Noether's Second Theorem and Strange Metals

Thanks to: NSF

Gabriele La Nave

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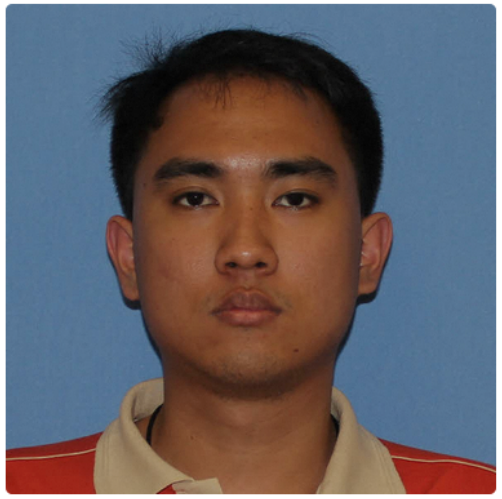
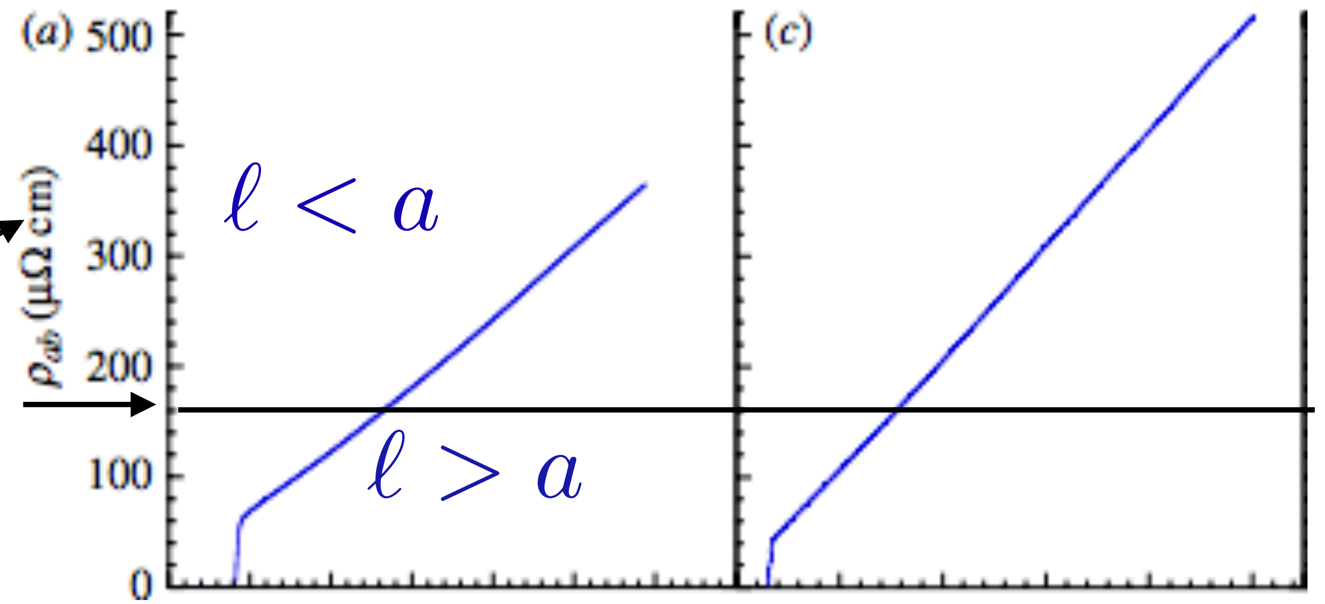
CIMP 2019
Adv. Th. Math. Phys. 2019



Kridsangaphong Limtragool

T-linear resistivity



$\rho > \rho_Q$

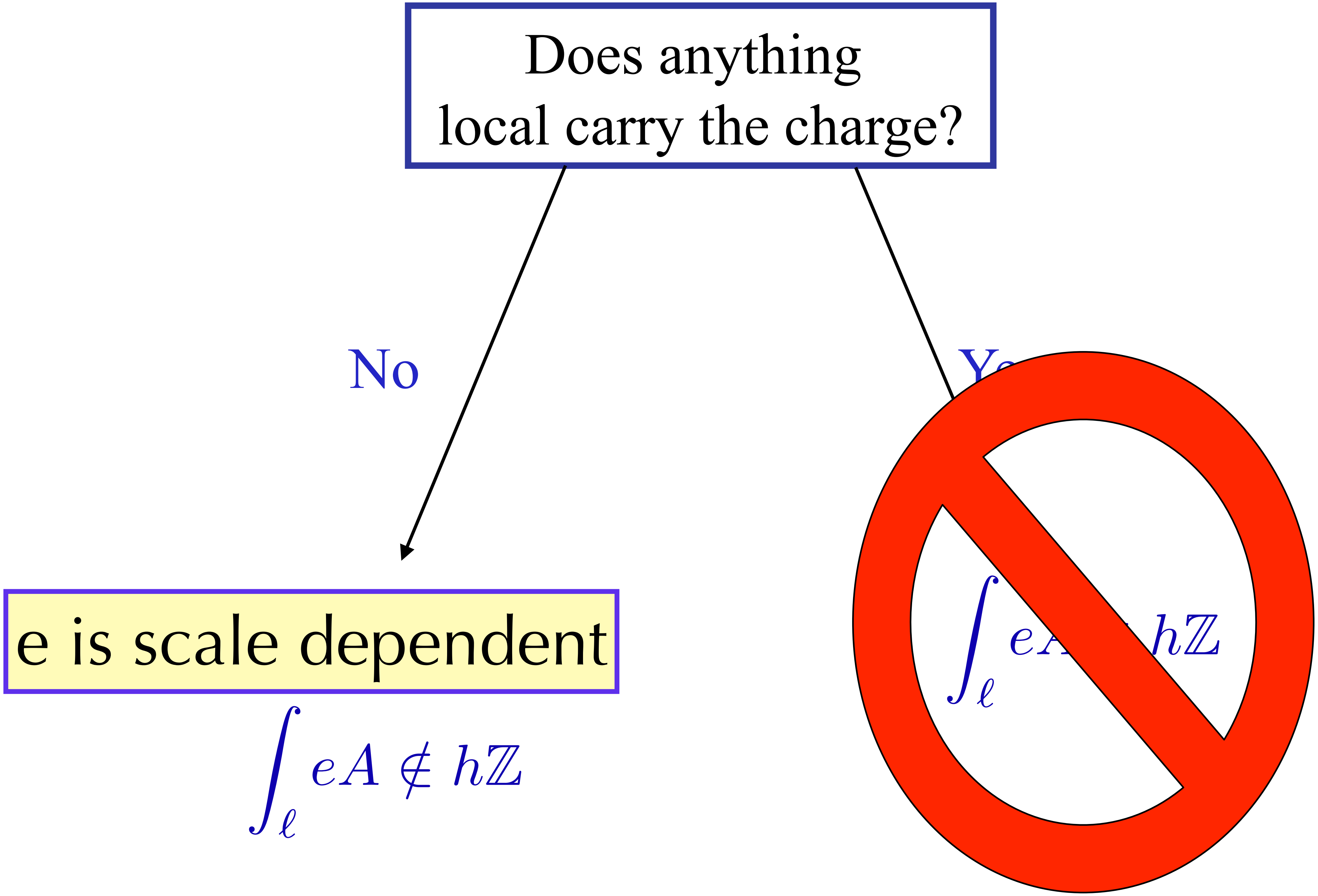
$$\rho > \rho_Q$$

Violates MIR limit

electrons not charge carriers

?

