Mottness and Holography: Spectral weight transfer and T-linear Resistivity

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PRL, 99, 46404 (2007);
PRB, 77, 14512 (2008); ibid, 77, 104524 (2008);
ibid, 79, 245120 (2009); RMP, 82, 1719 (2010)..., DMR/NSF

GGI Talk: Nov. 4, 2010
Dynamically Generated Gap

gauge, global symmetry breaking

chiral symmetry breaking, superconductivity
Dynamically Generated Gap

- gauge, global symmetry breaking
- chiral symmetry breaking, superconductivity
Dynamically Generated Gap

No symmetry breaking

emergent bound states not in UV: QCD(pions), vulcanized rubber, Mott insulators
interactions dominate: Strong Coupling Physics

\[ \frac{U}{t} = 10 \gg 1 \]
$U/t = 10 \gg 1$

interactions dominate: Strong Coupling Physics

2D Hubbard Model

YBa$_2$Cu$_3$O$_7$
Cuprate Superconductors
$H = -t \sum_{i,j} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$
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\[ H = -t \sum_{i,j} c_i^{\dagger} c_j + U \sum_i n_{i\uparrow} n_{i\downarrow} \]

- Mott insulator
- antiferromagnetism
- Strange Metal
- pseudogap
- d-wave superconductivity
\[ H = -t \sum_{i,j} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \]
Mott insulator

\[ \rho \sim aT \]

AF

temperature

Mott insulator

pseudo-gap

\[ \infty \]

\[ \infty \]

d-wave superconductor

\[ \infty \]

"strange" metal

0

conventional metal?

parent

under doped

optimally doped

over doped

doping

\[ x \]
Mott insulator $\rho \sim aT$
Mott insulator $\rho \sim aT$
How does Fermi Liquid Theory Breakdown?

$6,000,000$ question?

$\rho \sim aT$

AF

Mott insulator

temperature

pseudo-gap

$d$-wave superconductor

parent

under doped

optimally doped

over doped

doping

“strange” metal

conventional metal?
T-linear resistivity
T-linear resistivity

quantum criticality
T-linear resistivity

quantum criticality

\[ \frac{1}{\tau_{\text{scatt}}} \propto T \quad \Rightarrow \quad \rho \propto T \]
T-linear resistivity

quantum criticality

$\frac{1}{\rho \text{ scatt}} \propto T$
1.) one critical length scale
2.) charge carriers are critical
3.) charge conservation
\[ S \rightarrow S_{\text{whatever}} + \int d\tau d^d x A^\mu j_\mu \]

Vector potential

current
\[ S \rightarrow S_{\text{whatever}} + \int d\tau d^d x A^\mu j_\mu \]

**Vector potential**

**current**

**Charge conservation:** \( d_A = 1 \)
\[ \ln Z = \frac{L^d \beta}{\xi d \xi_t} F(\delta \xi^d, \{ A^i_{\lambda} \xi^d A \}) \]

\[ A^i_{\lambda} = A^i (\omega = \lambda \xi_t^{-1}) \]
\[ \ln Z = \frac{L^d \beta}{\xi d \xi_t} F(\delta \xi^d \delta, \{ A^i_\lambda \xi^d A \}) \]

\[ A^i_\lambda = A^i(\omega = \lambda \xi_t^{-1}) \]

\[ \sigma_{ij} = \frac{1}{L^d \beta \omega} \frac{\delta^2 \ln Z}{\delta A^i(-\omega) \delta A^j(\omega)} \]
\( \sigma(T) \propto T^{(d-2)/z} \)

\( E \propto p^z \)

dynamical exponent
E \propto p^z
\sigma(T) \propto T^{(d-2)/z}
\rho \propto T
\text{only if } z < 0

d=3

E \propto \rho^z \quad \text{dynamical exponent}

\sigma(T) \propto T^{(d-2)/z}

\rho \propto T \quad \text{only if } z < 0

One-parameter Scaling breaks down

\[ \rho \propto T \quad \rightarrow \quad \text{new length scale} \]

\[ \text{new degree of freedom} \]
How to break Fermi liquid theory in $d=2+1$?
Polchinski, Shankar, others

\[ p = k + l, \]

