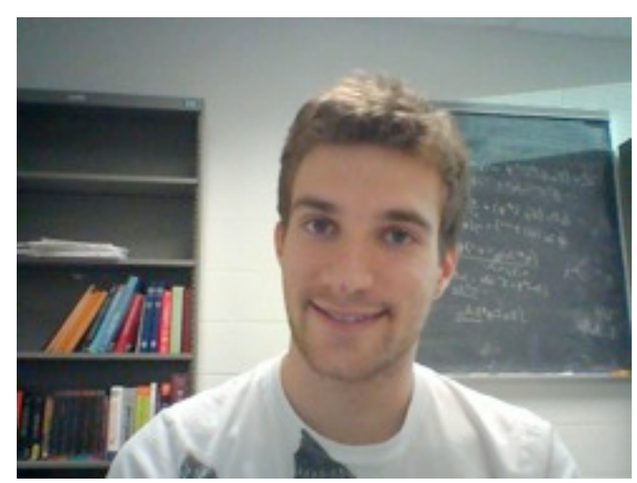
Unparticles and Mottness

Thanks to: NSF, EFRC (DOE)

Kiaran dave

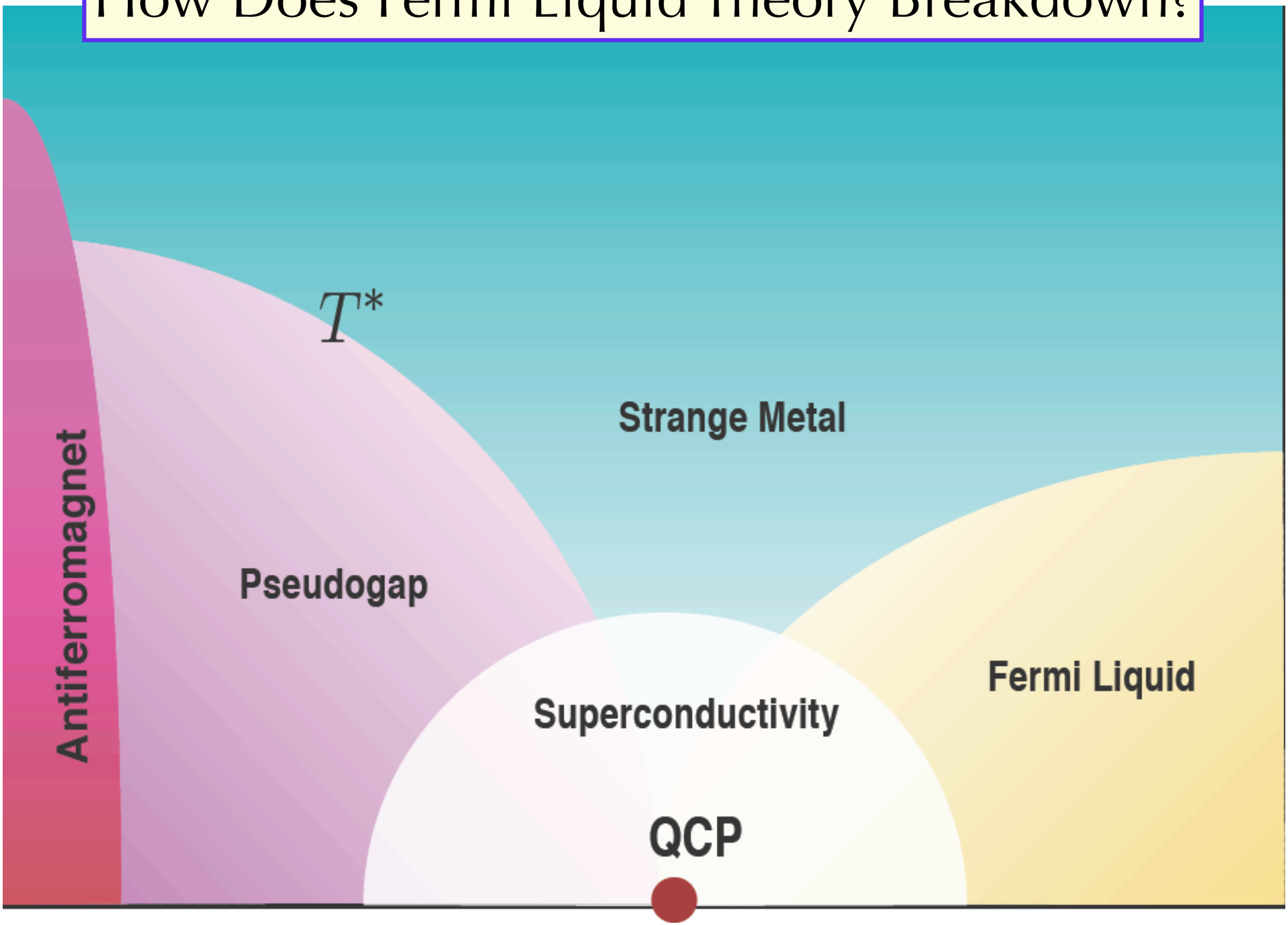
Charlie Kane



Brandon Langley

J. A. Hutasoit

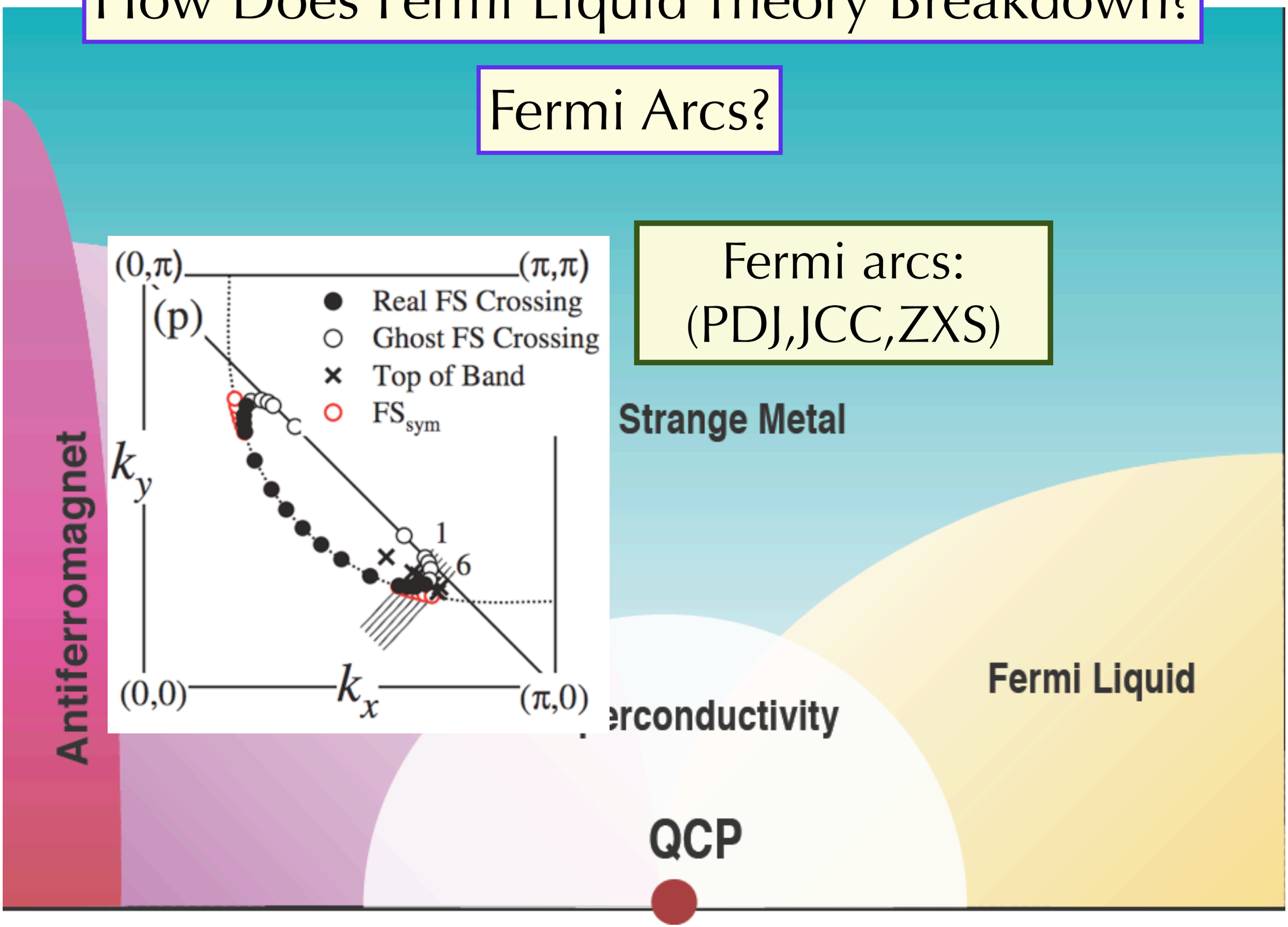
How Does Fermi Liquid Theory Breakdown?



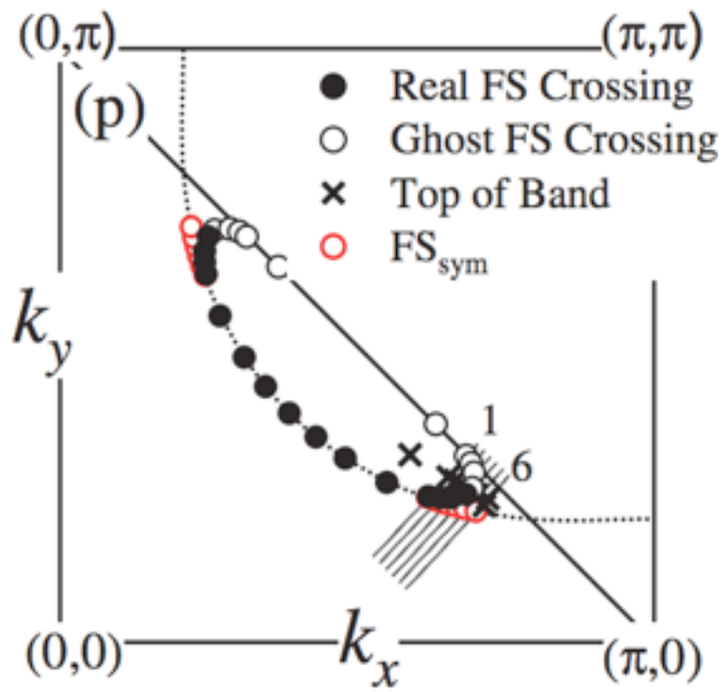
How Does Fermi Liquid Theory Breakdown?

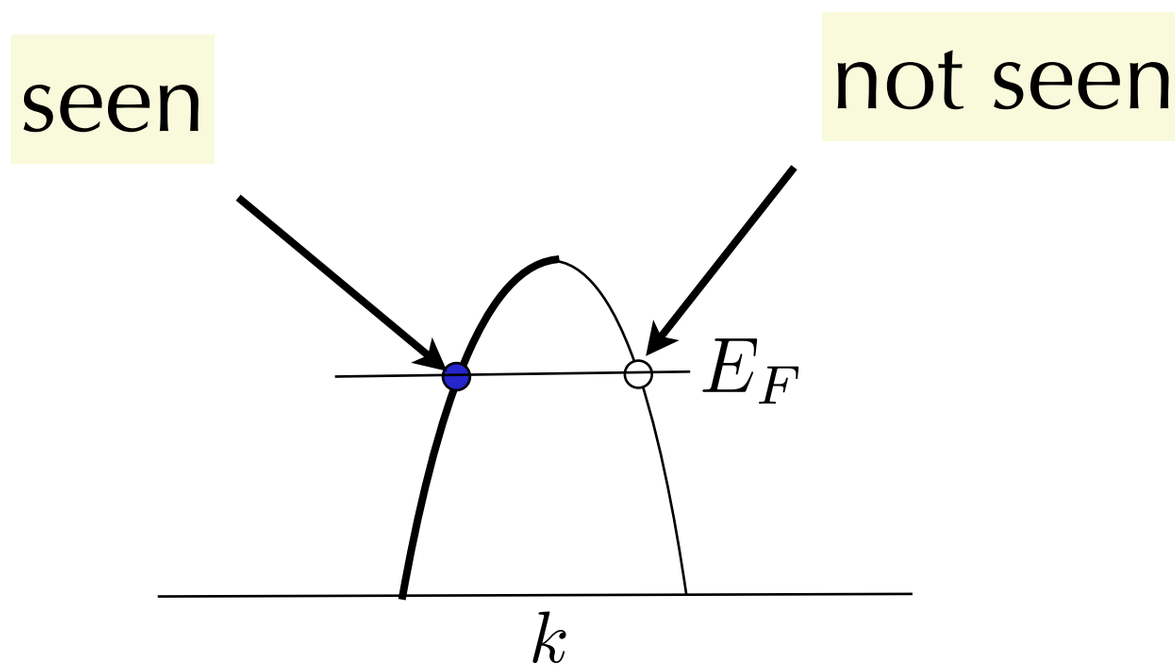
Fermi Arcs?

Fermi arcs:
(PDJ,JCC,ZXS)

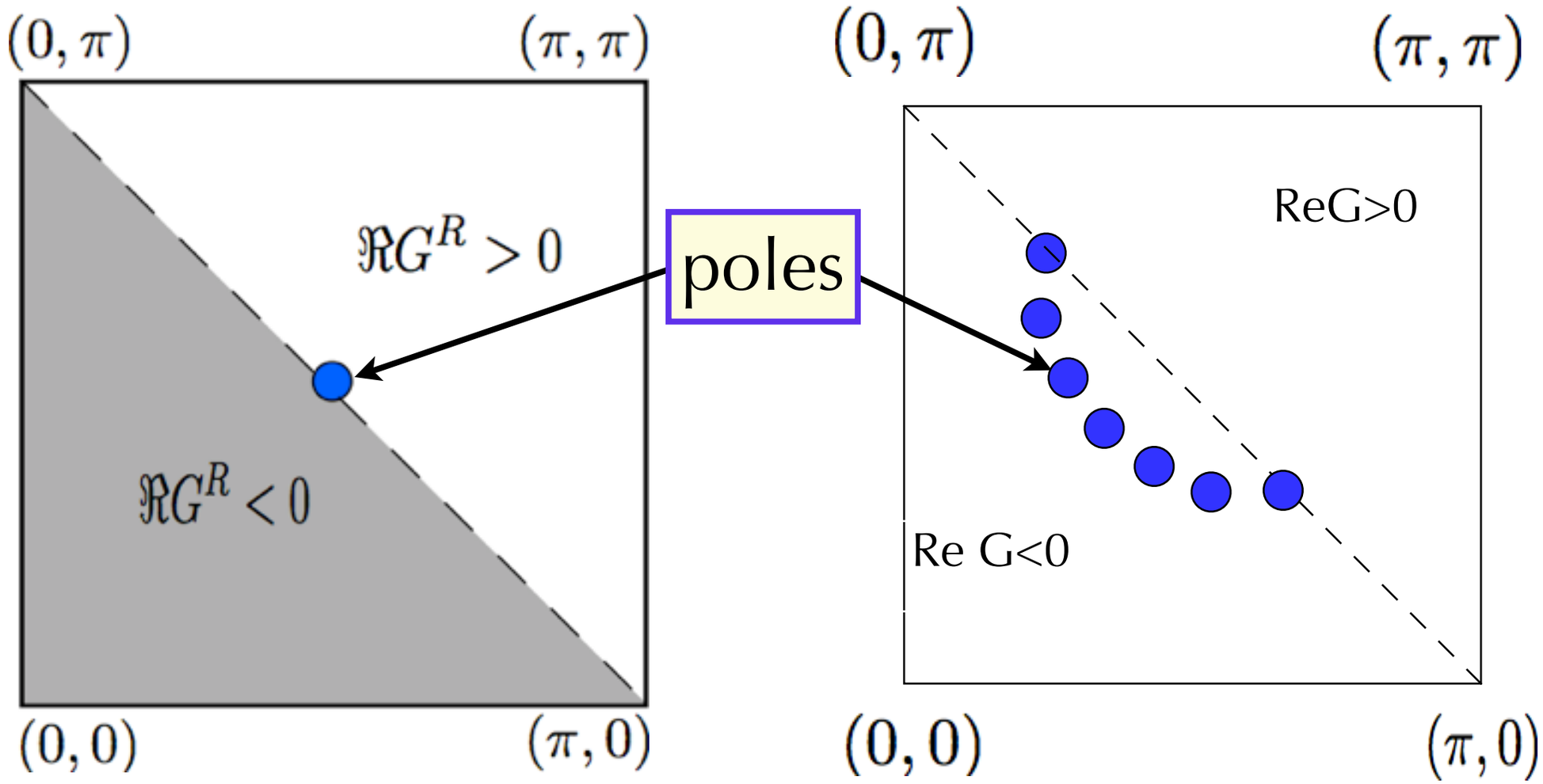


Why are Fermi Arcs Strange?

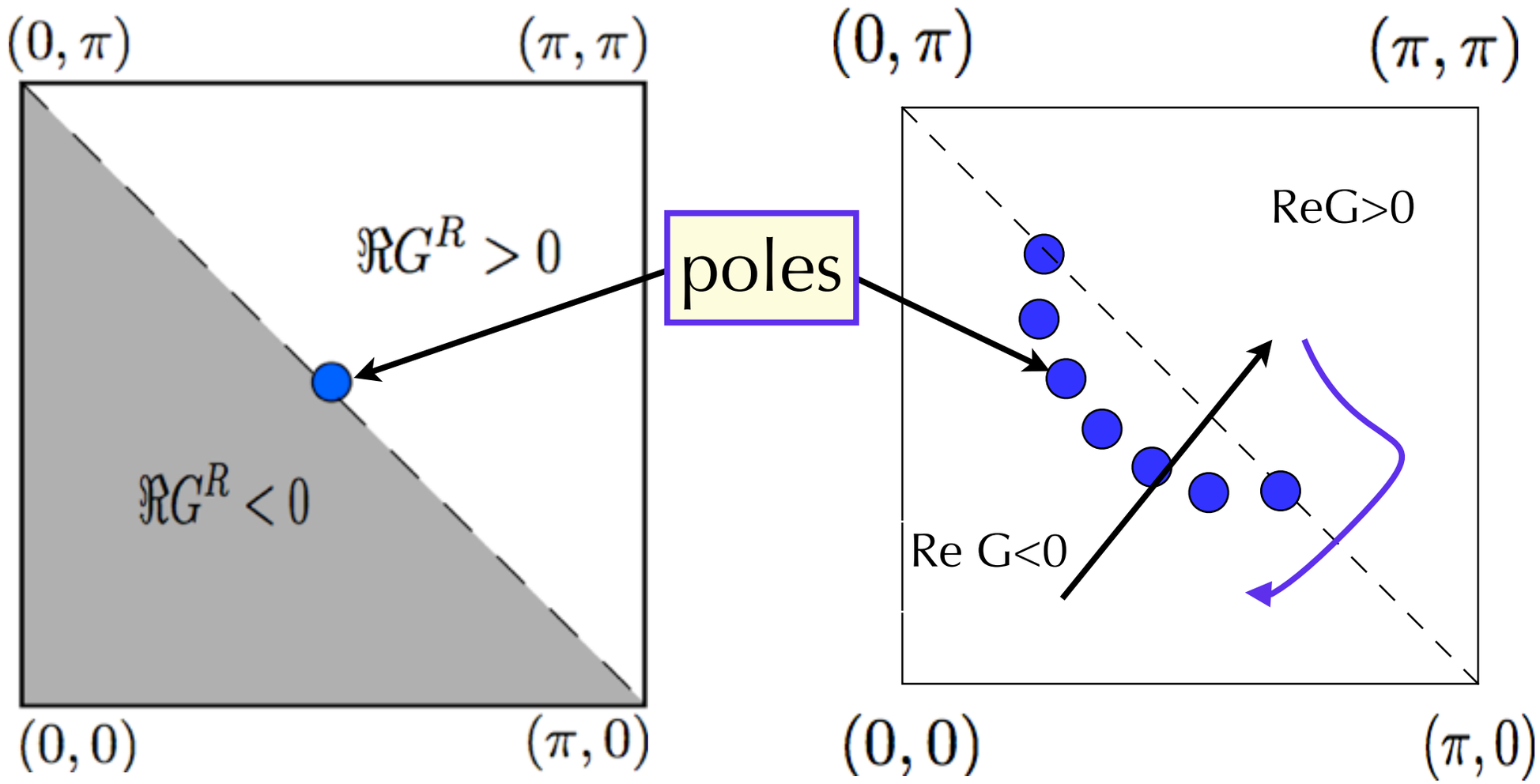




Problem

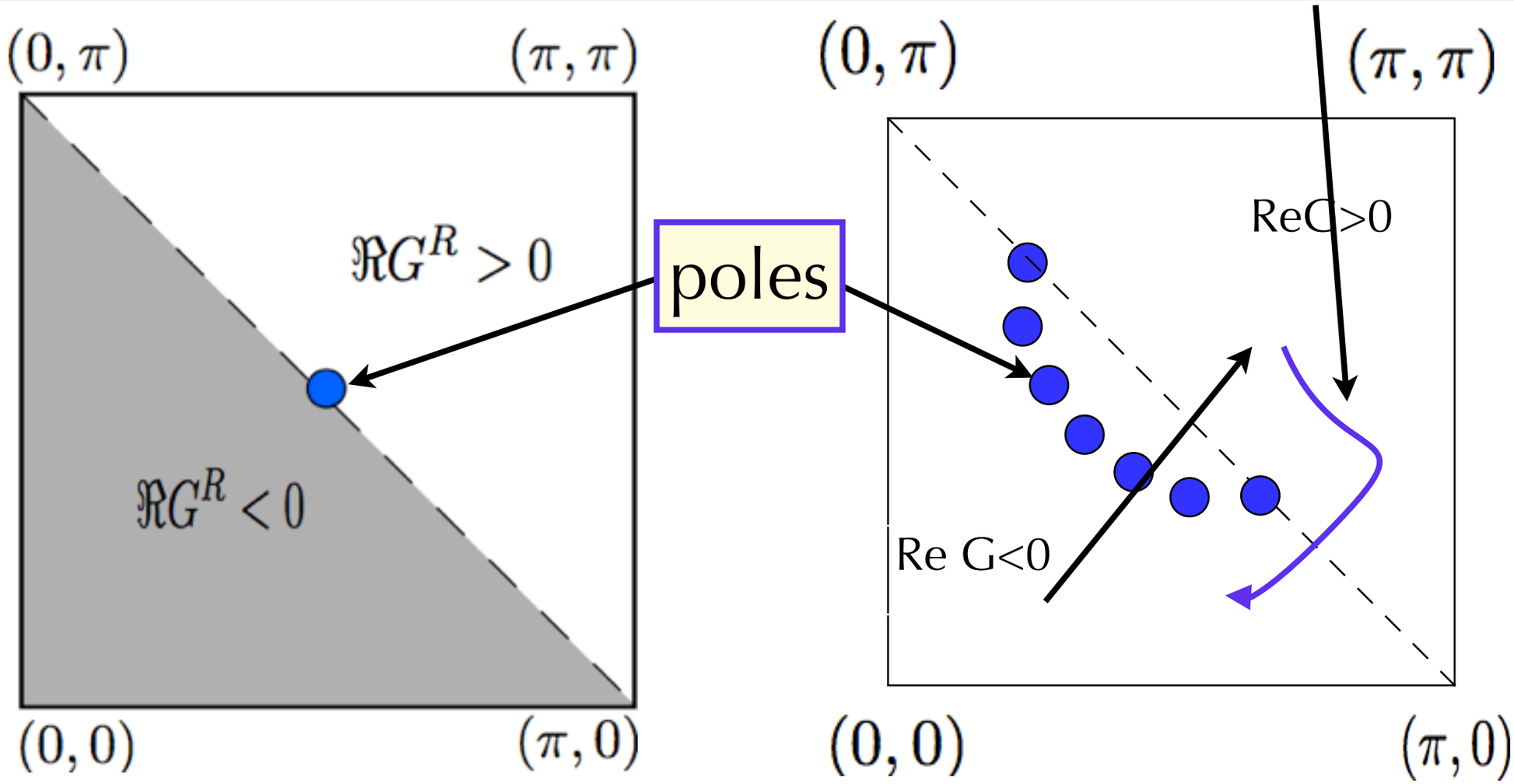


Problem



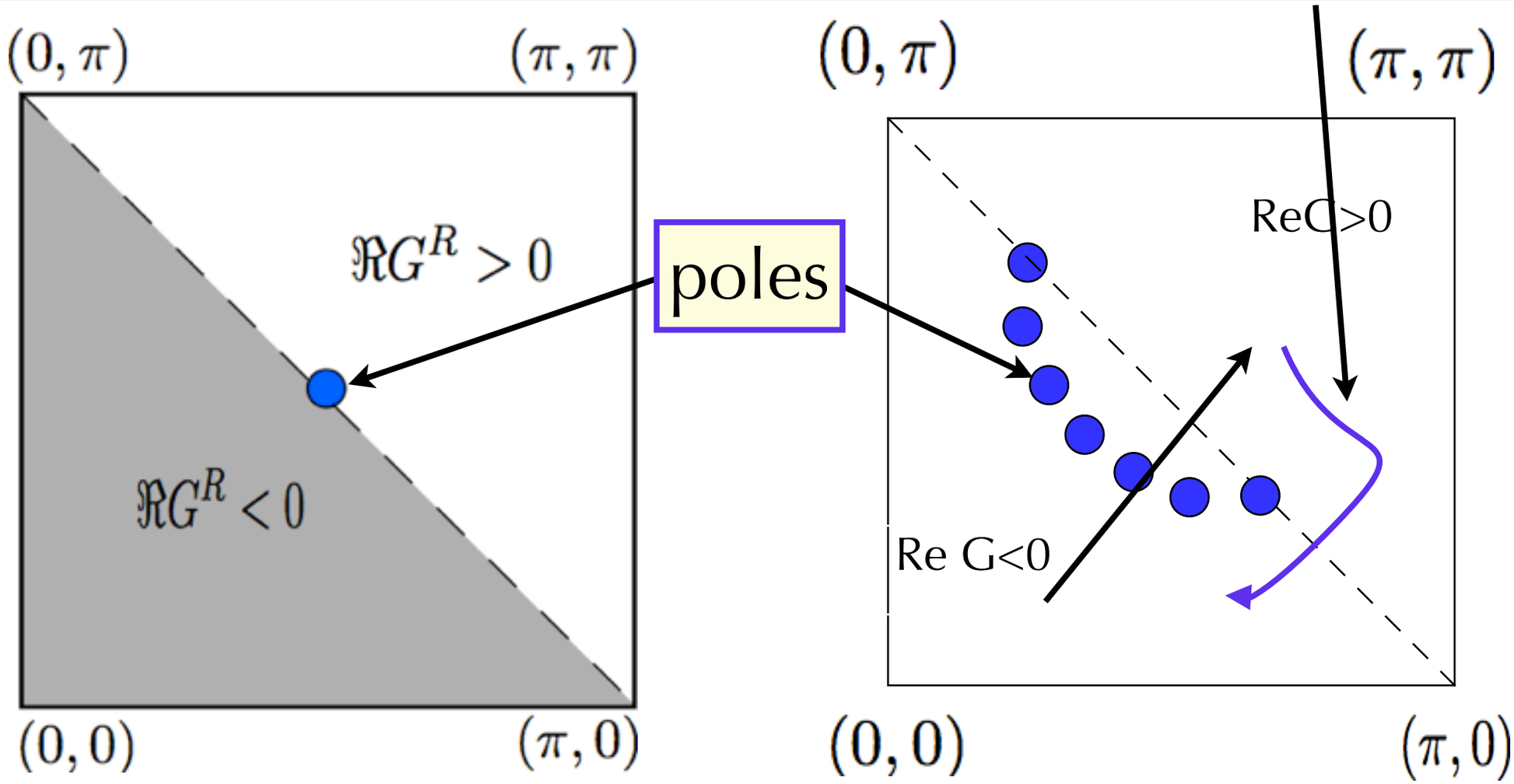
Problem

How to account for the sign change without poles?



Problem

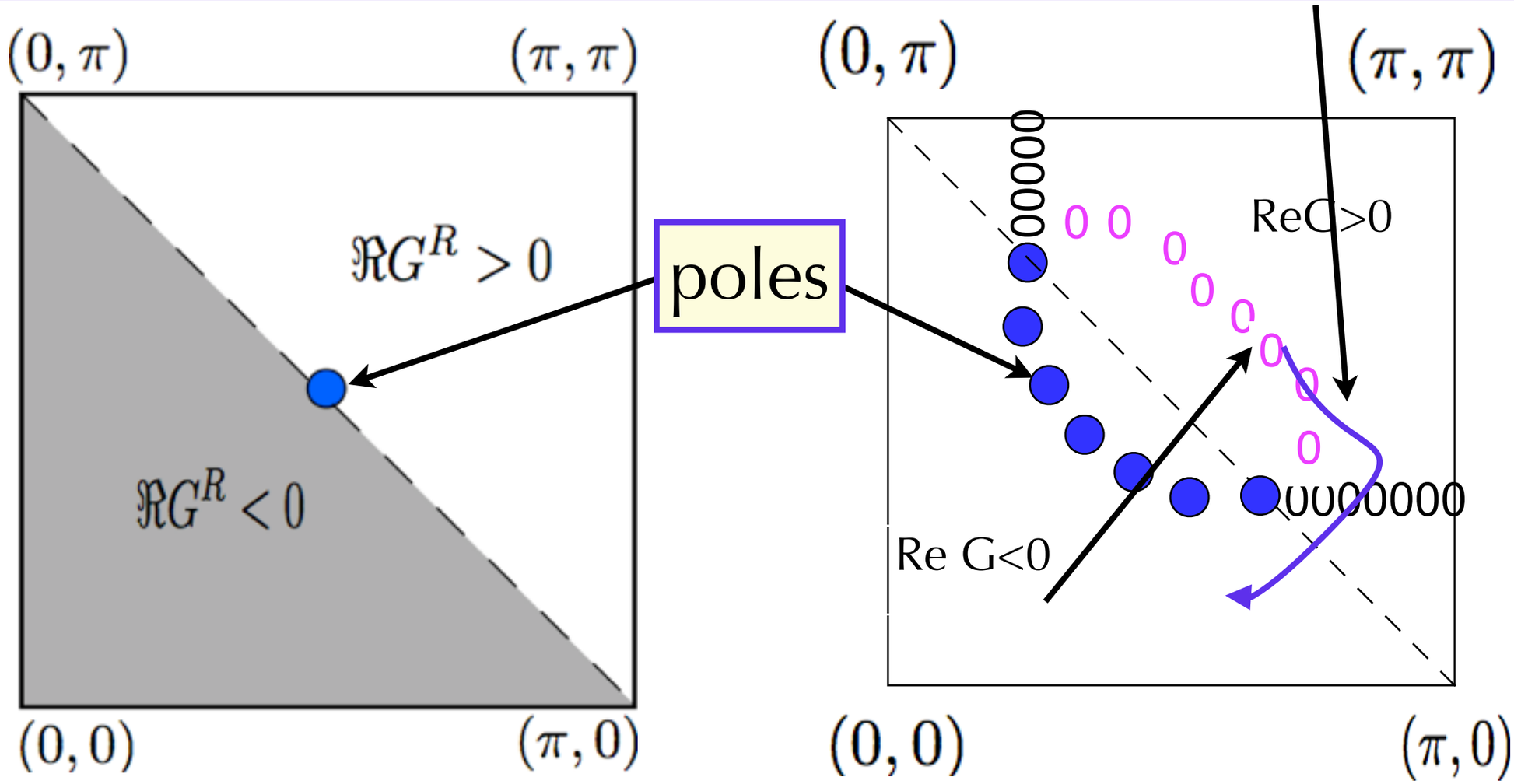
How to account for the sign change without poles?



Only option: DetG=0! (zeros)

Problem

How to account for the sign change without poles?



Only option: DetG=0! (zeros)

Fermi Arcs

zeros + poles

Luttinger, Dzyaloshinskii, Yang, Rice, Zhang,Tsvelik...

n=zeros + poles

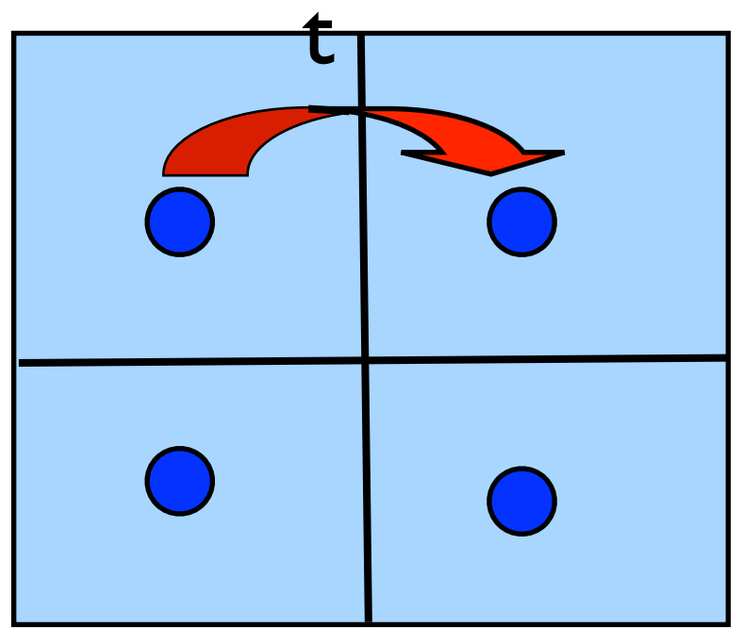
what are zeros?

are they (like poles) conserved?

Mott mechanism (not Slater)

NiO insulates

d^8 ?