Is a new gauge principle operative?


Does anything local carry the charge?
No
Yes
e is scale dependent
$\int_\ell eA \notin h\mathbb{Z}$
$\int_\ell eA \in h\mathbb{Z}$

Can you mess with A?

Noether's First Theorem

$U(1)$ ⟷ $qA \to qA - q\partial_\mu \Lambda$

$$\psi' = e^{iq\Lambda}\psi$$

$$[q\Lambda] = 0$$

$$[qA] = 1$$

$$[A] = 1$$

$$S = \int d^d x (J_\mu A^\mu + \cdots)$$

$$S \to S + \int d^d x J_\mu \partial \Lambda$$

fixes dimension of current

$$[d^d x J A] = 0$$

$$[J] = d - 1$$

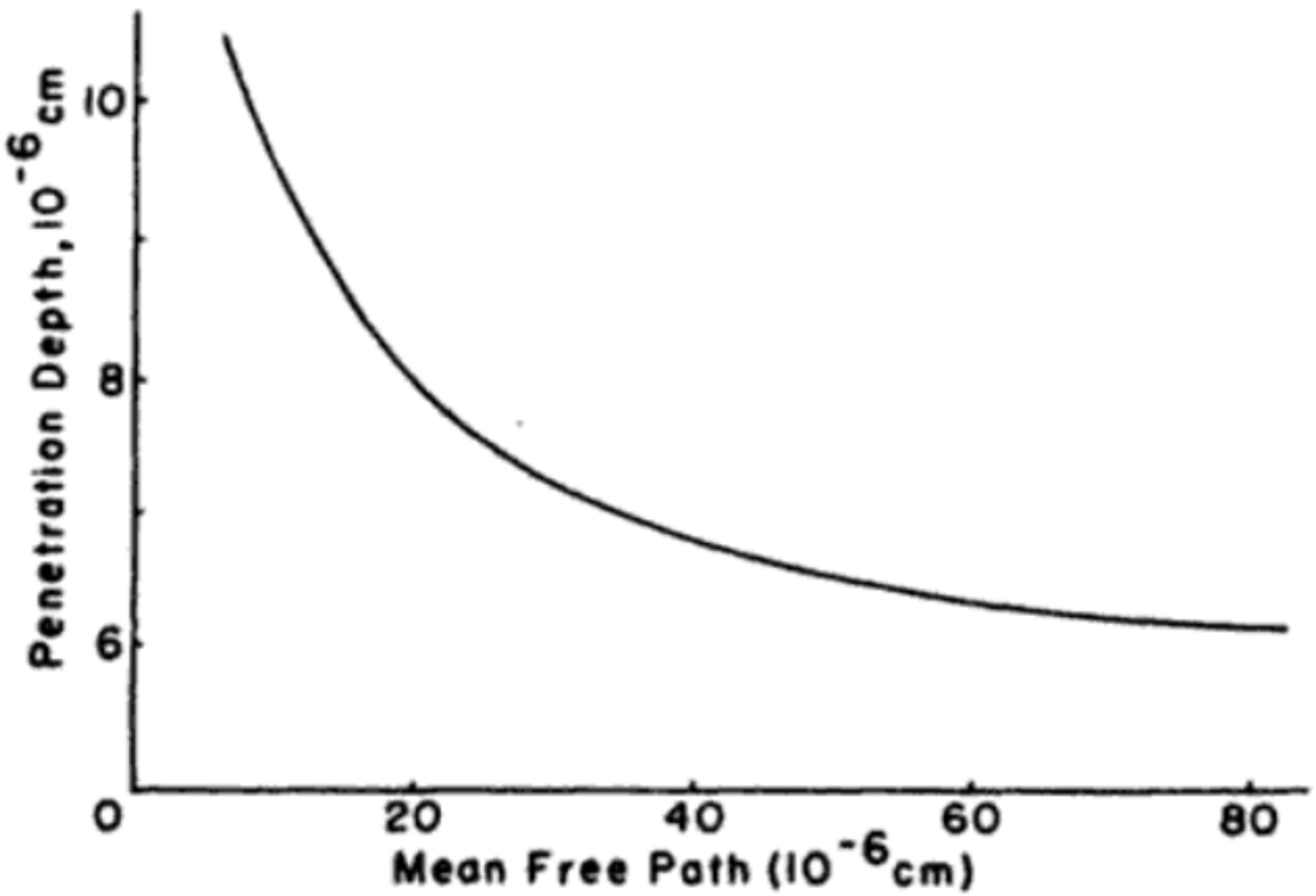
$$\partial_\mu J^\mu = 0$$

Noether's Thm. I

current conservation

Are there exceptions?

Pippard's problem



$$J_s \neq \frac{-c}{4\pi\lambda^2} A$$

London Eq.

failure of local London relations

Pippard Current

Pippard kernel

$$J_s = -\frac{3}{4\pi c\xi_0\lambda}\int \frac{(\vec{r}-\vec{r}')((\vec{r}-\vec{r}')\cdot\vec{A}(\vec{r}'))e^{-(\vec{r}-\vec{r}')/\xi(\ell)}}{(\vec{r}-\vec{r}')^4}d^3\vec{r}'$$

non-local

Units of Current

$$J_\mu(\mathbf{x}) = \frac{\delta \mathbf{L_m}}{\delta \mathbf{A}_\mu} = -\int \mathbf{C}^{\mu\nu}(\mathbf{x},\mathbf{x}')(\mathbf{A}_\nu(\mathbf{x}') - \partial_\nu \phi(\mathbf{x}'))\mathbf{d^3x}'$$

