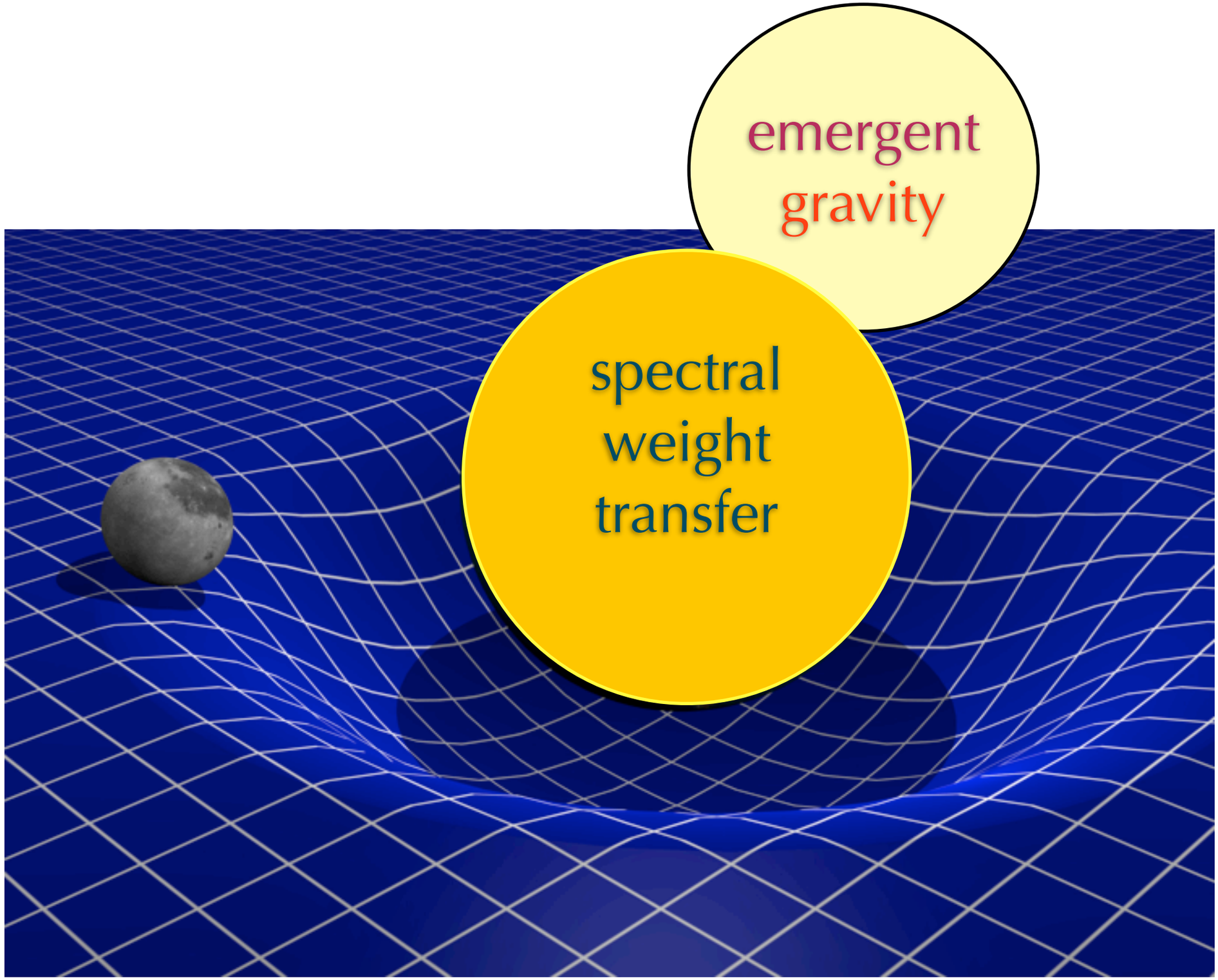
Mottness and Holography: A Discrete Marriage

Thanks to:
NSF, EFRC (DOE)

Seungmin Hong

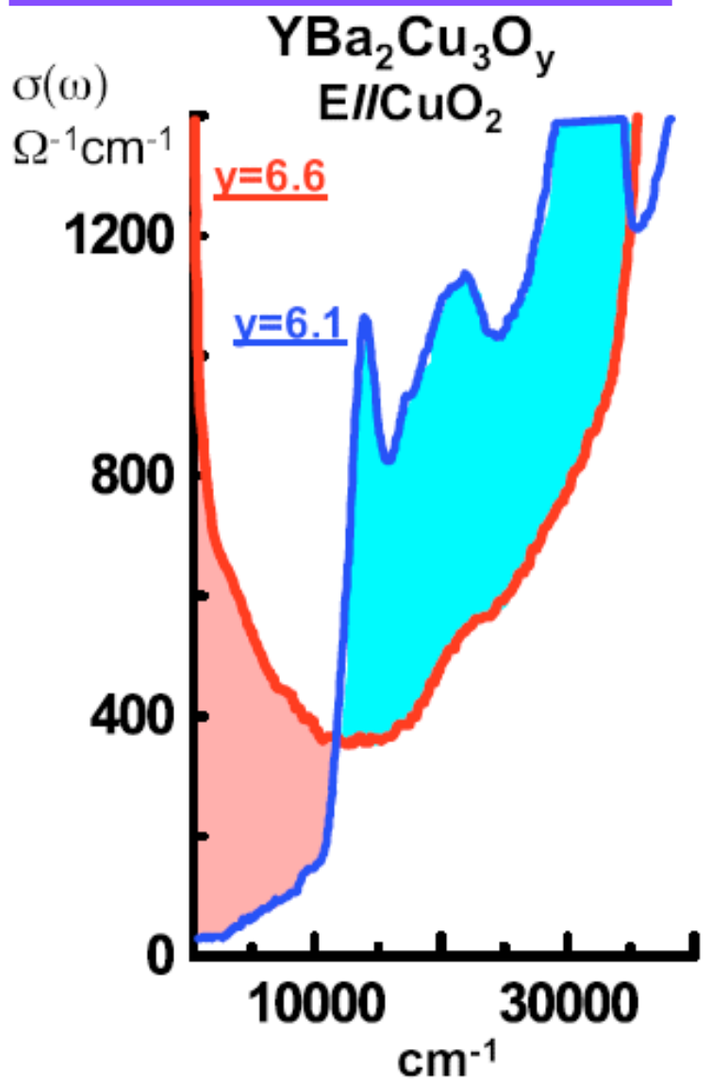
M. Edalati

R. G. Leigh


emergent
gravity
spectral
weight
transfer

spectral weight transfer

2-particle probe



spectral weight transfer

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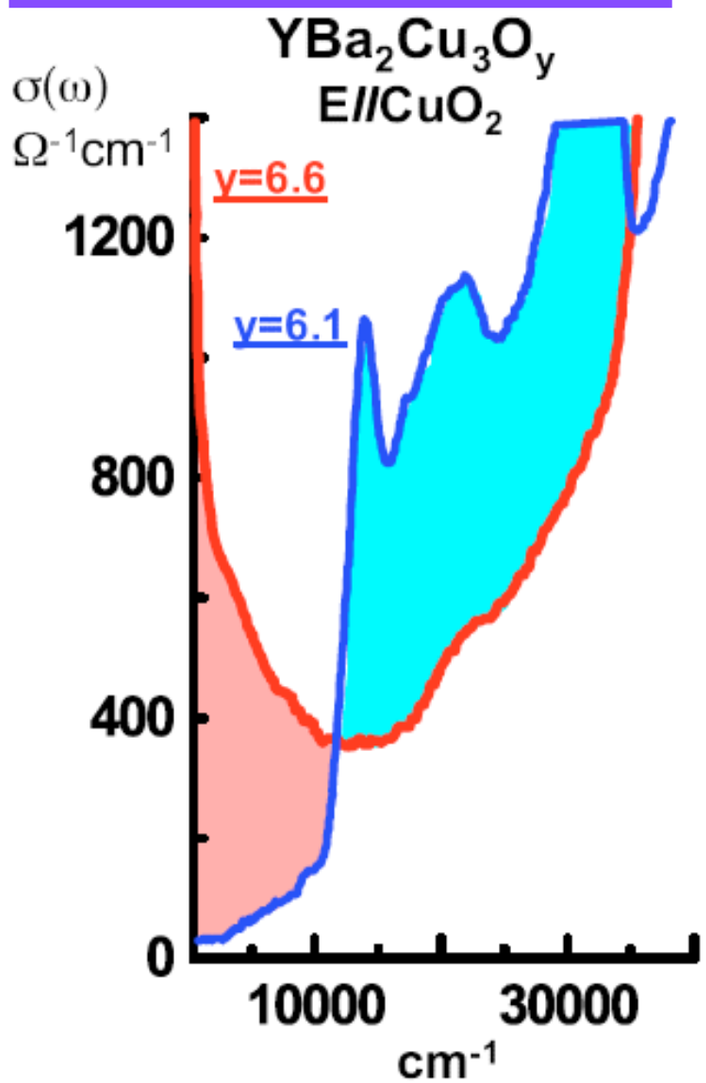
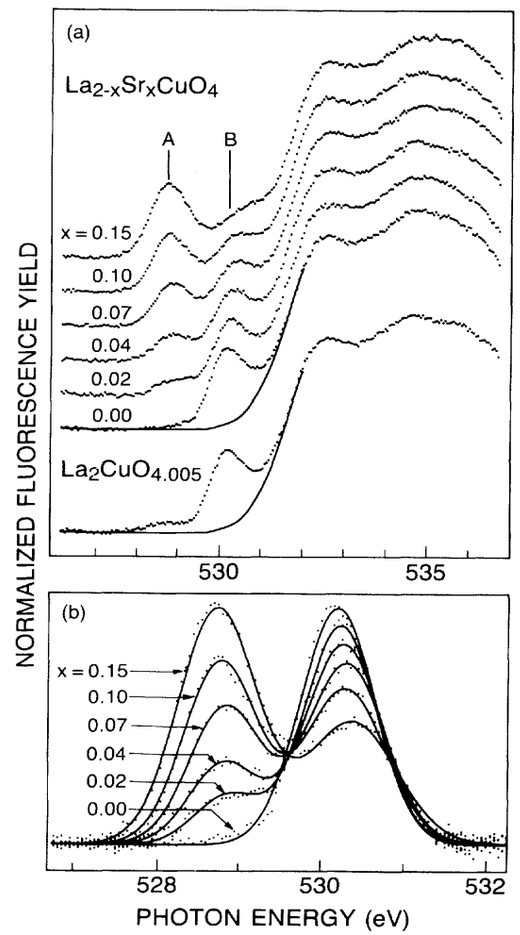
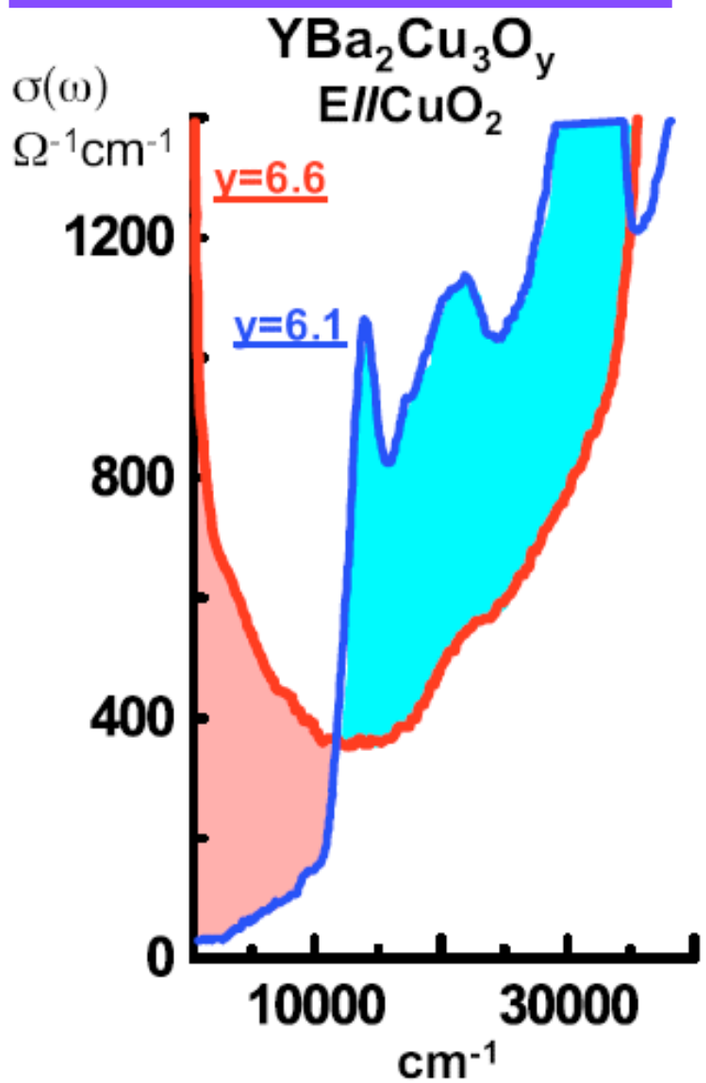


1-particle

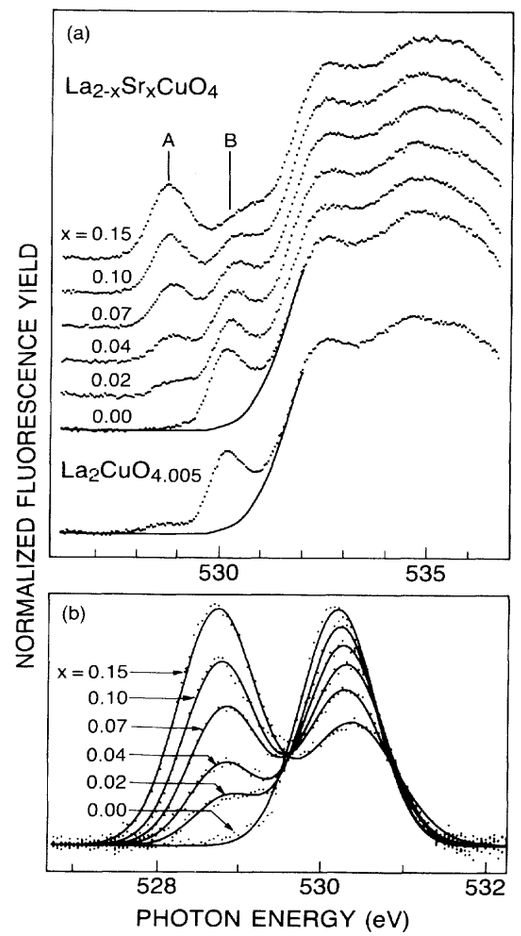


spectral weight transfer

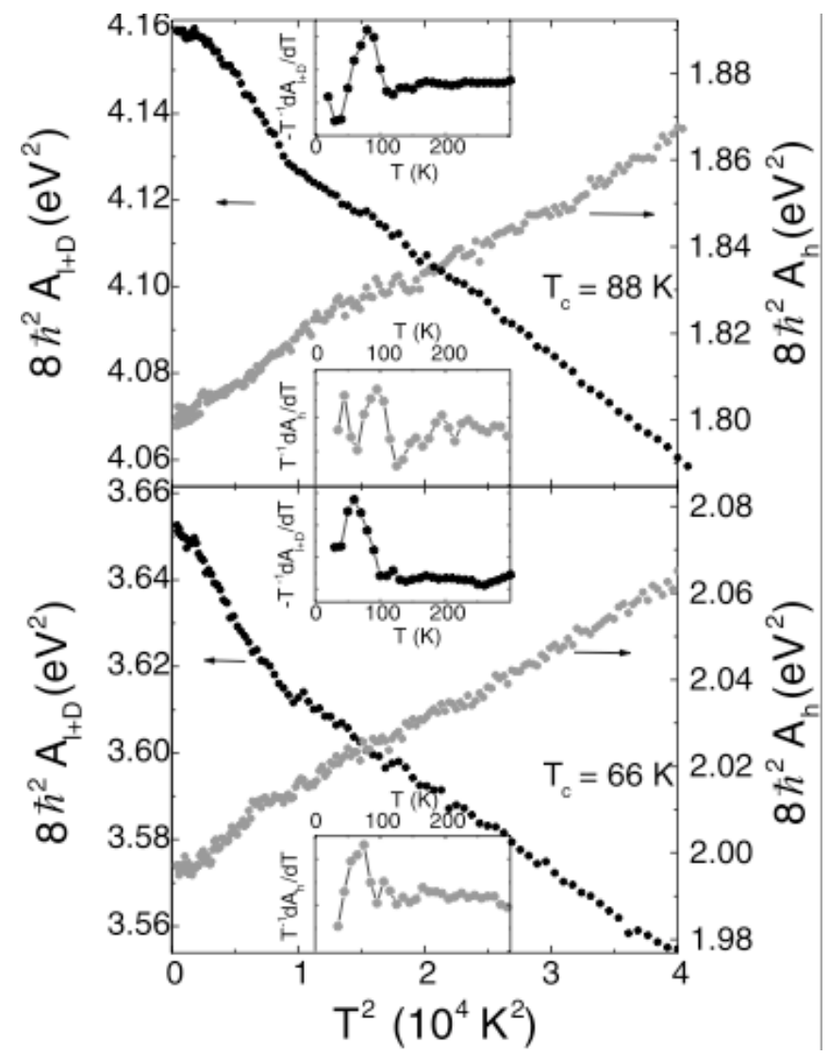
2-particle probe



1-particle



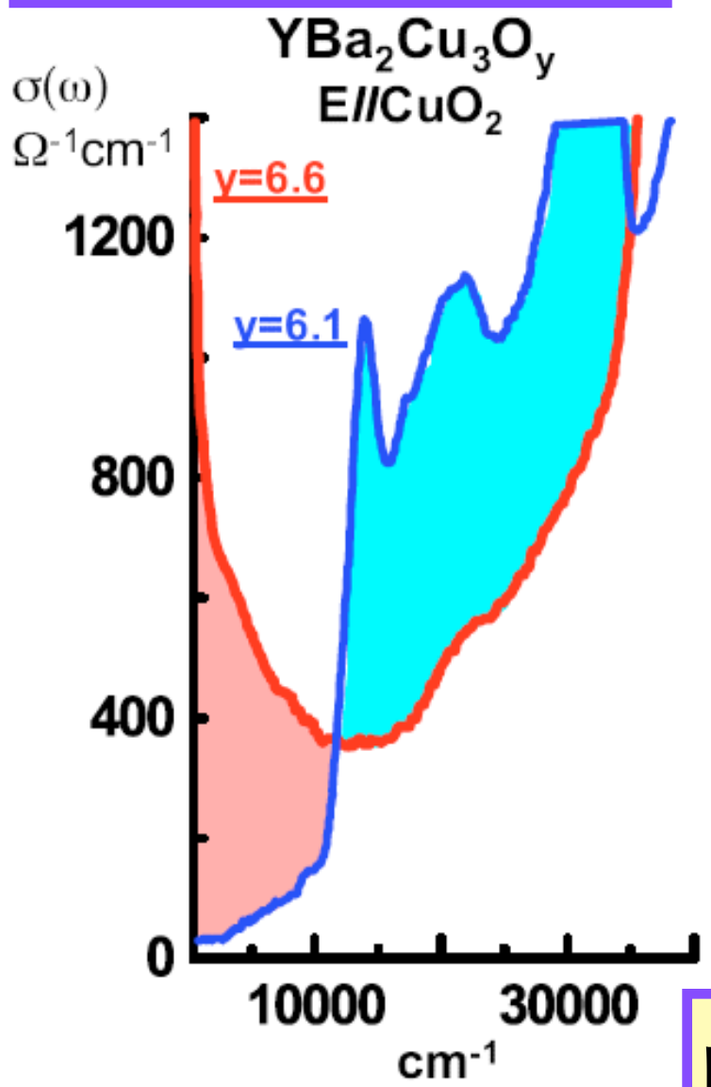
SC state



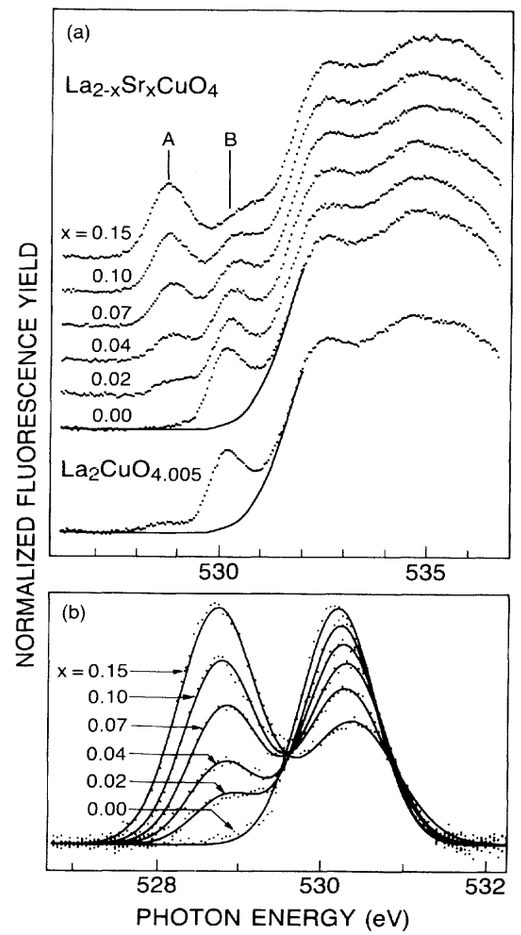
non-BCS

spectral weight transfer

2-particle probe



1-particle



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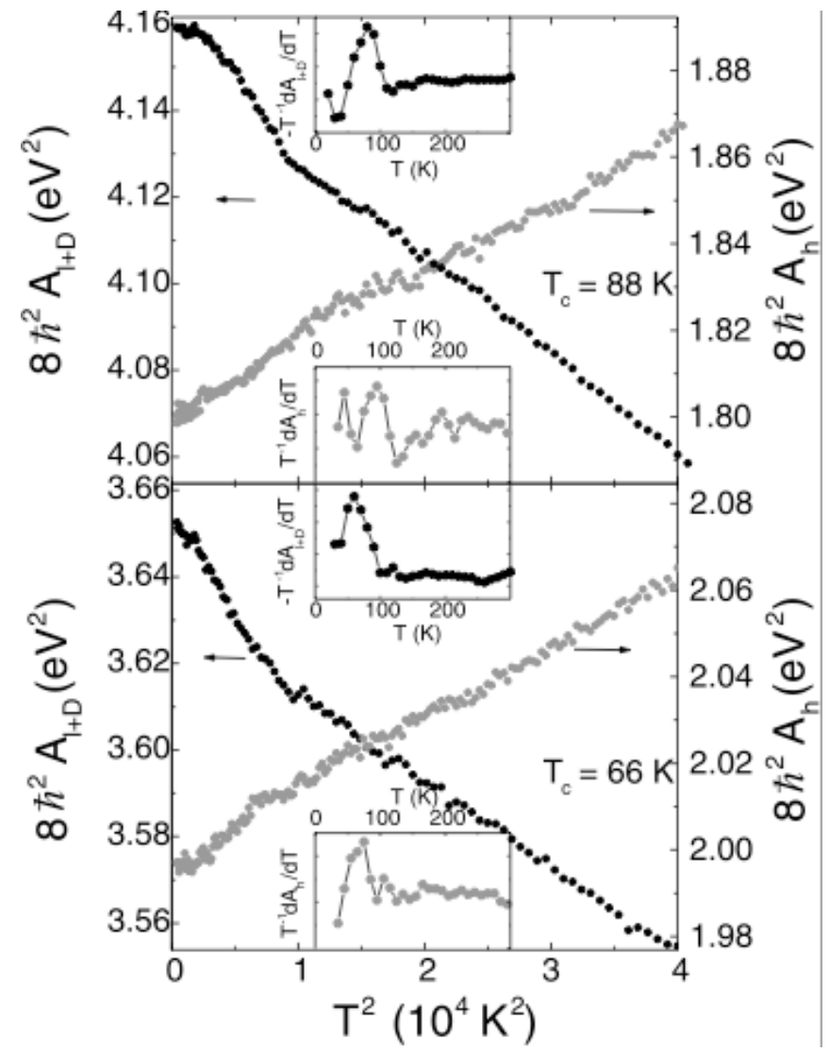
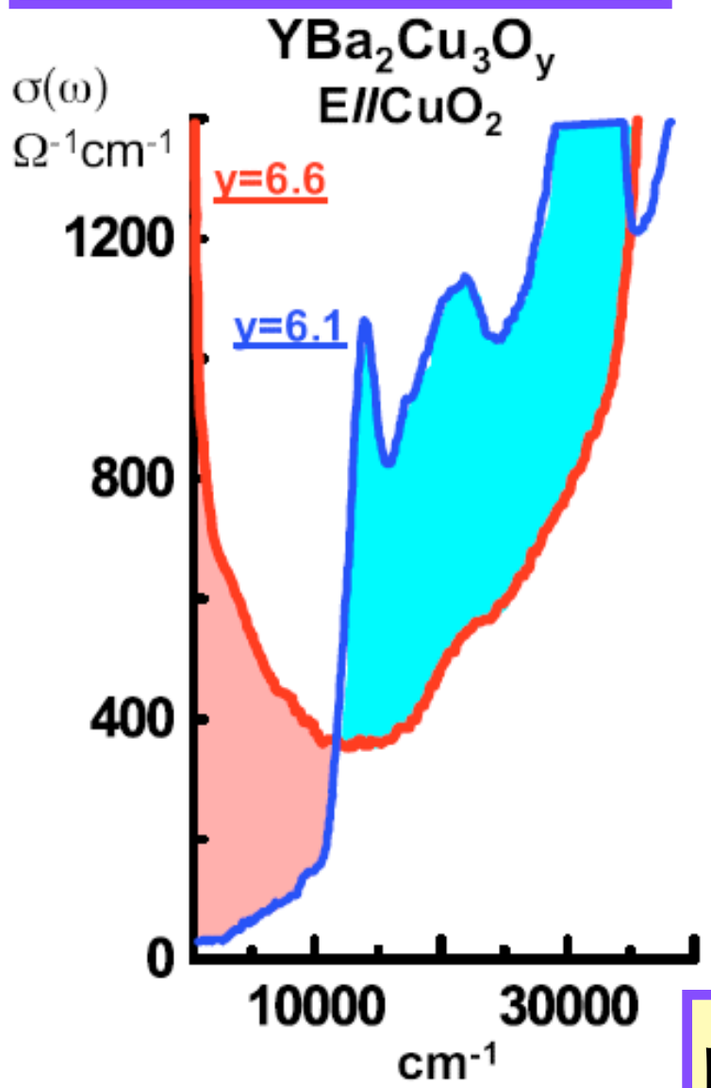


physics of a non-rigid band:
UV-IR mixing

non-BCS

spectral weight transfer

2-particle probe



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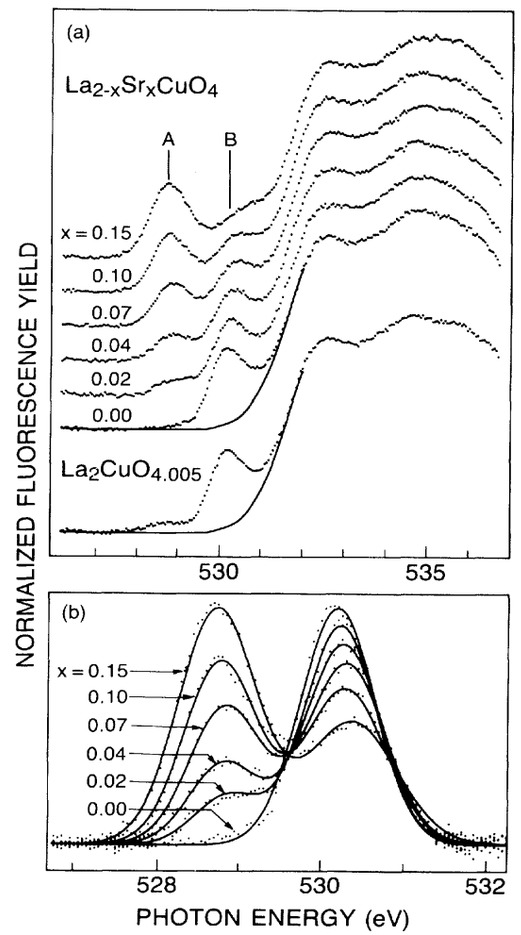

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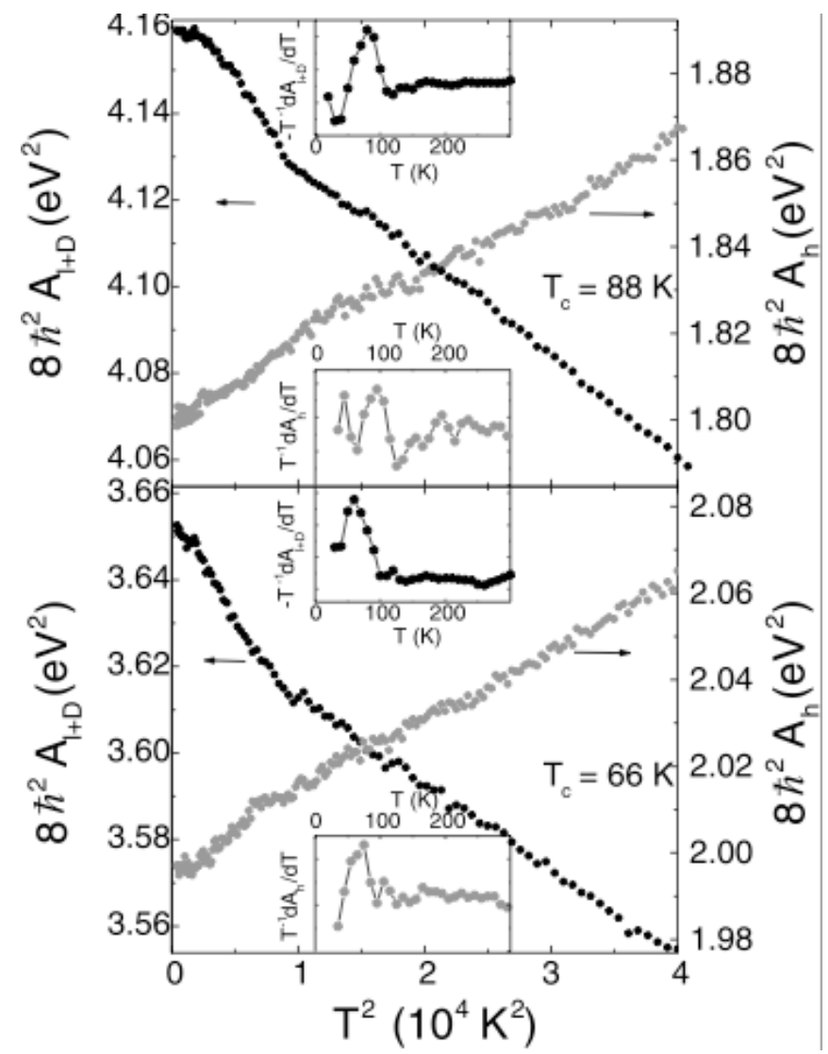


non-BCS

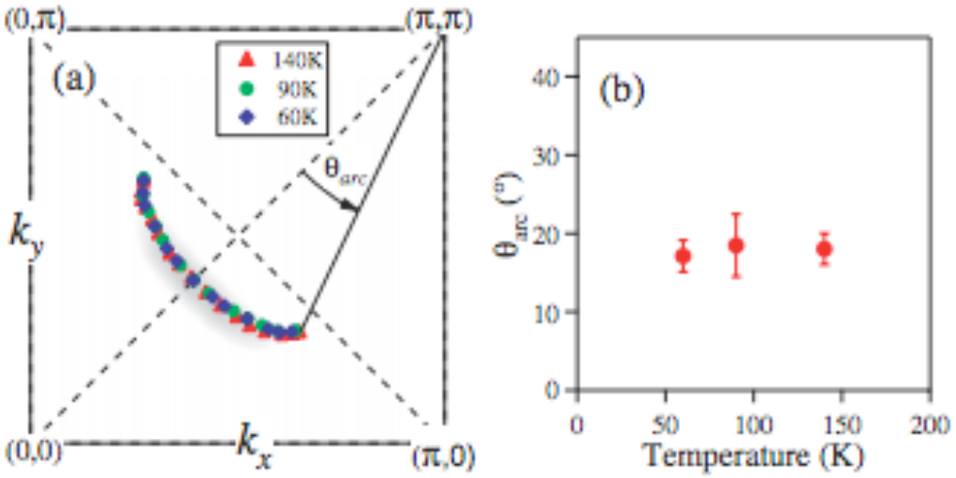
physics of a non-rigid band: UV-IR mixing

Wilsonian program

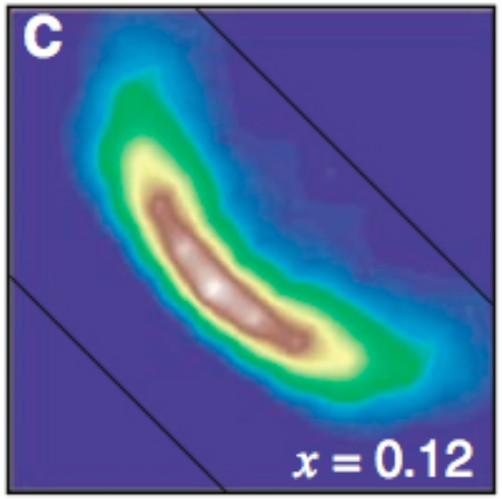
holography

Is spectral weight transfer important?

