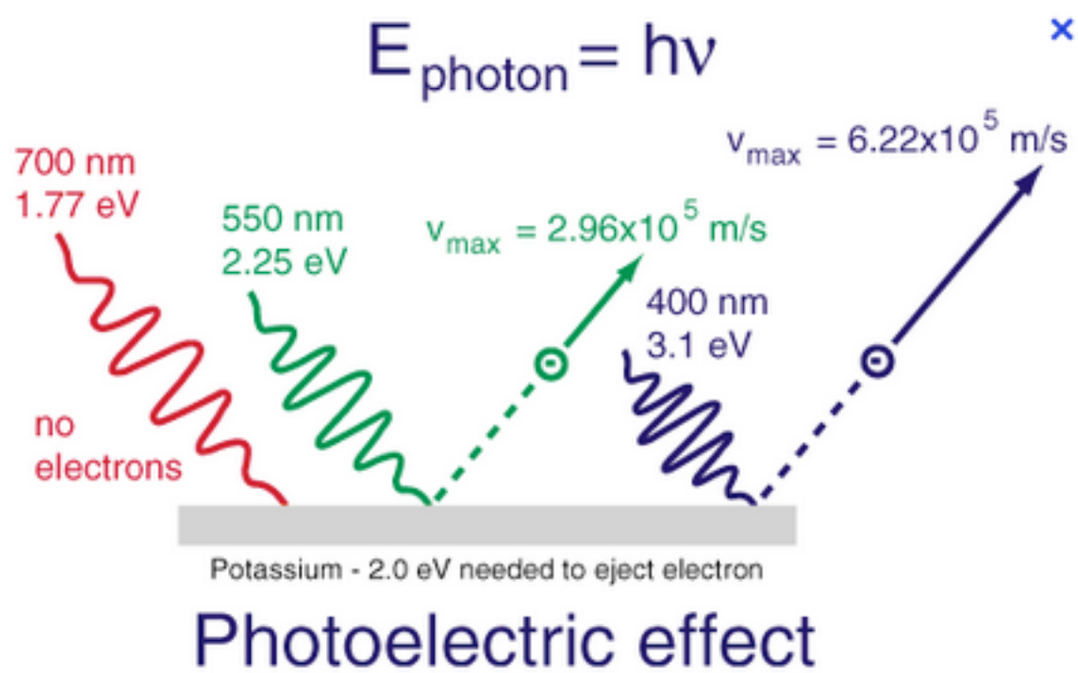
can n be deduced entirely from the IR(low-energy) scale?

Luttinger's `theorem'

Each particle has an energy

Each particle has an energy

particles

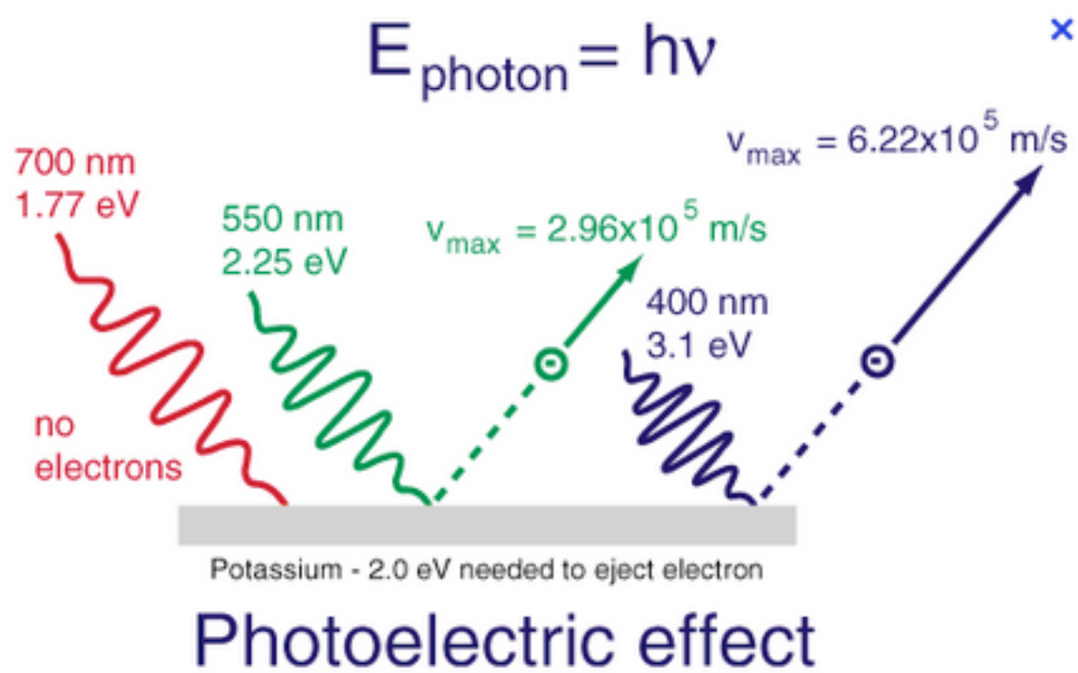


Photoelectric effect

energy $E = \varepsilon_p$

Each particle has an energy

particles



energy $E = \varepsilon_p$

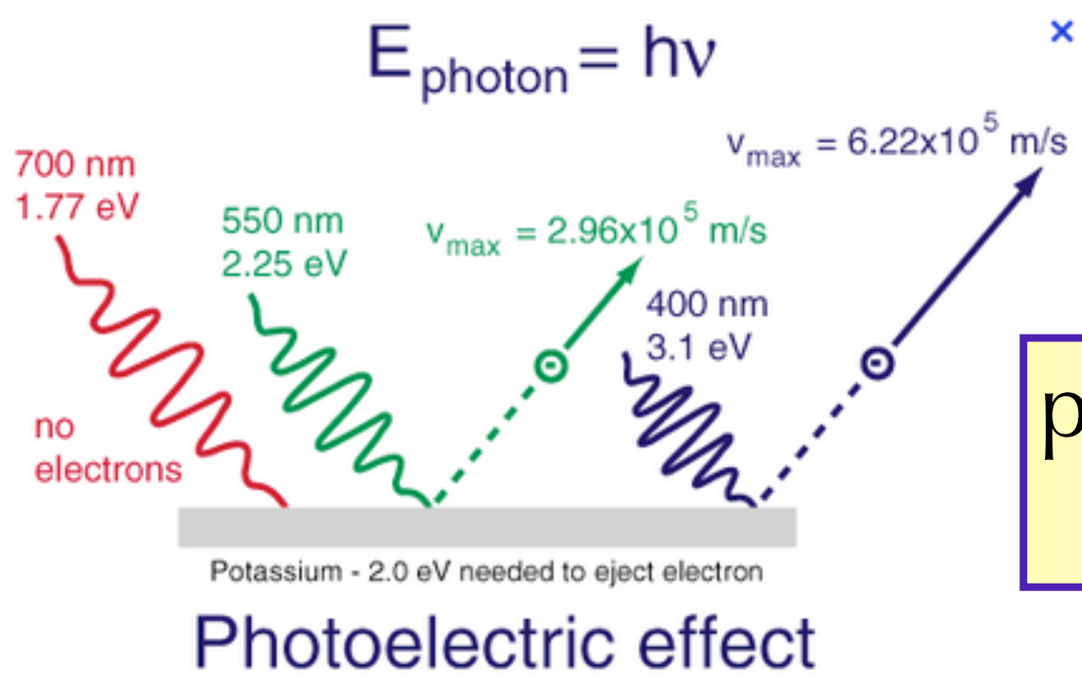
Green function (propagator)

$$G(E) = \frac{1}{E - \varepsilon_p}$$

particle=infinity

Each particle has an energy

particles



particle count is deducible from Green function

energy $E = \varepsilon_p$

Green function (propagator)

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Luttinger's Theorem

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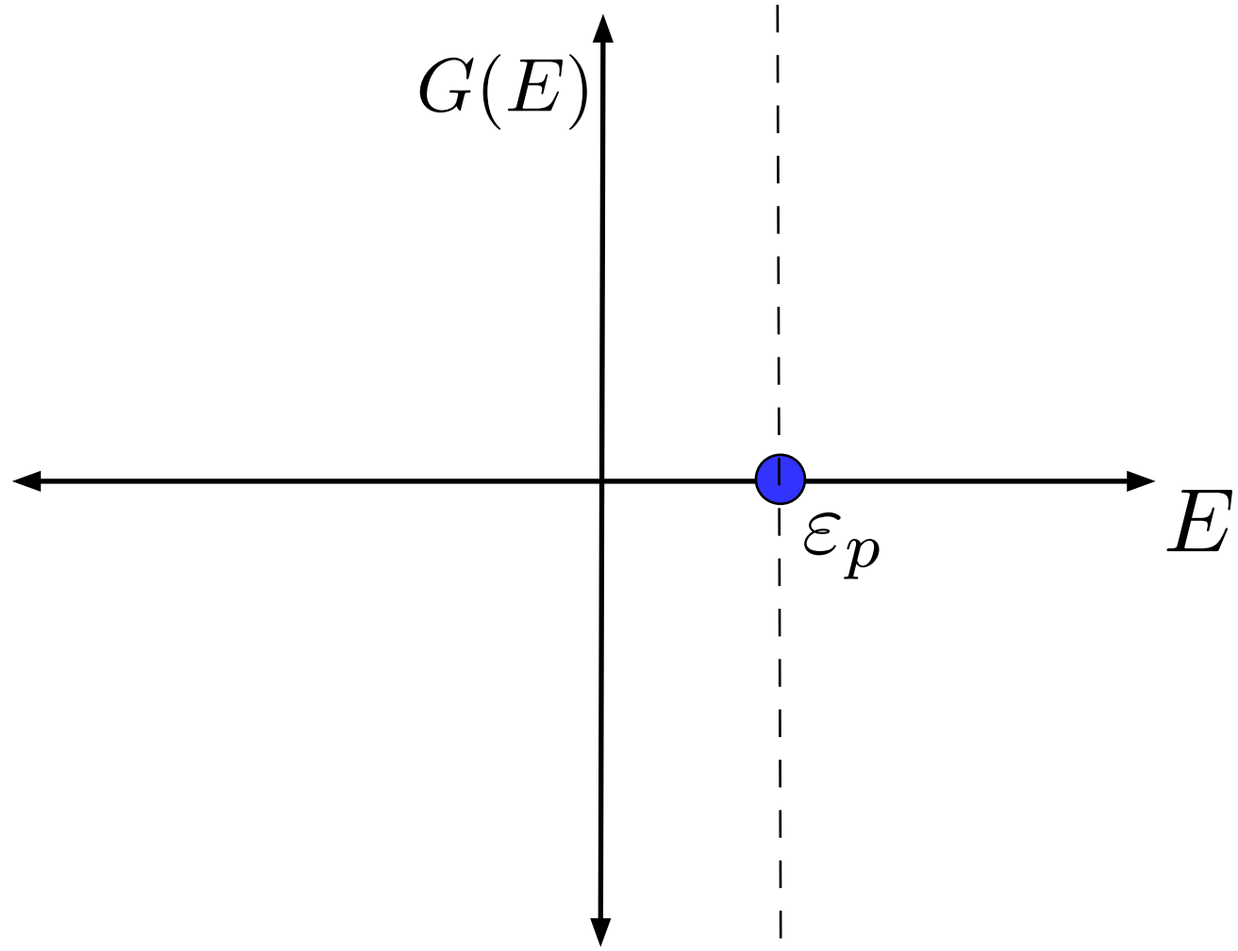

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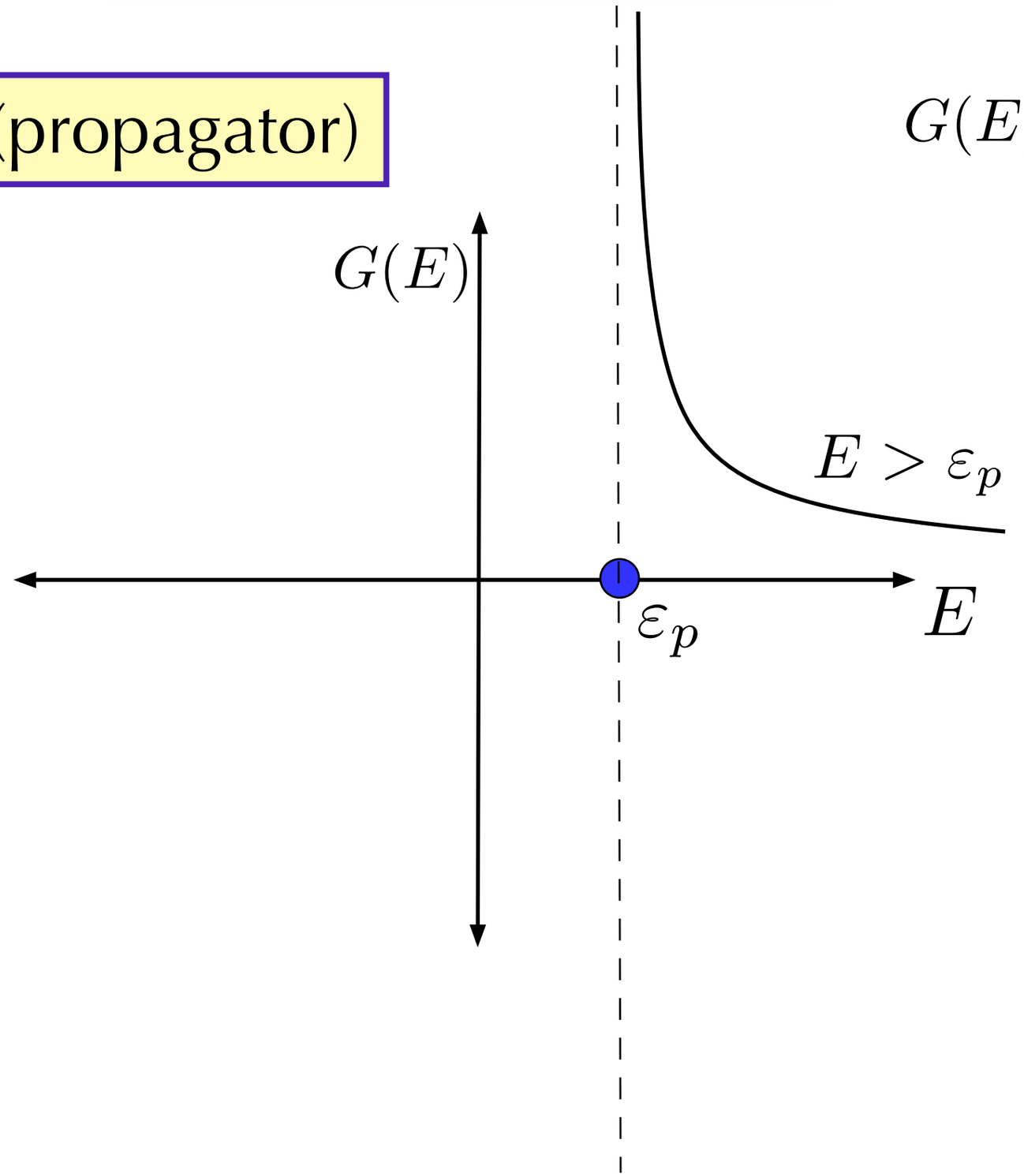

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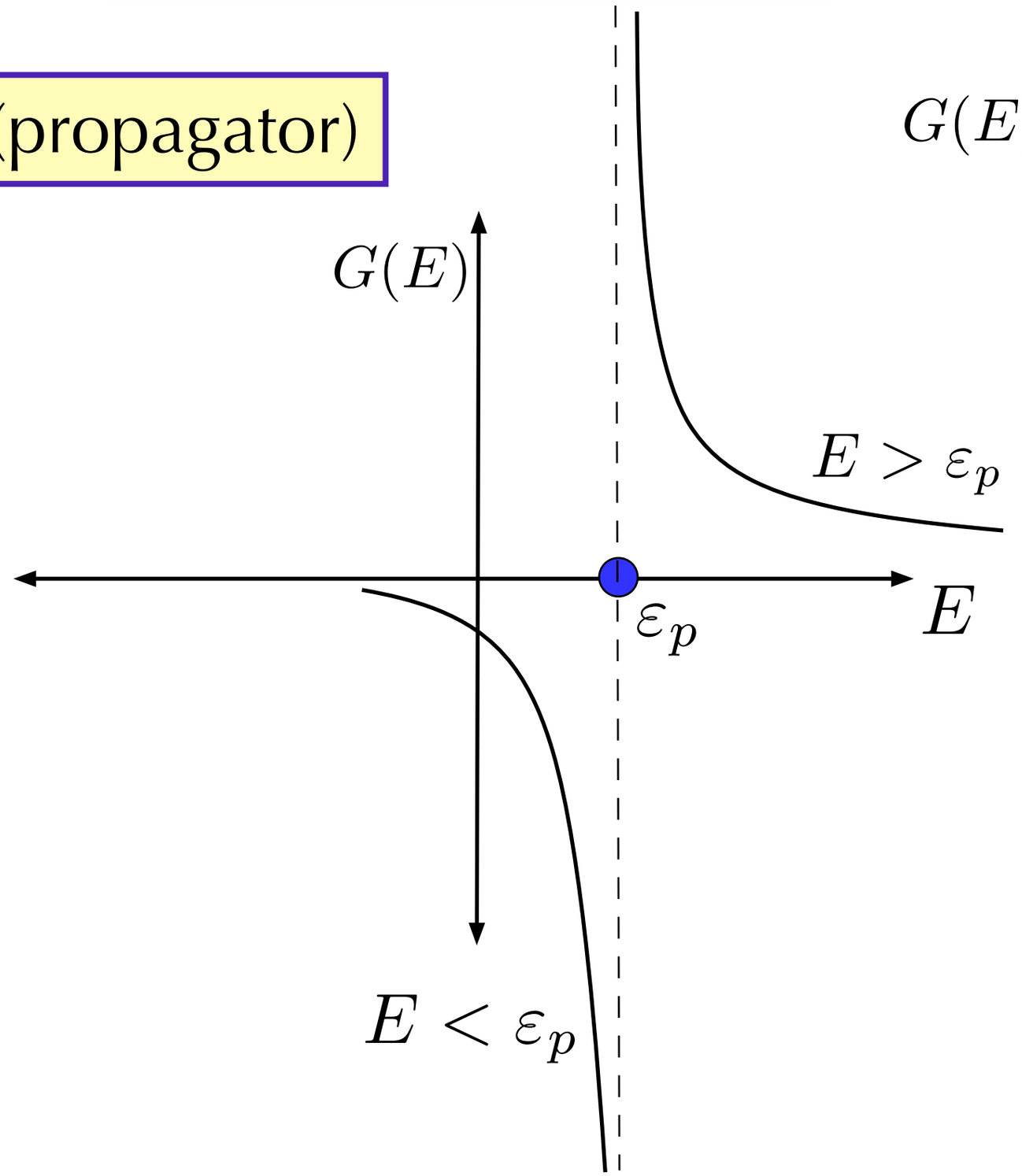

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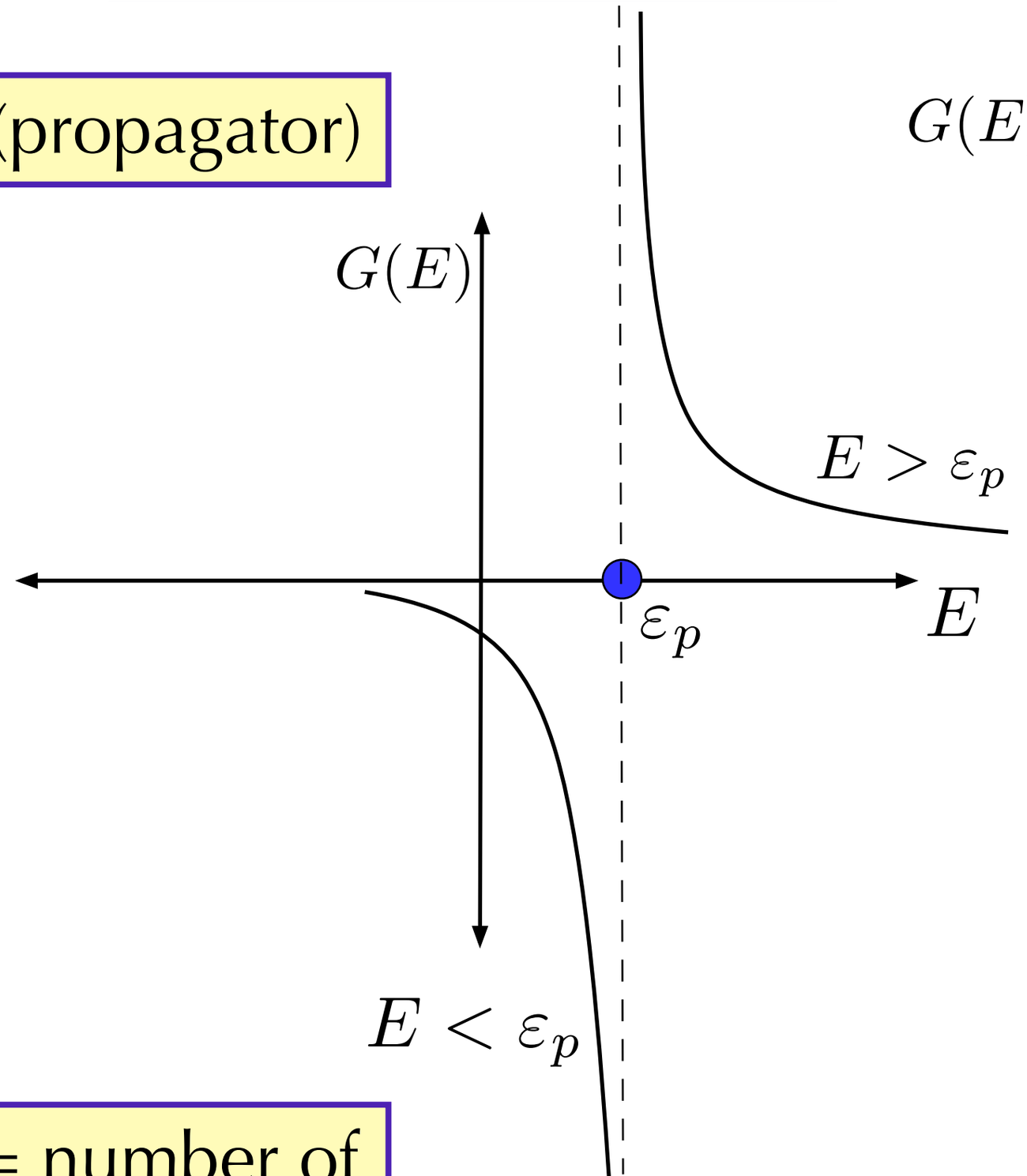

particle density= number of sign changes of G

Counting sign changes?

Counting sign changes?

$$\Theta(x)$$

Counting sign changes?

$$\Theta(x) = \begin{cases} 0 & x < 0 \\ & \end{cases}$$

Counting sign changes?

$$\Theta(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases}$$

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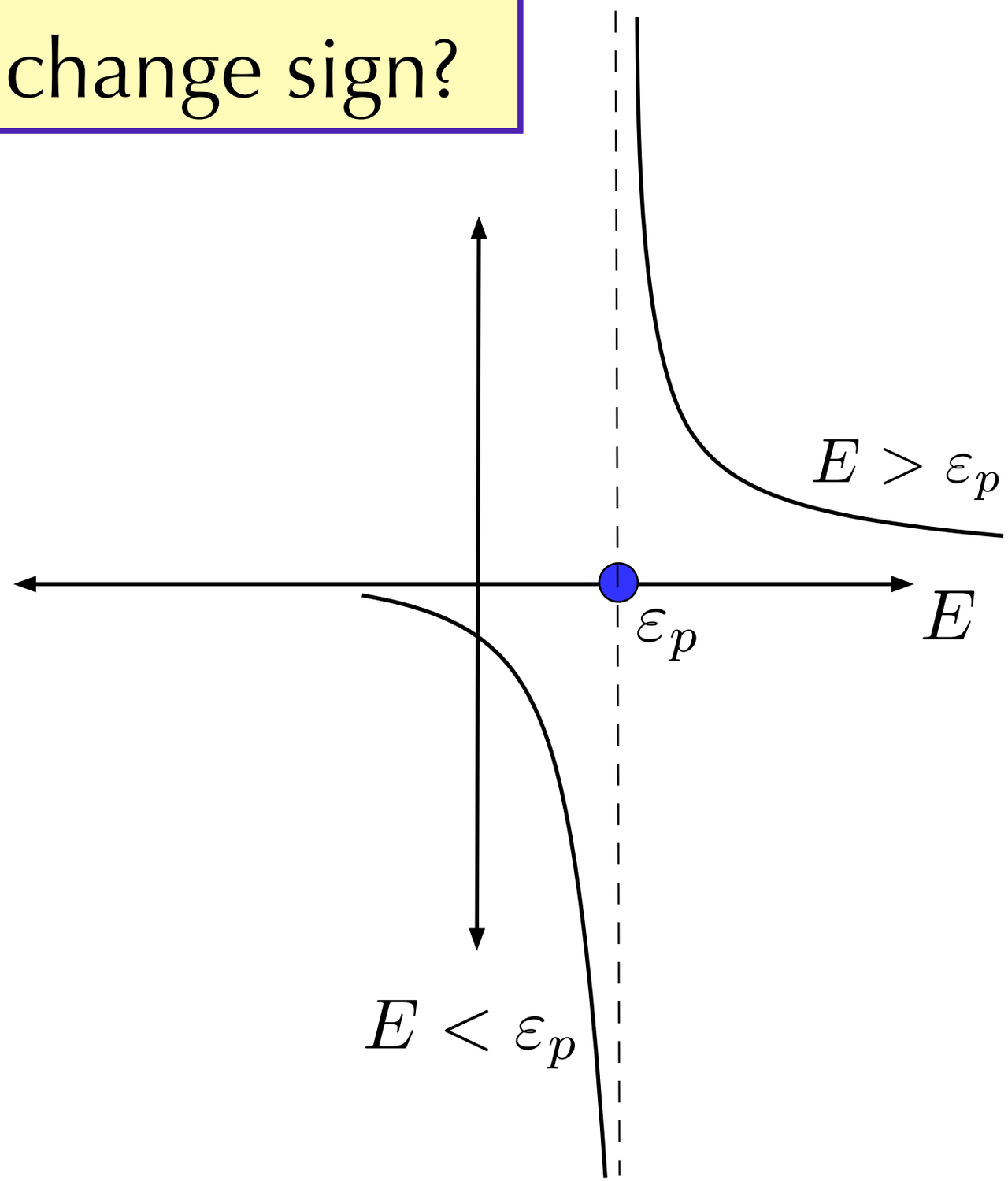
$$n = 2 \sum_{\mathbf{p}} \Theta(G(E = 0, \mathbf{p}))$$

Luttinger Theorem for electrons

How do functions change sign?



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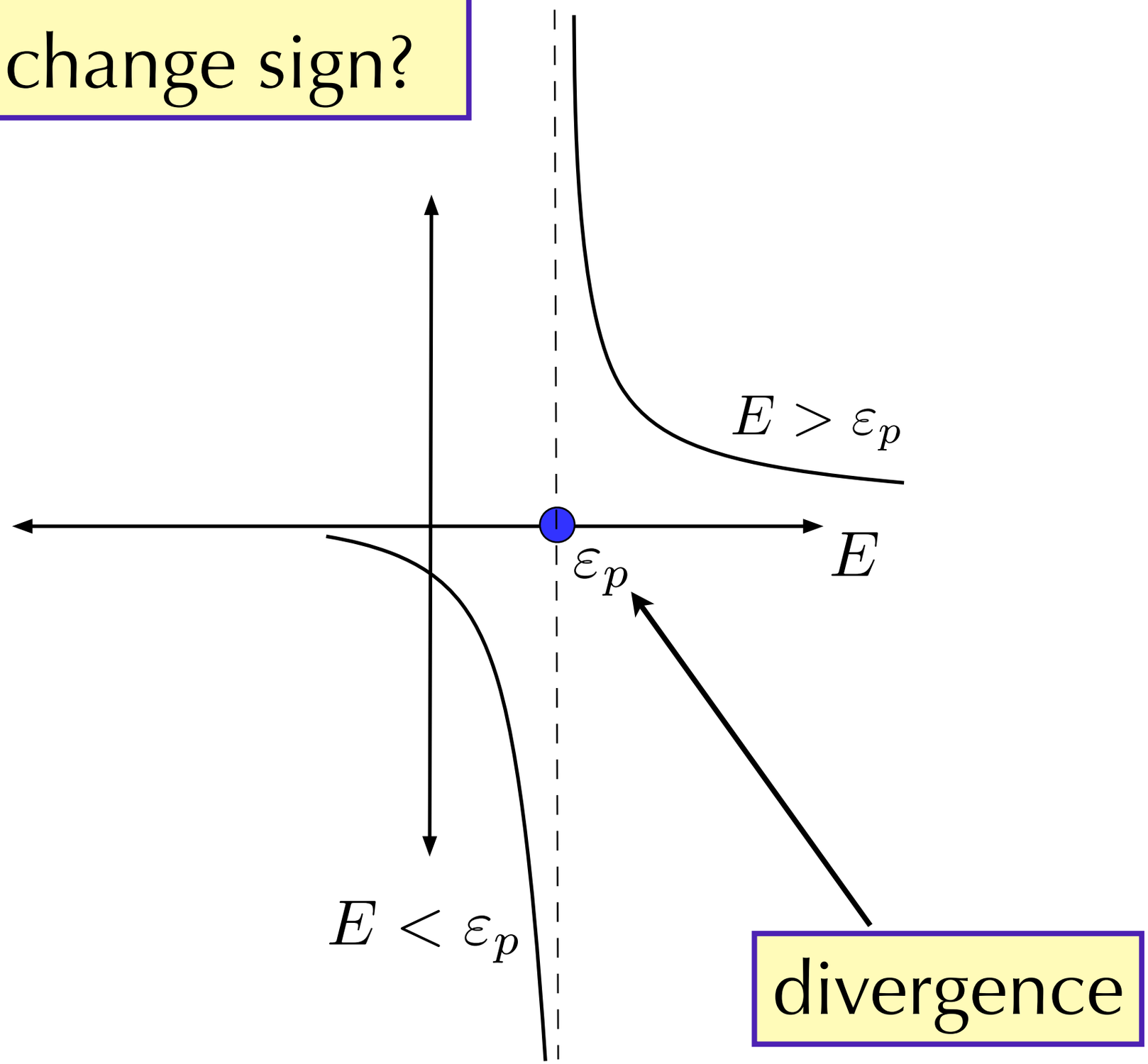
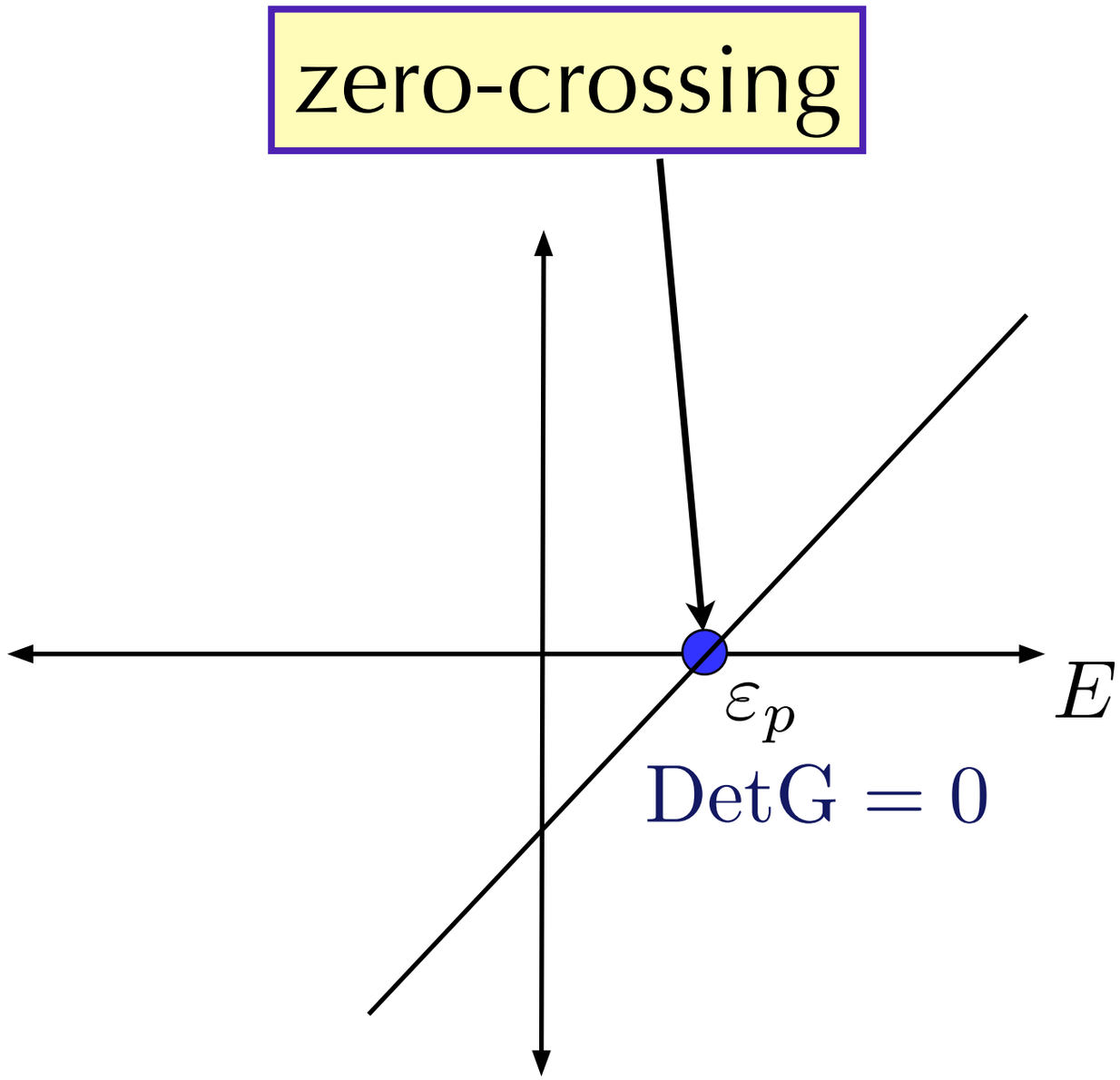
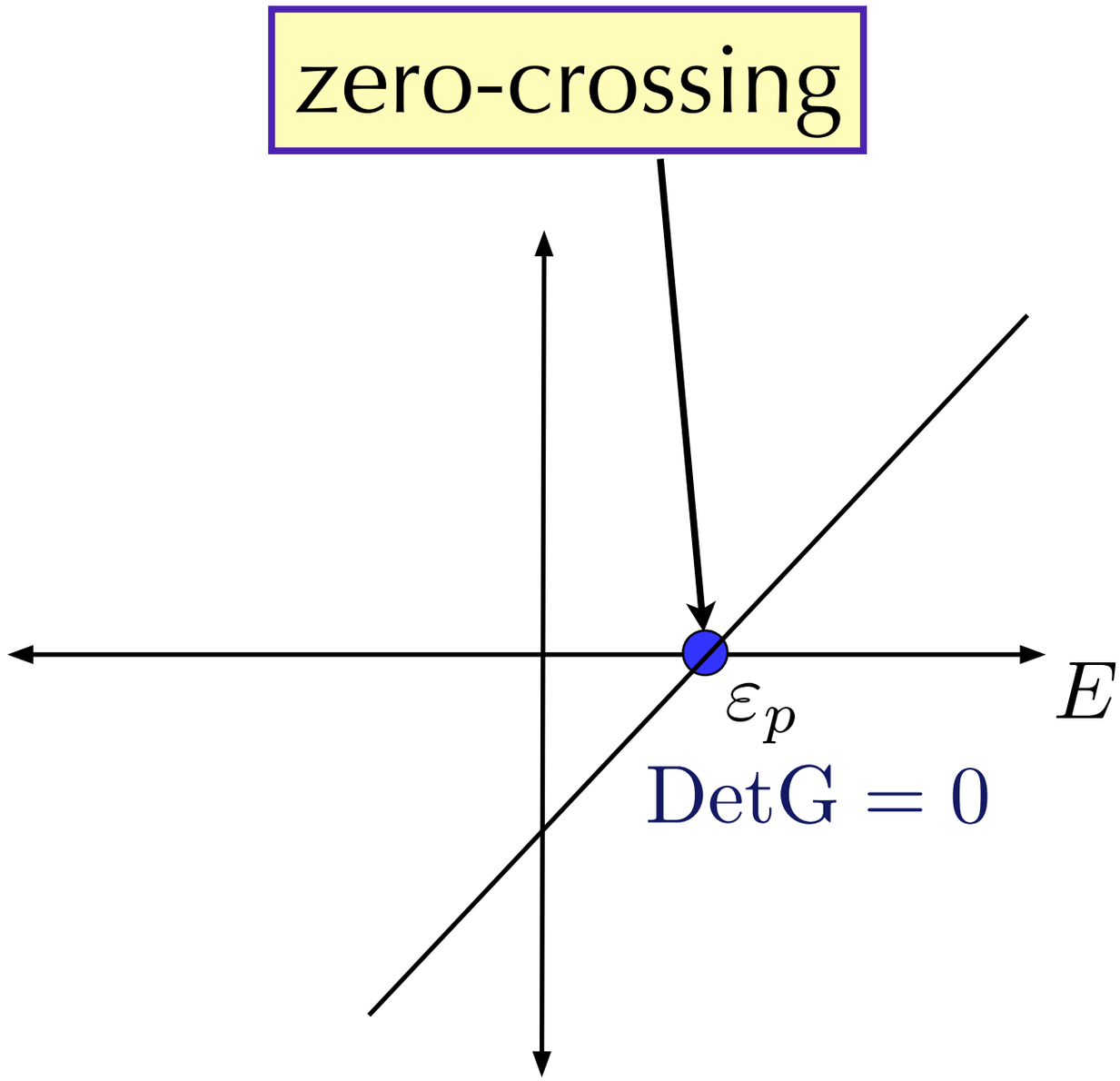


Is there
another way?

Yes


zero-crossing
E
ε_p
$\mathrm{DetG} = 0$



no divergence is necessary

closer look at Luttinger's theorem

$$n = 2 \sum_{\mathbf{p}} \Theta(G(E = 0, \mathbf{p}))$$

closer look at Luttinger's theorem

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divergences+ zeros

closer look at Luttinger's theorem

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divergences+ zeros

how can zeros affect the particle count?

PHYSICAL REVIEW B **68**, 085113 (2003)

Some consequences of the Luttinger theorem: The Luttinger surfaces in non-Fermi liquids and Mott insulators

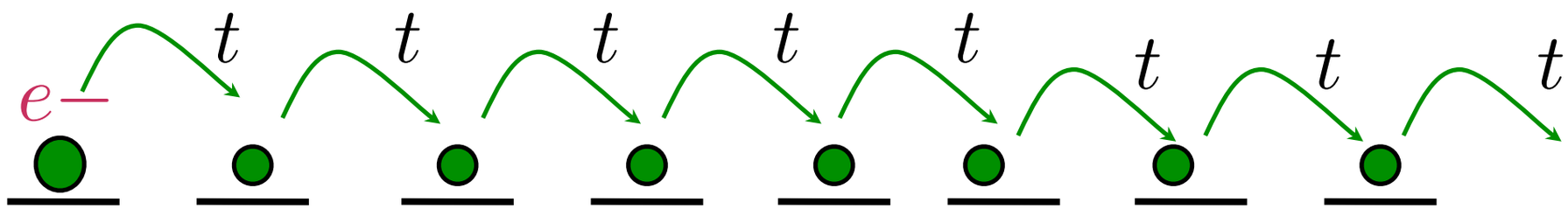
Igor Dzyaloshinskii
Department of Physics and Astronomy, University of California, Irvine, California 92697, USA
(Received 30 January 2003; published 27 August 2003)

The proof just presented is good for any state of our system: FL, NFL, or MI [in other words, either of poles or zeros can be used to change the sign of G_r in Eq. (1)]. The only way to incapacitate the Luttinger theorem in form (1) is to assume that the limit $T \to 0$ is discontinuous. Actually, one has to require that the whole line $T=0$ is a line of phase transitions.

