Superconductivity and Mottness: Exact Results

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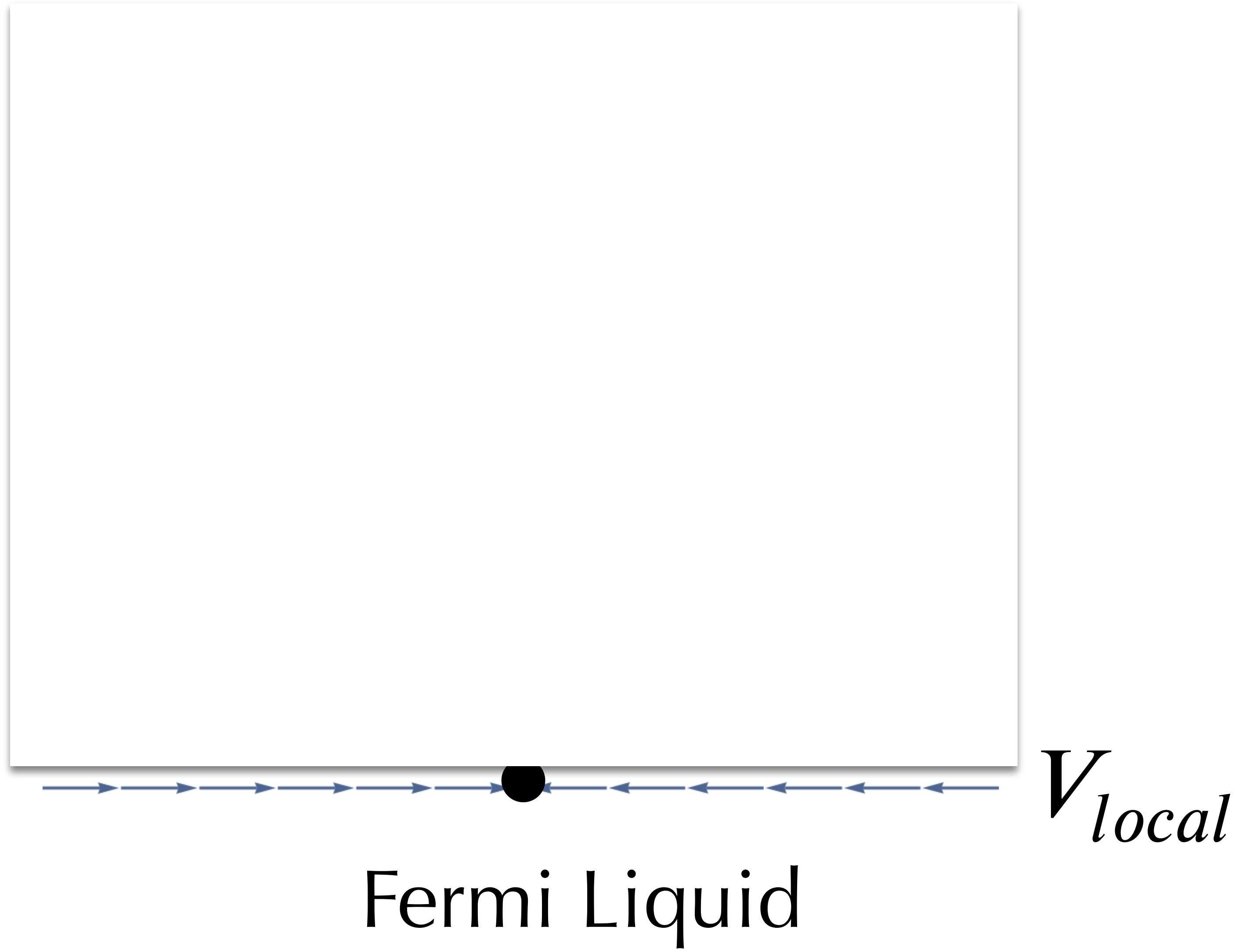
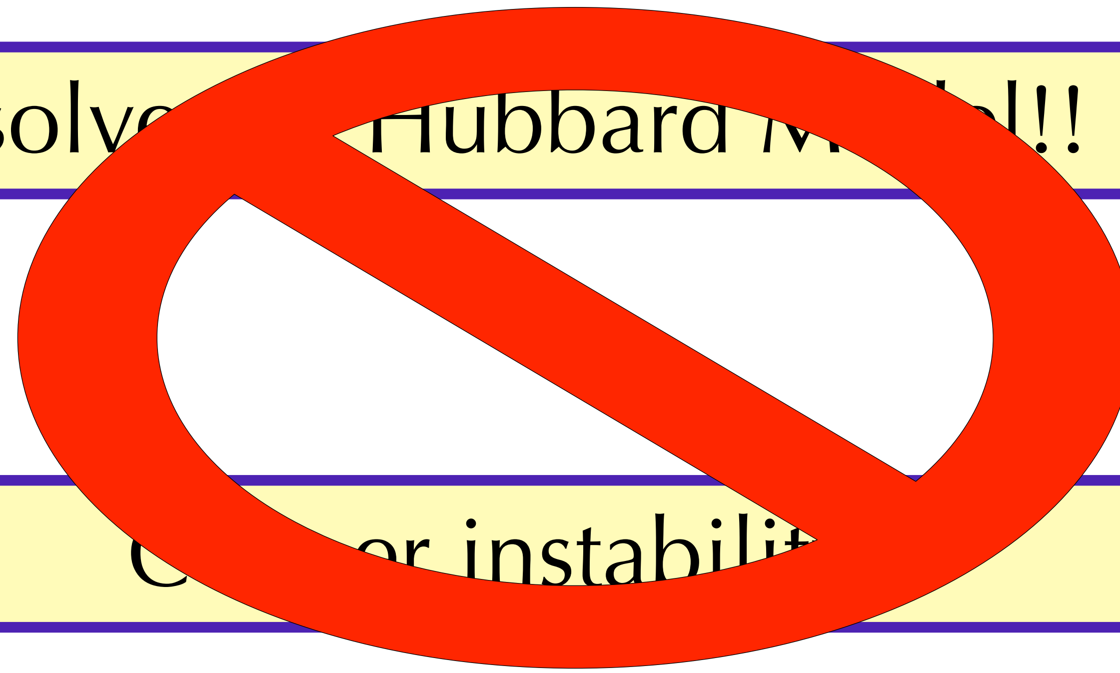
Luke Yeo	Edwin Huang	G. La Nave	Jinchao Z.
			



fixed point beyond FL?

quartic interacting theory?

solv[illegible] Hubbard M[illegible]l!!



C[illegible]er instabilit[illegible]

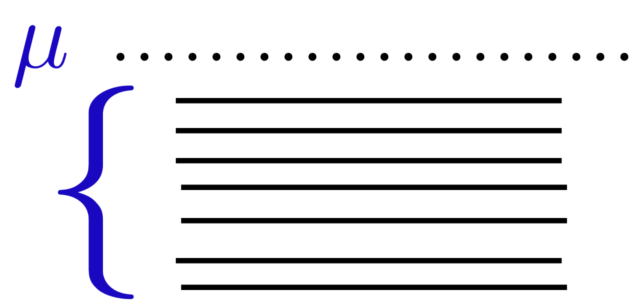
Is there a simpler approach that has Mottness?

Yes!

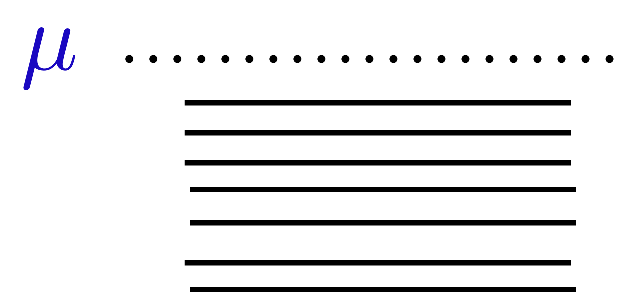
Fermi liquids

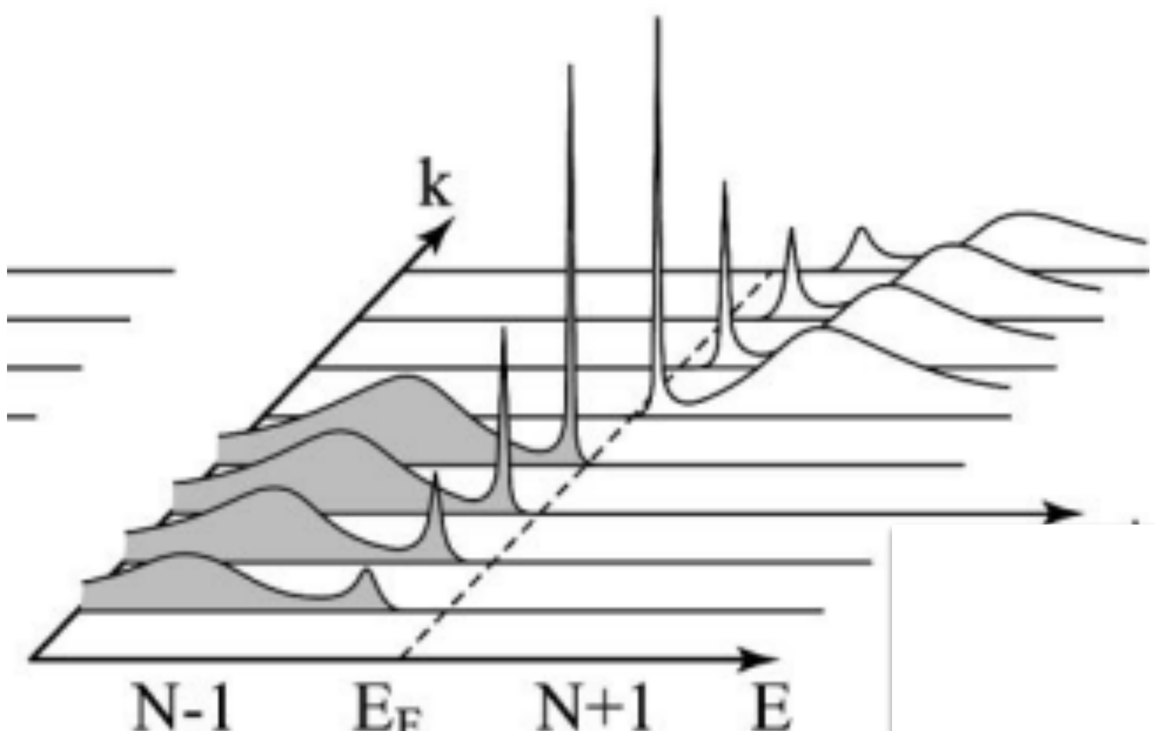
NFL

doubly occupied



Is single occupancy below chemical potential possible?



