Unparticles and Mottness

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How Does Fermi Liquid Theory Breakdown?
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Fermi Arcs?

Fermi arcs: (PDJ, JCC, ZXS)
Why are Fermi Arcs Strange?

- Real FS Crossing
- Ghost FS Crossing
- Top of Band
- $FS_{sym}$

$k_y$

$(0, \pi)$
$(\pi, \pi)$

$p$

$(0,0)$
$(\pi,0)$

$E_F$

$k$
Problem

\[ \Re G^R > 0 \]

\[ \Re G^R < 0 \]

\[ (0, \pi) \quad (\pi, \pi) \]

\[ (0, 0) \quad (\pi, 0) \]

\[ \text{poles} \]

\[ \text{Re } G > 0 \]

\[ \text{Re } G < 0 \]
Problem

$\Re G^R > 0$

$\Re G^R < 0$

$\Re G < 0$

$\Re G > 0$
Problem

How to account for the sign change without poles?
Problem

How to account for the sign change without poles?

Only option: $\text{Det}G=0!$ (zeros)
Problem

How to account for the sign change without poles?

Only option: DetG=0! (zeros)
Fermi Arcs

zeros + poles

Luttinger, Dzyaloshinskii, Yang, Rice, Zhang, Tsvelik...

n=zeros + poles
what are zeros?

are they (like poles) conserved?
NiO insulates $d^8$?

Mott mechanism (not Slater)

Monday, September 16, 13
NiO insulates $d^8$? perhaps this costs energy

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$U \gg t$

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Mott mechanism (not Slater)

$U \gg t$

$\mu = 0$
NiO insulates $d^8$? perhaps this costs energy

Mott mechanism (not Slater)

$\mu = 0$

no change in size of Brillouin zone

$U \gg t$
Mott Problem

Kramers-Kronig
Mott Problem

\[ \text{Kramers-Kronig} \]

\[ = \text{below gap} + \text{above gap} \]
Mott Problem

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \frac{1}{\omega - \mu + i \omega \tau} \right) d\omega \]

= below gap + above gap

\[ \mu = 0 \]

\[ \text{Im} G = 0 \]

Kramers-Kronig
Mott Problem

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \frac{1}{\omega} \right) d\omega \]

\[ = \text{below gap} + \text{above gap} = 0 \]
Mott Problem

\[ \text{Det} G(k, \omega = 0) = 0 \]  
(single band)

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \begin{array}{c} \mu = 0 \\ \omega \end{array} \right) d\omega \]  
= below gap + above gap

\[ \text{Im} G = 0 \]  
Kramers-Kronig
Mott Problem

\[ \text{Re} G(0, p) = \int_{-\infty}^{\infty} \left( \mu = 0 \right) \left( \omega \right) d\omega \]

= below gap + above gap

= 0

\[ \text{Det} G(k, \omega = 0) = 0 \] (single band)

\[ \text{Det} \text{Re} G(0, p) = 0 \] Mottnes
Mott Problem

\[ \text{Re} \, G(0, p) = \int_{-\infty}^{\infty} (\omega) d\omega \]

\[ = \text{below gap} + \text{above gap} = 0 \]

\[ \text{Im} \, G = 0 \]

\[ \text{Det} \, G(k, \omega = 0) = 0 \]

\[ \text{DetRe} \, G(0, p) = 0 \]

Mottnes

not true in MF theories

Kramers-Kronig
sign changes of Green function

zero-crossing

\[ \text{Det} G(\omega = 0, \vec{p}) = 0 \]

poles (qp)

\[ E > \varepsilon_p \]

\[ E < \varepsilon_p \]
$\text{Det}G(\omega = 0, \vec{p}) = 0$

$n = 2 \sum_k \Theta(\Re G(\mathbf{k}, \omega = 0))$

Luttinger’s theorem
singularities of $\ln G$

$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d p \int_{-\infty}^{0} d\xi \ln \frac{G_R(\xi, p)}{G^*_R(\xi, p)}$$

poles+zeros
(all sign changes)
singualrities of \( \ln G \)

\[
n = \frac{2i}{(2\pi)^{d+1}} \int d^d p \int_0^0 d\xi \ln \frac{G_R(\xi, p)}{G_R^*(\xi, p)}
\]

\[
n = 2 \sum_k \Theta(\Re G(k, \omega = 0))
\]

poles+zeros
(all sign changes)
singularities of $\ln G$

$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d p \int_{-\infty}^{0} d\xi \ln \frac{G^R(\xi, p)}{G^*_R(\xi, p)}$$

$$n = 2 \sum_k \Theta(\Re G(k, \omega = 0))$$

poles+zeros
(all sign changes)