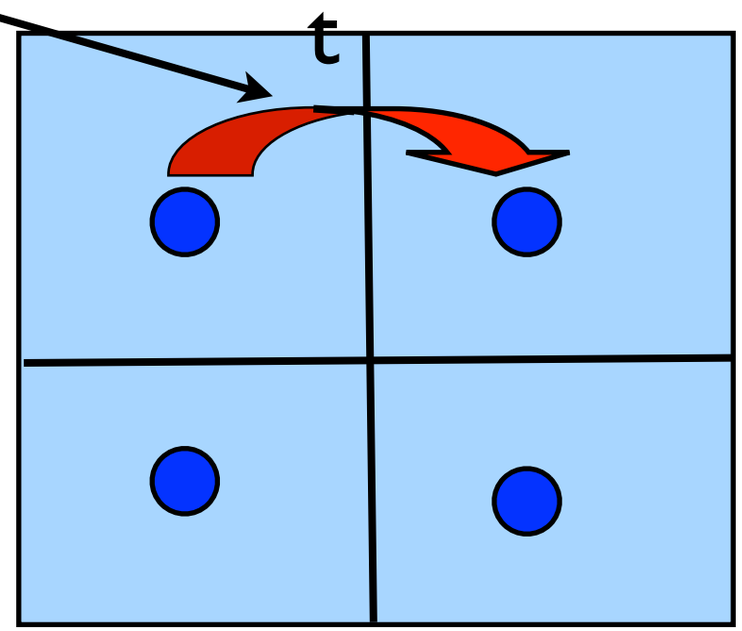


Mott mechanism (not Slater)

NiO insulates

d^8 ?

perhaps this costs energy

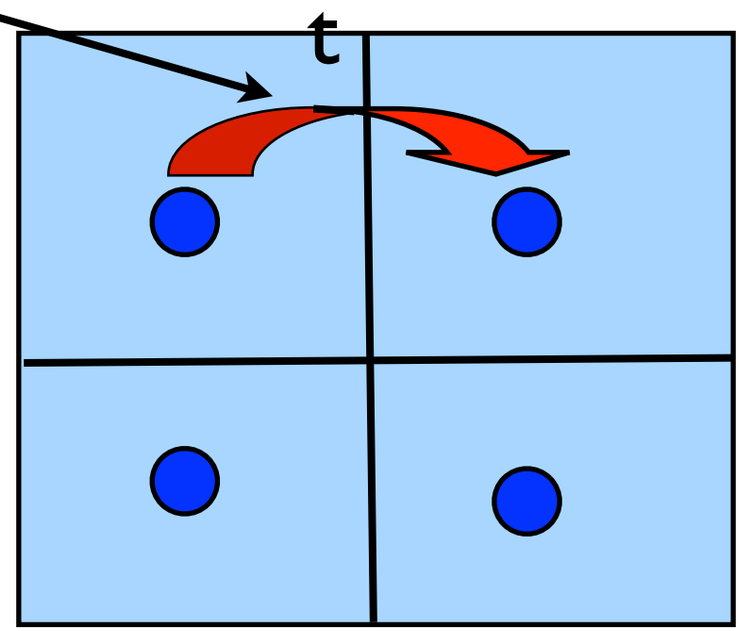


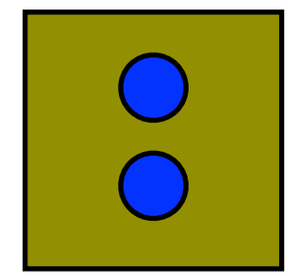
Mott mechanism (not Slater)

NiO insulates

d^8 ?

perhaps this costs energy





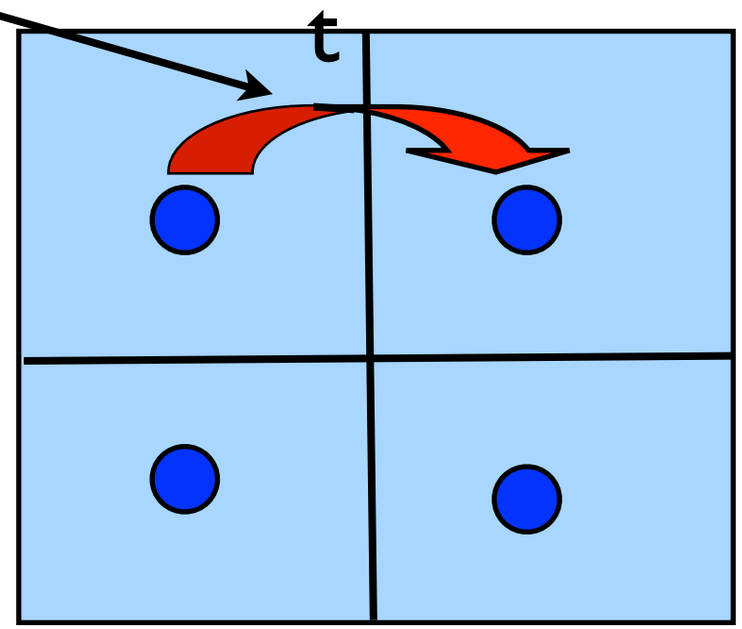
$$U \gg t$$

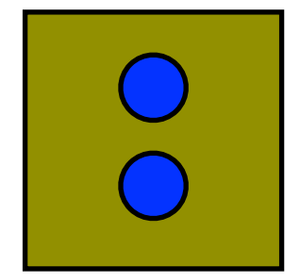
Mott mechanism (not Slater)

NiO insulates

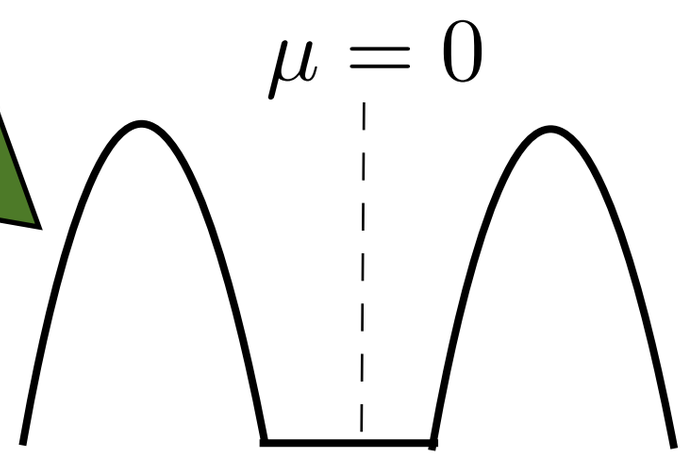
d^8 ?

perhaps this costs energy





$U \gg t$

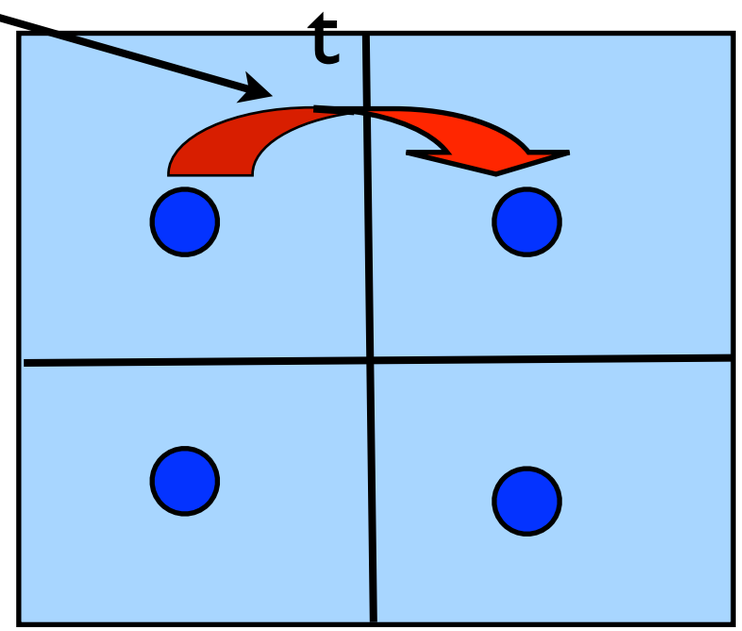


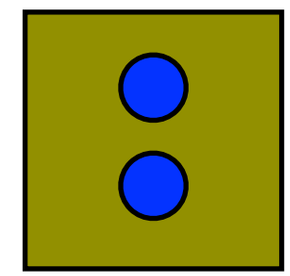
Mott mechanism (not Slater)

NiO insulates

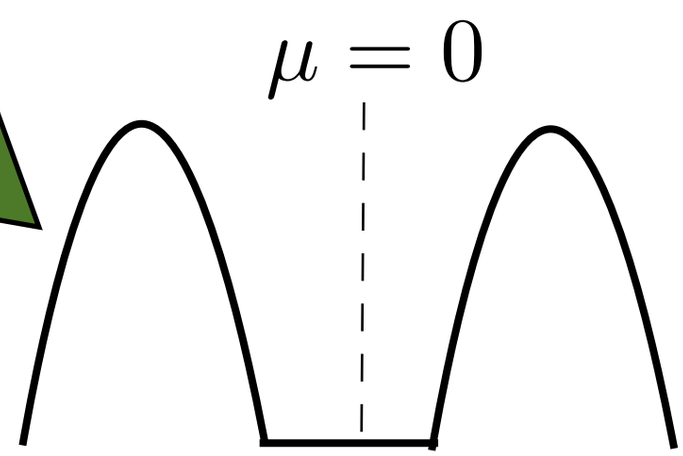
d^8 ?

perhaps this costs energy





$U \gg t$



no change in size of Brillouin zone

Mott Problem

Kramers
-Kronig

Mott Problem

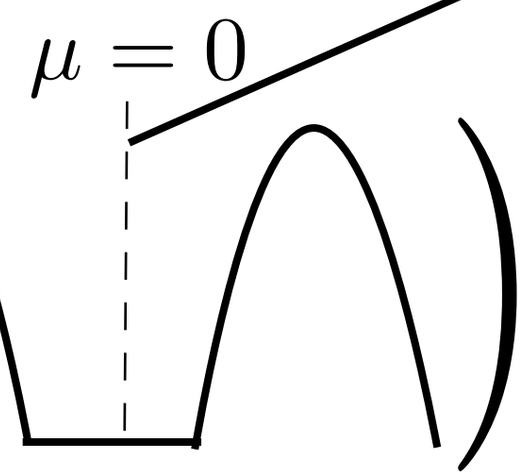
Kramers
-Kronig

= below gap+above gap

Mott Problem

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{\left(\quad \right)}{\omega} d\omega$$



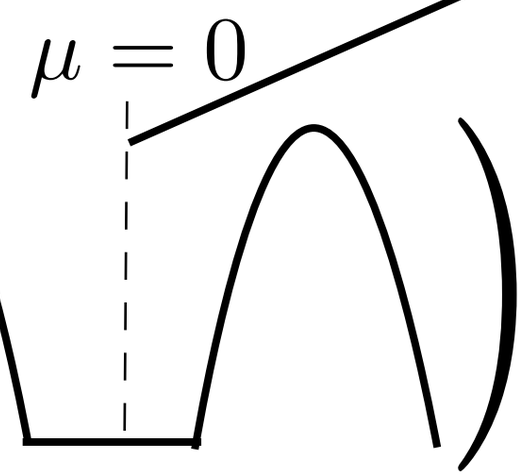
Kramers -Kronig

= below gap+above gap

Mott Problem

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{\left(\quad \right)}{\omega} d\omega$$



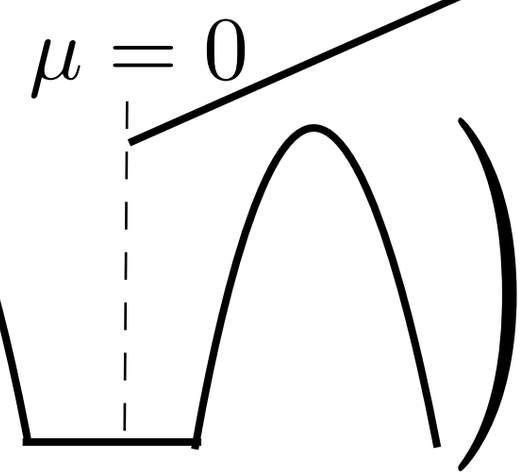
Kramers -Kronig

= below gap+above gap = 0

Mott Problem

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{\left(\quad \right)}{\omega} d\omega$$



Kramers -Kronig

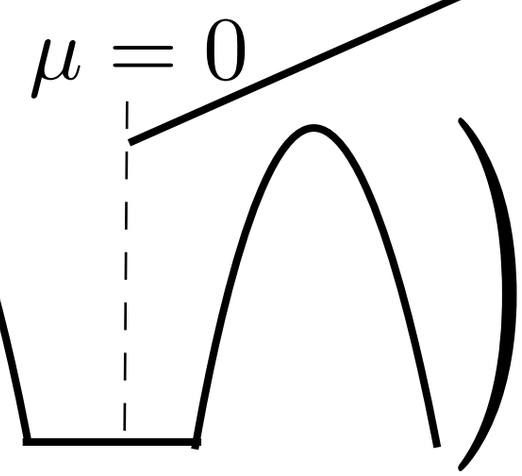
= below gap+above gap = 0

$$\mathrm{Det}G(\mathbf{k}, \omega = \mathbf{0}) = \mathbf{0}$$ (single band)

Mott Problem

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{\left(\quad \right)}{\omega} d\omega$$



Kramers -Kronig

= below gap+above gap = 0

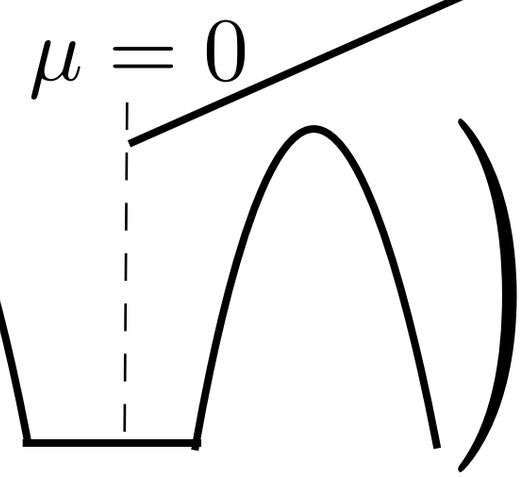
$$\mathrm{Det}G(\mathbf{k}, \omega = \mathbf{0}) = \mathbf{0}$$ (single band)

DetReG(0,p)=0 Mottnes

Mott Problem

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{\left(\quad \right)}{\omega} d\omega$$



Kramers -Kronig

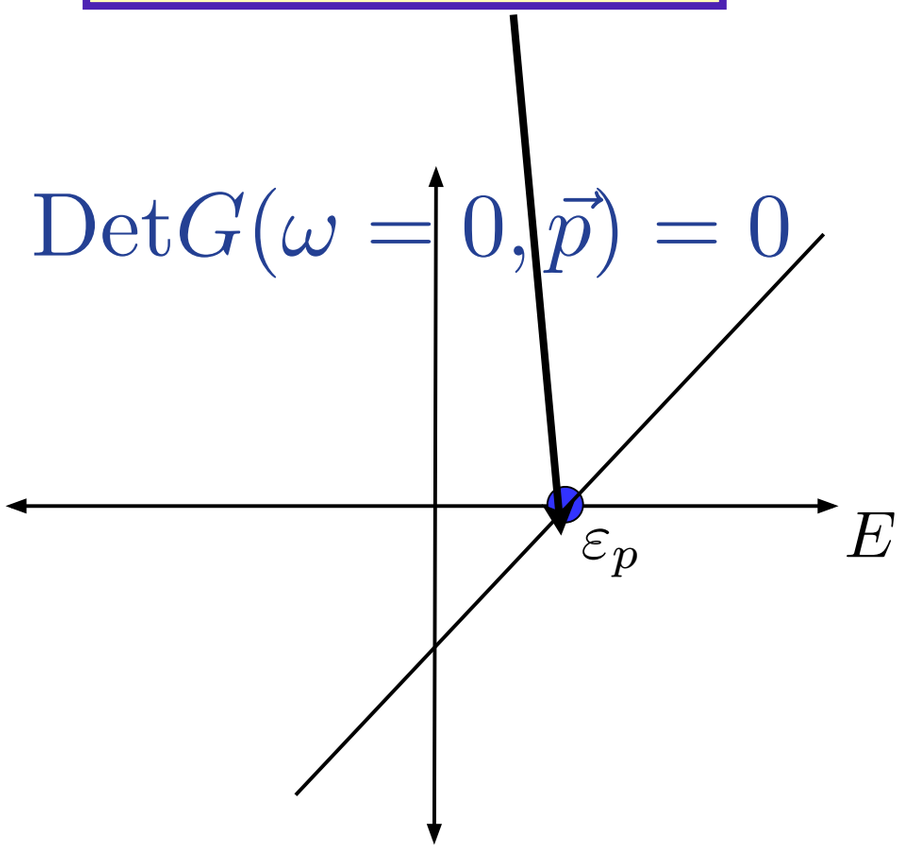
= below gap+above gap = 0

$$\mathrm{Det}G(\mathbf{k}, \omega = \mathbf{0}) = \mathbf{0}$$ (single band)

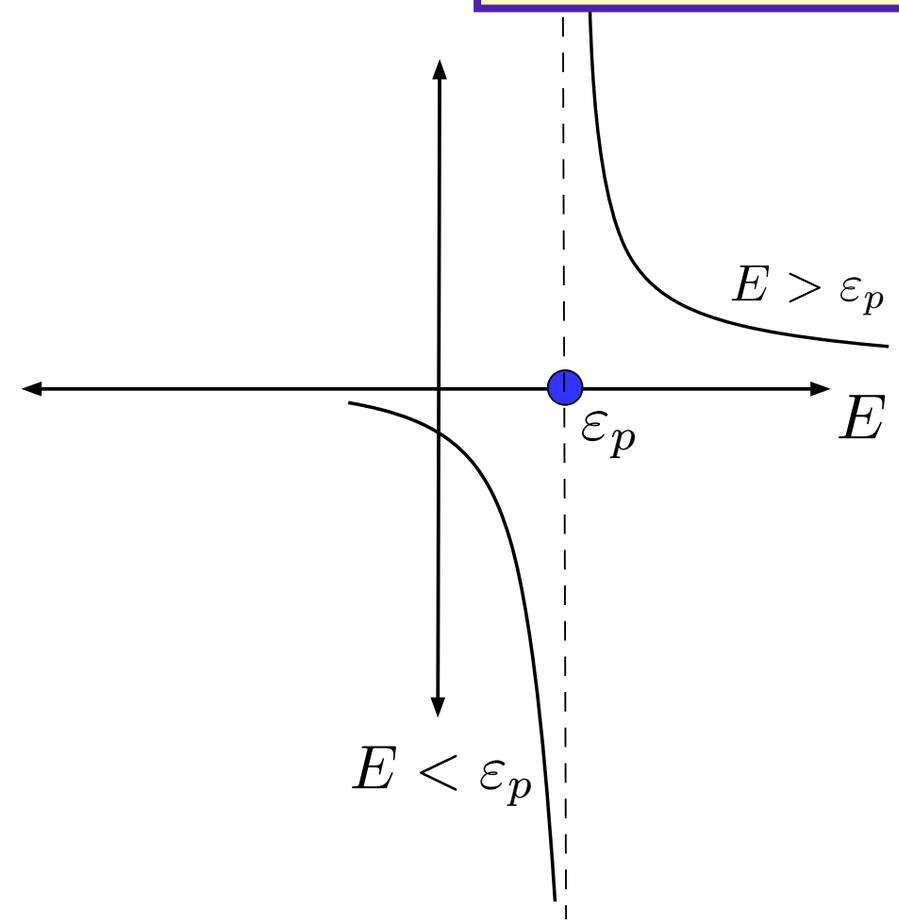
DetReG(0,p)=0 | Mottnes | not true in MF theories

sign changes of Green function

zero-crossing



poles (qp)

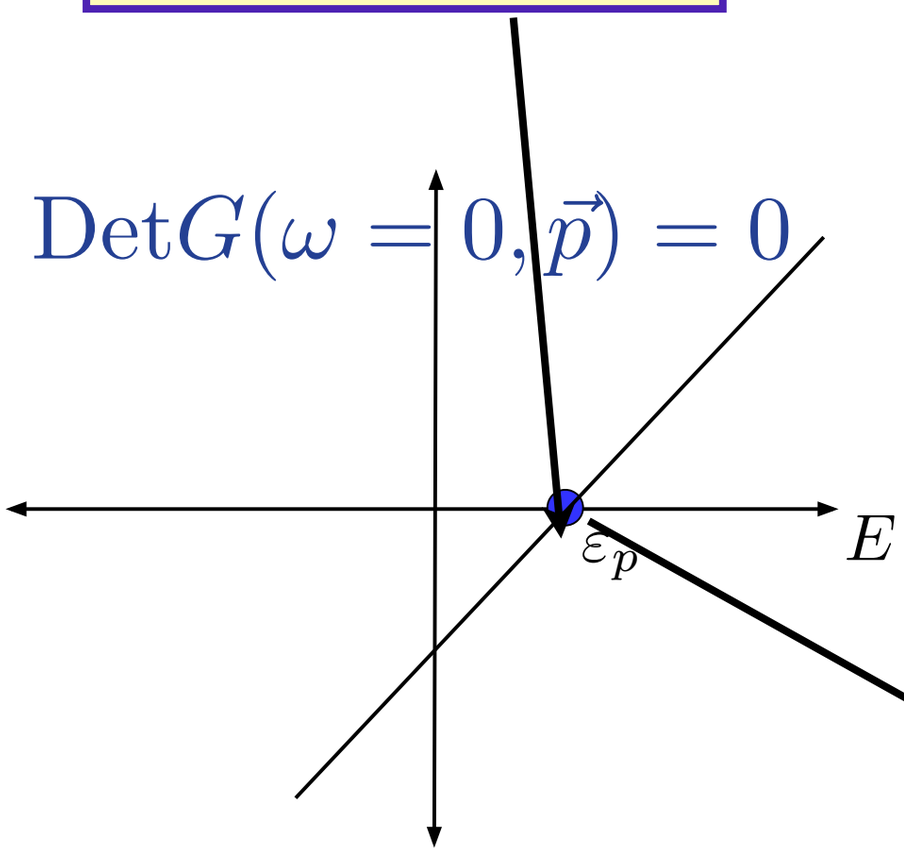


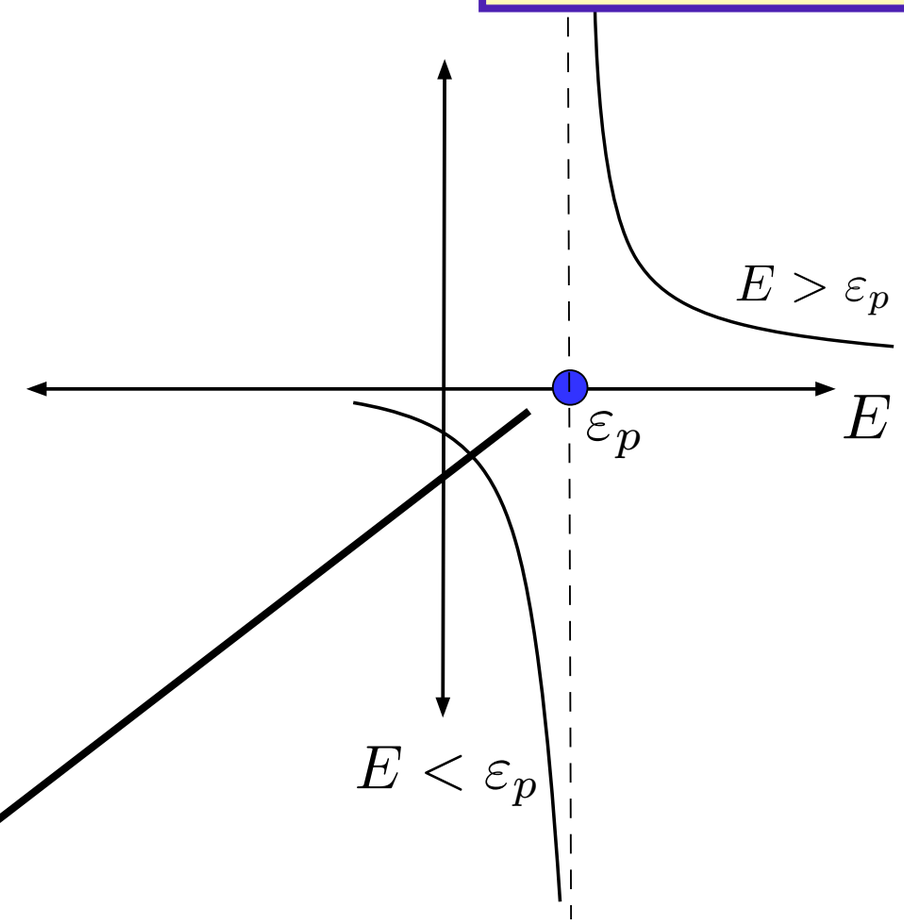
sign changes of Green function

zero-crossing

poles (qp)

$$\mathrm{Det}G(\omega = 0, \vec{p}) = 0$$





$$n = 2\sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

Luttinger's theorem

singularities of $\ln G$

$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d\mathbf{p} \int_{-\infty}^{0} d\xi \ln \frac{G^R(\xi, \mathbf{p})}{G_R^*(\xi, \mathbf{p})}$$

poles+zeros
(all sign changes)

singularities of $\ln G$

$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d\mathbf{p} \int_{-\infty}^{0} d\xi \ln \frac{G^R(\xi, \mathbf{p})}{G^*_R(\xi, \mathbf{p})}$$

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

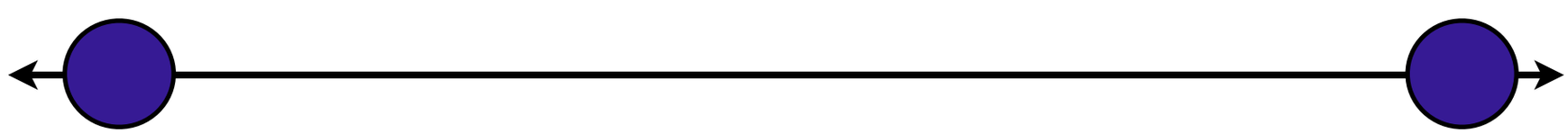
poles+zeros
(all sign changes)

singularities of $\ln G$

$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d\mathbf{p} \int_{-\infty}^{0} d\xi \ln \frac{G^R(\xi, \mathbf{p})}{G^*_R(\xi, \mathbf{p})}$$

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

poles+zeros
(all sign changes)



Fermi Liquids

Mott Insulators

PHYSICAL REVIEW B **68**, 085113 (2003)

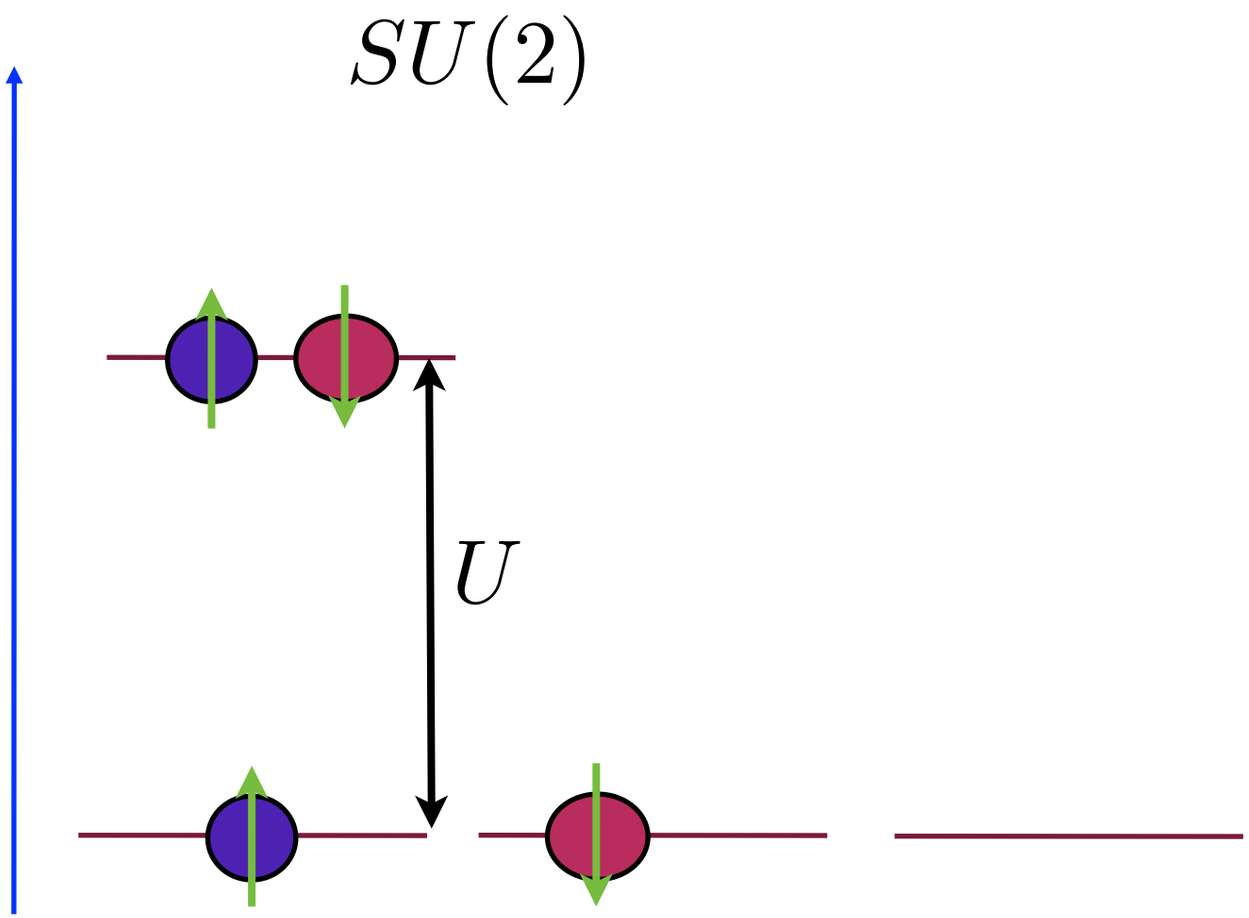
Some consequences of the Luttinger theorem: The Luttinger surfaces in non-Fermi liquids and Mott insulators

Igor Dzyaloshinskii
Department of Physics and Astronomy, University of California, Irvine, California 92697, USA
(Received 30 January 2003; published 27 August 2003)

The proof just presented is good for any state of our system: FL, NFL, or MI [in other words, either of poles or zeros can be used to change the sign of G_r in Eq. (1)]. The only way to incapacitate the Luttinger theorem in form (1) is to assume that the limit $T \to 0$ is discontinuous. Actually, one has to require that the whole line $T=0$ is a line of phase transitions.