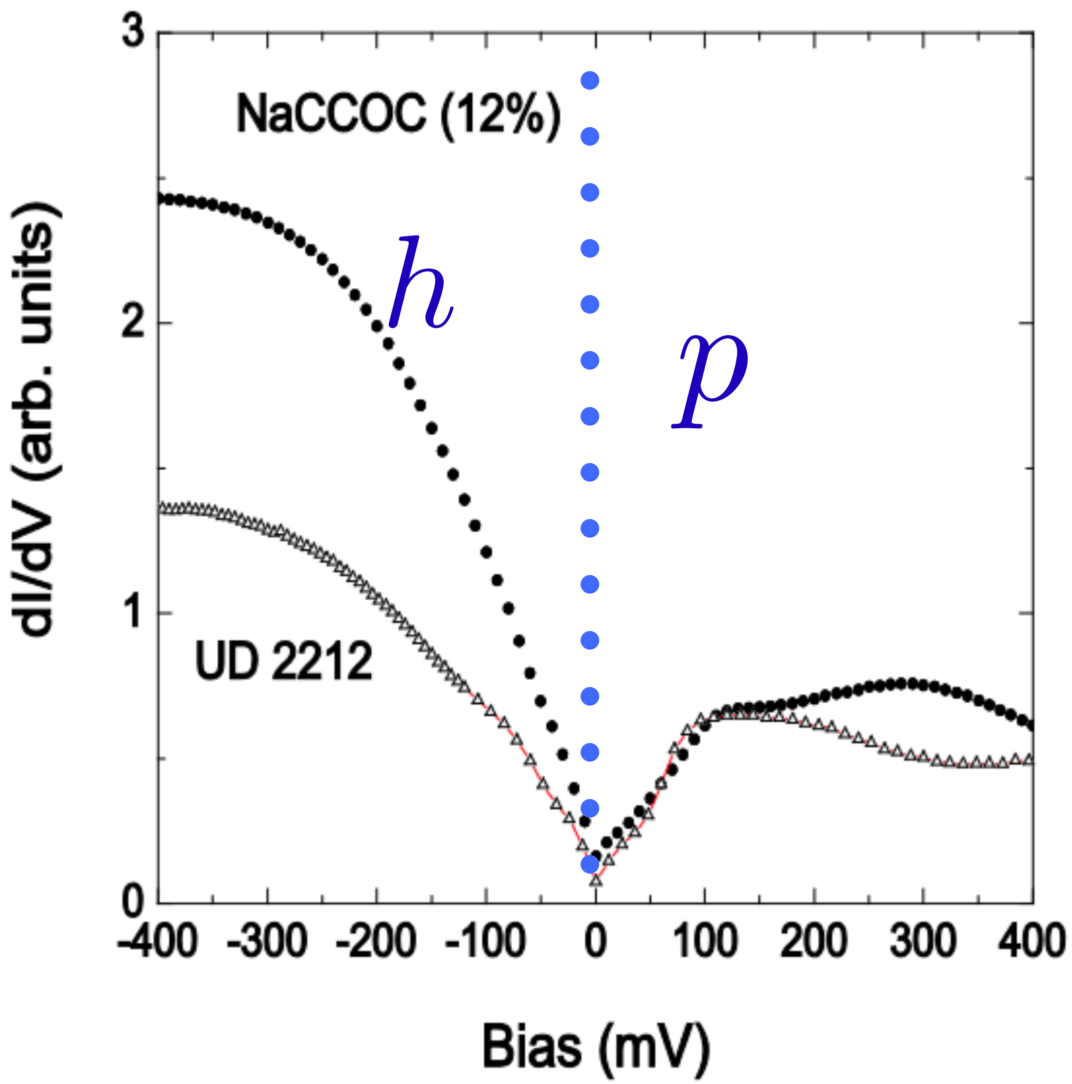


Fermi-liquid system

with time-reversal symmetry in tact?

p-h symmetry

p-h asymmetry



LAST WORDS ON THE CUPRATES

P W Anderson, Princeton University

theory. I remain baffled by the almost universal refusal of theorists to confront this evident fact of hole-particle asymmetry head on. Its meaning is that the first step of any single occupancy

?

particle-hole asymmetry

Anderson
Haldane
2000

3 citations

Fermi liquids

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + \cdots \quad (\cdots \to 0)$$

$(n_{p\uparrow}, n_{p\downarrow})$ conserved currents

$(c_{p\uparrow}, c_{p\downarrow}, \mathrm{h.c.})$ 4 objects

$O(4)$

$\mathrm{Det M} = 1$ → SO(4) proper rotations

$\mathrm{Det M} = -1$ → improper rotations

$$\mathrm{Det}\, M = \pm 1 \implies Z_2 = O(4) \div SO(4)$$

Improper Rotations

Majorana basis

$$\begin{pmatrix} c_{p\uparrow} \\ c^\dagger_{p\uparrow} \\ c_{p\downarrow} \\ c^\dagger_{p\downarrow} \end{pmatrix} \longrightarrow \begin{pmatrix} c_{p\uparrow} + c^\dagger_{p\uparrow} \\ i(c_{p\uparrow} - c^\dagger_{p\uparrow}) \\ c_{p\downarrow} + c^\dagger_{p\downarrow} \\ i(c_{p\downarrow} - c^\dagger_{p\downarrow}) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} c_{p\uparrow} + c^\dagger_{p\uparrow} \\ i(c_{p\uparrow} - c^\dagger_{p\uparrow}) \\ c_{p\downarrow} + c^\dagger_{p\downarrow} \\ i(c_{p\downarrow} - c^\dagger_{p\downarrow}) \end{pmatrix}$$

$$c_{p\downarrow} \rightarrow c^\dagger_{p\downarrow}$$

p-h transformation

$$\epsilon(p) = \epsilon_F$$

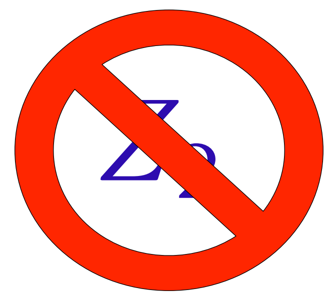
Fermi Surface

$$H = 0$$

$$\left.\begin{aligned} n_{p\uparrow} &\to 1 - n_{p\uparrow} \\ n_{p\downarrow} &\to n_{p\downarrow} \end{aligned}\right\} Z_2$$

at Fermi surface only

How to destroy Fermi liquids?



$$H = \sum_{p,\sigma} \left(\epsilon(p) - \epsilon_F\right) n_{p\sigma} + U n_{p\uparrow} n_{p\downarrow}$$

odd under Z_2

scaling dimension

$[n_{p\uparrow} n_{p\downarrow}] = -2$

relevant interaction

New fixed point!

Hatsugai-Kohmoto or Baskaran model

General HK Model

$$\sum_k (\xi_k (n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow})$$

Solvable Mott transition:U>W

$$G_{k\sigma}(i\omega_n \to z) = \frac{1 - \langle n_{k\bar{\sigma}} \rangle}{z - \xi_k} + \frac{\langle n_{k\bar{\sigma}} \rangle}{z - (\xi_k + U)} \neq \frac{1}{z - \omega_k}$$