Fermi Liquids  Mott Insulators
The proof just presented is good for any state of our system: FL, NFL, or MI [in other words, either of poles or zeros can be used to change the sign of $G_r$ in Eq. (1)]. The only way to incapacitate the Luttinger theorem in form (1) is to assume that the limit $T \to 0$ is discontinuous. Actually, one has to require that the whole line $T = 0$ is a line of phase transitions.
Is this famous theorem from 1960 correct?
simple problem: $n=1$

$SU(2)$

$\mu$

$U$
simple problem: \( n=1 \)

\[
SU(2)
\]

\[
\begin{align*}
\mu & \quad \text{and} \\
-\frac{U}{2} & \quad \uparrow \quad U
\end{align*}
\]
simple problem: $n=1$

$SU(2)$

$U/2$

$\mu$

$-U/2$

$U$
simple problem: \( n=1 \)

\[ G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} \]
simple problem: $n=1$

$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$
simple problem: \( n=1 \)

\[
G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0
\]

\[
n = 2\theta(0) = 1
\]
\[ G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2} \]
\[ n = 2\theta \left( \frac{2\mu}{\mu^2 - \left( \frac{U}{2} \right)^2} \right) \]

\[ G(\omega = 0) = \frac{2\mu}{\mu^2 - \left( \frac{U}{2} \right)^2} \]
\[
n = 2\theta \left( \frac{2\mu}{\mu^2 - \left( \frac{U}{2} \right)^2} \right)
\]

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G(\omega = 0) = \frac{2\mu}{\mu^2 - \left( \frac{U}{2} \right)^2}
\]
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\( n = 2\theta \left( \frac{2\mu}{\mu^2 - \left( \frac{U}{2} \right)^2} \right) \)

\[
G(\omega = 0) = \frac{2\mu}{\mu^2 - \left( \frac{U}{2} \right)^2}
\]

incompressible but \( \frac{\partial n}{\partial \mu} \neq 0 \)

A. Rosch, 2007
fix chemical potential

\[ \lim_{T \to 0} \mu(T) \]
fix chemical potential

\[ \lim_{T \to 0} \mu(T) \]

n=1
fix chemical potential

\[ \lim_{T \to 0} \mu(T) \]

n=1

does this fix all the problems?
No
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails
A model with zeros but Luttinger fails

no hopping => no propagation (zeros)
A model with zeros but Luttinger fails

no hopping => no propagation (zeros)
\[ H = \frac{U}{2} (n_1 + \cdots + n_N)^2 \]
no particle-hole symmetry

\begin{itemize}
\item a) $\frac{9}{2} U$
\item $2U$
\item $\frac{1}{2} U$
\item $0$
\item b) $\frac{3}{2} U$
\item $\frac{5}{2} U$
\end{itemize}
no particle-hole symmetry

\[
\lim_{T \to 0} \mu(T)
\]

Diagram:

- a) \( \frac{9}{2}U \)
- b) \( \frac{1}{2}U \)
- \( 2U \)
- \( \frac{1}{2}U \)
- \( 0 \)

Axes:

- \( \omega \)
- \( \mu \)

Points:

- \( \frac{1}{2}U \)
- \( \frac{3}{2}U \)
- \( \frac{5}{2}U \)
\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]
\[ G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n + 1) - K(n)} \left( \frac{2n - N}{N} \right) \]

\[ > 0 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]
Luttinger’s theorem

\[ n = N \Theta (2n - N) \]

\[ 0, 1, 1/2 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \quad 0, 1, 1/2 \]

\[ N = 3 \]
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \]
\[ N = 3 \]
\[ 2 = 3 \]
Luttinger’s theorem

\[ n = N \Theta (2n - N) \]

\[ n = 2 \]
\[ N = 3 \]

0, 1, 1/2
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\( n = 2 \)

\( N = 3 \)

\( \{0, 1, 1/2\} \)

\text{even}
Luttinger’s theorem

\[ n = N \Theta(2n - N) \]

\[ n = 2 \]

\[ N = 3 \]

\[ 0, 1, 1/2 \]

even

odd
Luttinger’s theorem

\[ n = N \Theta (2n - N) \]

\[ n = 2 \]

\[ N = 3 \]