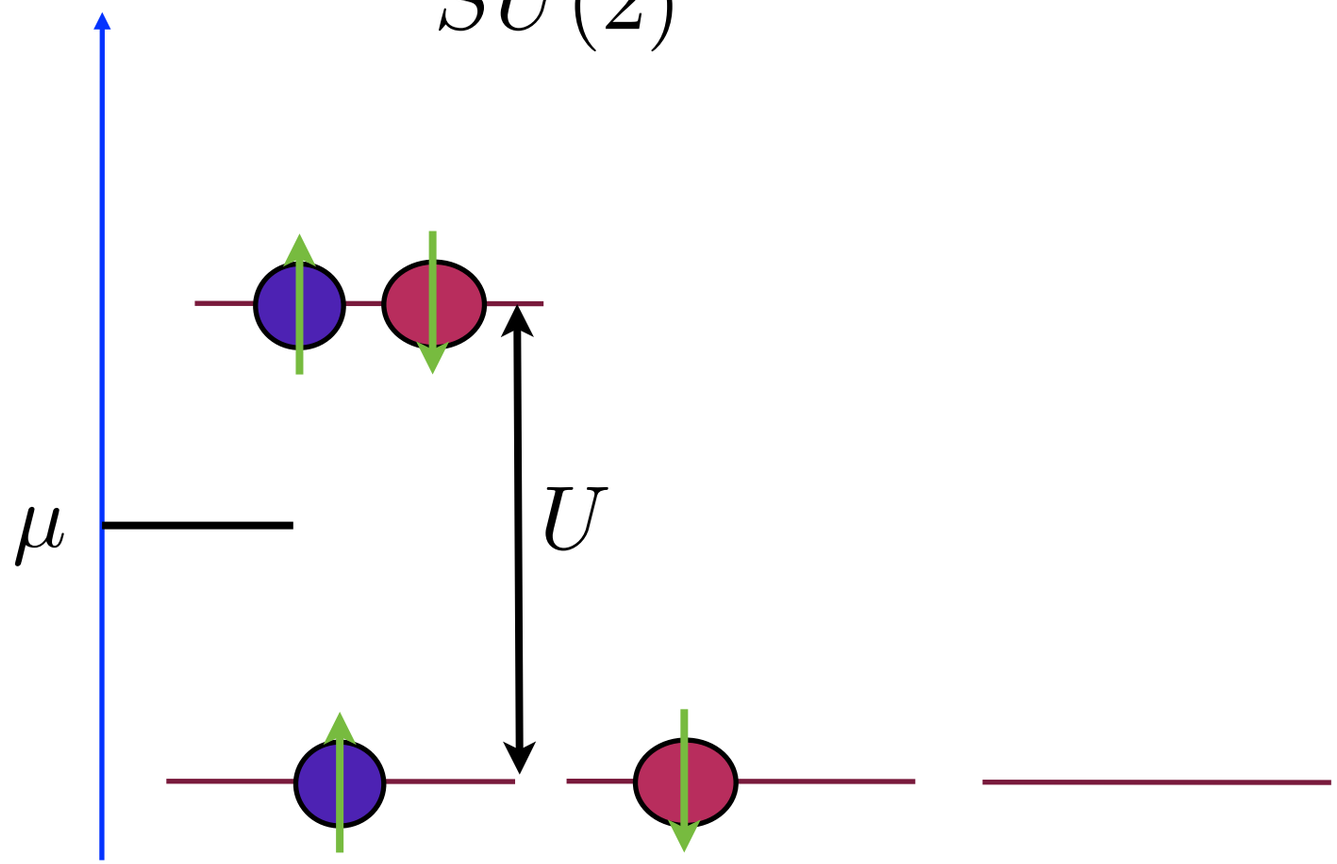
Is this famous theorem from 1960 correct?

simple problem: n=1

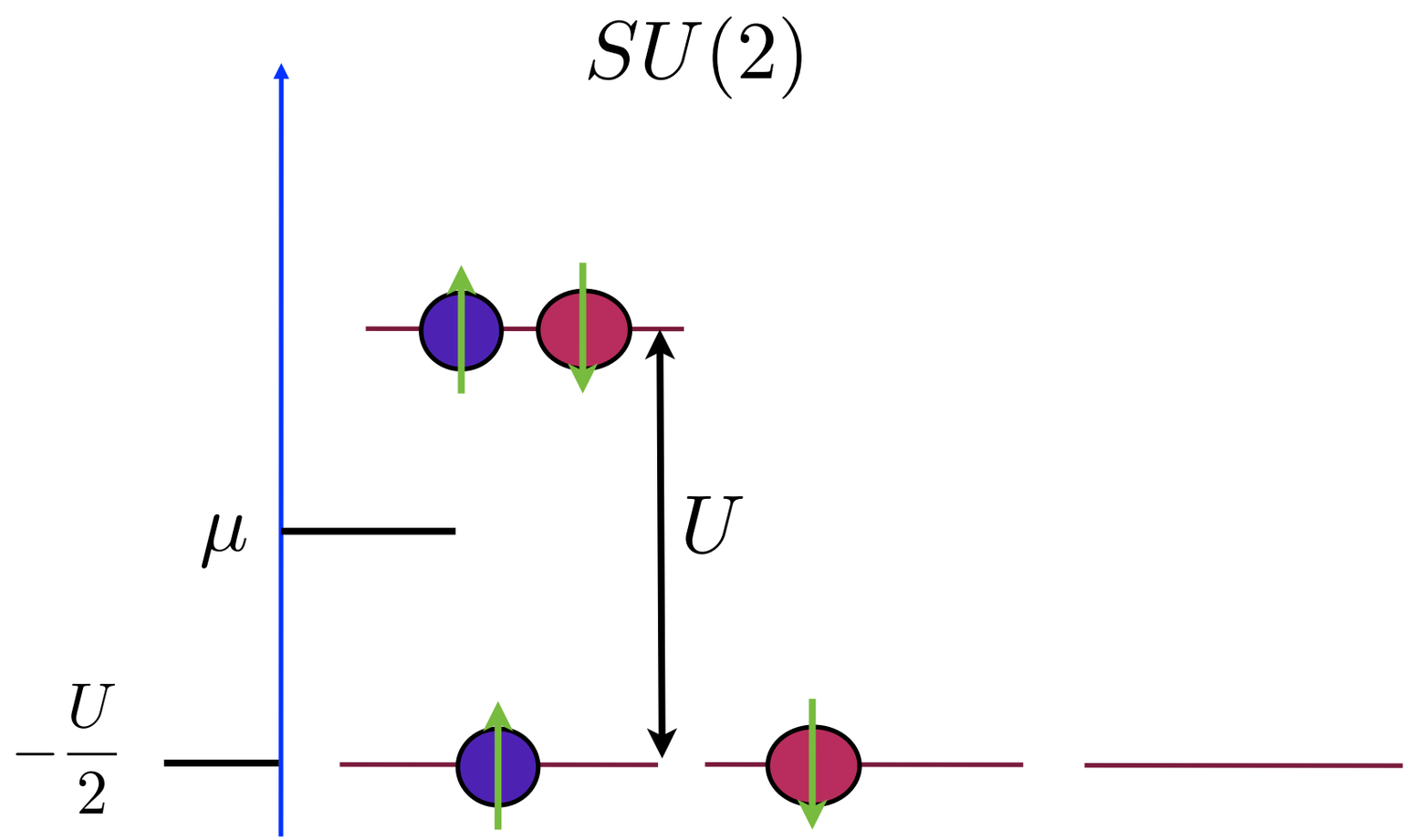


simple problem: n=1

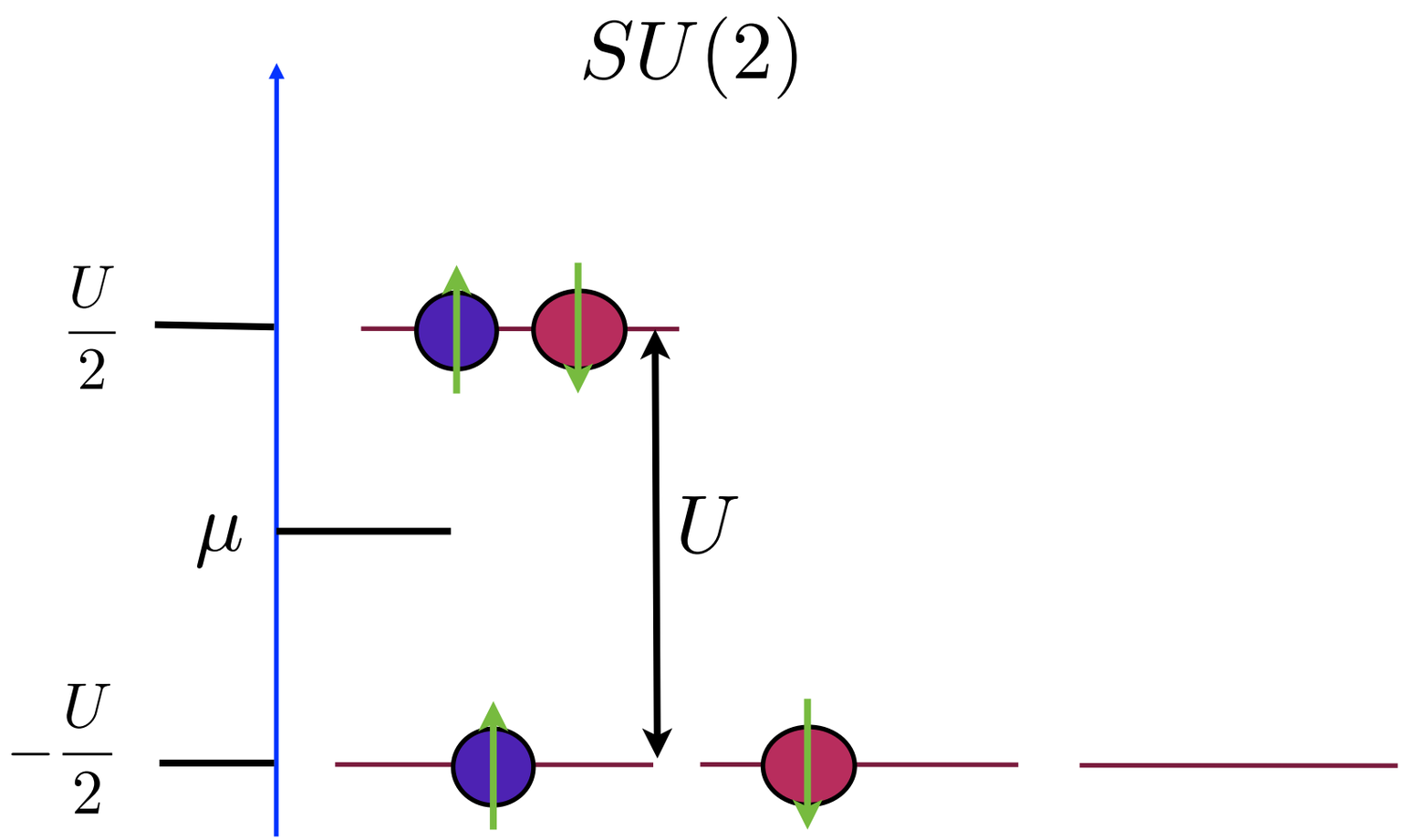
$SU(2)$



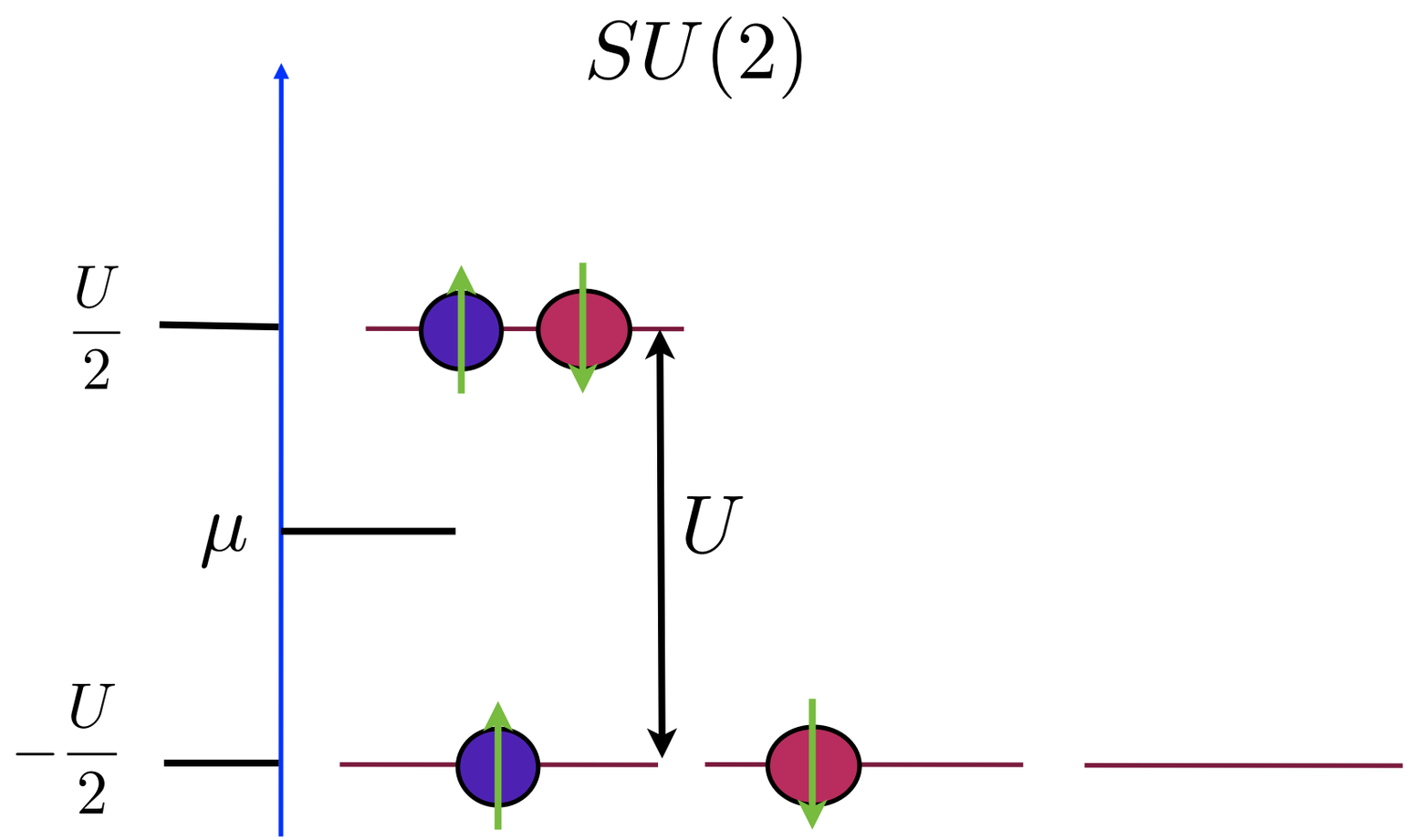
simple problem: n=1



simple problem: n=1

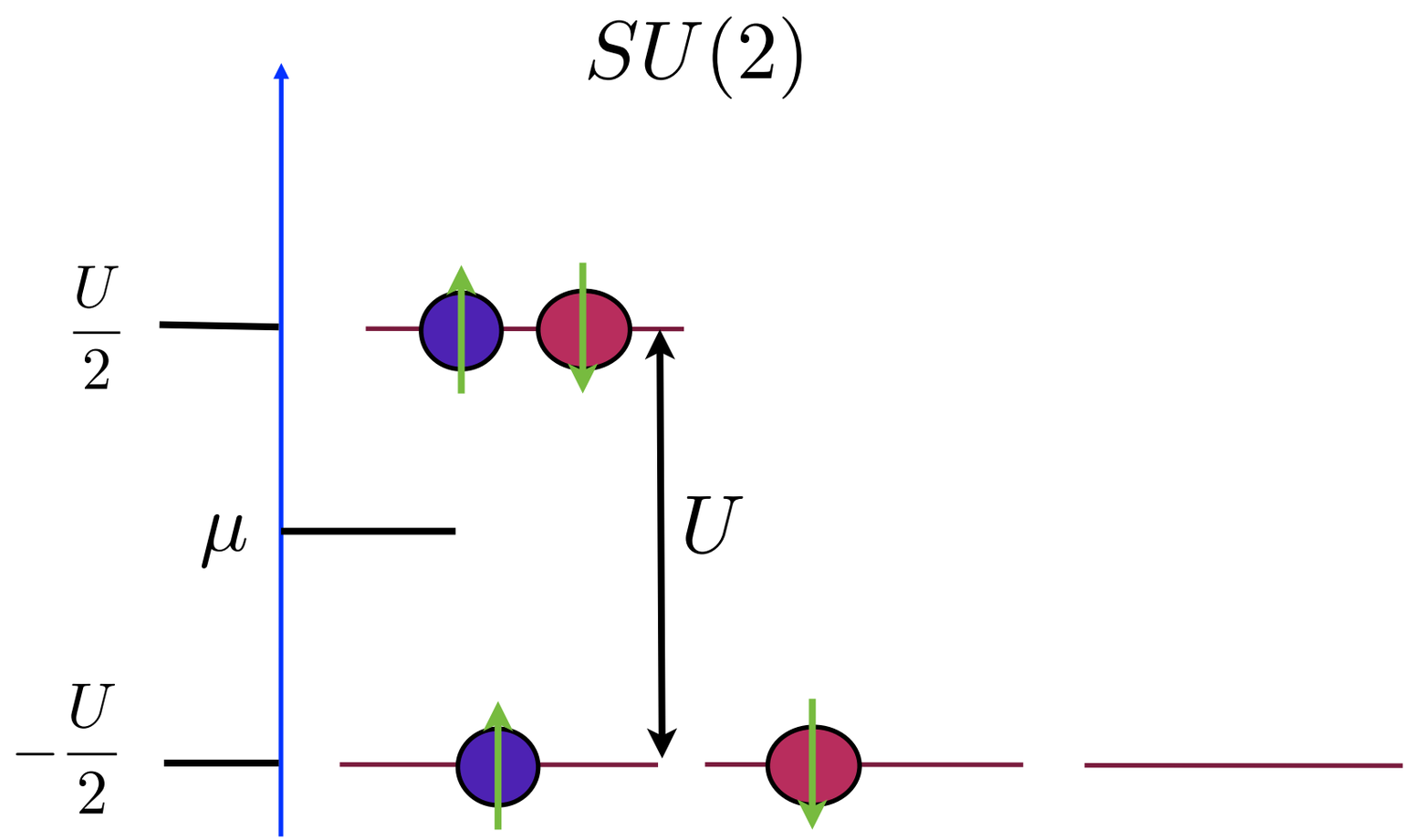


simple problem: n=1



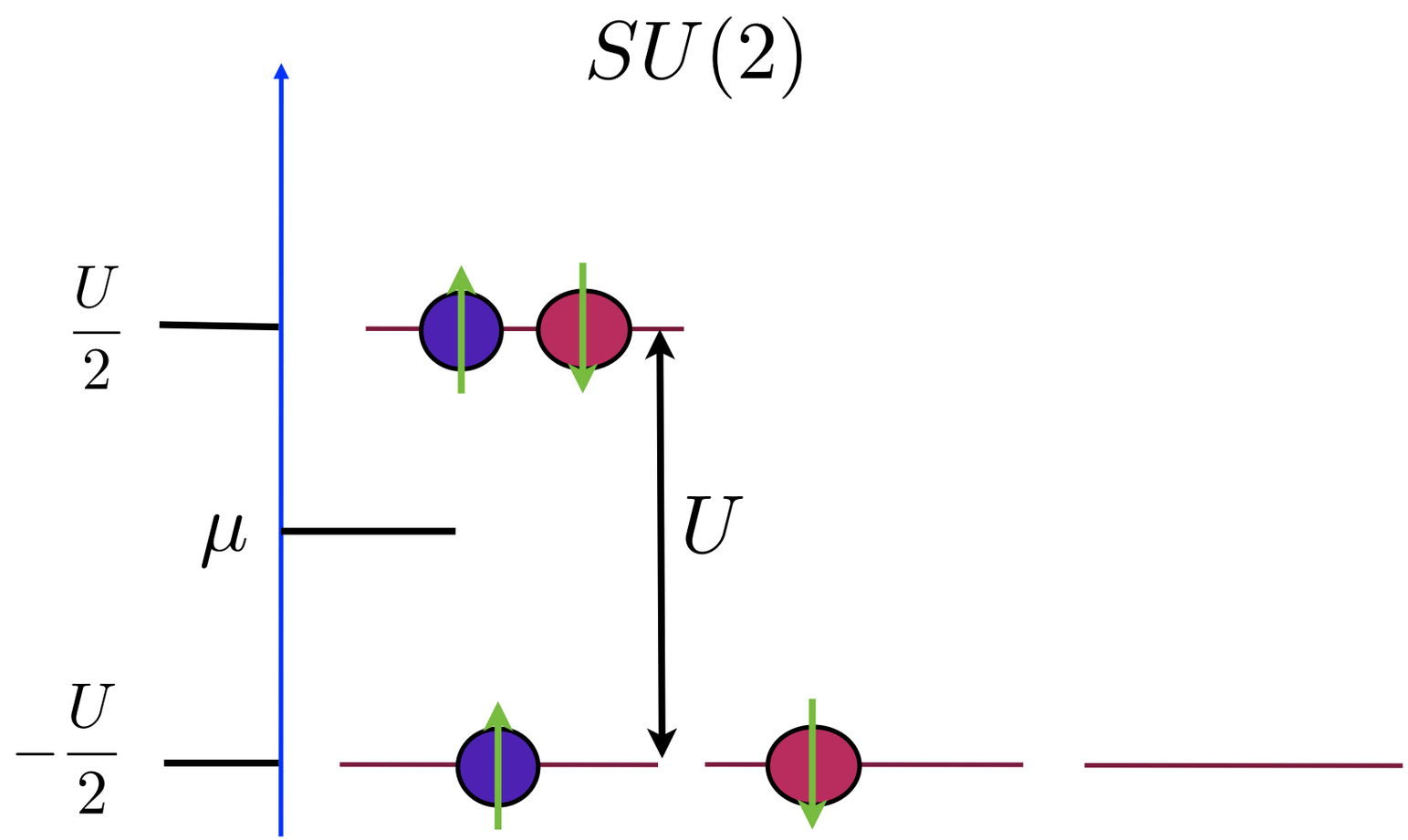
$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2}$$

simple problem: n=1



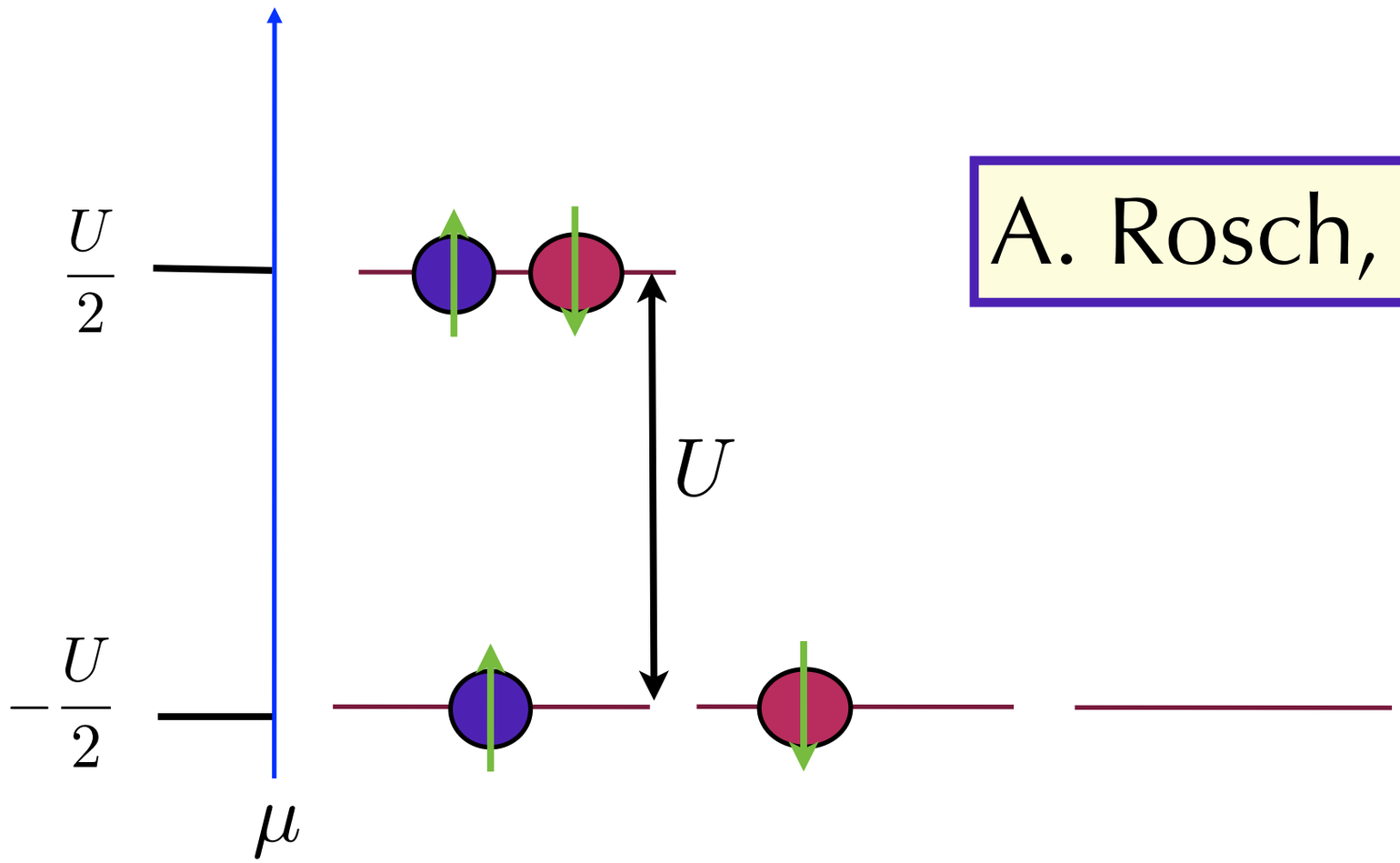
$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$

simple problem: n=1



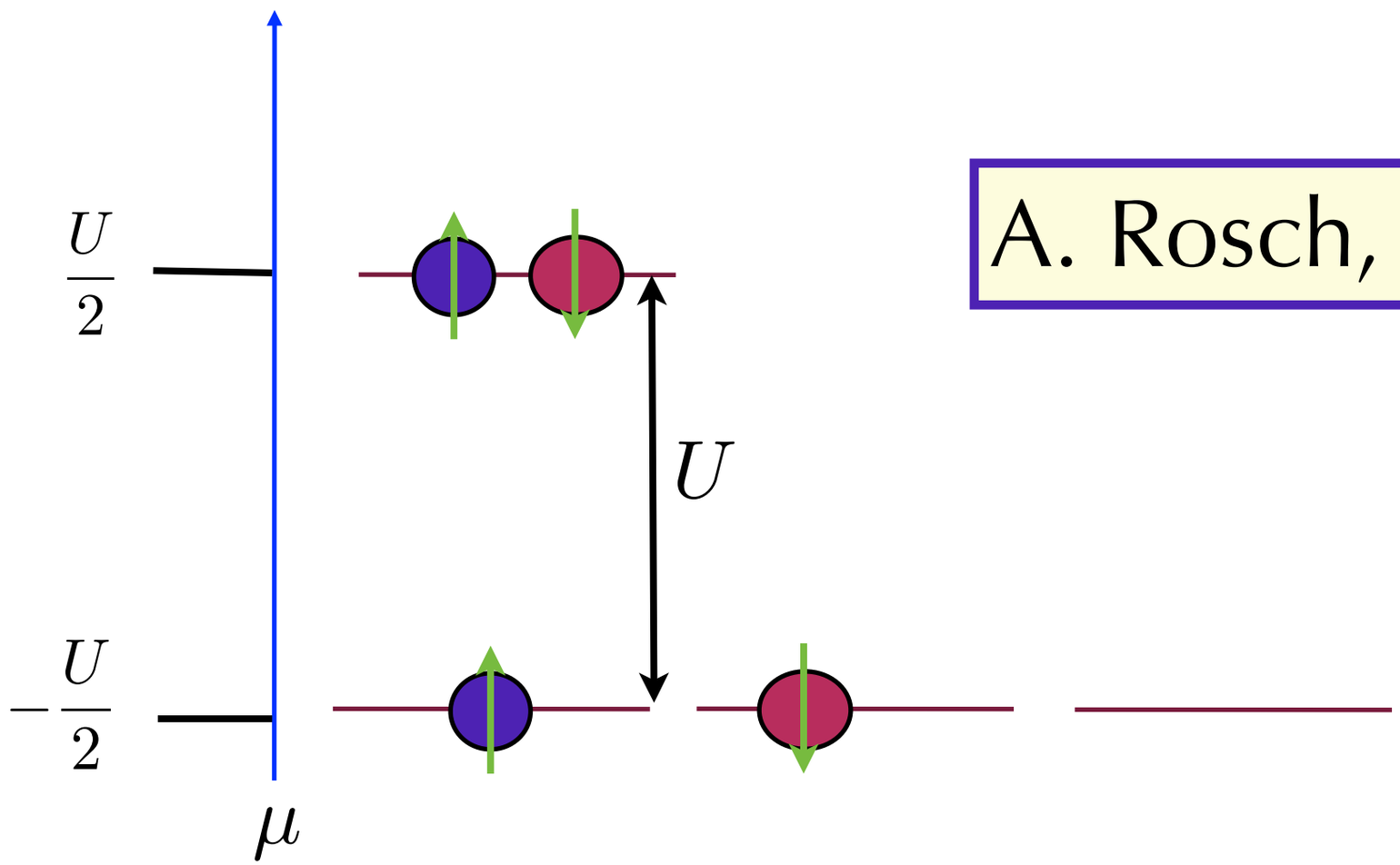
$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$

$$n = 2\theta(0) = 1$$



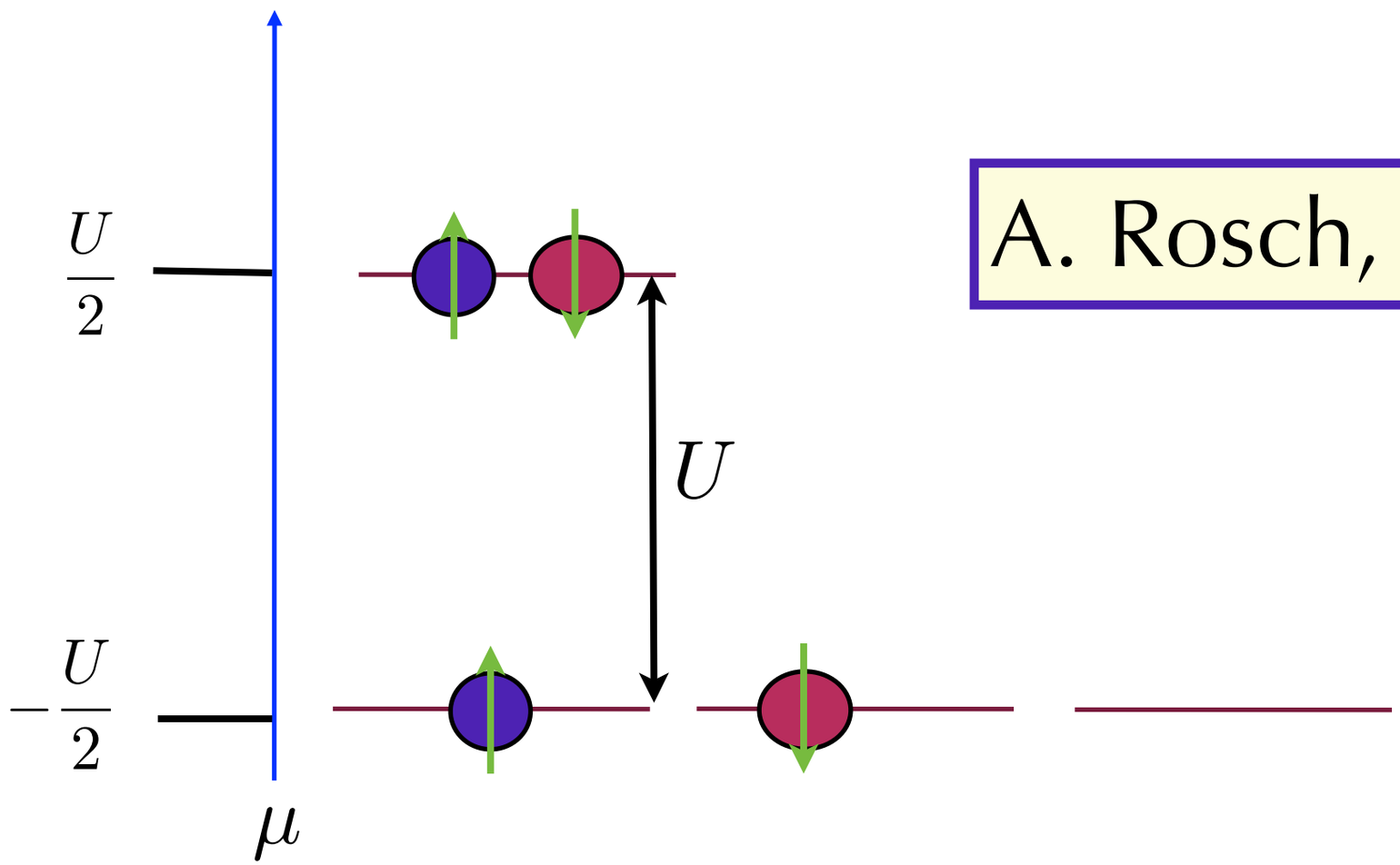
A. Rosch, 2007

$$G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



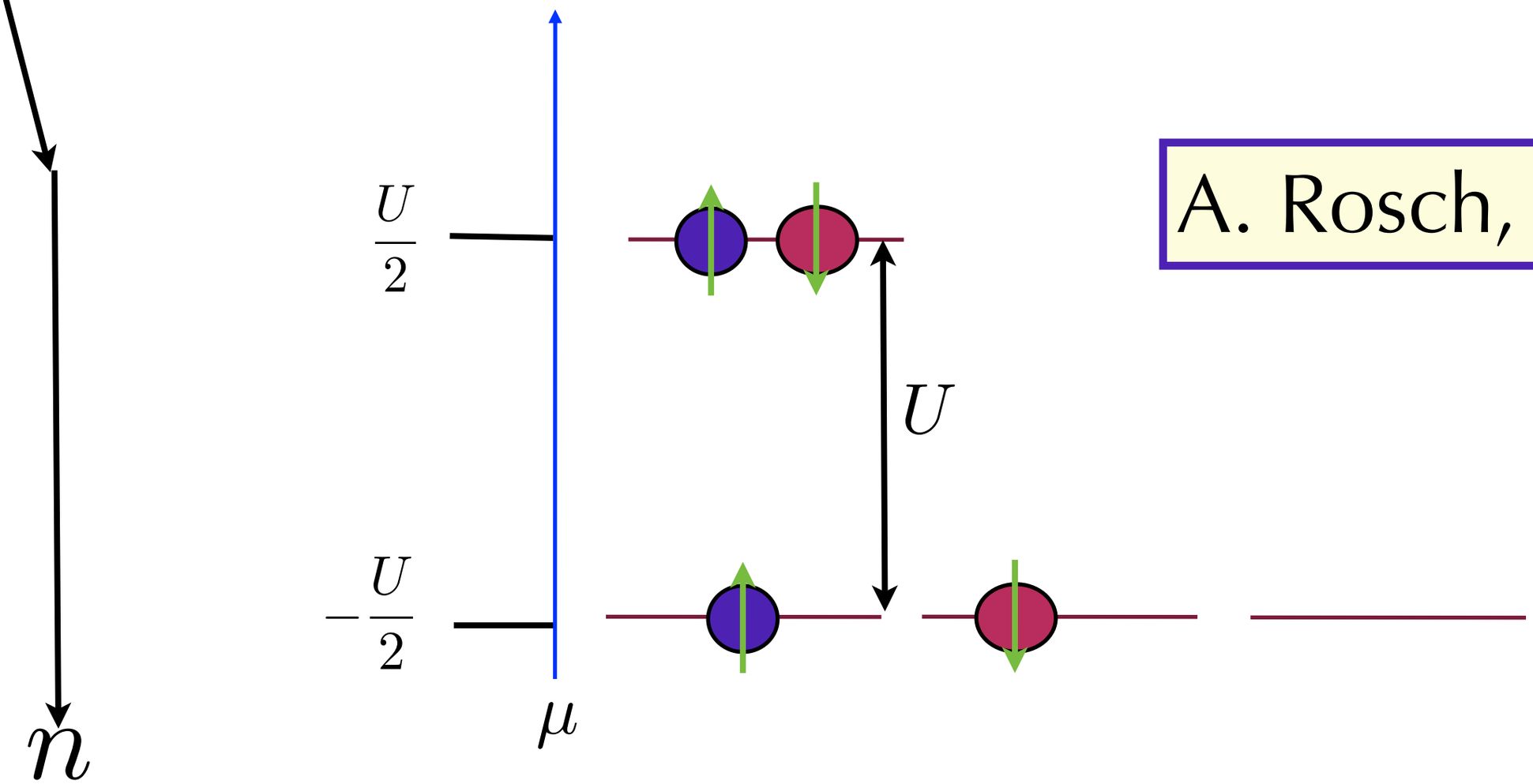
A. Rosch, 2007

$$n = 2\theta\left(\frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}\right) \Leftarrow G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