1.) e- charge carriers

2.) Fermi surface

\[ \int dt \, d^2k_1 \, dl_1 \, d^2k_2 \, dl_2 \, d^2k_3 \, dl_3 \, d^2k_4 \, dl_4 \, V(k_1, k_2, k_3, k_4) \psi_\sigma^\dagger(p_1) \psi_\sigma(p_3) \psi_\sigma^\dagger(p_2) \psi_\sigma'(p_4) \delta^3(p_1 + p_2 - p_3 - p_4). \]

No relevant short-range 4-Fermi terms in \( d \geq 2 \)
Polchinski, Shankar, others

\[ p = k + l, \]

1.) e- charge carriers

2.) Fermi surface

\[ \int dt \, d^2k_1 \, dl_1 \, d^2k_2 \, dl_2 \, d^2k_3 \, dl_3 \, d^2k_4 \, dl_4 \, V(k_1, k_2, k_3, k_4) \psi^\dagger_\sigma(p_1)\psi_\sigma(p_3)\psi^\dagger_\sigma(p_2)\psi_\sigma(p_4)\delta^3(p_1 + p_2 - p_3 - p_4). \]

No relevant short-range 4-Fermi terms in \( d \geq 2 \)

Exception: Pairing
Landau Correspondence

Free system

How does this break down?

low-energy: one-to-one correspondence

Interacting system
Atomic Limit of Hubbard Model

\[ H_{\text{Hubb}} \rightarrow U \sum_i n_{i\uparrow} n_{i\downarrow} \]

\( x=\# \text{ of holes} \)
Atomic Limit of Hubbard Model

\[ H_{\text{Hubb}} \rightarrow U \sum_i n_{i\uparrow} n_{i\downarrow} \]

\[ G(\omega, k) = \frac{1 + x}{\omega - \mu + U/2} + \frac{1 - x}{\omega - \mu - U/2} \]

\( x = \# \text{ of holes} \)
Atomic Limit of Hubbard Model

**spectral weight: x-dependent**

\[ H_{\text{Hubb}} \rightarrow U \sum_i n_{i\uparrow}n_{i\downarrow} \]

\[ G(\omega, k) = \frac{1 + x}{\omega - \mu + U/2} + \frac{1 - x}{\omega - \mu - U/2} \]

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Atomic Limit of Hubbard Model

spectral weight: \( x \)-dependent

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H_{\text{Hubb}} \rightarrow U \sum_i n_i^\uparrow n_i^\downarrow
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G(\omega, k) = \frac{1 + x}{\omega - \mu + U/2} + \frac{1 - x}{\omega - \mu - U/2}
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\( x = \# \text{ of holes} \)

PES

IPES

density of states
Atomic Limit of Hubbard Model

spectral weight: $x$-dependent

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density of states

PES

IPES

$\varepsilon_F$
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density of states

PES \hspace{1cm} \text{IPES}
Atomic Limit of Hubbard Model

spectral weight: \( x \)-dependent

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H_{\text{Hubb}} \rightarrow U \sum_i n_{i\uparrow} n_{i\downarrow}
\]

\[
G(\omega, k) = \frac{1 + x}{\omega - \mu + U/2} + \frac{1 - x}{\omega - \mu - U/2}
\]

\[
\xi_{i\sigma}^\dagger = c_{i\sigma}^\dagger (1 - n_{i\sigma})
\]

\[
1 + x
\]

\[
1 - x
\]

\[
\varepsilon_F
\]

PES

IPES

x=\# of holes

density of states
Atomic Limit of Hubbard Model

spectral weight: $x$-dependent

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$$\xi_{i \sigma} = c_{i \sigma}^\dagger (1 - n_{i \bar{\sigma}})$$

$$\eta_{i \sigma} = c_{i \sigma}^\dagger n_{i \bar{\sigma}}$$

$x =$ # of holes

density of states

PES

IPES

$\varepsilon F$
atomic limit

total weight = 1 + x = # of ways electrons can be added in lower band

intensity of lower band = # of electrons the band can hold

no problems yet!
quantum Mottness: $U$ finite

$U \gg t$
quantum Mottness: $U$ finite $U \gg t$

double occupancy in ground (all) state!!
quantum Mottness: $U$ finite

$U \gg t$

double occupancy in ground (all) state!!

