`Virasoro Algebra' for the Strange Metal

thanks to

Gabriele La Nave

NSF

Kridsangaphong Limtragool

strange metal: experimental facts

Quantum critical behaviour in a high-T_c superconductor

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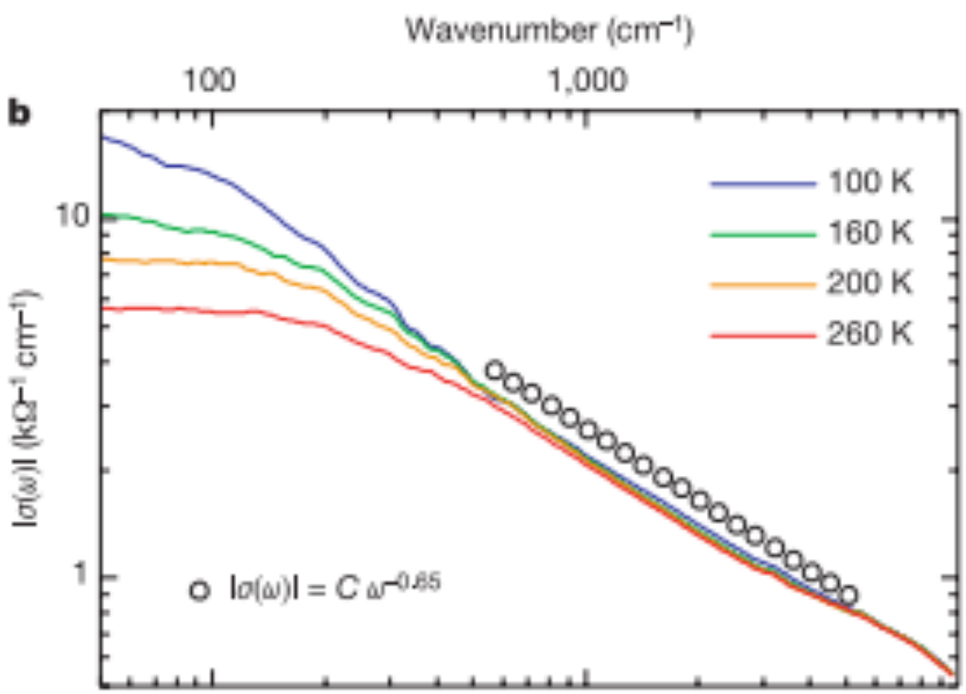
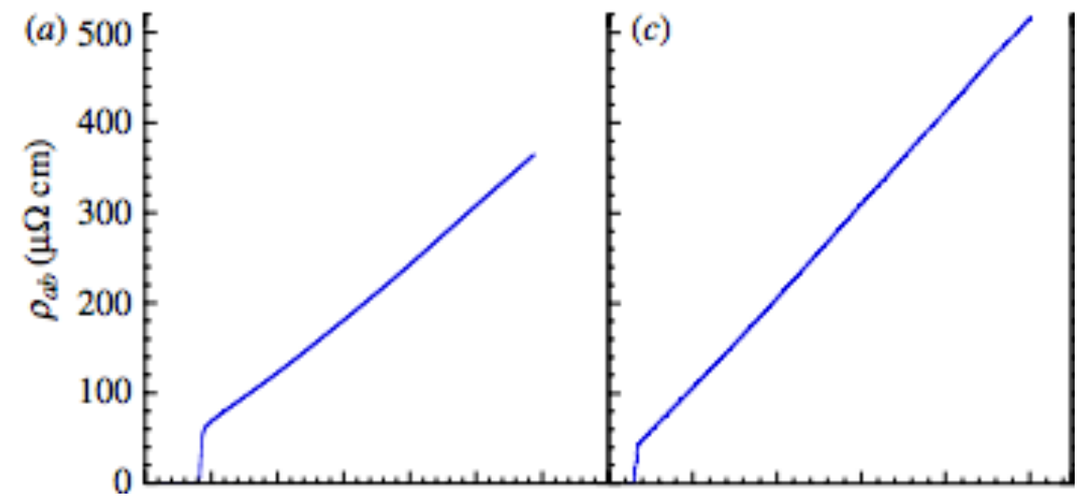


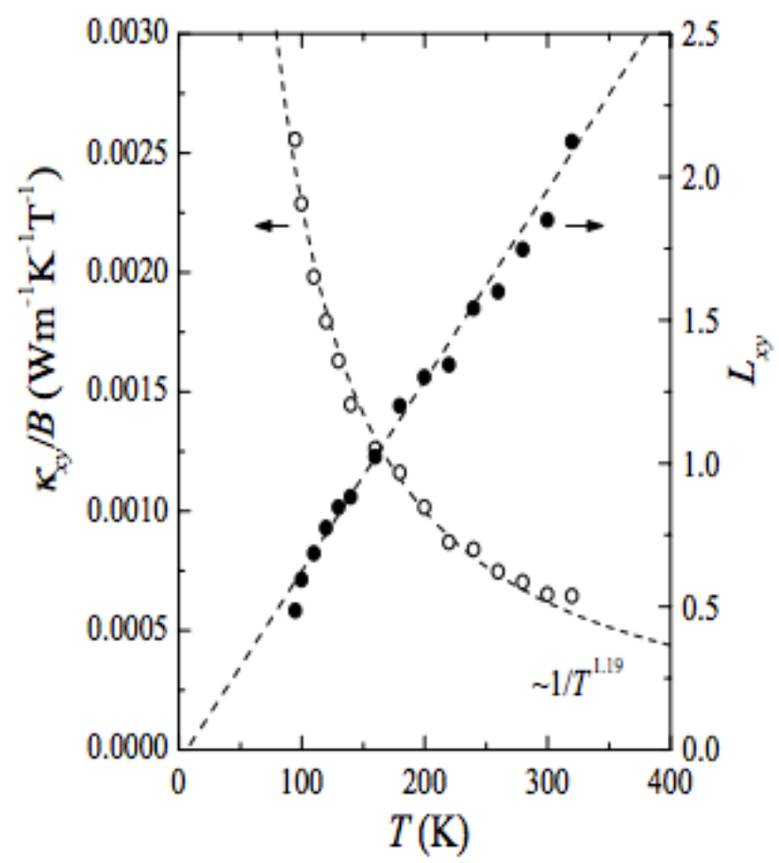
$$\sigma(\omega) = C\omega^{-\frac{2}{3}}$$