$$[J] = d - d_C - d_A$$

anomalous dimension

Standard Result

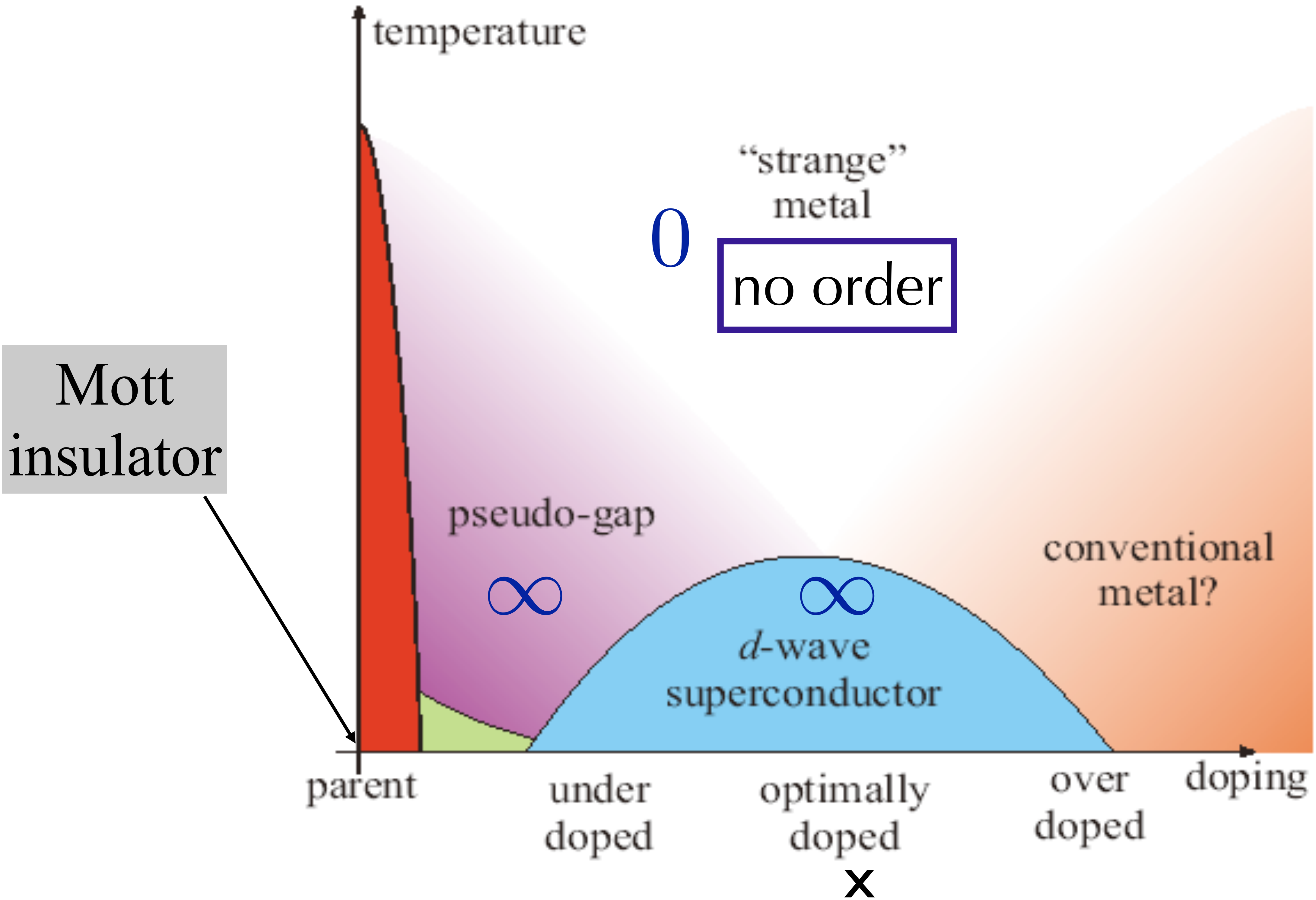
$$\delta(x_0 - y_0)[J_\mu(x), \phi(y)] = \delta^d(x-y)\delta\phi(y)$$

$$[J] = d - 1$$

Are there other examples of currents with anomalous dimensions?

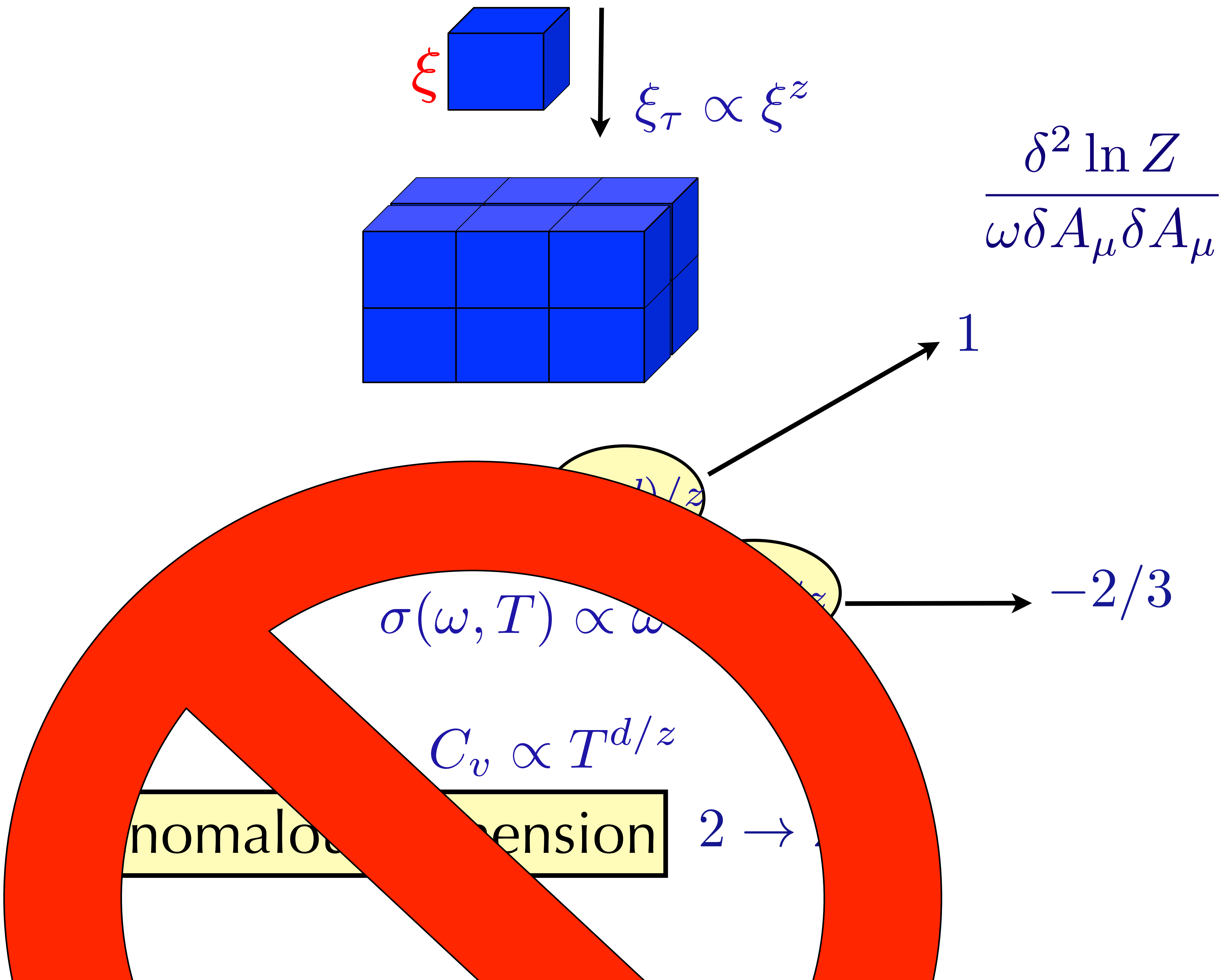
underlying electricity and magnetism?

is symmetry breaking necessary?


temperature
"strange"
metal
0
no order
Mott
insulator
pseudo-gap
∞
∞
d-wave
superconductor
conventional
metal?
parent
under
doped
optimally
doped
over
doped
doping
x

why is the problem hard?

