Demystifying the Strange Metal in High-temperature Superconductors: Composite Excitations

Thanks to: T.-P. Choy, R. G. Leigh, S. Chakraborty, S. Hong

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PRB, 77, 14512 (2008); ibid, 77, 104524 (2008);
ibid, 79, 245120 (2009); ibid, 80, 132505 (2009)...

DMR/NSF-ACIF
High temperature superconductor?
High temperature superconductor?

Unusually Good Metal
Matthias Rules for Superconductivity
Matthias Rules for Superconductivity
Matthias Rules for Superconductivity

1.) cubic structures
2.) avoid oxygen
3.) avoid magnetism
4.) avoid insulators
Matthias Rules for Superconductivity

1.) cubic structures
2.) avoid oxygen
3.) avoid magnetism
4.) avoid insulators
5.) don’t talk to theorists!!
Y Ba\textsubscript{2}Cu\textsubscript{3}O\textsubscript{7}

Cuprate Superconductors
Cu-O plane

Orthorhombic: asymmetric in xy (a-b) plane

YBa$_2$Cu$_3$O$_7$
Cuprate Superconductors
**High temperature superconductors:**
- discovered 1986
- copper oxygen planes
- theory not understood!

**Conventional or Low-Tc superconductors:**
- discovered 1911
- almost all metals (Hg, Pb, Nb, etc.)
- theory well understood

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**Graph:**
- Temperature $(T_c)$ (Kelvin) on the y-axis.
- Year of discovery on the x-axis from 1900 to 2000.

- Various compounds and elements with their respective $T_c$ values:
  - Hg/Ba/Ca/Cu/O
  - TI/Ba/Ca/Cu/O
  - Bi$_2$Sr$_2$CaCu$_2$O$_x$
  - YBa$_2$Cu$_3$O$_7$
  - (La, Sr)$_2$CuO$_4$
  - Hg, Pb, Nb, NbN, Nb$_3$Sn, Nb$_3$Ge

- Liquid N$_2$ marked on the graph.
What is left of Matthias' Rules?

1.) cubic structures
2.) avoid oxygen
3.) avoid magnetism
4.) avoid insulators
What is left of Matthias' Rules?

2.) avoid oxygen
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What is left of Matthias’ Rules?

3.) avoid magnetism

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What is left of Matthias' Rules?

4.) avoid insulators
What is left of Matthias' Rules?
New Problem: Mottness
Sir Neville Mott
Nobel Prize, 1977

Mott
Insulators
Mott Problem: NiO (Band theory failure)

Loomis floor plan (N rooms N occupants)
Mott Problem: NiO (Band theory failure)

Loomis floor plan (N rooms N occupants)

\[ \Delta E = U \gg K.E. \]
Mott Problem: NiO (Band theory failure)

Cuprates: 
U = 4eV 
t = 0.5eV

Loomis floor plan (N rooms N occupants)

Insulator ????

\[ \Delta E = U \gg K.E. \]
Half-filled band

Free electrons
Half-filled band

\[ U \gg t \]

charge gap

Free electrons
Half-filled band

Free electrons

Upper Hubbard band

Lower Hubbard band

$U \gg t$

Charge gap

Low-energy theory??
Doping a Mott insulator

x = fraction of empty rooms (holes)
Doping a Mott insulator

x = fraction of empty rooms (holes)
Doping a Mott insulator

$x =$ fraction of empty rooms (holes)
Doping a Mott insulator

\( x = \text{fraction of empty rooms (holes)} \)
Doping a Mott insulator

$x = \text{fraction of empty rooms (holes)}$

$X = \frac{3}{16}$
Mott insulator

AF

Doping

Parent

Under doped

Optimally doped

Over doped

Conventional metal?

$\rho \sim T^0$

$d$-wave superconductor

Pseudo-gap

$\infty$

Graph showing $\rho (10^{-3} \Omega \text{cm})$ vs. $T$ for various materials.
Mott insulator $\rho \sim T$
How does Fermi Liquid Theory Breakdown?