Fermi Arcs



P. Johnson, PRL 2011



Na-CCOC
(Shen, Science 2006)

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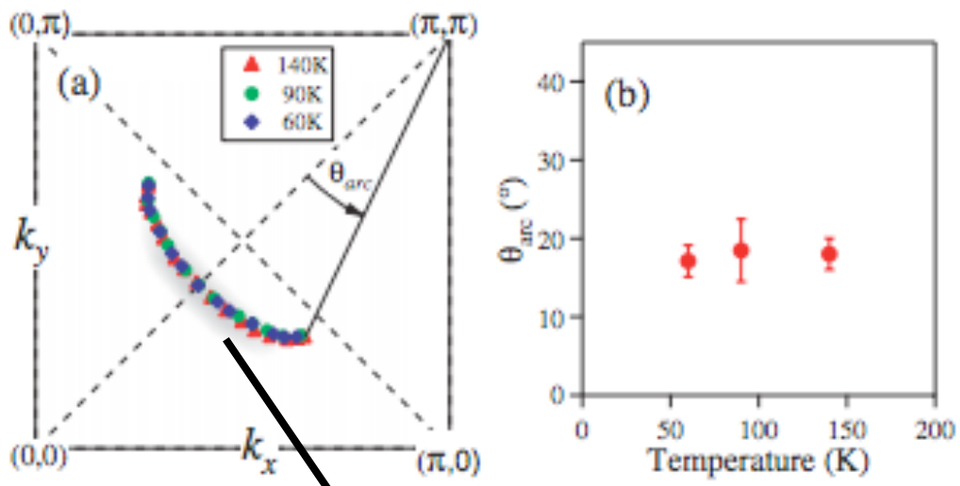

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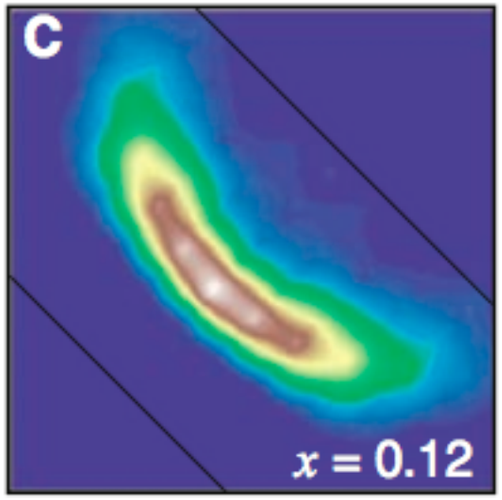

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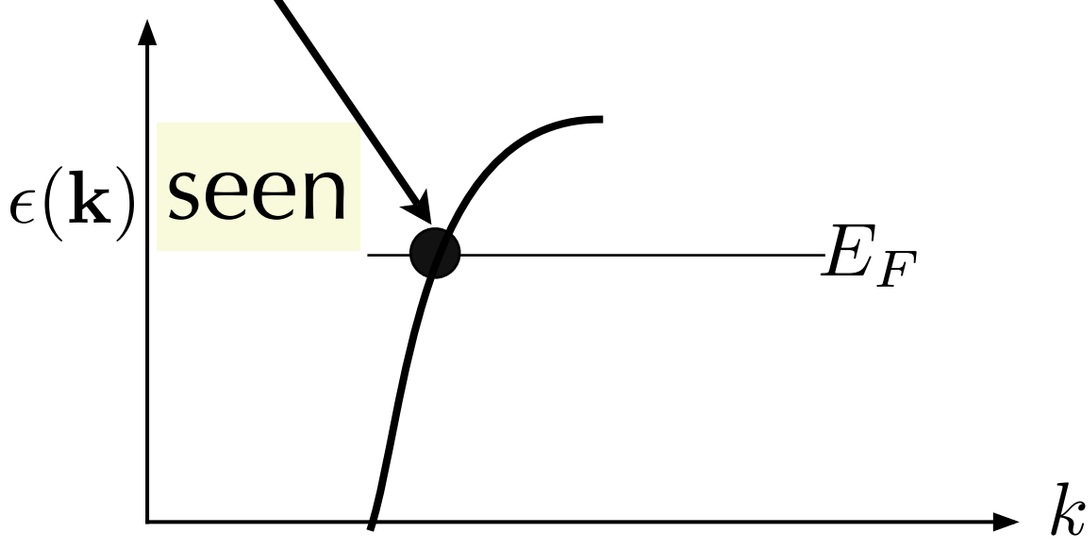

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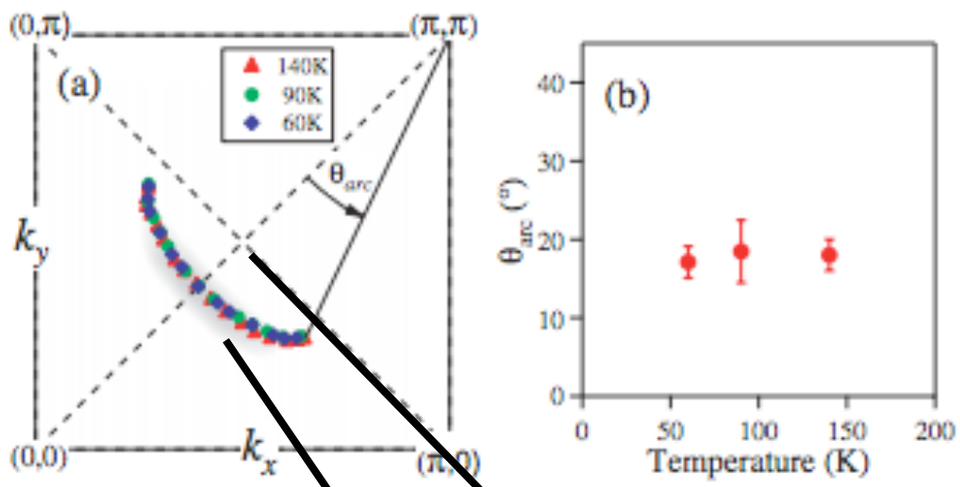

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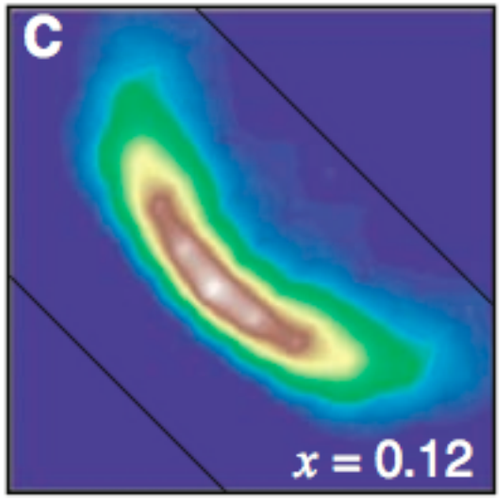

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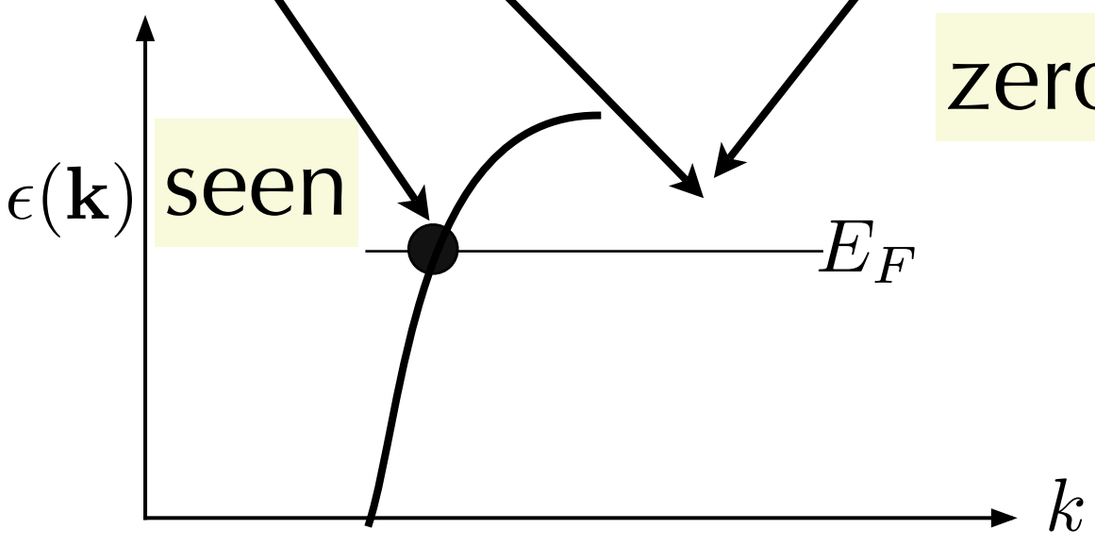
not seen

zeros (incoherence)



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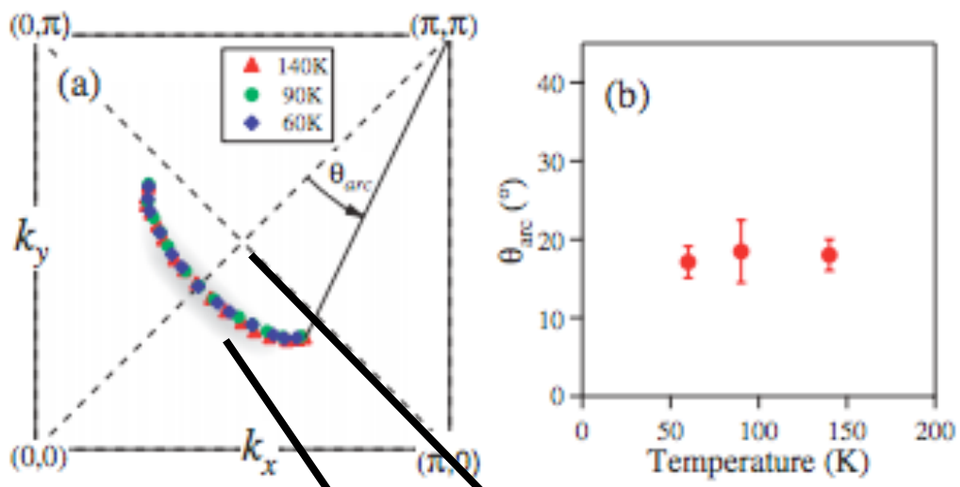

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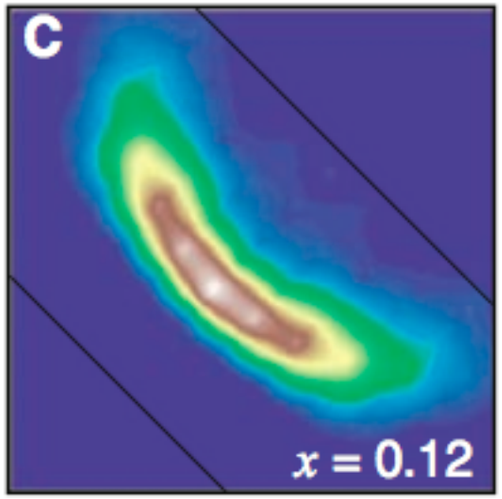

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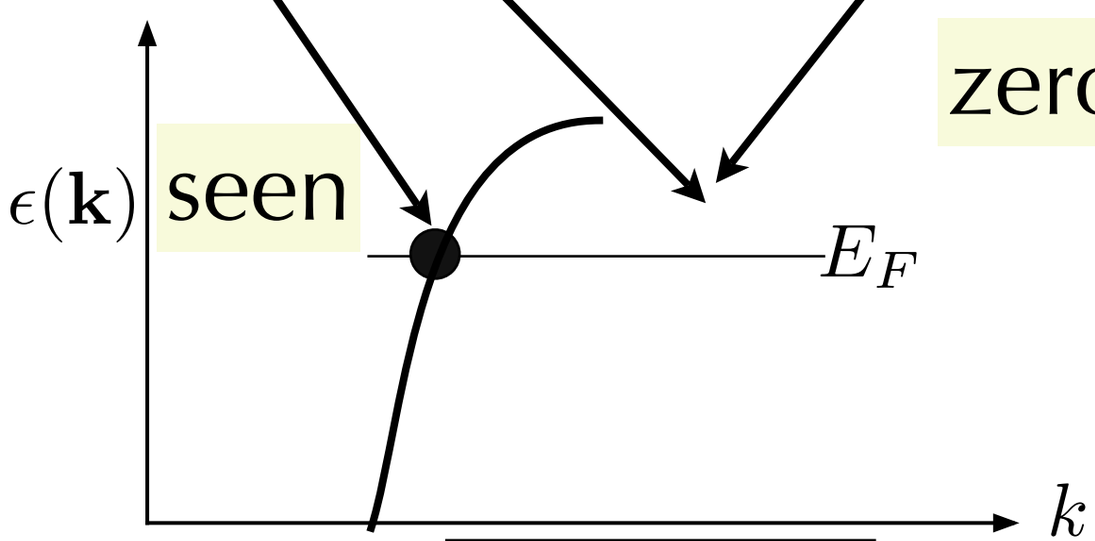

meaning?

can we explain this?

Where do the zeros come from?

$$G(\omega, \mathbf{k}) = \frac{\cos^2\theta}{\omega - \omega_+} + \frac{\sin^2\theta}{\omega - \omega_-} = \frac{Z}{\omega - \epsilon_k - \Sigma(\omega, \mathbf{k})}$$

Mixing between `two' bands

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phenomenology:
YRZ, Norman,
Sachdev, Imada,...

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Mott Problem

not rigid bands: spectral weight transfer

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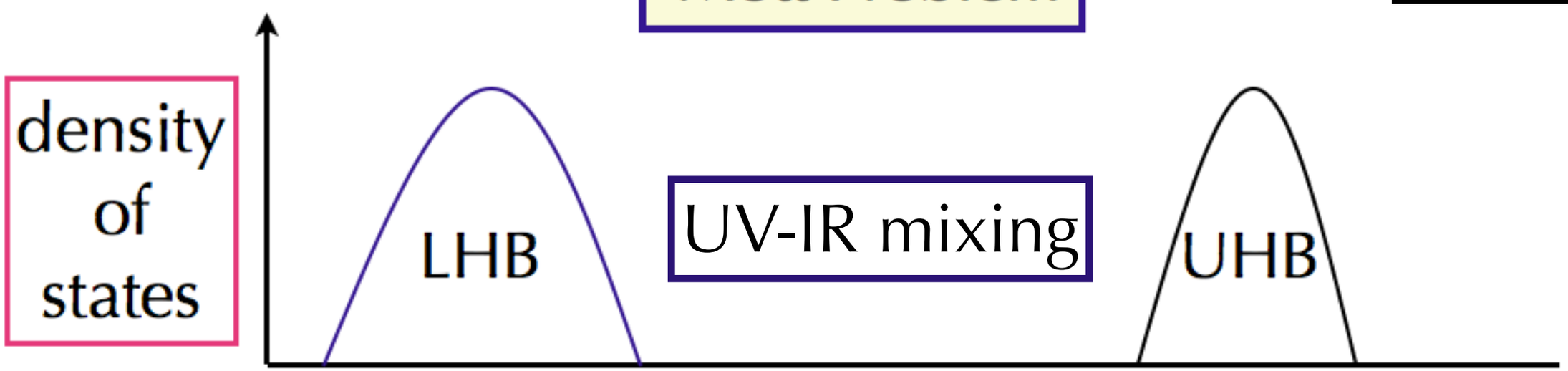
Mott Problem



not rigid bands: spectral weight transfer

zeros of Hubbard model: Exact Results

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atomic limit

static spectral weight

$$G(\omega, k) = \frac{1+x}{\omega - \mu + U/2} + \frac{1-x}{\omega - \mu - U/2}$$

zeros of Hubbard model: Exact Results

atomic limit

static spectral weight

$$\omega = \mu$$
$$x = 0$$

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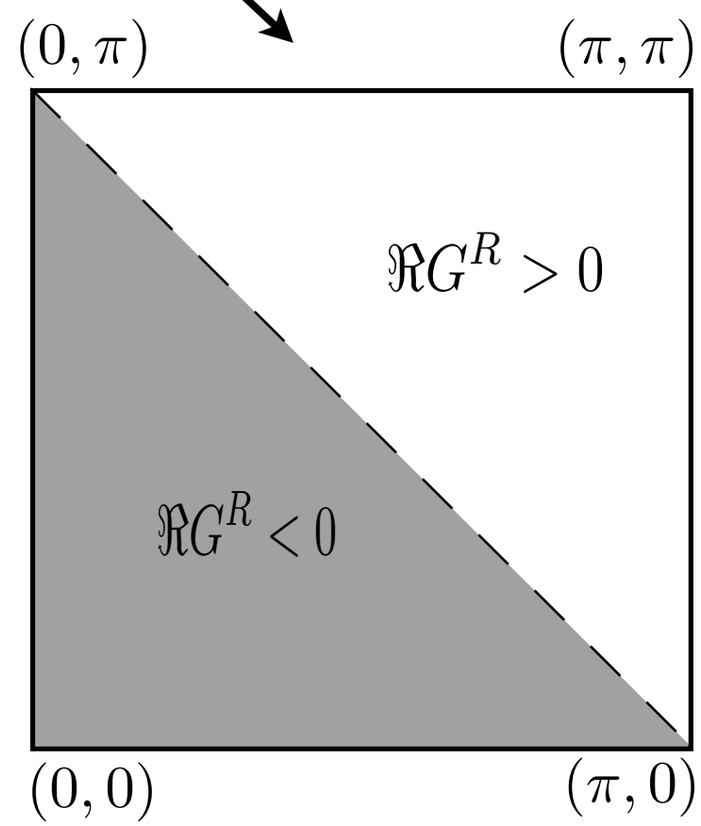
1/2-filling
p-h symmetry



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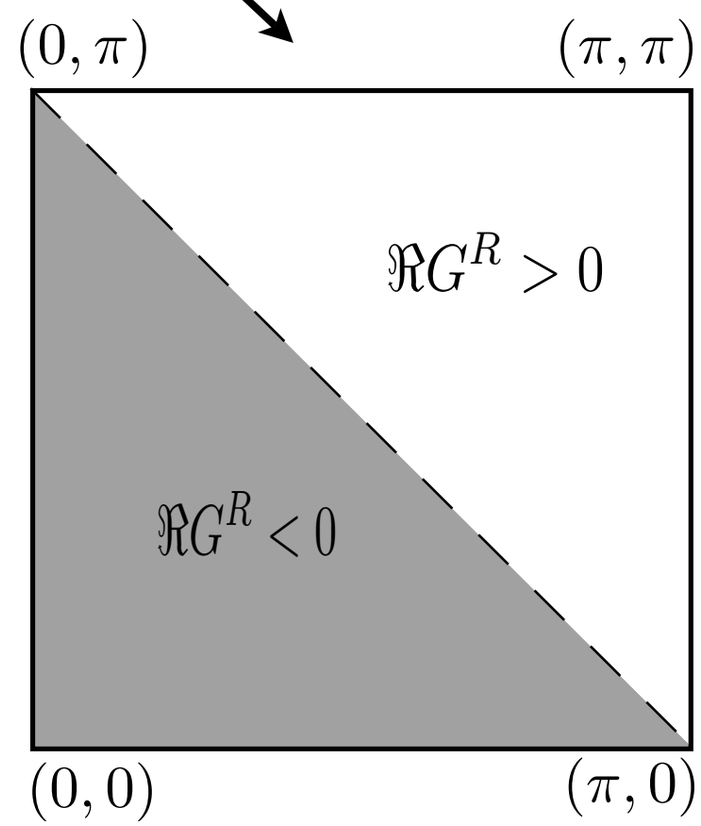

No Fermi arcs

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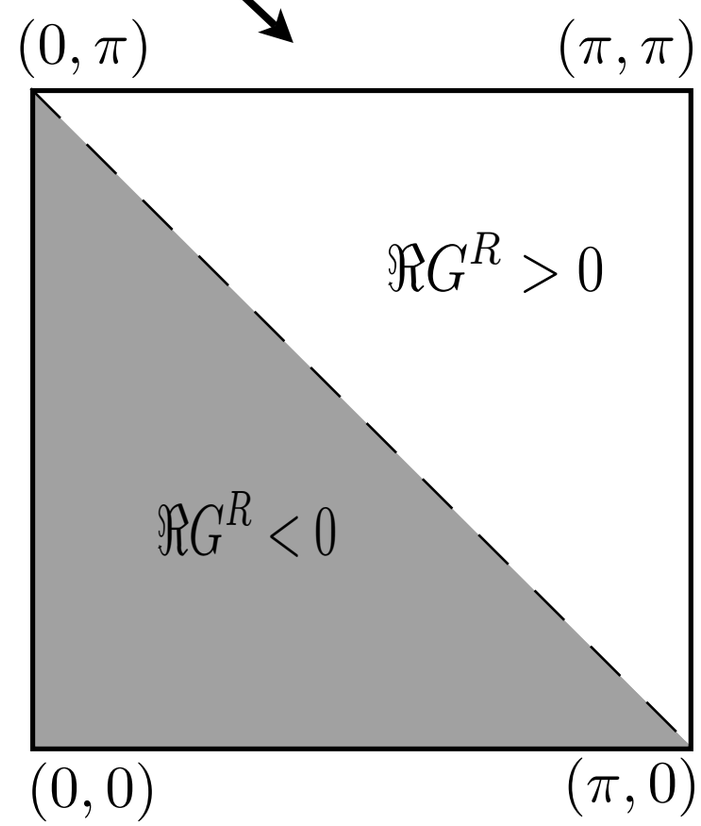
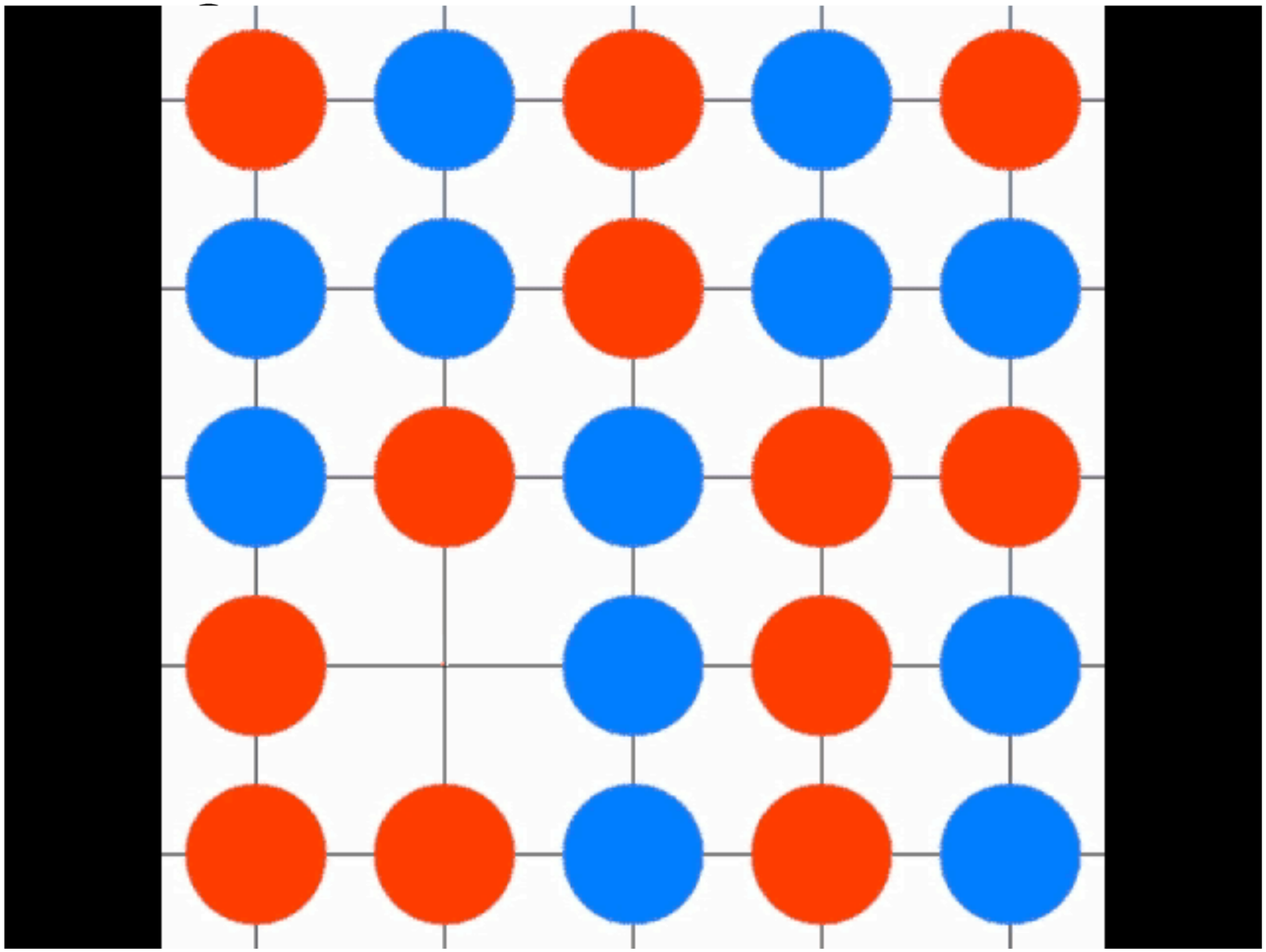


No Fermi arcs

dynamical spectral weight transfer

quantum Mottness: U finite

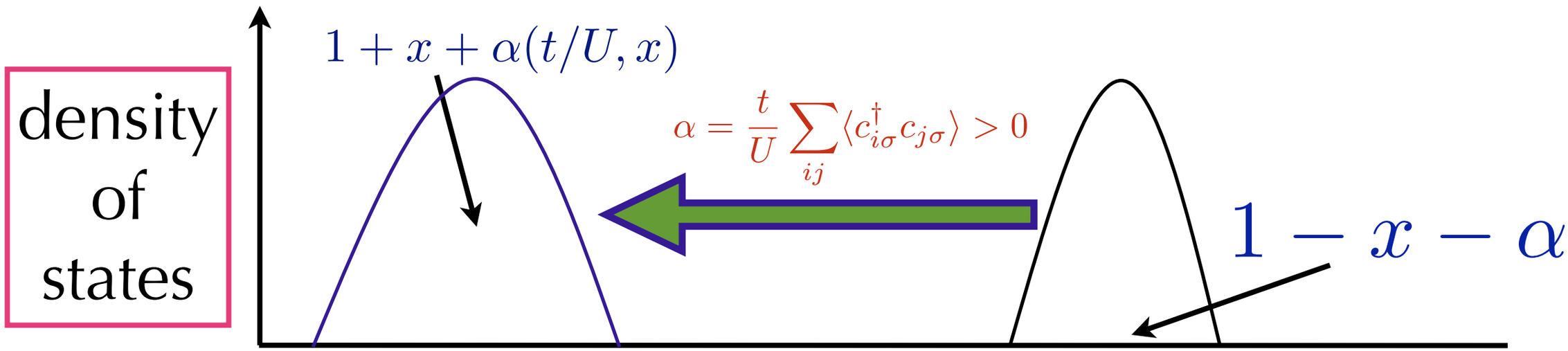
$U \gg t$



double occupancy in ground state!!

$W_{\mathrm{PES}} > 1 + x$

beyond the atomic limit: any real system



Harris & Lange, 1967

dynamical spectral weight transfer

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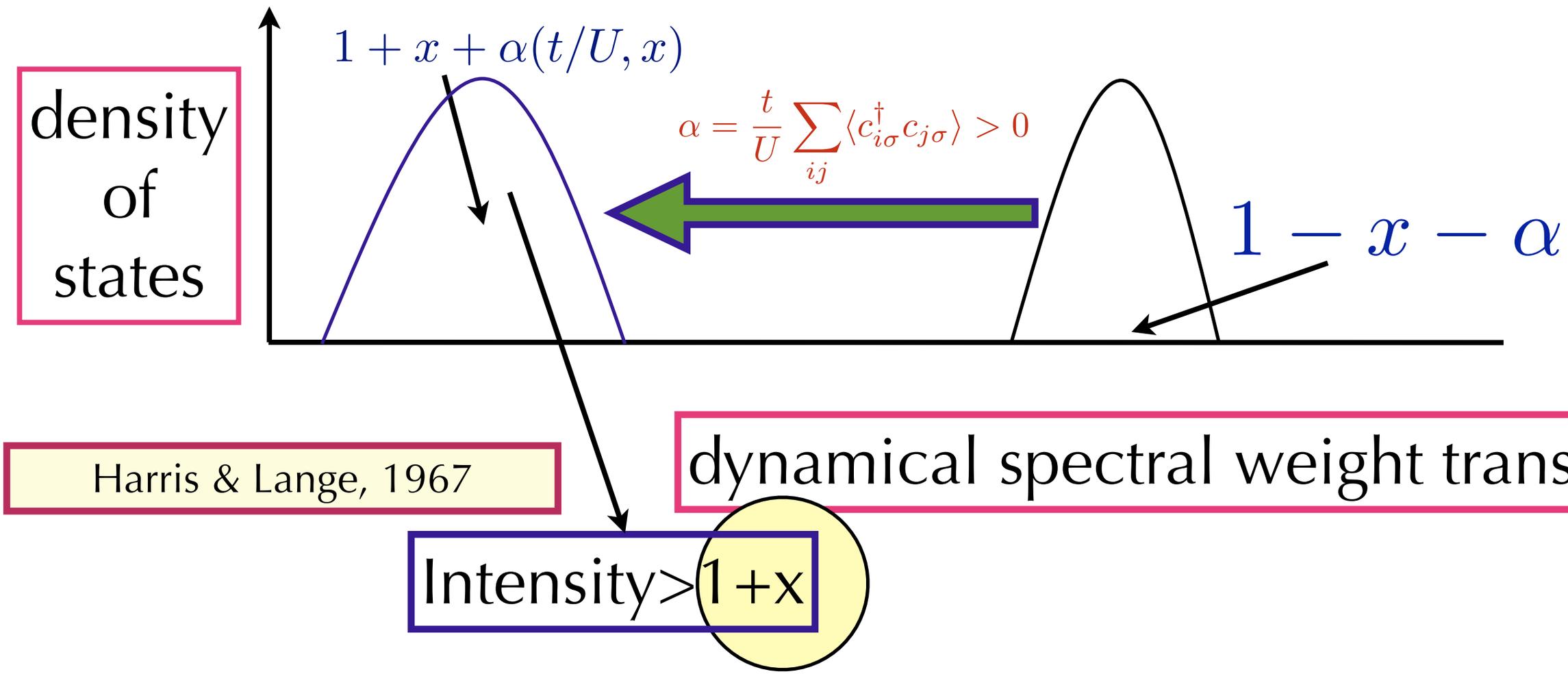
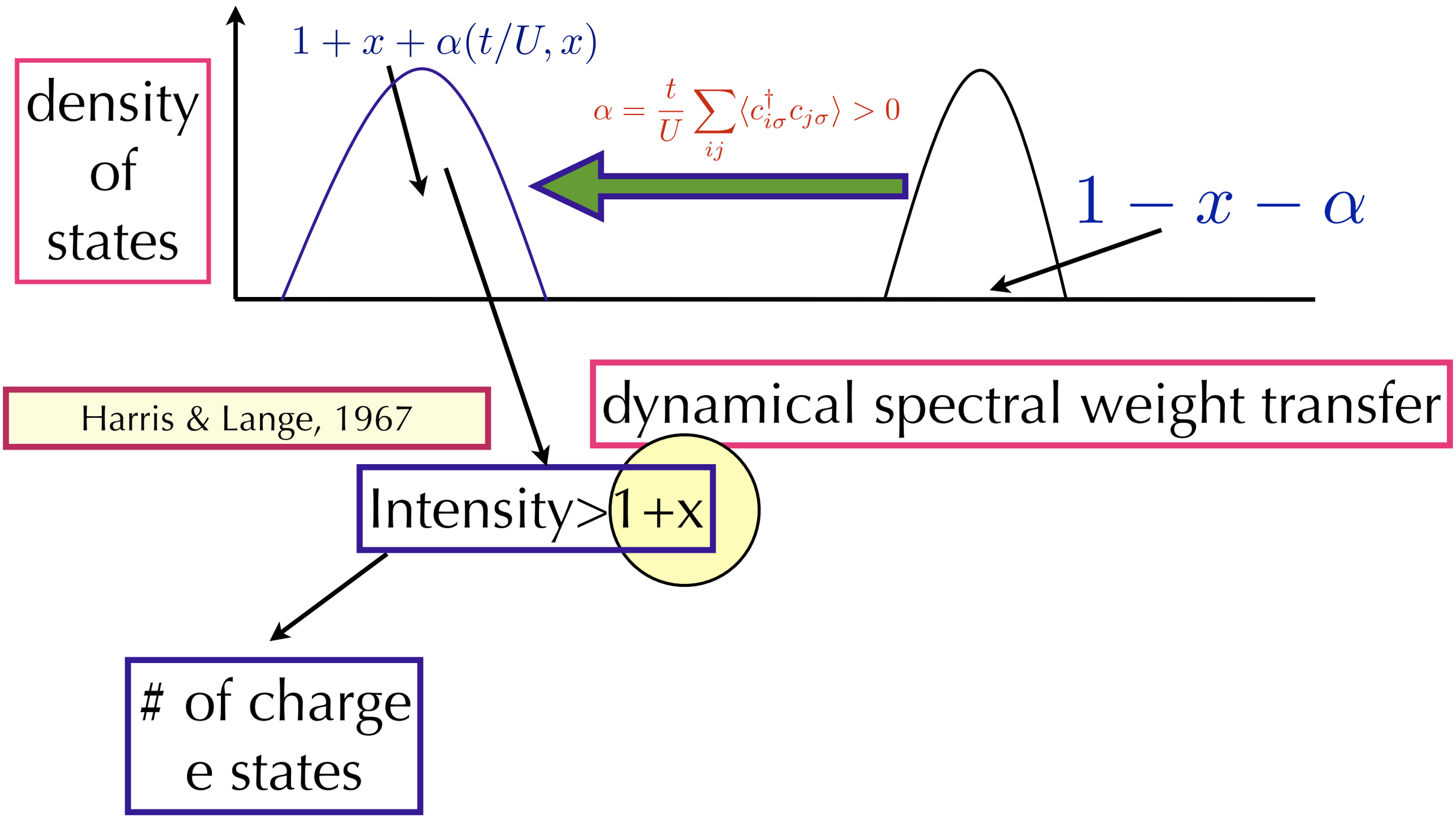


beyond the atomic limit: any real system



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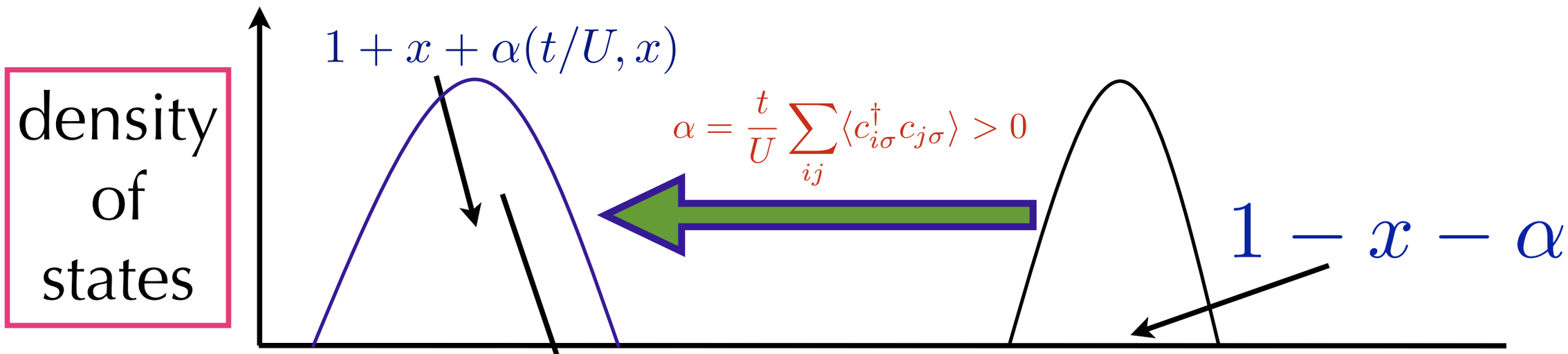