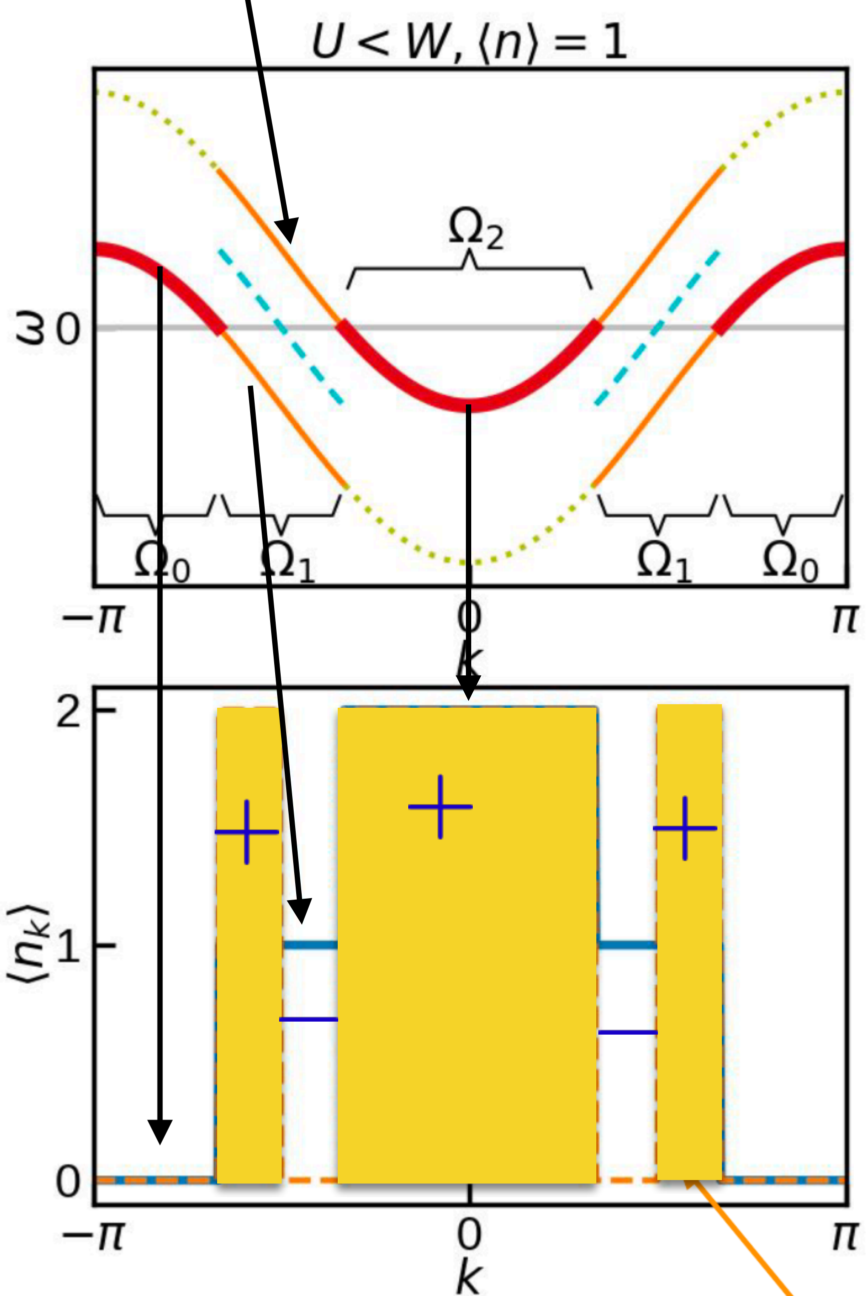
lower Hubbard band

upper Hubbard band

zeros

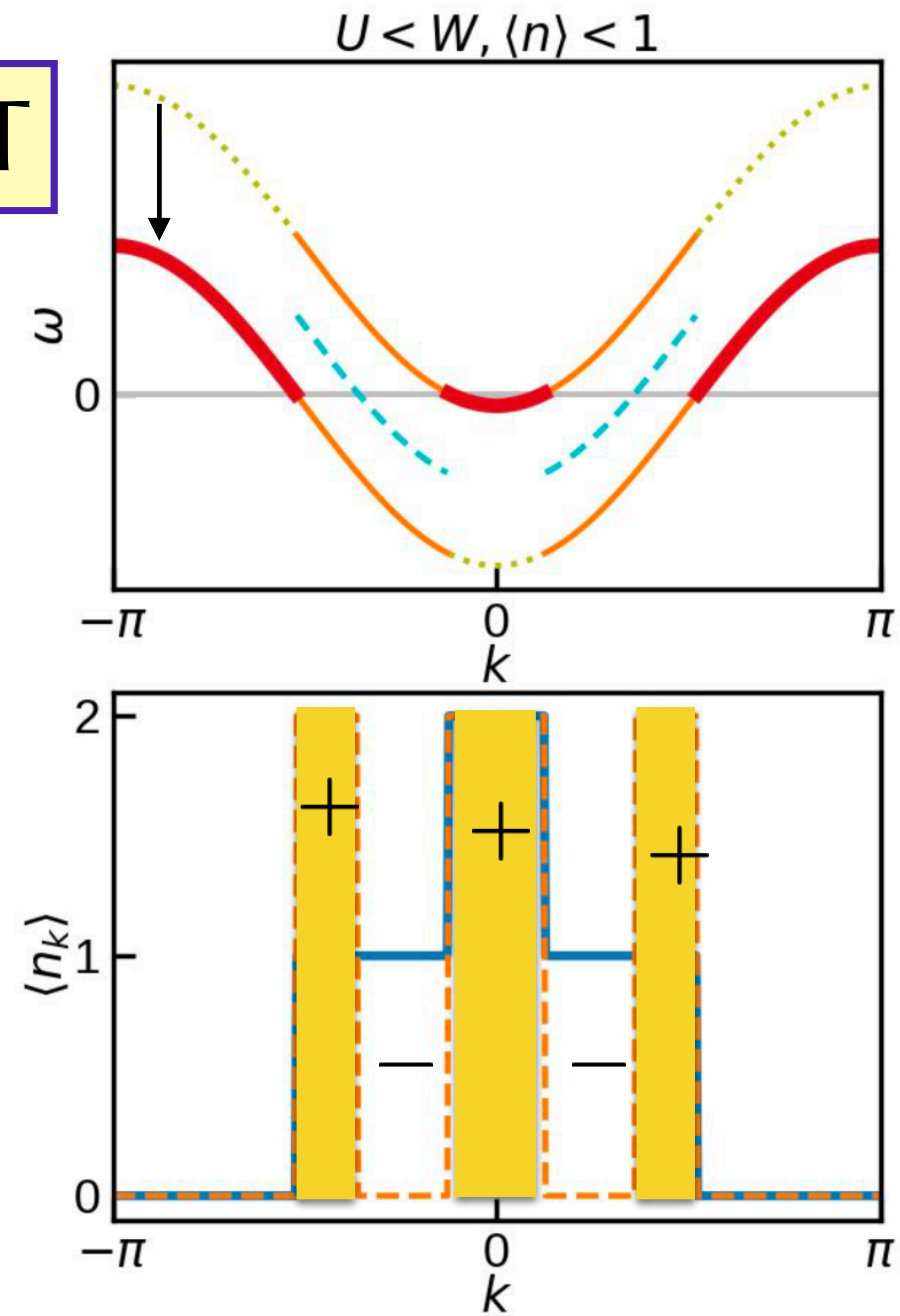
single occupancy

counting charges



$n_{\mathrm{Lutt}} = \langle n \rangle$

SWT



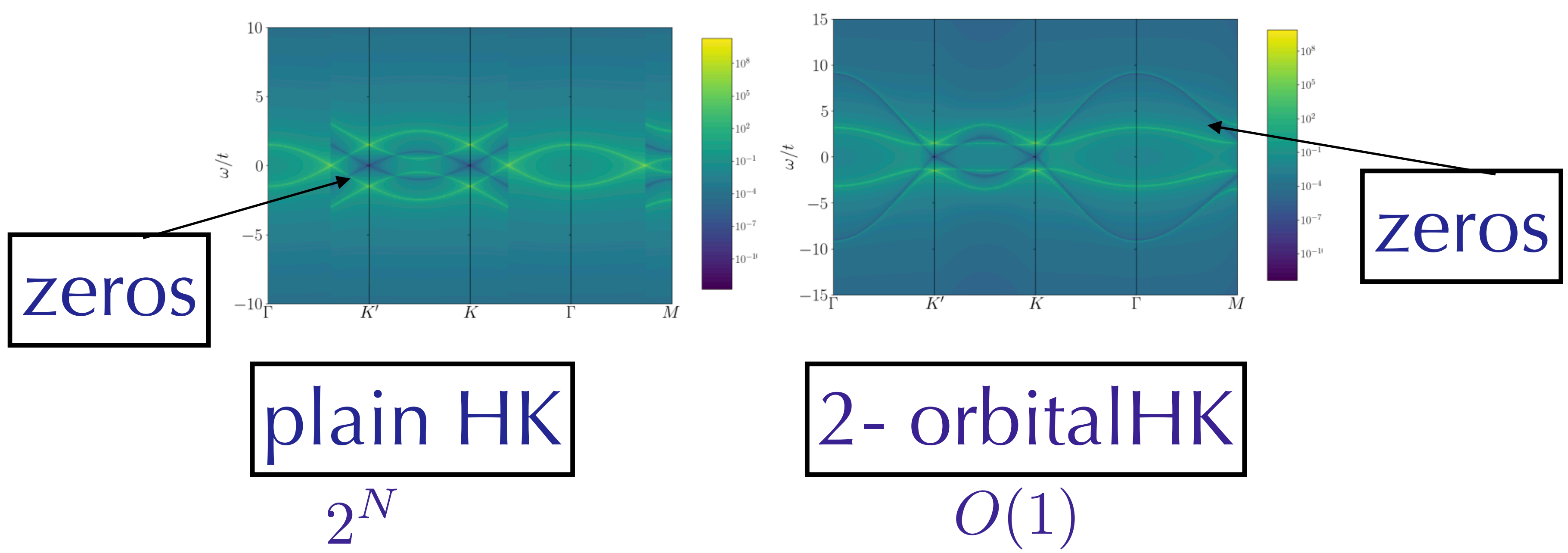
$n_{\mathrm{Lutt}} \neq \langle n \rangle$

zeros $\neq$ particles

Ground state stability, symmetry, and degeneracy in Mott insulators with long range interactions

Dmitry Manning-Coe* and Barry Bradlyn†
Department of Physics and Institute for Condensed Matter Theory,
University of Illinois at Urbana-Champaign, Urbana IL, 61801-3080, USA
(Dated: June 2, 2023)

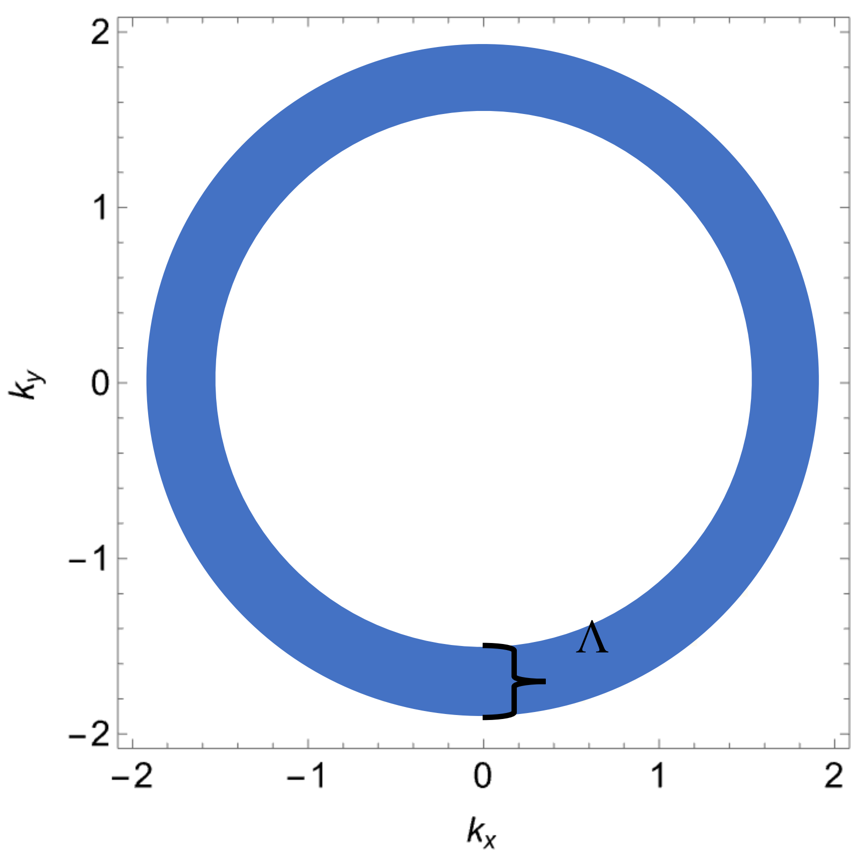
Recently, models with long-range interactions—known as Hatsugai-Kohmoto (HK) models—have emerged as a promising tool to study the emergence of superconductivity and topology in strongly correlated systems. Two obstacles, however, have made it difficult to understand the applicability of these models, especially to topological features: they have thermodynamically large ground state degeneracies, and they tacitly assume spin conservation. We show that neither are essential to HK models and that both can be avoided by introducing interactions between tight-binding states in the orbital basis, rather than between energy eigenstates. To solve these "orbital" models, we introduce a general technique for solving HK models and show that previous models appear as special cases. We illustrate our method by exactly solving graphene and the Kane-Mele model with HK interactions.