Even: 0, 1, 1/2

Odd: no solution

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does the degeneracy matter?

\[ e^- t t t t t t t t \]

\[ t = 0^+ \]
$$G_{ab}(\omega) = \text{Tr} \left( \left[ c_a^\dagger \frac{1}{\omega - H} c_b + c_b \frac{1}{\omega - H} c_a^\dagger \right] \rho(0^+) \right)$$
\[
G_{ab}(\omega) = \text{Tr} \left( \left[ c_a^{\dagger} \frac{1}{\omega} c_b + c_b \frac{1}{\omega - H} c_a^{\dagger} \right] \rho(0^+) \right)
\]
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ c_a^\dagger \frac{1}{\omega} \right] c_b + c_b \left[ \frac{1}{\omega - U} c_a^\dagger \right] \rho(0^+) \right) \]
$$G_{ab}(\omega) = \text{Tr} \left( \left[ c_a^\dagger \frac{1}{\omega} c_b + c_b \frac{1}{\omega - U} c_a^\dagger \right] \rho(0^+) \right)$$

$$G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega (\omega - U)}$$
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ c_a^\dagger \frac{1}{\omega} c_b + c_b \frac{1}{\omega - U} c_a^\dagger \right] \rho(0^+) \right) \]

\[ \rho_{ab} = \text{Tr} \left( c_a^\dagger c_b \rho(0^+) \right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle \]

\[ G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega (\omega - U)} \]
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ \begin{array}{c} 1 \\ \frac{1}{\omega} \end{array} \right] c_b + c_b \left[ \begin{array}{c} 1 \\ \frac{1}{\omega - U} \end{array} \right] c_a^\dagger \right) \rho(0^+) \]

\[ \rho_{ab} = \text{Tr} \left( c_a^\dagger c_b \rho(0^+) \right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle \]

\[ G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega(\omega - U)} \]

\[ 1/N \text{diag}(1, 1, 1, \cdots) \]

mixed state
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ \frac{1}{\omega - U} c_{a}^\dagger c_{b} + \frac{1}{\omega} c_{b} c_{a}^\dagger \right] \rho(0^+) \right) \]

\[ \rho_{ab} = \text{Tr} \left( c_{a}^\dagger c_{b} \rho(0^+) \right) = \langle u_0 | c_{a}^\dagger c_{b} | u_0 \rangle \]

\[ G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega (\omega - U)} \]

as long as SU(N) symmetry is intact

zeros in the wrong place
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ c_a^\dagger \frac{1}{\omega} c_b + c_b \frac{1}{\omega - U} c_a^\dagger \right] \rho(0^+) \right) \]

\[ \rho_{ab} = \text{Tr} \left( c_a^\dagger c_b \rho(0^+) \right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle \]

\[ G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega (\omega - U)} \]

1/\text{N} \text{diag}(1, 1, 1, \cdots)

mixed state

as long as SU(N) symmetry is intact

zeros in the wrong place

limiting procedure: \( \lim \limits_{t \to 0} \lim \limits_{\omega \to 0} \)
\[ G_{ab}(\omega) = \text{Tr} \left( \left[ c_a^\dagger \frac{1}{\omega} \right] c_b + c_b \frac{1}{\omega - U} c_a^\dagger \right) \rho(0^+) \]

\[ \rho_{ab} = \text{Tr} \left( c_a^\dagger c_b \rho(0^+) \right) = \langle u_0 | c_a^\dagger c_b | u_0 \rangle \]

\[ G_{ab}(\omega) = \frac{\omega \delta_{ab} - U \rho_{ab}}{\omega (\omega - U)} \]

as long as SU(N) symmetry is intact

mixed state

zeros in the wrong place

limiting procedure: \( \lim_{t \to 0} \lim_{\omega \to 0} \)

limits do not commute
Problem

G=0
Problem

$G = 0$

$$G = \frac{1}{E - \varepsilon_p - \Sigma}$$
Problem

$G = 0$

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

lifetime of a particle vanishes
Problem

\[ G = 0 \]

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

\[ \Im \Sigma < \varepsilon_p \]

lifetime of a particle vanishes
Problem

G = 0

\[ G = \frac{1}{E - \varepsilon_p - \Sigma} \]

\( \exists \Sigma < \varepsilon_p \)

lifetime of a particle vanishes

\( \infty \)

no particle
what went wrong?
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

integral does not exist

if \( \Sigma \to \infty \)
what went wrong?