Harris & Lange, 1967

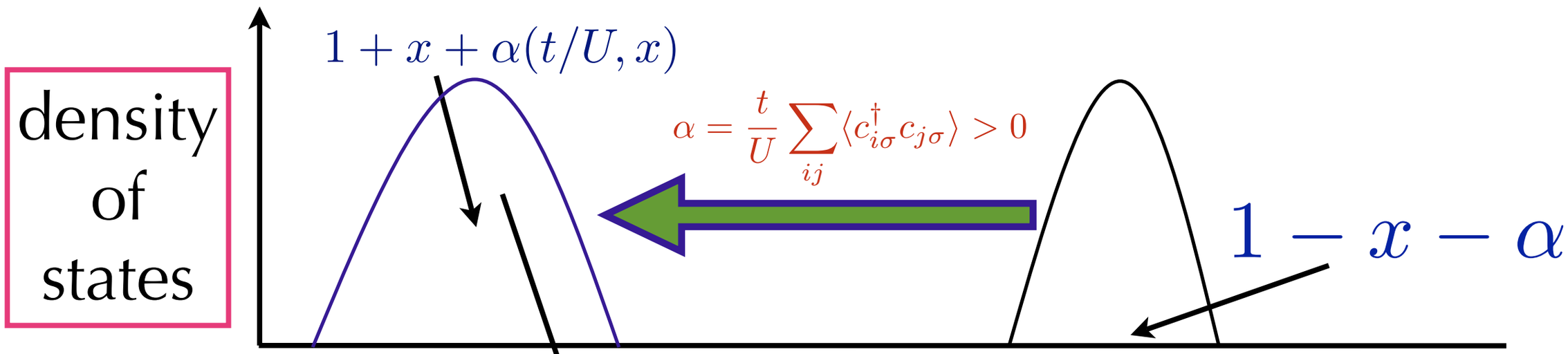
dynamical spectral weight transfer

Intensity>1+x

# of charge e states

# of electron states in lower band

beyond the atomic limit: any real system



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dynamical spectral weight transfer

Intensity>1+x

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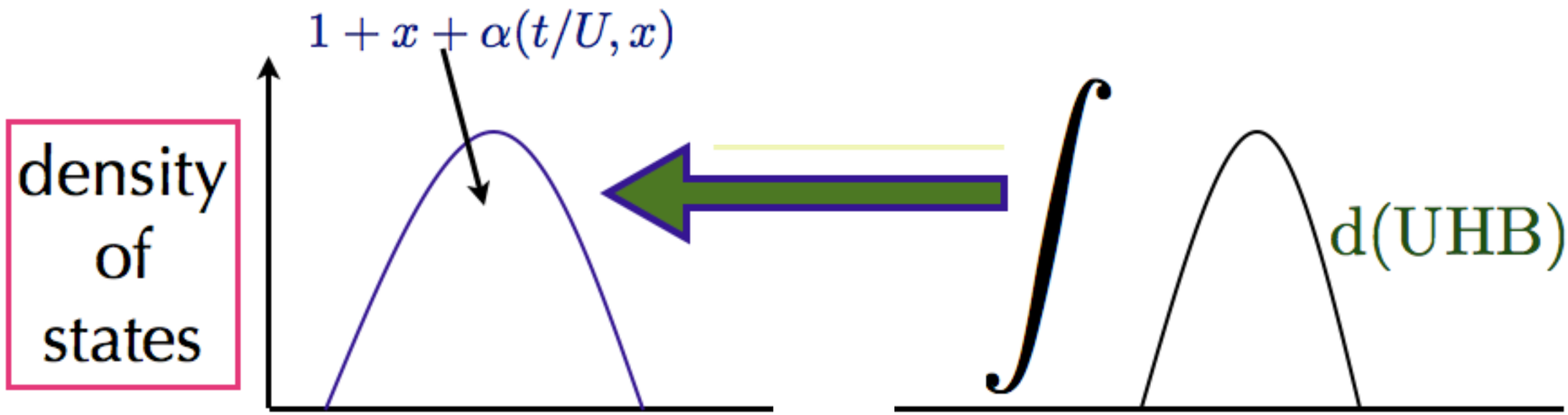
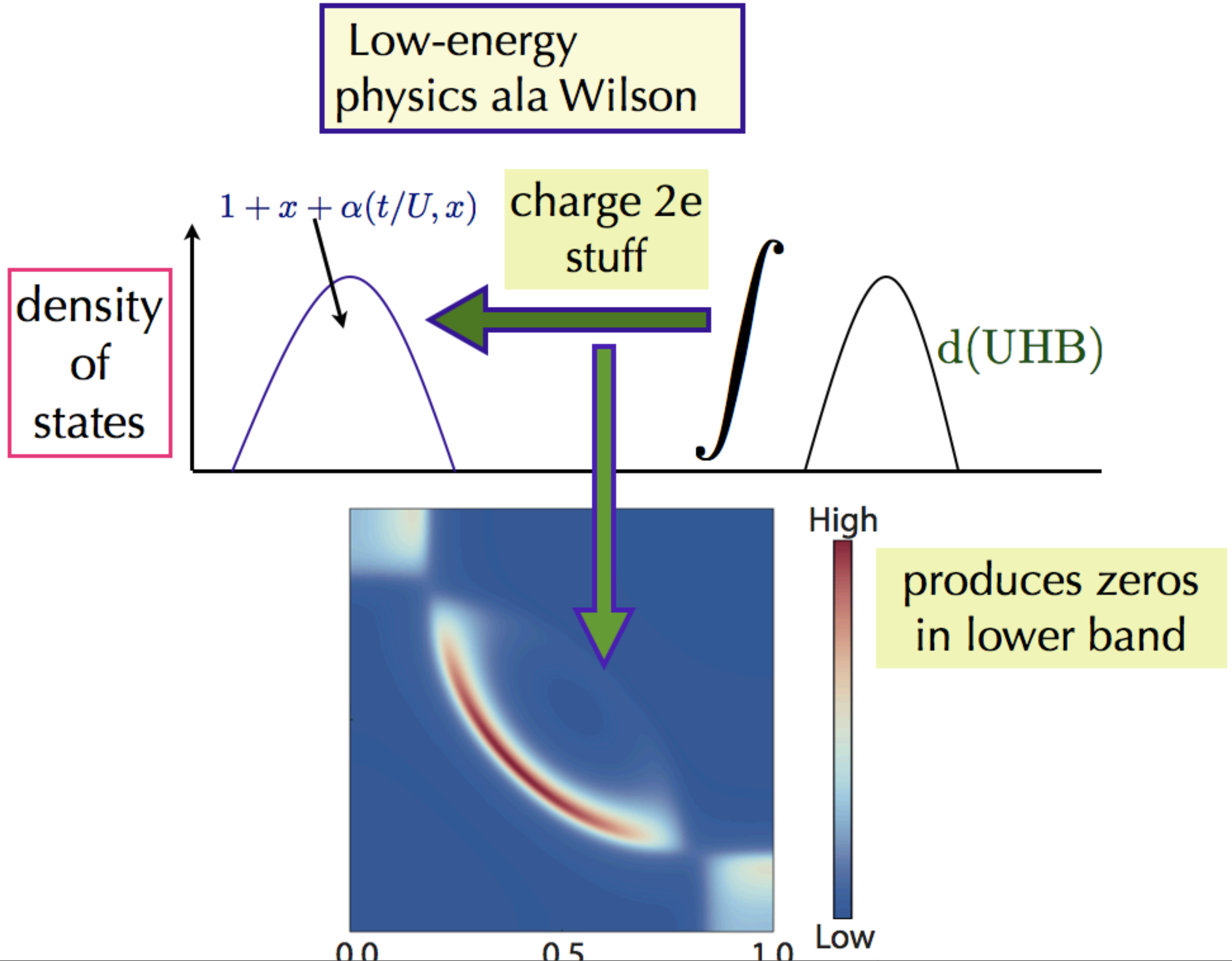
# of electron states in lower band

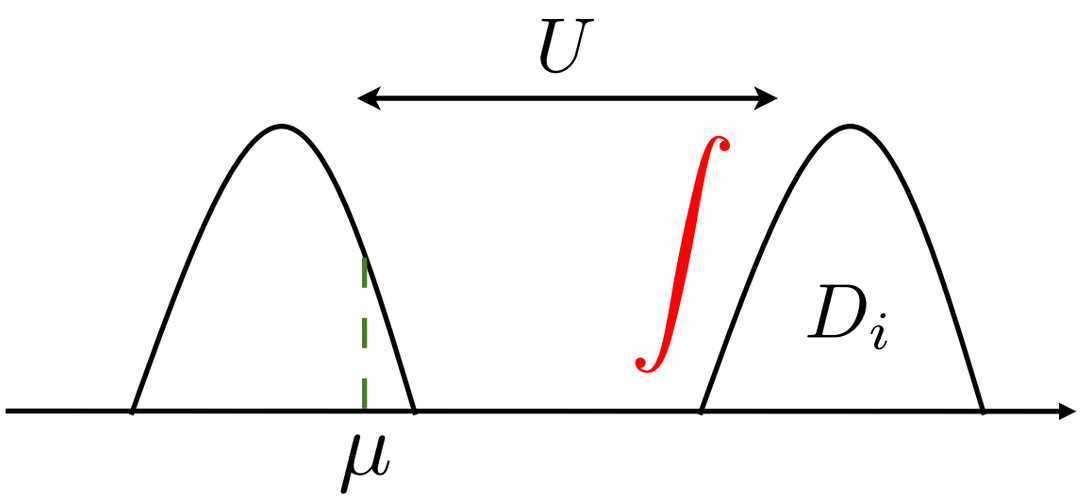
not exhausted by counting electrons alone?

Low-energy physics ala Wilson

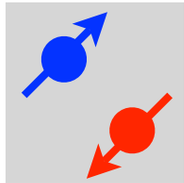



Low-energy physics ala Wilson
$1 + x + \alpha(t/U, x)$
charge 2e stuff
density of states
d(UHB)
High
Low
0.0
0.5
1.0
produces zeros in lower band

Key Idea: Extended Hilbert Space $\mathcal{H}_c \otimes \mathcal{H}_D$ + Integrate out D's



D_i



Fermionic (one per site)

Unphysical Hilbert space

$$\delta(D - \theta c_\uparrow c_\downarrow)$$

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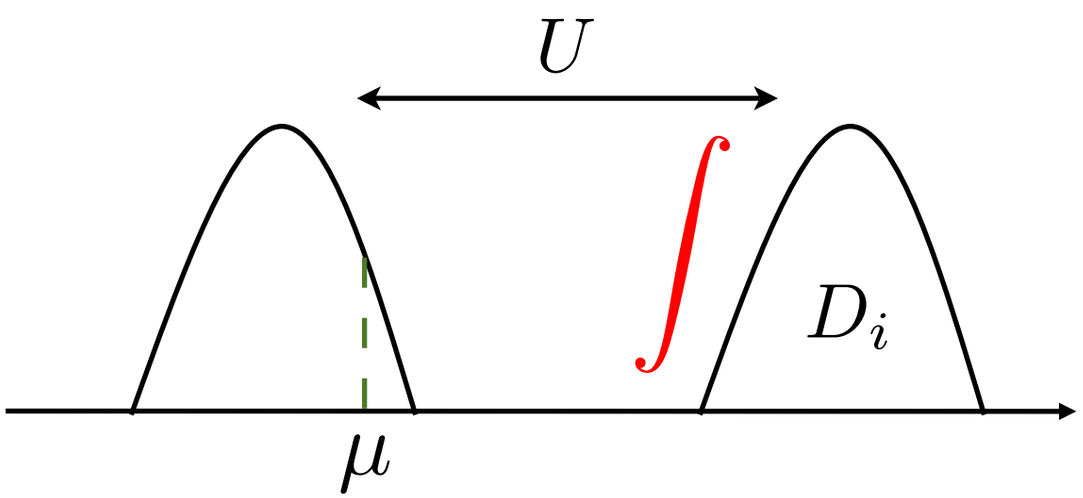

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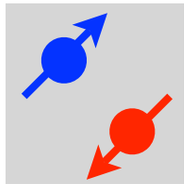


Fermionic (one per site)

Unphysical Hilbert space

$$\delta(D - \theta c_\uparrow c_\downarrow)$$

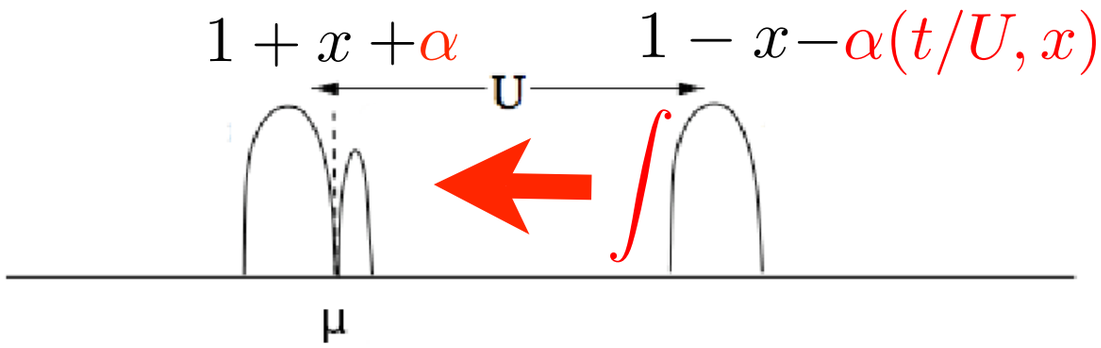
$$\theta \varphi_i^\dagger$$

charge 2e boson

$$\mathcal{L} = \overbrace{\mathcal{L}_{\text{projected}}(c) + \mathcal{L}_{\text{spin}}}^{\text{t-J model}} + \boxed{\mathcal{L}_{\text{charge}}(\varphi, c, c^\dagger)}$$

$$1 + x$$

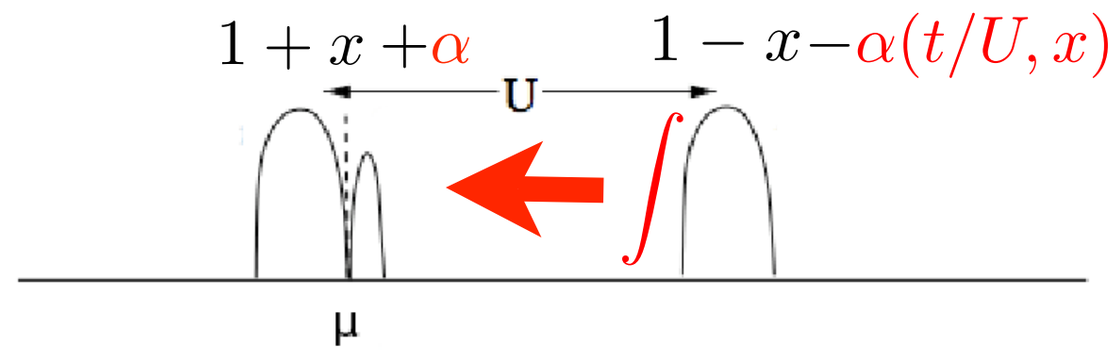
charge dynamics due to mixing



t-J model

$$\mathcal{L} = \overbrace{\mathcal{L}_{\text{projected}}(c)}^{1+x} + \mathcal{L}_{\text{spin}} + \boxed{\mathcal{L}_{\text{charge}}(\varphi, c, c^\dagger)}$$

charge dynamics due to mixing



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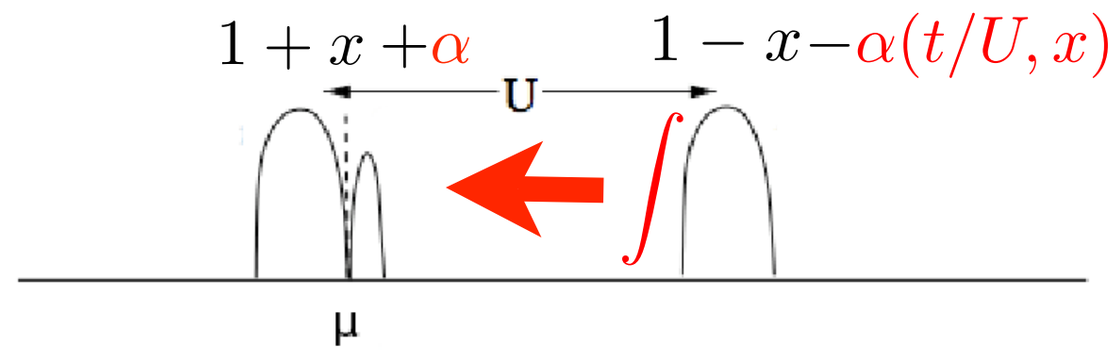
quadratic

spectral function

charge dynamics due to mixing



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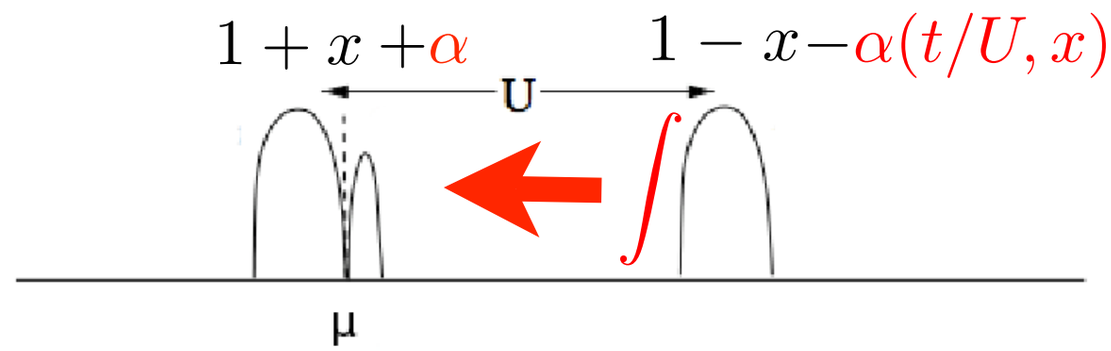

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quadratic

spectral function

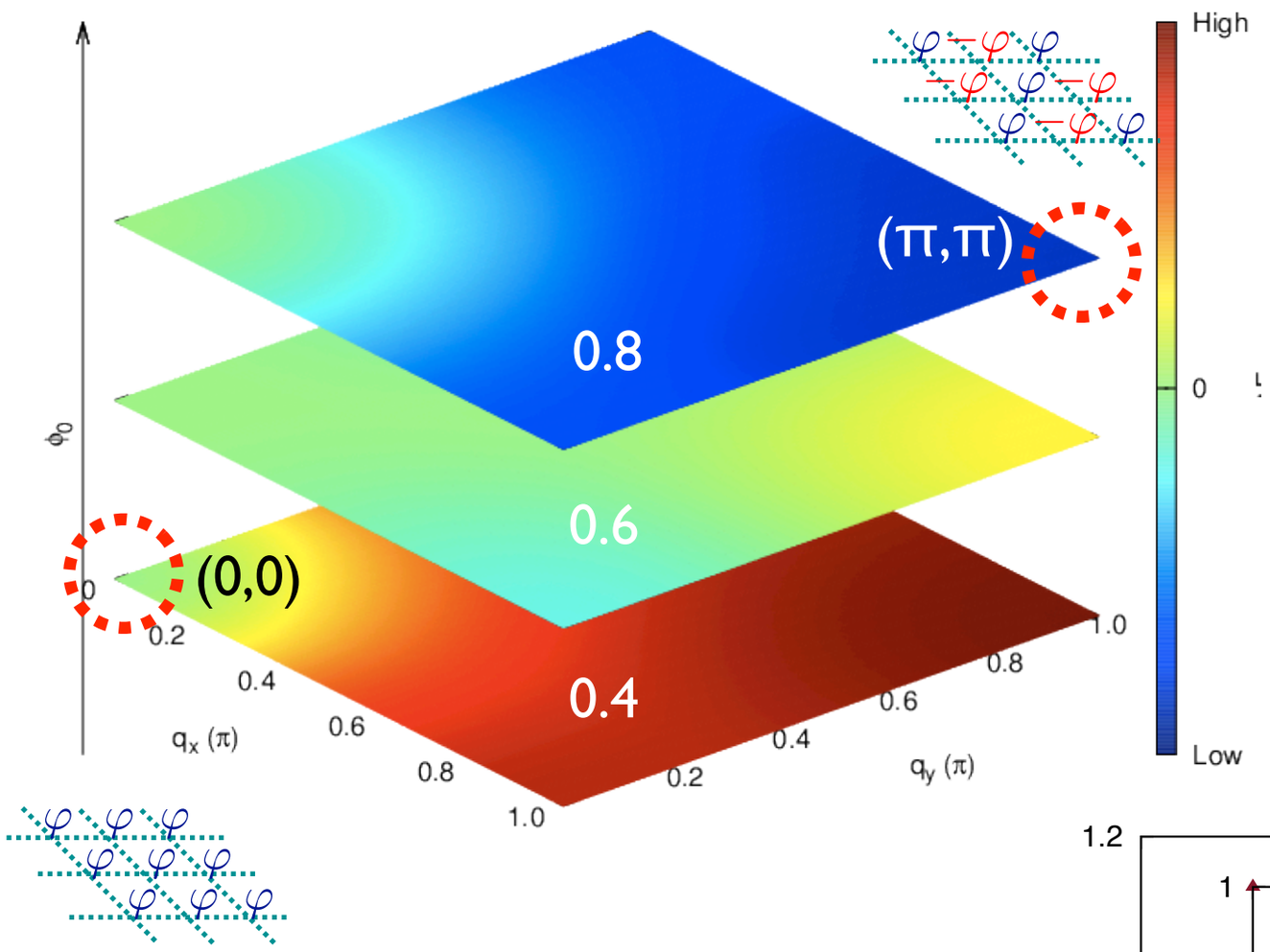
$$\varphi_i \to |\varphi_0|\, e^{i\mathbf{q}\cdot\mathbf{r}_i}$$

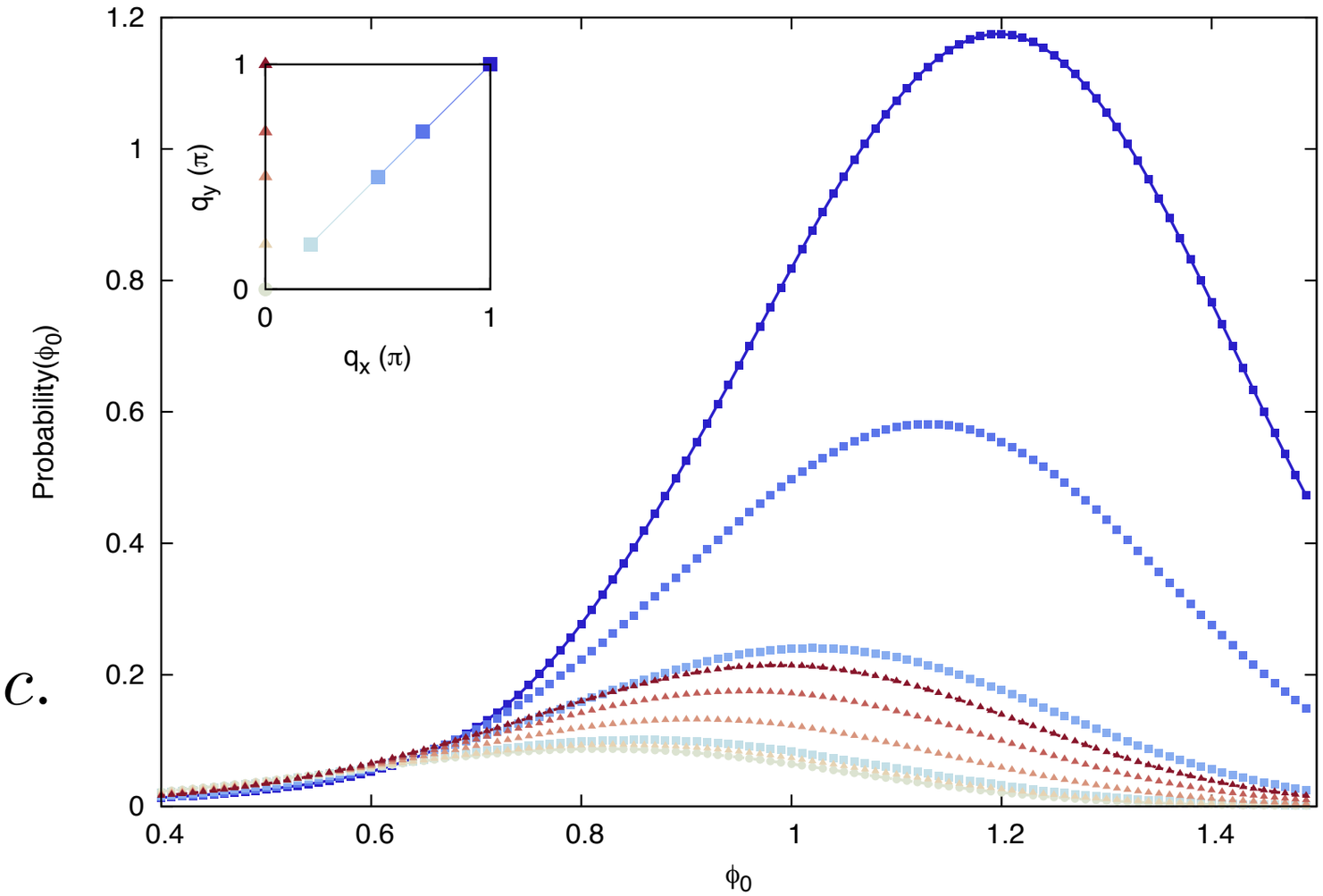
$$\mathcal{F} = -\frac{1}{\beta}\ln\mathcal{Z} = \sum_{\mathbf{q}}\sum_{\varphi_0} F[\varphi_0 e^{i\mathbf{q}\cdot\mathbf{r}}]$$

Free energy minimization

$$\varphi_i \to |\varphi_0|\, e^{i\mathbf{q}\cdot\mathbf{r}_i}$$

$$F[\varphi_0 e^{i\mathbf{q}\cdot\mathbf{r}}] - F[\varphi_0]$$





Probability

(π,π) is most likely

$$\mathcal{L}_{\text{charge}}(\varphi, c, c^\dagger) \sim \varphi c_\uparrow^\dagger c_\downarrow^\dagger + c.c.$$

$$G(i\omega_n, \mathbf{k}) = \int \mathbf{d}|\varphi||\varphi|\mathbf{P}(\varphi)\mathcal{G}(\mathbf{i}\omega_\mathbf{n}, \mathbf{k})|_{\varphi_\mathbf{q}=\delta_{\tilde{\mathbf{q}},\pi}|\varphi|}$$

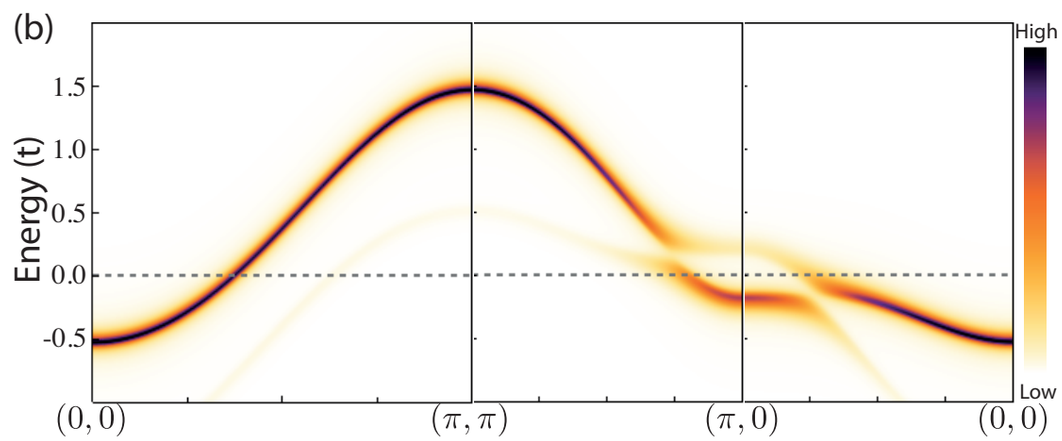
$$\mathcal{G}(i\omega_n, \mathbf{k}) = \frac{\widetilde{\mathbf{g}}_\mathbf{t}}{\mathbf{i}\omega_\mathbf{n} - \mu - \widetilde{\mathbf{g}}_\mathbf{t}\epsilon_\mathbf{k} - \mathbf{\Sigma}_\pm(\mathbf{i}\omega_\mathbf{n}, \mathbf{k})} + \frac{\mathbf{t}}{\mathbf{U}}(\cdots)$$

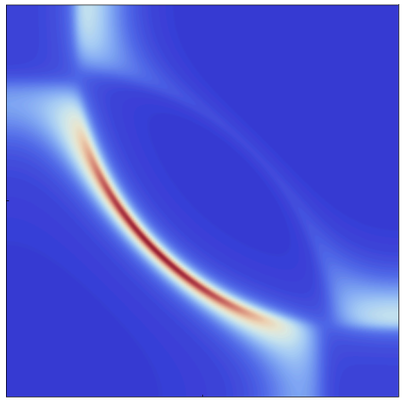
$$, \Sigma_\pm(i\omega_n, \mathbf{k}) = \frac{\mathbf{s}^\mathbf{2}_{\mathbf{k},\mathbf{q}}\varphi_\mathbf{q}\varphi^*_\mathbf{q}}{\mathbf{i}\omega_\mathbf{n} - \mu \pm \widetilde{\mathbf{g}}_\mathbf{t}\epsilon_{\mathbf{q}-\mathbf{k}}},$$