$W_{PES} > 1 + x$
Harris & Lange, 1967

\[ \alpha = \frac{t}{U} \sum_{ij} \langle c_i^{\dagger} c_j \rangle > 0 \]

\[ 1 + x + \alpha(t/U, x) \]

the rest of this state lives at high energy
\[ \alpha = \frac{t}{U} \sum_{ij} \langle c^\dagger_{i\sigma} c_{j\sigma} \rangle > 0 \]

\[ 1 + x + \alpha(t/U, x) \]
$$\alpha = \frac{t}{U} \sum_{ij} \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle > 0$$

$$1 + x + \alpha(t/U, x)$$

$$1 - x - \alpha$$
\[ \alpha = \frac{t}{U} \sum_{ij} \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle > 0 \]

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\[ 1 - x - \alpha \]

Intensity > 1 + x

New non-electron degrees of freedom
Can this system be a Fermi liquid?

\[
\alpha = \frac{t}{U} \sum_{ij} \langle c_i^{\dagger} c_j \rangle > 0
\]

\[
1 + x + \alpha(t/U, x)
\]

\[
1 - x - \alpha
\]

Intensity > 1 + x

New non-electron degrees of freedom

Can this system be a Fermi liquid?
No
what are the low-energy fermionic excitations?

\[ 1 + x + \alpha \]

\[ 2 \]

\[ 1 - x - \alpha \]
what are the low-energy fermionic excitations?

\[ 1 + x + \alpha \]

\[ 1 - x - \alpha \]
what are the low-energy fermionic excitations?

2 re-definition: \( x' = x + \alpha \)

\[ 1 + x + \alpha \]

\[ 1 - x - \alpha \]

\[ \varepsilon F \]
what are the low-energy fermionic excitations?

re-definition: \[ x' = x + \alpha \]

get rid of dynamical mixing
what are the low-energy fermionic excitations?

2) re-definition: \( x' = x + \alpha \)

get rid of dynamical mixing
what are the low-energy fermionic excitations?

$$x' = x + \alpha$$

FLT breaks down

get rid of dynamical mixing
\[ 1 - x - \alpha \]

\[ 2(x + \alpha) \]

\[ \varepsilon F \]

\[ 1 + x + \alpha \]
The number of fermionic quasiparticles is less than the number of bare electrons, so the Fermi-liquid (FL) theory breaks down.

New degrees of freedom are introduced when the number of fermionic quasiparticles is less than the number of bare electrons.
The number of fermionic qp is less than the number of bare electrons, which causes the FL theory to break down. There are new degrees of freedom per electron per spin greater than 1.
number of fermionic qp 
< number of bare 
electrons => FL theory 
breaks down

ways to add a particle 
but not an electron 
(gapped spectrum)

# of addition states 
per e per spin > 1

new degrees 
of freedom

1 - x - \alpha

\varepsilon_F

1 + x + \alpha

2(x + \alpha)
ways to add a particle but not an electron (gapped spectrum)

breakdown of electron quasi-particle picture: Mottness

number of fermionic qp < number of bare electrons => FL theory breaks down

new degrees of freedom

# of addition states per e per spin > 1

ways to add a particle but not an electron (gapped spectrum)
No proof exists?
Mottness is ill-defined

Same Physics at half-filling

$U > U_c$
No proof exists?
Mottness is ill-defined

Same Physics at half-filling

$U > U_c$
A Critique of Two Metals

R. B. Laughlin

idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.
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Church of weak coupling

Beliefs:
Mott gap is heresy?
HF is the way!
No UHB and LHB!
\[ \Delta = 0.6\text{eV} > \Delta_{\text{dimerization}} \]  
(Mott, 1976) \[ \frac{\Delta}{T_{\text{crit}}} \approx 20 \]

Recall, \[ eV = 10^4 K \]

of spectral weight to high energies beyond any ordering scale
\[ \Delta = 0.6eV > \Delta_{\text{dimerization}} \] (Mott, 1976) \[ \frac{\Delta}{T_{\text{crit}}} \approx 20 \]

Transfer of spectral weight to high energies beyond any ordering scale.