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

T-linear resistivity



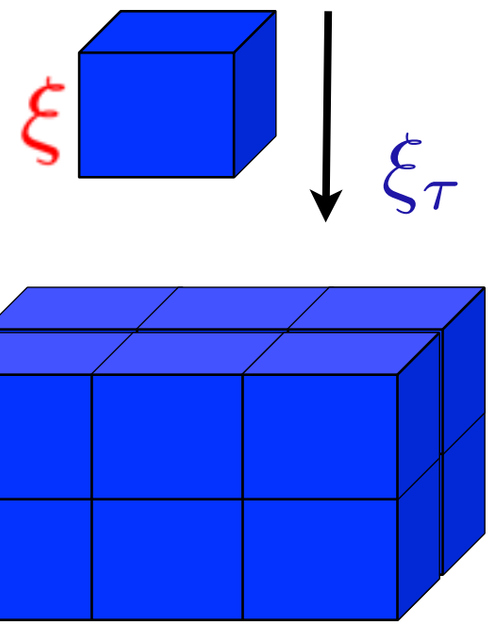
$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$



$$\frac{\text{Theories of cuprates}}{\text{Theories of strange metal}} = \infty$$

why is the problem hard?

single-parameter scaling



$$\xi_\tau \propto \xi^z$$

$$\frac{\delta^2 \ln Z}{\omega \delta A_\mu \delta A_\mu}$$

$$\rho \propto T^{(2-d)/z} \longrightarrow 1$$

$$\sigma(\omega, T) \propto \omega^{(d-2)/z} \longrightarrow -2/3$$

$$C_v \propto T^{d/z}$$

anomalous dimension $\quad 2 \to 2d_A$

dimension of vector potential

gauge invariance

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial_\mu \mathcal{G}$$

has no units

$$[A_\mu] = \frac{1}{L} = 1$$

vector potential cannot have an anomalous dimension

conserved current

dimensions of current

$$\phi(x) \to \phi(x) + \delta\phi(x)$$

$$[J_0(x), \phi(y)] = \delta\phi(x)\delta^d(x-y)$$

$$[J_0] = d$$

$$\nabla \cdot J_\mu = \partial_t J_0$$

$$\xi_\tau \approx \xi$$

$$[J_\mu] = d$$

can the dimension of the current change?

$$\nabla \cdot J_\mu = \partial_t J_0$$

$$\xi_\tau \propto \xi^z$$

$$[J_\mu] = d - \theta + z - 1$$

hyperscaling violation (Kiritsis, et al.)

