

Demystifying the Strange Metal in High-temperature

Superconductors: Composite Excitations

Thanks to: T.-P. Choy, R. G. Leigh, S. Chakraborty, S. Hong
PRL, 99, 46404 (2007);
PRB, 77, 14512 (2008); *ibid*, 77, 104524 (2008));
ibid, 79, 245120 (2009); *ibid*, 80, 132505 (2009)..., DMR/
NSF-ACIF

High temperature superconductor?

High temperature superconductor?

Unusually
Good Metal

Matthias Rules for Superconductivity

Matthias Rules for Superconductivity

Matthias, Bernd T.



Scanned at the American
Institute of Physics

Matthias Rules for Superconductivity

1.) cubic structures

2.) avoid oxygen

3.) avoid magnetism

4.) avoid insulators

Matthias, Bernd T.



Scanned at the American
Institute of Physics

Matthias Rules for Superconductivity

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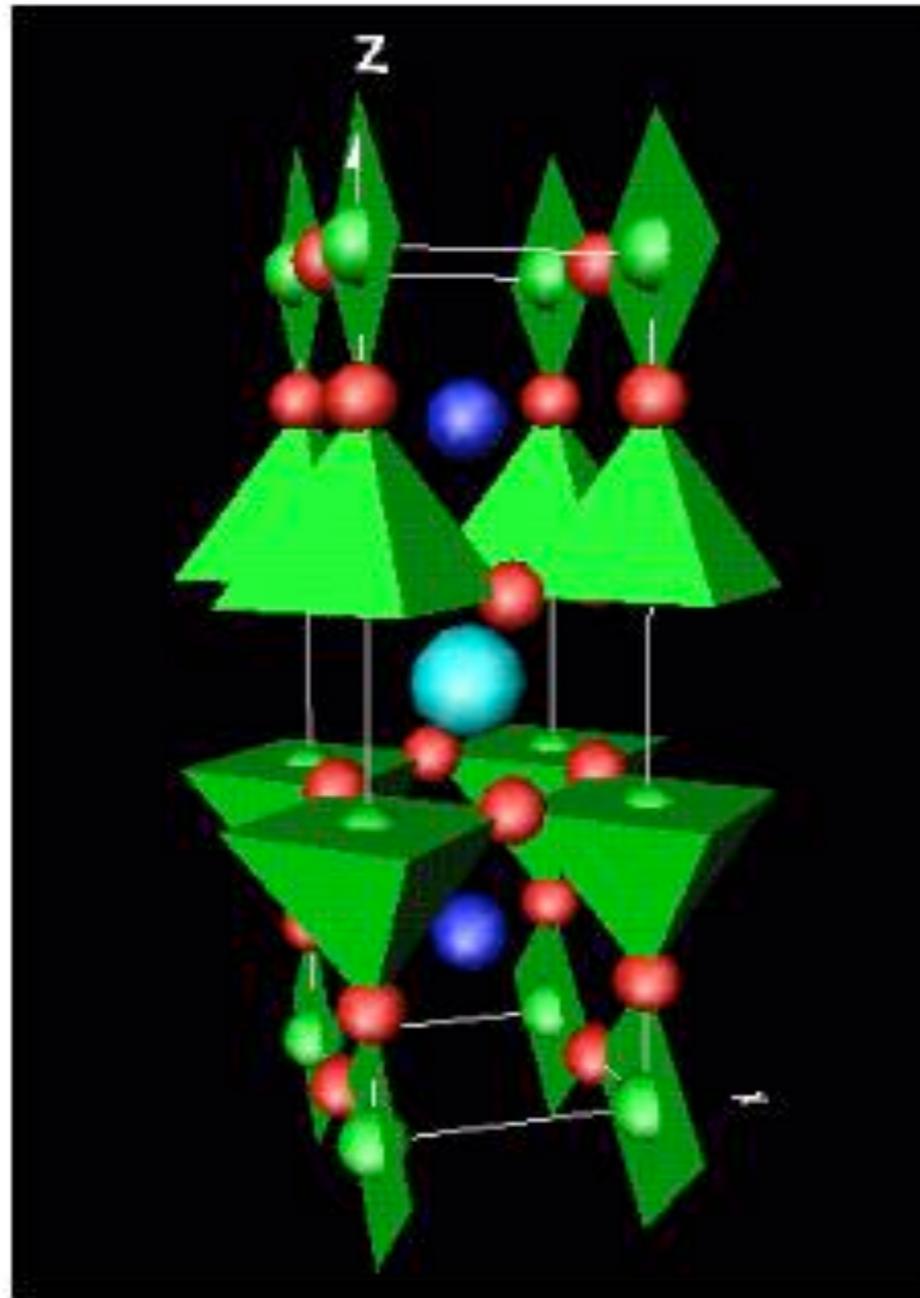
4.) avoid insulators

5.) don't talk to
theorists!!

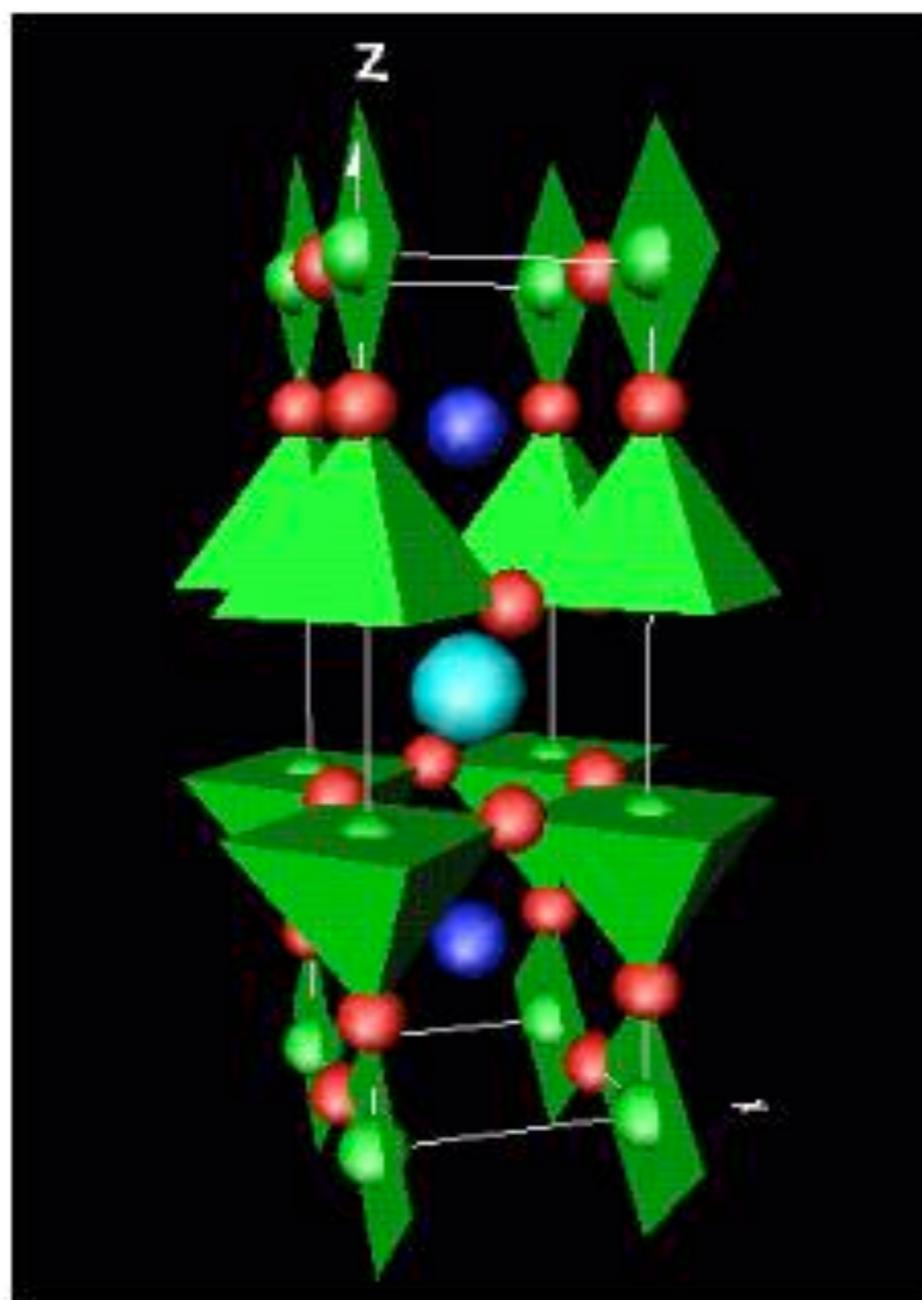
Matthias, Bernd T.



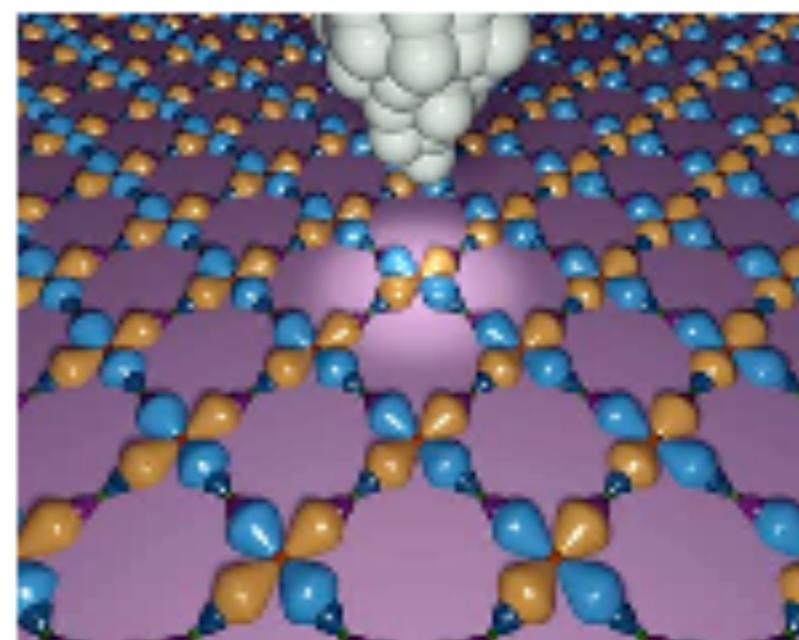
Scanned at the American
Institute of Physics



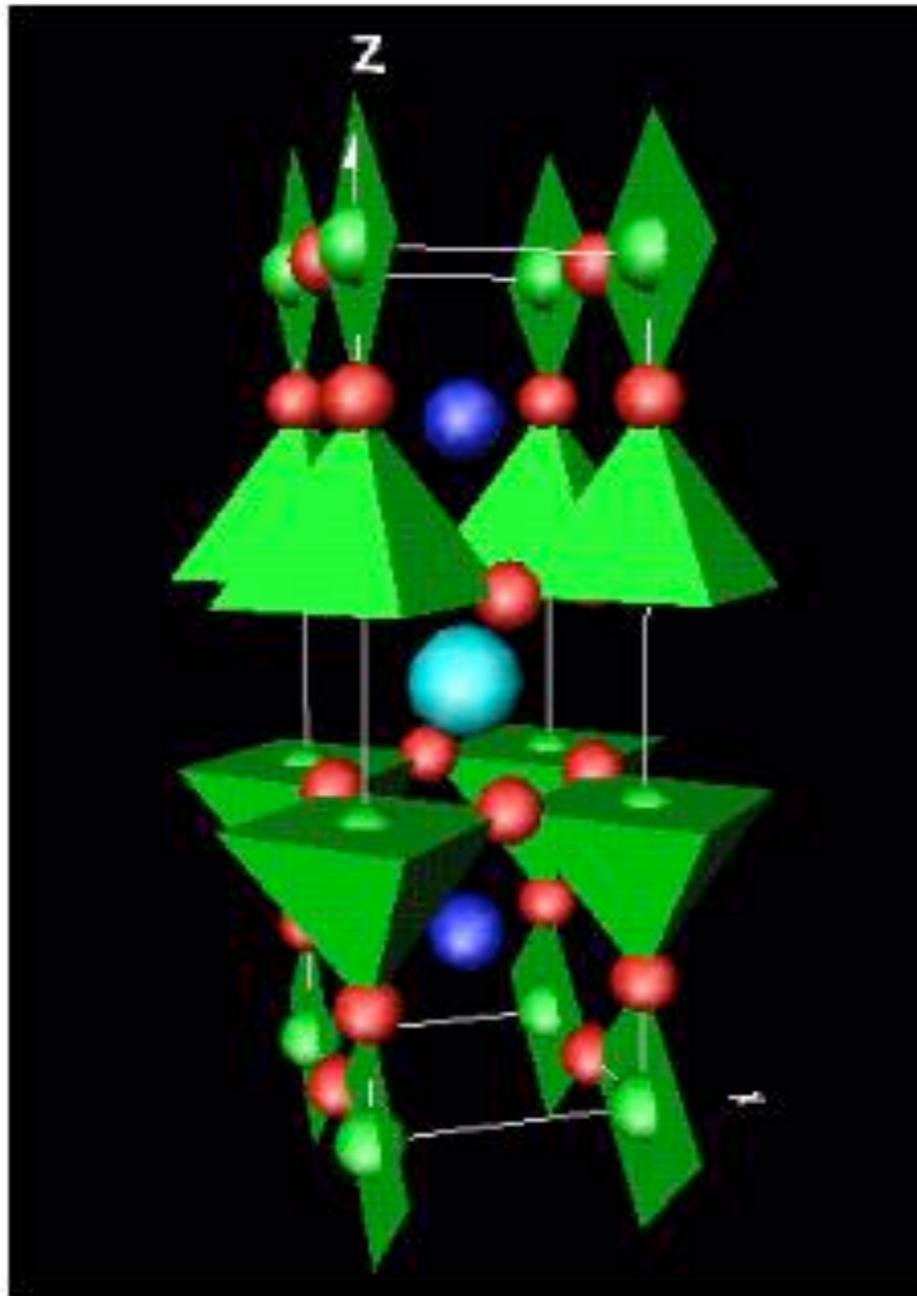
$\text{YBa}_2\text{Cu}_3\text{O}_7$
Cuprate Superconductors



Cu-O plane

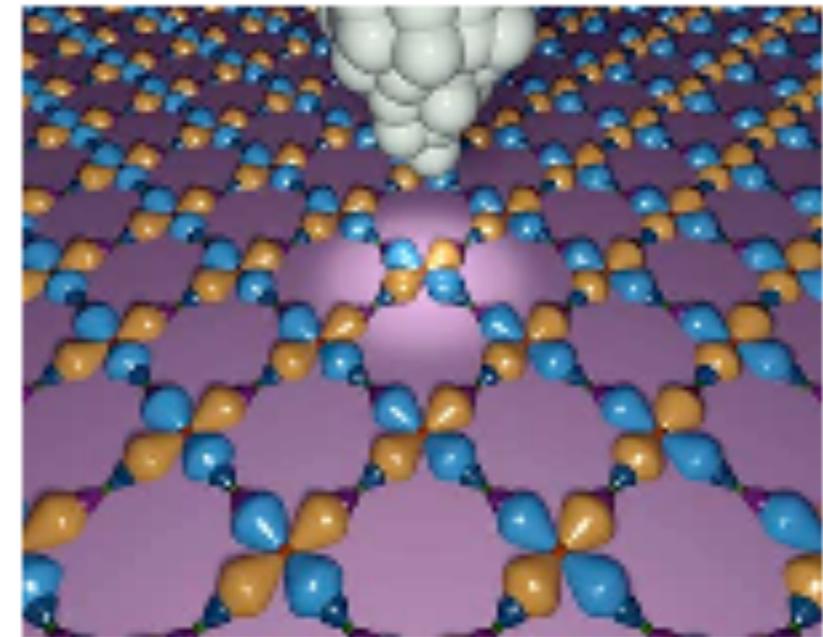


$\text{YBa}_2\text{Cu}_3\text{O}_7$
Cuprate Superconductors

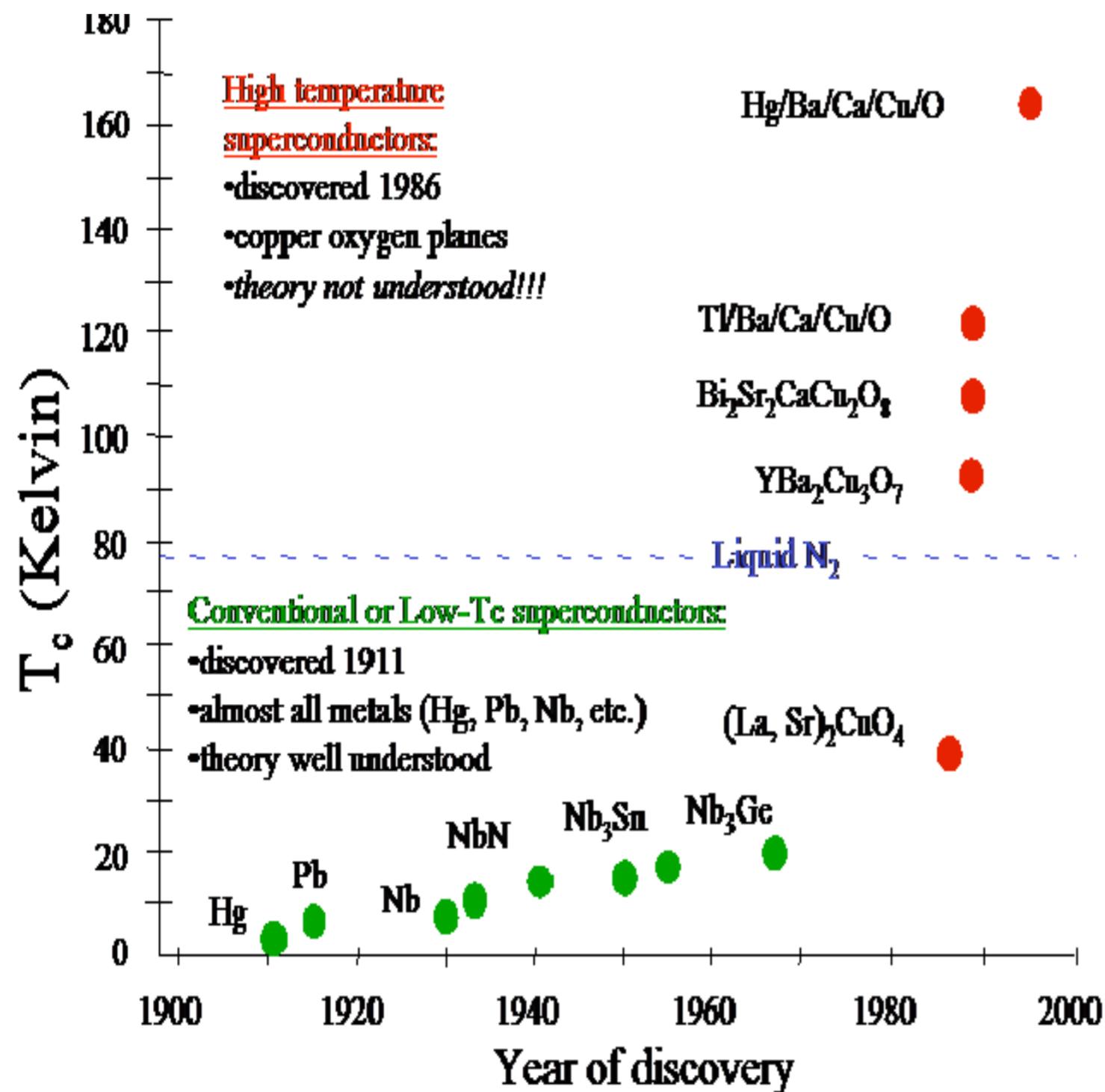


$\text{Y Ba}_2 \text{Cu}_3 \text{O}_7$
Cuprate Superconductors

Cu-O plane



Orthorhombic:
asymmetric in xy (a-b)
plane



What is left of Matthias' Rules?

1.) cubic structures

2.) avoid oxygen

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What is left of Matthias' Rules?

New Problem: Mottness

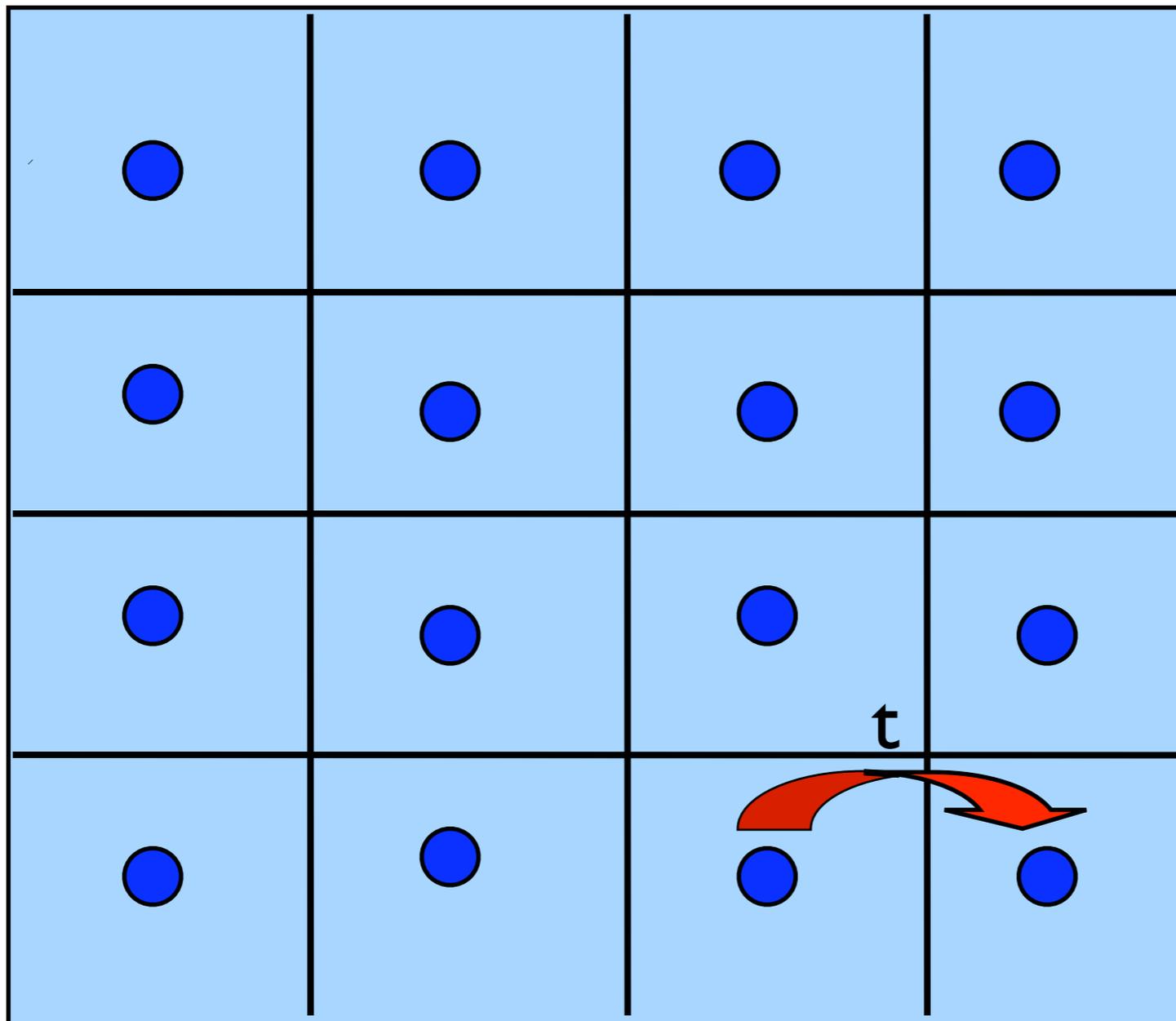
Sir Neville Mott
Nobel Prize, 1977

Mott
Insulators

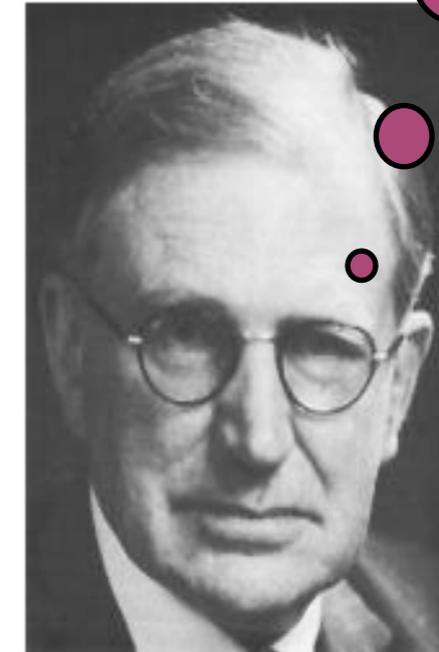


**Mott Problem:
NiO (Band
theory failure)**

Loomis floor plan (N rooms N occupants)



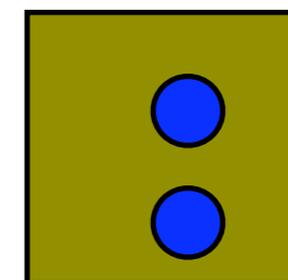
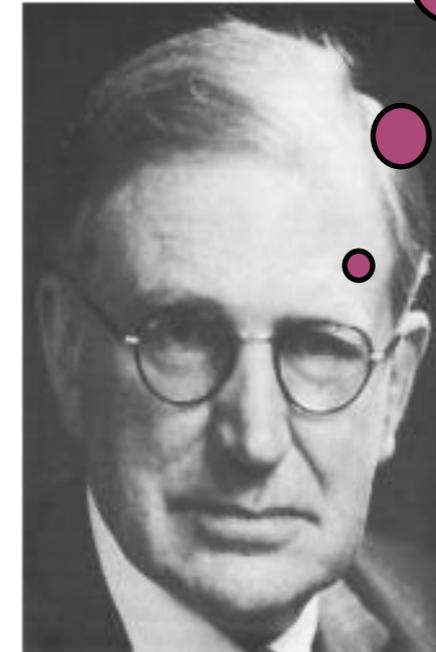
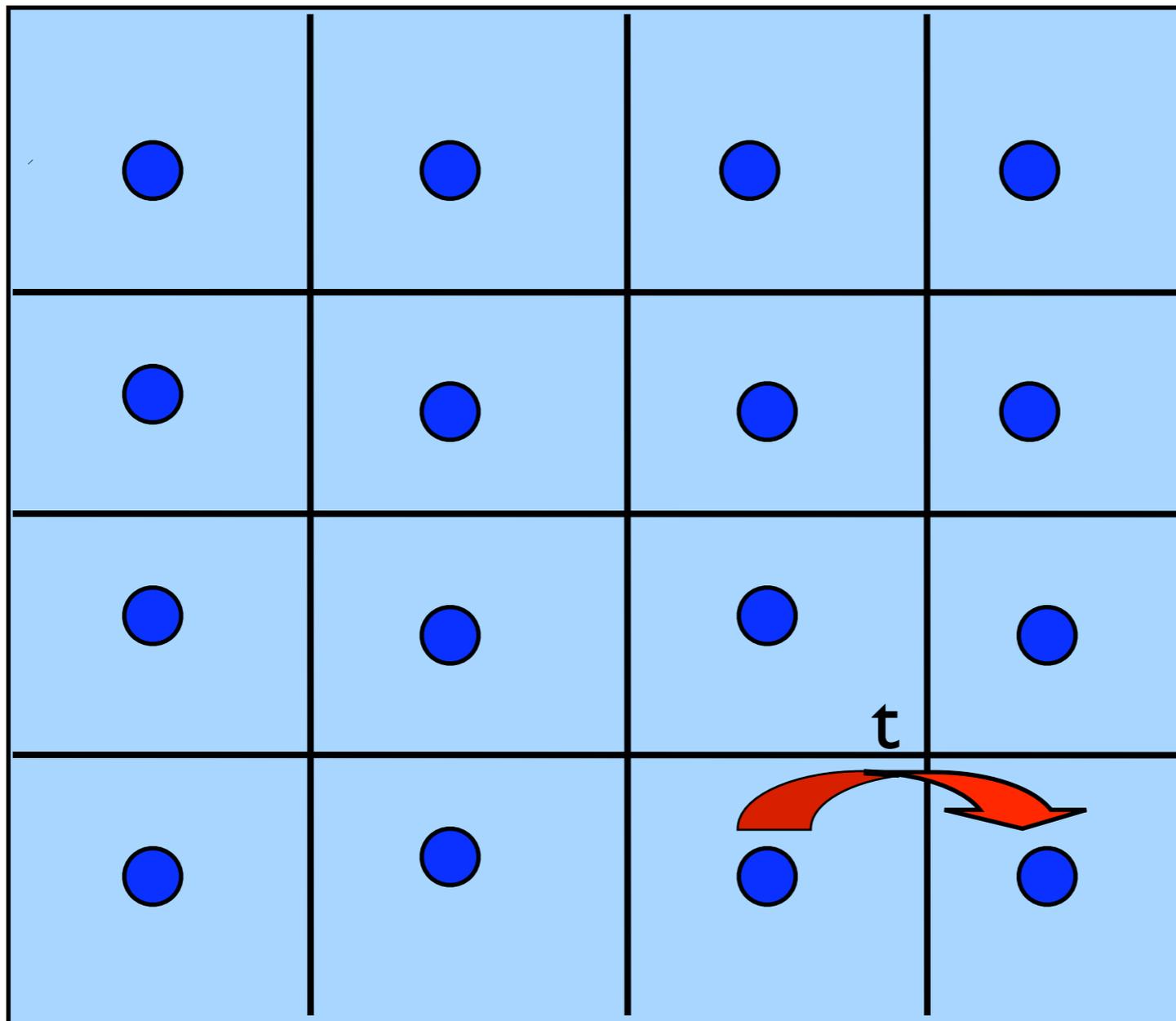
Insulator
???????



**Mott Problem:
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Insulator
???????



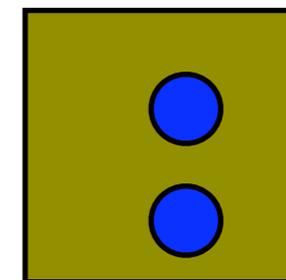
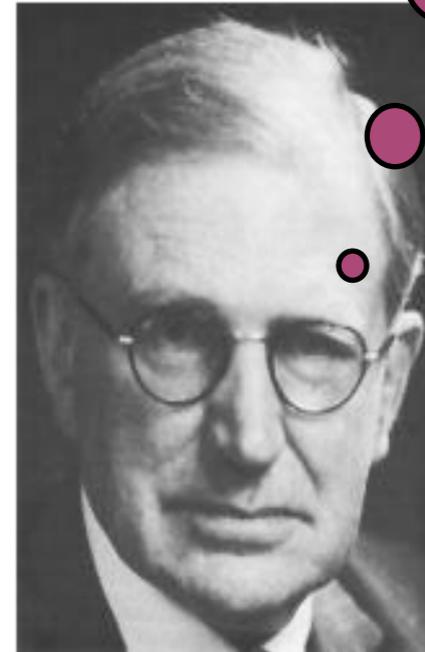
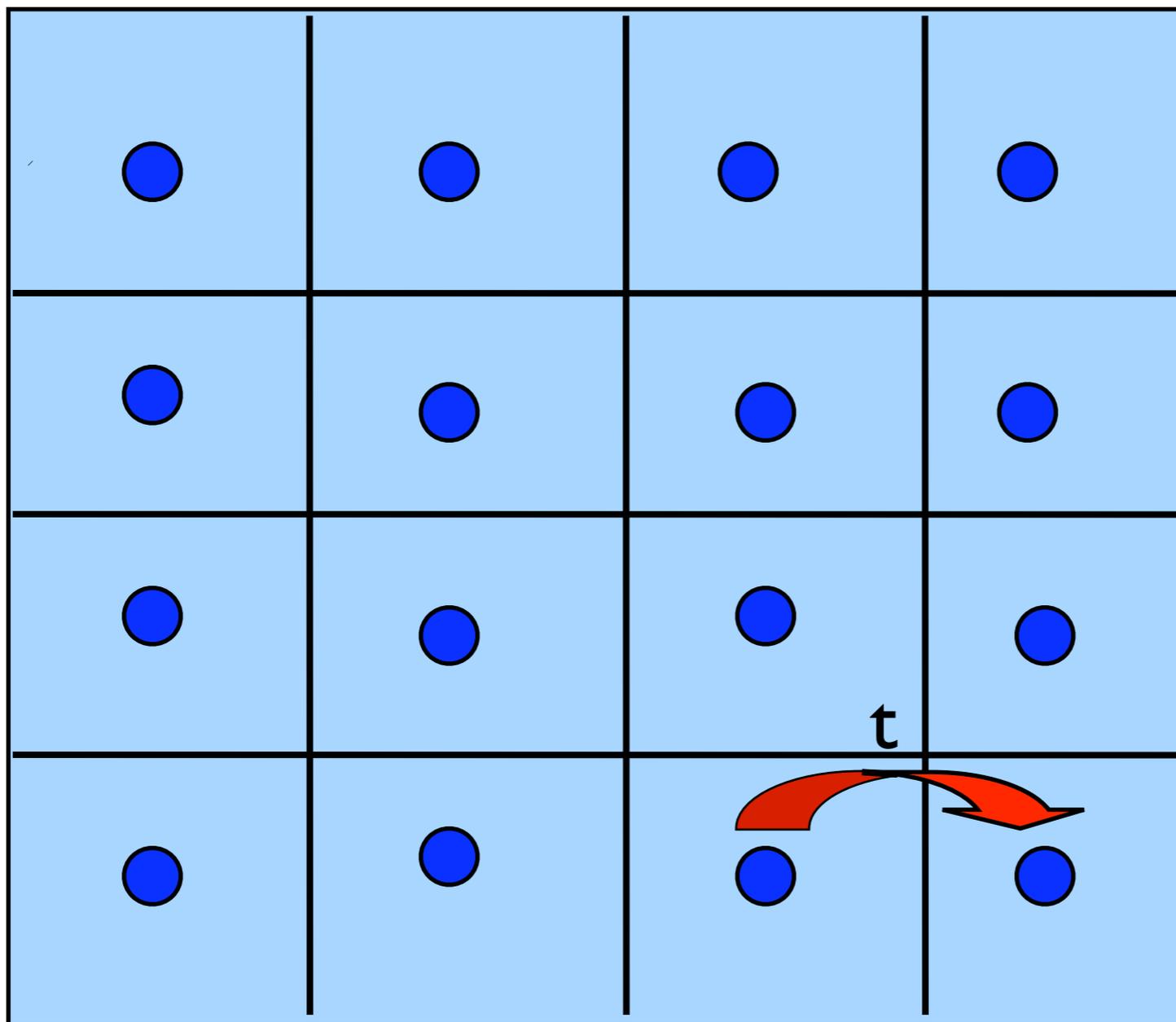
$$\Delta E = U \gg K.E.$$

**Mott Problem:
NiO (Band
theory failure)**

Cuprates:
 $U=4eV$
 $t=.5eV$

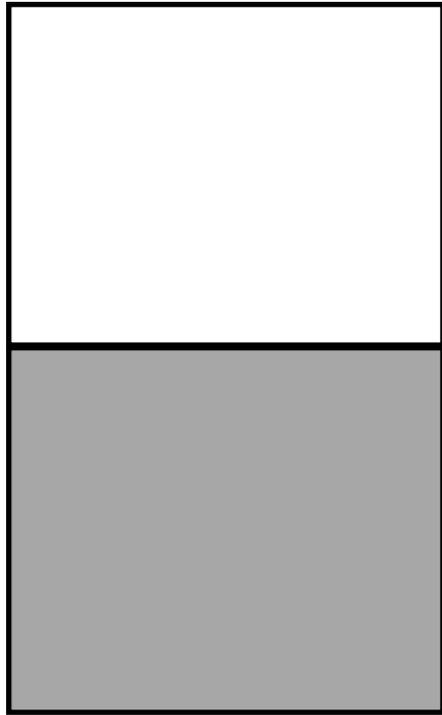
Insulator
???????