Mott insulator

$\rho \sim T$

$6,000,000$ question?
T-linear Resistivity?
Metals: $\rho \approx T^2$
Metals: \( \rho \approx T^2 \)

Fermi sea
Metals: $\rho \approx T^2$

Fermi sea
Metals: $\rho \approx T^2$

Fermi sea

Two degrees of freedom
Metals: $\rho \approx T^2$

Fermi sea

Two degrees of freedom

$$\frac{\hbar}{\tau} \approx \frac{\epsilon^2}{\epsilon_F} \propto \frac{T^2}{\epsilon_F}$$
T-linear Resistivity

\[ \frac{\mathcal{H}}{\tau} \equiv \#k_B T \]

Planckian limit of dissipation
T-linear Resistivity

\[ \frac{\hbar}{\tau} \equiv \# k_B T \]

Planckian limit of dissipation

breakdown of standard Fermi liquid picture
Cuprates: The Perfect Storm

Fermi Liquid Theory

Band Theory

BCS
Cuprates: The Perfect Storm
collective failure
collective failure

"I'm not into this detail stuff. I'm more concepty."
collective failure

"I'm not into this detail stuff. I'm more concepty."

New Concept
composite or bound states not in UV theory

Strong Coupling
composite or bound states not in UV theory

Strong Coupling

QCD
composite or bound states not in UV theory

Strong Coupling

QCD

\begin{align*}
\text{poly(isoprene)} & + \text{sulfur} \\
\text{vulcanization}
\end{align*}
composite or bound states not in UV theory

Strong Coupling

QCD

\[
\text{poly(isoprene)} + \text{sulfur}
\]

vulcanization

emergent low-energy physics
weakly interacting
identify propagating charge degrees of freedom in the normal state of a high-temperature superconductor
dynamical transition
dynamical transition

control parameter
dynamical transition

bound

control parameter

T
dynamical transition

bound

unbound (strange metal)
dynamical transition

bound

unbound (strange metal)

bound

(pseudogap)
dynamical transition

dictionary:
- green circle: hole
- blue circle: 2e boson

bound (pseudogap)
unbound (strange metal)
How to break Fermi liquid theory in $d=2+1$?
Polchinski (and others)

\[ p = k + l, \]

1.) e- charge carriers

2.) Fermi surface

\[ \int dt \, d^2k_1 \, dl_1 \, d^2k_2 \, dl_2 \, d^2k_3 \, dl_3 \, d^2k_4 \, dl_4 \, V(k_1, k_2, k_3, k_4) \psi_\sigma^\dagger(p_1)\psi_\sigma(p_3)\psi_\sigma^\dagger(p_2)\psi_\sigma^\prime(p_4)\delta^3(p_1 + p_2 - p_3 - p_4). \]

No relevant short-range 4-Fermi terms in \( d \geq 2 \)
Polchinski (and others)

\[ p = k + l, \]

1.) e- charge carriers

2.) Fermi surface

\[
\int dt \, d^2 k_1 \, dl_1 \, d^2 k_2 \, dl_2 \, d^2 k_3 \, dl_3 \, d^2 k_4 \, dl_4 \, V(k_1, k_2, k_3, k_4) \\
\psi_{\sigma}^\dagger(p_1)\psi_{\sigma}(p_3)\psi_{\sigma'}^\dagger(p_2)\psi_{\sigma'}(p_4)\delta^3(p_1 + p_2 - p_3 - p_4).
\]

No relevant short-range 4-Fermi terms in \( d \geq 2 \)

Exception: Pairing
does gauge/gravity duality help?

McGreevy, Liu
does gauge/gravity duality help?

McGreevy, Liu

\( AdS_4 \) \( \longrightarrow \) \( AdS_2 \times R^2 \)

\( \rho \propto T \)
No
So what does one add to break this correspondence?

\[ H = -t \sum_{ij\sigma} g_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \]
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So what does one add to break this correspondence?

