hyperbolic spacetime
hyperbolic spacetime

local CFT (operator locality)
hyperbolic spacetime

1-1 state correspondence

local CFT
(operator locality)
which theories?
Yang-Mills large $N$ which theories?
which theories?

Yang-Mills large $N$  

SYK model
which theories?

- Yang-Mills large $N$
- SYK model
- $N$ D3-branes
which theories?

Yang-Mills large N

+ N D3-branes

SYK model

\[ O_j = \sum_{jklm} J_{jklm} x_k x_l x_m \]
which theories?

\[ \text{Yang-Mills large } N \]
\[ + \]
\[ N \text{ D3-branes} \]

\[ \mathcal{O}_j = \sum_{jklm} J_{jklm} X_k X_l X_m \]

completely different limits
which theories?

- Yang-Mills large $N$
- SYK model

\[ \mathcal{O}_j = \sum_{jklm} J_{jklm} XkXlXm \]

N D3-branes

completely different limits

geodesic completeness
\[ ds^2_{\mathbb{H}} = \frac{1}{y^2} (dx^2 + dy^2) \quad \mathbb{H}^2 \]
\[ ds_\mathbb{H}^2 = \frac{1}{y^2} (dx^2 + dy^2) \quad \mathbb{H}^2 \]

non-zero Christoffel symbols

\[
\begin{align*}
\Gamma^x_{xy} &= \Gamma^x_{yx} = -\frac{1}{y} \\
\Gamma^y_{xx} &= -\Gamma^y_{yy} = \frac{1}{y}
\end{align*}
\]
\[ ds^2_{\mathbb{H}} = \frac{1}{y^2} (dx^2 + dy^2) \quad \mathbb{H}^2 \]

\[ \begin{cases} 
\Gamma^x_{xy} = \Gamma^x_{yx} = -\frac{1}{y} \\
\Gamma^y_{xx} = -\Gamma^y_{yy} = \frac{1}{y} 
\end{cases} \]

non-zero Christoffel symbols

godesics
\[ ds_{\mathbb{H}}^2 = \frac{1}{y^2} (dx^2 + dy^2) \quad \mathbb{H}^2 \]

\[
\left\{ \begin{array}{c}
\Gamma^x_{xy} = \Gamma^x_{yx} = -\frac{1}{y} \\
\Gamma^y_{xx} = -\Gamma^y_{yy} = \frac{1}{y}
\end{array} \right. 
\]

non-zero Christoffel symbols

geodesics

cover all spacetime

Saturday, June 11, 16
\( ds^2 = -e^{-2|y|/L} g_{\mu\nu} dx^\mu dx^\nu + dy^2 \)
$$\text{what if?}$$

$$ds^2 = -e^{-2|y|/L} g_{\mu \nu} dx^\mu dx^\nu + dy^2$$

$$\text{singularity}$$
what if?

\[ ds^2 = -e^{-2|y|/L} g_{\mu\nu} dx^\mu dx^\nu + dy^2 \]

singularity

\( \Gamma_{\mu\nu}^\rho \) ill-defined (GI)
\[ ds^2 = -e^{-2|y|/L} g_{\mu\nu} dx^\mu dx^\nu + dy^2 \]

singularity

\[ \Gamma^\rho_{\mu\nu} \]

ill-defined (GI)

+ non-compactness
\[ ds^2 = -e^{-2|y|/L} g_{\mu\nu} dx^\mu dx^\nu + dy^2 \]

what if?

singularity

\[ \Gamma^\rho_{\mu\nu} \]

ill-defined (GI)

+ non-compactness

boundary locality
Type IIB String theory
Type IIB String theory

N D3-branes

$AdS_5 \times S^5$

local CFT
Type IIB String theory

N D3-branes

$AdS_5 \times S^5$

local CFT

geneodesic complete

non-local theories
Type IIB String theory

N D3-branes

$AdS_5 \times S^5$

local CFT

geodesic complete

non-local theories

$O_j = \sum_{jklm} J_{jklm} X_k X_l X_m$

SYK model
does he feel his weight?
No
GR

No

Equivalence principle
no local measurement can ever tell you about a uniform gravitational field
any theory with gravity has less observables than a theory without it!
any theory with gravity has less observables than a theory without it!

how can a local CFT emerge at the boundary?
any theory with gravity has less observables than a theory without it!

how can a local CFT emerge at the boundary?