\[ \delta I[G] = \int d\omega \Sigma \delta G \]

if \( \Sigma \to \infty \)

integral does not exist

No Luttinger theorem!
Luttinger’s theorem
\[ n = 2 \sum_k \Theta(\mathcal{R}G(k, \omega = 0)) \]
\[ n = 2 \sum_{k} \Theta(\Re G(k, \omega = 0)) \]
\[ n = 2 \sum_k \Theta(\Re G(k, \omega = 0)) \]

only true for poles  
Gaussian fixed point
\[ n = 2 \sum_{k} \Theta(\mathcal{RG}(k, \omega = 0)) \]

- tautology
- only true for poles
- Gaussian fixed point

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experimental confirmation of violation?
zeros do not affect the particle density

experimental data (LSCO)

$\kappa_F$

$1 - x_{FS}$

`Luttinger’ count
Experimental data (LSCO) do not affect the particle density. 

`Luttinger' count

$\kappa_F$

$1 - x_{FS}$

Bi2212

Zeros do not affect the particle density.
Zeros do not affect the particle density. Each hole is not a single k-state.

Experimental data (LSCO) with $k_F$ and $1 - x_{FS}$ for Bi2212. The 'Luttinger' count is indicated in the graph with data points from Yang et al. (2011) and He et al. (2011).
how to count particles?
how to count particles?

some charged stuff has no particle interpretation
what is the extra stuff?
\[ \Sigma(\omega = 0, \mathbf{p}) = 0 \quad \text{Fermi liquid} \]

\[ \Sigma(\omega = 0, \mathbf{p}) = \infty \quad \text{new fixed point} \]
\[ \Sigma(\omega = 0, p) = 0 \]
Fermi liquid

\[ \Sigma(\omega = 0, p) = \infty \]
new fixed point

scale invariance
strongly correlated matter

new fixed point (scale invariance)
strongly correlated matter

new fixed point (scale invariance)
strongly correlated matter

new fixed point (scale invariance)

unparticles (IR) (H. Georgi)
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

mass
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ \Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

mass
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\[ x \rightarrow x/\Lambda \]

\[ \phi(x) \rightarrow \phi(x) \]

\[ \Lambda^2 \left( \frac{1}{2} \partial_\mu \phi \partial_\mu \phi \right) \]

\[ m^2 \phi^2 \]
massive free theory

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + m^2 \phi^2 \]

\( x \rightarrow x/\Lambda \\
\phi(x) \rightarrow \phi(x) \\
\Lambda^2 \left( \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right) \)

no scale invariance

mass
\[ \mathcal{L} = \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \]
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]
\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]

\[ x \rightarrow x / \Lambda \]

\[ m^2 / \Lambda^2 \rightarrow m^2 \]
\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) dm^2 \]

theory with all possible mass!

\[
\begin{align*}
\phi & \rightarrow \phi(x, m^2 / \Lambda^2) \\
x & \rightarrow x / \Lambda \\
m^2 / \Lambda^2 & \rightarrow m^2 \\
\mathcal{L} & \rightarrow \Lambda^4 \mathcal{L}
\end{align*}
\]

scale invariance is restored!!
\[
\mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2
\]

theory with all possible mass!

\[
\phi \rightarrow \phi(x, m^2/\Lambda^2)
\]

\[
x \rightarrow x/\Lambda
\]

\[
m^2/\Lambda^2 \rightarrow m^2
\]

\[
\mathcal{L} \rightarrow \Lambda^4 \mathcal{L}
\]

scale invariance is restored!!

not particles
unparticles

\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

theory with all possible mass!

\[ \phi \rightarrow \phi(x, m^2 / \Lambda^2) \]
\[ x \rightarrow x / \Lambda \]
\[ m^2 / \Lambda^2 \rightarrow m^2 \]
\[ \mathcal{L} \rightarrow \Lambda^4 \mathcal{L} \]

scale invariance is restored!!