single-parameter scaling



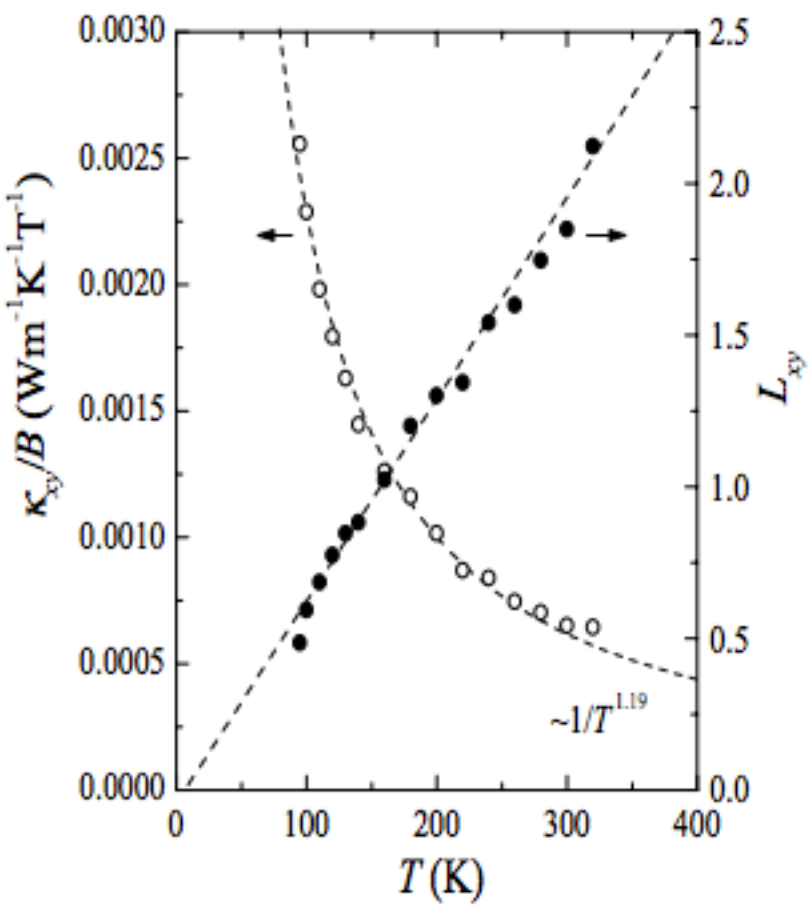
strange metal explained!

Hall Angle

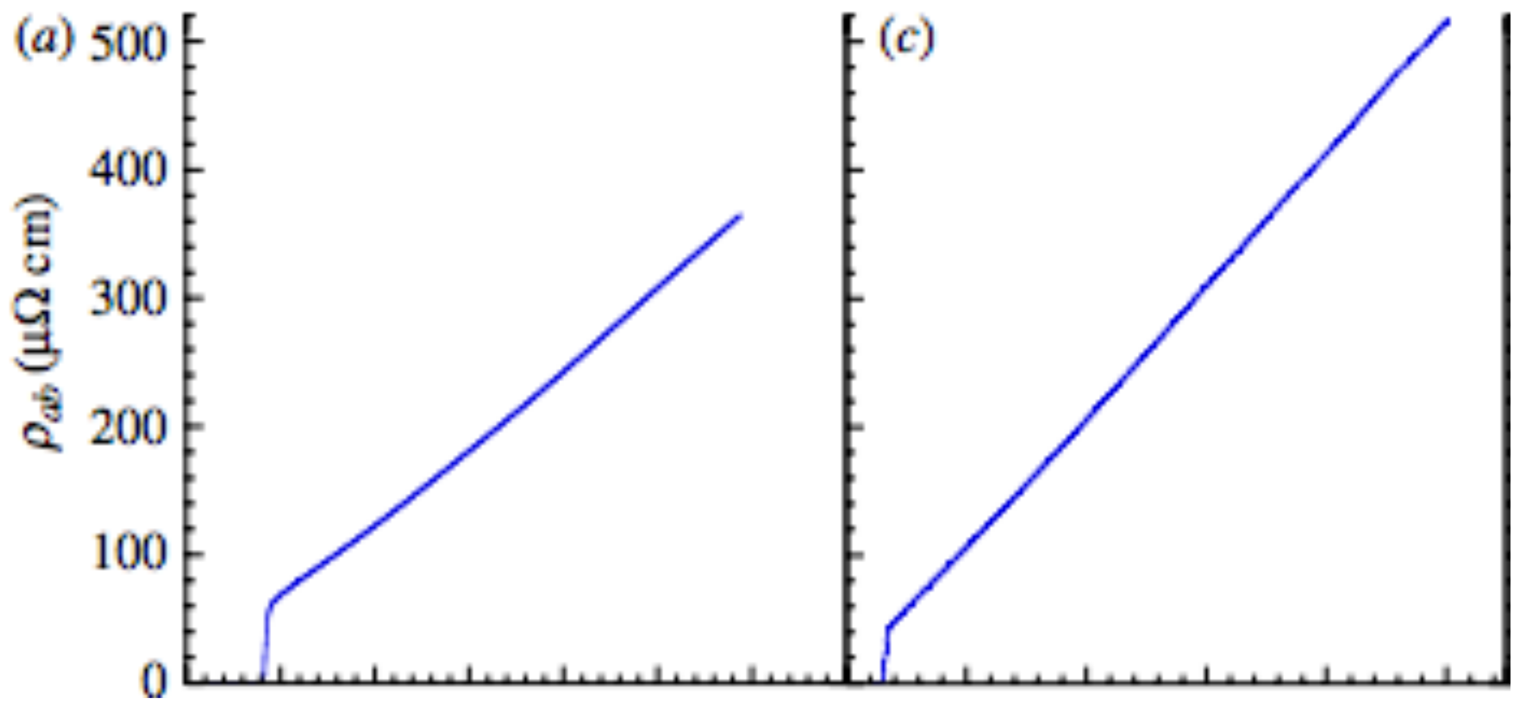
$$\cot\theta_H \equiv \frac{\sigma_{xx}}{\sigma_{xy}} \approx T^2$$

