Is Strongly Correlated Electron Matter Full of Unparticles?

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2 = 3
goal

\[2 \neq 3\]
Properties all particles share?
Properties all particles share?

- charge
- fixed mass!
Properties all particles share?

- charge
- fixed mass!
- conserved (gauge invariance)
Properties all particles share?

- charge
  - conserved (gauge invariance)
- fixed mass!
  - not conserved
mass sets a scale
mass sets a scale

mass breaks scale invariance
what is scale invariance?

invariance on all length scales
\[ f(x) = x^2 \]
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \]

scale change
$f(x) = x^2$

$f(x/\lambda) = (x/\lambda)^2$  

scale invariance

scale change
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \]  \hspace{1cm} \text{scale change}

\[ f(x) = x^2 \lambda^{-2} g(\lambda) \]  \hspace{1cm} \text{scale invariance}

\[ 1 \]
\[ f(x) = x^2 \]

\[ f(x/\lambda) = (x/\lambda)^2 \]  \hspace{1cm} \text{scale change}

\[ f(x) = x^2 \lambda^{-2} g(\lambda) \]

\[ g(\lambda) = \lambda^2 \]  \hspace{1cm} \text{scale invariance}

\[ 1 \]
\[ f(x) = ax^2 + bx^3 \]
\[ x \rightarrow x/\Lambda \]
\[ f(x) = ax^2 + bx^3 \]

\[ x \rightarrow x/\Lambda \]

\[ f(x) \rightarrow \Lambda^{-2}(ax^2 + bx/\Lambda) \]
\( f(x) = ax^2 + bx^3 \)

\[ x \rightarrow x/\Lambda \]

\[ f(x) \rightarrow \Lambda^{-2}(ax^2 + bx/\Lambda) \]

\textbf{not scale invariant}
Free field theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \]

\[ x \rightarrow \frac{x}{\Lambda} \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ \mathcal{L} \rightarrow \Lambda^2 \mathcal{L} \]
free field theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ \mathcal{L} \rightarrow \Lambda^2 \mathcal{L} \quad \text{scale invariant} \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]
The massive free theory is given by the action:

$$\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2$$

where $x \rightarrow x/\Lambda$ and $\phi(x) \rightarrow \phi(x)$. 

The mass is indicated as $m^2$. 

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massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \to x/\Lambda \]
\[ \phi(x) \to \phi(x) \]

mass
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

mass

\[ x \to x/\Lambda \]

\[ \phi(x) \to \phi(x) \]

\[ m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ \Lambda^2 \left( \frac{1}{2} \partial_\mu \phi \partial_\mu \phi \right) \]

\[ m^2 \phi^2 \]
massive free theory

\[ L = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\( x \rightarrow x/\Lambda \)
\( \phi(x) \rightarrow \phi(x) \)

\( \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \)

mass

no scale invariance

\( m^2 \phi^2 \)
how should we think about field theory?
how should we think about field theory?
how should we think about field theory?
what if $m$ depends on the scale?
what if \( m \) depends on the scale?
what if $m$ depends on the scale?

$L_{m_i+7} \; L_{m_i+6} \; L_{m_i+5} \; L_{m_i+4} \; L_{m_i+3} \; L_{m_i+2} \; L_{m_i+1} \; L_{m_i}$
sum over all mass

\[ \mathcal{L} = \sum_i \mathcal{L}(m_i) \]
\[ \mathcal{L} = \left( \partial_{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^2 \phi^2(x, m) \right) \]
\[ \mathcal{L} = \int_0^\infty (\partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m)) \, dm^2 \]
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial^{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]

\[ x \rightarrow x / \Lambda \]
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial_{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^{2} \phi^{2}(x, m) \right) \, dm^{2} \]

\[ \phi \rightarrow \phi(x, m^{2} / \Lambda^{2}) \]

\[ x \rightarrow x / \Lambda \]

theory with all possible mass!
\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]

\[ x \rightarrow x / \Lambda \]

theory with all possible mass!  

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

\[ \phi \rightarrow \phi(x, m^2/\Lambda^2) \]

\[ x \rightarrow x/\Lambda \]

theory with all possible mass!

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

\[ \phi \rightarrow \phi(x, m^2/\Lambda^2) \]

\[ x \rightarrow x/\Lambda \]

theory with all possible mass!