Is this famous theorem from 1960 correct?

A model with zeros but Luttinger fails

A model with zeros but Luttinger fails



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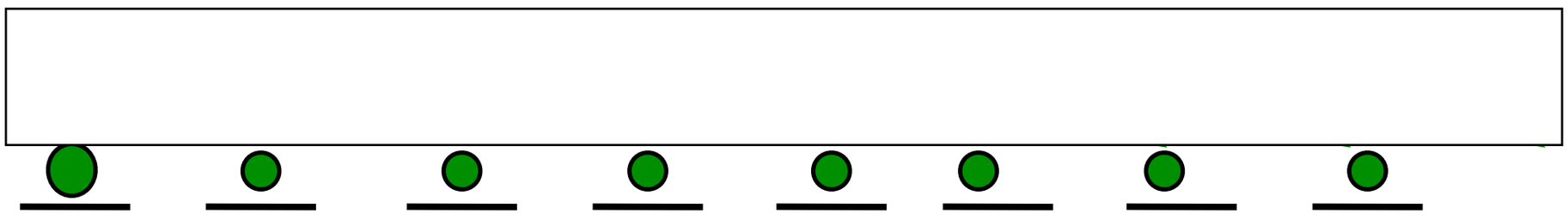

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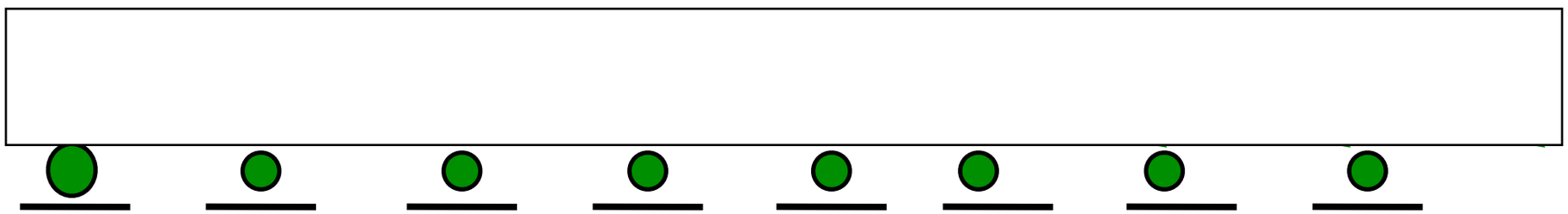
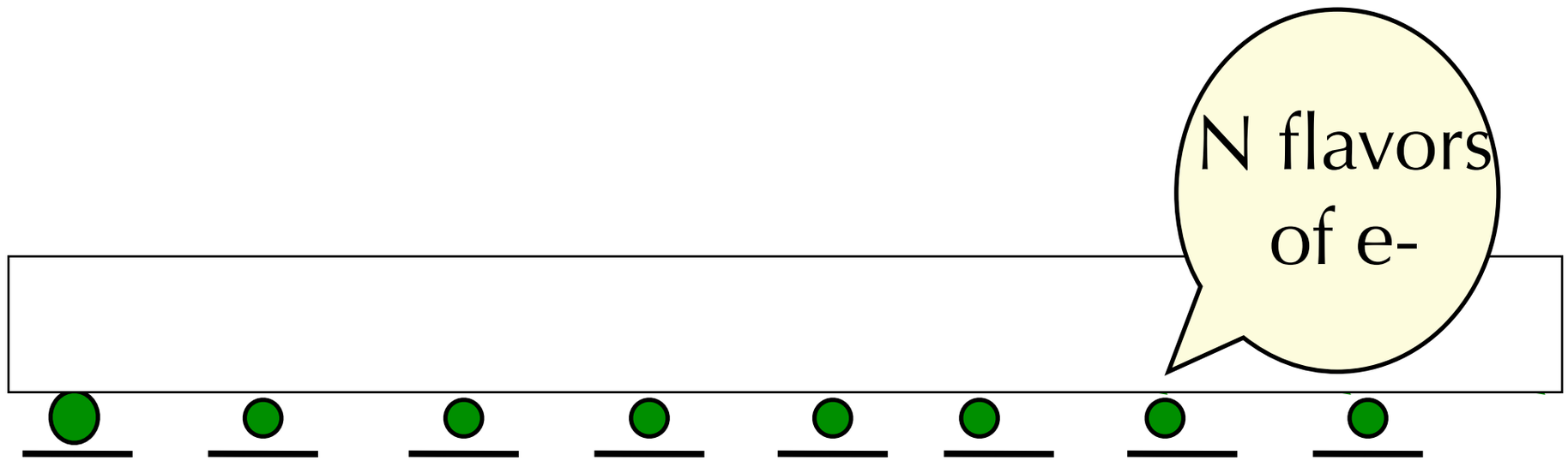


no hopping=> no propagation (zeros)

A model with zeros but Luttinger fails



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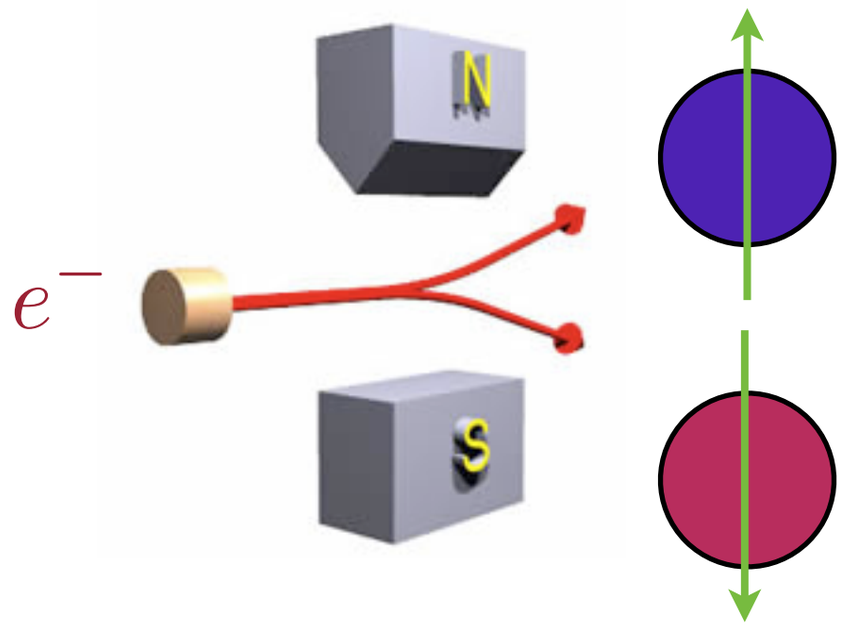
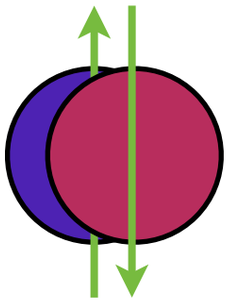
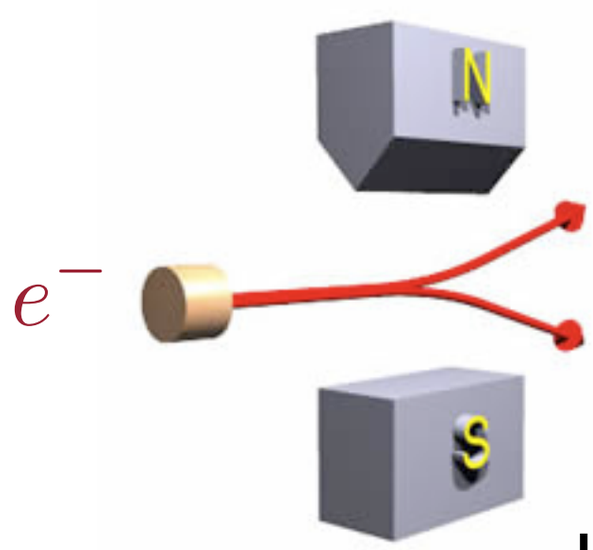


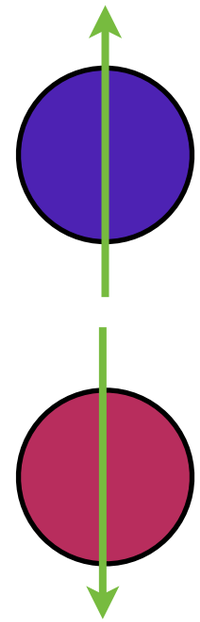
spin

$SU(2)$





e^-



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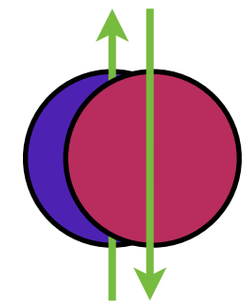
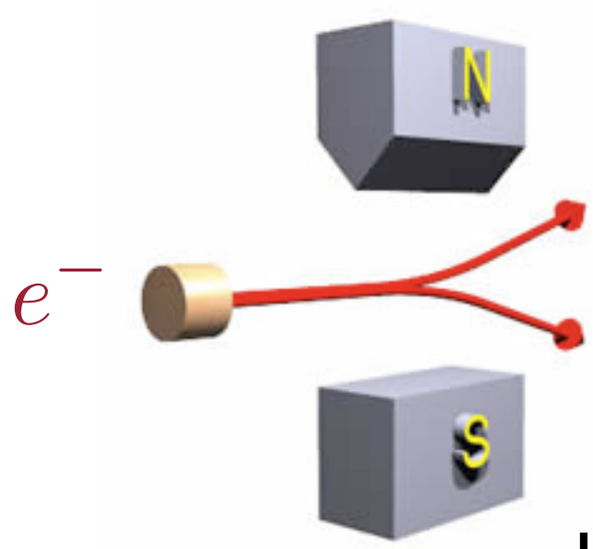
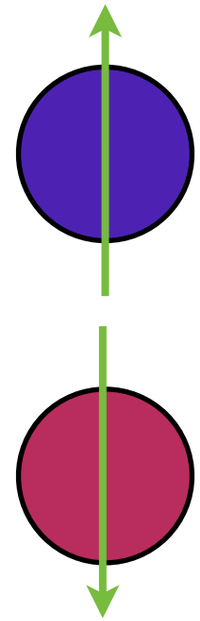


generalization

N flavors of spin

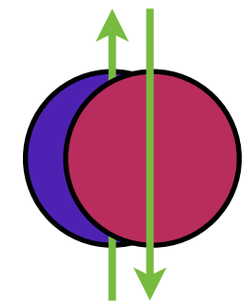
$SU(N)$





spin

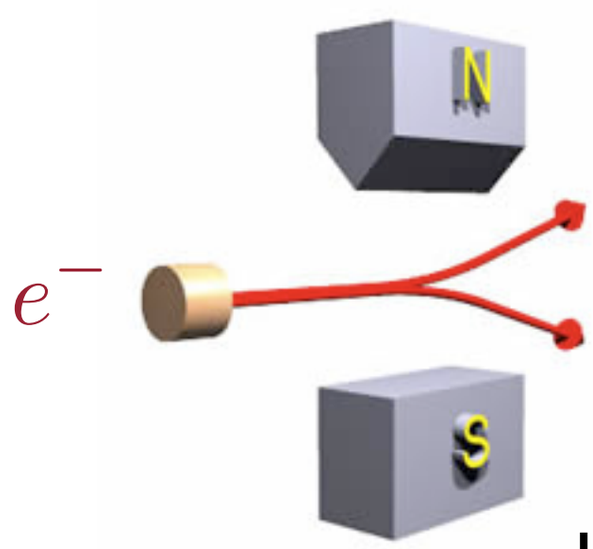
$SU(2)$

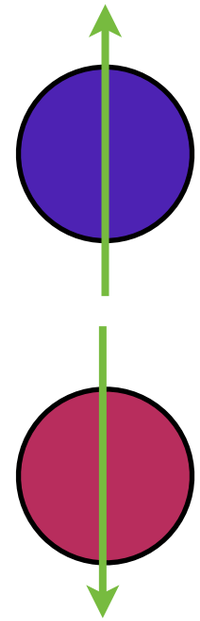


generalization

N flavors of spin $SU(N)$

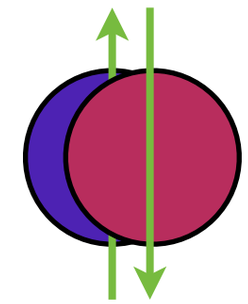
$N = 5$





spin

$SU(2)$



generalization

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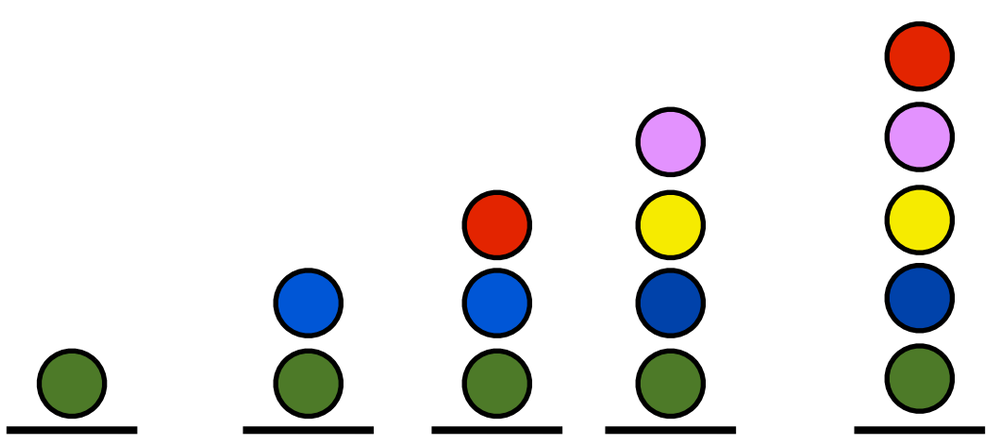

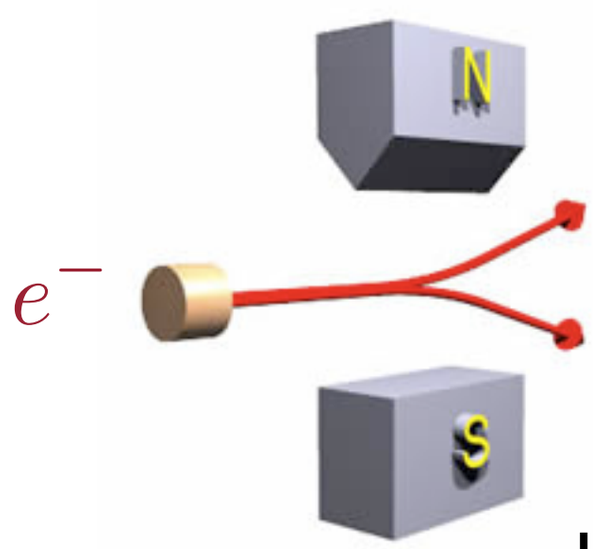

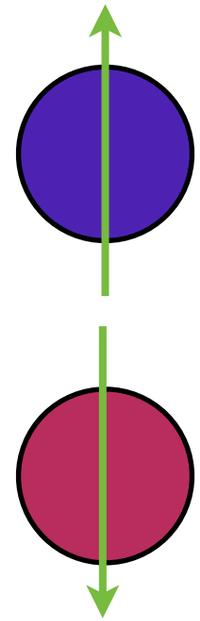

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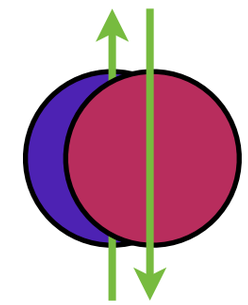

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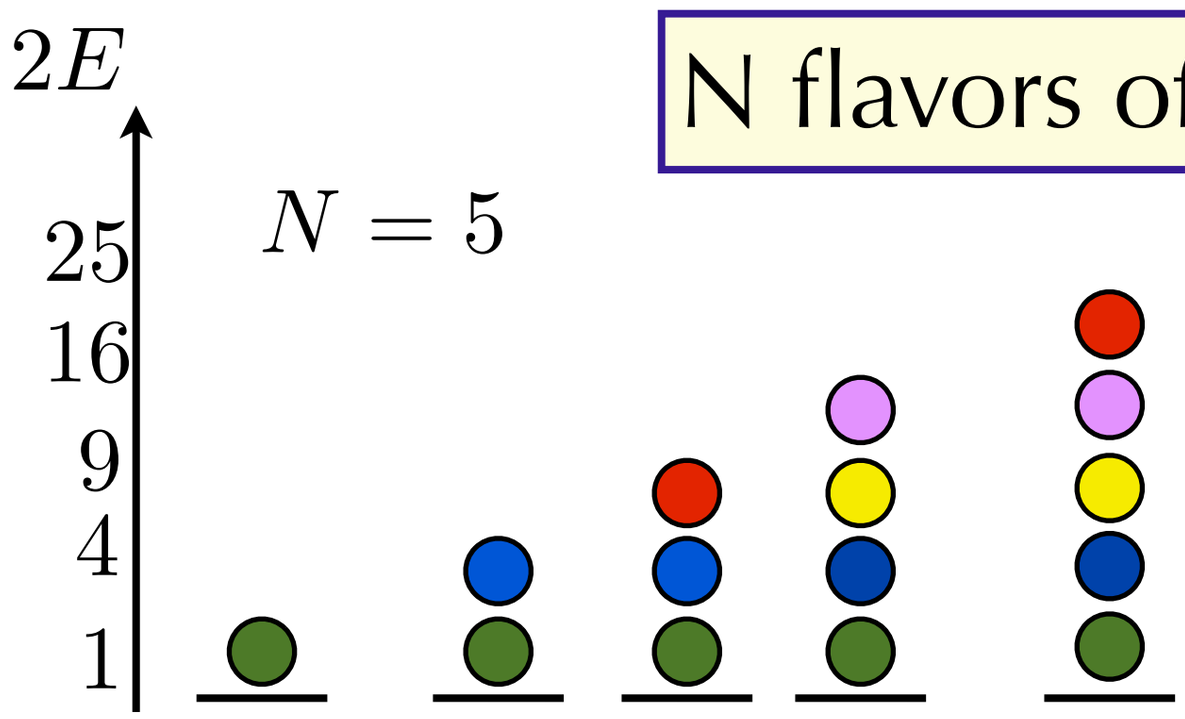
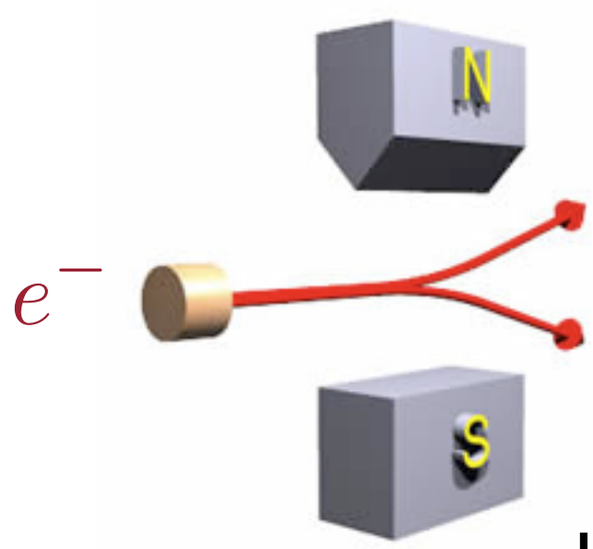
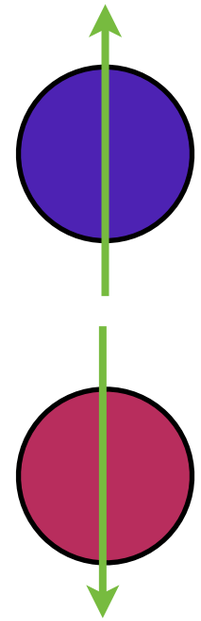
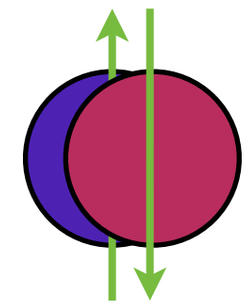