N D3-branes
standard holography

\[ S = S(g_{\mu\nu}, A_\mu, \phi, \cdots) \]
standard holography

\[ S = S(g_{\mu\nu}, A_\mu, \phi, \cdots) \]
standard holography

\[ S = S(g_{\mu\nu}, A_{\mu}, \phi, \cdots) \]

\[ (\partial_{\mu} \phi)^2 + m^2 \phi^2 \]
standard holography

\[ S = S(g_{\mu\nu}, A_\mu, \phi, \cdots) \]

\[ (\partial_\mu \phi)^2 + m^2 \phi^2 \]

Operators

\[ \int d^4x \phi_0 \mathcal{O} \]
standard holography

\[ S = S(g_{\mu\nu}, A_\mu, \phi, \cdots) \]

\[ (\partial_\mu \phi)^2 + m^2 \phi^2 \]

AdS=CFT claim:

\[ \langle e^{\int_{S^d} \phi_0 \mathcal{O}} \rangle_{\text{CFT}} = Z_S(\phi_0) \]
$Z_S(\phi_0)$
\( Z_S(\phi_0) \)

super-gravity partition function averaged over all double-pole metrics that impose boundary conformality
Why should the boundary be conformal?

\[ Z_S(\phi_0) \]

super-gravity partition function averaged over all double-pole metrics that impose boundary conformality
AdS metric: Euclidean signature

\[ ds^2 = \frac{dz^2 + \sum_i dx_i^2}{z^2} \]

what is the length of this segment?
AdS metric: Euclidean signature

\[ ds^2 = \frac{dz^2}{z^2} + \sum_i \frac{dx_i^2}{z^2} \]

what is the length of this segment?

\[ \Delta s = \int_0^{z_0} \frac{dz}{z} = \ln(z_0/0) = \infty \]
metric at boundary is not well defined

\[ z^2 ds^2 = dz^2 + \sum_i dx_i^2 \] solves problem
metric at boundary is not well defined

\[ z^2 ds^2 = dz^2 + \sum_i dx_i^2 \]

solves problem

\[ ds^2 \rightarrow e^{2w} ds^2 \]

works for any real \( w \)
Metric at boundary is not well defined

\[ z^2 ds^2 = dz^2 + \sum_i dx_i^2 \]

Solves problem

\[ ds^2 \rightarrow e^{2w} ds^2 \]

Works for any real \( w \)

Boundary can only be specified conformally
\[ \langle e^{\int_{S^d} \phi_0 \mathcal{O}} \rangle_{\text{CFT}} = Z_S(\phi_0) \]

requires boundary conformality
requires boundary conformality

\[ \langle e^{\int_{S^d} \phi_0 \mathcal{O}} \rangle_{\text{CFT}} = Z_S(\phi_0) \]

\( \mathcal{O} \) should be conformal
requires boundary conformality

\[ \langle e^{\int_{S^d} \phi_0} \mathcal{O} \rangle_{\text{CFT}} = Z_S(\phi_0) \]

\( \mathcal{O} \) should be conformal

what is \( \mathcal{O} \)?
requires boundary conformality

\[ \langle e^{\int_{S^d} \phi_0} \sigma \rangle_{\text{CFT}} = Z_S(\phi_0) \]

\( \sigma \) should be conformal

what is \( \sigma \)?

composite operator in interacting theory
requires boundary conformality

\[ \langle e^{\int S_{d} \phi_0} \rangle_{\text{CFT}} = Z_{S}(\phi_0) \]

\( \mathcal{O} \) should be conformal

what is \( \mathcal{O} \)?

composite operator in interacting theory

\[ \mathcal{O} = C_{\mathcal{O}} \lim_{z \to 0} z^{-\Delta} \phi(x, z) \]