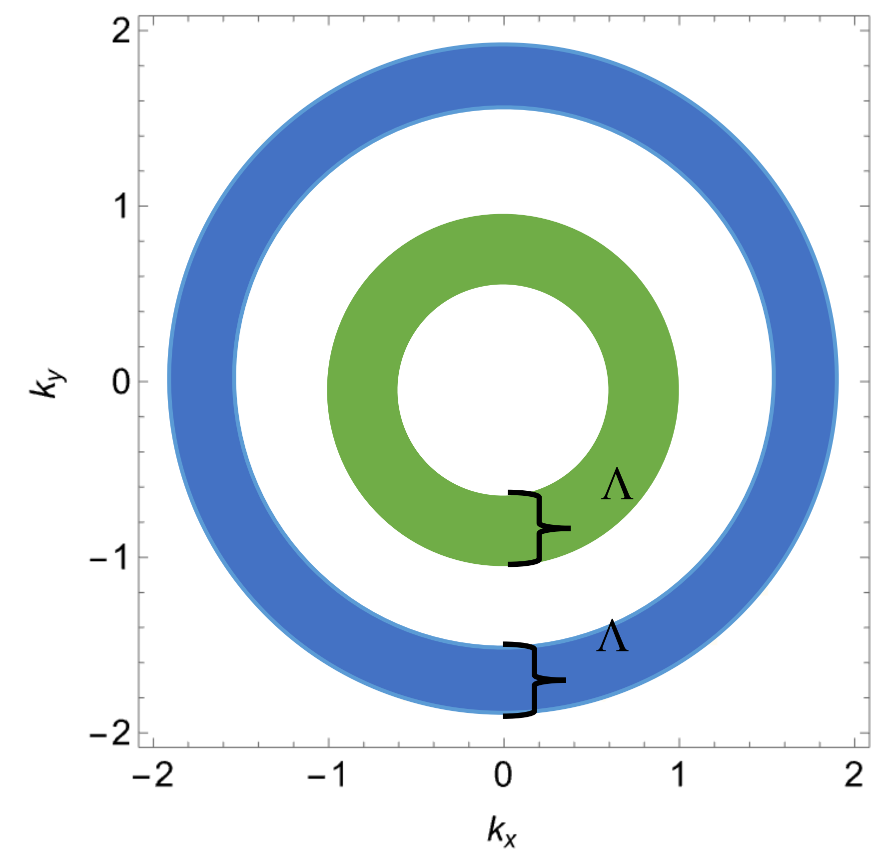
$$H_{\mathrm{HK}} + V_{\mathrm{local}}$$

$$\delta S_4 = \frac{1}{4}\int u(4,3,2,1)\bar{\psi}(4)\bar{\psi}(3)\psi(2)\psi(1)\times$$

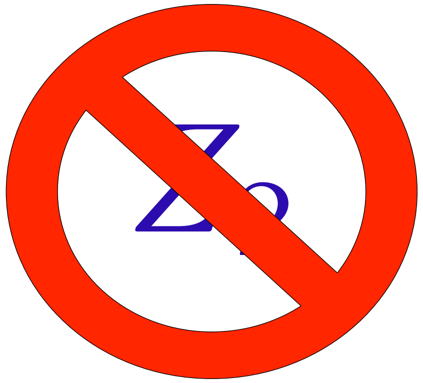
$$\prod_{i=1}^{4}\frac{d^d k_i d\omega_i}{(2\pi)^{d+1}}\delta(\mathbf{k_1}+\mathbf{k_2}-\mathbf{k_3}-\mathbf{k_4})\delta(\omega_1+\omega_2-\omega_3-\omega_4)$$



$$\Lambda \to \Lambda/s$$
$$q' = qs$$
$$\omega' = \omega/s$$
$$\psi'(k',\omega') = s^{-3/2}\psi(k,\omega)$$
$$U' = s^2 U$$



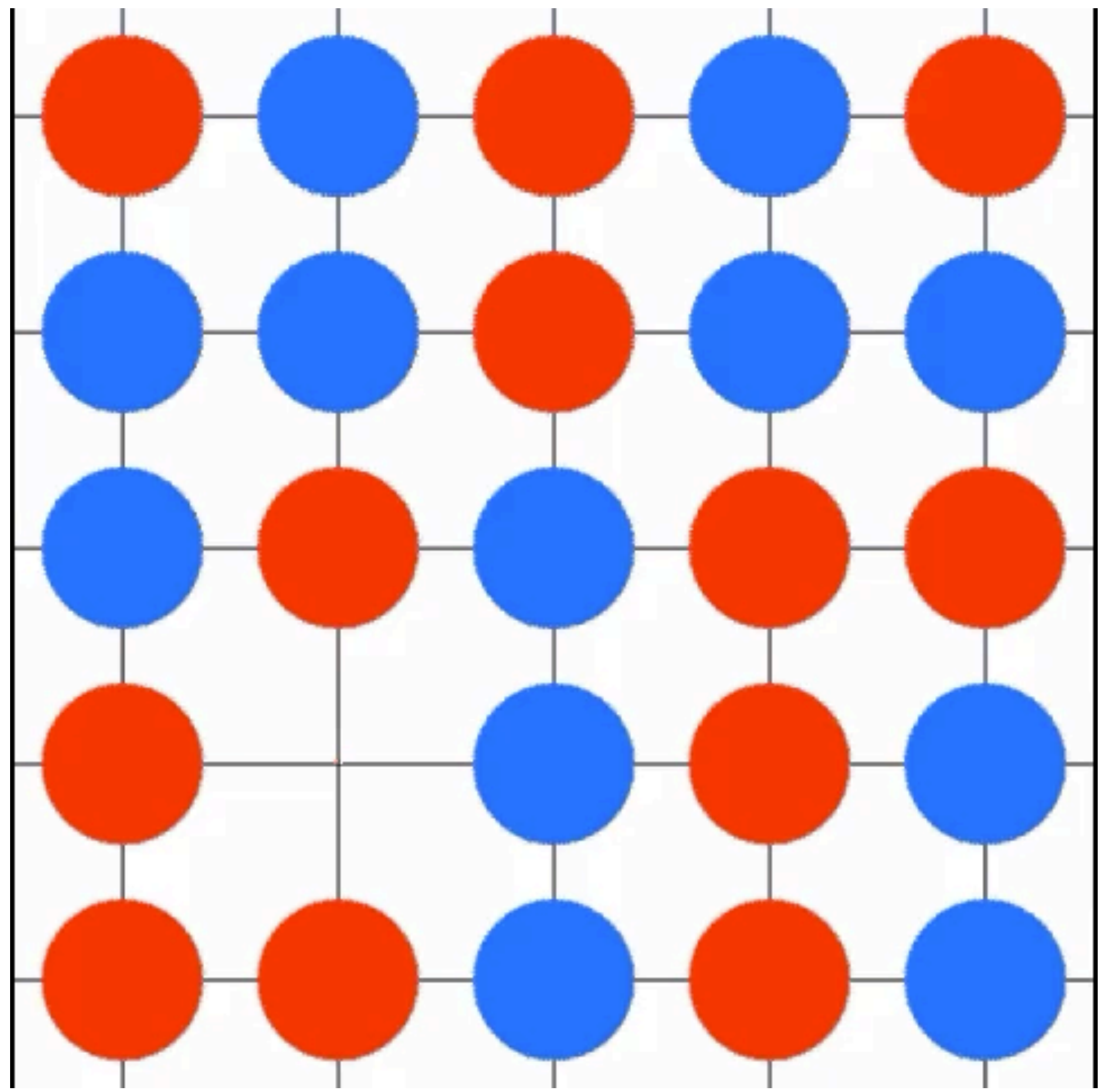
Local (Hubbard) part of 4-Fermion interactions are still irrelevant



	FL	HK
quantum numbers	k	k
stability: V_{local}	$\beta(V_{\text{local}}) = 0$	$\beta(V_{\text{local}}) = 0$
stability: V_{SC}	$\beta(V_{\text{SC}}) \to \infty$	?

what does the HK model leave out??