spin

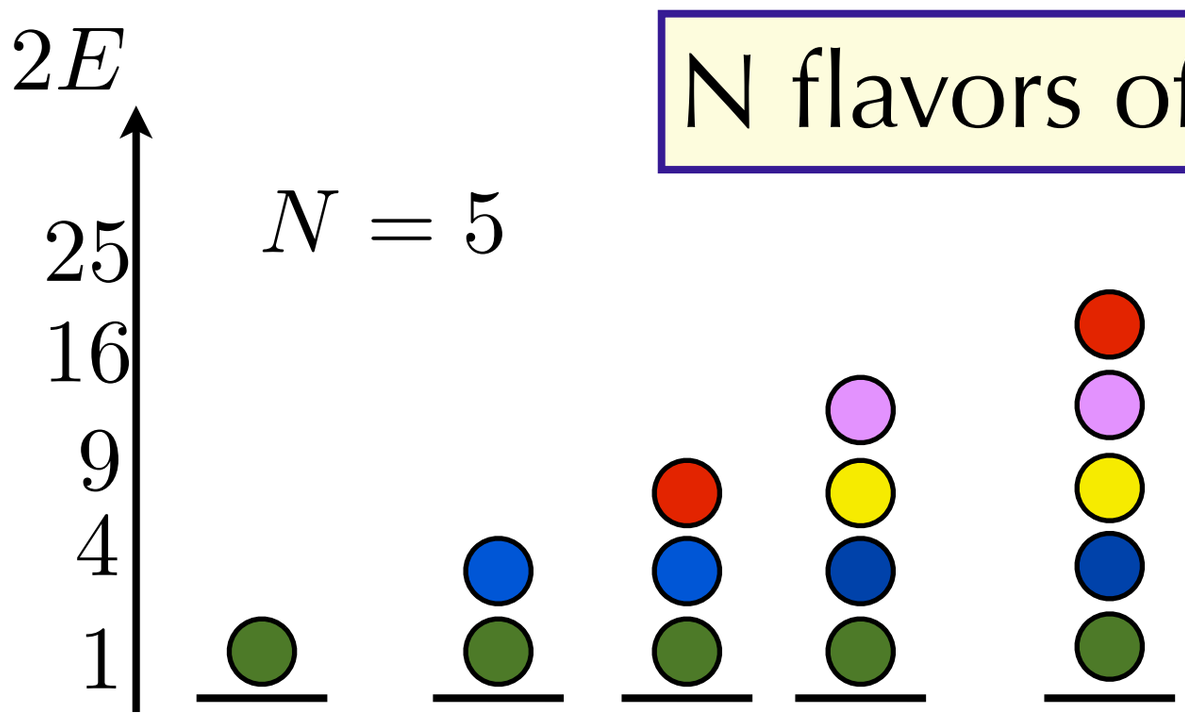
$SU(2)$



generalization

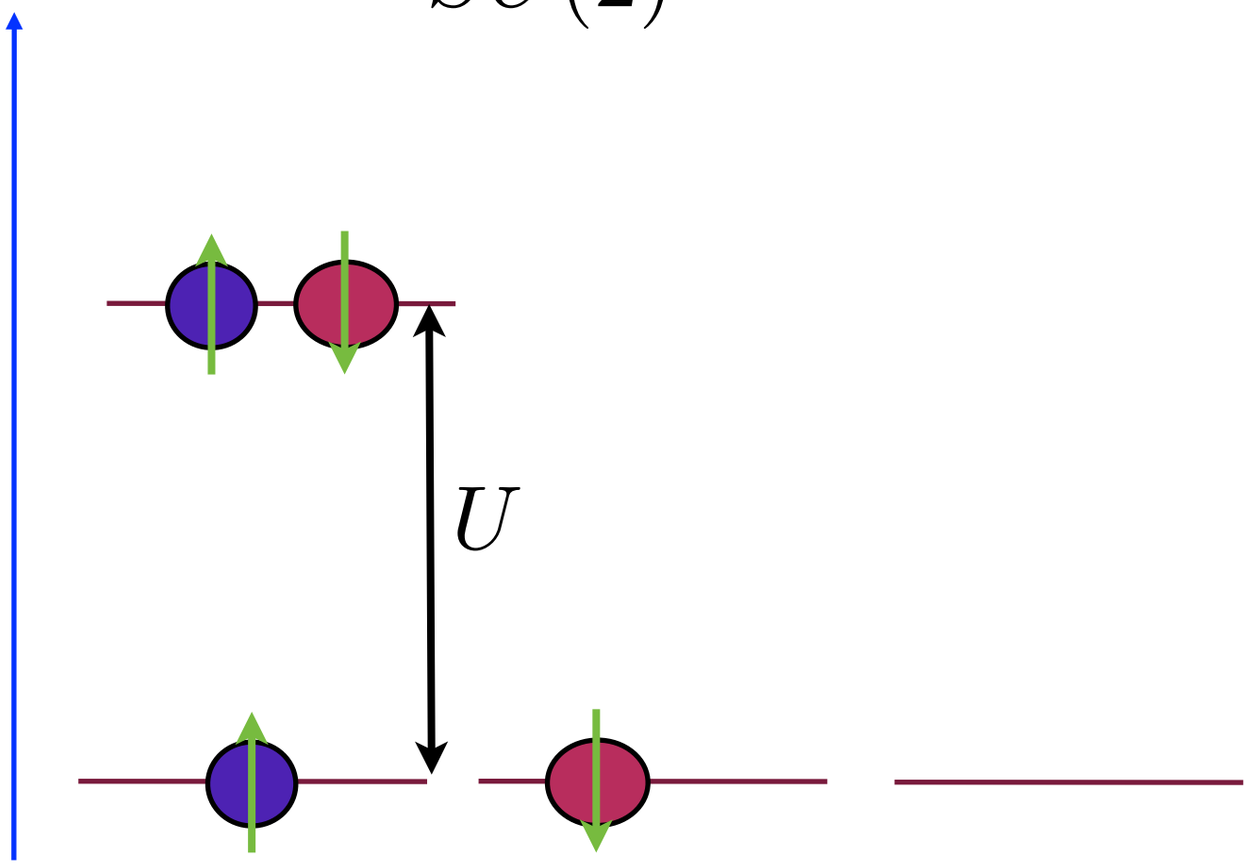
N flavors of spin

$SU(N)$

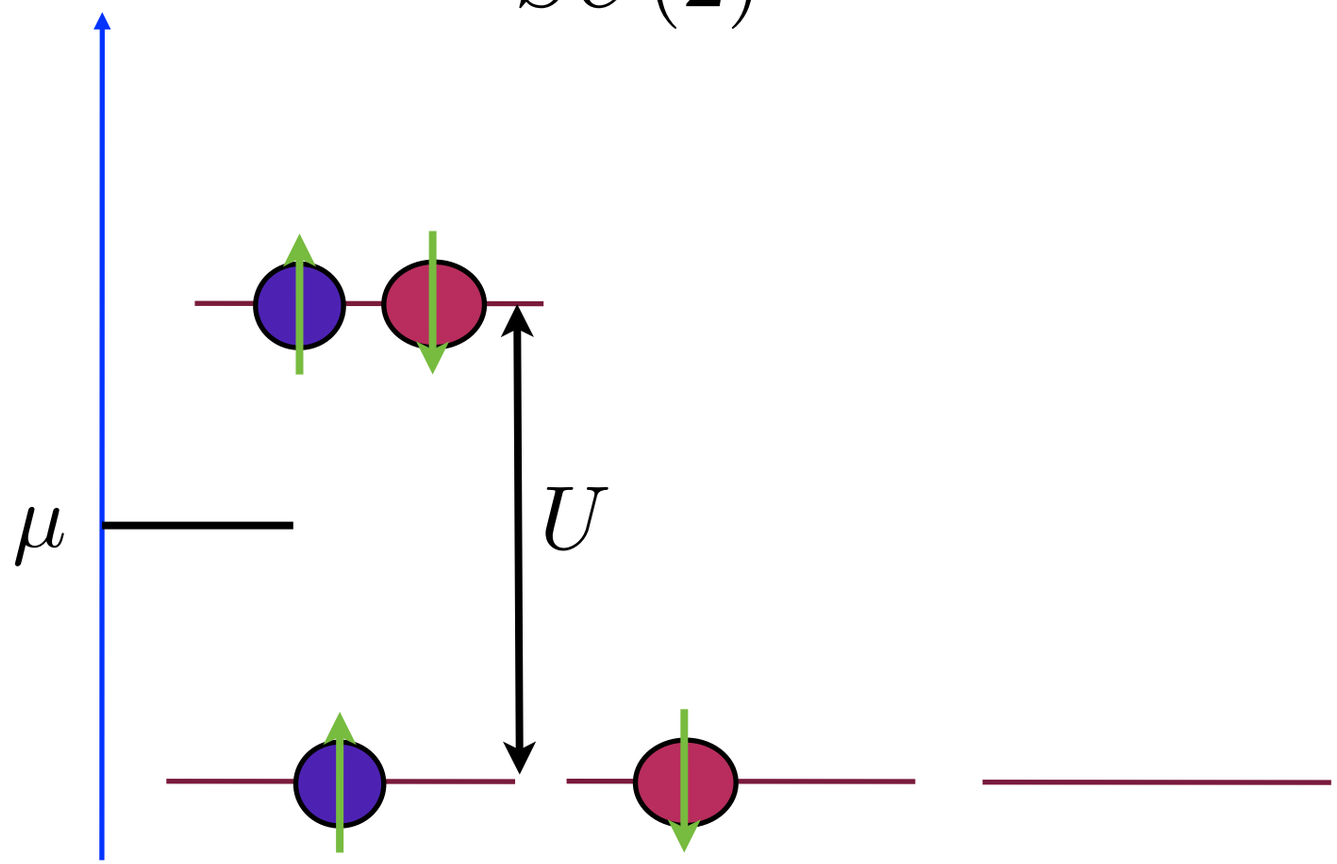


$$H = \frac{U}{2}(n_1 + \cdots n_N)^2$$

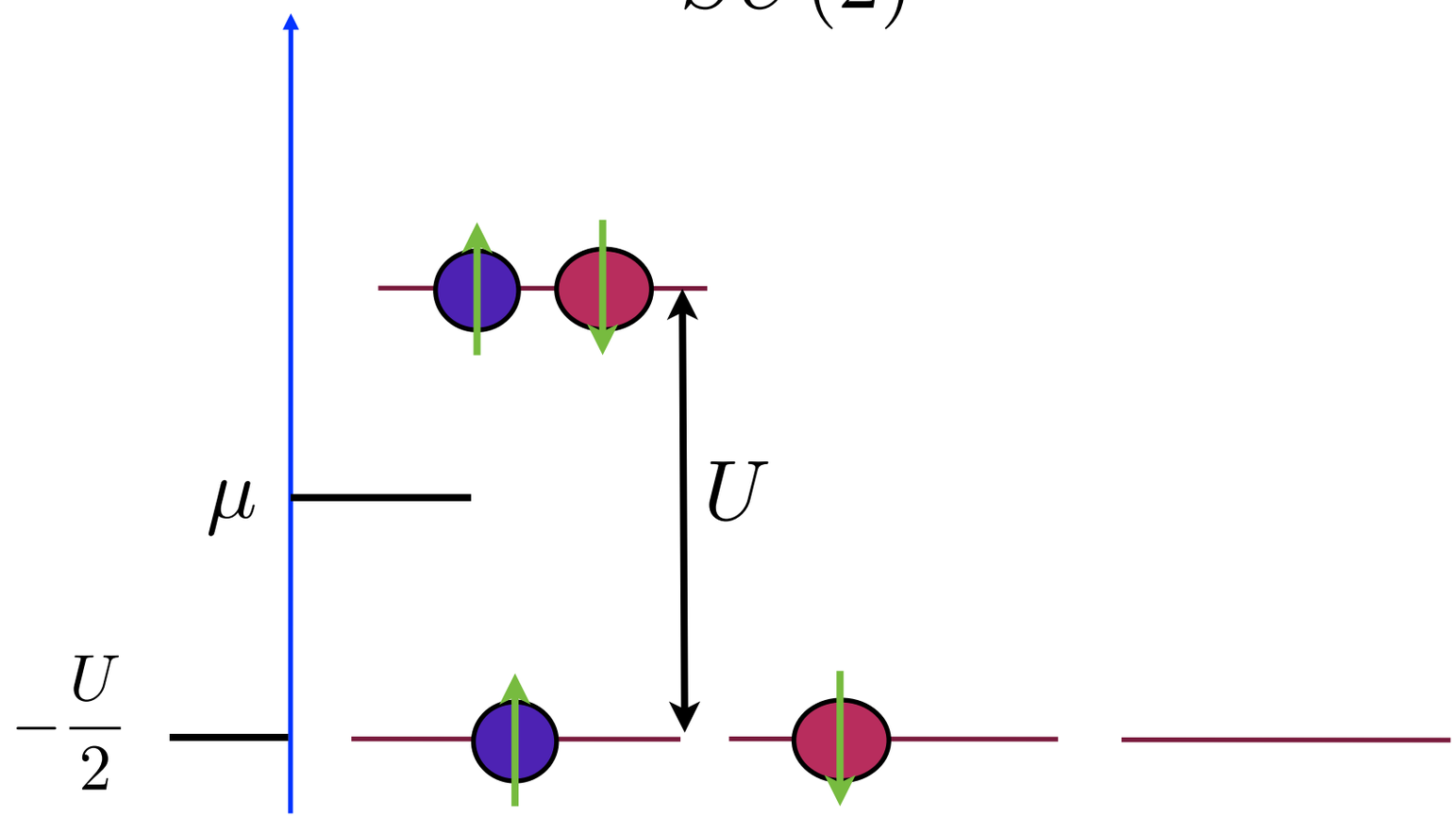
$SU(2)$



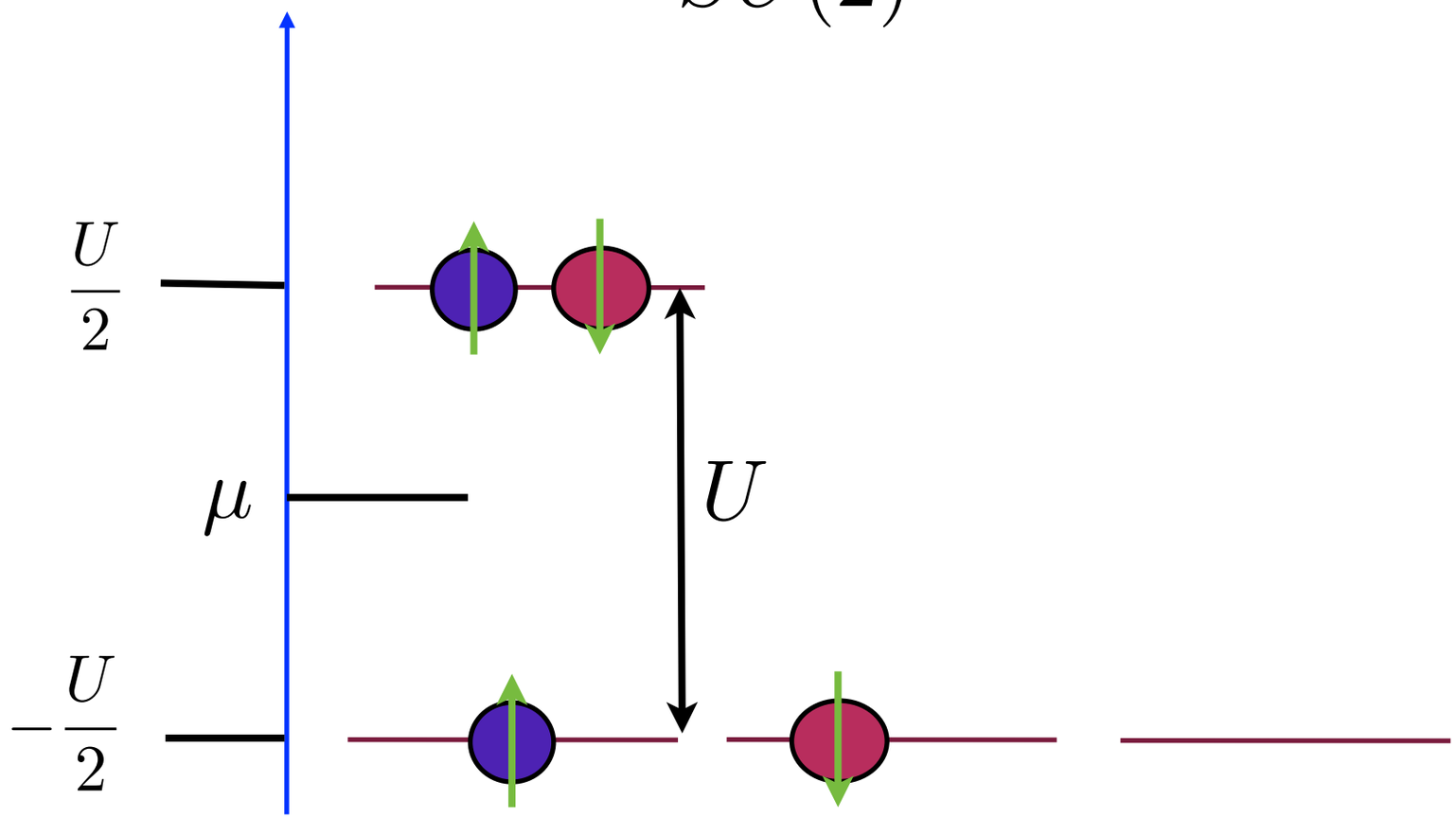
$SU(2)$



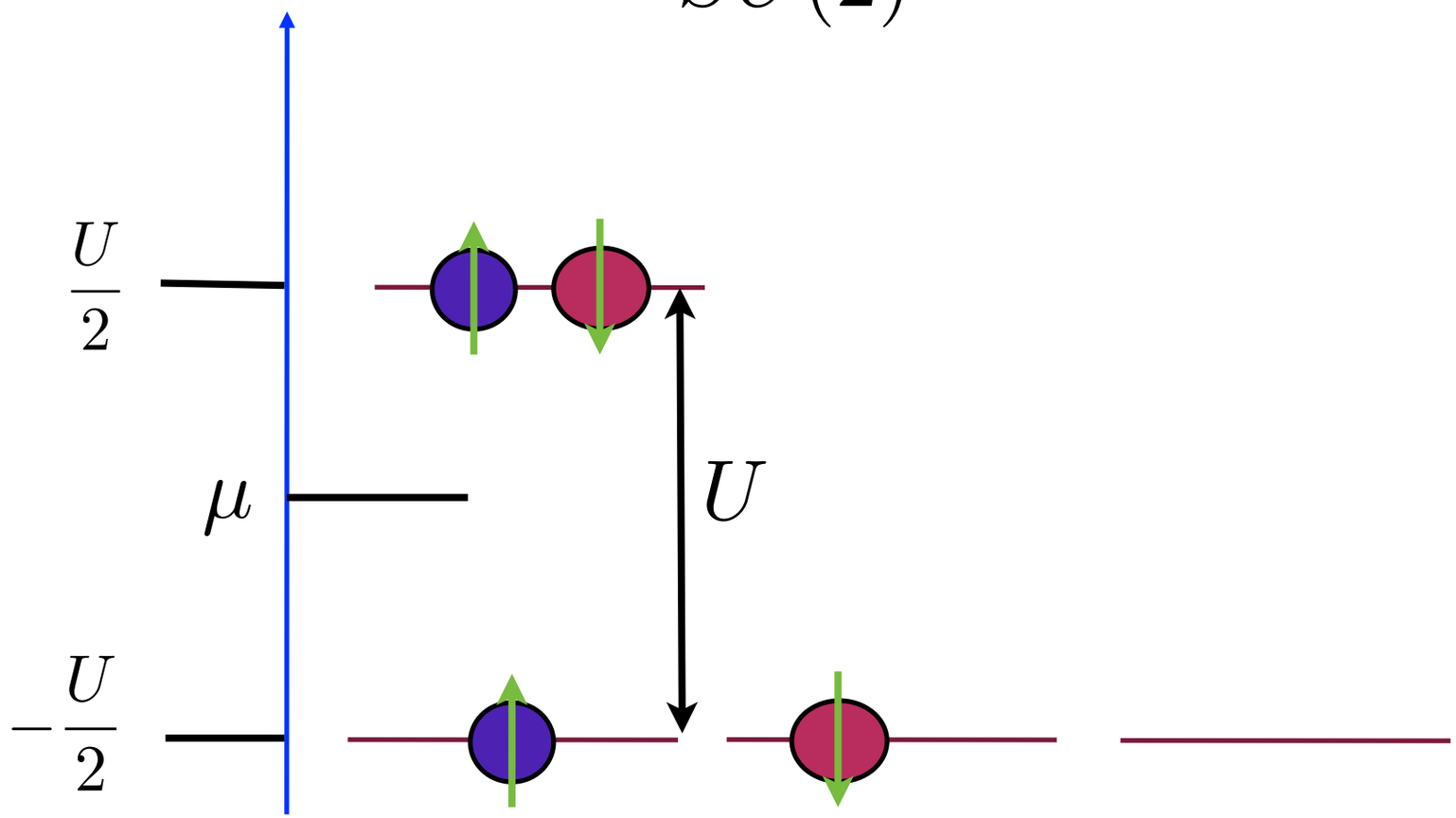
$SU(2)$



$SU(2)$

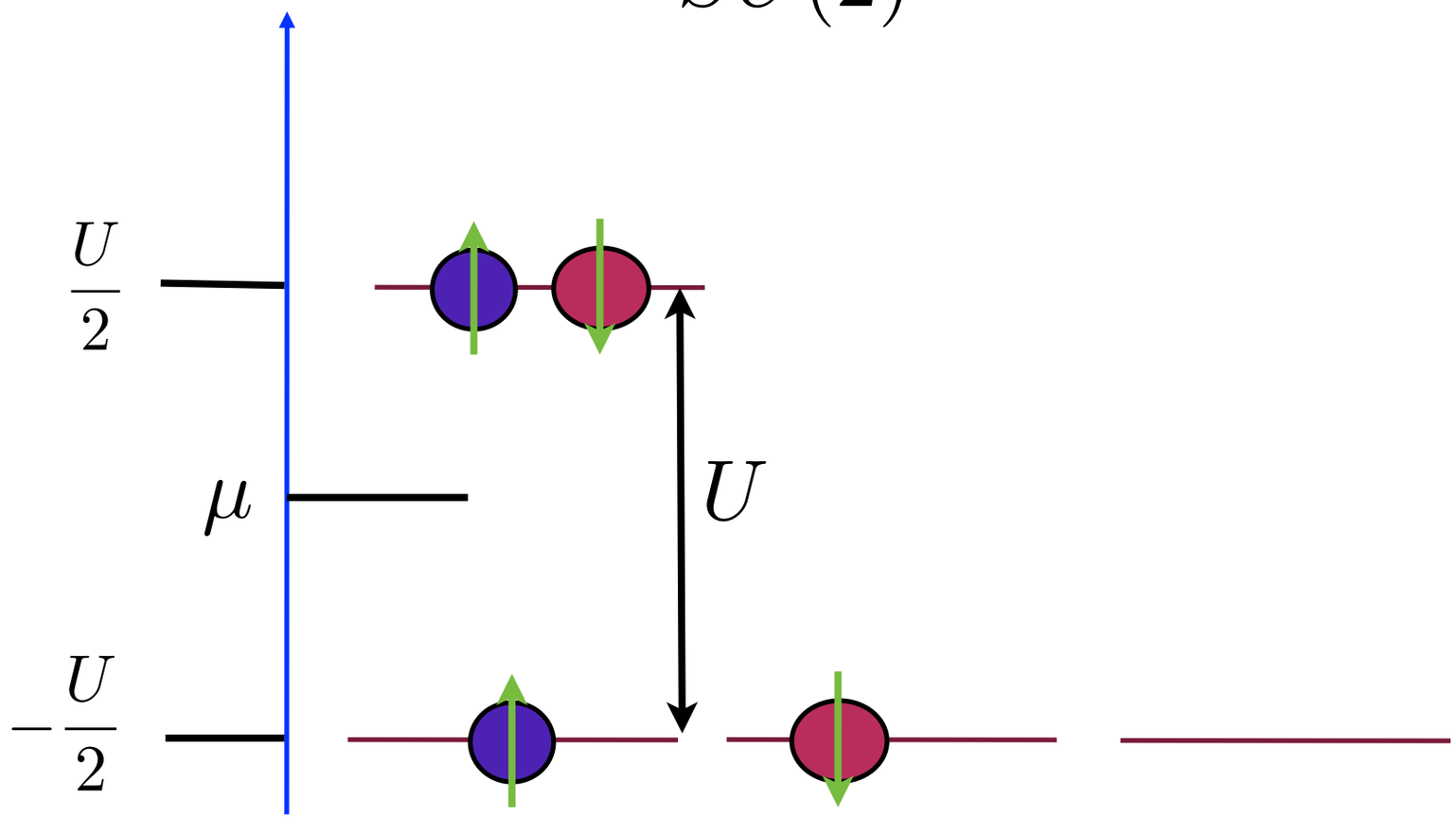


$SU(2)$



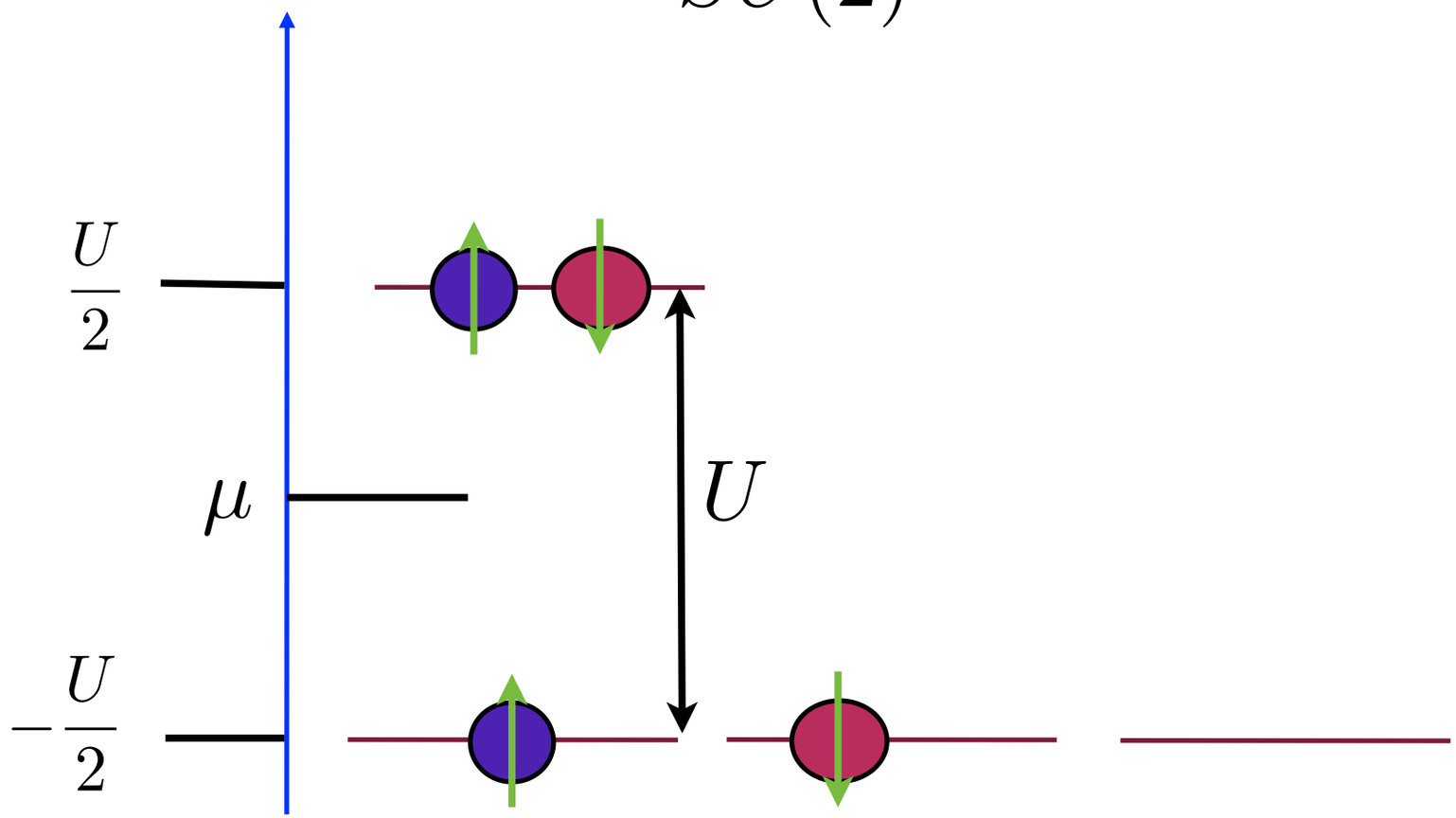
$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2}$$

$SU(2)$



$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$

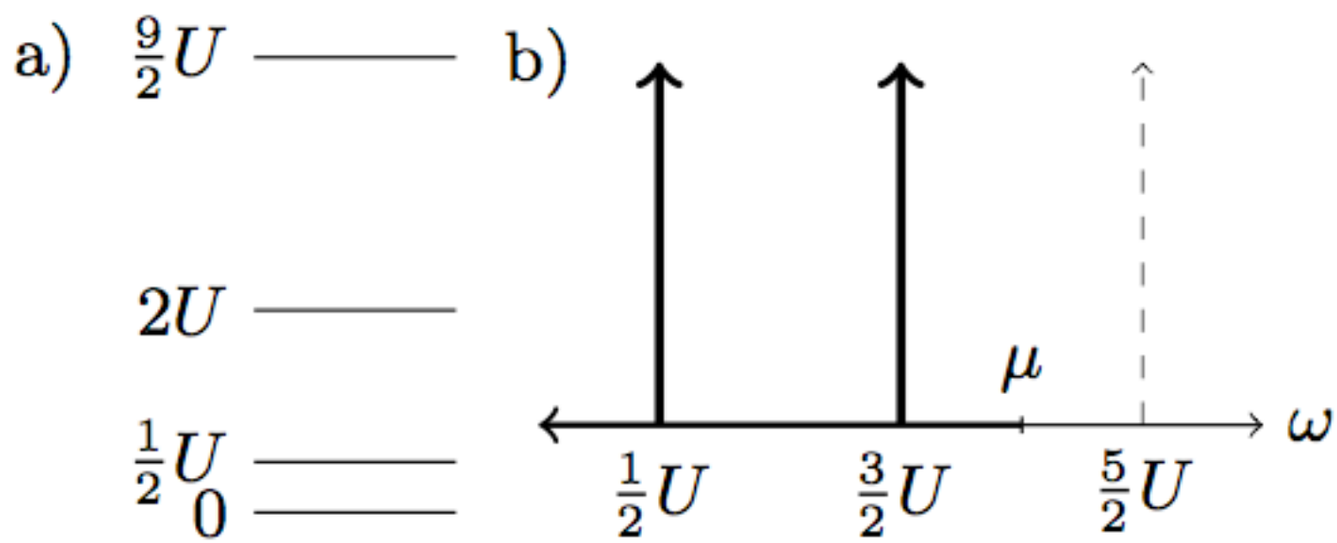
$SU(2)$



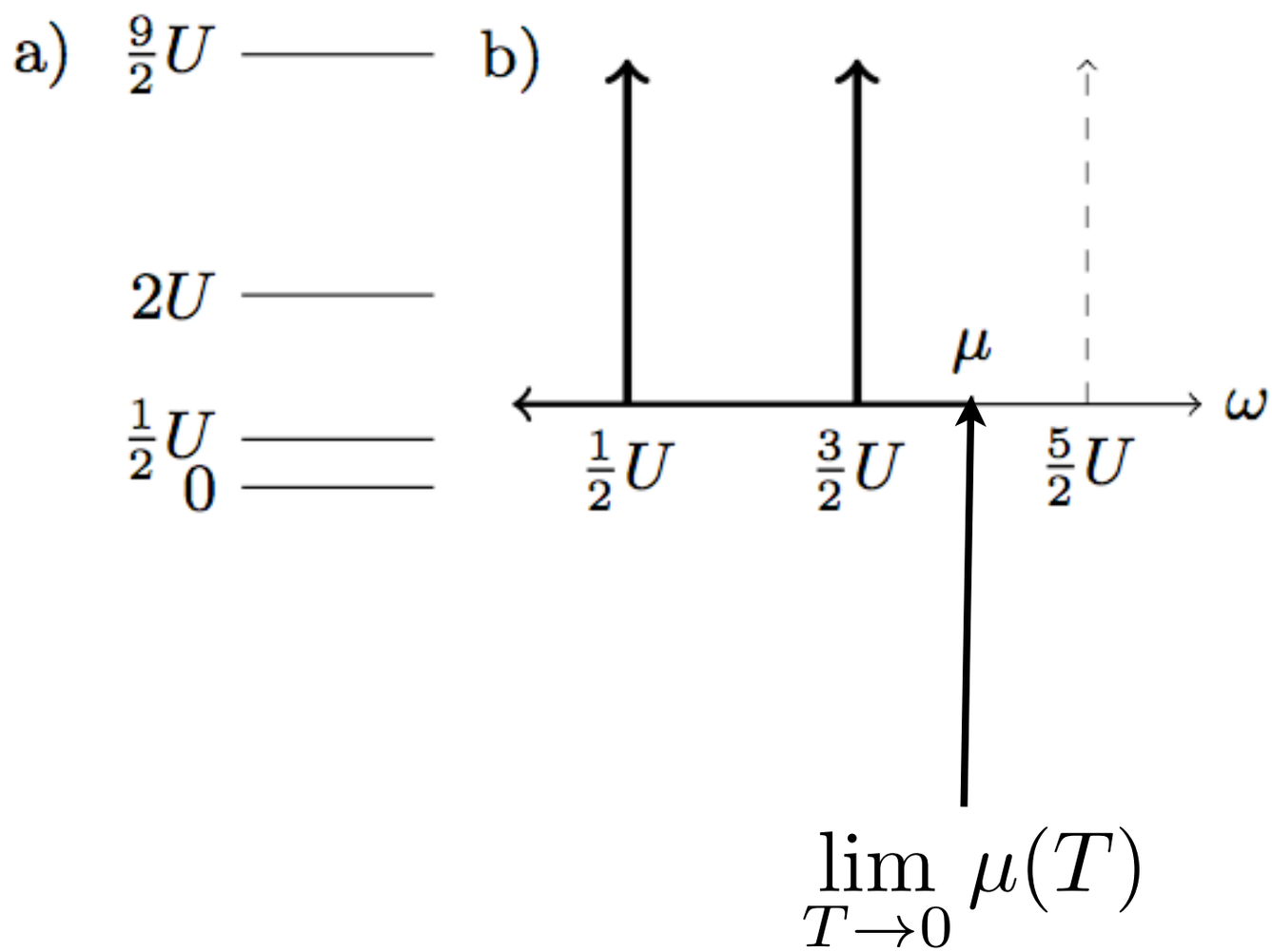
$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$

$$n = 2\theta(0) = 1$$

SU(N) for N=3: no particle-hole symmetry



SU(N) for N=3: no particle-hole symmetry



$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n+1) - K(n)} \left(\frac{2n - N}{N} \right)$$

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

< 0

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

< 0 $\qquad$ > 0

a zero must exist

< 0 $\qquad$ > 0

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

Luttinger's theorem

$$n = N\Theta(2n - N)$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

$n = 2$

$N = 3$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

$$n = 2$$

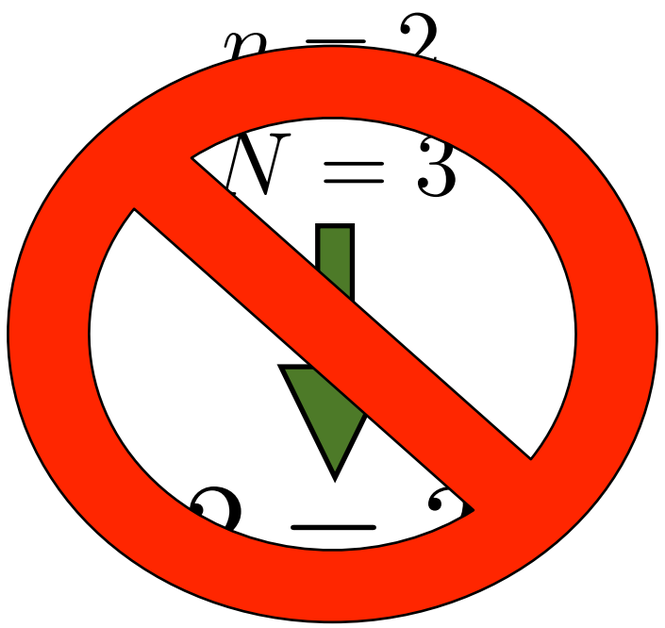
$$N = 3$$

$$\Downarrow$$

$$2 = 3$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$



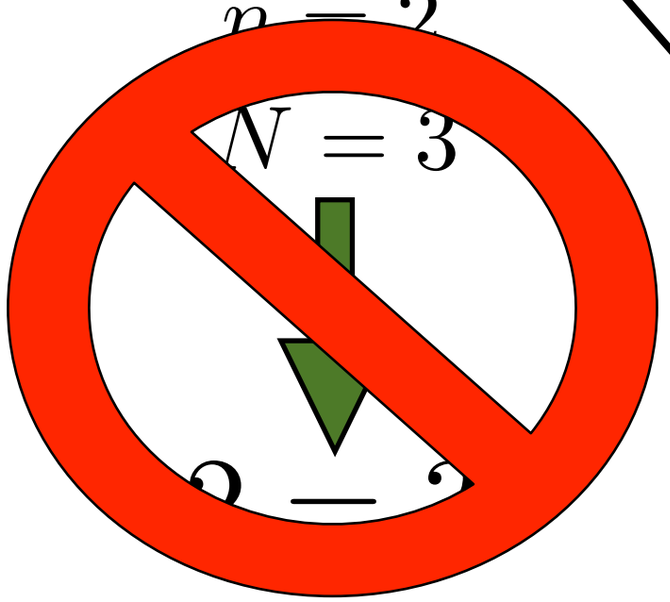
Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

even

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

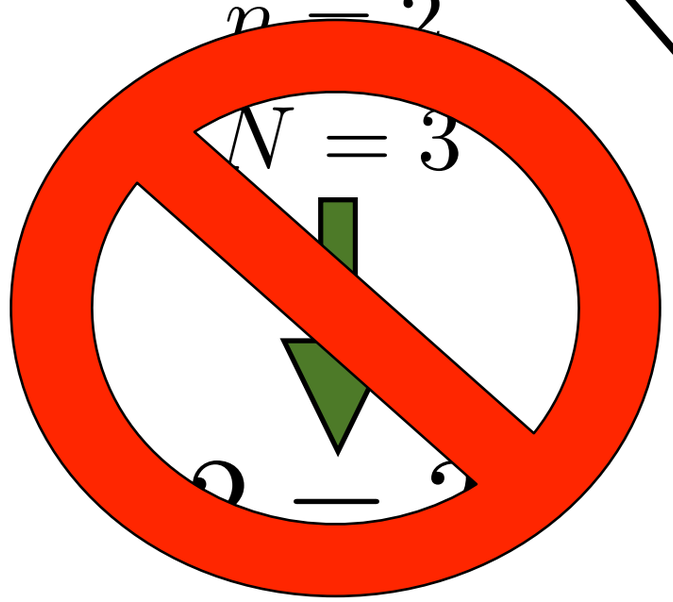


even

odd

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

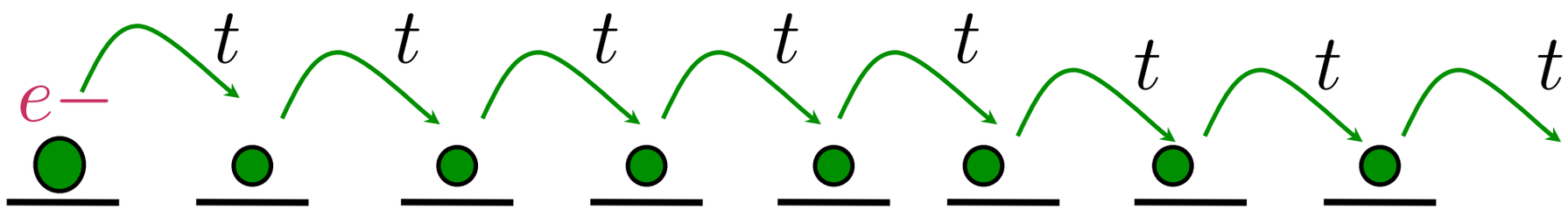


even

odd

no solution

does the degeneracy matter?



$$t = 0^+$$

No

No

as long as SU(N) symmetry is intact

$$\rho_{ab} = \mathrm{Tr}\left(c_a^\dagger c_b \rho(0^+)\right) = 1/N \mathrm{diag}(1,1,1,...)$$

mixed state

pure state

Luttinger's theorem

Problem

Problem

G=0

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p} = 0$$

??

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p} = 0$$

??

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n+1) - K(n)} \left(\frac{2n - N}{N} \right)$$

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p} = 0$$

??

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n+1) - K(n)} \left(\frac{2n - N}{N} \right)$$

must diverge

Problem

$$G = \frac{1}{E - \varepsilon_p - \Sigma}$$

$$\mathrm{Det} G = 0$$

Problem

$$G = \frac{1}{E - \varepsilon_p - \Sigma}$$

∞ self energy diverges

$$\mathrm{Det}G = 0$$

if Σ is infinite

if Σ is infinite

lifetime of a particle vanishes

if Σ is infinite

lifetime of a particle vanishes

$\Im\Sigma < \epsilon_p$

if Σ is infinite

lifetime of a particle vanishes

no particle

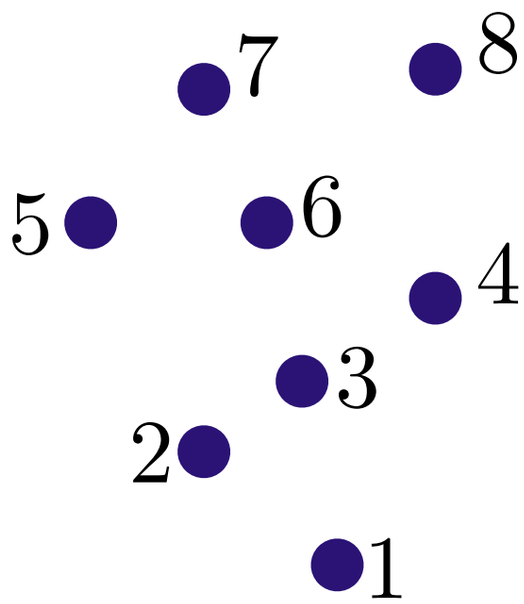
$\Im\Sigma < \epsilon_p$

zeros and particles are incompatible

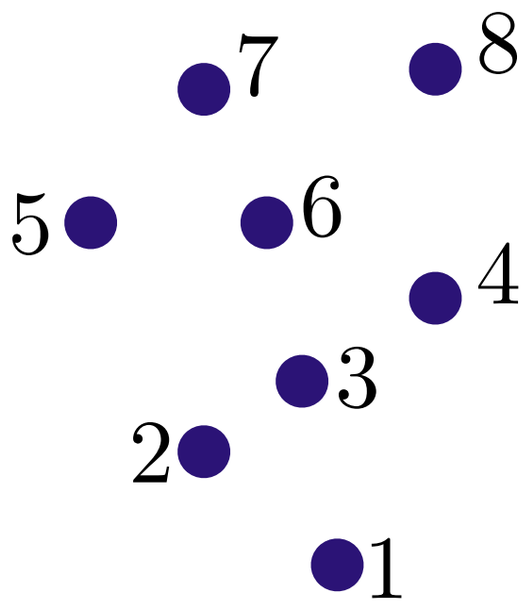
zeros and particles are incompatible

Luttinger theorem is wrong!

how to count particles?

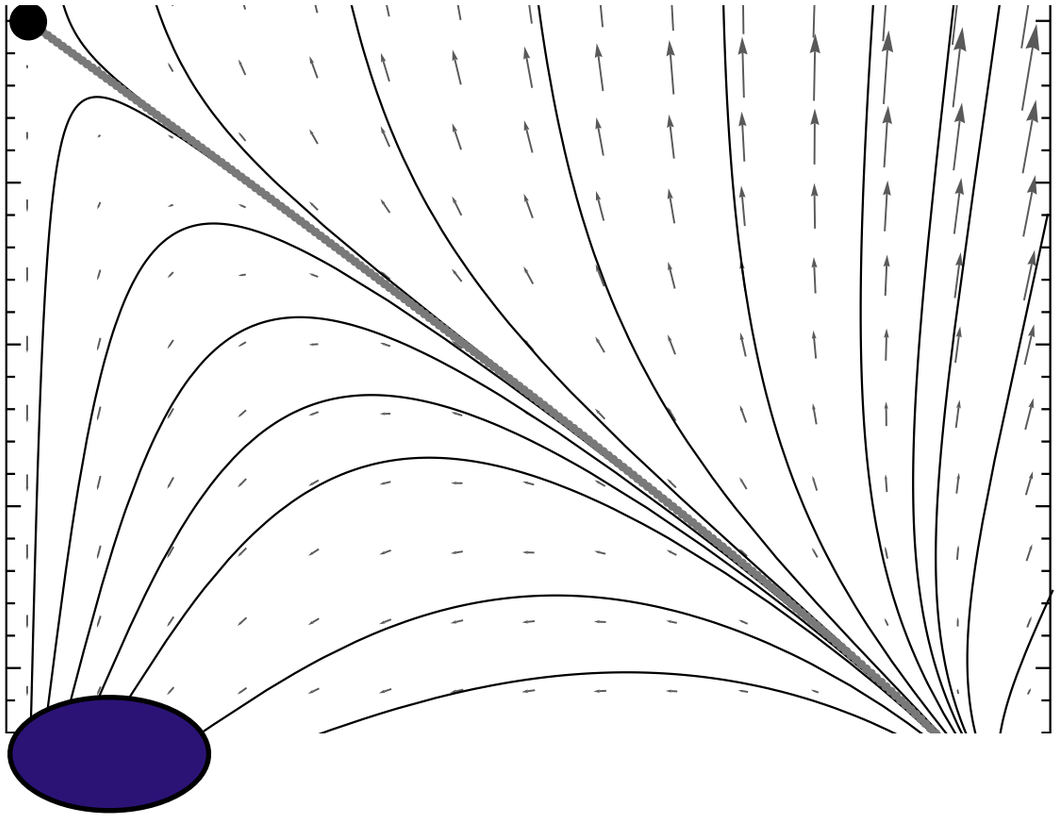


how to count particles?

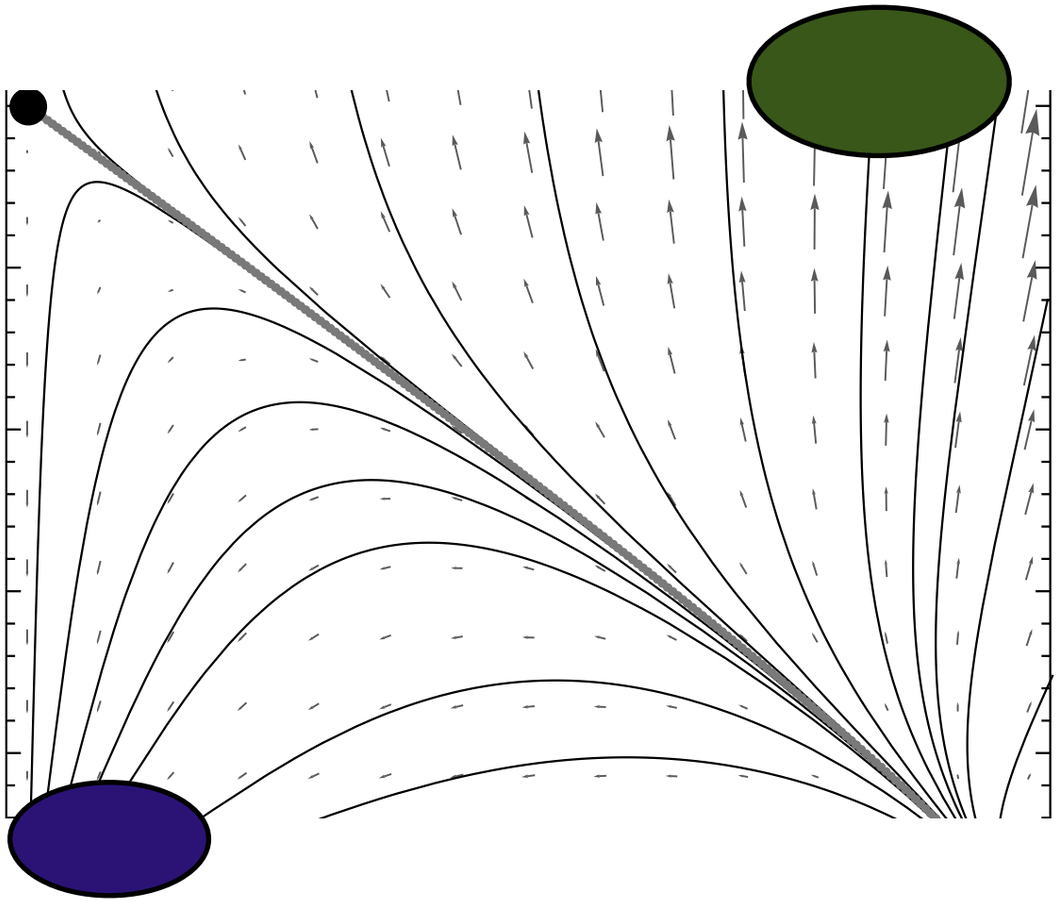


some charged stuff
has no particle interpretation

what is the extra stuff?



Fermi liquids
(particles/electrons)



Fermi liquids
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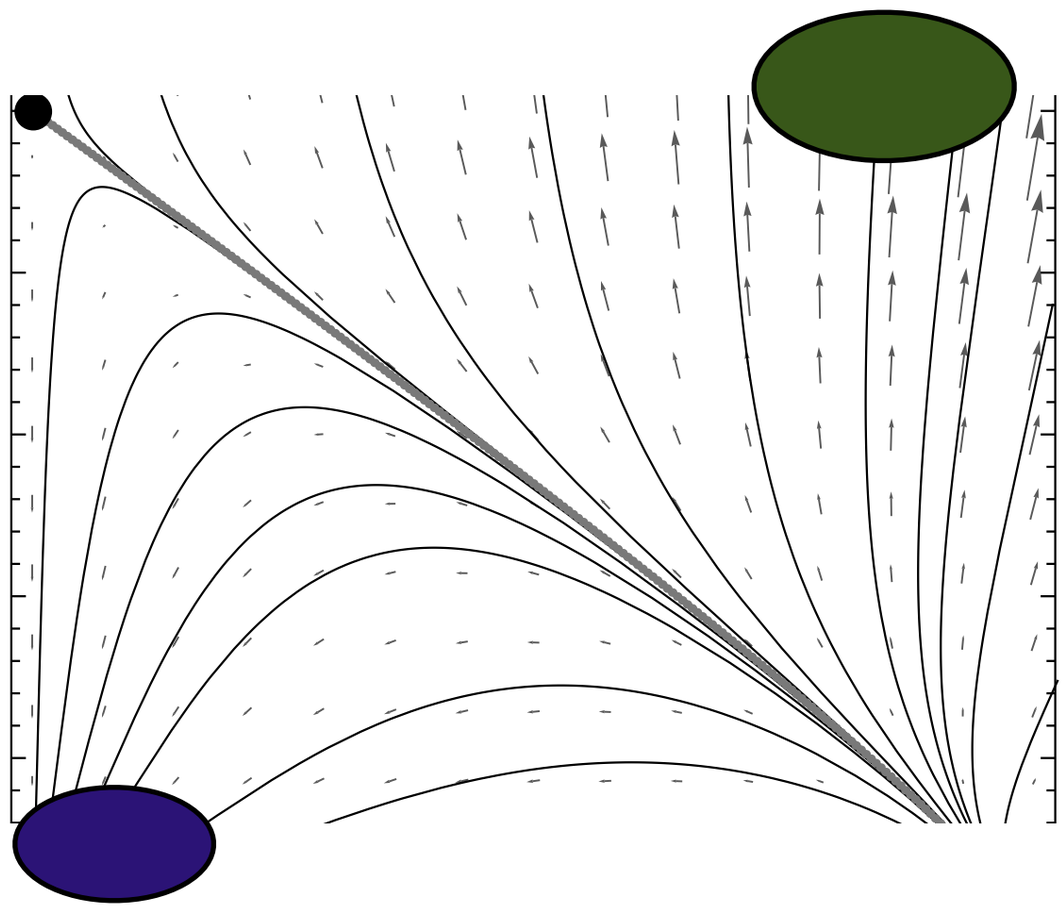
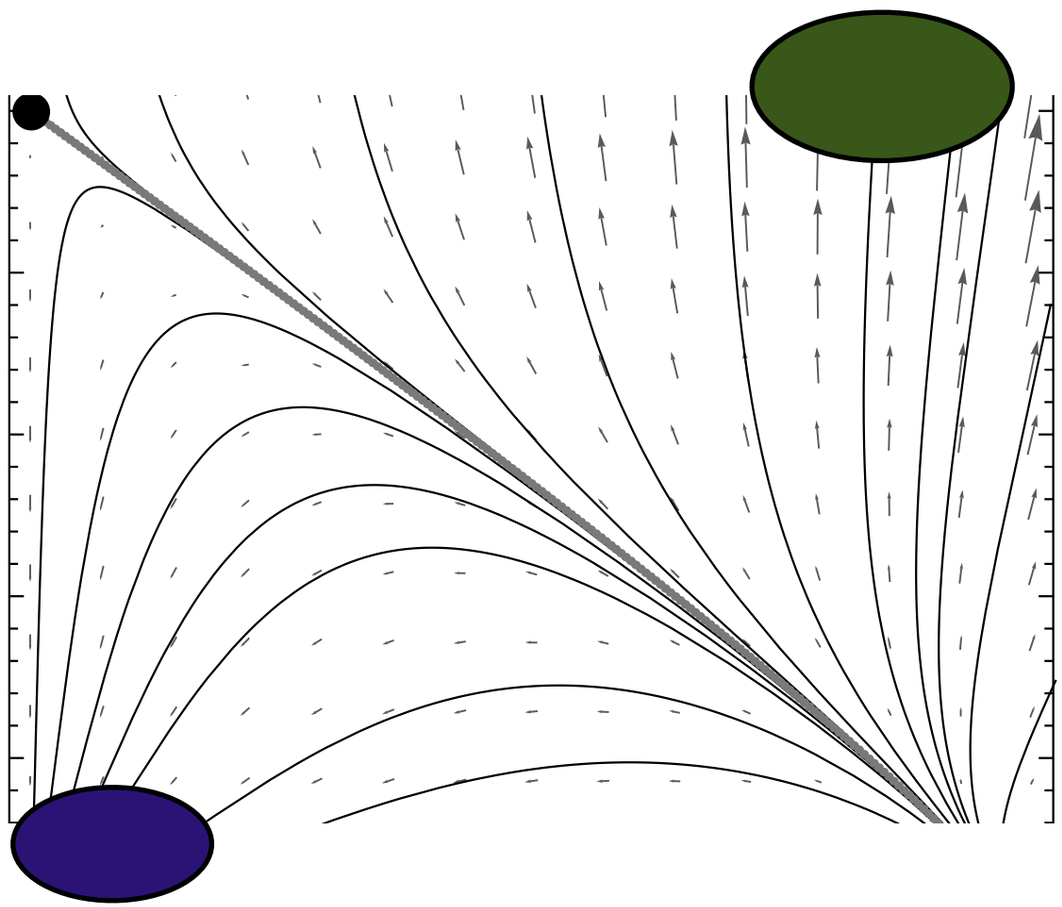


new fixed point
(?-particle stuff)

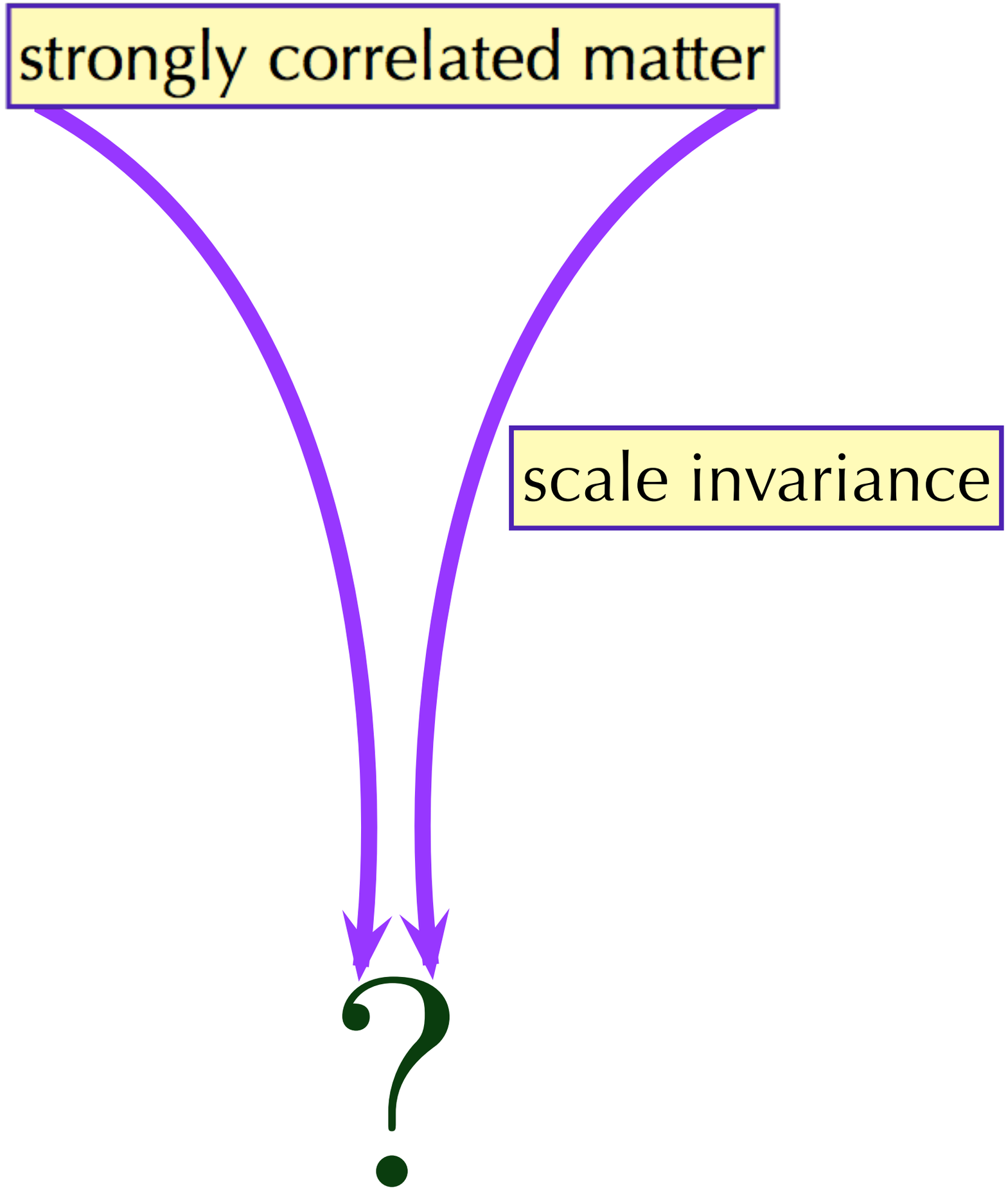
Fermi liquids
(particles/electrons)