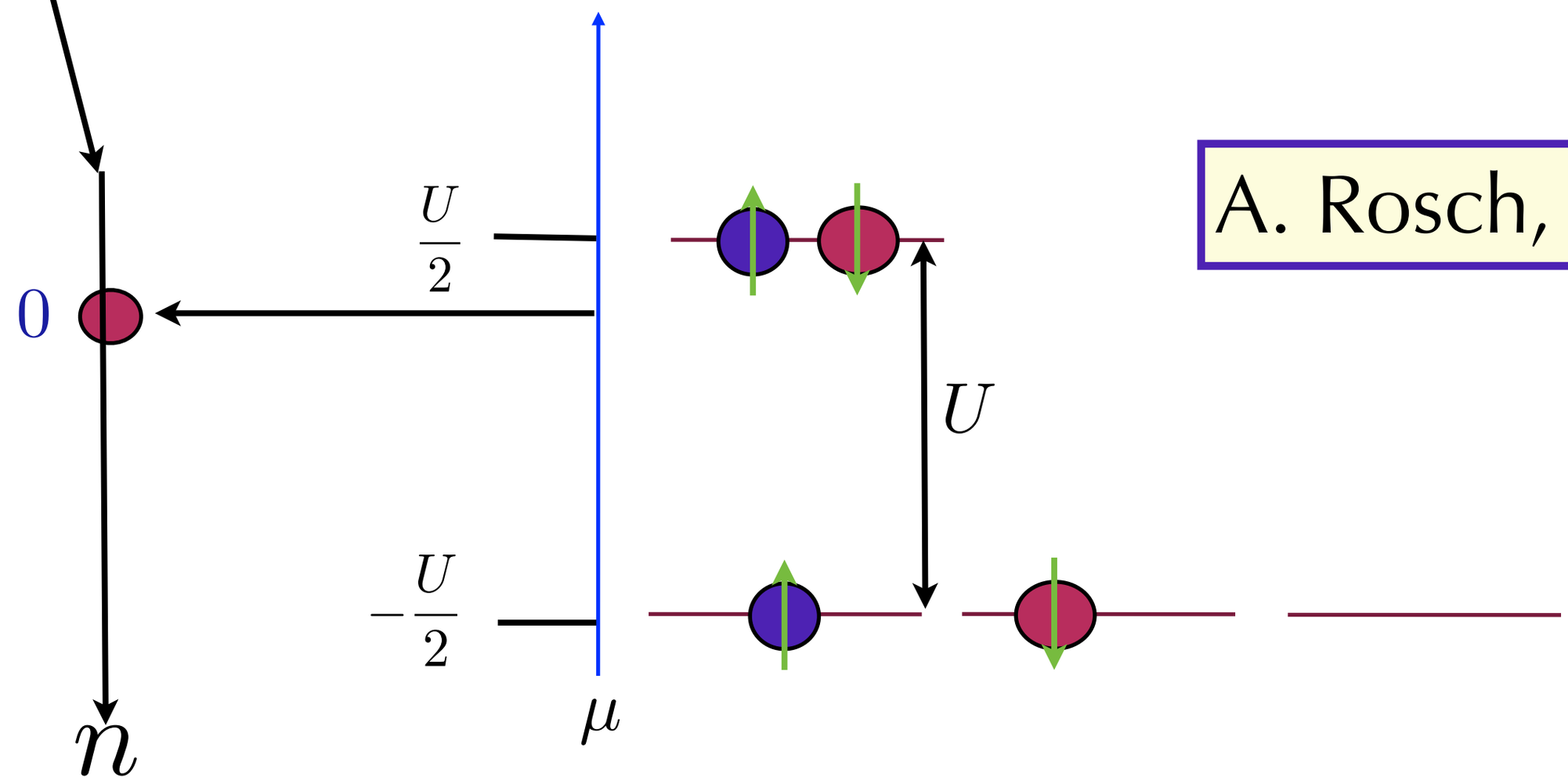
A. Rosch, 2007

$$n = 2\theta\left(\frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}\right) \Longleftarrow G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



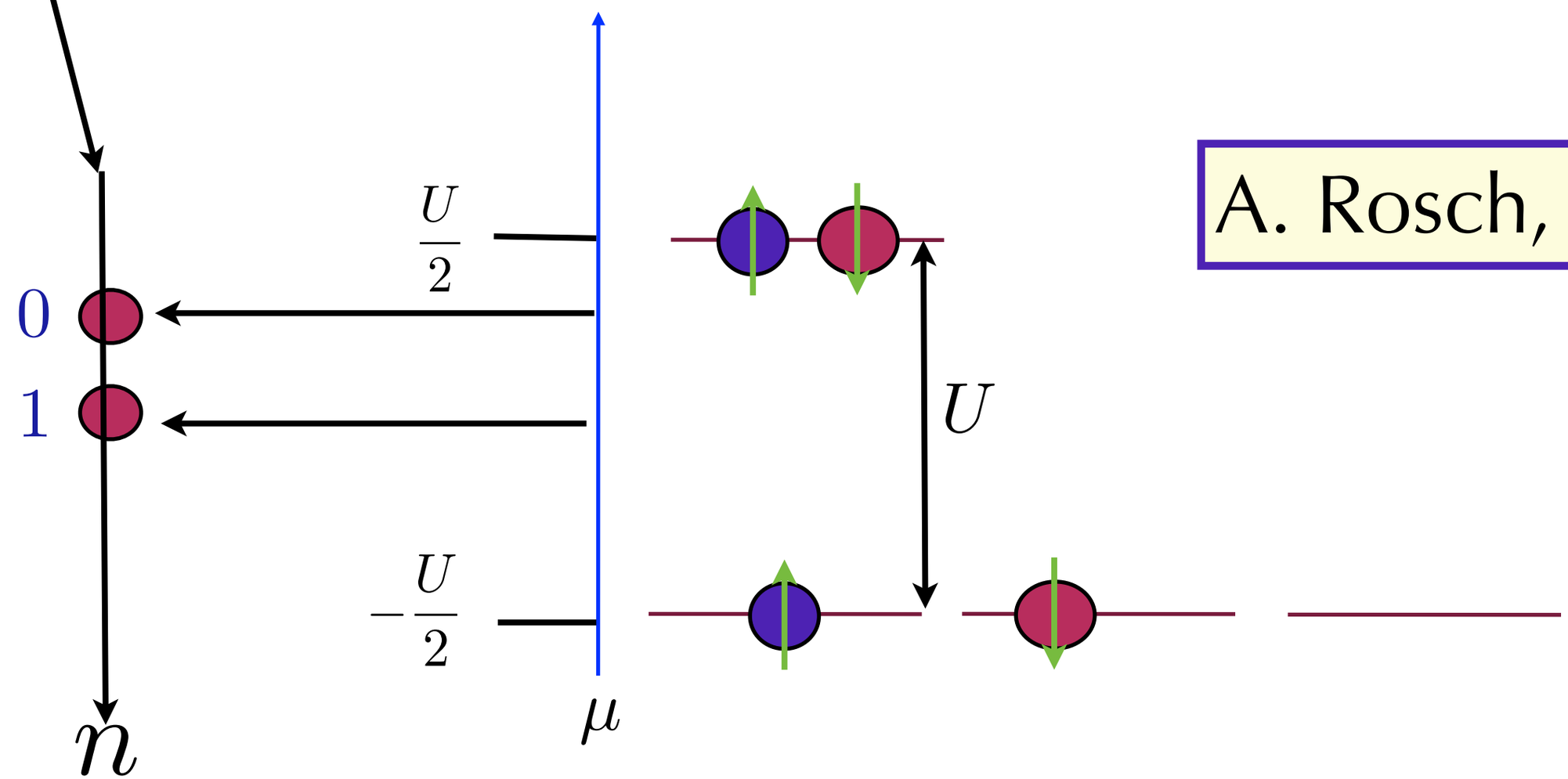
A. Rosch, 2007

$$n = 2\theta\left(\frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}\right) \Longleftarrow G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



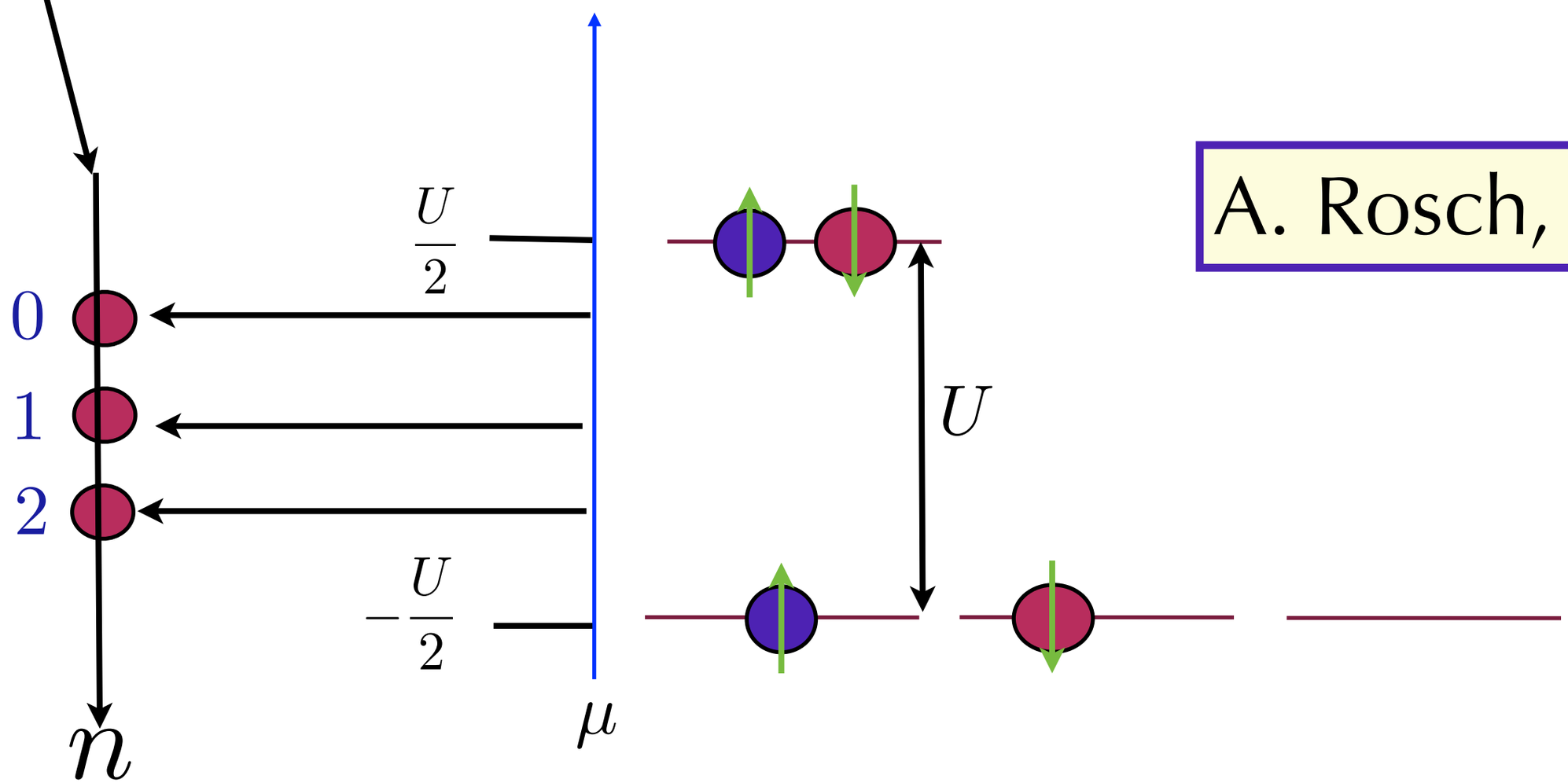
A. Rosch, 2007

$$n = 2\theta\left(\frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}\right) \Leftarrow G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



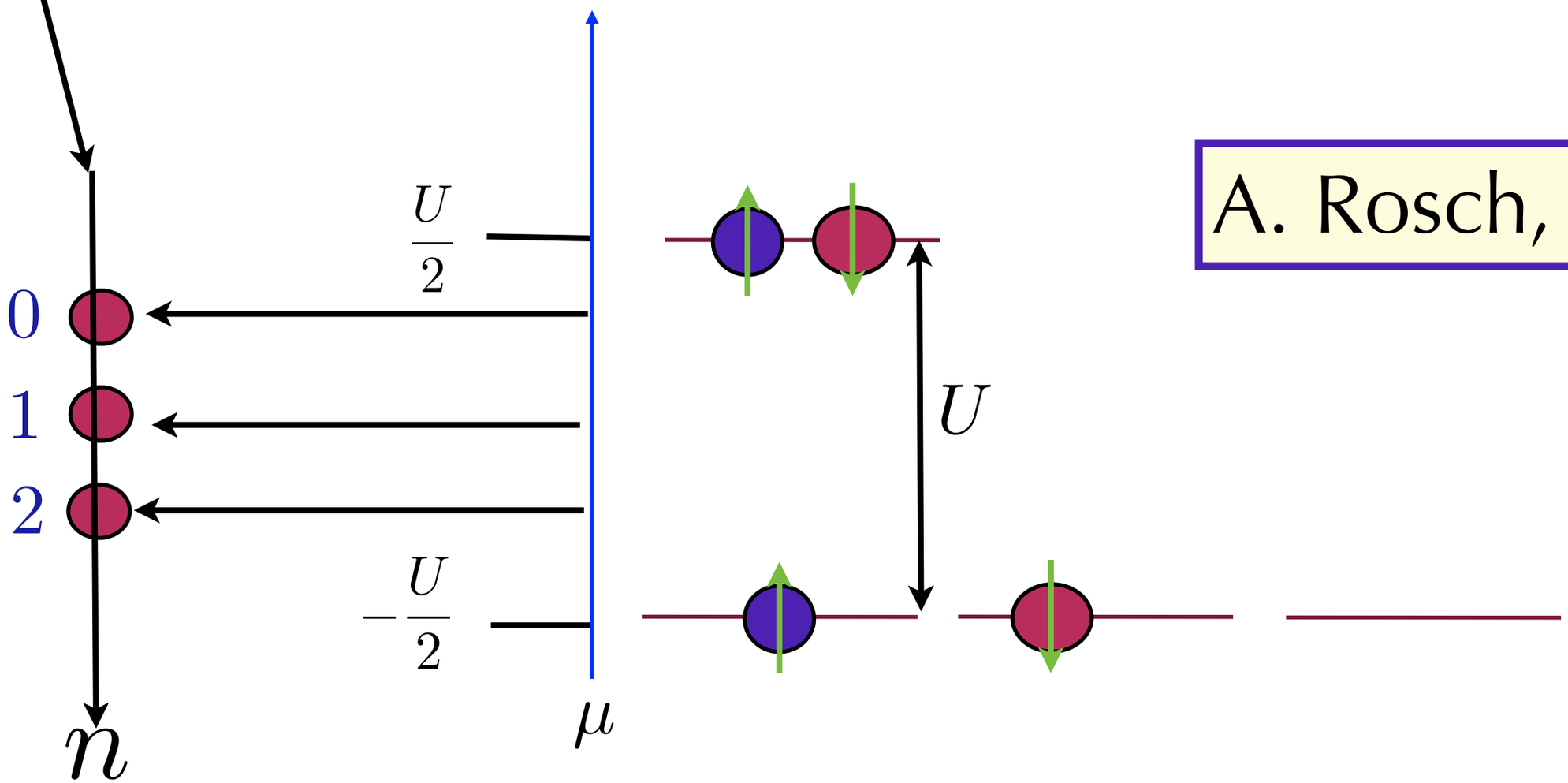
A. Rosch, 2007

$$n = 2\theta\left(\frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}\right) \Leftarrow G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



A. Rosch, 2007

$$n = 2\theta\left(\frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}\right) \Longleftarrow G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



A. Rosch, 2007

incompressible but $\frac{\partial n}{\partial \mu} \neq 0$

fix chemical potential

$$\lim_{T \to 0} \mu(T)$$

fix chemical potential

$$\lim_{T \to 0} \mu(T)$$

n=1

fix chemical potential

$$\lim_{T \to 0} \mu(T)$$

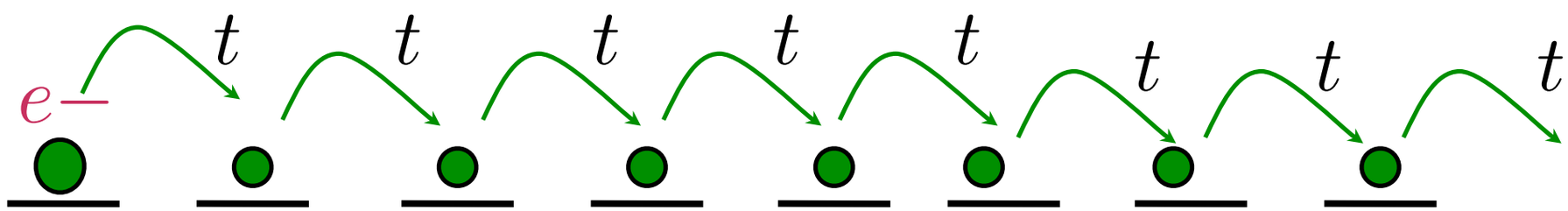
n=1

does this fix all the problems?

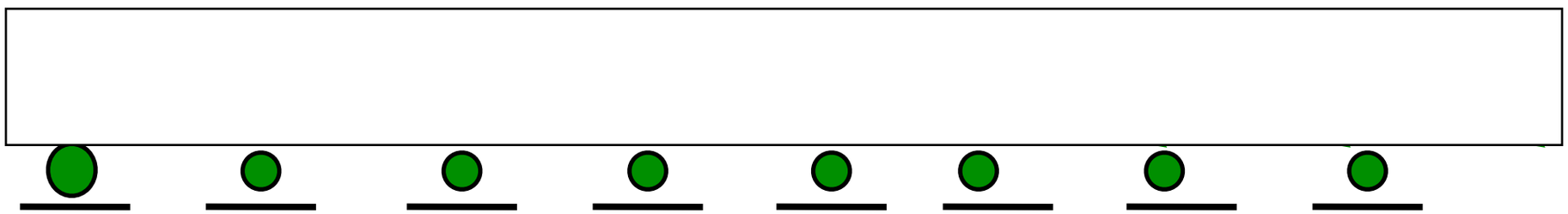
No

A model with zeros but Luttinger fails

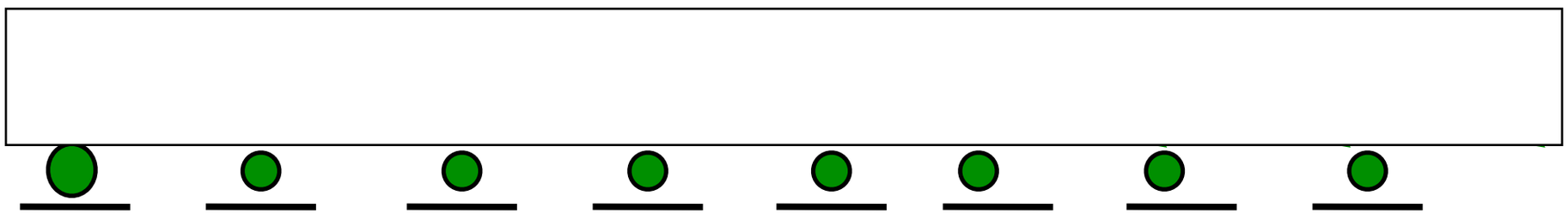
A model with zeros but Luttinger fails



A model with zeros but Luttinger fails

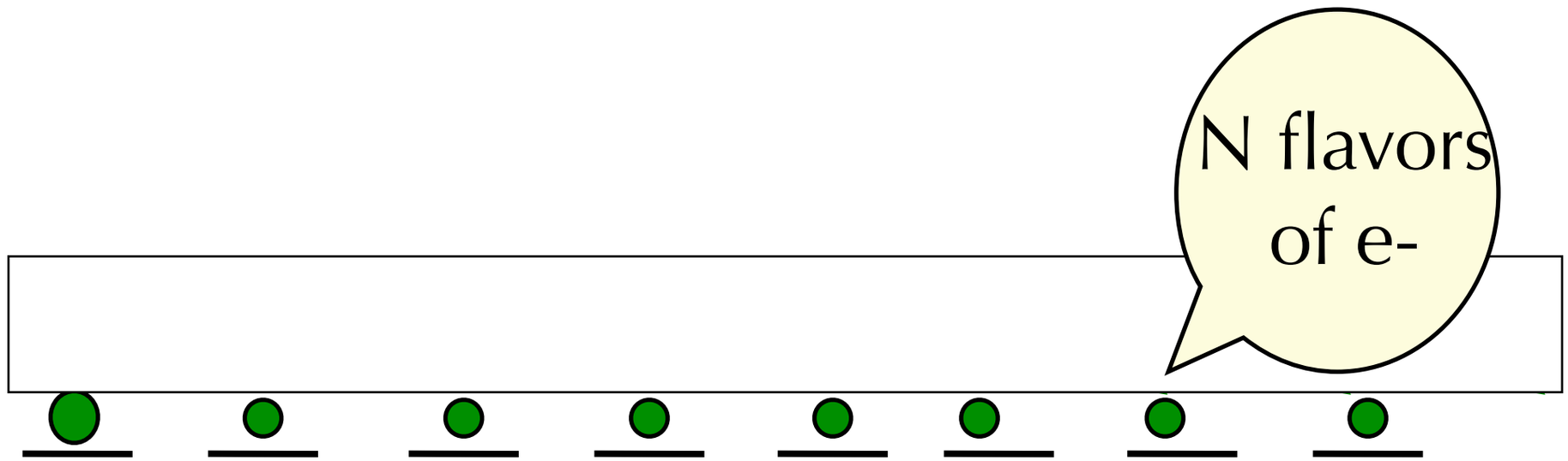


A model with zeros but Luttinger fails



no hopping=> no propagation (zeros)

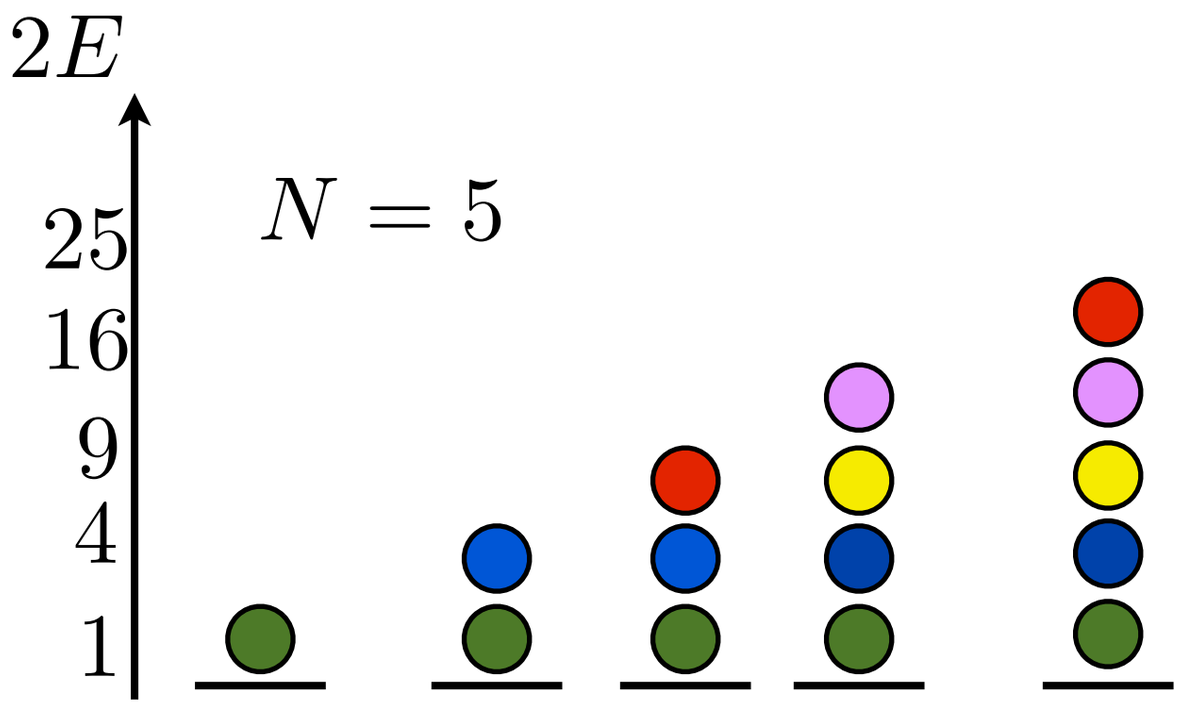
A model with zeros but Luttinger fails



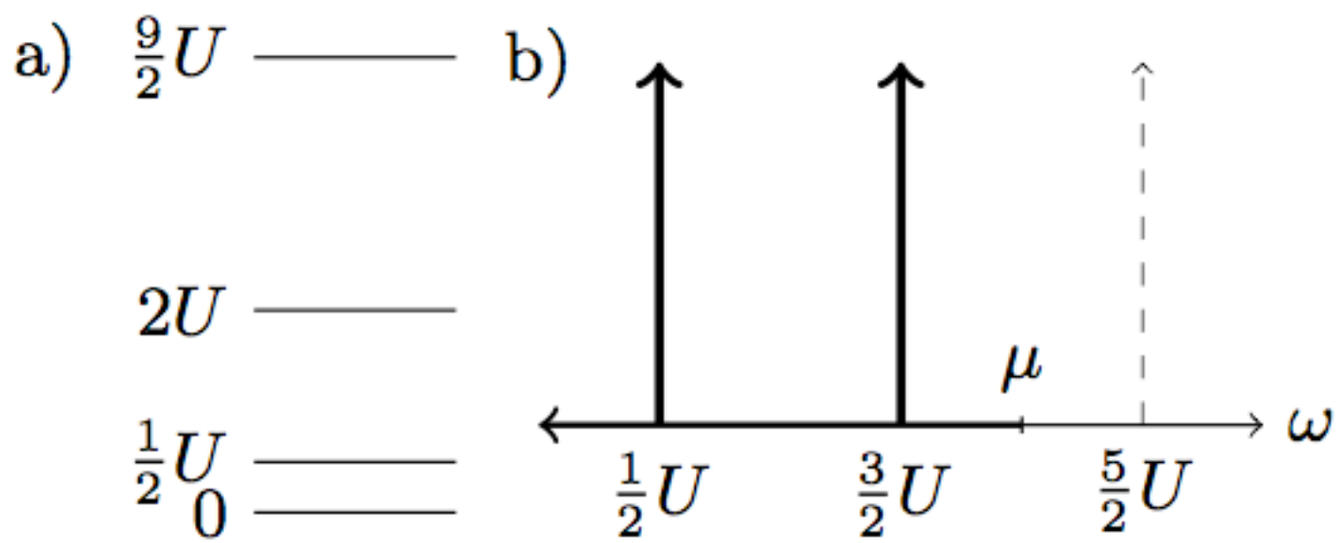
no hopping=> no propagation (zeros)

$$SU(N)$$

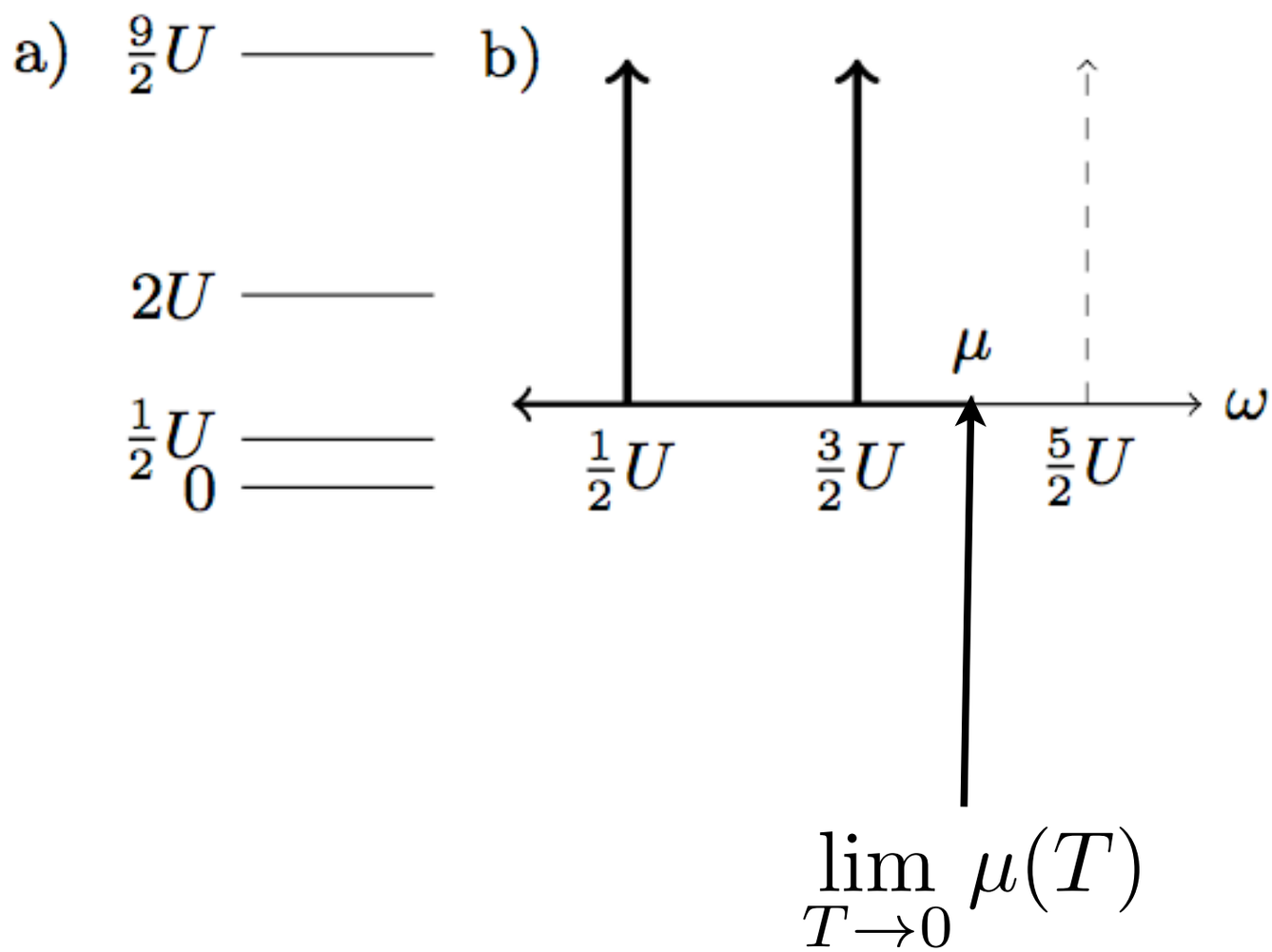
$$H = \frac{U}{2}(n_1 + \cdots n_N)^2$$



no particle-hole symmetry



no particle-hole symmetry



$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n+1) - K(n)} \left(\frac{2n - N}{N} \right)$$

$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{\underbrace{K(n+1) - K(n)}_{> 0}} \left(\frac{2n - N}{N} \right)$$

Luttinger's theorem

$$n = N\Theta(2n - N)$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

$n = 2$

$N = 3$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

$$n = 2$$

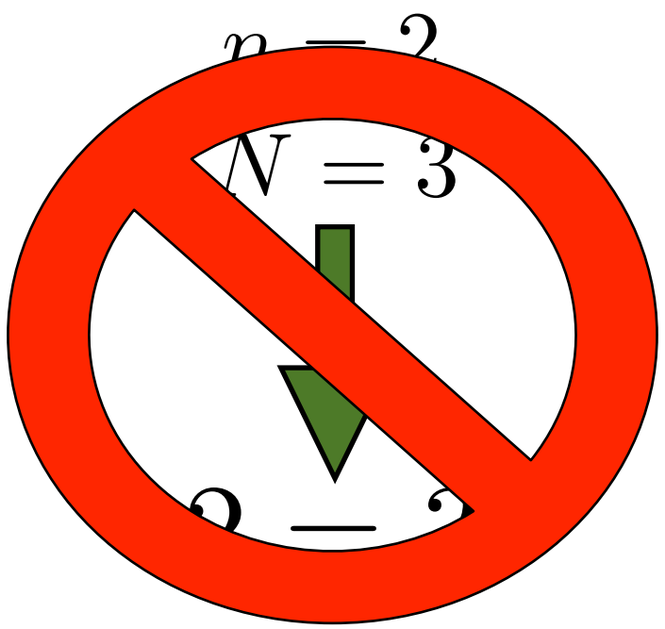
$$N = 3$$

$$\Downarrow$$

$$2 = 3$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$



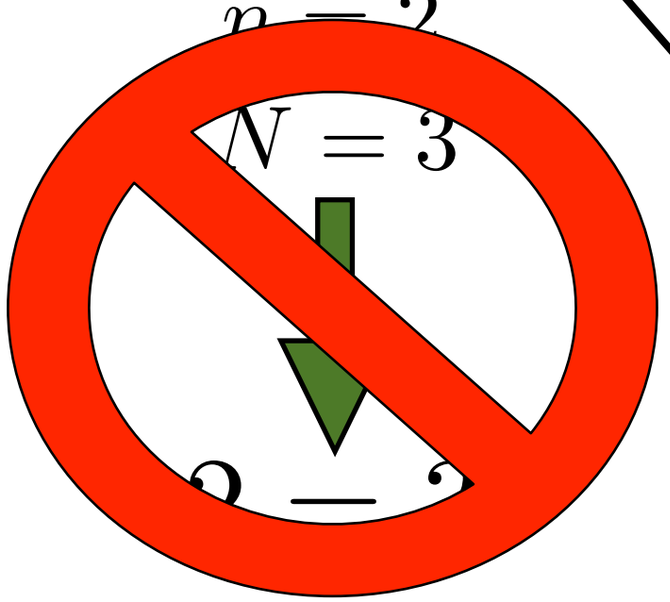
Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

even

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

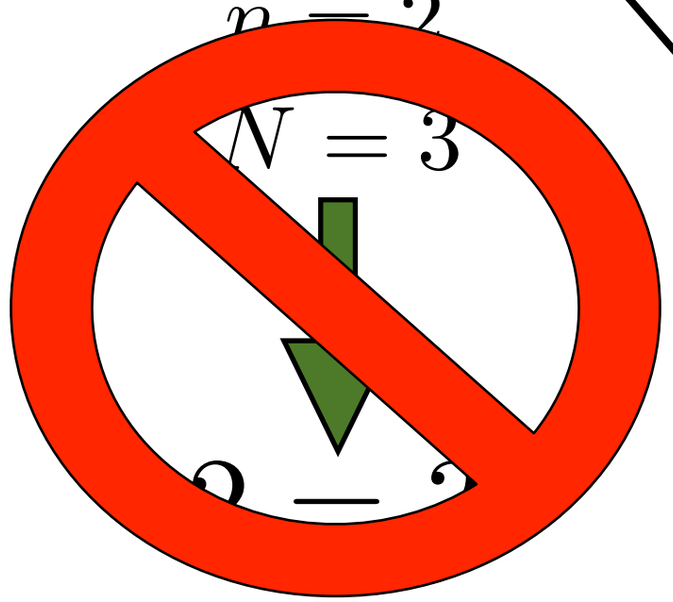


even

odd

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,\,1,\,1/2}$$

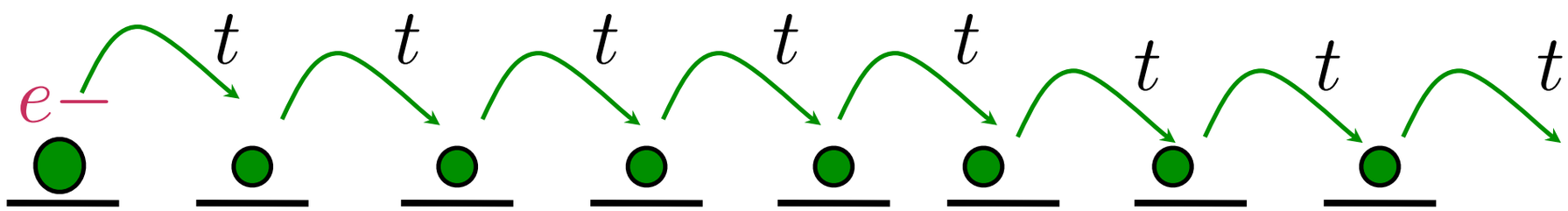


even

odd

no solution

does the degeneracy matter?



$$t = 0^+$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \frac{1}{\omega - H} c_b + c_b \frac{1}{\omega - H} c_a^\dagger\right] \rho(0^+)\right)$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \frac{1}{\omega - H} c_a^\dagger\right] \rho(0^+)\right)$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$G_{ab}(\omega) = \frac{\omega\delta_{ab} - U\rho_{ab}}{\omega(\omega - U)}$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$\rho_{ab} = \mathrm{Tr}\left(c_a^\dagger c_b \rho(0^+)\right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle$$

$$G_{ab}(\omega) = \frac{\omega\delta_{ab} - U\rho_{ab}}{\omega(\omega - U)}$$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$\rho_{ab} = \mathrm{Tr}\left(c_a^\dagger c_b \rho(0^+)\right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle$$

$$G_{ab}(\omega) = \frac{\omega\delta_{ab} - U\rho_{ab}}{\omega(\omega - U)}$$

$$1/N\mathrm{diag}(1, 1, 1, \cdots)$$

mixed state

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$\rho_{ab} = \mathrm{Tr}\left(c_a^\dagger c_b \rho(0^+)\right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle$$

$$G_{ab}(\omega) = \frac{\omega\delta_{ab} - U\rho_{ab}}{\omega(\omega - U)}$$

$$1/N\mathrm{diag}(1, 1, 1, \cdots)$$

mixed state

as long as SU(N) symmetry is intact

zeros in the wrong place

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$\rho_{ab} = \mathrm{Tr}\left(c_a^\dagger c_b \rho(0^+)\right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle$$

$$G_{ab}(\omega) = \frac{\omega\delta_{ab} - U\rho_{ab}}{\omega(\omega - U)}$$

$$1/N\mathrm{diag}(1, 1, 1, \cdots)$$

mixed state

as long as SU(N) symmetry is intact

zeros in the wrong place

limiting procedure: $\lim_{t\to 0} \lim_{\omega\to 0}$

$$G_{ab}(\omega) = \mathrm{Tr}\left(\left[c_a^\dagger \boxed{\frac{1}{\omega}} c_b + c_b \boxed{\frac{1}{\omega - U}} c_a^\dagger\right] \rho(0^+)\right)$$

$$\rho_{ab} = \mathrm{Tr}\left(c_a^\dagger c_b \rho(0^+)\right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle$$

$$G_{ab}(\omega) = \frac{\omega\delta_{ab} - U\rho_{ab}}{\omega(\omega - U)}$$

$$1/N\mathrm{diag}(1, 1, 1, \cdots)$$

mixed state

as long as SU(N) symmetry is intact

zeros in the wrong place

limiting procedure: $\lim_{t\to 0} \lim_{\omega\to 0}$

limits do not commute

Problem

G=0

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p - \Sigma} \rightarrow \infty$$

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p - \Sigma} \rightarrow \infty$$

lifetime of a particle vanishes

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p - \Sigma}$$

∞

$\Im\Sigma < \epsilon_p$

lifetime of a particle vanishes

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p - \Sigma}$$

$\Sigma \to \infty$

$\Im\Sigma < \epsilon_p$

lifetime of a particle vanishes

no particle

what went wrong?

what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$

what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$

$$\Big\Downarrow \text{ if } \Sigma \to \infty$$

what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$

$$\Big\Downarrow \text{ if } \Sigma \to \infty$$

integral does not exist

what went wrong?

$$\delta I[G] = \int d\omega \Sigma \delta G$$

if $\Sigma \to \infty$

integral does not exist

No Luttinger theorem!