Loomis floor plan (N rooms N occupants)



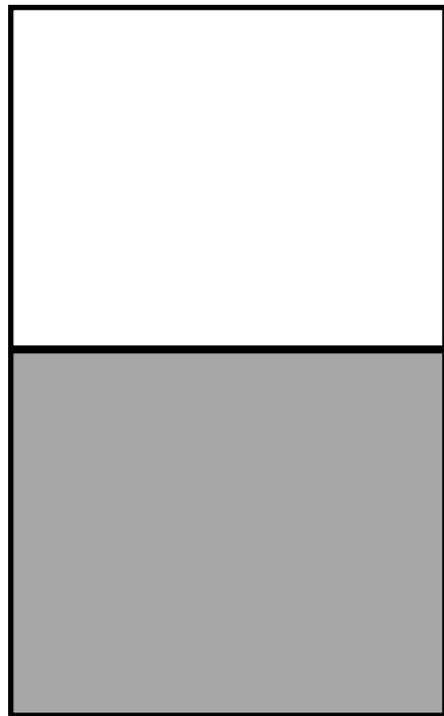
$$\Delta E = U \gg K.E.$$

Half-filled
band



Free electrons

Half-filled
band



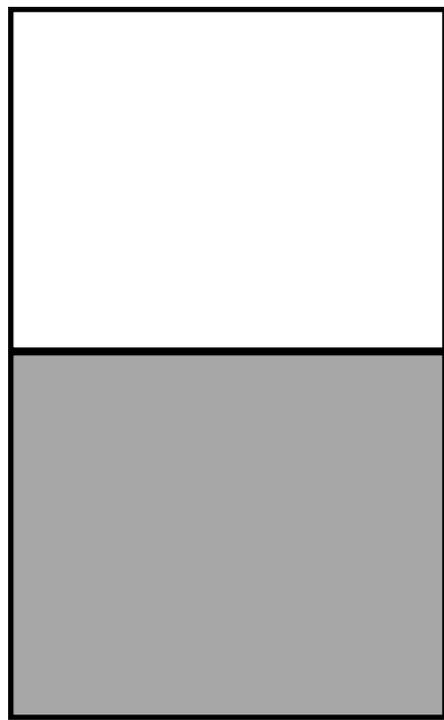
$$U \gg t$$



charge gap

Free electrons

Half-filled band



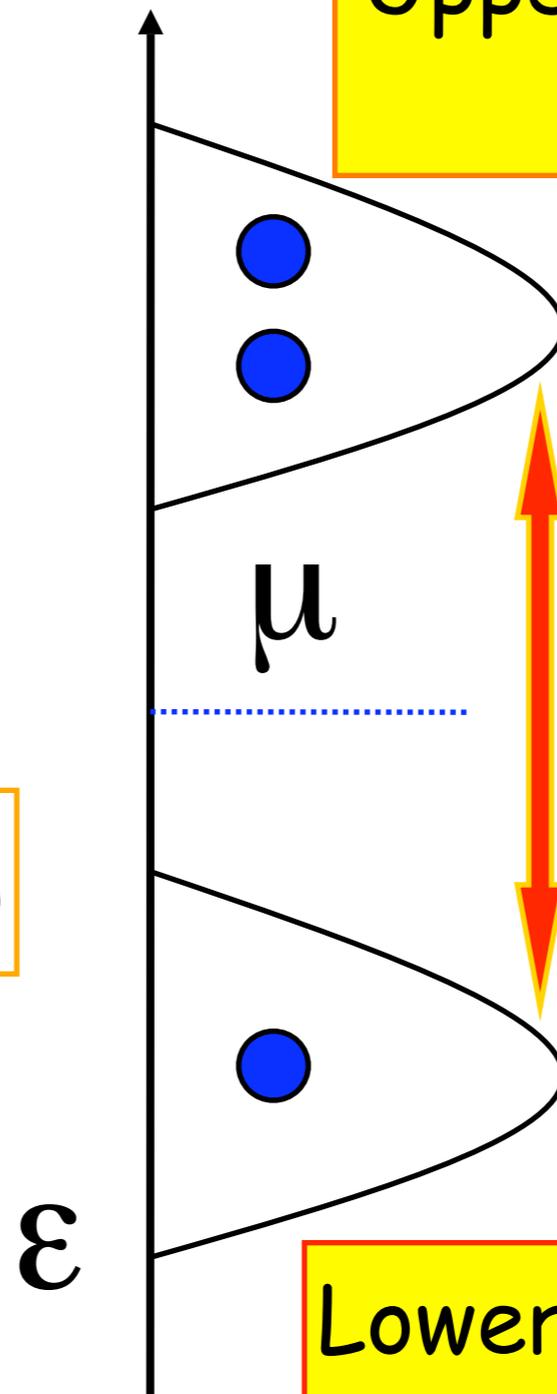
$$U \gg t$$



charge gap

Free electrons

Upper Hubbard band

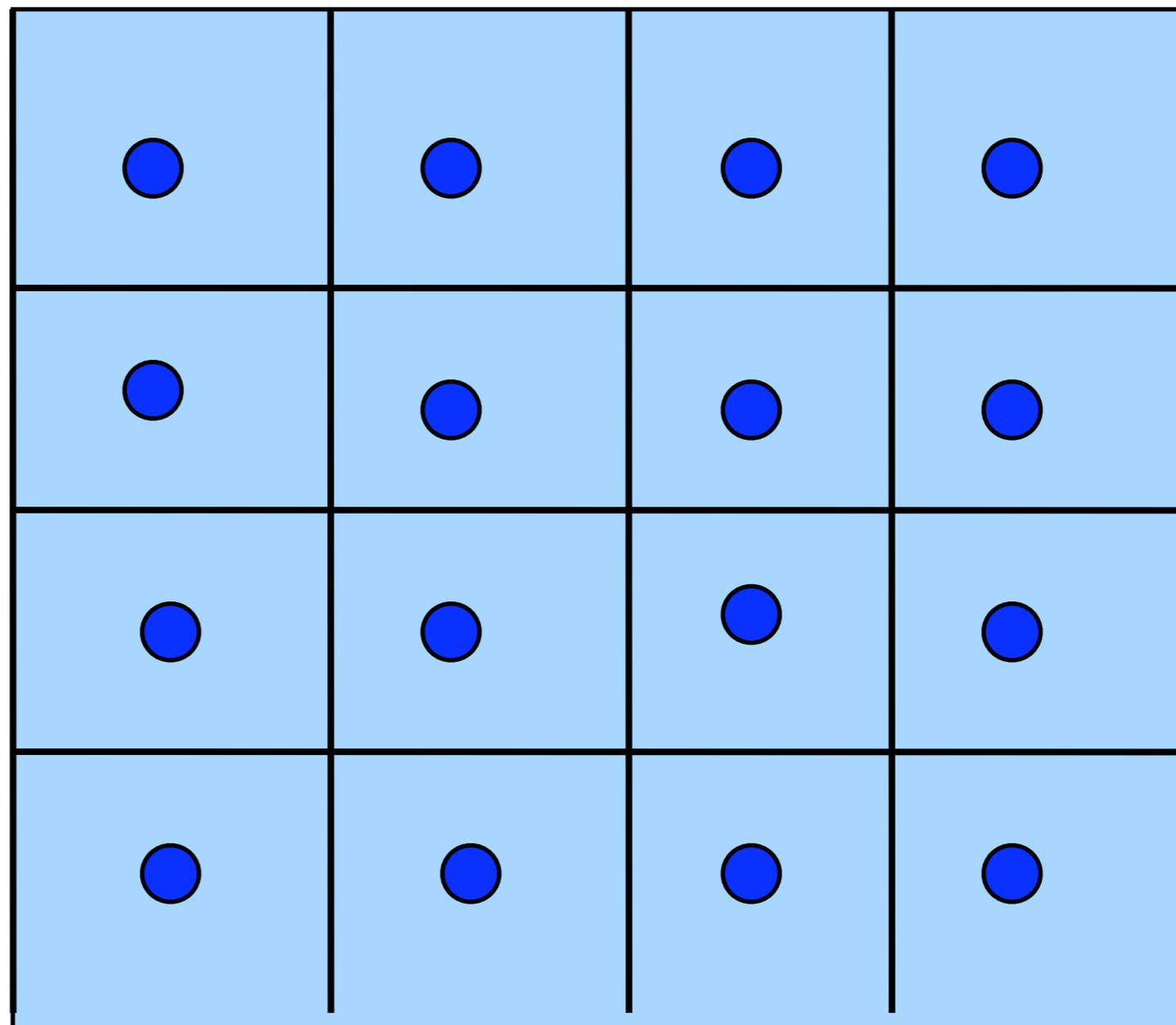


Lower Hubbard band

Low-energy theory??

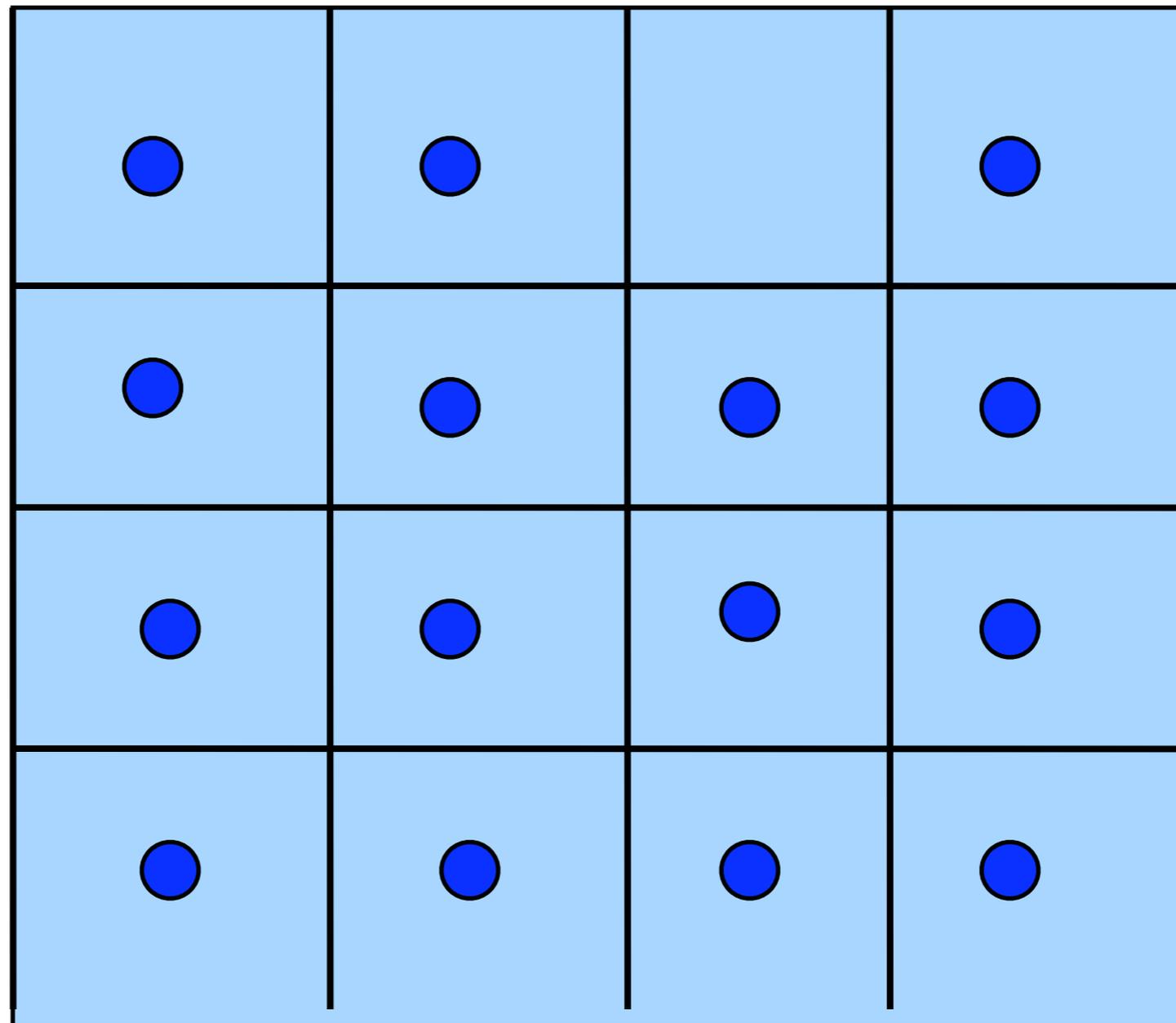
Doping a Mott insulator

x = fraction
of empty
rooms
(holes)



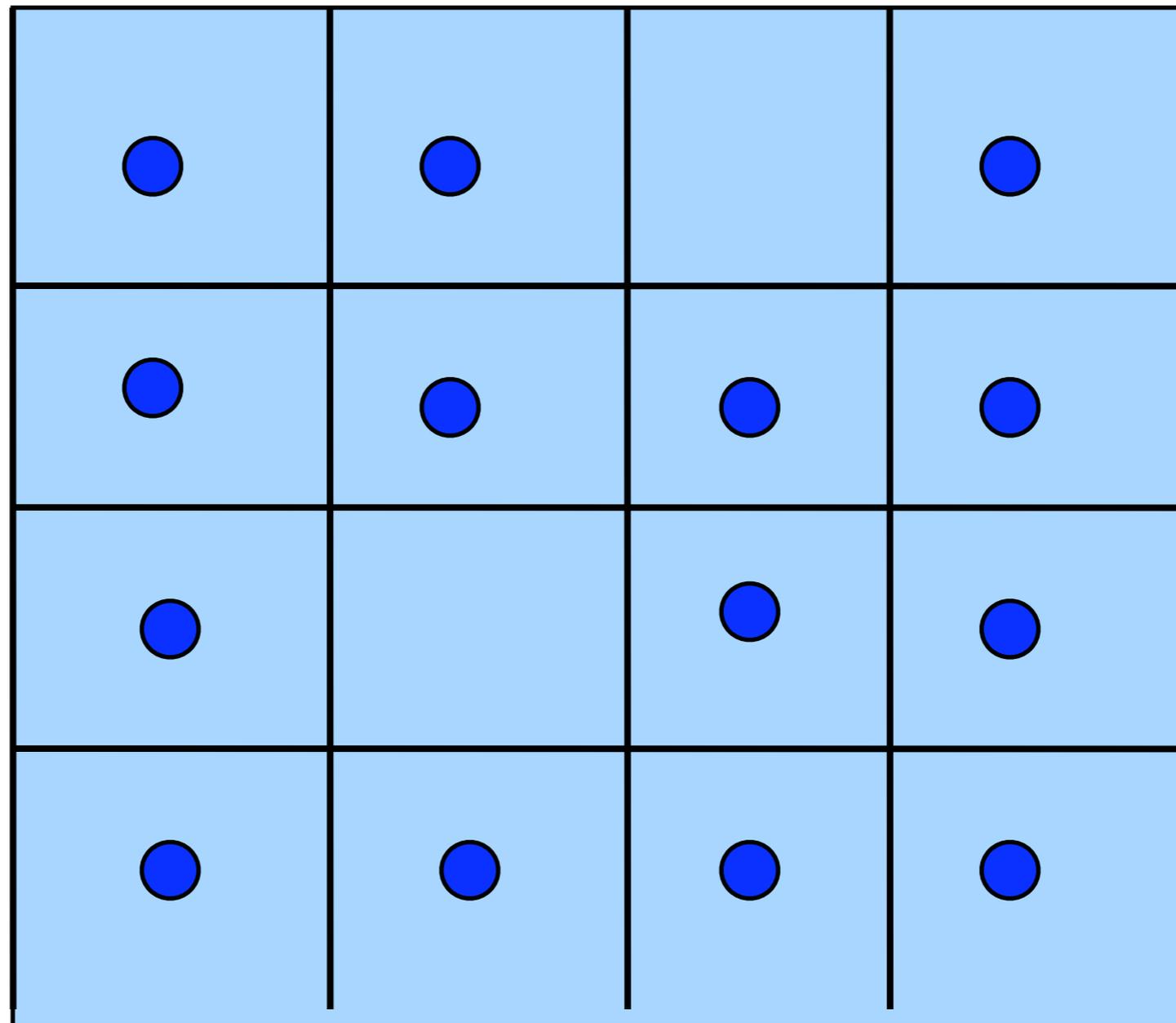
Doping a Mott insulator

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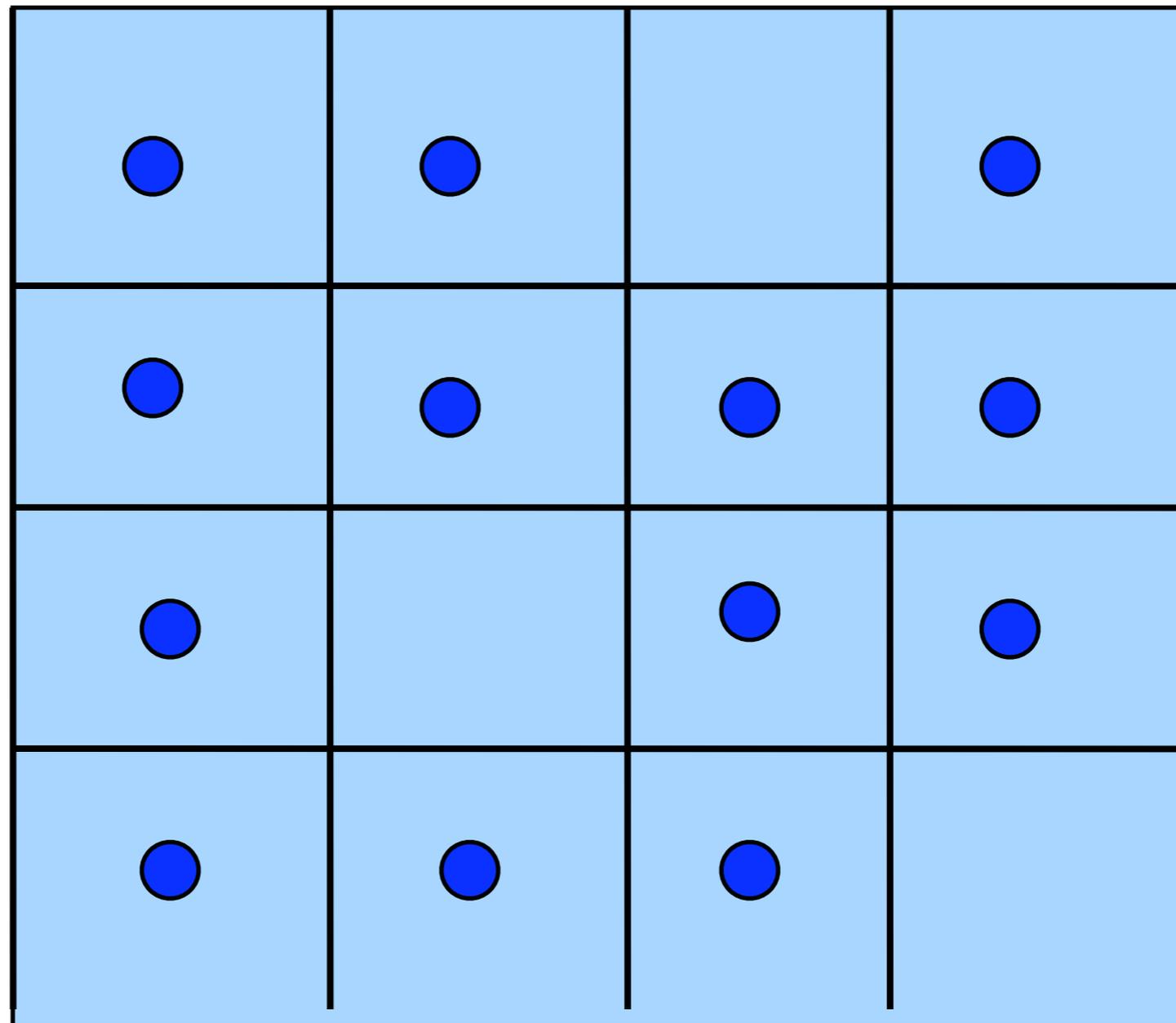
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Doping a Mott insulator

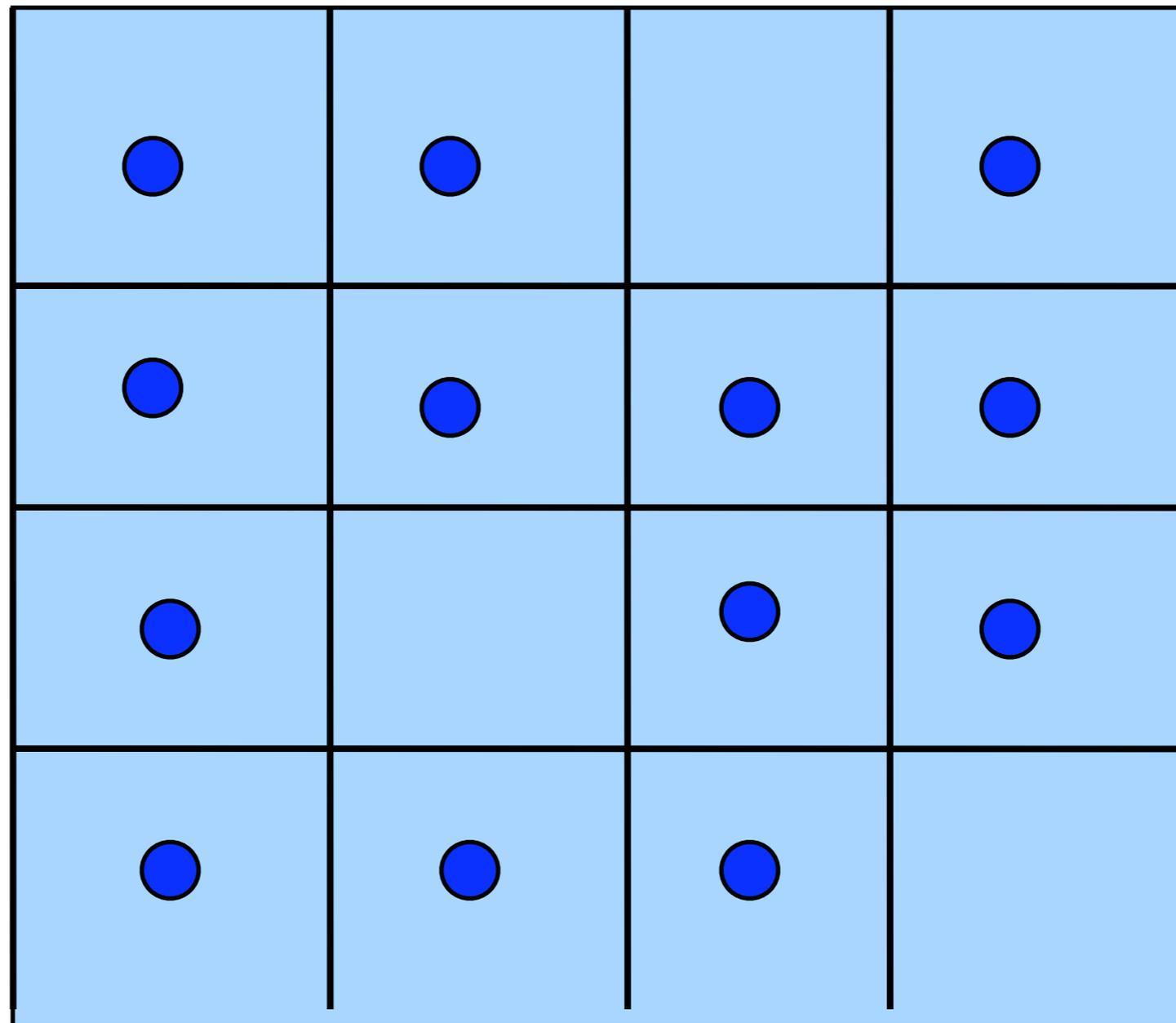
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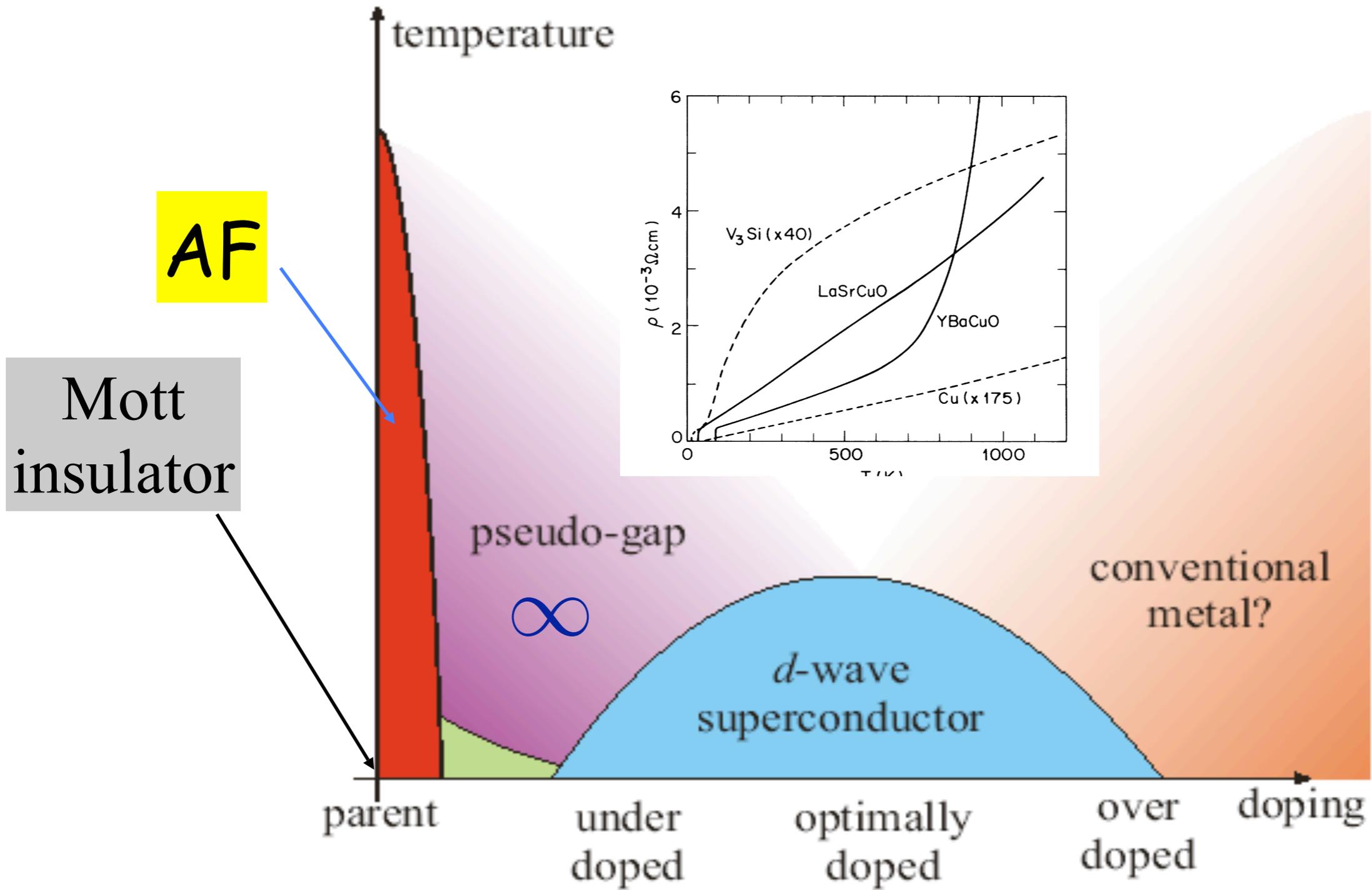


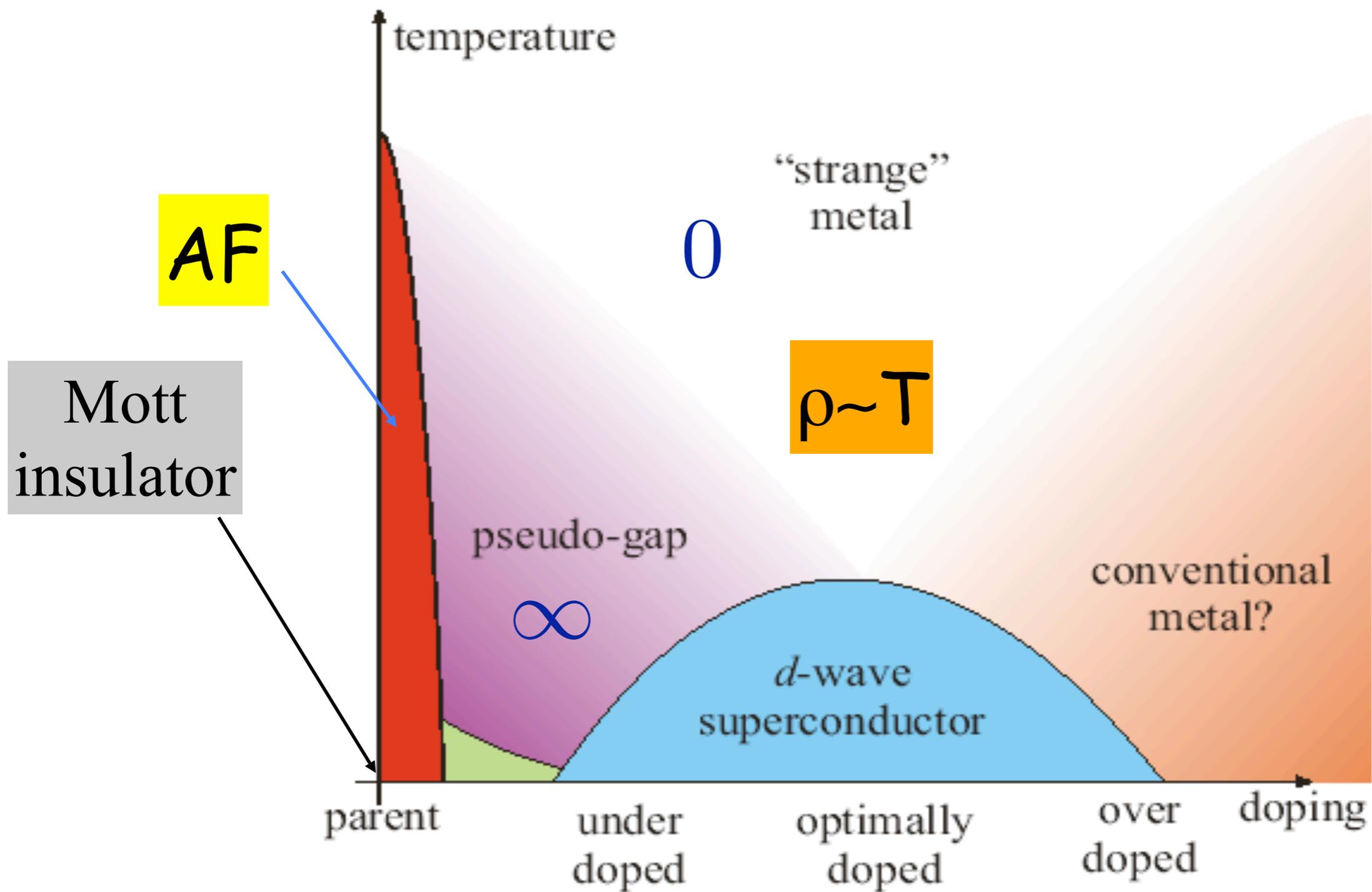
Doping a Mott insulator

x = fraction
of empty
rooms
(holes)

$$X = 3/16$$

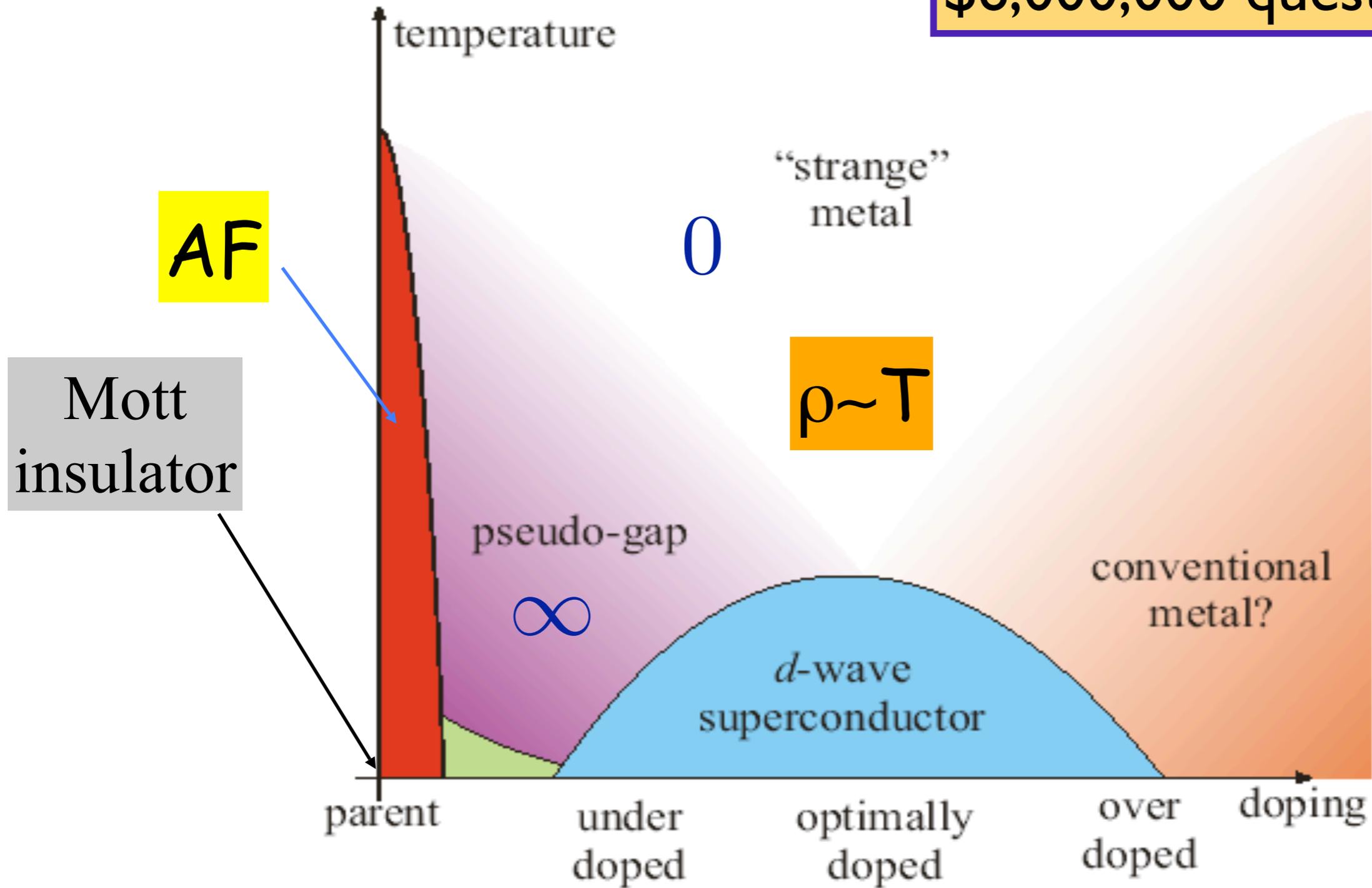






How does Fermi Liquid Theory Breakdown?

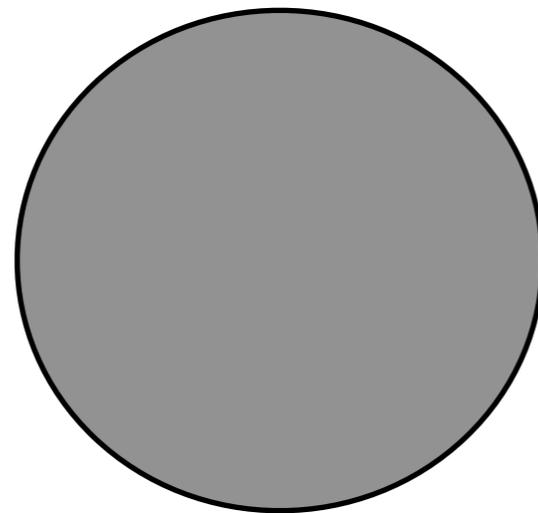
\$6,000,000 question?



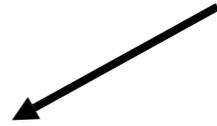
T-linear Resistivity?

Metals: $\rho \approx T^2$

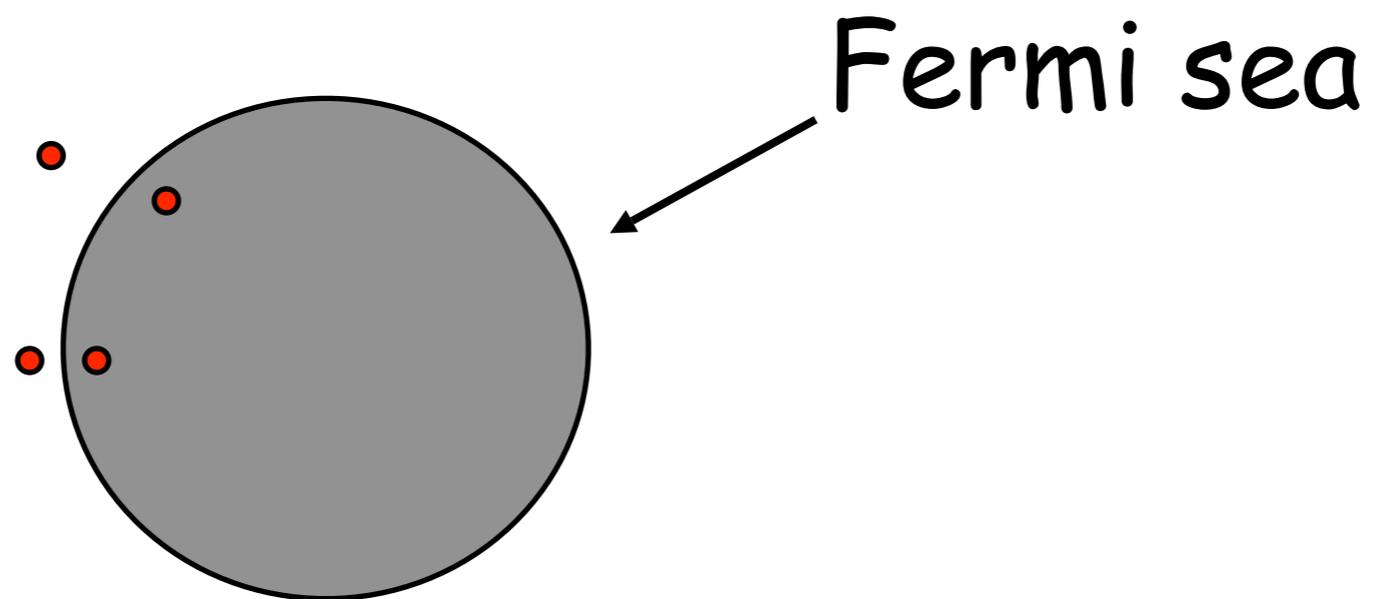
Metals: $\rho \approx T^2$



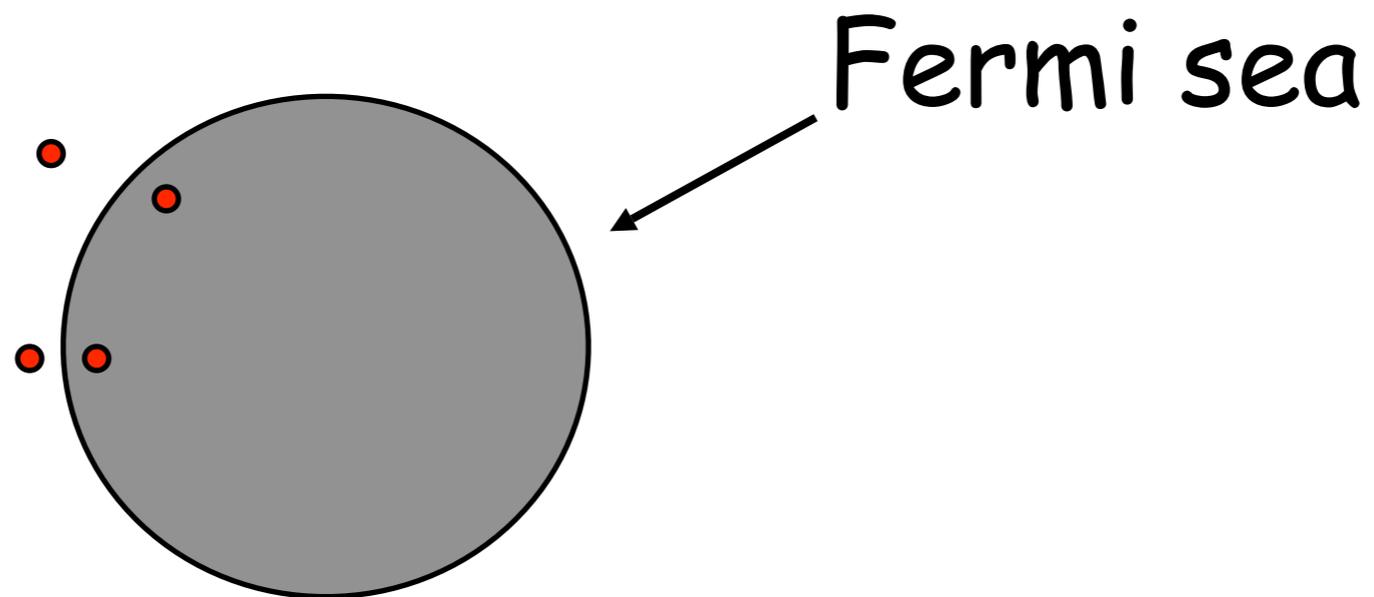
Fermi sea



Metals: $\rho \approx T^2$

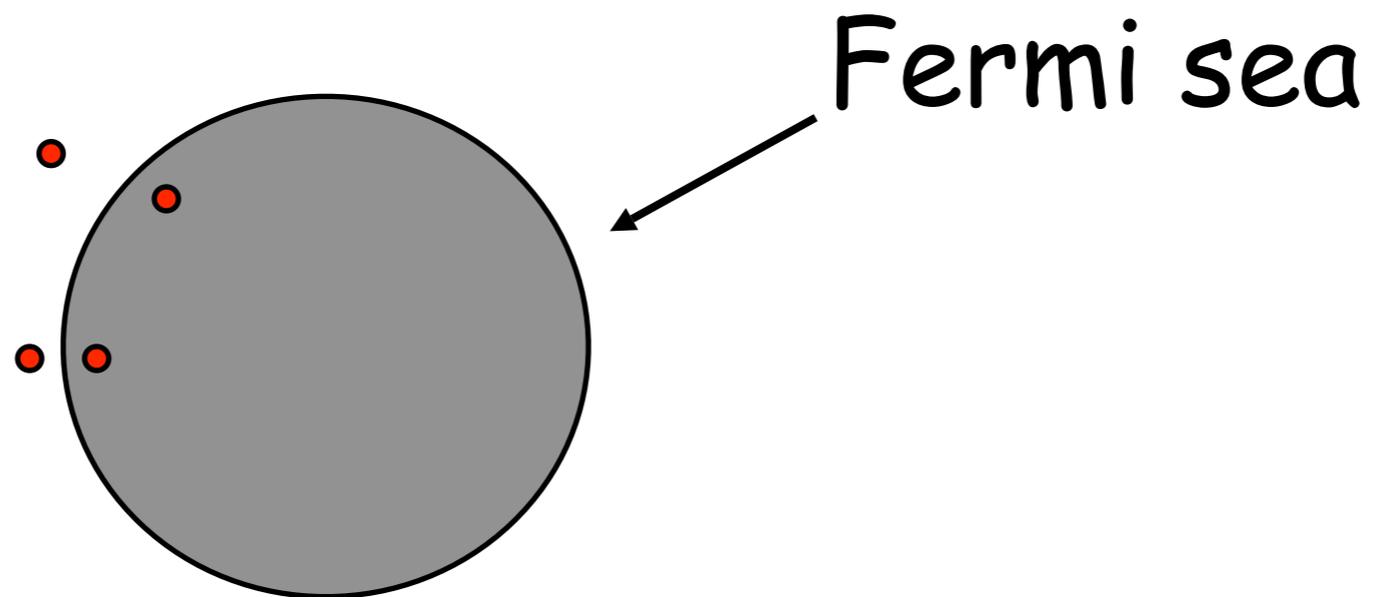


Metals: $\rho \approx T^2$



Two degrees of freedom

Metals: $\rho \approx T^2$



Two degrees of freedom

$$\frac{\hbar}{\tau} \approx \frac{\epsilon^2}{\epsilon_F} \propto \frac{T^2}{\epsilon_F}$$

T-linear Resistivity

$$\frac{\hbar}{\tau} \equiv \# k_B T$$

Planckian limit of dissipation

T-linear Resistivity

$$\frac{\hbar}{\tau} \equiv \# k_B T$$

Planckian limit of dissipation

breakdown of standard
Fermi liquid
picture

Cuprates: The Perfect Storm

**Fermi Liquid
Theory**

Band Theory

BCS



Cuprates: The Perfect Storm



collective failure

collective failure

"I'm not into this detail stuff. I'm more concepty."



collective failure

"I'm not into this detail stuff. I'm more concepty."