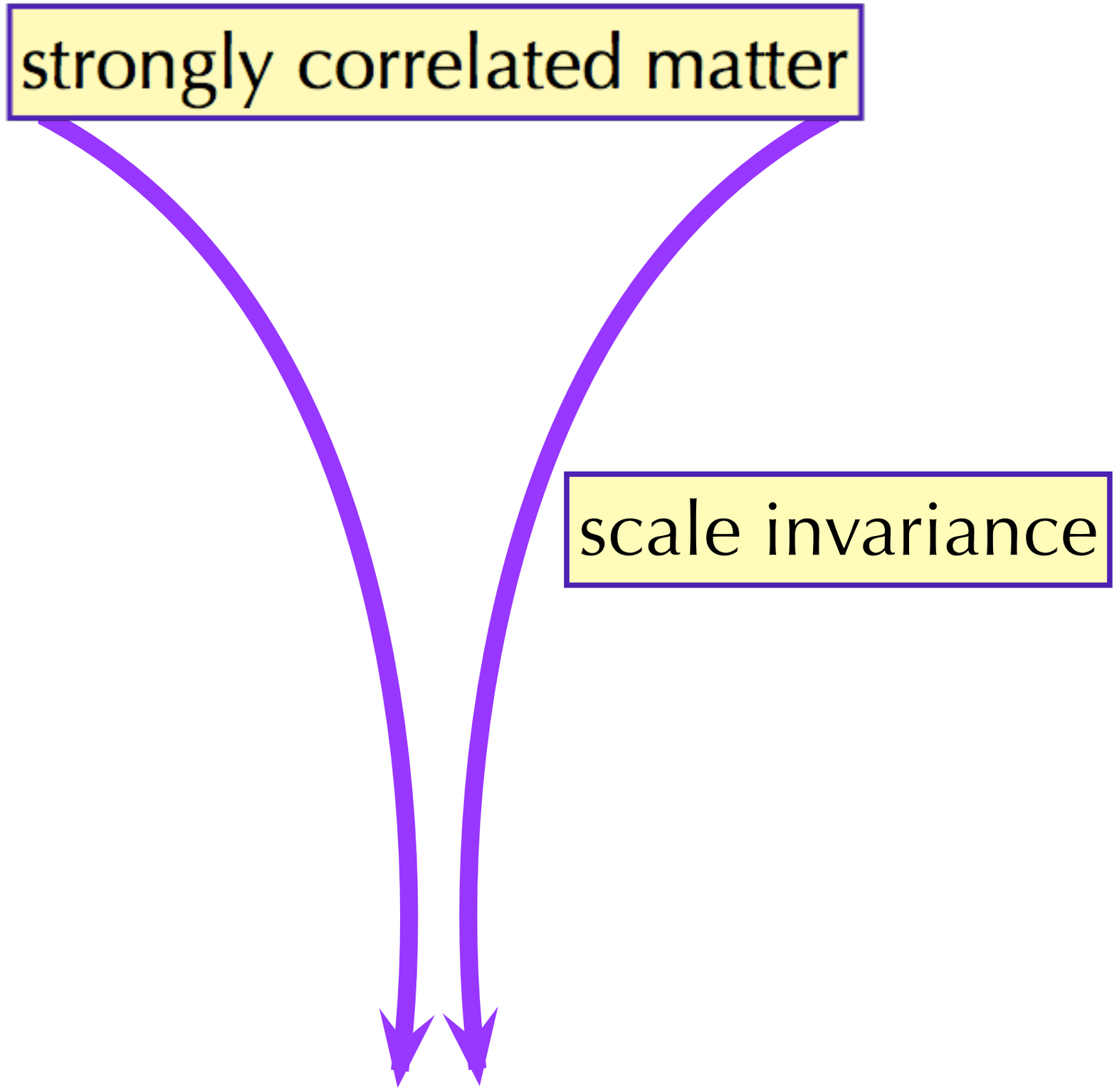
scale invariance

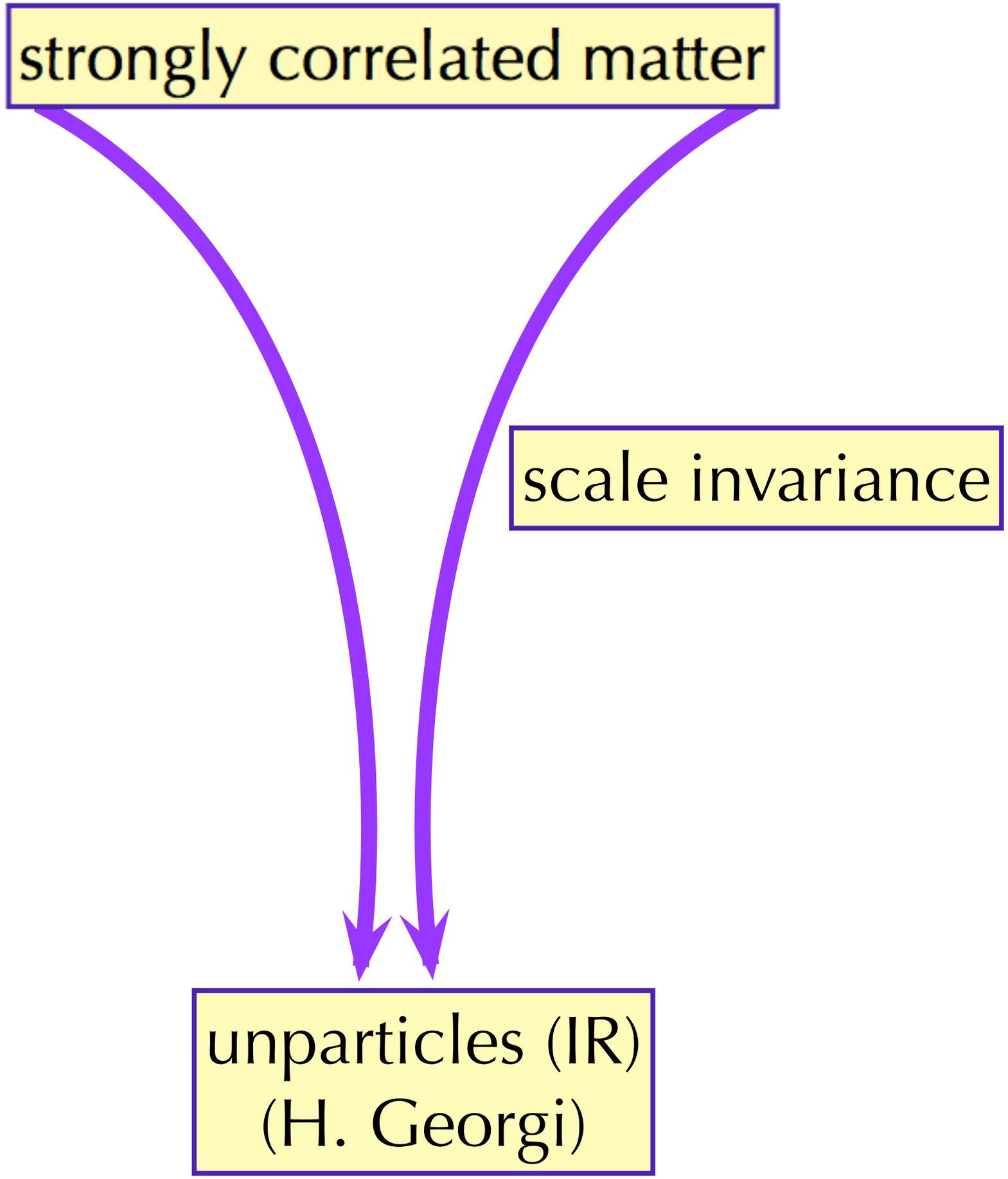


new fixed point
(?-particle stuff)

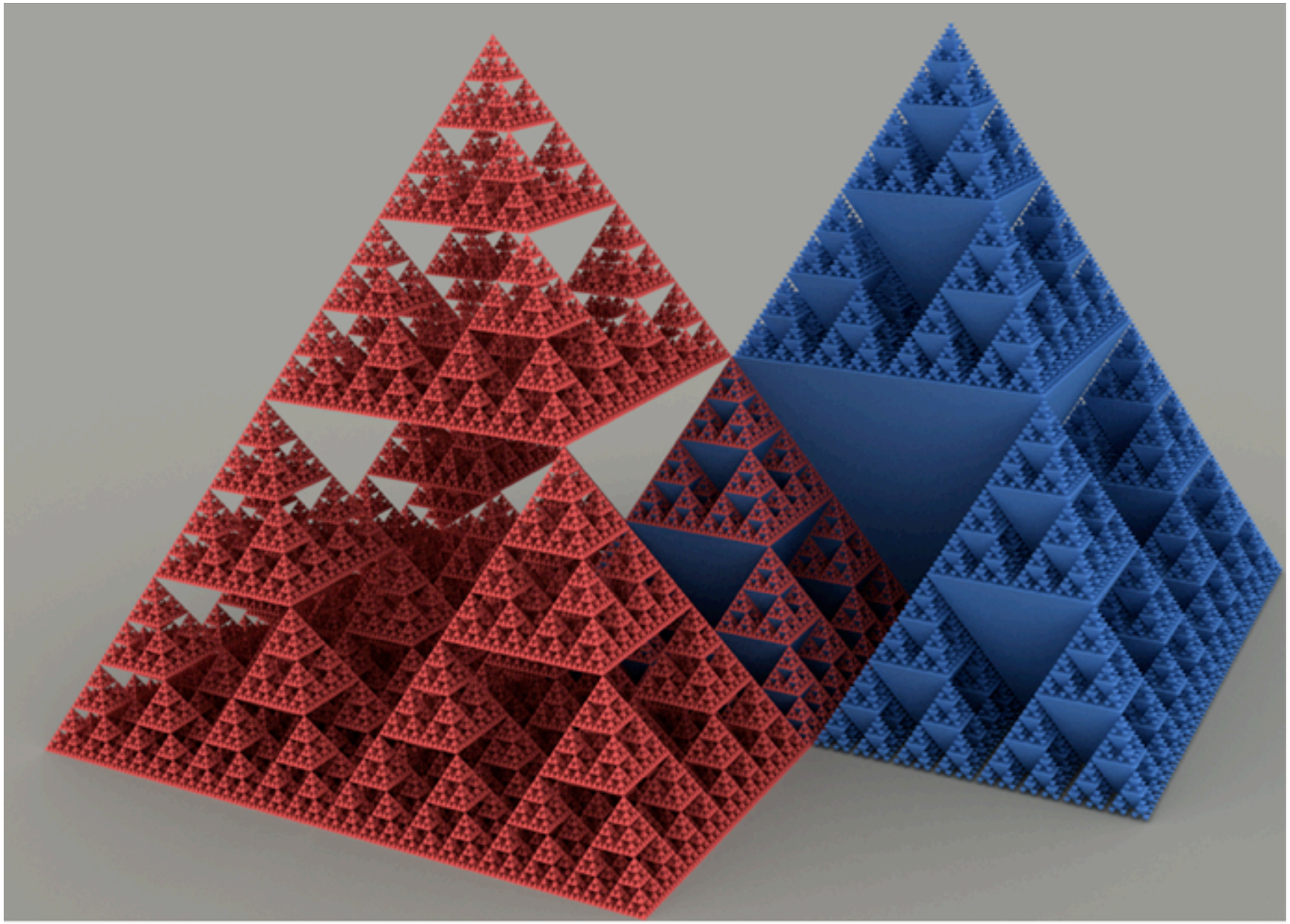
Fermi liquids
(particles/electrons)


strongly correlated matter
scale invariance
?


strongly correlated matter
scale invariance

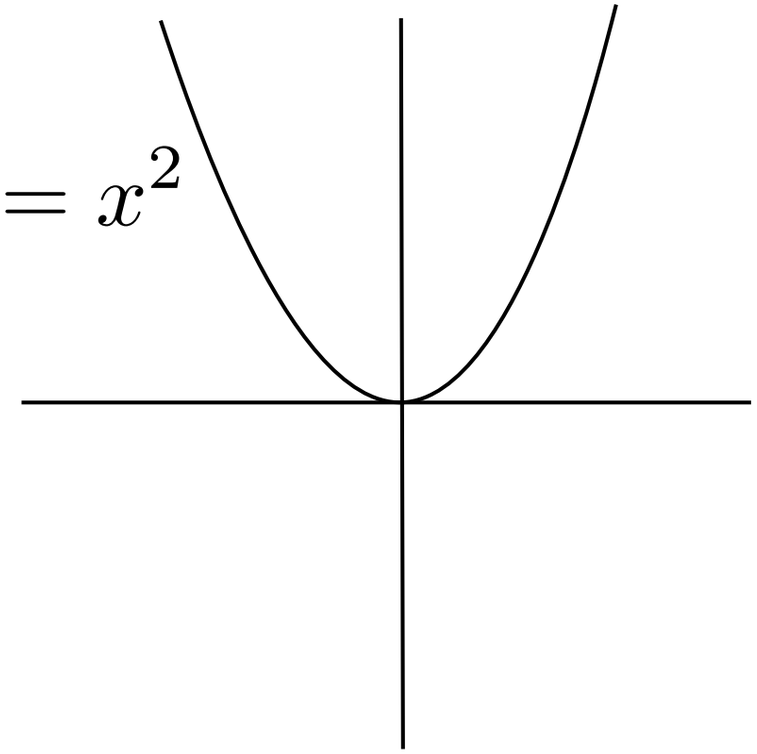

strongly correlated matter
scale invariance
unparticles (IR)
(H. Georgi)

what is scale invariance?

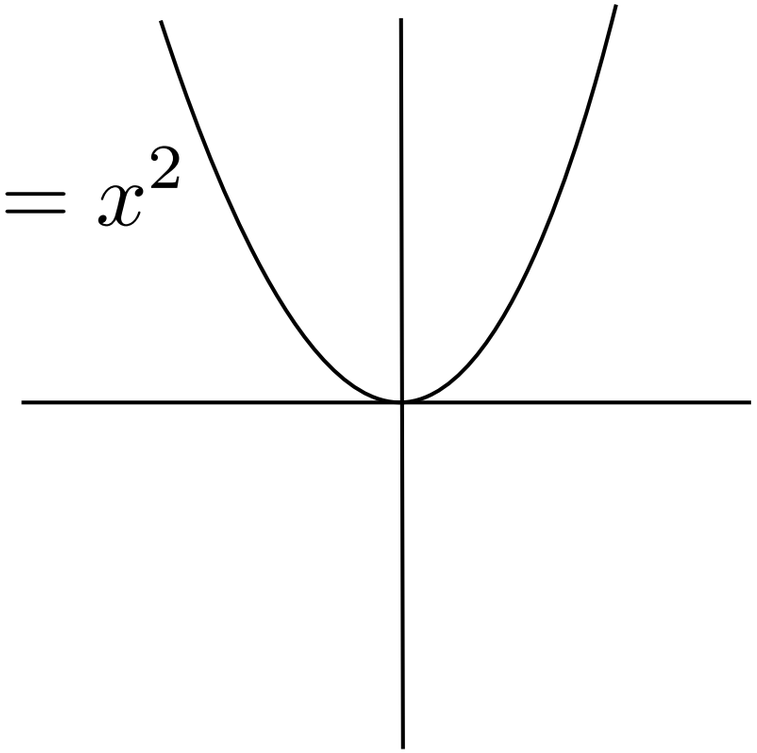


invariance on all length scales

$$f(x) = x^2$$



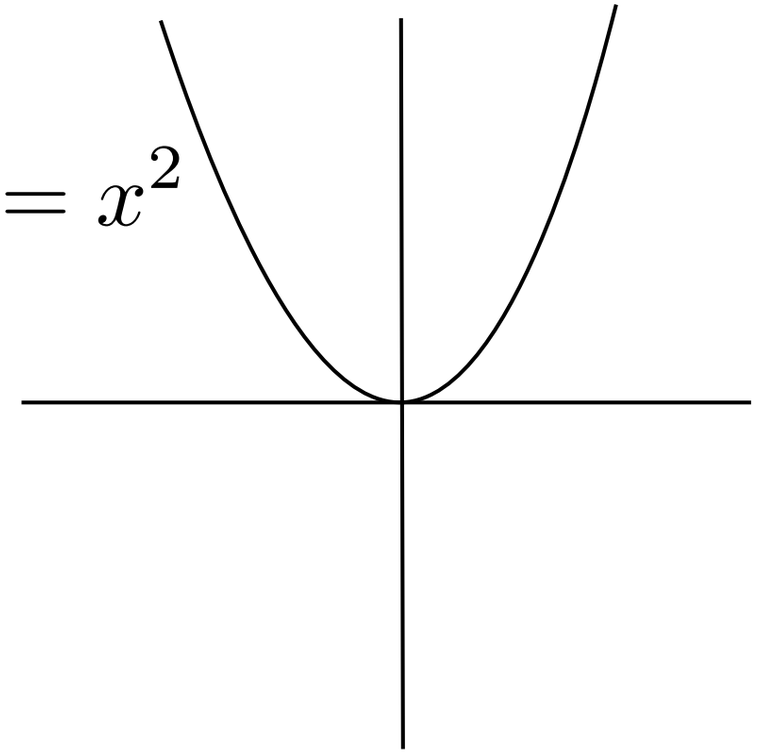
$$f(x) = x^2$$



$$f(x/\lambda) = (x/\lambda)^2$$

scale change

$$f(x) = x^2$$

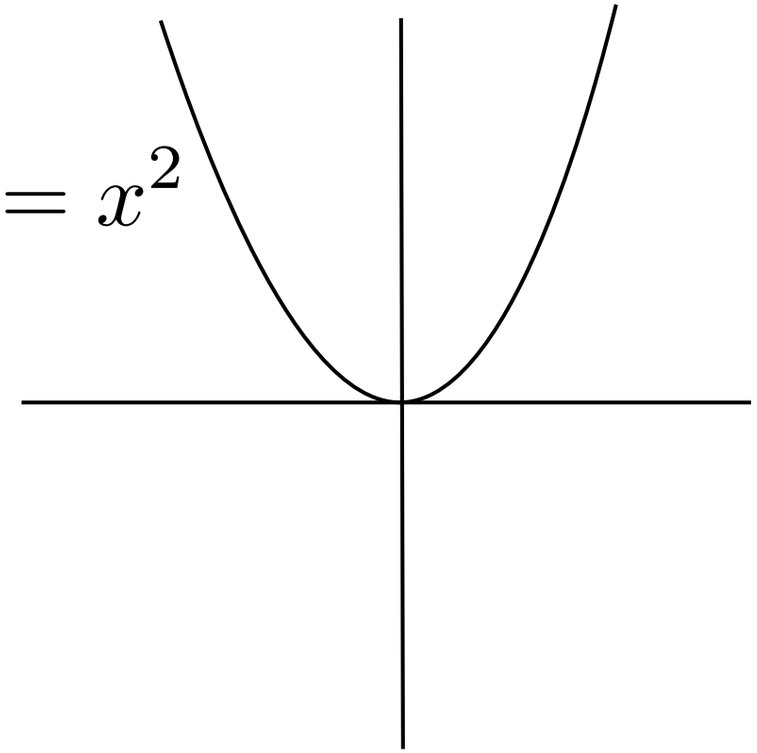


$$f(x/\lambda) = (x/\lambda)^2$$

scale change

scale invariance

$$f(x) = x^2$$



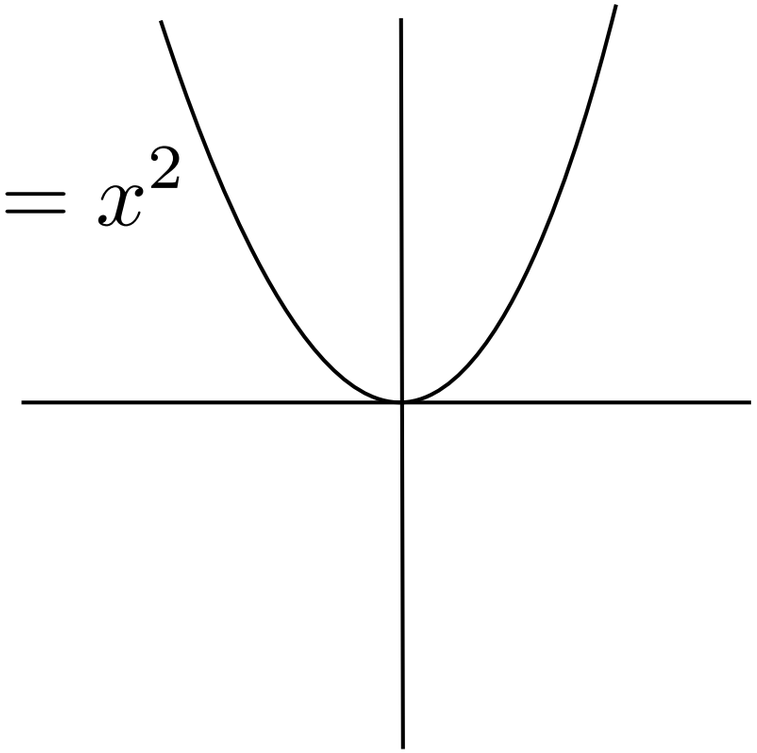
$$f(x/\lambda) = (x/\lambda)^2$$

scale change

scale invariance

$$f(x) = x^2 \underbrace{\lambda^{-2} g(\lambda)}_{\longrightarrow\ 1}$$

$$f(x) = x^2$$



$$f(x/\lambda) = (x/\lambda)^2$$

scale change

scale invariance

$$f(x) = x^2 \underbrace{\lambda^{-2} g(\lambda)}_{\xrightarrow{\;g(\lambda) = \lambda^2\;}\; 1}$$

$$f(x) = ax^2 + bx^3$$

$$x \to x/\Lambda$$

$$f(x) = ax^2 + bx^3$$

$$x \to x/\Lambda$$

$$f(x) \to \Lambda^{-2}(ax^2 + bx/\Lambda)$$

$$f(x) = ax^2 + bx^3$$

$$x \to x/\Lambda$$

$$f(x) \to \Lambda^{-2}(ax^2 + bx/\Lambda)$$

not scale invariant

free field theory

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi$$

$$\Big\downarrow \quad \begin{array}{l} x \to x/\Lambda \\ \phi(x) \to \phi(x) \end{array}$$

$$\mathcal{L} \to \Lambda^2 \mathcal{L}$$

free field theory

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi$$

$$\downarrow \quad x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

$$\mathcal{L} \to \Lambda^2 \mathcal{L}$$ scale invariant

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi + m^2 \phi^2$$

massive free theory

mass

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$$x \to x/\Lambda$$
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massive free theory

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$$m^2\phi^2$$

massive free theory

mass

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$$x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

$$\Lambda^2\left(\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\right)$$

$$m^2\phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

$$\Lambda^2\left(\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\right)$$

$$m^2\phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

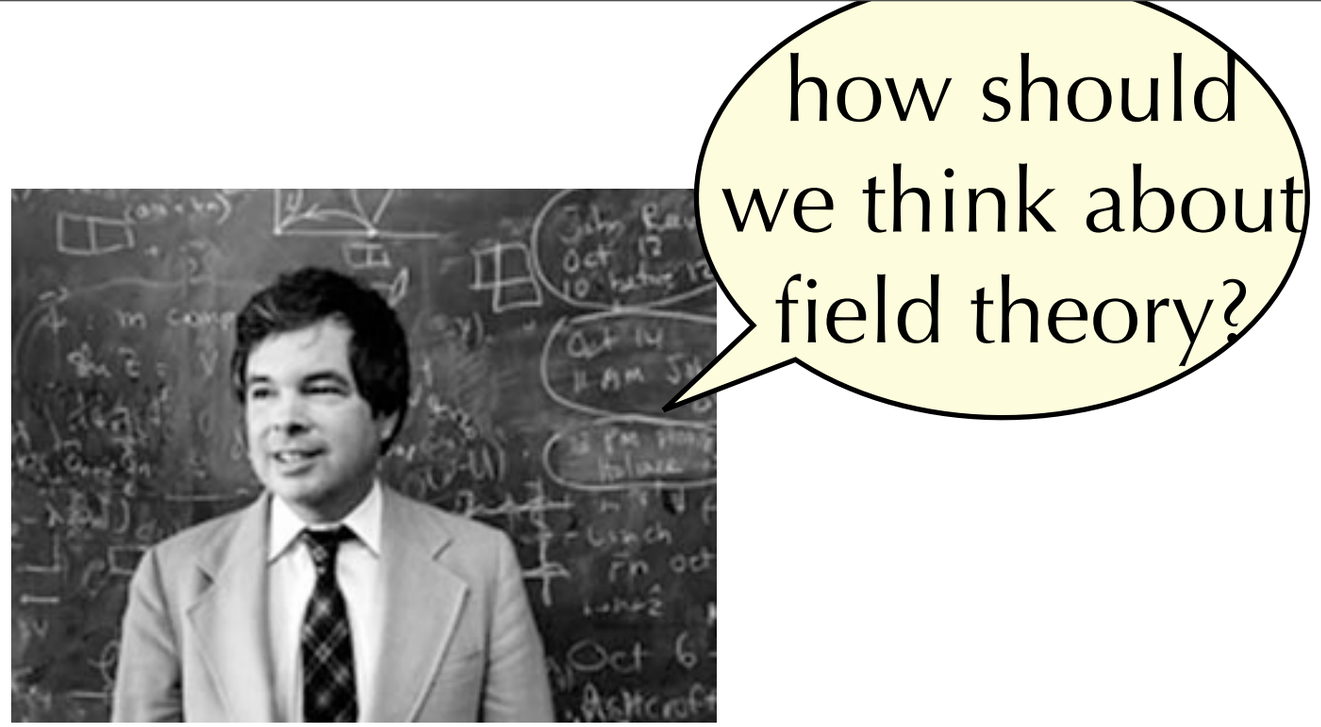
$$x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

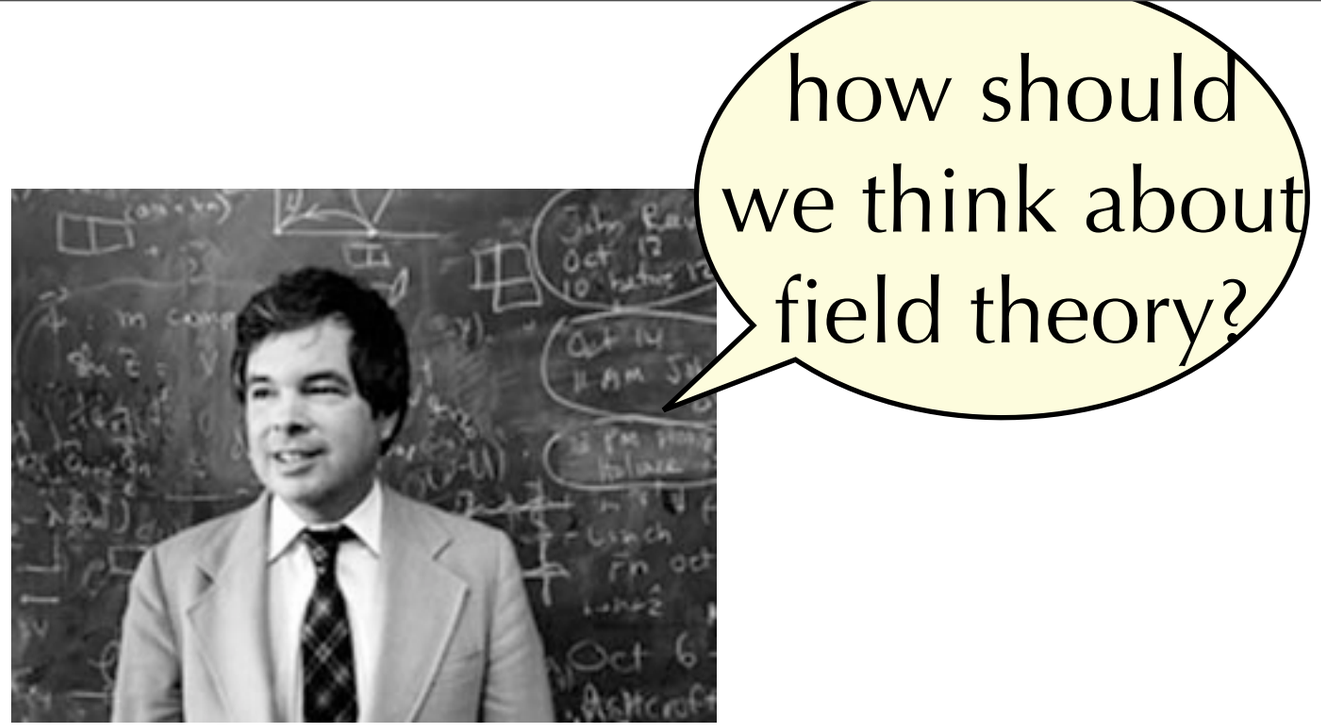
$$\Lambda^2\left(\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\right)$$

$$m^2\phi^2$$

no scale invariance

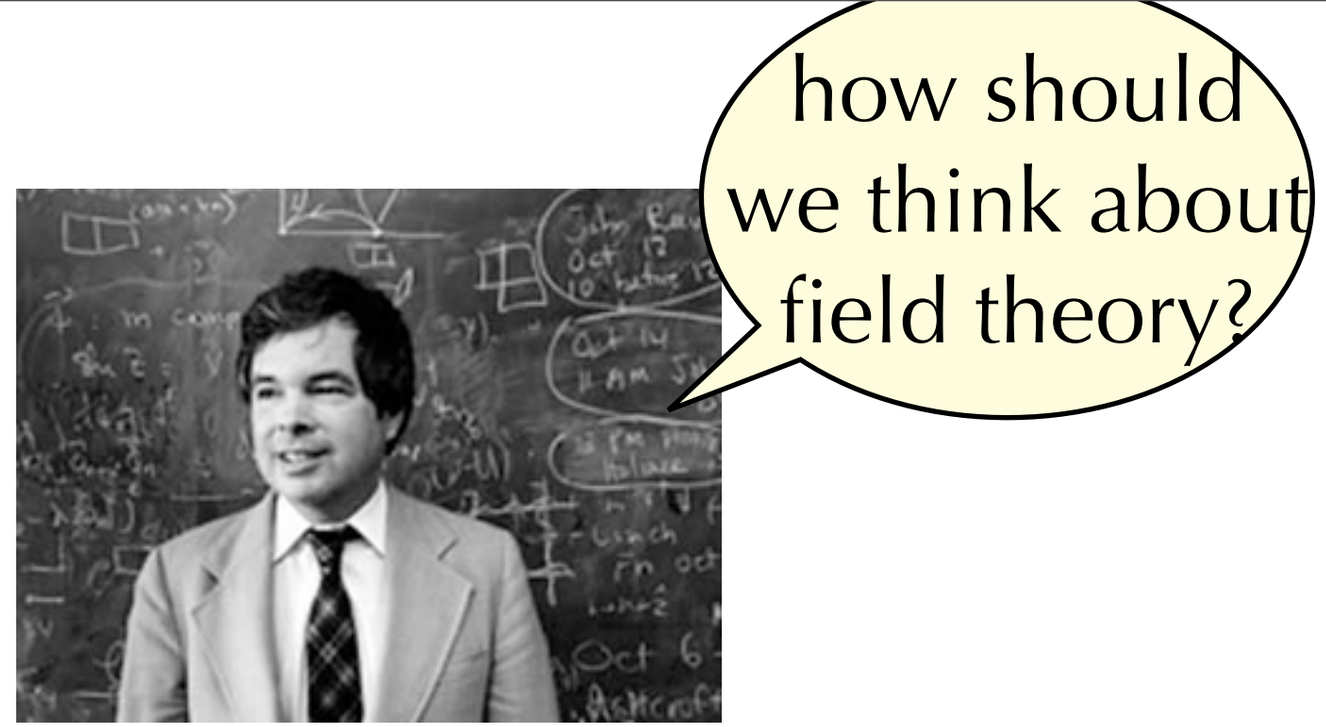


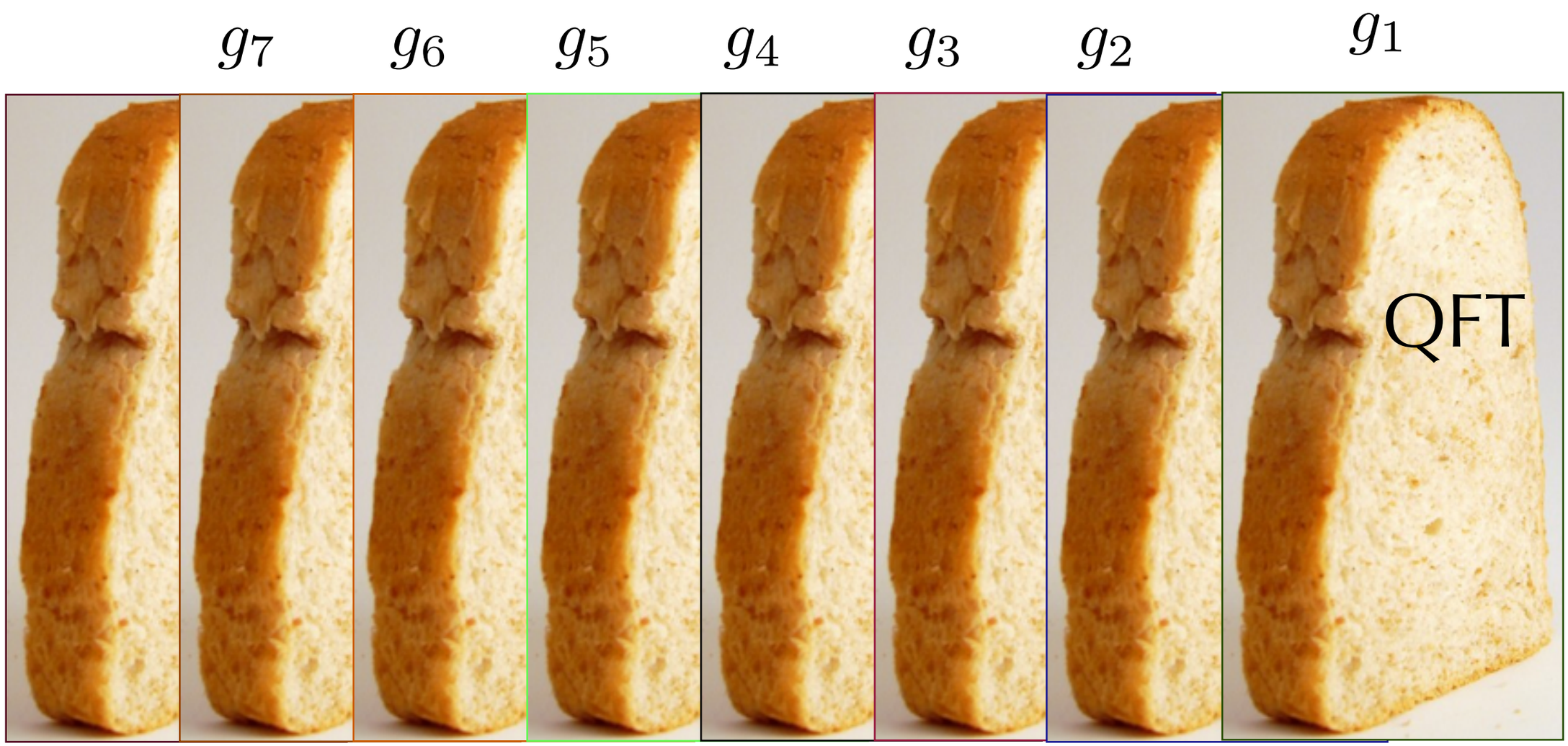
QFT

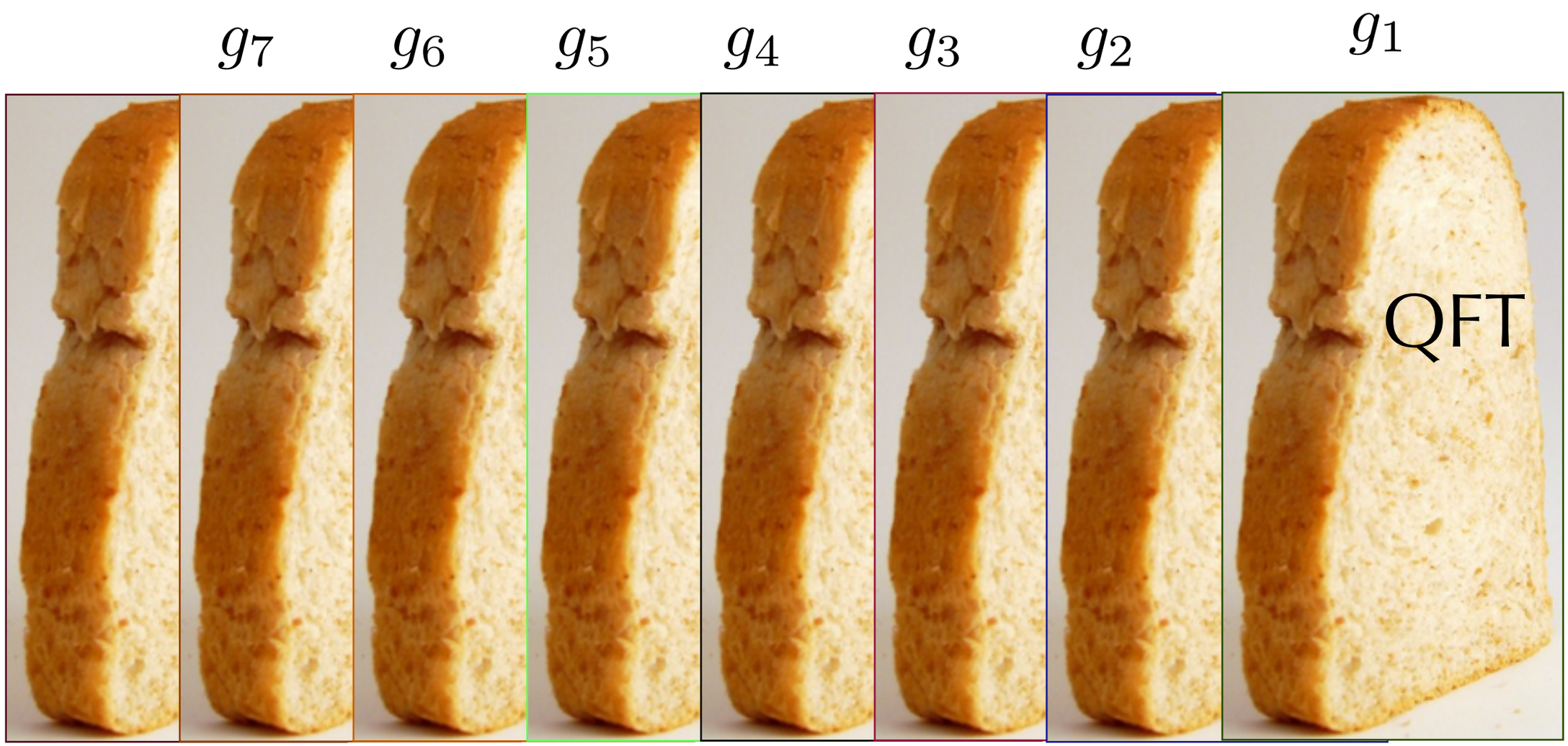


g_7 g_6 g_5 g_4 g_3 g_2 g_1

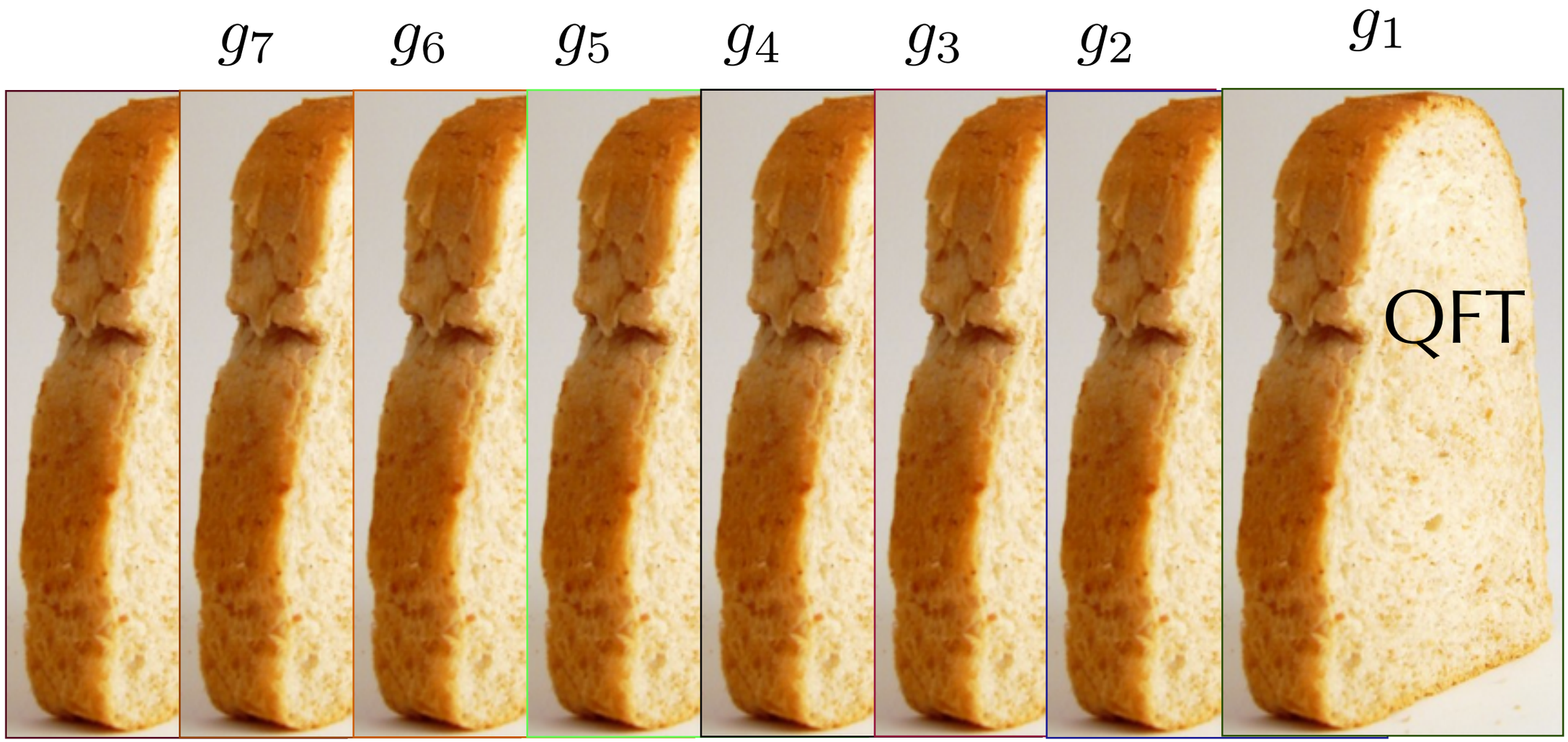
QFT

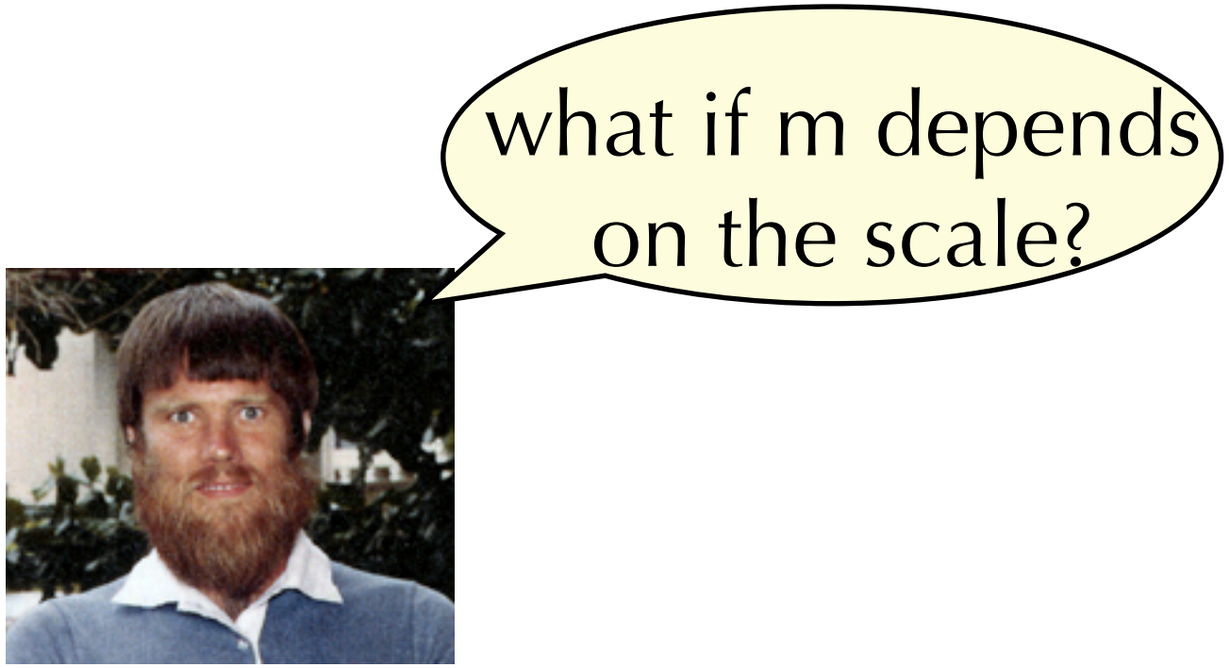

how should we think about field theory?