Luttinger's theorem

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

only true for poles

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

only true for poles

Gaussian fixed point

$$n = 2\sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

tautology

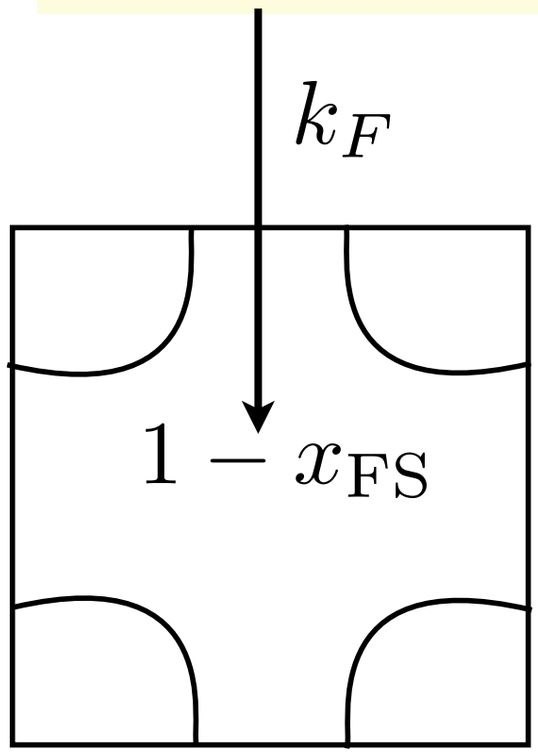
only true for poles

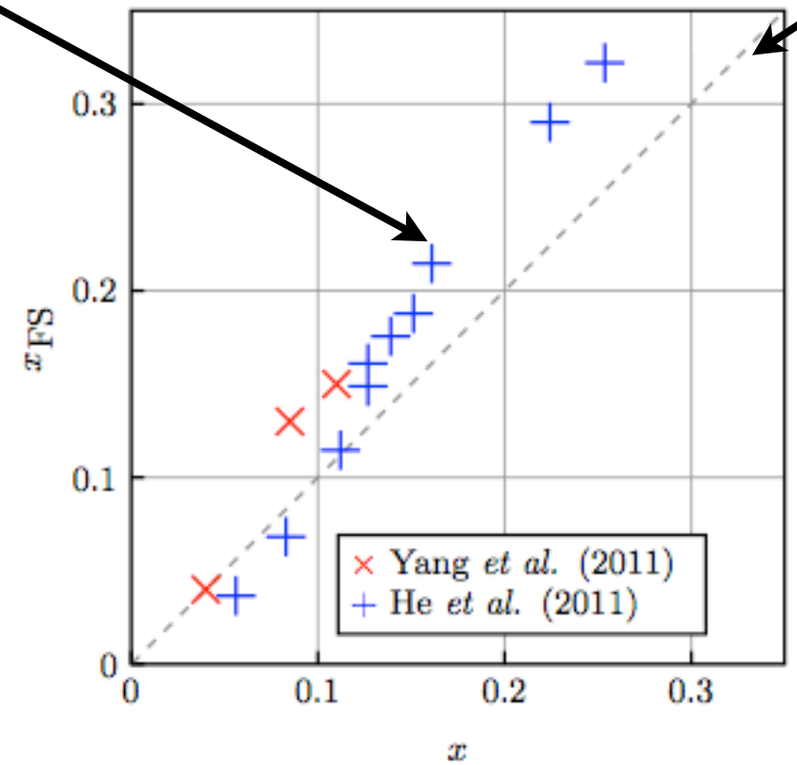
Gaussian fixed point

experimental confirmation of violation?

experimental data (LSCO)

`Luttinger' count

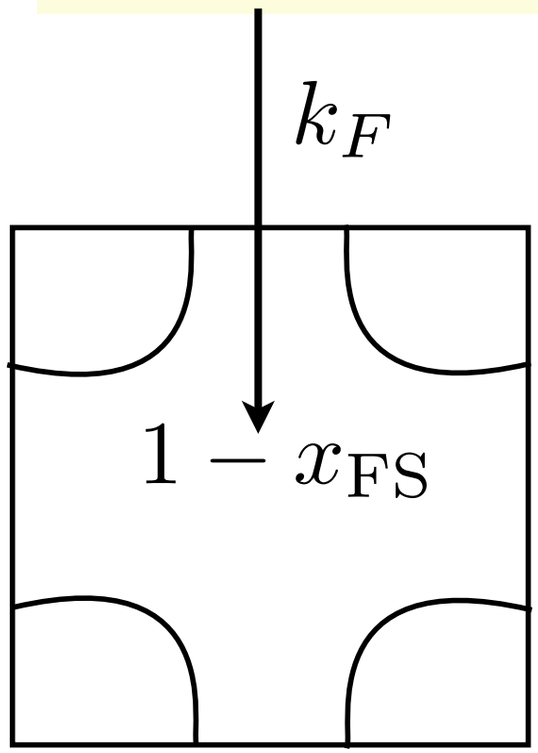


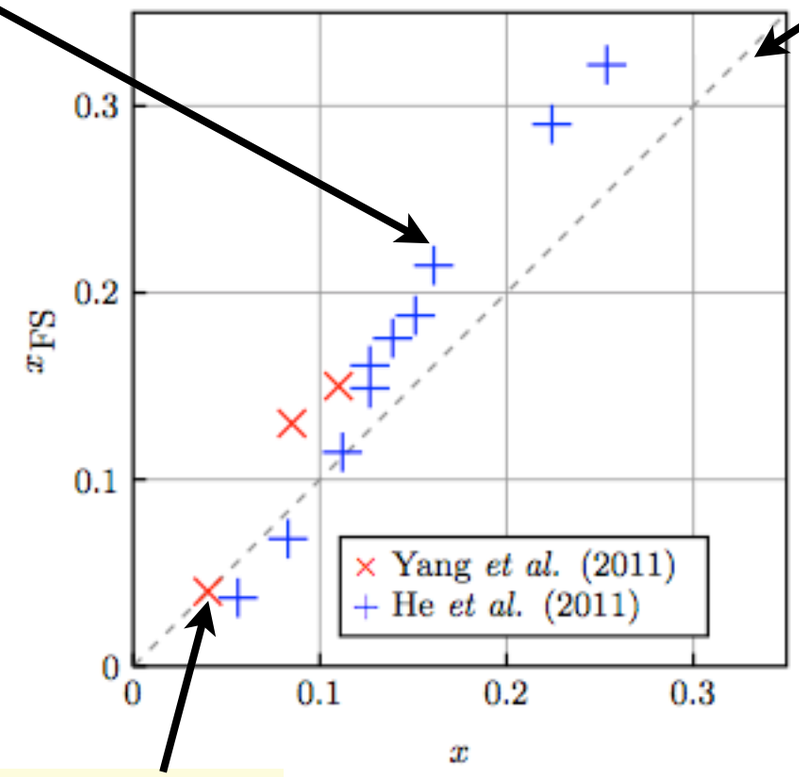


zeros do not
affect the particle density

experimental data (LSCO)

`Luttinger' count



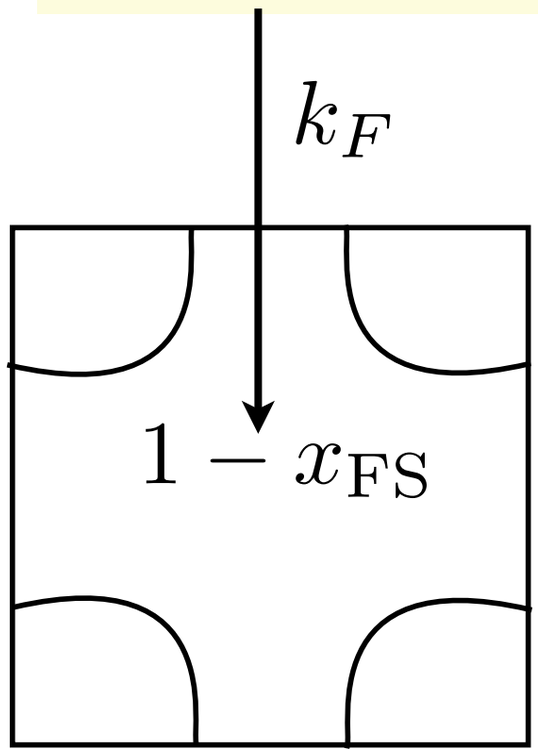


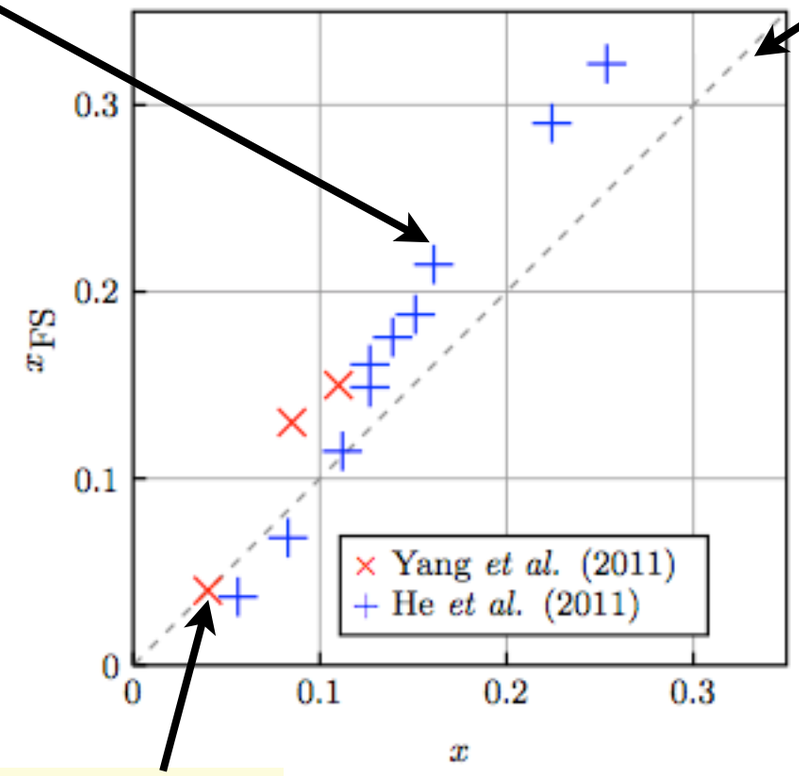
Bi2212

zeros do not affect the particle density

experimental data (LSCO)

`Luttinger' count



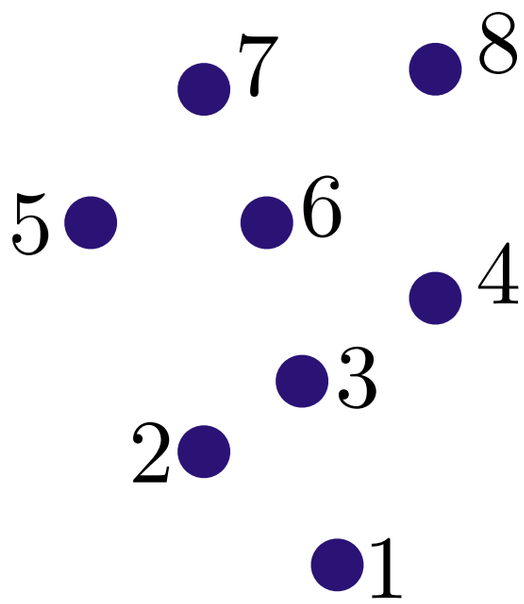


Bi2212

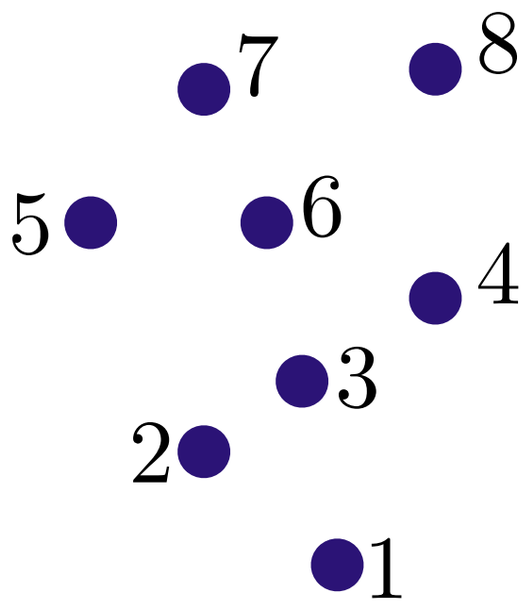
zeros do not
affect the particle density

each hole $\neq$ a single k-state

how to count particles?

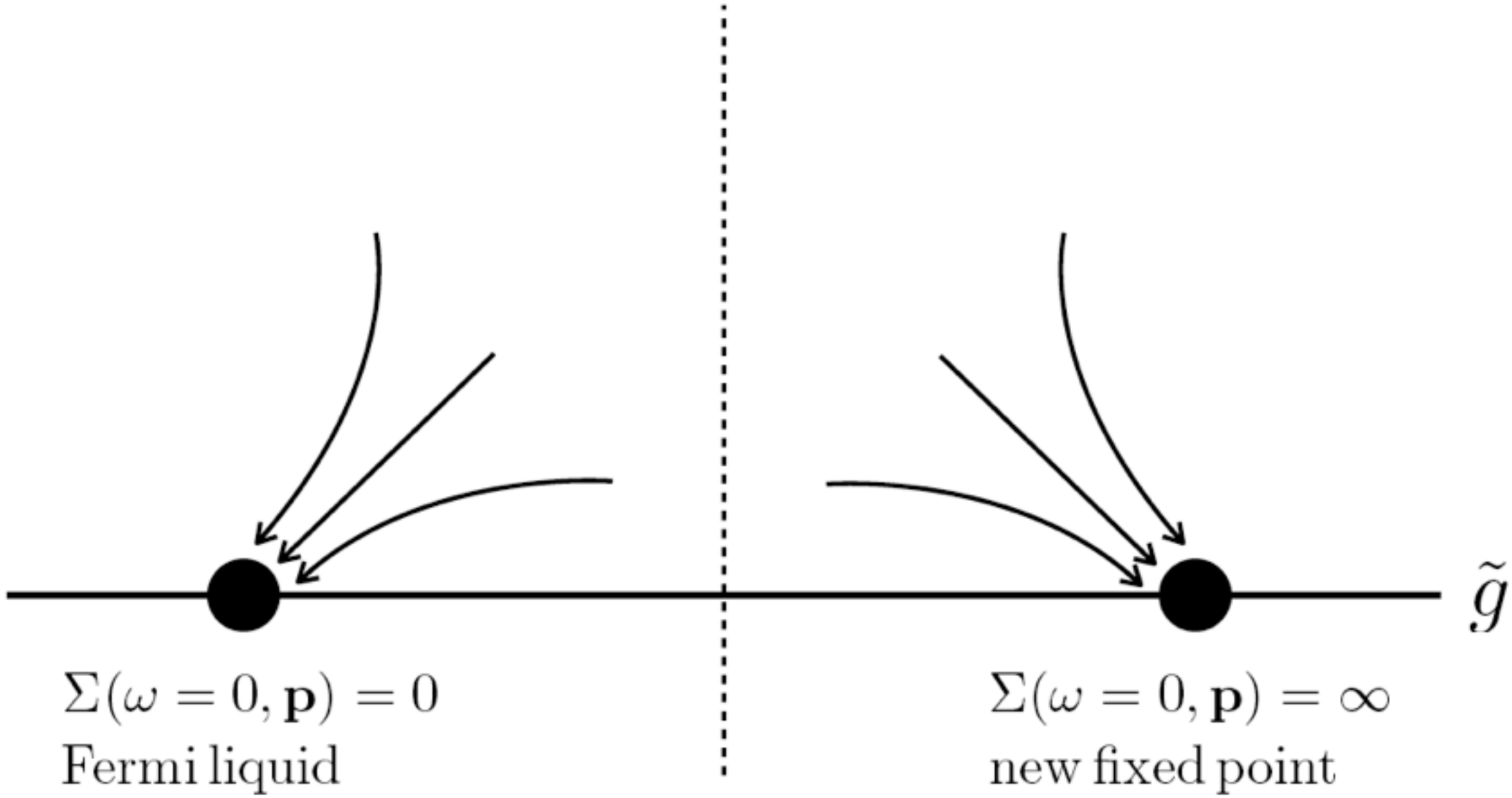


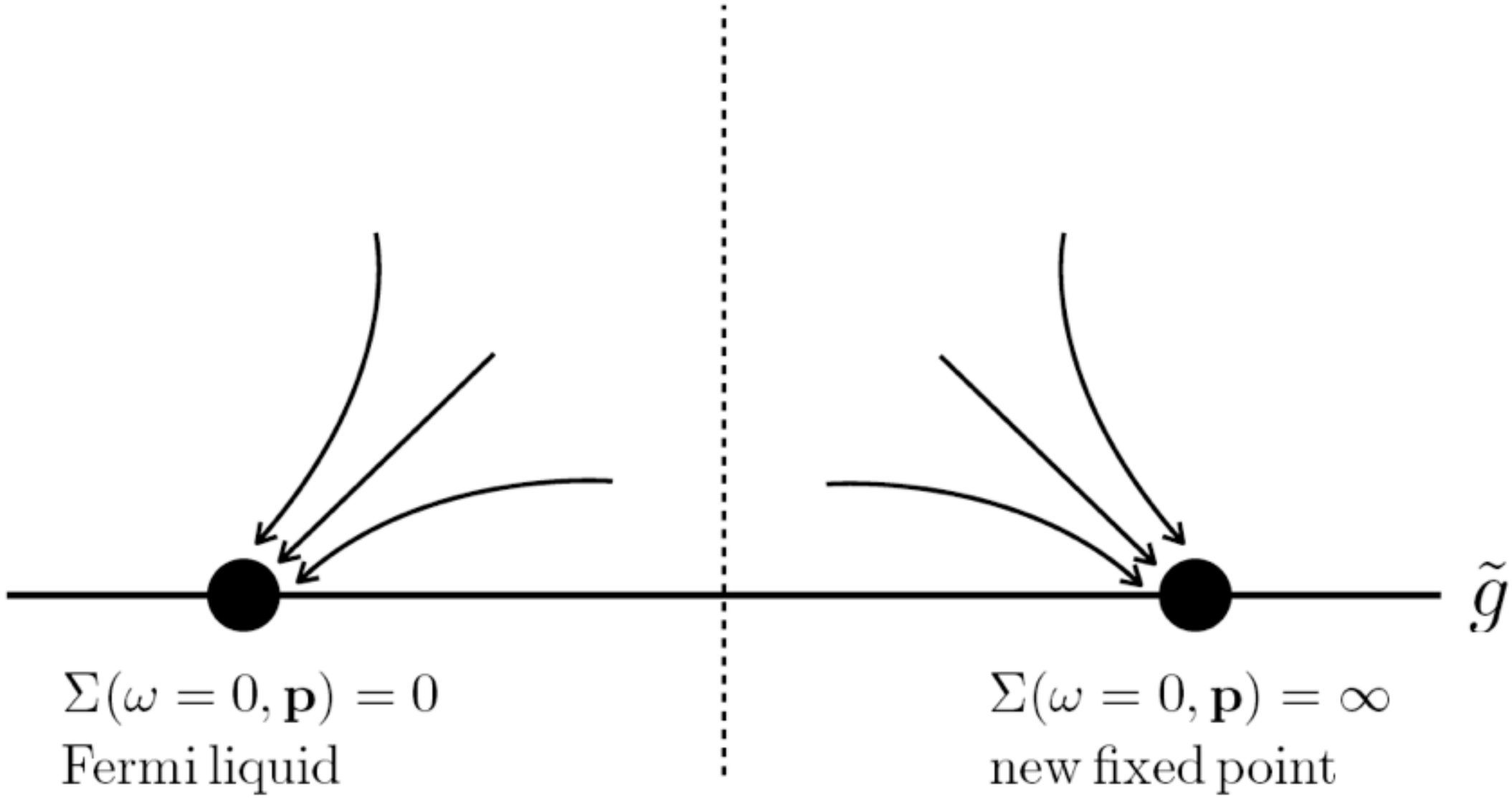
how to count particles?



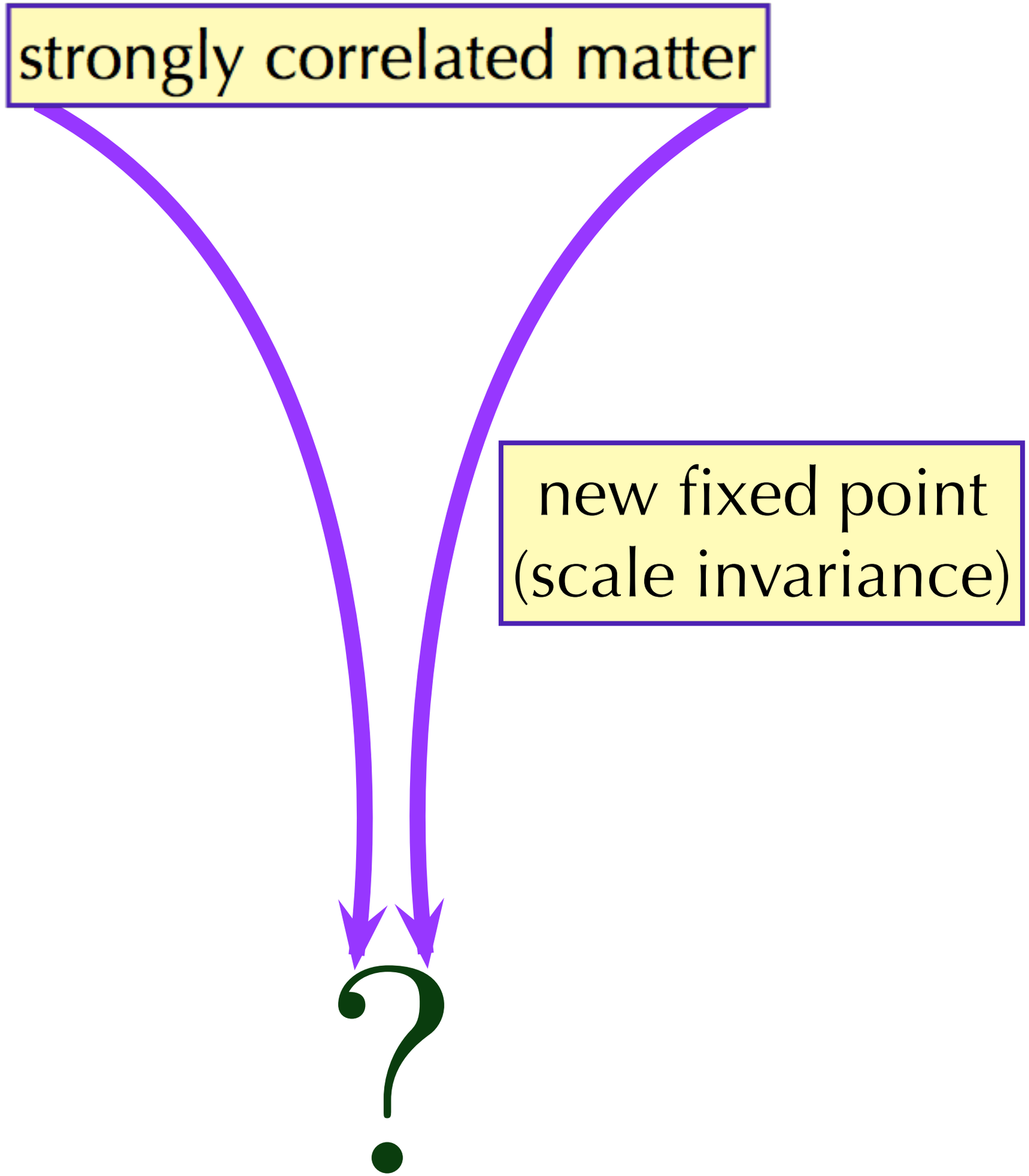
some charged stuff
has no particle interpretation

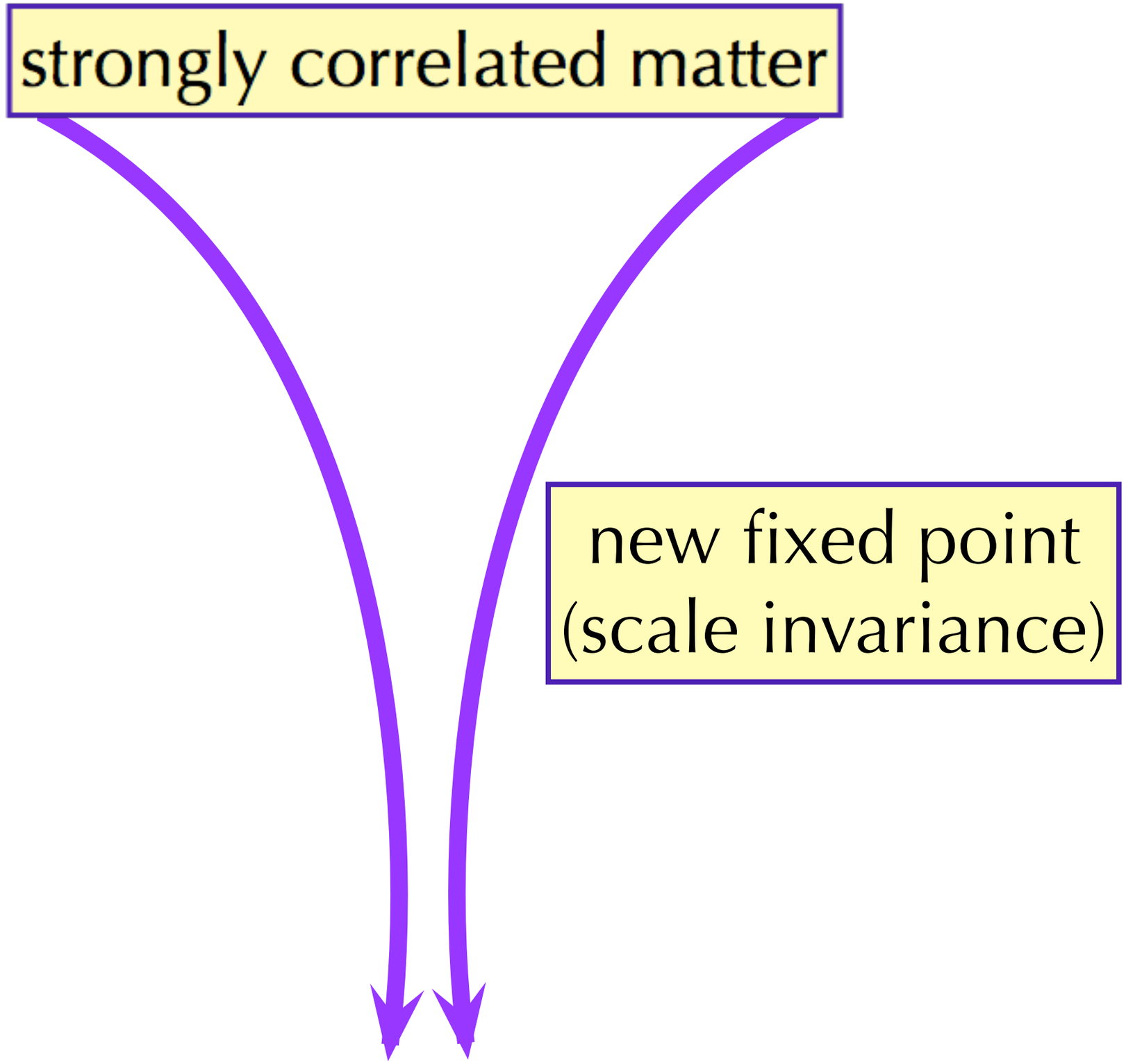
what is the extra stuff?