New Concept

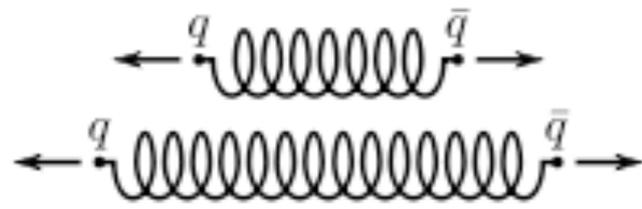
composite or bound states not in UV theory

Strong Coupling

composite or bound states not in UV theory

Strong Coupling

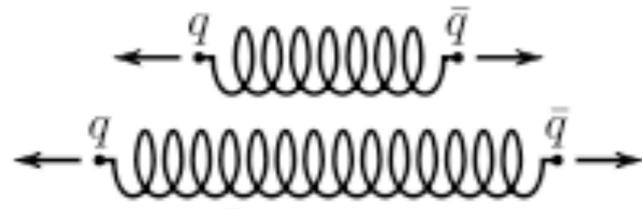
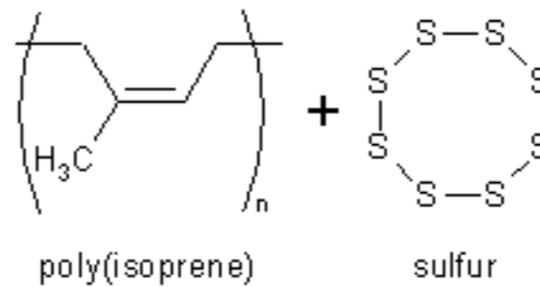
QCD



composite or bound states not in UV theory

Strong Coupling

QCD



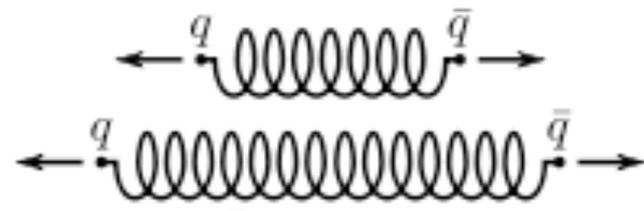
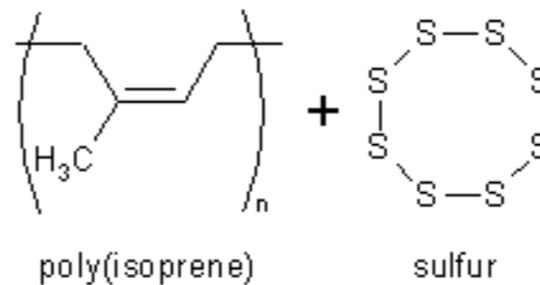
vulcanization



composite or bound states not in UV theory

Strong Coupling

QCD



vulcanization

emergent
low-energy physics









weakly interacting

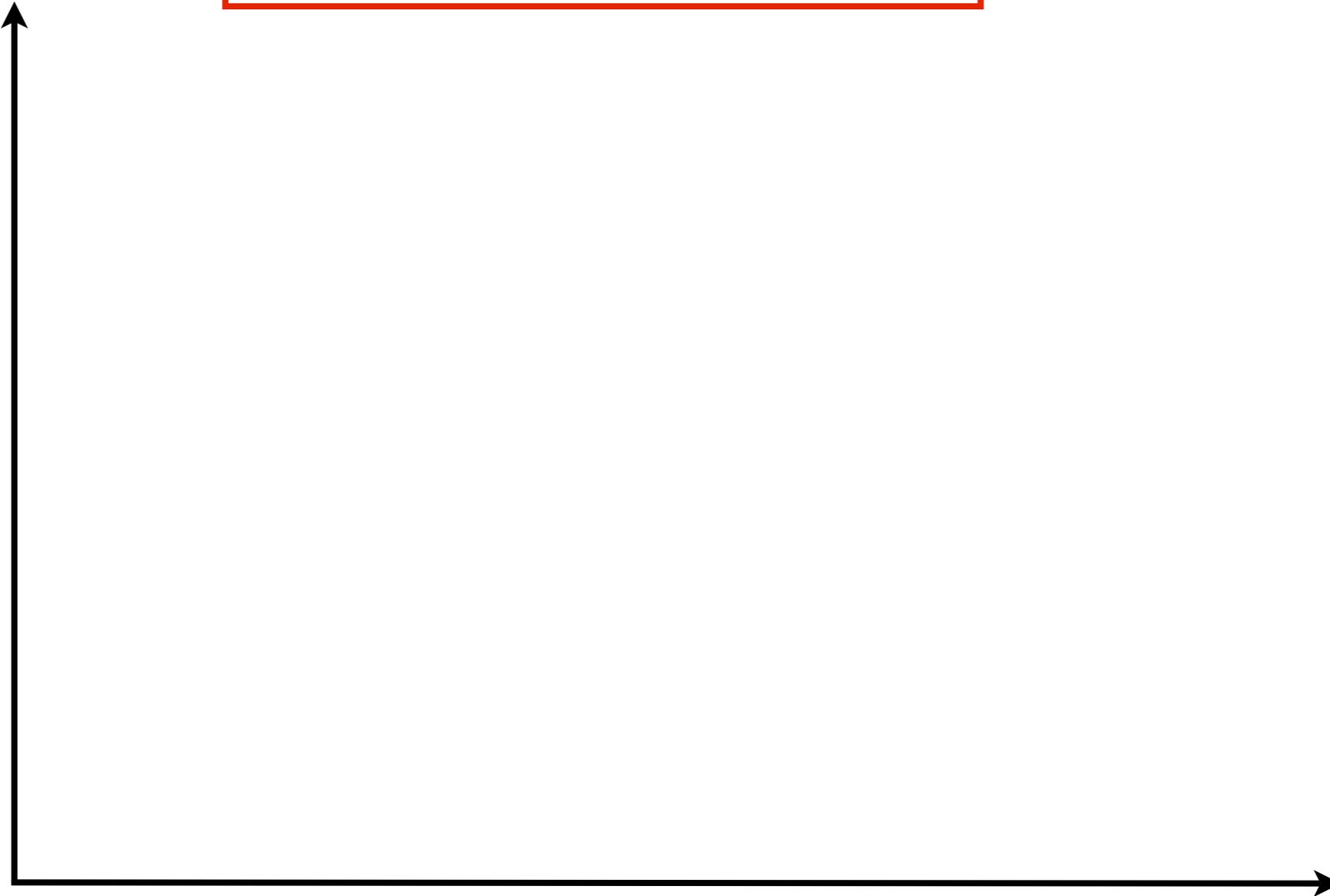
goal:

identify propagating charge degrees of freedom
in the normal state of
a high-temperature superconductor

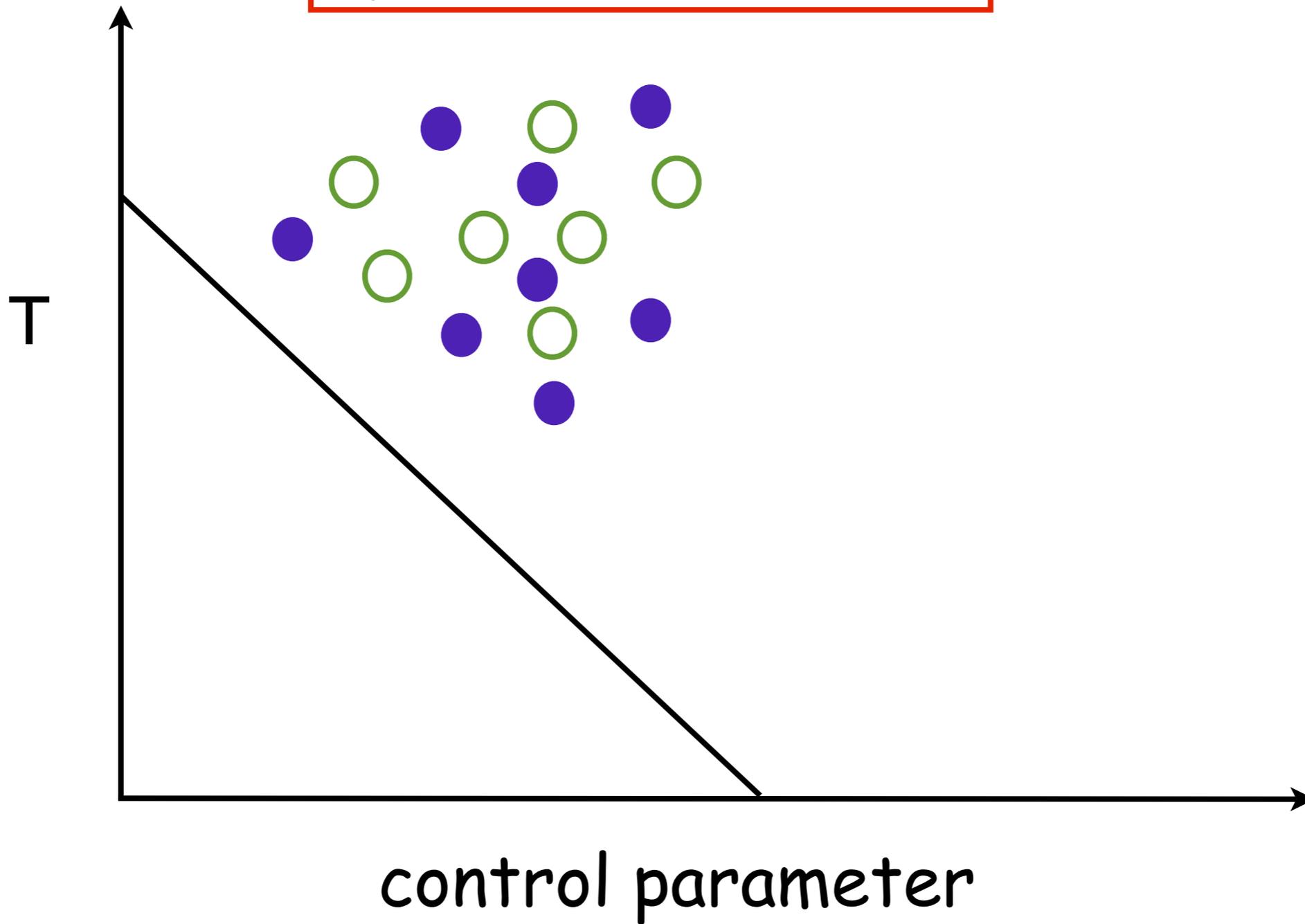
dynamical transition

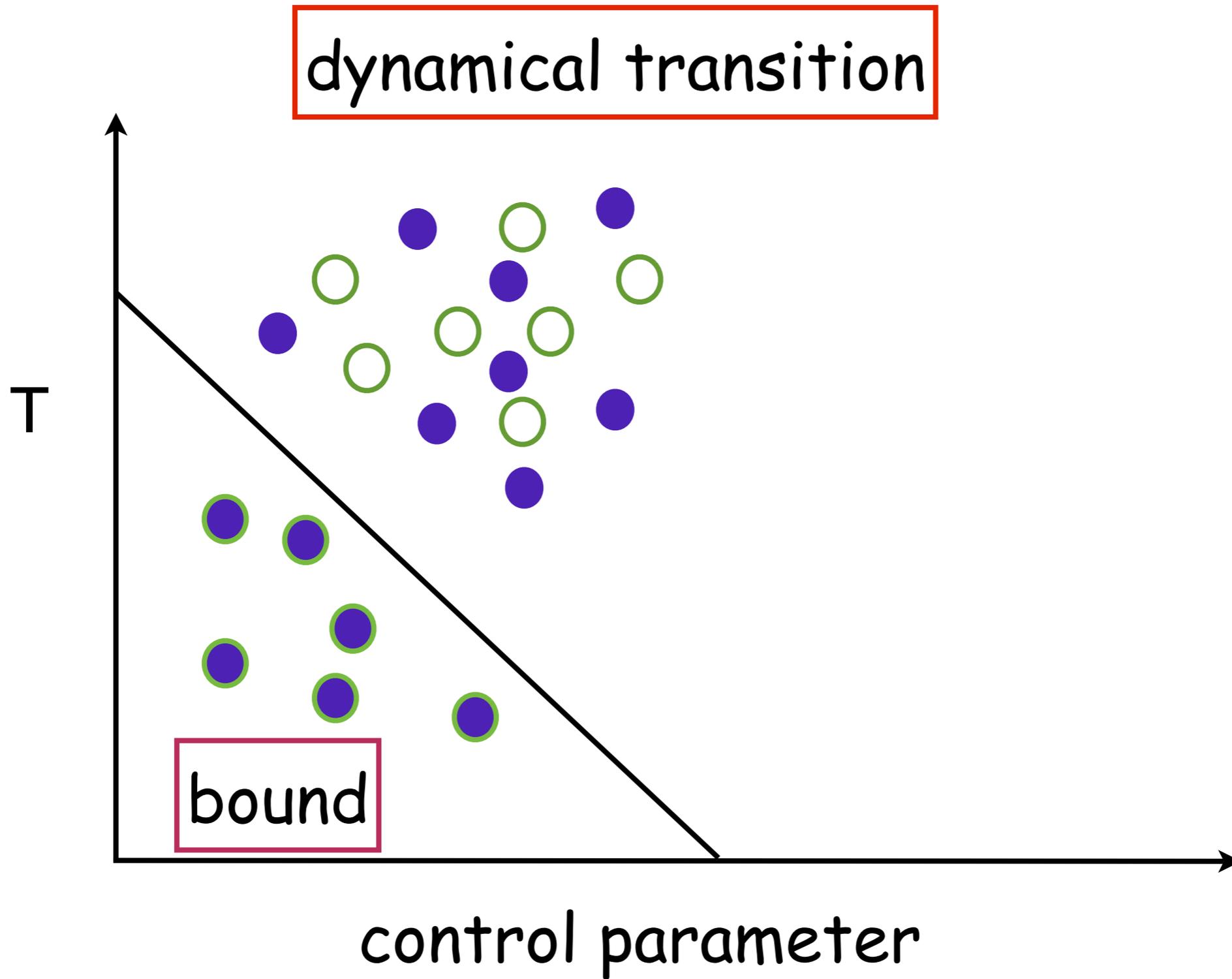
T

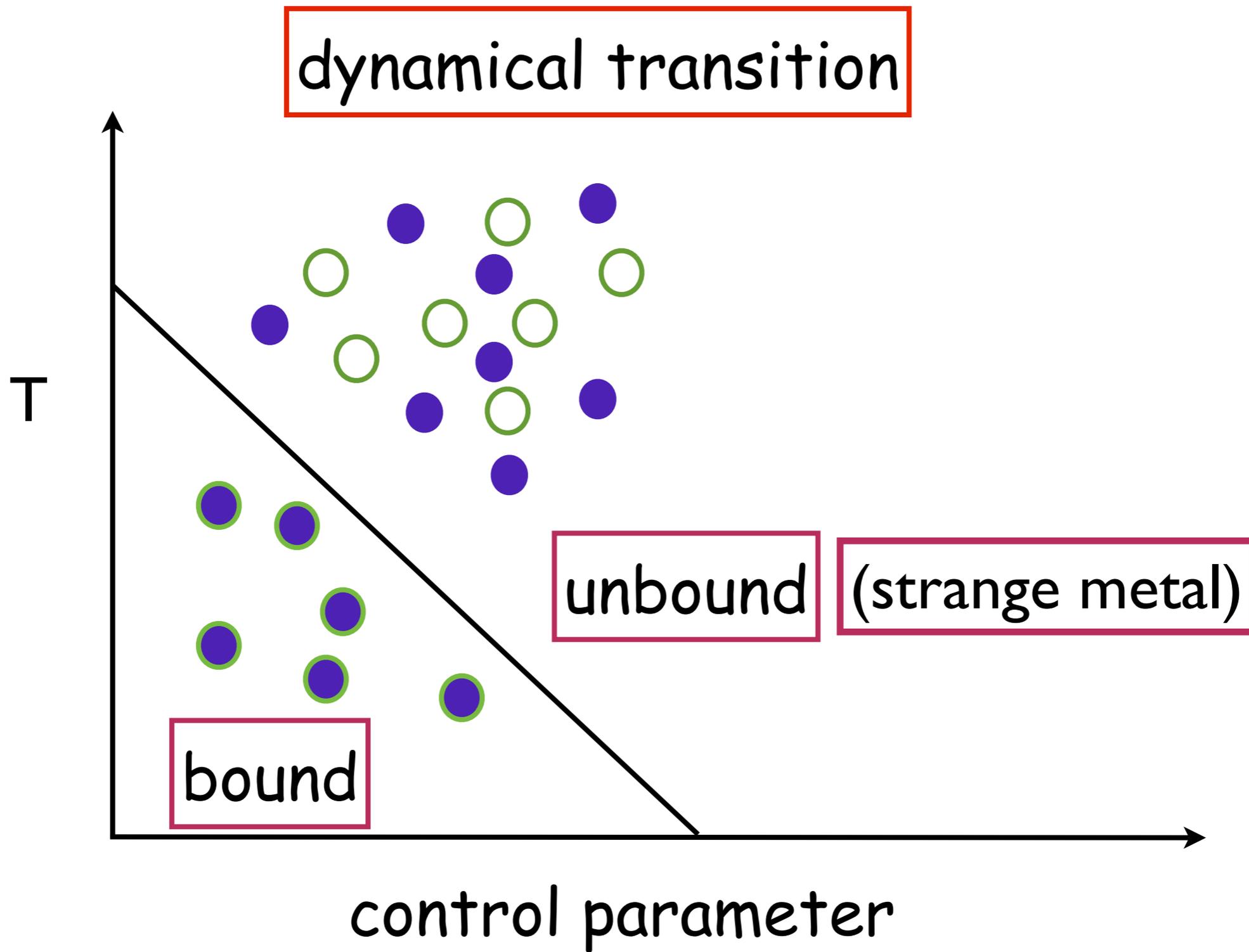
control parameter

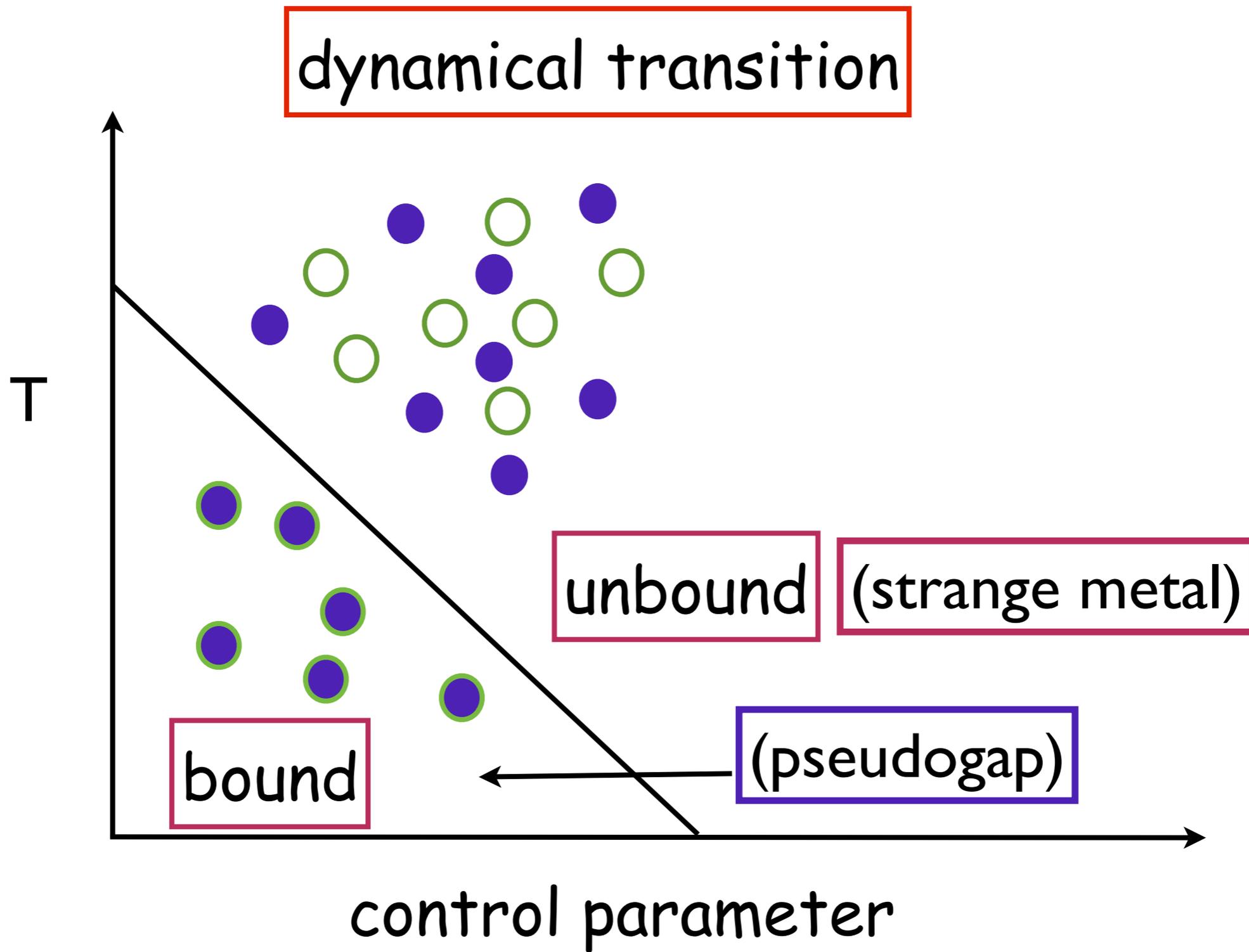


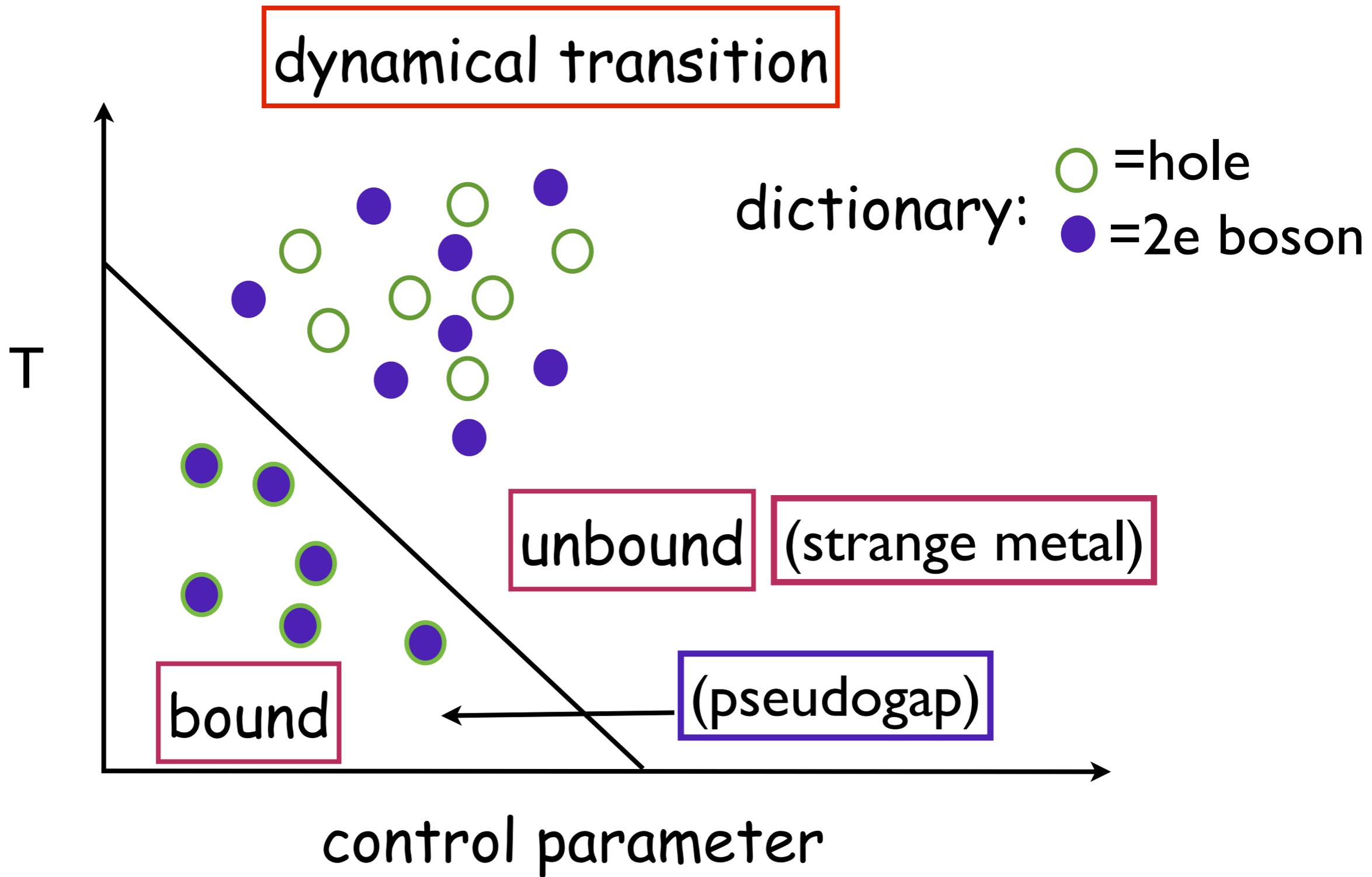
dynamical transition





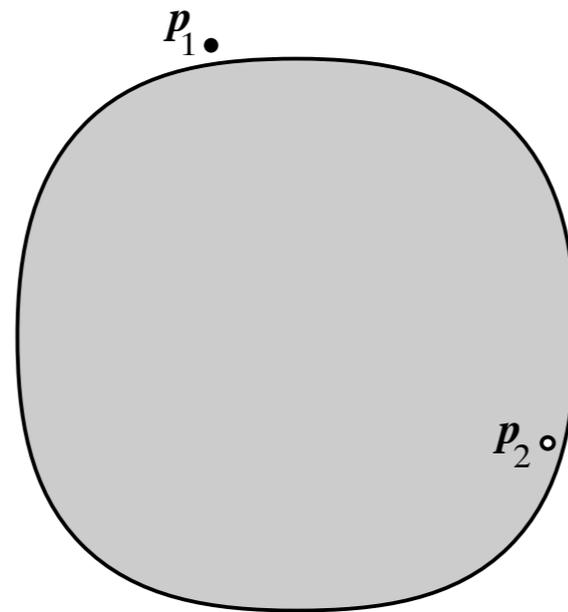






How to break Fermi liquid
theory in $d=2+1$?

Polchinski (and others)



$$\mathbf{p} = \mathbf{k} + \mathbf{l},$$

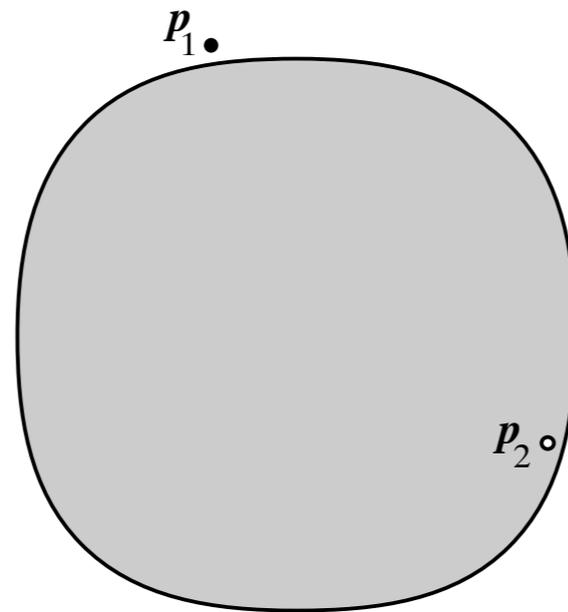
1.) e- charge carriers

2.) Fermi surface

$$\int dt d^2\mathbf{k}_1 d\mathbf{l}_1 d^2\mathbf{k}_2 d\mathbf{l}_2 d^2\mathbf{k}_3 d\mathbf{l}_3 d^2\mathbf{k}_4 d\mathbf{l}_4 V(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \psi_{\sigma}^{\dagger}(\mathbf{p}_1) \psi_{\sigma}(\mathbf{p}_3) \psi_{\sigma'}^{\dagger}(\mathbf{p}_2) \psi_{\sigma'}(\mathbf{p}_4) \delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4).$$

No relevant short-range 4-Fermi terms in $d \geq 2$

Polchinski (and others)



$$\mathbf{p} = \mathbf{k} + \mathbf{l},$$

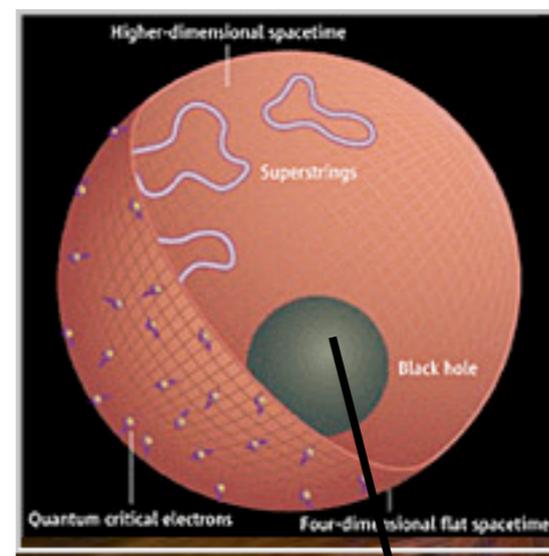
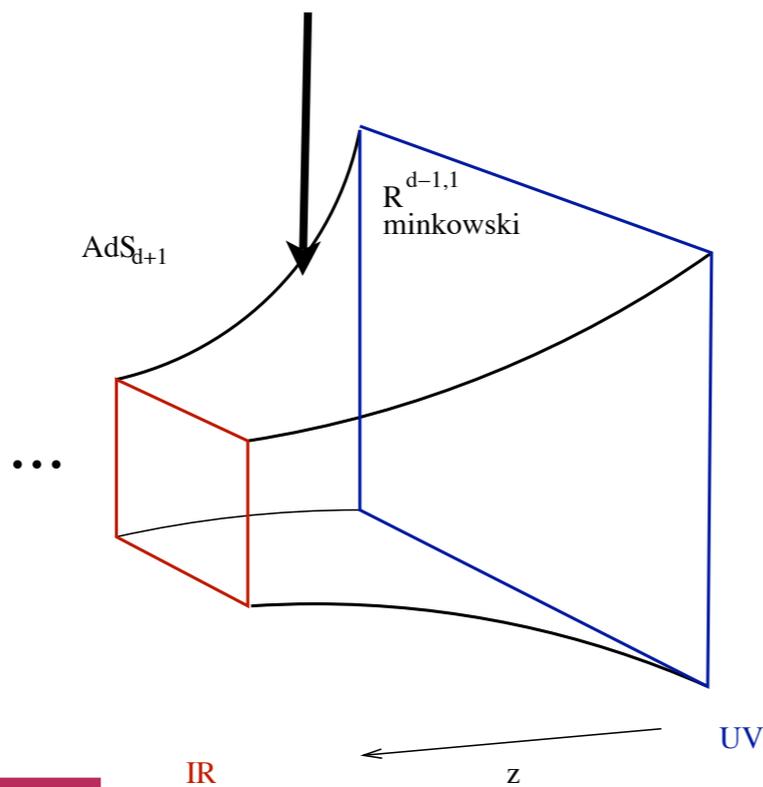
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No relevant short-range 4-Fermi terms in $d \geq 2$
Exception: Pairing

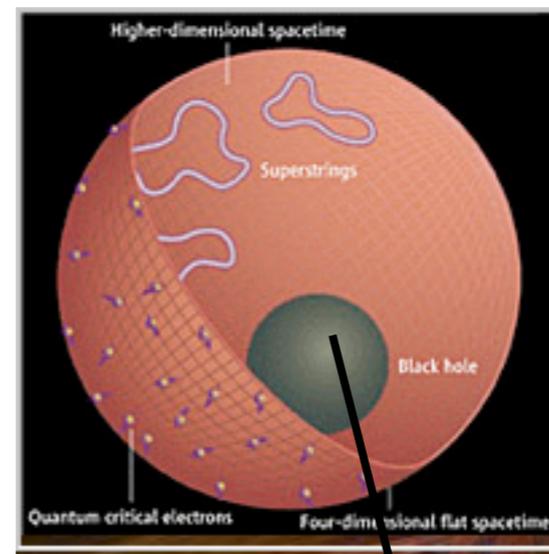
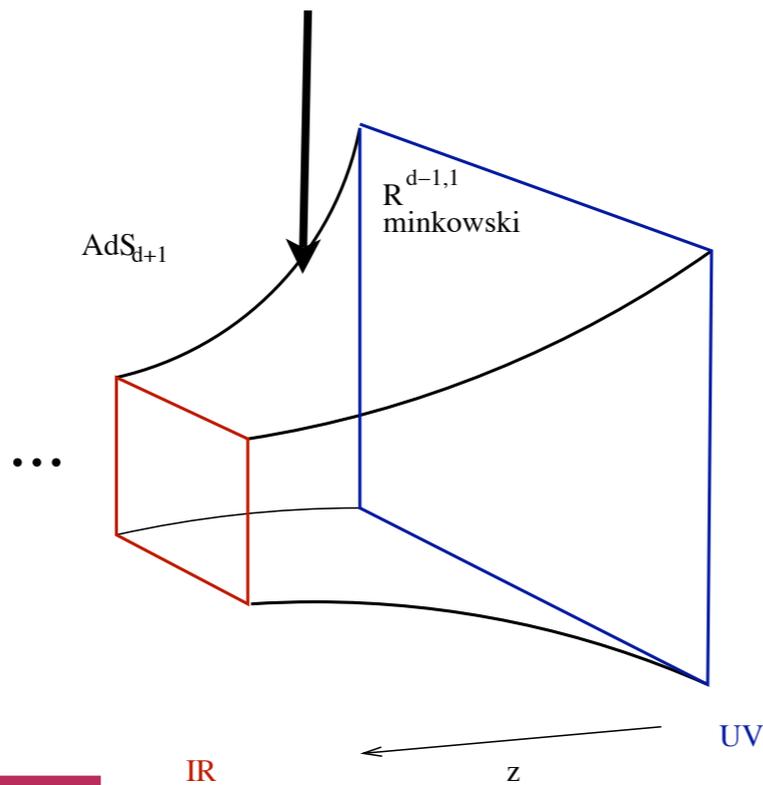
does gauge/gravity duality help?



McGreevy, Liu

$AdS_4 \xrightarrow{\rho \propto T} AdS_2 \times R^2$

does gauge/gravity duality help?



McGreevy, Liu

AdS_4

$AdS_2 \times R^2$

Mechanism??

$$\rho \propto T$$

No

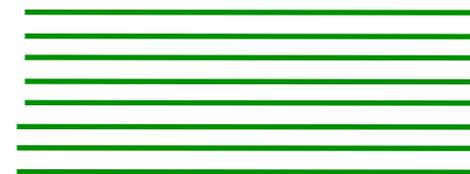
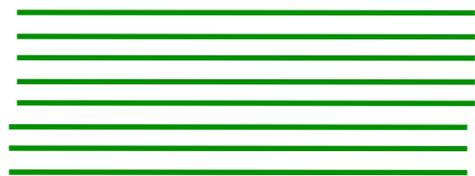
So what does one add
to break this
correspondence?

$$H = -t \sum_{ij\sigma} g_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Interacting
system

Free system

low-energy:
one-to-one
correspondence



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$$H = -t \sum_{ij\sigma} g_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Interacting
system

new degrees
of freedom!

Free system

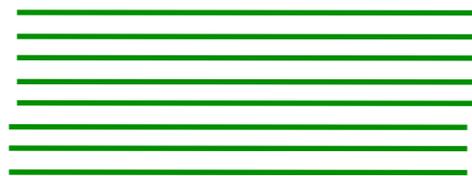
one energy:
one-one
correspondence



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$$H = -t \sum_{ij\sigma} g_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Free system



no energy:
one-to-one
correspondence



new degrees
of freedom!



So what does one add
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correspondence?