Free system

Low energy: one-to-one correspondence

New degrees of freedom!

\[ H = -t \sum_{ij\sigma} g_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \]
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So what does one add to break this correspondence?

\[ H = -t \sum_{ij\sigma} g_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \]
\[ U = \infty \]
$U = \infty$

No dynamics with doubly-occupied sector: each electron blocks 2 states
atomic limit: $x$ holes

total weight = $1+x$ = number of ways electrons can be added in lower band

No problems yet!
atomic limit

intensity of lower band = # of electrons the band can hold
$U \gg t$

$U$ finite
U finite $U \gg t$

double occupancy in ground state!!
$U \text{ finite}$

$U \gg t$

$W_{\text{PES}} > 1 + x$

double occupancy in ground state!!
What does $W_{PES} > 1 + x$ mean??
What does $W_{PES} > 1 + x$ mean?  new degrees of freedom
What does $W_{PES} > 1 + x$ mean??

new degrees of freedom

H&L, 1967

$1 + x + \alpha(t/U, x)$
What does $W_{\text{PES}} > 1 + x$ mean?

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new degrees of freedom

the rest of this state lives at high energy
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# of ways of adding electrons remains $1 + x$
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Has nothing to do with electron addition

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$\alpha \neq 0 \implies \frac{dn_e}{d\mu} = 0$
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Has nothing to do with electron addition

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Conserved charge

$n_e = n_{qp} + n_{\text{newstuff}} > n_{qp}$
What does $W_{\text{PES}} > 1 + x$ mean??

H&L, 1967

$1 + x + \alpha(t/U, x)$

The rest of this state lives at high energy

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Has nothing to do with electron addition

$$\alpha \neq 0 \implies \frac{dn_e}{d\mu} = 0$$

Hole screened with double occupancy: local

Conserved charge

$$n_e = n_{qp} + n_{\text{new stuff}} > n_{qp}$$
What are the low-energy quasi-particles?

spectral weights are equal
low-energy qp picture

weight $> 1 + x$, conserved charge $= 1 - x$ $\Rightarrow$

$n_{qp} < 1 - x$
low-energy qp picture

weight > 1 + x, conserved charge = 1 - x =>

\[ n_{qp} < 1 - x \]
low-energy qp picture

weight > 1 + x, conserved charge = 1 - x

\[ n_{qp} < 1 - x \]
low-energy qp picture

\[ 1 - x - \alpha < 1 - x \]

weight \(>1+x\), conserved charge \(=1-x\Rightarrow n_{qp} < 1 - x\)
low-energy qp picture

\[ x' = x + \alpha \]

weight > 1 + x, conserved charge = 1 - x \implies n_{qp} < 1 - x
dynamical spectral weight transfer: beyond $U = \infty$
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weight of low-energy band is bigger than electron weight
dynamical spectral weight transfer: beyond $U = \infty$

weight of low-energy band is bigger than electron weight

number of electron qp $< \text{number of bare electrons} \Rightarrow$ FL theory breaks down
dynamical spectral weight transfer: beyond $U = \infty$

- Weight of low-energy band is bigger than electron weight
- Number of electron qps < number of bare electrons $\Rightarrow$ FL theory breaks down
- # of addition states per e per spin > 1
dynamical spectral weight transfer: beyond $U = \infty$

- Weight of low-energy band is bigger than electron weight
- Number of electron qps is smaller than number of bare electrons, so FL theory breaks down
- Number of addition states per e per spin > 1
- Ways to add a particle but not an electron (gapped spectrum)
dynamical spectral weight transfer: beyond $U = \infty$

- Weight of low-energy band is bigger than electron weight
- Number of electron qps is less than the number of bare electrons $\Rightarrow$ FL theory breaks down
- Number of addition states per e per spin $> 1$
- Ways to add a particle but not an electron (gapped spectrum)

Breakdown of electron quasi-particle picture: Mottness
is the empty site mobile??