Recall, \( eV = 10^4 K \)

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Transfer of spectral weight to high energies beyond any ordering scale

Recall, \( eV = 10^4 K \)

Fermi-liquid analogy

\[ L_{FL} \propto (\omega - \epsilon_k) |\psi_k|^2 \]

Mott Problem?

\[ L_{MI} = (\omega - E_{LHB}(k)) |\eta_k|^2 + (\omega - E_{UHB}(k)) |\tilde{\eta}_k|^2 \]
half-filling: Mott gap
doping: SWT, pseudogap?
charge 2e boson

$W_{PES} > 1 + x$
How?
Effective Theories:

\[ S(\phi) \text{ at half-filling} \]

Integrate Out high Energy fields

\[ \phi = \phi_L + \phi_H \]

\[ e^{-S_{\text{eff}}[\phi_L]} = \int d\phi_H \exp -S(\phi_L, \phi_H) \]

Low-energy theory of MI
Half-filling

\[ N(\omega) \]

Integrate out both
Key idea: similar to Bohm/Pines

Extend the Hilbert space:
Associate with U-scale new
Fermionic oscillators

\[ N(\omega) \]

\[ \frac{U}{2} \tilde{D}_i^\dagger \tilde{D}_i \]

\[ \frac{U}{2} D_i^\dagger D_i \]
$D_i^\dagger$ transforms as a fermion.

Fermionic constraint

one per site (fermionic)

transforms as a boson
$D_i^\dagger$ transforms as a boson.

Fermionic constraint

one per site (fermionic)

transforms as a boson
one per site (fermionic)

Fermionic constraint

transforms as a boson

`supersymmetry'
$D_i^\dagger$ transforms as a boson

Fermionic constraint

one per site (fermionic)

`supersymmetry`

$\delta(D_i - \theta c_{i\uparrow} c_{i\downarrow})$

Grassmann
\[ D_i^\dagger \text{Fermionic constraint} \]

one per site (fermionic)

transforms as a boson

`supersymmetry`

\[ \delta(D_i - \theta c_i^\uparrow c_i^\downarrow) \]

Grassmann

\[ \bar{\theta} \varphi \text{ constraint field} \]
Dual Theory

solve constraint
\[ \varphi (Q_\varphi = 2e) \]

integrates over heavy fields

Exact low-energy theory (IR limit)

UV limit

\[ \int d^2 \theta \bar{\theta} \vartheta L_{\text{Hubb}} = \sum_{i, \sigma} c_i^\dagger \dot{c}_{i, \sigma} + H_{\text{Hubb}}, \]
Dual Theory

solve constraint
\( \varphi (Q_\varphi = 2e) \)

UV limit

\[ \int d^2 \theta \, \bar{\theta} \theta L_{\text{Hubb}} = \sum_{i,\sigma} c_{i,\sigma}^{\dagger} \dot{c}_{i,\sigma} + H_{\text{Hubb}}, \]

Exact low-energy theory (IR limit)

\[ L_{\text{UV}}^{hf} = \int d^2 \theta \left[ iD^{\dagger} \dot{D} - i\tilde{D}^{\dagger} \tilde{D} - \frac{U}{2} (D^{\dagger} D - \tilde{D} \tilde{D}^{\dagger}) \right. \]
\[ + \frac{t}{2} D^{\dagger} \theta b + \frac{t}{2} \bar{\theta} b \tilde{D} + h.c. + s\bar{\theta} \varphi^{\dagger} (D - \theta c_{\uparrow} c_{\downarrow}) \]
\[ + \check{s} \bar{\theta} \check{\varphi}^{\dagger} (\tilde{D} - \theta c_{\uparrow} c_{\downarrow}) + h.c. \]
dynamics of $\varphi$
Exact IR Lagrangian