Green Function and Self Energy

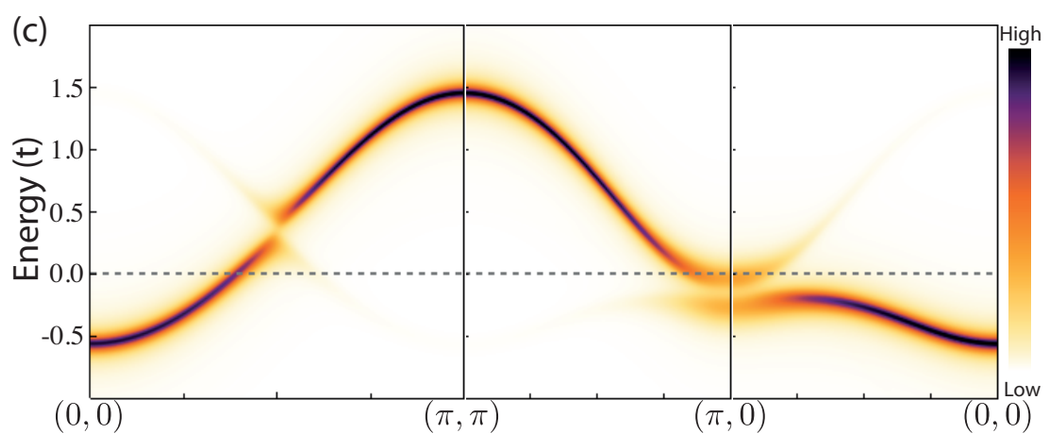
$$\varphi_i \to |\varphi_0|\, e^{i\mathbf{q}\cdot\mathbf{r}_i} \qquad \mathbf{q} = (\pi, \pi)$$

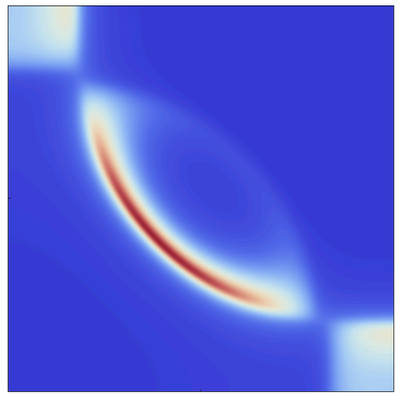
$$G(\omega, \mathbf{k}) = -\mathrm{FT}\,\langle \mathcal{T}_\tau \tilde{c}_{i\sigma}(\tau)\tilde{c}^\dagger_j(0)\rangle = \frac{g_t}{\omega - \epsilon_{\mathbf{k}} - \Sigma_{\mathbf{k}}} + O(t/U)$$





No Fermi Arcs





Fermi Arcs Incoherent feature

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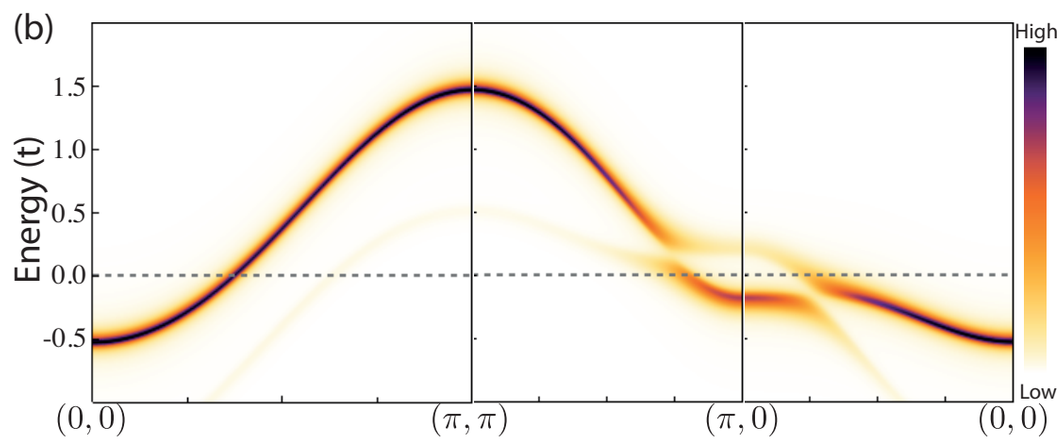
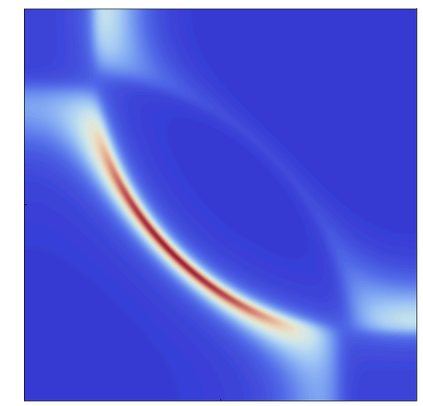
Self energy (particle - particle)

$$\varphi c^\dagger_\uparrow c^\dagger_\downarrow \to \langle\varphi\rangle c^\dagger_\uparrow c^\dagger_\downarrow$$

$$\Sigma_{\mathbf{k}} = \frac{|\varphi_0|^2}{\omega + \epsilon_{\mathbf{q}-\mathbf{k}}}$$





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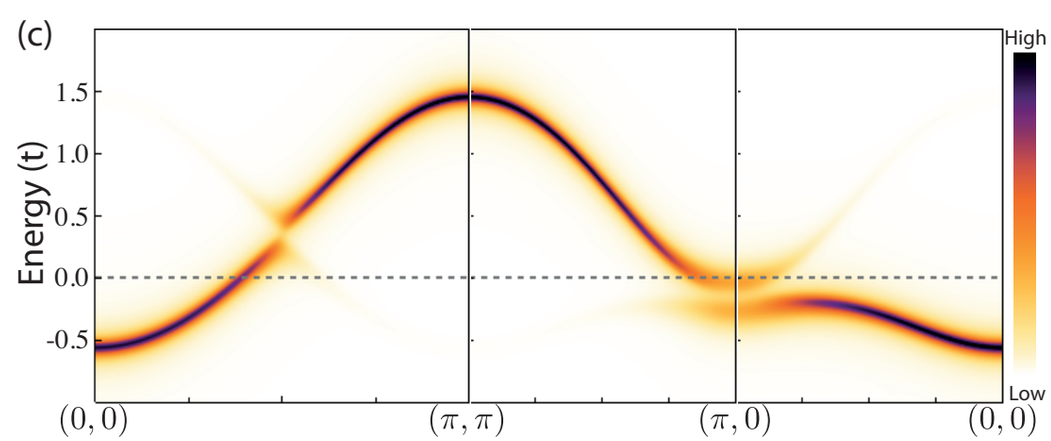

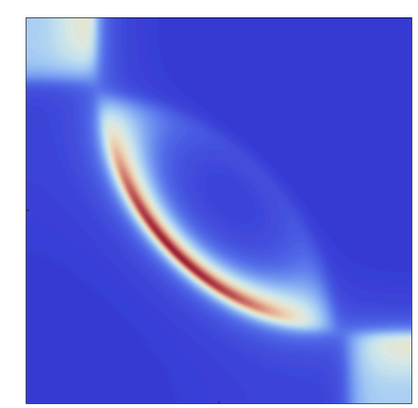

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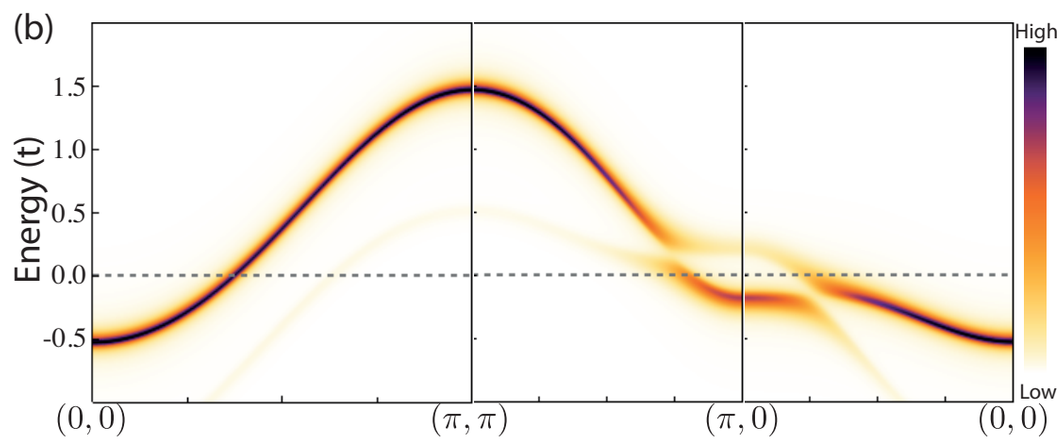

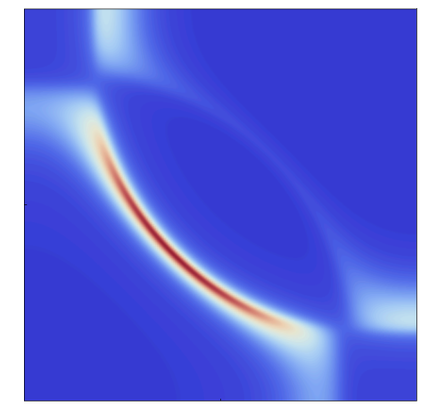

No Fermi Arcs

Self energy (particle - hole)

$$\varphi c^\dagger_\uparrow c^\dagger_\downarrow \to (\varphi c^\dagger_\uparrow)\cdot c^\dagger_\downarrow$$

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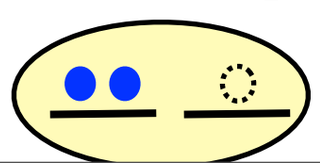
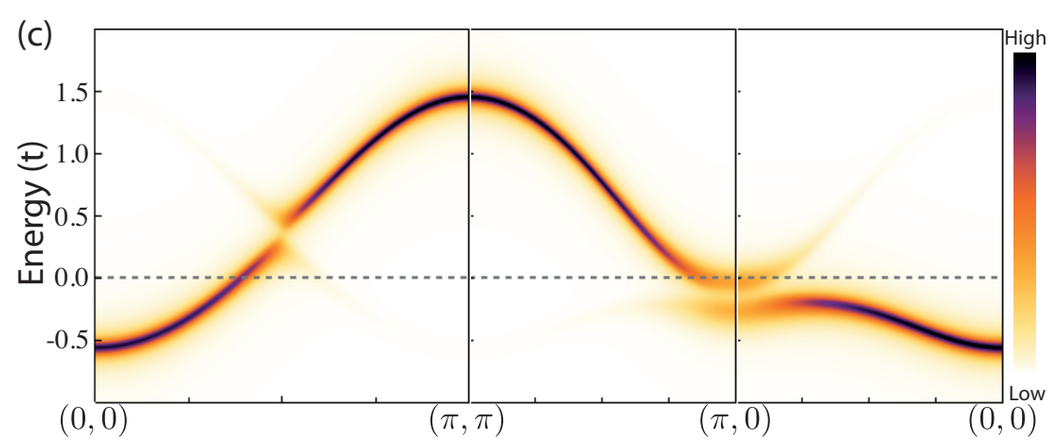
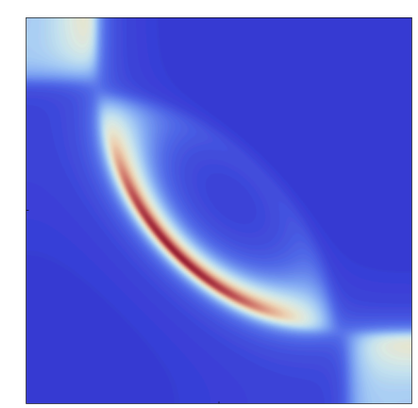
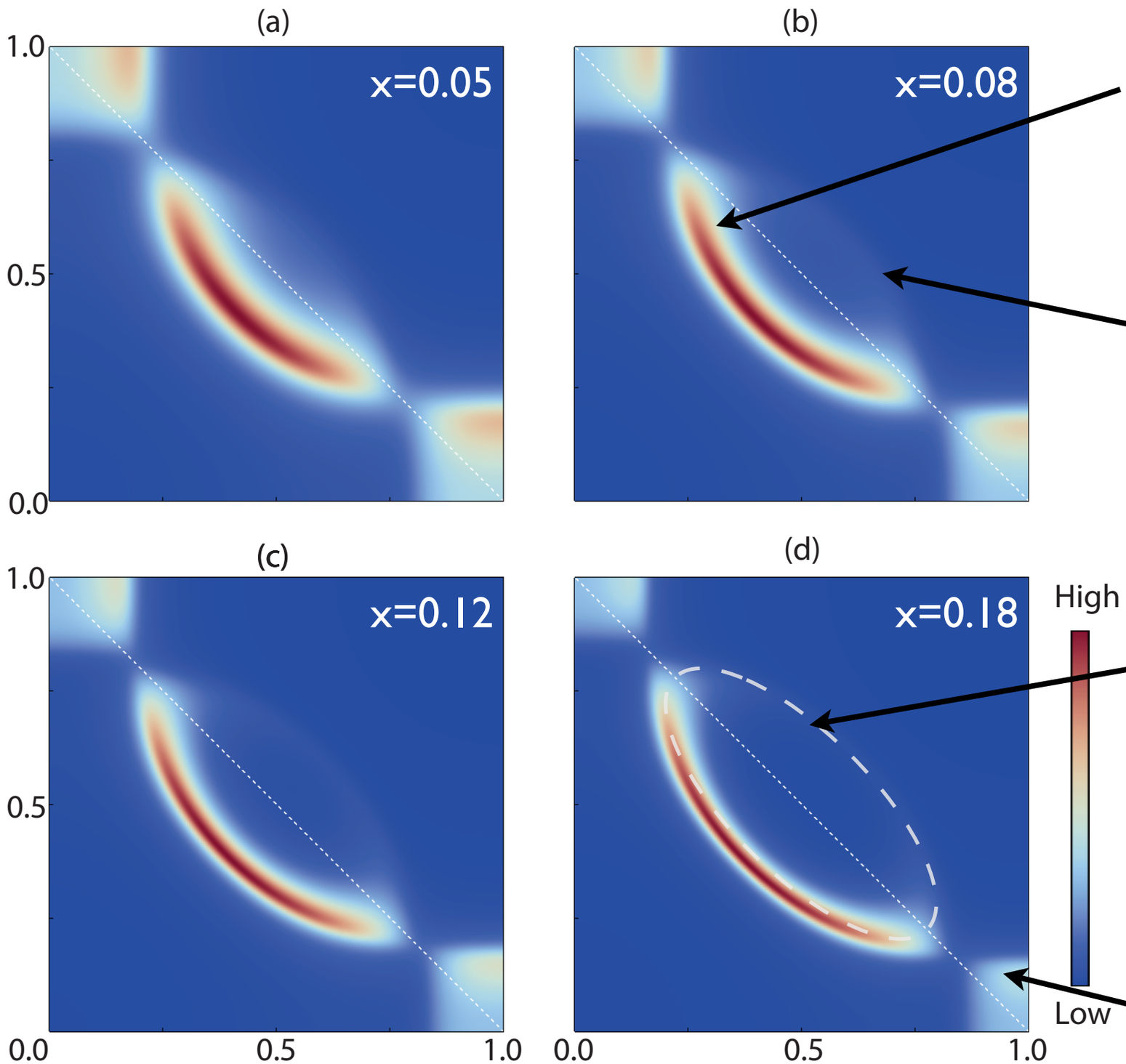






Fermi Arcs Incoherent feature

Results



Fermi Arc

$(1 - n_{\bar{\sigma}})c_\sigma$

Shadow band

$\varphi c_\uparrow^\dagger c_\downarrow^\dagger \rightarrow (\varphi c_\uparrow^\dagger) \cdot c_\downarrow^\dagger$

Pocket size ~ x

Electron pocket ?

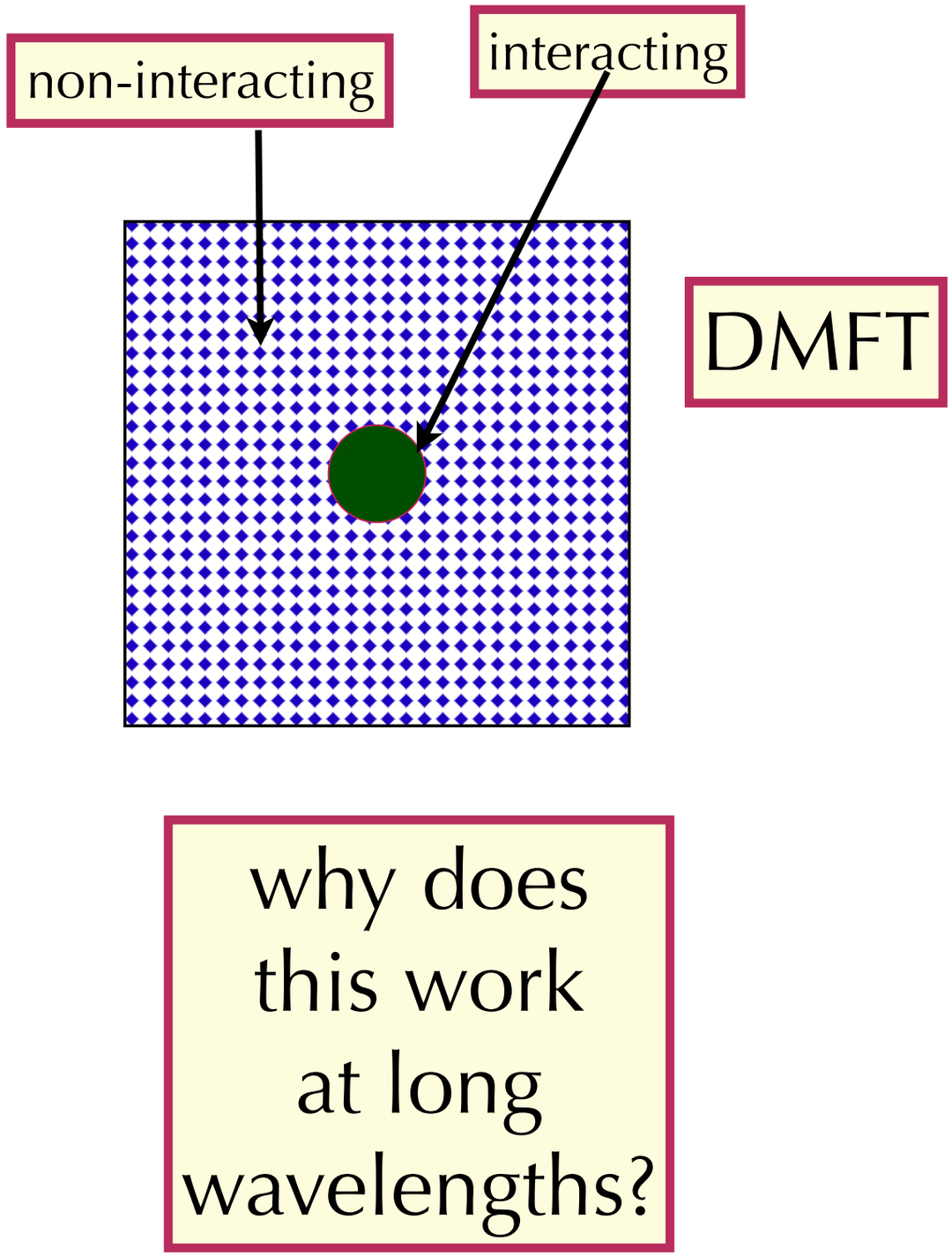
Fermi arcs

composite or bound states not in UV theory

Fermi arcs

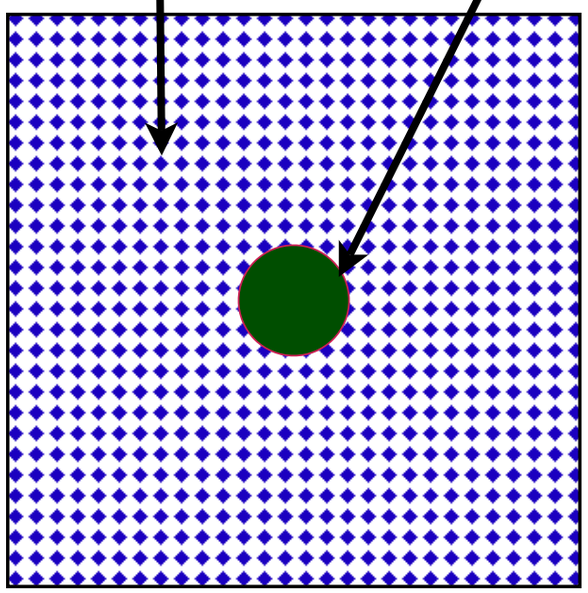
composite or bound states not in UV theory

What computational tools do we have for spectral functions for strongly correlated electron systems?


non-interacting
interacting
DMFT
why does
this work
at long
wavelengths?

non-interacting

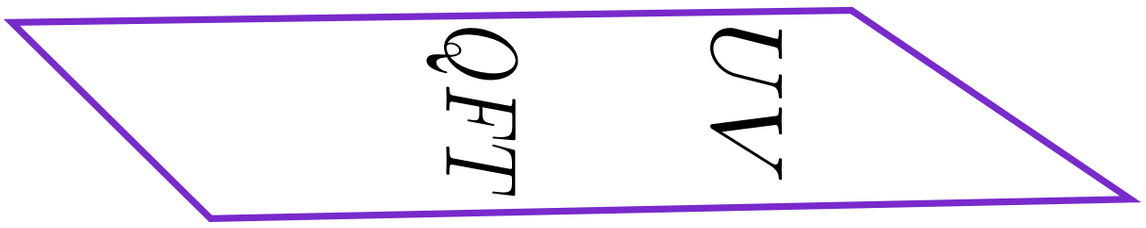
interacting

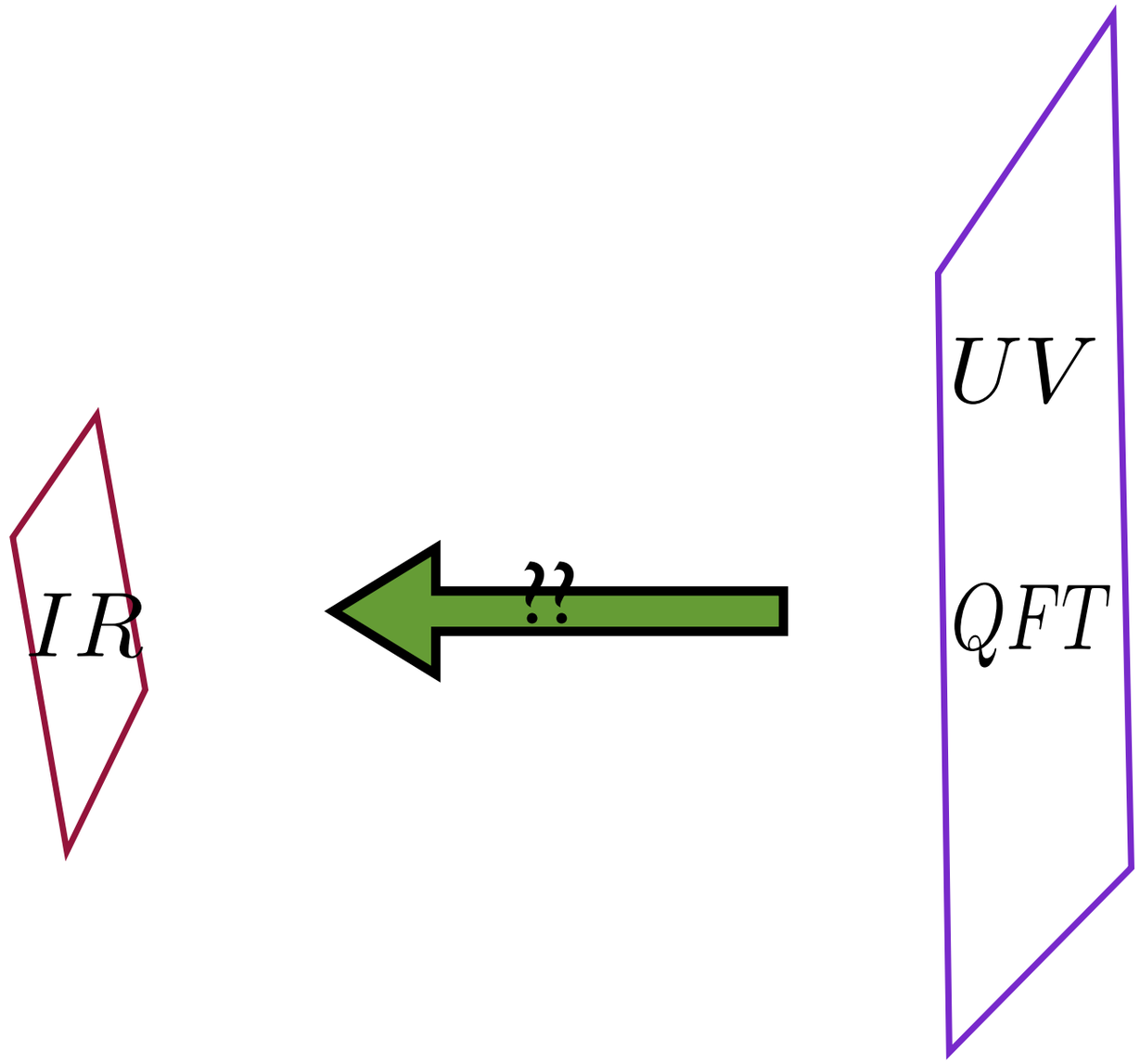


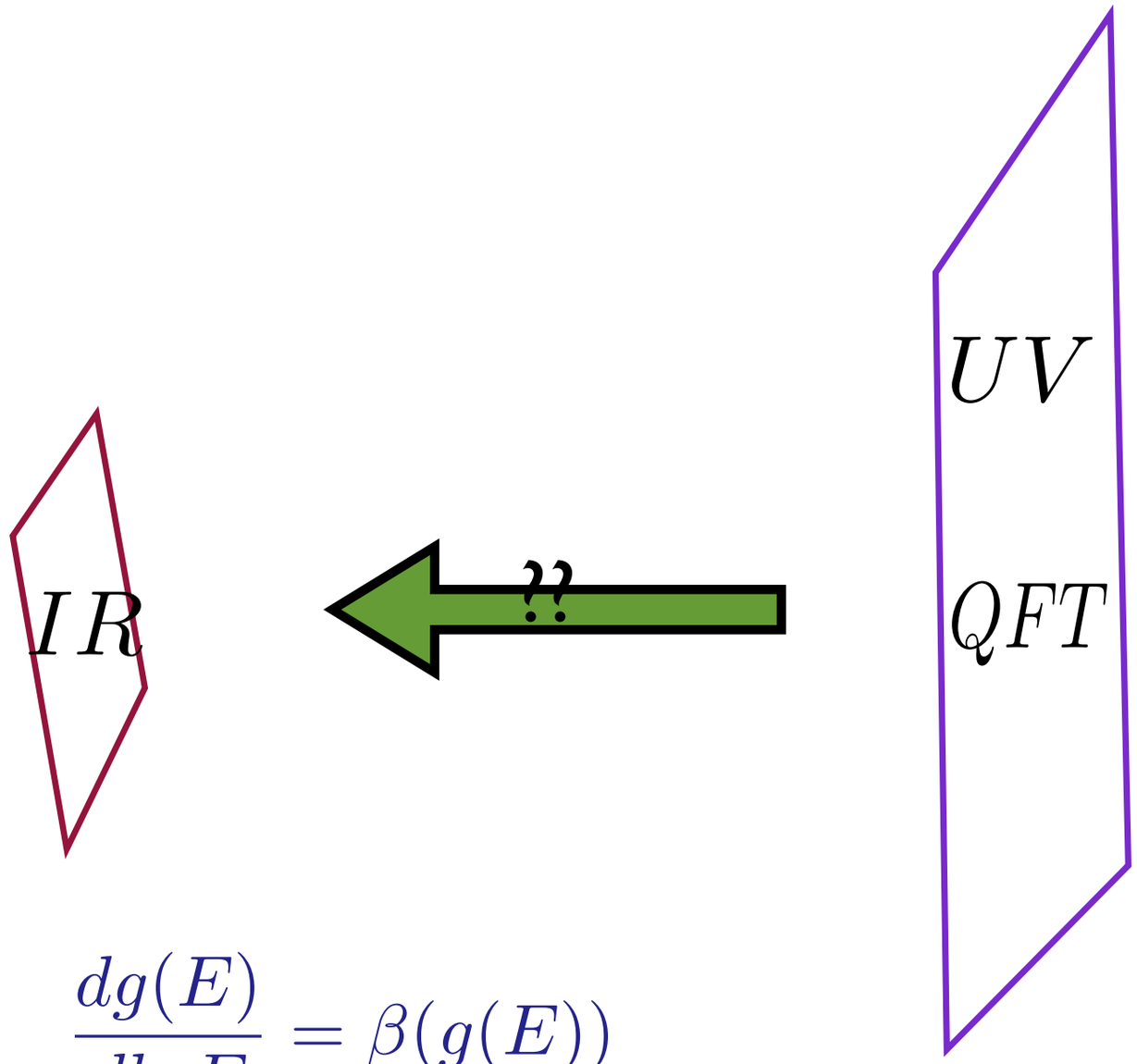
DMFT

the only difference
between this and
a theory is that this is not
a theory

why does
this work
at long
wavelengths?


UV
QFT

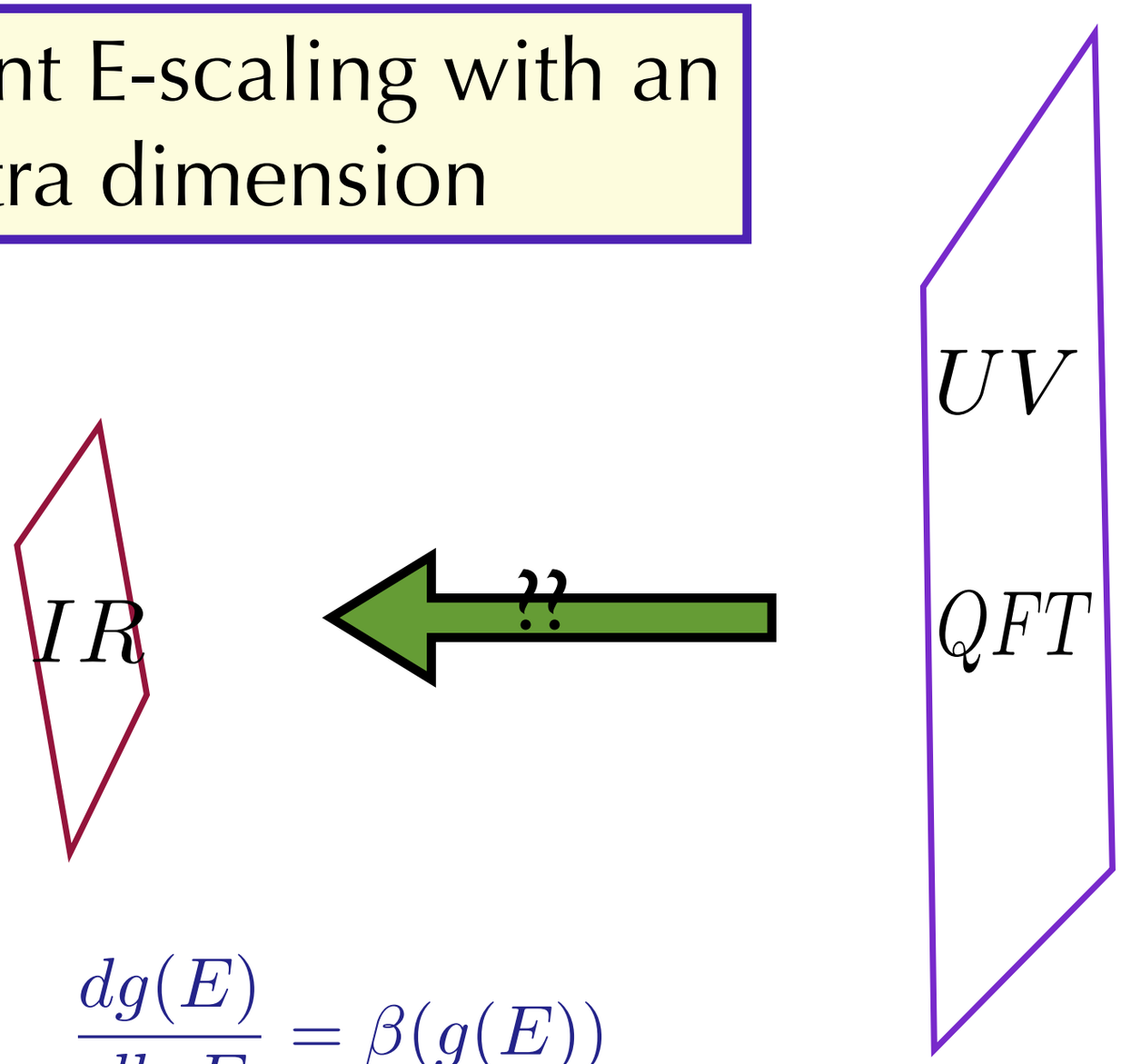

UV
QFT
??
IR



$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

implement E-scaling with an extra dimension

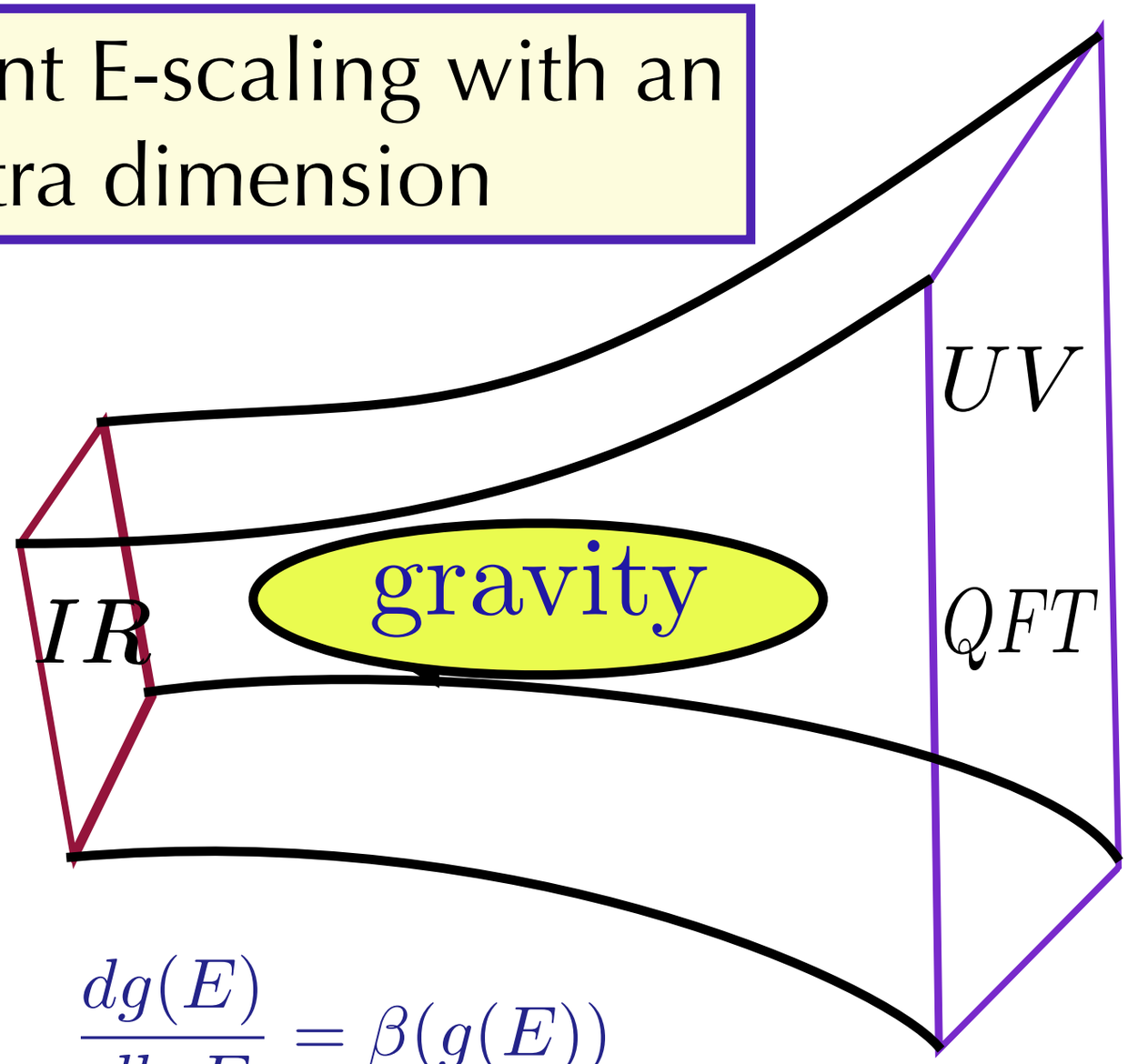


$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

gauge-gravity duality
(Maldacena, 1997)

implement E-scaling with an extra dimension



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locality in energy

what's the geometry?