Hall Lorenz ratio

$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$



T-linear resistivity



all explained if

$$[J_\mu] = d - \theta + \Phi + z - 1$$

$$[A_\mu] = 1 - \Phi$$

$$\Phi = -2/3$$

Hartnoll/Karch

strange metal

$$[J_\mu] = d - \theta + \Phi + z - 1$$

$$[A_\mu] = 1 - \Phi$$

$$\Phi = -2/3$$

$$[E] = 1 + z - \Phi$$

$$[B] = 2 - \Phi$$

note $\pi r^2 B \neq$ flux

$$\oint A \cdot d\ell \notin h\mathbb{Z}$$

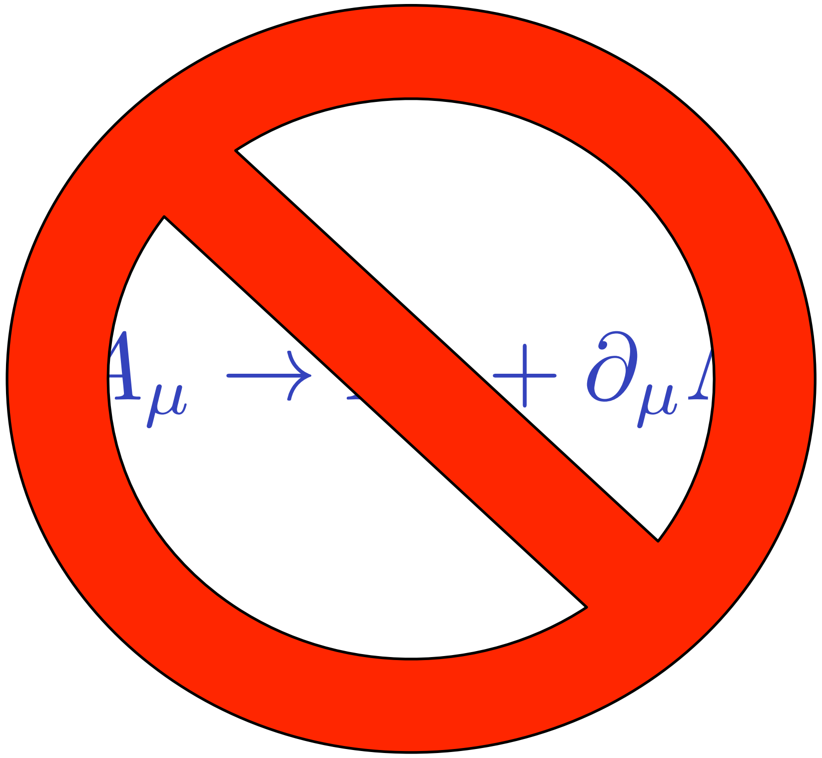
?

How is this possible - - if at all?

what is the new gauge principle?

if

$$[A_\mu] \neq 1$$



Noether's Second Theorem: precursor

hint

$$\partial_\mu J^\mu = 0$$

current conservation

what if

$$[\partial_\mu, \hat{Y}] = 0$$

new current

$$[\tilde{J}] = d - 1 - D_Y$$

gauge symmetry

$$\partial_\mu \hat{Y} J^\mu = \partial_\mu \tilde{J}^\mu = 0$$

possible gauge transformations

$$S = -\frac{1}{4}\int d^d x F^2$$

$$S = \frac{1}{2}\int \frac{d^d k}{2\pi^d} A_\mu(k)[k^2\eta^{\mu\nu} - k^\mu k^\nu]A_\nu(k)$$

$$M_{\mu\nu}k^\nu = 0$$

zero eigenvector

$$ik_\mu \to \partial_\nu$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

family of zero eigenvalues

$$M_{\mu\nu} \underbrace{f k^{\nu}}_{} = 0$$

generator of gauge symmetry

1.) rotational invariance

2.) A is still a 1-form

3.) $[f, k_{\mu}] = 0$

only choice

$$f \equiv f(k^2)$$

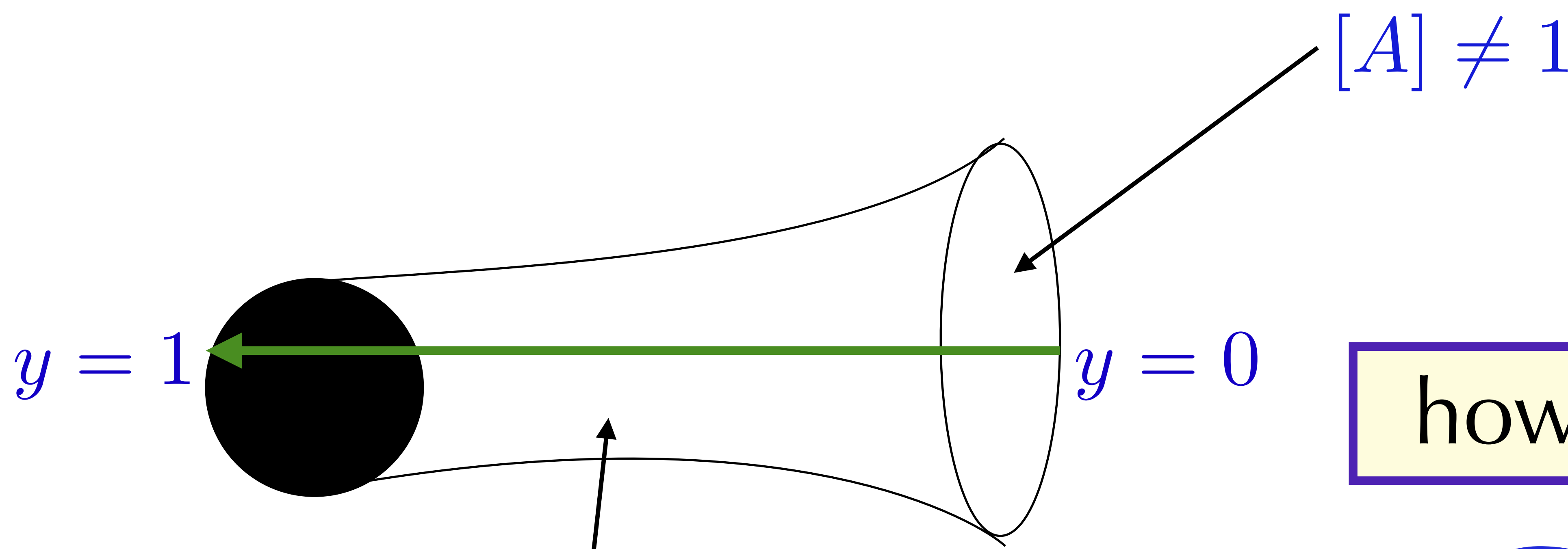
$$\Big\downarrow \; (-\Delta)^{\gamma}$$

$$A_\mu \to A_\mu + (-\Delta)^{\frac{\gamma-1}{2}} \partial_\mu \Lambda$$

$[A_\mu] = \gamma$

what kind of E&M has such gauge transformations?

claim: extra dimension



how?

?

$$S = \int dV_d dy \left(y^a F^2 + \cdots \right)$$

eom $d(y^a \star dA) = 0$

Karch:1405.2926
Gouteraux: 1308.2084

if holography is RG then
how can it lead to an
anomalous dimension?