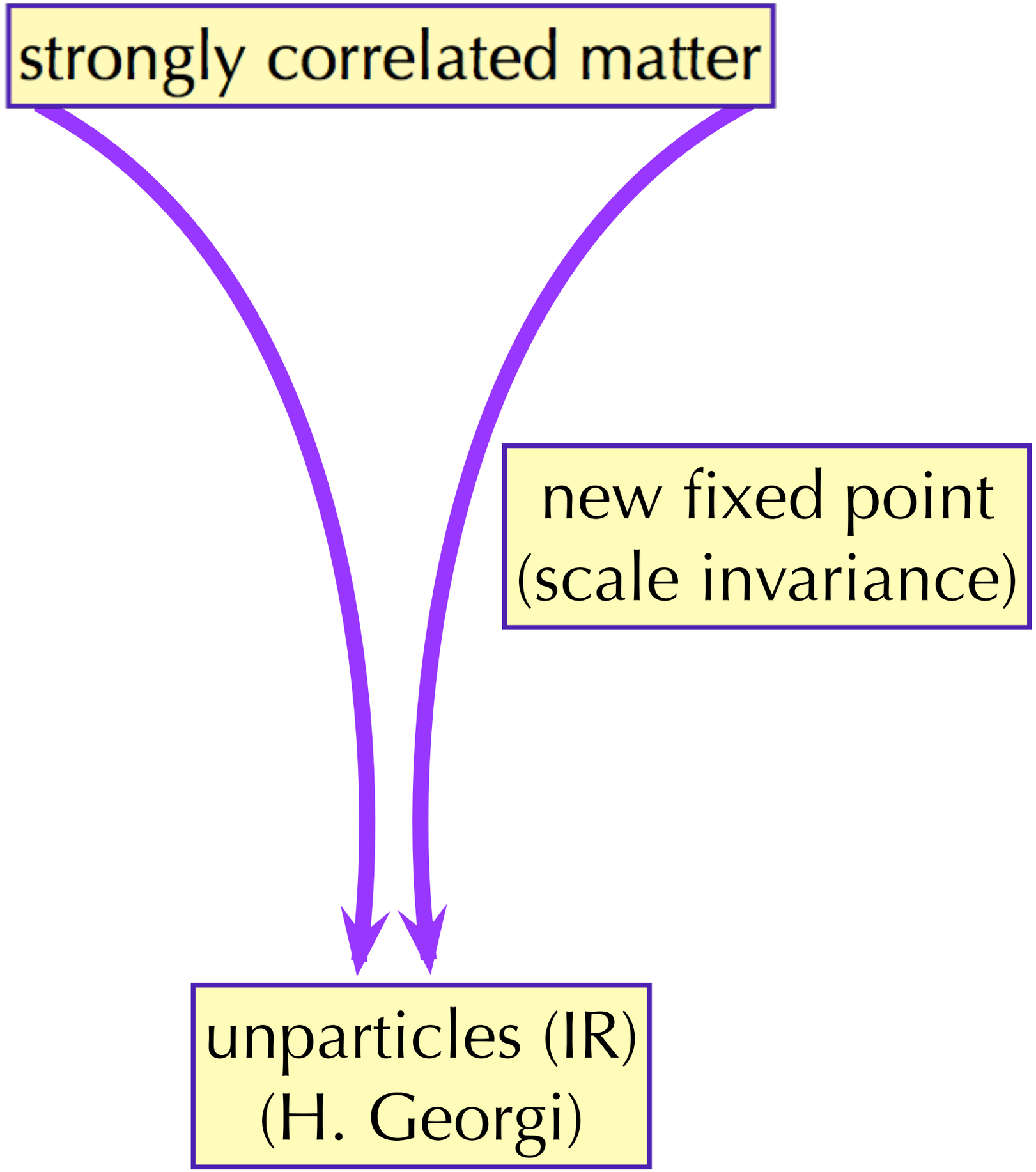

$\tilde{g}$
$\Sigma(\omega = 0, \mathbf{p}) = 0$
Fermi liquid
$\Sigma(\omega = 0, \mathbf{p}) = \infty$
new fixed point



scale invariance


strongly correlated matter
new fixed point
(scale invariance)
?


strongly correlated matter
new fixed point
(scale invariance)


strongly correlated matter
new fixed point
(scale invariance)
unparticles (IR)
(H. Georgi)

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi + m^2 \phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$
$$\phi(x) \to \phi(x)$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$
$$\phi(x) \to \phi(x)$$

$$m^2\phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

$$\Lambda^2\left(\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\right)$$

$$m^2\phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

$$\Lambda^2\left(\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\right)$$

$$m^2\phi^2$$

massive free theory

mass

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2$$

$$x \to x/\Lambda$$

$$\phi(x) \to \phi(x)$$

$$\Lambda^2\left(\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\right)$$

$$m^2\phi^2$$

no scale invariance

$$\mathcal{L} = \quad \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right)$$

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2 \phi^2(x,m)\right) dm^2$$

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

theory with all possible mass!

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} \to \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} \to \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

not particles

unparticles

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} \to \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

not particles

propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

$$\downarrow$$

$$d_U - 2$$

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

$$G \propto (E - \varepsilon_p)^{d_U - 2}$$

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

$$G \propto (E - \varepsilon_p)^{d_U - 2}$$

unparticles=fractional number of massless particles

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

$$G \propto (E - \varepsilon_p)^{d_U - 2}$$

unparticles=fractional number of massless particles

$d_U > 2$

zeros

n massless particles

$$G \propto (E - \varepsilon_p)^{n-2}$$

$$G \propto (E - \varepsilon_p)^{d_U - 2}$$

unparticles=fractional number of massless particles

$d_U > 2$

zeros

no simple sign change

no particle interpretation

no particle interpretation

$$\phi_U \neq \int B(m^2)\phi(m^2)dm^2$$

$$Z \neq \int \mathcal{D}\phi_U \exp\left\{\frac{i}{2}\int d^d p \phi_U(p)\, G_U^{-1} \phi_U(-p)\right\}$$

no particle interpretation

$$\phi_U \neq \int B(m^2)\phi(m^2)dm^2$$

$$d_U\,?$$

what really is the summation over mass?

what really is the summation over mass?

mass=energy

what has been done?

Stephanov, Terning, 2008/09

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2 \phi^2(x,m)\right) dm^2$$

what has been done?

Stephanov, Terning, 2008/09

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2 \phi^2(x,m)\right) dm^2$$

resembles action on AdS with

what has been done?

Stephanov, Terning, 2008/09

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

resembles action on AdS with

$$m^2_{AdS} = \frac{d_U(d_U - d)}{R^2}$$

what has been done?

Stephanov, Terning, 2008/09

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

resembles action on AdS with

$$m^2_{AdS} = \frac{d_U(d_U - d)}{R^2}$$

d_U is a free parameter

can something more exact be done?

can something more exact be done?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

can something more exact be done?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1}$$

can something more exact be done?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1} \qquad m^2_{\mathrm{AdS}} = \frac{d_U(d_U - d)}{R^2}$$

can something more exact be done?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1}$$

$$\boxed{\frac{1}{R^2}} = \frac{d_U(d_U - d)}{R^2}$$

can something more exact be done?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1} \qquad \boxed{\frac{1}{R^2}} = \frac{d_U(d_U - d)}{R^2}$$

$$\mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[\frac{1}{2}\frac{z^2}{R^2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{\phi^2}{2R^2}\right]$$

can something more exact be done?

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1} \qquad \boxed{\frac{1}{R^2}} = \frac{d_U(d_U - d)}{R^2}$$

$$\mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[\frac{1}{2}\frac{z^2}{R^2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{\phi^2}{2R^2}\right]$$

can be absorbed with AdS metric

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2}\int d^{4+2\delta}x\, dz\, \sqrt{-g}\left(\partial_a\Phi\partial^a\Phi + \frac{\Phi^2}{R^2}\right)$$

$$\sqrt{-g} = (R/z)^{5+2\delta}$$

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2}\int d^{4+2\delta}x\, dz\, \sqrt{-g}\left(\partial_a\Phi\partial^a\Phi + \frac{\Phi^2}{R^2}\right)$$

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \qquad \sqrt{-g} = (R/z)^{5+2\delta}$$

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2}\int d^{4+2\delta}x\, dz\, \sqrt{-g}\left(\partial_a\Phi\partial^a\Phi + \frac{\Phi^2}{R^2}\right)$$

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \qquad \sqrt{-g} = (R/z)^{5+2\delta}$$

unparticle lives in

$$d = 4 + 2\delta \qquad \delta \leq 0$$

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2}\int d^{4+2\delta}x\, dz\, \sqrt{-g}\left(\partial_a\Phi\partial^a\Phi + \frac{\Phi^2}{R^2}\right)$$

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \qquad \sqrt{-g} = (R/z)^{5+2\delta}$$

unparticle lives in

$$d = 4 + 2\delta \qquad \delta \leq 0$$

generating functional for unparticles

Claim: $Z_{\mathrm{QFT}} = e^{-S_{\mathrm{ADS}}^{\mathrm{on-shell}}(\phi(\phi_{\partial \mathrm{ADS}=\mathrm{J}_{\mathcal{O}}}))}$

$$S = \frac{1}{2}\int d^d x\, g^{zz}\sqrt{-g}\,\Phi(z,x)\partial_z\Phi(z,x)\Big|_{z=\epsilon}$$

Claim: $Z_{\rm QFT} = e^{-S_{\rm ADS}^{\rm on-shell}(\phi(\phi_{\partial {\rm ADS}={\rm J}_{\mathcal{O}}}))}$

$$S = \frac{1}{2}\int d^d x\, g^{zz}\sqrt{-g}\,\Phi(z,x)\partial_z\Phi(z,x)\Big|_{z=\epsilon}$$

$$\langle \Phi_U(x)\Phi_U(x')\rangle = \frac{1}{|x-x'|^{2d_U}}$$

Claim: $Z_{\rm QFT} = e^{-S^{\rm on-shell}_{\rm ADS}(\phi(\phi_{\partial {\rm ADS}={\rm J}_{\mathcal{O}}}))}$

$$S = \frac{1}{2}\int d^d x\, g^{zz}\sqrt{-g}\,\Phi(z,x)\partial_z\Phi(z,x)\Big|_{z=\epsilon}$$

$$\langle \Phi_U(x)\Phi_U(x')\rangle = \frac{1}{|x-x'|^{2d_U}}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

Claim: $Z_{\rm QFT} = e^{-S^{\rm on-shell}_{\rm ADS}(\phi(\phi_{\partial {\rm ADS}={\rm J}_{\mathcal{O}}}))}$

$$S = \frac{1}{2}\int d^d x\, g^{zz}\sqrt{-g}\,\Phi(z,x)\partial_z\Phi(z,x)\Big|_{z=\epsilon}$$

$$\langle \Phi_U(x)\Phi_U(x')\rangle = \frac{1}{|x-x'|^{2d_U}}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

scaling dimension is fixed

$$G_U(p) \propto p^{2(d_U - d/2)}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

$$G_U(0) = 0$$

unparticle (AdS) propagator has zeros!

$$G_U(p) \propto p^{2(d_U - d/2)}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

$$\downarrow$$

$$G_U(0) = 0$$

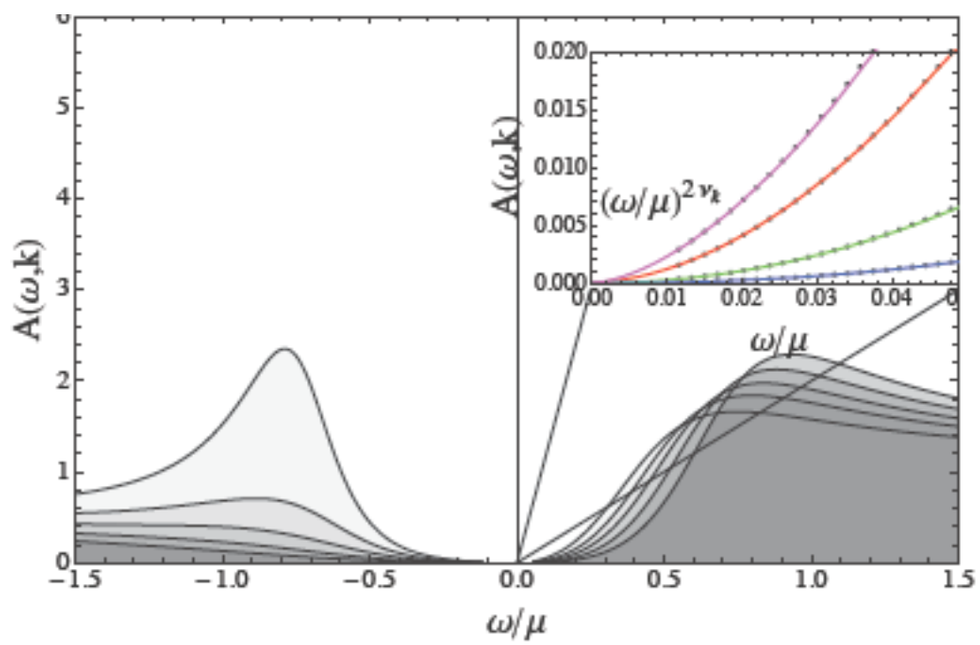
Similar Problem

Universal fermionic spectral functions from string theory

Jerome P. Gauntlett,[1] Julian Sonner,[1,2] and Daniel Waldram[1]

[1] *Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.*
[2] *D.A.M.T.P. University of Cambridge, Cambridge, CB3 0WA, U.K.*

We carry out the first holographic calculation of a fermionic response function for a strongly coupled $d = 3$ system with an explicit $D = 10$ or $D = 11$ supergravity dual. By considering the supersymmetry current, we obtain a universal result applicable to all $d = 3$ $N = 2$ SCFTs with such duals. Surprisingly, the spectral function does not exhibit a Fermi surface, despite the fact that the system is at finite charge density. We show that it has a phonino pole and at low frequencies there is a depletion of spectral weight with a power-law scaling which is governed by a locally quantum critical point.



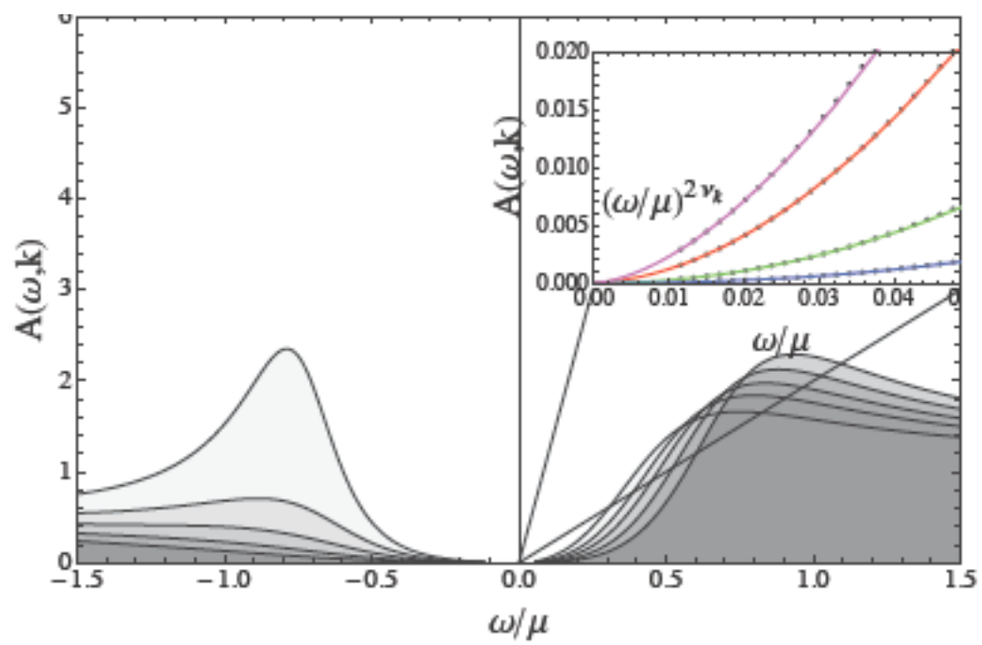
Similar Problem

Universal fermionic spectral functions from string theory

Jerome P. Gauntlett,[1] Julian Sonner,[1,2] and Daniel Waldram[1]

[1] *Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.*
[2] *D.A.M.T.P. University of Cambridge, Cambridge, CB3 0WA, U.K.*

We carry out the first holographic calculation of a fermionic response function for a strongly coupled $d = 3$ system with an explicit $D = 10$ or $D = 11$ supergravity dual. By considering the supersymmetry current, we obtain a universal result applicable to all $d = 3$ $N = 2$ SCFTs with such duals. Surprisingly, the spectral function does not exhibit a Fermi surface, despite the fact that the system is at finite charge density. We show that it has a phonino pole and at low frequencies there is a depletion of spectral weight with a power-law scaling which is governed by a locally quantum critical point.



$$(\not{D} - m - \frac{i}{2}F^{\mu\nu}\Gamma_{\mu\nu})\psi_\rho + iF\mu\nu\Gamma_\mu\Gamma_\rho\psi_\nu = 0$$

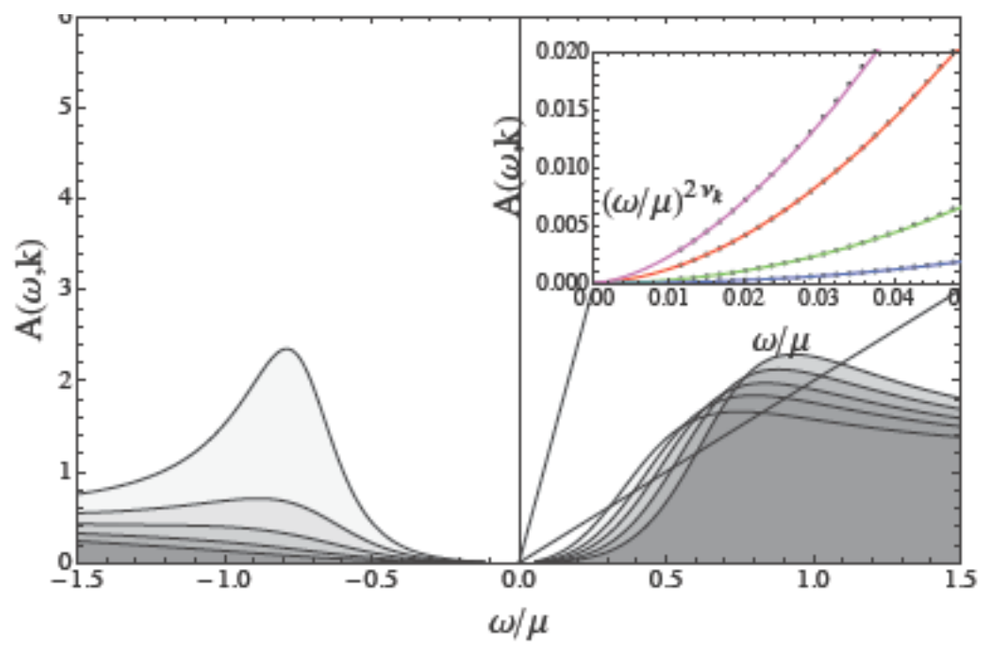
Similar Problem

Universal fermionic spectral functions from string theory

Jerome P. Gauntlett,[1] Julian Sonner,[1,2] and Daniel Waldram[1]

[1] *Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.*
[2] *D.A.M.T.P. University of Cambridge, Cambridge, CB3 0WA, U.K.*

We carry out the first holographic calculation of a fermionic response function for a strongly coupled $d = 3$ system with an explicit $D = 10$ or $D = 11$ supergravity dual. By considering the supersymmetry current, we obtain a universal result applicable to all $d = 3$ $N = 2$ SCFTs with such duals. Surprisingly, the spectral function does not exhibit a Fermi surface, despite the fact that the system is at finite charge density. We show that it has a phonino pole and at low frequencies there is a depletion of spectral weight with a power-law scaling which is governed by a locally quantum critical point.



also Gubser, et al.

$$(\not{D} - m - \frac{i}{2}F^{\mu\nu}\Gamma_{\mu\nu})\psi_\rho + iF\mu\nu\Gamma_\mu\Gamma_\rho\psi_\nu = 0$$

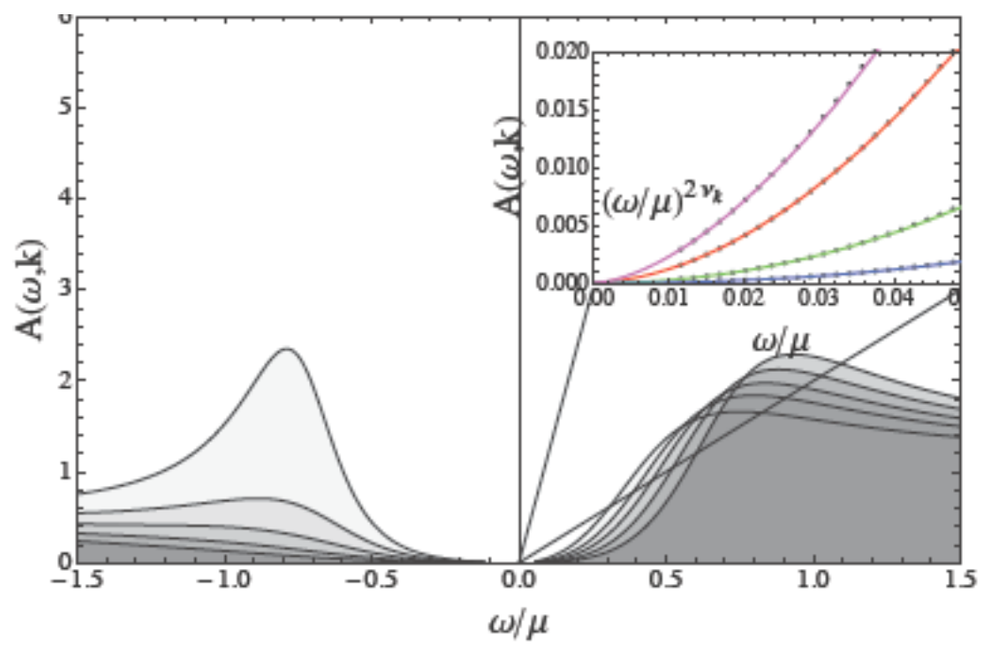
Similar Problem

Universal fermionic spectral functions from string theory

Jerome P. Gauntlett,[1] Julian Sonner,[1,2] and Daniel Waldram[1]

[1] *Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.*
[2] *D.A.M.T.P. University of Cambridge, Cambridge, CB3 0WA, U.K.*

We carry out the first holographic calculation of a fermionic response function for a strongly coupled $d = 3$ system with an explicit $D = 10$ or $D = 11$ supergravity dual. By considering the supersymmetry current, we obtain a universal result applicable to all $d = 3$ $N = 2$ SCFTs with such duals. Surprisingly, the spectral function does not exhibit a Fermi surface, despite the fact that the system is at finite charge density. We show that it has a phonino pole and at low frequencies there is a depletion of spectral weight with a power-law scaling which is governed by a locally quantum critical point.



also Gubser, et al.

$$(\not{D} - m - \boxed{\frac{i}{2}F^{\mu\nu}\Gamma_{\mu\nu}})\psi_\rho + iF\mu\nu\Gamma_\mu\Gamma_\rho\psi_\nu = 0$$

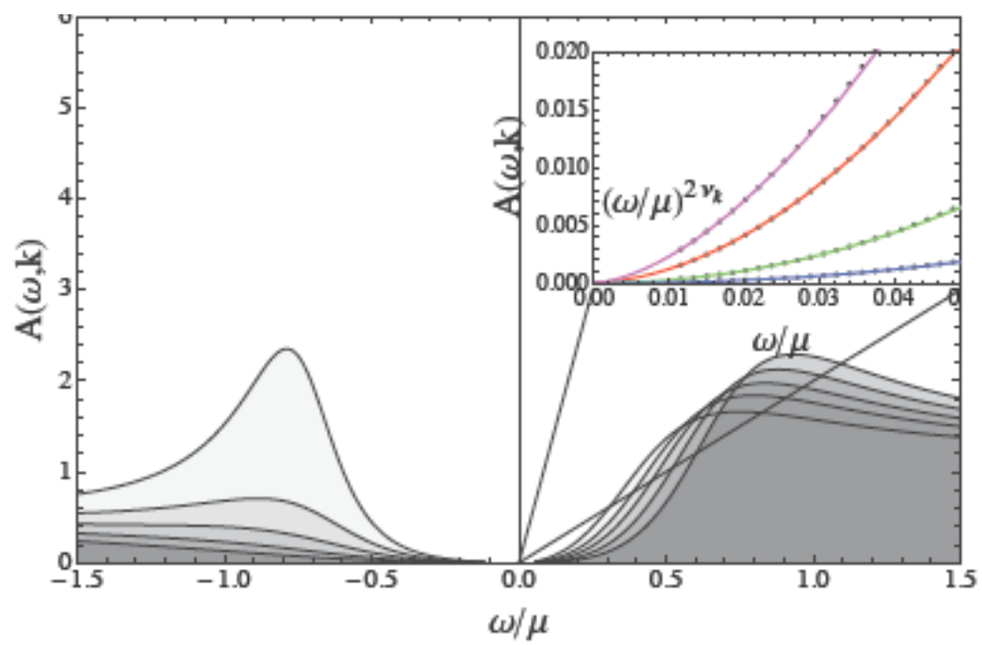
Similar Problem

Universal fermionic spectral functions from string theory

Jerome P. Gauntlett,[1] Julian Sonner,[1,2] and Daniel Waldram[1]

[1] *Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.*
[2] *D.A.M.T.P. University of Cambridge, Cambridge, CB3 0WA, U.K.*

We carry out the first holographic calculation of a fermionic response function for a strongly coupled $d = 3$ system with an explicit $D = 10$ or $D = 11$ supergravity dual. By considering the supersymmetry current, we obtain a universal result applicable to all $d = 3$ $N = 2$ SCFTs with such duals. Surprisingly, the spectral function does not exhibit a Fermi surface, despite the fact that the system is at finite charge density. We show that it has a phonino pole and at low frequencies there is a depletion of spectral weight with a power-law scaling which is governed by a locally quantum critical point.



fixed by supersymmetry

also Gubser, et al.

$$(\not{D} - m - \frac{i}{2}F^{\mu\nu}\Gamma_{\mu\nu})\psi_\rho + iF\mu\nu\Gamma_\mu\Gamma_\rho\psi_\nu = 0$$

bottom-up: with variable Pauli term

$F_{\mu\nu}\Gamma^{\mu\nu}$

consider $\sqrt{-g}i\bar{\psi}(\not{D} - m - ip\not{F})\psi$

bottom-up: with variable Pauli term

$F_{\mu\nu}\Gamma^{\mu\nu}$

consider $\sqrt{-g}i\bar{\psi}(\not{D} - m - ip\not{F})\psi$

$$-\psi_{I\pm}^{(0)\prime\prime}(\zeta) = i\sigma_2\left(1 + \frac{qe_d}{\zeta}\right) - \frac{L_2}{\zeta}\left[m\sigma_3 + \left(pe_d \pm \frac{kL}{r_0}\right)\sigma_1\right]\psi_{I\pm}^{(0)}(\zeta),$$

bottom-up: with variable Pauli term

$F_{\mu\nu}\Gamma^{\mu\nu}$

consider $\sqrt{-g}i\bar{\psi}(\not{D} - m - ip\not{F})\psi$

$$-\psi_{I\pm}^{(0)\prime\prime}(\zeta) = i\sigma_2\left(1 + \frac{qe_d}{\zeta}\right) - \frac{L_2}{\zeta}\left[m\sigma_3 + \left(pe_d \pm \frac{kL}{r_0}\right)\sigma_1\right]\psi_{I\pm}^{(0)}(\zeta),$$

$$e_d = 1/\sqrt{2d(d-1)}$$

p changes scaling dimension

$$m_k^2 = m^2 + \left(pe_d \pm \frac{kL}{r_0}\right)^2$$

bottom-up: with variable Pauli term

$F_{\mu\nu}\Gamma^{\mu\nu}$

consider $\sqrt{-g}i\bar{\psi}(\not{D} - m - ip\not{F})\psi$

$$-\psi_{I\pm}^{(0)\prime\prime}(\zeta) = i\sigma_2\left(1 + \frac{qe_d}{\zeta}\right) - \frac{L_2}{\zeta}\left[m\sigma_3 + \left(pe_d \pm \frac{kL}{r_0}\right)\sigma_1\right]\psi_{I\pm}^{(0)}(\zeta),$$

$$e_d = 1/\sqrt{2d(d-1)}$$

p changes scaling dimension

$$m_k^2 = m^2 + \left(pe_d \pm \frac{kL}{r_0}\right)^2$$

$$\nu_k^{\pm} = \sqrt{m_{k\pm}^2 L_2^2 - q^2 e_d^2 - i\epsilon},$$

bottom-up: with variable Pauli term

$F_{\mu\nu}\Gamma^{\mu\nu}$

consider $\sqrt{-g}i\bar{\psi}(\not{D} - m - ip\not{F})\psi$

$$-\psi_{I\pm}^{(0)\prime\prime}(\zeta) = i\sigma_2\left(1 + \frac{qe_d}{\zeta}\right) - \frac{L_2}{\zeta}\left[m\sigma_3 + \left(pe_d \pm \frac{kL}{r_0}\right)\sigma_1\right]\psi_{I\pm}^{(0)}(\zeta),$$

$$e_d = 1/\sqrt{2d(d-1)}$$

p changes scaling dimension

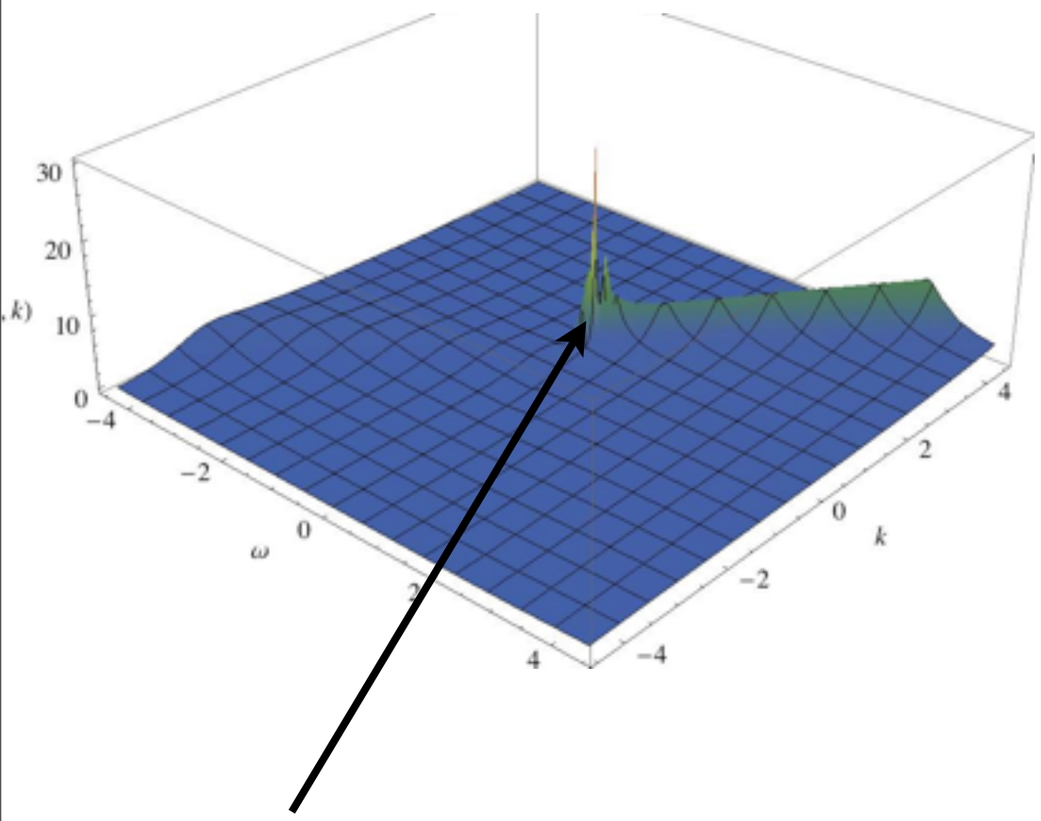
$$m_k^2 = m^2 + \left(pe_d \pm \frac{kL}{r_0}\right)^2$$

$$\nu_k^{\pm} = \sqrt{m_{k\pm}^2 L_2^2 - q^2 e_d^2 - i\epsilon},$$

increasing p should suppress the spectral weight at 0

How is the spectrum modified?

P=0

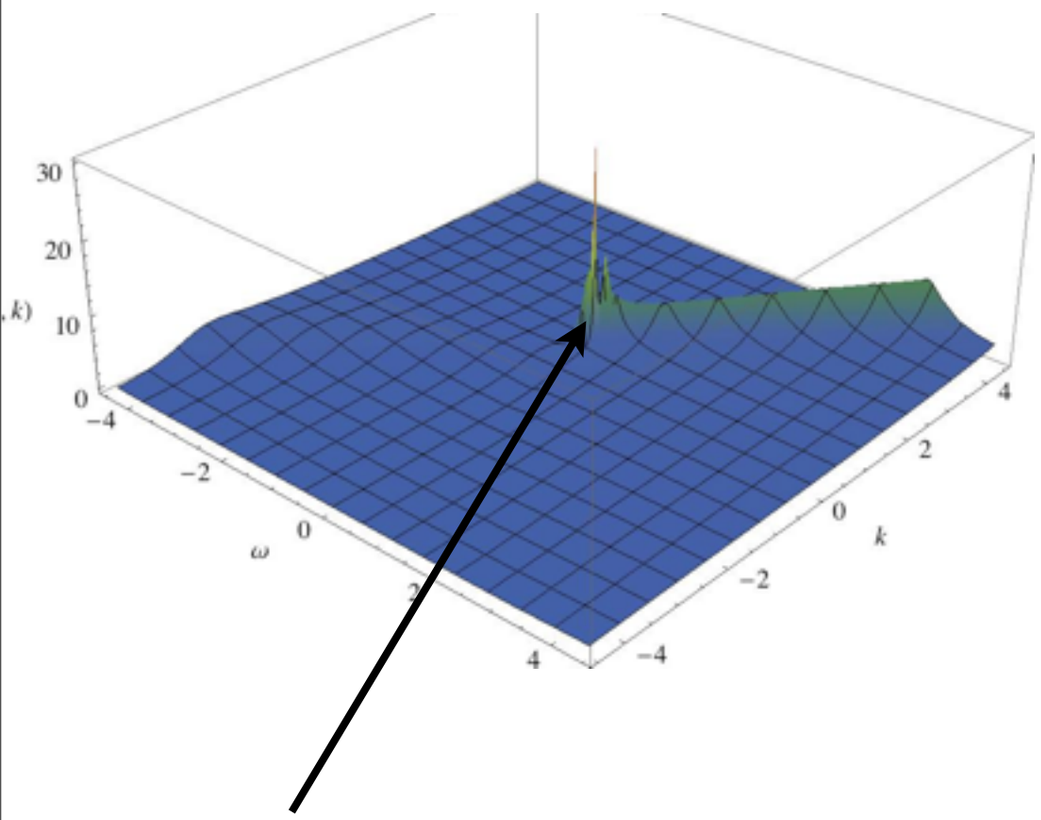


Fermi surface peak

How is the spectrum modified?