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$$\frac{dg(E)}{dlnE} = \beta(g(E)) = 0$$

scale invariance
(continuous)

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$$E \to \lambda E$$
$$x^\mu \to x^\mu / \lambda$$

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solve Einstein equations

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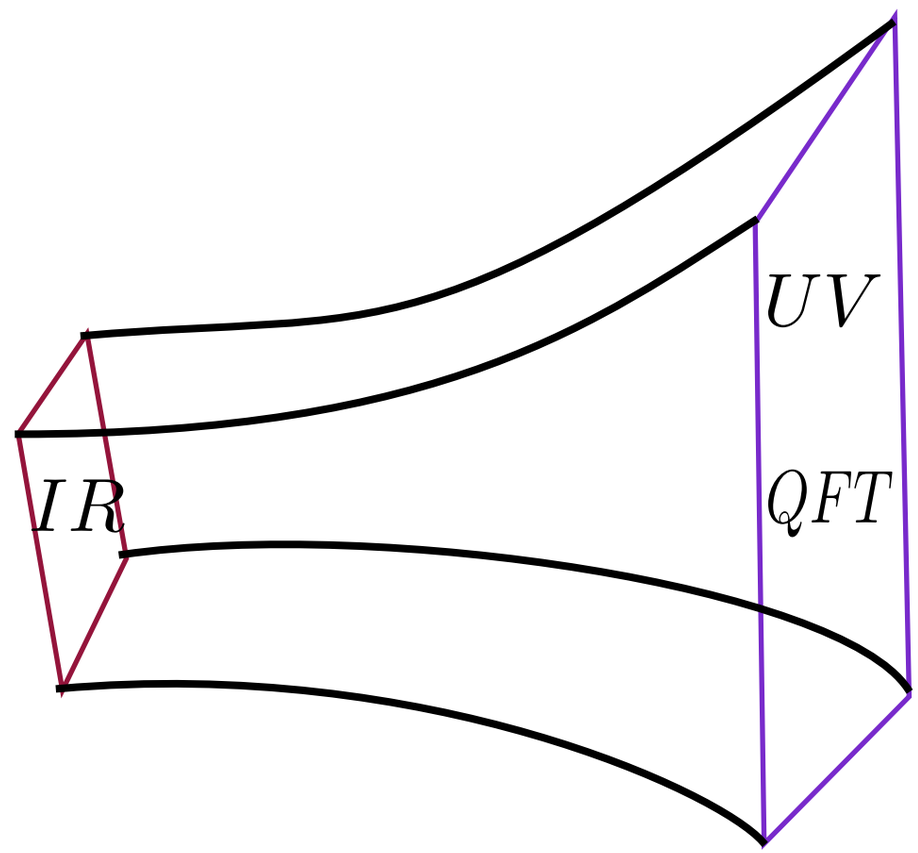
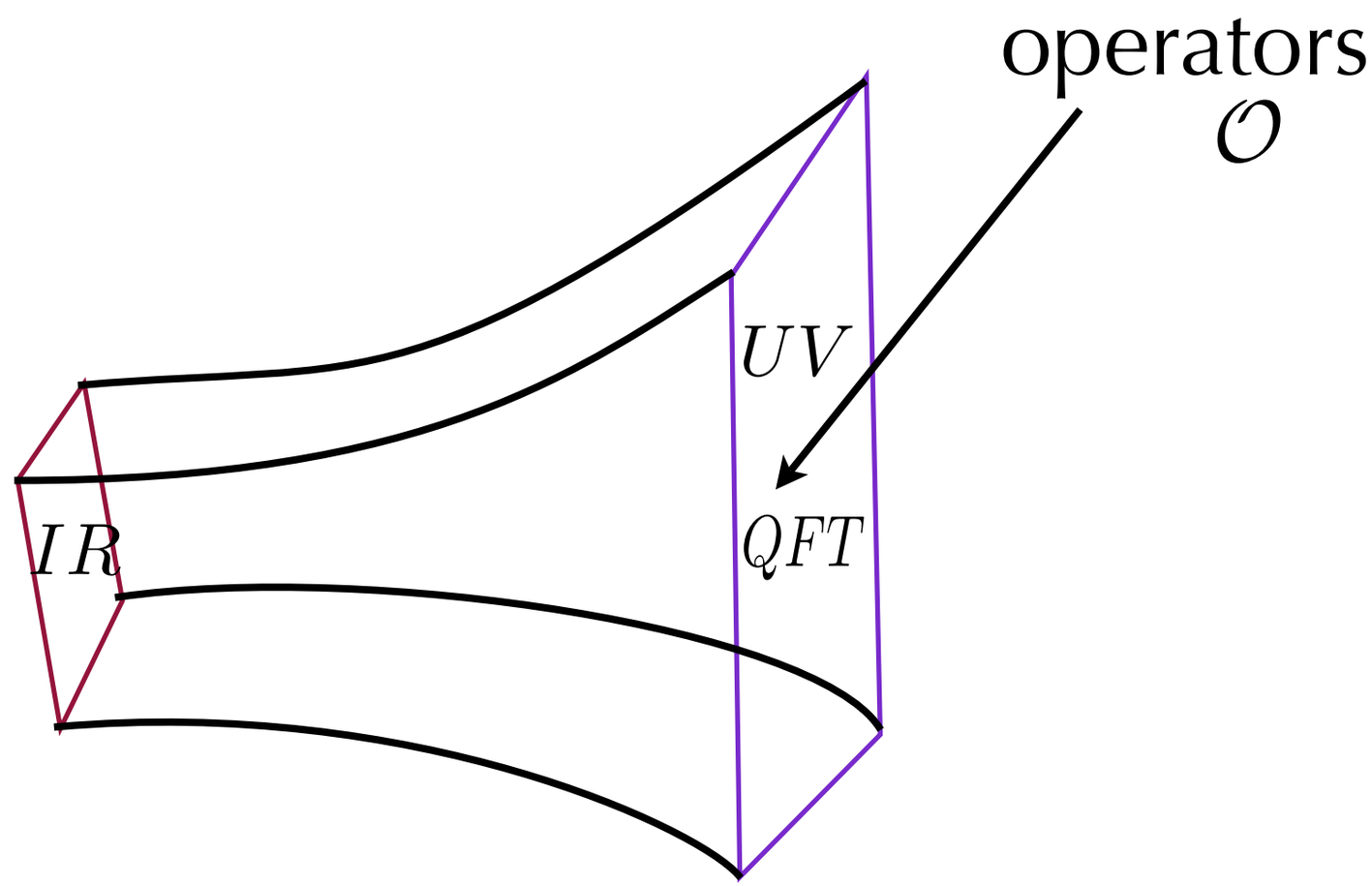
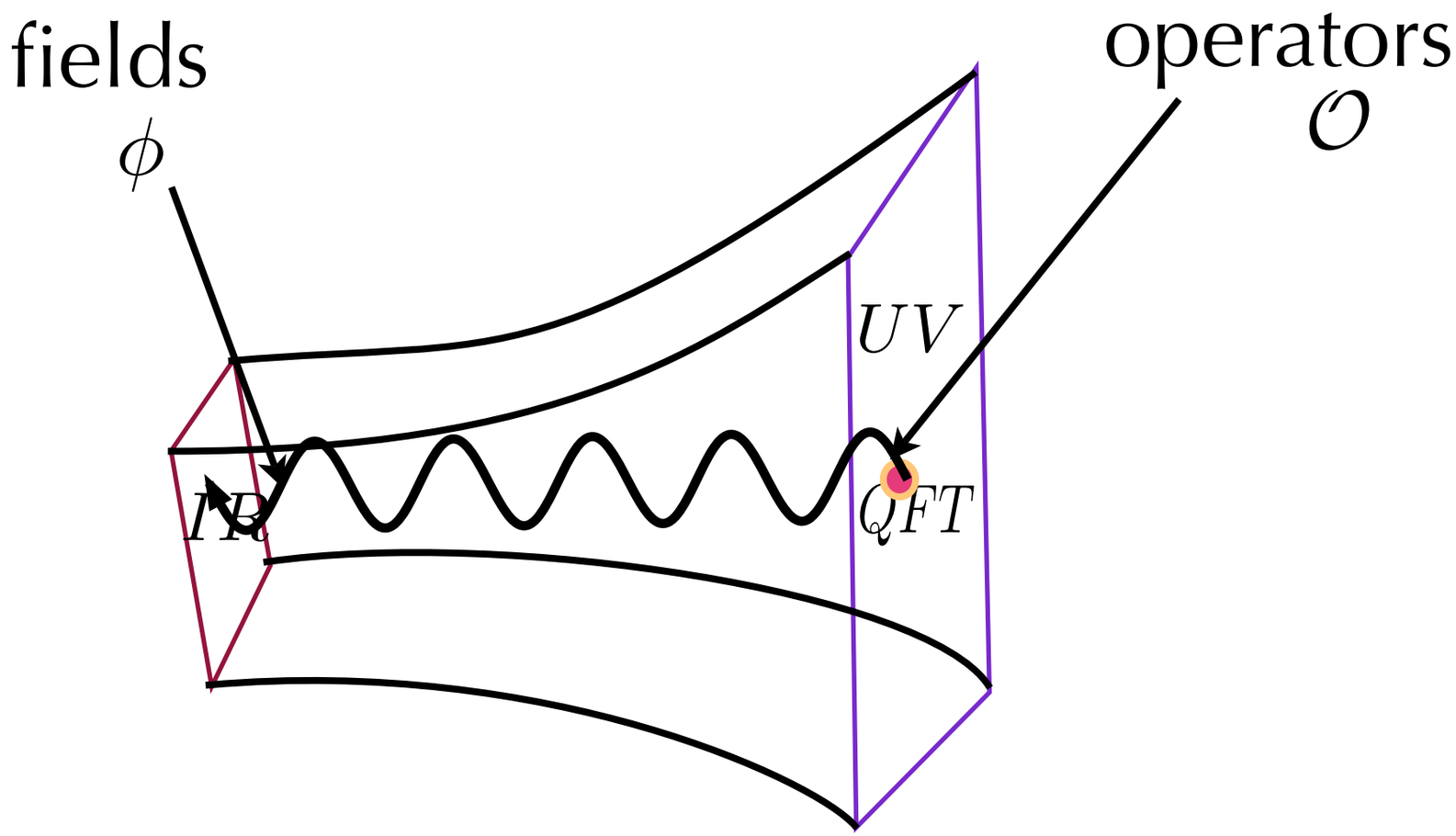
solve Einstein equations

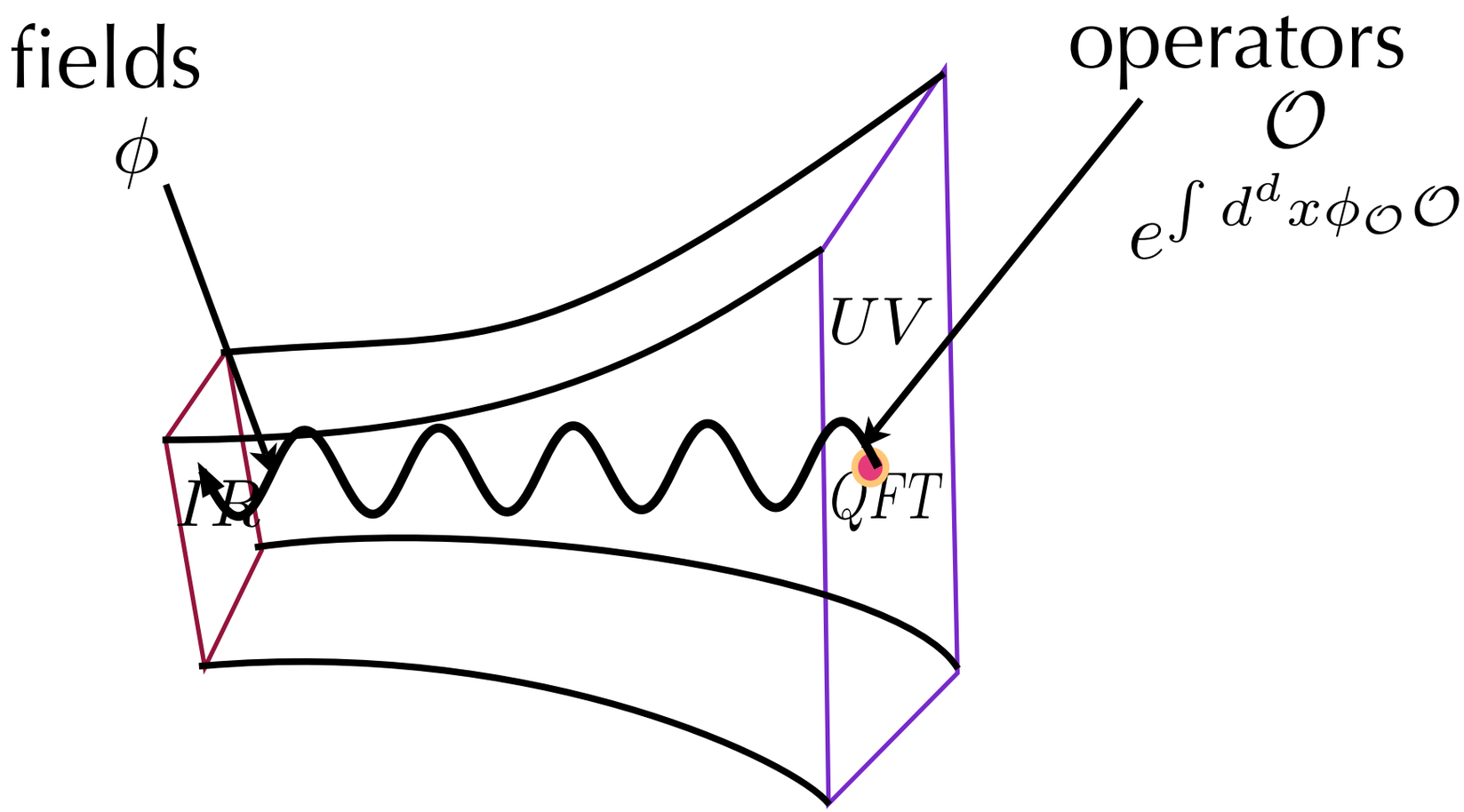
$$ds^2 = \left(\frac{u}{L}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{L}{E}\right)^2 dE^2$$

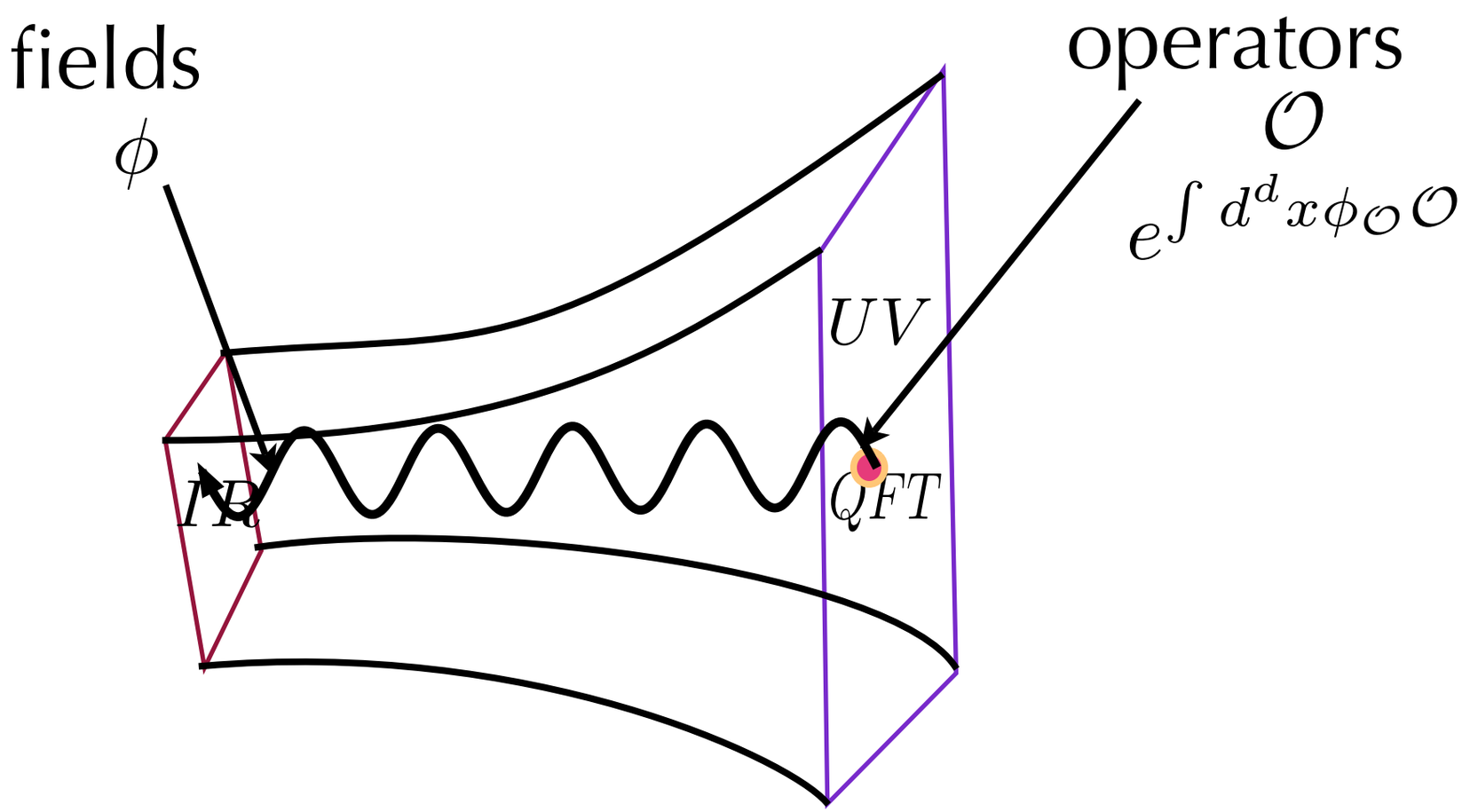
anti de-Sitter space


UV
IR
QFT


operators
$\mathcal{O}$
UV
QFT
IR

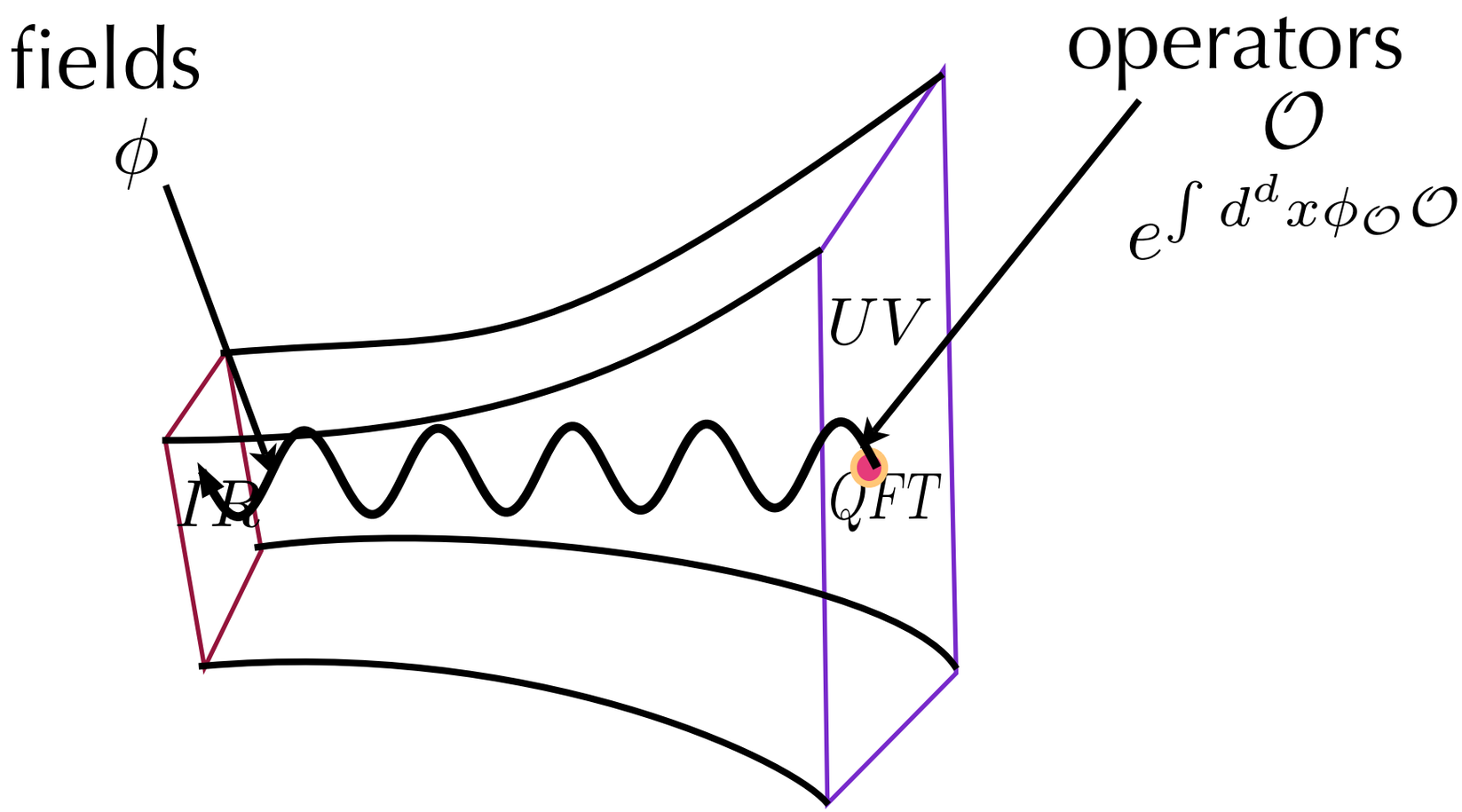

fields
ϕ
operators
$\mathcal{O}$
UV
IR
QFT


fields
ϕ
operators
$\mathcal{O}$
$e^{\int d^d x \phi_{\mathcal{O}} \mathcal{O}}$
UV
IR
QFT



Claim: $Z_{\mathrm{QFT}} = e^{-S^{\mathrm{on-shell}}_{\mathrm{ADS}}(\phi(\phi_{\partial \mathrm{ADS=J}_{\mathcal{O}}}))}$

RG=GR



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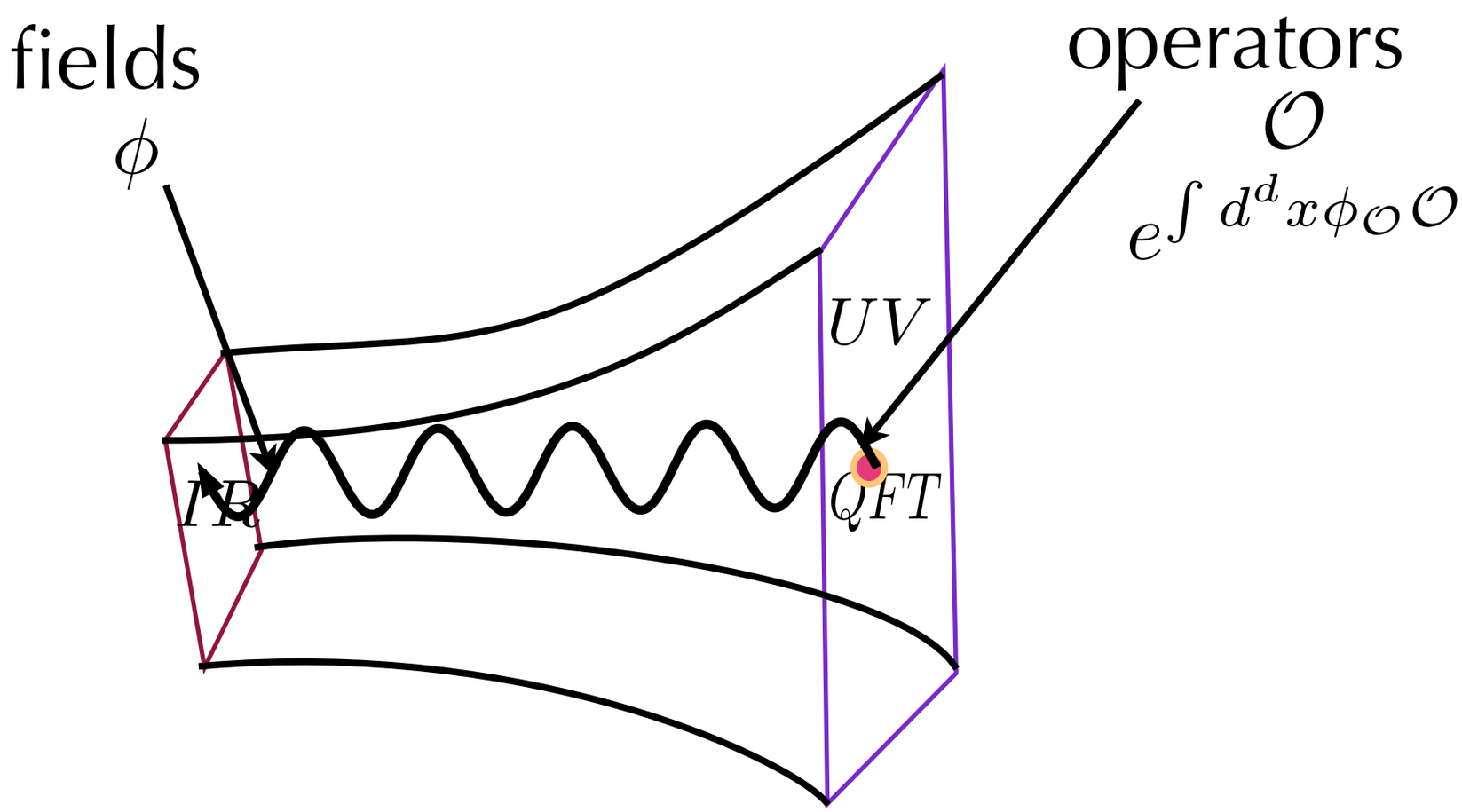
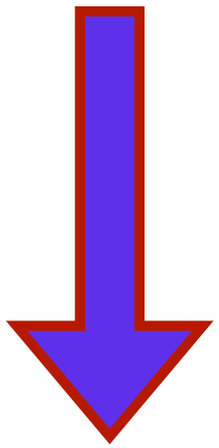
radial direction

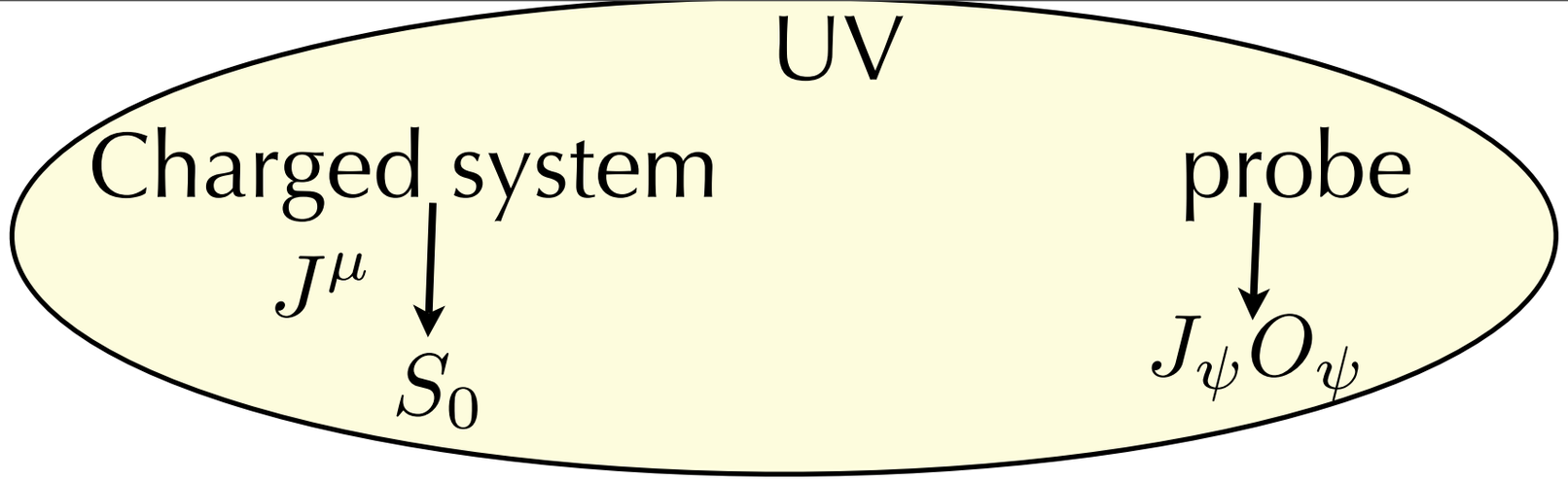


Claim: $Z_{\mathrm{QFT}} = e^{-S^{\mathrm{on-shell}}_{\mathrm{ADS}}(\phi(\phi_{\partial \mathrm{ADS=J}_{\mathcal{O}}}))}$

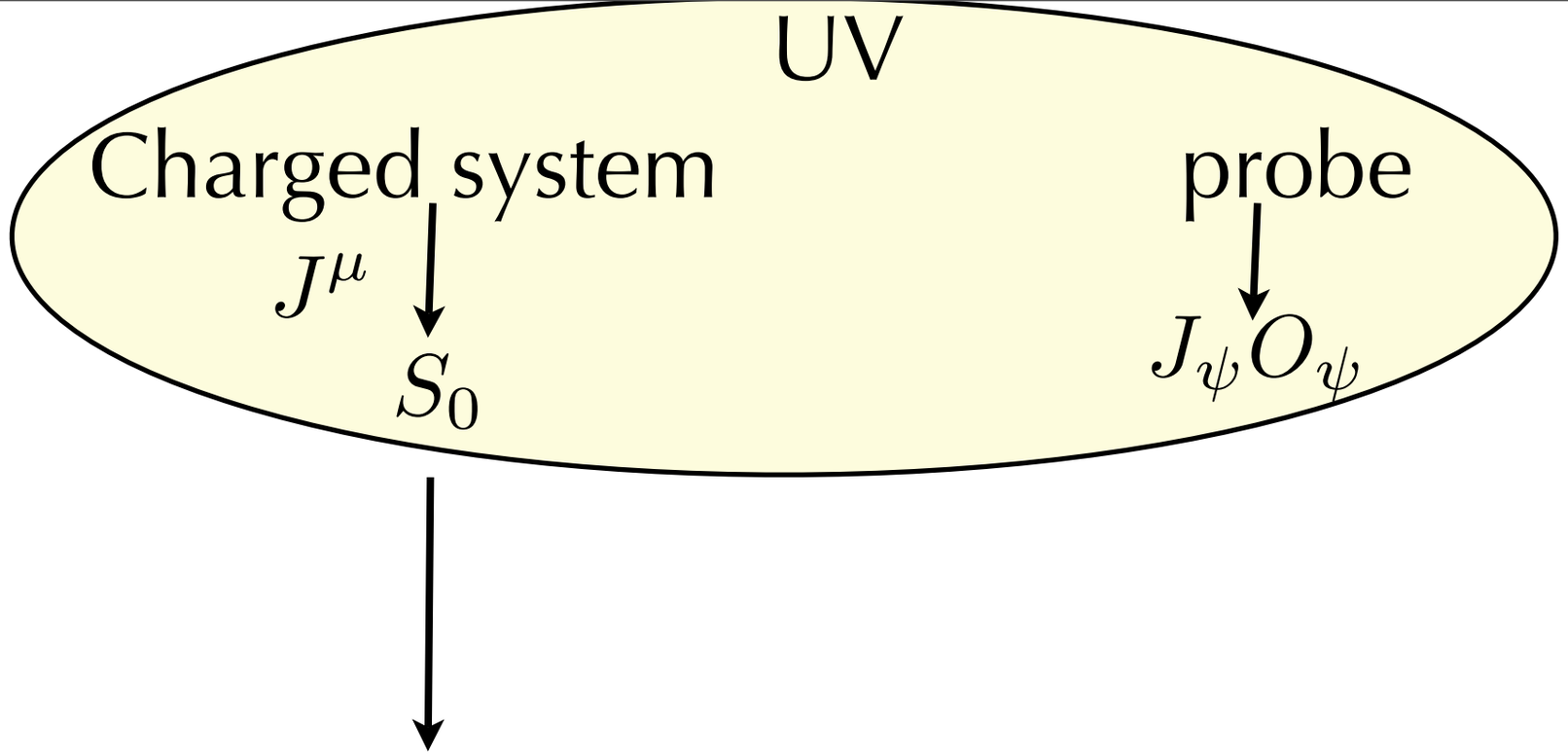
What gravitational theory gives rise to a gap in ImG without spontaneous symmetry breaking?



dynamically generated gap: Mott gap
(for probe fermions)



what has been done?
MIT, Leiden, McMaster,...