Polcinski: 1010.6134
can $O$ be determined exactly in some cases?
redo Witten’s massive scalar field calculation explicitly

\[ S_\phi = \frac{1}{2} \int d^{d+1}u \sqrt{g} (|\nabla \phi|^2 + m^2 \phi^2) \]

to establish correspondence
redo Witten’s massive scalar field calculation explicitly

\[ S_\phi = \frac{1}{2} \int \sqrt{g} \left( |\nabla \phi|^2 + m^2 \phi^2 \right) dV_g \]

to establish correspondence

\[ \langle e^{\int S_\phi} \mathcal{O} \rangle_{\text{CFT}} = Z_S(\phi_0) \]

\[ (-\nabla)^\gamma \phi_0 \] Reisz fractional Laplacian
\((-\Delta)^\gamma f(x) = C_{d,s} \int_{\mathbb{R}^d} \frac{f(x) - f(\xi)}{|x - \xi|^{d+2\gamma}} d\xi\)
Reisz fractional Laplacian

\[ (-\Delta)^{\gamma} f(x) = C_{d,s} \int_{\mathbb{R}^d} \frac{f(x) - f(\xi)}{|x - \xi|^{d+2\gamma}} d\xi \]
\[ S_\phi = \frac{1}{2} \int d^{d+1} u \sqrt{g} \left( |\nabla \phi|^2 + m^2 \phi^2 \right) \]

integrate by parts
\[ S_\phi = \frac{1}{2} \int d^{d+1}u \sqrt{g} \left( |\nabla \phi|^2 + m^2 \phi^2 \right) \]

**integrate by parts**

\[ S_\phi = \frac{1}{2} \int dV_g \left( -\phi \partial_\mu \phi + m^2 \phi^2 + \phi \partial_\mu \phi \right) \]
\[ S_\phi = \frac{1}{2} \int d^{d+1}u \sqrt{g} \left( |\nabla \phi|^2 + m^2 \phi^2 \right) \]

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integrate by parts

\[ S_\phi = \frac{1}{2} \int dV_g \left( -\phi \partial_\mu \phi + m^2 \phi^2 + \phi \partial_\mu \phi \right) \]

equations of motion

\[-\Delta \phi - s(d - s)\phi = 0 \quad -\Delta \phi = \nabla_i \nabla^i \phi \]

\[ m^2 = -s(d - s) \]

bound

\[ m^2 \geq -d^2 / 4 \]

BF bound
solutions

\[ \phi = F z^{d-s} + G z^s, \quad F, G \in \mathcal{C}^\infty(\mathbb{H}), \]
\[ F = \phi_0 + O(z^2), \quad G = g_0 + O(z^2) \]
solutions
\[ \phi = F z^{d-s} + G z^s, \quad F, G \in C^\infty(\mathbb{H}), \]
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restriction
\[ \phi_0 = \lim_{z \to 0} \phi \]  
boundary of AdS_{d+1}
solutions
\[ \phi = F z^{d-s} + G z^s, \quad F, G \in \mathcal{C}^\infty(\mathbb{H}), \]
\[ F = \phi_0 + O(z^2), \quad G = g_0 + O(z^2) \]

restriction
\[ \phi_0 = \lim_{z \to 0} \phi \quad \text{boundary of AdS}_{d+1} \]

\[ S_\phi = \frac{1}{2} \int dV_g \left( -\phi \partial_\mu \phi + m^2 \phi^2 + \phi \partial_\mu \phi \right) \]
\[ \int_{z > \epsilon} dV_g \phi \partial_\mu \phi \]
restriction \int_{z>\epsilon} \left( |\partial \phi|^2 - s(d-s)\phi^2 \right) dV_g = -d \int_{z=0} \phi_0 g_0
restriction \[ \text{pf} \int_{z > \epsilon} \left( |\partial \phi|^2 - s(d - s)\phi^2 \right) dV_g = -d \int_{z=0} \phi_0 g_0 \]

finite part from integration by parts
restriction

\[ \text{pf} \int_{z>\epsilon} \left( |\partial \phi|^2 - s(d-s)\phi^2 \right) dV_g = -d \int_{z=0} \phi_0 \ g_0 \]

finite part from integration by parts

use Caffarelli-Silvestre extension theorem (2006)

\[ g(x,0) = f(x) \]