+ t/U
Mott insulator is a metal, 
2.) no magnetic order

is the empty site mobile??

if yes, then 1.)
Mott insulator is a metal, 
2.) no magnetic order
is the empty site mobile??

if yes, then 1.) Mott insulator is a metal, 2.) no magnetic order

+ $t/U$
Mott Problem: what is the dynamical degree of freedom that makes this happen?

new bound states

Kohn, Mott, Castellani, others
Mott Problem: what is the dynamical degree of freedom that makes this happen?

localisation criterion

free \quad U \quad stuck

new bound states

Kohn, Mott, Castellani, others

No proof exists? Mottness is ill-defined
A Critique of Two Metals

R. B. Laughlin

idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.
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Fermi-liquid analogy

\[ L_{FL} \propto (\omega - \epsilon_k) |\psi_k|^2 \]

Mott Problem?

\[ L_{MI} = (\omega - E_{LHB}(k)) |\eta_k|^2 + (\omega - E_{UHB}(k)) |\tilde{\eta}_k|^2 \]
composite excitation: bound state

half-filling: Mott gap

doping: SWT, pseudogap?

charge 2e boson
How?
Effective Theories:

Integrate Out high Energy fields

$S(\phi)$ at half-filling

$\phi = \phi_L + \phi_H$

$e^{-S_{\text{eff}}[\phi_L]} = \int d\phi_H \exp -S(\phi_L, \phi_H)$

Low-energy theory of MI
Half-filling

Integrate out both

$N(\omega)$
Key idea: similar to Bohm/Pines

Extend the Hilbert space:
Associate with U-scale new Fermionic oscillators

\[ U \tilde{D}^\dagger_i \tilde{D}_i \]

\[ U D^\dagger_i D_i \]

\[ N(\omega) \]
Impose Constraint:

$D_i^+$

How is this possible with Fermions?
$D_i^\dagger$ Fermionic

one per site (fermionic)

transforms as a boson
One fermion per site transforms as a boson.
one per site (fermionic)
transforms as a boson
`supersymmetry'
$D_i^\dagger$ Fermionic

one per site (fermionic)

transforms as a boson

`supersymmetry`

$\delta(D_i - \theta c_i^{\uparrow}c_i^{\downarrow})$

Grassmann
String theory: \[ X^\mu(\sigma, \theta) = X^\mu(\sigma) + \theta \gamma^\mu(\sigma) \]

Fermionic

one per site (fermionic)

transforms as a boson

\[ \delta(D_i - \theta c_i^\uparrow c_i^\downarrow) \]

`supersymmetry'

Grassmann
In the sense, we have inserted unity into the Hubbard model path integral in a rather complicated fashion. To this end, we have to solve the constraint and integrate over heavy fields. Hence, the only way in which a low-energy theory of the Hubbard model exists is if the energy scale for heavy fields is small enough. This is typically achieved by taking the high-energy limit of the theory.

Mathematically, this is expressed as:

\[
\int d^2 \theta \overline{\theta} \theta L_{\text{Hubb}} = \sum_{i,\sigma} c_{i,\sigma}^{\dagger} \dot{c}_{i,\sigma} + H_{\text{Hubb}},
\]
Exact low-energy Lagrangian

\[ L = \#L_{\text{bare}}(\text{electrons}) + \#L_{\text{bare}}(\text{bosons}) \]

\[ + f(\omega)L_{\text{int}}(c, \phi) + \tilde{f}(\omega)L_{\text{int}}(c, \tilde{\phi}) \]

- \( f(\omega) = 0 \)
- dispersion of propagating modes
- composite excitations
- New interactions beyond spin physics
\[ L = \#L_{\text{bare}}(\text{electrons}) + \#L_{\text{bare}}(\text{bosons}) \]

\[ + f(\omega)L_{\text{int}}(c, \varphi) + \tilde{f}(\omega)L_{\text{int}}(c, \tilde{\varphi}) \]

\[ \Psi^\dagger \Psi \]

\[ \tilde{\Psi}^\dagger \tilde{\Psi} \]

quadratic form:
composite or bound excitations of \[ \varphi^\dagger c_i \sigma \]
composite excitations determine spectral density