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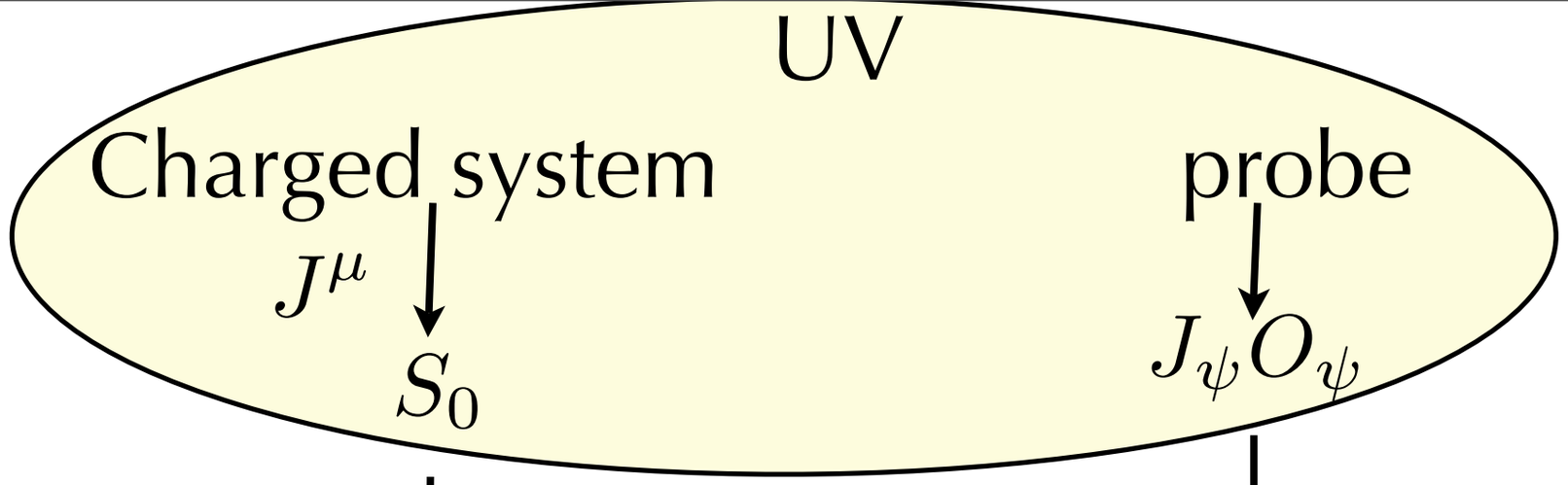
RN-AdS

$ds^2,\ A_t$



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$ds^2,\ A_t$

ψ Dirac Eq.

UV

Charged system $\xrightarrow{J^\mu}$ S_0

probe $\xrightarrow{J_\psi O_\psi}$

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in-falling boundary conditions

$$\psi(r \to \infty) \approx a r^m + b r^{-m}$$

Retarded Green function: $G = \dfrac{b}{a}$ =f(UV (k_F), IR (q,m))

a=0 defines FS

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$$F_{\mu\nu}\Gamma^{\mu\nu}$$

consider

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QED anomalous magnetic moment of an electron
(Schwinger 1949)

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QED anomalous magnetic moment of an electron
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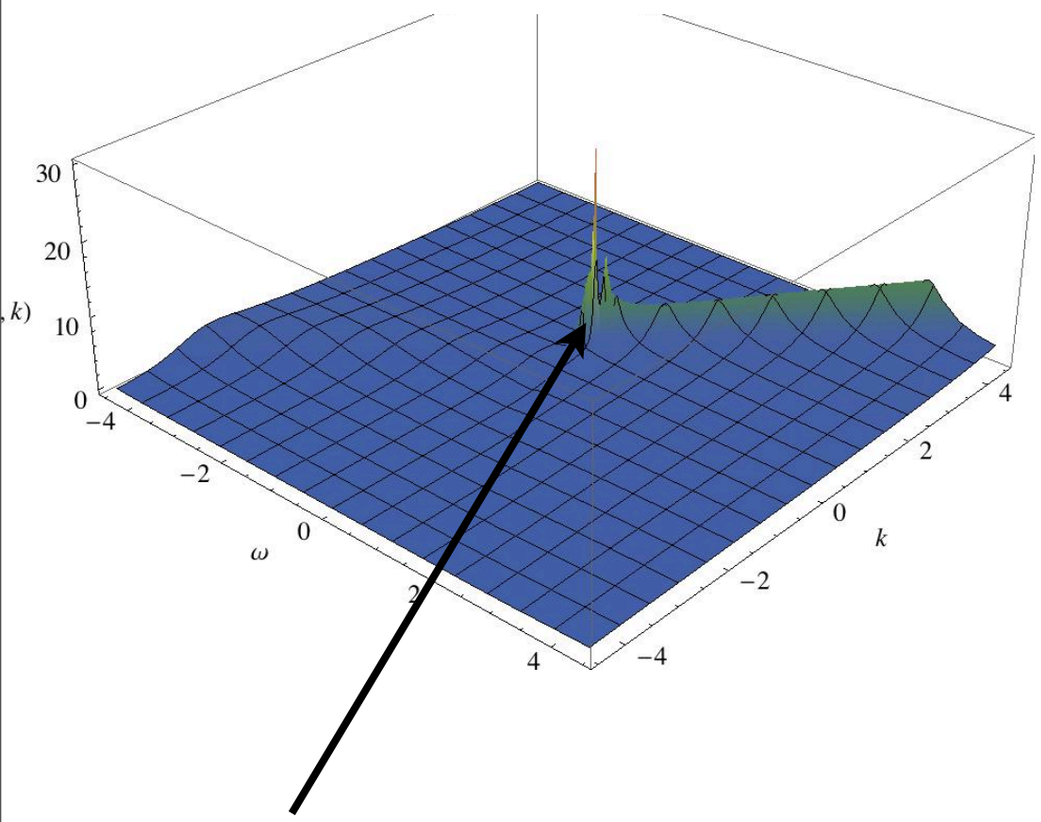
what happens at the boundary?

How is the spectrum modified?

P=0, MIT,
Leiden, McMaster



Fermi
surface
peak

How is the spectrum modified?

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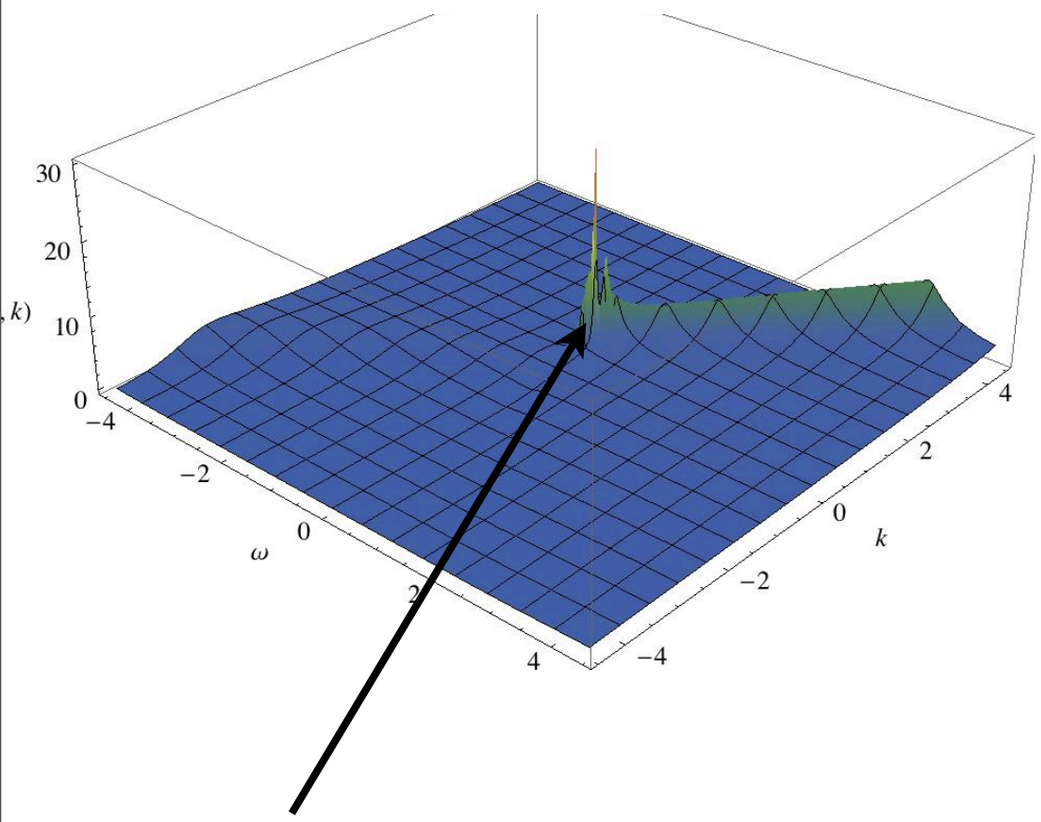

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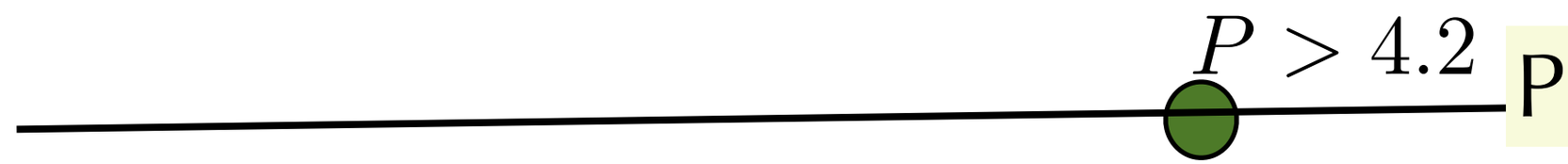

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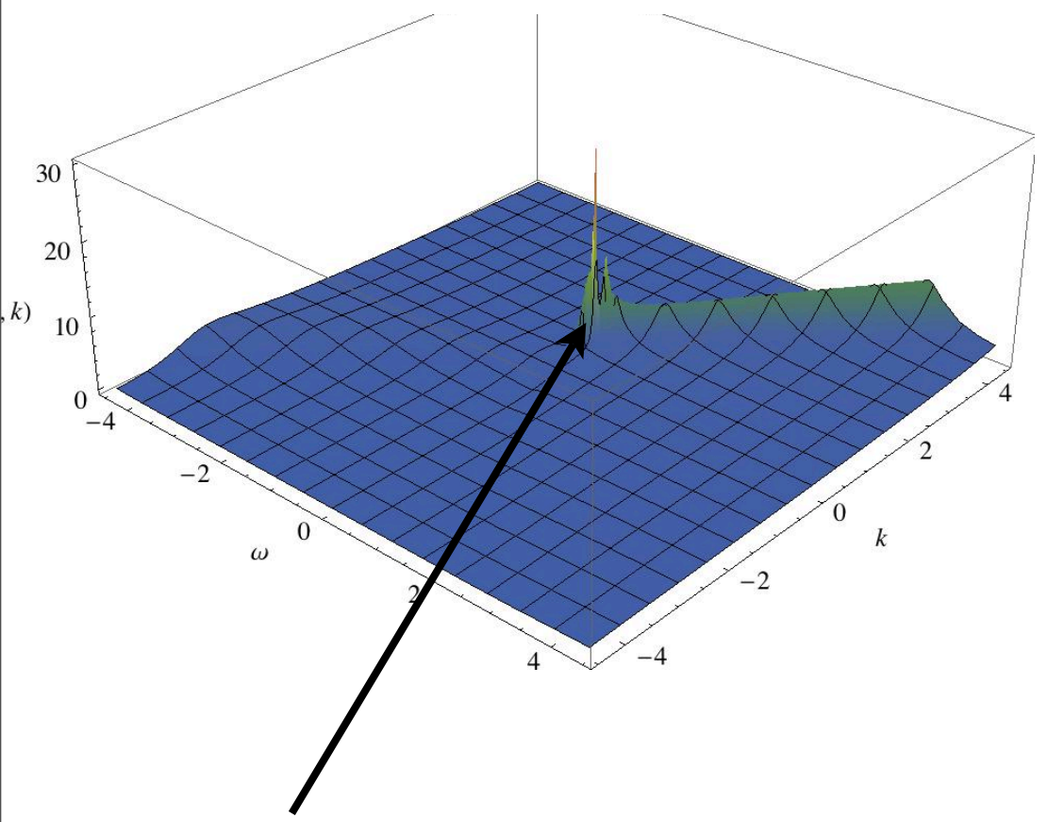

Fermi surface peak

How is the spectrum modified?

$P > 4.2$ P

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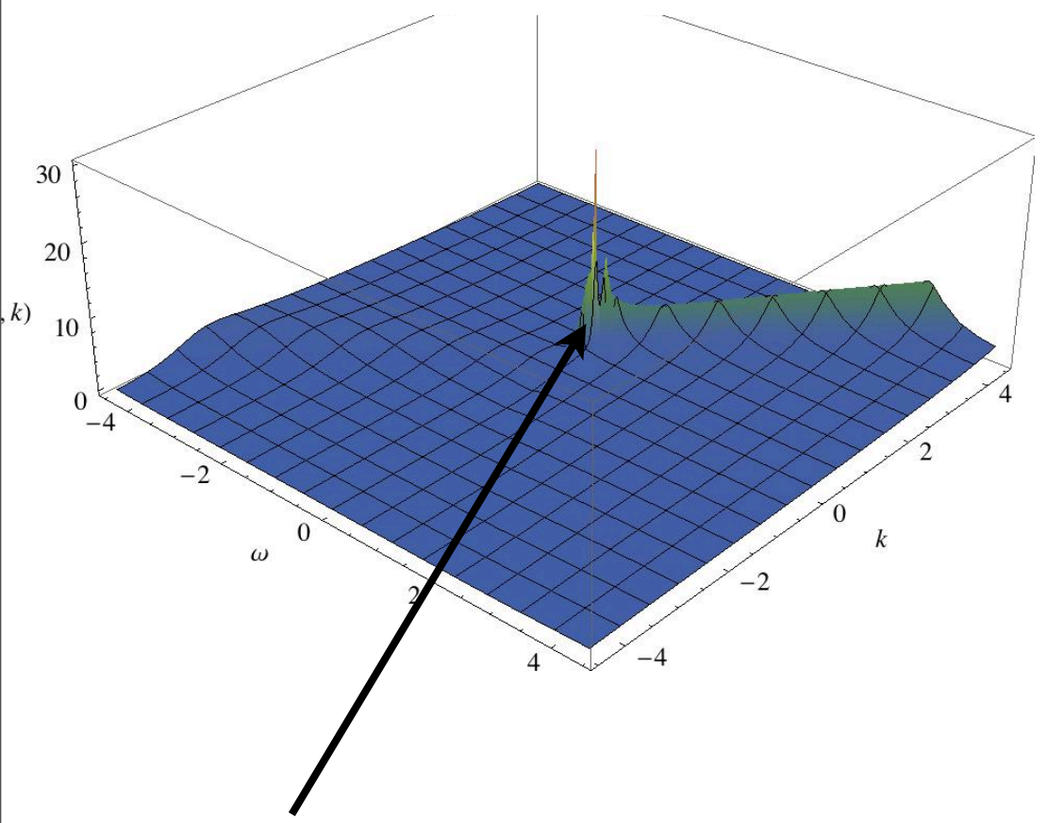

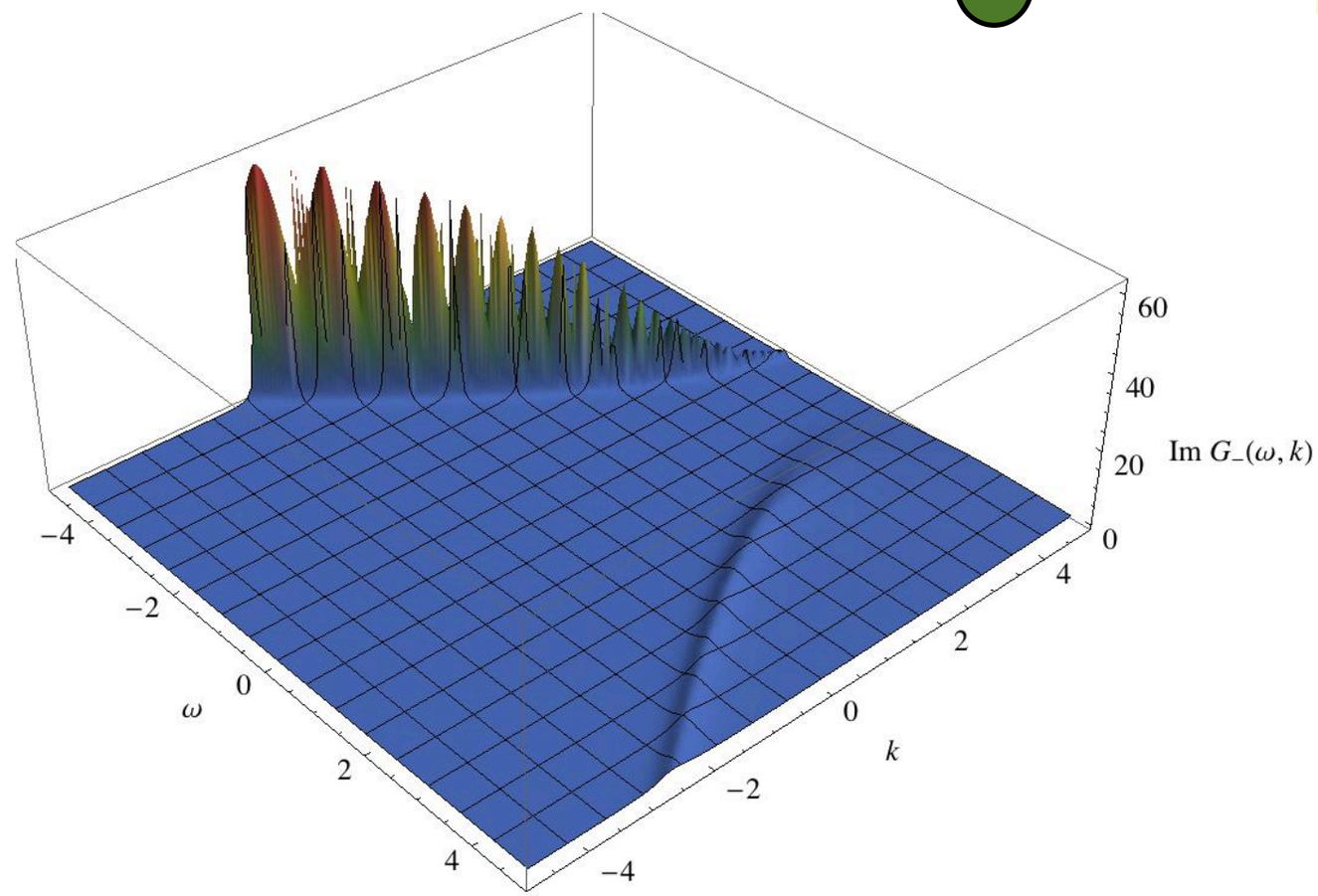

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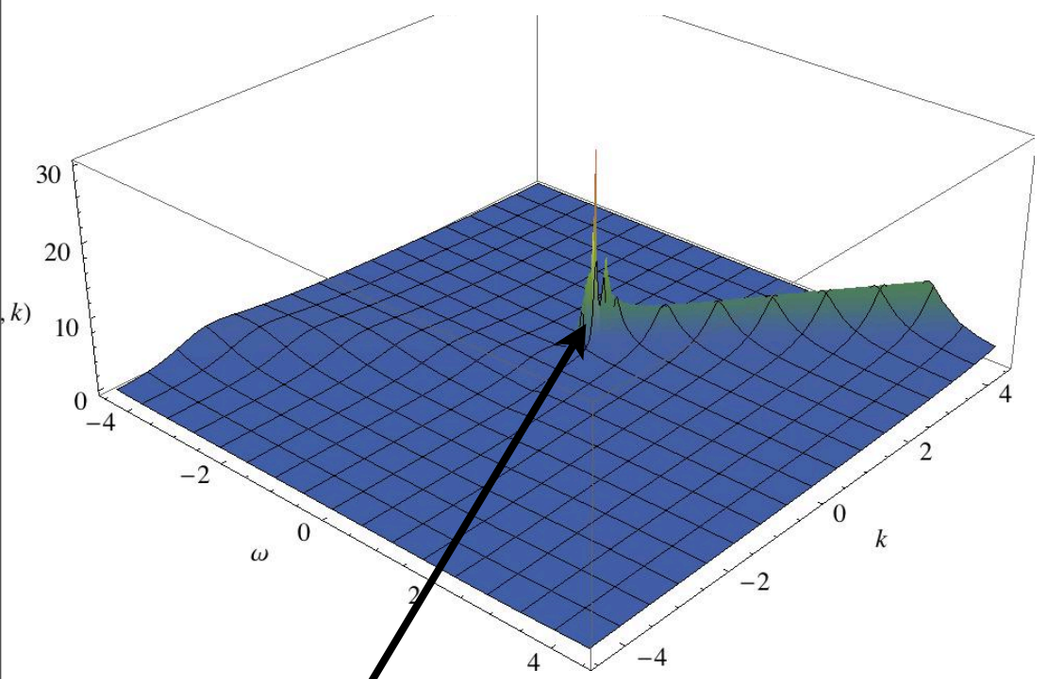
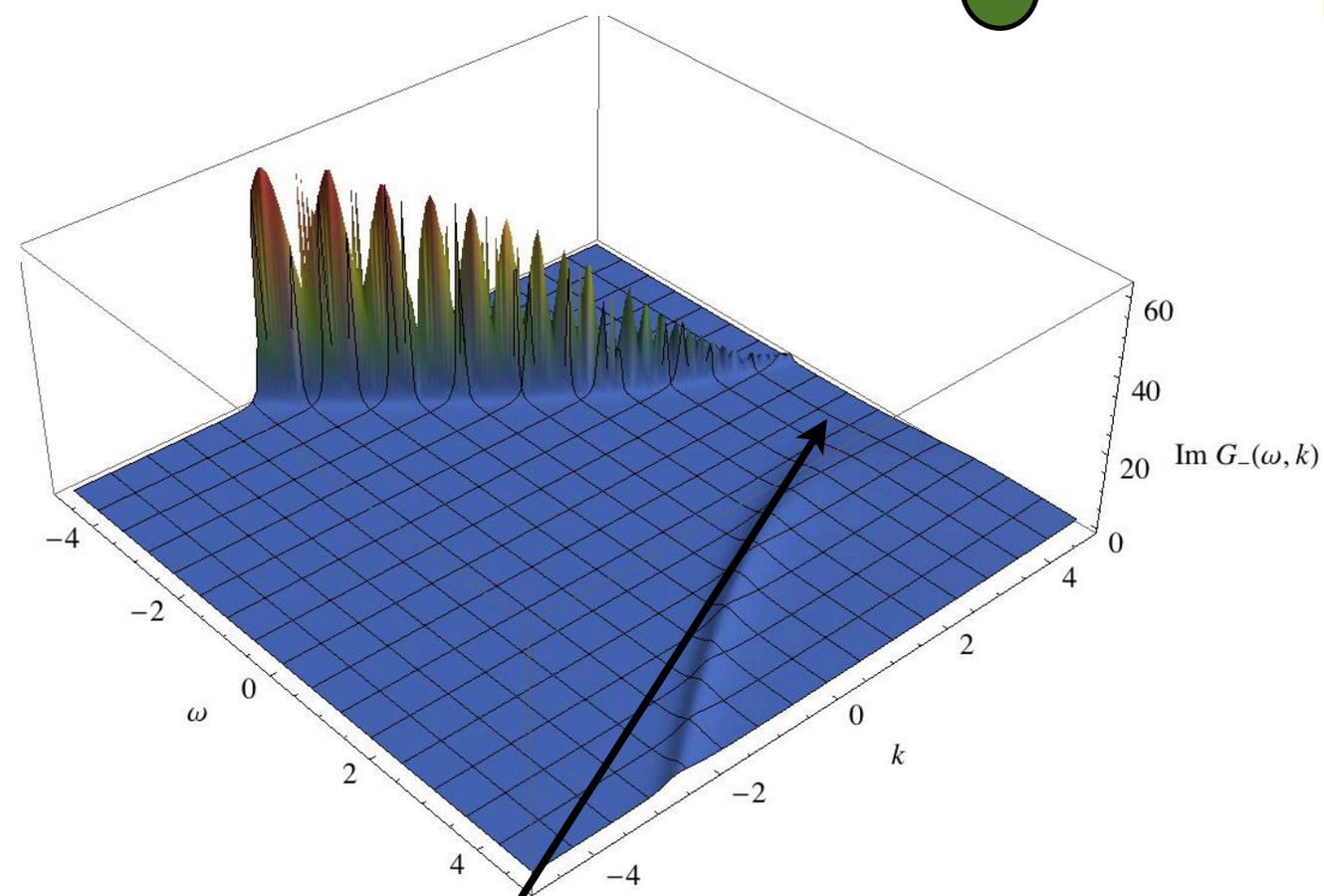




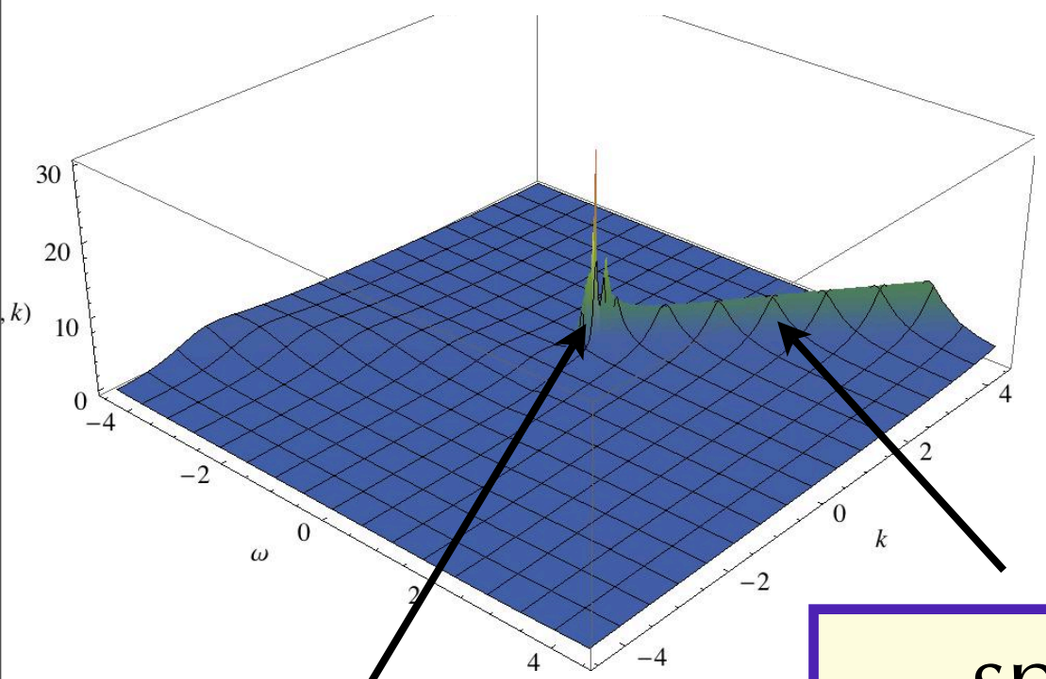
Fermi surface peak

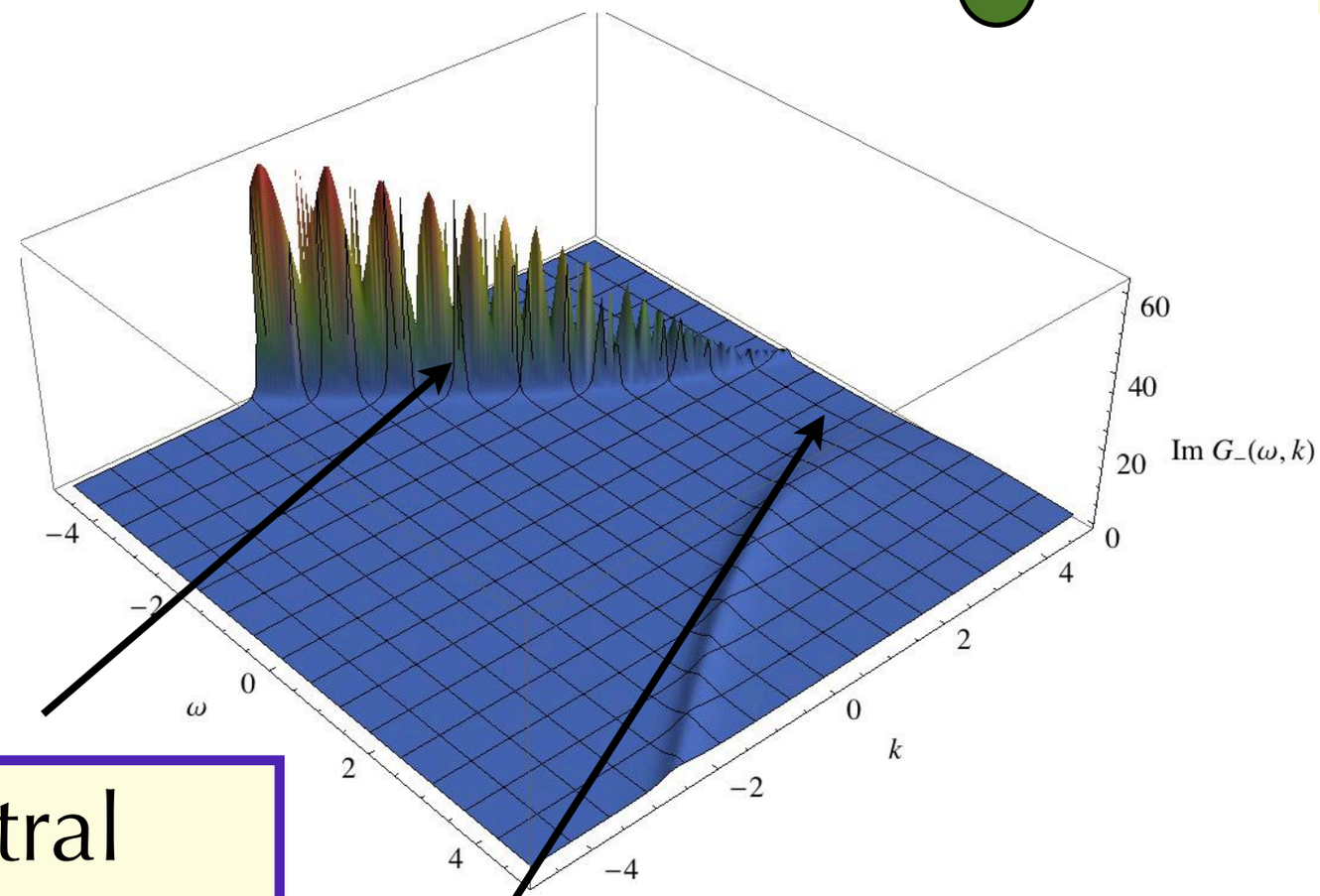
dynamically generated gap:

How is the spectrum modified?