bare fields have no dynamics

\[ L_{\text{IR}}^{hf} \rightarrow 2 \frac{|s|^2}{U} |\varphi_\omega|^2 + 2 \frac{|\tilde{s}|^2}{U} |\tilde{\varphi}_-\omega|^2 + \frac{t^2}{U} |b_\omega|^2 \]

\[ \begin{aligned} &+ s \gamma_{\tilde{p}}^{(\tilde{k})} (\omega) \varphi_\omega^{\dagger} c_{\tilde{k}/2+\tilde{p},\omega/2+\omega',\uparrow} c_{\tilde{k}/2-\tilde{p},\omega/2-\omega',\downarrow} \\
&+ \tilde{s}^* \gamma_{\tilde{p}}^{(\tilde{k})} (\omega) \tilde{\varphi}_-\omega^{\dagger} c_{\tilde{k}/2+\tilde{p},\omega/2+\omega',\uparrow} c_{\tilde{k}/2-\tilde{p},\omega/2-\omega',\downarrow} + h.c. \end{aligned} \]

\[ \gamma_{\tilde{p}}^{(\tilde{k})} (\omega) = \frac{U - t \varepsilon_{\tilde{p}}^{(\tilde{k})} - 2\omega}{U} \sqrt{1 + 2\omega/U} \]

\[ \tilde{\gamma}_{\tilde{p}}^{(\tilde{k})} (\omega) = \frac{U + t \varepsilon_{\tilde{p}}^{(\tilde{k})} + 2\omega}{U} \sqrt{1 - 2\omega/U} \]

\[ \varepsilon_{\tilde{p}}^{(\tilde{k})} = 4 \sum_\mu \cos(k_\mu a/2) \cos(p_\mu a) \]

bosons and fermions are strongly coupled

turn-on of spectral weight governed by composite excitations (CEXONS)
\[ \gamma_{\vec{k}}(\omega) = \frac{U - t\varepsilon_{\vec{p}}^{(k)} - 2\omega}{U} \sqrt{1 + 2\omega/U} \]

\[ \tilde{\gamma}_{\vec{k}}(\omega) = \frac{U + t\varepsilon_{\vec{p}}^{(k)} + 2\omega}{U} \sqrt{1 - 2\omega/U}. \]

\[ \Delta = U - 4dt \]

Composite excitations determine spectral density.

\[ \gamma = 0 \quad \tilde{\gamma} = 0 \]

Each momentum has SD at two distinct energies.
hole-doping?
Extend the Hilbert space: Associate with U-scale a new Fermionic oscillator
Electron spectral function

$t^2/U \sim 60\text{meV}$
Electron spectral function

$t^2/U \approx 60\text{meV}$
Electron spectral function

\[
\Lambda = 1 - \chi
\]

\[
\Lambda_{\mu^-}
\]

\[2/U \sim 60 \text{meV}\]
Electron spectral function

Conserved charge: \[ Q = \sum_i c_i^\dagger c_i + 2 \sum_i \varphi_i^\dagger \varphi_i \]

Two bands!!

Spin-charge separation?
Origin of two bands

Two charge e excitations

$\varphi_i \sigma_i \tilde{c}$

$\varphi_i \sigma_i \tilde{c}$

$\varphi_i \sigma_i \tilde{c}$

$\varphi_i$ is confined (no kinetic energy)

New bound state

Pseudogap
two types of charges

'free'

bound
direct evidence
direct evidence

charge carrier density:
direct evidence

charge carrier density:


\[ n_{\text{Hall}}(x, T) = n_0(x) + n_1(x) \exp\left(-\frac{\Delta(x)}{T}\right), \]


\( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \)
direct evidence
charge carrier density:

\begin{align*}
\sum_{\text{below}} r & = 0 \\
\frac{1}{n_{\text{eff}}} & = n_0(x) + n_1(x) \exp(-\Delta(x)/T),
\end{align*}

exponentially suppressed: confinement


\( \text{La}_{2-x} \text{Sr}_x \text{CuO}_4 \)
Our Theory

Exponential $T$-dependence
Our Theory

exponential T-dependence

no model-dependent free parameters: just $t/U$
strange metal: breakup (deconfinement) of bound states