\[ \Delta_x g + \frac{a}{\ell^2} \partial_z g + \partial^2_{\ell^2} g = 0 \]
restriction \[ \text{pf} \int_{z > \epsilon} \left( |\partial \phi|^2 - s(d - s)\phi^2 \right) dV_g = -d \int_{z=0} \phi \, g_0 \]

finite part from integration by parts

use Caffarelli-Silvestre extension theorem (2006)

\[ g(x, 0) = f(x) \]

\[ \Delta_x g + \frac{a}{z} \partial_z g + \partial_{z}^2 g = 0 \]

\[ \lim_{z \to 0^+} z^a \frac{\partial g}{\partial z} = C_{d, \gamma} (-\nabla)^\gamma f \]

\[ \gamma = \frac{1 - a}{2} \]
restriction \int_{z>\epsilon} (|\partial \phi|^2 - s(d-s)\phi^2) \, dV_g = -d \int_{z=0} \phi_0 \, g_0

finite part from integration by parts

use Caffarelli-Silvestre extension theorem (2006)

\begin{align*}
g(x, 0) &= f(x) \\
\Delta_x g + \frac{a}{z} \partial_z g + \partial_z^2 g &= 0 \\
\lim_{z \to 0^+} z^a \frac{\partial g}{\partial z} &= C_{d, \gamma} (-\nabla)^\gamma f \\
\gamma &= \frac{1 - a}{2}
\end{align*}

non-local
\[ \lim_{z \to 0^+} z^a \frac{\partial g}{\partial z} \]
\[ \lim_{z \to 0^+} z^\alpha \frac{\partial g}{\partial z} \]

\[ C_{d, \gamma} (-\nabla)^\gamma f \]
\[
\lim_{z \to 0^+} z^a \frac{\partial g}{\partial z} = C_{d, \gamma} \left( -\nabla \right)^\gamma f
\]

\[
g(z = 0, x) = f(x)
\]

\[
\gamma = \frac{1 - a}{2}
\]
solves massive scalar problem
\[ g = z^{\gamma - d/2} \phi \]

\[ \gamma := \frac{\sqrt{d^2 + 4m^2}}{2} \]
\[ g = z^{\gamma - d/2} \phi \]

\[ \gamma := \frac{\sqrt{d^2 + 4m^2}}{2} \]

\[ \mathcal{O} = (-\Delta)^\gamma \phi_0 \]

solves CS extension problem

solves massive scalar problem

the \( \mathcal{O} \) for massive scalar field
\[ \mathcal{O} = C_\mathcal{O} \lim_{z \to 0} z^{-\Delta} \phi(x, z) \]

consistency with Polcinski

use Caffarelli/Silvestre
\[ \mathcal{O} = C_0 \lim_{z \to 0} z^{-\Delta} \phi(x, z) \]

\[ \lim_{z \to 0^+} z^\alpha \frac{\partial g}{\partial z} = C_{d, \gamma} (-\nabla)^\gamma f \]

\[ \gamma = \frac{1 - a}{2} \]
\[ \mathcal{O} = (-\Delta)^{\gamma} \phi_0 \rightarrow |x - x'|^{-d-2\gamma} \]

2-point correlator
\[ \mathcal{O} = (-\Delta)^\gamma \phi_0 \rightarrow |x - x'|^{-d-2\gamma} \]

2-point correlator

\[ \langle e^{\int_{S^d} \phi_0} \mathcal{O} \rangle_{\text{CFT}} = Z_S(\phi_0) \]

AdS-CFT correspondence but operators are non-local!!
simpler proof:

Reisz fractional Laplacian

$(-\Delta)^\gamma f(x) = C_{d,s} \int_{\mathbb{R}^d} \frac{f(x) - f(\xi)}{|x - \xi|^{d+2\gamma}} d\xi$
simpler proof:

Reisz fractional Laplacian

\((-\Delta)^{\gamma} f(x) = C_{d,s} \int_{\mathbb{R}^d} \frac{f(x) - f(\xi)}{|x - \xi|^{d+2\gamma}} d\xi\)

pseudo-differential operator

\[
\left( (\overline{-\nabla})^s f(\xi) = |\xi|^{2s} \hat{f}(\xi) \right)
\]
simpler proof:

Reisz fractional Laplacian

\[ (-\Delta)^\gamma f(x) = C_{d,s} \int_{\mathbb{R}^d} \frac{f(x) - f(\xi)}{|x - \xi|^{d+2\gamma}} d\xi \]

pseudo-differential operator

\[ (\overline{-\nabla})^s f(\xi) = |\xi|^{2s} \hat{f}(\xi) \]

undo convolution

\[ I(\phi) \propto \int dxdx' \frac{\phi_0(x)\phi_0(x')}{|x - x'|^{2(\lambda+d)}} \]
d+1 gravity
bulk conformality

\[ S = S_{\text{gr}}[g] + S_{\text{matter}}(\phi) \]

\[ S_{\text{matter}} = \int_M d^{d+1}x \sqrt{g} \mathcal{L}_m \]

conformal sector
bulk conformality

\[ S = S_{\text{gr}}[g] + S_{\text{matter}}(\phi) \]

\[ S_{\text{matter}} = \int_M d^{d+1}x \sqrt{g} \mathcal{L}_m \]

\[ \mathcal{L}_m := |\partial \phi|^2 + \left( m^2 + \frac{d - 1}{4d} R(g) \right) \phi^2 \]
bulk conformality

\[ S = S_{\text{gr}}[g] + S_{\text{matter}}(\phi) \]

\[ S_{\text{matter}} = \int_M d^{d+1}x \sqrt{g} \mathcal{L}_m \]

\[ \mathcal{L}_m := |\partial \phi|^2 + \left( m^2 + \frac{d-1}{4d} R(g) \right) \phi^2 \]

``conformal mass''

scalar curvature

conformal sector
on Riemannian \((M,g)\) manifold of dimension \(N=d+1\)

\[
L_g = -\Delta_g + \frac{N - 2}{4(N - 1)} R_g = -\Delta_g + \frac{d - 1}{4d} R_g
\]
on Riemannian \((M, g)\) manifold of dimension \(N = d + 1\)

**conformal Laplacian**

\[
L_g = -\Delta_g + \frac{N - 2}{4(N - 1)} R_g = -\Delta_g + \frac{d - 1}{4d} R_g
\]

**conformal change**

\[
A_w(\varphi) = e^{-bw} A(e^{aw} \varphi)
\]

\[
\hat{g} = y^2 g
\]
on Riemannian $(M,g)$ manifold of dimension $N = d+1$

conformal Laplacian

\[ L_g = -\Delta_g + \frac{N - 2}{4(N - 1)} R_g = -\Delta_g + \frac{d - 1}{4d} R_g \]

conformal change

\[ A_w(\varphi) = e^{-bw} A(e^{aw} \varphi) \]

\[ \hat{g} = y^2 g \]

\[ L_g(\varphi) = y^{\frac{d+3}{2}} L_{\hat{g}} \left( y^{-\frac{d-1}{2}} \varphi \right) \]
\[ L_g = -\Delta_g + \frac{N - 2}{4(N - 1)} R_g = -\Delta_g + \frac{d - 1}{4d} R_g \]
\[ L_g = -\Delta_g + \frac{N - 2}{4(N - 1)} R_g \]
\[ R_{g_H} = -d(d + 1) \]
\[ L_{g_H} = -\Delta_{g_H} - \frac{d^2 - 1}{4} \]
$L_g = -\Delta_g + \frac{N - 2}{4(N - 1)} R_g = -\Delta_g + \frac{d - 1}{4d} R_g$

$R_{g_{\mathbb{H}}} = -d(d + 1)$

$L_{g_{\mathbb{H}}} = -\Delta_{g_{\mathbb{H}}} - \frac{d^2 - 1}{4}$

$m^2 - \frac{d^2 - 1}{4} = -s(d - s)$
hyperbolic metric

\[ L_g = -\Delta_g + \frac{N-2}{4(N-1)} R_g = -\Delta_g + \frac{d-1}{4d} R_g \]

\[ R_{gH} = -d(d+1) \]

\[ L_{gH} = -\Delta_g - \frac{d^2-1}{4} \]

\[ m^2 - \frac{d^2-1}{4} = -s(d-s) \]

\[ s = \frac{d}{2} + \frac{\sqrt{4m^2+1}}{2} \quad \Rightarrow \quad m^2 > -1/4 \]

stability independent of dimensionality
construct $\mathcal{O}$

\[-\Delta_g \phi + \frac{d-1}{4d} R_g \phi = m^2 \phi\]

\[-\Delta \phi + \left( m^2 - \frac{d^2-1}{4} \right) \phi = 0\]
\[-\Delta_g \phi + \frac{d - 1}{4d} R_g \phi = m^2 \phi\]

\[-\Delta \phi + \left( m^2 - \frac{d^2 - 1}{4} \right) \phi = 0\]