construct `boundary' theory explicitly

$$S = \int dV_d dy \left(y^a F^2 + \cdots \right)$$

eom

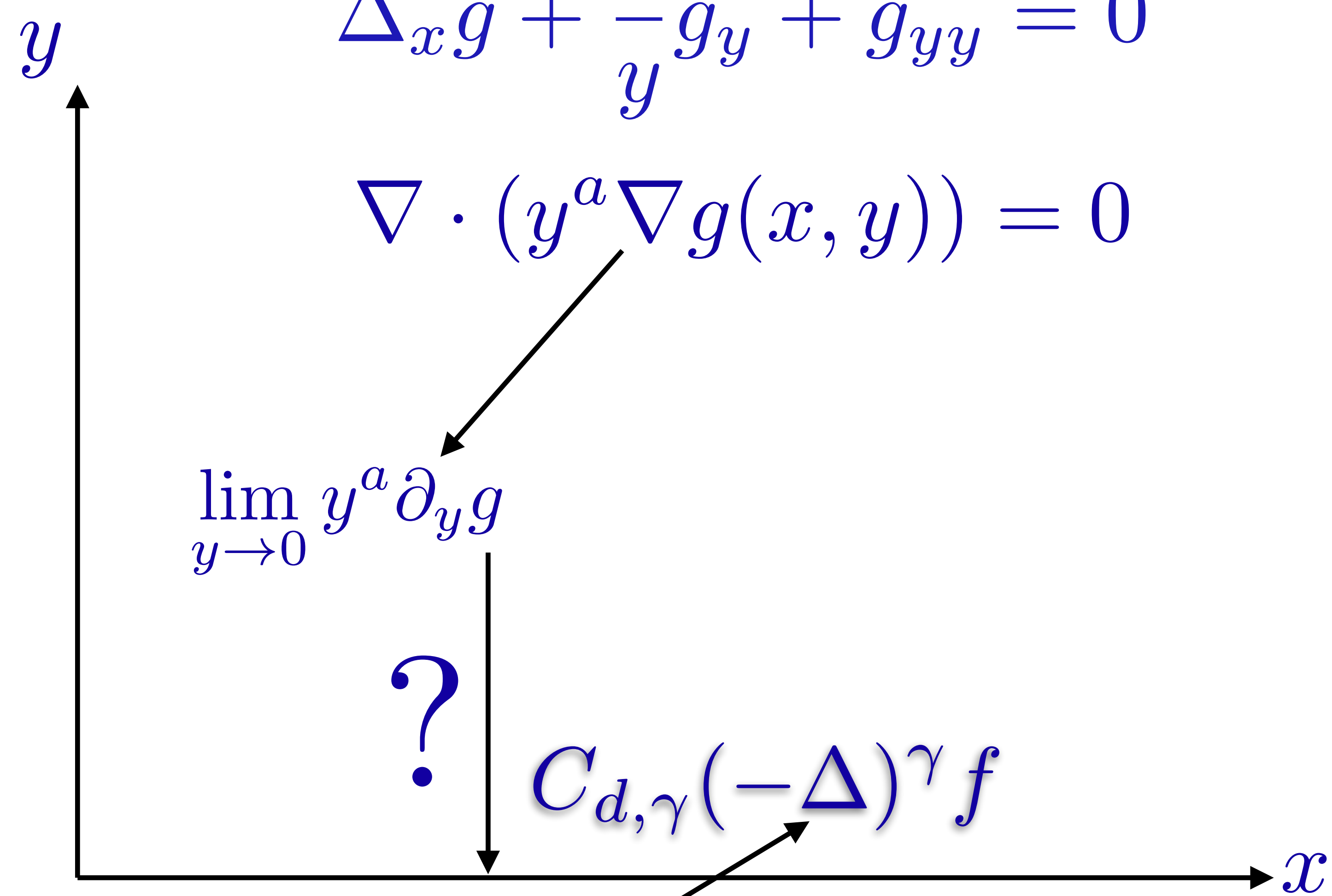
$$d(y^a \star dA) = 0$$

Caffarelli-Silvestre extension theorem (2006)

$$g(x, y = 0) = f(x)$$

$$\Delta_x g + \frac{a}{y} g_y + g_{yy} = 0$$

$$\nabla \cdot (y^a \nabla g(x, y)) = 0$$



fractional Laplacian

$$g(z = 0, x) = f(x)$$

$$\gamma = \frac{1 - a}{2}$$

closer look

$$\nabla \cdot (y^a \nabla u) = 0$$

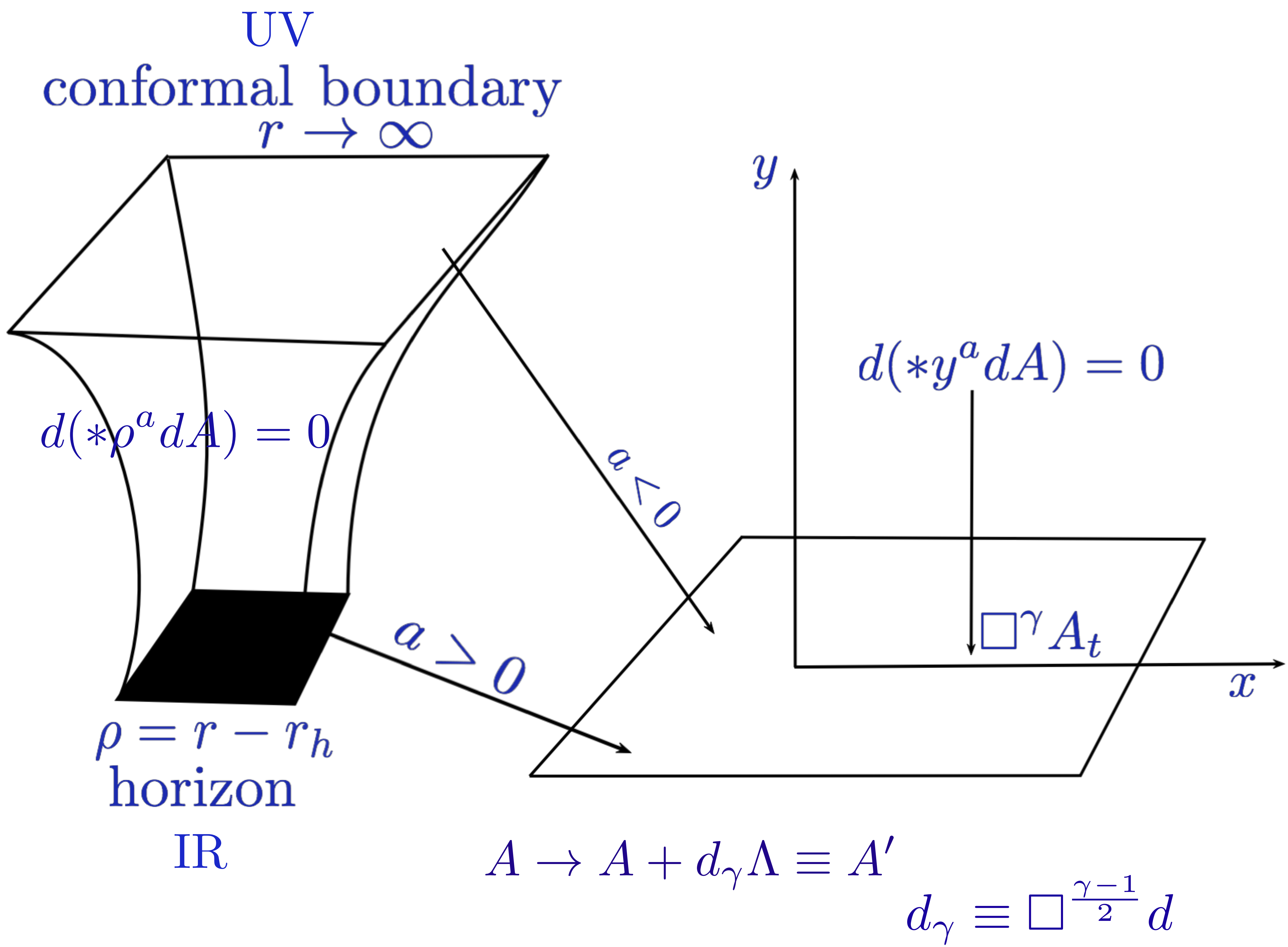
scalar field
(use CS theorem)

$$d(y^a \star dA) = 0$$

holography

similar equations

generalize CS theorem to p-forms
GL,PP:1708.00863
(CIMP, 366, 199 (2019)))