not particles
\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^2|\gamma|
\]
propagator

\[
\left( \int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon} \right)^{-1} \propto p^{2|\gamma|}
\]

\[d_U - 2\]
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{dU-2} \]
\( n \) massless particles

\[
G \propto \left( E - \varepsilon_p \right)^{n-2}
\]

\[
G \propto \left( E - \varepsilon_p \right)^{d_U-2}
\]

unparticles=fractional number of massless particles
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U - 2} \]

unparticles=fractional number of massless particles

\[ d_U > 2 \]

zeros
n massless particles

\[ G \propto (E - \varepsilon_p)^{n-2} \]

\[ G \propto (E - \varepsilon_p)^{d_U - 2} \]

unparticles=fractional number of massless particles

\[ d_U > 2 \]

zeros

no simple sign change
no particle interpretation
no particle interpretation

\[ \phi U \neq \int B(m^2) \phi(m^2) dm^2 \]
\[ Z \neq \int \mathcal{D}\phi_U \exp \left \{ \frac{i}{2} \int d^d p \phi_U(p) G^{-1}_U \phi_U(-p) \right \} \]

no particle interpretation

\[ \phi_U \neq \int B(m^2) \phi(m^2) dm^2 \]
\[ dU \]
what really is the summation over mass?
what really is the summation over mass?

mass=energy
\[ \mathcal{L} = \int_0^\infty \left( \partial^\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]
what has been done?  

\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial_{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^{2} \phi^{2}(x, m) \right) dm^{2} \]

resembles action on AdS with

Stephanov, Terning, 2008/09
what has been done?

\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial^{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

resembles action on AdS with

\[ m^2_{\text{AdS}} = \frac{d_U (d_U - d)}{R^2} \]
\[ \mathcal{L} = \int_0^\infty \left( \partial_\mu \phi(x, m) \partial_\mu \phi(x, m) + m^2 \phi^2(x, m) \right) \, dm^2 \]

resembles action on AdS with

\[ m_{AdS}^2 = \frac{d_U (d_U - d)}{R^2} \]

d_\_U is a free parameter
can something more exact be done?
can something more exact be done?

\[ \mathcal{L} = \int_{0}^{\infty} \left( \partial_{\mu} \phi(x, m) \partial_{\mu} \phi(x, m) + m^2 \phi^2(x, m) \right) m^{2\delta} \, dm^2 \]
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\[ m = z^{-1} \quad m_{\text{AdS}}^2 = \frac{d_U (d_U - d)}{R^2} \]
can something more exact be done?

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\[ \frac{1}{R^2} = \frac{d_U (d_U - d)}{R^2} \]
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$$m = z^{-1}$$

$$\frac{1}{R^2} = \frac{d_U (d_U - d)}{R^2}$$

$$\mathcal{L} = \int_0^\infty dz \frac{2R^2}{z^{5+2\delta}} \left[ \frac{1}{2} \frac{z^2}{R^2} \eta^{\mu \nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{\phi^2}{2R^2} \right]$$
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can be absorbed with AdS metric
action on $AdS_{5+2\delta}$

\[ S = \frac{1}{2} \int d^{4+2\delta}x \, dz \, \sqrt{-g} \left( \partial_a \Phi \partial^a \Phi + \frac{\Phi^2}{R^2} \right) \]

\[ \sqrt{-g} = (R/z)^{5+2\delta} \]
action on $AdS_{5+2\delta}$

$$S = \frac{1}{2} \int d^{4+2\delta}x \, dz \, \sqrt{-g} \left( \partial_\alpha \Phi \partial^\alpha \Phi + \frac{\Phi^2}{R^2} \right)$$

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unparticle lives in

\[ d = 4 + 2\delta \quad \delta \leq 0 \]
action on $AdS_{5+2\delta}$

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unparticle lives in

$$d = 4 + 2\delta \quad \delta \leq 0$$

generating functional for unparticles
Claim: \[ Z_{\text{QFT}} = e^{-S_{\text{on-shell}}^{\text{ADS}}(\phi(\phi \partial_{\text{ADS}} = J \phi))} \]

\[ S = \frac{1}{2} \int d^d x \ g^{zz} \sqrt{-g} \ \Phi(z, x) \partial_z \Phi(z, x) \bigg|_{z=\epsilon} \]
Claim: \( Z_{\text{QFT}} = e^{-S_{\text{on-shell}}^{\text{on-shell}}(\phi(\phi \partial_{\text{ADS}} = J_\phi))} \)