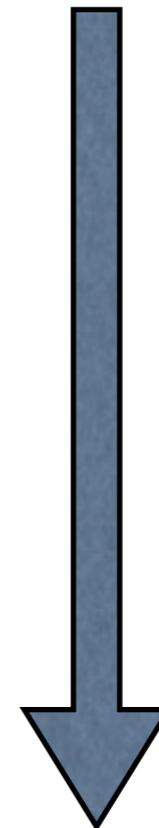
$$H = -t \sum_{ij\sigma} g_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

UV-IR
mixing

new degrees
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Free system

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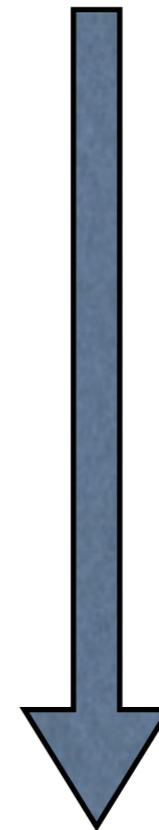
new degrees
of freedom!

dynamically
generated

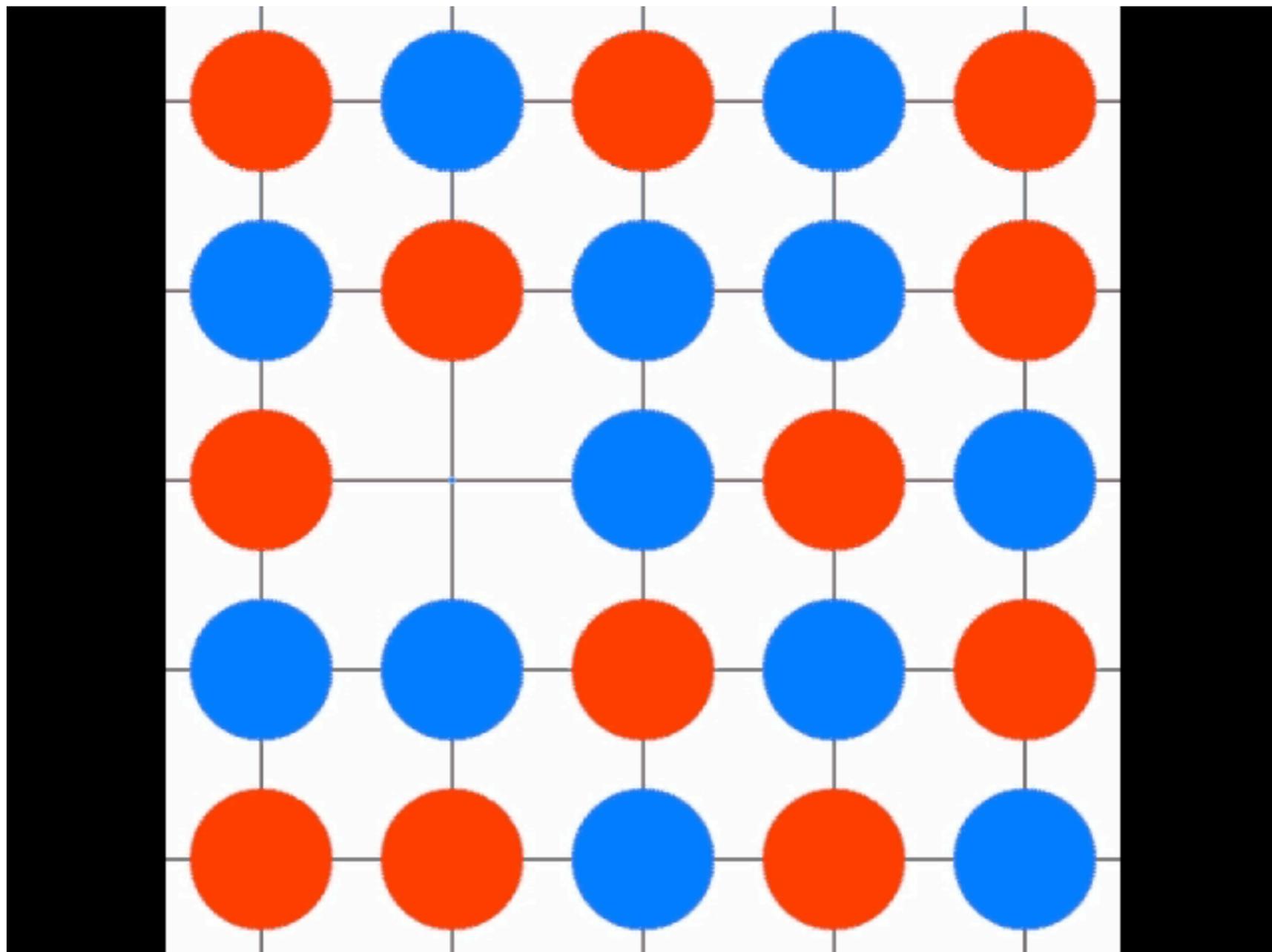
Free system



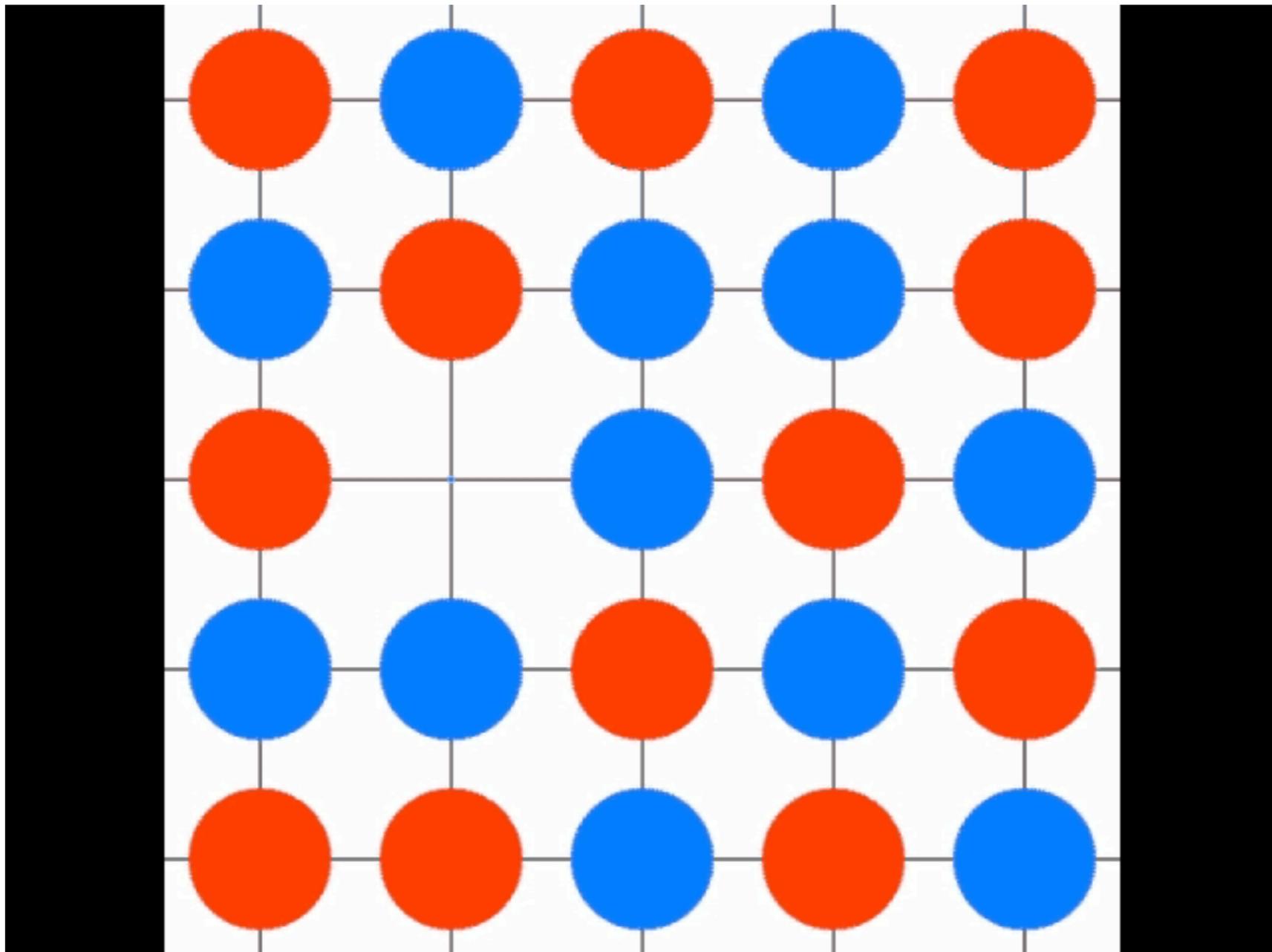
low energy:
one-to-one
correspondence



$$U = \infty$$

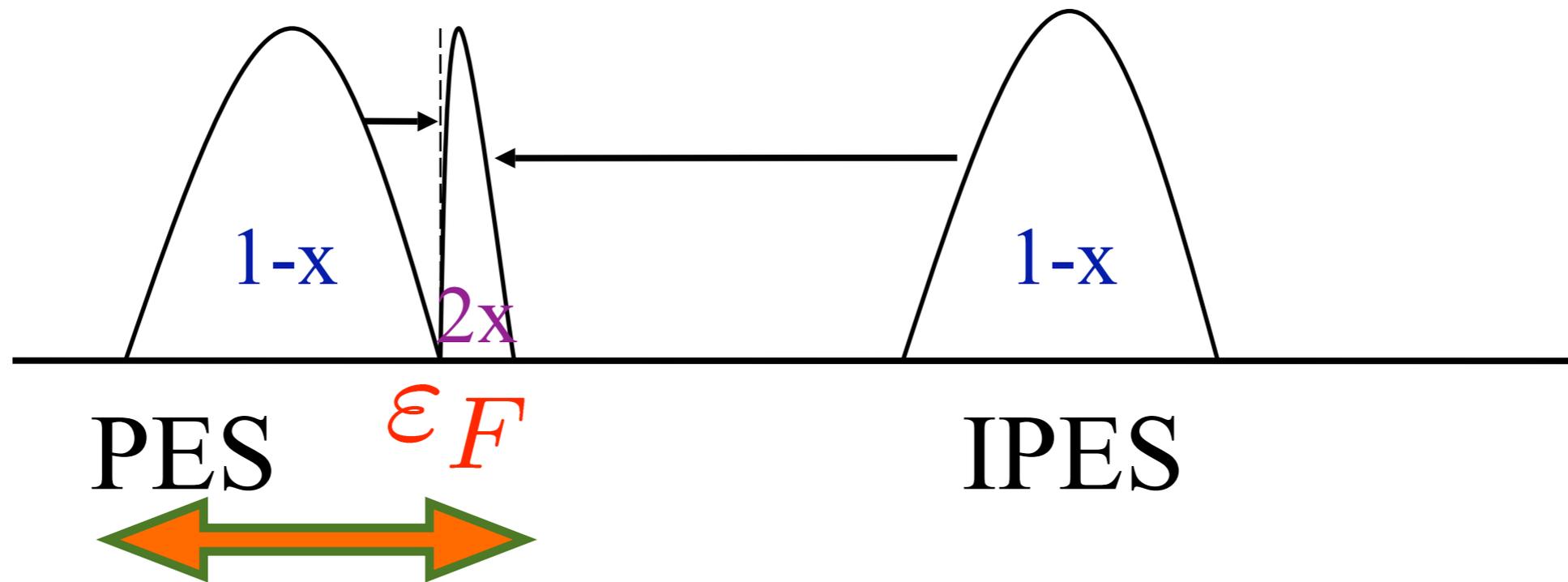


$$U = \infty$$



No dynamics with doubly-occupied sector:
each electron blocks 2 states

atomic limit: x holes



total weight = $1+x$ = # of ways electrons can be added in lower band

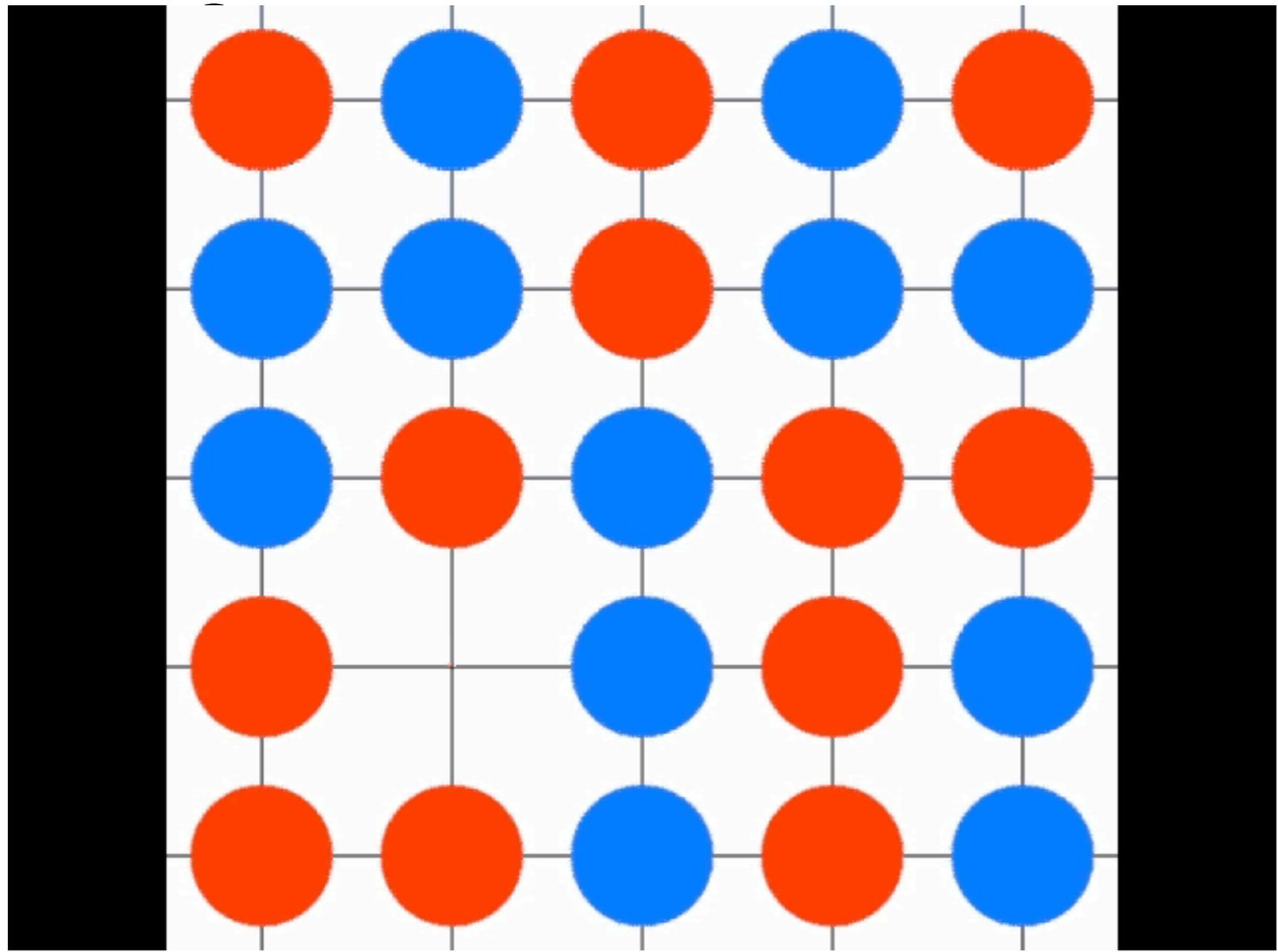
No problems yet!

atomic limit

intensity of lower band = # of electrons the
band can hold

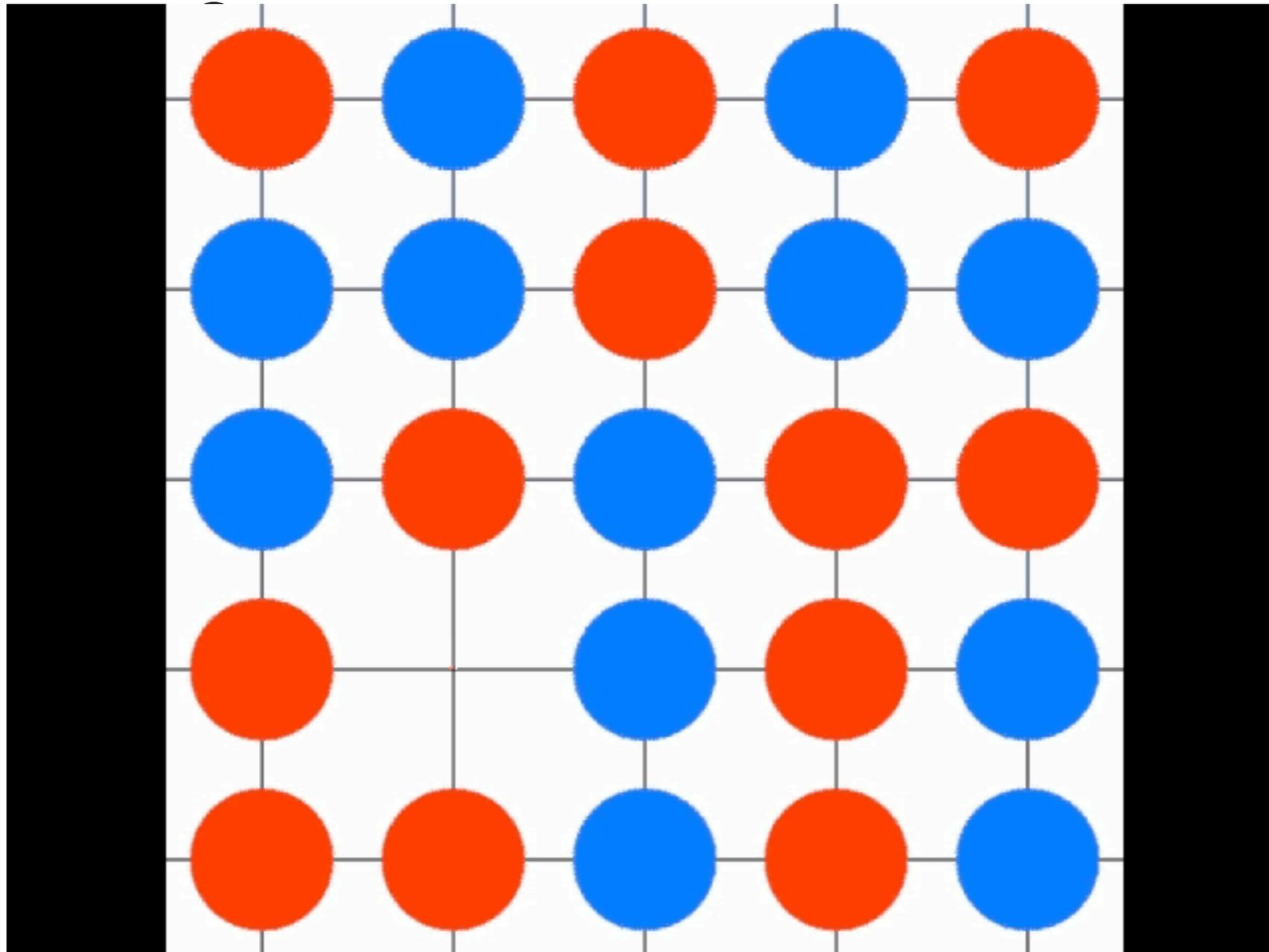
U finite

$$U \gg t$$



U finite

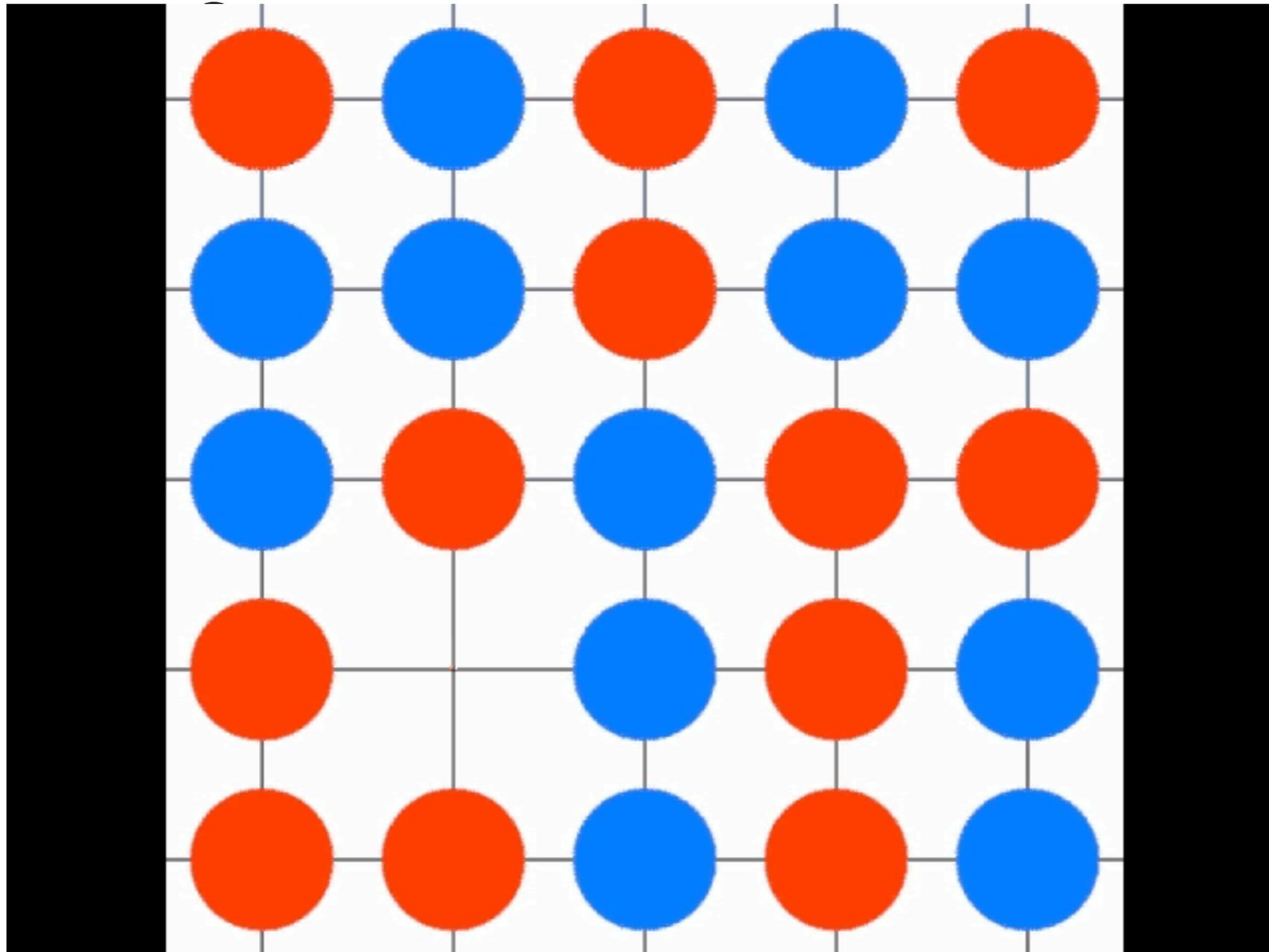
$$U \gg t$$



double occupancy in ground state!!

U finite

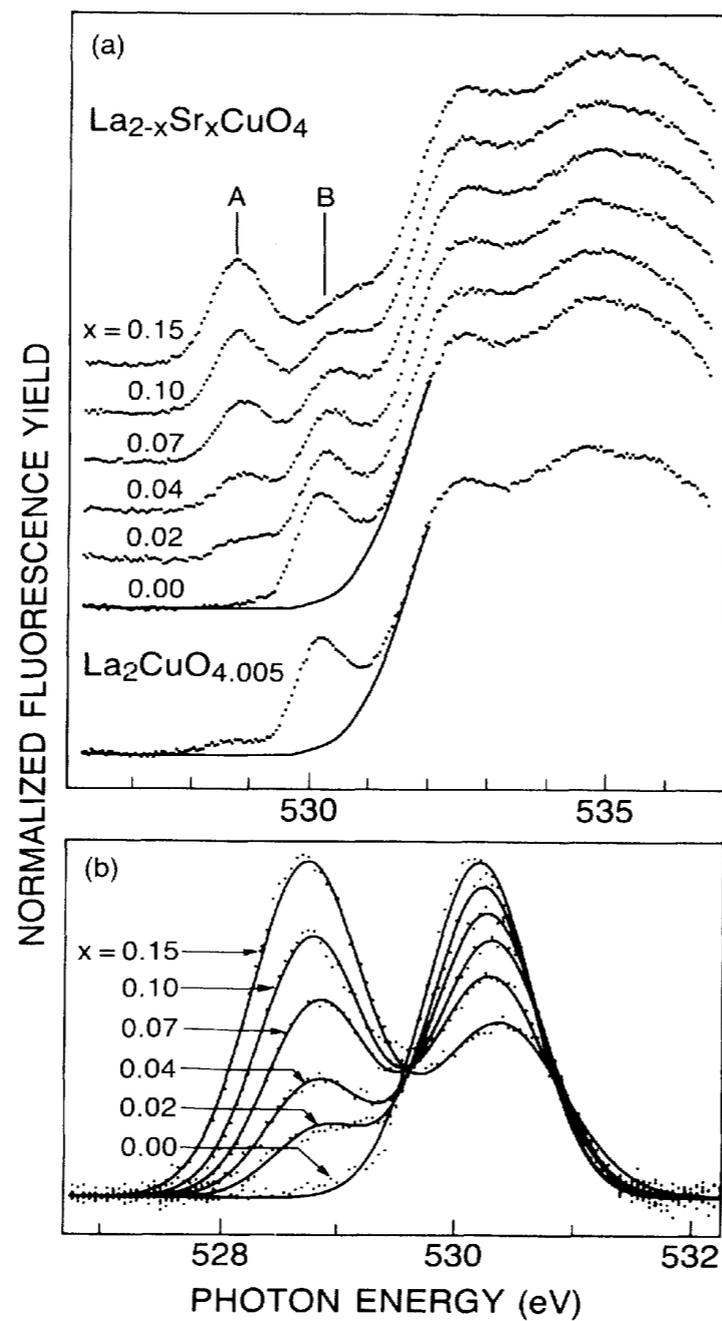
$$U \gg t$$



double occupancy in ground state!!

$$W_{\text{PES}} > 1 + x$$

C. T. Chen,
Batlogg,
et al. 1990



What does

$$W_{\text{PES}} > 1 + x$$

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new degrees of freedom

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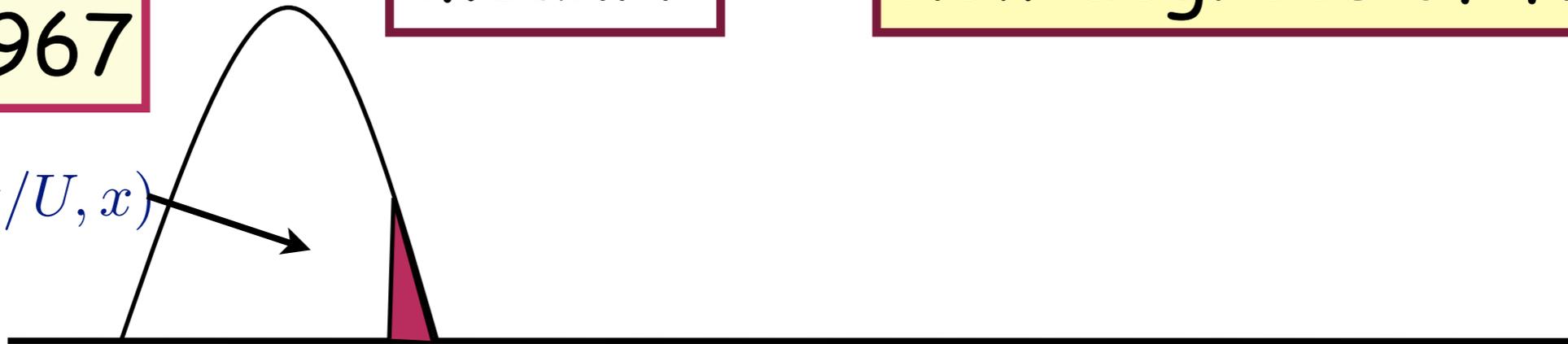
$$W_{\text{PES}} > 1 + x$$

mean??

new degrees of freedom

H&L, 1967

$$1 + x + \alpha(t/U, x)$$



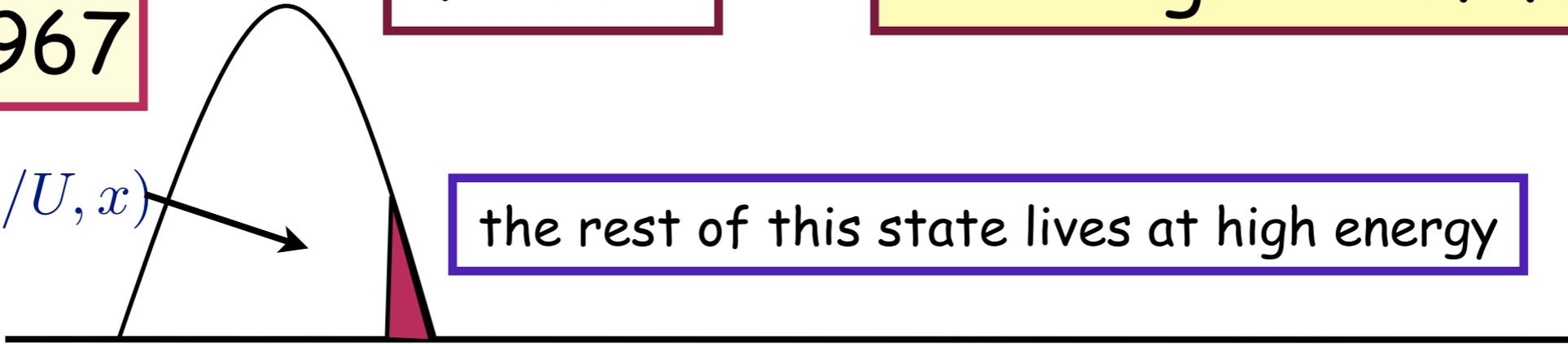
What does

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new degrees of freedom

H&L, 1967

$$1 + x + \alpha(t/U, x)$$


the rest of this state lives at high energy

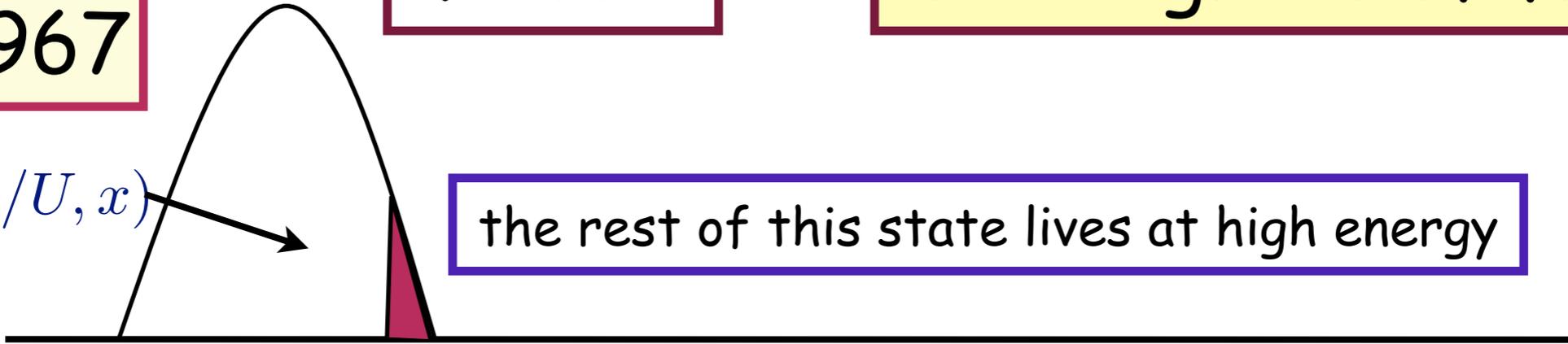
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mean??

new degrees of freedom

H&L, 1967

$$1 + x + \alpha(t/U, x)$$


the rest of this state lives at high energy

of ways of adding electrons remains $1+x$

What does

$$W_{\text{PES}} > 1 + x$$

mean??

new degrees of freedom

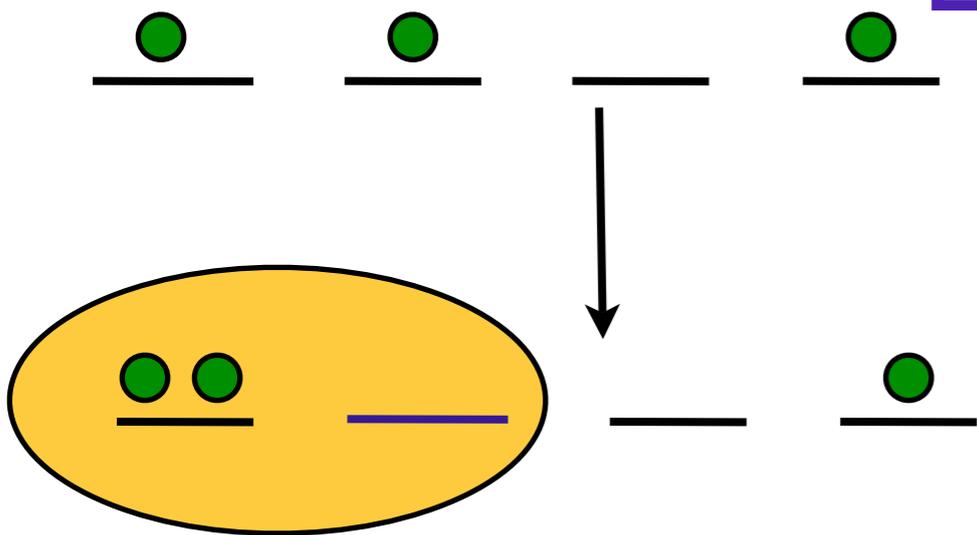
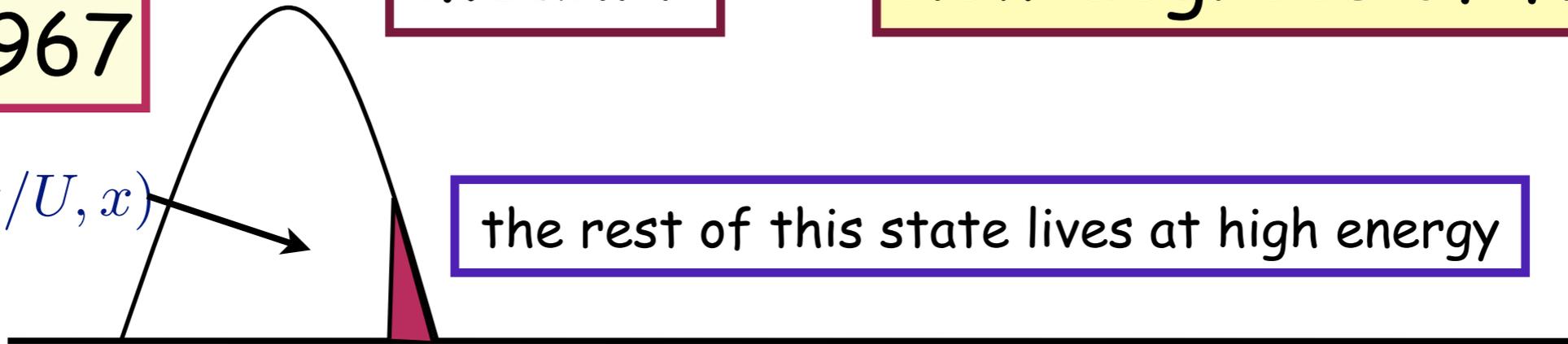
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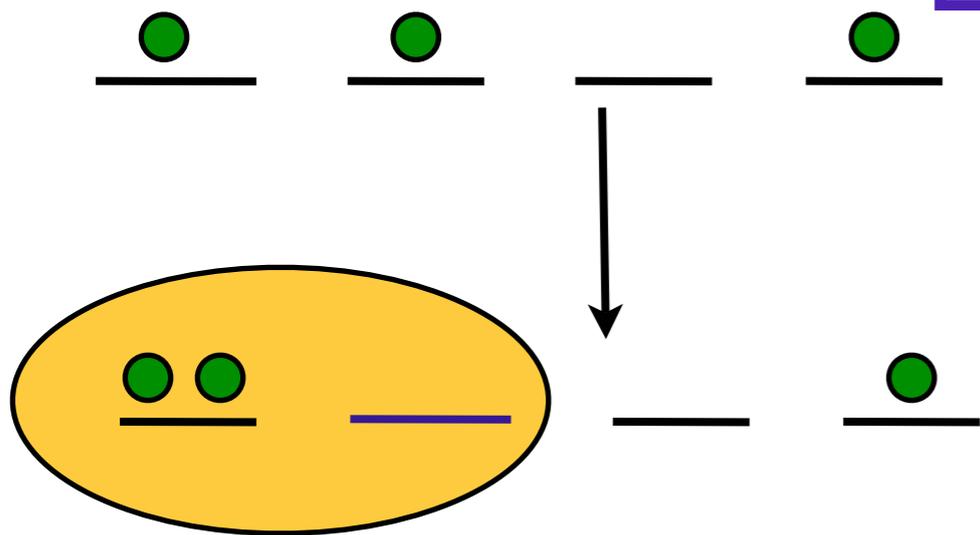
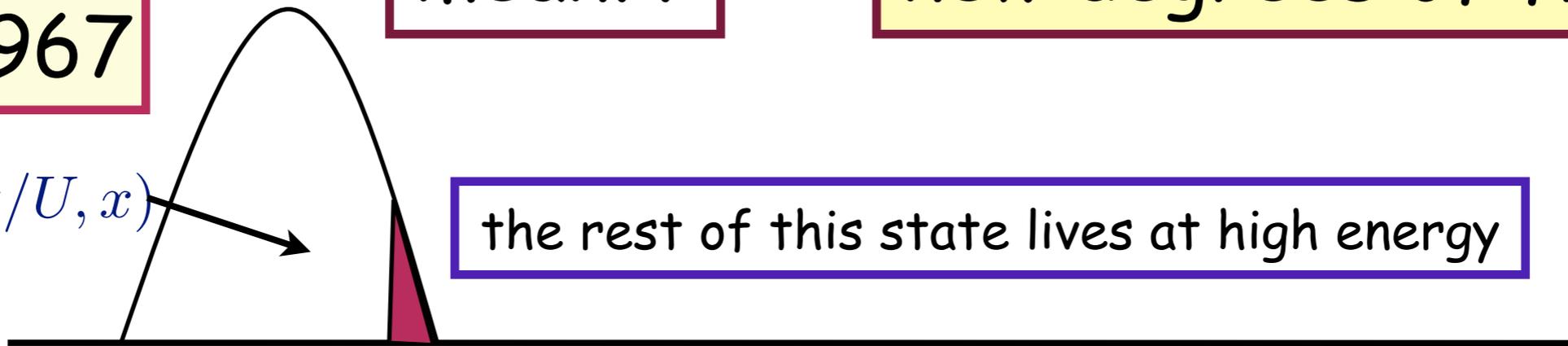
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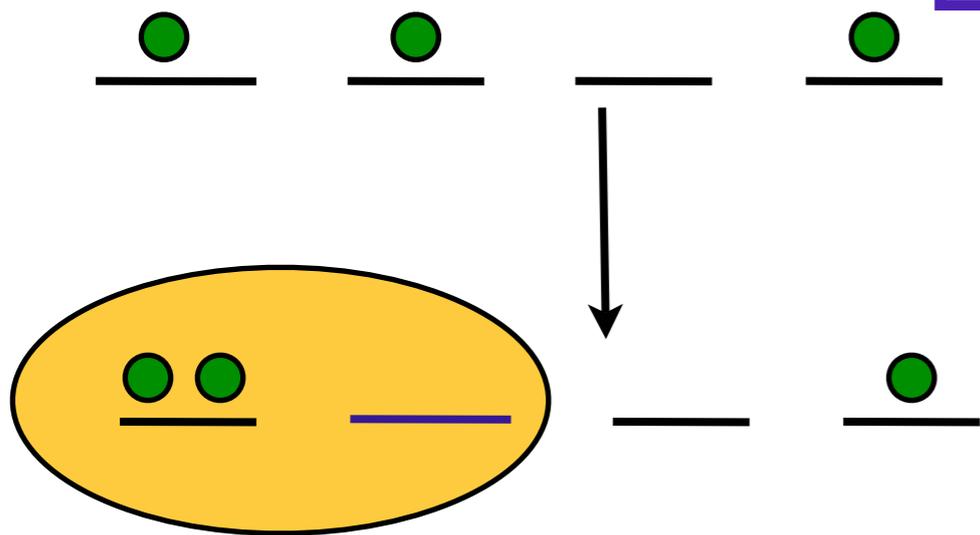
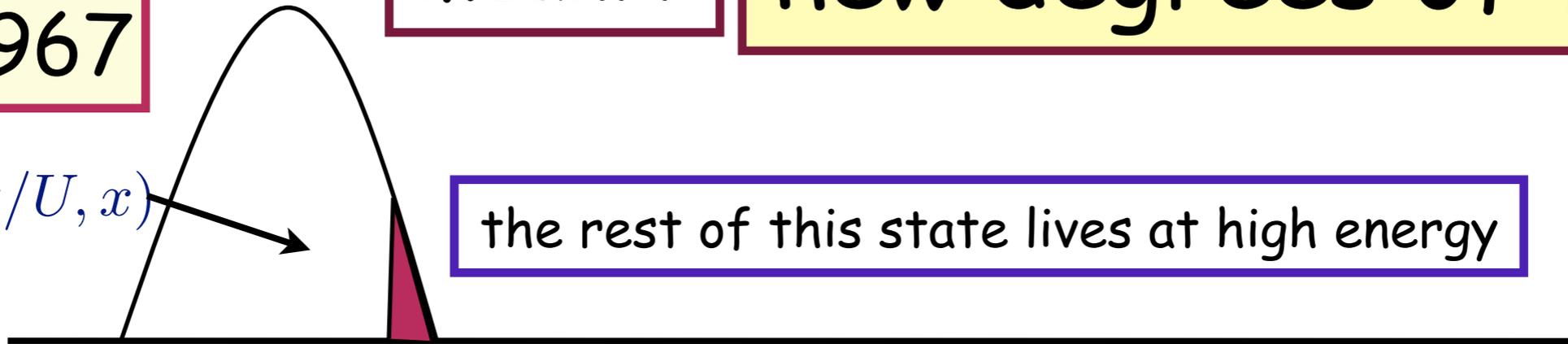
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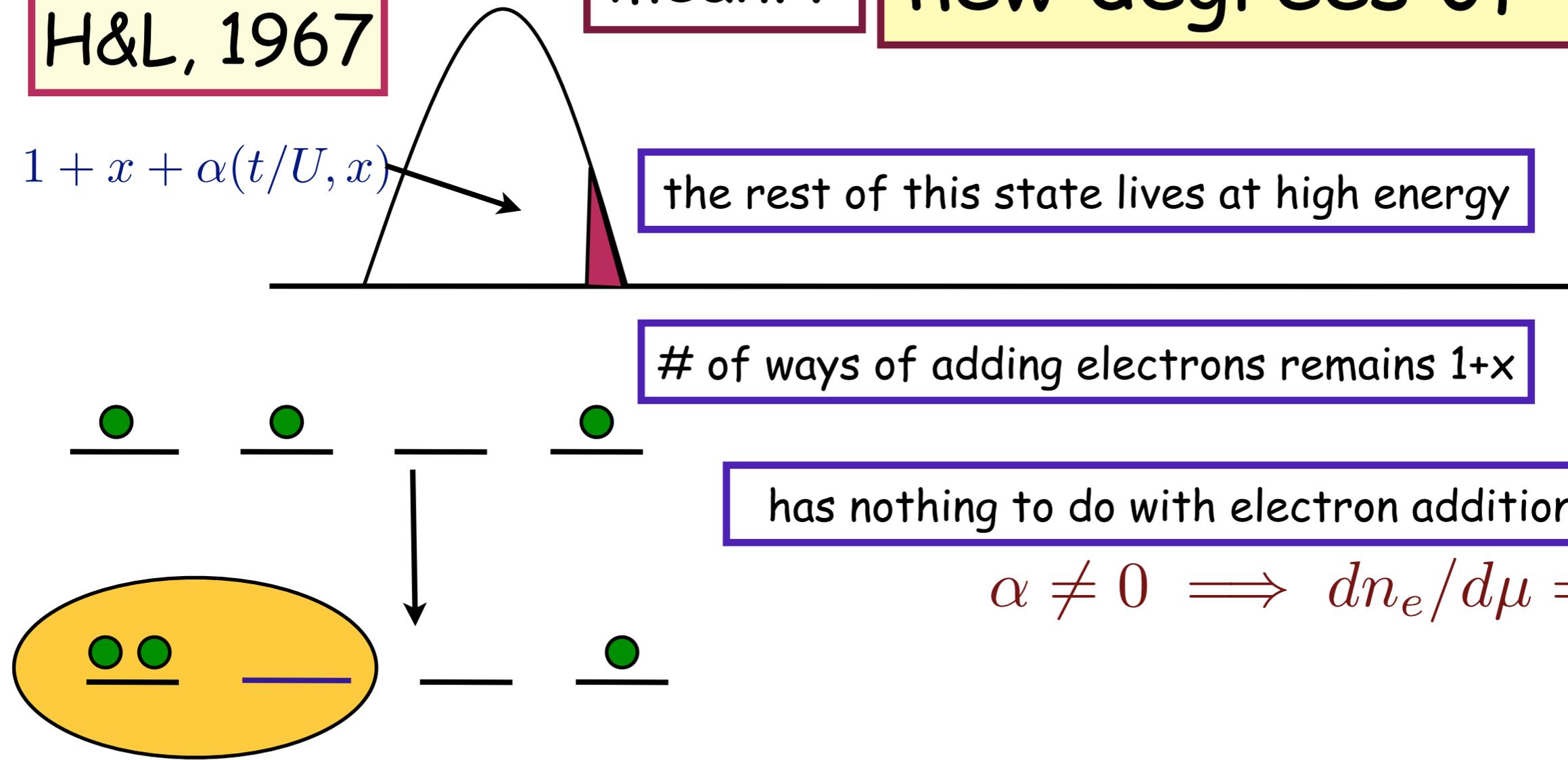
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conserved charge