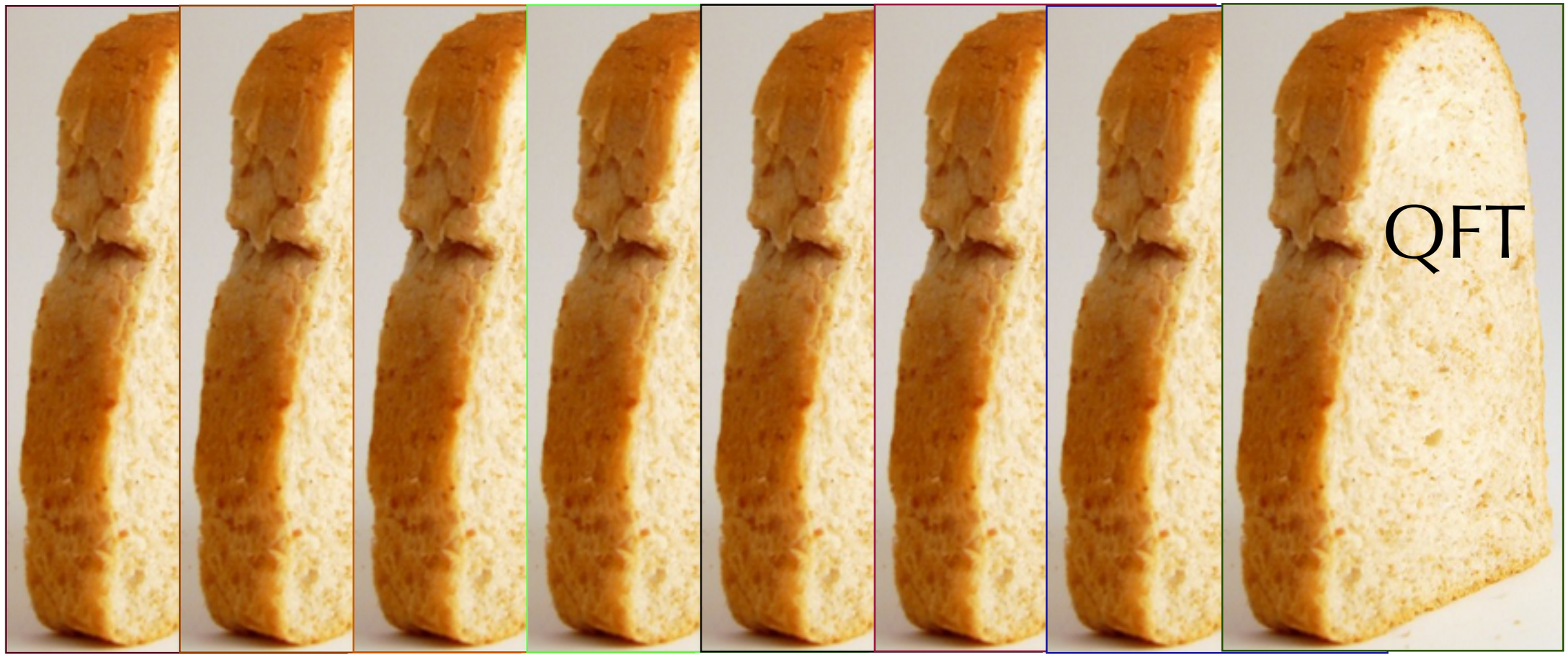

g_7
g_6
g_5
g_4
g_3
g_2
g_1
QFT

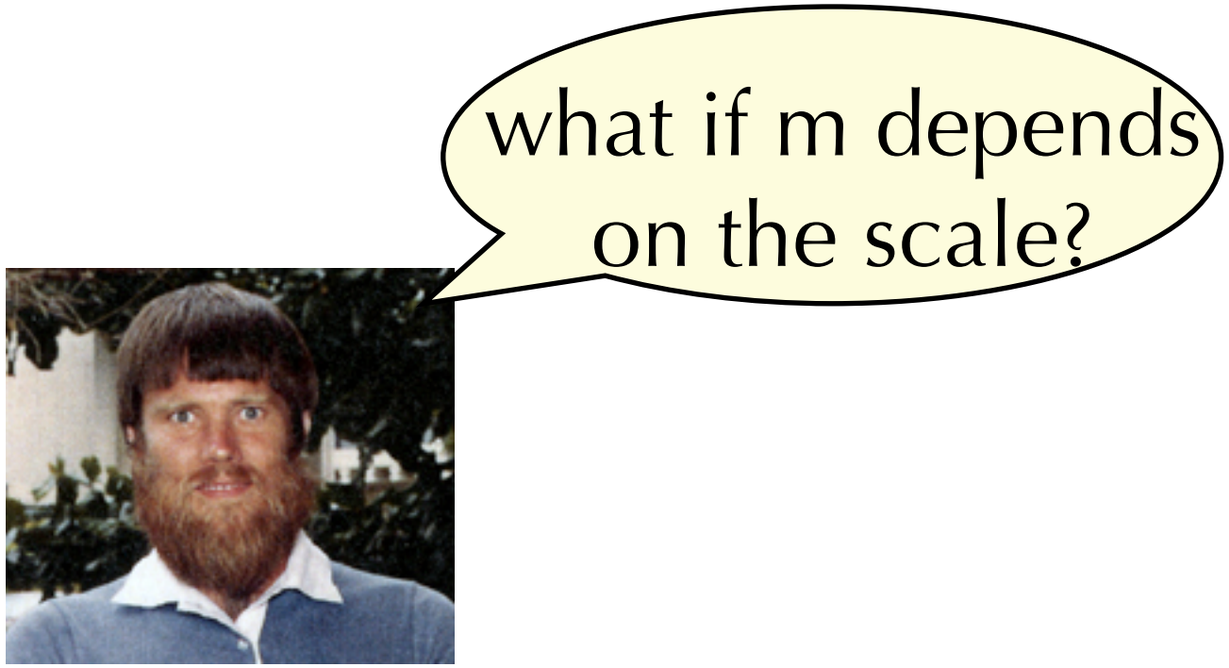

g_7
g_6
g_5
g_4
g_3
g_2
g_1
QFT

what if m depends on the scale?

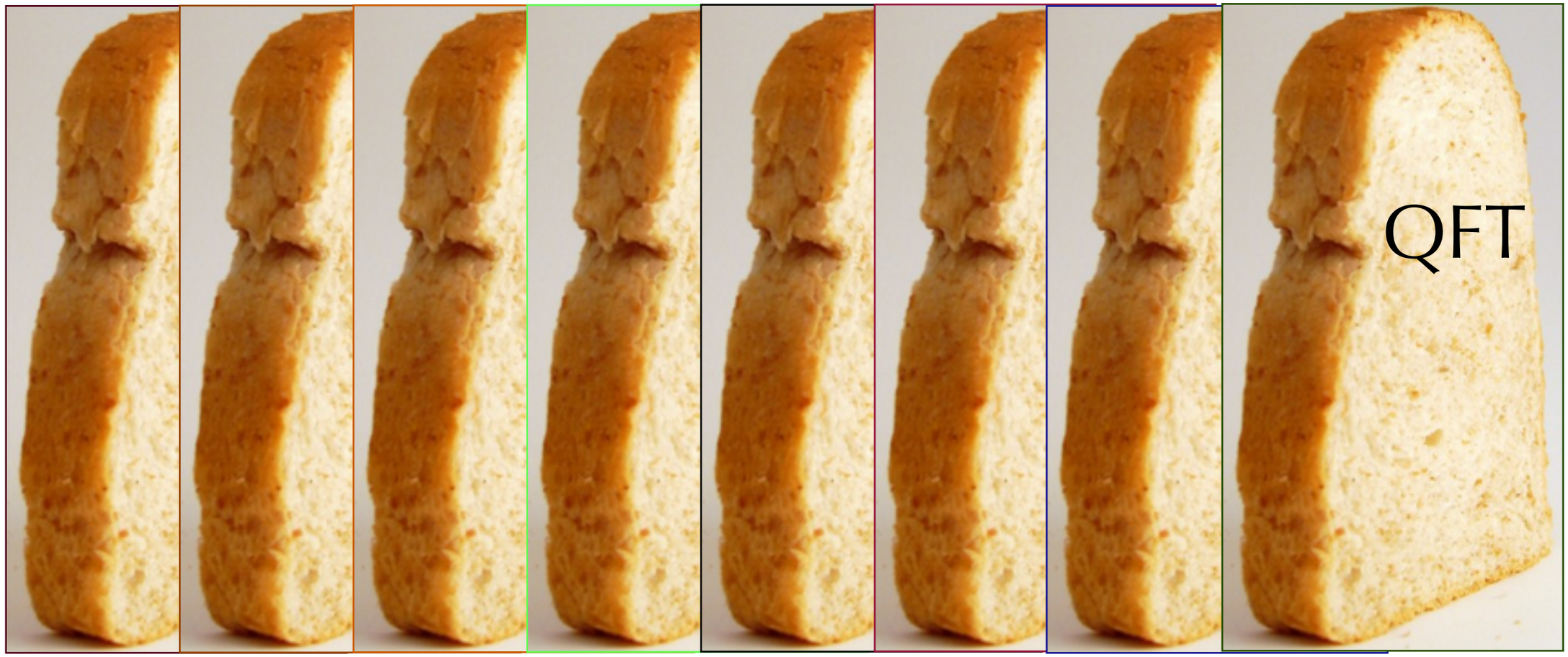



what if m depends on the scale?


QFT



$\mathcal{L}_{m_{i+7}}$ $\mathcal{L}_{m_{i+6}}$ $\mathcal{L}_{m_{i+5}}$ $\mathcal{L}_{m_{i+4}}$ $\mathcal{L}_{m_{i+3}}$ $\mathcal{L}_{m_{i+2}}$ $\mathcal{L}_{m_{i+1}}$ $\mathcal{L}_{m_i}$



sum over all mass

$$\mathcal{L} = \sum_i \mathcal{L}(m_i)$$

$$\mathcal{L} = \quad \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right)$$

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2 \phi^2(x,m)\right) dm^2$$

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theory with all possible mass!

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theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

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$$\mathcal{L} \to \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

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not particles

unparticles

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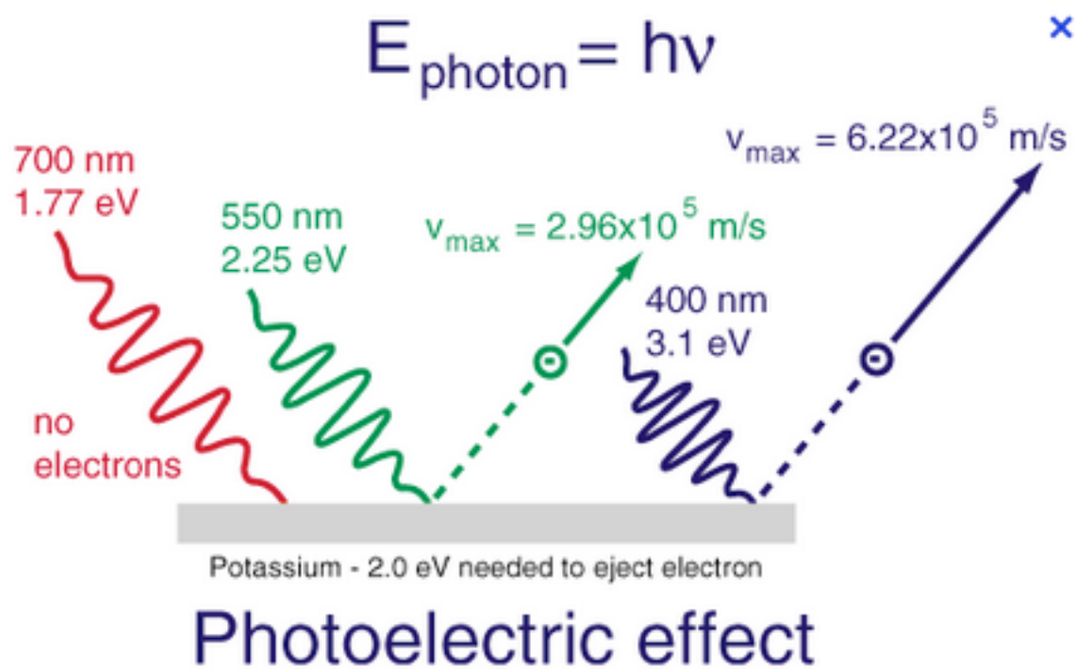
scale-invariant stuff is weird

particles



energy $E = \varepsilon_p$

Green function (propagator)

$$G(E) = \frac{1}{E - \varepsilon_p}$$

particle=infinity

scale-invariant stuff is weird

scale-invariant stuff is weird

unparticles

scale-invariant stuff is weird

unparticles

$$G \propto (E - \varepsilon_p)^{d_U - 2}$$

$d_U > 2$

unparticles=zero propagator

no particle interpretation

mass-distribution

$$\phi_U \neq \int B(m^2)\phi(m^2)dm^2$$

unparticle field
(non-canonical)

particle field

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

n massless particles

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unparticles=fractional number of massless particles

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unparticles=fractional number of massless particles

$d_U < 2$

almost Luttinger liquid
no pole at p_F

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$d_U < 2$

zeros

no simple sign change

almost Luttinger liquid

no pole at p_F

scale invariance

$$G(\Lambda p, \Lambda \omega) = \Lambda^{\alpha} G(p, \omega)$$

scale invariance

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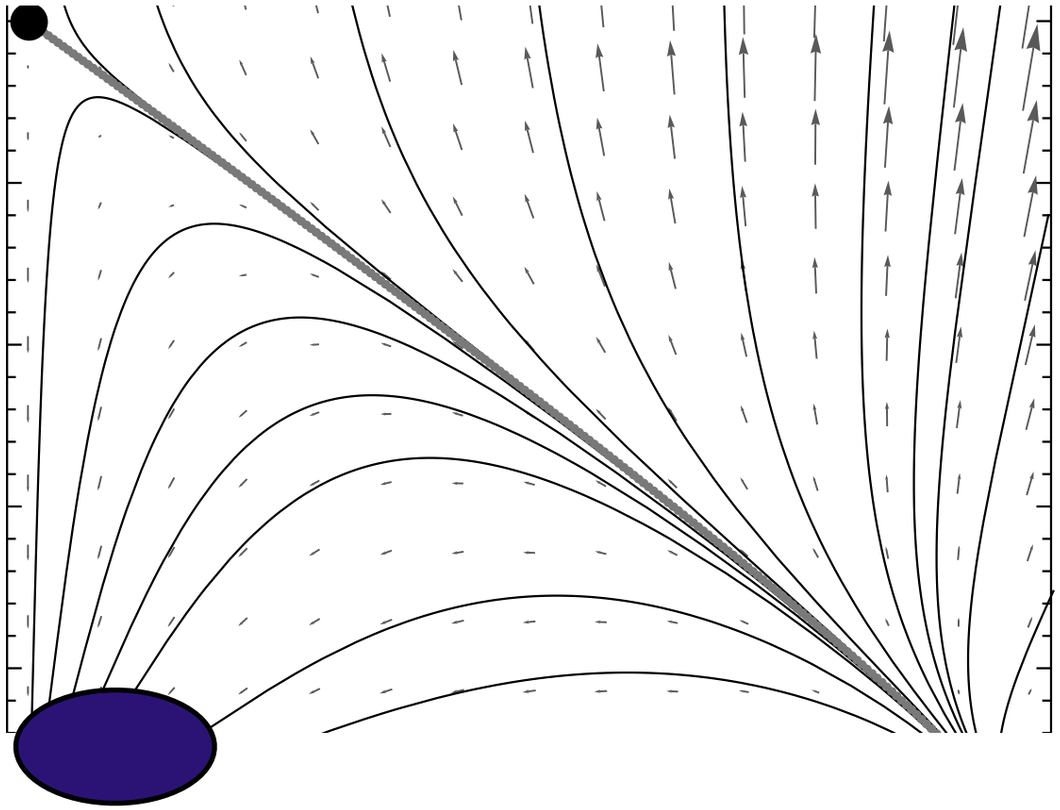

scale invariance

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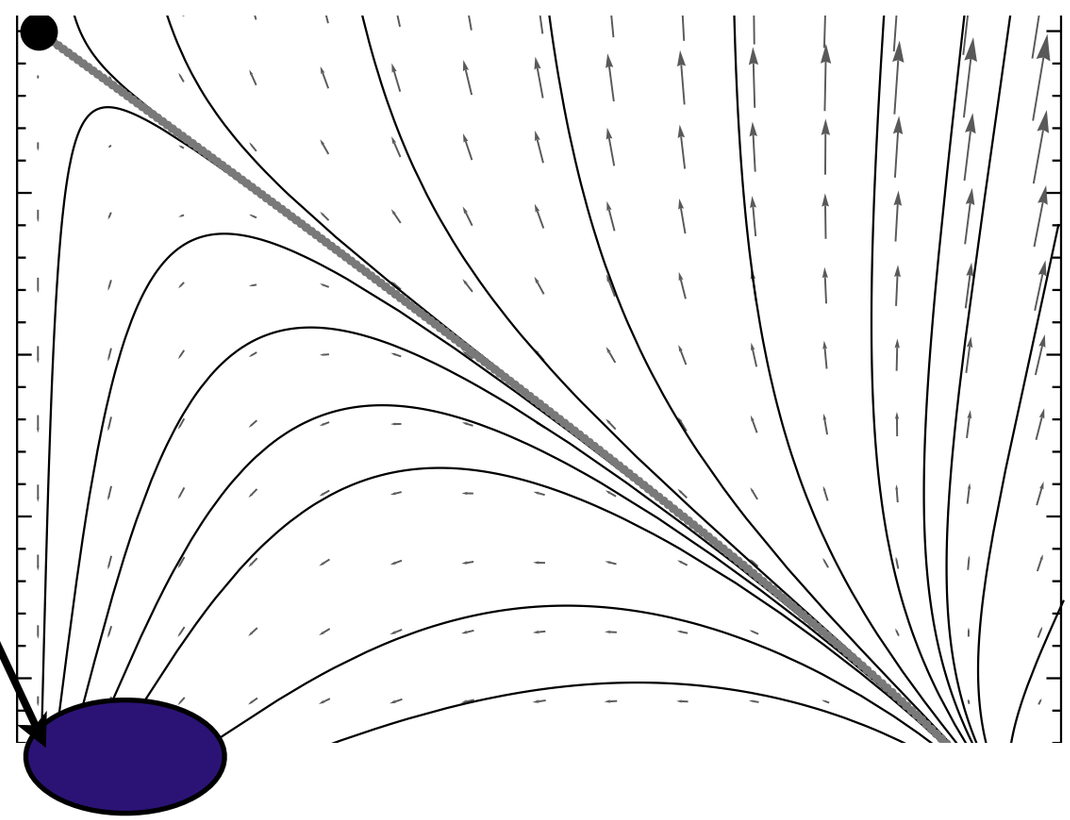
$\alpha = -1$

Fermi liquids



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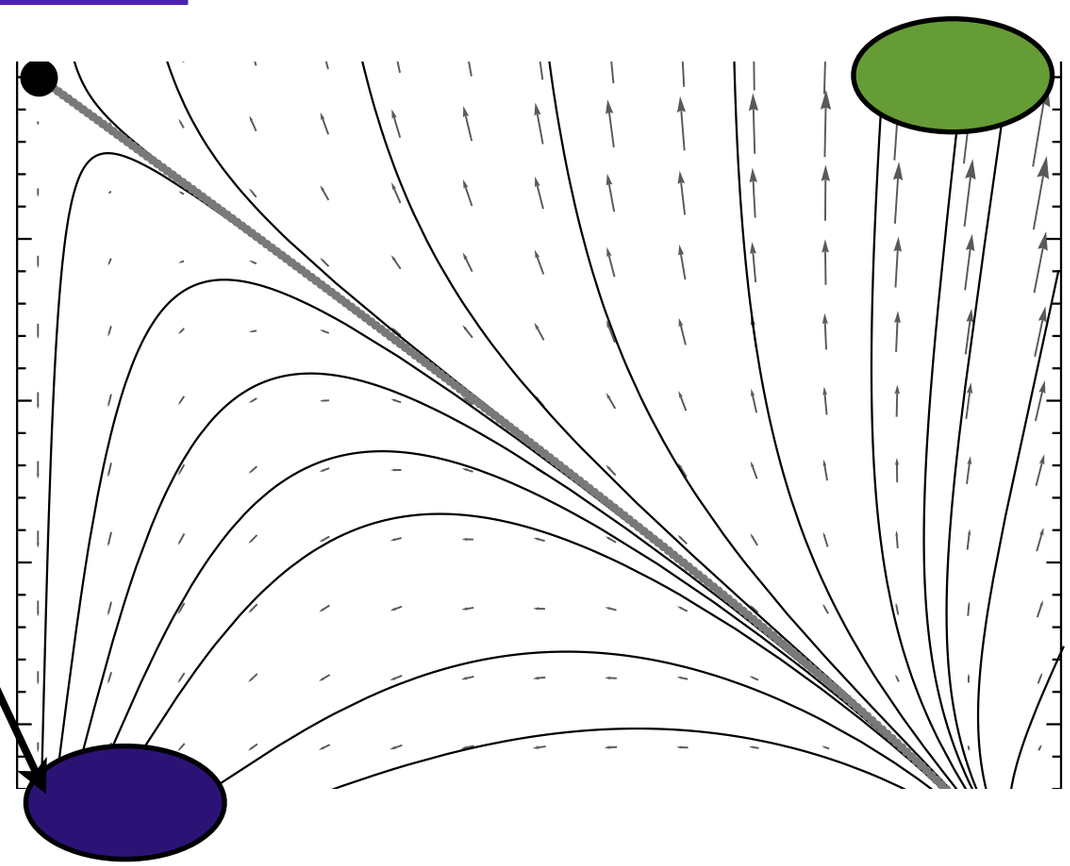


scale invariance

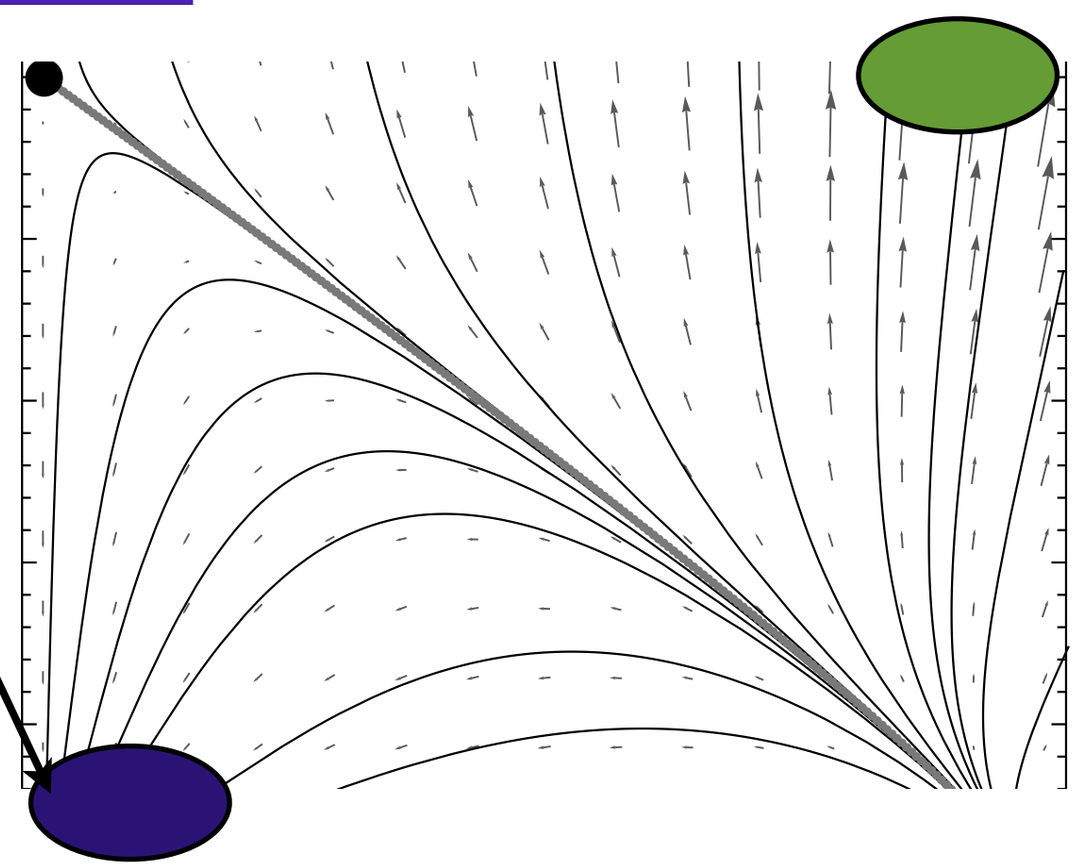
$$G(\Lambda p, \Lambda\omega) = \Lambda^{\alpha} G(p, \omega)$$

$\alpha = -1$

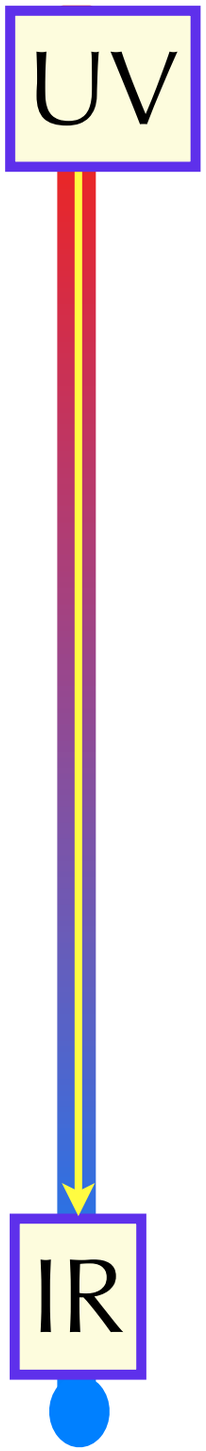
$\alpha > -1$

Fermi liquids

Un-Fermi liquids



Where do we expect to find zeros?



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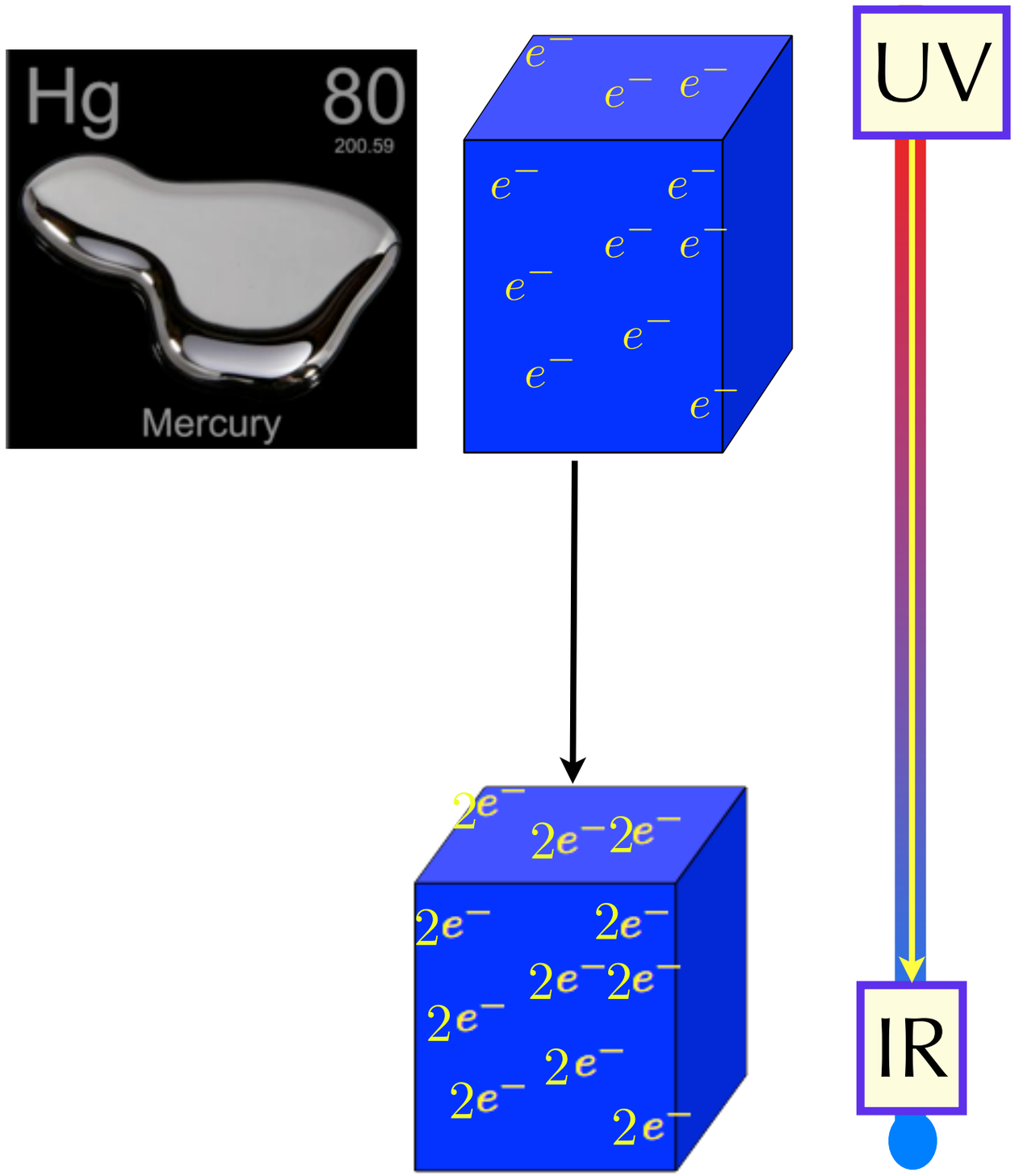

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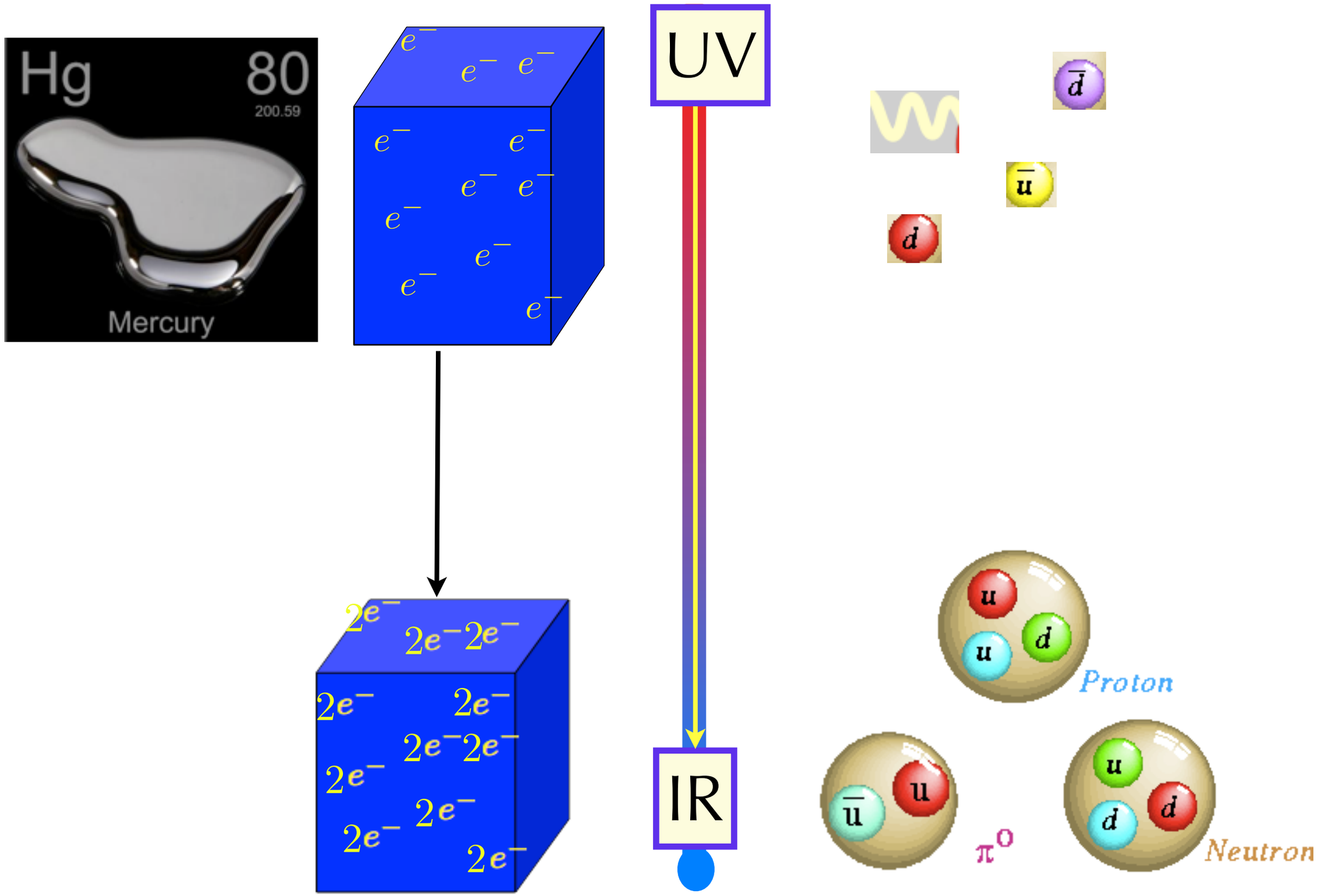
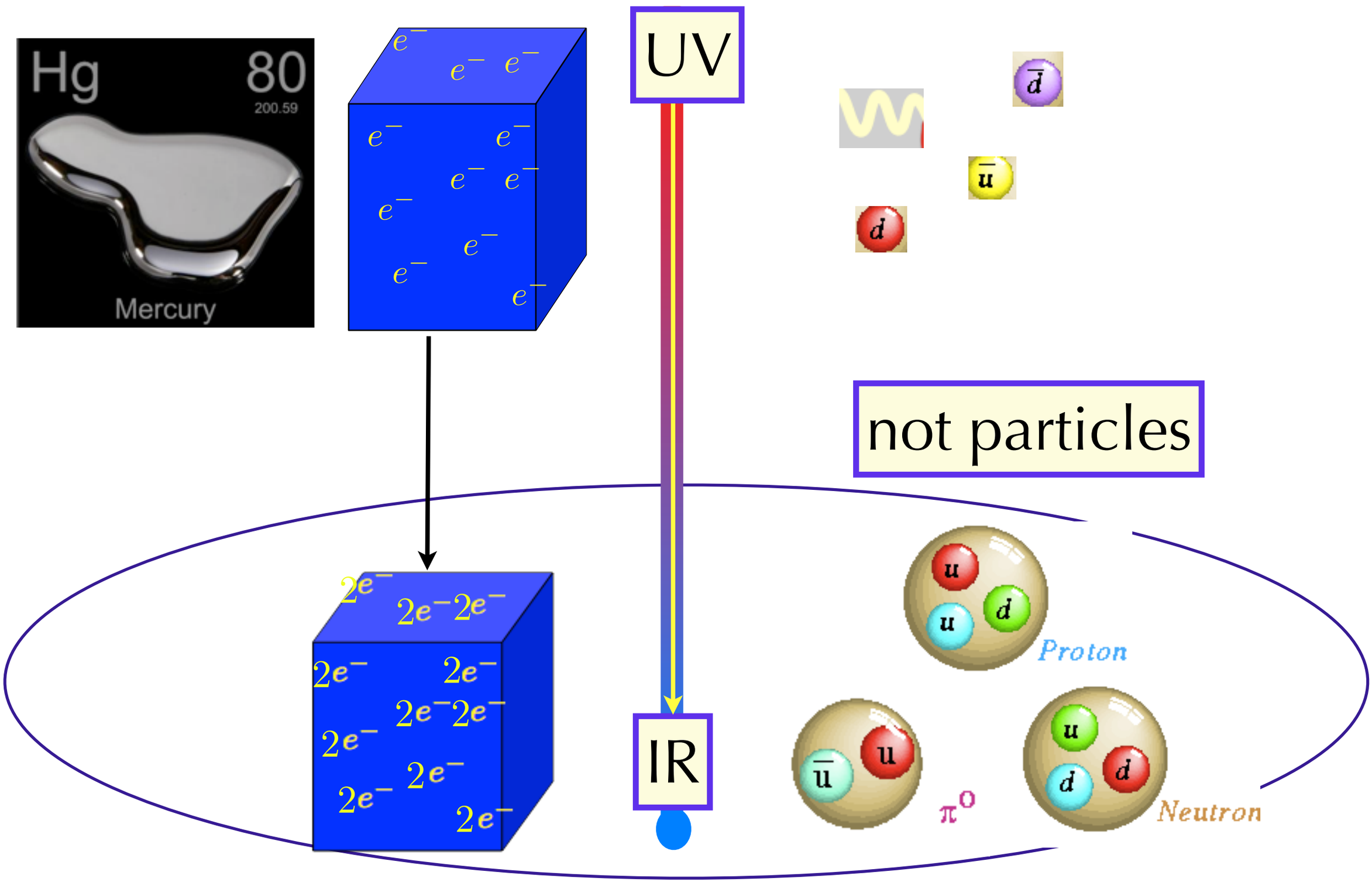
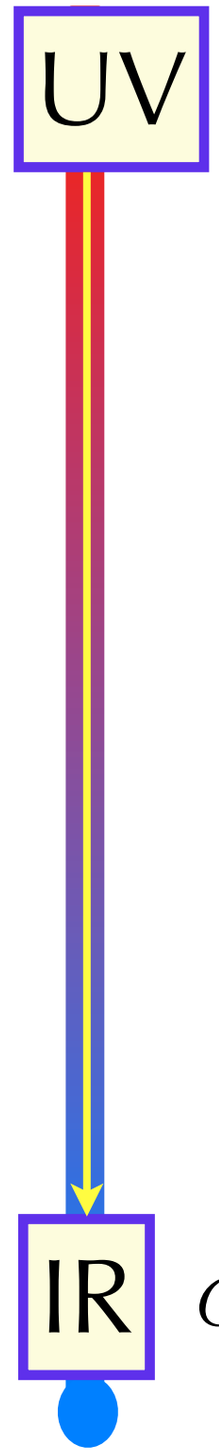


Where do we expect to find zeros?