\[ \gamma^{(\vec{k})}(\omega) = U - t \varepsilon_{p}^{(\vec{k})} - 2\omega \left(1 + 2\omega/U\right)^{1/2} \]

\[ \tilde{\gamma}^{(\vec{k})}(\omega) = U + t \varepsilon_{p}^{(\vec{k})} + 2\omega \left(1 - 2\omega/U\right)^{1/2}. \]

\[ \Delta = U - 4dt \]

each momentum has SD at two distinct energies
hole-doping
Extend the Hilbert space: 
Associate with U-scale a new Fermionic oscillator

\[ N(\omega) \]

\[ \varepsilon \]
charge 2e boson field (non-propagating)

\[ H_{IR}^R = H_{t-J} \]

\[ -\frac{t^2}{U} \sum_i \varphi_i^\dagger \varphi_i - t \sum_j \varphi_j^\dagger c_{j,\uparrow} c_{j,\downarrow} + \frac{t^2}{U} \sum_{i,j} \varphi_i^\dagger b_i + h.c. , \]

Non-projective

\[ b_i = \sum_j g_{i,j} (c_{i,\downarrow} c_{j,\uparrow} - c_{i,\uparrow} c_{j,\downarrow}) \]
Non-projective charge 2e boson field (non-propagating)

\[
H_{IR}^h = H_{t-J}
\]

\[
-\frac{t^2}{U} \sum_i \varphi_i^\dagger \varphi_i - t \sum_j \varphi_j^\dagger c_j,\uparrow c_j,\downarrow + \frac{t^2}{U} \sum_{i,j} \varphi_i^\dagger b_i + h.c.
\]

\[
b_i = \sum_j g_{ij} \left( c_i\downarrow c_j\uparrow - c_i\uparrow c_j\downarrow \right)
\]

\(\varphi_i\) is an emergent degree of freedom
Not made out of the elemental fields
Electron spectral function

$t^2/U \sim 60\text{meV}$
Electron spectral function

$t^2/U \sim 60\text{meV}$
This implies that the weight in the unoccupied part of the LHB is \(2(x + \alpha)\). The fermionic degrees of freedom that contribute to the formation of the LHB and the half-filled Fermi level is a sum of two components,

\[
Q = \sum_i c_i^\dagger c_i + 2 \sum_i \phi_i^\dagger \phi_i,
\]

where \(\phi_i\) are the bosonic operators.

Electron spectral function

\[\Lambda = 1 - x\]

\[\Lambda = \mu^+\]

\[\sqrt{U} \sim 60\text{meV}\]
Electron spectral function

Conserved charge: \[ Q = \sum_i c_i^\dagger c_i + 2 \sum_i \varphi_i^\dagger \varphi_i \]
Two bands!!

Spin-charge separation?
Origin of two bands

Origin of two bands

Two charge e excitations

\[ \varphi_i \, c_i \bar{\sigma} \]

New bound state

Origin of two bands

Two charge $e$ excitations

$\varphi_i \, c_{i\sigma}$

$\varphi_i$ is confined (no kinetic energy)

New bound state

Origin of two bands

Two charge e excitations

$\varphi_i^{\dagger} c_{i\sigma}$

$\varphi_i$ is confined (no kinetic energy)

New bound state

$C_{i\sigma}$

Pseudogap

two types of charges

'free'

bound
direct evidence
direct evidence

charge carrier density:
direct evidence

charge carrier density:

\[ n_{\text{Hall}}(x, T) = n_0(x) + n_1(x) \exp(-\Delta(x)/T), \]

direct evidence

charge carrier density:

\[ n_{\text{Hall}}(x, T) = n_0(x) + n_1(x) \exp\left(-\Delta(x)/T\right), \]


exponentially suppressed: confinement
Our Theory

no model-dependent free parameters: just $t/U$

exponential $T$-dependence
Our Theory

\[ \Delta(x) (eV) \]

\[ n_H(T) \]

exponential T-dependence

no model-dependent free parameters: just \( t/U \)
Like Mott gap, Pseudogap is a bound-state problem with new IR modes.
strange metal: breakup (deconfinement) of bound states
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strange metal: breakup (deconfinement) of bound states