$$n_e = n_{\text{qp}} + n_{\text{newstuff}} > n_{\text{qp}}$$



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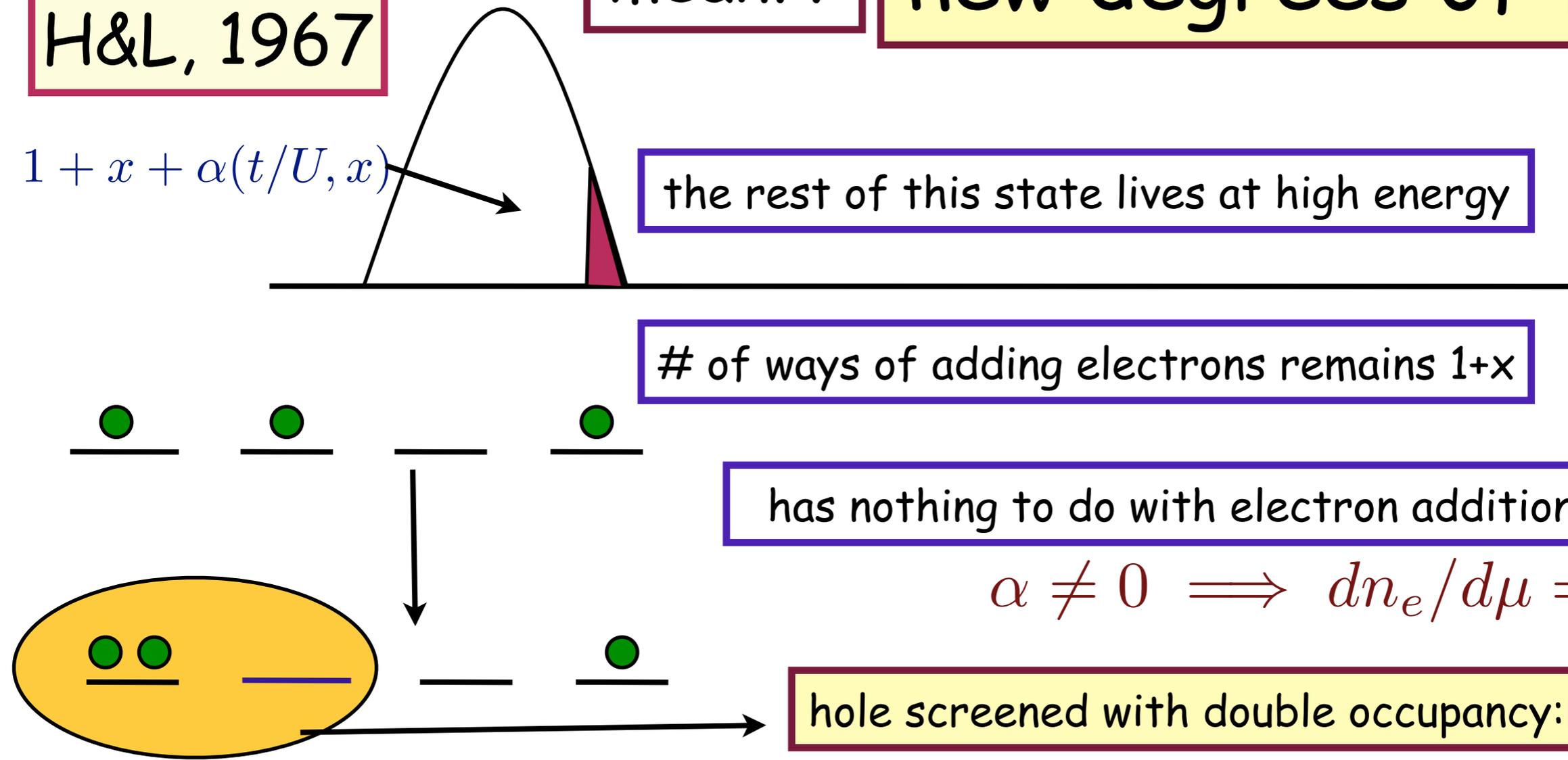
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hole screened with double occupancy: local

conserved charge

$$n_e = n_{\text{qp}} + n_{\text{newstuff}} > n_{\text{qp}}$$

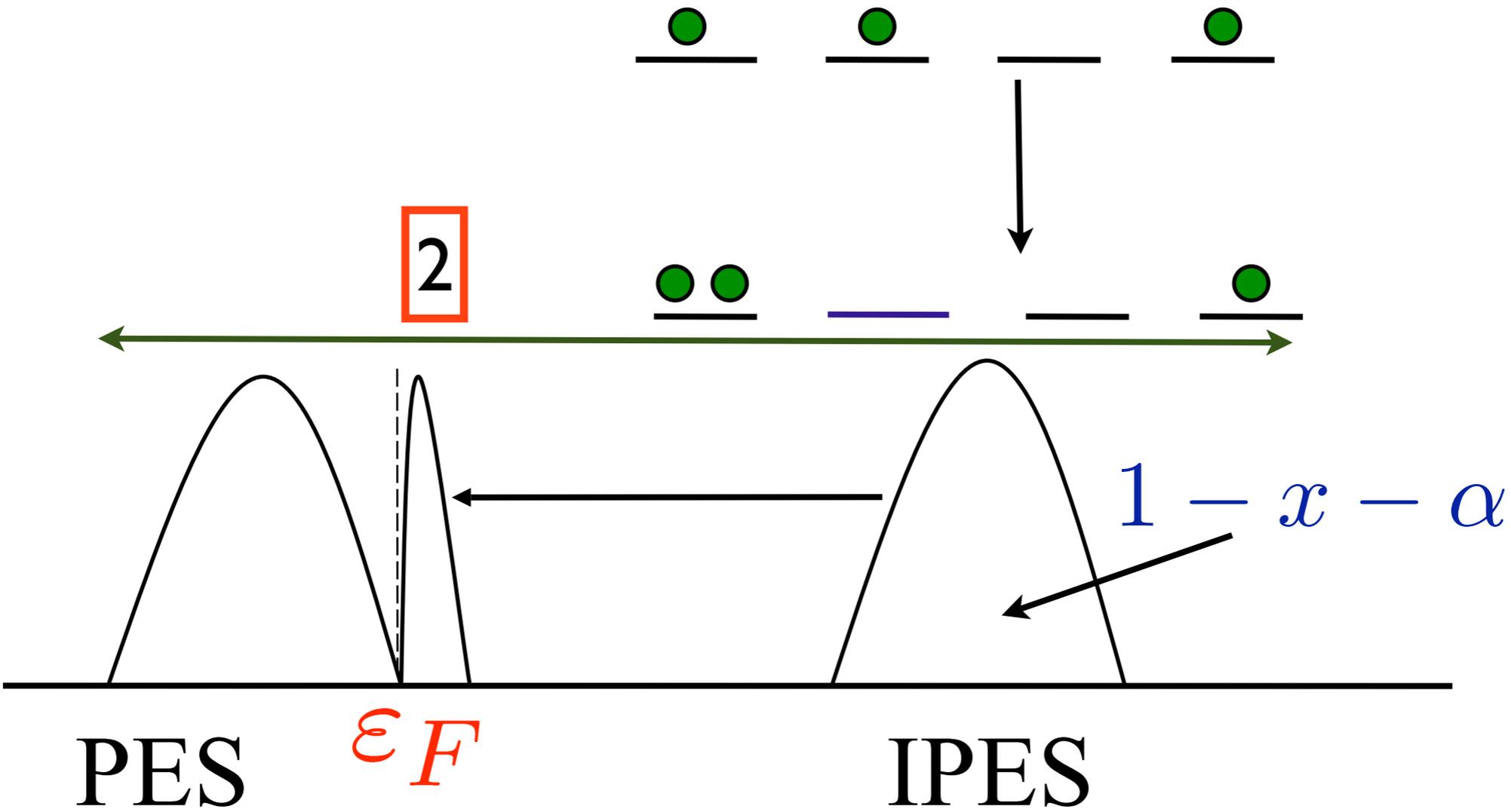


What are the low-energy quasi-particles?



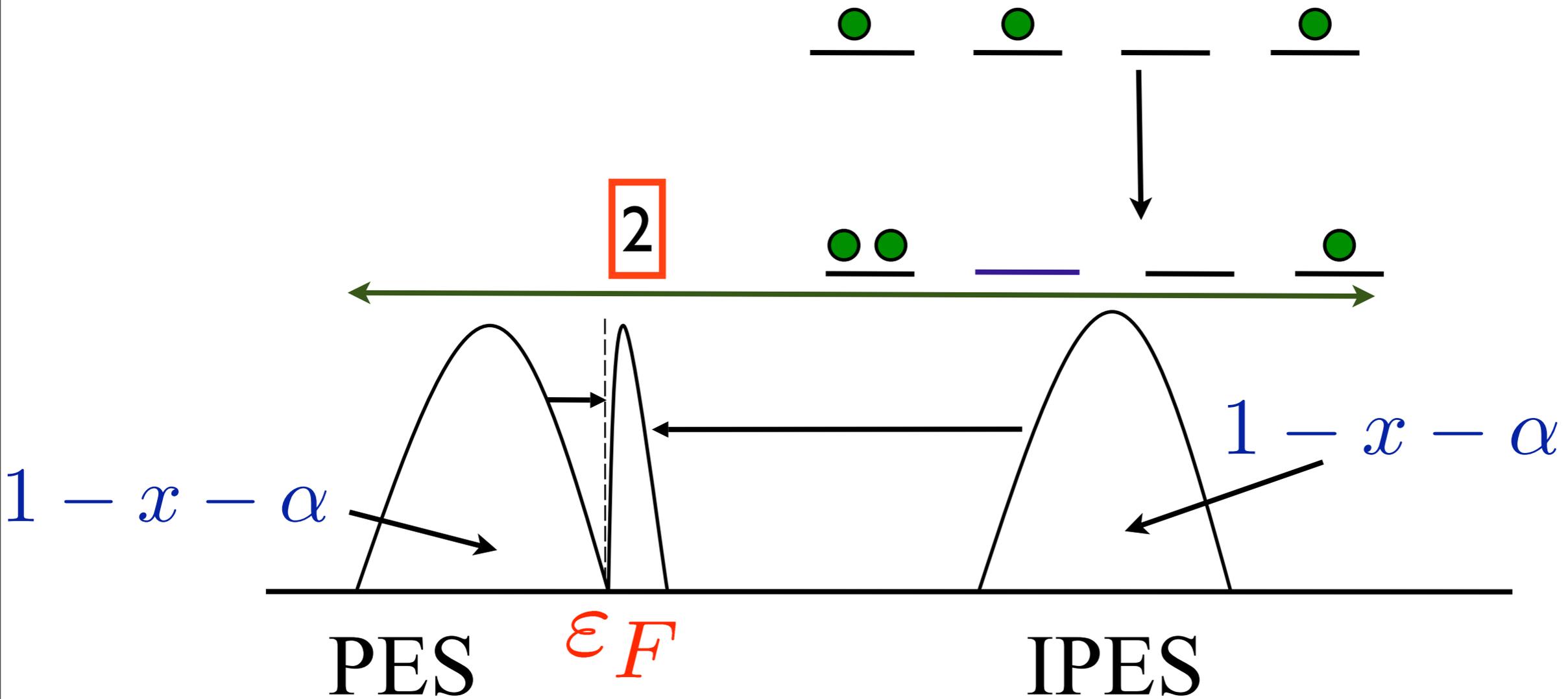
spectral weights are equal

low-energy qp picture



weight $> 1+x$, conserved
 charge $= 1-x \Rightarrow$
 $n_{qp} < 1 - x$

low-energy qp picture

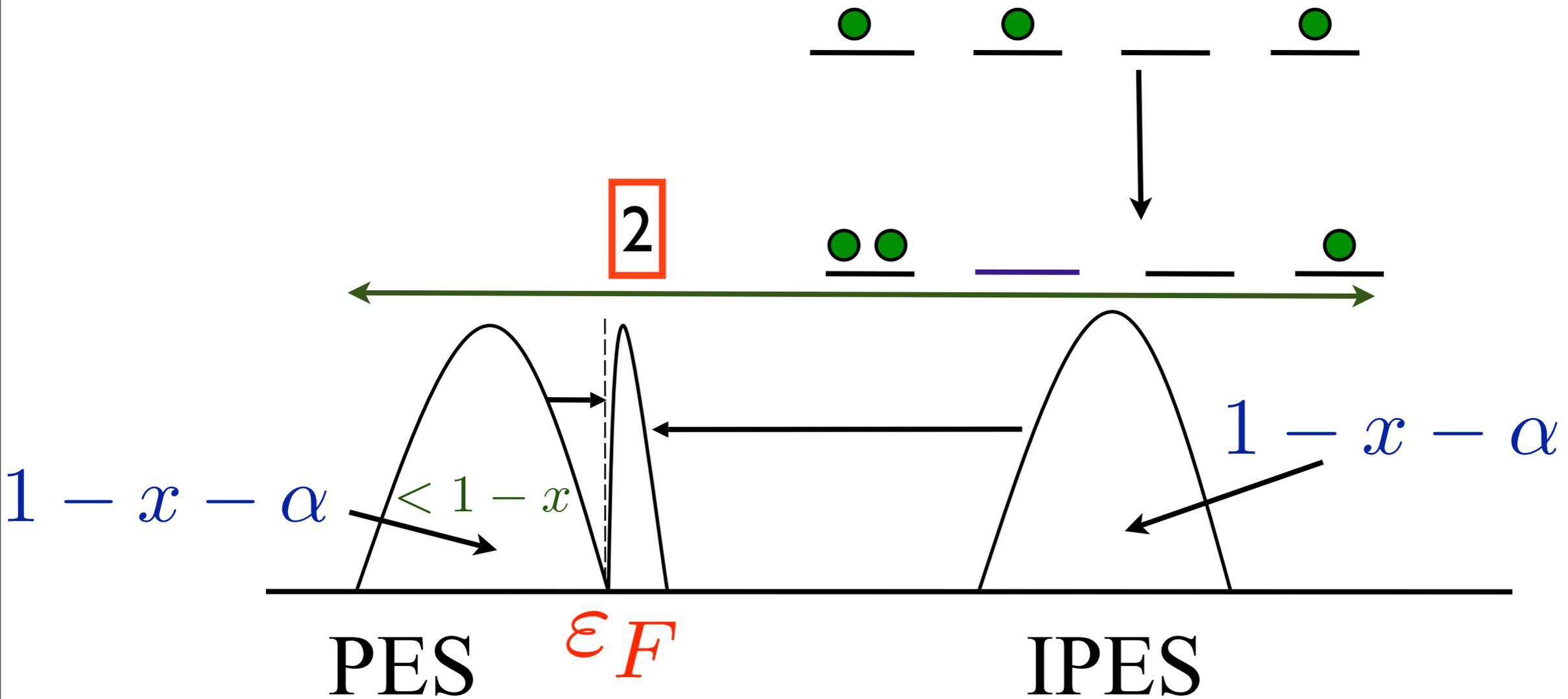


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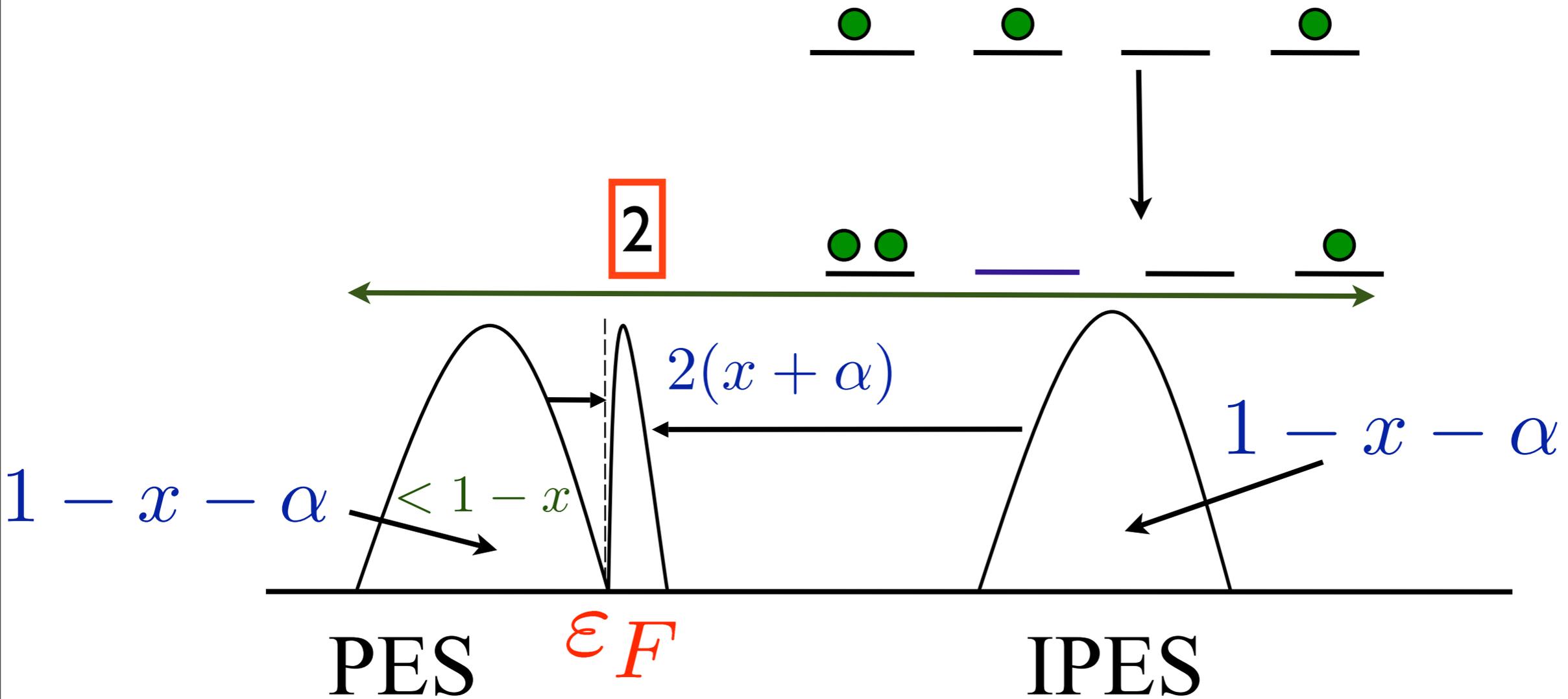


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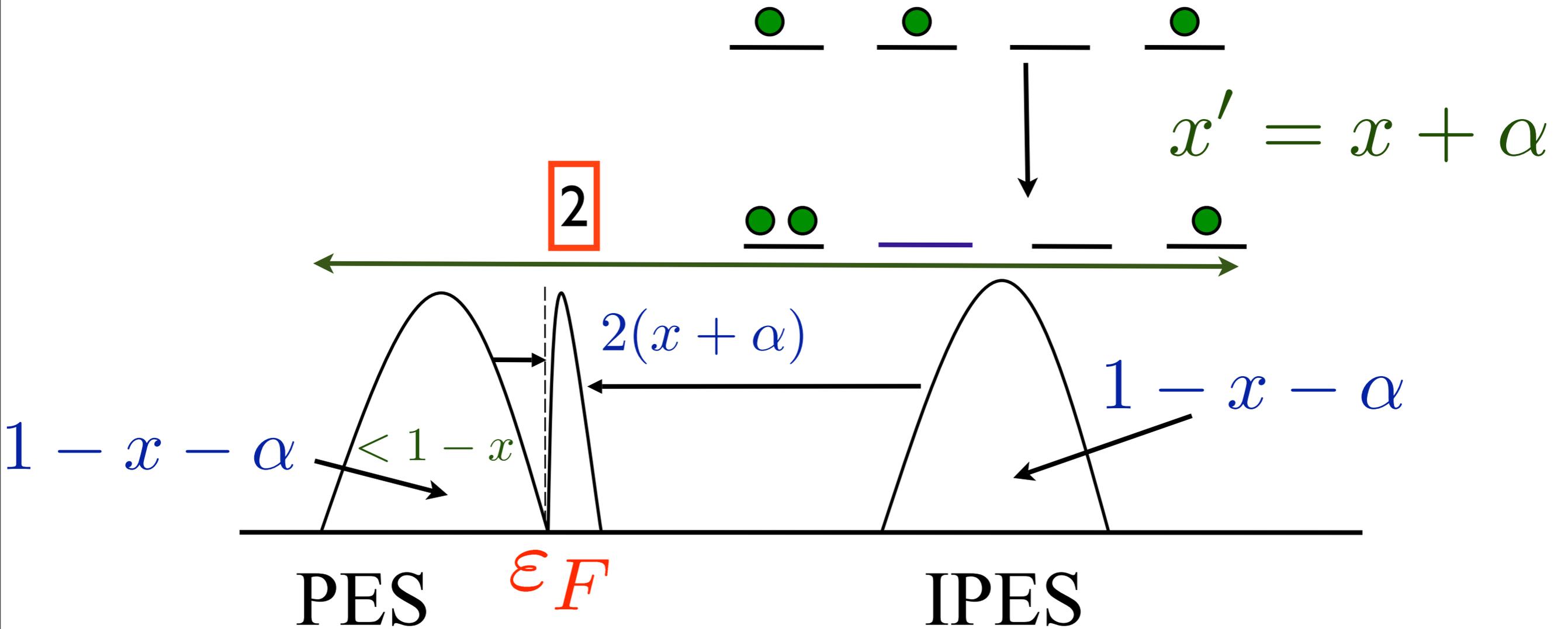


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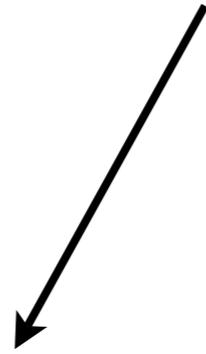
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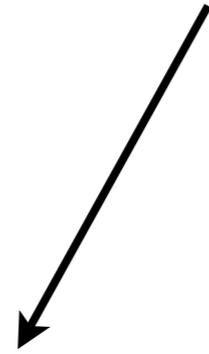
dynamical spectral weight transfer: beyond $U = \infty$

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weight of low-energy band
is bigger than electron weight

dynamical spectral weight transfer: beyond $U = \infty$

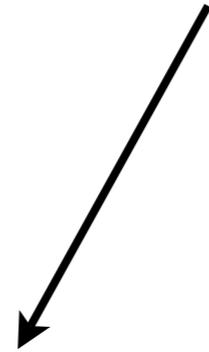


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number of electron qp
< number of bare
electrons \Rightarrow FL theory
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per e per spin > 1



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ways to add a particle
but not an electron
(gapped spectrum)

dynamical spectral weight transfer: beyond $U = \infty$

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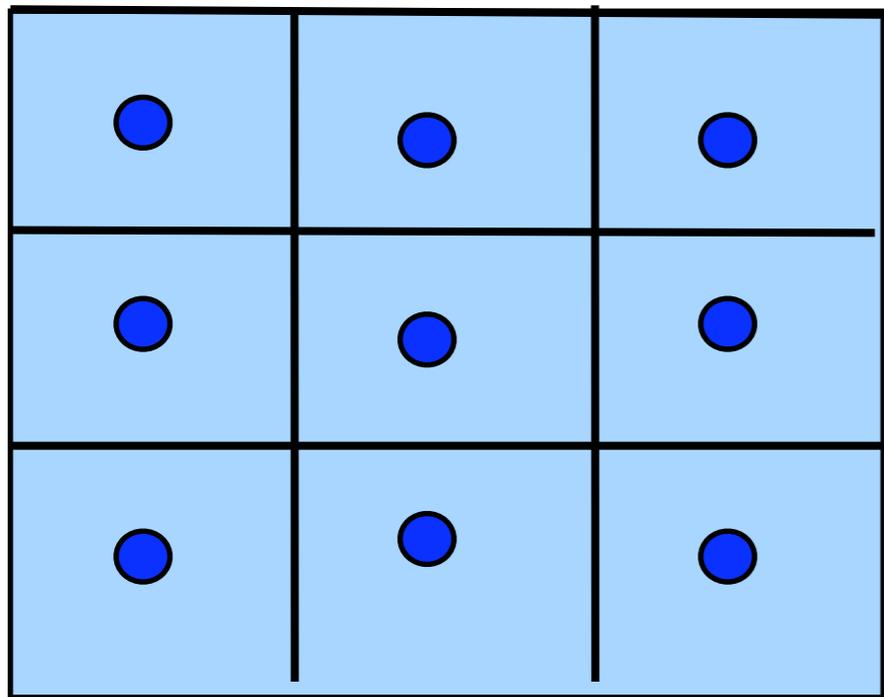
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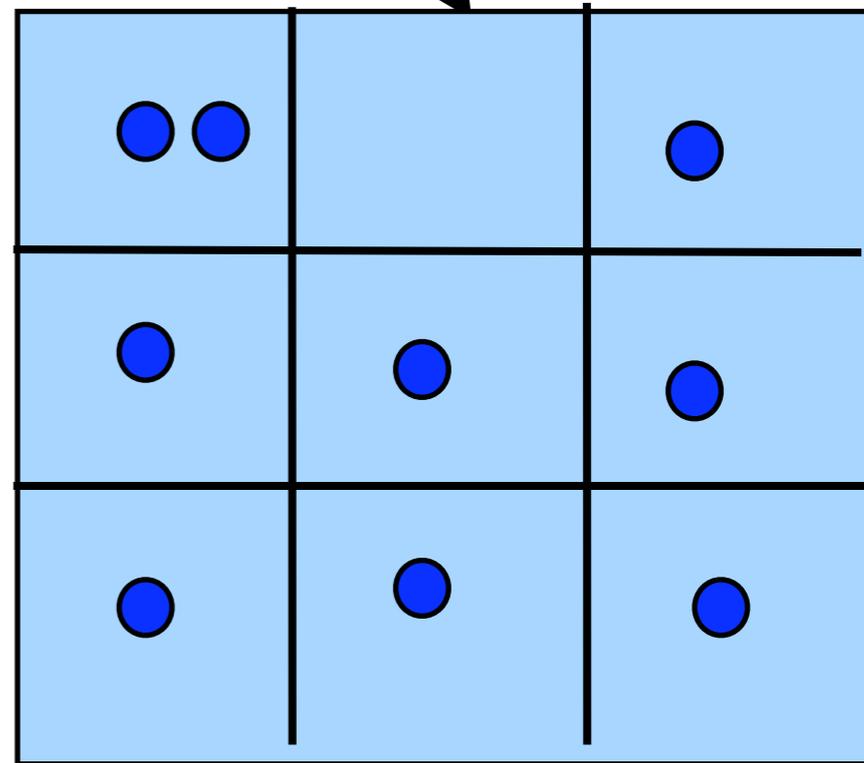
ways to add a particle
but not an electron
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breakdown of electron quasi-particle picture: Mottness

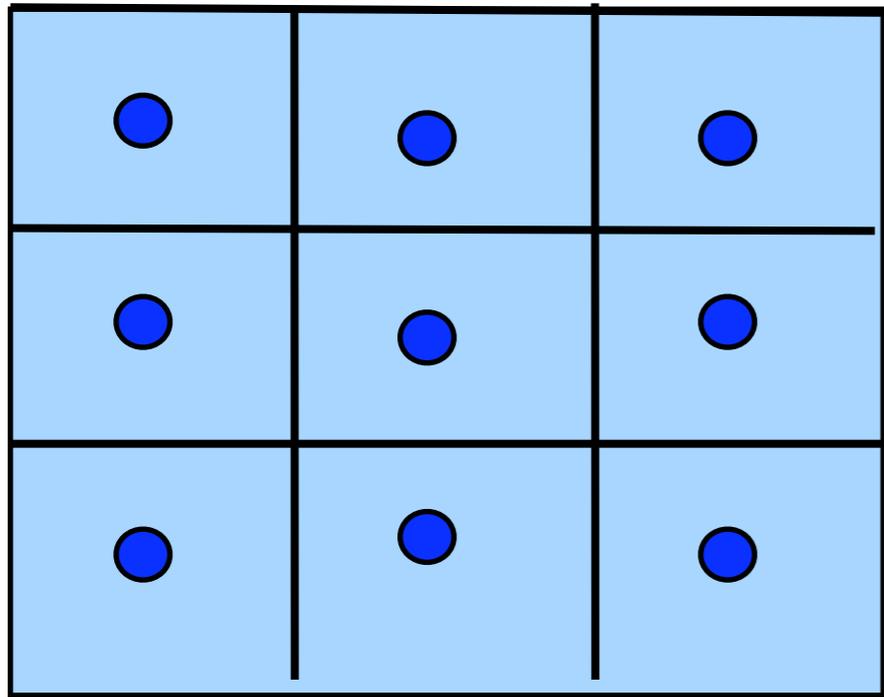
is the empty site
mobile??