$G(p,\omega) \neq 0$



no physical quark or electron states

$G(p_L, \omega = 0) = 0$

$$G_{\rm BCS}(\omega, p) = \frac{\omega + v(|p| - p_F)}{\omega^2 - \Delta^2 - v^2(|p| - p_F)^2}$$

$$G_{1+1\rm QCD} = \frac{-ik_-}{m^2 - g^2/\pi + 2k_+k_- + g^2|k_-|/\pi\lambda - i\epsilon}$$

where else do we expect to see zeros?

where else do we expect to see zeros?

strongly correlated systems

where else do we expect to see zeros?

strongly correlated systems

Mott insulators

What is a Mott Insulator?

NiO insulates

d^8 ?

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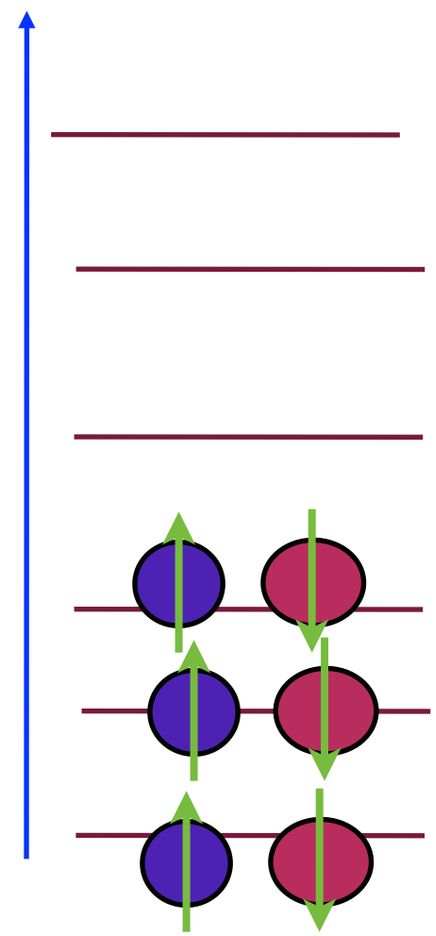

EMPTY STATES=
METAL

What is a Mott Insulator?

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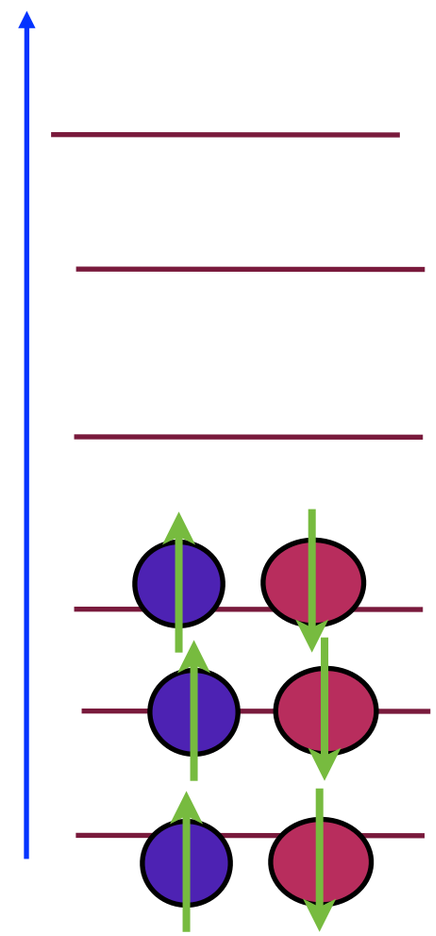
d^8 ?



EMPTY STATES=
METAL

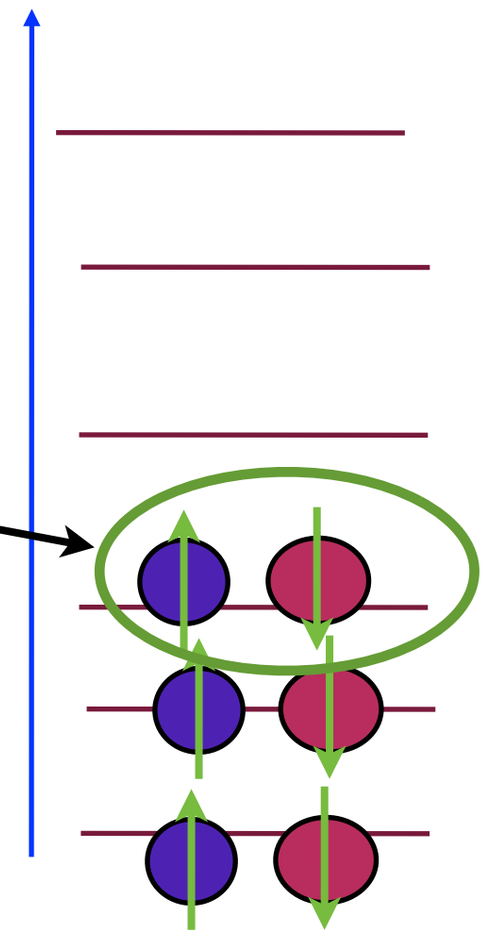
band theory fails!

What is a Mott Insulator?

NiO insulates

d^8 ?

perhaps this costs energy

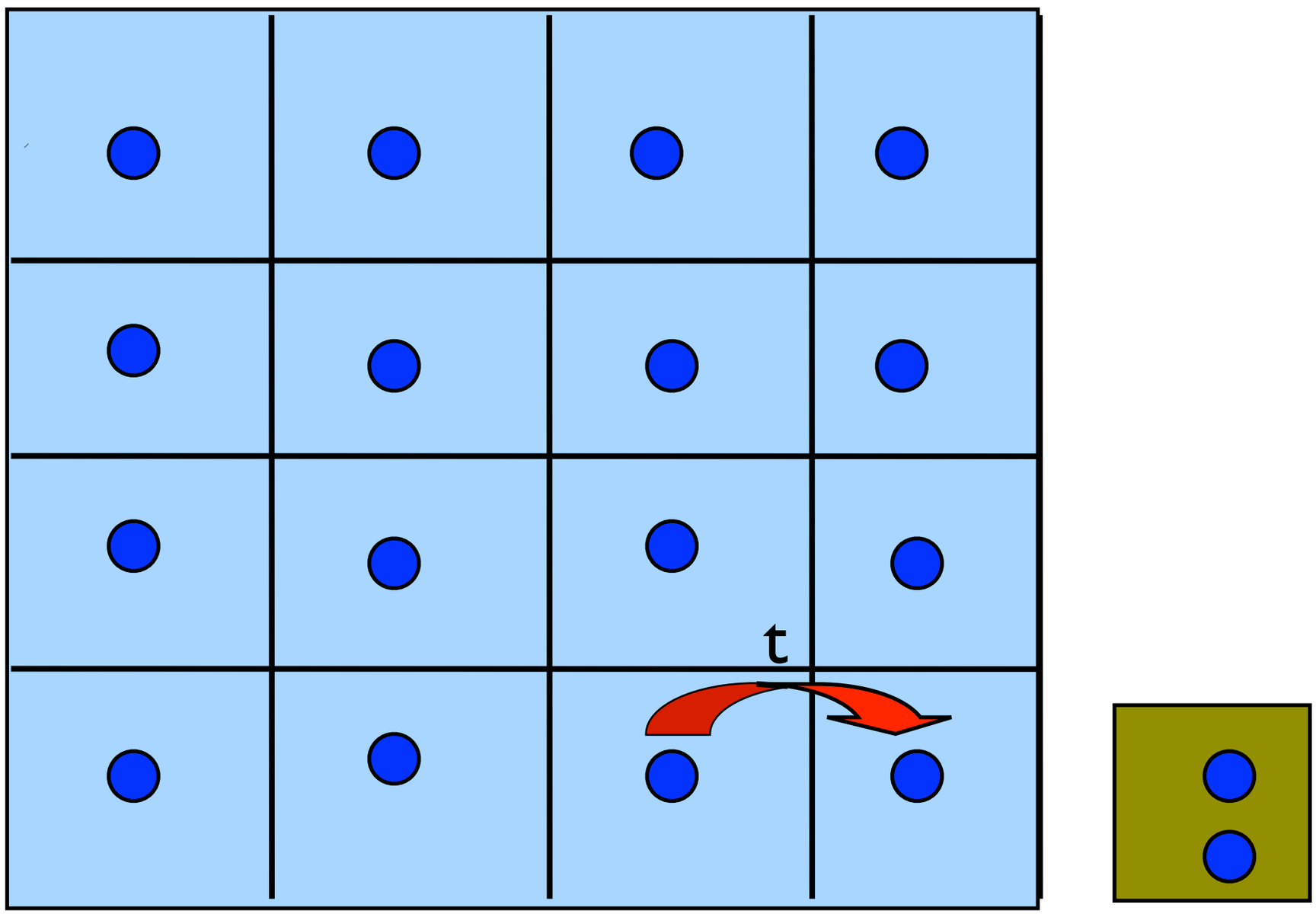


EMPTY STATES= METAL

band theory fails!

Mott Problem: NiO (Band theory failure)

(N rooms N occupants)



$U \gg t$

Mott Problem: NiO (Band theory failure)

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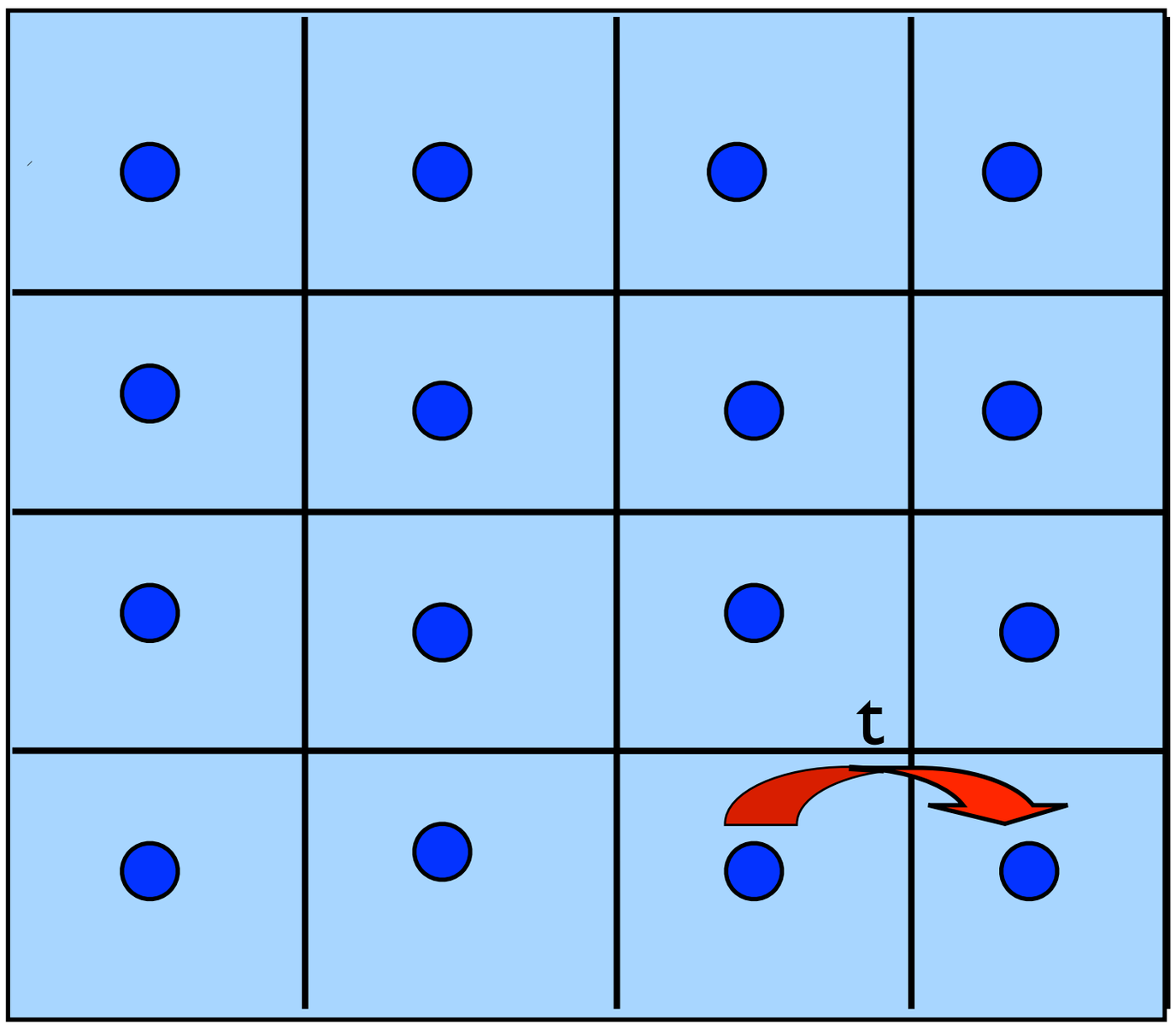
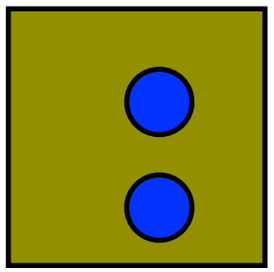


$$\mathrm{Det}G(\omega = 0, \mathbf{p}) = \mathbf{0}$$



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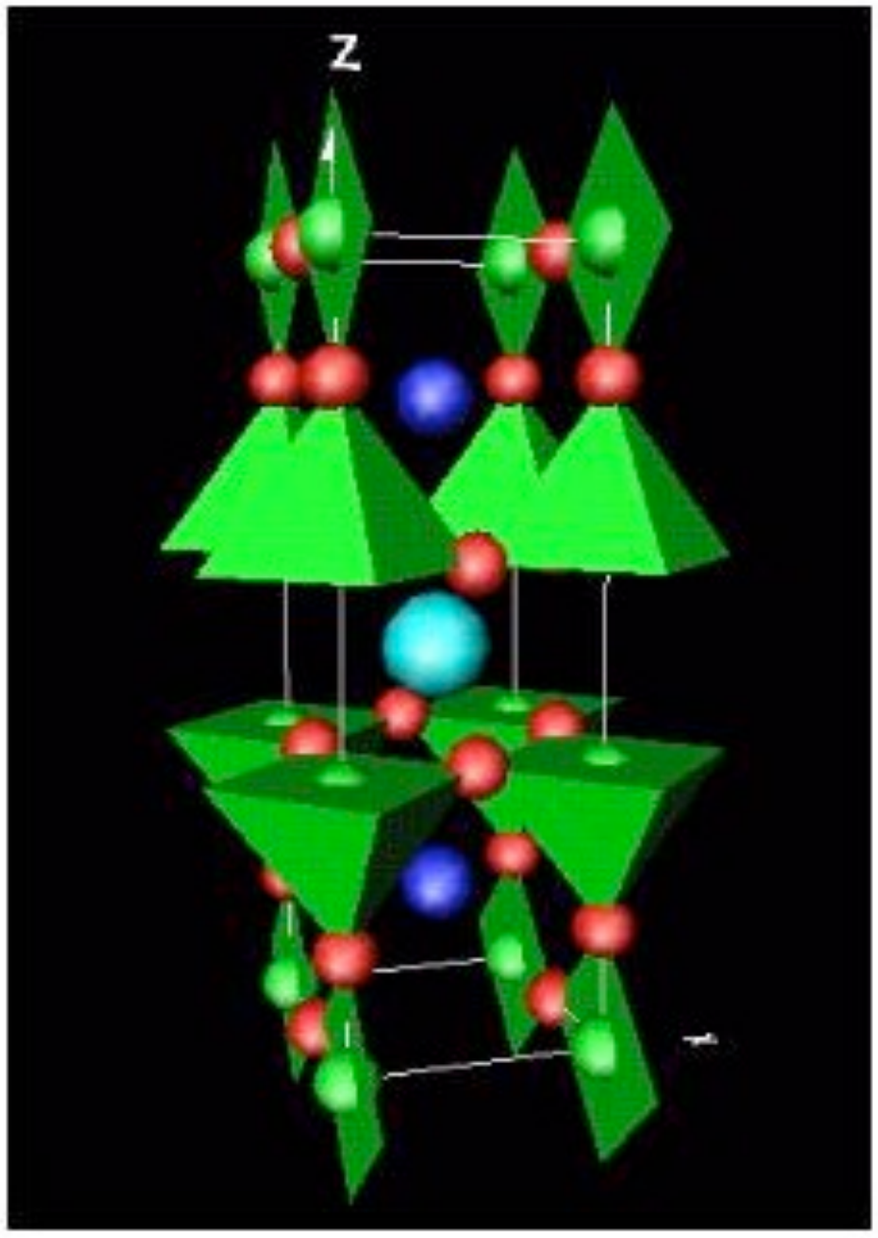
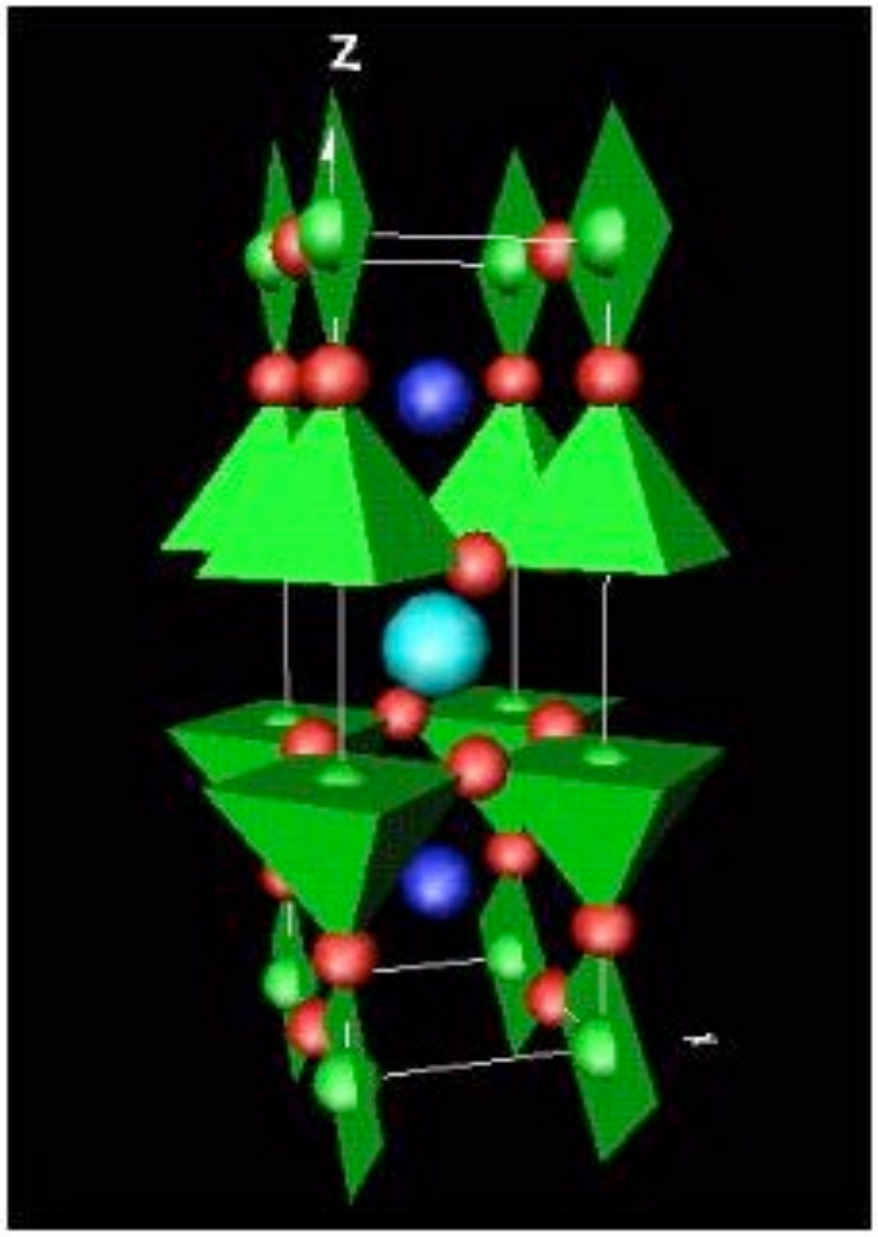


$YBa_2Cu_3O_7$

Cuprate Superconductors



$Y\,Ba_2Cu_3O_7$
Cuprate Superconductors

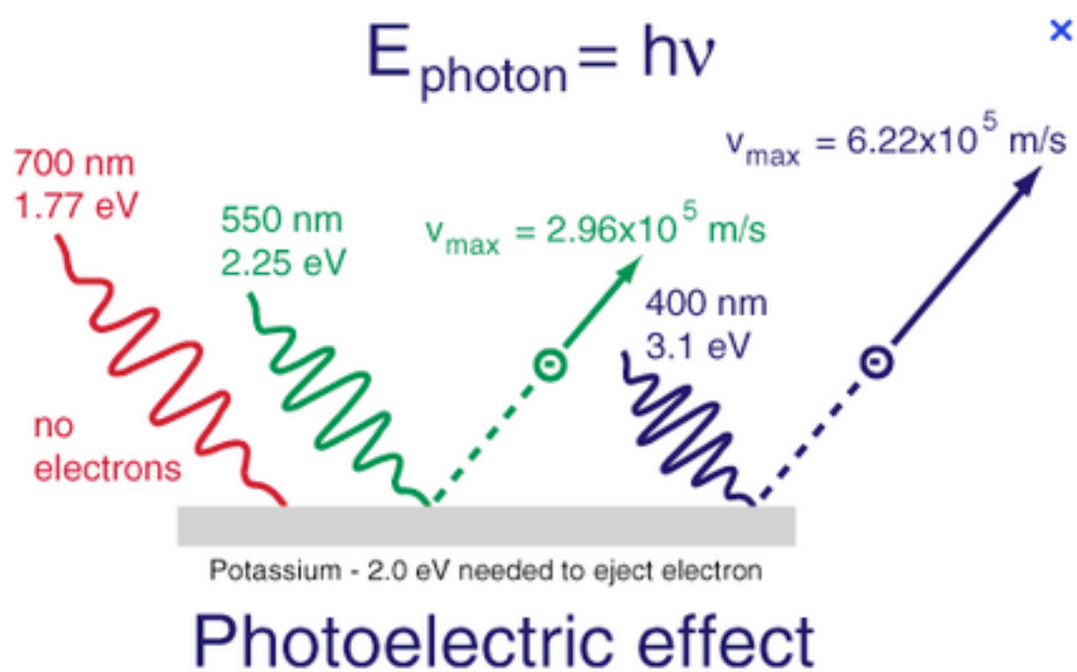
$$U/t = 10 \gg 1$$

interactions dominate:
Strong Coupling Physics

photoemission

photoemission

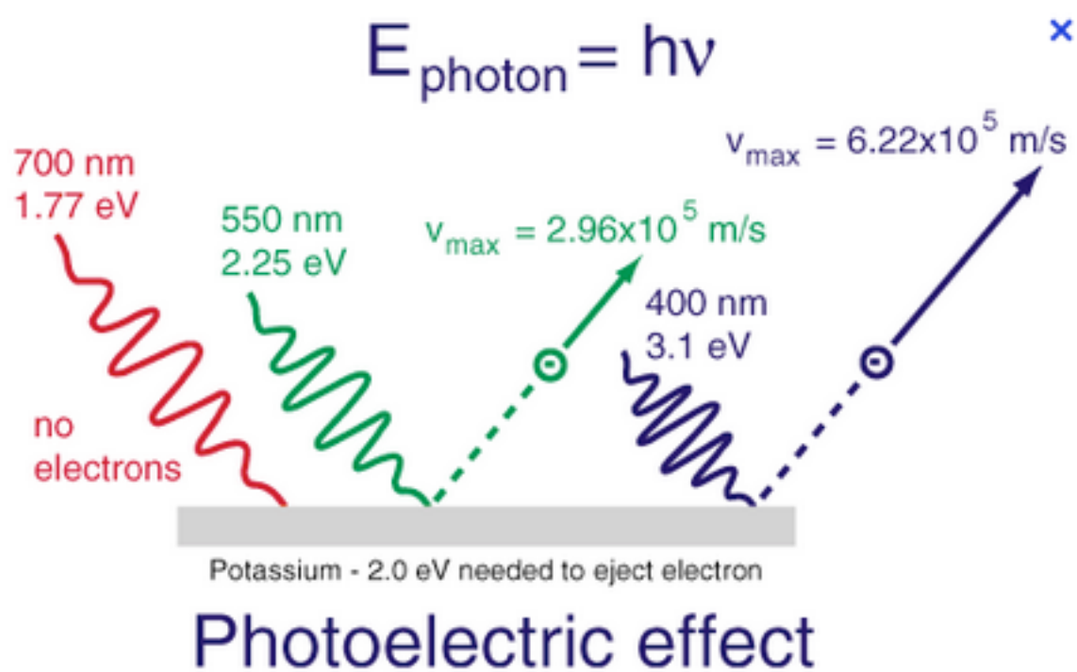
particles



energy $E = \varepsilon_p$

photoemission

particles



energy $E = \varepsilon_p \approx p^2$

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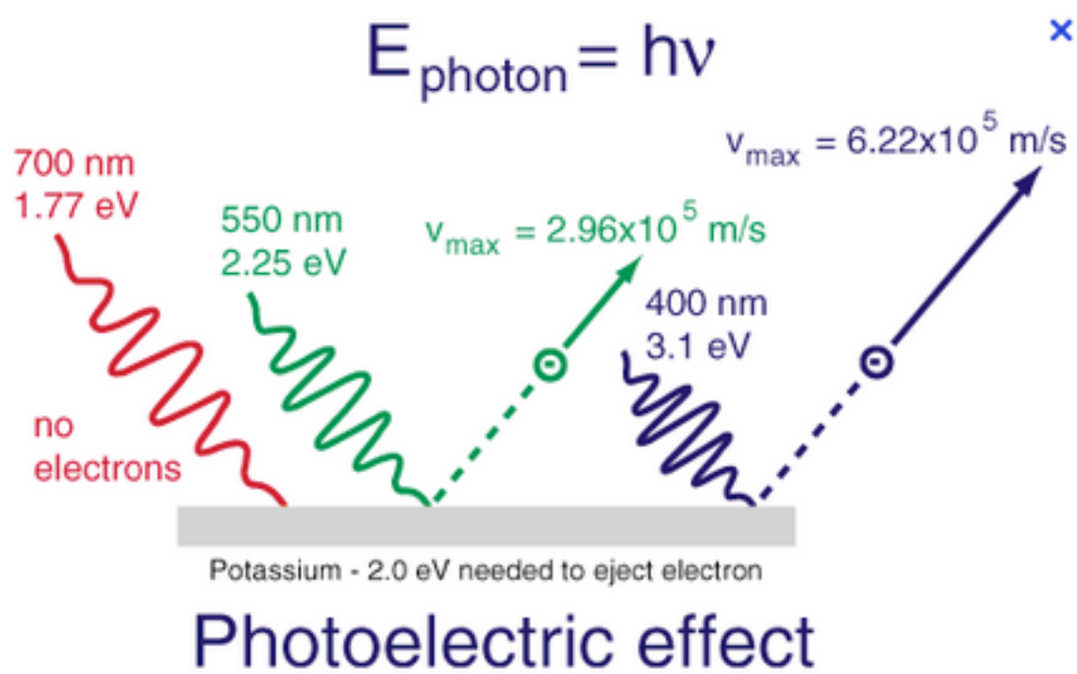

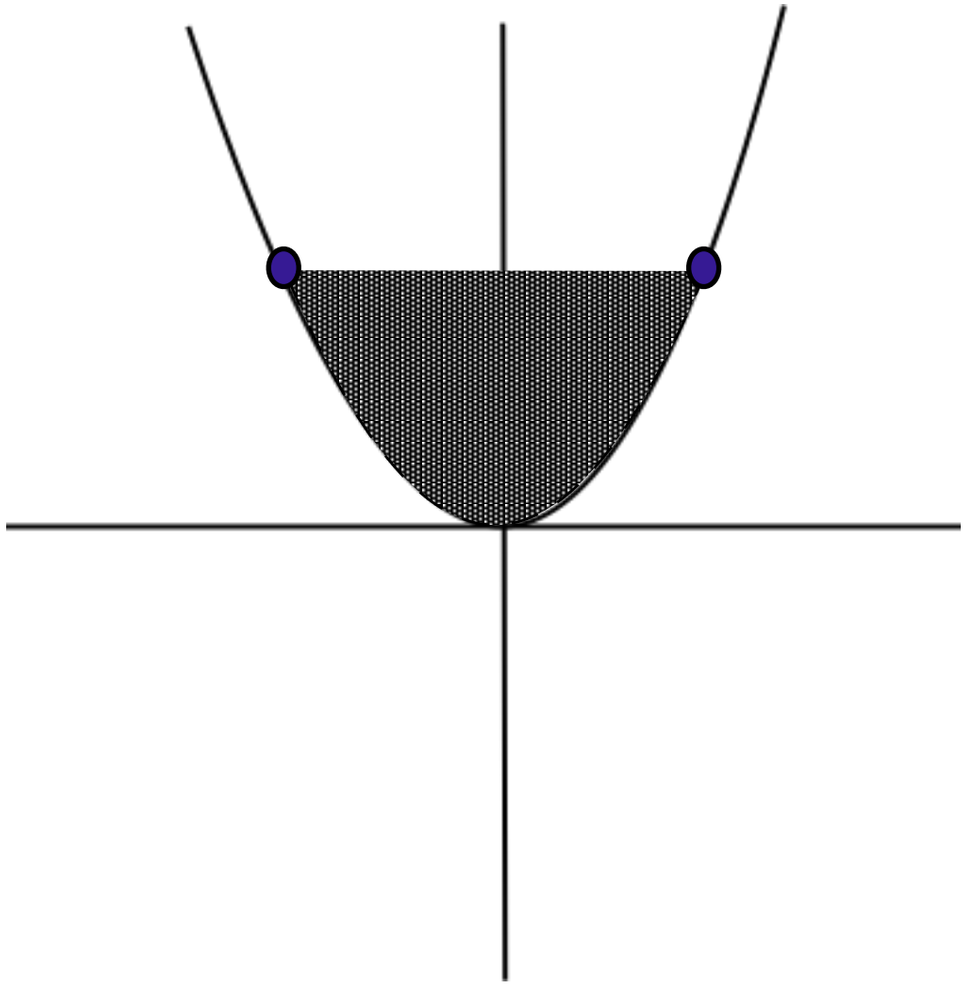

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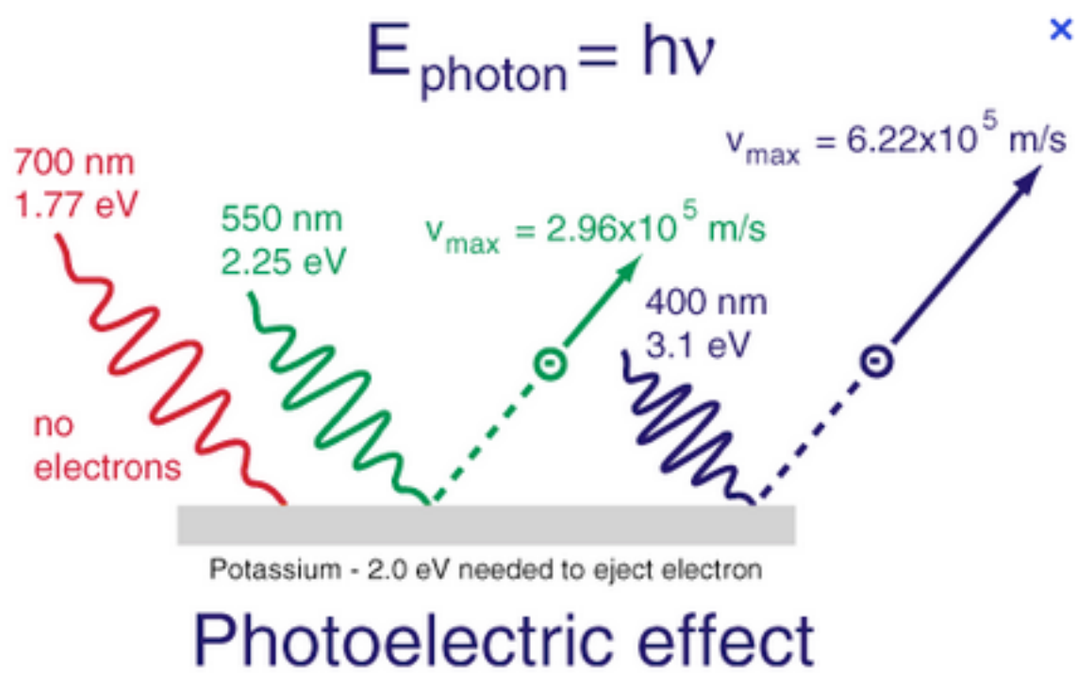

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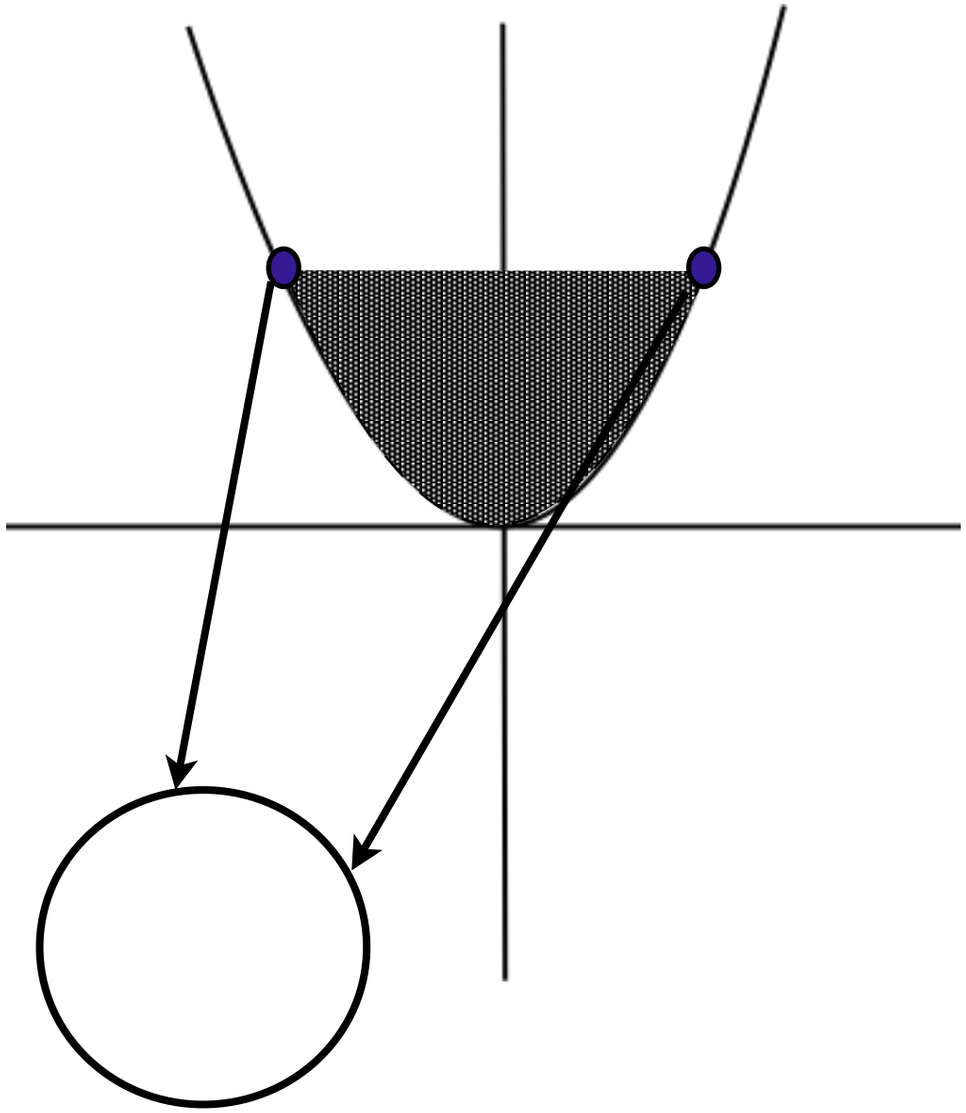
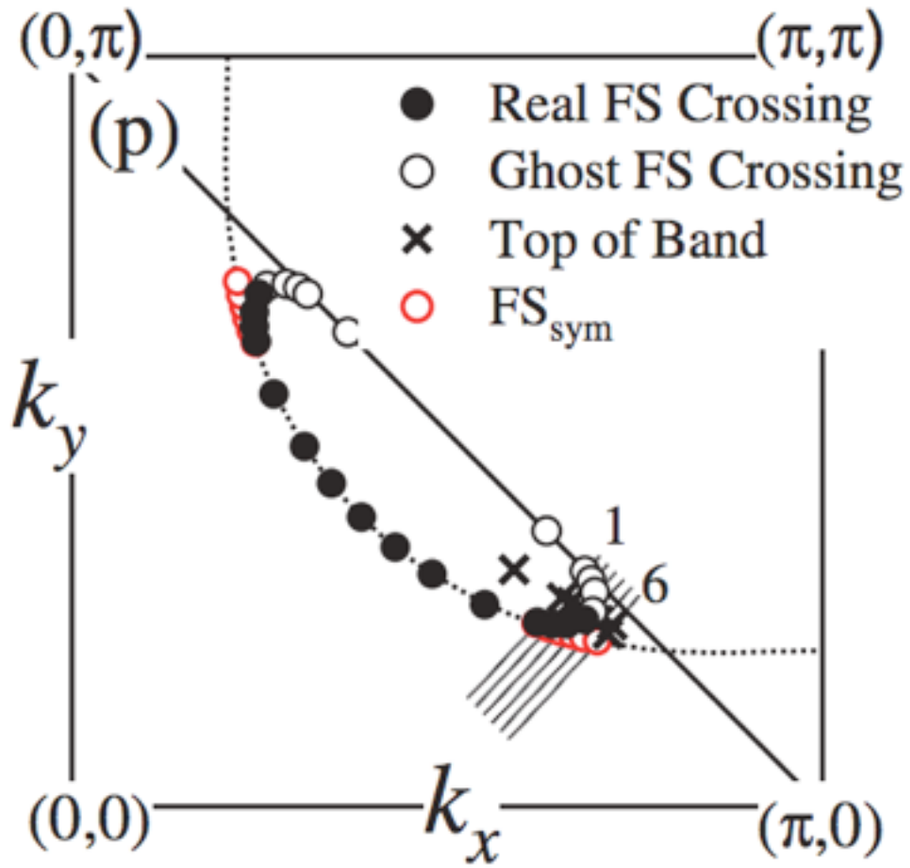


two crossings:
closed surface of excitations

what is seen experimentally?