**solutions**

\[\gamma = \sqrt{4m^2 + 1}\]

\[\phi = F y^{\frac{d}{2} - \gamma} + G y^{\frac{d}{2} + \gamma}, \quad F, G \in C^\infty(\mathbb{H}), \quad F = \phi_0 + O(y^2), \quad G = g_0 + O(y^2)\]
\[-\Delta_g \phi + \frac{d - 1}{4d} R_g \phi = m^2 \phi\]

\[-\Delta \phi + \left( m^2 - \frac{d^2 - 1}{4} \right) \phi = 0\]

solutions
\[\gamma = \sqrt{4m^2 + 1}\]
\[\phi = F y^{\frac{d}{2} - \gamma} + G y^{\frac{d}{2} + \gamma}, \quad F, G \in \mathcal{C}^\infty(\mathbb{H}), \quad F = \phi_0 + O(y^2), \quad G = g_0 + O(y^2)\]

redefinition
\[g = y^{\gamma - \frac{d}{2}} \phi, \quad \lim_{y \to 0} y^{1 - 2\gamma} \frac{\partial g}{\partial y} = 2\gamma g_0\]
$S(s)F = G|_M$

source \hspace{2cm} response

$P_\gamma[d\tau^2, h] := d_\gamma S\left(\frac{d+1}{2} + \gamma\right), \quad d_\gamma = 2^{2\gamma} \frac{\Gamma(\gamma)}{\Gamma(-\gamma)}$
$P_\gamma \in (-\Delta_{\hat{g}})^\gamma + \Psi_{\gamma-1}$

pseudo-differential operator
\[ P_\gamma \in (-\Delta_{\hat{g}})^\gamma + \Psi_{\gamma-1} \]

pseudo-differential operator

**in general** \[ P_k = (-\Delta)^k + \text{lower order terms} \]
in general \[ P_k = (-\Delta)^k + \text{lower order terms} \]

\[
P_1 = -\Delta + \frac{d - 1}{4(d - 1)} R_g
\]
scattering problem

\[ P_\gamma f = d_\gamma S \left( \frac{d}{2} + \gamma \right) = d_\gamma \ h \]
scattering problem

\[ P_\gamma f = d_\gamma S \left( \frac{d}{2} + \gamma \right) = d_\gamma h \]

\[ \text{pf } \int_{y > \epsilon} [\partial \phi]^2 - \left( s(d - s) + \frac{d - 1}{4d} R(g) \right) \phi^2 \] \[ dV_g = -d \int_{\partial X} dV_h \int P_\gamma [g^+, \hat{g}] f \]
scattering problem

\[ P_\gamma f = d_\gamma S \left( \frac{d}{2} + \gamma \right) = d_\gamma h \]

\[ \text{pf } \int_{y>\epsilon} [|\partial \phi|^2 - \left(s(d - s) + \frac{d-1}{4d} R(g)\right) \phi^2] dV_g = -d \int_{\partial X} dV_h f P_\gamma [g^+, \hat{g}] f \]
d+1 gravity
d-dimensional non-local `QFT'
d+1 gravity
What about Maldacena conjecture?
What about Maldacena conjecture?

Type IIB String `action'

\[
S = \int d^{10}x \sqrt{-g} \left( e^{-2\phi} (R + 4|\nabla \phi|^2) - \frac{2e^{2\alpha \phi}}{(D - 2)} F^2 \right)
\]
What about Maldacena conjecture?