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S = \frac{1}{2} \int d^d x \ g^{zz} \sqrt{-g} \ \Phi(z, x) \partial_z \Phi(z, x) \bigg|_{z=\epsilon}
\]

\[
\langle \Phi_U(x) \Phi_U(x') \rangle = \frac{1}{|x - x'|^{2d_U}}
\]
Claim: \[ Z_{\text{QFT}} = e^{-S_{\text{on-shell}}^{\text{ADS}}(\phi(\phi \partial \text{ADS} = J \mathcal{O}))} \]

\[ S = \frac{1}{2} \int d^d x \ g^{zz} \sqrt{-g} \Phi(z, x) \partial_z \Phi(z, x) \bigg|_{z=\epsilon} \]

\[ \langle \Phi_U(x) \Phi_U(x') \rangle = \frac{1}{|x - x'|^{2d_U}} \]

\[ d_U = \frac{d}{2} + \frac{\sqrt{d^2 + 4}}{2} > \frac{d}{2} \]
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scaling dimension is fixed
\[ G_U(p) \propto p^{2(d_U - d/2)} \]

\[ d_U = \frac{d}{2} + \frac{\sqrt{d^2 + 4}}{2} > \frac{d}{2} \]

\[ G_U(0) = 0 \]
unparticle (AdS) propagator has zeros!

\[ G_U(p) \propto p^{2(d_U - d/2)} \]

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Universal fermionic spectral functions from string theory

Jerome P. Gauntlett, Julian Sonner, and Daniel Waldram

1 Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.
2 D.A.M.T.P. University of Cambridge, Cambridge, CB3 0WA, U.K.

We carry out the first holographic calculation of a fermionic response function for a strongly coupled $d = 3$ system with an explicit $D = 10$ or $D = 11$ supergravity dual. By considering the supersymmetry current, we obtain a universal result applicable to all $d = 3$ $N = 2$ SCFTs with such duals. Surprisingly, the spectral function does not exhibit a Fermi surface, despite the fact that the system is at finite charge density. We show that it has a phonino pole and at low frequencies there is a depletion of spectral weight with a power-law scaling which is governed by a locally quantum critical point.
Similar Problem

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\[
(\mathcal{D} - m - \frac{i}{2} F^{\mu\nu} \Gamma_{\mu\nu}) \psi_\rho + iF_{\mu\nu} \Gamma_\mu \Gamma_\rho \psi_\nu = 0
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bottom-up: with variable Pauli term

Consider

\[ \sqrt{-g}i\bar{\psi}(\not{D} - m - ip\not{F})\psi \]
bottom-up: with variable Pauli term

consider \( \sqrt{-g} i \bar{\psi} (\mathcal{D} - m - ipF) \psi \)

\[-\psi''_{I_{\pm}}(\zeta) = i \sigma_2 \left( 1 + \frac{q e_d}{\zeta} \right) - \frac{L_2}{\zeta} \left[ m \sigma_3 + \left( p e_d \pm \frac{k L}{r_0} \right) \sigma_1 \right] \psi^{(0)}_{I_{\pm}}(\zeta), \]
bottom-up: with variable Pauli term

consider \[ \sqrt{-g} i \psi (\slashed{D} - m - ipF) \psi \]

\[-\psi^{(0)''}_I(\zeta) = i \sigma_2 \left( 1 + \frac{q e_d}{\zeta} \right) - \frac{L_2}{\zeta} \left[ m_3 + \left( p e_d \pm \frac{k L}{r_0} \right) \sigma_1 \right] \psi^{(0)}_I(\zeta), \]

\[ e_d = \frac{1}{\sqrt{2d(d-1)}} \]

\[ m^2_k = m^2 + \left( p e_d \pm \frac{k L}{r_0} \right)^2 \]

p changes scaling dimension
Consider \( \sqrt{-g} i \bar{\psi} (\not{D} - m - ipF) \psi \)

\[
-\psi_{I_{\pm}}^{(0)}''(\zeta) = i \sigma_2 \left( 1 + \frac{q e_d}{\zeta} \right) - \frac{L_2}{\zeta} \left[ m \sigma_3 + \left( p e_d \pm \frac{k L}{r_0} \right) \sigma_1 \right] \psi_{I_{\pm}}^{(0)}(\zeta),
\]

\[
e_d = 1 / \sqrt{2d(d - 1)}
\]

\[
m_k^2 = m^2 + \left( p e_d \pm \frac{k L}{r_0} \right)^2
\]

\[
\nu_k^{\pm} = \sqrt{m_k^{2} L_2^2 - q^2 e_d^2} - i \epsilon,
\]