\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
scale-invariant stuff is weird
scale-invariant stuff is weird

particles

\[ E_{\text{photon}} = h \nu \]

\[ v_{\text{max}} = 6.22 \times 10^5 \text{ m/s} \]

Energy

\[ E = \varepsilon_p \]
scale-invariant stuff is weird

particles

\[ E_{\text{photon}} = h\nu \]

\[ v_{\text{max}} = 6.22 \times 10^5 \, \text{m/s} \]

\[ v_{\text{max}} = 2.96 \times 10^5 \, \text{m/s} \]

\[ 400 \, \text{nm} \quad 3.1 \, \text{eV} \]

\[ 550 \, \text{nm} \quad 2.25 \, \text{eV} \]

\[ 700 \, \text{nm} \quad 1.77 \, \text{eV} \]

Photoelectric effect

energy \[ E = \varepsilon_p \]

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle=\infty
scale-invariant stuff is weird

particles

$E = \varepsilon_p$

unparticles

$G(E) \propto (E - \varepsilon_p)^d_U$

$d_U > 0$

Green function (propagator)

$G(E) = \frac{1}{E - \varepsilon_p}$

particle=infinity
scale-invariant stuff is weird

particles

\[ E = \varepsilon_p \]

unparticles

\[ G(E) \propto (E - \varepsilon_p)^{d_U} \]

\[ d_U > 0 \]

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle=infinity

unparticles=zero propagator
counting particles
counting particles
counting particles
counting particles

1 2 3 4 5 6 7 8
counting particles

is there a more efficient way?
Each particle has an energy
Each particle has an energy

particle count is deducible from the Green function
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \epsilon_p} \]
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

\[ E > \varepsilon_p \]

\[ E < \varepsilon_p \]
Luttinger’s Theorem

Green function (propagator)

\[ G(E) = \frac{1}{E - \varepsilon_p} \]

particle density = number of sign changes of \( G \)

\( E > \varepsilon_p \)

\( E < \varepsilon_p \)
Counting sign changes?
Counting sign changes?

$\Theta(x)$
Counting sign changes?

\[ \Theta(x) = \begin{cases} 0 & x < 0 \end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 
\end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 \\
1/2 & x = 0 
\end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
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1/2 & x = 0 
\end{cases} \]
Counting sign changes?

\[ \Theta(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 0 \\
1/2 & x = 0 
\end{cases} \]

counts sign changes

\[ n = 2 \sum_p \Theta(G(E = 0, p)) \]

Luttinger Theorem for electrons
Is this famous theorem from 1960 correct?
How do functions change sign?
How do functions change sign?

$E \leq \varepsilon_p$

$E > \varepsilon_p$

$E < \varepsilon_p$

divergence
Is there another way?
zero-crossing
zero-crossing

no divergence is necessary
closer look at Luttinger’s theorem

\[ n = 2 \sum_{\mathbf{p}} \Theta(G(E = 0, \mathbf{p})) \]
closer look at Luttinger’s theorem

\[ n = 2 \sum_{p} \Theta(G(E = 0, p)) \]

divergences + zeros
closer look at Luttinger’s theorem

\[ n = 2 \sum_p \Theta(G(E = 0, p)) \]

divergences+ zeros

how can zeros affect the particle count?
zeros arise from unparticles
so what gives?
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails

no hopping => no propagation (zeros)
A model with zeros but Luttinger fails

no hopping => no propagation (zeros)
generalization

N flavors of spin
$e^{-}$

$N = 5$

$N$ flavors of spin

generalization

spin
$N = 5$
$e^-$

Spin generalization

$N$ flavors of spin

$2E$

$N = 5$

$25$

$16$

$9$

$4$

$1$
$H = \frac{U}{2} (n_1 + \cdots n_N)^2$
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ \{ \text{0, 1, 1/2} \]
Luttinger’s theorem

\[ n = N \Theta (2n - N) \]

\[ n = 2, 0, 1, 1/2 \]

\[ N = 3 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \quad 0, 1, 1/2 \]

\[ N = 3 \]

\[ 2 = 3 \]
Luttinger’s theorem

\[ n = N \Theta (2n - N) \]

\[ n = 2 \]

\[ N = 3 \]

\[ 0, 1, 1/2 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

even

\( n = 2 \)

\( N = 3 \)