Type IIB String `action'

\[ S = \int d^{10}x \sqrt{-g} \left( e^{-2\phi}(R + 4|\nabla \phi|^2) - \frac{2e^{2\alpha \phi}}{(D - 2)}F^2 \right) \]

\[ D = 7 \]

extremal solution

\[ ds_L^2 = H^{-1/2}(r) \eta_{\mu\nu} dx^{\mu} dx^{\nu} + H^{1/2}(r) \delta_{mn} dx^{m} dx^{n} \]
What about Maldacena conjecture?

Type IIB String `action’

\[ S = \int d^{10} x \sqrt{-g} \left( e^{-2\phi} (R + 4|\nabla \phi|^2) - \frac{2e^{2\alpha \phi}}{(D-2)} F^2 \right) \]

\[ D = 7 \]

extremal solution

\[ ds^2_L = H^{-1/2}(r) \eta_{\mu \nu} dx^\mu dx^\nu + H^{1/2}(r) \delta_{mn} dx^m dx^n \]

\[ H = 1 + \frac{L^4}{r^4}, \quad L^4 = 4\pi g N \alpha'^2, \quad r^2 = \delta_{mn} x^m x^n \]
What about Maldacena conjecture?

Type IIB String `action’

\[
S = \int d^{10}x \sqrt{-g} \left( e^{-2\phi} (R + 4|\nabla \phi|^2) - \frac{2e^{2\alpha\phi}}{(D - 2)} F^2 \right)
\]

\[D = 7\]

extremal solution

\[
d s_L^2 = H^{-1/2}(r) \eta_{\mu\nu} dx^\mu dx^\nu + H^{1/2}(r) \delta_{mn} dx^m dx^n
\]

D3-branes

\[H = 1 + \frac{L^4}{r^4}, \quad L^4 = 4\pi g N \alpha'^2, \quad r^2 = \delta_{mn} x^m x^n\]
What about Maldacena conjecture?

Type IIB String `action’

\[ S = \int d^{10}x \sqrt{-g} \left( e^{-2\phi}(R + 4|\nabla \phi|^2) - \frac{2e^{2\alpha \phi}}{(D - 2)}F^2 \right) \]

\[ D = 7 \]

extremal solution

\[ ds^2_L = H^{-1/2}(r) \eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}(r) \delta_{mn}dx^m dx^n \]

horizon at \( r=0 \)

\[ H = 1 + \frac{L^4}{r^4}, \quad L^4 = 4\pi g N \alpha'^2, \quad r^2 = \delta_{mn}x^m x^n \]

D3-branes
rescale AdS metric

\[ ds^2 \rightarrow ds_L^2 \]
rescale AdS metric

\[ ds^2 \rightarrow ds_L^2 \]

\[ \Box_{ds^2}^{conf}(\phi) + m^2 \phi = L^2 \left( \Box_{ds_L^2}^{conf} + \frac{m^2}{L^2} \right) \phi \]
rescale AdS metric

\[ ds^2 \to ds^2_L \]

\[ \Box^{conf}_{ds^2} (\phi) + m^2 \phi = L^2 \left( \Box^{conf}_{ds^2_L} + \frac{m^2}{L^2} \right) \phi \]

\[ \left( \Box^{conf}_{ds^2_L} + \frac{m^2}{L^2} \right) \phi = 0 \]
what determines the exponent?

\[ (-\Delta)^\gamma \quad \gamma = \sqrt{\frac{4m^2}{L^2}} + 1 \]
what determines the exponent?

\[ (-\Delta)^\gamma \gamma = \frac{\sqrt{4 \frac{m^2}{L^2}} + 1}{2} \]

\[ \lim_{L \to +\infty (N \to \infty)} \gamma = \frac{1}{2} \]

non-locality vanishes
more generally

\[ \mathbb{R}^{3,1} \times K_6 \]

\[ ds^2 = f^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + f^{1/2} \delta_{mn} dx^m dx^n \]
more generally

\[ ds^2 = f^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + f^{1/2} \delta_{mn} dx^m dx^n \]

\[ \Delta f = (2\pi)^4 \alpha'^2 g \rho \]
more generally

\[ ds^2 = f^{-1/2} \eta_{\mu \nu} dx^\mu dx^\nu + f^{1/2} \delta_{mn} dx^m dx^n \]

\[ \Delta f = (2\pi)^4 \alpha'^2 g \rho \]

\[ N \delta(r) \]

density of D3-branes
\[ y = \epsilon \]

\[ f(y_0) = f(\epsilon) = 0 \]