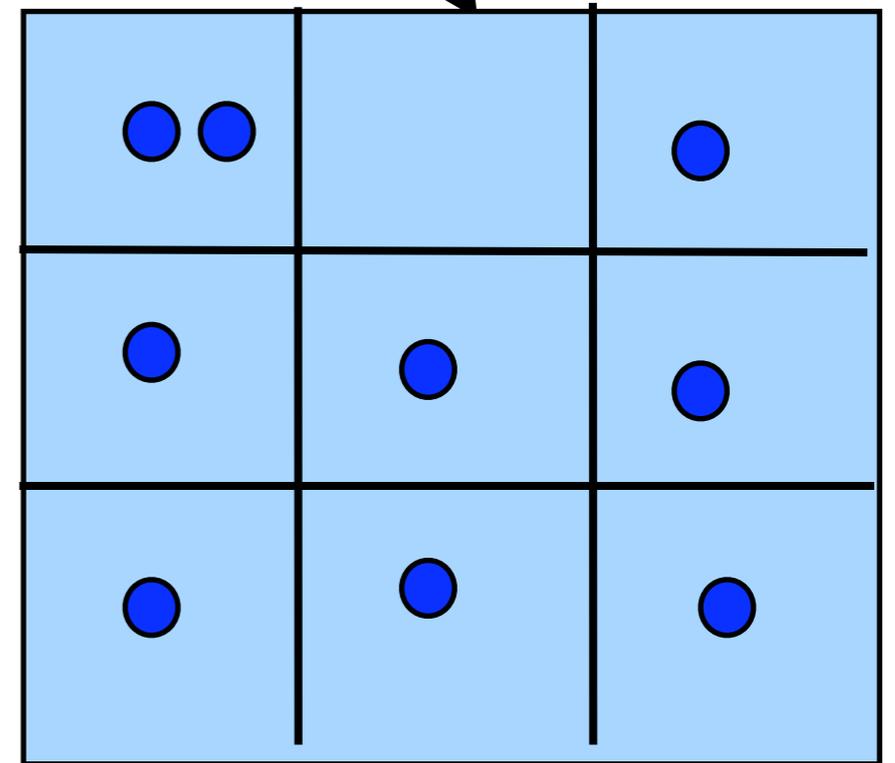
+ t/U



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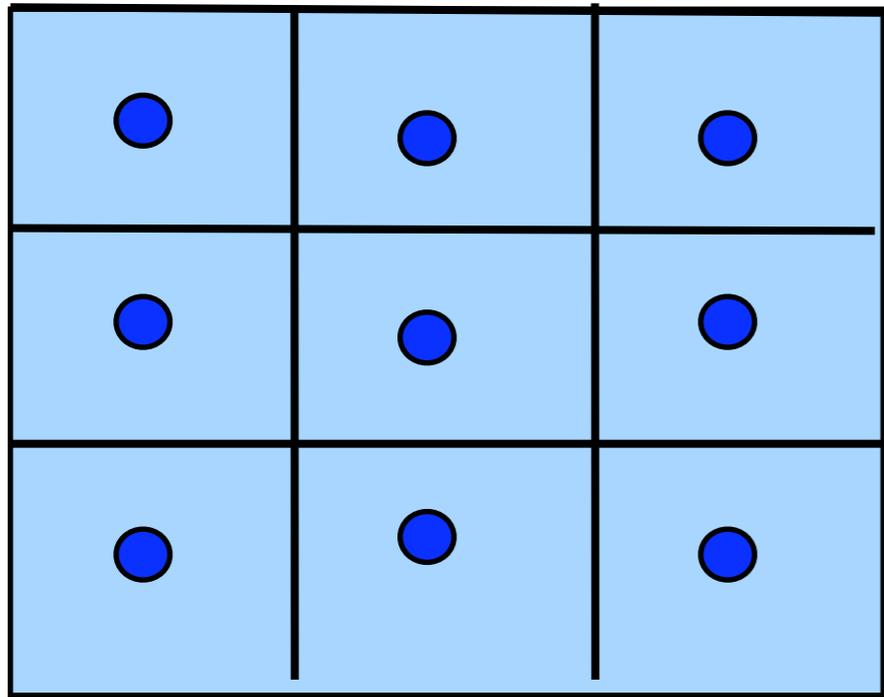


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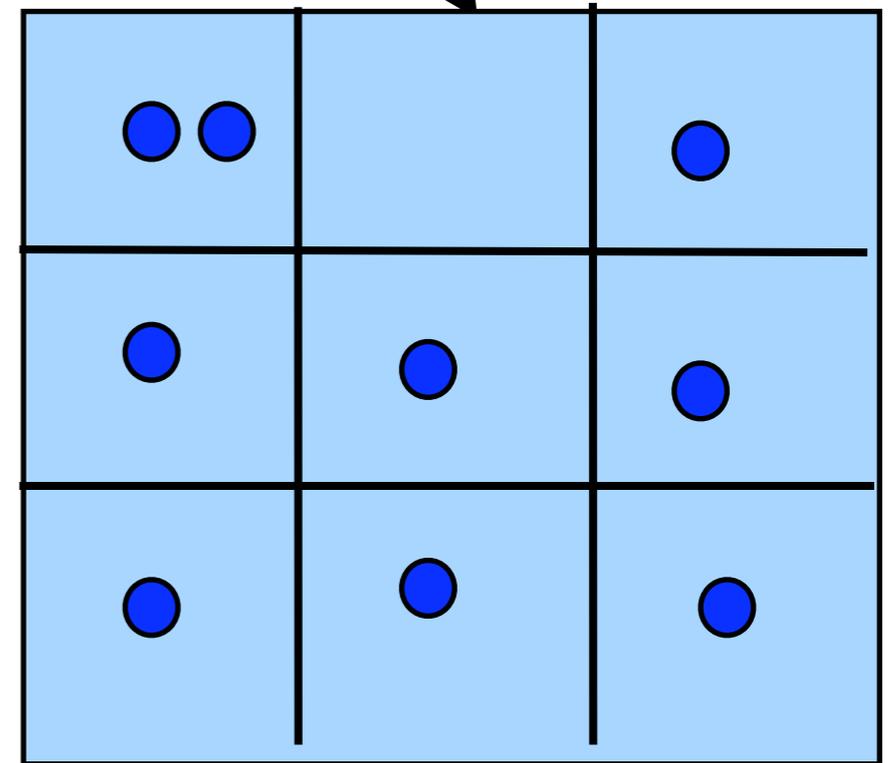


if yes, then 1.)
Mott insulator is a metal,
2.) no magnetic order

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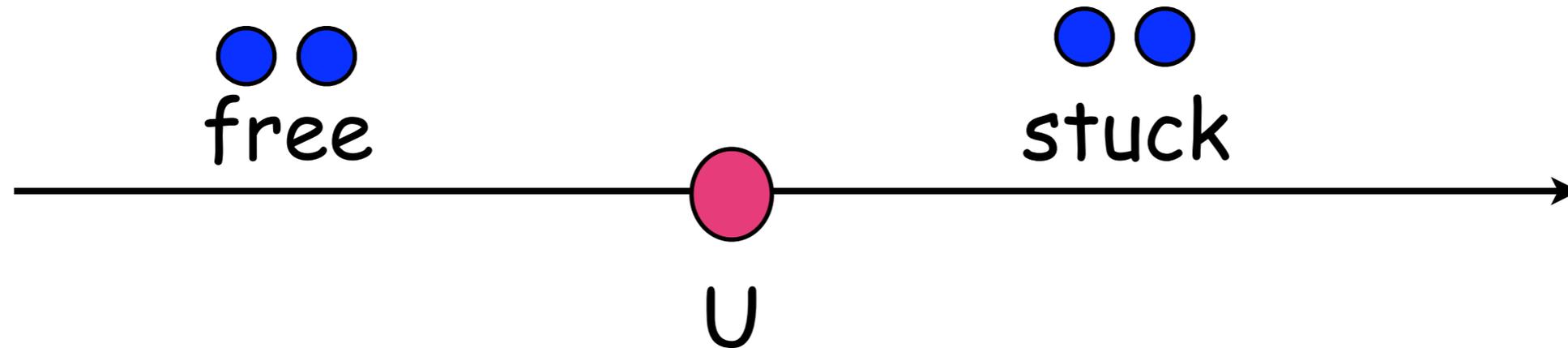


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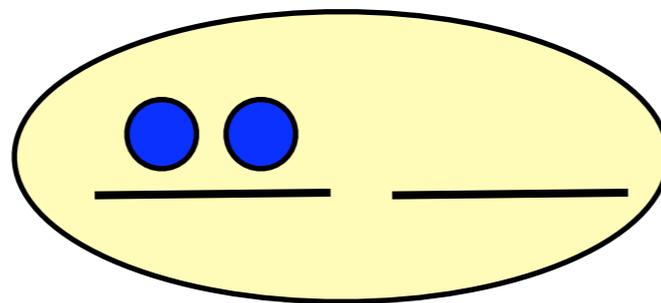
localisation criterion



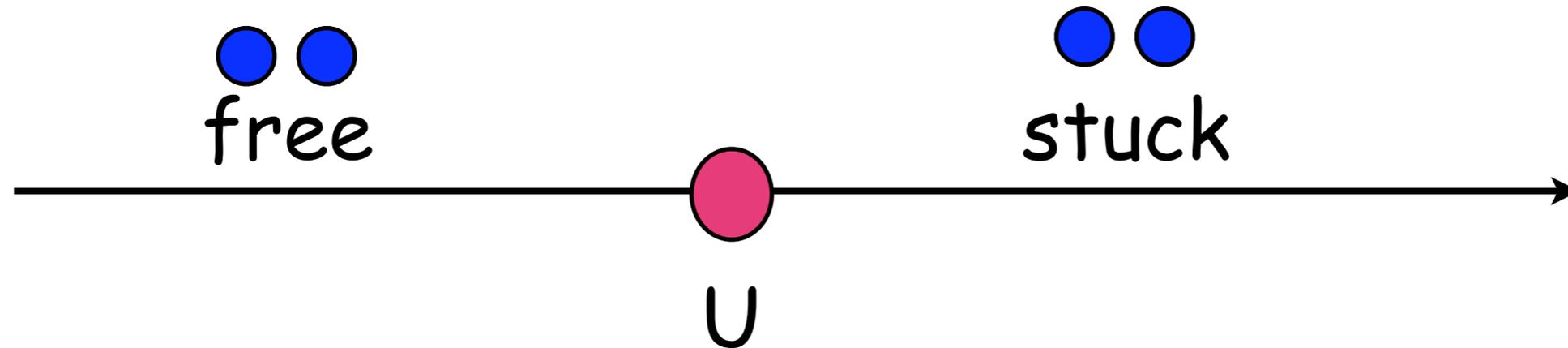
Mott Problem: what is the dynamical degree of freedom that makes this happen?

new bound states

Kohn, Mott,
Castellani,
others



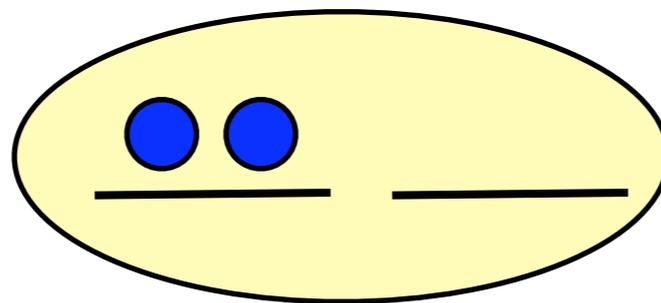
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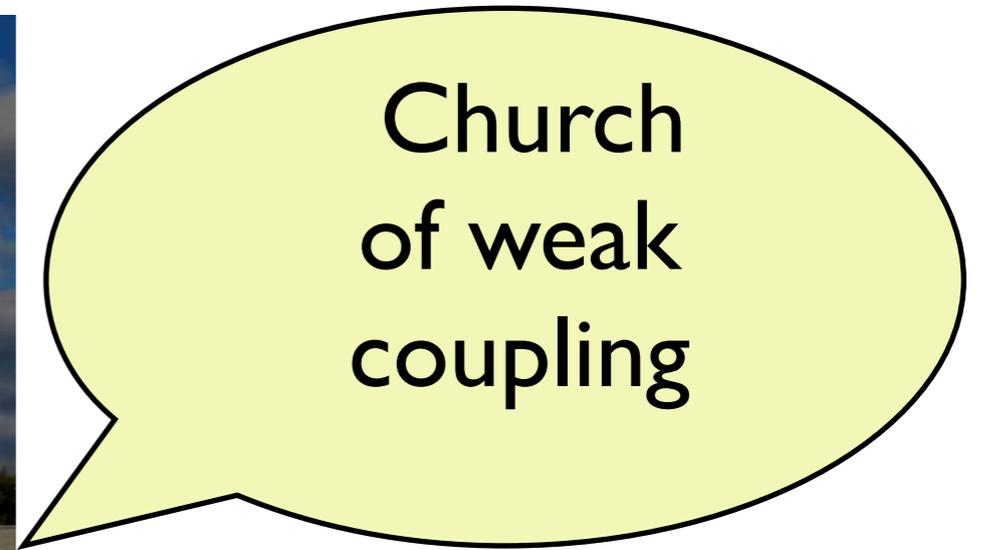


No proof exists?
Mottness is ill-defined

A Critique of Two Metals

R. B. Laughlin

idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.



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Church
of weak
coupling

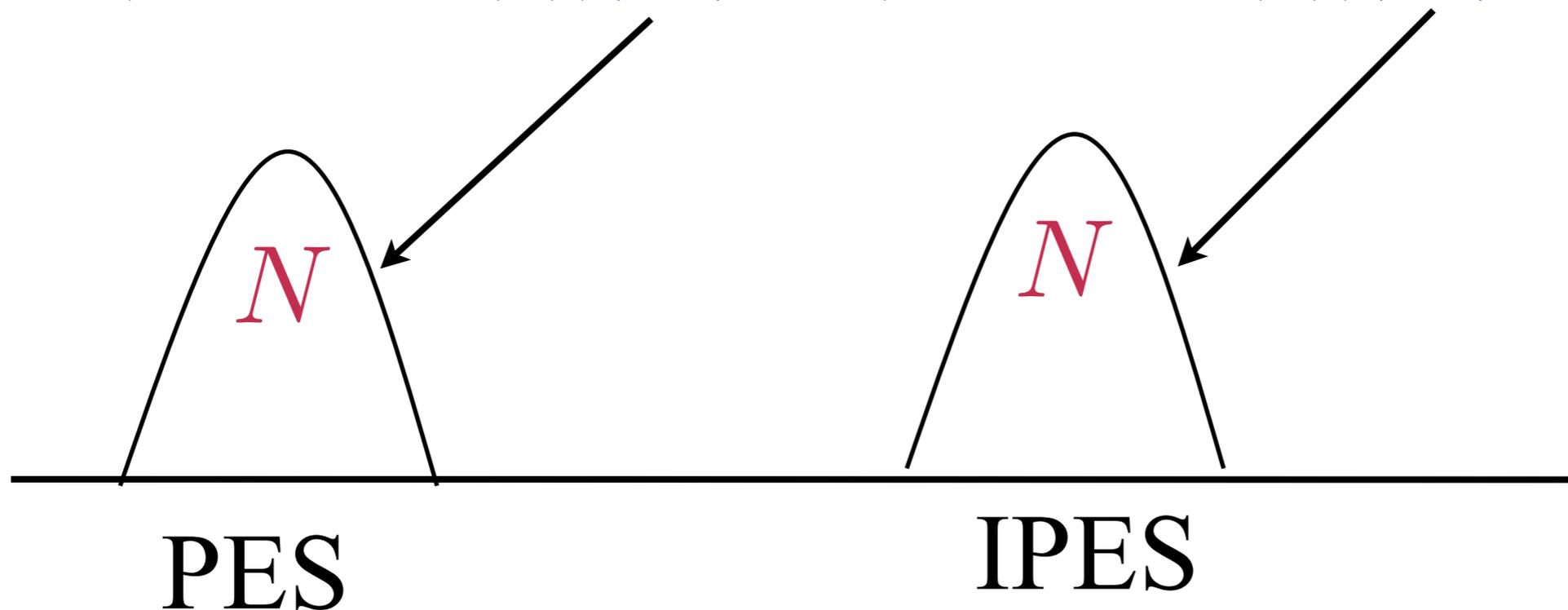
Beliefs:
Mott gap is heresy?
HF is the way!
No UHB and LHB!

Fermi-liquid analogy

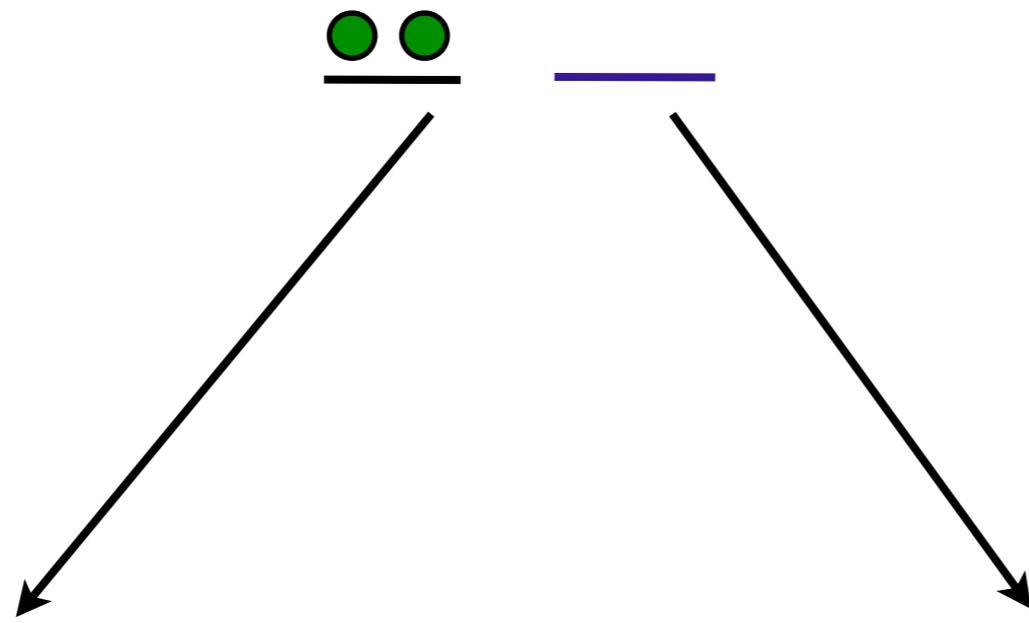
$$L_{\text{FL}} \propto (\omega - \epsilon_k) |\psi_k|^2$$

Mott Problem?

$$L_{\text{MI}} = (\omega - E_{\text{LHB}}(k)) |\eta_k|^2 + (\omega - E_{\text{UHB}}(k)) |\tilde{\eta}_k|^2$$



composite excitation: bound state



half-filling:
Mott gap

doping:
SWT, pseudogap?

charge $2e$ boson

How?

Effective Theories:

$S(\phi)$ at half-filling

Integrate
Out high
Energy fields

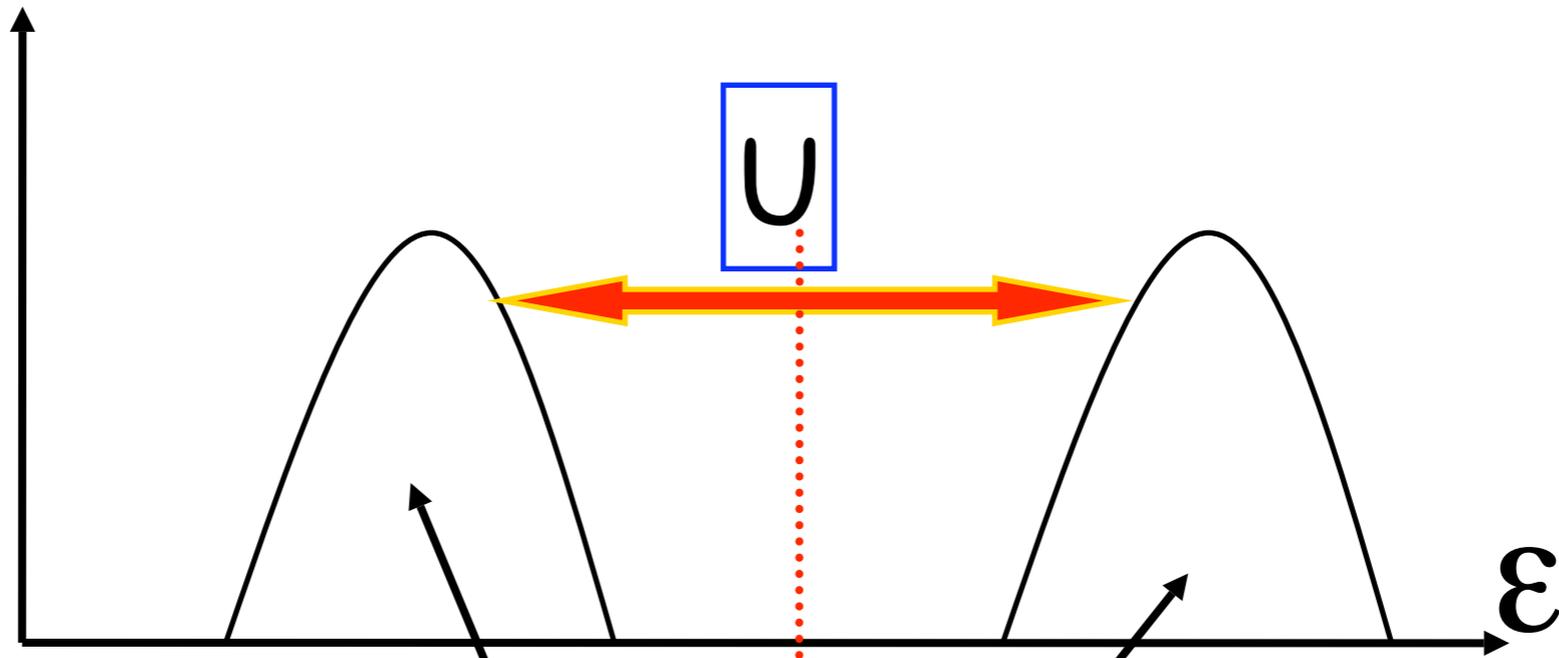
$$\phi = \phi_L + \phi_H$$

$$e^{-S_{\text{eff}}[\phi_L]} = \int d\phi_H \exp -S(\phi_L, \phi_H)$$

Low-energy theory of M I

Half-filling

$N(\omega)$

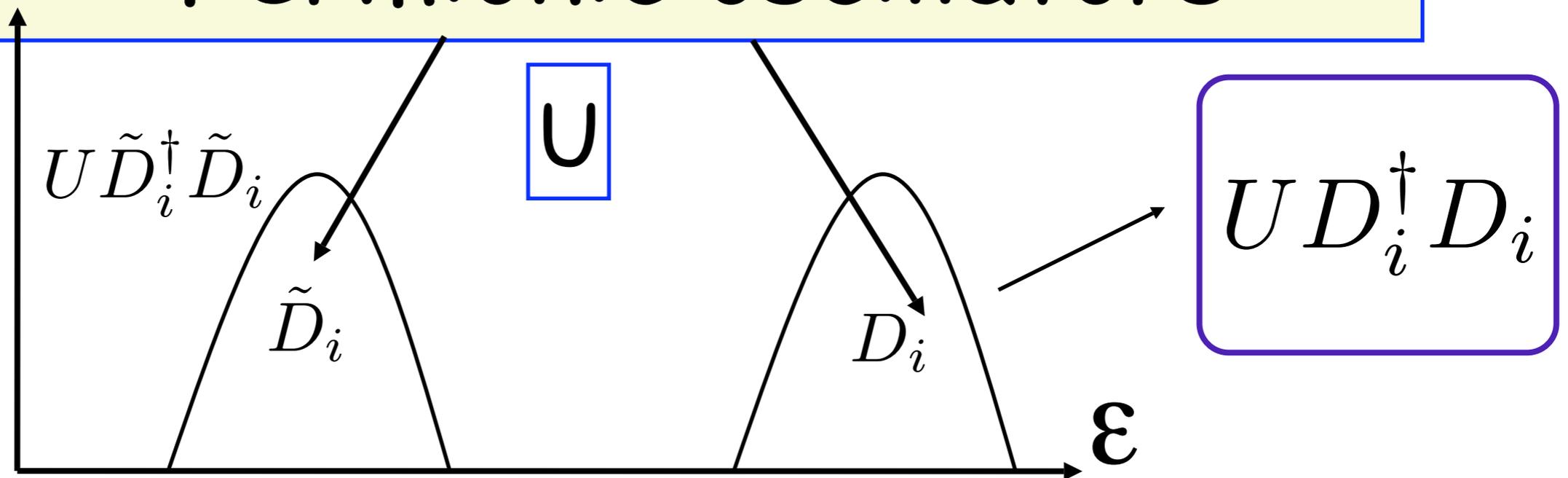


Integrate out both

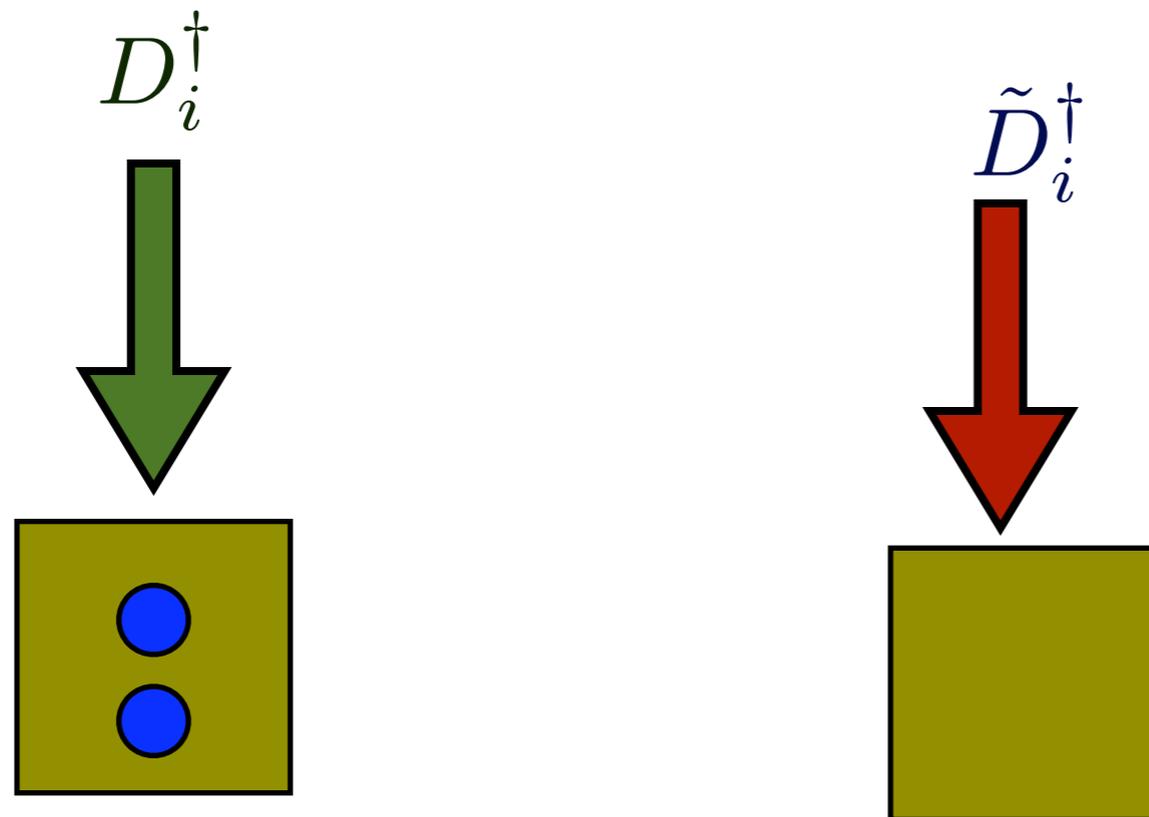
Key idea: similar to Bohm/Pines

Extend the Hilbert space:
Associate with U -scale new
Fermionic oscillators

$N(\omega)$



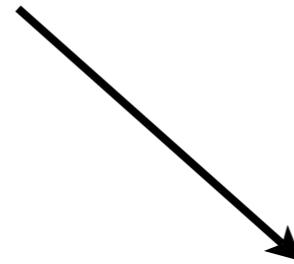
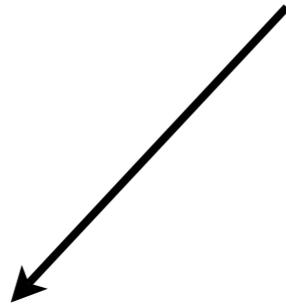
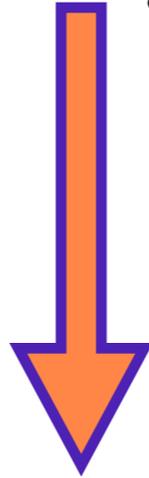
Impose Constraint:



How is this possible with
Fermions?

$$D_i^\dagger$$

Fermionic

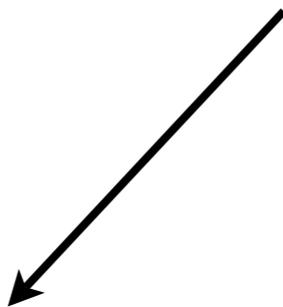
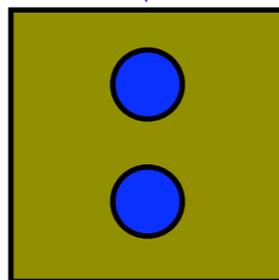
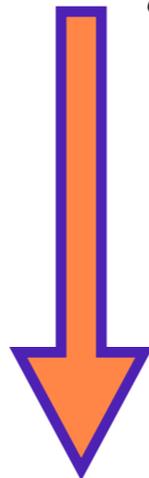


one per site
(fermionic)

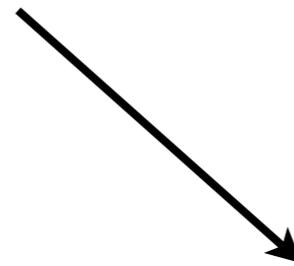
transforms as a boson

$$D_i^\dagger$$

Fermionic



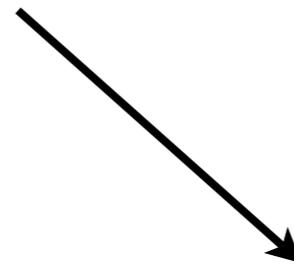
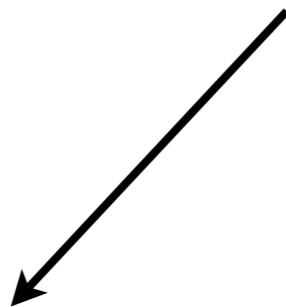
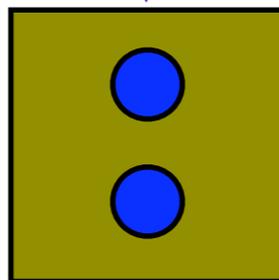
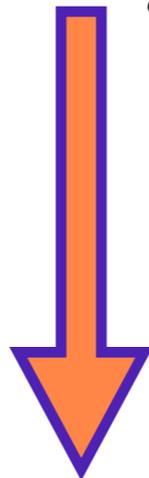
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transforms as a boson

$$D_i^\dagger$$

Fermionic

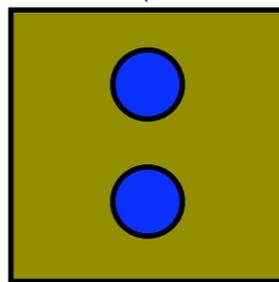
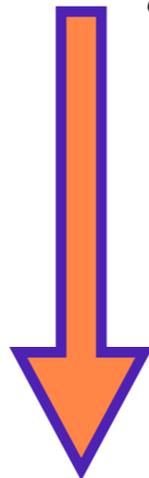


one per site
(fermionic)

transforms as a boson

'supersymmetry'

D_i^\dagger Fermionic



one per site
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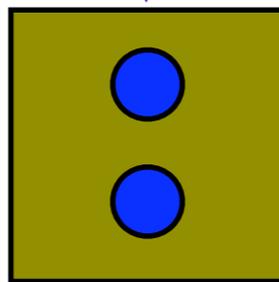
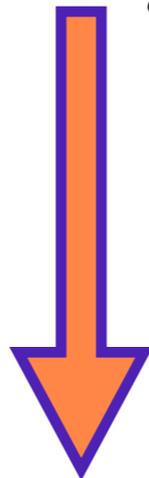
transforms as a boson

'supersymmetry'

Grassmann

$$\delta(D_i - \theta c_{i\uparrow} c_{i\downarrow})$$

D_i^\dagger Fermionic



one per site
(fermionic)

transforms as a boson

'supersymmetry'

Grassmann

$$\delta(D_i - \theta c_{i\uparrow} c_{i\downarrow})$$

String theory:

$$X^\mu(\sigma, \theta) = X^\mu(\sigma) + \theta \gamma^\mu(\sigma)$$

Dual Theory

solve constraint

UV limit

$$\int d^2\theta \bar{\theta}\theta L_{\text{Hubb}} = \sum_{i,\sigma} c_{i,\sigma}^\dagger \dot{c}_{i,\sigma} + H_{\text{Hubb}},$$

integrate over
heavy fields

Exact low-energy
theory (IR limit)

Exact low-energy Lagrangian

$$L = \#L_{\text{bare}}(\text{electrons}) + \#L_{\text{bare}}(\text{bosons})$$

$$+ f(\omega)L_{\text{int}}(c, \varphi) + \tilde{f}(\omega)L_{\text{int}}(c, \tilde{\varphi})$$

$$f(\omega) = 0$$

dispersion

of propagating
modes

composite excitations

New interactions beyond
spin physics

$$L = \#L_{\text{bare}}(\text{electrons}) + \#L_{\text{bare}}(\text{bosons})$$

$$+ f(\omega) L_{\text{int}}(c, \varphi) + \tilde{f}(\omega) L_{\text{int}}(c, \tilde{\varphi})$$


$$\Psi^\dagger \Psi$$

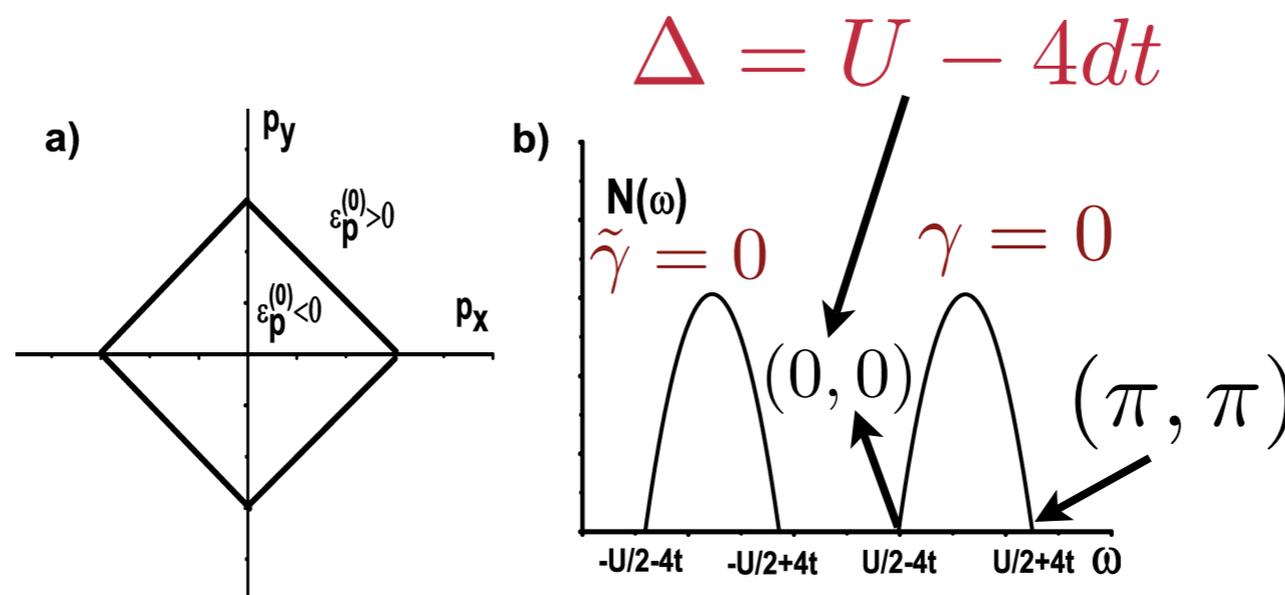

$$\tilde{\Psi}^\dagger \tilde{\Psi}$$

quadratic form:
composite or bound
excitations of
 $\varphi^\dagger c_{i\sigma}$

composite excitations determine spectral density

$$\gamma_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U - t\varepsilon_{\vec{p}}^{(\vec{k})} - 2\omega}{U} \sqrt{1 + 2\omega/U}$$

$$\tilde{\gamma}_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U + t\varepsilon_{\vec{p}}^{(\vec{k})} + 2\omega}{U} \sqrt{1 - 2\omega/U}.$$

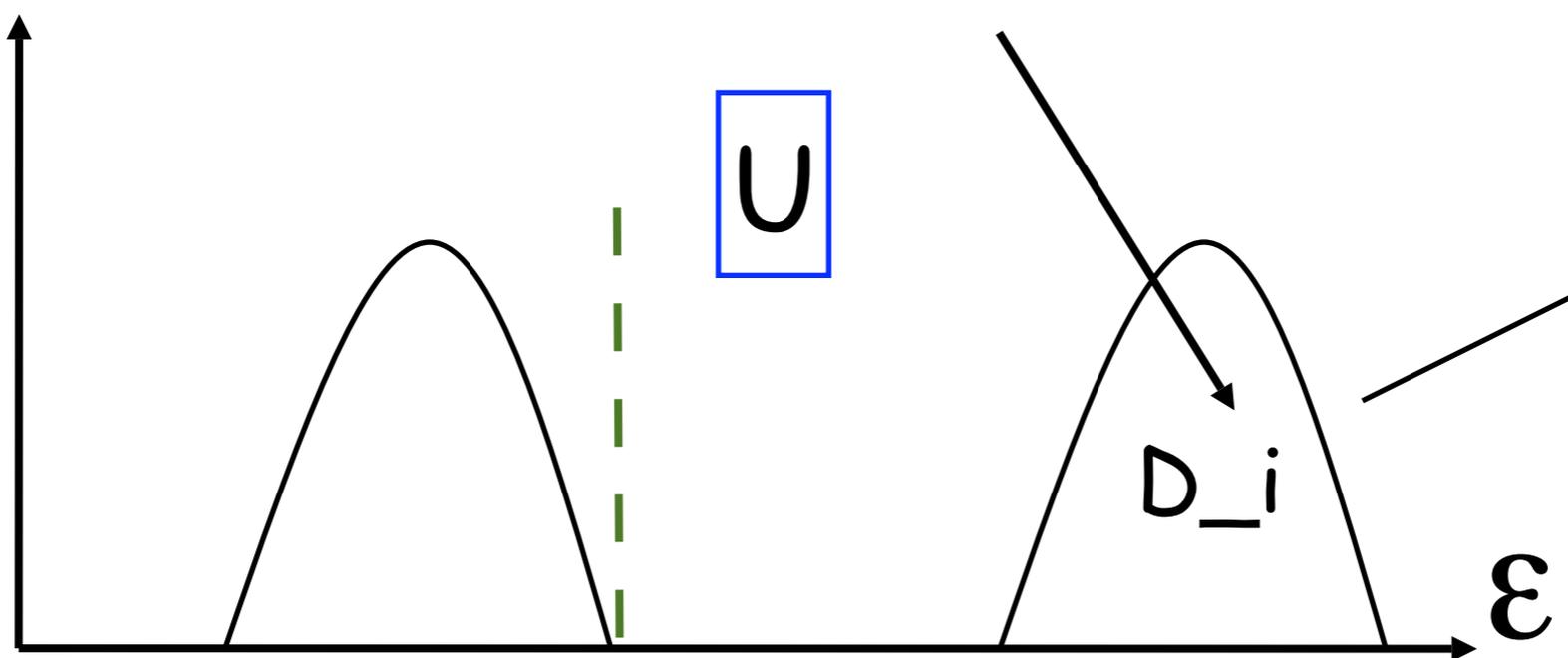


each momentum has SD at two distinct
energies

hole-doping

Extend the Hilbert space:
Associate with U-scale a new
Fermionic oscillator

$N(\omega)$



$$U D_i^\dagger D_i$$

charge $2e$ boson field (non-propagating)

$$H_h^{IR} = H_{t-J}$$

$$-\frac{t^2}{U} \sum_i \varphi_i^\dagger \varphi_i - t \sum_j \varphi_j^\dagger c_{j,\uparrow} c_{j,\downarrow} + \frac{t^2}{U} \sum_{i,j} \varphi_i^\dagger b_i + h.c. ,$$

Non-projective

$$b_i = \sum_j g_{ij} (c_{i\downarrow} c_{j\uparrow} - c_{i\uparrow} c_{j\downarrow})$$

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$$H_h^{IR} = H_{t-J}$$

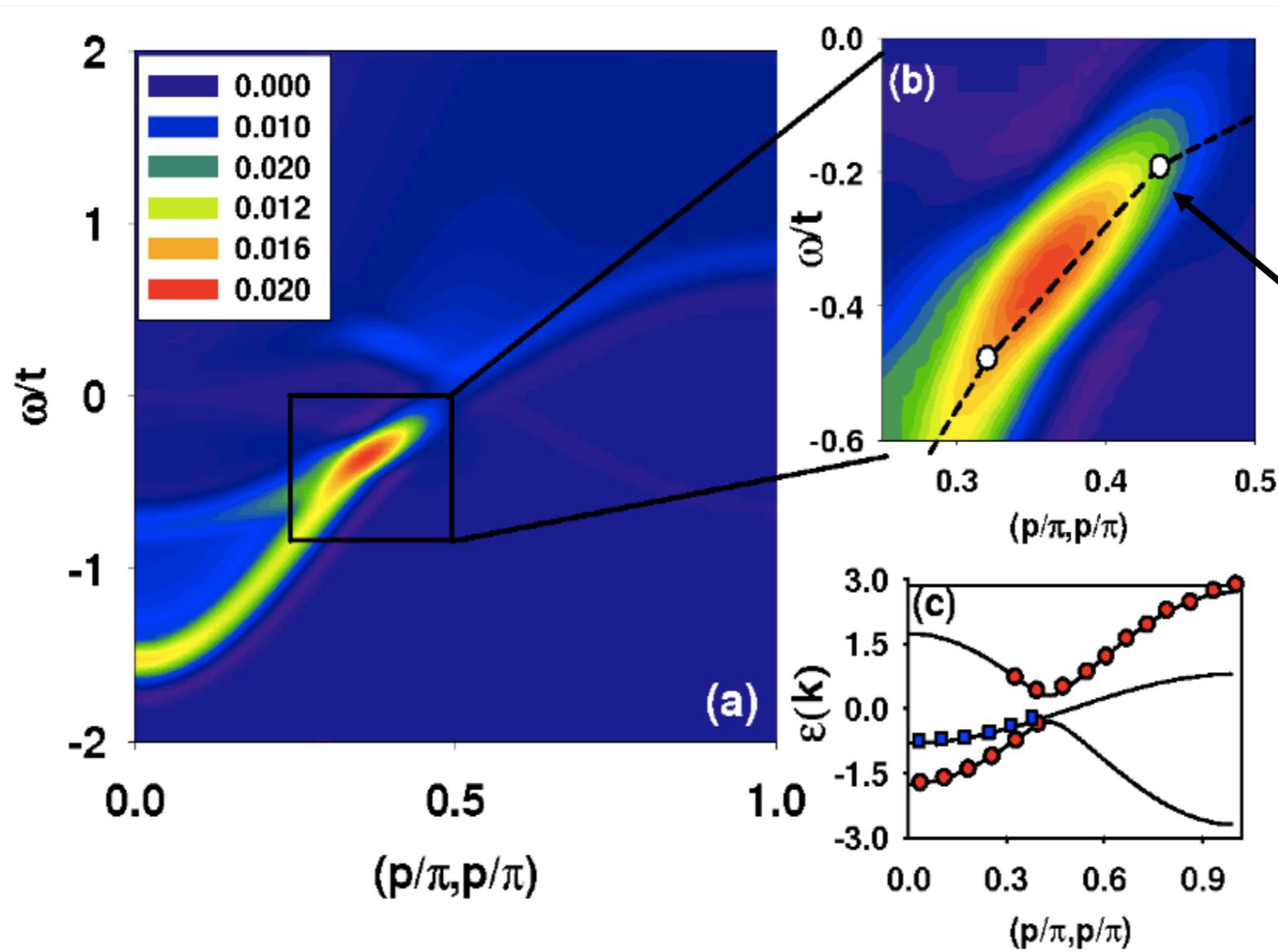
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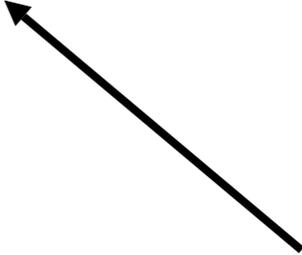
φ_i is an emergent degree of freedom
Not made out of the elemental fields