UV
conformal boundary
$r \to \infty$
$d(*\rho^a dA) = 0$
$a < 0$
$a > 0$
$\rho = r - r_h$
horizon
IR
y
x
$d(*y^a dA) = 0$
$\Box^\gamma A_t$
$A \to A + d_\gamma \Lambda \equiv A'$
$d_\gamma \equiv \Box^{\frac{\gamma-1}{2}} d$

boundary action:
fractional Maxwell
equations

$$\Box^{\gamma} A_{\perp} = J$$

boundary action has
`anomalous dimension'
(non-locality)

$$F \to d_{\gamma} A = \partial_{\mu} \Box^{(\gamma-1)/2} A_{\nu} - \partial_{\nu} \Box^{(\gamma-1)/2} A_{\mu}$$

if holography is RG then how can it lead to an anomalous dimension?

$$S = \int dV_d dy \left(y^a F^2 + \cdots \right)$$

$$[A] = 1 - a/2$$

dimension of A is fixed by the bulk theory: not really anomalous dimension answer

?

$$[\partial_\mu, \hat{Y}] = 0$$

$$[d, \Box^\gamma] = 0$$

$$J \to \Box^\gamma J \qquad [J] = d - 1 - \gamma$$

Ward identities

$$C^{ij}(k) \propto (k^2)^\gamma \left(\eta^{ij} - \frac{k^i k^j}{k^2}\right).$$

standard Ward identity

$$k_i C^{ij}(k) = 0 \quad \Longrightarrow \quad \partial_i C^{ij}(k) = 0$$

but

$$k^{\gamma-1} k_\mu C^{\mu\nu} = 0 \quad \Longrightarrow \quad \partial_\mu (-\Delta)^{\frac{\gamma-1}{2}} C^{\mu\nu} = 0$$

inherent ambiguity in E&M

family of zero eigenvalues

$$M_{\mu\nu} f k^{\nu} = 0$$

most fundamental conservation law

$$\partial^{\mu} \underbrace{(-\nabla^2)^{(\gamma-1)/2} J_{\mu}}_{J'_{\mu}} = 0$$

Noether's Second Theorem

$$\sum \psi_i \delta u_i = \delta f -$$

$$- \frac{d}{dx}\left\{\sum\left[\binom{1}{1}\frac{\partial f}{\partial u_i^{(1)}}\delta u_i + \binom{2}{1}\frac{\partial f}{\partial u_i^{(2)}}\delta u_i^{(1)} + \cdots + \binom{\kappa}{1}\frac{\partial f}{\partial u_i^{(\kappa)}}\delta u_i^{(\kappa-1)}\right]\right\} +$$

$$+ \frac{d^2}{dx^2}\left\{\sum\left[\binom{2}{2}\frac{\partial f}{\partial u_i^{(2)}}\delta u_i + \binom{3}{2}\frac{\partial f}{\partial u_i^{(3)}}\delta u_i^{(1)} + \cdots + \binom{\kappa}{2}\frac{\partial f}{\partial u_i^{(\kappa)}}\delta u_i^{(\kappa-2)}\right]\right\} +$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$+ (-1)^\kappa \frac{d^\kappa}{dx^\kappa}\left\{\sum \binom{\kappa}{\kappa}\frac{\partial f}{\partial u_i^{(\kappa)}}\delta u_i\right\} \tag{6}$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda + \partial_\mu \partial_\nu G^\nu + \cdots ,$$

$$A \to A + d_\gamma \Lambda \equiv A'$$

$$d_\gamma \equiv (-\Delta)^{\frac{\gamma-1}{2}} d$$

$$\Delta \to \Box$$

Noether's Second Theorem and Ward Identities for Gauge Symmetries

Steven G. Avery[a], Burkhard U. W. Schwab[b]

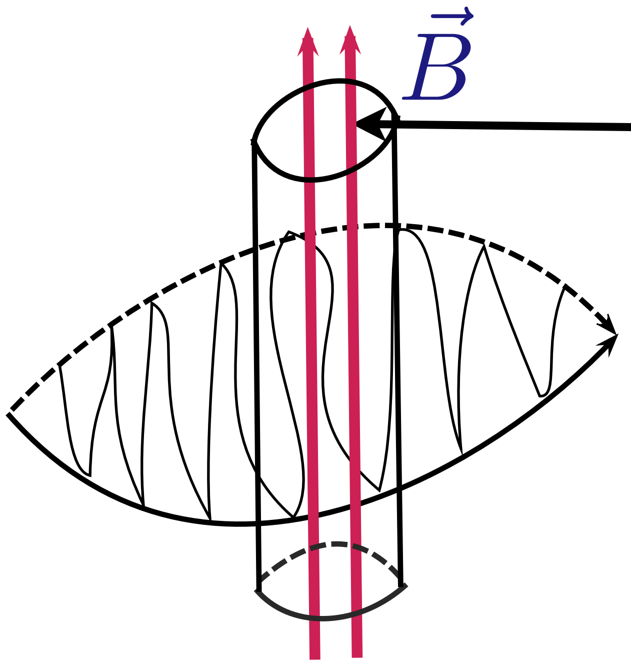
For simplicity, we focus on the case when the transformation may be written in the form[6]

$$\delta_\lambda \phi = f(\phi)\,\lambda + f^\mu(\phi)\partial_\mu \lambda, \tag{10}$$

but it is straightforward to consider transformations, as Noether did, involving arbitrarily high derivatives of λ. (Although, the authors know of no physically interesting examples.) Let us start with

arxiv:1510.07038

experiments?



magnetic flux

$\pi r^2 B$

should be dimensionless

$$[B] = 2 - \Phi = 2 + 2/3 \neq 2$$

what's the resolution?

correct dimensionless quantity

$$a_i \equiv [\partial_i, I_i^{\alpha}{}_{\alpha} A_i] = \partial_i I_i^{\alpha}{}_{\alpha} A_i$$

$\Delta^{-\alpha}$

what's the relationship?

$$\oint_{\partial\Sigma} a \qquad\qquad \oint_{\partial\Sigma} A$$

$$\text{Norm} \oint_{\partial\Sigma} a = \frac{1}{\Gamma(3/2-\gamma)} \oint_{\partial\Sigma} A$$

not an integer

obstruction theorem to charge quantization (NST)

$$A_\mu \to A_\mu + \partial_\mu \Lambda + \partial_\mu \partial_\nu G^\nu + \cdots ,$$