Fermi arcs: no double crossings (PDJ,JCC,ZXS)



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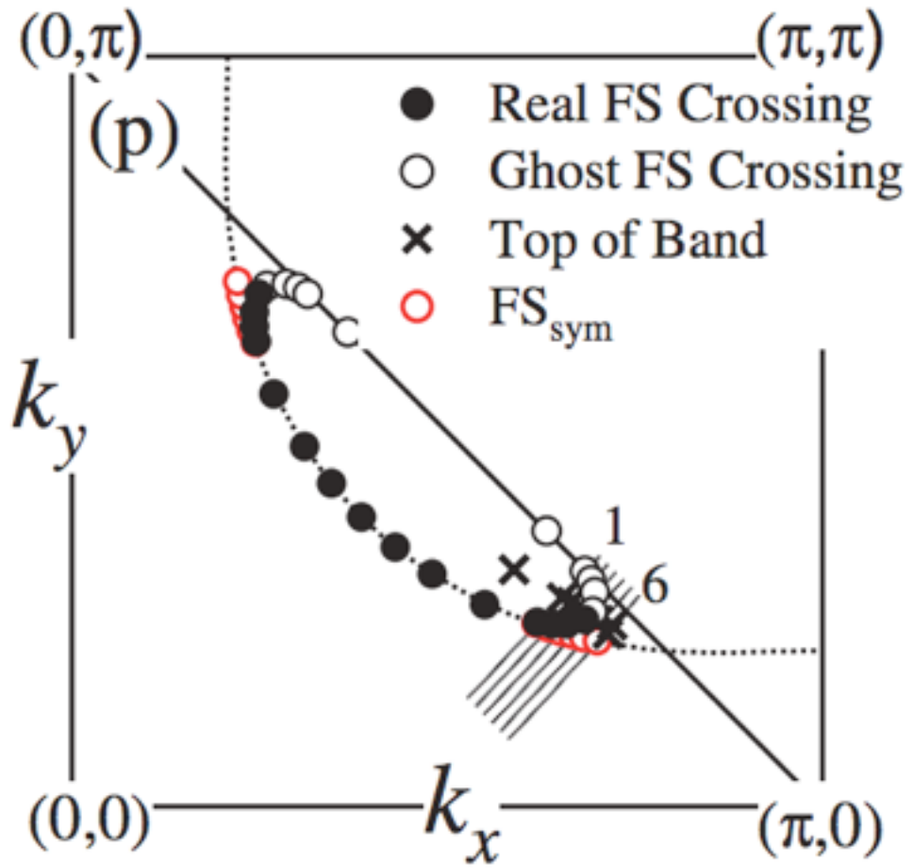
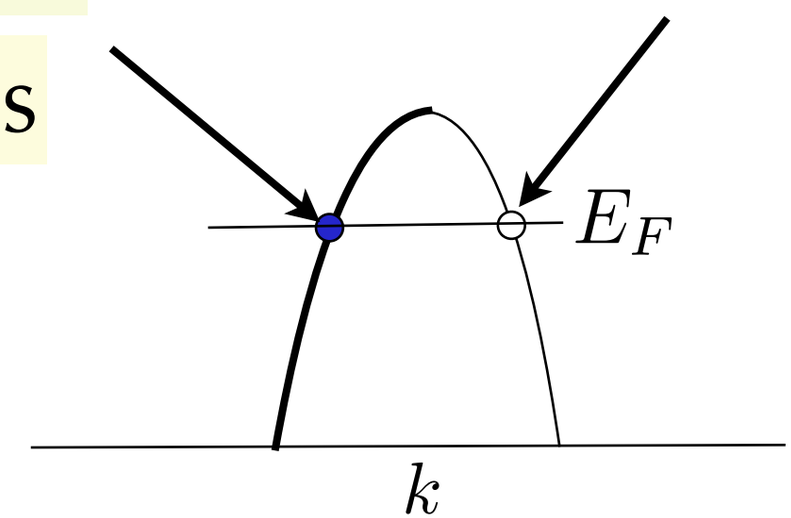


seen

infinities

not seen

zeros



zeros do not
affect the particle density

zeros do not
affect the particle density

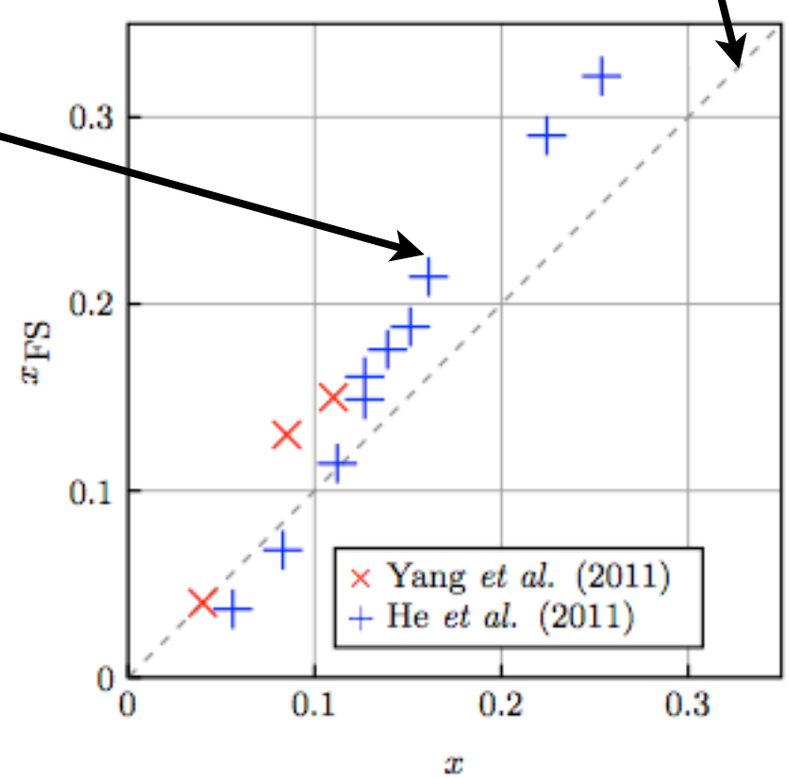
↓

no Luttinger theorem

zeros do not
affect the particle density

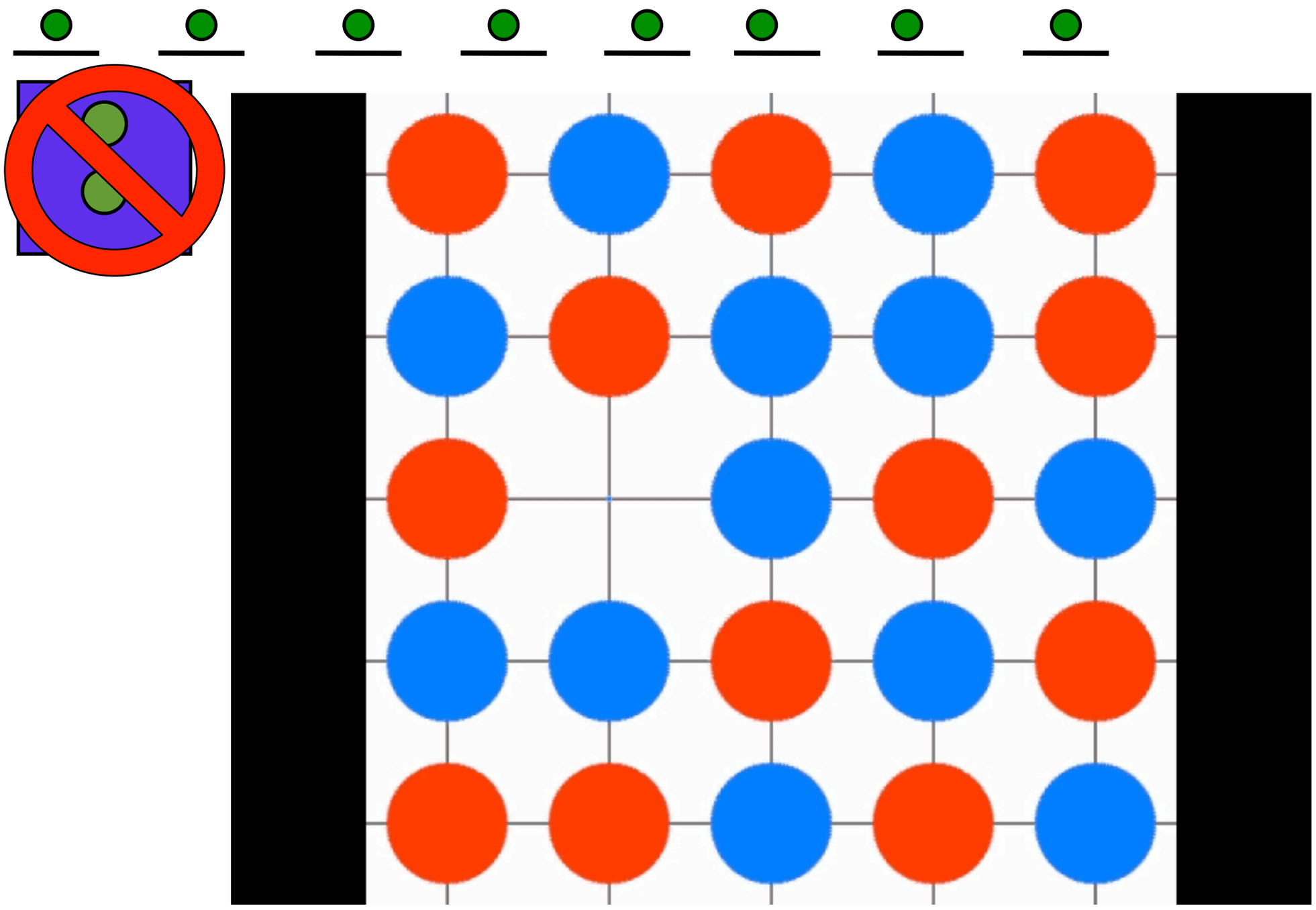
no Luttinger theorem

experimental
data



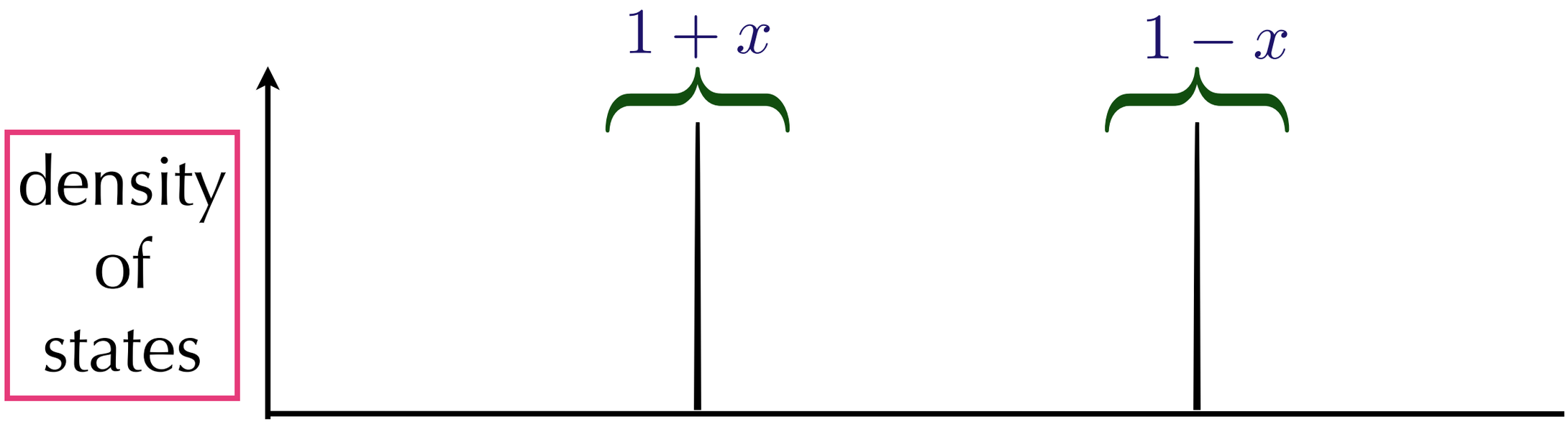
where do these zeros come from?

classical (atomic) limit



warm-up

no hopping: x holes



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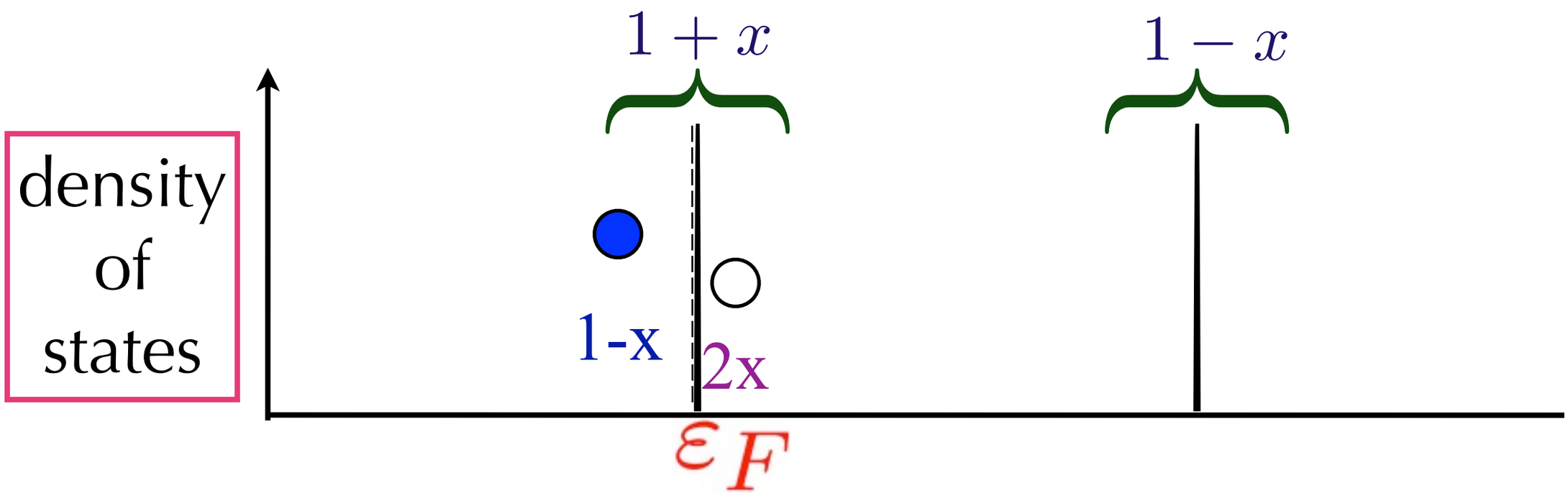

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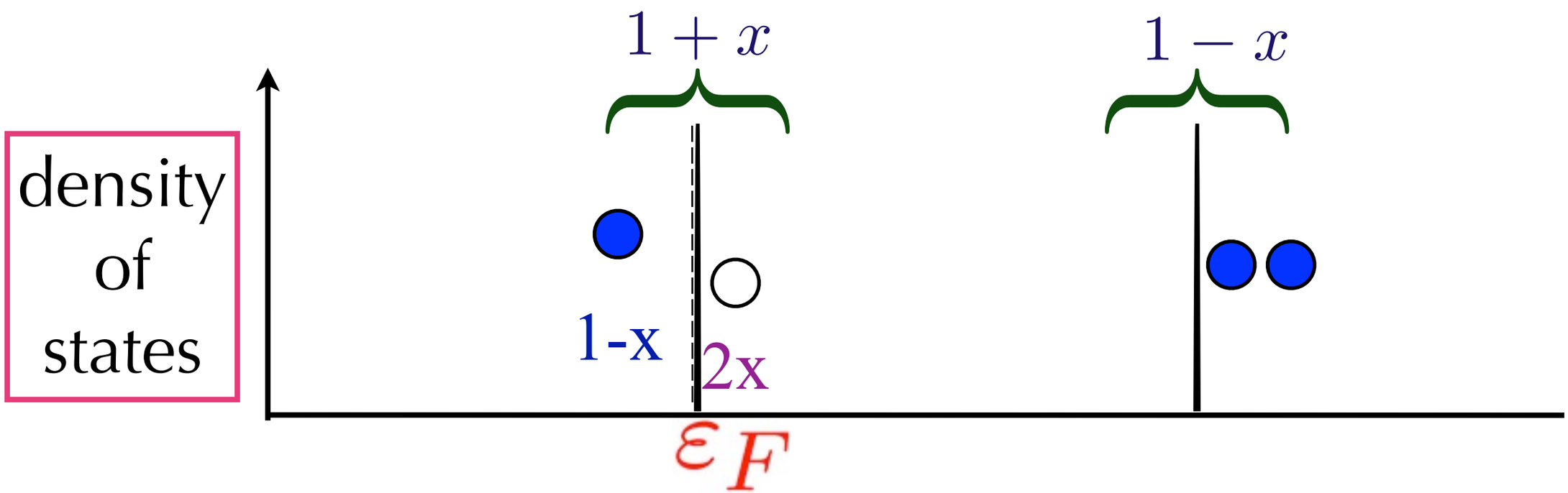

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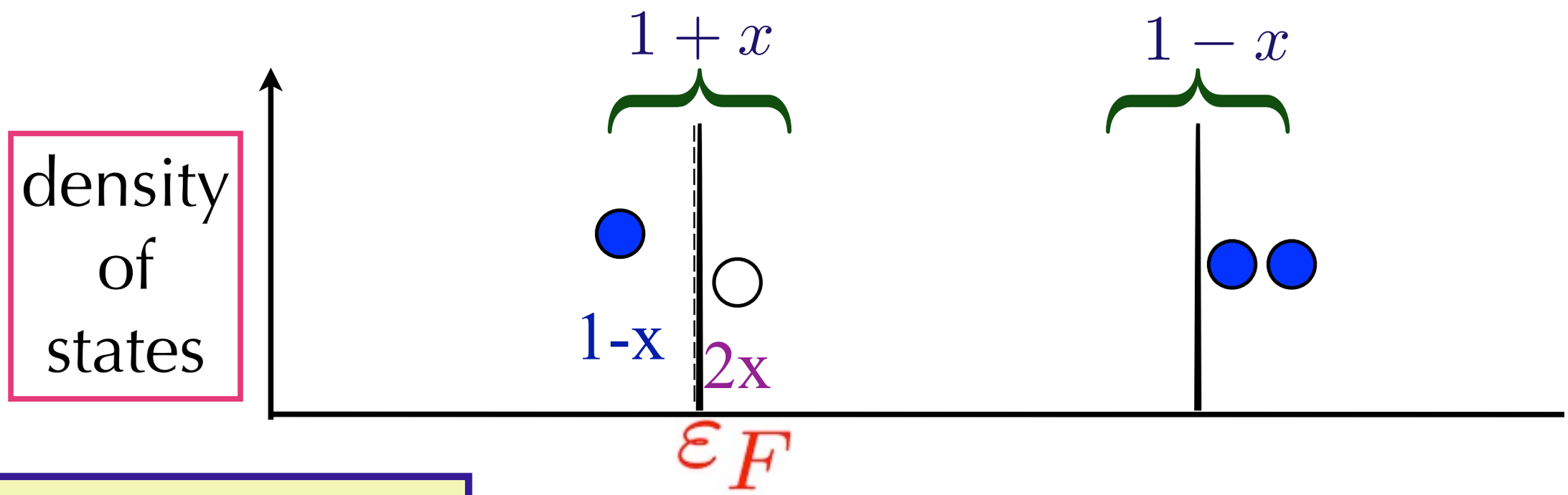
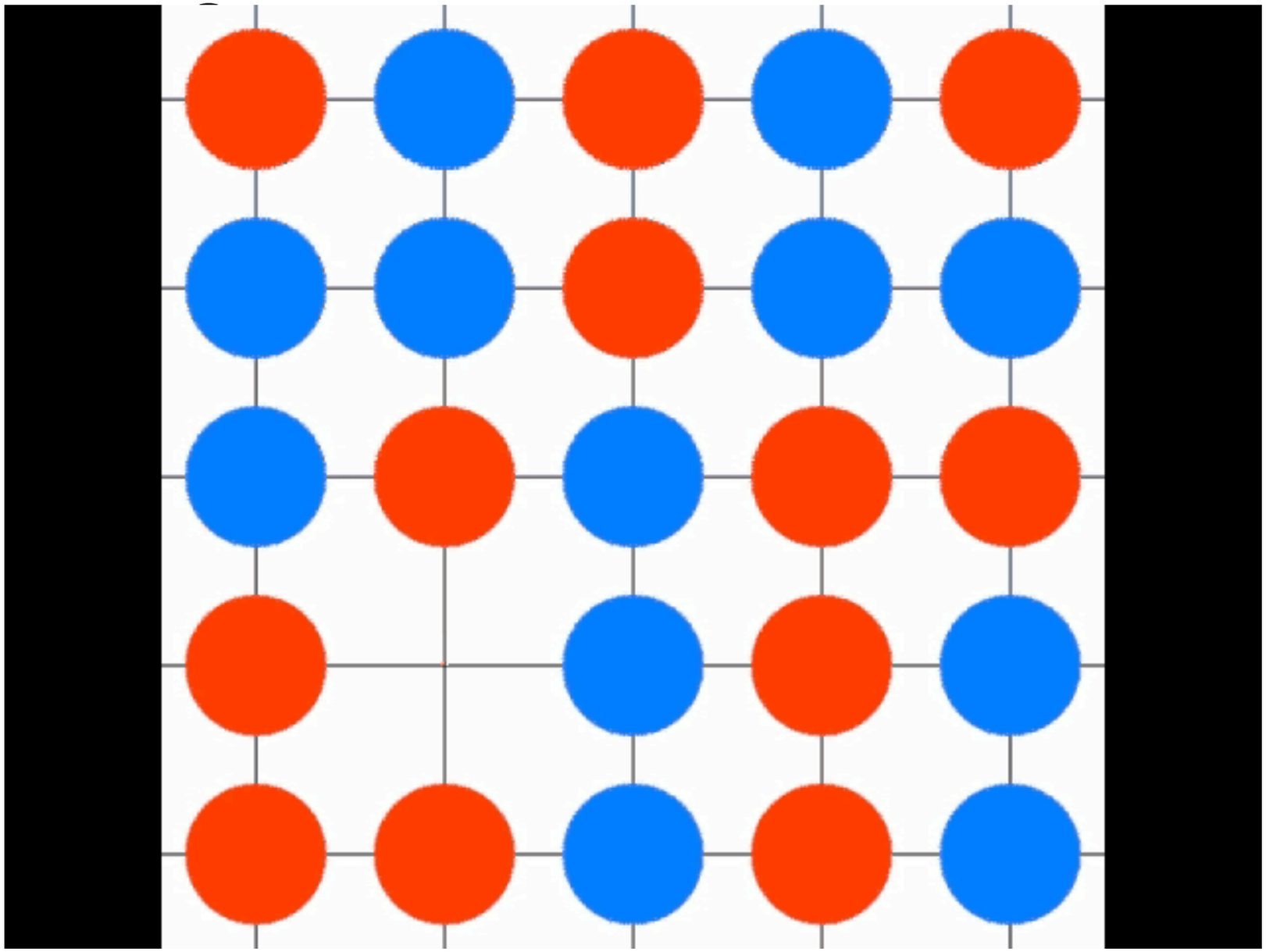


spectral weight: x-dependent

$$G = \frac{1+x}{\omega+\mu+U/2} + \frac{1-x}{\omega+\mu-U/2}$$

quantum Mottness: U finite

$U \gg t$



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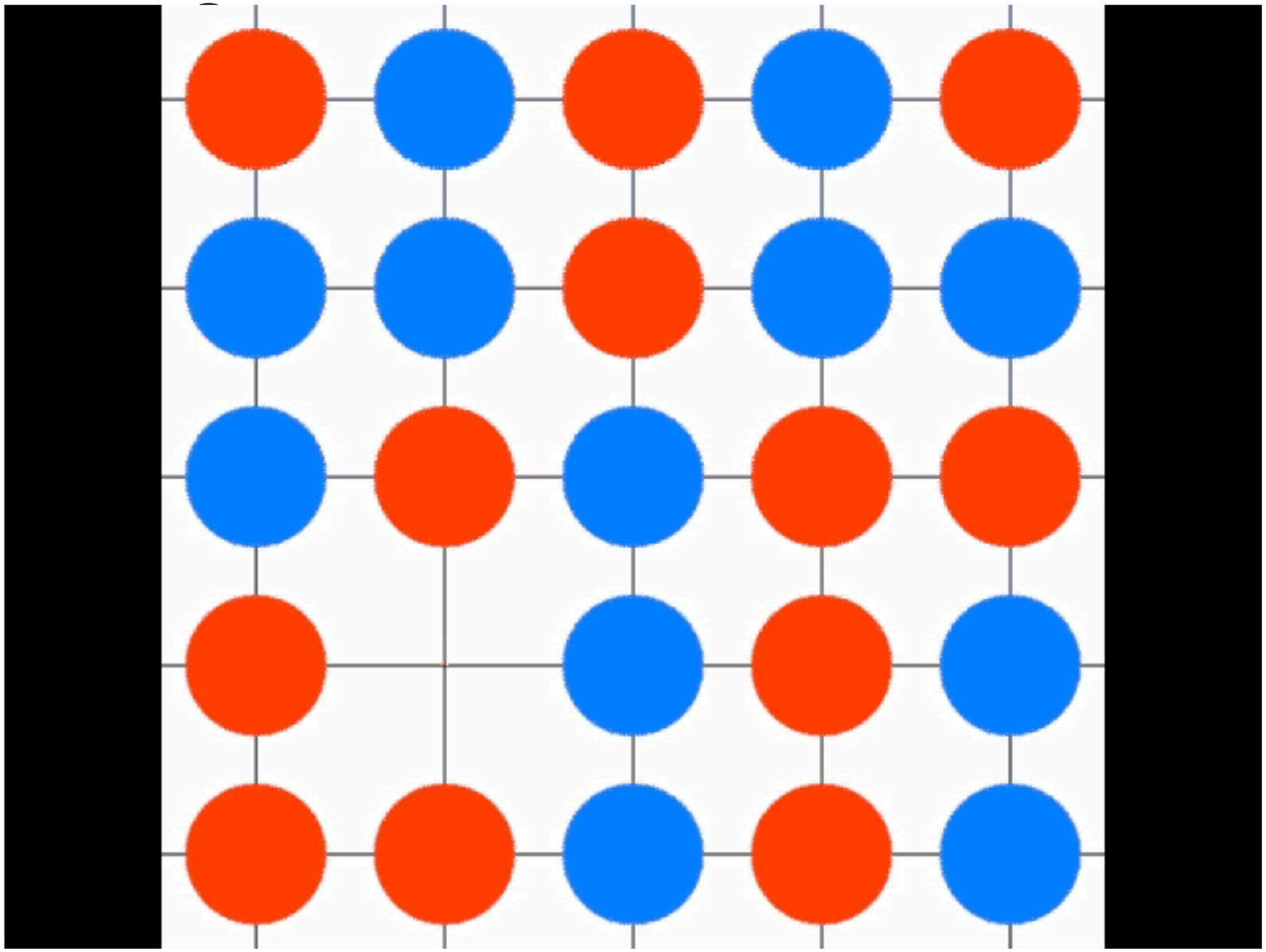

double occupancy in ground state!!

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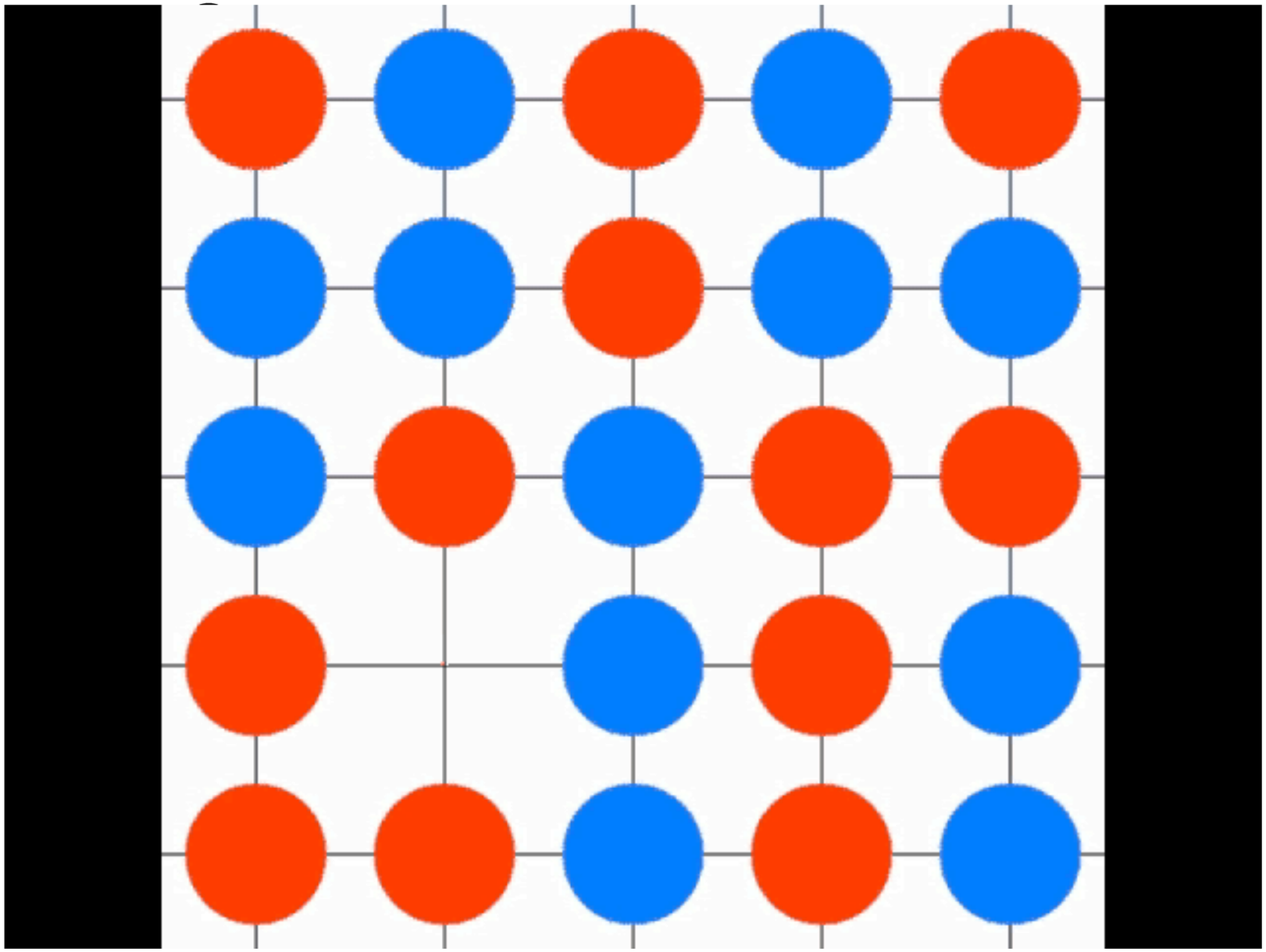
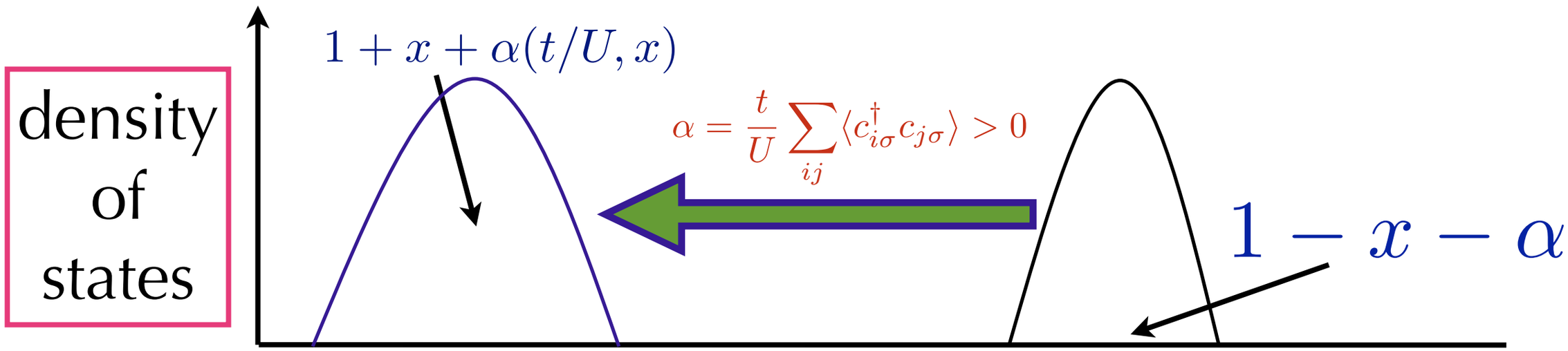


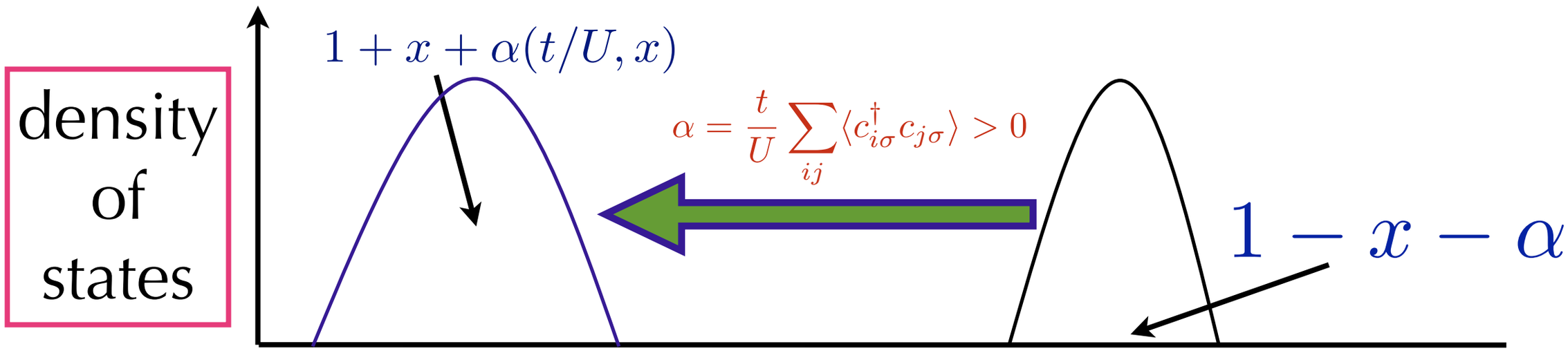
double occupancy in ground state!!