P

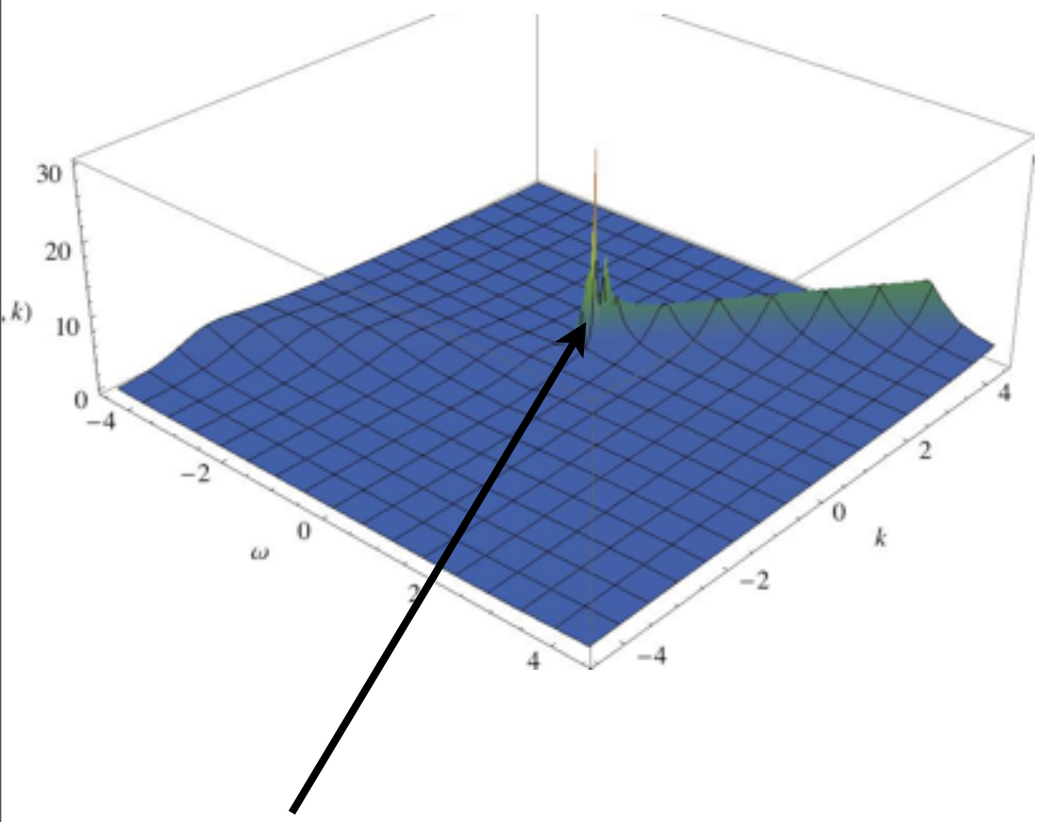
P=0



Fermi surface peak

How is the spectrum modified?

P=0



$$-1.54 < p < -0.53$$

$$1 > \nu_{k_F} > 1/2$$

$$\Re\omega \propto k - k_F$$

$$\Im\omega \propto (k - k_F)^{2\nu_{k_F}}$$

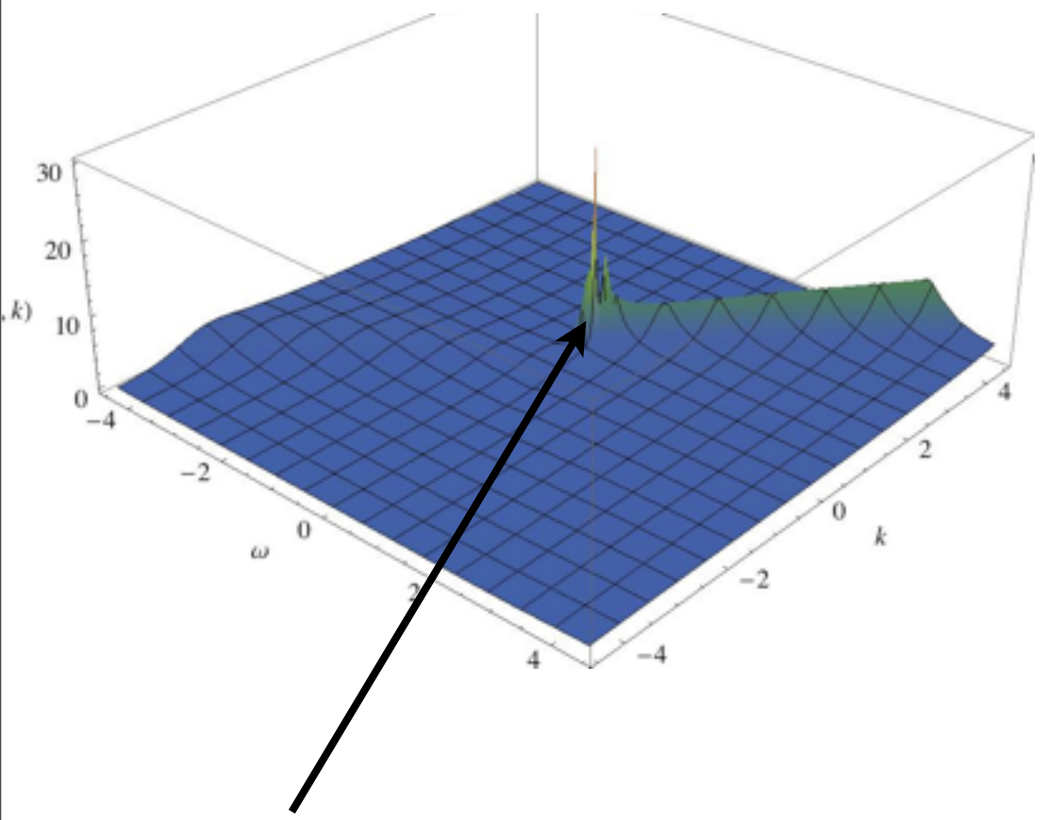
`Fermi Liquid'

Fermi surface peak

How is the spectrum modified?

P

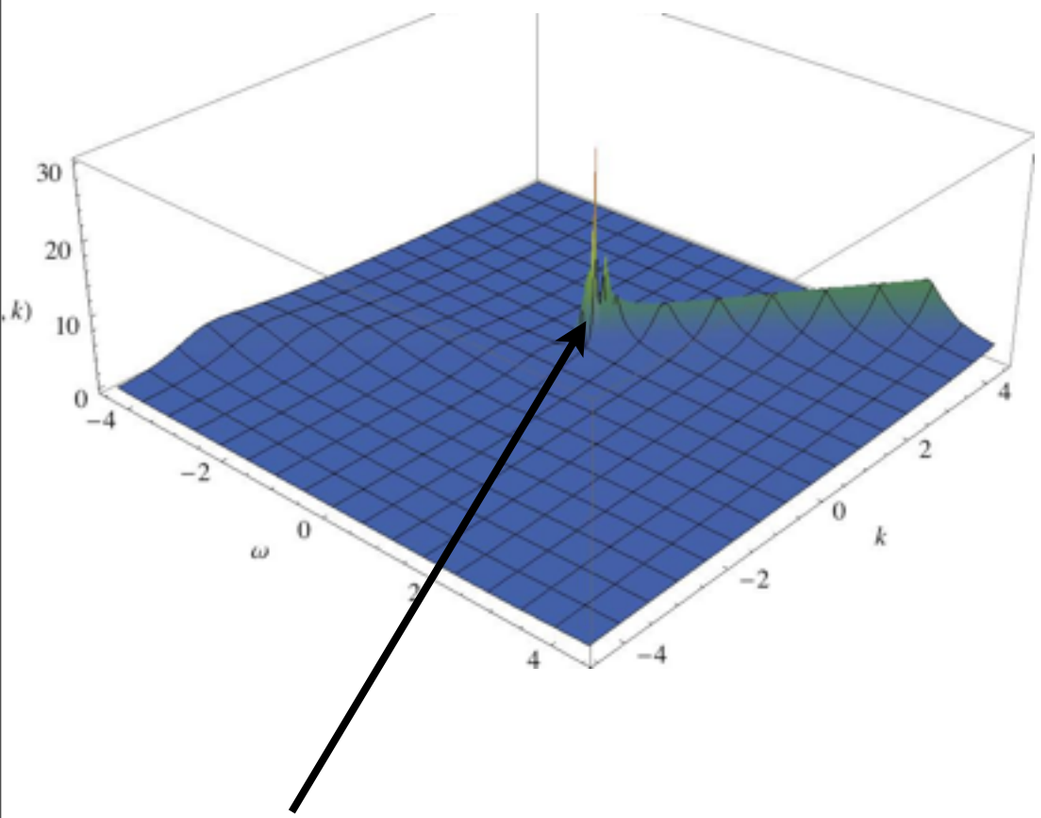
P=0



Fermi surface peak

How is the spectrum modified?

P=0



$p = -0.53$

$\nu_{k_F} = 1/2$

MFL

$-0.53 < p < 1/\sqrt{6}$

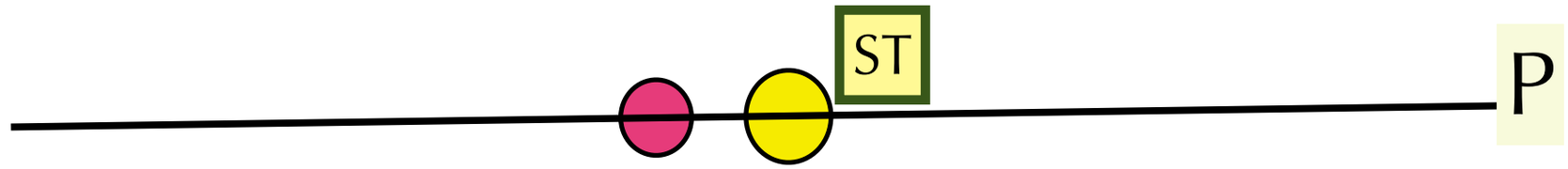
$1/2 > \nu_{k_F} > 0$

$\Re\omega = \Im\omega \propto (k - k_F)^{1/(2\nu_{k_F})}$

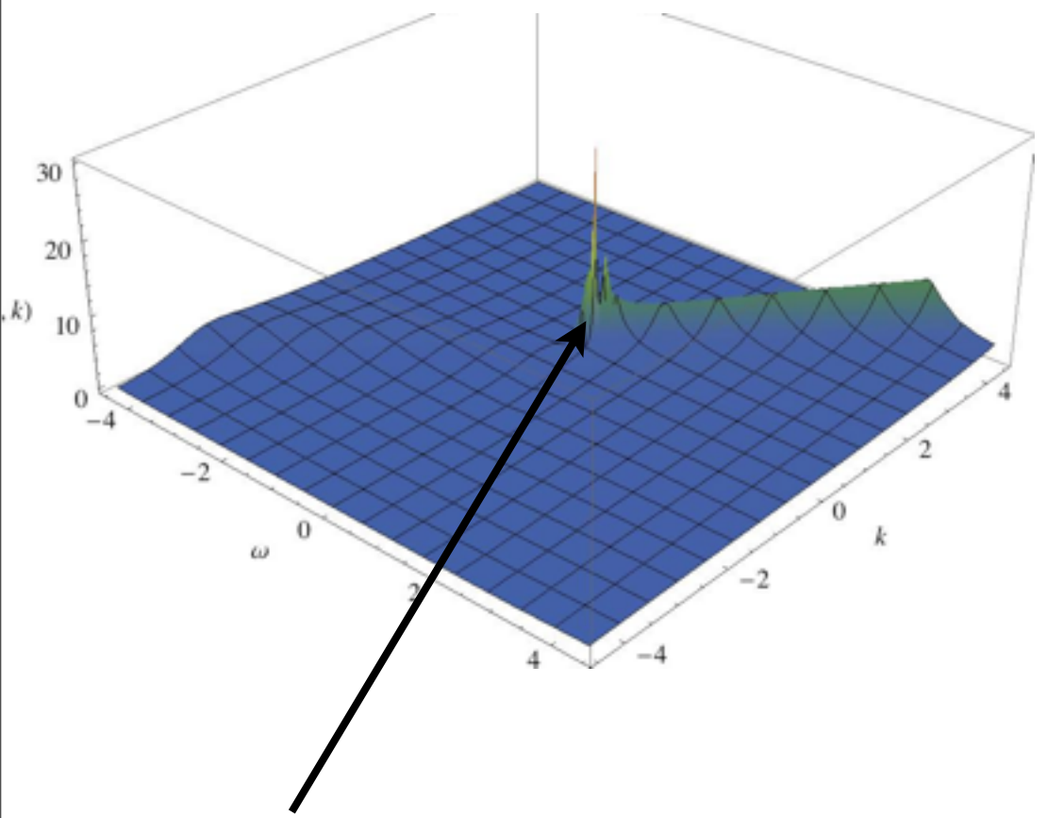
NFL

Fermi surface peak

How is the spectrum modified?



P=0



$p = -0.53$

$\nu_{k_F} = 1/2$

MFL

$-0.53 < p < 1/\sqrt{6}$

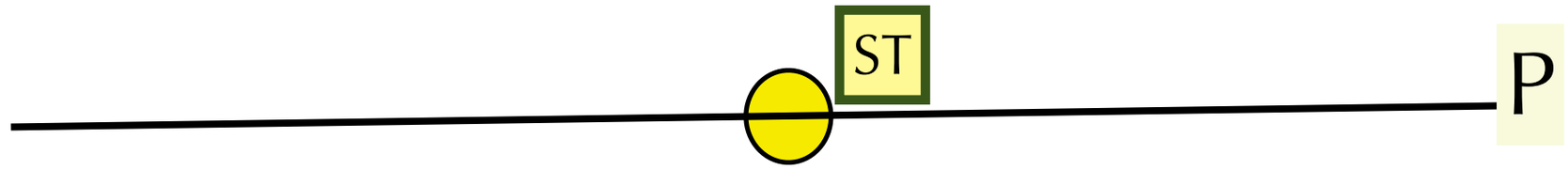
$1/2 > \nu_{k_F} > 0$

$\Re\omega = \Im\omega \propto (k - k_F)^{1/(2\nu_{k_F})}$

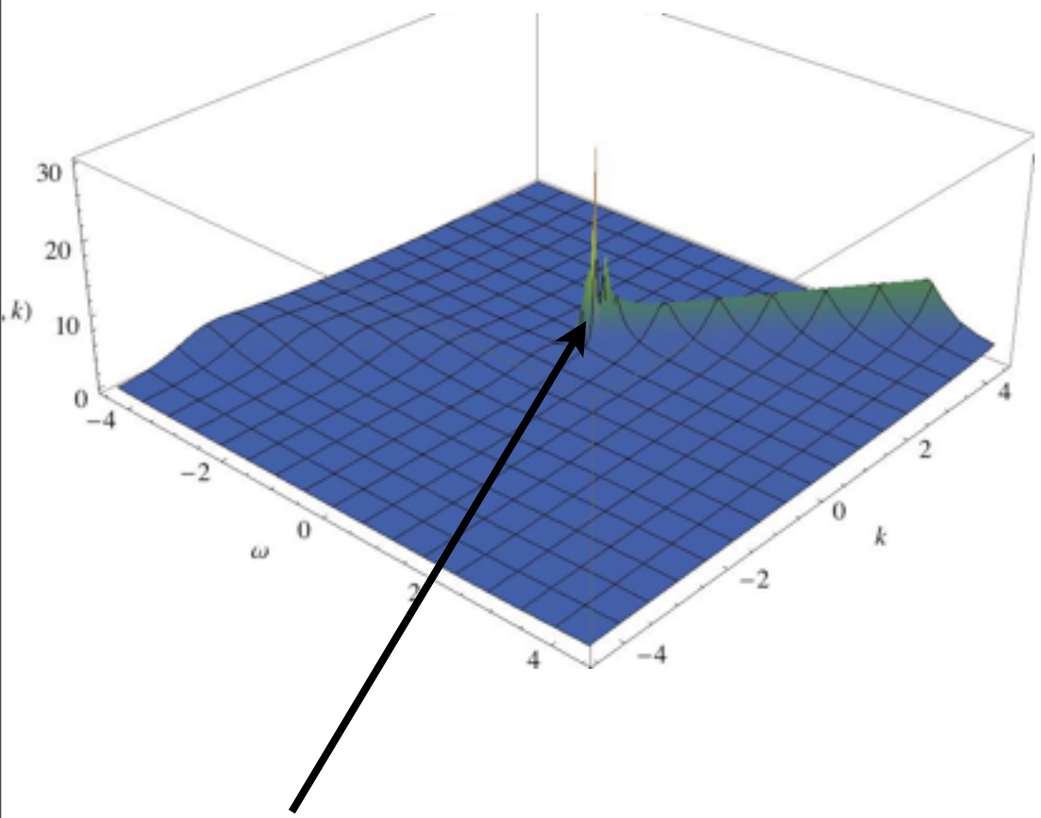
NFL

Fermi surface peak

How is the spectrum modified?

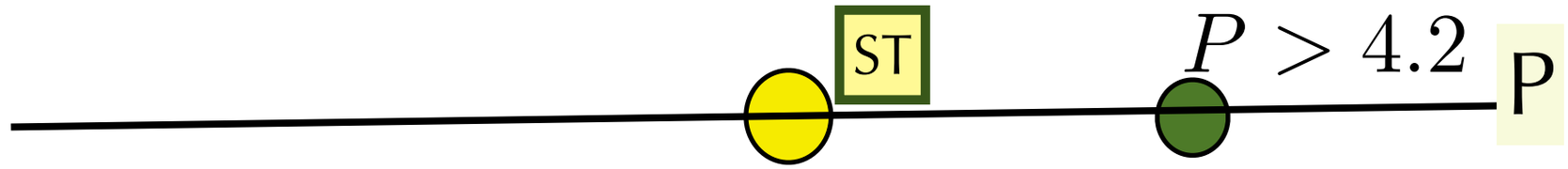


P=0

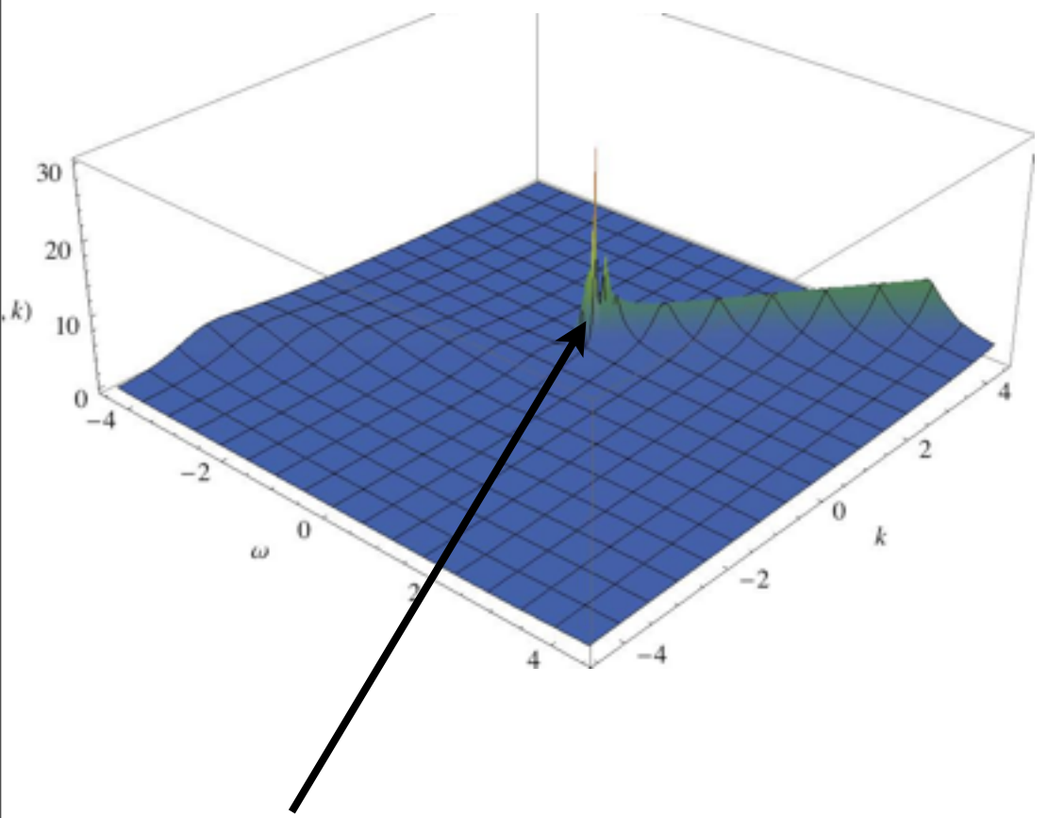


Fermi surface peak

How is the spectrum modified?

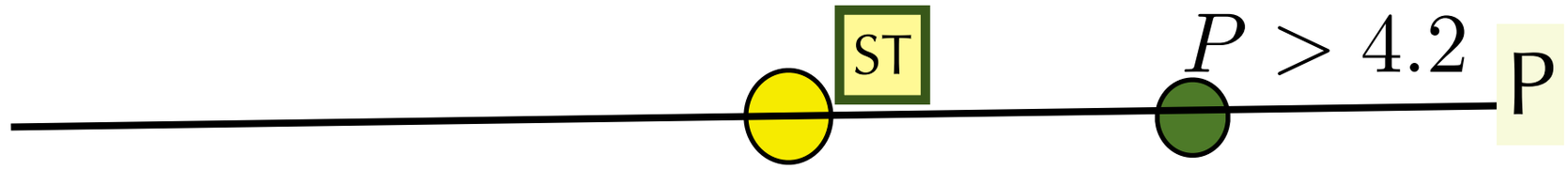


P=0

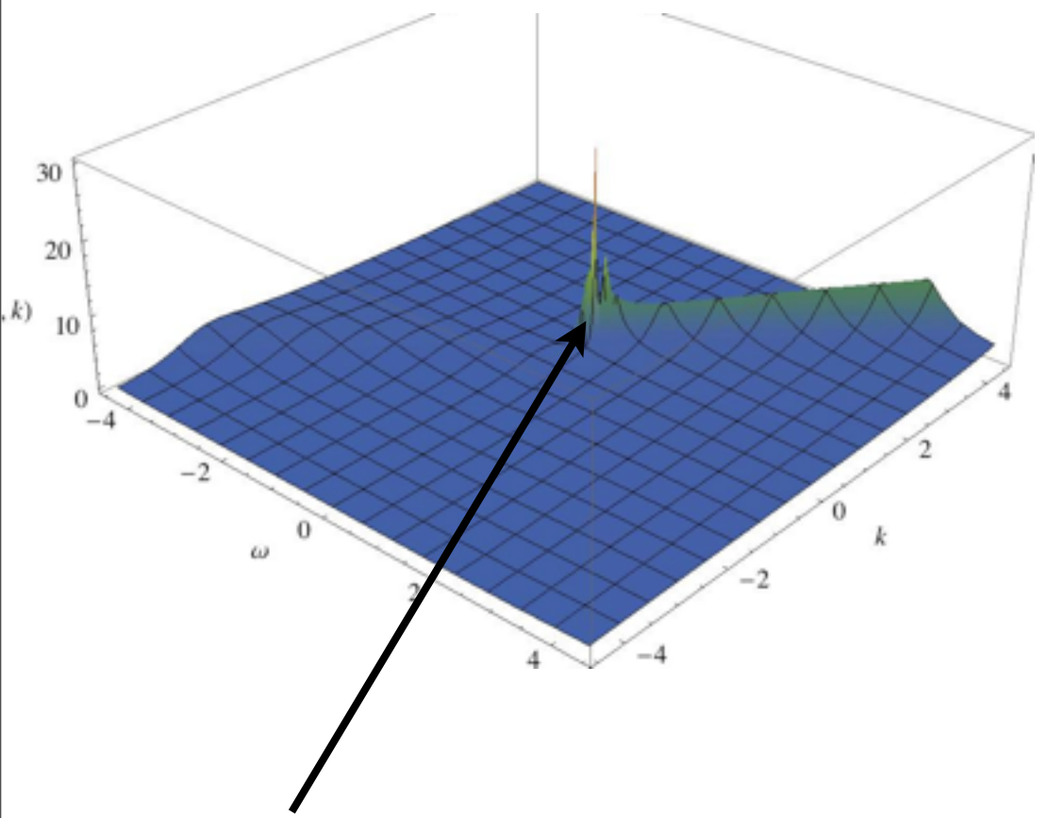


Fermi surface peak

How is the spectrum modified?



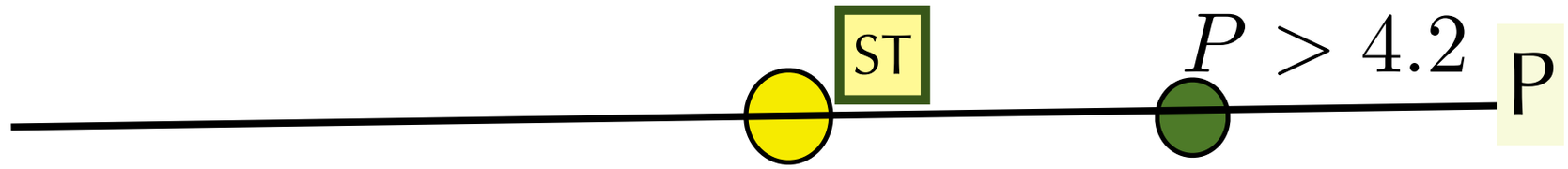
P=0



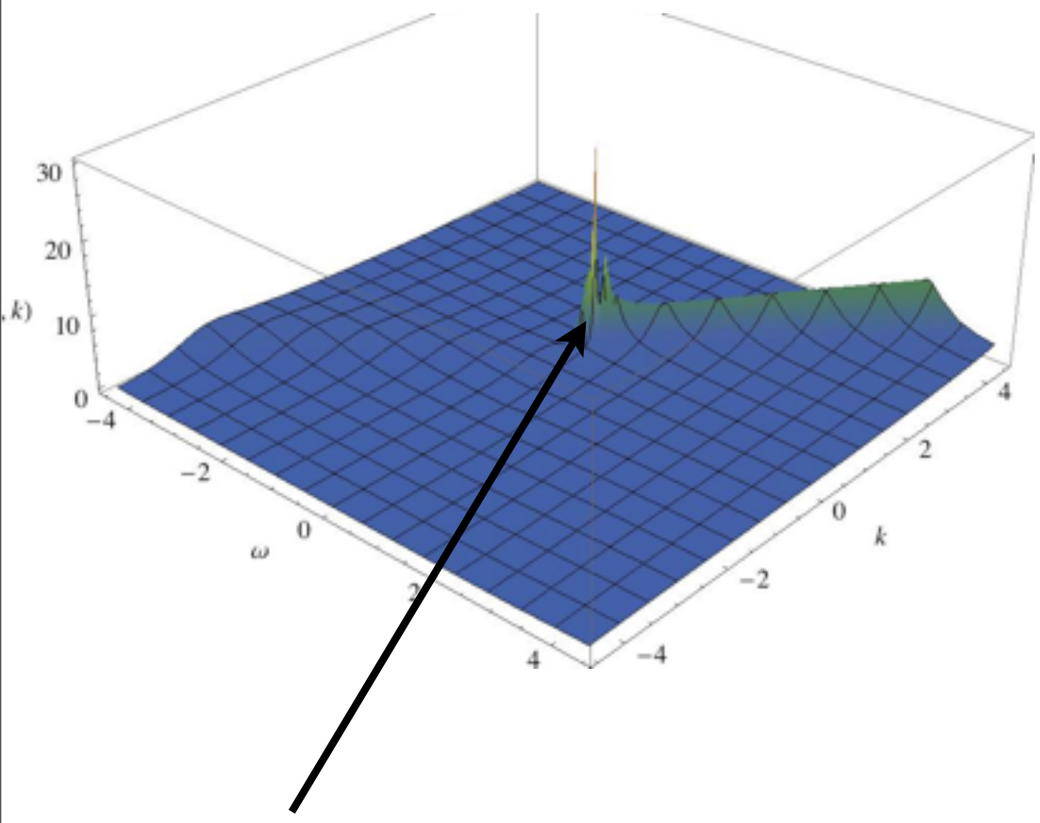
Fermi surface peak

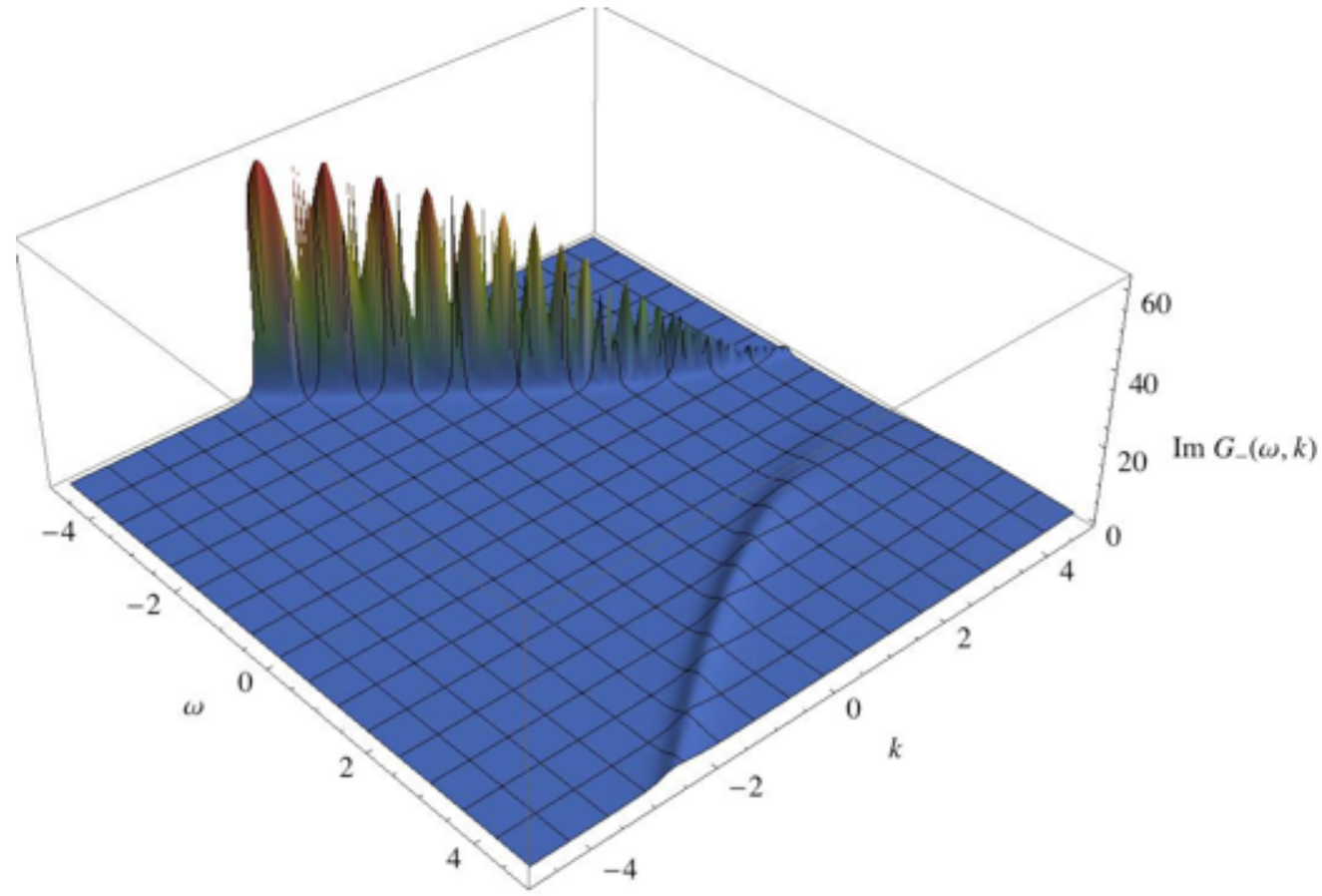
matches unparticle picture with large d_U

How is the spectrum modified?