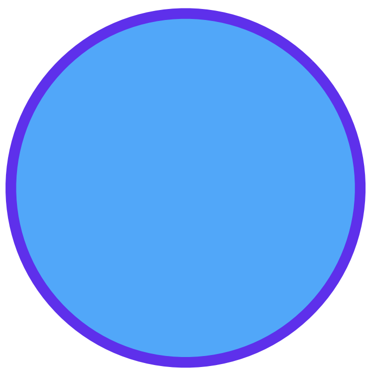
$$[H_t, H_U] \neq 0$$



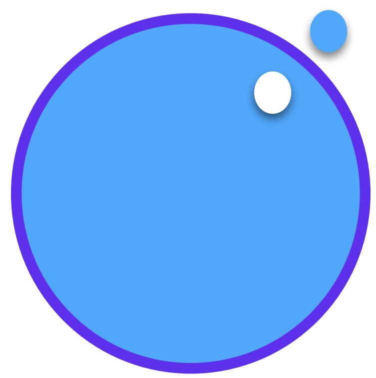
dynamical spectral weight transfer

Mottness

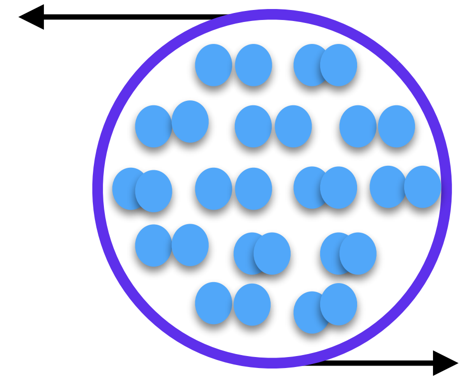
Fermi gas

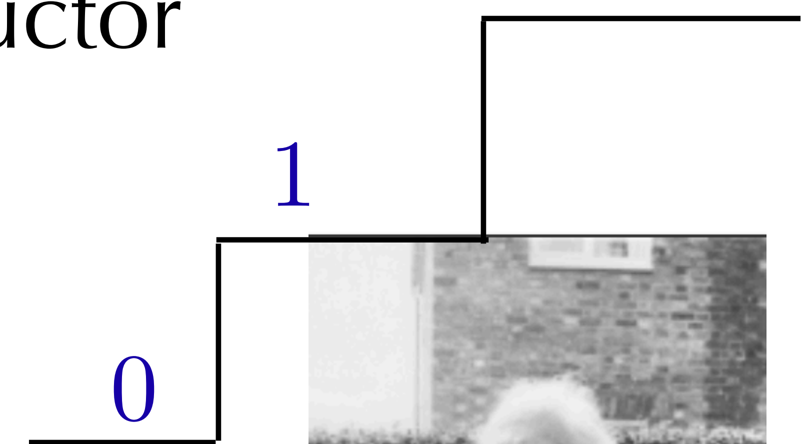


Fermi liquid



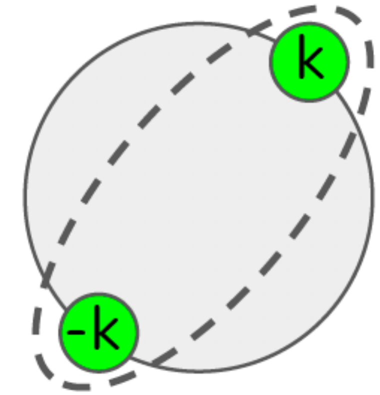
BCS superconductor



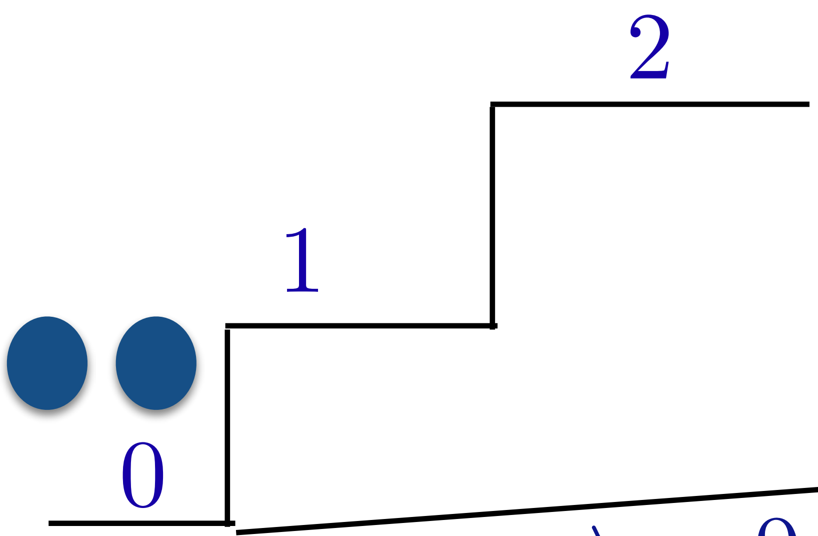


Superconductivity?

Cooper Instability



$$H = H_{\mathrm{HK}} - gH_p$$



$$|\psi\rangle = \sum_{k\in\Omega_0} \alpha_k b_k^\dagger |\mathrm{GS}\rangle$$

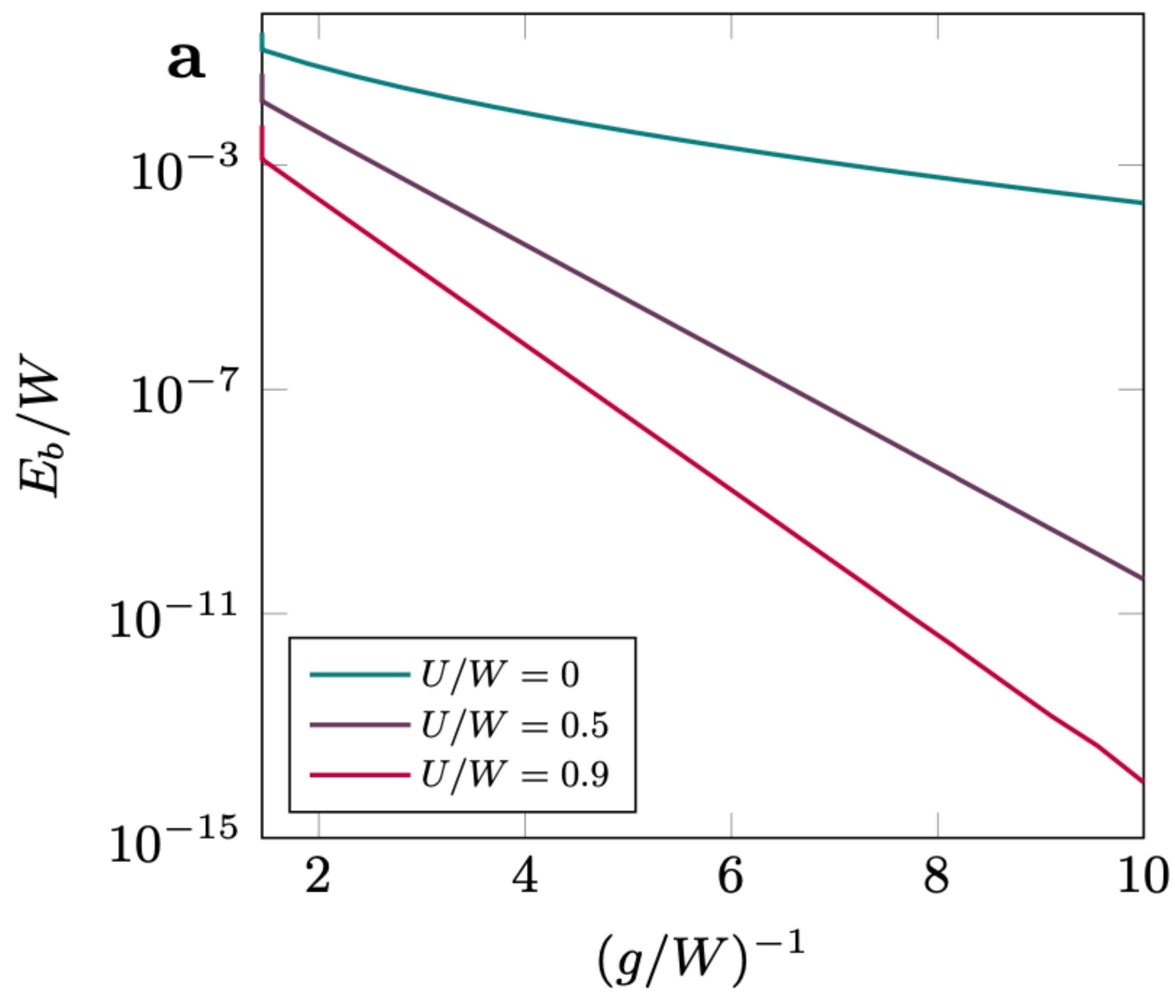
$$= \sum_{k,k'} b_k^\dagger b_{k'}$$

$$\langle n_{k\sigma}\rangle = 0$$

$$E_b = \langle \mathrm{GS}|\mathrm{H}|\mathrm{GS}\rangle - \langle\psi|\mathrm{H}|\psi\rangle \lessgtr 0$$

Cooper Instability

$$E_b = -E \sim W(1-(U/W)^2)e^{-\pi W\sqrt{1-(U/W)^2}/g}$$



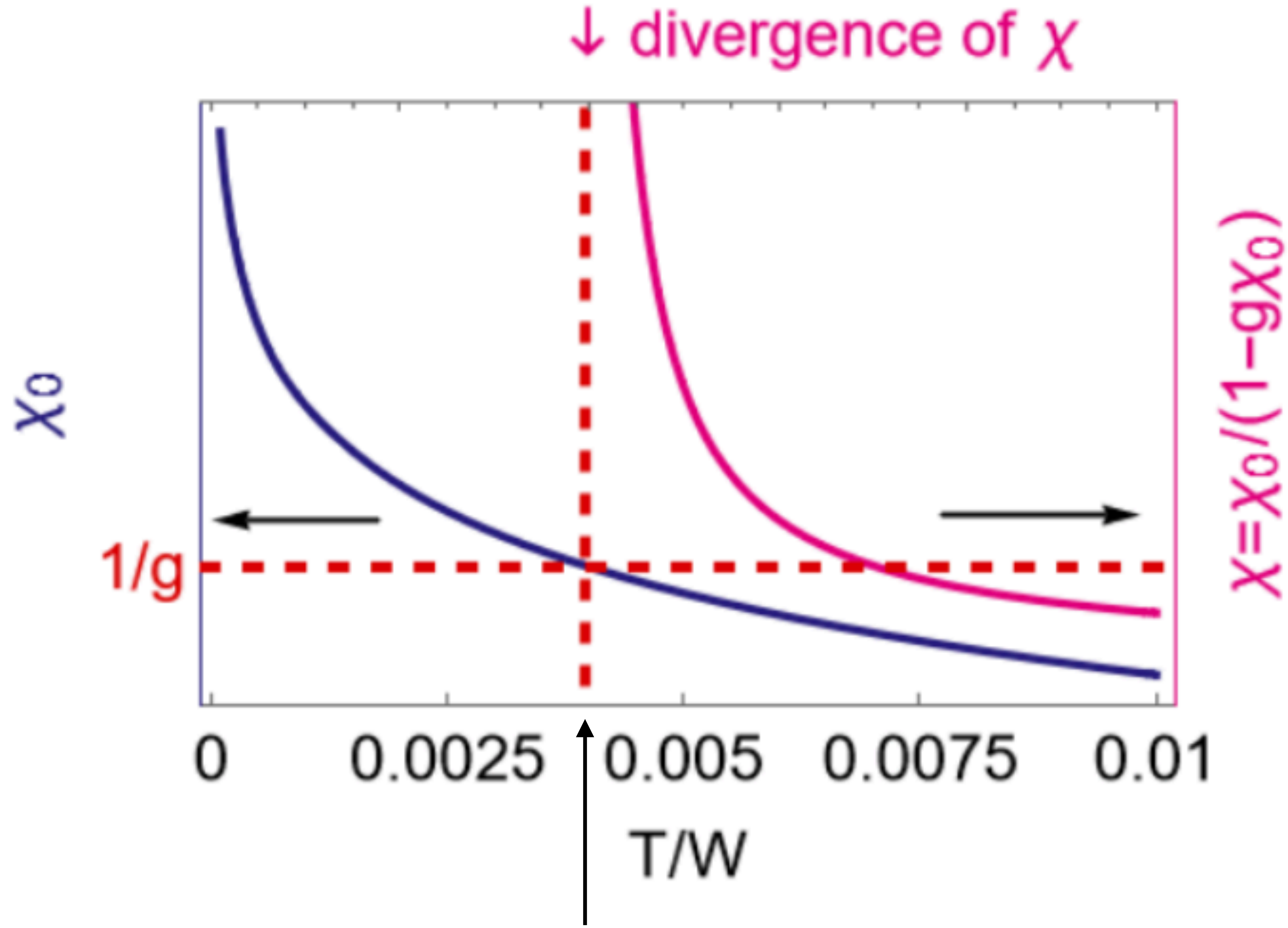
Pair Susceptibility

$$\chi(i\nu_n) \equiv \frac{1}{L^d}\int_0^\beta d\tau e^{i\nu_n\tau}\langle T\Delta(\tau)\Delta^\dagger\rangle_g$$