$$d \to d - \theta$$

allowable by spacetime symmetries

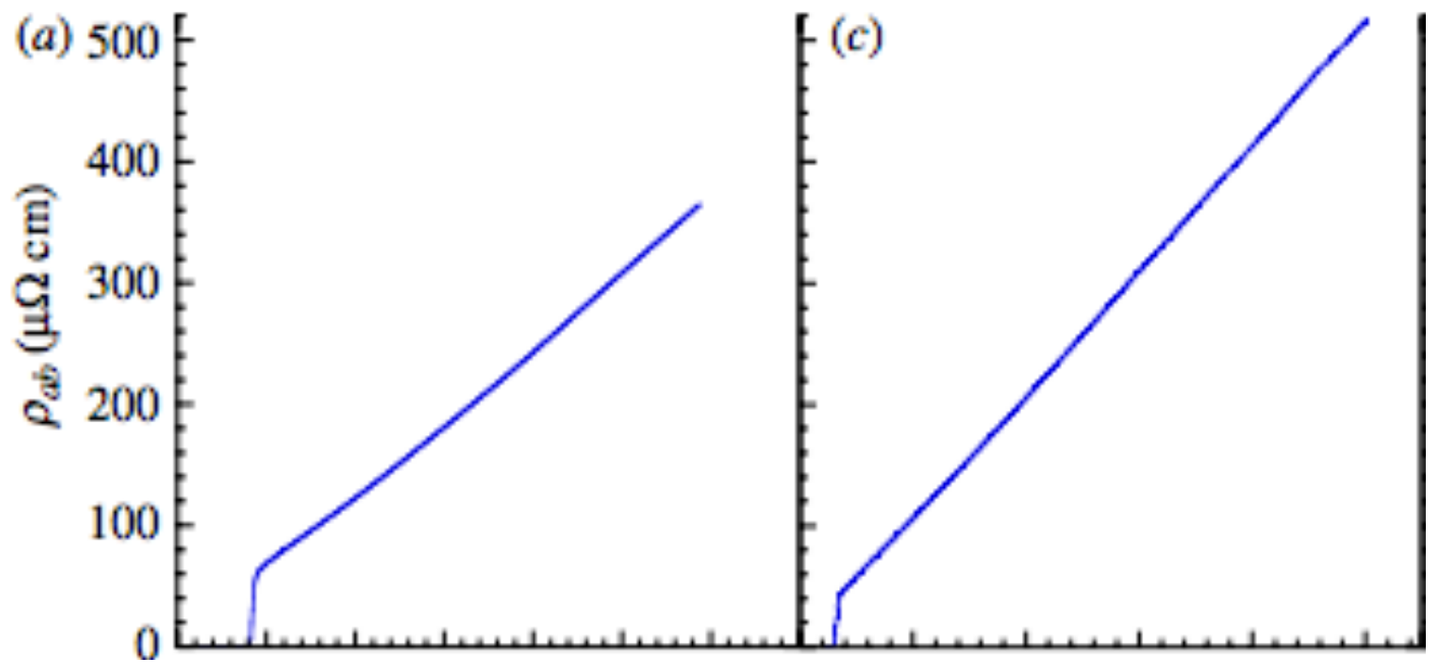
$$[A_\mu] = 1$$

strange metal: experimental facts

Hall Angle

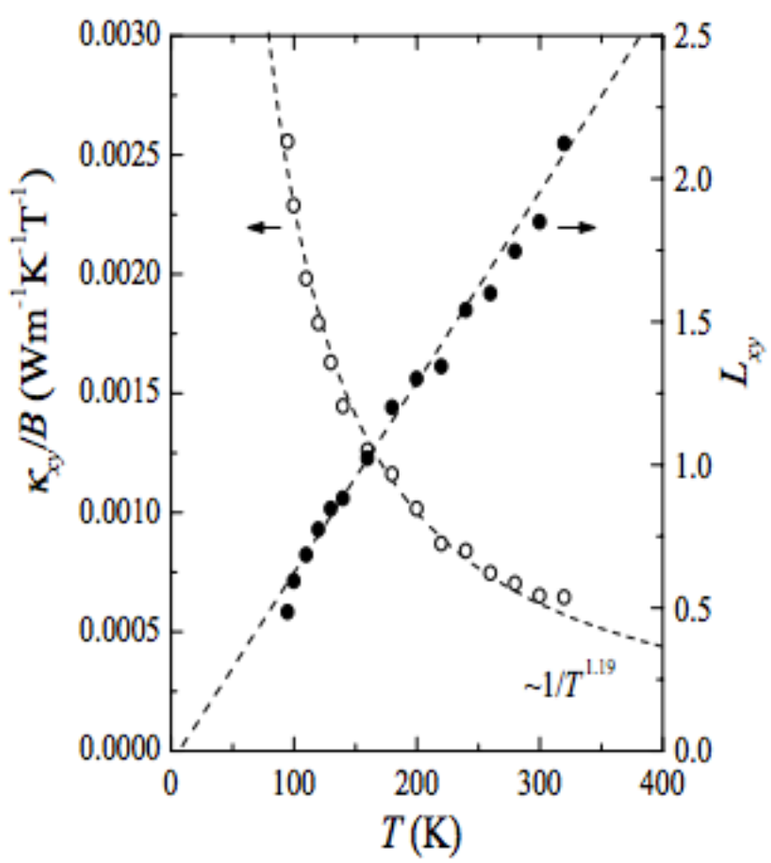
$$\cot\theta_H \equiv \frac{\sigma_{xx}}{\sigma_{xy}} \approx T^2$$

T-linear resistivity



Hall Lorenz ratio

$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$



all explained if

$$[J_\mu] = d - \theta + \Phi + z - 1$$

Hartnoll/Karch

$$[A_\mu] = 1 - \Phi$$

$$\Phi = -2/3$$

current algebra with an anomalous dimension?

is there a direct experimental probe of this anomaly?

how can A have an anomalous dimension?

what else can be explained with the anomalous dimension?

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with the anomalous dimension?

Quantum critical behaviour in a high-T_c superconductor

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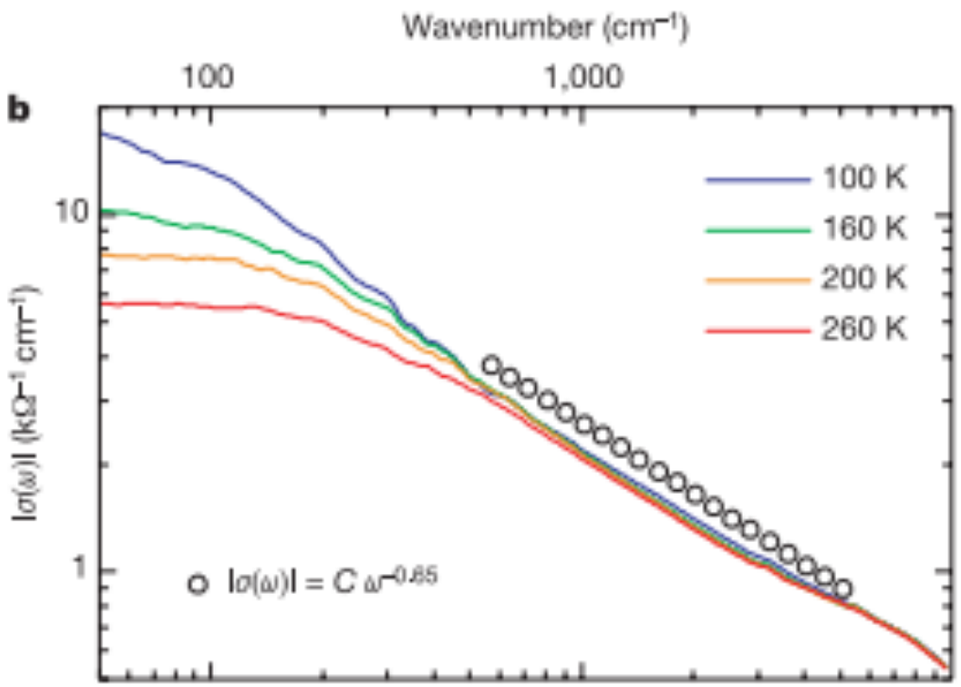


Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

$$\sigma(\omega) = C\omega^{\gamma-2}e^{i\pi(1-\gamma/2)}$$
$$\gamma = 1.35$$

take experiments seriously

$$\sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i}$$

continuous mass(scale invariant unparticles)

$$\sigma(\omega) = \int_0^M \frac{\rho(m) e^2(m) \tau(m)}{m} \frac{1}{1 - i\omega\tau(m)} dm$$

variable masses for everything

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

$$e(m) = e_0 \frac{m^b}{M^b}$$

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

Karch, 2015

$$\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0}{M^{a+2b+c}} \int_0^M dm \frac{m^{a+2b+c-2}}{1 - i\omega\tau_0 \frac{m^c}{M^c}}$$

perform integral

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

$$\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_0^{\omega\tau_0} dx \; \frac{x^{-\frac{1}{3}}}{1-ix}$$

$$\Big\downarrow \; \omega\tau_0 \to \infty$$

$$\boxed{\sigma(\omega) = \frac{1}{3}(\sqrt{3}+3i)\pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M\omega^{\frac{2}{3}}}}$$

$$\boxed{\begin{array}{c} \tan\sigma = \sqrt{3} \\ 60^\circ \end{array}}$$

are anomalous dimensions necessary

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

$$e(m) = e_0 \frac{m^b}{M^b}$$

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

hyperscaling violation

anomalous dimension

momentum loss

$$c = 1 \qquad\qquad b = 0$$

$$a + 2b = 2/3 \quad \Rightarrow \quad a = 2/3$$

No

but the Lorenz ratio is not a constant

$$L_H = \frac{\kappa_{xy}}{T\sigma_{xy}} \sim T \equiv T^{-2\Phi/z}$$

Hartnoll/Karch

$$\Phi = bz = -2/3$$

$$\rho \propto T^{-2\Phi/z}$$

AC+DC require anomalous dimension

how can A have an anomalous dimension?

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial^{\alpha}_\mu \mathcal{G}$$

$$F_{\mu\nu} = \partial^{\alpha}_{[\mu} A_{\nu]}$$

$$\nabla^\alpha \cdot J_\mu = \partial_t J_0$$

both J and A have anomalous dimensions!

what kind of an operator

is ∂_μ^α

V open set

if $f(x) = 0,\ x \in V$

and $\hat{T} f(x) = 0,\ x \in V$

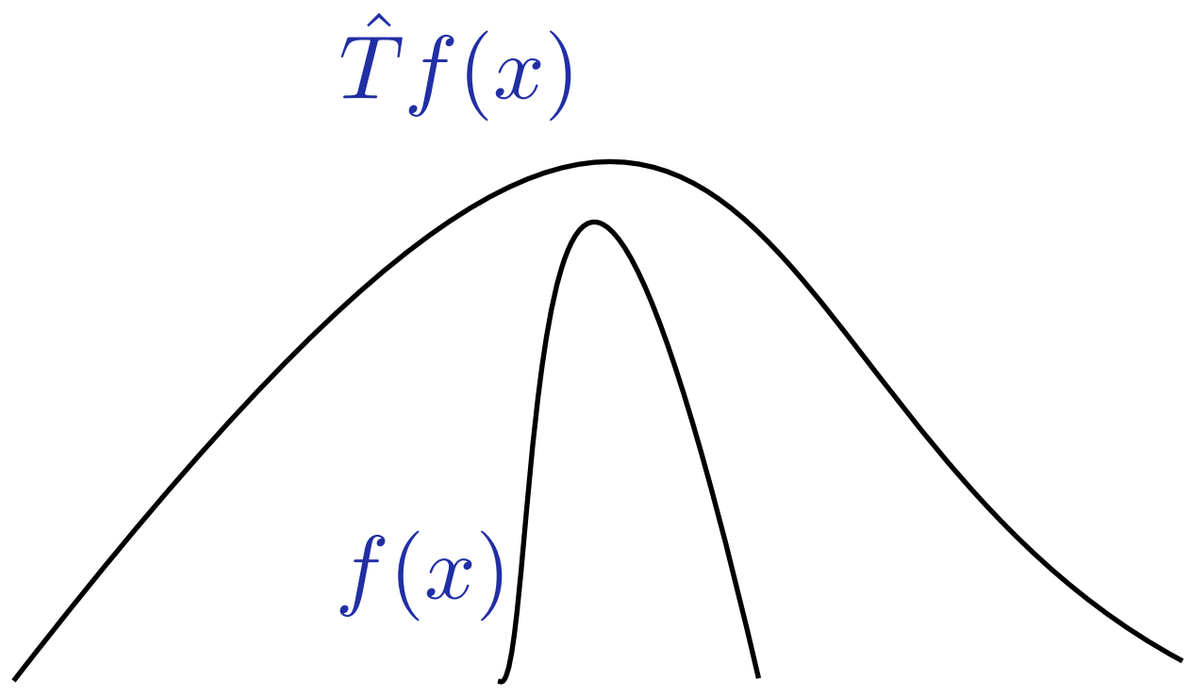
for only $f = 0$

then $\hat{T}$ is antilocal

$$\partial_y^\alpha y^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} y^{\beta-\alpha} \Rightarrow \partial_y^\alpha c = \frac{y^{-\alpha}}{\Gamma(1-\alpha)}$$