P=0





Fermi surface peak

Edalati,Leigh, PP PRL, 106 (2011)

matches unparticle picture with large d_U

Superconducting Instability

ladder approximation

$$1 = \lambda T \sum_{n\vec{k}} |w_{n\vec{k}}|^2 G_U\left(\omega_n, \vec{k}\right) G_U\left(-\omega_n, -\vec{k}\right),$$

$$1 \quad = \quad g\left(\frac{T}{W}\right)^{4d_U - d} \qquad , T/W < 1$$

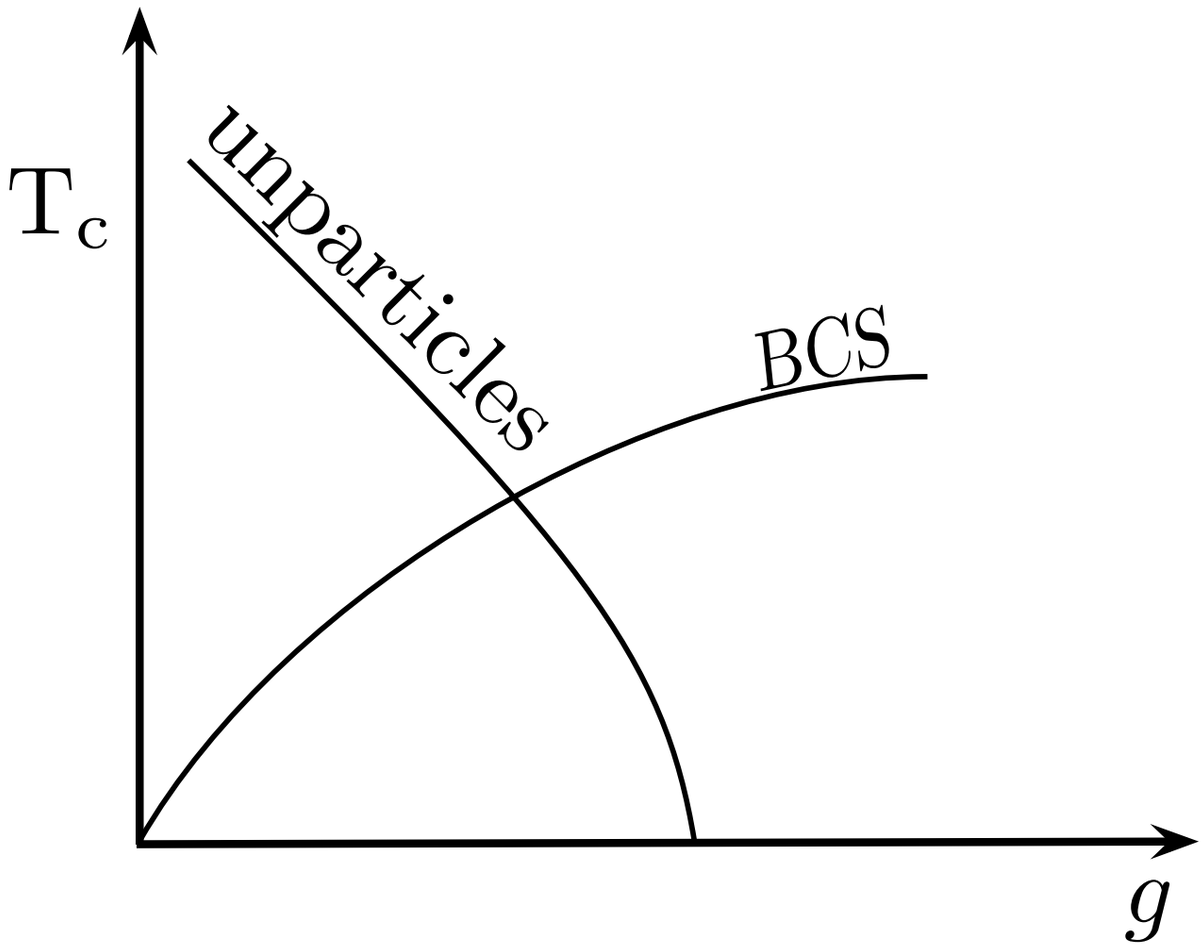
$$1 \quad = \quad g\left(\frac{T}{W}\right)^{4d_U - d} \qquad , T/W < 1$$

zeros condition $d_U > d/2$

$$1 \quad = \quad g\left(\frac{T}{W}\right)^{4d_U - d} \qquad , T/W < 1$$

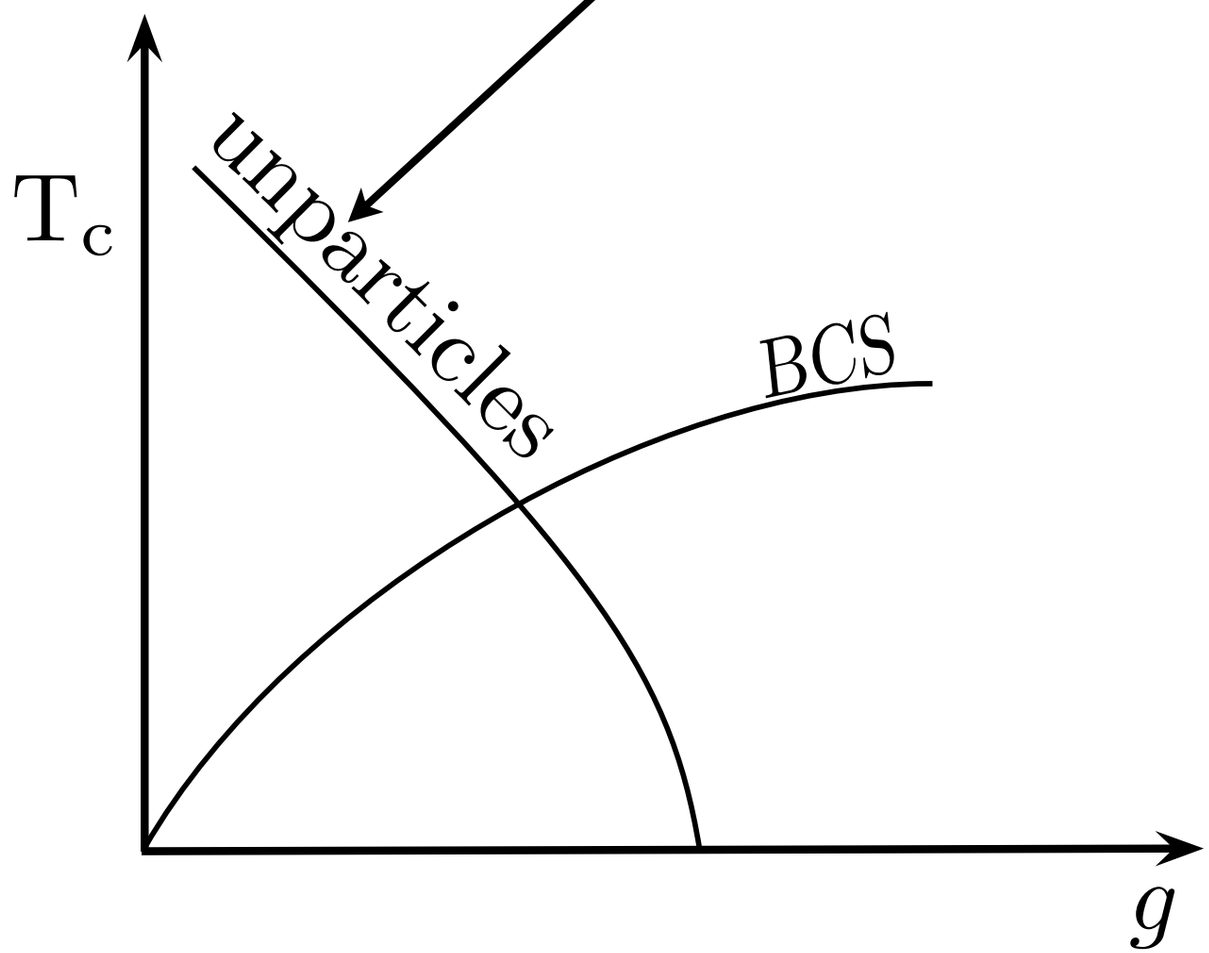
zeros condition $d_U > d/2$

$$\frac{d\ln g}{d\ln\beta} \quad = \quad 4d_U - d > 0$$



tendency towards pairing (any instability which establishes a gap)

$$\frac{d \ln g}{d \ln \beta} = 4d_U - d > 0$$



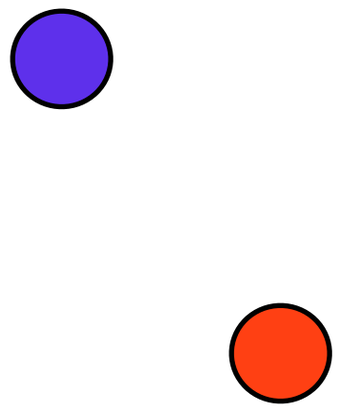
tendency towards pairing (any instability which establishes a gap)

interchanging unparticles

fractional (d_U) number of massless particles

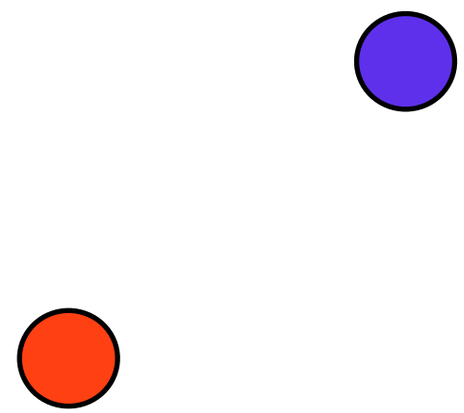
interchanging unparticles

fractional (d_U) number of massless particles



interchanging unparticles

fractional (d_U) number of massless particles



interchanging unparticles

fractional (d_U) number of massless particles

interchanging unparticles

fractional (d_U) number of massless particles

$$e^{i\pi d_U} \neq -1, 0$$

interchanging unparticles

fractional (d_U) number of massless particles

$$e^{i\pi d_U} \neq -1, 0$$

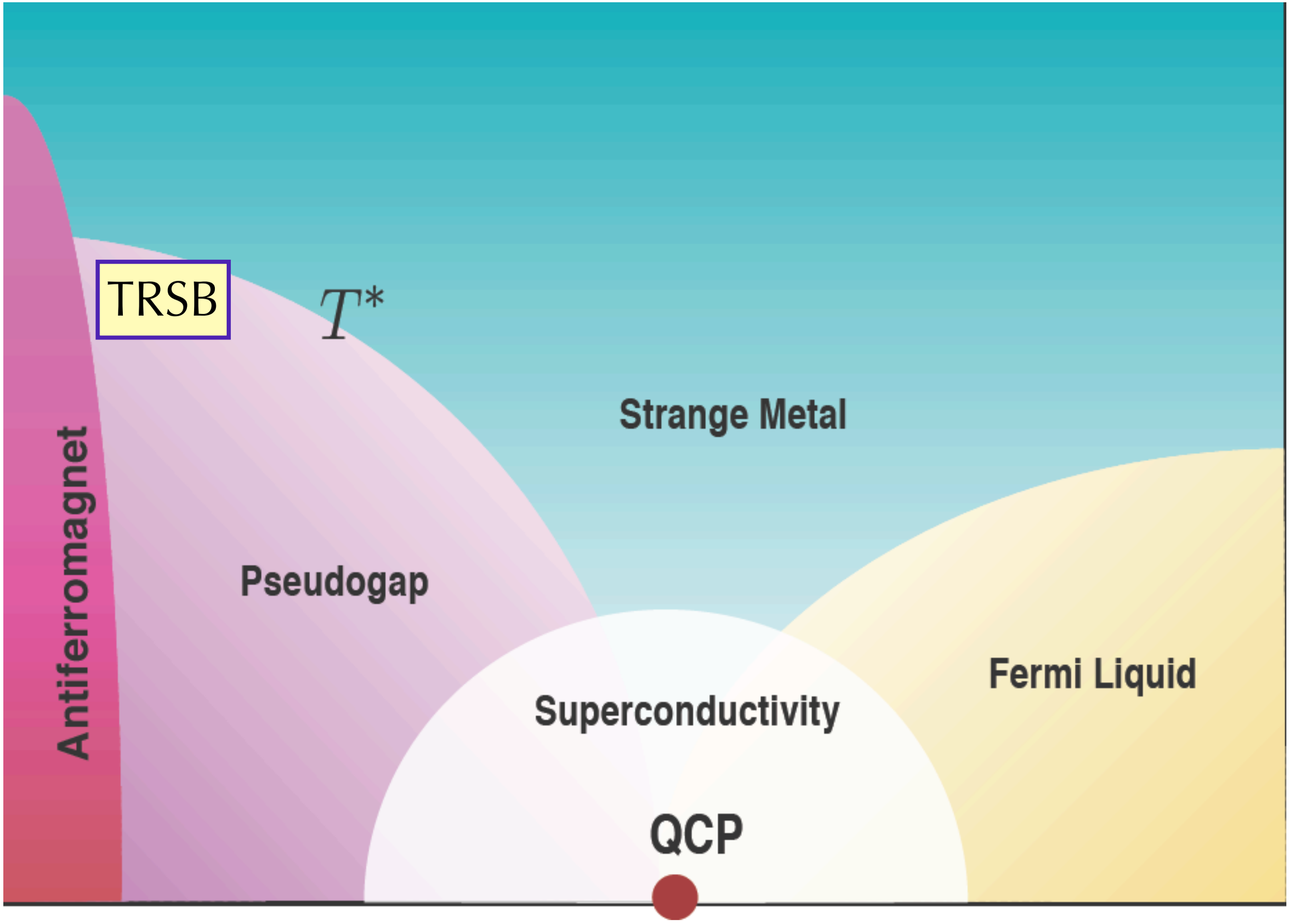
fractional statistics in d=2+1

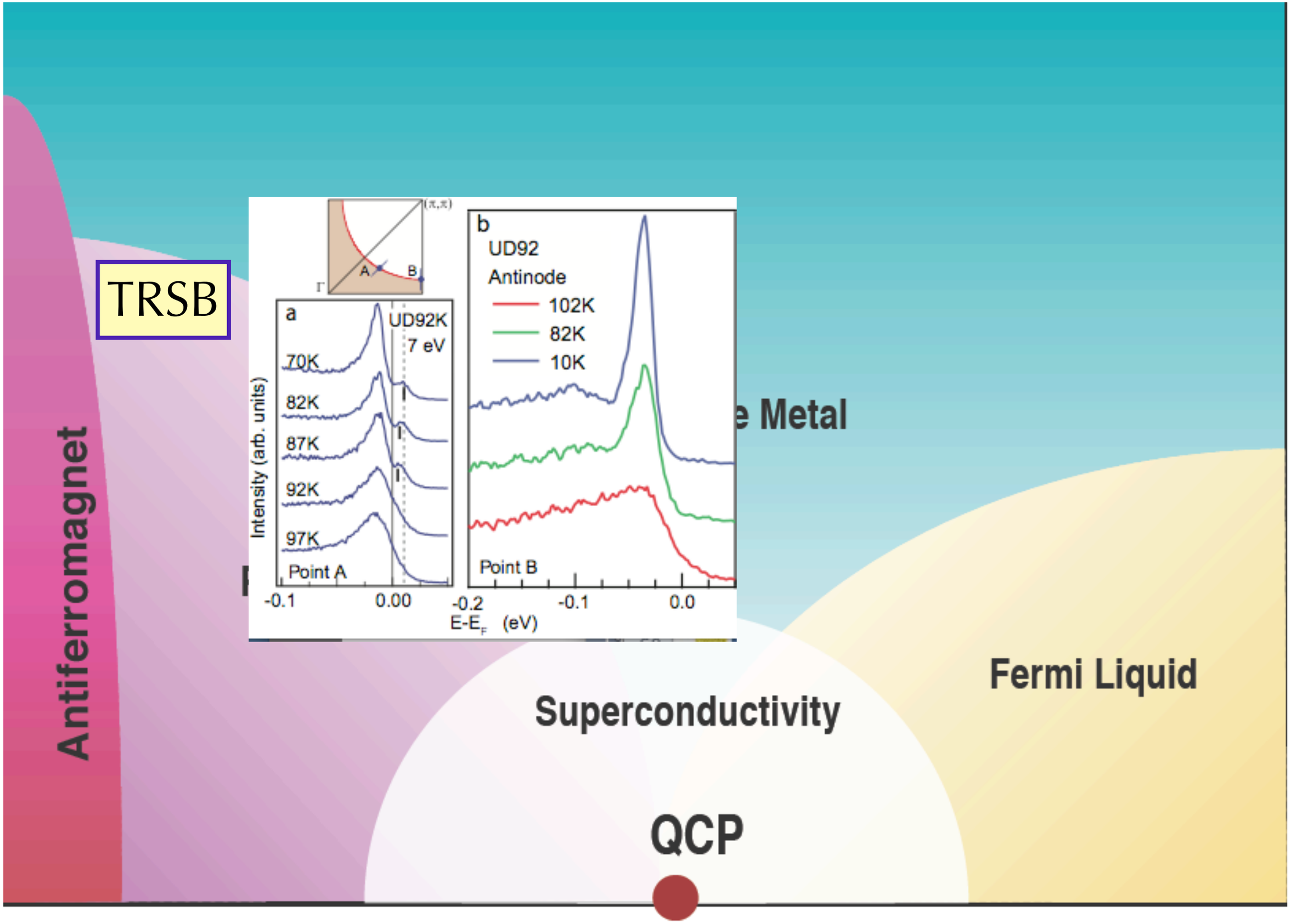
$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

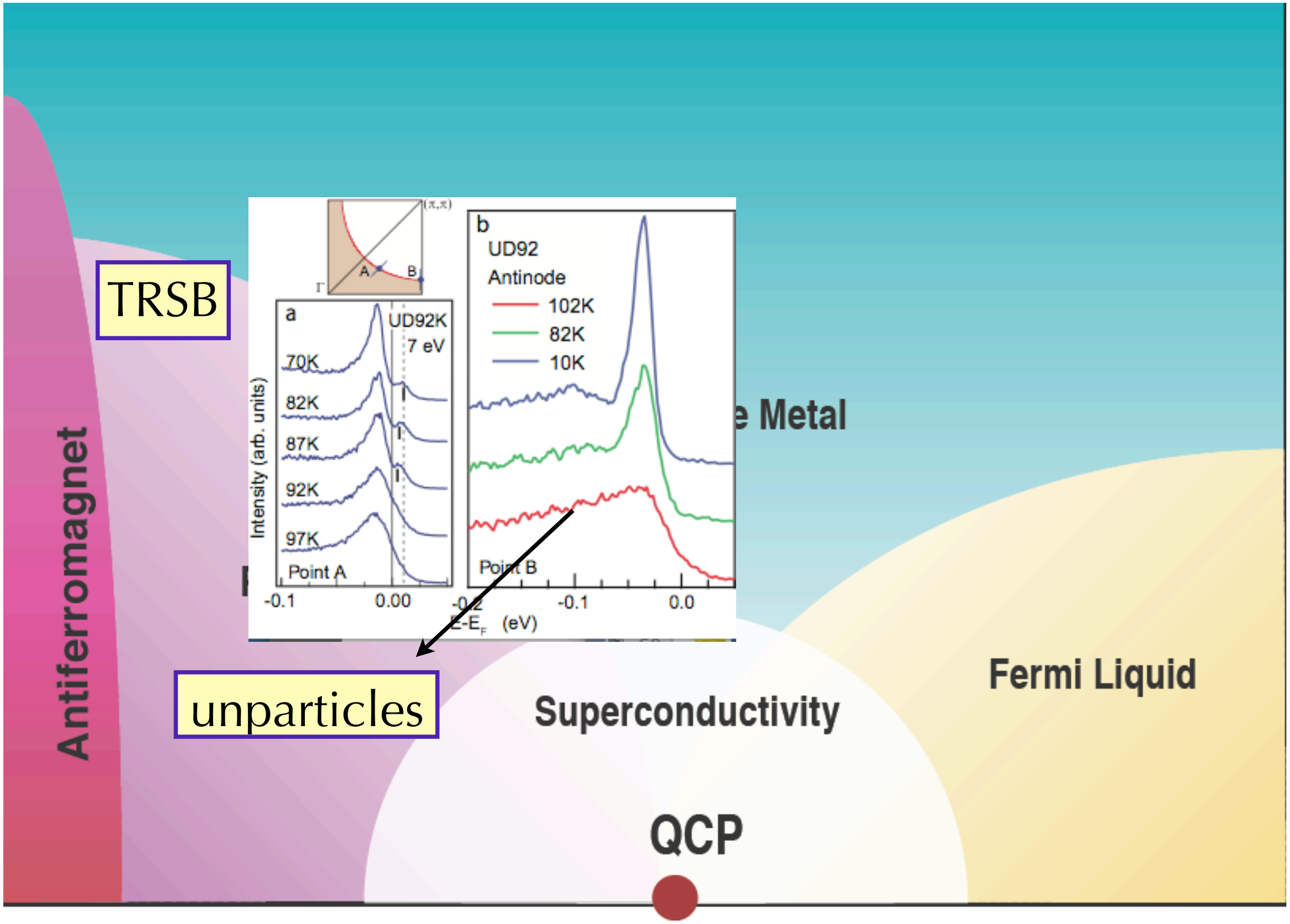
$$e^{i\pi d_U} \neq e^{-i\pi d_U}$$

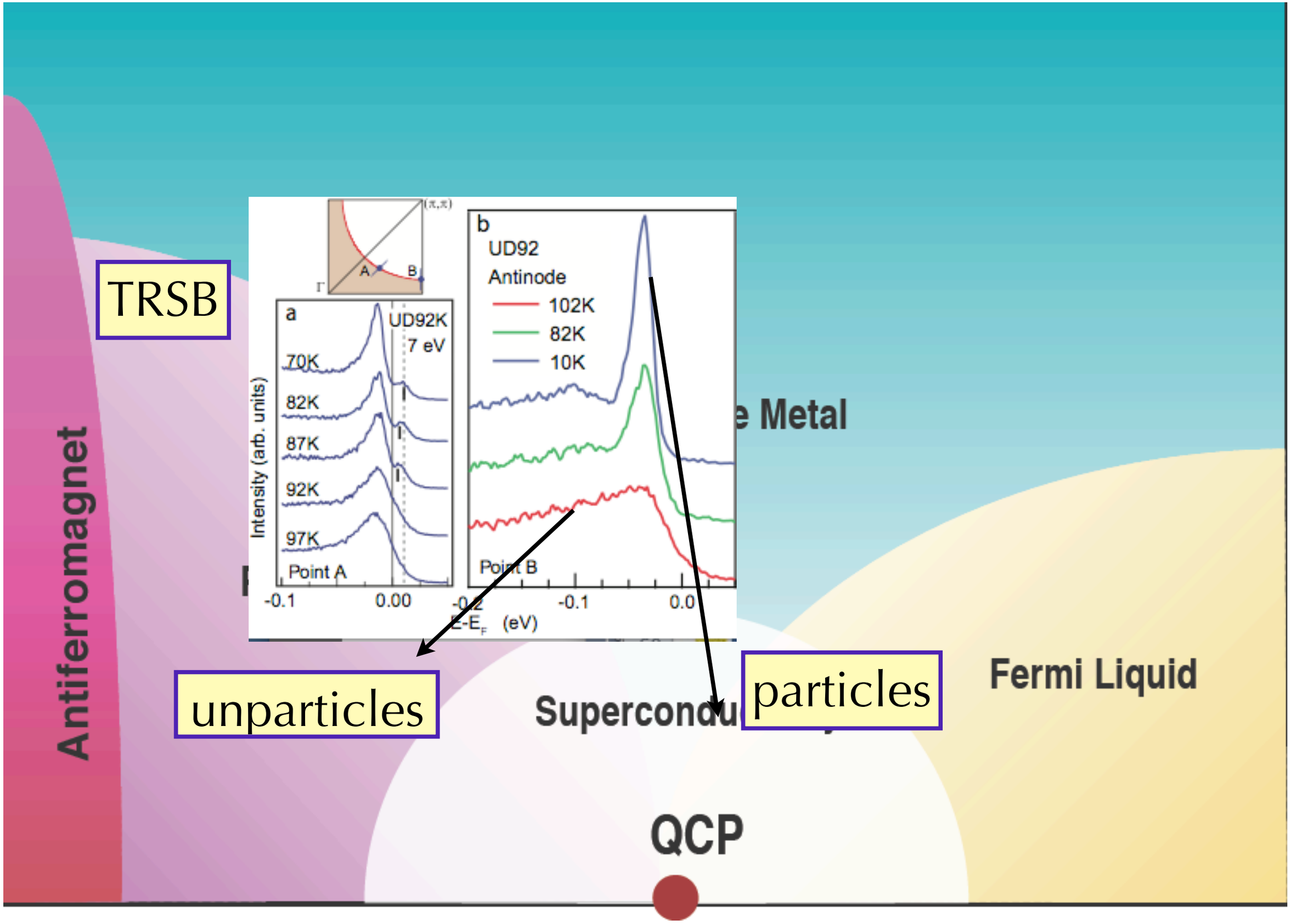
$$\Downarrow$$

time-reversal symmetry breaking
from unparticle (zeros=Fermi arcs) matter

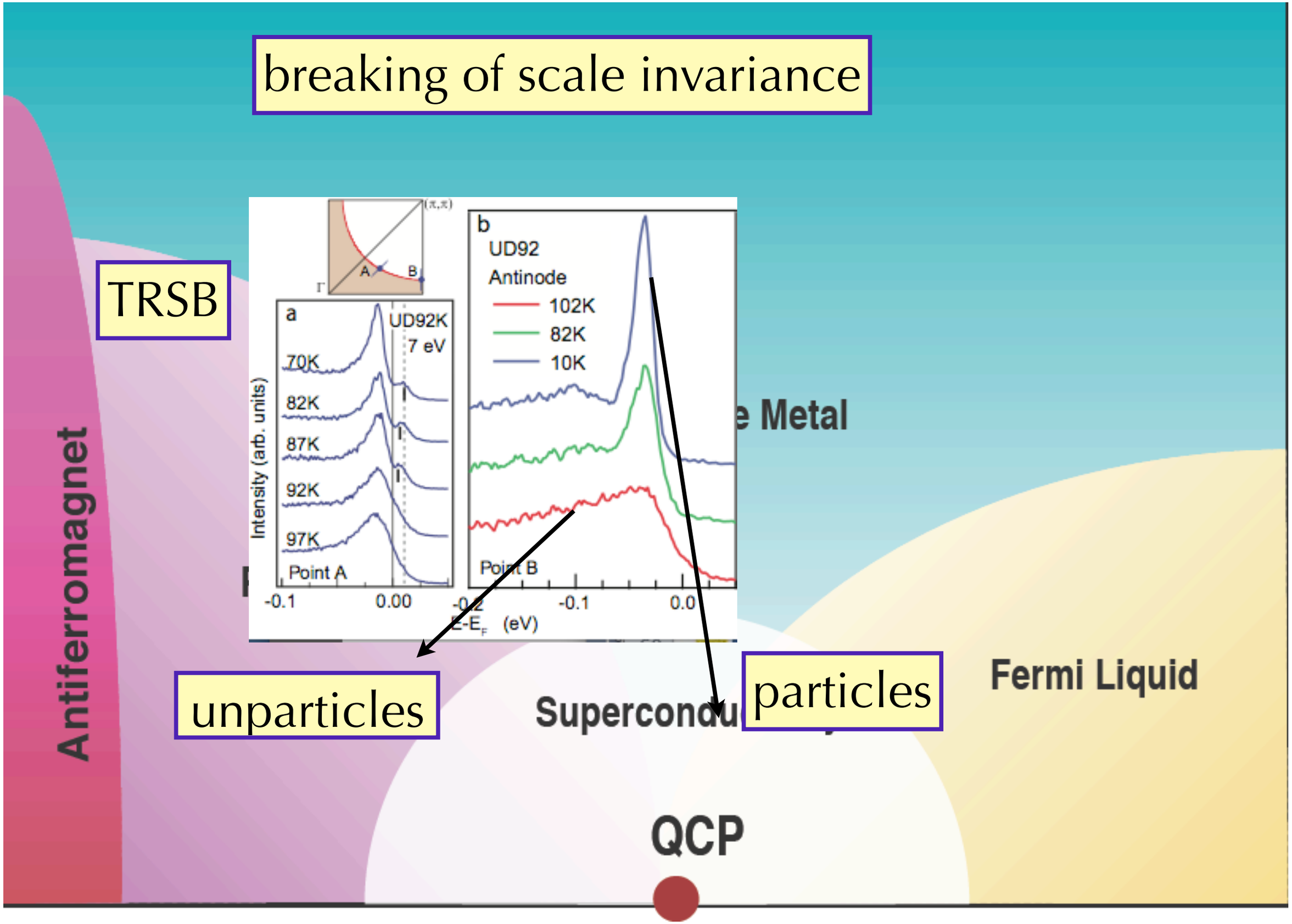

TRSB
T^*
Strange Metal
Antiferromagnet
Pseudogap
Fermi Liquid
Superconductivity
QCP


TRSB
Antiferromagnet
e Metal
Fermi Liquid
Superconductivity
QCP
(π,π)
Γ
A
B
a
UD92K
7 eV
70K
82K
87K
92K
97K
Point A
Intensity (arb. units)
-0.1
0.00
b
UD92
Antinode
102K
82K
10K
Point B
-0.2
-0.1
0.0
E-E_F (eV)

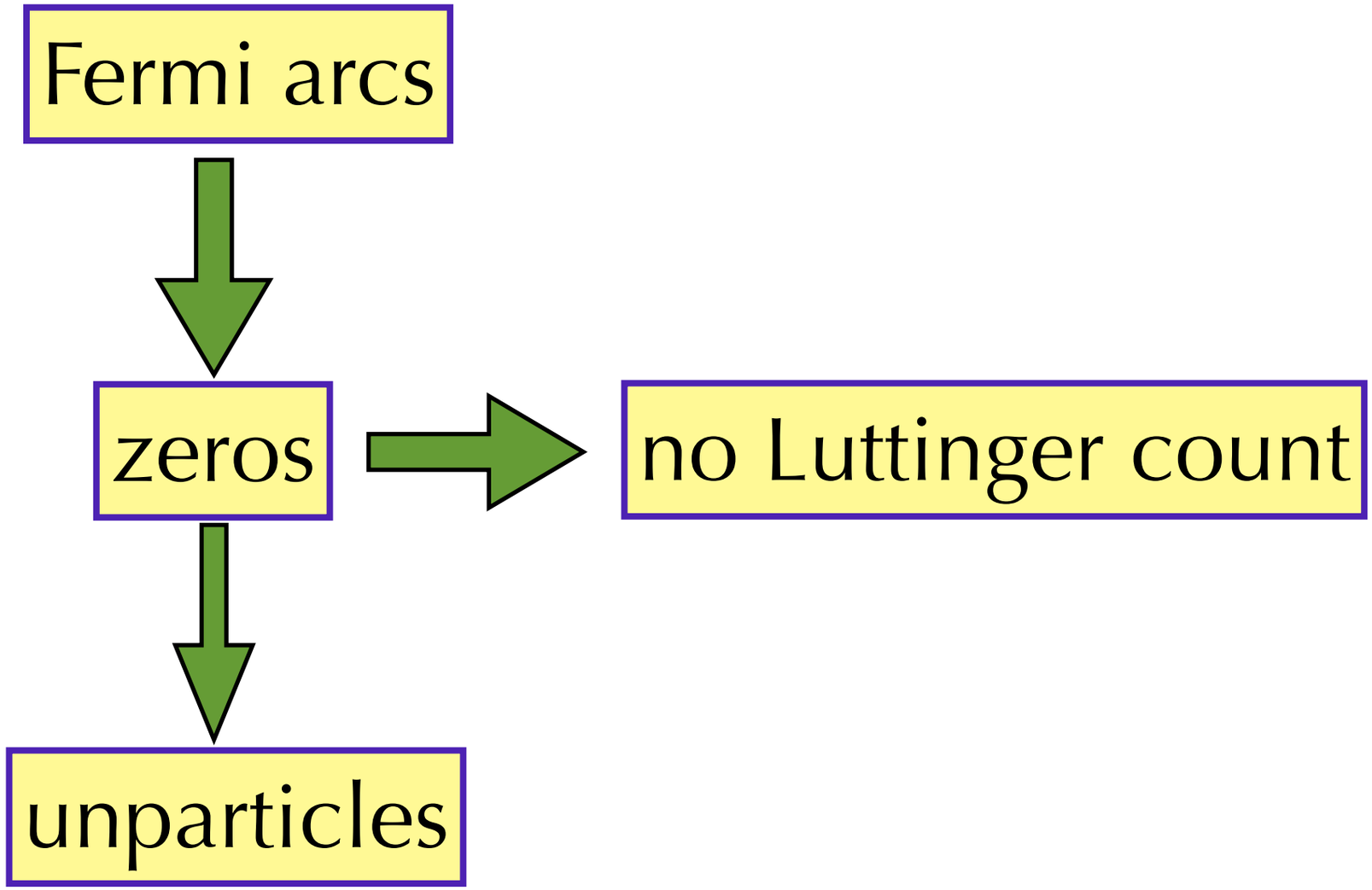

TRSB
Antiferromagnet
Metal
Fermi Liquid
Superconductivity
QCP
unparticles
(π,π)
Γ
A
B
a
UD92K
7 eV
70K
82K
87K
92K
97K
Point A
Intensity (arb. units)
-0.1
0.00
b
UD92
Antinode
102K
82K
10K
Point B
-0.2
-0.1
0.0
E-E_F (eV)

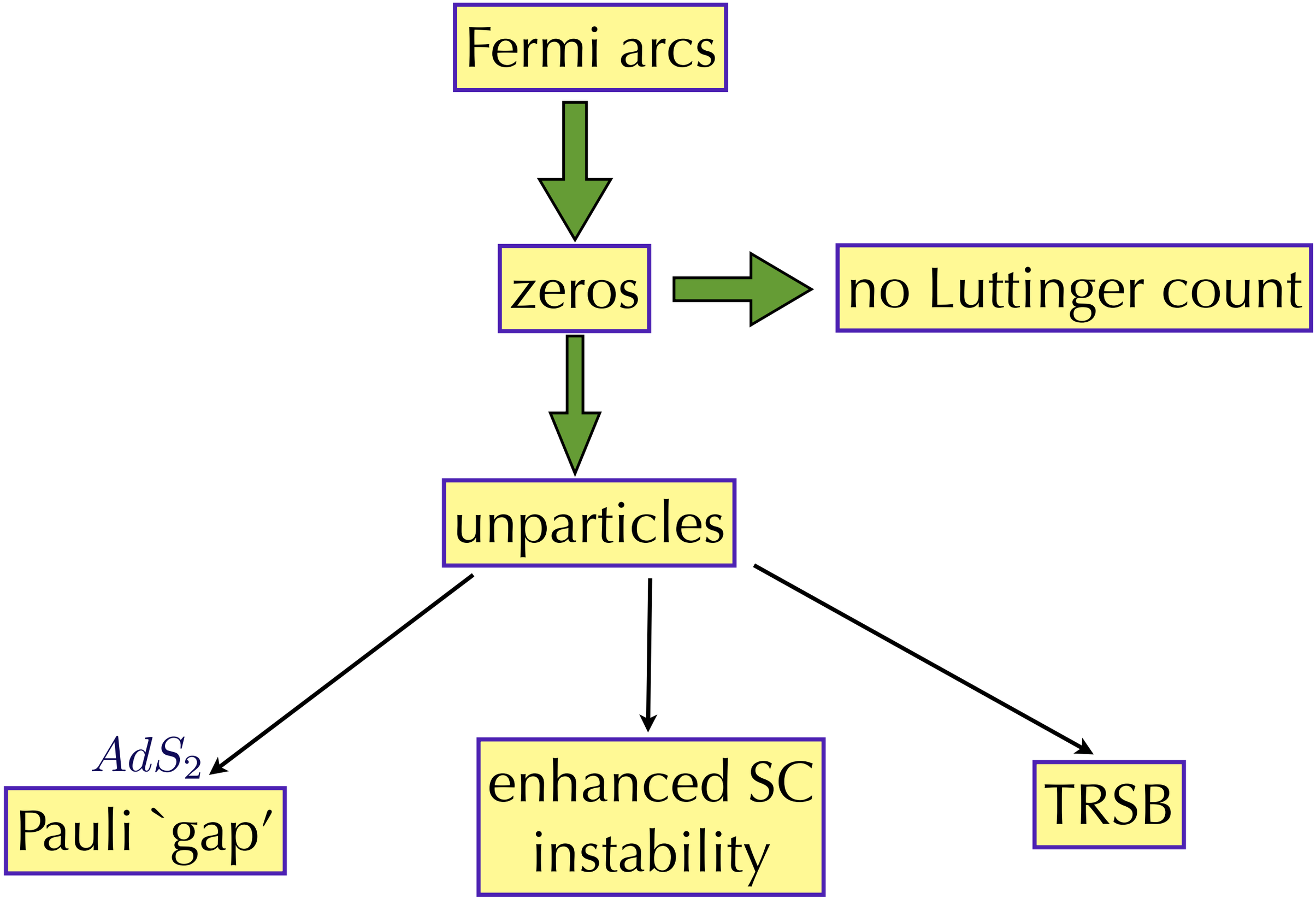

TRSB
Antiferromagnet
Metal
Fermi Liquid
Supercondu
QCP
unparticles
particles
(π,π)
Γ
A
B
a
UD92K
7 eV
70K
82K
87K
92K
97K
Point A
Intensity (arb. units)
-0.1
0.00
b
UD92
Antinode
102K
82K
10K
Point B
-0.1
0.0
E-E_F (eV)

breaking of scale invariance



Fermi arcs


Fermi arcs
zeros
no Luttinger count
unparticles


Fermi arcs
zeros
no Luttinger count
unparticles
AdS_2
Pauli `gap'
enhanced SC instability
TRSB