\( 0, 1, 1/2 \)
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ p = 2 \]

\[ N = 3 \]

\[ 0, 1, 1/2 \]

even

odd

Monday, October 1, 2012
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\begin{align*}
&n = 2 \\
&N = 3
\end{align*}

\{0, 1, 1/2\}

even \quad odd

no solution
Luttinger’s theorem
Problem
Problem

G = 0
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p} = 0 \]
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p} = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p} = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]
Problem

\[ G = \frac{1}{E - \varepsilon_p} = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]
if \Sigma \text{ is infinite}
if $\Sigma$ is infinite

lifetime of a particle vanishes
if $\Sigma$ is infinite

lifetime of a particle vanishes

$\exists \Sigma < \epsilon_p$
if $\Sigma$ is infinite

lifetime of a particle vanishes

no particle

$\exists \Sigma < \epsilon_p$
zeros and particles are incompatible
zeros and particles are incompatible

Luttinger theorem is wrong!
how to count particles?
how to count particles?

some charged stuff has no particle interpretation
where do we expect to see zeros?
where do we expect to see zeros?

strongly correlated systems
Y$_{2}$Ba$_{3}$Cu$_{7}$O$_{7}$
Cuprate Superconductors
interactions dominate: Strong Coupling Physics

\[ \frac{U}{t} = 10 \gg 1 \]
photoemission
photoemission particles

\[ E_{\text{photon}} = h \nu \]

700 nm 1.77 eV
550 nm 2.25 eV
no electrons

\[ v_{\text{max}} = 2.96 \times 10^5 \text{ m/s} \]

Potassium - 2.0 eV needed to eject electron

Photoelectric effect

energy \[ E = \varepsilon_p \]
photoemission

particles

\[ E_{\text{photon}} = h\nu \]

\[ E = \varepsilon_p \approx p^2 \]

Photoelectric effect

Potassium - 2.0 eV needed to eject electron

700 nm 1.77 eV

550 nm 2.25 eV

no electrons

400 nm 3.1 eV

\( v_{\text{max}} = 6.22 \times 10^5 \text{ m/s} \)
photoemission

particles

\[ E_{\text{photon}} = h \nu \]

\[ v_{\text{max}} = 6.22 \times 10^5 \text{ m/s} \]

Photoelectric effect

energy

\[ E = \varepsilon_p \approx p^2 \]
Photoemission

Particles

\[ E_{\text{photon}} = h\nu \]

Energy

\[ E = \varepsilon_p \approx p^2 \]

Two crossings:
Closed surface of excitations
what is seen experimentally?
Fermi arcs: no double crossings (PDJ, JCC, ZXS)
what is seen experimentally?

Fermi arcs: no double crossings (PDJ, JCC, ZXS)

seen
infinities

not seen
zeros

\( E_F \)

\( k_x \to 1.0 \)

\( k_y \)
zeros do not affect the particle density
zeros do not affect the particle density

no Luttinger theorem
zeros do not affect the particle density

no Luttinger theorem

experimental data
do
particles + unparticles = infinities + zeros?
what really is the summation over mass?

\[ \mathcal{L}_{m_i+7} \quad \mathcal{L}_{m_i+6} \quad \mathcal{L}_{m_i+5} \quad \mathcal{L}_{m_i+4} \quad \mathcal{L}_{m_i+3} \quad \mathcal{L}_{m_i+2} \quad \mathcal{L}_{m_i+1} \quad \mathcal{L}_{m_i} \]
mass=energy
high energy (UV)

low energy (IR)

related to sum over mass

QFT
High energy (UV)

Low energy (IR)

\[ \frac{dg(E)}{dlnE} = \beta(g(E)) \]

Related to sum over mass

Locality in energy
implement E-scaling with an extra dimension

low energy (IR)

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]

locality in energy

related to sum over mass

QFT

high energy (UV)
implement E-scaling with an extra dimension

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]

related to sum over mass

locality in energy
implement E-scaling with an extra dimension

gauge-gravity duality (Maldacena, 1997)

\[ \frac{dg(E)}{d\ln E} = \beta(g(E)) \]

related to sum over mass

locality in energy
\[ \mathcal{L} = \int_0^{\infty} \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]
unparticles

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

classical gravity in a d+1 curved spacetime
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

classical gravity in a d+1 curved spacetime

gauge theory in \( AdS_4 \)
High $T_c$
unparticles
variable mass
$\text{UPt}_3$