∂_μ^α anti-local operator

$\hat{T}$ is antilocal



physical consequences of anomalous dimension for $\mathcal{A}_\mu$

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial_\mu^{\alpha} \mathcal{G}$$

$$F_{\mu\nu} = \partial^{\alpha}_{[\mu} A_{\nu]}$$

$$\vec{\nabla}^{\alpha} \times \vec{A} = \vec{B}$$

no Stokes' theorem

$$\oint \vec{A} \cdot d\ell \neq \int_S B \cdot d\vec{S}$$

Aharonov-Bohm Effect must change

physical gauge connection

$$a_i \equiv [\partial_i, I_i^\alpha A_i] = \partial_i I_i^\alpha A_i$$

$$A_\mu \to A_\mu + \partial_\mu^\alpha \Lambda$$

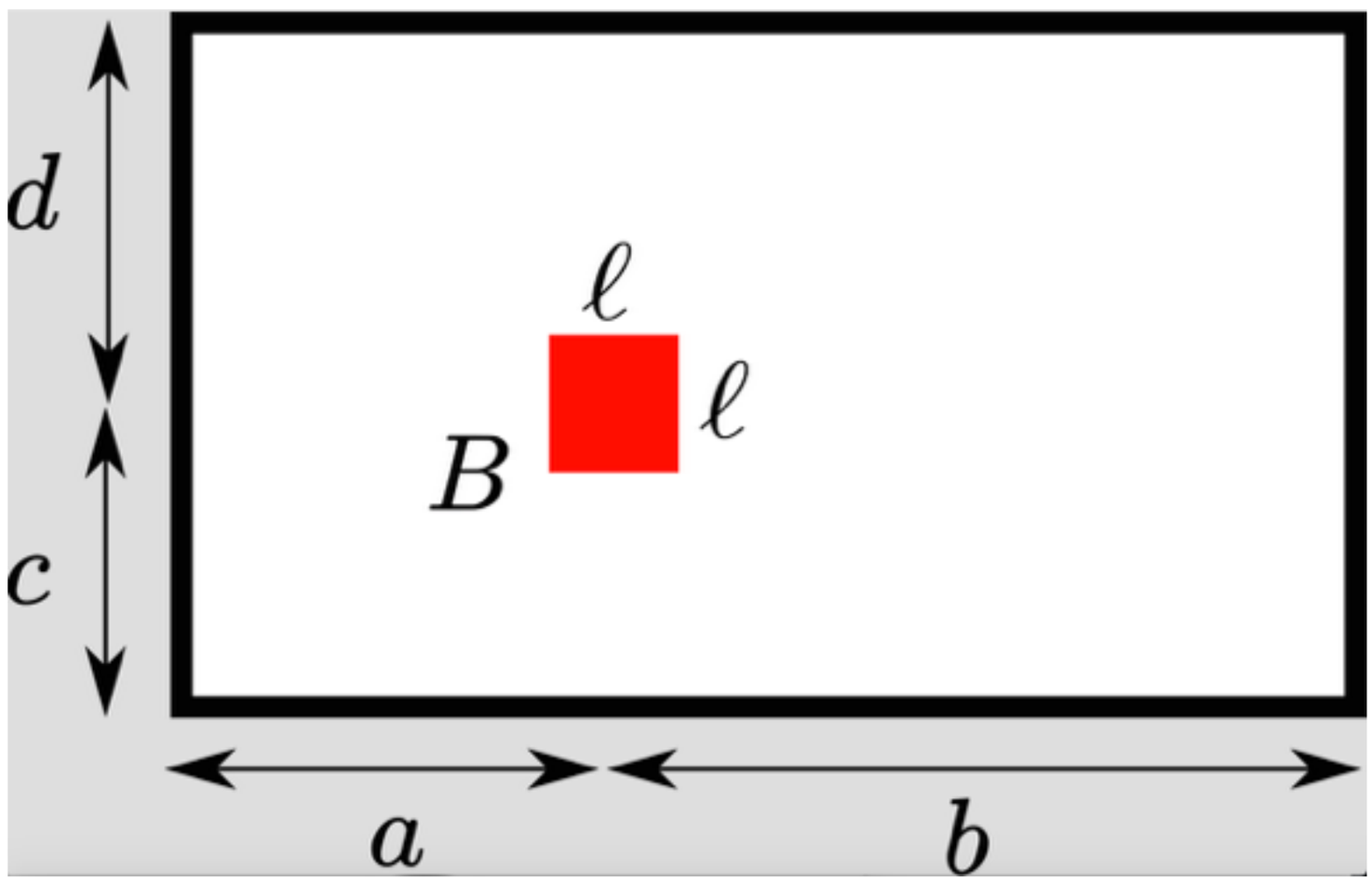
$$a_\mu \to a_\mu + \partial_\mu \Lambda$$

$$-\frac{\hbar^2}{2m}(\partial_i - i\frac{e}{\hbar}a_i)^2\psi = i\hbar\partial_t\psi.$$