\( p \) changes scaling dimension
bottom-up: with variable Pauli term

consider

\[ \sqrt{-g} i \bar{\psi} (\not{D} - m - i p F) \psi \]

\[-\psi_{I \pm}^{(0)''}(\zeta) = i \sigma_2 \left( 1 + \frac{q e_d}{\zeta} \right) - \frac{L_2}{\zeta} \left[ m \sigma_3 + \left( p e_d \pm \frac{k L}{r_0} \right) \sigma_1 \right] \psi_{I \pm}^{(0)}(\zeta), \]

\[ \nu_{k \pm} = \sqrt{m_{k \pm}^2 + \frac{k L}{r_0}^2 - q^2 e_d^2 - i \epsilon}, \]

\[ m_{k \pm}^2 = m^2 + \left( p e_d \pm \frac{k L}{r_0} \right)^2 \]

\[ e_d = \frac{1}{\sqrt{2d(d-1)}} \]

p changes scaling dimension

increasing p should suppress the spectral weight at 0
How is the spectrum modified?

$P=0$

Fermi surface peak
How is the spectrum modified?

P=0

Fermi surface peak
How is the spectrum modified?

\[ -1.54 < p < -0.53 \]

\[ 1 > \nu_{k_F} > 1/2 \]

\[ \Re \omega \propto k - k_F \]

\[ \Im \omega \propto (k - k_F)^{2\nu_{k_F}} \]

‘Fermi Liquid’
How is the spectrum modified?

$P=0$

Fermi surface peak
How is the spectrum modified?

P=0

\[ p = -0.53 \]
\[ \nu_{k_F} = 1/2 \]

-0.53 < p < 1/\sqrt{6}
\[ 1/2 > \nu_{k_F} > 0 \]
\[ \Re \omega = \Im \omega \propto (k - k_F)^{1/(2\nu_{k_F})} \]

Fermi surface peak

MFL

NFL
How is the spectrum modified?

$p = 0$

$\nu_{k_F} = 1/2$

$-0.53 < p < 1/\sqrt{6}$

$1/2 > \nu_{k_F} > 0$

$\Re \omega = \Im \omega \propto (k - k_F)^{1/(2\nu_{k_F})}$

Monday, September 16, 13
How is the spectrum modified?

P=0

Fermi surface peak
How is the spectrum modified?

P=0

Fermi surface peak

P > 4.2
How is the spectrum modified?

\[ P = 0 \]

matches unparticle picture with large \( d_U \)
How is the spectrum modified?

$P = 0$

Fermi surface peak

matches unparticle picture with large $d_U$

Edalati, Leigh, PP PRL, 106 (2011)
Superconducting Instability

ladder approximation

\[ 1 = \lambda T \sum_{n\vec{k}} |w_{n\vec{k}}|^2 G_U (\omega_n, \vec{k}) G_U (-\omega_n, -\vec{k}) , \]
\[ 1 = g \left( \frac{T}{W} \right)^{4d_U - d}, \quad T/W < 1 \]
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**zeros condition**

\[ d_U > d/2 \]
\[ 1 = g \left( \frac{T}{W} \right)^{4d_U - d}, \quad T/W < 1 \]

zeros condition \[ d_U > d/2 \]

\[ \frac{d \ln g}{d \ln \beta} = 4d_U - d > 0 \]
tendency towards pairing (any instability which establishes a gap)
tendency towards pairing (any instability which establishes a gap)
interchanging unparticles

fractional \((d_U)\) number of massless particles
interchanging unparticles

fractional (d_U) number of massless particles
interchanging unparticles

fractional (d_U) number of massless particles
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fractional \((d_U)\) number of massless particles

\[
e^{i\pi d_U} \neq -1, 0
\]
interchanging unparticles

fractional \((d_U)\) number of massless particles

\[ e^{i\pi d_U} \neq -1, 0 \]

fractional statistics in \(d=2+1\)
\[
\begin{align*}
d_U &= d + \frac{\sqrt{d^2 + 4}}{2} > \frac{d}{2} \\
\end{align*}
\]

\[
e^{i\pi d_U} \neq e^{-i\pi d_U}
\]

**time-reversal symmetry breaking from unparticle (zeros=Fermi arcs) matter**
breaking of scale invariance

unparticles

particles

TRSB
Fermi arcs
Fermi arcs

zeros

unparticles

no Luttinger count
Fermi arcs

zeros → no Luttinger count

unparticles

$AdS_2$

Pauli `gap'

enhanced SC instability

TRSB