Electron spectral function

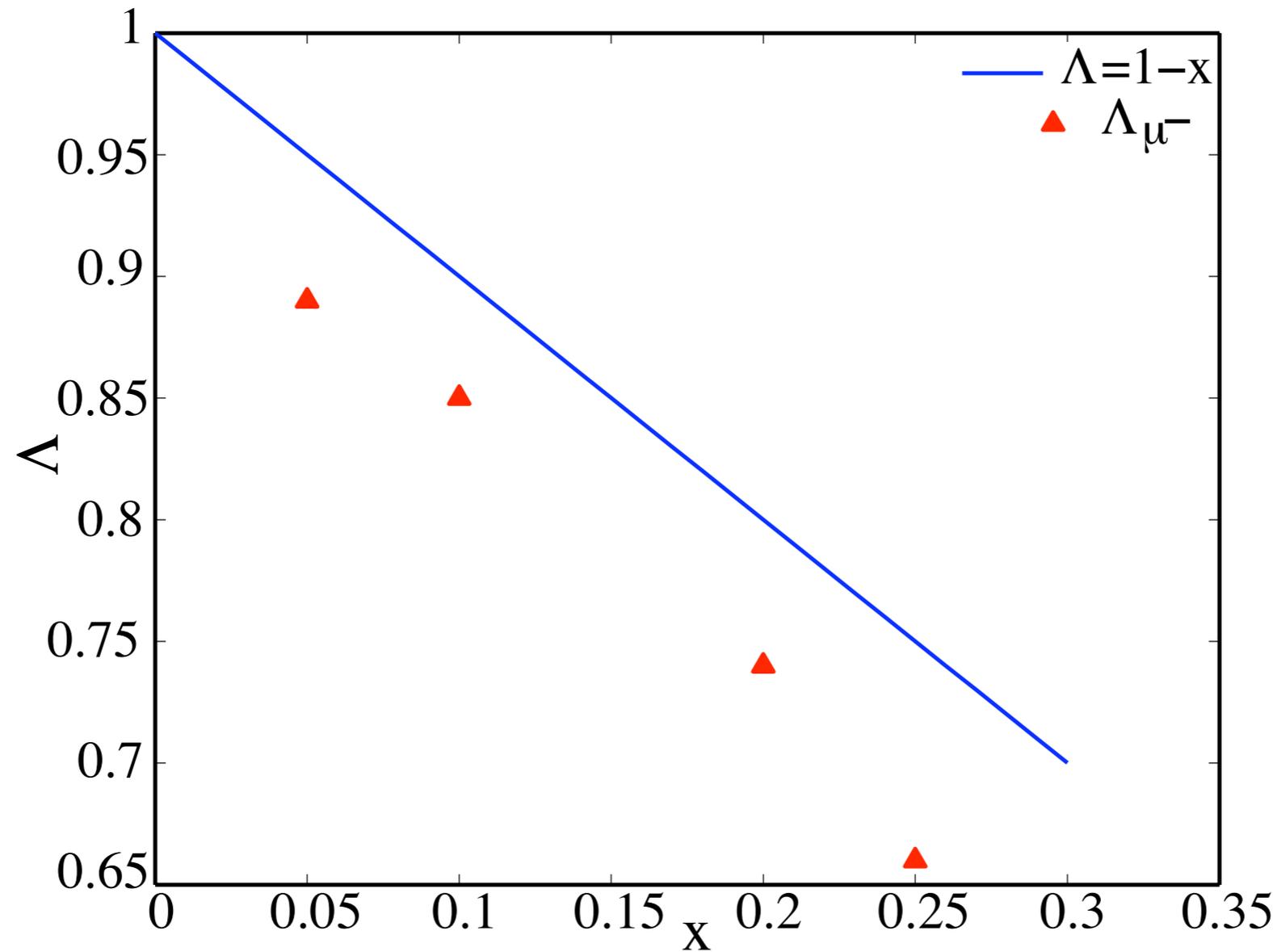


$$t^2/U \sim 60 \text{ meV}$$

Electron spectral function

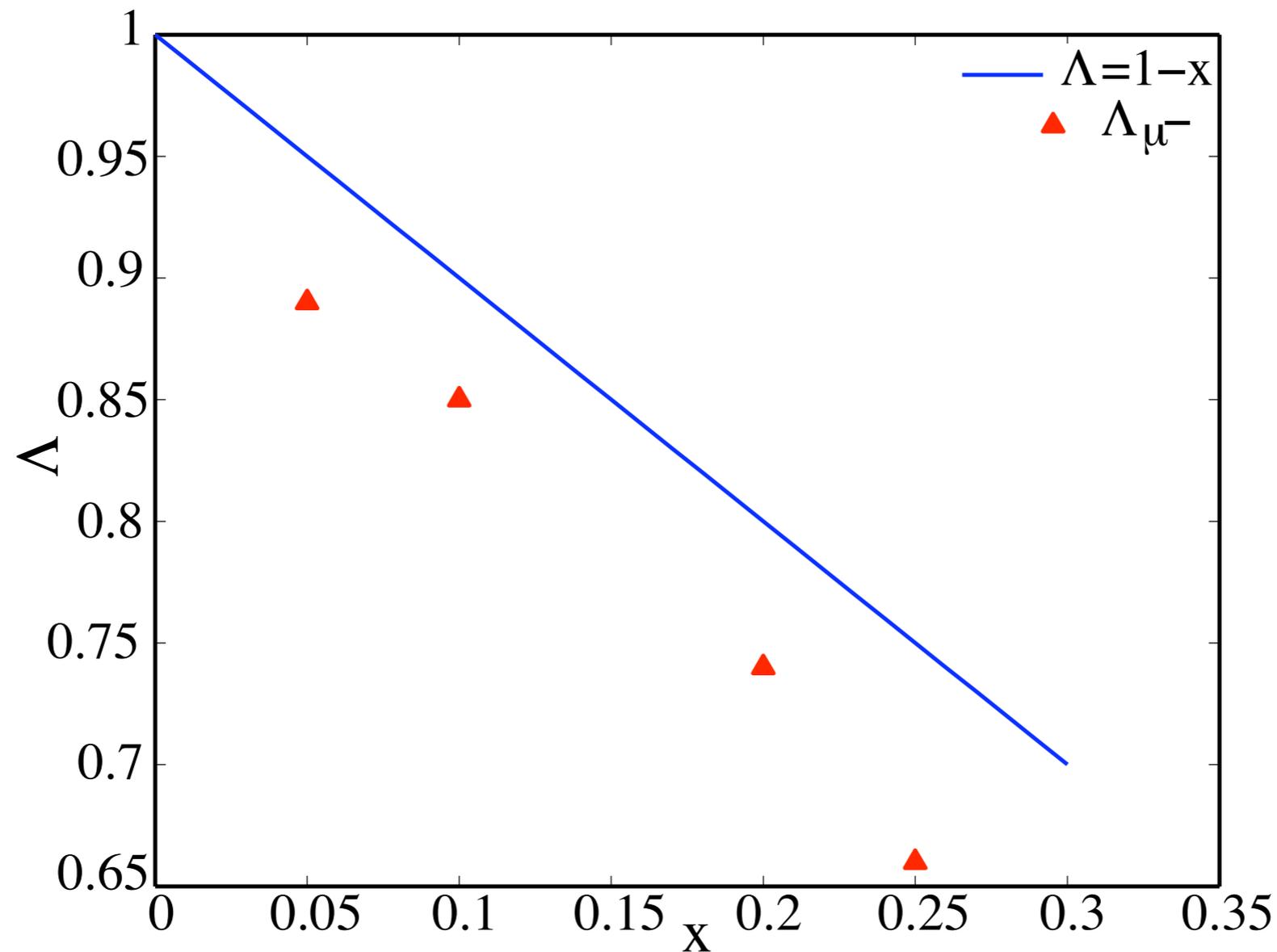

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Electron spectral function



$\Delta^2/U \sim 60 \text{ meV}$

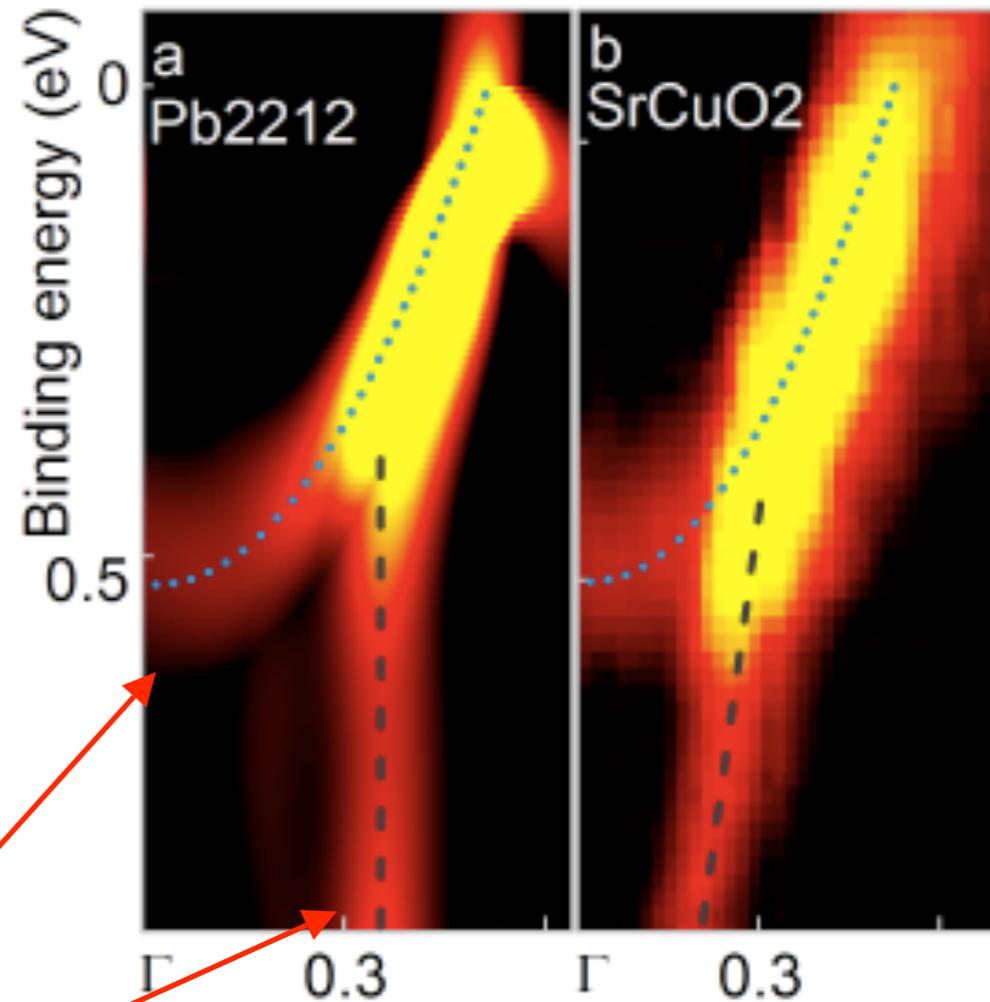
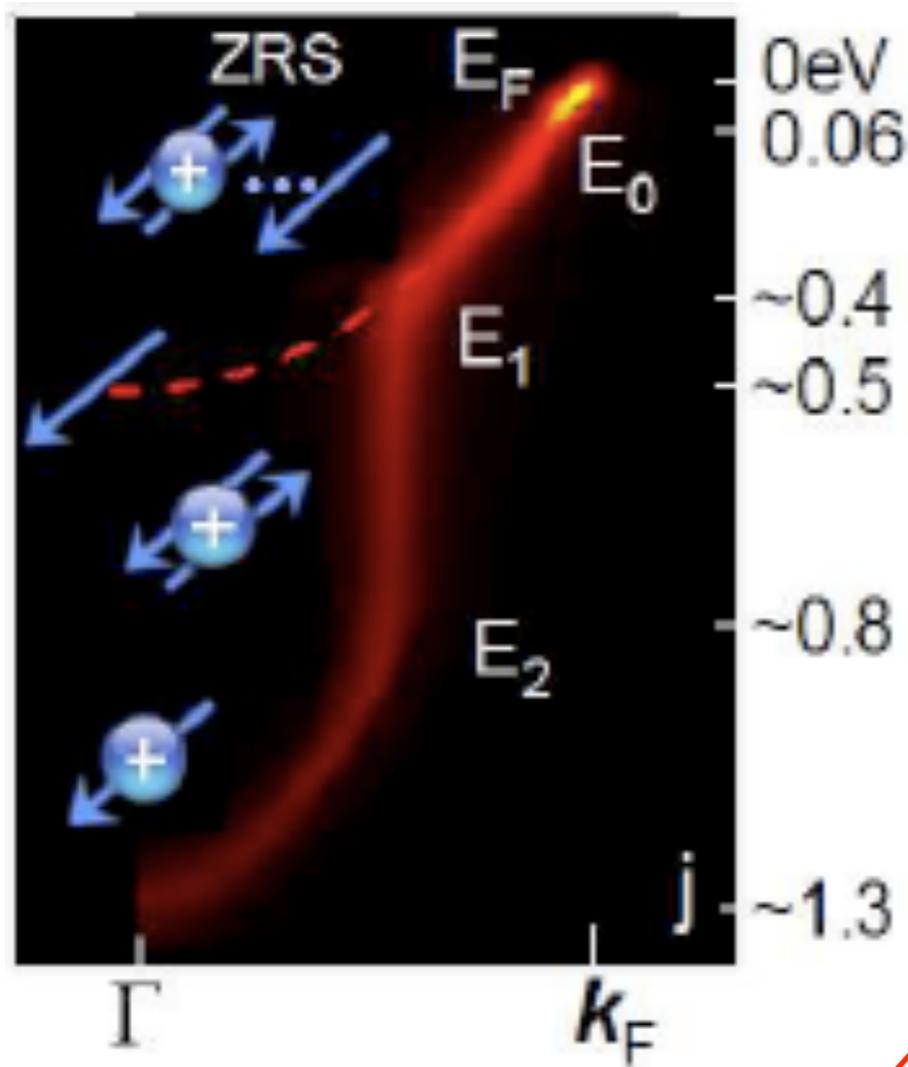
Electron spectral function



$\Delta^2/U \sim 60 \text{ meV}$

Conserved charge:
$$Q = \sum_i c_i^\dagger c_i + 2 \sum_i \varphi_i^\dagger \varphi_i$$

Graf, et al. PRL vol. 98, 67004 (2007).



Two bands!!

Spin-charge separation?

Graf, et al. PRL vol. 98, 67004 (2007).

Origin of two bands

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Two charge e excitations

$c_{i\sigma}$

$\varphi_i^\dagger c_{i\bar{\sigma}}$

New bound state

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Two charge e excitations

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New bound state

Pseudogap

two types of charges

'free'

bound

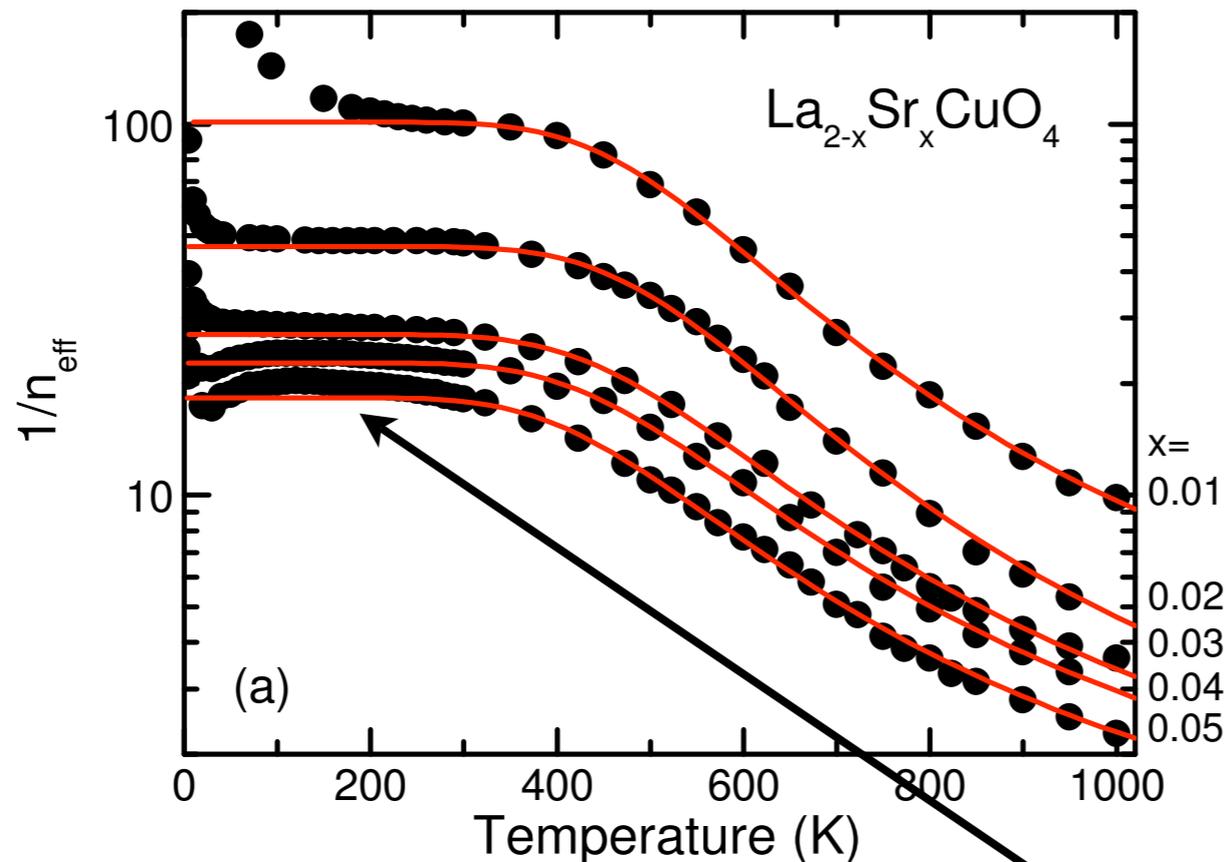
direct evidence

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charge carrier density:

direct evidence

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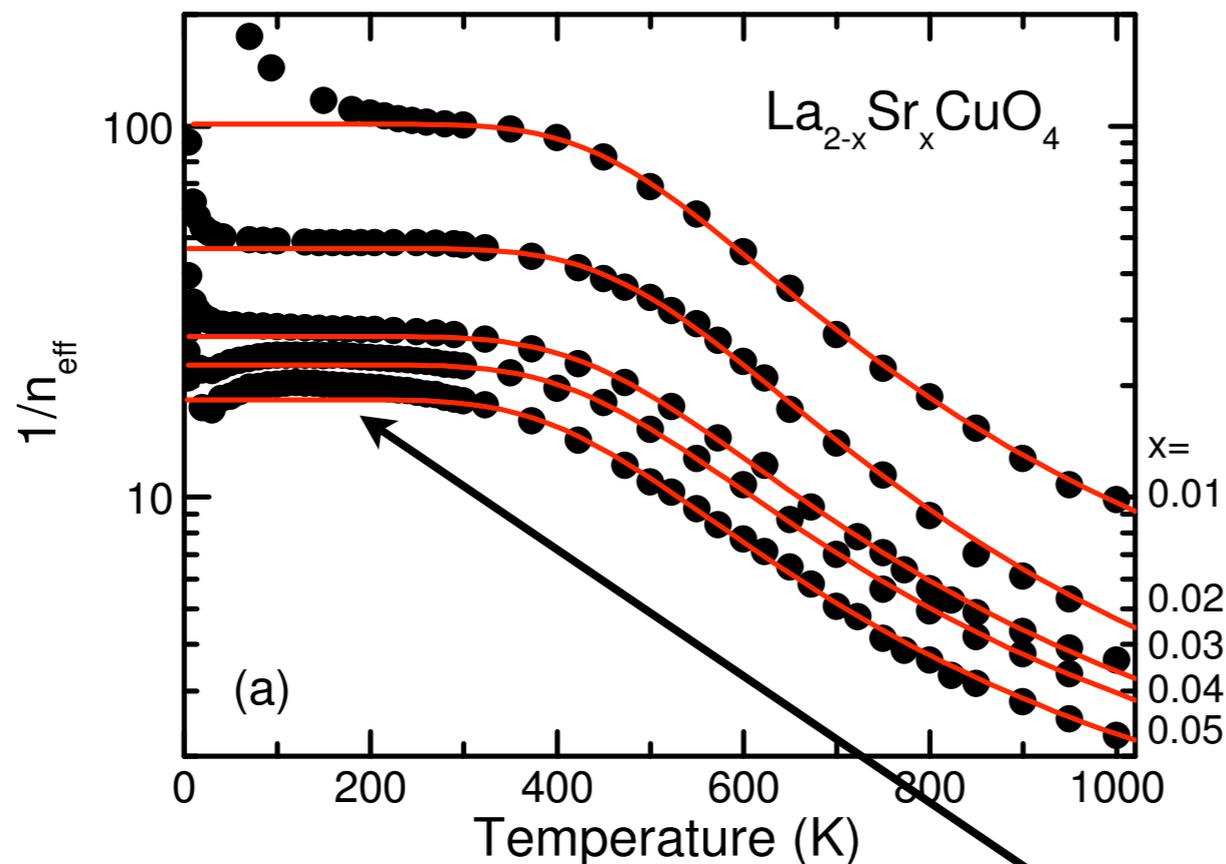


Ono, et al., Phys. Rev. B 75, 024515
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$$n_{\text{Hall}}(x, T) = n_0(x) + n_1(x) \exp(-\Delta(x)/T), \quad \text{PRL, vol. 97, 247003 (2006).}$$

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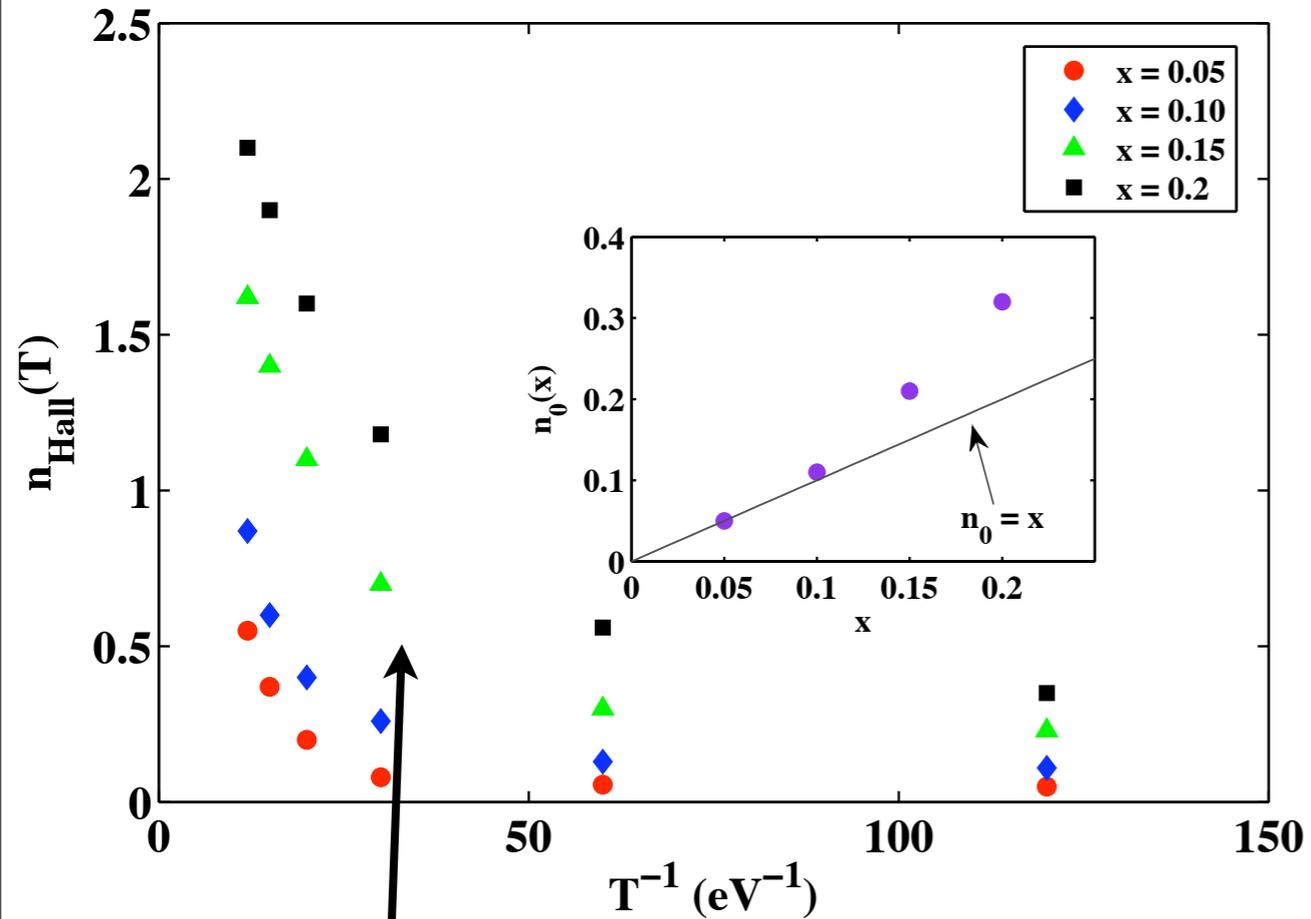


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$$n_{\text{Hall}}(x, T) = n_0(x) + n_1(x) \exp(-\Delta(x)/T), \quad \text{PRL, vol. 97, 247003 (2006).}$$

exponentially suppressed: confinement

Our Theory

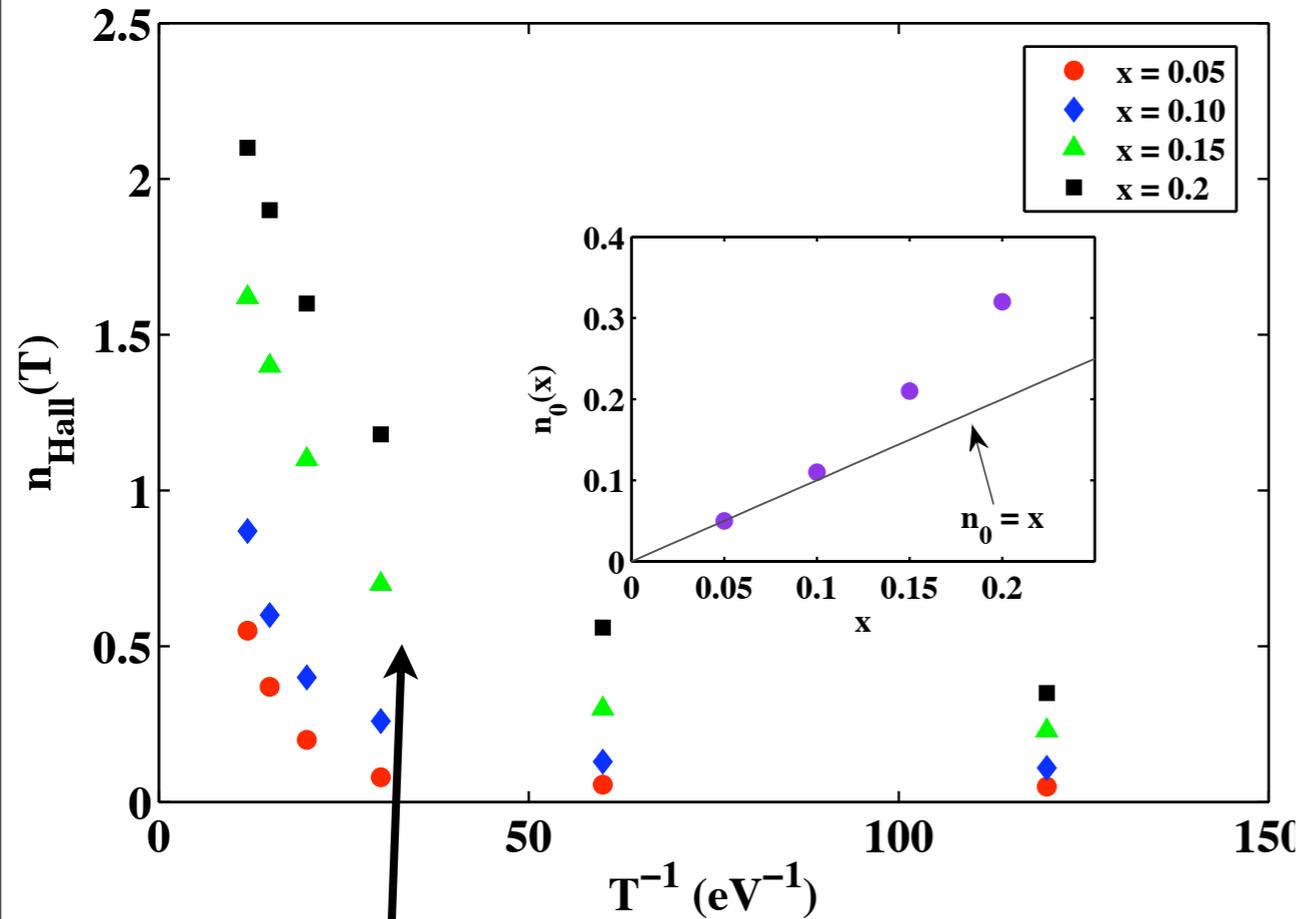


exponential
T-dependence

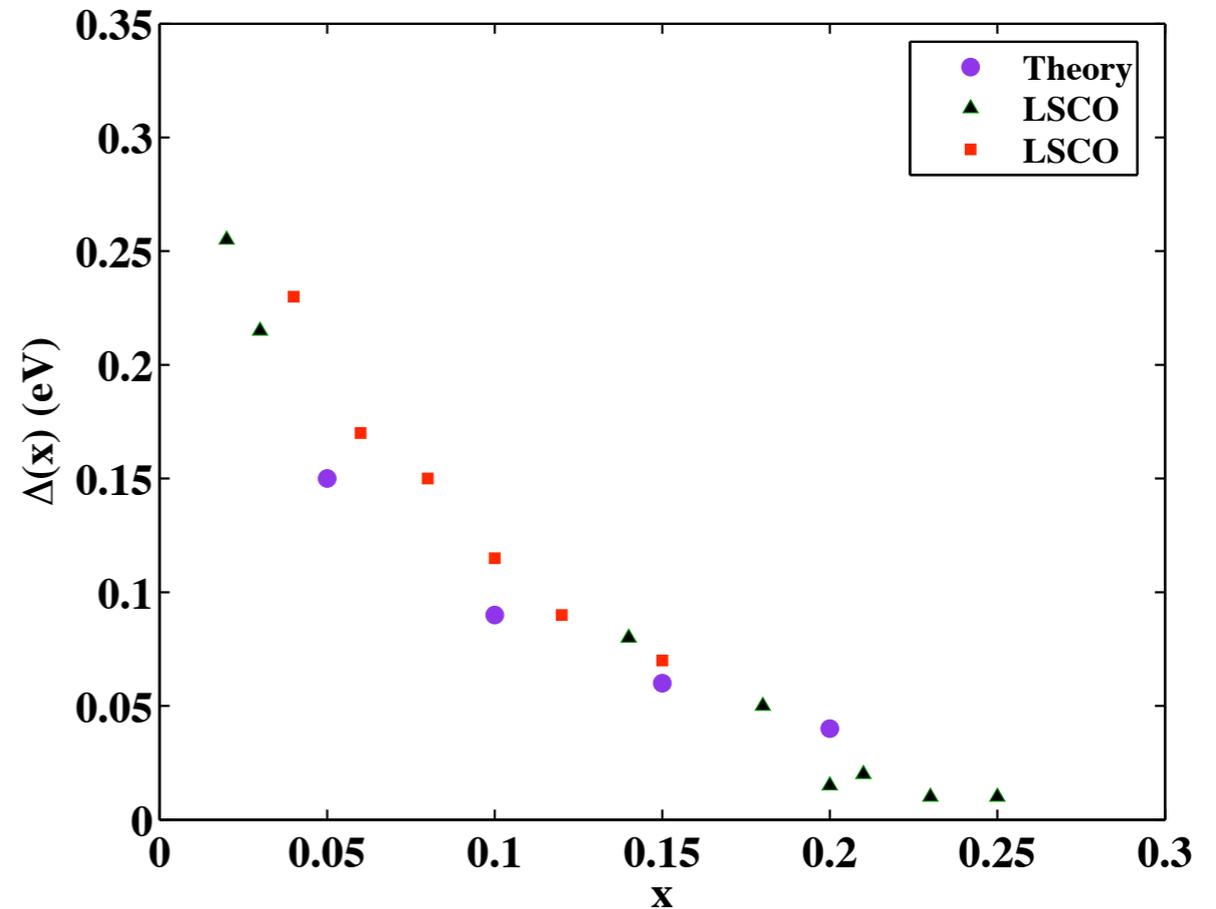
no model-dependent
free parameters: just
 t/U

Our Theory

gap



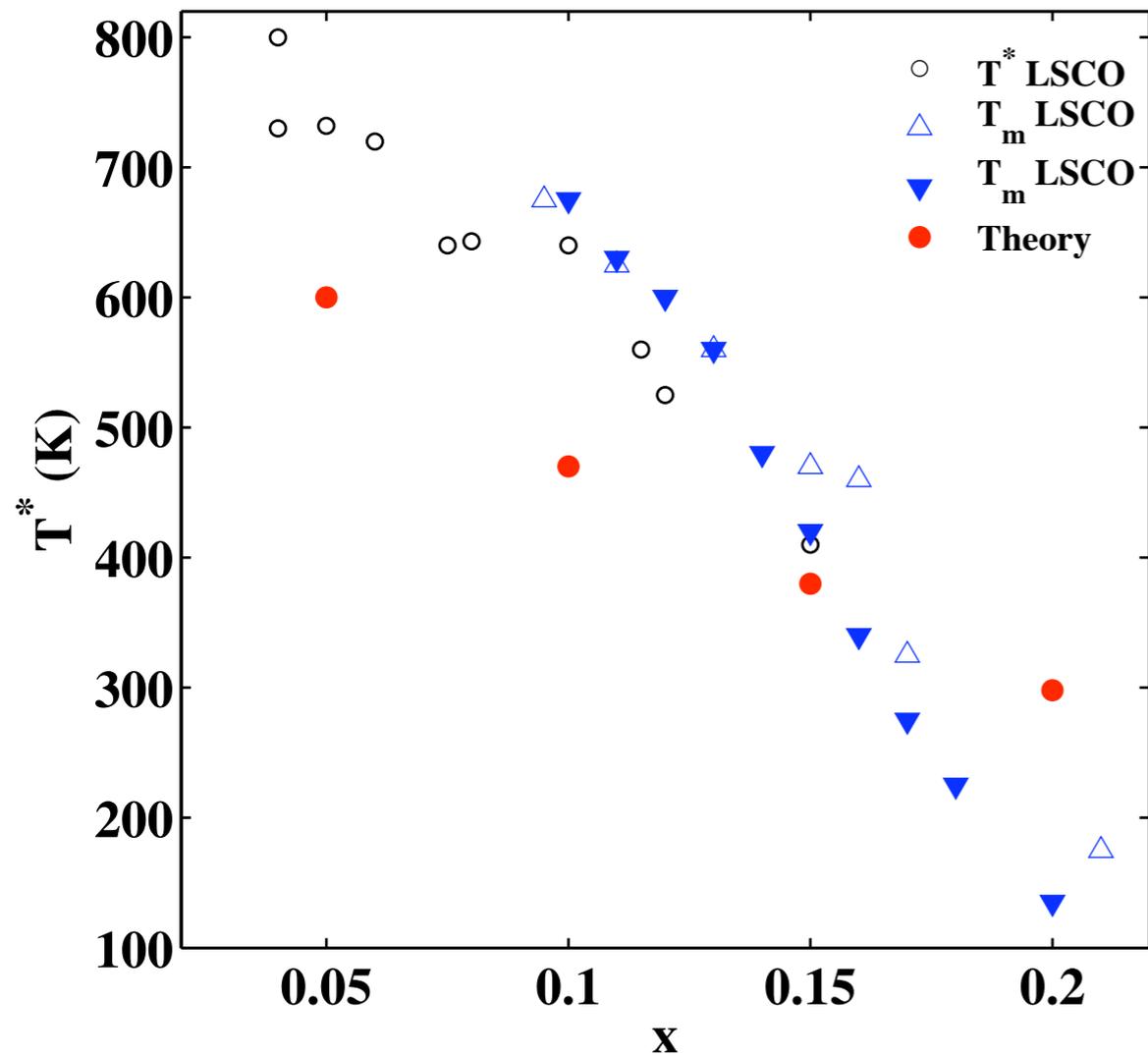
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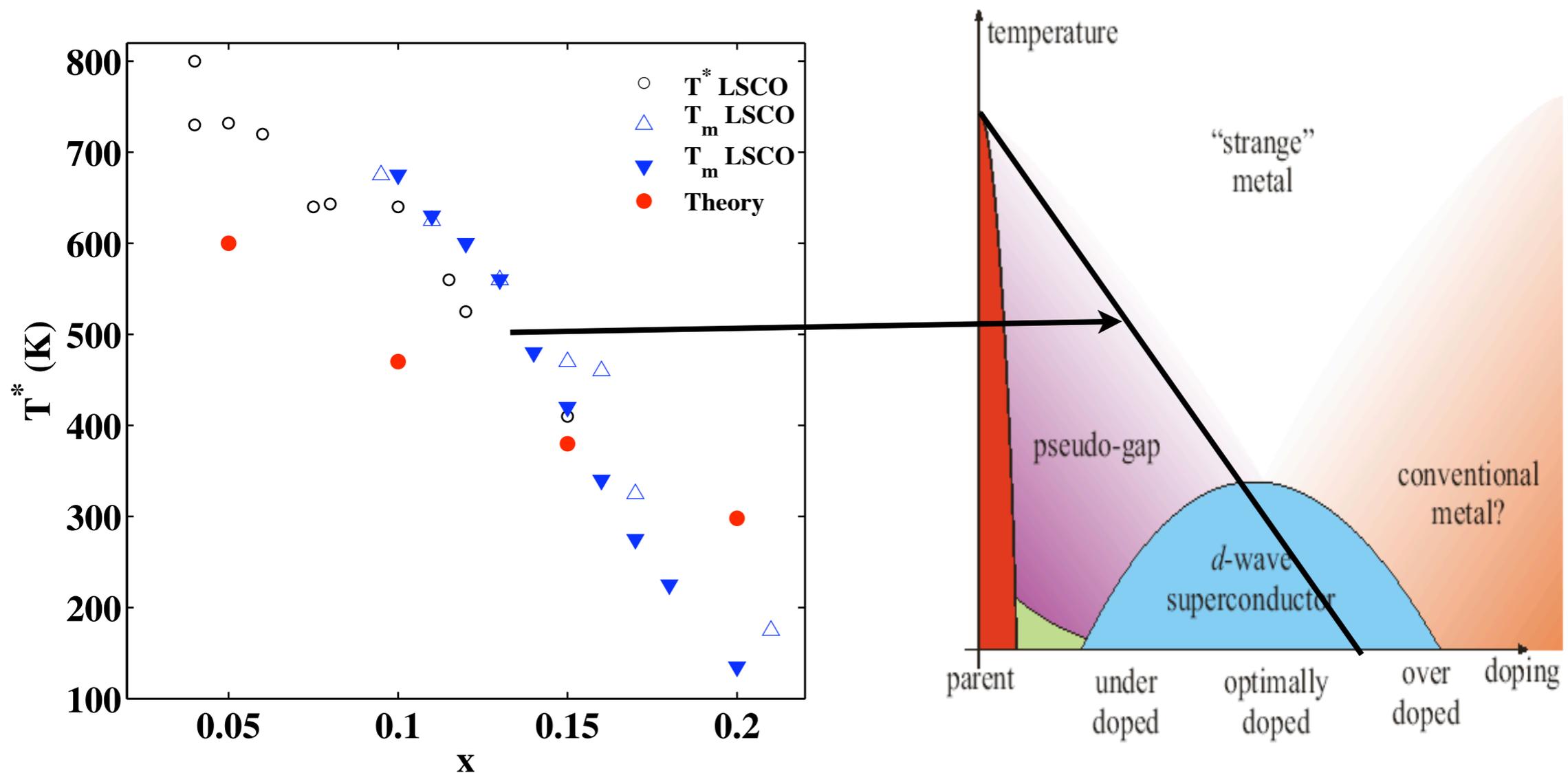
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Like Mott gap,
Pseudogap is a bound-state
problem with new IR modes