compute AB phase

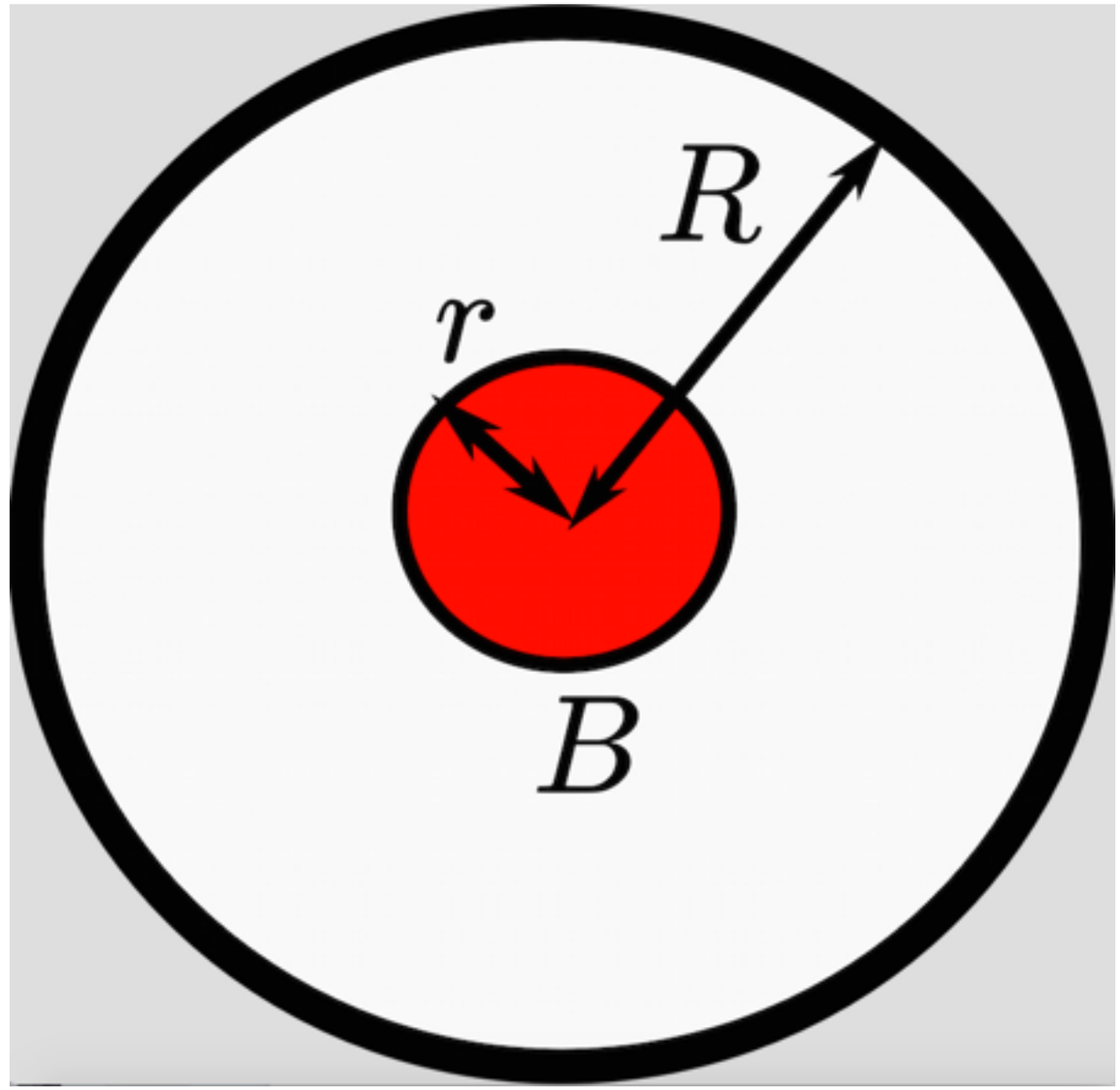
compute AB phase

$$\Delta\Phi = \frac{e}{\hbar}\oint \vec{a}(r')\cdot d\vec{\ell}'$$



use fractional calculus

$$\Delta\phi_{\mathrm{R}} = \frac{eB\ell^2}{\hbar}\left(\frac{b^{\alpha-1}d^{\alpha-1}}{\Gamma^2(\alpha)}\right) c \gg l, d \gg l$$



$$\Delta\phi_{\mathrm{D}} = \frac{e}{\hbar}\pi r^2 B \boxed{R^{2\alpha-2}\left(\frac{\sqrt{\pi}2^{1-\alpha}\Gamma(2-\alpha)\Gamma(1-\frac{\alpha}{2})}{\Gamma(\alpha)\Gamma(\frac{3}{2}-\frac{\alpha}{2})}\sin^2\frac{\pi\alpha}{2}\,{}_2F_1(1-\alpha,2-\alpha;2;\frac{r^2}{R^2})\right)}$$

is the correction large?

$$\alpha = 1 + 2/3 = 5/3$$

$$\Delta\Phi_R = \frac{eB\ell^2}{\hbar} L^{-5/3}/(0.43)^2$$

yes!

is this just a game?

is there a consistent algebra for fractional currents?