$P > 4.2$ P

P=0, MIT,
Leiden, McMaster





spectral
weight transfer

Fermi
surface
peak

dynamically generated gap:

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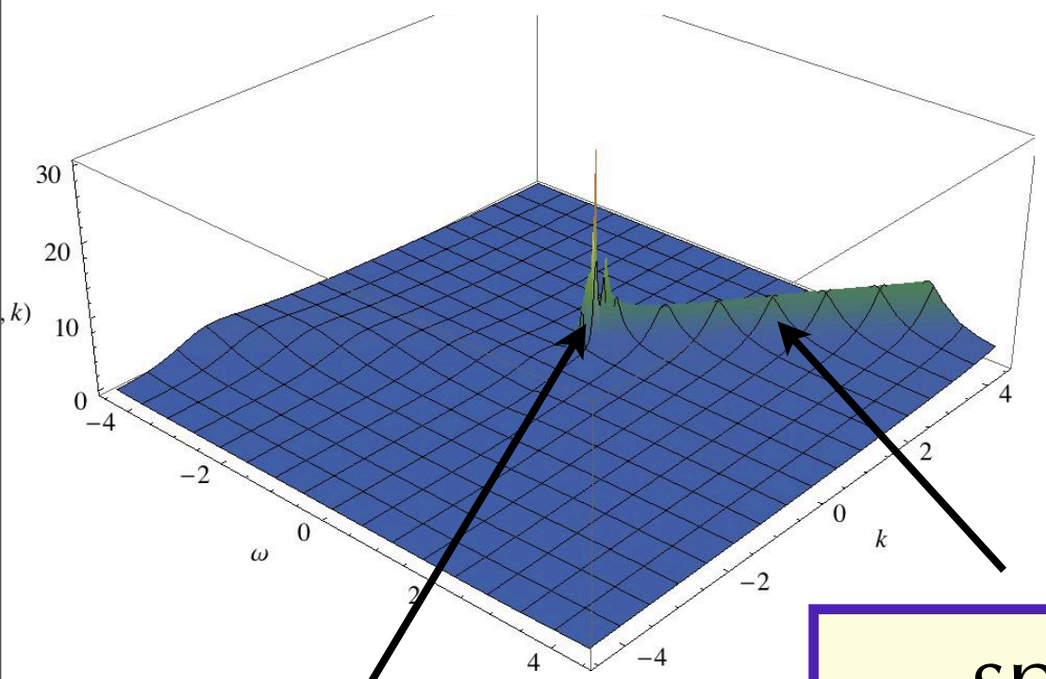

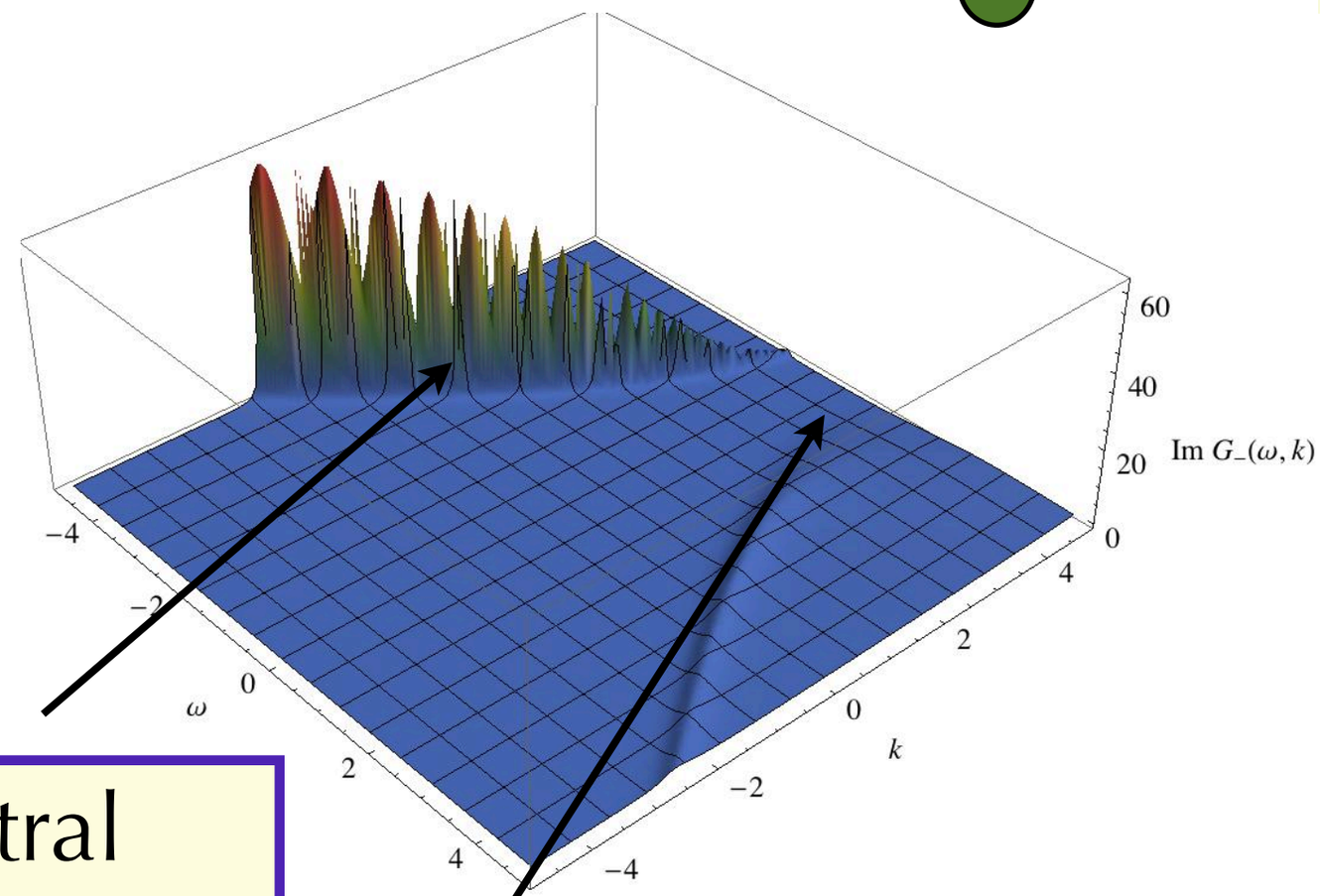

spectral
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Fermi
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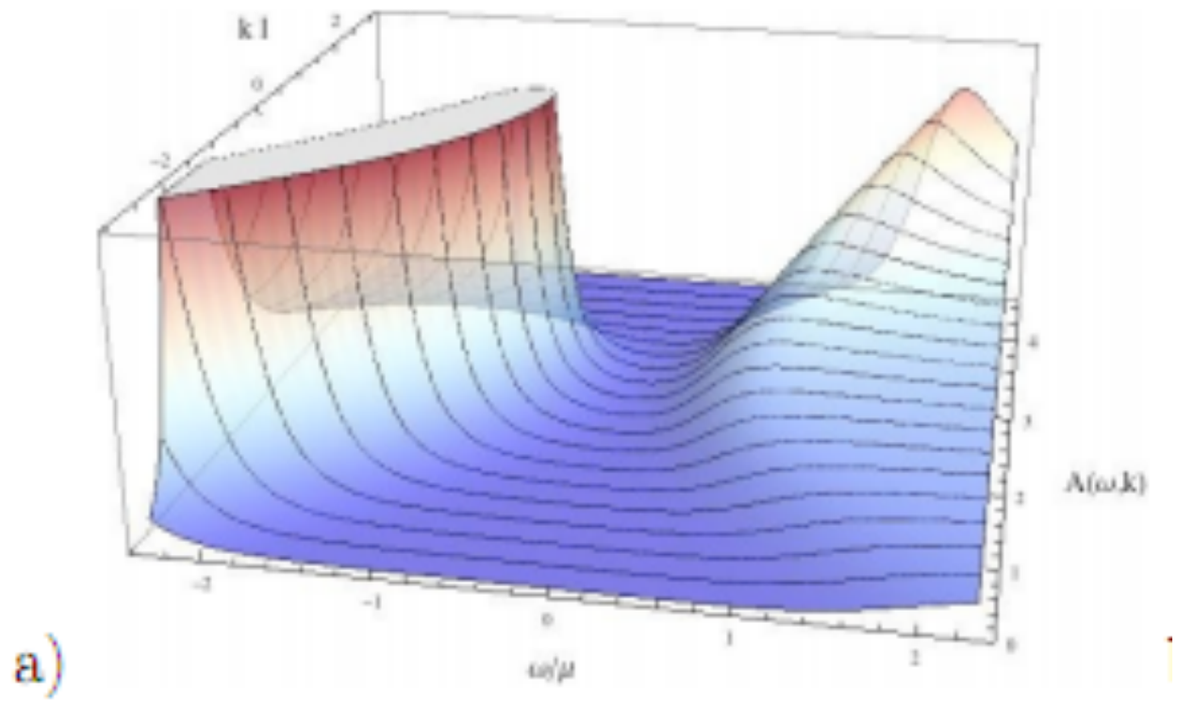
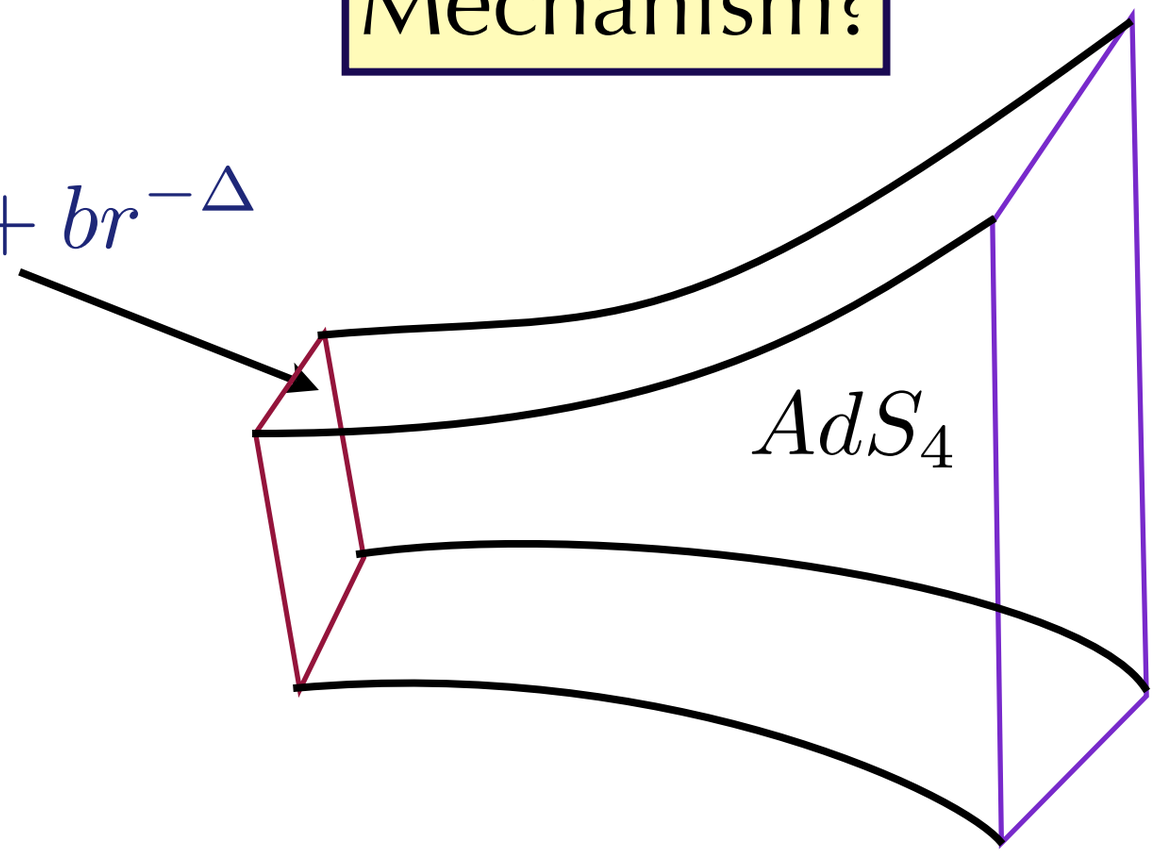
Gubser, Gauntlett, 2011 `similar' results



Sonner, 2011 (top-down gravitino model)

Mechanism?

$$\psi \propto ar^{\Delta} + br^{-\Delta}$$

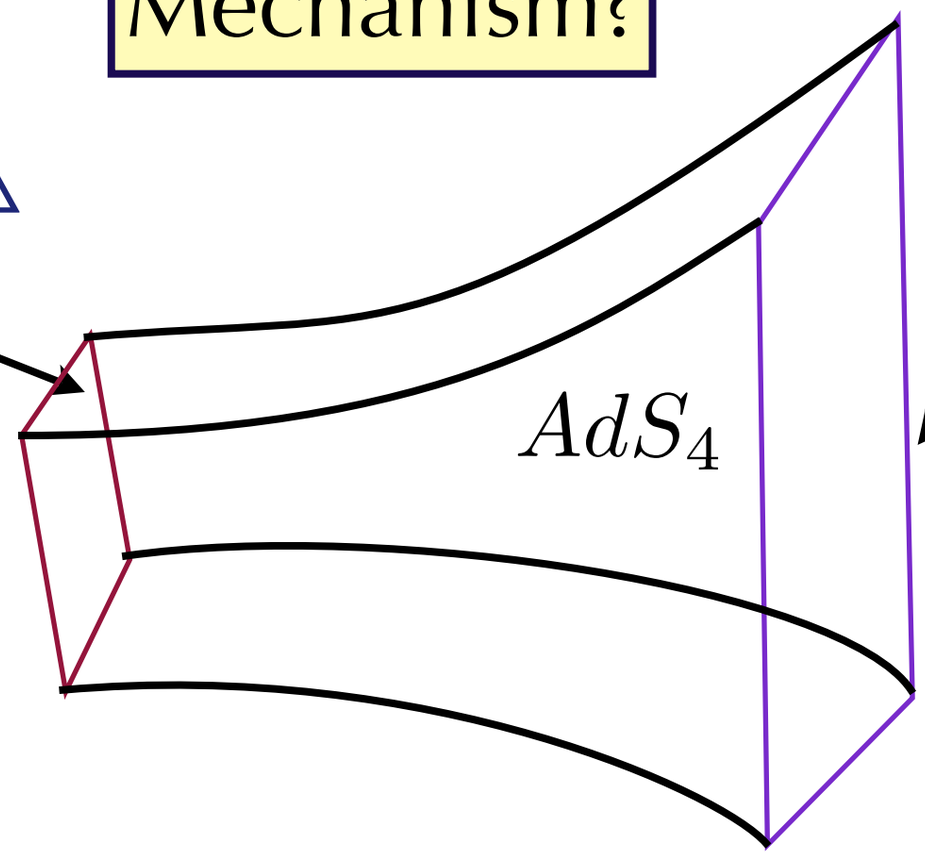


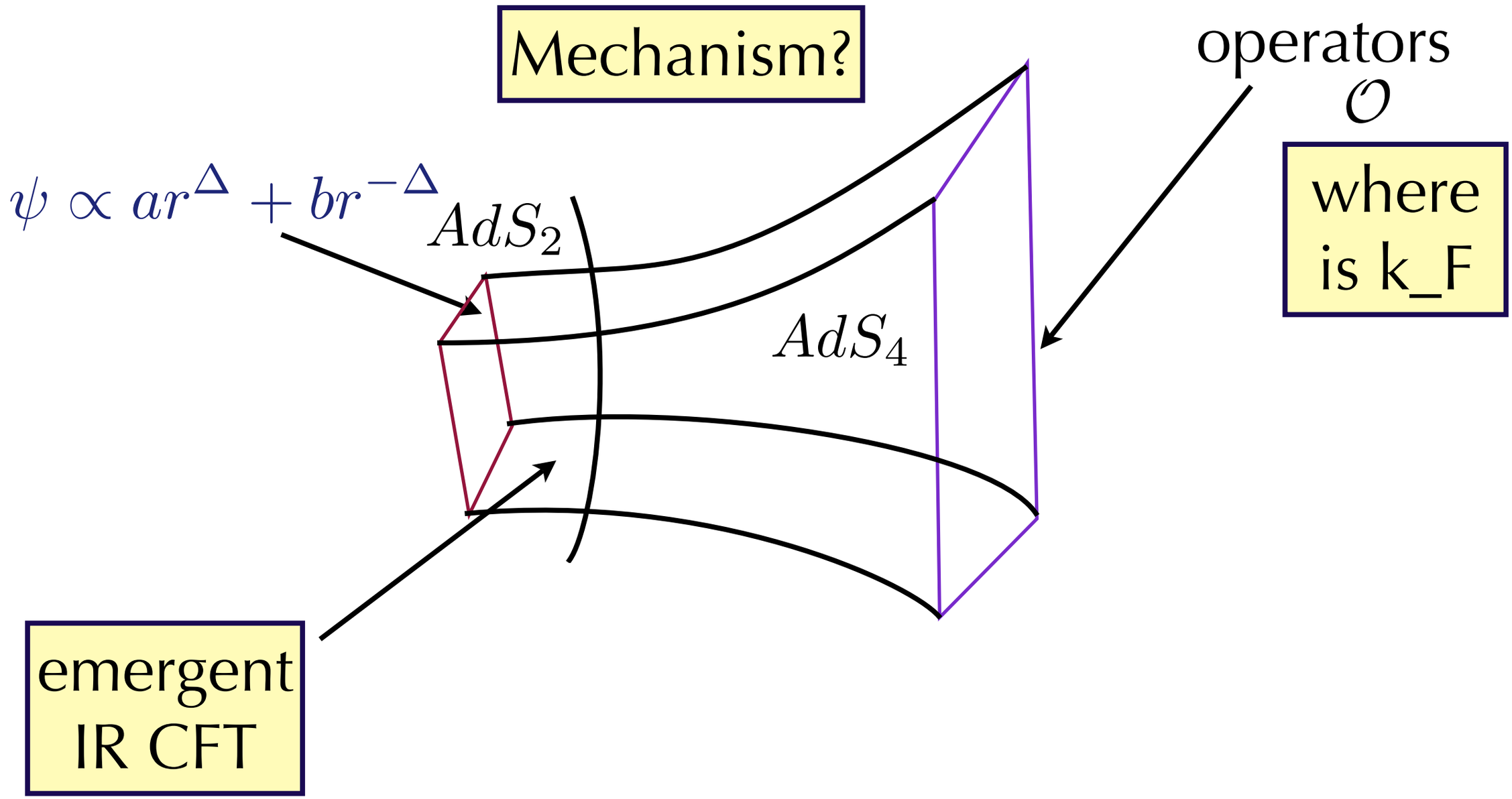
Mechanism?

operators $\mathcal{O}$

where is k_F

$\psi \propto ar^{\Delta} + br^{-\Delta}$




Mechanism?
operators
$\mathcal{O}$
where is k_F
$\psi \propto ar^{\Delta} + br^{-\Delta}$
AdS_2
AdS_4
emergent IR CFT

Mechanism?

$\psi \propto ar^{\Delta} + br^{-\Delta}$

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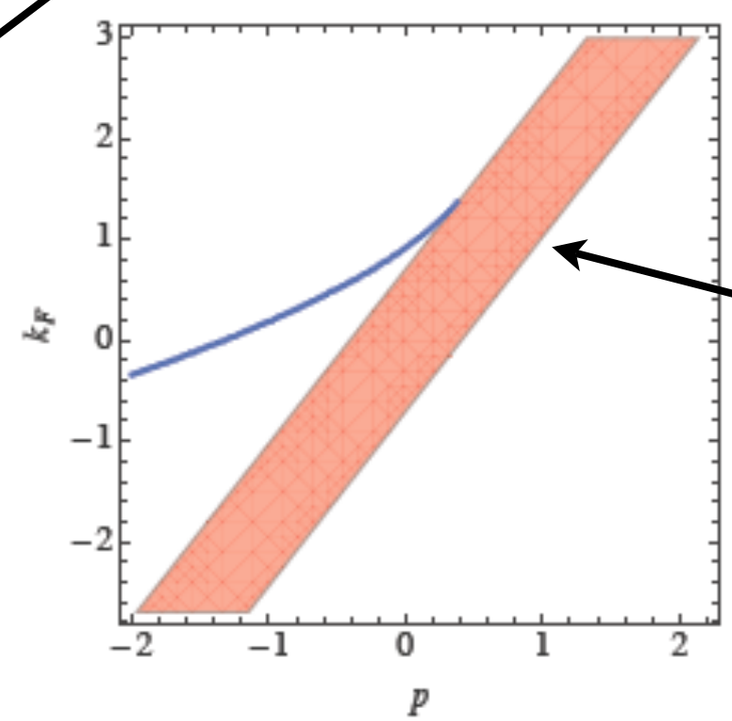
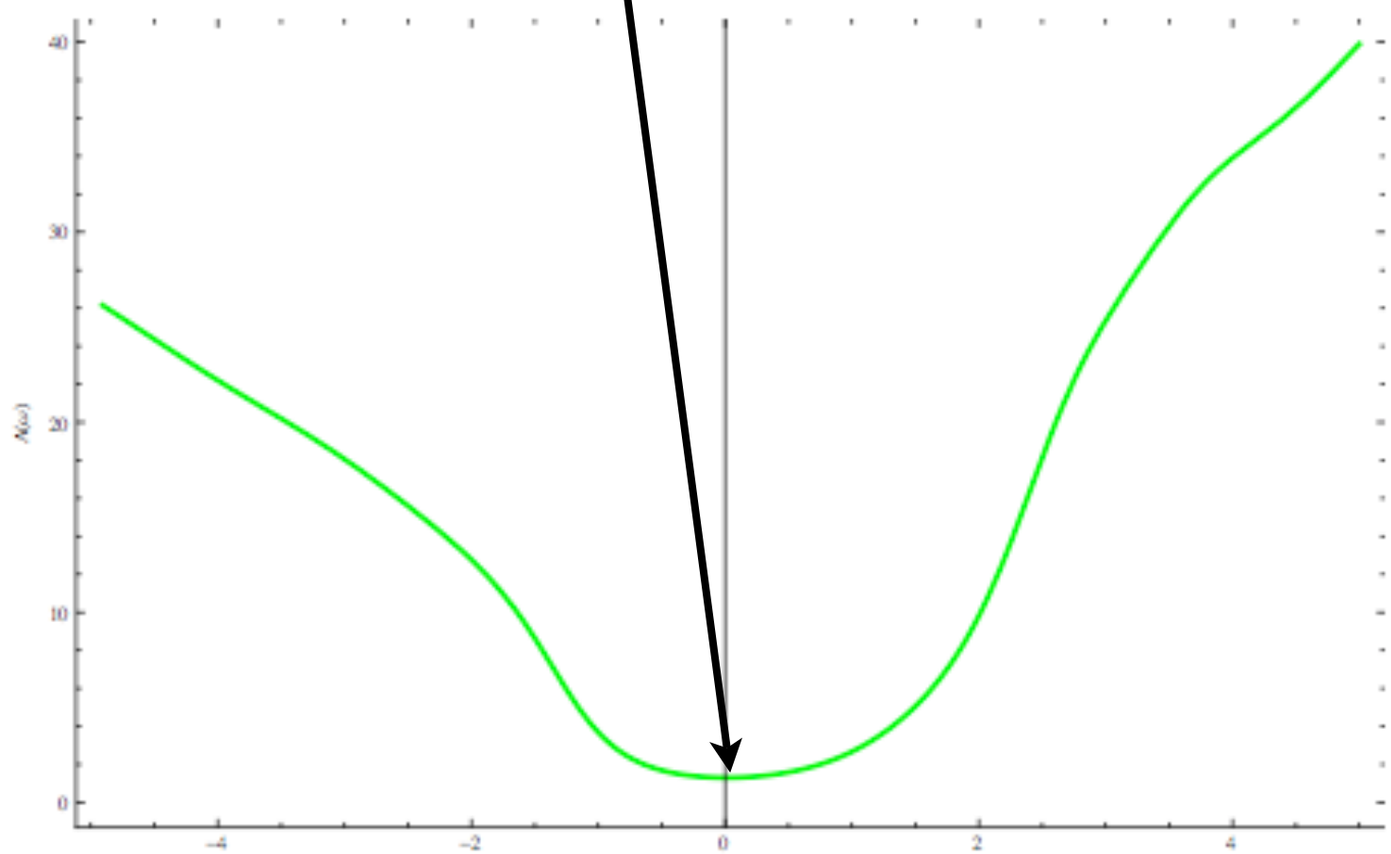


log-oscillatory

k_F moves into log-oscillatory region: IR $\mathcal{O}_{\pm}$ acquires a complex dimension

Schwarzschild (no log-oscillatory region)

finite density of states



$$G(\omega, k) = G(\omega\lambda^n, k)$$

Discrete scale invariance (DSI)

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$$\mathcal{O} = \mu(\lambda)\mathcal{O}(\lambda r)$$

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$$1 = \mu\lambda^{\Delta}$$

$$G(\omega, k) = G(\omega\lambda^n, k)$$

Discrete scale invariance (DSI)

$$\mathcal{O} = \mu(\lambda)\mathcal{O}(\lambda r)$$

$$e^{2\pi i n} = 1 = \mu\lambda^{\Delta}$$

$$G(\omega, k) = G(\omega\lambda^n, k)$$

Discrete scale invariance (DSI)

$$\mathcal{O} = \mu(\lambda)\mathcal{O}(\lambda r)$$

$$e^{2\pi i n} = 1 = \mu\lambda^{\Delta}$$

$$\Delta = -\frac{\ln\mu}{\ln\lambda} + \frac{2\pi i n}{\ln\lambda}$$

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n=0:CSI

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n=0:CSI

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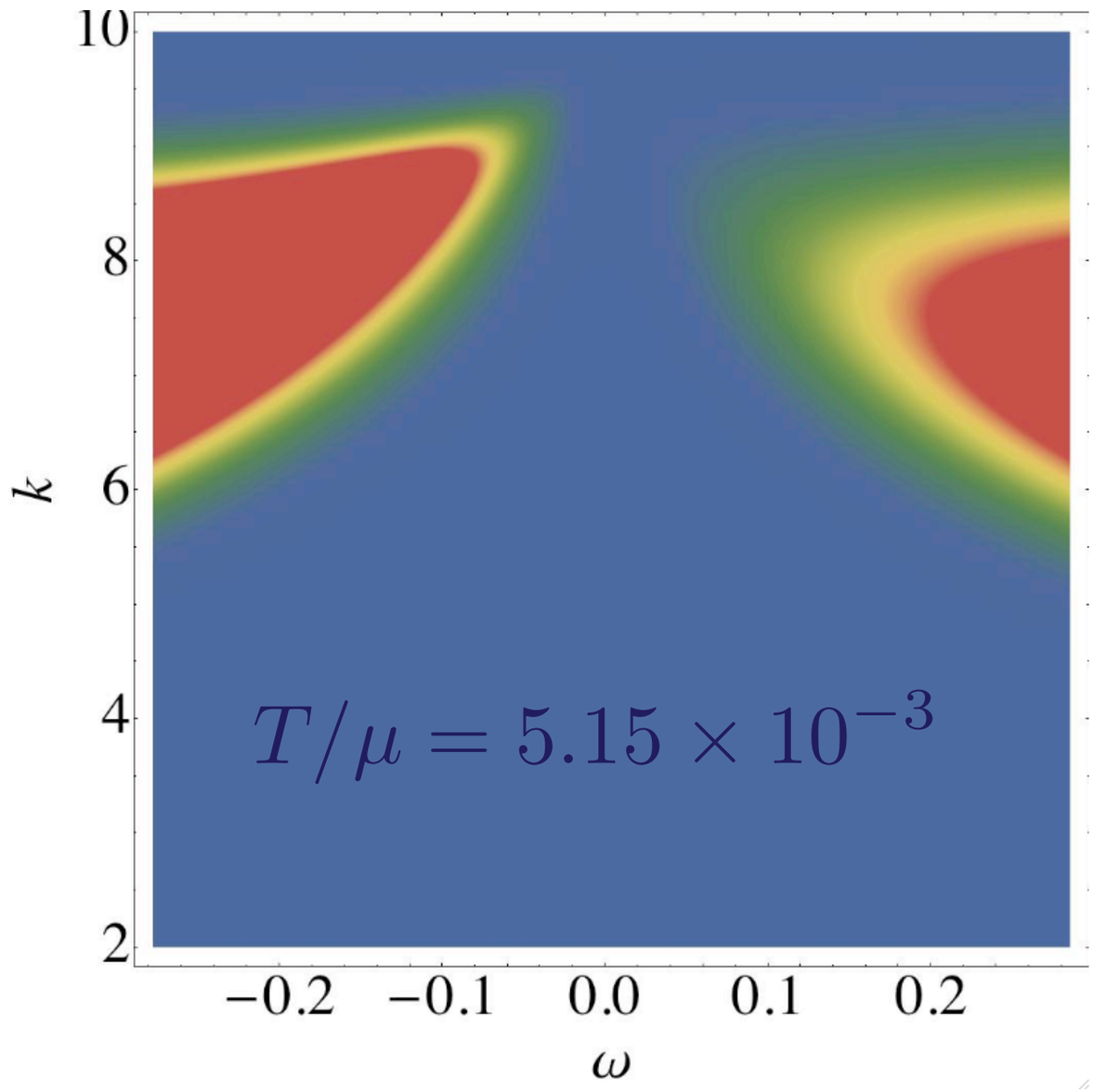
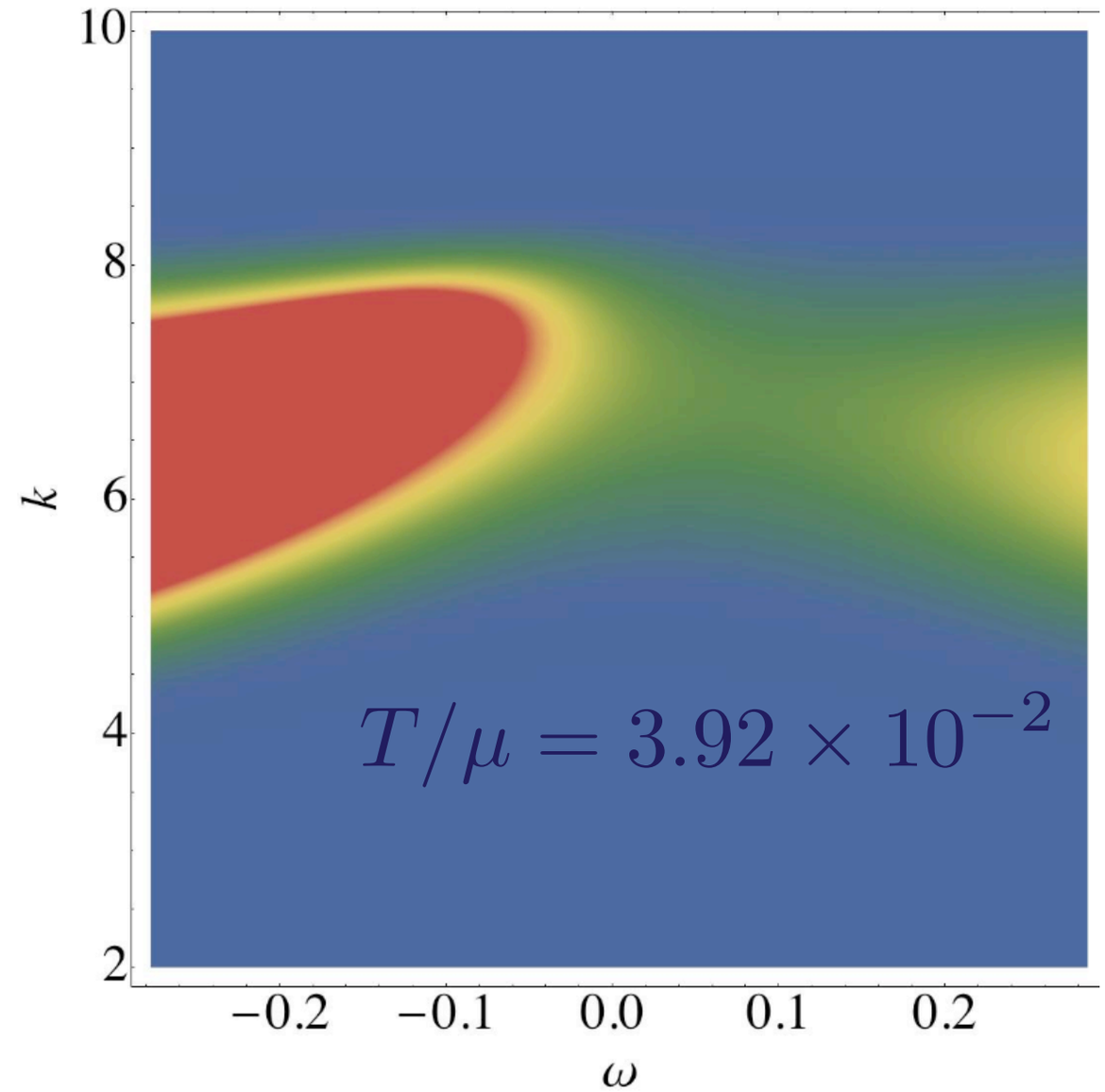
scaling dimension depends on scale