D3-branes

\( f \) is a harmonic function
$f$ is a harmonic function

\[ f(y_0) = f(\epsilon) = 0 \]

D3-branes

requires absolute-value singularity
|y| singular metrics (GI)
$|y|$ singular metrics (GI)

Randall-Sundrum

$$ds^2 = -e^{-2|y|/L}g_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$y \in [-\pi R, \pi R]$$
\[ |y|\] singular metrics (GI)

**Randall-Sundrum**

\[
ds^2 = -e^{-2|y|/L} g_{\mu\nu} dx^\mu dx^\nu + dy^2
\]

\[ y \in [-\pi R, \pi R]\]

massive-particle action at Brane at \(\pi R\)

\[
\int d^4 x \sqrt{-g} \left( g^{\mu\nu} \hat{\partial}_\mu \hat{\phi} \hat{\partial}_\nu \hat{\phi} + m^2 e^{-2\pi R/L} \hat{\phi}^2 \right),
\]

\[ \hat{\phi} = e^{-\pi R/L} \phi \]
\[ \lim_{R/L \to \infty} m^2 e^{-2\pi R/L} \to 0 \]
\[
\lim_{{R/L \to \infty}} m^2 e^{-2\pi R/L} \rightarrow 0
\]

\[
\gamma = \frac{1}{2}
\]

**non-locality vanishes**
\[ m^2 = -\frac{1}{\alpha'} + \frac{(\ln \epsilon)^2}{(2\pi \alpha')^2} \]
\[ m^2 = -\frac{1}{\alpha'} + \frac{(\ln \epsilon)^2}{(2\pi \alpha')^2} \]

\[ |\ln \epsilon| > 2\pi \sqrt{\alpha'} \]

positive mass
\[ m^2 = -\frac{1}{\alpha'} + \frac{(\ln \epsilon)^2}{(2\pi \alpha')^2} \]

positive mass

\[ |\ln \epsilon| > 2\pi \sqrt{\alpha'} \]

non-locality vanishes
Branes in Type IIB
string theory
eliminate non-local boundary
interactions
are there any consequences for the enganglement entropy?
yes
minimal surface avoids the D3-brane
a.) \( y = \epsilon \)

\( y = y_0 \)

**minimal surface avoids the D3-brane**
what happens as brane approaches boundary?

\[ y = y_0 \]

\[ y = \epsilon \]

\[ b. \]
what happens as brane approaches boundary?

$b.)$

$y = y_0$

$y = \epsilon$

minimal surface must avoid brane
what happens as brane approaches boundary?

minimal surface must avoid brane
$\epsilon = 0$

metric doubles $S^1/\mathbb{Z}_2$

c.)

$y = y_0$

wall singularity

entropy vanishes $R/L = \infty$
higher-dimensional minimal surfaces can avoid singularities
higher-dimensional minimal surfaces can avoid singularities

is this how the entanglement entropy should be formulated??
application: gauge fields with anomalous dimensions

\[ F_{\mu\nu} F^{\mu\nu} + m^2 A_y^2 \]
application: gauge fields with anomalous dimensions

\[ F_{\mu\nu} F^{\mu\nu} + m^2 A_y^2 \]

\[ A_{\mu}^{\perp} \partial_{\mu} A_{\mu}^{\perp} \]

\[ \gamma = \sqrt{d^2 + m^2 - 1/2} \]
application: gauge fields with anomalous dimensions

\[ F_{\mu\nu} F^{\mu\nu} + m^2 A_y^2 \]

\[ A_\mu^\perp \partial_\mu^\perp A_\mu^\perp \]

\[ \gamma = \sqrt{d^2 + m^2 - 1/2} \]

dynamical `Higgs' mode
gauge-gravity correspondence

= local CFT
gauge-gravity correspondence

N D3-branes

= local CFT
gauge-gravity correspondence

N D3-branes + = local CFT

entanglement entropy?
gauge-gravity correspondence

\[ N \text{ D3-branes} + \quad = \quad \text{local CFT} \]

entanglement entropy?

SYK model is different