Yes

Virasoro algebra

$$L_n := -z^{n+1}\frac{\partial}{\partial z}$$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$

Witt algebra

central extension

conformal transformations on unit disk

$$\mathcal{V} \to \mathcal{W} \to 1$$

Fractional Virasoro algebra

generators

$$L_n^a = -z^{a(n+1)} \left(\frac{\partial}{\partial z}\right)^a \qquad \bar{L}_n^a := -\bar{z}^{a(n+1)} \left(\frac{\partial}{\partial \bar{z}}\right)^a$$

$$[L_n, L_m](z^{ak}) = \left(\frac{\Gamma(a(k+n)+1)}{\Gamma(a(k-1+n)+1)} - \frac{\Gamma(a(k+m)+1)}{\Gamma(a(k-1+m)+1)}\right) L_{n+m}(z^{ak})$$

$$= (A^a_{n,m}(k) \otimes L_{n+m})(z^{ak})$$

$$1 \to \mathcal{H} \to \mathcal{V}_a \to \mathcal{W}_a \to 1$$

$$A^1_{n,m}(k) = n - m$$

Fractional Virasoro algebra

cohomology group

$$H^2(\mathcal{W}_a, \mathcal{H}) = Z^2(\mathcal{W}_a, \mathcal{H})/B^2(\mathcal{W}_a, \mathcal{H})$$

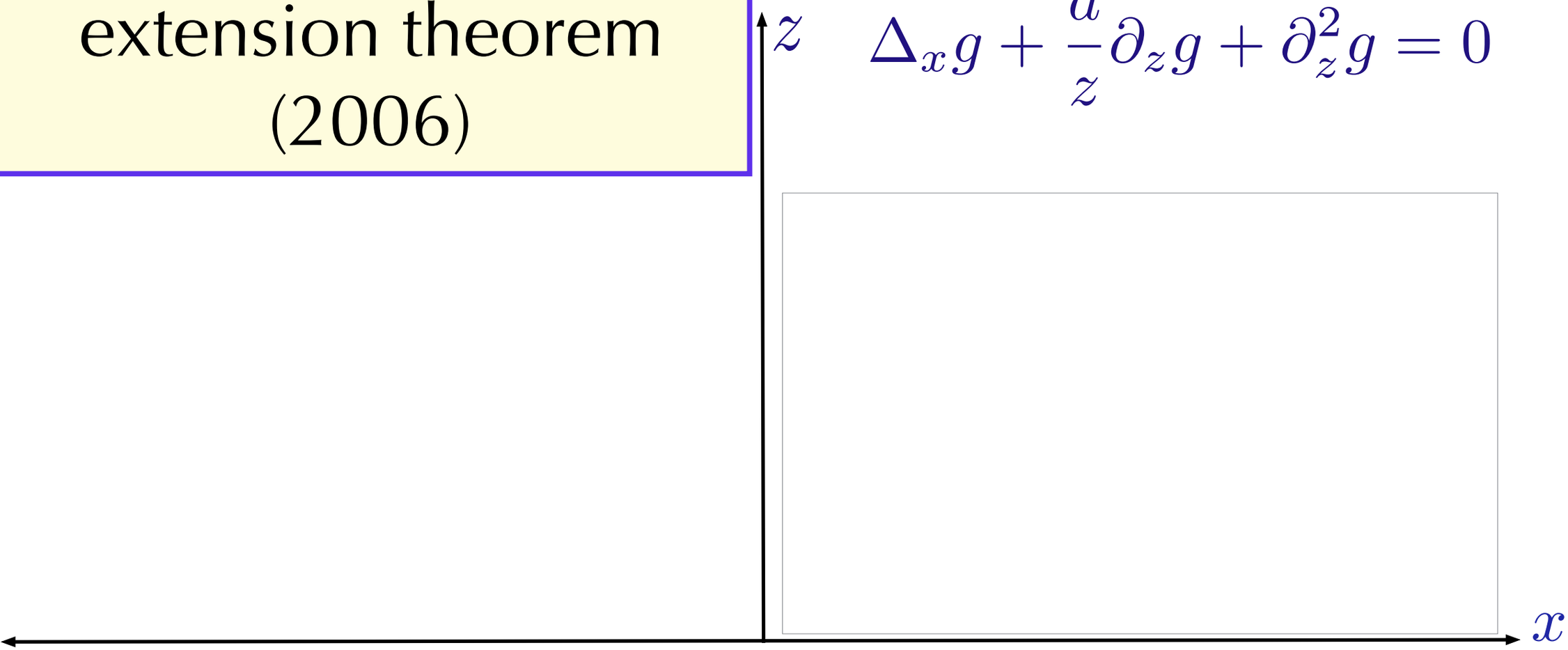
algebra for conformal non-local actions

origin of non-local actions

use Caffarelli-Silvestre extension theorem (2006)

$$g(x,0) = f(x)$$

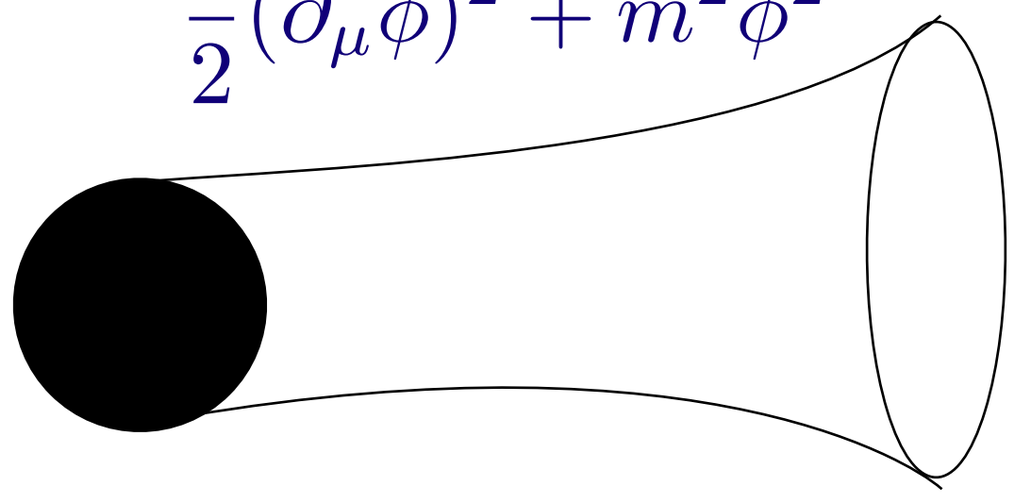
$$\Delta_x g + \frac{a}{z}\partial_z g + \partial_z^2 g = 0$$