$$= \frac{\chi_0}{1 - g\chi_0}$$

$g\chi_0 = 1$

solve for T_c

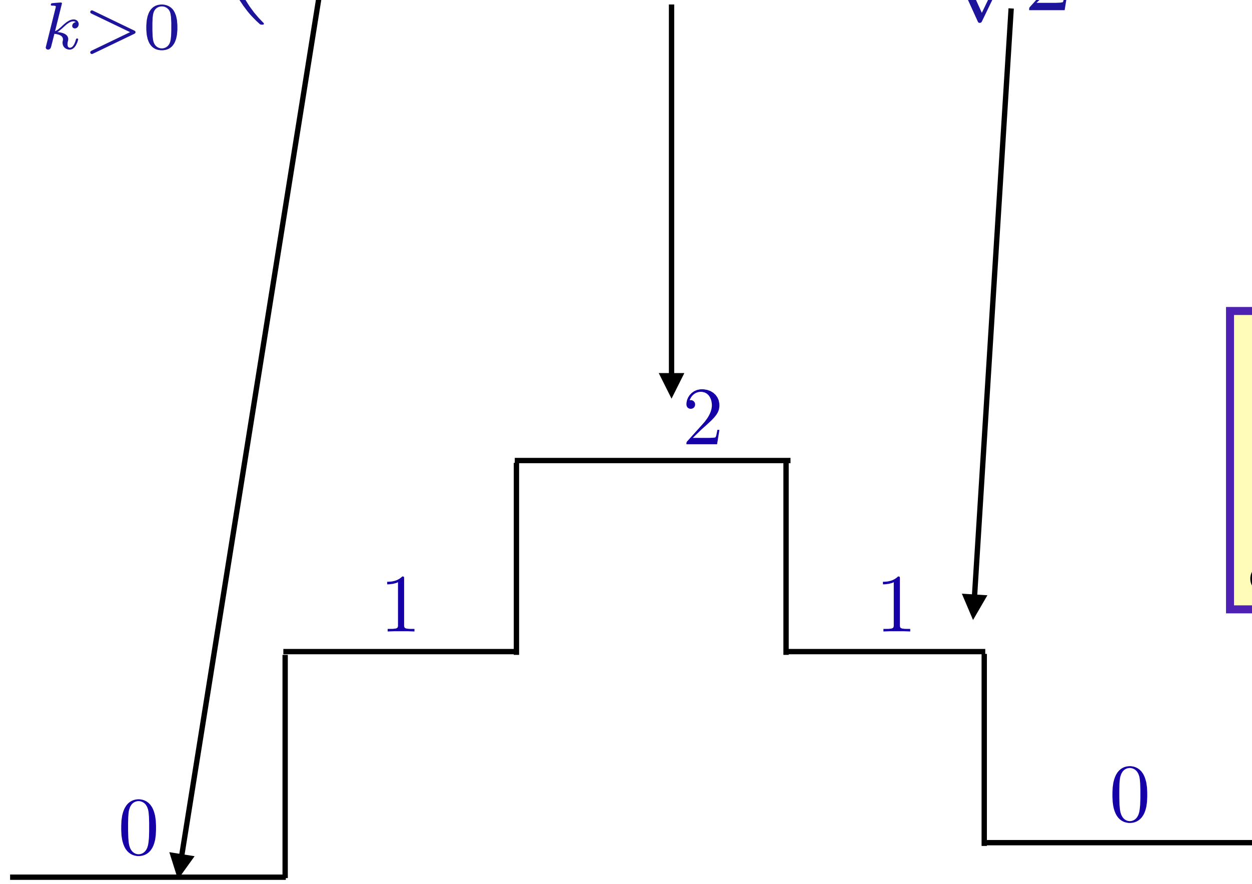


$$T_c$$

$$T_c = (W-U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}}.$$

variational MF wave function

$$|\psi\rangle = \prod_{k>0} \left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}} (b_k^\dagger + b_{-k}^\dagger) \right) |0\rangle$$



HK generalization

three variational parameters

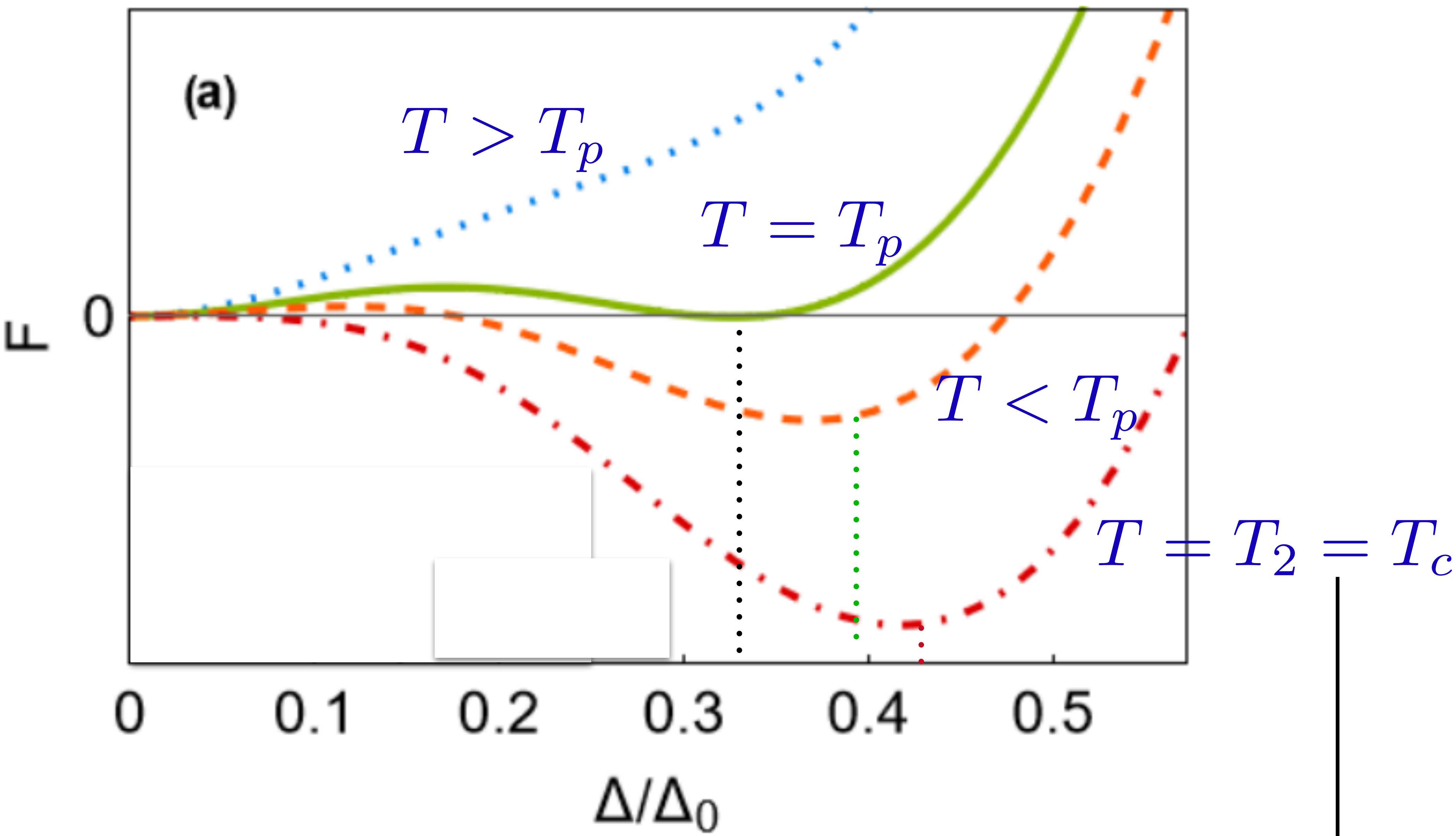
$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation

$\Delta \ll U, W$

$$\Delta = (W - U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

compute free energy



$t_G \approx 10^{-11}$

$$\lim_{g \to 0} \frac{\Delta}{T_c} \to \infty$$

MF theory
is accurate!

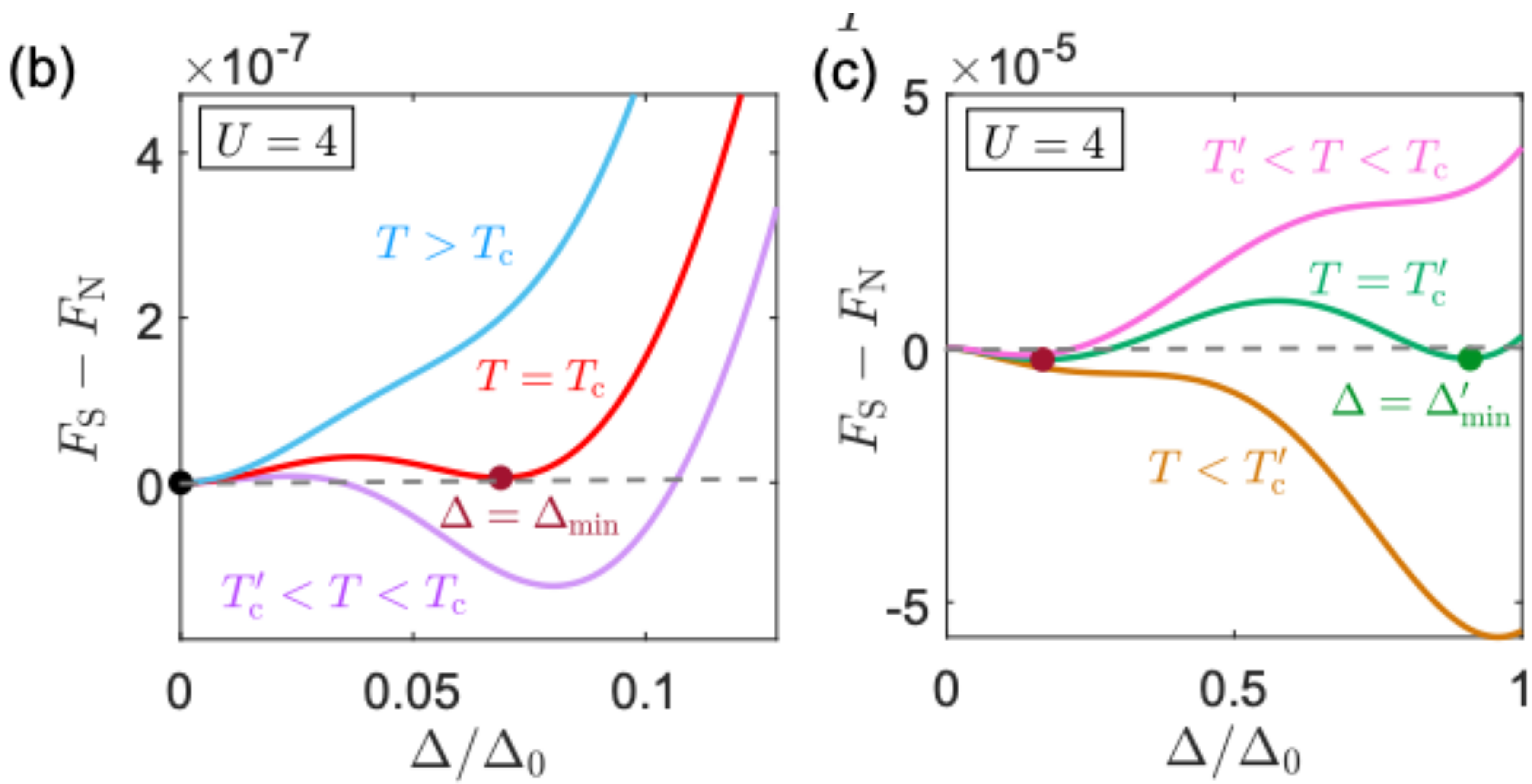
Two-stage superconductivity in the Hatsugai-Kohomoto-BCS model