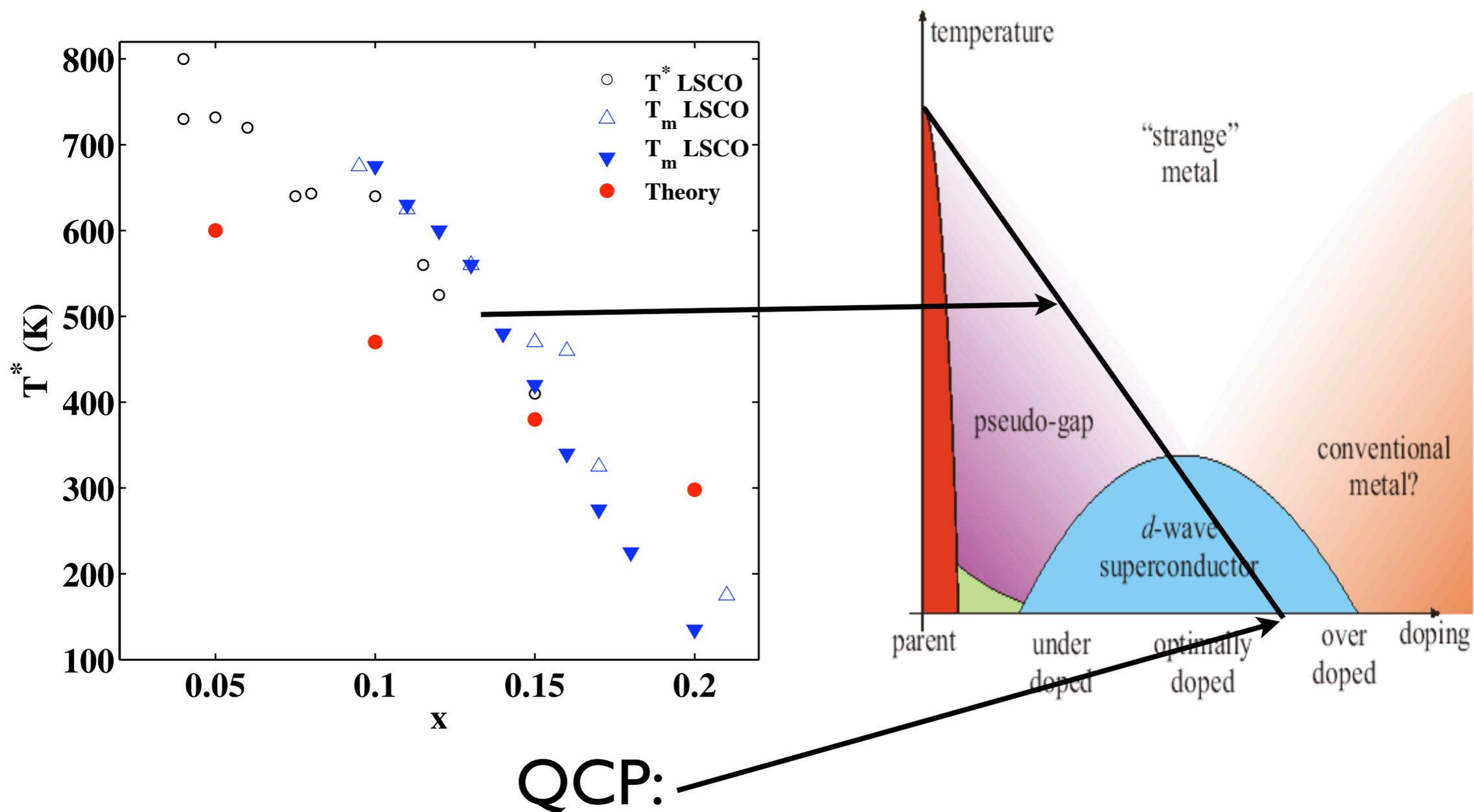
strange metal: breakup (deconfinement) of bound states



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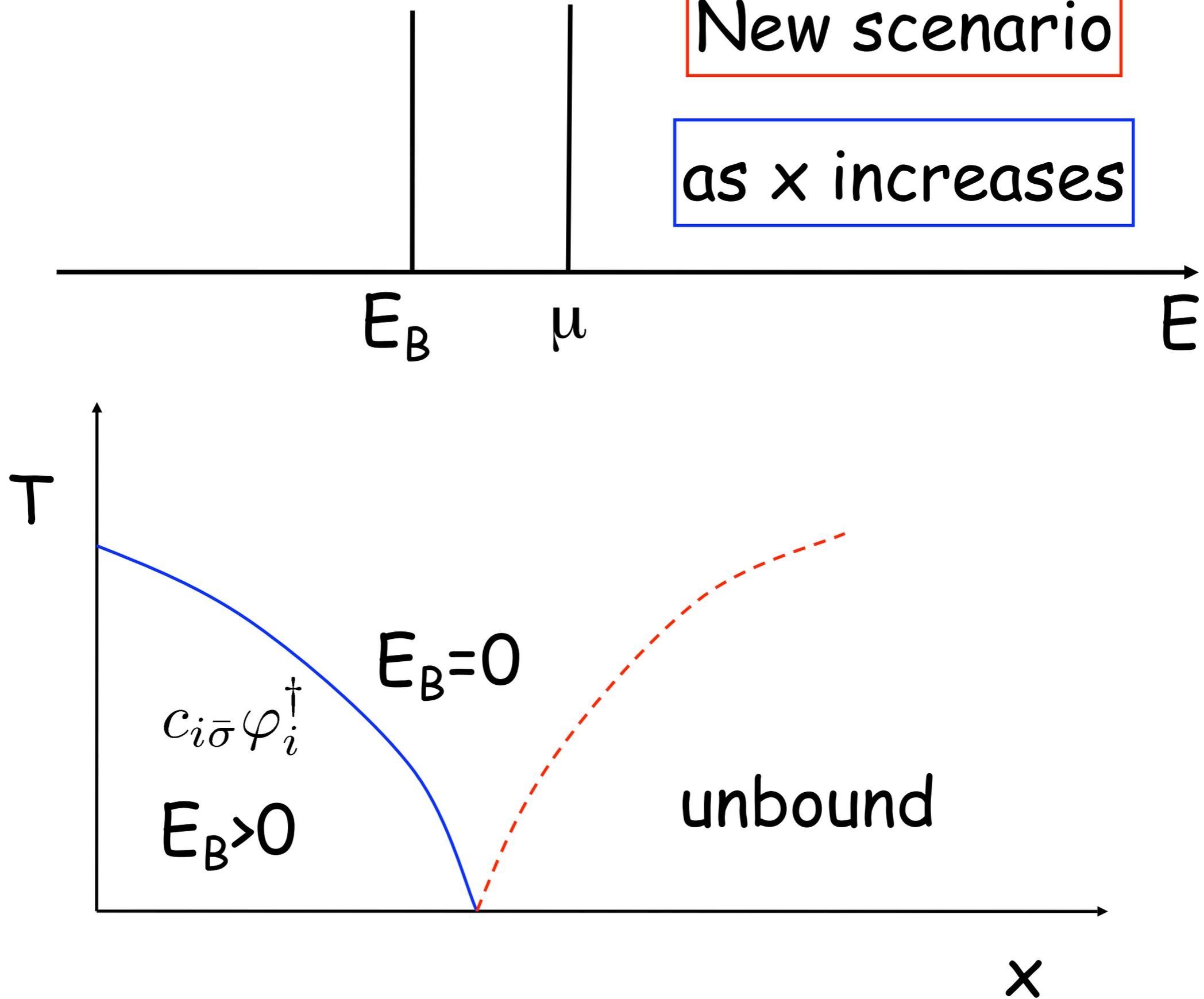
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T-linear resistivity

New scenario

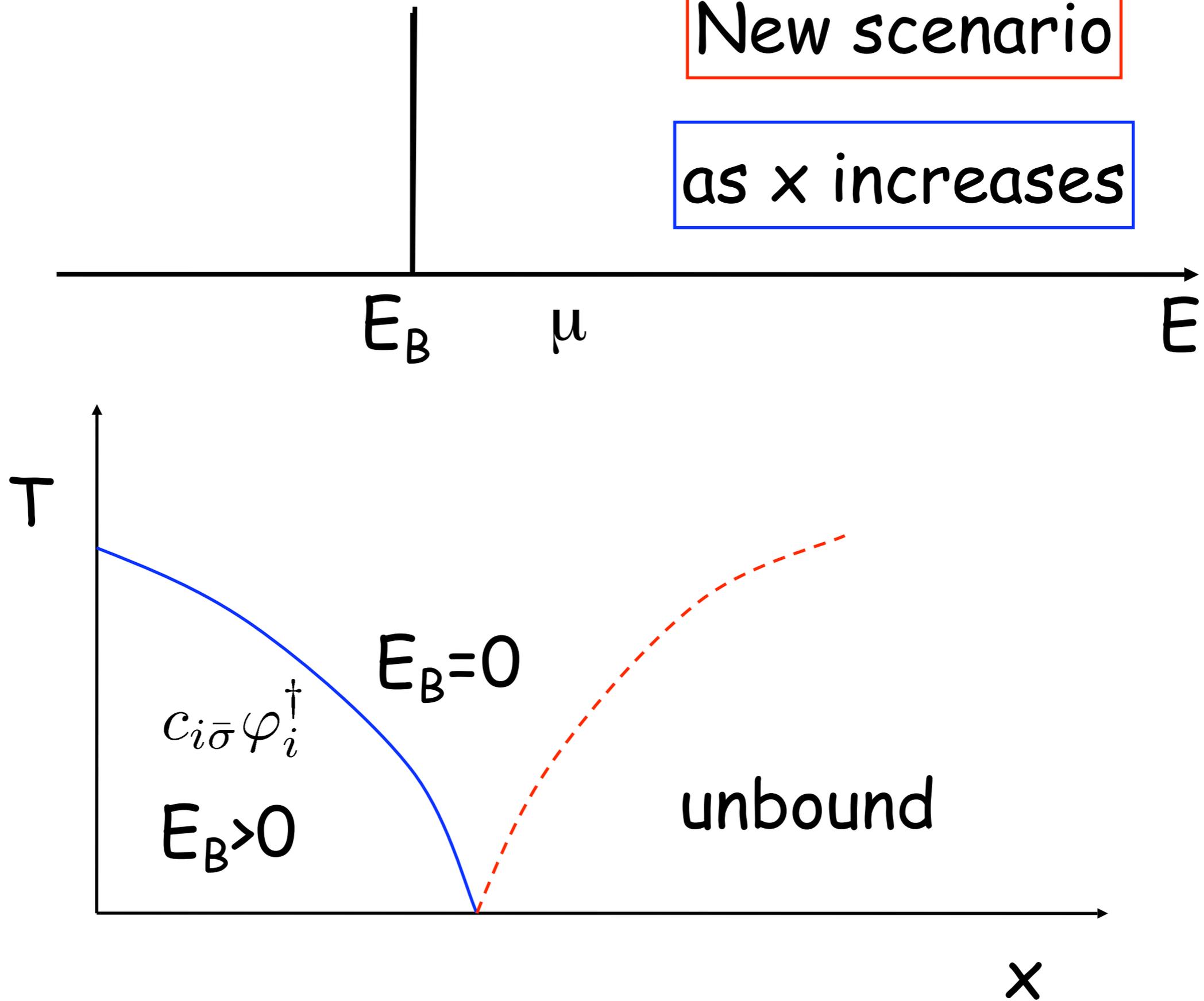
as x increases



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New scenario

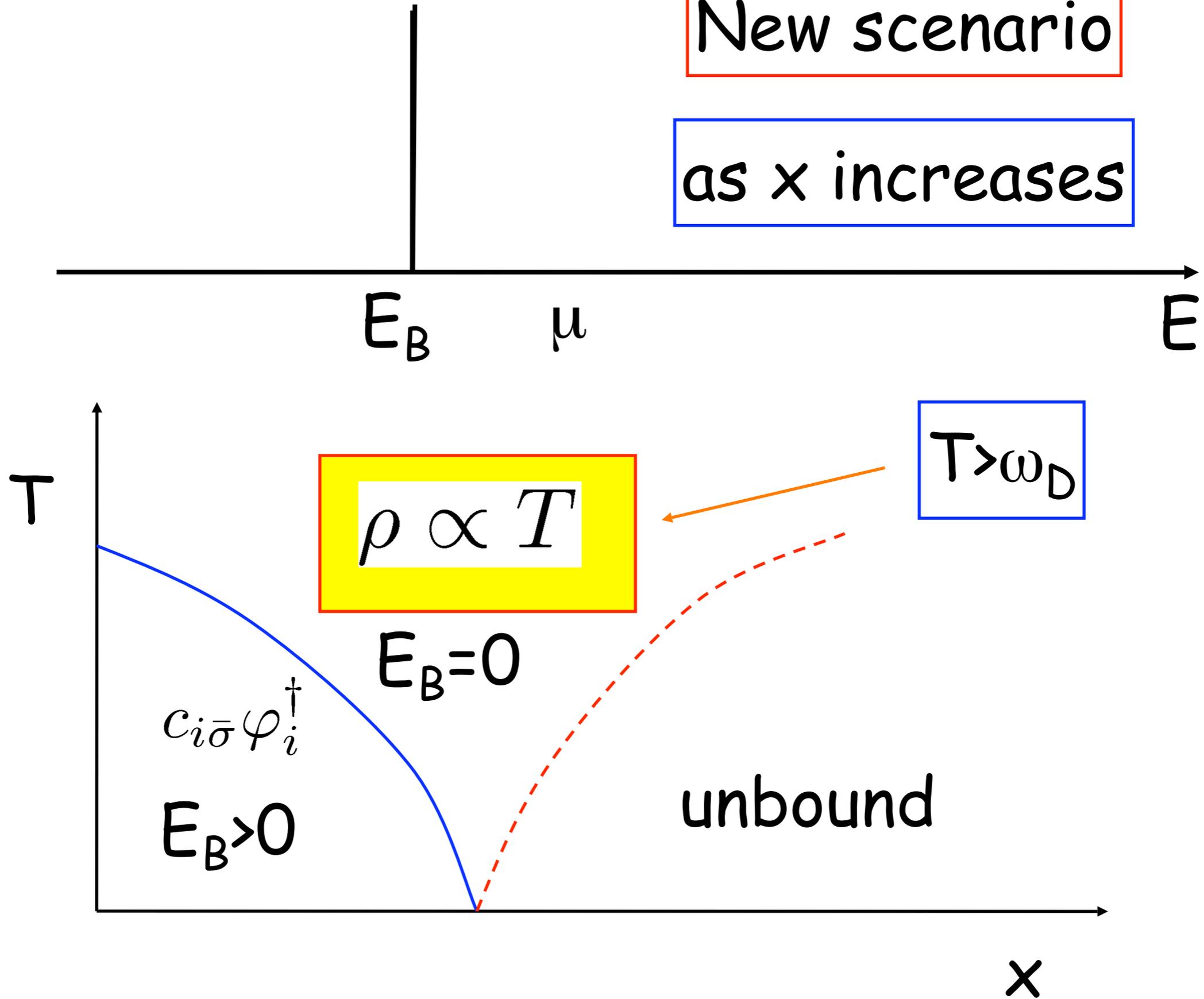
as x increases

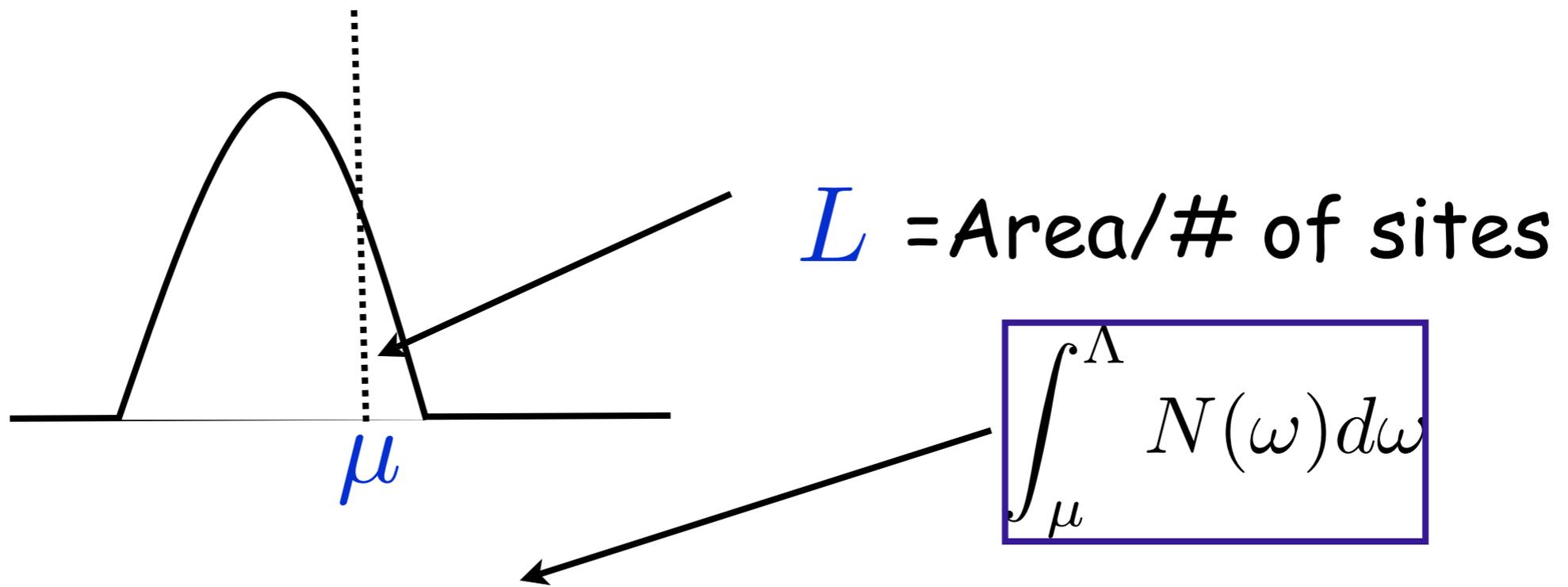


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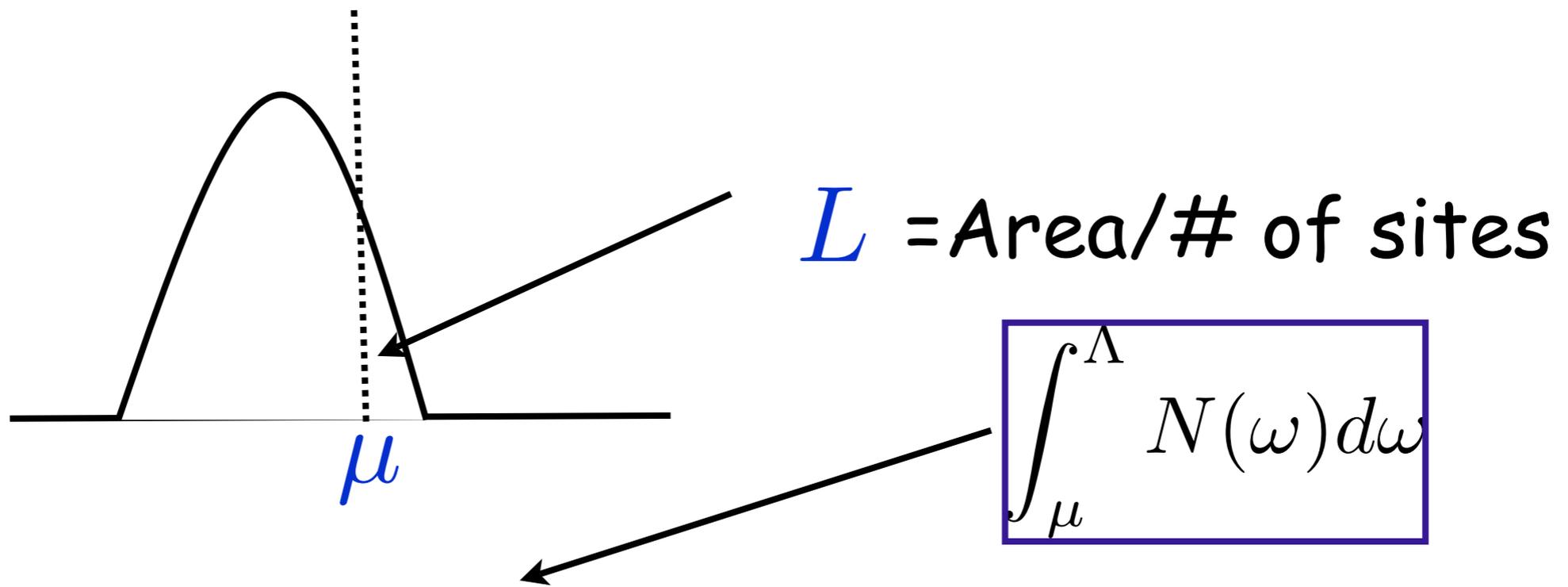
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$L = \# \text{ of single-particle addition states (qp)}$

$n_h = \# \text{ of ways electrons can be added} = \# \text{ of holes created by the dopants}$



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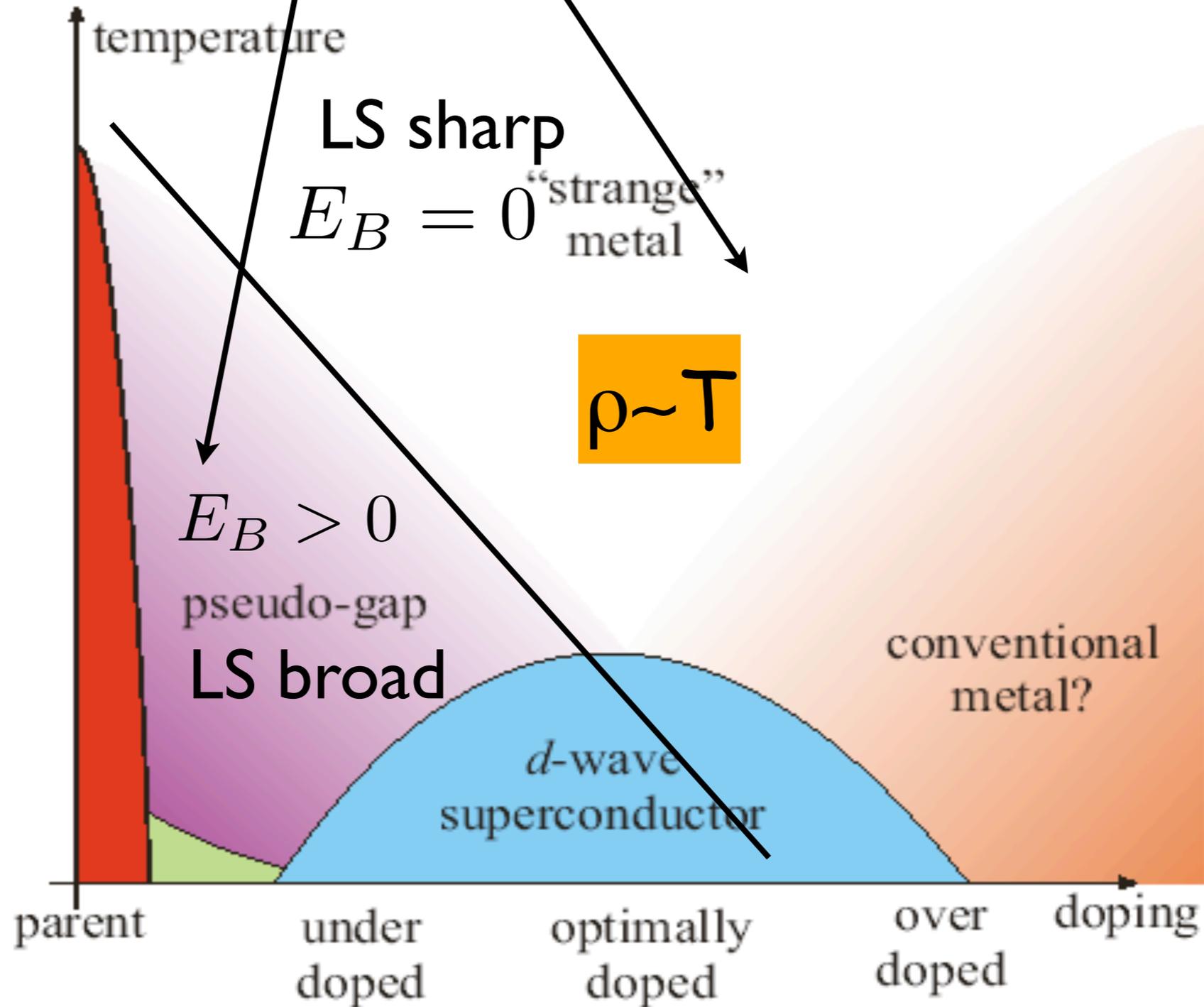
Equivalent in Fermi liquid

Experimental Prediction:

$$L/n_h > 1$$

$$L/n_h = 1$$

More states at lower temperature



Experiments

How did I start working with Rob?

PP, CC, PRL, vol. 95, 107002 (2005)

PP, CC, PRL, vol. 95, 107002 (2005)

quantum criticality

$$\sigma(T) \propto T^{(d-2)/z}$$

PP, CC, PRL, vol. 95, 107002 (2005)

quantum criticality

$\sigma(T) \propto T^{(d-2)/z}$ $\xrightarrow{d=3}$ $\rho \propto T$

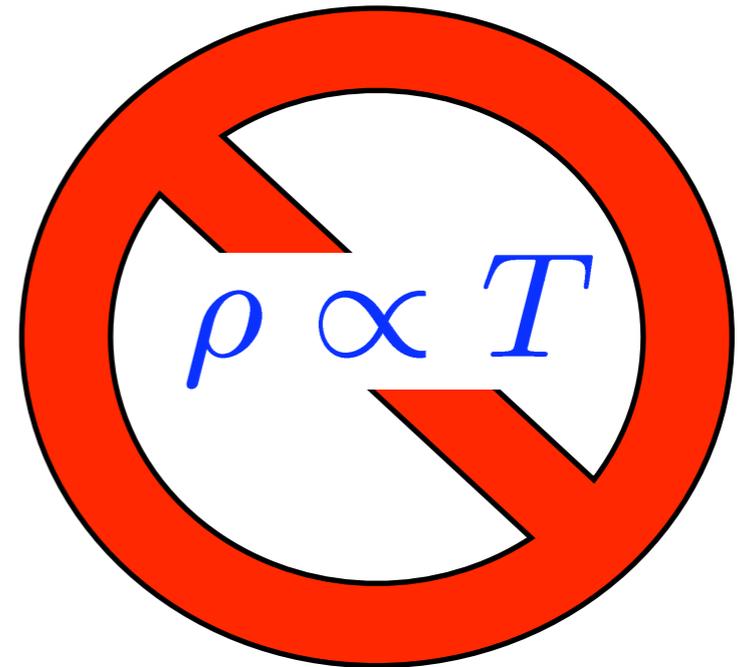
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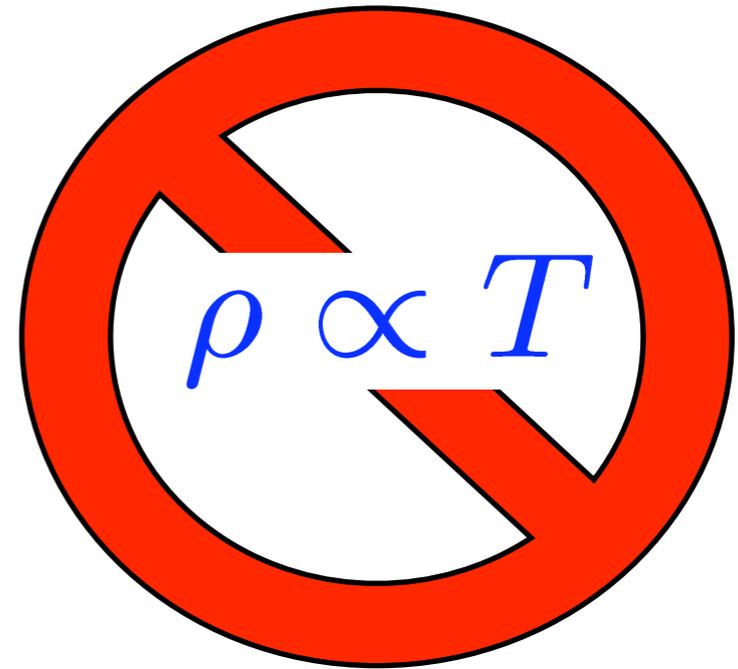
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$$E \propto p^z$$

dynamical exponent

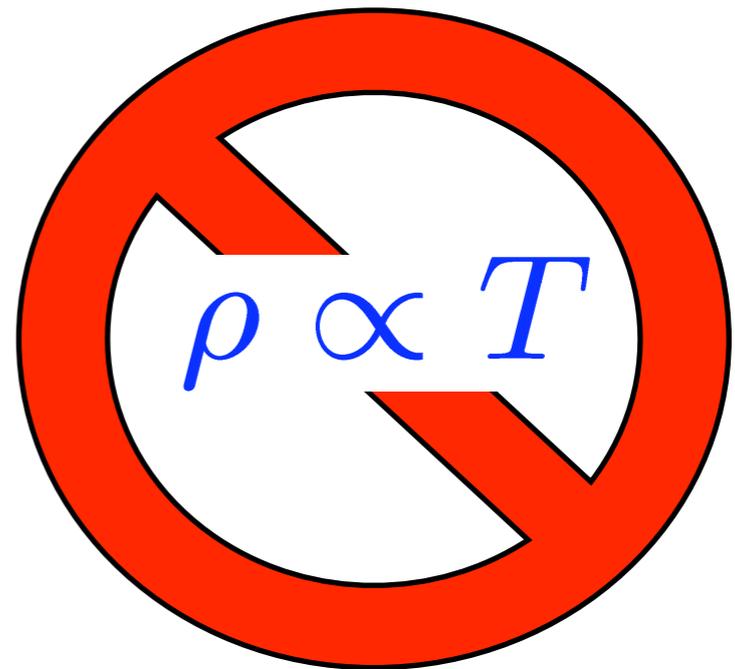
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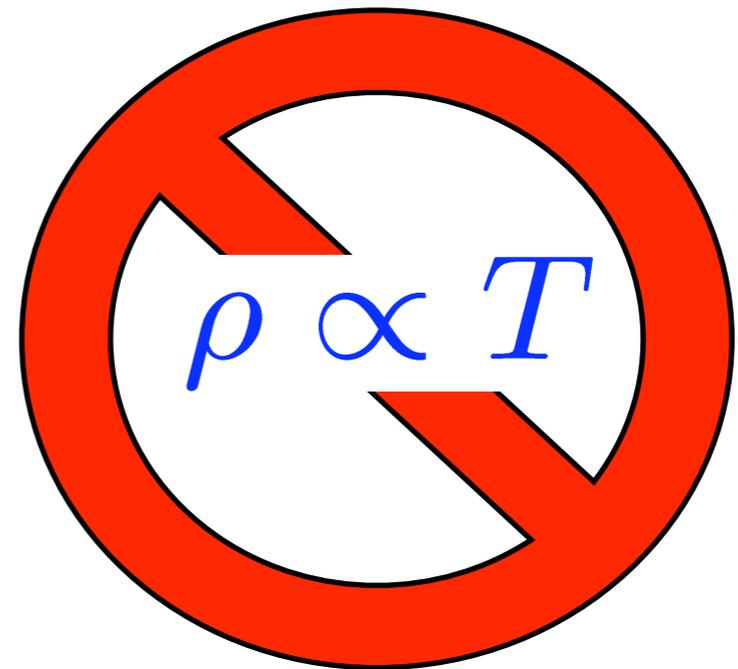


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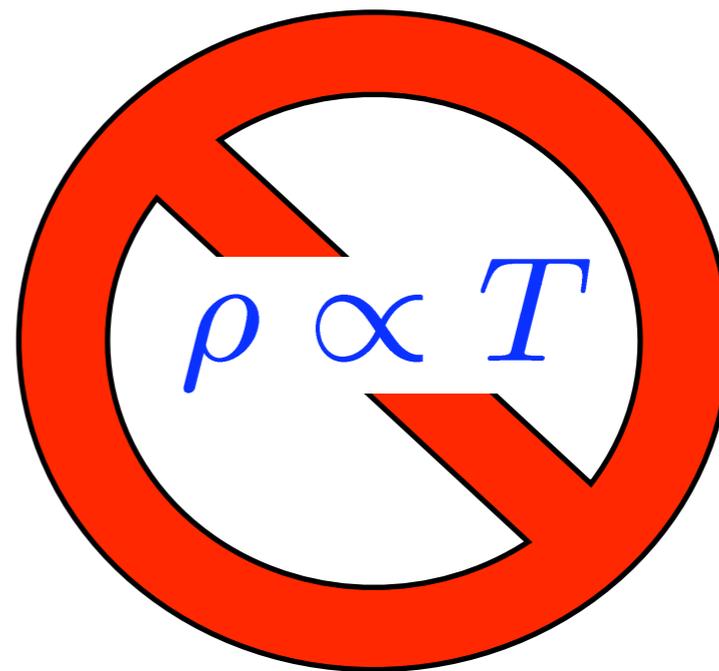
UV-IR mixing

PP, CC, PRL, vol. 95, 107002 (2005)

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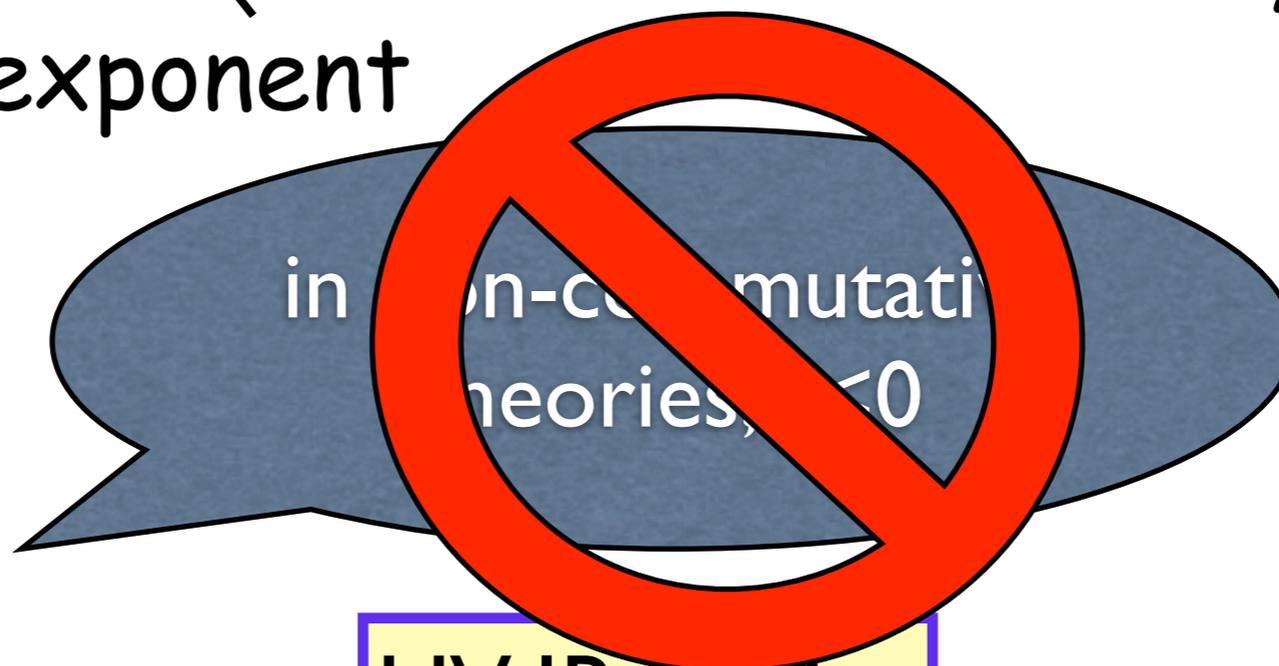
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UV-IR mixing

Motttness: Strong Coupling



low-energy reduction

Mottness: Strong Coupling



low-energy reduction

bare particles

Mottness: Strong Coupling

low-energy reduction

~~bare particles~~

Mottness: Strong Coupling

low-energy reduction

~~bare particles~~

Pseudogap=confinement

Mottness: Strong Coupling

low-energy reduction

~~bare particles~~

composite or bound states not in UV theory

Pseudogap=confinement

Predictions:

$$x \rightarrow x + \alpha$$

STM asymmetry: $2(x + \alpha)/(1 - x - \alpha)$

QO oscillations: $V_{\text{hp}} - V_{\text{ep}} = 2(x + \alpha)$

Superfluid density: $\rho_s \propto x + \alpha$

pairing from Mott scale

optical conductivity: $n_{\text{eff}} \propto (x + \alpha) > x$

Thanks to R. G. Leigh and Ting-Pong Choy, Shiladitya Chakraborty, Seunming Hong and DMR-NSF/ACIF