$W_{\mathrm{PES}} > 1 + x$

beyond the atomic limit: any real system



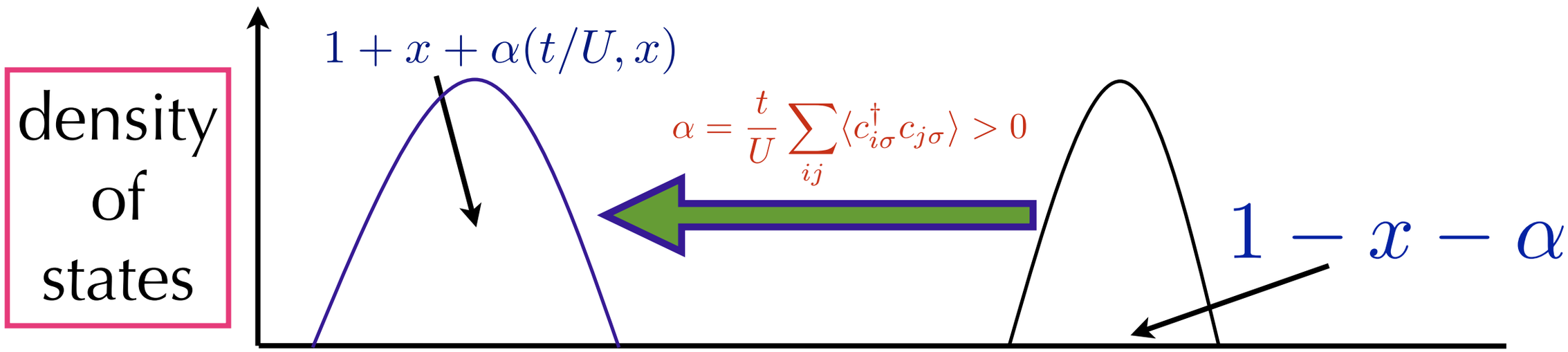
beyond the atomic limit: any real system



Harris & Lange, 1967

dynamical spectral weight transfer

beyond the atomic limit: any real system

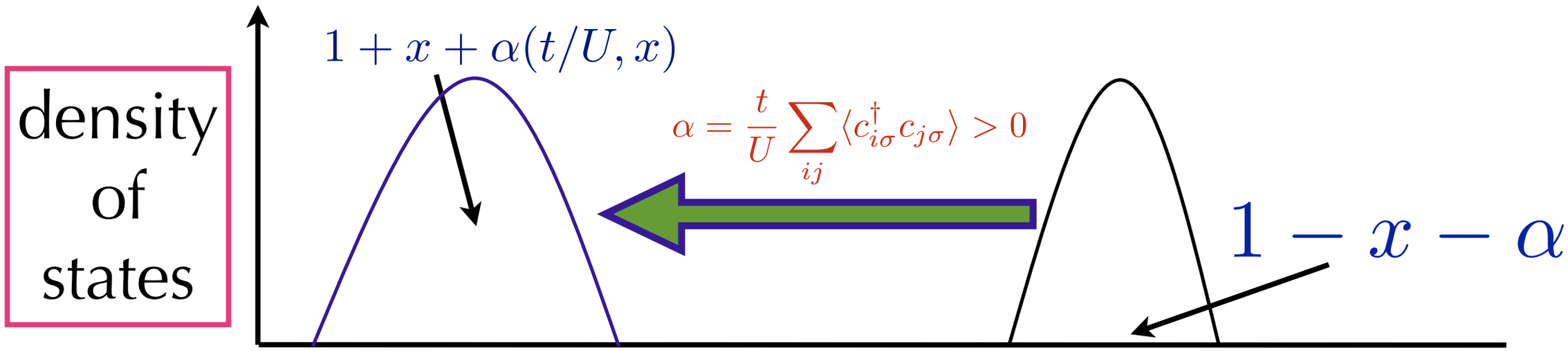


Harris & Lange, 1967

dynamical spectral weight transfer

Intensity>1+x

beyond the atomic limit: any real system



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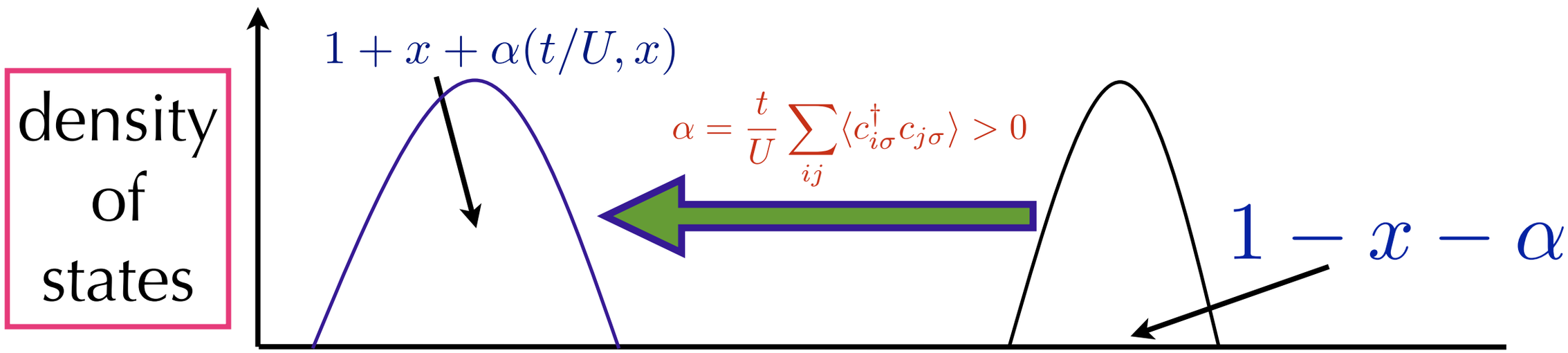
dynamical spectral weight transfer

Intensity>1+x

# of charge e states

beyond the atomic limit: any real system



Harris & Lange, 1967

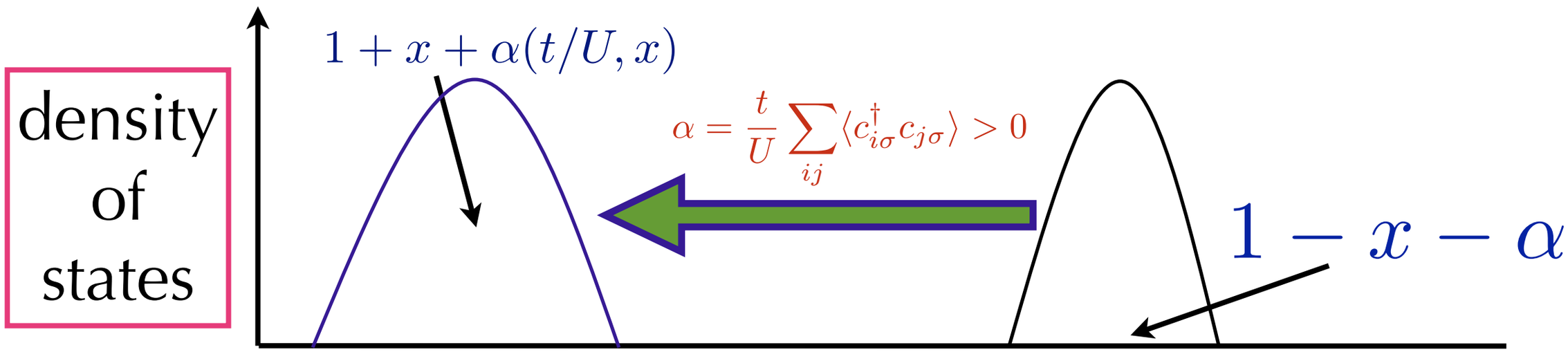
dynamical spectral weight transfer

Intensity>1+x

# of charge e states

# of electron states in lower band

beyond the atomic limit: any real system



Harris & Lange, 1967

dynamical spectral weight transfer

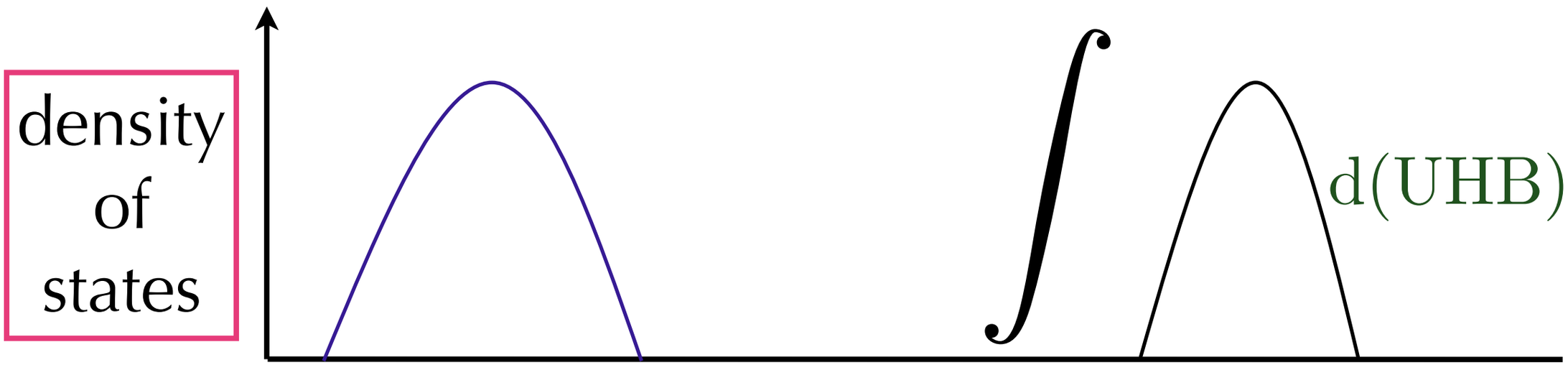
Intensity>1+x

# of charge e states

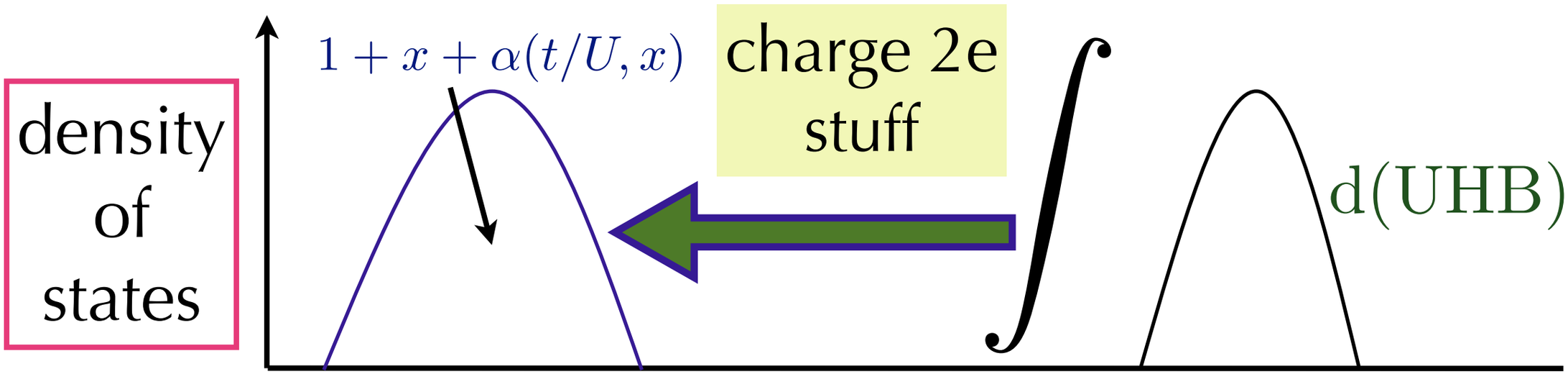
# of electron states in lower band

not exhausted by counting electrons alone?

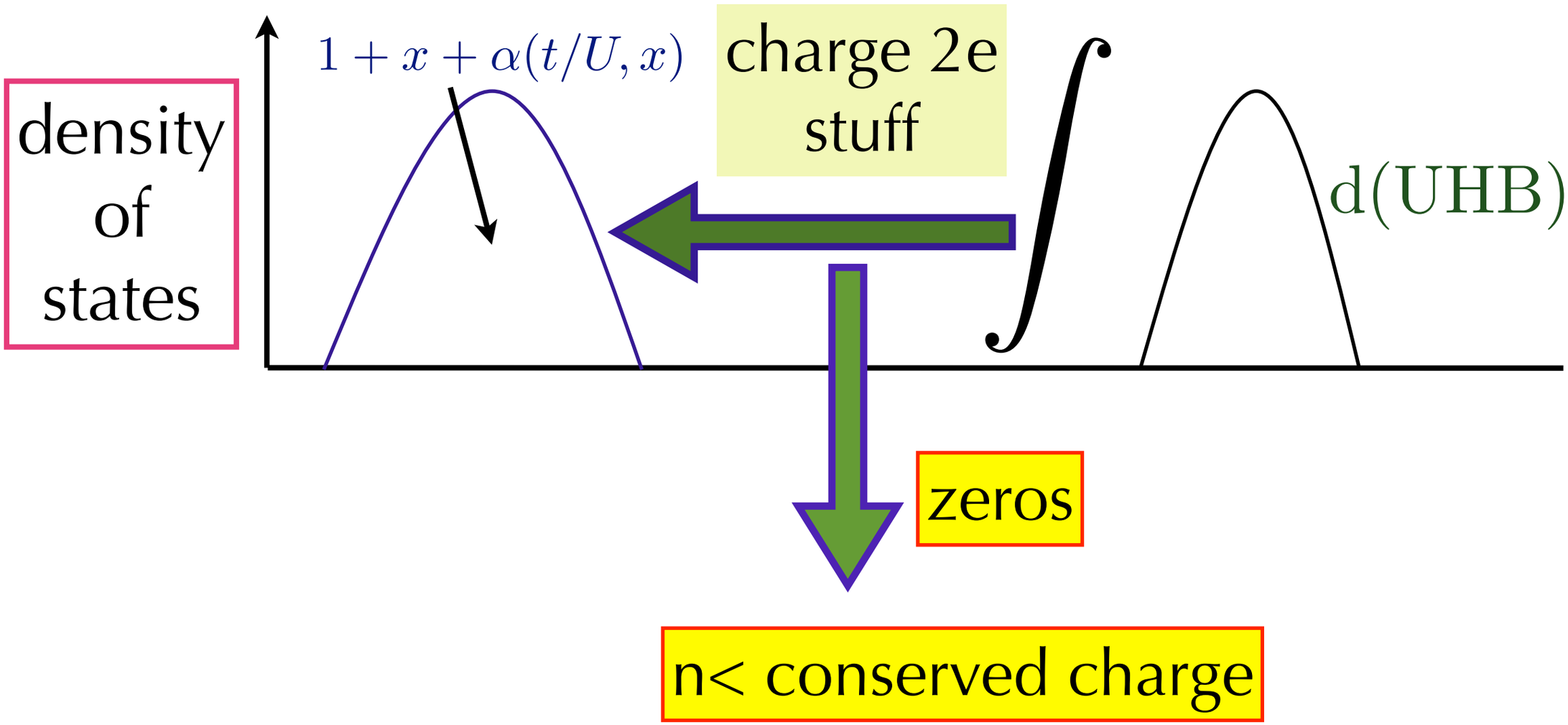
Wilsonian analysis



Wilsonian analysis

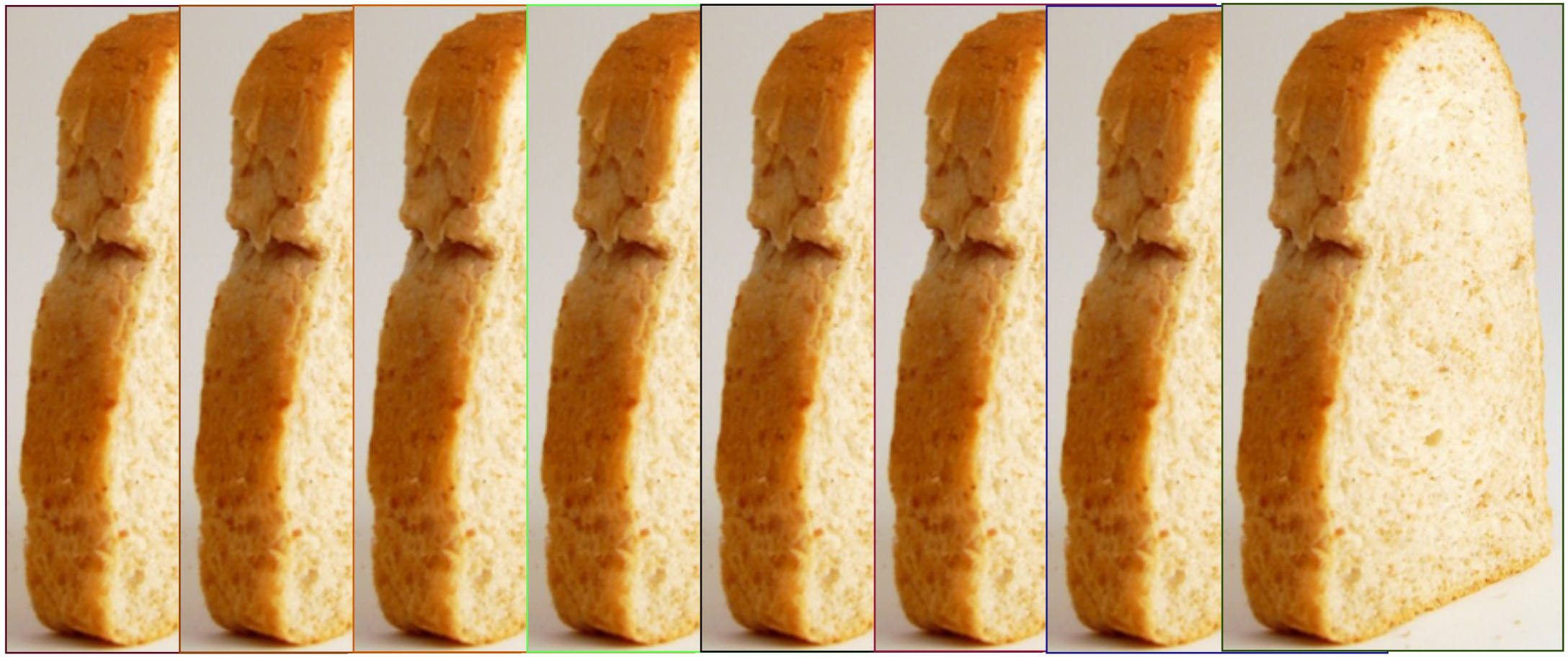


Wilsonian analysis



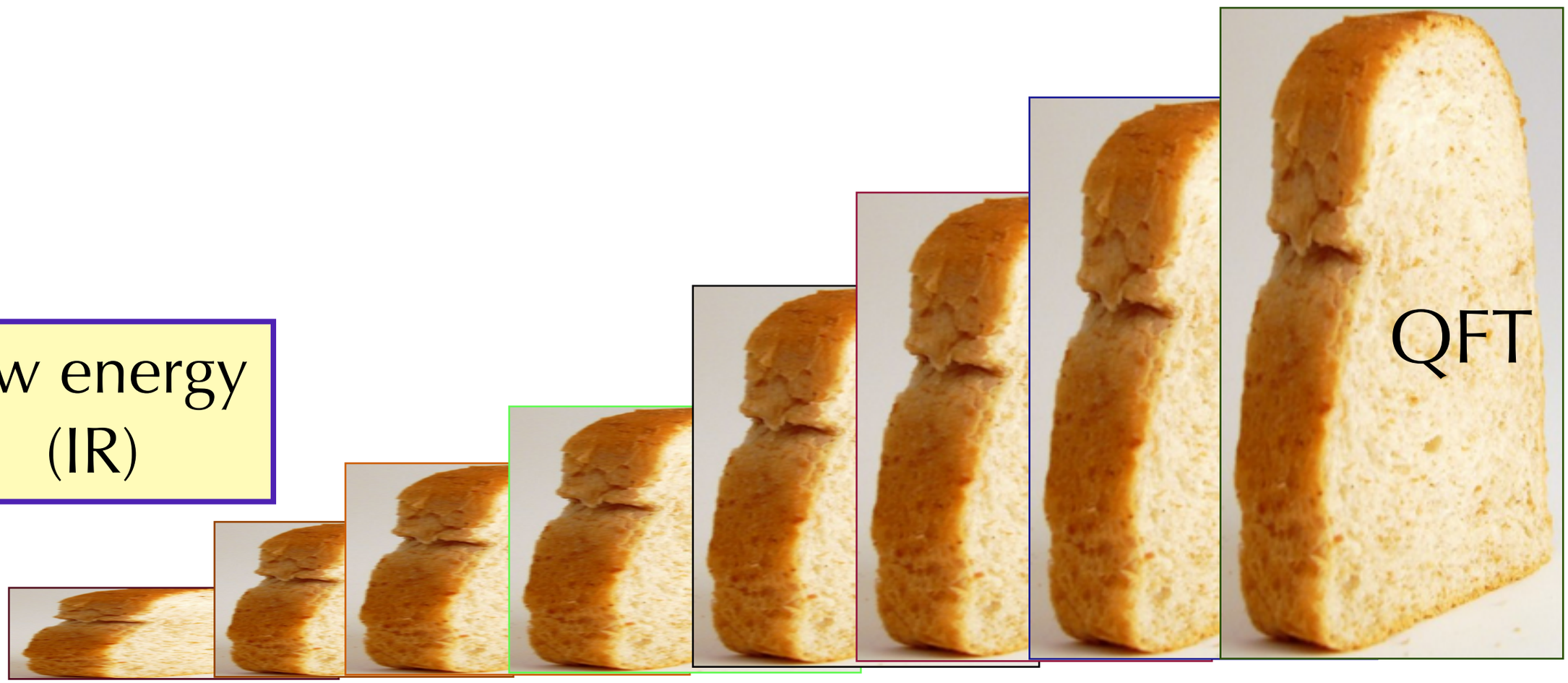
what really is the summation over mass?

$\mathcal{L}_{m_{i+7}}$ $\mathcal{L}_{m_{i+6}}$ $\mathcal{L}_{m_{i+5}}$ $\mathcal{L}_{m_{i+4}}$ $\mathcal{L}_{m_{i+3}}$ $\mathcal{L}_{m_{i+2}}$ $\mathcal{L}_{m_{i+1}}$ $\mathcal{L}_{m_i}$



mass=energy

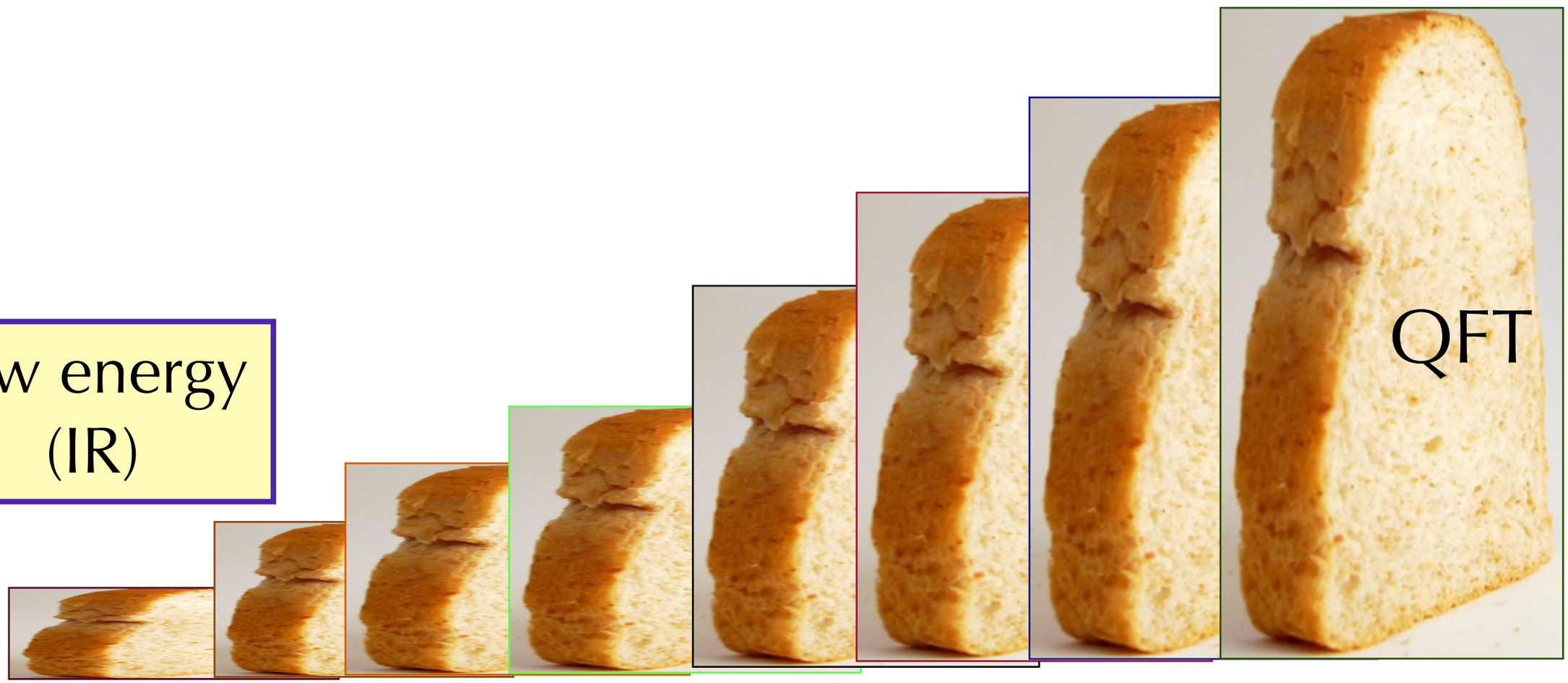
high energy
(UV)



low energy
(IR)

related to sum
over mass

high energy
(UV)



low energy
(IR)

$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

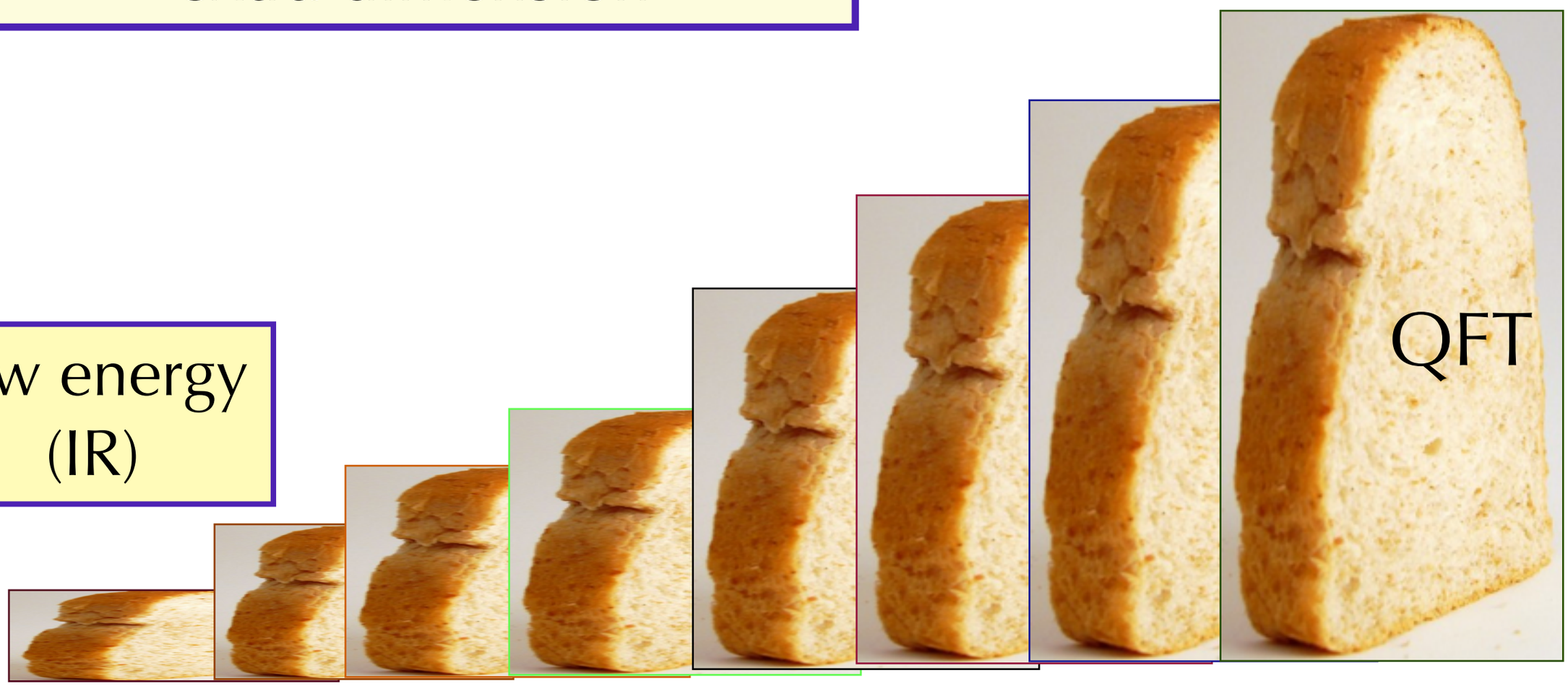
related to sum
over mass

locality in energy

implement E-scaling with an extra dimension

high energy (UV)

low energy (IR)



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QFT

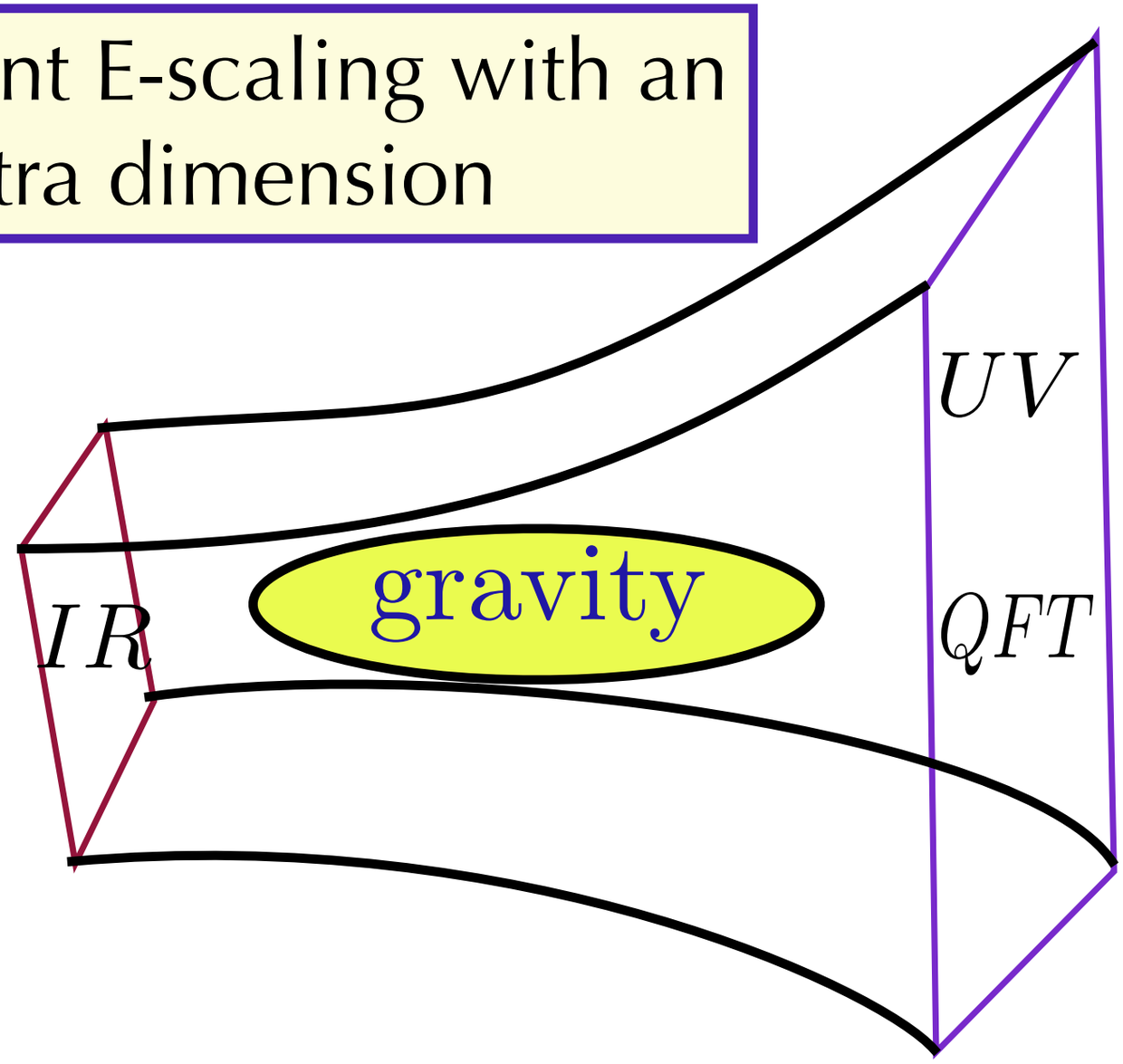
$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

related to sum over mass

locality in energy

gauge-gravity duality
(Maldacena, 1997)

implement E-scaling with an extra dimension



QFT

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locality in energy

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(continuous)

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$$ds^2 = \left(\frac{u}{L}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{L}{E}\right)^2 dE^2$$

anti de-Sitter space

unparticles

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2 \phi^2(x,m)\right) dm^2$$

unparticles

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classical gravity in a d+1 curved spacetime

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classical gravity in a d+1 curved spacetime

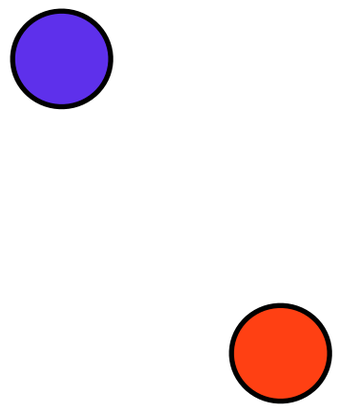
gauge theory in AdS_4

interchanging unparticles

fractional (d_U) number of massless particles

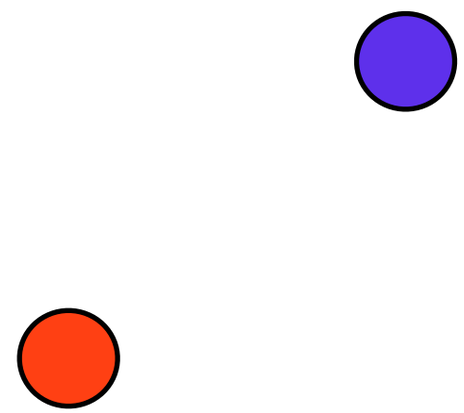
interchanging unparticles

fractional (d_U) number of massless particles



interchanging unparticles

fractional (d_U) number of massless particles



interchanging unparticles

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interchanging unparticles

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$$e^{i\pi d_U} \neq -1, 0$$

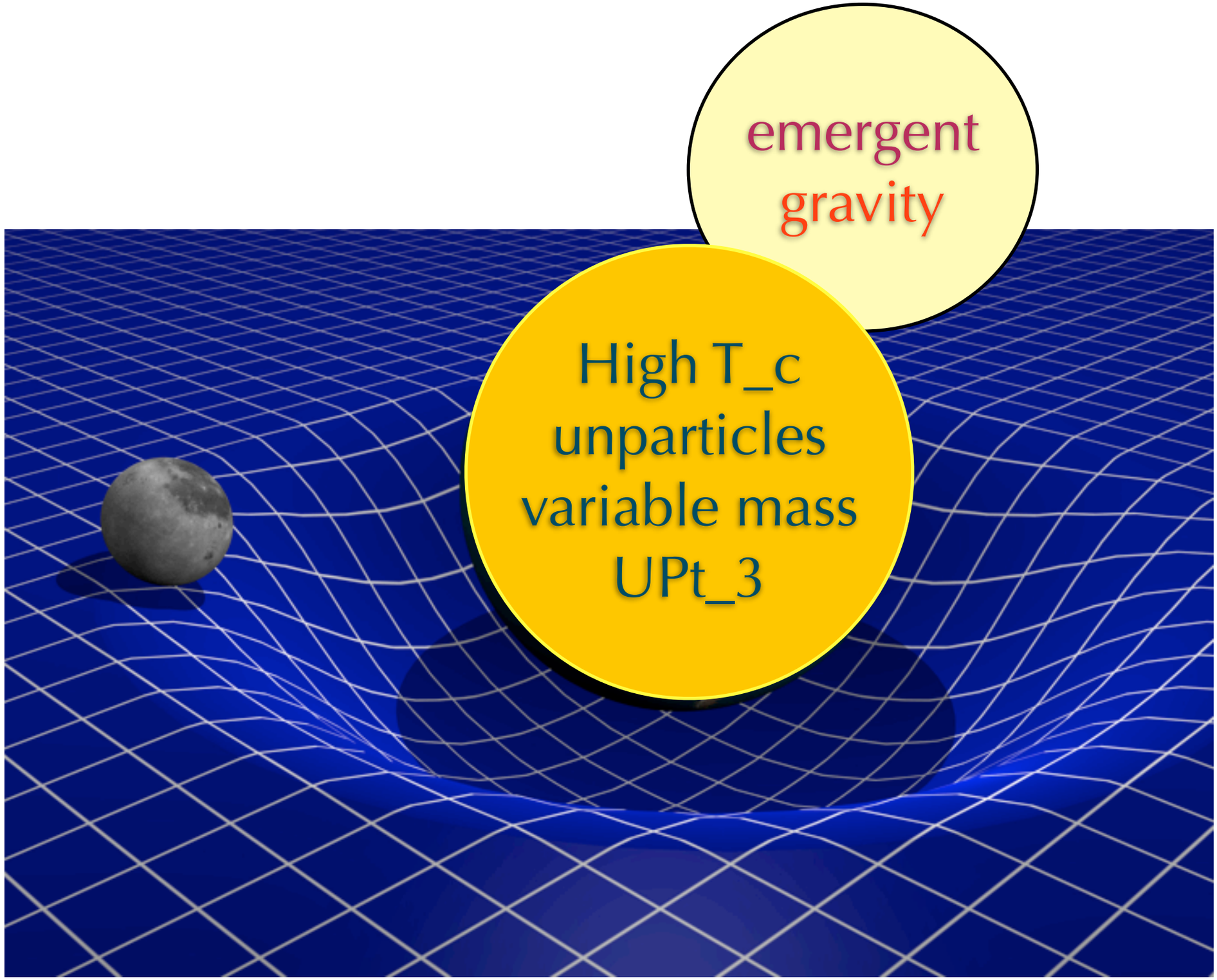
interchanging unparticles

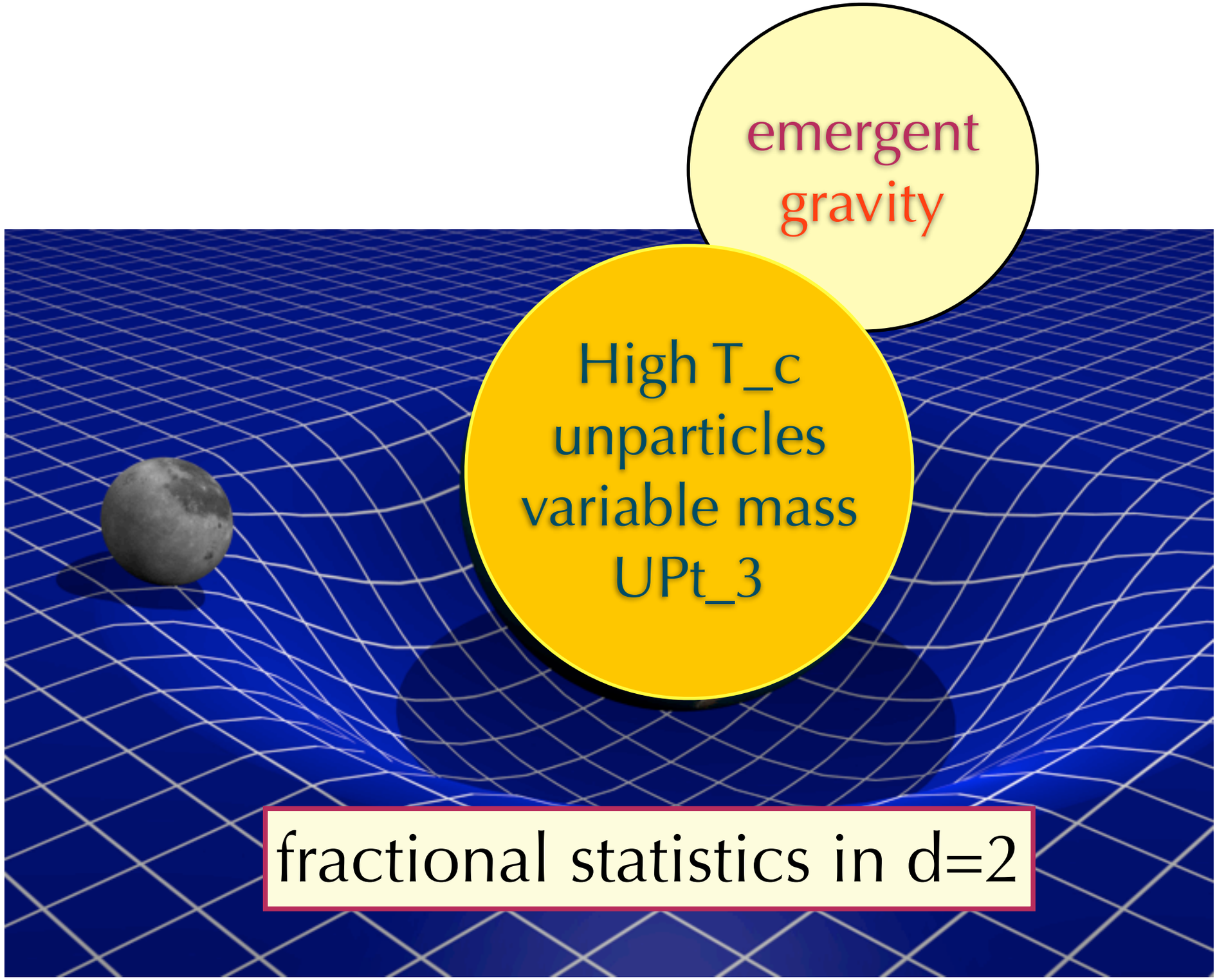
fractional (d_U) number of massless particles

$$e^{i\pi d_U} \neq -1, 0$$

fractional statistics in d=2

High T_c
unparticles
variable mass
UPt_3


emergent
gravity
High T_c
unparticles
variable mass
UPt_3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fractional statistics in d=2