QCP:
T-linear resistivity

New scenario
as x increases
T-linear resistivity

New scenario

as $x$ increases
T-linear resistivity

New scenario as $x$ increases

Diagram: Graph showing $E_B$ and $\mu$ axes with $E$ on the right. The graph illustrates different regions:
- Bound $E_B > 0$
- Unbound $E_B = 0$

Symbols and expressions:
- $c_{i\sigma}^\dagger$, $\varphi_i c_{i\bar{\sigma}}$
- $\rho \sim T$
Mottness from holography

\[ H = -t \sum_{i,j} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \]

\( AdS_{d+1} \)

RN black hole
Mottness from holography

\[ H = -t \sum_{i,j} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \]
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Mottness from holography

\[ H = -t \sum_{i,j} c^\dagger_i \sigma c_j \sigma + U \sum_i n_{i\uparrow} n_{i\downarrow} \]

AdS\(_{d+1}\)

RN black hole
$\sqrt{-g}i\bar{\psi}(D - m)\psi$

AdS-RN

bottom-up schemes

`non-Fermi liquids'
\[
\sqrt{-g} \psi (D - m) \psi
\]

**AdS-RN**

**bottom-up schemes**

`non-Fermi liquids`

Mott Insulator
Consider fermions in RN AdS_{d+1} coupled to a gauge field through a dipole interaction.

$$\sqrt{-g} \bar{\psi} (D - m) \psi$$

Mott Insulator

`non-Fermi liquids'

bottom-up schemes
near horizon

radial Dirac Equation

$$\psi_{I\pm}(\zeta) = \psi_{I\pm}^{(0)}(\zeta) + \omega \psi_{I\pm}^{(1)}(\zeta) + \omega^2 \psi_{I\pm}^{(2)}(\zeta) + \cdots$$

$$-\psi_{I\pm}^{(0)''}(\zeta) = i\sigma_2 \left( 1 + \frac{q e_d}{\zeta} \right) - \frac{L_2}{\zeta} \left[ m \sigma_3 + \left( p e_d \pm \frac{k L}{r_0} \right) \sigma_1 \right] \psi_{I\pm}^{(0)}(\zeta),$$

$$e_d = 1/\sqrt{2d(d-1)}$$

$$m_k^2 = m^2 + \left( p e_d \pm \frac{k L}{r_0} \right)^2$$

p shifts momenta up and down through mass coupling
\[ \psi_{I_{\pm}}^{(0)}(\zeta) \]

\[ \mathcal{O}_\pm \]

**IR CFT**

**scaling dimension**

\[ \delta_\pm = \nu_\pm + \frac{1}{2} \]

\[ \nu_\pm = \sqrt{m_k^2 L_2^2 - q^2 e_d^2} - i\epsilon \]
How is the spectrum modified?
How is the spectrum modified?

P=0

Fermi surface peak
How is the spectrum modified?

$P = 0$

$P > 4.2$

Fermi surface peak
How is the spectrum modified?

$P=0$

spectral weight transfer

Fermi surface peak

dynamically generated gap:

$P > 4.2$
How is the spectrum modified?

Fermi surface peak

spectral weight transfer

dynamically generated gap:

\[ A(k, \omega) \]
How is the spectrum modified?

Fermi surface peak

spectral weight transfer

dynamically generated gap:

\( P > 4.2 \)
spectral weight
transfer
UV-IR mixing
Mott gap

Total weight varies with $n$
Mott gap

Total weight varies with $n$
Total weight varies with n
Finite Temperature Mott transition

\[ \frac{\Delta}{T_{\text{crit}}} \approx 10^3 \]

\[ \frac{\Delta}{T_{\text{crit}}} \approx 20 \] vanadium oxide
quasi-normal modes: where are the peaks?

$k=1 \ p=5$
quasi-normal modes: where are the peaks?

$k=1 \ p=5$
quasi-normal modes: where are the peaks?

\[ k=1 \quad p=5 \]
quasi-normal modes: where are the peaks?
quasi-normal modes: where are the peaks?

no leaking to +Im\omega: no instability
dynamical
spectral
weight
transfer
Hubbard

dynamical spectral weight transfer