$$\lambda_n = \lambda^n$$

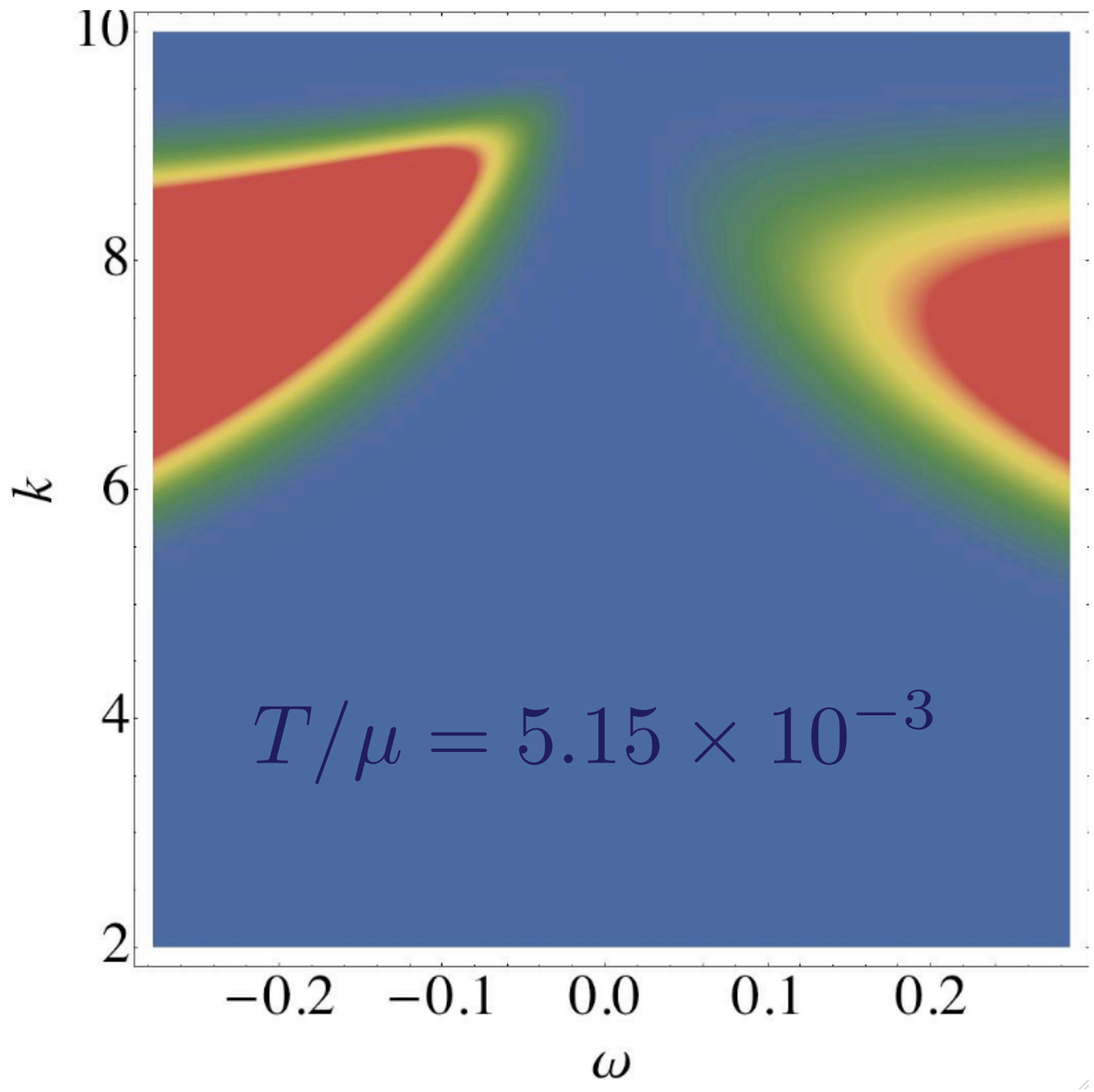
magnification

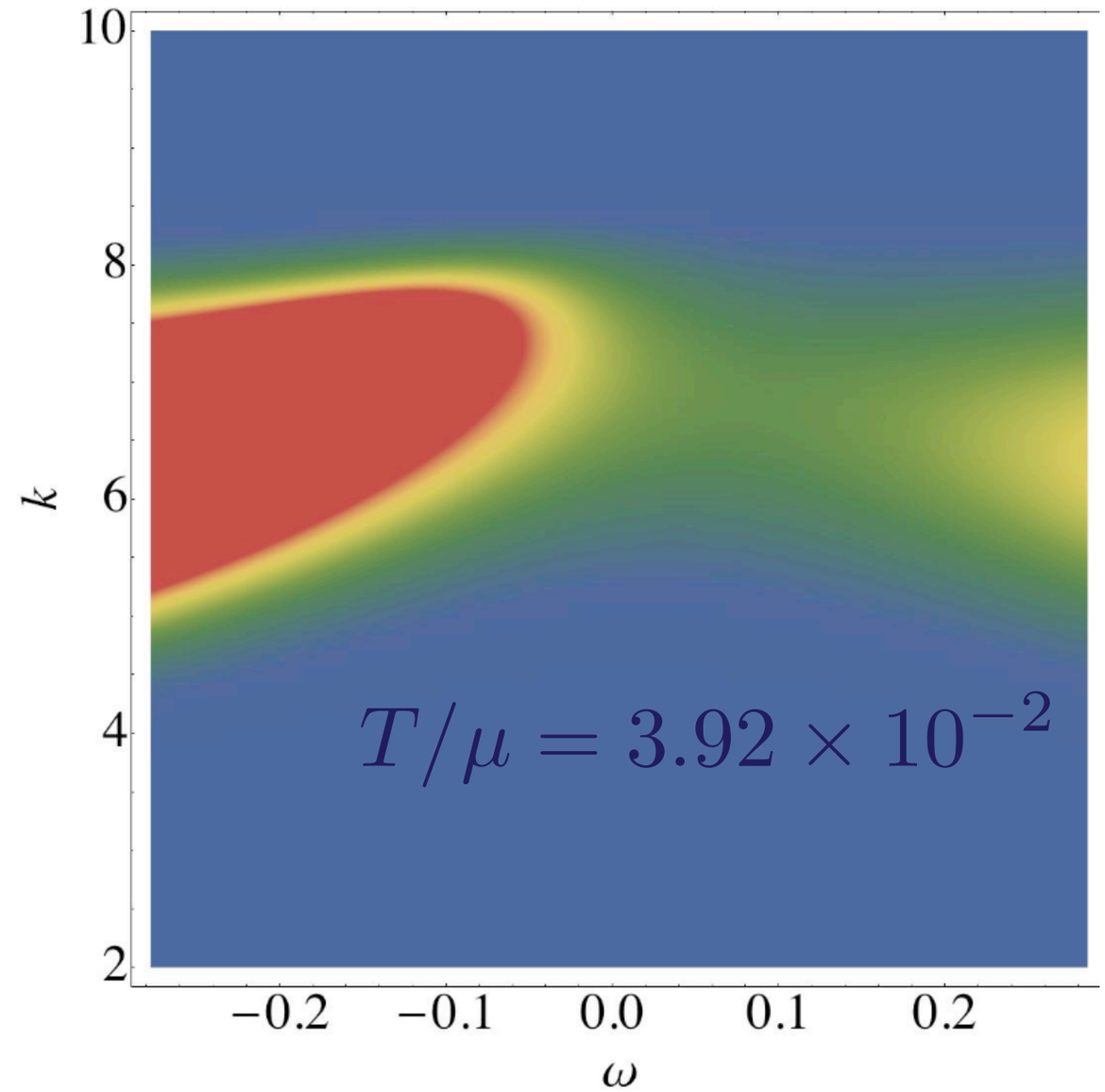
Finite Temperature Mott transition





Finite Temperature Mott transition

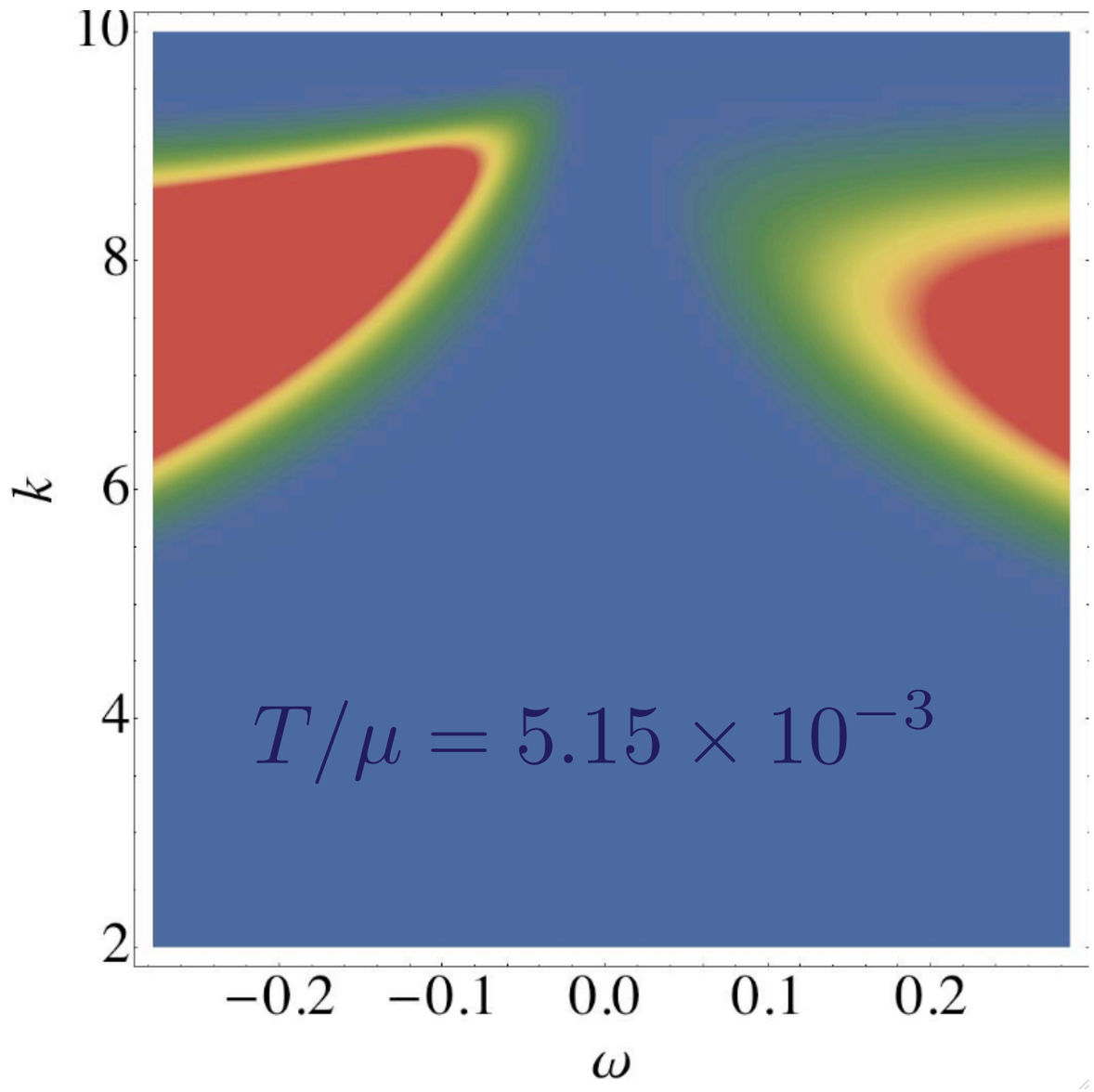


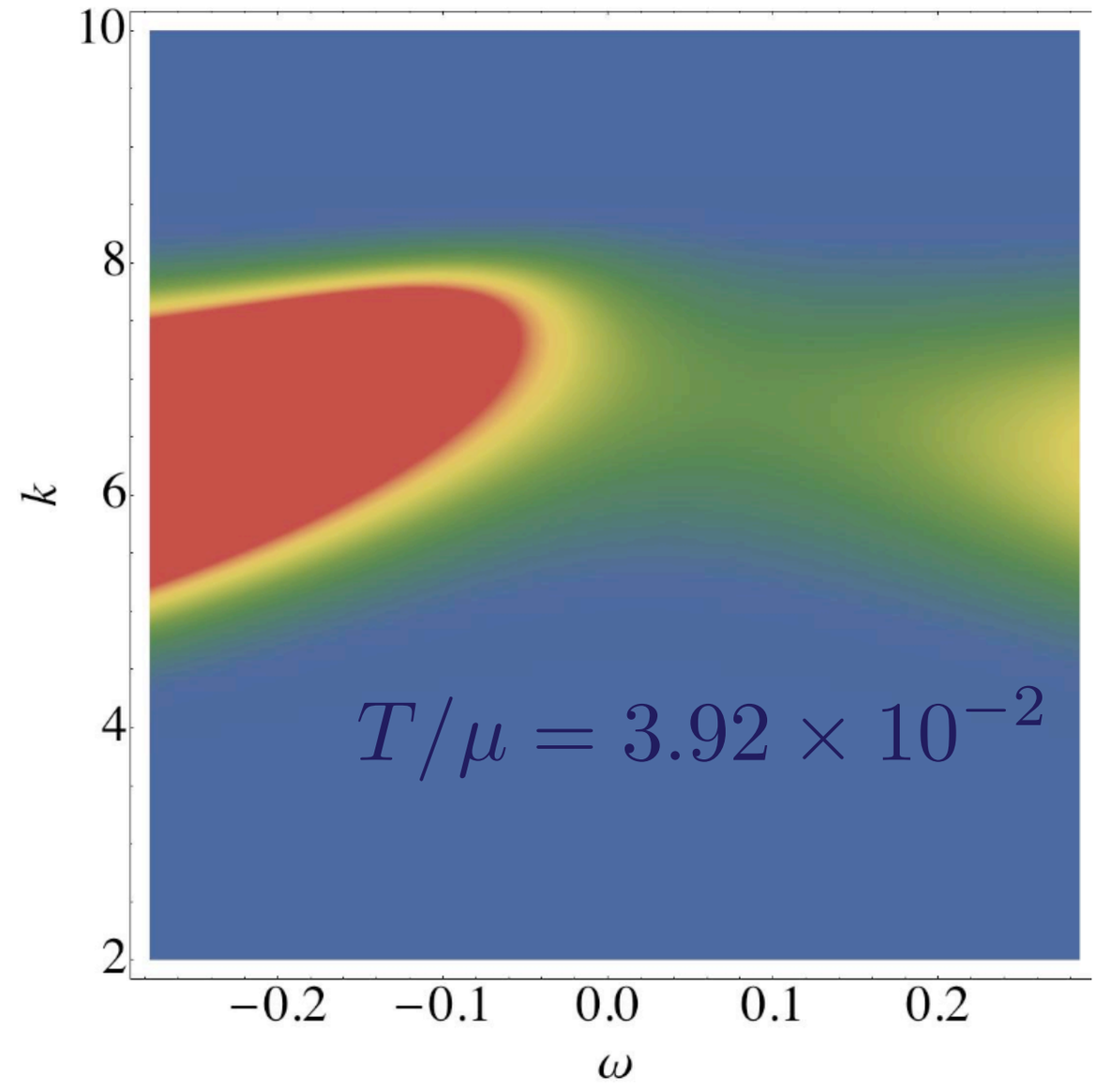


$$\frac{\Delta}{T_{\rm crit}} \approx 20$$

vanadium oxide

Finite Temperature Mott transition

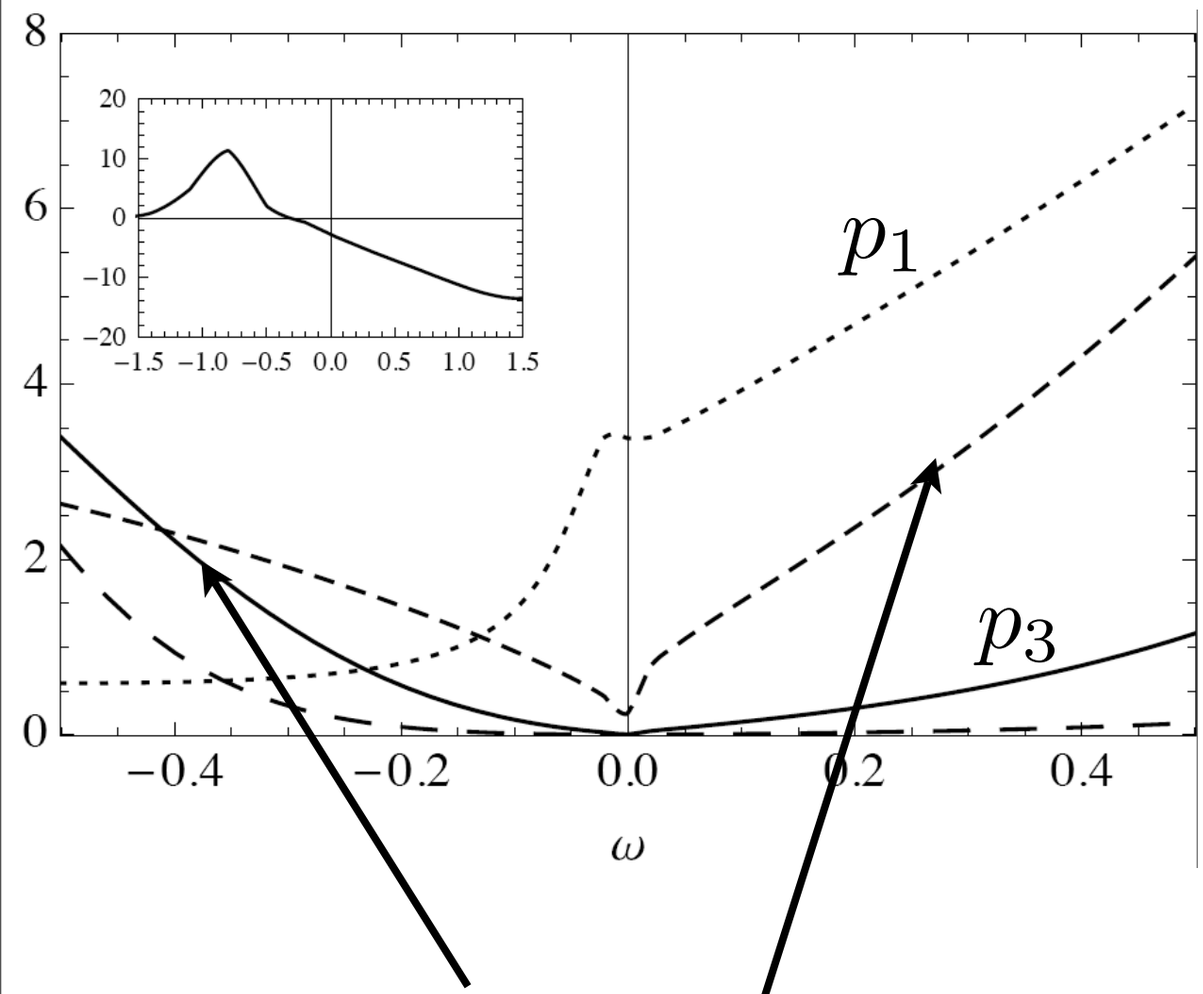




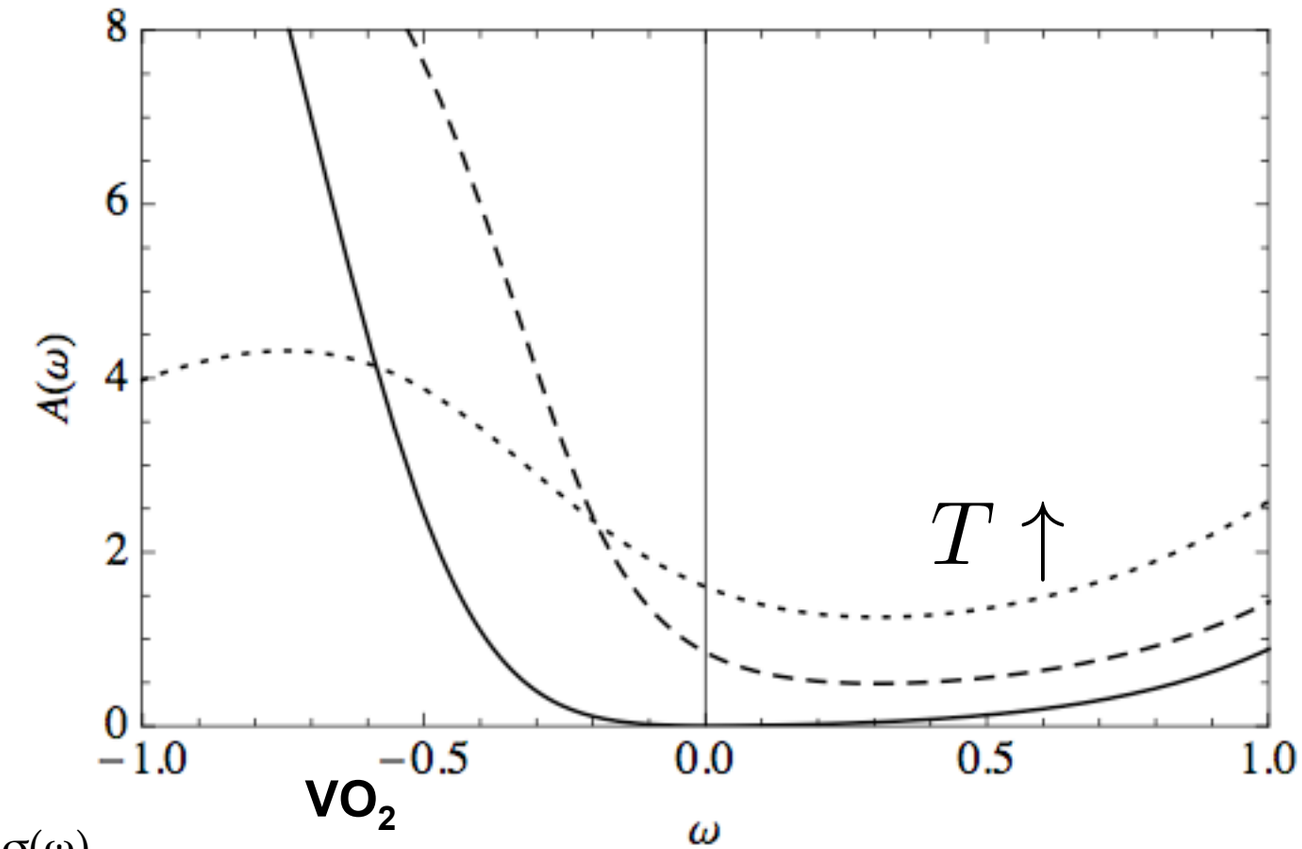
$$\frac{\Delta}{T_{\text{crit}}} \approx 10$$

$$\frac{\Delta}{T_{\text{crit}}} \approx 20$$

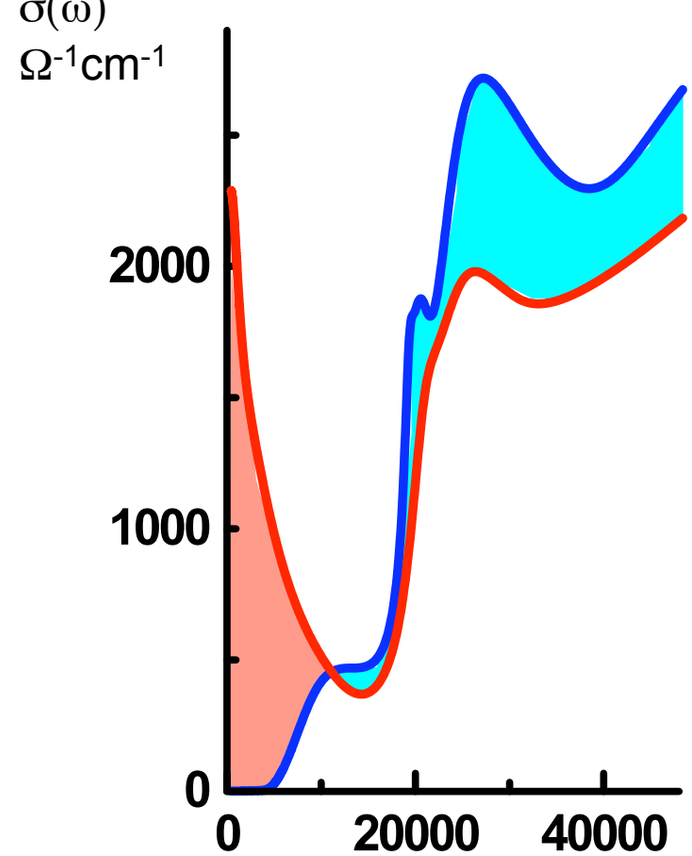
vanadium oxide



spectral weight transfer
UV-IR mixing



VO_2



is CSI->DSI the symmetry that is ultimately broken in the Mott problem?

a.) yes

b.) no

holography

a.) yes	√
b.) no	

	holography	VO_2, cuprates,...
a.) yes	√	
b.) no		

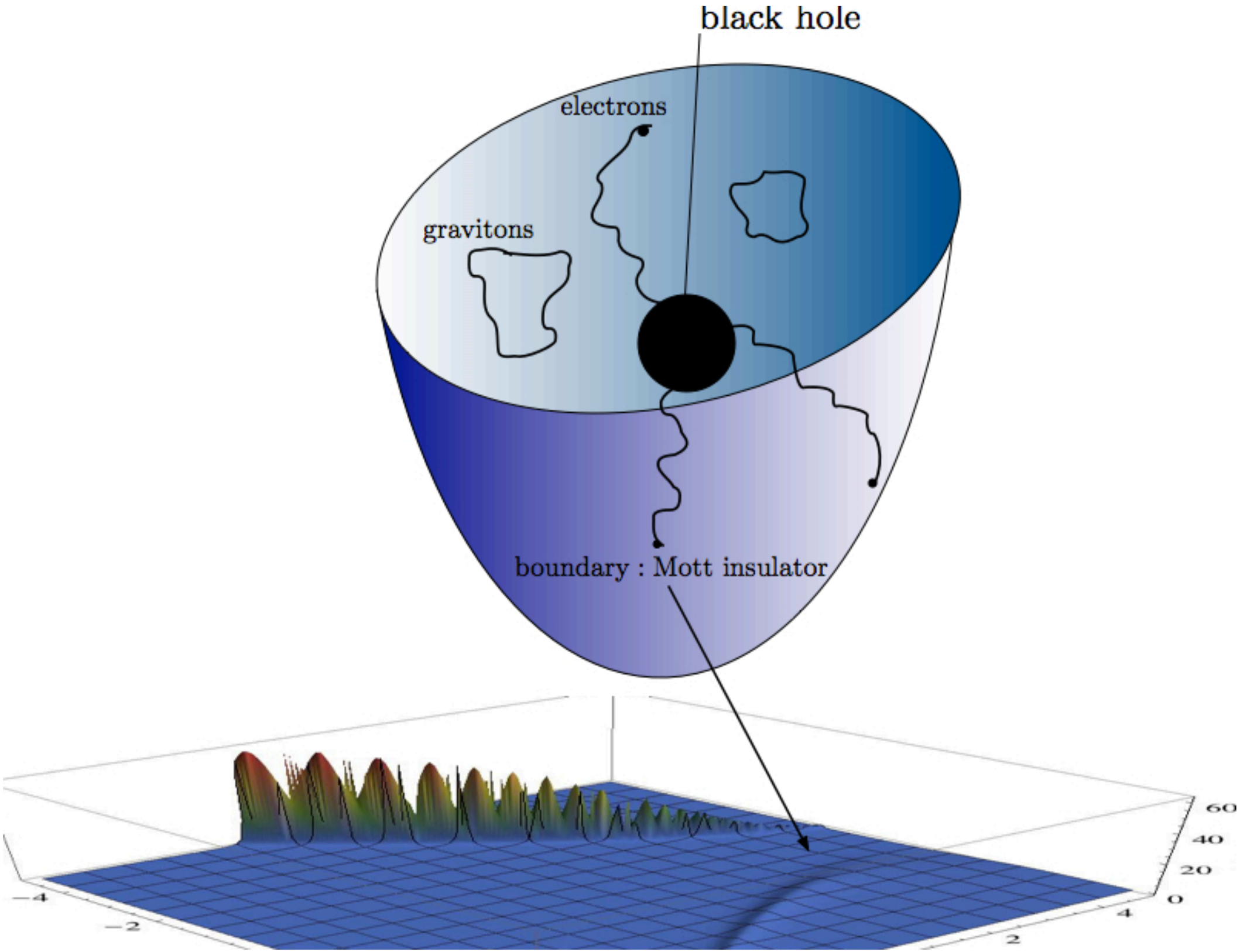
	holography	VO_2, cuprates,...
a.) yes	√	
b.) no		

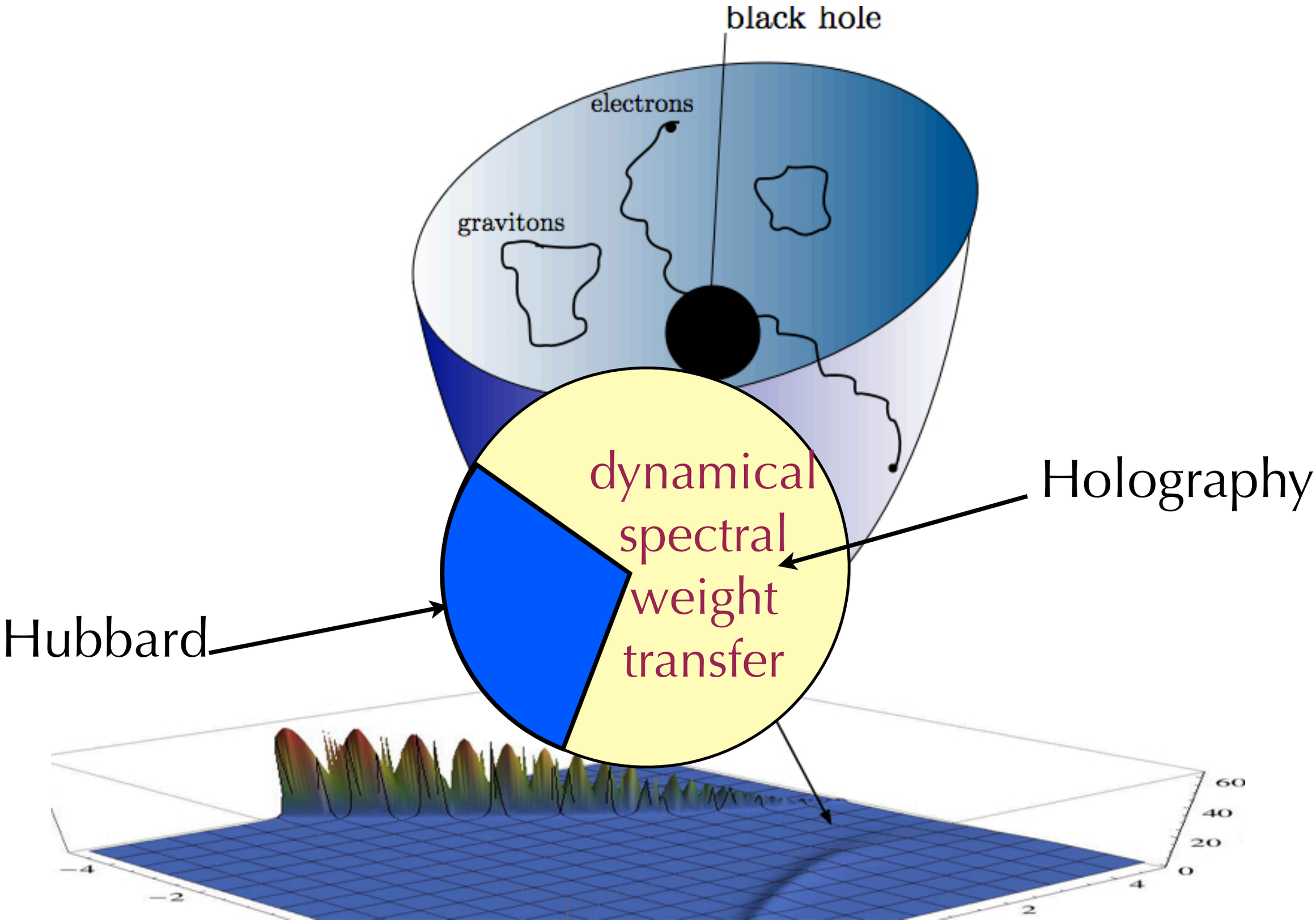
Does scaling in VO_2 obey: $\Lambda_{\mathrm{IR}} = \Lambda_{\mathrm{UV}} e^{-\pi/\sqrt{g_c - g}}$

	holography	VO_2, cuprates,...
a.) yes	√	
b.) no		

Does scaling in VO_2 obey: $\Lambda_{\mathrm{IR}} = \Lambda_{\mathrm{UV}} e^{-\pi/\sqrt{g_c - g}}$

if yes: holography has solved the Mott problem


black hole
electrons
gravitons
boundary : Mott insulator
60
40
20
0
−4
−2
2
4


black hole
electrons
gravitons
dynamical
spectral
weight
transfer
Holography
Hubbard
60
40
20
0
−4
−2
2
4