Yu Li,[1] Vivek Mishra,[1] Yi Zhou,[2,3,4] and Fu-Chun Zhang[1,4,*]

[1]Kavli Institute for Theoretical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China
[2]Beijing National Laboratory for Condensed Matter Physics & Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China
[3]Songshan Lake Materials Laboratory, Dongguan, Guangdong 523808, China
[4]CAS Center for Excellence in Topological Quantum Computation, University of Chinese Academy of Sciences, Beijing 100190, China

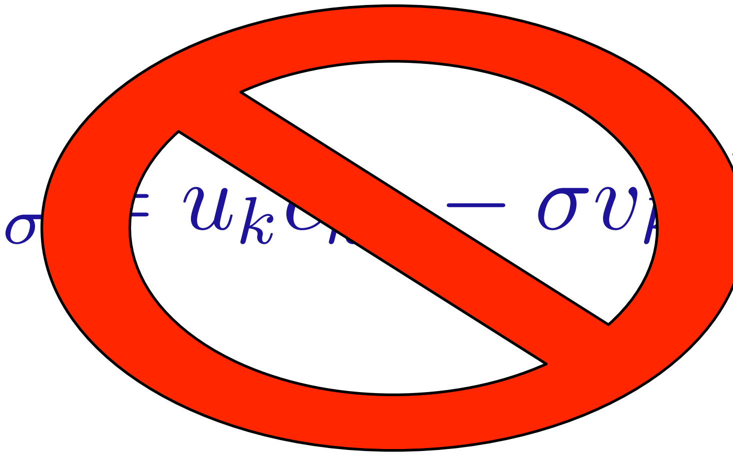
(Dated: July 7, 2022)

https://arxiv.org/pdf/2207.01904.pdf



Bogoliubov excitations

$$\gamma_{k\sigma} |\psi_{\mathrm{BCS}}\rangle = 0$$

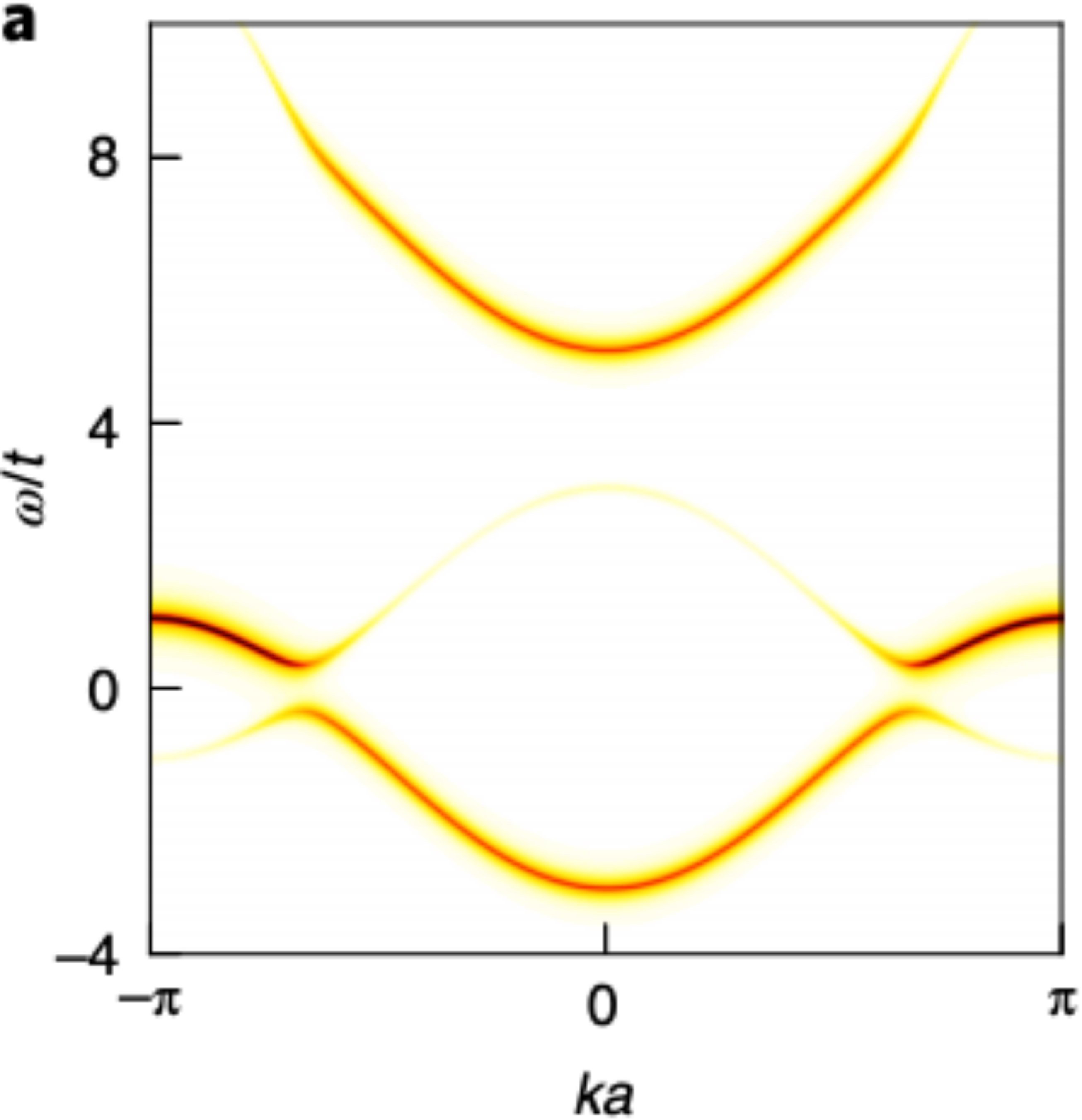


$$\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c^\dagger_{-k\overline{\sigma}}$$

PYHons excitations

$$\gamma^l_{k\sigma} \propto \sqrt{2} x_k \zeta^\dagger_{k\sigma} - \sigma z_k \zeta_{-k\overline{\sigma}}$$

$$\gamma^u_{k\sigma} \propto z_k \eta^\dagger_{k\sigma} - \sigma \sqrt{2} y_k \eta_{-k\overline{\sigma}}$$