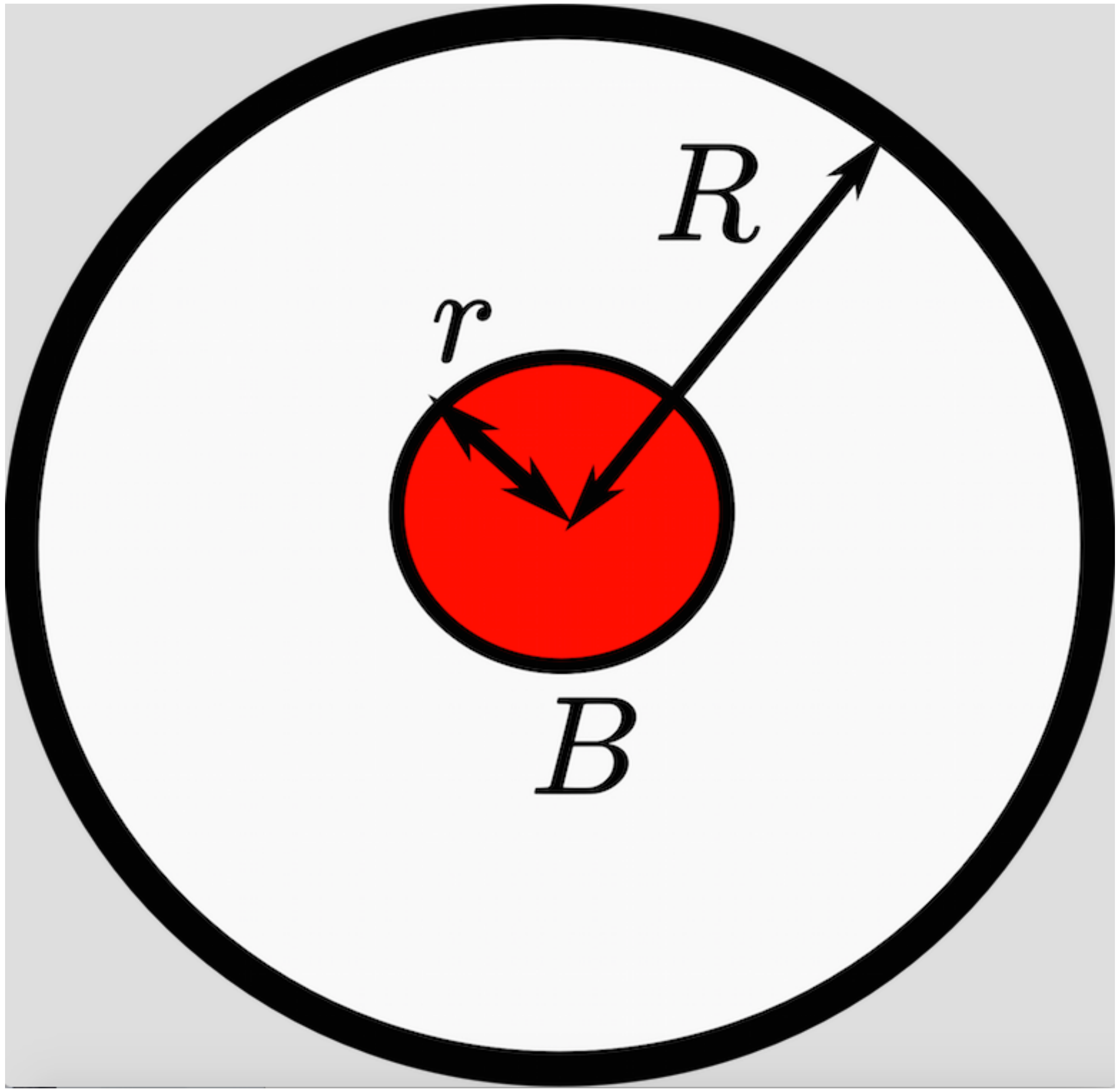
$$A \to A + d_\gamma \Lambda \equiv A'$$

$$d_\gamma \equiv (-\Delta)^{\frac{\gamma-1}{2}} d$$

$\Downarrow$ J'_μ

charge ill-defined (new landscape problem)

New Aharonov-Bohm Effect



$$\Delta\phi_{\mathrm{D}} = \frac{e}{\hbar}\pi r^2 B R^{2\alpha-2}\left(\frac{\sqrt{\pi}2^{1-\alpha}\Gamma(2-\alpha)\Gamma(1-\frac{\alpha}{2})}{\Gamma(\alpha)\Gamma(\frac{3}{2}-\frac{\alpha}{2})}\sin^2\frac{\pi\alpha}{2}\,{}_2F_1(1-\alpha,2-\alpha;2;\frac{r^2}{R^2})\right)$$

is the correction large?

$\alpha = 1 + 2/3 = 5/3$

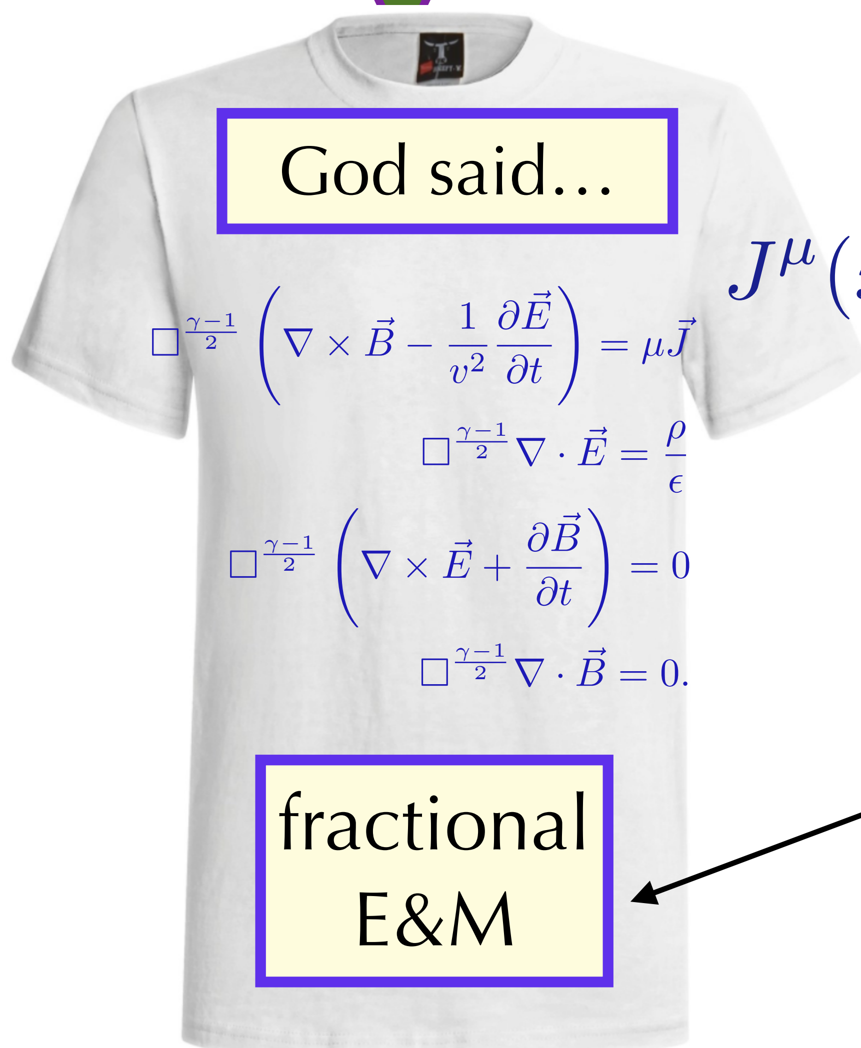
$$\Delta\Phi_R = \frac{eB\ell^2}{\hbar} L^{-5/3}/(0.43)^2$$

yes!

if in the strange metal

$$[A_\mu] = d_A \neq 1$$

Pippard Kernel



$$J^\mu(x) = -\int d^dx' C_{\mu\nu}(|x - x'|)A^\nu$$

$$[J] \neq d - 1$$

$$[A] \neq 1$$

$$\omega = ck$$

in SC!

$$U(1) \to \mathbb{Z}_2$$