$$g(z=0,x) = f(x)$$

$$\gamma = \frac{1-a}{2}$$

$$\frac{1}{2}(\partial_\mu \phi)^2 + m^2\phi^2$$



$$\mathcal{O} = C_{\mathcal{O}} \lim_{z\to 0} z^{-\Delta}\phi(x,z)$$

ϕ

solves massive scalar problem

$g = z^{\gamma - d/2}\phi$

solves CS extension problem

exact form of boundary operator

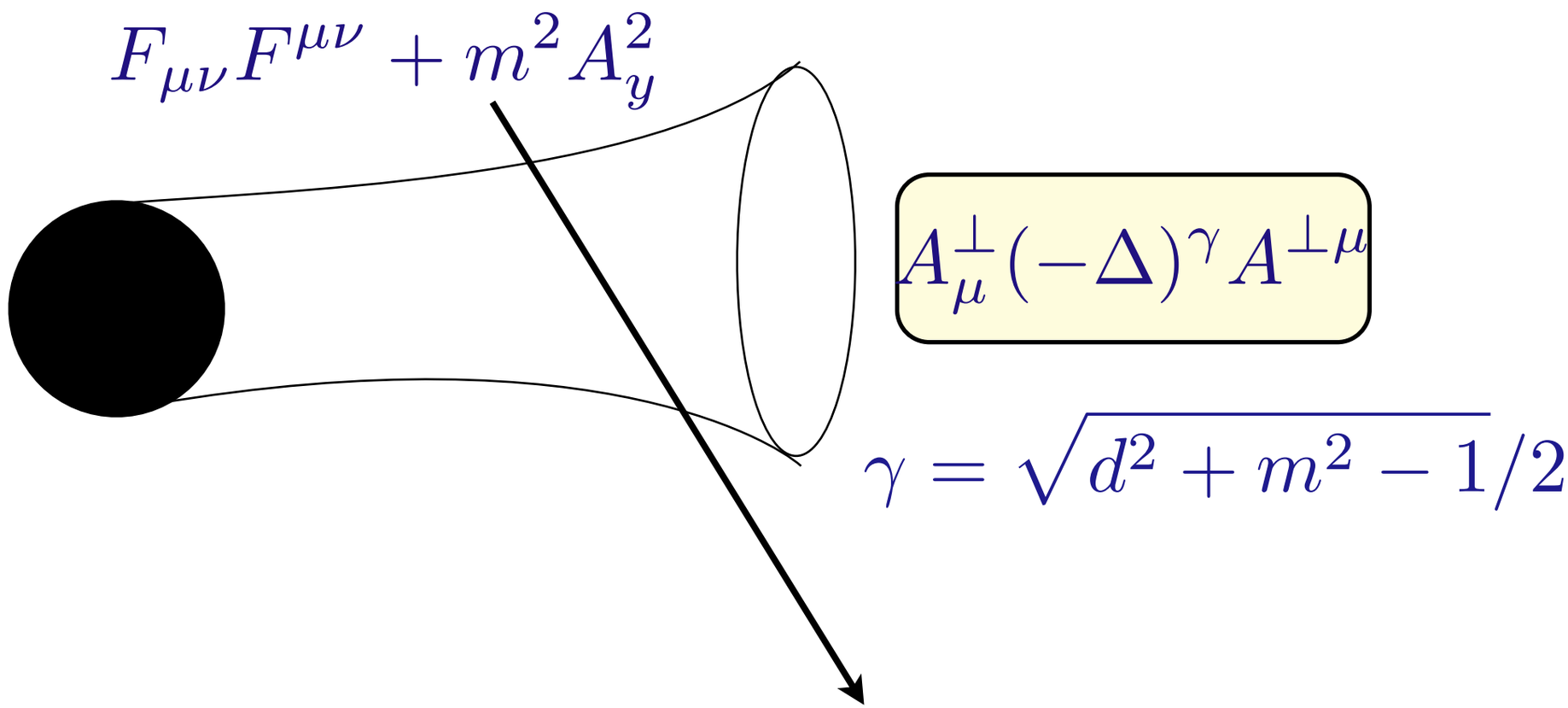
$$\mathcal{O} = C_{\mathcal{O}} \lim_{z \to 0} z^{-\Delta} \phi(x, z)$$
$$= C_{\mathcal{O}} \lim_{z \to 0} z^{-\Delta+1} \partial_z \phi(x, z)$$

$$\mathcal{O} = (-\Delta)^{\gamma} \phi_0$$

use Caffarelli/ Silvestre

the $\mathcal{O}$ for massive scalar field

application: gauge fields with anomalous dimensions



dynamical `Higgs' mode

additional length scale

strange metal

combine AC+DC transport

$[J] = \Phi$

fraction not d

non-local action

$[A_\mu] = d_A \neq 1$

probe by Aharonov-Bohm effect on underdoped cuprates

new Virasoro algebra