QCP:
New scenario as $x$ increases

$E_B > 0$ unbound

$C_i \bar{\sigma}_i \varphi^+_i$

$E_B = 0$

$T$-linear resistivity
New scenario as $x$ increases

$E_B > 0$ unbound

$E_B = 0$

$c_i \bar{\sigma} \varphi_i^\dagger$

$E_B > 0$

T-linear resistivity
New scenario as $x$ increases

$E_B > 0$ unbound

$E_B = 0$

$\rho \propto T$

$T > \omega_D$

$T$-linear resistivity
\[ L = \text{Area/\# of sites} \]

\[ \int_{\mu}^{\Lambda} N(\omega) d\omega \]

\[ L = \# \text{ of single-particle addition states (qp)} \]

\[ \eta_h = \# \text{ of ways electrons can be added} = \# \text{ of holes created by the dopants} \]
\( L = \text{Area/\# of sites} \)

\[ \int_{\mu}^{\Lambda} N(\omega) d\omega \]

\( L = \# \text{ of single-particle addition states (qp)} \)

\( \eta_h = \# \text{ of ways electrons can be added} = \# \text{ of holes created by the dopants} \)

Equivalent in Fermi liquid
$\rho \sim T$

Experimental Prediction:

$L/n_h > 1$

$L/n_h = 1$

More states at lower temperature

$E_B > 0$

$pseudo$-gap

$E_B = 0$

"strange" metal

LS sharp

LS broad

$d$-wave superconductor

conventional metal?

temperatur
Experiments
How did I start working with Rob?
\[ \sigma(T) \propto T^{(d-2)/z} \]

quantum criticality
quantum criticality

\[ \sigma(T) \propto T^{(d-2)/z} \]

\[ \rho \propto T \]

only if \( z < 0 \)
quantum criticality

\[ \sigma(T) \propto T^{(d-2)/z} \]

only if \( z < 0 \)

\[ \rho \propto T \]
quantum criticality

\[ \sigma(T) \propto T^{(d-2)/z} \]

\[ E \propto \rho^z \]

dynamical exponent

only if \( z < 0 \)

\[ \rho \propto T \]

\[ d = 3 \]

quantum criticality

\[ \sigma(T) \propto T^{(d-2)/z} \]

\[ E \propto p^z \]

dynamical exponent

only if \( z < 0 \)

\[ \rho \propto T \]

in non-commutative theories, \( z < 0 \)

\[ \sigma(T) \propto T^{(d-2)/z} \]

\[ E \propto p^z \]

quantum criticality

dynamical exponent

only if \( z < 0 \)

in non-commutative theories, \( z < 0 \)

UV-IR mixing

quantum criticality

\[ \sigma(T) \propto T^{(d-2)/z} \]

dynamical exponent

\[ E \propto \rho^z \]

in non-commutative theories, \( z < 0 \)

UV-IR mixing

\[ \rho \propto T \]

only if \( z < 0 \)

\[ d = 3 \]
Mottness: Strong Coupling

low-energy reduction
Mottness: Strong Coupling

low-energy reduction

bare particles
Mottness: Strong Coupling

low-energy reduction

bare particles
Mottness: Strong Coupling

low-energy reduction

Pseudogap=confinement
Mottness: Strong Coupling

low-energy reduction

bare particles

composite or bound states not in UV theory

Pseudogap=confinement
Predictions:

\[ x \rightarrow x + \alpha \]

STM asymmetry: \[ \frac{2(x + \alpha)}{1 - x - \alpha} \]

QO oscillations: \[ V_{hp} - V_{ep} = 2(x + \alpha) \]

Superfluid density: \[ \rho_s \propto x + \alpha \]

pairing from Mott scale

optical conductivity: \[ n_{eff} \propto (x + \alpha) > x \]

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