PYHon band

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$

H. J. A. Molegraaf[1], C. Presura[1], D. van der Marel[1,*], P. H. Kes[2], M. Li[2]

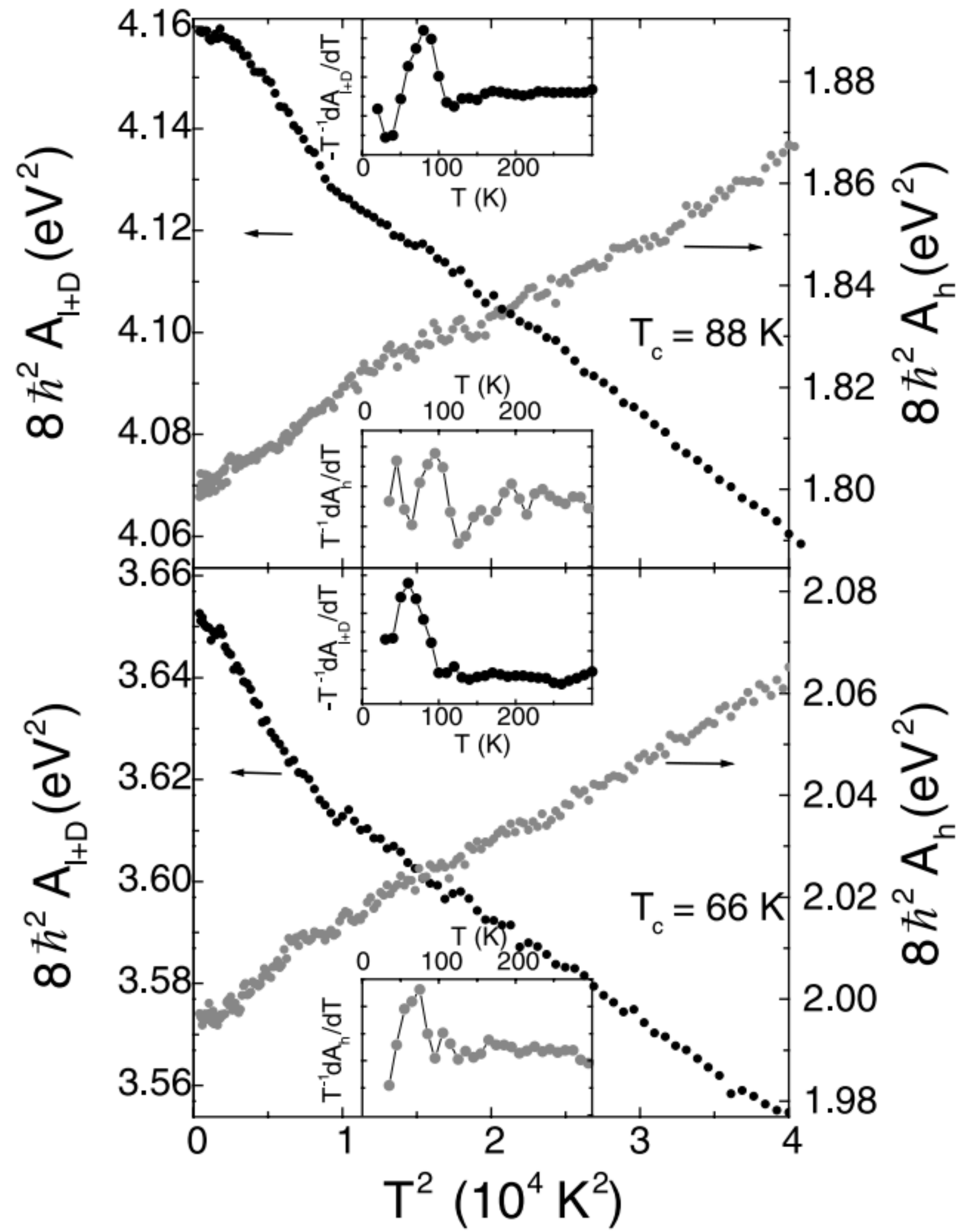
+ See all authors and affiliations

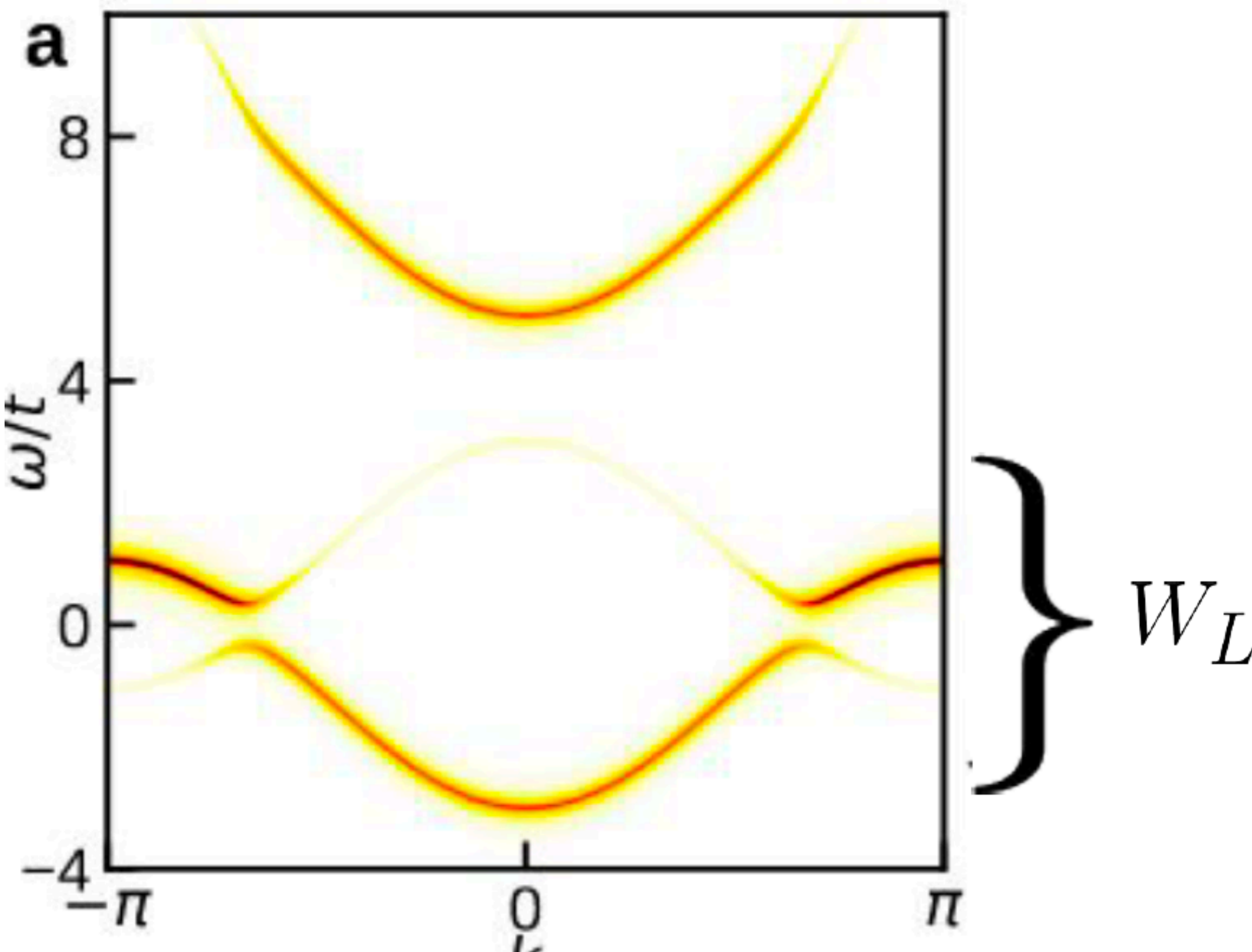
Science 22 Mar 2002:
Vol. 295, Issue 5563, pp. 2239-2241
DOI: 10.1126/science.1069947

$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$\frac{\Delta A_l}{A_l} \propto 3\%$$




a
8
4
0
−4
ω/t
−π
0
π
k
W_L

why?

$$H = H_{\mathrm{HK}} + H_p$$

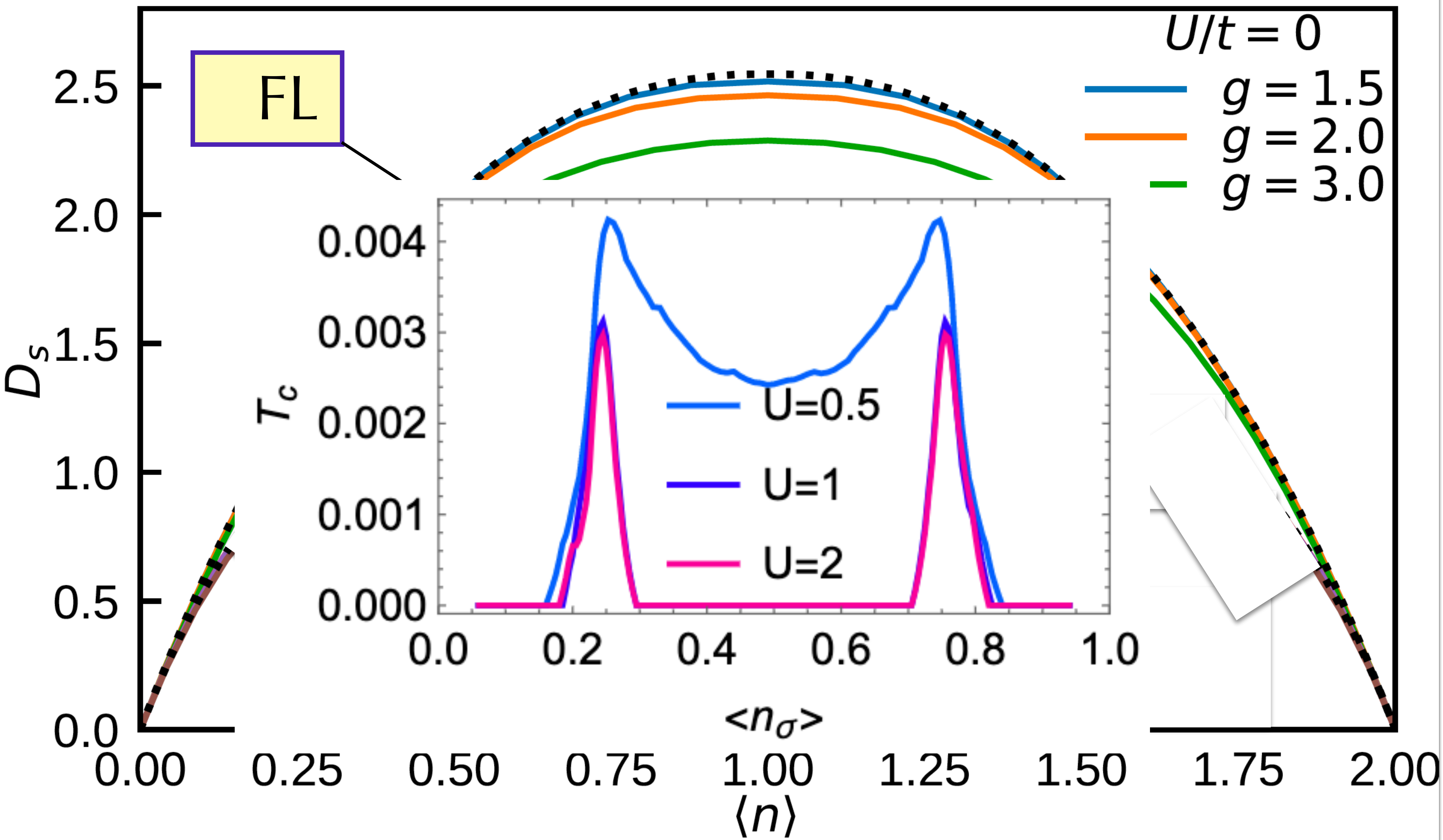
$$[H_{\mathrm{HK}}, H_p] \neq 0$$

↓

dynamical spectral weight transfer

Superfluid Density

Mottness-induced suppression



observable	Superconductivity BCS/FL	Mottness PYHZ/HK
$\chi \to \infty$	T_c	$T_c (= T_2)$
$\Delta \neq 0$	T_c	$T_p (> T_2)$
$\lim_{g \to 0} 2\Delta_0 / k_B T_c$	3.52	∞
quasi − particles	Bogoliubons	PYHons
t_G(Ginzburg)	$\approx 10^{-12}$	$\approx 10^{-11}$
$1/TT_1$	HS peak	no HS peak
Landau Expansion	$a = \alpha t,\ b > 0$	$a = \alpha t,\ b < 0$ $c > 0$
$E_{\rm cond}/N(0)\Delta^2$	-1	$[-2, -1]$

$\not{Z}_2$

	FL	HK
quantum numbers	k	k
stability: V_{local}	$\beta(V_{\text{local}}) = 0$	$\beta(V_{\text{local}}) = 0$
stability: V_{SC}	$\beta(V_{\text{SC}}) \to \infty$	$\beta(V_{\text{SC}}) \to \infty$?